Channel Flow Behaviour During Mixed Convection at Low Reynolds Numbers

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Graduate Program in Mechanical and Materials Engineering  
A thesis submitted in partial fulfillment of the requirements for the degree in Doctor of Philosophy  
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(Thesis format: Integrated Article)

by

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of the requirements for the degree of
Doctor of Philosophy

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Abstract

The effect of mixed convection on low Reynolds numbers flow inside a horizontal square channel heated from below has been investigated experimentally. The channel flow rate ranged from 0.0210 kg/s to 0.0525 kg/s which correspond to Reynolds numbers between 300 and 750 in the absence of heating. The channel bottom surface temperature was controlled and varied from 30 °C to 55 °C (Grashof number ranged between $6.37 \times 10^6$ and $3.86 \times 10^7$). Planer Particle Image Velocimetry (PIV) technique was used to measure two-dimensional velocity fields in the channel mid-vertical plane and two horizontal planes close to the bottom heated wall. Stereo-PIV technique was used to measure velocity fields in the channel cross plane. Natural convection was dominant over forced convection at all measurement conditions and $Gr/Re^2$ ranged between 9 and 206. Mean and turbulent velocity fields were computed and analyzed. It was found that buoyancy-induced secondary flow alters the mean flow behaviour. The mean streamwise velocity in the vertical plane was asymmetric and skewed towards the bottom heated surface. Back flow was observed near the top unheated wall when $Gr/Re^2 > 55$ and its magnitude increased with the increase of bottom surface temperature and/or decrease in the flow rate. Turbulence was induced mainly due to buoyancy-induced secondary flow. The streamwise and spanwise turbulent velocity magnitudes were largest close to the bottom heated surface. The underlying physical processes associated with the low Reynolds number mixed convection were investigated by applying the POD technique on the turbulent velocity fields and through the characterization of turbulent coherent structures formed in the flow. POD analysis revealed intriguing features within the vertical turbulent velocity fields. Local convective cells at different mode levels were observed near the bottom heated wall. Coherent structures were formed extensively in all planes due to the interactions between rising plumes, falling parcels and shear flow. The results showed that the bottom wall temperature had a direct effect on the number of coherent structures generated and their associated characteristics. The flow development was also investigated at five different locations along the channel in the vertical mid-plane. It was found that the flow development length increased with the increase of bottom wall temperature and/or decrease of the flow rate (i.e. increasing $Gr/Re^2$).
Keywords

Mixed convection; low Reynolds numbers; square channel; Particle Image Velocimetry (PIV); Stereo-PIV; coherent structures; flow development; turbulence; Proper Orthogonal Decomposition (POD).
Co-Authorship Statement

I hereby declare co-authorship in the following chapters:


Chapter 4 will be submitted for publication under the co-authorship of A. Elatar and K. Siddiqui.

Chapter 5 will be submitted for publication under the co-authorship of A. Elatar and K. Siddiqui.
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<td>$a_n$</td>
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<td>$c_p$</td>
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<tr>
<td>$q_s$</td>
<td>Surface heat flux, kW/cm$^2$</td>
</tr>
<tr>
<td>$Q_o$</td>
<td>Kinematic heat flux, cm °C/s</td>
</tr>
<tr>
<td>$Q$</td>
<td>Second invariant of velocity gradient tensor, 1/s</td>
</tr>
<tr>
<td>$Ra$</td>
<td>Rayleigh number</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$Ri$</td>
<td>Gradient Richardson number</td>
</tr>
<tr>
<td>$S$</td>
<td>Symmetric component of velocity gradient tensor, 1/s</td>
</tr>
</tbody>
</table>
T  Temperature, °C

\( t \)  Time, s

U  Mean streamwise velocity, cm/s

\( u_* \)  Fractional velocity, cm/s

\( u' \)  Streamwise turbulent velocity, cm/s

\( \overline{u'u'} \)  Reynolds stress, cm\(^2\)/s\(^2\)

\( v' \)  Vertical turbulent velocity, cm/s

W  Mean spanwise velocity, cm/s

\( w' \)  Spanwise turbulent velocity, cm/s

\( w_* \)  Convective velocity, cm/s

\( y_* \)  Length scale, cm

**Greek symbols**

\( \gamma \)  Specific gravity

\( \nu \)  Kinematic viscosity, cm\(^2\)/s

\( \rho \)  Density, kg/m\(^3\)

\( \beta \)  Coefficient of thermal expansion, 1/°C

\( \phi \)  Basis function (mode)

\( \varphi_{TKE} \)  Coherent structure fractional turbulent kinetic energy

\( \lambda \)  Eigen value

\( \Omega \)  Antisymmetric component of velocity gradient tensor, 1/s

\( \omega \)  Vorticity, 1/s

\( \nabla u \)  Velocity gradient tensor, 1/s
### Subscript

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>Channel</td>
</tr>
<tr>
<td>rms</td>
<td>Root mean square</td>
</tr>
<tr>
<td>p</td>
<td>Particle</td>
</tr>
<tr>
<td>t</td>
<td>Terminal</td>
</tr>
<tr>
<td>TKE</td>
<td>Turbulent kinetic energy</td>
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<tr>
<td>max</td>
<td>Maximum</td>
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</table>
Chapter 1
Introduction

1.1 Introduction

Convective heat transfer exists widely in different industrial applications including heat exchangers, HVAC systems and electronics cooling. The main convection heat transfer mechanisms are natural convection and forced convection. Mixed convection is the heat transfer mode where both natural and forced convection mechanisms coexists and contribute in the heat transfer process. During mixed convection, the relative contribution of both natural and forced convection is quantified in terms of $Gr/Re^2$ [1] where $Re$ is Reynolds number and $Gr$ is Grashof number which represents the ratio between buoyancy and viscous forces and is defined as,

$$Gr = g\beta\Delta T d^3/\nu^2$$  \hspace{1cm} (1.1)

Where $\beta$ is the coefficient of thermal expansion, $d$ is the characteristic length and $\nu$ is the kinematic viscosity [2]. Natural convection is dominant over forced convection when $Gr/Re^2 >> 1$ while forced convection is dominant if $Gr/Re^2 << 1$ [1]. Mixed convection may occur in low or high Reynolds number flows. There are several industrial applications where low Reynolds number mixed convection is found such as chemical vapor deposition (CVD), food process industry and biomedical applications. Low Reynolds number mixed convection is also found in the emerging solar thermal energy sector. In these types of applications, the thermal fluid is heated at low Reynolds and high Grashof numbers to increase the fluid temperature.

During natural convection, turbulence can be induced at certain conditions. The term that quantifies the instability of natural convection is Rayleigh number and is defined as

$$Ra = g\beta\Delta T d^3/\alpha\nu = Gr \times Pr$$  \hspace{1cm} (1.2)

Where $\alpha$ is the thermal diffusivity [2]. Rayleigh number presents the balance between the driving buoyancy force and the two opposing processes (i.e. viscosity and diffusion) that
work on stabilizing the motion. The critical value of Rayleigh number is 1708 for the case of two rigid boundaries [2]. Below this value, the fluid heated from below is stable to small disturbances. With increasing Rayleigh number, the flow starts the transition into turbulent flow. At a given Prandtl number, as Rayleigh number increases, the flow becomes time dependent with regular oscillation; the oscillation is in the form of hot or cold spots that advects with the original cellular motion. The oscillation frequency increases with the increase of Rayleigh number and the flow eventually becomes turbulent [3]. It was suggested by Rossby [4] that the transition to turbulence occurs when \( Ra = 14000 \ Pr^\alpha \) where \( \alpha \approx 0.6 \) for \( Pr >> 1 \).

Mixed convection can occur in flows that are originally laminar or turbulent in the absence of heating. For the case of mixed convection in originally laminar flow in the absence of heating, turbulence can be induced in the flow due to natural convection [5] if Rayleigh number (i.e. Grashof number) is high enough as described earlier. For originally turbulent flow, in addition to the turbulence induced due to shear instability, turbulence can also occur due to natural convection. In this case, shear induced turbulence works against buoyancy induced turbulence which results in a reduction of the turbulence magnitude [5]. Turner [2] suggested that for mixed convection in turbulent flows, within the classical inertial subrange, a buoyancy subrange exists where turbulent kinetic energy due to shear is removed over a range of wave numbers due to working against buoyancy forces. Richardson number is the term that quantifies the turbulence generated due to buoyancy to the turbulence due to shear and is defined as,

\[
Ri = -g \frac{\partial \rho}{\partial y} / \rho \left( \frac{\partial u}{\partial y} \right)^2
\]  

(1.3)

Where \( \frac{\partial \rho}{\partial y} \) is the vertical density gradient and \( \frac{\partial u}{\partial y} \) is the vertical gradient of the streamwise velocity [2].

During mixed convection, secondary flow is generated in the form of rising plumes of warm less dense flow and falling parcels of cold denser flow. These two streams are mainly driven by buoyancy forces that are generated due to natural convection. The co-
existence of this secondary flow in addition to the mean shear flow creates complexity in the flow behaviour. As a result, the channel flow characteristics are altered (i.e. turbulence is generated in low Reynolds number flow). The interaction between these streams produces vortices in different orientations. Several studies focused on analysing the flow behaviour mainly in the channel cross plane and observed the longitudinal vortices and studied their behaviour along the channel length [6] and at different boundary conditions and channel aspect ratios [7]. The attention given to these vortices is stemmed from the role these vortices can play as they are responsible for large scale transport of heat, mass and momentum [8]. The vertical secondary flow behaviour in the channel cross plane was found to strongly depend on both Grashof and Reynolds numbers. Lin and Lin [9] found that with the increase of Grashof number, the flow changes from a steady vortex flow to reach a chaotic turbulent state at $Gr = 5 \times 10^6$. They observed that decreasing Reynolds number yields similar trends.

Mixed convection strongly influences laminar internal flows by affecting both the velocity and temperature fields [10]. During mixed convection, internal flows undergo a hydrodynamic and thermal developing stage before reaching a fully developed stage. The boundary conditions play a significant role in altering channel flow structures. Also, the onset of instability is strongly dependent on the increase of Grashof number and/or decrease of Reynolds numbers [11]. The buoyancy-driven secondary flow develops along the channel length and hence, the flow circulation is enhanced. Consequently, Nusselt number is augmented. However, further downstream, the secondary flow works on diminishing itself. This is due to fluid mixing which in turn reduces the temperature gradient [7]. These characteristics manifest the importance of the entry region during low Reynolds numbers mixed convection.

1.2 Motivation

Mixed convection received considerable attention in 1970s and 1980s [1]. Several studies investigated in particular low Reynolds numbers mixed convection inside horizontal
channels at different boundary conditions and aspect ratios. The main focus was on quantifying bulk properties, mainly, Nusselt number and investigating its behaviour.

It is clear that in the applications where heat is transferred via convection, the ultimate goal is to maximize the heat transfer efficiency. The flow hydrodynamic behaviour plays an important role in controlling the rate of heat transfer. Nevertheless, at low Reynolds numbers, the role the turbulent flow field plays in the heat transfer process is not well understood. There is a scarcity of studies characterizing the turbulent flow in low Reynolds numbers mixed convection. Although mean flow behaviour is reported in several studies, the literature still lacks detailed investigation of the mean flow behaviour under these conditions, in particular during hydrodynamic and thermal flow development. Although the mixed convection in low Reynolds number channel flows presented a three-dimensional problem, few studies have actually conducted a detailed three-dimensional analysis to depict a comprehensive picture of the flow behaviour. A detailed literature review is included in the following chapters.

The performance of the heat transfer mechanism inside low Reynolds numbers mixed convection flows depends on the flow dynamics. Thus, understanding the flow dynamics would certainly help in optimizing the heat transfer process. Moreover, investigating the underlying physical process(es) during low Reynolds numbers mixed convection would provide a better insight into this complex fluid dynamical problem.

1.3 Objectives

The objectives of this study are to:

I. Investigate the mean and turbulent flow characteristics during mixed convection at high Grashof numbers and low Reynolds numbers.
II. Investigate the flow behaviour in the near wall region.
III. Explore the underlying physical processes.
IV. Study the flow development under such conditions.
The focus of the current study is to characterize both mean and turbulent flow behaviours during low Reynolds numbers and high Grashof numbers mixed convection.

Achieving these objectives would help in understanding the flow behaviour during low Reynolds numbers and high Grashof numbers mixed convection inside channels. This would give a better insight of the flow characteristics, in particular, the turbulent flow field. Moreover, the turbulent vortices and their associated properties can be quantified which provide a clear understanding of their importance under such conditions. This would help in design improvements of applications that involve low Reynolds and high Grashof numbers mixed convection.

1.4 Experimental Procedures

1.4.1 Experimental Setup

The main objective of this research as mentioned earlier is to investigate the flow behavior during mixed convection at low Reynolds numbers. Due to the complexity of the flow field, in order to obtain a better understanding of the fundamental thermo-fluid processes, square channel geometry was selected with the heat addition from the bottom wall. Such configuration would allow the implementation of Particle Image Velocimetry (PIV) technique for detailed velocity measurements which is vital for the in-depth investigation of the underlying processes. The square channel constructed with full optical access from the top and side walls to conduct this research in a laboratory environment is shown in Fig. 1.1. This optical access would allow velocity measurements in vertical, horizontal and cross planes of the channel to properly characterize all three velocity components. The heat addition was through the bottom wall whose temperature was controlled. Maintaining a constant temperature condition at the heated wall was relatively easy to implement than the constant heat flux condition. The four barrels shown in the figure are used as reservoir for water (working fluid) and to ensure uniform inlet conditions during measurements.
1.4.2 Measurement Planes

The flow field is measured at four different planes as seen in Fig. 1.2(a). The first plane is the vertical plane at \( z/D_h = 0.5 \) where \( D_h \) is the channel hydraulic diameter. The flow field in this plane would give a perception of the flow behavior along the channel height in the streamwise direction. Due to the importance of the flow behavior in the near wall region where the secondary flow is induced, two measurement planes parallel to the bottom heated wall at \( y/D_h = 0.05 \) and 0.25 are considered to measure the flow field in the channel horizontal (spanwise) plane. The vertical and horizontal planes were \( x/D_h \sim 29 \) downstream of the channel entrance to ensure that the flow is fully developed in the absence of heating. For the three measurement planes, the field of view is \( 1.3 \times D_h \) wide. The forth measuring plane is the channel cross plane at \( x/D_h = 29 \). The flow field in this plane along with the results from the other measurement planes would help in depicting a three-dimensional picture of the flow field under such conditions. To study the flow development along the channel length, the flow field was measured at five locations along the channel test section as seen in Fig. 1.2(b). The first location is upstream edge
coincides with the strip heaters upstream tip. For each section, the measurement plane is 1.3×D_h wide and the distance between two measurement locations is 2×D_h.

![Diagram of measuring planes](image1)

**Figure 1.2:** Schematic of (a) measuring planes, (b) flow development measuring locations.

### 1.4.3 Measurement Technique

Particle Image Velocimetry (PIV) is a non-intrusive measuring technique that can measure the flow velocity in a plane (PIV and stereo-PIV) or in a volume (volumetric PIV) with high spatial resolution. In the Planer PIV technique, the flow under investigation is seeded with tracer particles that follow the fluid motion. A laser light
sheet is used to illuminate the plane of measurement to make these particles visible while keeping the background dark. The technique works as follows; a laser pulse illuminates the measurement plane at a given time \( t_1 \) and particles are exposed to the first camera frame. The first image recorded the position of the particles at this given time. A second laser pulse illuminates the measurement plane again after a given time difference \( \Delta t \) from the first laser pulse and the particles are exposed again to the second camera frame. The second image recorded the particles position at \( t_2 = t_1 + \Delta t \). A synchronizer is used to synchronize between the laser pulses and the camera frames. It also controls the time difference \( \Delta t \) between the first and second laser pulses. This way a series of image pairs are obtained. An instantaneous velocity field is obtained by processing an image pair (First and second consecutive camera images). The first image of the image pair is divided into large numbers of interrogation windows. The second image is divided into larger regions called search windows. Typically, the dimension of the search window is twice the dimension of the interrogation window. The velocity vector in each interrogation window region is computed by cross-correlating the interrogation region in the first image with the search region in the second image. The average particles displacement within the interrogation region between the two images is determined by locating the highest peak in the cross correlation plane. The velocity vectors can be calculated based on the particles displacement and the time difference between the images \( \Delta t \). The cross correlation technique is performed on all interrogation windows to obtain the velocity field.

Stereo-PIV is the measuring technique that is able to measure the in-plane velocity component (i.e. perpendicular to the measuring plane) in addition to the two velocity components measured in the standard Planer PIV technique. Stereo-PIV works the same way as the human vision where each eye see the object from a different angle and the brain process the image from each eye to construct the third dimension of the object. Stereo-PIV uses two cameras positioned at an angle to the normal of the measuring plane as seen in Fig. 1.3(b). This configuration is known as the rotational system which is the most common and recommended to use \[13\]. The translation system is another configuration where the two cameras axes are parallel \[13\]. The two cameras focus on the
same object plane. To overcome the focusing problem in the rotational systems, Scheimpflug mount adapters are used to fulfill the Scheimpflug condition where the image plane, the lens plane and the object plane are nominally collinear [12] as seen in Fig. 1.3(b).

For each camera, an image pair is processed in the same manner as in planar PIV as described earlier. After that, the reconstruction step takes place in which the displacement from each image plane is mapped into the object plane [13]. There are two reconstruction methods used, geometric reconstruction and calibration-based construction. The geometric construction requires a complete knowledge of the recording configuration geometry and also the ability to mathematically model the geometry. Therefore, this type of reconstruction is complex and not widely used [13]. The common construction method is the calibration-based construction. In this method, before conducting the measurements, a calibrated target is positioned in the object plane and coincides with the light sheet [13]. In the current research, a 3-D calibration method is implemented using a 3-D calibration target. The calibration target is typically a plate that contains a collection of dots organized in a Cartesian grid [13]. A calibration image is captured by each camera. The location of grid points in the image plane is determined by template matching. The relation between the three dimensional object field position and its two dimensional image field from each camera is obtained using polynomial expression [13]. The mapping functions yielded from the calibration stage is utilized to reconstruct the particle displacements from each image plane into the object plane with a three displacement components [13]. For more details about the StereoPIV technique, see Prasad [13]. In the current research, the cameras are looking at an angle to the channel side walls, accordingly, astigmatism would occur which distorts the image quality. To avoid that, two right angle glass prisms were attached to both channel side walls in the vicinity of the measurement plane and filled with clear tap water [14](see Fig. 1.3b).

Water was seeded with silver-coated glass spheres, with the mean diameter of 15 µm and a specific gravity of 1.65. These glass spheres were used as the tracer particles for the PIV measurements. It was necessary to examine how precisely these particles are
following the flow motion as they are not neutrally buoyant. The characteristic particles
response time was calculated by,

\[ t_p = \frac{u_t}{g} \]  

(1.4)

Where \( t_p \) is the particle response time, \( u_t \) is the particle terminal velocity and \( g \) is the
gravitational acceleration [15]. The particle terminal velocity can be calculated by,

\[ u_t = \frac{(\gamma-1)D^2g}{18v} \]  

(1.5)

Where \( \gamma \) is the specific gravity, \( D \) is the seed particle diameter and \( v \) is the kinematic
viscosity of water [16]. The terminal velocity was found to be 0.01 mm/sec. The
estimated response time was 0.01 ms. To examine the suitability of the particles response
time in the current research, the response time was compared with the Kolmogrov time
scale, which is the lowest possible time-scale for fluid motion [17] for the given
experimental conditions. The Kolmogorov time-scale was estimated to be approximately
800 ms. The particle response time was approximately four orders of magnitudes smaller
than the Kolmogrov time scale which shows that the given particles accurately follow the
fluid motion [15 & 18].

To quantify the temperature field at each experimental condition, temperatures were
measured in the same plane as for the velocity measurements in a separate set of
experiments under identical conditions. A rake of nine T-type thermocouples with
accuracy of \( \pm 0.5 \) °C was positioned vertically in the measurement plane. The
thermocouples were clustered at different distances to capture the thermal boundary layer
as seen in Fig. 1.3(c). The inlet and outlet water temperatures were measured during PIV
as well as temperature field measurements using two T-type thermocouples located at the
beginning of the inlet section and at the end of channel end section. Three DAQ cards
(National Instrument NI9211) 4 channels each were used to acquire the data from all
thermocouples via LabView software. Temperatures were recorded for five minutes at a sampling rate of 3 Hz.
1.5 Proper Orthogonal Decomposition

One of the objectives of the current research work is to investigate the underlying physical process(es) within the flow. Proper orthogonal decomposition known as POD is an advanced tool that capable of investigating the underlying physical process by finding the dominant structures. The spatial distribution of flow energy content at different orthogonal modes can be obtained by POD analysis. It allows determining the spatial scales and the associated energy of dominant turbulent flow structures. The orthogonal modes or basis functions $\phi^n(\bar{x})$ are derived by decomposing the turbulent velocity fields. These modes individually or in combination describe different coherent turbulent structures. The snapshot method proposed by Sirovich [19] was used in the current work with the planar PIV turbulent velocity data using the algorithm developed by Doddipatla [20]. The velocity is expanded in this algorithm into a sum of spatial and temporal components as [21]:

Figure 1.3: Schematic of (a) PIV setup, (b) stereo-PIV camera setup, (c) position of thermocouples in the rake; thermocouples 1-4 were 1 mm apart, thermocouples 5-7 were 2 mm apart, thermocouples 7-8 were 7.5 mm apart, and thermocouples 8-9 were 17.5 mm apart.
\[ \bar{u}(\bar{x}, t) = \sum_{n=1}^{N} a^n(t) \phi^n(\bar{x}) \] (1.6)

Where \( N \) is the number of snapshots and \( a^n(t) \) is the temporal coefficient.

The basis functions \( \phi^n(\bar{x}) \) are obtained in such a way that the minimum error between the original velocity and the reconstructed velocity field square is minimal [22] as follows,

\[ \| \bar{u}(\bar{x}, t) - \sum_{n=1}^{N} a^n(t) \phi^n(\bar{x}) \|^2 \rightarrow \text{min} \] (1.7)

To achieve the minimum error, equation (5) is reduced into an eigenvalue problem [21]:

\[ \int \langle \bar{u}(\bar{x}, t) \bar{u}^*(\bar{x}', t) \rangle \cdot \phi^n(\bar{x}') d\bar{x}' = \lambda^n \phi^n(\bar{x}) \] (1.8)

where \( \lambda^n \) is the eigenvalue representing the energy associated with mode \( n \) and \( \langle \bar{u}(\bar{x}, t) \bar{u}^*(\bar{x}', t) \rangle \) is the spatial autocorrelation matrix at zero time lag. * denoted the complex conjugate. The eigenvectors are then used to determine different POD modes by projecting the velocity field onto them.

The POD modes are orthogonal and the temporal coefficients \( a^n(t) \) are obtained using the POD modes by projecting the velocity field onto the POD modes [21-24]:

\[ a^n(t) = \int \bar{u}(\bar{x}, t) \cdot \phi^n(\bar{x}) d\bar{x} \] (1.9)

1.6 Thesis Layout

The first chapter is introducing the fundamental foundation for mixed convection in addition to examples for the industrial applications where the phenomenon occurs. The motivation behind this type of work is introduced based on the gap that exists in the literature. The objectives of this project and the scope of work are included as well. The second chapter focuses on characterizing the mean and turbulent flow behaviours in the vertical mid plane of the channel. The third chapter investigates the flow behaviour in the near wall region due to its importance in the heat transfer process. Both mean and turbulent flow behaviour in two horizontal planes close to the bottom heated wall are
quantified. These results are coupled with the results obtained in the vertical plane in the near wall region to depict the flow in a three-dimensional manner. The fourth chapter concentrates on characterizing the turbulent vortices that formed in all measurement planes (vertical plane, cross plane and the two horizontal planes) via a thorough statistical analysis. Due to the importance of the flow development inside channels as described earlier, the fifth chapter takes a different path and investigates the flow development by examining both mean and turbulent flow characteristics as the flow advances along the heated section of the channel.

The underlying physical processes are explored using Proper Orthogonal Decomposition (POD). The analysis is conducted on the results obtained in chapters two and five. This technique provides the information needed to understand the underlying physical processes by dissecting the turbulent flow energies and plotting the associated flow structures. The final chapter brings the key conclusions from each of the preceding chapters for a thorough and comprehensive understanding of the flow behaviour.

1.7 References


Chapter 2

The effect of mixed convection on the structure of channel flow at low Reynolds numbers\(^1\)

2.1 Introduction

Mixed convection heat transfer where both forced as well as free convection modes exist, can be found in several industrial applications. Two main factors that control the heat transfer mechanism and consequently the flow behaviour are Grashof number ($Gr$) and Reynolds number ($Re$). Mixed convection occurs in both high and low Reynolds number flows where the latter is typically in the laminar regime in the absence of heating. Low Reynolds number mixed convection is important in electronics cooling, food process industry, chemical and nuclear reactors, and biomedical applications. Recently, new applications of low Reynolds number mixed convection are emerging in the green energy sector where the solar thermal systems that convert solar energy into heat operate at low Reynolds numbers and high Grashof numbers.

Several studies investigated the low Reynolds number mixed convection inside channels. Different channel geometries, orientations and boundary conditions were examined. Generally, the main focus of these previous studies was on quantifying the bulk properties such as Nusselt number or coefficient of friction and investigating its variation along the channel heating section (e.g. Mahaney et al. [1] and Maughan and Incropera [2]). Some studies were focused on the flow visualization and identification of different flow patterns emerged due to convection (e.g. Lin and Lin [3] and Wang et al. [4]). The low Reynolds number flow in the laminar regime when exposed to heat, generates turbulence and hence, the investigation of turbulent flow behaviour is crucial to understand the underlying physical processes associated with the mixed convection. To the best of authors’ knowledge, no previously reported studies had conducted a detailed

\(^{1}\) The chapter is a journal article submitted to the International Journal of Heat and Fluid Flow
investigation of the turbulent flow behaviour in low Reynolds number mixed convection channel flows.

Gajusingh and Siddiqui [5] experimentally studied the effect of wall heating on the flow characteristics in the near wall region inside a square channel. Their main focus was on the region immediately adjacent to the bottom heated surface. They studied how heat transfer would affect the flow dynamics in the near wall region for originally laminar and turbulent flows. They found that the buoyancy generated turbulence for originally laminar flow while for turbulent flow buoyancy dampened turbulence. They argued that for originally turbulent flow, turbulence is dampened due to working against buoyancy. They quantified the instability due to stratification using Richardson number. They argued that for originally laminar flow, the instability produced by heating enhances turbulence while for originally turbulent flow, instability due to heating would reduce turbulence magnitude.

Several studies investigated the effect of bottom wall heating on the Nusselt number in a horizontal rectangular channel [1-3 & 6-8] and a horizontal tube [9] at low Reynolds numbers. They all observed enhancement of Nusselt number along the channel length and attributed this to the secondary flow enhancement which disrupts the thermal boundary layer. A decay in Nusselt number was observed at the channel entrance region [2,3 & 8] where the forced convection was the dominant mode as the buoyancy-driven secondary flow was not developed yet. Osborne and Incropera [6&7] experimentally investigated the effect of buoyancy on convection heat transfer inside horizontal channels with top and bottom heated walls for laminar, transient and turbulent flow regimes. Their main focus was to quantify Nusselt number at the top and bottom walls. At the top wall, forced convection was dominant in all flow regimes and the Nusselt number values were lower than that for the bottom wall. They argued that for the transient regime at the top wall, the laminarizing effect on the flow conditions due to stably stratified temperature distribution is responsible for the decrease in the Nusselt number values. For laminar flow regime, forced convection was dominant at the top wall region and a thermally stable boundary layer was formed preventing ascending plumes from the bottom wall to
penetrate this region. They proposed a correlation to quantify Nusselt number. They concluded that the convection heat transfer in one plate was not influenced by the other plate. It has been reported in the previous studies that an increase in the Grashof number accelerates the onset of the secondary flow and consequently the mixed convection, while an increase in the Reynolds number delays the onset of secondary flow and consequently the mixed convection [2,7 & 8]. Patil and Babu [10] studied laminar mixed convection in a horizontal square duct with uniform wall temperatures. They found that the average Nusselt number at a given Reynolds number increased with the increase of $Gr/Re^2$ and Prandtl number of the fluid. They also observed that the influence of $Gr/Re^2$ on Nusselt number is higher at low Reynolds number than in high Reynolds number.

