August 2013

Stochastic Simulation of Multiple-Station Ground Motions and Its Applications

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Graduate Program in Civil and Environmental Engineering

A thesis submitted in partial fulfillment of the requirements for the degree in Doctor of Philosophy

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STOCHASTIC SIMULATION OF MULTIPLE-STATION GROUND MOTIONS AND ITS APPLICATIONS

(Thesis format: Integrated Article)

by

Taojun Liu

Graduate Program in Engineering Science
Department of Civil and Environmental Engineering

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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ABSTRACT

Approaches to simulate single- and multiple-component multiple-station ground motion records with target spatial coherency and spatial correlation structures are developed for scenario events in this study. To develop the approaches, spatial correlation of the Fourier amplitude spectrum for a random orientation and for two orthogonal directions is assessed, and empirical equations are suggested. Moreover, spatial coherency of ground motions for two orthogonal horizontal directions is estimated using actual records from seven seismic events. Empirical coherency function is suggested for the components of records in two orthogonal horizontal directions at single and multiple recording stations. It was also found that the coherency for the records along the major and minor principal axes at a recording station is similar to that for two randomly oriented orthogonal directions.

Based on the proposed approaches in this thesis, spatial correlated and coherent ground motions can be simulated for randomly oriented uni-directional excitations at considered sites for a scenario event. For the simulation, it is considered that the reference Fourier amplitude spectrum for scenario events can be defined by using the stochastic point-source method or the stochastic finite-fault method. It is shown that the estimated spatial correlation and coherency from the simulated records match well the target spatial correlation and coherency. Furthermore, the application of the simulated records for seismic risk assessment of a group of buildings is presented. The results indicate that the spatial correlation of Fourier amplitude spectrum must be considered in estimating the distribution of the aggregated seismic loss of spatially distributed group of buildings.
Spatial correlated and coherent ground motions are simulated for two orthogonal horizontal directions at considered sites for a scenario event. Again, it is shown that the estimated spatial correlation and coherency from the simulated records adequately match the target spatial correlation and coherency.

Keywords

Ground motion; Stochastic simulation; Point-source; Finite-fault; Spatial coherency; Spatial correlation; Bi-directional excitation; Seismic hazard; Seismic risk assessment.
CO-AUTHORSHIP STATEMENT

The material presented in Chapter 2, 3, 4 and 5 of this thesis have been published or submitted for potential publication in peer-reviewed journals.

A version of Chapter 2 is published in the *Bulletin of Seismological Society of America* co-authored by T. J. Liu, G. M. Atkinson, H. P. Hong, and K. Assatourians.

Chapter 3 contains the material of the paper published in the *Bulletin of Seismological Society of America* co-authored by T. J. Liu and H. P. Hong.

Chapter 4 contains the material of a manuscript submitted to *Earthquake Spectra* co-authored by T. J. Liu and H. P. Hong for potential publication.

A version of Chapter 5 will be submitted for possible publication in a peer-reviewed journal co-authored with the advisor.
DEDICATION

To Xiamo and Yi
ACKNOWLEDGMENTS

I would like to express my most sincere gratitude to my supervisor, Professor Hanping Hong, for his continuous advice, support and encouragement throughout the course of my Ph.D. program. His vast knowledge, enthusiasm and dedication to research were guiding me on a daily basis during the last four years. He was serious, sharp and precise to every discussion we had, which I believe are the crucial qualities for a researcher. He also has a great sense of humor which made the research experience more enjoyable. I am honored to be his student.

My special thanks go to Professor Gail M. Atkinson and Dr. Karen Assatourians, who are my co-authors for the material presented in Chapter 2. My course project report would not turn out to be a publication without Professor Atkinson’s encouragement. Her comments and suggestions for other chapters are acknowledged. Karen’s expertise on stochastic simulations helped my understanding on the stochastic models and he is always there to answer my questions. It was a great experience working with them.

I would like to thank my thesis committee members and examiners, Professors Abouzar Sadrekarimi, Wenxing Zhou, Kristy Tiampo, and Peter Stafford, for reviewing this thesis. I am certain that the quality of my thesis is improved by their minds and hands.

I also would like to extend my appreciation to my colleagues at Western and my friends in Canada. Their presence colored my personal life.