Flow visualizations provided an insight into the type of the secondary flow induced by the wall heating in low Reynolds number mixed convection. The reported studies were focused on the flow visualization along the channel length [6,7 &11-13] and in the cross stream direction [3,11, 14 & 15]. Wang et al. [13] identified four different flow patterns for mixed convection along the heated test section of a horizontal square channel with bottom heated wall using shadowgraph technique. Grashof number ranged from $2.8 \times 10^6$ to $2.5 \times 10^7$, Reynolds ranged from 100 to 1000. They found that the flow patterns changed with the Reynolds number and Grashof number. Based on the flow patterns they argued that the flow passes through four different flow regimes along the channel heated section: laminar forced convection, laminar mixed convection, transient mixed convection and turbulent free convection. Lin and Lin [3] experimentally investigated the unsteady mixed convection for air in a bottom heated horizontal rectangular channel in the cross stream direction using smoke tracer. The Reynolds numbers in the range from 9 to 186 and Grashof numbers up to $5 \times 10^6$ were considered. They found that increasing Grashof number and/or decreasing Reynolds number alter the flow structure from periodic into quasiperiodic and even chaotic flow.

Nandakumar et al. [16] investigated the flow structure in the cross stream direction for different horizontal channel geometries heated from below for Grashof number up to $5 \times 10^5$. Longitudinal vortex patterns of two or four vortices were observed and the
bifurcation of the vortices was found to depend on the Grashof number and the channel aspect ratio. Huang and Lin [17] numerically investigated laminar mixed convection in a horizontal rectangular duct heated from below. Their main focus was on studying the effect of buoyancy-inertia ratio on the cross stream flow behaviour. They found that with an increase of $Gr/Re^2$, the cross stream flow behaviour shifts from a steady vortex flow at $Gr/Re^2 < 4$ into a chaotic flow at $Gr/Re^2 > 25$.

Reverse flow was reported in many studies that investigated laminar mixed convection channel flows [9,11-13 &18]. Wang et al. [13] numerically studied mixed convection in a horizontal pipe for air at $Re = 100$ and $Gr = 2 \times 10^4$. They observed reverse flow near the top wall at the entrance region, which distorted the mean velocity behaviour. However, the reverse flow diminished with the distance from the entrance region. They argued that the secondary flow development and the axial conduction are responsible for the reverse flow. Toriyama and Ichimiya [11] numerically and experimentally investigated laminar mixed convection in a horizontal square channel for two conditions considering isothermal walls. For the first condition, wall temperature was higher than the inlet water temperature while for the second case, wall temperature was lower than the inlet water temperature. A reverse flow appeared at the top wall for the first case while for the second case, the reverse flow appeared at the bottom wall.

Despite several studies, there is a scarcity of detailed investigation of the turbulent flow structure in mixed convection channel flows at low Reynolds numbers. Such studies are important as they provide a better insight into the underlying physical processes that occurs in such flows. The present study is focused on experimentally investigating the impact of bottom wall heating on the flow structure inside a horizontal square channel at low Reynolds and high Grashof numbers. A detailed qualitative as well as quantitative analysis of both the mean and turbulent flow fields have been conducted to obtain better understanding of the fundamental flow processes associated with the mixed convection channel flow.
2.2 Experimental Setup

A 7 cm × 7 cm square channel was built for the experiments. The channel consists of three sections as shown in Fig. 2.1(a). The inlet section has a ½ inch diameter inlet followed by a divergent section that transitions into the 7 cm × 7 cm square cross section. The length of the inlet section was 70 cm. A honeycomb was placed inside the square section to straighten the flow and damp any disturbance before entering the test section. The inlet section was made of aluminum and contains a bleed valve to remove any air trapped inside the channel and a pressure gauge to monitor the inlet fluid pressure. The inlet section was connected to the test section using two aluminum flanges. The test section was 150 cm long and has a 7 cm × 7 cm square cross section. The top and side walls were made of ½ inch non-tempered glass for visual access to the channel and the bottom wall was made of ½ inch aluminum plate. The aluminum bottom surface was coloured in black with a marker to eliminate any light reflection during the experiments.

The test section was supported by two 5 cm high and 1.3 cm thick aluminum plates. Two strip heaters (1500W- 250V) 1.3 m in length were installed in parallel directly underneath the bottom aluminum surface 10 cm downstream of the test section entrance. The temperature of the bottom wall was controlled by a temperature controller (ZESTA-ZCP513) through a feedback loop from a thermocouple embedded in the bottom wall close to the measurement location.

The end section was 30 cm in length and connected to the downstream end of the test section by two aluminum flanges. The section contains a bleeder valve to remove any trapped air in the channel and a pressure gauge to monitor the exit fluid pressure. It has a convergent end section with a 1/2 inch diameter exit opening.

Clean tap water was used as the working fluid. As the water was being continuously heated through the channel in a closed loop, the water temperature tended to build up with time. Four barrels, 200 Litres each, were coupled together in series and used as a water reservoir; This reservoir can supply water at room temperature throughout the experiment without a need to recycle i.e. water was circulated one time only during a
given set of experiments. Due to a constant room temperature, the inlet water temperature maintained a constant value of around 24.5 °C. The outlet water temperature ranged from 25.5 °C at the lowest wall temperature (30 °C) and highest flow rate (0.0525 kg/s) to 36.5 °C at the highest wall temperature (55 °C) and lowest flow rate (0.0210 kg/s). Air bubbles present in the tap water had to be removed to obtain good quality results. Therefore, water was stored in the barrels for two days with periodic stirring to remove air bubbles. A magnetic pump (Little Giant, 5 MD) installed downstream of the barrels was used to circulate the water through the loop. A flow meter with a control valve (FL4205, Omega Engineering) was installed between the pump and the channel to control the water flow rate (see Fig. 2.1a).

Four mass flow rates 0.0210, 0.0315, 0.0420 and 0.0525 Kg/s were used in the experimental runs. The Reynolds numbers correspond to these flow rates in the absence of heating are 300,450,600 and 750 for reference. At each flow rate, experiments were conducted at different bottom wall temperatures which were 30, 35, 40, 45, 50 and 55 °C. The corresponding Grashof numbers ranged from $6.37 \times 10^6$ to $3.86 \times 10^7$. For a given set of experiments, the measurements were taken 30 minutes after adjusting the flow rate and the heater to allow steady state to be reached where the outlet water temperature was steady.

Particle image velocimetry (PIV) technique was used for measuring two-dimensional velocity fields. Fig. 2.1(b) shows the schematic of the PIV setup. The measurements were taken 130 cm downstream of the test section in the mid vertical plane of the channel. The PIV system comprised of a 120 mJ Nd:YAG laser (SoloPIV 120XT 532nm) to illuminate the measurement plane, a CCD camera (VA-4M32, Vieworks) with the resolution of 2336×1752 pixels to capture the images in the measurement plane. The camera was horizontally positioned i.e., 2336 pixel in horizontal and 1752 pixel in vertical. An image acquisition system (CORE-DVR, IO industries) connected to a PC was used to record images.
A four-channel pulse generator (555-4C, Berkeley Nucleonics Corporation) was used to control the laser pulses timing and synchronizing them with the camera frames. Water was seeded with silver-coated glass spheres, with the mean diameter of 15 µm. These glass spheres are used as the tracer particles for the PIV measurements. At each experimental run, 3000 images were captured at a sampling rate of 30 Hz. This resulted in 1500 instantaneous velocity fields at a rate of 15Hz.

PIV measuring technique computes velocity vectors by cross correlating two consecutive images (i.e. an image pair). Interrogation window in the first image is being correlated with the search region in the second image of the image pair. In the present study, the interrogation windows size (i.e. the region over which each velocity vector was calculated) was set as 32×32 pixels and the search windows was set as 64×64 pixels. The nominal resolution of the velocity field was increased to 16×16 pixels (0.68×0.68 mm) by using a 50% overlap of interrogation windows. Spurious velocity vectors were identified
and then corrected by using a scheme based on the local median test proposed by Siddiqui et al. [19]. The spurious vectors detected and corrected were well below 1%.

The uncertainty of the PIV velocity measurements was calculated at the highest flow rate and highest bottom wall temperature as the largest velocity gradients occur at these conditions. The uncertainty was estimated based on the criteria and data from Cowen and Monismith [20] and Prasad et al. [21]. The maximum error in velocity measurements was estimated to be ±0.071 cm/s which was less than 6.7% of the bulk flow velocity.

Temperatures were measured in the same plane and under the same conditions as for the velocity measurements in a separate set of experiments. A rake of nine T-type thermocouples with accuracy of ±0.5 ºC was positioned vertically 130 cm downstream of the test section. The spacing of the thermocouples was set in a way that most of the thermocouples were clustered near the bottom wall to resolve the thermal boundary layer. The exact positioning of each thermocouple is show in Fig. 2.1(c). Inlet and outlet water temperatures were also measured using T-type thermocouples located at the upstream and downstream ends of the channel, respectively. A 12 channel data acquisition module (National Instruments NI 9211) was used to acquire the temperature data via LabVIEW data acquisition software. The data was recorded for five minutes at a rate of 3Hz. The sampling rate of thermocouples was set based on the response time of the thermocouples. Thermocouples were calibrated using a mercury-bulb thermometer.

2.3 Results

In mixed convection where both forced and natural convection modes are present, one parameter that quantifies the relative contributions of buoyancy versus inertial forces is $Gr/Re^2$. The natural convection mode is considered to be dominant if $Gr/Re^2 >> 1$, while the forced convection mode is dominant if $Gr/Re^2 << 1$ [22]. In the present study, $Gr/Re^2$ ranged from about 10 at the highest flow rate and lowest wall temperature to 206 at the lowest flow rate and highest wall temperature. This shows that the natural convection mode was dominant at all cases.
The bottom wall heating induces buoyancy-driven secondary flow. This flow is in the form of rising plumes of warm fluid from the bottom wall and falling parcels of cooler fluid from the top unheated wall. In the present conditions, there are two sources of turbulence generation; the buoyancy and the shear flow. The contribution of each source to the turbulence generation depends on their relative strengths, which is typically quantified in terms of the Richardson number \((Ri)\) which is the ratio between the turbulence generation due to buoyancy and the turbulence generation by the mean shear. The gradient Richardson number was computed using the measured temperature and velocity data as,

\[
Ri = -g \frac{\partial \rho}{\partial y} / \rho \left( \frac{\partial u}{\partial y} \right)^2
\]  

(2.1)

Where \(\frac{\partial \rho}{\partial y}\) is the vertical density gradient and \(\frac{\partial u}{\partial y}\) is the vertical gradient of the mean streamwise velocity [23]. A negative value of the Richardson number implies the presence of unstable stratification. The values of Richardson number at the lowest and highest flow rates and wall temperatures are presented in table 2.1. The results show that for all cases, the flow was unstably stratified \((Ri < 0)\) as expected. For a given wall temperature, the magnitude of the Richardson number decreased with an increase in the flow rate, while for a given flow rate, the magnitude of Richardson number increased with the wall temperature.

Table 2.1: Various parameters for the maximum and minimum wall temperatures and flow rates.

<table>
<thead>
<tr>
<th>Mass flow rate (kg/s)</th>
<th>0.0210</th>
<th>0.0525</th>
<th>0.0210</th>
<th>0.0525</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall temperature (ºC)</td>
<td>30</td>
<td>30</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>Grashof number</td>
<td>(6.37 \times 10^6)</td>
<td>(6.61 \times 10^6)</td>
<td>(3.86 \times 10^7)</td>
<td>(3.45 \times 10^7)</td>
</tr>
<tr>
<td>(Gr/Re^2)</td>
<td>53</td>
<td>9</td>
<td>206</td>
<td>37</td>
</tr>
<tr>
<td>Richardson number</td>
<td>-0.43</td>
<td>-0.08</td>
<td>-0.84</td>
<td>-0.13</td>
</tr>
</tbody>
</table>
This indicates that at a given wall temperature, the turbulence due to buoyancy is higher in magnitude at lower flow rate and as the flow rate increases, the contribution of buoyancy-induced turbulence decreases. Similarly, at a given flow rate, an increase in wall temperature strengthens the buoyancy and hence enhances the buoyancy-produced turbulence.

To illustrate the dynamical and complex flow structure due to the interaction of shear and buoyancy-driven flows, turbulent velocity fields at different flow rates and heating conditions are plotted in Fig. 2.2. Turbulent velocity fields were computed by subtracting the time-averaged velocity from the instantaneous velocity fields at each grid point. Fig. 2.2(a) shows the turbulent velocity field for \( \dot{m} = 0.0315 \text{ kg/s} \) at the lowest heated wall temperature \( T_{wall} = 30 \, ^{\circ}\text{C} \). The plot shows the rising plumes of warm less dense fluid parcels near the heated wall. The falling parcels of cooler dense fluid are also visible in the plot in the mid channel region. The plot shows the interaction of rising and falling parcels of fluid which formed complex flow structure. The plot also shows that multiple vortices are formed due to this complex interaction with clockwise as well as counter clockwise orientations. It is also observed that at this condition, the magnitude of turbulent velocities is very weak in the upper half of the channel. This is likely due to the reason that the thermal plumes are relatively weak due to the low heating condition and the corresponding buoyancy-driven turbulence associated with these plumes.

As a result, most of the turbulent kinetic energy of these plumes is utilized in the complex interactions discussed above and they become very weak as they reach the upper region of the channel. Fig. 2.2(b) \( (\dot{m} = 0.0315 \text{ kg/s} \) at \( T_{wall} = 40 \, ^{\circ}\text{C} \)) shows that as the wall heating increases, the turbulence becomes stronger in magnitude and spreads throughout the channel domain. It implies the presence of a strong cross-stream component of turbulent velocity (perpendicular to the measurement plane). This behaviour manifests the existence of three-dimensional turbulent flow within the channel. The plot also shows that the magnitude of cross-stream velocity component is strong enough to diverge the falling parcels. The turbulent velocity field in this plot appears to be dominated by the falling parcels within the measurement domain, however, rising plume is observed at the
bottom left side of the velocity field. The generation of a vortex due to the interaction of falling and rising fluid parcels is also visible.

Figure 2.2: Snapshots of turbulent velocity vector fields at, (a) \( \dot{m} = 0.0315 \text{ kg/s}, T_{wall} = 30^\circ \text{C} \), (b) \( \dot{m} = 0.0315 \text{ kg/s}, T_{wall} = 40^\circ \text{C} \), (c) \( \dot{m} = 0.042 \text{ kg/s}, T_{wall} = 45^\circ \text{C} \), (d) \( \dot{m} = 0.0315 \text{ kg/s}, T_{wall} = 55^\circ \text{C} \).

Velocity field in Fig. 2.2(c) (\( \dot{m} = 0.042 \text{ kg/s at } T_{wall} = 45^\circ \text{C} \)) shows a different flow structure. The plot shows the formation of multiple vortices immediately above the bottom heated wall due to the interaction of rising and falling parcels of fluid. A strong rising plume in the middle section is prominent in the plot. The plume reached the top wall although it was diverted in the mid-height region by the strong cross-plane flow dispersing in the measurement plane. Strong sweeping flow is also visible in the upper
right and bottom left sections of the plot. The turbulent velocity field in Fig. 2.2(d) ($\dot{m} = 0.0315$ kg/s at $T_{wall} = 55$ °C) further highlights the complex interaction of flow throughout the measurement domain and the vortex formation at multiple heights as the wall temperature increases.

The plots in Fig. 2.2 show that the wall heating induces strong buoyancy-driven flow and the interaction of rising and falling fluid parcels forms a complex and dynamic three-dimensional flow structure. The plots also show that the bulk movement of the rising and falling fluid parcels is not organized, however, their interactions locally generate organized patterns such as vortices. It is also observed that at relatively low wall heating conditions, the strong turbulent motions are mainly restricted near the heated wall, however, as the wall heat flux increases, the turbulence becomes stronger and its influence reaches the upper unheated wall. The fluid parcels lose heat as they approach the upper wall, become denser and fall towards the bottom wall. The results also indicate that the buoyancy-driven flow also induced three-dimensional flow patterns within the domain.

The mean velocity field for each case was obtained by time-averaging the instantaneous velocities at each grid point. To illustrate the overall structure of the mean velocity within the measurement domain, the contours of the mean streamwise velocity field are presented in Fig. 2.3 at the lowest and highest flow rates and wall temperatures. The figure shows that at the lowest flow rate and lowest wall temperature (Fig. 2.3a), the mean streamwise velocity magnitude increased sharply with the distance and then gradually decreased to zero at the upper wall. However, as the wall temperature increased to the maximum value ($T_{wall} = 55$ °C), the maximum mean velocity remained almost at the same location but the mean velocity became negative in the upper quarter of the channel i.e. the mean flow direction in this region was opposite to the bulk channel flow (Fig. 2.3b). The mean velocity magnitude in the lower section of the channel was higher at the highest wall temperature. At the maximum flow rate, the trends and magnitudes of the mean velocity are quite similar at both low and high wall temperatures (Figs. 2.3c & 2.3d). Comparison of the plots at the lowest and highest flow rates shows that at the
lowest wall temperature, the flow structure is quite similar at both flow rates although the velocity magnitude is significantly higher at the highest flow rate, as expected.

Figure 2.3: Contours of mean streamwise velocity at (a) $\dot{m}=0.021$ kg/s, $T_{wall} = 30$ °C (b) $\dot{m}=0.021$ kg/s, $T_{wall} = 55$ °C (c) $\dot{m}=0.052$ kg/s, $T_{wall} = 30$ °C (d) $\dot{m}=0.052$ kg/s, $T_{wall} = 55$ °C. The colorbar is in cm/s.

However, the flow structure at the highest temperature is quite different. It is observed that the negative velocity trends present at the lowest flow rate diminished and the mean velocity became unidirectional at the highest flow rate.
The mean velocity profiles were computed by spatially averaging the time-averaged velocities at each height. The mean streamwise velocity profiles normalized by the domain-averaged streamwise velocity are presented in Fig. 2.4 for all flow rates and wall temperature conditions. In the classical Poiseuille flow, the mean streamwise velocity exhibits a parabolic profile. However, the plots in Fig. 2.3 and 2.4 clearly demonstrate that bottom heating altered the conventional parabolic velocity profiles for all cases. At the three higher flow rates, the mean velocity profiles became asymmetric and skewed towards the bottom heating wall. The location of the maximum mean velocity was around \( y/D \sim 0.1 \) and was consistent for all cases.

Figure 2.4: Normalized mean streamwise velocity profiles for various wall temperatures at (a) \( \dot{m} = 0.021 \text{ kg/s} \) (b) \( \dot{m} = 0.0315 \text{ kg/s} \) (c) \( \dot{m} = 0.042 \text{ kg/s} \) (d) \( \dot{m} = 0.0525 \text{ kg/s} \).
The mean streamwise velocity trends at the lowest flow rate (Fig. 2.4a) are significantly different from that at the higher flow rates particularly at the higher heating conditions. The plot shows that at the lowest wall temperature \( T_{\text{wall}} = 30^\circ \text{C} \), the mean streamwise velocity behaviour is similar to that at the higher flow rates. However, as the wall temperature increased, reverse flow observed near the top (unheated) wall region.

The magnitude and extent of the reverse flow increased with an increase in the wall temperature. Some previous studies have also reported back flow near the top unheated wall [4, 9 & 12]. Wang et al. [4] reported a mean velocity profile similar to that observed in Fig. 2.4 in the presence of the back flow. The issue of the back flow will be discussed in detail in the discussion section.

The turbulent properties are typically normalized by the characteristic velocity scale which however is defined differently for the forced and natural convection. For forced convection, friction velocity \( u_* \) is used as the characteristic velocity scale which is defined as,

\[
  u_* = \sqrt{\nu \frac{du}{dy}}_{y=0} \quad (2.2)
\]

Where \( \nu \) is the kinematic viscosity and \( \frac{du}{dy} \) is the mean streamwise vertical velocity gradient at the wall [24]. For natural convection, a convective velocity scale, \( w_* \) is used by Deardorff [25] and Adrian et al. [26], which is defined as,

\[
  w_* = (\beta g Q_o y_*)^{1/3}, \quad Q_o = \frac{q_s}{\rho c_p} \quad (2.3)
\]

Where \( \beta \) is the thermal coefficient of expansion, \( g \) is the gravitational acceleration, \( Q_o \) is the kinematic heat flux, \( y_* \) is the length scale equal to the channel height, \( q_s^* \) is the surface heat flux, \( \rho \) is the density and \( c_p \) is the specific heat [26].

None of these velocity scales can be used in the present study as both forced and natural convection exist and their relative contribution changed with the change in the flow rate.
or the wall temperature. That is, the forced convection contribution increased with the flow rate while the natural convection contribution increased with the wall temperature. As both forced and natural convection modes are present in the mixed convection, the velocity scale should account for both of these modes. Scaling based on $w_*$ and $u_*$ has been proposed in the field of atmospheric sciences for mixed convection [27-29]. The basic form of the defined velocity scale was used which is defined as,

$$v_* = \sqrt{w_*^2 + u_*^2}$$  \hspace{1cm} (2.4)

The values of $v_*$ for all cases are presented in Table 2.2. The results show that the influence of the wall temperature was more profound on $v_*$ compared to the flow rate.

<table>
<thead>
<tr>
<th>Wall temperature (ºC)</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass flow rate (kg/s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.021</td>
<td>0.47</td>
<td>0.61</td>
<td>0.67</td>
<td>0.74</td>
<td>0.8</td>
<td>0.83</td>
</tr>
<tr>
<td>0.0315</td>
<td>0.49</td>
<td>0.58</td>
<td>0.71</td>
<td>0.76</td>
<td>0.83</td>
<td>0.89</td>
</tr>
<tr>
<td>0.0420</td>
<td>0.5</td>
<td>0.6</td>
<td>0.73</td>
<td>0.8</td>
<td>0.85</td>
<td>0.94</td>
</tr>
<tr>
<td>0.0525</td>
<td>0.51</td>
<td>0.65</td>
<td>0.72</td>
<td>0.785</td>
<td>0.86</td>
<td>0.92</td>
</tr>
</tbody>
</table>

The contours of Root-Mean-Square (RMS) streamwise turbulent velocity at the lowest and highest flow rates and wall temperatures are shown in Fig. 2.5. The results show that at the lowest flow rate and wall temperature, the streamwise turbulent velocity magnitude is low (Fig. 2.5a). Relatively strong magnitudes are observed in the middle of the channel and immediately above the heated wall. The velocity magnitude decreased towards the upper unheated wall. As the flow rate increased while the wall temperature remained the same, an overall increase in the streamwise turbulent velocity was observed (Fig. 2.5c). The largest magnitudes of turbulent velocity were found adjacent to the bottom heated
wall and the upper region of the channel core. The velocity magnitudes decreased sharply when approaching the upper unheated wall. Fig. 2.5 (b & d) show the velocity contours at the highest wall temperature at both lowest and highest flow rates, respectively. Both plots show that the wall temperature has a significant impact on the streamwise turbulent intensity in the channel. The peak turbulent velocity in the channel increased by almost 100% and 70% at the lowest and highest flow rates as the wall temperature increased from 30°C to 55°C.

Figure 2.5: Contours of RMS streamwise turbulent velocity at (a) $\dot{m} = 0.021$ kg/s, $T_{wall} = 30$ °C (b) $\dot{m} = 0.021$ kg/s at $T_{wall} = 55$ °C (c) $\dot{m} = 0.052$ kg/s at $T_{wall} = 30$ °C (d) $\dot{m} = 0.052$ kg/s at $T_{wall} = 55$ °C. The colorbar is in cm/s.
The overall magnitude of streamwise turbulent velocity was increased by 130% and 82%. Although the turbulent velocity magnitude increased with the wall temperature, the overall pattern of the streamwise turbulent velocity distribution remained almost the same for all cases. That is, very strong streamwise turbulent velocity magnitude was observed immediately above the heated bottom wall, which decreased over a shorter distance and then increased to a maximum magnitude in the channel core and then decreased sharply to an almost negligible magnitude at the upper unheated wall. The comparison also shows that at a given wall temperature, an increase in the flow rate did not significantly influence the turbulent velocity magnitude but contributed to the diffusion of strong turbulence over a wider domain.

The RMS streamwise turbulent velocity profiles normalized by $U_*$ are presented in Fig. 2.6 for all flow rates and wall temperature conditions. The figure shows that the pattern of streamwise turbulent velocity at all conditions are in general similar. That is, strong streamwise turbulent intensities in the region immediately above the heated wall and in the channel core and almost negligible magnitudes at the upper unheated wall. The results however show that the location of the enhanced turbulence in the channel core shifted with the flow rate.
Figure 2.6: Normalized RMS streamwise turbulent velocity profiles for various wall temperatures at (a) $\dot{m} = 0.021$ kg/s (b) $\dot{m} = 0.0315$ kg/s (c) $\dot{m} = 0.042$ kg/s (d) $\dot{m} = 0.0525$ kg/s.

At the lowest flow rate, the enhanced turbulence region was in the middle of the channel which shifted towards the upper wall with an increase in the flow rate. In classical channel flows in the absence of heating, the streamwise turbulent velocity magnitude peaks near the wall and then decreases towards the channel core. Present results show that in the presence of wall heating, strong turbulence is observed in the channel core, which is due to the complex interaction of the buoyancy-driven flow with itself (i.e. the interaction of the rising plumes and falling fluid parcels as shown in Fig. 2.2) and with the streamwise channel shear flow.

The contour plots of the vertical turbulent velocity fields are plotted in Fig. 2.7 for the four extreme cases (maximum and minimum flow rates and wall temperatures). The structure of the vertical turbulent velocity is quite similar for all these cases, i.e., the vertical turbulent velocity magnitude increased sharply at the bottom heated wall to a peak value and then gradually decreased to zero at the upper unheated wall. The strong magnitudes of vertical turbulent velocity are observed in the region $0.2 < y/D_h < 0.7$, indicating that the vertical turbulent velocities are significant over almost half of the channel height.
The plots also show that the influence of the bottom wall temperature on the vertical turbulent velocity magnitudes is more profound than the effect of the flow rate. That is, at a given flow rate, increasing the bottom wall temperature from 30 °C to 55 °C almost doubled the velocity magnitude whereas the increase of the flow rate at a given wall temperature had an insignificant effect on the velocity magnitude. It is also observed that the strongest vertical turbulent velocities are present in the lower half of the channel.
The normalized RMS vertical turbulent velocity profiles are presented in Fig. 2.8 for all flow rates and wall temperature conditions. The behaviour for all flow rates and heating conditions is relatively comparable where strong asymmetry is evident in all velocity profiles. The velocity magnitude increased sharply from the bottom surface up to a height of $y/D_h \sim 0.2$ which is followed by a decrease towards the upper wall.

Figure 2.8: Normalized RMS vertical turbulent velocity profiles for various wall temperatures at (a) $\dot{m} = 0.021$ kg/s (b) $\dot{m} = 0.0315$ kg/s (c) $\dot{m} = 0.042$ kg/s (d) $\dot{m} = 0.0525$ kg/s.

The Reynolds stress ($-\overline{u'v'}$) contours are plotted in Fig. 2.9 for the four extreme cases. The plots show positive Reynolds stress ($-\overline{u'v'}$) adjacent to both heated and unheated walls which became negative in the middle region of the channel. The comparison of Reynolds stress magnitude near both walls shows that the magnitude of positive Reynolds stress is relatively stronger near the heated wall. It is also found that the relative
strength of Reynolds stress near the heated wall compared to the unheated wall increased with an increase in the wall temperature as well as the flow rate, however, the enhancement is more profound due to wall heating.

Figure 2.9: Contours of Reynolds stress at (a) $\dot{m} = 0.021$ kg/s at $T_{wall} = 30$ °C (b) $\dot{m} = 0.021$ kg/s at $T_{wall} = 55$ °C (c) $\dot{m} = 0.052$ kg/s at $T_{wall} = 30$ °C (d) $\dot{m} = 0.052$ kg/s at $T_{wall} = 55$ °C. The colorbar is in cm²/s².

It is also observed that at both flow rates, the magnitudes of positive and negative Reynolds stresses are quite comparable at the low wall temperature while, the negative Reynolds stress magnitude was almost three times larger than that of the positive Reynolds stress at the high wall temperature. The normalized Reynolds stress profiles are
presented in Fig. 2.10 for all flow rates and wall temperature conditions. The plots show similar overall trend for all cases however, as the flow rate increased, the negative Reynolds stress tends to become more uniform in the channel core. Overall the region of negative Reynolds stress remained almost the same at different flow rates and wall temperatures. Gajusingh and Siddiqui [5] also reported negative Reynolds stress away from the heated wall in a channel flow at low flow rates that fall in the laminar regime in the absence of heating. At higher flow rates in the turbulent regime, classical Reynolds stress trends were observed.

![Figure 2.10: Normalized Reynolds stress profiles for various wall temperatures at (a) \( \dot{m} = 0.021 \text{ kg/s} \) (b) \( \dot{m} = 0.0315 \text{ kg/s} \) (c) \( \dot{m} = 0.042 \text{ kg/s} \) (d) \( \dot{m} = 0.0525 \text{ kg/s} \).]

2.4 Discussion

The results presented in the preceding section provide a qualitative and quantitative insight into a channel flow subjected to heating from the bottom wall at low flow rates
that falls within the laminar regime in the absence of heating. The mean velocity results show that the mean streamwise velocity profiles deviated significantly from the classical Poiseuille trend, indicating that at low Reynolds numbers and high Grashof numbers, the bottom wall heating significantly alters the mean velocity structure of the channel flow. The mean velocity peak shifted towards the bottom heated wall at all cases. One plausible explanation for the increase in the mean velocity near the heated wall could be the decrease in viscosity of water due to higher temperatures near the heated wall. This decrease in viscosity causes a reduction in the flow resistance and hence, the flow tends to follow the path of lower resistance. This increase in flow near the bottom wall automatically causes a decrease in the mean velocity towards the upper unheated wall to satisfy mass conservation. This explanation was further confirmed by the behaviour of mean vertical velocity component at different conditions (not shown here). Unlike the classical Poiseuille flow, the vertical component of velocity was found to be non-zero and predominantly negative (i.e. downward motion). As mentioned earlier, the low flow resistance made the bottom wall region a preferred flow path, which also continuously dragged some flow from above resulting in a mean downward vertical velocity.

The results also show a bulk back flow in the upper region of the channel. This back flow is observed at the lowest flow rate, and its magnitude increased with an increase in the wall temperature, and also at some higher flow rates at higher wall temperatures. It is observed that the back flow occurred when the $Gr/Re^2$ was greater than about 55 and the magnitude of the back flow increased in general with an increase in $Gr/Re^2$. The contour plots in Fig. 2.3 (b & d) show almost horizontal contour lines associated with the back flow indicating that the mean back flow observed at the measurement location is not a local phenomenon. As a part of a separate study in the same channel, we measured the flow fields at different locations along the test section at different flow rates and bottom wall temperatures and observed that in cases where the back flow phenomenon is present, the back flow extended over the entire heated section of the channel. In other words, the forward flow is found along the bottom heated wall while the back flow is found along the top unheated wall. This flow behaviour indicated the manifestation of a large convective cell that covered the entire heated section of the channel. The intensity of this
channel-scale cell increased with an increase in the bottom wall temperature. A thorough analysis of the PIV images and the corresponding instantaneous velocity fields from the present dataset showed that the local instabilities in the form of rising plumes and falling fluid parcels continuously interacted with the cell and the energy transfer due to this interaction is likely the source that provided sustenance to this convective cell. Fig. 2.11 shows the snapshots of instantaneous velocity fields depicting these interactions. Several previous studies also observed the back flow near the top wall [4,9,11 & 12]. However, only few of them provided a plausible explanation of this trend and attributed it to the positive streamwise pressure gradient caused by the vertical motion of the rising plumes and falling fluid parcels [4&11].

Figure 2.11: Snapshots of instantaneous velocity vector fields at $\dot{m} = 0.021$ kg/s and (a) $T_{wall} = 50$ °C (b) $T_{wall} = 55$ °C.

The streamwise and vertical turbulent velocity results show trends which are significantly different from the conventional unheated channel flows. It is observed that both turbulent velocity components have large amplitudes away from the wall. The profiles of the vertical turbulent velocities were found to be more organized with the peak magnitudes located in the region $0.2 < y/D < 0.4$ for all cases, while, the streamwise turbulent velocity profiles were less organized. The streamwise turbulent velocities showed two peaks; the strongest peak located immediately above the heated wall, whereas the
secondary peak located in the middle of the channel at the low flow rate and shifted to the upper half of the channel with an increase in the flow rate. The streamwise turbulent velocity was in general found to be relatively more uniform in the channel core.

The rising plumes are more intense close to the bottom heated wall and start to weaken with height due to decrease in the density fluctuations and an increase in viscosity [23]. Similarly, the falling parcels of cooler fluid accelerate as a result of the gravitational force. As depicted in Fig. 2.2, the interaction between the rising plumes and the falling parcels is most intense in the mid and lower sections of the channel. This is the region where the vertical turbulent velocity magnitudes are relatively large. As discussed earlier, a shear flow is also superimposed on the buoyancy-driven secondary flow (plumes and falling parcels). This interaction of the shear flow with the secondary flow is expected to manifest more profoundly in the streamwise turbulent velocity. As the shear flow is dominant in the bulk of the channel, it interacts with the rising plumes and the falling fluid parcels throughout this domain. Furthermore, this shear flow distorts the vertical motions, i.e. the mean shear sweeps the secondary flow. This sweeping behaviour occurred in the channel core region away from the bottom and top walls. These interactions would likely be the reason for strong and relatively uniform distribution of the streamwise turbulent velocities in this domain as seen in Fig. 2.6. The results also show that the contribution of the top unheated wall to the streamwise turbulent velocity is almost negligible as compared to that contributed by the bottom wall heating.

The contours and profiles of Reynolds stress in general show a different behavior than that in conventional unheated turbulent channel flows. In conventional turbulent channel flows, the Reynolds stress increases sharply to a peak value in the inner layer and then decreases to zero in the channel core. The present results show that in the region immediately adjacent to the heated wall, the Reynolds stress increases sharply to a peak value in the inner layer similar to the conventional channel flows. However, unlike conventional channel flows, the Reynolds stress become negative in the channel core and again become positive as approaching the upper unheated wall. The region of the negative Reynolds stress approximately coincides with the region of strong turbulent
velocity magnitudes. This indicates that in this region, the complex interaction of buoyancy induced rising plumes and falling parcels of cooler fluid with the shear flow resulted in the strong turbulent motions but at the same time these interactions caused a rapid energy and hence the momentum transfer from the energetic turbulent field.

The positive Reynolds stress is a measure of the momentum transfer by the turbulent velocity field [24]. Turner [23] argued that in the presence of buoyancy, the turbulent energy transfer is more rapid due to the stronger interaction of turbulent and buoyancy fields and proposed a buoyancy subrange within the inertial subrange, where the energy transfer rate is faster (-3 slope) than that in the classical inertial subrange (-5/3 slope). The spectral analysis of our data (not shown here) indicated that the spectra become steeper than -5/3 in the region of the negative Reynolds stress. Hence, it can be argued that the negative Reynolds stress implies the momentum transfer from the turbulent velocity field to the buoyancy field. The plots in Fig. 2.9 also show that the magnitude of negative Reynolds stress increased with the wall temperature but was almost independent of the flow rate. This further confirms that the presence of the negative Reynolds stress is due to the buoyancy field.

The results presented and discussed earlier have shown that the structure of the turbulent velocity fields is very complex due to the three-dimensional interactions of buoyancy-induced secondary flows among themselves and with the shear-driven flow. The vector plots in Fig. 2.2 and the various contour maps of turbulent properties provide a general overview of this structure. However, in order to get a deeper insight into the underlying physical mechanisms that lead to this complex turbulent flow, we performed the proper orthogonal decomposition (POD) analysis of the turbulent velocity fields.

### 2.4.1 Proper Orthogonal Decomposition

The spatial distribution of flow energy content at different orthogonal modes can be obtained by POD analysis. It allows determining the spatial scales and the associated energy of dominant turbulent flow structures. The mathematical formulation of POD was presented in chapter 1. In order to validate the algorithm used for the POD analysis, a
comparison was conducted between the original turbulent velocity fields obtained from PIV data and the turbulent velocity fields reconstructed by combining all POD modes with their temporal coefficients. Fig. 2.12 shows this comparison. As can be seen from the plots, both velocity fields are almost identical qualitatively and quantitatively, thus, validating the POD scheme used in this study.

The fractional energy distribution over the POD modes is presented in Fig. 2.13 for the highest and lowest bottom wall temperatures and flow rates. The figure shows that the fractional energy for all cases decreases with an increase in the POD mode, as expected. The plot also shows that up to mode 25, the fractional mode energy remains approximately the same over the given range of flow rates and wall temperatures. However, the mode energy starts to vary with the flow rate and wall temperature beyond mode 25.

![Figure 2.12: Turbulent velocity vector field at $\dot{m} = 0.0315 \text{ kg/s}$ and $T_{wall} = 45 \degree C$, (a) original (b) reconstructed POD energy vector field.](image)

It is observed that beyond mode 25, at a given flow rate, the fractional mode energy increases with an increase in the bottom wall temperature; and at a given wall temperature, the fractional mode energy increases with the flow rate.
The increase in the magnitude of the fractional mode energy with temperature at a given flow rate is possibly due to the increase in the strength of buoyancy-driven secondary flow, whereas, the increase in the magnitude of fractional mode energy with the flow rate at a given temperature is likely due to the increase in the shear produced turbulence (see Table 2.1). The low fractional energy at higher modes implies that most of the flow energy resides in lower modes (see modes 8-14 in Fig. 2.13). At the lowest wall temperature and the flow rate, 86% of the fractional mode energy resides in the first 25 modes, while, at the highest wall temperature and the flow rate, 74% of the fractional mode energy resides in the first 25 modes. These results indicate that an increase in both the buoyancy-induced as well as shear-driven flows redistributes the flow energy towards the higher modes (see Fig. 2.13).

The detailed analysis of POD energy patterns associated with streamwise and vertical turbulent velocities showed that the flow structure for all cases is quite similar at different modes. This indicates that the underlying mechanisms associated with the generation of the secondary flow and its interactions are similar however, the energy associated with these mechanisms changes with the Grashof and Reynolds numbers. For example, it is found that the overall flow energy at all modes increases with an increase in the wall
temperature at the given flow rate, and increases with an increase in the flow rate at the
given wall temperature.

The detailed analysis of POD energy patterns of vertical turbulent velocity at different
modes revealed some intriguing features which provides a better understanding of the
overall flow dynamics. The contour plots of the vertical turbulent flow energy at various
modes are presented in figure 2.14 for the flow rate of 0.042 kg/s and wall temperature of
45°C. Regert et al. [30] argued that a physical link exists between the dominant POD
structures and the real flow structure. Hence, the dominant flow structures presented in
Fig. 2.14 can be related to the real physical behaviour of the flow. The results at mode 2,
which represents almost an overall flow behaviour, showed two alternate positive and
negative contours. The positive contours represent the upward motion while the negative
contours represent the downward motion. This alternate upward and downward motion is
hereinafter referred to as a convective cell. Thus, the result at mode 2 showed the
presence of a large convective cell, which brings the warm lighter fluid from the bottom
heated wall towards the upper unheated wall and brings the cool and dense fluid from the
upper end towards the bottom. As the mode number increased, the cells become smaller
in size and strength. As the turbulent flow is comprised of the superposition of all modes,
the results in Fig. 2.14 show that buoyancy-driven flow induced these convective cells of
different scales and intensities. In general, these cells are more intense near the heated
wall and hence their influence on the fluid transportation is more dominant in the lower
half of the channel. The presence of these intense convective cells near the heated wall is
likely the reason for the enhanced vertical turbulent velocity in this region as observed in
Fig. 2.8. The contour plots of the POD modes of streamwise turbulent velocity (not
shown here) do not show similar cell-type features but rather a relatively uniform
distribution of positive and negative contours in the bulk of the channel domain. This
could be the reason for the relatively uniform magnitudes of streamwise turbulent
velocity in the bulk channel domain as shown in Fig. 2.5.
Figure 2.14: Contours of POD modes of vertical turbulent velocity at $\dot{m} = 0.042$ kg/s and $T_{wall} = 45$ °C for (a) mode 2 (b) mode 4 (c) mode 9 (d) mode 18 (e) mode 25 (f) mode 30.
The results presented in this paper provided a deeper insight into the mean and turbulent flow structure in a horizontal channel heated from below at low flow rates. Based on these results, the overall physical processes associated with this type of flow can be described as follows. When heat is added from the bottom wall, the mean streamwise velocity structure is modified and the velocity peak shifts towards the bottom wall due to the lower flow resistance. As the wall heating increases and the flow rate decreases, a channel-scale convective cell is induced which is sustained primarily by the local convective cells. The interactions of the rising plumes of warm fluid and the falling parcels of the cooler dense fluid generate complex turbulent velocity fields. The complexity further increases due to their interactions with the mean shear flow. These interactions caused the momentum transfer from the turbulent field to the buoyancy field i.e. the turbulence is utilized in working against buoyancy forces. The underlying mechanisms of these interactions are similar but their intensity changes with the wall heating and the flow rate.

2.5 Conclusion

The impact of bottom heating on the flow behaviour inside a horizontal square channel at low Reynolds numbers and high Grashof numbers has been investigated experimentally. The results showed that free convection is dominant over forced convection for all cases. Buoyancy–driven secondary flow was generated as a result of bottom heating in the form of rising plumes of warm fluid and falling parcels of cold fluid. Turbulence was generated in the flow due to the interaction between the buoyancy-driven secondary flow among themself and with the shear-driven flow. These interactions created complex three-dimensional flow. The mean streamwise velocity profiles were altered as a result of bottom heating where the maximum streamwise velocity magnitude was shifted in the lower half of the channel. A back flow along the upper unheated heated wall was induced when $Gr/Re^2 > 55$ and its magnitude increased with an increase in the wall temperature. The profiles of RMS streamwise turbulent velocity showed peak magnitude immediately above the heated wall, and relatively high magnitudes in the channel core. It was found
that increasing the shear flow diffuses the streamwise turbulence intensity in the channel core region. The vertical turbulent velocity structure was found to be similar for all cases and their magnitudes were relatively large in the mid and lower sections of the channel. The bottom wall temperature showed a stronger influence than the flow rate on the magnitudes of both turbulent velocity components. Reynolds stress increased sharply to a peak value in the inner layer in the region adjacent to the bottom heated wall before becoming negative in the channel core region. It was argued that the negative Reynolds stress indicates the momentum transfer from the turbulent velocity field to the buoyancy field. It was found that the negative Reynolds stress magnitude increases with the wall temperature and was almost independent of the flow rate. The POD analysis revealed the presence of convective cells primarily in the lower half of the channel. The analysis also showed that the underlying mechanisms associated with the generation of the secondary flow and its interactions are similar however, the energy associated with these mechanisms changes with the Grashof and Reynolds numbers.

2.6 Acknowledgment

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2.7 References


Chapter 3

The influence of bottom wall heating on the mean and turbulent flow behaviour in the near wall region during mixed convection

3.1 Introduction

Buoyancy is known for its influence on the hydraulic and thermal behaviour of the flow. Mixed convection heat transfer is the process where buoyancy through natural convection coexists with forced convection. Mixed convection can occur at both low and high Reynolds number flows. Grashof number ($Gr$) to Reynolds number ($Re$) ratio ($Gr/Re^2$) determines the relative contribution of natural and forced convection modes in the mixed convection regime [1]. The Grashof number is the ratio between buoyancy forces and viscous forces while, Reynolds number is the ratio between the inertial forces and viscous forces. Hence, the $Gr/Re^2$ represents the ratio between the buoyancy and inertial forces. Low Reynolds number mixed convection can be found in several industrial applications such as: electronic cooling, chemical and nuclear reactors, food process industry and biomedical applications. Mixed convection heat transfer has been studied extensively over the past few decades. However, majority of the previous work was focused on evaluating overall heat transfer behaviour by quantifying the Nusselt number ($Nu$).

Gajusingh and Siddiqui [2] experimentally studied the effect of wall heating on the flow characteristics in the near wall region inside a square channel. They found that the buoyancy generates turbulence for originally laminar flow while for originally turbulent flow, buoyancy dampens turbulence. They argued that for turbulent flow, turbulence is dampened due to working against buoyancy. Mahaney et al. [3] studied mixed convection inside rectangular duct with bottom heated wall and adiabatic side and top walls. They

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2 The chapter is a journal article submitted to International journal of Thermal Science
found that the secondary flow induced by natural convection was in the form of vortices due to the interactions between ascending plumes near the side walls and falling of cooler fluid. Nusselt number was found to increase downstream of the channel due to buoyancy-driven secondary flow enhancement. Osborne and Incropera [4-5] experimentally investigated the effect of buoyancy on convection heat transfer inside horizontal channels with heated top and bottom walls. For turbulent flow regime, they observed forced convection near the top wall and mixed convection near the bottom wall [4]. For laminar flow, they observed the presence of thermally stable boundary layer at the top blocking the thermal plumes originating from the bottom wall to reach the top wall [5].

Wang et al. [6] identified different flow patterns for mixed convection along the heated test section of a horizontal square channel with a bottom heated wall through flow visualization. They found that the flow passes through four different flow regimes along the channel heated section: laminar forced convection, laminar mixed convection, transient mixed convection and turbulent free convection. Ozsunar et al. [7] experimentally studied mixed convection inside a bottom heated rectangular channel with side insulated walls. Their results showed that the development of mixed convection mechanism is strongly affected by $Re$ and $Gr$ values. They found that an increase in the Grashof number accelerated the development of mixed convection due to the generation of secondary buoyancy-driven flow, while, an increase in the Reynolds number delayed it. Huang and Lin [8] numerically investigated laminar mixed convection in a horizontal rectangular duct heated from below. They studied the effect of buoyancy-inertia ratio on the cross stream flow behaviour. They found that with the increase of $Gr/Re^2$, the cross stream flow behaviour shifts from a steady vortex flow at $Gr/Re^2 < 4$ into a chaotic flow at the channel exit at $Gr/Re^2 > 25$. Lin and Lin [9] visualized the air flow structure in a cross stream plane inside a bottom heated horizontal rectangular channel and found that the flow structure changes from periodic to quasiperiodic or chaotic with an increase in Gr or decrease in Re. Nandakumar et al. [10] also focused on visualizing the flow structure in the cross stream direction for different horizontal channel geometries heated from below. Longitudinal vortex patterns of two or four vortices were observed and the
bifurcation of the vortices was found to depend on the Grashof number and channel aspect ratio.

In spite of numerous studies on mixed convection at low Reynolds numbers, the structure of turbulence and its role on the transport of heat inside the channel fluid domain in the mixed convection mode still lack fundamental understanding. In particular, the flow dynamics in the region close to the heated bottom wall, which plays a very crucial role in regulating the buoyancy-induced secondary flow and the heat transfer rate, has not been thoroughly investigated and characterized. Furthermore, due to the complex interaction of the buoyancy-induced secondary flow and the shear-driven flow, flow measurements in multiple planes provide a better characterization of the turbulent flow structure, which is likely three-dimensional in this case. The present study provides a qualitative as well as quantitative analysis of the flow near the heated wall in multiple planes with the aim of characterizing the three-dimensional mean and turbulent flow behaviour.

3.2 Experimental Setup

Experiments were conducted in a 7 cm × 7 cm square channel comprised of three sections; inlet section, test section and end section, as shown in Fig. 3.1(a). The inlet section 0.7 m long was made of aluminum and had a ½ inch diameter inlet, which housed a honeycomb to straighten the flow and damp any disturbances. The test section (1.5 m in length) has top and side walls made of ½ inch non-tempered glass and an aluminum bottom plate. Two strip heaters (1500W - 250V) 1.3 m in length were installed directly underneath the bottom aluminum plate in parallel. The upstream ends of the heaters were located 10 cm downstream of the test section inlet. Five thermocouples were embedded in the bottom wall along the channel to monitor the uniformity of the bottom heated wall. It was observed that the bottom wall has good temperature uniformity with the variations within ±1 °C. To control the bottom wall temperature, a temperature controller (ZESTA-ZCP513) was used through a feedback loop from a thermocouple embedded in the bottom wall close to the measurement location. The end section was 30 cm in length and had a ½ inch diameter exit. Both inlet and end sections have bleed valves to remove any
air trapped inside the channel, and pressure gauges to monitor the inlet and outlet fluid pressure.

Clean tap water was used as the working fluid. Four, 200 liters storage tanks (barrels) were used to store the water at the room temperature, which allowed supplying the water at the same inlet conditions throughout a given experimental run. In the present study, the inlet water temperature was maintained around 24.5 ºC. The water was stored in the barrels for two days with periodic stirring to remove air bubbles present in the tap water. To circulate the water through the loop, a magnetic pump (Little Giant, 5 MD) was used. The flow rate of water was controlled via a flow meter with a control valve (FL4205, Omega Engineering) (see Fig. 3.1a).

Four mass flow rates 0.0210, 0.0315, 0.0420 and 0.0525 kg/s were considered (corresponding Reynolds numbers in the absence of heating are 300, 450, 600 and 750, respectively, for reference). At each flow rate, experiments were conducted at different bottom wall temperatures which were 30, 35, and 40 ºC. The corresponding Grashof numbers ranged from $6.37 \times 10^6$ to $1.57 \times 10^7$. To allow steady state to be reached, for each set of experiments, the measurements were taken 30 minutes after adjusting the flow rate for a given bottom wall temperature.

Particle image velocimetry (PIV) technique was used for measuring two-dimensional velocity fields in one vertical and two horizontal planes (see Fig. 3.1b). The PIV system comprised of a 120 mJ Nd:YAG laser (SoloPIV 120XT 532nm) as the light source, a 2336×1752 pixels CCD camera (VA-4M32, Vieworks) to capture the images in the measurement plane along with an image acquisition system (CORE-DVR, IO industries) connected to a PC, and a four-channel pulse generator (555-4C, Berkeley Nucleonics Corporation) to control the laser pulses timing and synchronizing them with the camera frames. Silver-coated glass spheres with the mean diameter of 15 µm. were mixed into water, which acted as the tracer particles for the PIV measurements.
Since the bottom heated wall acts as the source of buoyancy-induced secondary flow, more detailed information about the flow structure in its vicinity is crucial to improve the understanding of the overall flow dynamics in the channel as well as the fundamental mechanisms associated with the wall heating. Therefore, the horizontal plane measurements were focused in the near bottom wall region, while the vertical plane covered the vertical extent of the entire channel. The measurements were taken 1.3 m downstream of the test section in the horizontal planes at two different heights ($y/D_h=0.05$ $y/D_h=0.25$) and in the vertical mid-plane of the channel at $(z/D_h=0.5)$, see Fig. 3.1(b), where $D_h$ is the hydraulic diameter of the channel. Hereinafter, the horizontal plane at $y/D_h=0.05$ will be referred to as the lower plane and the plane at $y/D_h=0.25$ will be referred to as the upper plane. The side wall at $z/D_h=0.0$ will be referred to as the right wall and the side wall at $z/D_h=1.0$ will be referred to as the left wall. At each experimental run, 3000 images were captured at a sampling rate of 30 Hz (100 seconds of recording time). This resulted in 1500 instantaneous velocity fields at a rate of 15 Hz. To
ensure that the recording time was long enough for time-averaging, the mean streamwise velocities were computed over different time durations from 20 seconds to 100 seconds. Comparison showed almost identical profiles of the mean streamwise velocity for 60, 80 and 100 seconds time durations confirming that the recorded time was long enough. The flow was hydrodynamically developed at this measurement location based on a correlation for the laminar entrance region [11], which was further confirmed by the mean streamwise velocity measurements over the given Reynolds number range in the same channel in the absence of heating, prior to these experiments.

PIV employs cross-correlation technique to compute velocity vector field from two consecutive images (an image pair). That is, correlating the interrogation window in the first image with the search region in the second image of the image pair. In the present study, the interrogation windows size was set as 32×32 pixels and the search window size was set as 64×64 pixels. That is, each velocity vector was calculated based on the interrogation window area (32×32 pixels or 1.36×1.36 mm). By using a 50% overlap of interrogation windows, the nominal resolution of the velocity field was increased to 16×16 pixels (0.68×0.68 mm). A scheme based on the local median test proposed by Siddiqui et al. [12] was used to identify and correct spurious velocity vectors. The spurious vectors detected and corrected were well below 1%.

The uncertainty in the PIV velocity measurements is introduced due to the errors arise from the tracer particle diameter, tracer particles density, velocity gradients, out of plane motion, the dynamic range, peak locking, and Adaptive Gaussian Window interpolation [13]. The case with the highest flow rate and the highest bottom wall temperature was considered for the error estimation as the largest velocity gradients occur at these conditions. Using the criteria and data from Cowen and Monismith [13] and Prasad et al. [14], the total uncertainty in the PIV velocity measurements was estimated. The maximum error in velocity measurements was estimated to be ±0.05 cm/s which was less than 4.6 % of the bulk flow velocity.

Temperatures were measured in the vertical plane at (z/D_Pt=0.5) and under the same conditions as for the velocity measurements in a separate set of experiments. A rake of
nine T-type thermocouples with the accuracy of ±0.5°C was used with variable spacing and positioned vertically at a distance of 130 cm downstream of the test section. Most of the thermocouples were clustered near the bottom wall to resolve the thermal boundary layer. The exact positioning of each thermocouple is shown in Fig. 3.1(c). T-type thermocouples were also located at the upstream and downstream ends of the channel to measure the inlet and outlet water temperatures, respectively. A 12-channel data acquisition module (National Instruments NI 9211) was used to acquire the temperature data via LabVIEW data acquisition software. The data was recorded for five minutes at a sampling rate of 3Hz. Thermocouples were calibrated using a mercury-bulb thermometer as a reference.

3.3 Results

As mentioned earlier, the relative contributions of forced and free convection modes in mixed convection can be quantified by the ratio $Gr/Re^2$. Natural convection is dominant when $Gr/Re^2 >> 1$ while forced convection becomes dominant when $Gr/Re^2 << 1$ [1]. In the present work, this ratio ranged between 9 at the lowest wall temperature and highest flow rate, and 107 at the highest wall temperature and lowest flow rate (see Table 3.1).

<table>
<thead>
<tr>
<th>Mass flow rate</th>
<th>Wall temperature (°C)</th>
<th>$Gr/Re^2$</th>
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<tbody>
<tr>
<td>0.0210</td>
<td>30</td>
<td>52.5</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>106.5</td>
</tr>
<tr>
<td>0.0315</td>
<td>30</td>
<td>24.7</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>37.2</td>
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<td></td>
<td>40</td>
<td>50</td>
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<tr>
<td>0.0420</td>
<td>30</td>
<td>14.2</td>
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<td></td>
<td>35</td>
<td>21.5</td>
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<td>40</td>
<td>28.6</td>
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<td>0.0525</td>
<td>30</td>
<td>9.3</td>
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<td></td>
<td>35</td>
<td>14</td>
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<td></td>
<td>40</td>
<td>19</td>
</tr>
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</table>

This indicates that natural convection was the dominant mode for all cases. Natural convection induces buoyancy-driven secondary flow in the form of rising plumes of
warm fluid from the bottom heated wall and falling parcels of cooler fluids from the top unheated wall.

The interactions of buoyancy-induced rising plumes and falling fluid parcels among themselves and with the shear flow results in a complex flow structure. The complexity of this flow structure in both the horizontal and vertical planes of the channel at different flow rates and heating conditions is illustrated in Fig. 3.2. Turbulent velocity fields were computed by subtracting the time-averaged velocity from the instantaneous velocity fields at each grid point of the measurement domain. Turbulent velocity vector field in x-z plane at \( y/D_h = 0.05 \) for \( m = 0.0420 \text{ kg/s} \) and \( T_{wall} = 30 ^\circ \text{C} \) is shown in Fig. 3.2(a). The plane is located very close to the heated wall which highlights the formation mechanism of the rising plumes. The plot shows that the flow converges along streaks of strong turbulent intensity (two such streaks are evident in the figure). As the warm fluid always rises, the velocity field in Fig. 3.2(a) indicates that for the given conditions, the rising plumes originate from the streaks which are formed by the convergence of the warm fluid. A visual inspection of the turbulent velocity fields indicate that these streaks are roughly aligned in the streamwise direction. It is also observed that they move very slowly in the downstream direction and also meander. The merger of streak was also found in some instances resulting in a longer streak. The plot also shows the dispersion of the flow along a line at \( x=2 \). This is likely associated with the falling cold fluid from the top, which disperses as it approaches the bottom wall. The dispersion is observed to occur along lines, which indicates that the cold fluid falls in the form of sheet as normally observed in natural convection [15]. As the wall temperature was increased more streaks of shorter lengths were observed (see Fig. 3.2b). An interesting observation in both Figs. 3.2(a) and 3.2(b) is the presence of two counter-rotating vortices attached to the streaks. These vortices mainly appear at the upstream end of the streaks. The turbulent velocity vectors in x-z plane at \( y/D_h = 0.25 \) for \( m = 0.315 \text{ kg/s} \) and \( T_{wall} = 30 ^\circ \text{C} \) are shown in Fig. 3.2(c). The figure shows that the velocity field was mainly comprised of flow patterns that emerge from source-like flow. This source-like flow can be interpreted as the flow approaching the plane from either the bottom (i.e. rising plume) or the top (falling cold fluid). As it is difficult to distinguish the direction of the approaching fluid in this plane,
the presence of multiple sources indicates that both streams are likely active in this plane and interacting with each other. From the visual inspection of the turbulent velocity fields, source-like flow patterns were also observed in the lower plane, which are likely associated with the falling fluid parcels reaching the bottom heated wall.

The turbulent velocity vector field at the same height and flow rate as in Fig. 3.2(c) is shown in Fig. 3.2(d) at the highest wall temperature ($T_{wall} = 40$ °C). The plot shows that as the wall temperature increases, turbulent velocity magnitudes become large which is likely due to the stronger buoyancy-driven secondary flow. The vector field still shows the complex flow structure due to the interactions of the rising and falling fluids as in Fig. 3.2(c). The turbulent velocity field in the vertical mid plane parallel to the side walls along the channel center line ($z/D_h = 0.5$) for $m = 0.315$ kg/s and $T = 35$ °C is shown in Fig. 3.2(e) to illustrate the impact of bottom wall heating on the overall flow structure in the channel’s vertical plane (i.e. bounded between the bottom heated wall and unheated upper wall). The plot shows rising plumes of warm fluid in the vicinity of the bottom heated wall. It also shows falling parcels of cooler fluid close to the bottom heated wall and along the right edge of the plot. The interaction between these two streams forms vortices as seen at different location in the domain. Fig. 3.2(f) shows the turbulent velocity vector plot at the same flow rate but higher wall temperature ($T_{wall} = 40$ °C).
Figure 3.2: Snapshots of turbulent velocity vector fields at $y/D_h=0.05$ for (a) $\dot{m} = 0.0420 \text{ kg/s}, T_{wall} = 30 ^\circ \text{C} (Gr/Re^2 = 14)$ and (b) $\dot{m} = 0.0420 \text{ kg/s}, T_{wall} = 40 ^\circ \text{C} (Gr/Re^2 = 28.6)$, at $y/D_h=0.25$ for (c) $\dot{m} = 0.0315 \text{ kg/s}, T_{wall} = 30 ^\circ \text{C} (Gr/Re^2 = 24.7)$ and (d) $\dot{m} = 0.0315 \text{ kg/s}, T_{wall} = 40 ^\circ \text{C} (Gr/Re^2 = 50)$, and at $z/D_h=0.5$ for (e) $\dot{m} = 0.0315 \text{ kg/s}, T_{wall} = 35 ^\circ \text{C} (Gr/Re^2 = 37.2)$ and (f) $\dot{m} = 0.0315 \text{ kg/s}, T_{wall} = 40 ^\circ \text{C} (Gr/Re^2 = 50)$. Note that the resolution of the turbulent velocity field is reduced in the plots for better illustration of the flow features.

The turbulent velocity vector fields shown in Fig. 3.2 at both the horizontal and vertical planes depict the turbulent flow structure along three axes under the effect of bottom wall heating. The plots show that the turbulent flow structure in the region near the heated wall is complex and three-dimensional in nature. This complexity and three-dimensionality is a result of the generation of buoyancy-driven secondary flow in the
form of rising plumes, which eventually transform into the falling parcels as they ascend in the channel, and their interaction with each other and with the mean shear flow. The plots also show that in the immediate vicinity of the heated wall, turbulent velocity vectors merge together to form streak of intense turbulent velocity. Comparison of turbulent velocity fields in different planes shows that the turbulent velocity field is most intense and active in the immediate vicinity of the wall, which regulates the formation of thermal plumes that in turn control the dynamics of the overall turbulent flow structure in the channel. In the following, detailed characterization of the mean and turbulent flows in the region near the heated wall is presented to obtain a better understanding of the underlying heat transfer process.

The average profiles of different velocity components were calculated in x-z (horizontal) and x-y (vertical) planes by spatially averaging in the streamwise direction the time-averaged velocities at each spanwise and vertical location, respectively. Since the focus of this paper is on the flow dynamics in the region near the heated wall where the data in horizontal planes were acquired, the vertical plane data only in the corresponding region ($0 < y/D_h < 0.3$) are presented as the data at greater height are not relevant to this study and will be presented and discussed elsewhere. The mean streamwise velocity profiles in the lower vertical plane, normalized by the average streamwise velocity in the channel ($U_c$), are plotted in Fig. 3.3 over the complete range of $Gr/Re^2$ considered in this study. The results in a given plane are split in two figures to cover the lower range ($9.3 < Gr/Re^2 < 28.6$) and higher range ($37.2 < Gr/Re^2 < 106.5$). As expected, the plot showed the boundary layer profile in the near wall region. However, for almost all conditions, the mean velocity increased sharply from the bottom wall to a height $y/D_h \sim 0.1$ and reached the maximum magnitude and then gradually decreased towards the channel core.
Figure 3.3: Normalized mean streamwise velocity profiles at $z/D_h=0.5$ for (a) lower and (b) higher ranges of $Gr/Re^2$.

This shift in the peak velocity from the channel centerline (classical Poiseuille flow) towards the bottom heated wall indicates the influence of bottom wall heating on the mean flow structure. Similar shift in the peak velocity was also reported in other studies [16&17].
An interesting observation in the plots is that the profiles collapsed very well at low magnitudes of $Gr/Re^2$, indicating that the mean streamwise flow structure and the normalized magnitudes are very similar in the lower vertical plane. However, at high magnitudes of $Gr/Re^2$, the profiles showed variations in the trend and the magnitude. That is, in this range, the normalized mean streamwise velocity magnitude increased with an increase in the $Gr/Re^2$ values and the velocity decreases more sharply towards the channel core (see Fig. 3.3b).

![Graphs showing velocity profiles](image)

Figure 3.4: Normalized mean streamwise velocity profiles at $y/D_h=0.05$ for (a) lower and (b) higher ranges of $Gr/Re^2$ and at $y/D_h=0.25$ for (c) lower and (d) higher ranges of $Gr/Re^2$. 
The mean normalized streamwise velocity profiles in the two horizontal planes are shown in Fig. 3.4. The plots show that the mean streamwise velocity has similar behaviour in both horizontal planes (i.e. near the heated wall).

However, the mean streamwise velocity structure in the spanwise direction shows $Gr/Re^2$ dependency. The results show that at high values of $Gr/Re^2$, the mean streamwise velocity magnitude is relatively low in the middle of the channel and increases in the spanwise direction up to a normalized distance of approximately 0.1-0.2 from each channel side wall and then sharply decreases to zero at the walls. However, as the magnitude of $Gr/Re^2$ decreases, the streamwise velocity becomes relatively uniform over most of the channel width ($0.1 < z/D_h < 0.9$), at these heights and decreases sharply to zero at the side walls. The plots also showed higher magnitudes of the mean streamwise velocity in the upper horizontal plane, as expected. The results in Figs. 3.3 and 3.4 indicate that when the buoyancy-induced secondary flow becomes strong relative to inertia, it could alter the mean streamwise flow structure.

The mean spanwise velocity profiles normalized by $U_c$ are plotted in Fig. 3.5 at both horizontal planes for the given range of $Gr/Re^2$. The plots show that in general, the spanwise velocity magnitude gradually increases from the right wall till it reaches a maximum negative value (i.e. directed towards the right wall) at $z/D_h \sim 0.1$ for the lower plane and $z/D_h \sim 0.25$ for the upper plane. It then gradually decreases to zero at the mid plane and gradually increases till it reaches a maximum positive value (i.e. directed towards the left wall) at $z/D_h \sim 0.9$ for the lower plane and $z/D_h \sim 0.75$ for the upper plane followed by a gradual decrease towards the left wall. The plots also show that the magnitudes of the velocity profiles are generally symmetric about the channel mid-span. In the lower horizontal plane however, these trends are not very distinct at high values of $Gr/Re^2$ but become more distinct as $Gr/Re^2$ decreases. In the upper horizontal plane, this trend is prominent over the entire range of $Gr/Re^2$. Comparison of the velocity magnitudes in Fig. 3.5 shows that the mean spanwise velocity magnitudes are in general higher in the lower plane. The results show that the peak velocity magnitudes in the lower plane are almost 100% larger than that in the upper plane. The trends of the mean
velocity components observed in Figs. 3.3, 3.4 and 3.5 are discussed in detail in the discussion section.

Figure 3.5: Normalized mean spanwise velocity profiles at $y/D_h = 0.05$ for (a) lower and (b) higher ranges of $Gr/Re^2$ and at $y/D_h = 0.25$ for (c) lower and (d) higher ranges of $Gr/Re^2$.

Conventionally, the frictional velocity $u_*$ is used as the characteristic velocity scale for forced convection [18] and $w_*$ is defined as the velocity scale for natural convection [19,20]. However, in mixed convection both forced and natural convection modes are present and hence a proper velocity scale for mixed convection would be the one that accounts for both modes of convection. Researchers have defined a turbulent velocity
scale by combining the two velocity scales described above in different forms (Zeman and Tennekes [21]; Driedonks [22] and Moeng and Sullivan [23]).

![Figure 3.6: Normalized RMS streamwise turbulent velocity profiles at $z/D_n=0.5$ for (a) lower and (b) higher ranges of $Gr/Re^2$.](image)

We used this velocity scale in the basic form as,
\[ v_* = \sqrt{w_*^2 + u_*^2} \]  

(3.1)

Fig. 3.6 shows the vertical profiles of the RMS (root mean square) streamwise turbulent velocity normalized by \( v_* \) at \( z/D_h = 0.5 \) for all cases. It can be seen that the trend of the streamwise turbulent velocity is overall similar for the entire range of \( Gr/Re^2 \). Strong streamwise turbulent velocity is found in the region adjacent to the bottom wall with the peak magnitude close to \( y/D_h = 0.05 \). The streamwise turbulent velocity showed some decrease in the magnitude with a further increase in height and beyond \( y/D_h = 0.15 \), it showed increasing trend for most cases. The results also showed that the magnitude of the streamwise turbulent velocity very close to the heated wall (0 < \( y/D_h < 0.1 \)) are approximately 20% higher at low values of \( Gr/Re^2 \), however, at greater heights the velocity magnitudes are quite comparable.

The spanwise profiles of the RMS streamwise turbulent velocity normalized by \( v_* \) are plotted in Fig. 3.7 at \( y/D_h = 0.05 \) and 0.25 for all cases. The profiles show similar behaviour for all cases in both planes. That is, the streamwise turbulent intensity is relatively uniform across almost the entire spanwise width of the channel and decreased sharply close to the side walls.
Figure 3.7: Normalized RMS streamwise turbulent velocity profiles at $y/D_h=0.05$ for (a) lower and (b) higher ranges of $Gr/Re^2$ and at $y/D_h=0.25$ for (c) lower and (d) higher ranges of $Gr/Re^2$.

This indicates that the streamwise turbulent velocities are well distributed throughout the spanwise width of the channel at these heights. The plots also show that the normalized magnitudes of streamwise turbulent velocity are relatively higher near the heated wall, particularly at lower values of $Gr/Re^2$, which is consistent with the results in Fig. 3.6. The variations in the velocity magnitudes in the spanwise direction as observed in the plots are likely due to the complex interactions of the buoyancy-driven secondary flow and the presence of streaks in the lower plane and the source-like flow patterns as can be seen in Fig. 3.2.

The vertical profiles of the normalized RMS vertical turbulent velocity at $z/D_h=0.5$ are presented in Fig. 3.8 for all conditions. The profiles show a similar behaviour for all cases where the velocity magnitude increases sharply from the bottom wall up to a height of $y/D_h=0.25$. The data at greater heights (not shown here) indicated a gradual decrease from this height towards the top unheated wall.
Figure 3.8: Normalized RMS vertical turbulent velocity profiles at $z/D_h=0.5$ for (a) lower and (b) higher ranges of $Gr/Re^2$.

The spanwise profiles of the normalized spanwise RMS turbulent velocity are plotted in Fig. 3.9 at $y/D_h = 0.05$ and 0.25, for all conditions. The plots show that the spanwise turbulent velocity magnitudes gradually increased from the side wall and generally have relatively uniform magnitudes in the channel mid horizontal plane. The plots also show higher variability of the velocity magnitudes in the lower horizontal plane which is closer...
to the heat wall and that the velocity profiles become more uniform in the channel mid horizontal plane with an increase in the distance from the heated wall. As mentioned earlier, this could likely be due to the complex interactions and the presence of streaks in the lower plane (see Fig. 3.2).

The spanwise turbulent velocity magnitudes in the lower range of $Gr/Re^2$ are relatively larger than that in the higher range of $Gr/Re^2$ in both planes. It is also observed that the spanwise turbulent velocity magnitudes in the lower plane are approximately 40% higher than that in the upper plane for the entire range of $Gr/Re^2$.

Figure 3.9: Normalized RMS spanwise turbulent velocity profiles at $y/D_h = 0.05$ for (a) lower and (b) higher ranges of $Gr/Re^2$ and at $y/D_h = 0.25$ for (c) lower and (d) higher ranges of $Gr/Re^2$. 
The turbulent velocity fields were measured over a range of $Gr/Re^2$ values. To investigate the impact of this ratio i.e. the relative contribution of buoyancy and inertia on the turbulence generation near the heated wall, the overall magnitude of all three turbulent velocity components (expressed as root-mean-square values) within the two horizontal planes ($y/D_h = 0.05$ and $0.25$) are plotted as a function of $Gr/Re^2$ in Fig. 3.10. Fig. 3.10(a) shows the magnitude of the streamwise component of turbulent velocity.

![Graph of streamwise component of turbulent velocity](image1)

![Graph of wall-normal component of turbulent velocity](image2)
Figure 3.10: Normalized RMS turbulent velocity in the near wall region versus $Gr/Re^2$, (a) streamwise turbulent velocity (b) spanwise turbulent velocity (c) vertical turbulent velocity.

The results show a clear dependency of streamwise turbulent velocity on the $Gr/Re^2$ near the heated wall and almost no dependency at a greater height. Near the heated wall, the streamwise turbulent velocity magnitude decreases with an increase in the $Gr/Re^2$ in the range $10 < Gr/Re^2 < 30$. However, for $Gr/Re^2 > 30$, the change in the streamwise turbulent velocity is not significant.

The spanwise turbulent velocity component in the lower plane near the heated wall also showed similar dependency in the range $10 < Gr/Re^2 < 50$ as seen in Fig. 3.10(b). The vertical turbulent velocity component however did not show a clear dependency on $Gr/Re^2$ in the given range (see Fig. 3.10c). The results show that the flow rate has relatively stronger influence on the normalized magnitudes of the streamwise and spanwise turbulent velocities. The lower values of $Gr/Re^2$ presented in the results primarily correspond to the higher flow rates at both high and low wall temperatures. It was found that an increase in the bottom wall temperature at these flow rates decreases the normalized magnitudes of turbulent velocities.
To obtain the local Nusselt number \((Nu)\), the local heat transfer coefficient was calculated by equating the local conduction heat transfer in the sublayer with the local convective heat transfer, using the fluid temperatures measured in the vertical plane, as shown earlier in Fig. 1(c). The Nusselt number was obtained from,

\[
Nu = \frac{h D_h}{k}
\]  

(3.2)

Where \(k\) is the fluid thermal conductivity and \(h\) is the coefficient of heat transfer. The Graetz number (Gz) was computed as,

\[
Gz = \left(\frac{D_h}{L}\right) Re Pr
\]  

(3.3)

Where \(L\) is the longitudinal distance from the test section entrance and \(Pr\) is the Prandtl number [28]. In the current study, the temperature measurements were conducted at \(L = 135\) cm. The increase in Gz is due to the increase of both \(Re\) and \(Pr\) values. The Nusselt number is plotted in Fig. 3.11 as a function of Gz at three different bottom wall temperatures. The plot shows that the Nusselt number increased with an increase in the Graetz number at all bottom wall temperature cases. It is also observed that at a given Graetz number, the Nusselt number increased with an increase in the bottom wall temperature. The results are also compared with the correlation proposed by Osborne and Incropera [5] for the low Reynolds numbers mixed convection inside a channel. That is,

\[
Nu = \left[1.656Gz + 0.012 Ra^{3/4}\right]^{1/3}
\]  

(3.4)

Where \(Ra\) is Rayleigh number [5]. The comparison shows a good agreement between the present data and the given correlation with the average difference of 2% and the maximum difference of 4%.
Figure 3.11: Local Nusselt number versus Graetz number for various flow rates at different wall temperatures. The Nusselt number values based on the correlation proposed by Osborne and Incorpera [5] (Equation 3.4) are also plotted for comparison.

3.4 Discussion

From the results shown in the previous section, it can be clearly seen that in mixed convection with a predominant free convection mode (i.e. low Reynolds number, high Grashof number flows), the flow structure is complex and three-dimensional near the bottom wall due to the heating. The turbulent velocity fields in Fig. 3.2 provide a pictorial description of this flow structure in different planes which can be combined to obtain a three-dimensional perspective of the flow field near the heated bottom wall. As observed, in the plane parallel and close to the heating wall (i.e. lower horizontal plane) the warm fluid converges along the streaks and rises while the cooler fluid falls in the form of sheets and disperses horizontally. During the horizontal dispersion, the fluid gains the heat from the wall, becomes lighter and converges along the streaks. Thus, these streams are sustained by the continuous supply of warm fluid. Another interesting observation was the two counter-rotating vortices attached to the upstream end of the streaks. As
mentioned earlier, these streaks travel in the downstream direction, therefore, it is safe to speculate that these vortices play an important role in the streak dynamics. One plausible explanation could be that these vortices feed the warm fluid to the streak which helps in its propulsion as well as to move the streaks in the downstream direction. As noticed through visual observations, the movement and meandering of these streaks is very slow. This could likely be due to the longer time scales of the buoyancy-induced flow.

The complex interaction of the rising plumes and the falling sheets in the vertical plane results in the flow dispersion in the horizontal planes as observed in the velocity field in the upper horizontal plane (Figs. 3.2 c & d), thus, forming complex three-dimensional flow. The complex three-dimensional flow structure observed in Fig. 3.2 is similar to that observed in natural convection flows (e.g. see Volino and Smith [15]; Bukhari and Siddiqui [24]). This similarity indicates that in the present conditions, the buoyancy plays the major role in the turbulence generation, while the impact of the shear flow is minor.

As mentioned earlier, the flow rates considered in this study are low (well within the conventional laminar regime in the absence of heating). In classical internal laminar flow in the absence of heating, the mean velocity has parabolic profile with the boundary layers on all the walls merge at the center of the channel. The above results show that the bottom wall heating significantly influences the mean velocity behaviour inside the channel. It induces asymmetry in the mean flow with the velocity peak shifted towards the bottom wall and suppresses the bottom boundary layer. One plausible explanation for this velocity shift is that close to the bottom heated wall, the flow has higher temperature compared to the upper region of the channel. Therefore, the viscosity in the lower section of the channel is smaller than that in the upper section, which results in less flow resistance in the lower section of the channel. Hence, the flow tends to follow the least resistance path. The results in Fig. 3.3 indicate that low flow inertia has some impact on the mean streamwise velocity behaviour. It is observed that in the lower range of $Gr/Re^2$ i.e. higher flow rates (0.0420 and 0.0525 kg/s), the mean streamwise velocity profiles collapsed within a narrow range (see Fig. 3.3a). However, in the higher range of $Gr/Re^2$ (lower flow rates), the mean streamwise velocity magnitudes show a monotonic increase.
with $Gr/Re^2$ (see Fig. 3.3b), and the profiles shape near the bottom heated wall are also different compared to the lower range of $Gr/Re^2$.

The mean streamwise velocity profiles in the two horizontal planes near the heated wall (Fig. 3.4) show a relatively uniform velocity distribution in the spanwise direction unlike the expected parabolic profile. In fact, these profiles look very similar to the mean velocity profiles in fully turbulent channel flows [25]. By definition, a turbulent flow has lower viscous effects compared to inertia which causes the uniform velocity profile in a channel. In typical unheated channel flows, the relative reduction of the viscous effects compared to inertial effects (i.e. the transition from laminar to fully turbulent flow) occurs by increase in inertia due to the increase in the fluid velocity while the fluid viscosity remains constant. In the present case, the opposite process occurs i.e. the viscous effects reduced in the lower section of the channel due to the wall heating and thus, the inertial effect become more dominant in that region. This resulted in a uniform velocity profile in the spanwise direction. A partial $Gr/Re^2$ dependency of the mean streamwise velocity similar to that in the vertical plane is observed in the spanwise plane. The streamwise and vertical turbulent velocity profiles in Figs. 3.6, 3.7 and 3.8 show a behaviour different from the classical turbulent velocity profiles inside a channel in the absence of heating. The interactions between rising plumes of warm fluid and falling parcels of cooler fluid resulted in complex turbulent flow patterns in both horizontal and vertical planes (see Fig. 3.2).

In both horizontal planes near the heated wall, the streamwise turbulence intensity is well distributed over almost the entire spanwise extent of the channel for all cases (see Fig. 3.7). As mentioned earlier in the results section, the variations in the velocity magnitudes in the spanwise direction as observed in the plots are likely due to the complex interactions of the buoyancy-driven secondary flow and the presence of streaks in the lower plane and the source-like flow patterns as can be seen in Fig. 3.2. The spanwise turbulent velocity profiles show parabolic trend i.e. the spanwise turbulent velocities are strongest in the middle of the channel and decreases gradually to zero at the side walls. The magnitudes of both streamwise and spanwise turbulent velocities in the lower plane
are stronger than that in the upper plane. This could be due to the strong dispersion of the cold falling fluid in the horizontal plane near the wall and the strong convergence of the warm fluid to form streaks as evident in Figs. 3.2(a) and 3.2(b). In the upper horizontal plane, the flow dispersion is due to the interaction of the warm rising plumes and the falling cold fluid however, the magnitudes of the dispersed flow in this plane is not as strong as that observed in the lower horizontal plane (see Figs. 3.2a to 3.2d). This explains the peak in streamwise turbulent velocity in the vertical plane close to the bottom heated wall as seen in Fig. 3.6. Due to the symmetry conditions, the unheated channel side walls have similar effect on both the streamwise and spanwise turbulent velocities.

The vertical turbulent velocity profiles have peak magnitudes at a height of $y/D_h \sim 0.25$ for all cases as seen in Fig. 3.8. The enhancement in the vertical turbulent velocity in this region can be attributed to the interactions of intense rising plumes and falling parcels. The intensity of the rising plumes is always higher near the bottom heated wall, which decreases away from the heated wall due to the decrease in density fluctuations and an increase in viscosity [26]. Similarly for the falling cold fluid parcels, they accelerate towards the bottom wall.

The results in the study provided an insight into the fundamental process of buoyancy generation and flow dynamics in the region adjacent to the heated wall. The flow rates considered in the study falls in the classical laminar regime in the absence of heating. The present results demonstrate that wall heating transitions the flow to fully turbulent even at low Reynolds numbers. This is attributed to the generation of strong buoyancy-induced secondary flow as well as the reduction of viscosity. The region near the heated wall plays an important role in the overall dynamics of the flow in the channel as it is the source of buoyancy-induced secondary flow. The results show that the bottom wall heating significantly alters the structure of both mean and turbulent velocities compared to that in the channel without heating. The turbulent velocities also show strong magnitudes near the bottom heated wall which could be attributed to the generation of the rising plumes and the interaction of rising plumes and falling fluid parcels. The results
also revealed the underlying process of the formation of the plumes near the heated wall. It is observed that these plumes are formed by the convergence of the warm fluid along the streaks. Over the vertical extent of the channel, these plumes interact with the falling cold parcels creating complex three-dimensional flow structure.

3.5 Conclusion

The flow structure under mixed convection heat transfer inside a horizontal square channel at low Reynolds numbers and over a range of Grashof numbers has been studied experimentally. Particle image velocimetry was used to measure two-dimensional velocity fields in various horizontal and vertical planes. Natural convection was found to be dominant over forced convection for all cases. Bottom wall heating induced buoyancy-driven secondary flow in the form of rising plumes of warm fluid and falling parcels of cold fluid. The flow structure was found to be complex and three-dimensional due to the interaction of rising plumes and falling fluid parcels. Both mean and turbulent velocity profiles were found to be altered due to bottom wall heating. Due to the lower viscosity near the bottom heated wall, the flow accelerated in this section causing a shift in the peak velocity from the centerline towards the heated wall. The mean streamwise velocity near the heated bottom wall was uniform in the spanwise plane due to uniform heating. The mean spanwise velocity showed axisymmetric trend near the bottom wall where the spanwise flow was primarily directed from the channel mid span towards the side walls. The mean spanwise velocities are almost an order of magnitude lower than the mean streamwise velocities. The mean streamwise velocities in all planes showed a partial dependency on the $Gr/Re^2$ ratio. In the lower range of $Gr/Re^2$, the mean streamwise velocity profiles collapsed well in all planes. However, in the higher range of $Gr/Re^2$, the velocity profiles showed an increase with $Gr/Re^2$.

The magnitudes of streamwise and spanwise turbulent velocity components in general are higher near the bottom heated wall and are of same order of magnitude. The streamwise and spanwise turbulent velocities near the bottom wall also showed a partial $Gr/Re^2$ dependency with larger magnitudes in the lower range of $Gr/Re^2$. The local
Nusselt number values were found to be in agreement with the correlation for low Reynolds number mixed convection in channel flows. The results also revealed the formation mechanism of the rising plumes and the dispersion of the falling fluid parcels near the bottom heated wall. That is, the cooler fluid falls in the form of sheets and disperses horizontally, while, the warm fluid converges and forms streaks which initiates the rising plumes.

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3.7 References


Chapter 4
The characteristics of coherent structures in low Reynolds number mixed convection flows

4.1 Introduction

Mixed convection occurs when both natural and forced convection coexist in the flow. The ratio between natural and forced convection, which is quantified by $Gr/Re^2$ [1] plays a significant role in the flow dynamics during mixed convection. Buoyancy-driven secondary flow is induced in the form of rising plumes of warm less dense fluid and falling parcels of cooler denser fluid. When mixed convection occurs in channel flows, the mean shear flow starts to interact with the buoyancy-driven secondary flow which results in a complex three-dimensional flow behaviour. One of the main characteristics of this type of flow is the strong presence of vortices at different directions. Vortices are commonly considered as coherent structures in turbulent flows [2]. Hussain [3] defined coherent structure as a connected turbulent fluid mass with phase-correlated vorticity over its spatial extent. Coherent structures play a significant role in the large-scale transport of mass, heat and momentum [4].

In order for the coherent structures to be investigated, they must be detected within the turbulent flow field. Several detection methods have been utilized. For instance, Chong et al. [5] employed regions of complex eigenvalues of the velocity gradient tensor to detect coherent structures. They suggested that the complex eigenvalues refer to closed or spiral streamlines. Hunt et al. [6] utilized the positive second invariant of velocity gradient tensor to detect coherent structures. The second invariant presents the balance between the rotational magnitude and shear strain rate. Hussain and Hayakawa [7] used the vorticity definition to detect coherent structures based on their rotational magnitude. Zhou et al. [8] stated that every detection scheme showed a powerful performance in capturing vortices under certain flow situations but each one has its own limitations.
Mixed convection inside channels has been studied extensively in both laminar and turbulent flows. Particularly, for laminar flows, the flow has been studied in both the cross stream and streamwise vertical planes through numerical and experimental investigations. The main focus of the majority of previous work was to investigate bulk properties such as the Nusselt number. A qualitative analysis of the mean flow was of particular interest in some studies. Mixed convection inside horizontal ducts with bottom heated wall has been studied for both air and water at different channel aspect ratios [9-12]. Mahaney et al. [9] numerically investigated mixed convection in a rectangular channel with side and top adiabatic walls with aspect ratios between 2 and 10. They studied the secondary flow development in the channel cross plane. Plumes were observed in the mid-plane and along the side walls which created multiple longitudinal vortices. They found that the number of plumes increased with the increase of Grashof number. They also found that longitudinal vortex advects cooler fluid from the core to the bottom surface which disrupts the boundary layer development and enhance heat transfer.

Huang and Lin [10] numerically studied the buoyancy-induced flow transition in a rectangular channel at a Reynolds number of 500. They specified four regions based on the ratio between buoyancy and inertial forces ($Gr/Re^2$). They found that for $Gr/Re^2 < 4$, the flow is steady with dominating two longitudinal vortices. For higher values of $Gr/Re^2$, the flow became unsteady at a certain distance from the channel entrance. This distance was found to decrease with an increase in $Gr/Re^2$. For $Gr/Re^2 \geq 25$, the flow was chaotic near the channel exit. Lin and Lin [11] experimentally studied unstable mixed convection for Reynolds numbers up to 186 and Grashof numbers up to $5 \times 10^5$. They found that the secondary flow structure in the channel cross-plane was altered by increasing Grashof numbers and/or by decreasing Reynolds numbers. They observed that at high Grashof numbers, the steady vortex flow started transition and reached a chaotic flow state at Grashof numbers greater than $2.9 \times 10^5$. This trend was also observed with the decrease of Reynolds numbers. They also measured the temperature in the spanwise direction and found that the time-averaged profiles were changed with the change of secondary flow structure in the cross plane. Koizumi and Hosokawa [12] studied the effect of side wall temperature on the secondary flow behaviour. They observed that the longitudinal
vortices in the cross plane are affected by both Reynolds number and side wall temperatures.

Incropera and Schutt [13] studied the flow development in the cross plane of a channel heated from top and bottom walls with insulated side walls. They observed that the secondary flow was initiated at the bottom corners of the channel and a flow recirculation started to grow in the streamwise direction which was also observed by Huang and Lin [10]. They found that the top wall heating had negligible effect on the thermal and hydrodynamic conditions at the bottom plate. Sillekens et al. [14] investigated mixed convection inside a square channel with heated side walls. They found that the secondary flow is an order of magnitude lower than the mean streamwise flow. Nandakumar and Weinitschke [15] numerically investigation the flow under mixed convection in a horizontal square channel and found that at Grashof numbers between $10^5$ and $10^6$, the mean flow in the channel cross plane changes from a stable two cell flow to a four cell unstable flow. The secondary flow during mixed convection was also investigated inside horizontal tubes with different boundary conditions [16-17]. Choi and Choi [16] numerically studied the secondary flow development in the streamwise direction at a Reynolds number of 250 and Grashof number range $10^6 - 10^7$. They conducted the study for Prandtl numbers of 0.7 and 5. The tube upper half was insulated while the lower half was exposed to constant heat flux. They found that the strength of the secondary flow is weaker for Prandtl number of 5 compared to 0.7. They also found that close to the developed region, the secondary motion vanished for the higher Prandtl number case and the temperature in the core region became stably stratified. They also concluded that a four vortex flow can occur by increasing Grashof and/or Prandtl numbers.

As the literature review showed, the secondary flow has been studied in both the developing and fully developed region for different boundary conditions. The majority of previous research conducted a qualitative investigation of the coherent structures in the instantaneous flow filed. The investigation was done mainly in the channel cross plane and primarily comprised of numerical simulations. The buoyancy-driven secondary flow induces turbulence even at low Reynolds numbers. There is a scarcity of studies
investigating the coherent structures in turbulent flow field during low Reynolds numbers mixed convection. The main goal of the current study is to investigate the characteristics of turbulent coherent structures during mixed convection inside bottom heated horizontal square channel at low Reynolds numbers and high Grashof numbers. As the flow is complex and three-dimensional, the study will be conducted in different planes of measurements to elucidate coherent structures in all directions. The focus will be on performing a statistical analysis on the generated turbulent coherent structures and their associated properties. This would provide a better understanding of the behaviour of coherent structures generated during mixed convection and the role they play in the transport processes.

4.2 Experimental Setup

The experiment was conducted in a horizontal square channel 7 cm × 7 cm which consists of three parts as seen in Fig. 4.1(a). The first part is the inlet section which is 70 cm in length and made out of Aluminum. It contains a honeycomb to straighten the flow and dampen any disturbance before entering the test section. The test section is 150 cm long and the lower surface was made of ½ inch Aluminum plate. The inner surface of the aluminum plate was colored in black to prevent any light reflection during the experiment. The side and top walls were made of ½ inch non-tempered glass to allow visual access into the channel. Two aluminum plates, 1.3 cm thick and 5 cm high were used to support the channel. Two strip heaters (1500W- 250V) 1.3 m in length were directly installed in parallel underneath the bottom surface of the test section. The upstream edge of heaters was 10 cm downstream of the test section entrance. The test section bottom surface temperature was controlled by a temperature controller (ZESTA-ZCP513) through a feedback loop from a thermocouple embedded in the bottom wall close to the measurement location. The third section of the channel is the end section which is 30 cm in length and was made of Aluminum. The inlet and end sections were connected to the test section by Aluminum flanges. Both inlet and end sections have 1/2 inch diameter inlet and exit openings. Bleed valves were installed in the inlet and exit
sections to remove any trapped air in the channel. Pressure gauges were installed to monitor the inlet and exit fluid pressure as seen in Fig. 4.1(a).

Clean tap water was used as the working fluid and circulated in a close loop. As the water was continuously being heated through the channel, its temperature tended to build up during the experiment. Therefore, four barrels, 200 Litres each, were connected in series and worked as the water reservoir. By using these barrels, water was circulated only one time throughout a set of experiments. The channel inlet water temperature was maintained around 24.5 ºC due to constant room temperature. To obtain good quality results, air bubbles presented in the water had to be removed. Therefore, the water was stored in the barrels for two days with periodic stirring before running the experiments. A magnetic pump (Little Giant, 5 MD) installed downstream of the reservoir was used to pump the water into the channel. The water flow rate was controlled by a flow meter with a control valve (FL4205, Omega Engineering) installed between the pump and the channel inlet (see Fig. 4.1a).

The experiments were conducted at four mass flow rates 0.0210, 0.0315, 0.0420 and 0.0525 kg/s which correspond to Reynolds numbers of 300,450,600 and 750, respectively in the absence of heating. For each flow rate, the experiments were conducted at different bottom wall temperatures of the test section. The temperatures considered were 30, 35, 40, 45, 50 and 55 ºC. The experiments operating conditions yielded Grashof number in the range from $6.37 \times 10^6$ to $3.86 \times 10^7$. The experiments were conducted at different sets. For each set of experiments, the bottom wall temperature was fixed while the channel flow rate changed. For a given set of experiment, measurements were conducted 30 minutes after setting the flow rate to allow steady state to be reached. Particle image velocimetry (PIV) technique was used to measure the two-dimensional velocity fields in a streamwise vertical plane and two horizontal planes 130 cm downstream from the test section entrance as seen in Fig. 4.1(b). The streamwise vertical plane was at ($z/D_h=0.5$) parallel to the side walls of the test section where $D_h$ is the hydraulic diameter of the channel.
Figure 4.1: Schematic of (a) the experimental setup, (b) measurement planes, (c) stereo-PIV camera setup, (d) position of thermocouples in the rake; thermocouples 1-4 were 1 mm apart, thermocouples 5-7 were 2 mm apart, thermocouples 7-8 were 7.5 mm apart, and thermocouples 8-9 were 17.5 mm apart.
The two horizontal planes were parallel to the bottom surface at two different heights ($y/D_h = 0.05$ and $0.25$). For the lower horizontal planes, when the bottom wall temperature exceeded $40 ^\circ C$, bubble formation started in the vicinity of the bottom surface. These bubbles caused a substantial noise in the recorded images. It was impossible to filter out this noise which in turn degraded the results quality drastically. Therefore, the surface temperatures considered when measuring the flow in the horizontal planes were $30$, $35$ and $40 ^\circ C$ only. Stereoscopic particle image velocimetry (stereo-PIV) technique was used to measure the three velocity components in the channel cross plane, $135$ cm downstream of the test section entrance. Fig. 4.1(b) shows the orientation and location of all measurement planes. The mean and turbulent velocity fields in the cross plane are shown in Appendix 2. The PIV setup comprised of a $120$ mJ Nd:YAG laser (SoloPIV 120XT 532nm) to illuminate the measurement plane, a $4$ Megapixel ($2336 \times 1752$ pixels resolution) CCD camera (VA-4M32, Vieworks) to record the images in the vertical and horizontal planes, an image acquisition system (CORE-DVR, IO industries) to record images and a four-channel pulse generator (555-4C, Berkeley Nucleonics Corporation) to control the laser pulses timing and to synchronize them with the camera frames. For stereo PIV, two identical CCD cameras with the resolution of $2048 \times 2048$ pixels (Flare 4M-180, IO industries) were used. The cameras were mounted on the bottom side of a camera rail and positioned at $30^\circ$ angle to capture the images in the cross plane (see Fig. 4.1c).

Two Scheimpflug mount adapters (LaVision) were used to connect the camera with the lens to keep the entire image in the plane of focus [18]. For stereo PIV, as the cameras are looking at an angle to the channel side walls (see Fig. 4.1c), astigmatism would occur which in turn distorts the image quality. Therefore, two right angle glass prisms were attached to both channel side walls in the vicinity of the measurement plane and filled with clear tap water to eliminate any astigmatism in the recorded images [18](see Fig. 4.1c). A two level calibration plate (La Vision) was used for aligning the two cameras with the laser sheet. Silver coated glass spheres with a mean diameter of $15 \mu m$ were used as tracer particles. For each run, $3000$ images were recorded at a frequency of $30$
Hz. The recorded images yielded 1500 instantaneous velocity fields at a frequency of 15 Hz.

For two-dimensional PIV measurements, the images were processed using an in-house code that runs on MATLAB software. For stereo-PIV, the images were processed using commercial software (Davis 8.0, La Vision). For all PIV images, cross correlation was utilized with an interrogation window of 32×32 pixels and search window of 64×64 pixels. By using a 50% overlap of interrogation windows, the nominal resolution was increased to 16×16 pixels (0.68×0.68 mm for horizontal and vertical planes and 0.65×0.65 mm for the cross-plane). Local median filter was utilized to identify and correct spurious velocity vectors. The spurious vectors detected and corrected were well below 1%.

Different sources of errors are involved in PIV technique. These sources are tracer particle diameter, tracer particles density, velocity gradients, out of plane motion, the dynamic range, peak locking, and Adaptive Gaussian Window interpolation [19]. The uncertainty of the PIV velocity measurements was calculated at the highest flow rate and highest bottom wall temperature as the largest velocity gradients occur at these conditions. The uncertainty was estimated based on the criteria and data from Cowen and Monismith [19] and Prasad et al. [20]. The maximum error in velocity measurements was estimated to be ±0.071 cm/s which was less than 6.7% of the bulk flow velocity.

For the same operating conditions (i.e. velocities and bottom surface temperatures), temperatures were measured in the streamwise vertical plane in a separate set of experiments. Nine T-type thermocouples were positioned vertically on a rake. The thermocouples accuracy was ±0.5 °C. The thermocouples were positioned at different spacing. That is, most of the thermocouples were clustered near the bottom heated surface to resolve the thermal boundary layer (see Fig. 4.1d). Two T-type thermocouples located at the upstream and downstream ends of the channel were used to measure the water inlet and outlet temperature respectively. The temperatures were acquired using a 12 channel data acquisition module (National Instruments NI 9211) via LabVIEW data
acquisition software. Temperatures were recorded for five minutes at a rate of 3Hz. Thermocouples were calibrated using a mercury-bulb thermometer.

4.3 Coherent Structures Detection Scheme

Different detection schemes were used in the past to detect coherent structures. For instance, Hussain and Hayakawa [7] and Siddiqui et al. [21] used vorticity threshold ($\omega$) to detect vortices. Chong et al. [5] used eigenvalues of the velocity gradient tensor $\nabla u$. They proposed that the region of complex eigenvalues refers to the vortex core. Hunt et al. [6] defined vortices as the region with positive second invariant of the velocity gradient tensor $\nabla u$ and defined it as

$$Q = \frac{1}{2}(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij})$$

(4.1)

Where $\Omega$ and $S$ are the antisymmetric and symmetric components of the velocity gradient tensor $\nabla u$ and defined as,

$$\Omega_{ij} = (u_{i,j} - u_{j,i})/2$$

(4.2)

$$S_{ij} = (u_{i,j} + u_{j,i})/2$$

(4.3)

The term Q represents the balance between the shear strain rate and the rotational magnitude [6]. Dubief and Delcayre [22] argued that positive Q isosurfaces represent the region where the strength of rotation overcomes the strain and hence could be considered as the vortex. Jeong and Hussain [2] proposed another criterion based on the eigenvalues of the symmetric tensor, $\Omega^2 + S^2$, and considered connected region with two negative eigenvalues ($\lambda_2$) of the symmetric tensor as the vortex core.

We tested three of these detection schemes on our data, which are vorticity threshold ($\omega$), second invariant (Q) [6] and the eigenvalues of the symmetric tensor [2]. The turbulent velocity fields were computed by subtracting the time-averaged velocity from the instantaneous velocity at each grid point in the given measurement domain. The turbulent velocity fields were used to compute the vorticity ($\omega$), Q, and $\lambda_2$ fields for each of the
above detection schemes, respectively. A threshold was applied to each of the \( \omega \), \( Q \) and \( \lambda_2 \) fields and all the connected grid points where the value was greater than the threshold were detected as a coherent structure, i.e. \( (\omega > \omega_T) \) for the vorticity scheme, \( (Q > Q_T) \) for the second invariant \( Q \) scheme and \( (\lambda_2 > \lambda_{2,T}) \) for the symmetric tensor scheme. Since the data obtained from PIV were spaced 16 pixels (0.65 in the cross plane and 0.68mm in the horizontal and streamwise vertical planes), one grid point could be identified as a coherent structure if the value at that grid point is greater than the threshold. To ensure that each detected coherent structure occupies at least two grid points, a size threshold was applied on the detected coherent structures [7&21]).

The optimum threshold values for each scheme, which are \( Q_T \), \( \lambda_{2,T} \) and \( \omega_T \), respectively, were selected based on the comparison of coherent structures identified by these schemes with that detected through visual inspection in the corresponding turbulent velocity fields. 1500 turbulent velocity fields were considered for this comparison.
The coherent structures detected through each scheme (based on the optimal threshold) were then compared again with the coherent structures identified in the corresponding turbulent velocity fields by visual inspection to determine the scheme that detected the coherent structures most accurately.

The comparison shows that the second invariant (Q) scheme detected 90% of the true coherent structures correctly while 6% of the detected coherent structures were false. Symmetric tensor scheme detected 81% true and 12% false coherent structures while vorticity scheme detected 72% true and 19% false coherent structures. The above comparison shows that the second invariant (Q) scheme detected the coherent structures most accurately in the present dataset. The optimum threshold values for this scheme is Q=0.15. To further confirm this threshold, we used an approach described by Dubief and Delcayre [22], which is based on conditional sampling technique to identify the most energetic coherent structures. Based on this approach, it was found that at Q=0.225, the most energetic structures are detected. In the present study, the objective was to detect all
potential coherent structures not just the most energetic ones, hence, the threshold of $Q=0.15$ was considered to be appropriate. An example of the coherent structures detected by the second invariant ($Q$) scheme with the optimum threshold is shown in Fig. 4.2, where a typical turbulent velocity field and the detected coherent structures are shown. The figure clearly demonstrates the accuracy of the detection scheme.

4.4 Results

The coherent structures were detected in the turbulent velocity data obtained in different measurement planes (see Fig. 4.1b) over a range of flow rates and wall temperatures. The corresponding $Gr/Re^2$ values ranged from approximately 9 to 206 for vertical and cross-plane measurements and 9 to 107 for horizontal plane measurements. The values of $Gr/Re^2$ are significantly higher than unity, which indicates that under these conditions, the natural convection is a dominant mode in the mixed convection [1]. Various properties of the detected coherent structures were computed along with the location of each structure.

4.4.1 Number of Coherent Structures

Fig. 4.3 shows the number of coherent structures detected per second per cm$^2$ at different conditions in all measurement planes. Note that for all figures, the data points are connected through lines to show the trend. Fig. 4.3(a) shows the number of coherent structures per second per unit area ($f_{cs}$) detected in the cross-plane (streamwise or longitudinal vortices). It is observed that at a given flow rate, the number of coherent structures increased with an increase in the wall temperature, but the increasing trends are not similar. At the lowest flow rate, the number of coherent structures increased almost linearly with the bottom wall temperature. However, as the flow rate increased, the increase in the number of coherent structures was found to be more rapid in the middle range of the bottom wall temperature. At the two highest flow rates, the number of coherent structures remained almost the same in the lower range of the wall temperature, it increased rapidly in the mid wall temperature range and then increased slowly in the upper range of the wall temperature. As the wall temperature increased from $30 \, ^\circ C$ to 55
°C, the number of coherent structures increased approximately by a factor of 60 and 2.5 at the lowest and highest flow rates, respectively. The results also show that as the flow rate increased from 0.021 kg/s to 0.0525 kg/s, the number of coherent structures increased approximately by a factor of 18 and 1.1 at the lowest and highest wall temperatures, respectively.

Fig. 4.3(b) shows the number of coherent structures in the streamwise vertical plane (spanwise vortices). Overall, the number of coherent structures increased with an increase in the bottom wall temperature, however, no specific flow rate dependency was observed. The results also show that the trends are similar at all flow rates. The number of coherent structures at the highest wall temperature was approximately on average 10 times higher than that at the lowest wall temperature. The number of coherent structures increased almost linearly in the lower range of bottom wall temperature (30 °C to 40 °C), then remained almost unchanged from 40°C to 45°C and then increased rapidly as the wall temperature increased from 45°C to 50°C and then remained almost constant.

The coherent structures in the two horizontal planes near the bottom heated wall are shown in Fig. 4.3(c). In the lower horizontal plane immediately adjacent to the bottom heated wall ($y/D_h=0.05$), the results show an almost linear increase in the number of coherent structures with the bottom wall temperature at a given flow rate. The trends are similar for all flow rates, however, at a given wall temperature, the number of coherent structures increased with an increase in the flow rate. In the upper horizontal plane relatively away from the bottom heated wall ($y/D_h=0.25$), the number of coherent structures remained almost the same between 30°C and 35°C but increased as the wall temperature increased to 40°C. The results in Fig. 4.3 show the impact of flow rate and wall temperature on the number of coherent structures in various planes. Comparison shows that in the lower wall temperature range (30°C ≤ $T_{wall}$ ≤ 40°C), the largest number of coherent structures is present in the lower horizontal plane.
(a)

(b)
Figure 4.3: Number of coherent structures per second per unit area ($f_{cs}$) in the (a) cross plane, (b) streamwise vertical plane, (c) horizontal planes (open symbols: lower horizontal plane, solid symbols: upper horizontal plane).

For the higher wall temperature range ($45^\circ C \leq T_{wall} \leq 55^\circ C$), the largest number of coherent structures is present in the streamwise vertical plane. In the lower horizontal plane, the coherent structures in the immediate vicinity of the bottom heated wall are approximately two to seven times more in numbers than that at a greater height at a given condition. The coherent structures in the cross-plane are relatively lower in numbers compared to the vertical and near-wall horizontal planes. This could be due to the reason that buoyancy-induced secondary flow, the shear flow and their interactions are responsible for the generation of coherent structures in the vertical and horizontal planes. On the other hand, the coherent structures in the cross-plane are primarily induced by the buoyancy-induced secondary flow only.

The results in Fig. 4.3 show the overall number of coherent structures in a given measurement plane. Buoyancy-induced flow plays a major role in the generation of coherent structures and since the buoyancy force acts in the vertical direction, the distribution of coherent structures in the vertical direction relative to the bottom heated wall provides a better insight into the generation and advection of coherent structures in
such channel flows. The detection scheme used in the present study records the location of each detected coherent structure, and hence the data were used to estimate the number of coherent structures observed in different vertical regions relative to the bottom heated wall (vertical and cross-planes only). For this purpose, a given measurement plane was divided into three regions as follows. Region I represents the thickness of the thermal boundary layer which corresponds to \(0 < y/D_h \leq 0.12\) as seen in Fig. 4.4. This region shows the coherent structures characteristics that are generated in the thermal boundary layer. Region II represents the region of strong vertical turbulent velocity (i.e. vertical turbulent velocity is 80% or higher of the peak turbulent velocity) which corresponds to \(0.12 < y/D_h \leq 0.4\). Region III represents the channel upper region where the vertical turbulent velocity is lower than 80% of the peak turbulent velocity and corresponds to \(0.4 < y/D_h \leq 1\).

![Figure 4.4: Schematic of the regions along the channel height in the channel cross plane.](image)

Figure 4.4: Schematic of the regions along the channel height in the channel cross plane.
Due to the large volume of data (vertical profiles at 24 different cases), the results are presented as a function of bottom wall temperature at a given flow rate and as a function of flow rate at a given bottom wall temperature to effectively summarize the trends.

The variation in the number of coherent structures in the cross-plane in various vertical regions at a given flow rate versus the bottom wall temperature is shown in Fig. 4.5(a) and at a given wall temperature versus the flow rate is shown in Fig. 4.5(b). The results show that at all temperatures, the largest number of coherent structures is present in the channel upper region (region III) and the number decreases closer to the heated wall except for the lowest two wall temperature. At the lowest two wall temperatures, the difference in the number of coherent structures in the three regions is not very large. At higher wall temperatures ($T_{wall} \geq 40$ °C), the number of coherent structures increased monotonically from region I to III and the number increased more rapidly with an increase in the bottom wall temperature. At the highest wall temperature, the number of coherent structures in region III is approximately three times the number of coherent structures in region I. At each height, the results show a monotonic increase in the number of coherent structures with an increase in the bottom wall temperature. At a given flow rate, the number of coherent structures is slightly increasing from region I to region II and the number of coherent structures in these regions is comparable for the three largest flow rates as seen in Fig. 4.5(b). In the top region of the channel, it is seen that the number of coherent structures decreased for the lower two flow rates while a sharp increase in the number of coherent structures in observed at the two highest flow rates. In region III, the number of coherent structures is increased by approximately a factor of 4.5 as the flow rate increased from 0.0210 kg/s to 0.0525 kg/s.

The variation of the number of coherent structures in the streamwise vertical plane is shown in Fig. 4.6(a) versus the bottom wall temperature at a given flow rate ($\dot{m} = 0.042$ kg/s). A trend different from the cross-plane is observed for all bottom wall temperatures. The results show that the largest number of coherent structures is found in region II where the vertical turbulent velocity is strongest. It can be seen that the number of coherent structures increases from region I to region II at all wall temperatures.
Figure 4.5: Distribution of number of coherent structures ($f_{cs}$) per second per unit area in the cross plane along the channel height at (a) $\dot{m}=0.042$ kg/s, (b) $T_{wall}=40$ °C.

One can observe that the number of coherent structures slightly decreases at the top region of the channel (region III) as seen in Fig. 4.6(a). In general, the number of
coherent structures in all three regions increased with an increase in the bottom wall temperature.

Figure 4.6: Distribution of number of coherent structures ($f_{cs}$) per second per unit area in the streamwise vertical plane along the channel height at (a) $\dot{m} = 0.042$ kg/s, (b) $T_{wall}=40$ °C.

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The number of coherent structures in region II of the streamwise vertical plane is increased approximately by an order of magnitude as the bottom wall temperature increased from 30 °C to 55 °C. Fig. 4.6(b) shows the number of coherent structures distribution as a function of flow rate at a given wall temperature ($T_{wall} = 40 °C$). The number of coherent structures increased sharply away from the bottom heated wall in a linear trend for all flow rates between region I and II. Then, the number of coherent structures decreased in region III. It can be seen from the figure that the flow rate had negligible effect on the number of coherent structures in the streamwise vertical plane in the lower half of the channel (region I and II). However, the number of coherent structures in the top region of the channel is showing a strong dependency on the channel flow rate where the number of coherent structures increased with the increase of the flow rate.

### 4.4.2 Vorticity of Coherent structures

The mean and maximum vorticity associated with each coherent structure were computed. The trends for both parameters were similar and hence results for only one parameter (maximum vorticity) are presented here.

Fig. 4.7 shows the maximum absolute vorticity of coherent structures ($\omega_{max}$) at different conditions in all measurement planes. The results were computed by averaging the maximum absolute vorticity of all detected coherent structures at a given condition. Fig. 4.7(a) depicts the average maximum vorticity of coherent structures in the cross-plane. The results show that at a given flow rate, similar to the number of coherent structures, the vorticity of coherent structures increased with an increase in the wall temperature and the trends are relatively similar. As the bottom wall temperature increased from 30 °C to 55 °C, the maximum vorticity, on average, increased by 33% and 23% at the lowest and highest flow rates, respectively. It is also observed that at a given wall temperature, the vorticity of coherent structures in general increased as the flow rate increased. This increase in maximum vorticity was around 13% and 3% for the lowest and highest wall temperatures, corresponding to a flow rate increase from 0.021 kg/s to 0.0525 kg/s, respectively.
Figure 4.7: Average maximum vorticity ($\omega_{\text{max}}$) of the coherent structures in the (a) cross plane, (b) streamwise vertical plane, (c) horizontal planes (open symbols: lower horizontal plane, solid symbols: upper horizontal plane).

The maximum vorticity of coherent structures in the streamwise vertical plane (Fig. 4.7b) was found to increase with an increase in the wall temperature except in temperature ranges 40 °C - 45 °C and 50 °C - 55 °C. The results also show no significant dependency on the flow rate. These trends are consistent with that of the number of coherent structures in the streamwise vertical plane (see Fig. 4.3b). The average increase in the maximum vorticity of coherent structure was about 50% as the bottom wall temperature increased from 30°C to 55°C. The coherent structures in the two horizontal planes near the bottom heated wall are shown in Fig. 4.7(c).

In the lower horizontal plane immediately adjacent to the bottom heated wall, the results show that the vorticity of coherent structures increased almost linearly with the bottom wall temperature at a given flow rate. The trends are similar for all flow rates, however, at a given wall temperature, the vorticity of coherent structures increased with an increase in the flow rate. In the upper horizontal plane relatively away from the bottom heated wall, the vorticity of coherent structures increased as the wall temperature increased and the rate of increase is higher between 35 °C and 40 °C. The maximum vorticity showed a
clear dependency on the flow rate at both planes. The plot shows that at a given condition, the maximum vorticity in the lower horizontal plane is 15% to 30% higher than that in the upper horizontal plane. Comparison of maximum vorticity in different plane shows that in the lower range of bottom wall temperatures ($30 \leq T_{\text{wall}} \leq 40$), the maximum vorticity magnitudes are largest in the lower horizontal plane while for the upper temperature range ($40 < T_{\text{wall}} \leq 55$), the coherent structures in the streamwise vertical plane have largest maximum vorticity magnitudes. The vorticity magnitudes of coherent structures in the cross-plane are lower compared to vertical and lower horizontal planes. These trends are similar to that observed for the number of coherent structures.

The variation of the averaged maximum vorticity of coherent structures in the vertical direction is shown in Fig. 4.8 in the cross plane. The results are presented in the same manner as in Fig. 4.5. Fig. 4.8(a) shows the vertical variation of the maximum vorticity of coherent structures in the cross-plane at a given flow rate over the bottom wall temperature range. The results show a consistent trend at all bottom wall temperatures. That is, the maximum vorticity values are high near the bottom wall (region I), which then decreased toward region II and then increased again in the channel upper region (region III) except for $T_{\text{wall}} = 30$ and 50 °C where the maximum vorticity in region II and III are approximately equal. This trend is completely different from the trend seen for the variation of the number of coherent structures in the same plane (see Fig. 4.5a). Furthermore, in a given region, the values of the maximum vorticity in general are found to increase with an increase in the bottom wall temperature. Fig. 4.8(b) shows the variation of maximum coherent structures in the vertical direction at a given wall temperature as a function of the flow rate. The trends seen in the figure are relatively similar to the trend seen in Fig. 4.8(a). The maximum vorticity decreased in region II where it has the lowest maximum vorticity, which then increased in the channel upper region. The distribution showed in general a dependency on the channel flow rate where the maximum vorticity increased with the increase in the flow rate.
Figure 4.8: Distribution of average maximum vorticity ($\omega_{max}$) of the coherent structures in the cross plane along the channel height at (a) $\dot{m} = 0.042$ kg/s, (b) $T_{wall} = 40$ °C.

The vertical variations of the maximum coherent structure vorticity in the streamwise vertical plane is shown in Fig. 4.9(a) versus bottom wall temperatures at a given flow rate. The results show that for the two lower bottom wall temperatures, the maximum
vorticity had different distribution along the channel height away from the bottom heated wall where the maximum vorticity is almost the same in regions II and III.

Figure 4.9: Distribution of average maximum vorticity ($\omega_{\text{max}}$) of the coherent structures in the streamwise vertical plane along the channel height at (a) $\dot{m} = 0.042 \text{ kg/s}$, (b) $T_{\text{wall}} = 40 ^\circ \text{C}$.
For the higher temperature range, the maximum vorticity increased with height in the channel lower region (regions I and II) followed by a decrease in the maximum vorticity in the channel upper region (region III). The rates of increase and decrease are higher for the two highest wall temperatures. The maximum vorticity variation did not show a monotonic change with bottom wall temperature especially in the channel upper region (region III). The effect of flow rate on the coherent structures maximum vorticity distribution is shown in Fig. 4.9(b) in the streamwise vertical plane. In general, the flow rate showed no effect on the maximum vorticity distribution. The results in Fig. 4.7 present the mean value of maximum coherent structure vorticity for a given run. Fig. 4.8 and 4.9 provide an insight into the variation of maximum coherent structure vorticity in the vertical direction.

To obtain a better insight into the maximum vorticity of all coherent structures present in the flow at a given condition, the distribution of maximum vorticity of coherent structures was computed for each case and is presented in the form of probability density function (PDF) for different measurement planes in Fig. 4.10, 4.11 and 4.12. PDFs are shown for different bottom wall temperatures at \( \dot{m} = 0.042 \text{ kg/s} \) and for different flow rates at \( T_{wall} = 40 \degree \text{C} \) in the cross, streamwise vertical and the two horizontal planes. This provides an insight into how the coherent structures maximum vorticity distribution changes over the measured range of bottom wall temperature and flow rate.

The PDFs for the cross plane are shown in Fig. 4.10(a) for different bottom wall temperatures. The PDFs shift to the right with the increase of bottom wall temperature except at \( T_{wall} = 50 \degree \text{C} \). The distribution peak decreased with the increase of the bottom wall temperature and the distribution extended towards the right tail indicating that the maximum vorticity of more coherent structures increased when the bottom wall temperature increased. This shows the strong effect of the bottom wall temperature on the coherent structures maximum vorticity distribution. The PDFs for different flow rates are shown in Fig. 4.910(b). The figure shows that the PDFs in general shift to the right with the increase of flow rate. However, the figure did not show a monotonic shift of PDFs with the flow rate increase. Thus, in the cross plane, the effect of flow rate on the
distribution of coherent structures maximum vorticity is less significant than the effect of the bottom wall temperature.

![Figure 4.10](image_url)

Figure 4.10: Probability density function (PDF) of average maximum vorticity ($\omega_{\text{max}}$) of the coherent structures in the cross plane at (a) $\dot{m} = 0.042$ kg/s, (b) $T_{\text{wall}} = 40$ °C.
Figure 4.11: Probability density function (PDF) of average maximum vorticity ($\omega_{\text{max}}$) of the coherent structures in the streamwise vertical plane at (a) $\dot{m} = 0.042$ kg/s, (b) $T_{\text{wall}} = 40 ^\circ\text{C}$.

Fig. 4.11 shows the PDFs in the streamwise vertical plane as a function of the bottom wall temperature and flow rate. The PDFs shift to the right with the increase of the bottom wall temperature as seen in Fig. 4.11(a). However, no clear dependency on the bottom wall temperature was observed. The flow rate showed almost no effect on the
PDFs as seen in Fig. 4.11(b) and the PDFs for different flow rates collapse in a narrow band. This trend is different from the trend seen in the cross plane in Fig. 4.10(b). In the horizontal planes (Fig. 4.12a), similar to the cross plane, the distribution strongly depends on the bottom wall temperature. At a particular plane, PDFs shift to the right with the increase of bottom wall temperature.

Figure 4.12: Probability density function (PDF) of average maximum vorticity ($\omega_{\text{max}}$) of the coherent structures in the horizontal planes at (a) $\dot{m} = 0.042$ kg/s, (b) $T_{\text{wall}} = 40$ °C. Thick line: lower horizontal plane, thin line: upper horizontal plane.
PDFs in the lower horizontal plane (i.e. in the vicinity of the bottom heated wall) are shifted to the right and become relatively flattened relative to the PDFs in the upper horizontal plane. Thus, over the given range of bottom wall temperature, the lower horizontal plane has more coherent structures with higher maximum vorticity compared to the upper horizontal plane. The effect of the flow rate on the PDFs as seen in Fig. 4.12(b) shows that the PDFs had almost no dependency on the flow rate in a particular plane. While, the PDFs shift to the right in the lower horizontal plane compared to the upper horizontal plane similar to the trend seen in Fig. 4.12(a). Comparison between the PDFs in all planes show that the variation of coherent structures maximum vorticity increased with the increase of bottom wall temperature, while, over the giving range of flow rates, distribution of coherent structures vorticity had no dependency on the flow rate. The right-end tail extended more profoundly in the streamwise vertical plane than the cross plane. This indicates that the bottom wall temperature has a higher influence on the vorticity strength of coherent structures in the streamwise vertical plane.

4.4.3 Turbulent kinetic energy of coherent structures

The average turbulent kinetic energy (TKE) of all coherent structures was computed and the mean of averaged turbulent kinetic energy of all coherent structures for each case is presented in Fig. 4.13 for all measurement planes. The results for the coherent structures in the channel cross plane are shown in Fig. 4.13(a). It is seen from the figure that at a given flow rate, the mean turbulent kinetic energy of the detected coherent structures increased with an increase in the bottom wall temperature. The rate of increase is relatively similar at all flow rates. The profiles showed that the mean turbulent kinetic energy slightly decreased with an increase of the bottom wall temperature from 30 °C to 35 °C then, it increased almost linearly with the bottom wall temperature. For the highest bottom wall temperature range (50 °C - 55 °C), the rate of increase at the two lower flow rates was less than that at the two higher flow rates. Over the given range of wall temperature increase, the mean turbulent kinetic energy was increased by about 100% and 56% at the lowest and highest flow rates, respectively. The results also showed a strong dependency of turbulent kinetic energy on the flow rate.
\( T_{\text{wall}} \)

(a)

(b)
At a given wall temperature, the mean turbulent kinetic energy increased almost monotonically with an increase in the flow rate. The mean turbulent kinetic energy of coherent structures increased by 90% and 40% as the flow rate increased from 0.021 kg/s to 0.0525 kg/s for the lowest and highest wall temperature respectively. The mean turbulent kinetic energy of coherent structures in the streamwise vertical plane is shown in Fig. 4.13(b). Results show that the turbulent kinetic energy increased almost linearly in the lower range of bottom wall temperature (30 °C to 40 °C) then from 40 °C to 45 °C, the mean turbulent kinetic energy slightly decreased for the two lower flow rates and remained almost constant for the two higher flow rates. As the wall temperature increased from 45 °C to 50 °C, the mean turbulent kinetic energy at the lowest and highest flow rates increased relatively fast but then decreased with a further increase in the wall temperature. However, at the middle two flow rates, the mean turbulent kinetic energy increased almost linearly from 45 °C to 55 °C. In the streamwise vertical plane, the mean turbulent kinetic energy of coherent structures showed no dependency on the flow rate which was also observed in the same plane for other properties shown earlier. On
average, the turbulent kinetic energy of coherent structures in the streamwise vertical plane approximately increased by a factor of 2.7 as the bottom wall temperature increased from 30 °C to 55 °C.

Fig. 4.13(c) shows the mean turbulent kinetic energy of the detected coherent structures in the two horizontal planes. In the lower horizontal plane, the mean turbulent kinetic energy increased with an increase in the bottom wall temperature. In this plane, the flow rate showed a strong effect on the mean turbulent kinetic energy at a given bottom wall temperature. However, in the upper horizontal plane, relatively away from the bottom wall, the mean turbulent kinetic energy of coherent structures is less dependent on the bottom wall temperature. In this plane, the mean turbulent kinetic energy remained almost constant between 30 °C and 35 °C for the two higher flow rates while increased for the two lower flow rates. With a further increase in the wall temperature, the mean turbulent kinetic energy increased at all flow rates and the rate of increase was higher for the two middle flow rates. The figure also shows that the turbulent kinetic energy is on average 36% higher in the lower horizontal plane compared to the upper horizontal plane. Furthermore, over the range of bottom wall temperatures (30 °C to 40 °C), the mean turbulent kinetic energy of the coherent structures is higher in the lower plane than the upper plane by a range from 15% to 56%. Comparison of the mean turbulent kinetic energy magnitudes of coherent structures in different planes show that the coherent structures in the streamwise vertical plane have the largest mean turbulent kinetic energy followed by those in the cross plane and lower horizontal plane.

The variation in the mean turbulent kinetic energy of coherent structures with height is shown in Fig. 4.14 versus the bottom wall temperatures and flow rates in the cross plane. Figure 4.14 (a) shows that at all bottom wall temperatures except for $T_{wall} = 50 \, ^\circ\text{C}$ the mean turbulent kinetic energy of coherent structures is lowest near the bottom wall and increased with an increase in the vertical distance from the wall. The results also did not show a clear dependency on the bottom wall temperature. Fig. 4.14(b) shows the mean turbulent kinetic energy distribution as a function of flow rate. In general, the trends seen in the figure are similar to the trends observed in Fig. 4.14(a). The flow rate shows a clear
effect at a particular region. That is, the mean turbulent kinetic energy increased monotonically with the increase of flow rate.

Figure 4.14: Distribution of mean kinetic energy (TKE) of the coherent structures in the cross plane along the channel height at (a) $\dot{m} = 0.042 \text{ kg/s}$, (b) $T_{\text{wall}} = 40 \degree \text{C}$.

Figure 4.14: Distribution of mean kinetic energy (TKE) of the coherent structures in the cross plane along the channel height at (a) $\dot{m} = 0.042 \text{ kg/s}$, (b) $T_{\text{wall}} = 40 \degree \text{C}$. 

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In the streamwise vertical plane (Fig. 4.15a), at $30 \, ^\circ{C} \leq T_{\text{wall}} \leq 40 \, ^\circ{C}$, the turbulent kinetic energy distribution profile showed in general an increasing trend with height, however, at other bottom wall temperatures, the mean turbulent kinetic energy of coherent structures was found to be higher in region II.

(a)

(b)

Figure 4.15: Distribution of mean kinetic energy (TKE) of the coherent structures in the streamwise vertical plane along the channel height at (a) $\dot{m}=0.042 \, \text{kg/s}$, (b) $T_{\text{wall}}=40 \, ^\circ{C}$.
Some increasing trends in the mean turbulent kinetic energy magnitudes in the channel lower half (regions I and II) are observed with an increase in the bottom wall temperature. On the other hand, the flow rate did not show a noticeable effect on the mean turbulent kinetic energy distribution as seen in Fig. 4.15(b). This is consistent with the effect of flow rate on the number of coherent structures and the maximum vorticity in the streamwise vertical plane as seen in Fig 4.6(b) and 4.9(b).

Figure 4.16: Probability density function (PDF) of mean kinetic energy (TKE) of the coherent structures in the cross planes at (a) $\dot{m}=0.042$ kg/s, (b) $T_{wall}=40$ ºC.
Probability density functions (PDFs) of the mean turbulent kinetic energy in all measurement planes are presented in Fig. 4.16, 4.17 and 4.18 for different values of bottom wall temperatures and flow rates. The PDFs in the cross plane did not show a dependency on bottom wall temperature as seen in Fig. 4.16(a). The PDFs shift in general to the right with an increase in bottom wall temperature but it did not have a clear dependency on the bottom wall temperature.

![Figure 4.17](image1.png)

Figure 4.17: Probability density function (PDF) of mean kinetic energy (TKE) of the coherent structures in the streamwise vertical planes at (a) $\dot{m} = 0.042 \text{ kg/s}$, (b) $T_{wall} = 40 \degree C$. 
The PDFs right tails extended which implied that increasing the bottom wall temperature helped in increasing the number of coherent structures with high mean turbulent kinetic energy. This trend is also observed in Fig. 4.16(b) for PDFs at different flow rates. Thus, the effect of the bottom wall temperature and flow rate on the coherent structures mean turbulent kinetic energy is similar in the cross plane. However, the PDFs shift monotonically to the right with the increase of flow rate. This trend was not observed with the increase of bottom wall temperature. In the streamwise vertical plane, the PDFs in Fig. 4.17(a) have a similar trend to the trend observed in Fig. 4.16(a) in the cross plane with the increase of bottom wall temperature. However, the PDFs in the streamwise vertical plane show a clear dependency on the bottom wall temperature and a monotonic shift of the PDFs is observed. The flow rates in the streamwise vertical plane did not show a clear influence on the PDFs as seen in Fig. 4.17(b) which is similar to the effect of flow rate on the maximum vorticity distribution as seen in Fig. 4.11(b). The PDFs in the horizontal planes are shown in Fig. 4.18(a) as a function of bottom wall temperature. For both planes, PDFs shift to the right and the PDFs right tail extended with the increase of bottom wall temperature. This trend is more profound in the lower horizontal plane. Also, the PDFs in the lower horizontal plane shifts to the right and become less skewed with the increase of flow rate as seen in Fig. 4.18(b). However, in the upper horizontal plane, the flow rate did not show a clear influence on the PDFs. Comparison between the PDFs in the upper and lower horizontal planes shows that over the given range of bottom wall temperatures or flow rates, the lower horizontal plane has more coherent structures with higher turbulent kinetic energy.

Figures 4.16 to 4.18 presented the mean kinetic energy of coherent structures at different conditions which provide an overview of the relative strength of coherent structures under different operating conditions. However, the strength of coherent structures relative to the turbulent flow field is not clear from these plots. Such information would be useful as it provides a quantitative assessment of the energy carried by these structures and this contribution is influenced by the operating conditions. Fig. 4.19 shows the fractional turbulent kinetic energy of the detected coherent structures relative to the turbulent kinetic energy of the flow field ($\phi_{TKE}$) for all measurement planes. This parameter was
computed as follows: at a given condition, in each turbulent velocity field, the sum of the total kinetic energy of coherent structures detected in that field was computed and divided by the sum of turbulent kinetic energy of the corresponding turbulent velocity field.

Figure 4.18: Probability density function (PDF) of mean kinetic energy (TKE) of the coherent structures in the horizontal planes at (a) $\dot{m} = 0.042 \text{ kg/s}$, (b) $T_{\text{wall}} = 40 ^\circ \text{C}$. Thick line: lower horizontal plane, thin line: upper horizontal plane.
The computed fractional kinetic energy of coherent structures in each turbulent velocity field was then averaged in time for all turbulent velocity fields at this particular condition. The results are presented in Fig. 4.19 for all measurement planes. It can be observed from the plots that the turbulent kinetic energy of the coherent structures generally remains less than 10% of the turbulent kinetic energy of the flow field in the cross and horizontal planes and less than 15% in the streamwise vertical plane. The trends of fractional turbulent kinetic energy in general are similar to that of $f_{cs}$, $\omega_{max}$ and TKE as seen in Figs. 4.3, 4.7 and 4.13. In the cross plane, as the bottom wall temperature increased from 30°C to 55°C, the fractional turbulent kinetic energy of coherent structures increased by about a factor of 47 and 2.5 for the lowest and highest flow rates respectively. The plot also shows that at a given bottom wall temperature, the fractional turbulent kinetic energy increased with the increase of flow rate by a factor of 20 and 1.1 for the lowest and highest bottom wall temperatures respectively. However, in the streamwise vertical plane, the fractional turbulent kinetic energy did not have a dependency on the flow rate but it increased on average by a factor of 12 with the increase of bottom wall temperature from 30 °C to 55 °C. This trend is consistent with other properties presented earlier in the same plane.

In the lower horizontal plane, at a given wall temperature and flow rate, the fractional turbulent kinetic energy is one to six times more than that in the upper horizontal plane. The fractional turbulent kinetic energy in the lower horizontal plane showed a strong dependency on the flow rate at a given wall temperature as seen in Fig. 4.19(c) where the fractional turbulent kinetic energy increased with an increase in the flow rate. In the lower horizontal plane i.e. in the vicinity of the bottom heated wall, the fractional turbulent kinetic energy in the lower temperature range possessed the largest values. In the upper temperature range, the largest fractional turbulent kinetic energy is found in the streamwise vertical plane.
(a)

(b)
Figure 4.19: Fractional turbulent kinetic energy ($\phi_{TKE}$) of the coherent structures in the (a) cross plane, (b) streamwise vertical plane, (c) horizontal planes (open symbols: lower horizontal plane, solid symbols: upper horizontal plane).

The lowest fractional turbulent kinetic energy is seen in the upper horizontal plane for the lower temperature range ($T_{wall} \leq 40 \, ^\circ C$) and in the cross plane for the higher temperature range ($T_{wall} \geq 45 \, ^\circ C$).

4.5 Discussion

The coherent structures found in the vertical and horizontal measurement planes are generated due to the interactions between rising plumes of warm fluid from below, falling parcels of cooler and dense fluid from above, and the streamwise shear flow. However, as explained earlier, in the cross plane, coherent structures are mainly formed due to the interactions between the rising plumes and falling parcels. The results in the preceding section present various characteristics of these coherent structures which provide a better understanding of the behaviour and dynamics of coherent structures generated in the low Reynolds number channel flows subjected to bottom wall heating. The most noticeable trend seen from all presented figures was the effect of bottom surface temperature on the characteristics of coherent structures. As the bottom wall temperature increased,
numerous coherent structures with higher vorticity and turbulent kinetic energy were generated. This is due to the reason that the increase in the bottom wall temperature enhanced the buoyancy force and consequently, the secondary flow. Thus, the interactions between the secondary flow streams (rising plumes and falling parcels) among themselves and with the shear flow became more intense. This resulted in the generation of more and energetic coherent structures.

It was seen that for the bottom wall temperature lower range, the largest number of coherent structures and their associated maximum vorticity and mean turbulent kinetic energy occurred in the lower horizontal plane in the vicinity of the bottom heated wall. The coherent structures in this plane were formed due to the interactions between the intense secondary flows that dispersed in the horizontal direction. The secondary flow in the lower horizontal plane is in the form of warm fluid that is traveling in the horizontal direction and gaining heat and the cool denser fluid that dispersed in the horizontal direction. For the higher wall temperature range, the number of coherent structures and their associated maximum vorticity and mean turbulent kinetic energy are larger in the streamwise vertical plane compared to the cross plane. As explained earlier for the difference in the number of coherent structures in both planes, this could be due to the existence of the shear flow in the streamwise vertical plane in addition to the secondary flow. This will increase the potential for coherent structure generation in the streamwise vertical plane and enhance their strength. On the other hand, the vortices formed in the cross plane are mainly due to the secondary flow. Therefore, the numbers of coherent structures, their maximum vorticity and mean turbulent kinetic energy are less. Furthermore, at a given flow rate and bottom wall temperature, the spanwise and vertical turbulent velocity magnitudes in the cross plane were lower than the streamwise turbulent velocity magnitude in the streamwise vertical plane by around 50%. The lower turbulent velocities in the cross plane could also be a plausible cause for relatively weaker and fewer coherent structures in that plane.

Results also show that the flow rate influenced the generation and characteristics of coherent structures but primarily in the cross plane and the lower horizontal plane near
the heated wall. It is observed that at a given wall temperature, the number and strength of coherent structures increased with an increase in the flow rate in these planes. The increase in flow rate implies an increase in the Reynolds number and hence at a given wall temperature, an increase in the Reynolds number reduces the relative contribution of free convection (i.e. a reduction in $Gr/Re^2$). No considerable influence of the flow rate on the coherent structures behaviour is observed in the streamwise vertical plane. A thorough investigation of the root mean square (RMS) turbulent velocity components in all planes showed behaviour consistent with that observed for the coherent structures. In the channel cross plane, the spanwise and vertical RMS turbulent velocities showed a monotonic increase in magnitude with the increase of the flow rate at a given wall temperature. This trend was also observed in the lower horizontal plane for the streamwise and spanwise turbulent velocity components. Thus, the coherent structures properties in the cross and lower horizontal planes showed a clear dependency on the flow rate. However, in the streamwise vertical plane, both streamwise and vertical turbulent velocity components did not show a clear flow rate dependency at a given wall temperature. However, the trends of the coherent structures properties are found to be similar to that of the vertical turbulent velocity in the vertical plane.

The distribution of coherent structures in the vertical direction showed an increasing trend in the cross plane. In the streamwise vertical plane, the largest number of coherent structures was generated in region II then the number decreases in the channel upper region. The increase from region I to region II in both the cross and streamwise vertical planes is likely due to the reason that the buoyancy-induced secondary flow is strongest in the channel lower half and their interactions resulted in the generation of large number of coherent structure. In the streamwise vertical plane, the strength of this secondary flow becomes weaker with the distance from the heated wall and hence a reduced number of coherent structures in region III. This can be explained as follows: for the rising plumes, the buoyancy force that is inducing this stream is higher close to the bottom heated wall because of the relatively higher flow temperature. However, the rising plumes start to lose its energy by working against the viscous and gravitational forces (Turner 1973). Moreover, the temperature of the plumes decrease due to mixing as they rise further away.
from the bottom heated wall which further degrades the buoyancy forces. For the falling parcels, they gain more momentum close to the bottom heated wall as they accelerate in the downward direction. Consequently, this justifies the largest number of coherent structures occurring close to the bottom heated wall.

The maximum vorticity of coherent structures showed opposite trends in the cross and streamwise vertical planes. In the cross plane, the maximum vorticity of coherent structures initially decreased with height from the bottom wall and then increased again in the upper region of the channel. However, in the streamwise vertical plane, the maximum vorticity increased with height from the channel bottom wall then it decreased in the channel upper region (region III). The increase of maximum vorticity in the streamwise vertical plane can be attributed to the strong turbulent velocity in region II. Also, the decrease in maximum vorticity in region III can be attributed to the drop of the secondary flow strength in the upward direction as explained before. However, the trend in the cross plane is unusual and there is no plausible physical explanation for this trend.

The turbulent kinetic energy of the coherent structures showed that in general, the most energetic coherent structures are present in the channel upper region (region III) in both cross and streamwise vertical planes, except at the highest two wall temperatures in the streamwise vertical plane. This could be due to the reason that thermal plumes gain momentum in the upper height of the channel and hence the coherent structures present in this section of the channel have higher energy.

The number of coherent structures that have higher maximum vorticity and mean turbulent kinetic energy increased with the increase of bottom wall temperature in all measurement planes as seen in the PDFs plots. This implies that increasing bottom wall temperature not only generated more coherent structures but also stronger. The PDFs also show that the variation of generated coherent structures maximum vorticity and mean turbulent kinetic energy decreased with the decrease of bottom wall temperature. Thus, it can be argued that the bottom wall temperature added more flow interactions but with different intensities. The flow rate showed a clear effect on the distribution of coherent structures with maximum vorticity in the cross plane and mean turbulent kinetic energy
in the cross and lower horizontal planes only with negligible effect in the other planes. This is consistent with the trends shown in the figures (4.7, 4.8, 4.9, 4.13, 4.14 and 4.15).

The coherent structures relative strength with respect to the turbulent velocity field was presented in all planes in Fig. 4.19. The coherent structures possessed up to 15% of the total flow field turbulent kinetic energy. It can be seen that the bottom surface temperature worked on enhancing the relative strength of the coherent structures in all planes. This is mainly due to the increased number of coherent structures and their associated turbulent kinetic energy with the increase of bottom surface temperature as seen in Fig. 4.3 and 4.13. Also, the maximum fractional turbulent kinetic energy for the lower temperature range is still found in the lower horizontal plane while for the upper temperature range, the maximum value is found in the streamwise vertical plane. The reason is similar to what have been proposed earlier for the same tendency seen for the number of coherent structures, maximum vorticity and turbulent kinetic energy.

4.6 Conclusions

The turbulent coherent structures generated in low Reynolds number channel flows during mixed convection have been detected and characterized. The velocity tensor second invariant (Q) scheme was used to detect coherent structures. Results show that the bottom wall temperature plays a major role in the generation of coherent structures and their associated properties in all measurement planes. It was found that increasing the bottom wall temperature increased the number of coherent structures and also enhanced their vorticity and turbulent kinetic energy. This was attributed to the enhancement of the buoyancy driven secondary flow due to the increase of bottom wall temperature.

For the lower bottom temperature range \( (T_{wall} \leq 40 \, ^\circ C) \), the strongest and numerous coherent structures were found in the lower horizontal plane in the vicinity of the bottom heated wall. This is due to the intense secondary flow that dispersed in the horizontal direction. In the higher wall temperature range, streamwise vertical plane has the strongest and most numerous coherent structures. The coherent structures in the vertical cross plane are in general weaker and smaller in number compared to that in the
streamwise vertical plane. This is likely due to the presence of the streamwise shear flow in the streamwise vertical plane which interacts with the secondary flow. The most energetic coherent structures were located in the channel upper region in both cross and streamwise vertical planes. The flow rate showed a strong influence on the number of coherent structures in the cross and lower horizontal planes. Increasing the flow rate at a given bottom wall temperature enhanced the number of coherent structures and their strength. The PDFs showed that more coherent structures with high maximum vorticity and mean turbulent kinetic energy were generated with the increase of bottom wall temperature.

4.7 Acknowledgment

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4.8 References


Chapter 5

Flow development during low Reynolds number mixed convection

5.1 Introduction

Heat transfer inside channels has several applications in industry such as HVAC systems, nuclear and chemical reactors and energy-sector related applications. Recently, applications in the solar thermal are evolving where the solar energy is converted into heat. Generally, the channel flow in these applications undergoes a developing stage where both the flow hydrodynamic and thermal behaviour progress in the flow direction. Some of the applications that involve heat transfer inside a channel operate at low Reynolds numbers and high Grashof numbers. Typically, at these operating conditions, mixed convection occurs where both natural and forced convection coexist. The ratio of Grashof to Reynolds numbers \((Gr/Re)^2\) characterizes the relative contribution of free and forced convection in the mixed convection regime [1]. Natural convection dominates over forced convection when the ratio is larger than unity. When mixed convection exists, buoyancy-driven secondary flow is generated and interacts with the mean shear-driven flow. These interactions have a significant role in the flow development at these conditions. There have been several studies on mixed convection in the entrance region of channels at low Reynolds numbers.

The effect of buoyancy induced convection on forced convection heat transfer between heated parallel plates was studied experimentally by Osborne and Incropera for transient and turbulent flow regimes [2] and laminar flow regime [3]. They found by visualizing the flow that an increase in the Reynolds number and/or a decrease in the channel height suppress the buoyancy-driven secondary flow. They also observed that natural convection enhanced the heat transfer at the bottom plate for all flow regimes. In the vicinity of the top plate, heat flux was found to have a negative effect on the Nusselt number during the transitional regime. Incropera and Schutt [4] studied laminar mixed convection in the
entrance region of a horizontal channel with top and bottom heated walls and adiabatic side walls. Reynolds number ranged from 100 to 1000 while Grashof number varied from $2.5 \times 10^5$ to $6.5 \times 10^6$. The mean flow development in the channel cross plane was examined. They observed that the flow circulation in that plane was enhanced in the streamwise direction due to the development of the secondary flow. However, further downstream, the flow circulation was reduced due to the dissipation of the secondary flow. They attributed that to the reduction of the flow temperature gradient. They found that Reynolds number did not change the effect of buoyancy on the flow structure, however, it changed the onset of instability location. In their study, fully developed hydrodynamic and thermal conditions up to $z/D_h=50$ were not reached where $D_h$ is the hydraulic diameter.

Mixed convection in the thermal entrance region of a channel heated from below was studied [5 – 9]. Maughan and Incropera [5] and Lin and Lin [6] conducted an experimental work inside a square duct with air as the working fluid. Similar to the other studies, they found that the generation of instability as the secondary flow developed enhanced the Nusselt number. They also found that the instability was enhanced as Grashof number increased while the onset of secondary flow was delayed with an increase in the Reynolds number. Before the onset on instability, they observed that forced convection was dominant which was coupled with a decrease in the Nusselt number in the streamwise direction. Development of laminar mixed convection flow was studied numerically by Mahaney et al. [7]. They investigated numerically the development of laminar mixed convection inside a rectangular duct with bottom heated wall and adiabatic side and top walls. Reynolds number in their study was 500 and Grashof number ranged from $4.5\times10^4$ to $7.5\times10^5$. In the thermally developing region, they found that Nusselt number was oscillating in the streamwise direction and became stable in the thermally fully developed region for $Gr < 5 \times 10^5$ whereas for larger values of Grashof number, fully developed conditions were not achieved. They attributed the oscillation to the successive strengthening and weakening of the secondary flow. Ozsunar et al. [8] experimentally studied mixed convection for air inside a rectangular channel
with the lower surface subjected to constant heat flux and insulated side walls. Their main interest was to investigate Nusselt number behaviour along the channel length. They scrutinized the onset of instability by the ratio between Nusselt numbers for mixed convection and forced convection cases. They found that an increase in the Grashof number and/or a decrease in the Reynolds number move the onset point of thermal instability in the upstream direction. They also indicated that the establishment of the fully developed flow delayed by an increase in the Grashof number. Wang et al. [9] visualized the flow pattern during mixed convection inside a horizontal square channel heated from top and bottom wall and insulated from the sides. Reynolds number ranged from 100 to 1000 while Grashof number ranged from $2.8 \times 10^6$ to $2.5 \times 10^7$. They observed that the flow passes four different regimes starting from the upstream end of the channel heated section: laminar forced convection, laminar mixed convection, transient mixed convection and turbulent free convection. They also found that the distance from the edge where the flow changes took place decreased with the increase of heat flux and/or with the decrease of the channel flow rate.

Many studies reported reverse flow during laminar mixed convection [10 – 13]. Toriyama and Ichimiya [10] studied laminar mixed convection in a horizontal square channel with isothermal walls. They observed the reverse flow close to the bottom wall when the wall temperature was lower than the inlet water temperature whereas the reverse flow was observed close to the top wall when the wall temperature was higher than the flow inlet temperature. Choi and Choi [11] numerically investigated the development of mixed convection in a horizontal tube subjected to uniform heat flux in the lower half while the upper half was insulated. Grashof number ranged between $10^6$ and $10^7$ and Reynolds number was 250. They studied Nusselt number development and came up with the same conclusions as other studies. They also observed reverse flow in the upper region which magnitude increased with an increase in the Grashof number. They claimed that adverse pressure gradient in the upper half induced the reverse flow. They found that downstream of the tube, the reverse flow diminished as the buoyancy driven secondary flow penetrated in the upper half of the channel.
Sankar et al. [14] numerically studied the flow development inside an entrance region of a horizontal square channel subjected to axial uniform heat flux. They observed that for Grashof numbers in the range from $1 \times 10^5$ to $2 \times 10^5$, the one-dimensional flow at the entrance evolves into a streamwise-periodic, three-dimensional flow. The mean streamwise velocity in the vertical mid plane was found to deviate from the parabolic shape at the channel entrance into asymmetric profiles in the streamwise direction at $Gr = 25000$. Mixed convection in the entrance region of an isothermally horizontal heated tube was studied both experimentally and numerically [15 – 17]. Hishida et al. [15] found that the streamwise mean velocity profiles in the vertical plane were distorted by the secondary flow up to a certain position, however, further downstream, the symmetric profiles were recovered due to the deterioration of the secondary flow. Yousef and Tarasuk [17] quantified the free convection intensity by the temperature gradient in the streamwise direction. They found that the free convection intensity has a maximum value near the entrance of the tube then it starts to decrease in the streamwise direction.

As seen from the above literature review, the previous studies that investigated the channel flow development during mixed convection focused mainly on studying the heat transfer behaviour along the channel length. Some studies deviated from this trend by visualizing the flow or by measuring the mean velocity profiles. There has been less consideration given to describe the fluctuating flow behaviour inside channels in the developing stage during laminar mixed convection. The focus of the current work is to investigate and characterize both mean and fluctuating characteristics of the developing flow during mixed convection at low Reynolds number and high Grashof numbers. This would ease the understanding of the evolution of the underlying physical process(es) along the channel under these conditions.

### 5.2 Experimental Setup

A 7 cm × 7 cm square channel was used to conduct the experiments. The channel was comprised of three sections; an inlet section, test section and end section as shown in Fig. 5.1(a). The inlet section had a length of 0.7 m and was made of aluminum. A honeycomb
was installed inside the inlet section to straighten the flow and dampen disturbances. The test section (1.5 m in length) had an aluminum bottom plate while the top and side walls were made of non-tempered glass. The bottom wall was heated using two strip heaters (1500W-250V) 1.3 m in length installed in parallel directly underneath the aluminum plate. A temperature controller (ZESTA-ZCP513) was used to control the bottom wall temperature through a feedback loop from a thermocouple embedded in the bottom wall close to the measurement location. The bottom section was coloured in black to eliminate any light reflections during the experiments. The end section was 30 cm in length. Bleed valves were installed in both inlet and end sections to remove any air trapped inside the channel. Pressure gauges were also mounted to monitor the inlet and outlet fluid pressures.

Clean tap water was stored in a water reservoir comprised of four, 200 Litres storage tanks connected in series. The water reservoir was able to supply water at room temperature throughout a given experiment. In the present study, the inlet water temperature was consistently around 24.5 ºC. Air bubbles were removed from the water by storing the water in barrels for two days with periodic stirring prior to the experiments. A magnetic pump (Little Giant, 5 MD) was used to circulate the water through the loop. The flow rate of water was controlled via a flow meter (see Fig. 5.1a). Three mass flow rates 0.0105, 0.0315 and 0.0525 kg/s were considered (corresponding Reynolds numbers in the absence of heating are 150, 450 and 750, respectively, for reference). At each flow rate, experiments were conducted at different bottom wall temperatures which were 35, 45, and 55 ºC. The corresponding Grashof numbers ranged from $8.1 \times 10^6$ to $4.6 \times 10^7$. For each set of experiments, the measurements were taken 30 minutes after setting the flow rate and bottom wall temperature to allow steady state to be reached.
Two-dimensional velocity fields in the vertical mid-plane of the channel, parallel to the side walls, were measured using particle image velocimetry (PIV) technique. The PIV system comprised of a 2336×1752 pixels CCD camera (VA-4M32, ViewWorks), a 120 mJ Nd:YAG laser (SoloPIV 120XT 532nm) as the light source and a four-channel pulse generator (555-4C, Berkeley Nucleonics Corporation) to control the laser pulses timing and synchronizing them with the camera frames (see Fig. 5.1b). The tracer particles for the PIV measurements were silver-coated glass spheres with the mean diameter of 15 µm. The camera was connected to an image acquisition system (CORE-DVR, IO industries) to store images. The image acquisition system was connected to a PC and was controlled by CoreView software installed on the PC.

To study the flow development, measurements were taken at five different locations along the test section of the channel starting from the upstream tip of the heaters (see Fig. 5.1a). The width of each measurement domain was 10 cm and the spacing between two
consecutive measurement domains was 14 cm. Hereinafter, the measurement domains from the most upstream location to the most downstream location will be referred to as sections I, II, III, IV and V, respectively (See Fig. 5.1a). A traverse system was used to precisely move the camera along the channel length and position it at each section. For each set of experiments at a given flow rate and wall temperature, measurement were made at all five locations without stopping the pump to guarantee consistency in the measuring conditions. For each experimental run, 3000 images were captured at a sampling rate of 30 Hz. This resulted in 1500 instantaneous velocity fields at a rate of 15 Hz.

Cross-correlation technique was used to compute velocity vector field from two consecutive images (an image pair), by correlating the interrogation window in the first image with the search region in the second image of the image pair. In the present study, the interrogation windows size was set as 32×32 pixels and the search window size was set as 64×64 pixels. That is, each velocity vector was calculated based on the interrogation window area (32×32 pixels or 1.36×1.36 mm). The nominal resolution of the velocity field was increased to 16×16 pixels (0.68×0.68 mm) by using a 50% overlap of interrogation windows. To identify and correct spurious velocity vectors, a scheme based on the local median test proposed by Siddiqui et al. [18] was used. The spurious vectors detected and corrected were well below 1%.

The errors arise from tracer particles density, particle diameter, particles out of plane motion, velocity gradients, peak locking, the dynamic range, and Adaptive Gaussian Window interpolation introduce uncertainty in the velocity vectors computed by PIV technique [19]. The case with the highest flow rate and bottom wall temperature was considered for the error estimation as the errors are expected to be the largest in this case. The total uncertainty in the PIV velocity measurements was estimated based on the criteria and data from Cowen and Monismith [19] and Prasad et al. [20]. The maximum error in velocity measurements was estimated to be ± 0.069 cm/s which was less than 6.5 % of the bulk flow velocity.
The fluid temperature was also measured at various heights from the bottom wall in all five measurement locations in a separate set of experiments and under the same conditions as for the velocity measurements. A rake of nine T-type thermocouples with the accuracy of ±0.5°C was used with variable spacing. To resolve the thermal boundary layer, most of the thermocouples were clustered near the bottom wall (see Fig. 5.1c). To measure the inlet and outlet water temperatures, T-type thermocouples were also installed at the upstream and downstream ends of the channel to measure the inlet and outlet temperatures. The temperature data was recorded for five minutes at a rate of 3Hz using a 12-channel data acquisition module (National Instruments NI 9211) via LabVIEW data acquisition software.

5.3 Results

Laminar flows inside channels undergo a developing phase from the channel entrance due to the growth of the velocity boundary layers in the downstream direction [21]. In the current study, the hydrodynamic flow development started from the channel entrance whereas, the thermal development started after entering the test section. The bottom heating induced mixed convection in the developing region of the originally laminar flow. Mixed convection is the mechanism where both natural and forced convection modes coexist in the flow. The relative contribution of both convection modes can be quantified by the ratio of Grashof to Reynolds numbers as \( \frac{Gr}{Re^2} \). If this ratio has the magnitude of unity, it implies that both modes have equal contribution but when it is greater than unity, the natural convection mode is dominant over forced convection and vice versa [1]. In the present study, this ratio on average, ranged from 18 at the highest flow rate and lowest bottom wall temperature to 997 at the lowest flow rate and highest bottom wall temperature, indicating that natural convection was dominant over forced convection along the heated section of the channel for all cases. Note that as the flow passes through the heated section, the mean fluid temperature increases which causes a reduction in the temperature difference between the heated wall and the bulk fluid and hence a local decrease in the \( \frac{Gr}{Re^2} \) values along the heated section of the channel. In
other words, the flow experiences a relative decrease in the contribution of natural convection as it passes over the heated bottom wall.

Table 5.1 presents the decrease in the $Gr/Re^2$ values along the heated section of the channel at different conditions. The results show that the largest decrease in $Gr/Re^2$ by about 30% occurred at the three highest $Gr/Re^2$ cases (i.e. lowest flow rate). The percentage reduction in $Gr/Re^2$ along the channel heated length decreased with an increase in the flow rate (i.e. at relatively lower $Gr/Re^2$ values). At the highest flow rate, the reduction in $Gr/Re^2$ along the channel heated length is about 7%. These results indicate that the influence of bottom wall heating on the flow development is expected to be high at the higher $Gr/Re^2$ values.

Table 5.1: Values of $Gr/Re^2$.

<table>
<thead>
<tr>
<th>Flow rate (kg/s)</th>
<th>Wall temperature ($^\circ$C)</th>
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Natural convection induces buoyancy-driven secondary flow. In the present case, the bottom heated wall induced unstable thermal stratification in the channel, which caused low density thermal plumes to originate from the bottom surface and ascend towards the upper unheated wall. These thermal plumes are replaced by the dense and cooler fluid descending from the upper section of the channel towards the bottom heated wall. As mentioned above, for the conditions considered in the present study, the natural convection was dominant and hence, the buoyancy-induced secondary flow was superimposed on the primary shear-driven channel flow. This secondary flow influenced the hydrodynamic development of the flow in the channel.

The mean velocity fields were calculated first by temporal averaging instantaneous velocity fields at each grid point and then spatial averaging the time-averaged velocity at each height. The mean streamwise velocity profiles at different channel locations normalized by the mean channel velocity \( U_c \) are shown in Figs. 5.2, 5.3, and 5.4 for different flow rates. At each flow rate, results are also presented for different heated wall temperatures. Fig. 5.2 shows the mean velocity profiles at the lowest flow rate of 0.0105 kg/s (corresponding Reynolds number in the absence of heating is 150).

The plots show a very different trend compared to the typical parabolic profile expected in the laminar channel flow in the absence of heating. It is observed that the mean flow along the channel length is shifted in the lower half of the channel with the velocity peak located near the channel bottom surface. In the channel upper half, back flow is present for all cases with the peak velocity near the upper wall of the channel. Previous studies have also reported the presence of back flow near the top unheated wall [10 – 13]. The plots show that the bottom wall heating influenced the flow development at all wall temperatures. At a given wall temperature, the flow development trends are present over almost the entire channel height except very close to the upper unheated wall. However, the trends are more prominent in the lower section of the channel.
It is observed that the positive velocity magnitude decreased and the negative velocity magnitude increased as the flow progressed along the channel heated section. Comparison of the velocity profiles at different wall temperatures indicates that the magnitudes of the peak positive and negative velocities increased with an increase in the bottom wall temperature. It is also observed that in the lower section of the channel, the mean streamwise velocity magnitude decreased along the channel as the wall temperature increased. The presence of the back flow in the upper section of the channel will be discussed later in the paper.

Fig. 5.3 shows the mean velocity profiles at the flow rate of 0.0315 kg/s (corresponding Reynolds number in the absence of heating is 450). The plots show a decrease in the normalized peak mean velocity magnitude compare to the lower flow rate case.
Comparison of plots at different wall temperature cases shows a clear influence of wall temperature on the development of mean flow structure along the channel. At the lowest wall temperature (35 °C), the mean velocity profiles varied with the distance in sections I and II. However, as the flow progressed further downstream, the mean flow profiles showed a similar behaviour although the velocity magnitude was lower than that in the upstream sections. With an increase in the wall temperature, the mean velocity profiles showed distinct variations with the distance along the channel. Trends were similar at the two higher wall temperature cases. That is, the magnitude of the mean streamwise velocity decreased monotonically with the distance from the upstream end of the heated section.

That is, as the heat continued to add along the channel it decelerated the streamwise flow. This deceleration of the streamwise flow is likely due to the presence of the strong buoyancy-induced secondary flow.
Figure 5.4: Normalized mean streamwise velocity profiles for $\dot{m} = 0.0525$ kg/s at various sections for bottom wall temperatures of (a) 35 °C (b) 45 °C (c) 55 °C.

It is also seen from Fig. 5.3 that the back flow is lower in magnitude at this flow rate and it increased with the increase of bottom wall temperature. Fig. 5.4 shows the mean streamwise velocity profiles at the highest flow rate of 0.0525 kg/s along the channel at different wall temperature conditions. The profiles collapsed relatively well at the two lower wall temperatures (some flow development effect in the channel core). The flow development effects became relatively significant at the highest wall temperature. The normalized peak mean velocity magnitude further decreased and no back flow observed at this flow rate.

Comparison of the mean velocity profiles at different cases show that the normalized peak velocity magnitude is largest at the highest $Gr/Re^2$ value which decreased monotonically with a decrease in the $Gr/Re^2$ value. The influence of back flow also decreased monotonically with a decrease in the $Gr/Re^2$ value. The comparison of the mean velocity variation along the channel heated length also showed strong $Gr/Re^2$ dependency. It is observed that the mean velocity varied by almost 100% from section I to section V at the highest $Gr/Re^2$ value (lowest flow rate and highest wall temperature)
and this variation decreased monotonically with the $Gr/Re^2$ value and at the lowest $Gr/Re^2$ value (highest flow rate and lowest wall temperature), the variation in the mean velocity from section I to section V is about 9%.

As above figures indicate, the mean flow structure along the channel length was altered due to the bottom wall heating which is related to the buoyancy-induced secondary flow.

To obtain a better perception of this secondary flow induced due to bottom wall heating and its development along the channel, the snapshots of the turbulent velocity fields at five sections along the heated length of the channel are shown in Fig. 5.5. The turbulent velocities were computed by subtracting the mean velocity from the instantaneous velocity at each grid point. The plots provide a clear depiction of the growth of the turbulent velocity field induced primarily by buoyancy. At the upstream end (see section I), the buoyancy effects were very weak and limited to the immediate vicinity of the heated wall. Above this, the turbulent velocity magnitudes were negligible and the flow was predominantly laminar. With an increase in the distance from the upstream heated end, the buoyancy-induced flow and hence the turbulence continued to grow. At the downstream end of section I, the turbulent velocity field extended up to the mid-height of the channel. The spatial extent of the turbulent velocity field continued to increase and by section IV it covered almost the entire cross-section of the channel.

At a further downstream location, the turbulence was fully established throughout the channel (see Section V). The plots also show the rising plumes of warm fluid originated at the bottom surface, which started to interact with the shear-driven flow (for example, see Fig. 5.5a). Falling parcels of cooler dense fluid were also observed in different sections. The rising plumes and falling parcels interact with each other and numerous vortices were generated particularly in the lower half of the channel due to this interaction (see Figs. 5.5b and 5.5c). Plots also show the presence of source-like flow i.e. the spanwise flow dispersing in the measuring plane (for example see Figs. 5.5b & 5.5e), which indicates that the turbulent flow under such condition is three-dimensional in nature.
Figure 5.5: Snapshots of turbulent velocity vector fields for $\dot{m} = 0.0315$ kg/s and $T_{wall} = 45$ °C at section (a) I (b) II (c) III (d) IV (e) V.

The frictional velocity $u_*$ is usually used as the characteristic velocity scale to normalized turbulent properties for forced convection and is defined as,

$$u_* = \sqrt{\nu \frac{du}{dy}
\bigg|_{y=0}}$$  \hspace{1cm} (5.1)

Where $\nu$ is the kinematic viscosity and $\frac{du}{dy}$ is the mean streamwise vertical velocity gradient at the wall (Pope [22]). For natural convection, a natural convection velocity scale $w_*$ was proposed by Adrian et al. [23] and Deardoff [24] to normalize turbulent properties, which is defined as,

$$w_* = (\beta g Q_o y_*)^{1/3}, \quad Q_o = \frac{q_s^*}{\rho c_p}$$  \hspace{1cm} (5.2)

Where $\beta$ is the thermal coefficient of expansion, $g$ is the gravitational acceleration, $Q_o$ is the kinematic heat flux, $y_*$ is the length scale equal to the channel height, $q_s^*$ is the
surface heat flux, $\rho$ is the density and $c_p$ is the specific heat. For mixed convection, both natural and forced convection are present simultaneously and therefore, neither of the two characteristic velocities can be used alone. Researchers have defined a turbulent velocity scale based on the two velocity scales described above in different forms (Zeman and Tennekes [25]; Driedonks [26] and Moeng and Sullivan [27]). We used the basic form of this velocity scale which is defined as,

$$v_* = \sqrt{w_*^2 + u_*^2}$$

The normalized Root-Mean-Square (RMS) streamwise turbulent velocity profiles at different sections are presented in Fig. 5.6 at the lowest flow rate ($\dot{m} = 0.0105$ kg/s) and various wall temperatures. The profiles showed a clear progression of the streamwise turbulent velocity along the channel at different conditions i.e. the streamwise turbulent velocity magnitude increased monotonically with the distance along the channel particularly in the lower half of the channel. It is found that the average normalized streamwise turbulent velocity increased along the channel heated section by approximately 110%, 130% and 120% for the bottom wall temperatures of 35, 45 and 55°C, respectively. At the lowest wall temperature (Fig. 5.6a), at the entrance of the heated section, the streamwise turbulent velocity has high magnitude near the heated wall, which decreased sharply to a very low magnitude at $y/D_h = 0.3$ and remained low above this height. As the flow progressed downstream, the turbulent velocity magnitude in the lower section of the channel ($0.15 < y/D_h < 0.5$) continued to increase.

Similar structure is observed at the higher wall temperatures (Figs. 5.6 b & c) in the lower half of the channel however, the thickness of the enhanced turbulence layer reduced with an increase in the wall temperature although the magnitudes of the normalized peak velocity remained comparable. In the channel upper half, however, the flow structure changed with the wall temperature. The results show an increase in the turbulent velocity magnitudes near the upper wall with an increase in the wall temperature. This enhanced turbulence near the upper wall is likely due to the mean back flow whose magnitude increased with an increase in the wall temperature.
(a) and (b)
As the flow rate increases ($m = 0.0315 \text{ kg/s}$), the streamwise turbulent velocity magnitude enhanced in the channel core region as seen in Fig. 5.7. At the lower two wall temperatures, except for the most upstream location (I), the turbulent velocity profiles are similar and collapsed very well (see Figs. 5.7a & 5.7b) indicating that the streamwise turbulent velocity was developed. However, at the highest wall temperature, the gradual progression of the turbulent velocity is observed indicating that the flow is undergoing development. At the highest flow rate ($m = 0.0525 \text{ kg/s}$), the streamwise turbulent velocity magnitude was enhanced further particularly in the upper half of the channel (see Fig. 5.8). The profiles in sections II-V collapsed reasonably well at all wall temperatures implying that the flow was development. Comparison of results in Figs. 5.6-5.8 show that similar to the mean velocity, both wall temperature and flow rate have impact on the development of the streamwise turbulent velocity field.

At very low flow rate, the flow was under the developmental phase over a wide range of wall temperature from $35\degree \text{C}$ to $55\degree \text{C}$ (see Fig. 5.6). However, as the flow rate increased the flow development achieved faster at low wall temperatures (see Fig. 5.7).
Figure 5.7: Normalized RMS streamwise turbulent velocity profiles for $\dot{m} = 0.0315$ kg/s at various sections for bottom wall temperatures of (a) 35 °C (b) 45 °C (c) 55 °C.

At relatively high flow rate, the flow development became almost independent of the wall temperature within the considered range. Results also show that the overall magnitude of the streamwise turbulent velocity in the channel has strong flow rate dependency and negligible wall temperature dependency. It is observed that the streamwise turbulent velocity magnitude increased with an increase in the flow rate at a given wall temperature.

The profiles of the RMS vertical turbulent velocity at different sections are shown in Fig. 5.9 at the lowest flow rate ($\dot{m} = 0.0105$ kg/s) and various wall temperatures. The results show similar trend of the vertical turbulent velocity at different sections and wall temperatures in the lower half of the channel i.e. the velocity increased sharply from the bottom heated wall to a peak value in the near-wall region and then decreased towards the middle and upper sections of the channel. Similar to the streamwise turbulent velocity, the vertical turbulent velocity at this flow rate also showed a clear flow progression and a monotonic enhancement of the vertical turbulent velocity towards the downstream section.
Figure 5.8: Normalized RMS streamwise turbulent velocity profiles for $\dot{m} = 0.0525$ kg/s at various sections for bottom wall temperatures of (a) 35 °C (b) 45 °C (c) 55 °C.

The average normalized vertical turbulent velocity increased along the heated section at this flow rate by approximately 155%, 170% and 180% at the bottom wall temperatures of 35, 45 and 55°C, respectively. It is also observed that the peak velocity location was shifted away from the heated wall in the downstream direction. Similar to the streamwise turbulent velocity, the vertical turbulent velocity magnitude increased near the upper wall with the increase of wall temperature, which as mentioned earlier is likely due to the back flow.

As the flow rate increased, the overall magnitude of vertical turbulent velocity increased in the channel. At the lowest flow rate, the vertical turbulent velocity magnitude was in general negligible in the upper section of the channel. However, at the flow rate of ($\dot{m} = 0.0315$ kg/s), the vertical turbulent velocities were extended to the upper end of the channel (see Fig. 5.10). As the flow rate further increased, the vertical turbulent velocity magnitude in the upper half of the channel increased (see Fig. 5.11), however, at all flow rates, the location of the peak velocity remained in the lower half of the channel.
Figure 5.9: Normalized RMS vertical turbulent velocity profiles for $\dot{m} = 0.0105$ kg/s at various sections for bottom wall temperatures of (a) 35 °C (b) 45 °C (c) 55 °C.

The flow development trends of the vertical turbulent velocity were found to be similar to that of streamwise turbulent velocity i.e. the flow generally reached the developed state in section II at higher flow rate and lower wall temperature. Furthermore, the peak and the overall magnitudes of vertical turbulent velocity increased with an increase in the flow rate.

In general, the flow rate showed a strong effect on the enhancement of the streamwise and vertical turbulent velocities along the channel heated section. It was found that at a given bottom wall temperature, the highest enhancement of turbulence (both velocity components) occurred at the lowest flow rate. For instance, at a wall temperature of 45°C, the average streamwise turbulent velocity enhanced along the heated section by approximately 130%, 90% and 60% for flow rates of 0.0105, 0.0315 and 0.0525 kg/s, respectively.
(a)

(b)
Figure 5.10: Normalized RMS vertical turbulent velocity profiles for $\dot{m} = 0.0315$ kg/s at various sections for bottom wall temperatures of (a) 35 °C (b) 45 °C (c) 55 °C.

Also, the average vertical turbulent velocity was enhanced at the same wall temperature along the heated section by about 170%, 70% and 60% for the same flow rates. It was also found that this trend of monotonic decrease of the average turbulent velocity enhancement with the increase of the flow rate is consistent for all bottom wall temperatures. On the other hand, the bottom wall temperature showed a relatively weaker but not consistent effect on both turbulent velocities enhancement along the heated section at a given flow rate.

The above results provide general structures of the mean and turbulent flow that undergoes developmental phase inside the channel heated section at low Reynolds and high Grashof numbers. For deeper understanding of the flow development under these conditions, Proper orthogonal decomposition (POD) analysis has been conducted on the turbulent velocity fields.
Figure 5.11: Normalized RMS vertical turbulent velocity profiles for $\dot{m} = 0.0525$ kg/s at various sections for bottom wall temperatures of (a) 35 °C (b) 45 °C (c) 55 °C.

5.4 Proper Orthogonal Decomposition

The Proper Orthogonal Decomposition (POD) technique was initially proposed by Lumly [28] to characterize the coherent structures of turbulent flows. Conceptually, POD analysis decomposes a set of velocity fields into the sum of weighted average basis functions or modes $\phi^n(\bar{x})$. The POD mathematical formulation was described in detail in chapter 1. For the purpose of validating the code, a comparison was done between an original turbulent velocity field obtained from PIV experiment and reconstructed turbulent velocity field from POD results. The reconstructed turbulent velocity field was obtained by combining all POD modes along with their temporal coefficients. The two velocity vector fields (not shown here) showed a qualitative as well as quantitative similarity. The difference between the original and reconstructed turbulent velocity fields was on average less than 1%, which therefore validates the scheme used to obtain the POD data.
The POD modes were computed for streamwise and vertical turbulent velocity components for all conditions. The POD modes of streamwise turbulent velocity did not show any organized structure. However, a detailed analysis showed interesting features in the POD modes of vertical turbulent velocity which provide a deeper insight into the underlying physical mechanisms that regulate the convection heat transfer process along the heated section of the channel. Analysis also showed that at a given wall temperature, change in flow rate has a more significant influence on the POD mode structures than the change in wall temperature at a given flow rate. A representative set of results are presented in the following at a wall temperature of 45°C and flow rates of 0.0105 kg/s, 0.0315 kg/s and 0.0525 kg/s.

Fig. 5.12 shows the energy distribution of POD modes at different sections of the channel at these conditions. At the lowest flow rate, the results show that for the first mode, the flow energy in the entrance of the heated section is higher than the downstream sections but becomes approximately equal in the POD mode range 2-6. In the mode range of approximately 8-100, the energy of the modes in the downstream section is significantly higher than that in the entrance section. In further higher modes, the fractional energy is very low and hence negligible influence. As the flow rate increased, the fractional energy trends remains similar but the difference in fractional energy between the entrance and downstream sections decreases in the mode range 8-100. As the flow rate further increases, the fractional energy in the mode range 1-100 becomes almost the same in all sections. These results indicate that at a given heated wall temperature, the impact of the change in flow rate is most significant in the entrance region of the heated section. Higher flow rate increases the energy of the flow feature at the intermediate modes in the entrance region.

Contour maps of various POD modes of vertical turbulent velocity are plotted in Fig. 5.13 at locations I, III and V for $T_{wall} = 45$ °C and flow rates of 0.0105 kg/s, 0.0315 kg/s and 0.0525 kg/s. These results would provide a better perception and understanding of the influence of flow rate on the underlying flow structures and how they evolve during the passage over a heated wall. First mode shows a strong energetic feature in the
entrance region. At the low flow rate, the feature is restricted very close to the bottom heated wall (within $y/D_h = 0.25$), but with an increase in the flow rate, the extent of the feature grows and at the highest flow rate, it covered almost half of the channel height at the end of section I. The positive value corresponds to the vertical turbulent velocity in the upward direction. Hence, the pattern in mode 1 indicates a bulk upward flow that corresponds to the warm thermal plume originated at the heated wall. Further downstream, pair of positive and negative patterns is observed which indicates the rise of warm fluid and fall of cool denser fluid.

(a)

(b)
Figure 5.12: POD fractional energy ($\lambda_n/E$) distribution across all modes at $T_{wall} = 45$ °C for mass flow rate of (a) 0.0105 kg/s (b) 0.0315 kg/s (c) 0.0525 kg/s.

Such a configuration represents the formation of a convective cell. At the lowest flow rate this convective cell is energetic and restricted to the lower quarter of the channel but with an increase in flow rate, the cell grows while its energy distributed over the cell size. Further downstream (section V), the overall patterns did not change significantly except at the middle flow rate.

At mode 5, the structure of the convective cells appeared to be more prominent. At the lowest flow rate, the cells size slightly increased along the heated section of the channel but they remained near the heated wall. With an increase in the flow rate, these cells grew in size and occupied the entire channel at the highest flow rate in section III. The cell structure however, disrupted in section V at the medium and high flow rates. The higher mode structure of convective cells is clearly depicted in the contour maps at modes 10 and 20. The results show that at higher modes the convective cells appear in the form a continuous series adjacent to the bottom heated wall. The results clearly demonstrate the development of the convective cell along the heated section. The height of the convective cells in the developing region almost coincides with the region of high turbulence. The
(b)
Figure 5.13: Contours of POD energy patterns associated with vertical turbulent velocity at $T_{wall} = 45$ °C for (a) mode 1, (b) mode 5, (c) mode 10 and (d) mode 20 at $\dot{m} = 0.0105$ kg/s (first column from left), $\dot{m} = 0.0315$ kg/s (second column) and $\dot{m} = 0.0525$ kg/s (third column). First row: section I, second row: section III and third row: section V (rows along x-axis and columns along y-axis).
results also show that the flow rate has a very significant impact on the underlying structures of the convective cell along the channel length. It can be argued that the strongest magnitude of vertical turbulent velocity in the lower half that is observed at almost all conditions (Figs. 5.9, 5.10 and 5.11) is due to the presence of these convective cells.

5.5 Discussion

The results presented in the previous sections provide a better understanding of the flow development inside a channel under the effect of bottom wall heating at low Reynolds numbers. The snapshots of the turbulent velocity fields along the channel heated section show how the turbulence was generated and influenced by the buoyancy-driven secondary flow. The secondary flow is seen in the form of rising plumes of warm less dense fluid and falling parcels of cooler heavier fluid. It is observed that the turbulence was generated from the upstream end of the heated section and enhanced in the streamwise direction (Fig. 5.5). The mean streamwise velocity profiles display how mixed convection at low Reynolds numbers and high Grashof numbers alters the mean flow behaviour. The velocity profiles were found to be different from the classical laminar parabolic shape velocity profiles inside a channel in the absence of heating. The mean streamwise velocity peak is shifted towards the lower heated wall as a result of bottom heating. This shift can be attributed to lower viscosity in the near wall region as a result of bottom heating which yields less flow resistance. Therefore, the flow always tends to follow the path of least resistance.

The results show that the mean flow development was influenced by both flow rate and the bottom wall temperature. At a given flow rate, the increase in the bottom wall temperature, extended the flow development length. The results also show in general a decrease in the mean velocity magnitude near the heated wall as the flow advanced along the heated section of the channel. Furthermore, the instantaneous and turbulent velocity fields in the channel cross plane (not shown here) clearly showed the three-dimensionality of the flow due to generation of the buoyancy-driven secondary flow.
Sankar et al. [14] also found that the one dimensional flow in the channel entrance evolves into three-dimensional flow in the streamwise direction. Thus, another plausible cause of the reduction in the mean streamwise velocity along the channel heated section could be the increase in flow three dimensionality with the distance, which induced mean flow in the cross plane which resulted in a redistribution of the mean streamwise velocity in the channel cross plane. At a given wall temperature, the increase in flow rate reduced the flow development length. The change in flow rate and bottom wall temperature can be collectively expressed in terms of the change in $Gr/Re^2$ values. The present results showed that as the $Gr/Re^2$ value increased, the flow development length and the mean velocity magnitude near the heated wall increased monotonically.

It is also observed that the back flow is induced in the channel in the upper region (close to the unheated wall) at $Gr/Re^2$ values greater than 50. The backflow indicates the formation of a large convective cell that occupies the entire heated section of the channel at these conditions. A thorough analysis of the instantaneous velocity fields at all locations revealed that the local buoyancy-driven secondary flow in the form of rising plumes of warm fluid and falling parcels of cold one continuously interacts with the back flow. Hence, the energy transferred from the secondary flow to the back flow is likely the source of its sustenance. The enhancement of back flow with the increase of $Gr/Re^2$ confirms this explanation. That is, as explained earlier, the secondary flow (i.e. raising plumes and falling parcels) is enhanced as a result of bottom heating, which accordingly strengthens the back flow. One plausible explanation for the occurrence of back flow at high $Gr/Re^2$ values is that the inertial force is relatively weak and the buoyancy-induced secondary flow is dominant at these conditions, which supported the formation of large convective cell and hence backflow. With a decrease in $Gr/Re^2$, the inertial effects started to become significant which suppressed the back flow development. Mean streamwise velocity profiles at ($m = 0.0315 \text{ kg/s}$) further confirm this explanation. That is, weaker back flow is seen at this flow rate and its magnitude is enhanced with the increase of wall temperature.

The streamwise and vertical RMS turbulent velocity profiles show that the buoyancy driven secondary flow (i.e. rising plumes and falling parcels) induces turbulence starting
from the upstream end of the channel heated section. This effect is clearly evident in the contour maps of the POD structures. In the present conditions, the streamwise turbulent velocities are found to be less organized than the vertical turbulent velocity profiles, which could likely be due to the presence of organized local convective cells in the POD modes of vertical turbulent velocity. As mentioned earlier, the POD modes of streamwise turbulent velocity did not show any organized structure.

The development of the streamwise and vertical turbulent velocity fields showed trends similar to that of the mean flow development. That is, the flow development length increased with a decrease in the flow rate and/or increase in the bottom wall temperature. During the developmental phase, the magnitudes of both turbulent velocity components increased monotonically with the distance along the heated section of the channel.

As mentioned in the experimental setup section, the vertical variation of the fluid temperature was obtained by temperature measurements at 9 vertical locations (see Fig. 5.2c) in all five sections. Table 5.2 presents the percentage increase in the average fluid temperature along the channel relative to the wall temperature for all measurement conditions. The average flow temperatures at section I and V are $T_I$ and $T_V$ respectively. It can be seen from the table that the percentage increase in the fluid temperature increased with an increase in the bottom wall temperature and/or a decrease in the flow rate.

<table>
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<th>0.0525</th>
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<td>Wall temperature ($^\circ$C)</td>
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<td>35 45 55</td>
<td>35 45 55</td>
</tr>
<tr>
<td>$(T_V - T_I) / T_{wall}$</td>
<td>17 17.5 19.3</td>
<td>12.2 15.7 19.4</td>
<td>7.5 10.9 10</td>
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This trend clearly demonstrates that the percentage increase in the fluid temperature is directly linked with the flow development under these conditions. That is, the percentage increase in the fluid temperature increased with the flow development length.
This implies that the rate of temperature increase and the flow dynamics during the developmental stage are interdependent.

The streamwise turbulent velocity profiles showed local peaks adjacent to the heated bottom wall. These peaks are found along the heated section of the channel for all cases, which shows that the mechanism of turbulence generation in this region is the same regardless of the location, flow rate, or bottom wall temperature. The structure of streamwise turbulent velocity in the channel core was found to be influenced by the flow rate. At the low flow rate, the streamwise turbulent velocity has higher magnitudes in the lower half of the channel. As the flow rate increased, the streamwise turbulent velocity magnitudes tend to increase towards the channel core and at the highest flow rate, the upper half of the channel has very strong streamwise turbulent velocities. These trends are found to be consistent for all flow rates at a given wall temperature.

Similar to the streamwise turbulent velocity, the structure of the vertical turbulent velocity was also found to be influenced by the flow rate. At the lowest flow rate, the vertical turbulent velocity was restricted in the lower half of the channel but with an increase in the flow rate, its magnitude tends to grow and at the highest flow rate, extended into the upper section of the channel. However, the vertical turbulent velocity was found to be relatively strong in the lower half of the channel for all cases. The maximum vertical turbulent velocity is found in the region $0.1 < y/D_h < 0.4$ for all cases along the channel heated section. This is likely due to the presence of strong local convective cells in this region as shown in Fig. 5.13. These cells are induced due to the interaction between the rising plumes and falling parcels in this region. The rising plumes are strongest near the bottom heated wall and decreases in energy with height due to viscous and gravity resistance (Turner [29]) while the falling fluid parcels accelerate in the downward direction. Thus, the local convective cells formed near the bottom heated wall are strong.
5.6 Conclusions

The flow development has been studied inside a horizontal square channel heated from below during low Reynolds number mixed convection. The mean and turbulent flow developing length showed a strong dependency on the flow rate and the wall temperature. It was found that the developing length increased with the increase of bottom wall temperature and/or the decrease of flow rate (i.e. increasing $Gr/Re^2$). The mean streamwise velocity was found to decelerate in the streamwise direction near the bottom heated wall, and the rate of deceleration was higher for the higher $Gr/Re^2$ values. The deceleration was attributed to the three dimensionality of the flow that was enhanced in the streamwise direction with the increase of $Gr/Re^2$.

It was observed that turbulence was generated from the upstream end of the heated section and grew spatially in the streamwise direction. During the developmental phase, the magnitudes of both streamwise and vertical turbulent velocities were enhanced as the flow advanced along the heated section. The rate of turbulence enhancement along the channel was suppressed with the flow rate increase and/or bottom wall temperature decrease (i.e. decreasing $Gr/Re^2$).

It was found that the flow developing behaviour had a direct relation with the percentage increase in the flow temperature along the channel. POD results for the vertical turbulent velocity fields showed the underlying flow mechanisms that are associated with the phenomena. Convective cells were observed close to the bottom heated wall and they grew along the channel length. The flow rate showed a strong influence on these convective cells.

5.7 Acknowledgments

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5.8 References


Chapter 6

Conclusions

An experimental study was conducted to characterize the mean and turbulent velocity fields in a channel flow at low Reynolds numbers subjected to bottom wall heating. Planer and Stereo PIV techniques were used to measure velocity fields in different measurements planes. Five flow rates and six bottom wall temperatures were considered in the study. The two-dimensional velocity fields were measured in the vertical mid-plane and two horizontal planes near the bottom heated wall and the three-dimensional velocity fields were measured in the channel cross plane. For the flow development study, three flow rates and three bottom wall temperatures were considered. The two dimensional velocity fields were measured at five different locations along the channel heated section. The flow rates considered in the study covered the laminar flow regime in the absence of heating (i.e. $750 \geq Re \geq 150$). The underlying physical processes were investigated using POD as well as the turbulent coherent structures generated inside the channel.

6.1 Discussion Summary, and Conclusion

For all measurement conditions, $Gr/Re^2$ values were higher than unity and thus, natural convection was dominant over forced convection. The bottom wall heating caused unstable stratification in the channel and hence buoyancy-driven secondary flow was induced in the form of rising plumes of warm lighter fluid and falling parcels of cool heavier fluid. The flow was complex and three-dimensional due to the generation of secondary flow and their interactions with themselves and also with the shear flow. Bottom heating altered the mean streamwise velocity structure and the profiles were skewed towards the bottom wall. This was attributed to lower viscosity in the channel lower half which in turn reduced the flow resistance. Back flow occurred near the top unheated wall when $Gr/Re^2 > 55$ and its magnitude increased with the increase of $Gr/Re^2$ i.e. an increase in the bottom wall temperature and/or a decrease in the flow rate. It was found that the back flow existed along the channel heated section forming a large
convective cell and plausibly extracted its energy from the secondary flow. Near the bottom heated wall, the streamwise velocity profiles were uniform in the spanwise plane due to uniform heating. The streamwise velocity profiles in the vicinity of the bottom heated wall showed a dependency on $Gr/Re^2$. The spanwise velocity profiles near the bottom heated wall were one order of magnitude lower than the streamwise velocity. Furthermore, the spanwise velocity showed an axisymmetric behaviour near the heated wall with the flow directed from the channel mid span towards the side walls.

Turbulence was generated in the flow due to the buoyancy-induced instabilities (or secondary flow). Turbulent velocity fields in the lower horizontal plane revealed for the first time, the formation mechanism of rising plumes and dispersion of falling parcels. The turbulent velocity profiles were different from the profiles in the classical turbulent flows inside channels. It was observed that the streamwise turbulent velocity had a peak value in the vicinity of the bottom heated wall. The turbulence was negligible close to the top unheated wall. Bottom wall temperature showed more influence than the flow rate on both turbulent velocity components. It was found also that the shear flow worked on diffusing the streamwise turbulence intensity in the channel core region. The buoyancy-driven secondary flow was more intense (i.e. higher momentum) closer to the bottom heated wall and therefore, the vertical turbulent velocity was found to be larger in the mid and lower sections of the channel.

The underlying physical processes associated with the low Reynolds number mixed convection were investigated by applying the POD technique on the turbulent velocity fields and through the characterization of turbulent coherent structures formed in the flow. POD analysis revealed intriguing features within the vertical turbulent velocity fields. Local convective cells at different mode levels were observed near the bottom heated wall. The convective cells maintained relatively high energy up to higher modes indicating the presence of strong energy extraction and energy cascading processes. The results also showed that the formation of these convective cells is similar at different conditions but their energy changes with Grashof and Reynolds numbers.
The coherent structures generated within the turbulent velocity fields in all measurement planes were investigated statistically. The coherent structures were generated due to the interactions between rising plumes, falling parcels and the shear flow. However, the coherent structures generated in the channel cross plane were mainly due to the interactions of the secondary flow only. The results showed that the bottom wall temperature had a direct effect on the number of coherent structures generated and their associated characteristics. For all measurement planes, more coherent structures were generated by increasing the bottom wall temperature. Also, the maximum vorticity and the mean turbulent kinetic energy of the coherent structures were enhanced with the increase of the bottom wall temperature. The flow rate showed an influence only in the cross and lower horizontal planes. The strength of the secondary and shear flows that involved in the coherent structures generation varied in different planes. Thus, the number of coherent structures and their characteristics showed a variation in all measurement planes. For the lower bottom wall temperature range ($T_{\text{wall}} \leq 40 \, ^\circ\text{C}$), more coherent structure were formed in the lower horizontal plane followed by the streamwise vertical plane and then the cross plane. The associated characteristics of the coherent structures showed the same trend. Due to the involvement of the shear flow in the coherent structures generation in the streamwise vertical plane in addition to the secondary flow, more coherent structures were formed in this plane and they were stronger compared to that in the cross plane. Along the channel height, more coherent structures were generated in the region close to the bottom heated wall. However, the most energetic coherent structures were found in the channel core region. This was attributed to larger number of plumes that existed in the near wall region, however, only stronger plumes (i.e. higher momentum) reached the channel core region. The bottom wall temperature increased the number of coherent structures with higher maximum vorticity and mean turbulent kinetic energy.

The flow development along the channel heated section was investigated by measuring the mean and turbulent velocity fields in the channel mid-vertical plane at five different locations. The mean streamwise velocity was found to decelerate in the streamwise direction in the lower section of the channel and the rate of deceleration increased with
the increase of $Gr/Re^2$ values. Turbulence was generated from the upstream end of the channel and was propagated in the vertical direction along the channel length due to the enhancement of the buoyancy-driven secondary flow. The mean and turbulent flow developing length showed a strong dependency on the flow rate and bottom wall temperature. It was found that the developing length increased with the increase of bottom wall temperature and/or the decrease of flow rate (i.e. increasing $Gr/Re^2$). POD results showed that the flow rate contributes to the growth of the local convective cells.

### 6.2 Future Recommendations

The current results presented a detailed analysis of the mean and turbulent channel flow behaviour during low Reynolds number mixed convection. The results showed the complexity of the flow interactions which induced three-dimensionality to the flow. Thus, it is highly recommended to study the flow in a real three dimensional manner under the same conditions. One of the state of the art measuring techniques is volumetric PIV which can measure the velocity field inside a flow volume. This would certainly provide a better insight into the three dimensional structure of both mean and turbulent flow fields. Moreover, the results would help in studying the three dimensional coherent structures and their dynamics.

Another aspect is the heat transfer within the flow and how the flow temperatures are linked to the mean and turbulent flow characteristics. Thermal efficiency is the most important design parameter for any application that involves heat transfer. Thus, measuring the flow temperatures in the whole field of view is strongly recommended. Laser induced fluorescence (LIF) technique provides non-intrusive temperature measurements in a plane. Simultaneous PIV and LIF measurements in a plane are recommended as they provide an insight into the interactions and direction relation between the velocity and temperature fields.
6.3 Contributions

The original contributions from the present study to the scientific knowledge are listed below:

- First detailed characterization of the turbulent velocity field in channel flows subjected to bottom wall heating at low Reynolds numbers
- Investigation of the underlying flow mechanisms through POD analysis
- First detailed characterization of the coherent structures present in the channel flows at low Reynolds numbers during mixed convection
- Detailed investigation of the development of mean flow field and the first investigation of the development of turbulent flow field in low Reynolds number channel flows subjected to bottom wall heating
- Identification for the first time, of the presence of longitudinal streaks near the heated wall that initiate thermal plumes.
APPENDIX 1: PIV Error Calculation

The total error in Particle Image Velocimetry can be calculated by adding all the errors caused from different sources. These error sources are particle diameter, seeding density, out of plane motion, velocity gradient, dynamic range, peak locking and Adaptive Gaussian Window interpolation [1]. The errors from different sources were calculated based on Figs. 5 (a-f) provided by Cowen and Monismith [1]. According to Cowen and Monismith [1], the total error is the sum of mean and RMS errors of the above error sources. They referred to the random uncertainty in locating both the correlation peak and particle image as RMS error. This type of error is caused by random noise during imaging process.

The silver coated glass spheres used as tracer particles with mean diameter of 15µm. Thus, the tracer particle diameter is 0.355 pixels. Fig. 5(a) in Cowen and Monismith [1] provides the error due to particle diameter. Since the smallest particle diameter in the figure is 1 pixel, the error due to particle size was calculated based on this diameter and is equal to

\[ \varepsilon_u = (-0.03) + 0.095 = 0.065 \text{ pixels} \]  

(1)

Fig. 13 in Prasad et al. [2] which shows the bias and peak locking errors as a function of particle diameter was used as an approximation for estimating the error corresponding to a particle diameter of 0.355 pixels. The figure shows that the error associated with a particle diameter of 0.355 pixels is larger by 40% compared to the error associated with a particle diameter of 1 pixel. In the study of Prasad et al. [2], the centre of mass cross-correlation procedure is susceptible to peak locking. On the other hand, the current work used three point Gaussian estimation which has a reduced peak locking error. Thus, additional error in particle diameter was estimated to be 30%. The final error estimation for particle diameter is

\[ \varepsilon_u = 0.065 \times 1.3 = 0.0845 \text{ pixels} \]  

(2)
PIV error was calculated based on the largest average velocity gradient across all directions in all measurement planes. The largest velocity gradient was \( \frac{\partial u}{\partial x} = 0.02 \) pixels/pixel in the vertical mid plane at the highest flow rate and bottom wall temperature. Thus, the error estimation will be conducted using that velocity gradient. The error due to velocity gradients was estimated based on Fig. 5(e) in Cowen and Monismith [1]. The total error due to velocity gradient is the sum of both mean and RMS errors which is equal to

\[
\varepsilon_u = (-0.02) + 0.06 = 0.04 \text{ pixels}
\]  \hspace{1cm} (3)

Thus the total error from the velocity gradient and particle diameter is

\[
\varepsilon_u = 0.04 + 0.0845 = 0.1245 \text{ pixels}
\]  \hspace{1cm} (4)

To estimate the error due to out of plane motion, the displacement in the traverse direction was calculated. First, based on the results obtained in the cross plane, the velocities in the vertical and spanwise direction are comparable. Thus, it is safe to assume that the displacement in the out of plane motion (i.e. spanwise direction) is equal to the displacement in the vertical direction and equals to summing the mean and standard deviation of the vertical displacement

\[
\Delta z = \bar{\Delta z} + \sigma_z = 1.74 \text{ pixels}
\]  \hspace{1cm} (5)

For the vertical plane, 1.74 pixels equals to 0.0738 mm. Since the laser sheet thickness is 2 mm, this out of plane displacement can be neglected.

The error attributed to Adaptive Gaussian Window interpolation (AGW) was calculated from Fig. 5(f) in Cowen and Monismith [1]. The error was 0.08 pixels for 1500 vector fields and the total error is

\[
\varepsilon_u = 0.1245 + 0.08 = 0.2045 \text{ pixels}
\]  \hspace{1cm} (6)

It can be assumed that the velocity gradient produces equal errors in both the streamwise and vertical direction, thus the total error can be calculated as follows.
\[ E_{\text{total}} = \sqrt{2\varepsilon_{\text{u}}^2} = 0.2892 \text{ pixels} \quad (7) \]

By converting the error into cm/s, the total error will be 0.071 cm/s

The largest error in the PIV results is estimated at the highest flow rate and bottom wall temperature case where the streamwise velocity gradient in the vertical plane shows the largest magnitude. Thus, the maximum uncertainty in the velocity measurement is 6.67% of the bulk channel velocity.

References


Figure A2.1: Mean velocity field in the cross plane with contours of the streamwise velocity at (a) $\dot{m}=0.021$ kg/s, $T_{wall} = 30 ^\circ$C (b) $\dot{m}=0.021$ kg/s, $T_{wall} = 55 ^\circ$C (c) $\dot{m}=0.052$ kg/s, $T_{wall} = 30 ^\circ$C (d) $\dot{m}=0.052$ kg/s, $T_{wall} = 55 ^\circ$C. The colorbar is in cm/s.
Figure A2.2: Contours of mean streamwise velocity at (a) $\dot{m}=0.021$ kg/s, $T_{\text{wall}} = 30$ °C (b) $\dot{m}=0.021$ kg/s, $T_{\text{wall}} = 55$ °C (c) $\dot{m}=0.052$ kg/s, $T_{\text{wall}} = 30$ °C (d) $\dot{m}=0.052$ kg/s, $T_{\text{wall}} = 55$ °C. The colorbar is in cm/s.
Figure A2.3: Contours of mean spanwise velocity at (a) $\dot{m}=0.021 \text{ kg/s, } T_{\text{wall}} = 30 \degree \text{C}$ (b) $\dot{m}=0.021 \text{ kg/s, } T_{\text{wall}} = 55 \degree \text{C}$ (c) $\dot{m}=0.052 \text{ kg/s, } T_{\text{wall}} = 30 \degree \text{C}$ (d) $\dot{m}=0.052 \text{ kg/s, } T_{\text{wall}} = 55 \degree \text{C}$. The colorbar is in cm/s.
Figure A2.4: Contours of mean vertical velocity at (a) $\dot{m}=0.021$ kg/s, $T_{wall} = 30$ ºC (b) $\dot{m}=0.021$ kg/s, $T_{wall} = 55$ ºC (c) $\dot{m}=0.052$ kg/s, $T_{wall} = 30$ ºC (d) $\dot{m}=0.052$ kg/s, $T_{wall} = 55$ ºC. The colorbar is in cm/s.
Figure A2.5: Contours of RMS streamwise turbulent velocity at (a) $\dot{m}=0.021$ kg/s, $T_{wall} = 30 ^\circ$C (b) $\dot{m}=0.021$ kg/s, $T_{wall} = 55 ^\circ$C (c) $\dot{m}=0.052$ kg/s, $T_{wall} = 30 ^\circ$C (d) $\dot{m}=0.052$ kg/s, $T_{wall} = 55 ^\circ$C. The colorbar is in cm/s.
Figure A2.6: Contours of RMS spanwise turbulent velocity at (a) $\dot{m}=0.021$ kg/s, $T_{\text{wall}} = 30 ^\circ$C (b) $\dot{m}=0.021$ kg/s, $T_{\text{wall}} = 55 ^\circ$C (c) $\dot{m}=0.052$ kg/s, $T_{\text{wall}} = 30 ^\circ$C (d) $\dot{m}=0.052$ kg/s, $T_{\text{wall}} = 55 ^\circ$C. The colorbar is in cm/s.
Figure A2.7: Contours of RMS vertical turbulent velocity at (a) $\dot{m} = 0.021$ kg/s, $T_{wall} = 30$ °C (b) $\dot{m} = 0.021$ kg/s, $T_{wall} = 55$ °C (c) $\dot{m} = 0.052$ kg/s, $T_{wall} = 30$ °C (d) $\dot{m} = 0.052$ kg/s, $T_{wall} = 55$ °C. The colorbar is in cm/s.
# Curriculum Vitae

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