The financial support of the Natural Science and Engineering Research Council of Canada, the University of Western Ontario and China Scholarship Council are gratefully acknowledged.
I am grateful to my mother, Xiamo, for allowing and encouraging me to pursue my Ph.D. degree in Canada. I will never be able to make it up to her for what she has done for me. I thank her for the unconditional love. I am also grateful to my family, for always supporting me.

Finally, I would like to thank my beloved fiancé, Yi, for her understanding, support and seamless love. She is always able to cheer me up and she made me happier than I could imagine. I could not complete my Ph.D. program without her.
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LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviations

ASCE  American Society of Civil Engineers

cov  Coefficient of variation

FAS  Fourier Amplitude Spectrum

FEMA  Federal Emergency Management Agency

GMPE  Ground motion prediction equation

NBCC  National Building Code of Canada

NEHRP  National Earthquake Hazard Reduction Program

NGA  Next Generation Attenuation (database)

NIBS  National Institute of Building Science

NRCC  National Research Council of Canada

PEER  Pacific Earthquake Engineering Research (Center)

PGA  Peak ground acceleration

SA  Spectral acceleration

SDOF  Single degree of freedom (system)

UHS  Uniform hazard spectra

Symbols

CHAPTER 2

A  Ground-motion measure in the GMPE

$c_0$, $c_1$, $c_2$, $c_3$  Coefficients in ground motion prediction equation

$f$  Frequency

$g$  Gravitational acceleration
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<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$h$</td>
<td>Fictitious depth term that builds in near-source saturations due to finite-fault effects</td>
</tr>
<tr>
<td>$M$</td>
<td>Moment magnitude</td>
</tr>
<tr>
<td>$Q$</td>
<td>Quality factor (anelastic attenuation term in stochastic finite-fault simulation method)</td>
</tr>
<tr>
<td>$R$</td>
<td>Source-to-site distance</td>
</tr>
<tr>
<td>$R_{jb}$</td>
<td>Closest horizontal distance from site to surface projection of the rupture</td>
</tr>
<tr>
<td>$R_{rup}$</td>
<td>Closest distance from the recording site to the fault rupture plane</td>
</tr>
<tr>
<td>$T$</td>
<td>Natural vibration period</td>
</tr>
<tr>
<td>$V_{\text{ref}}$</td>
<td>Reference $V_{S30}$</td>
</tr>
<tr>
<td>$V_{S30}$</td>
<td>Average shear-wave velocity of the top 30 m of earth</td>
</tr>
<tr>
<td>$Y(T)$</td>
<td>Ground-motion measure, such as PGA or SA</td>
</tr>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$\alpha, \beta$</td>
<td>Model parameters for empirical spatial correlation function</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Separation distance between two stations</td>
</tr>
<tr>
<td>$\Delta\sigma$</td>
<td>Stress parameter</td>
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<tr>
<td>$\varepsilon(T)$</td>
<td>Intraevent residuals</td>
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<tr>
<td>$\varepsilon_2(T)$</td>
<td>Additional intraevent variability</td>
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<td>$\varepsilon_{\text{sim}}(T)$</td>
<td>Intraevent residuals of simulated records</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>High frequency decay parameter</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Explanatory parameter</td>
</tr>
<tr>
<td>$\sigma_d(\Delta, T)$</td>
<td>Standard deviation of the differences of intraevent residuals for a separation distance bin</td>
</tr>
<tr>
<td>$\sigma_\delta(T)$</td>
<td>Standard deviation of intraevent residuals</td>
</tr>
<tr>
<td>$\sigma_{\text{sim}}(T)$</td>
<td>Standard deviation of intraevent residuals of simulated records</td>
</tr>
<tr>
<td>$\sigma_{\text{sim}}'(T)$</td>
<td>Standard deviation of intraevent residuals of simulated records with additional variability</td>
</tr>
</tbody>
</table>
\( \rho_s(\Delta, T) \) \hspace{1cm} \text{Intraevent spatial correlation coefficient}

**CHAPTER 3**

\( A \) \hspace{1cm} \text{Parameter in the coherency function}

\( A(f(z)) \) \hspace{1cm} \text{Site amplification term}

\( a \) \hspace{1cm} \text{Spatial correlation model parameter for } \ln(r_{A_j})

\( B \) \hspace{1cm} \text{Parameter in the coherency function}

\( b \) \hspace{1cm} \text{Spatial correlation model parameter for } \ln(r_{A_j})

\( C \) \hspace{1cm} \text{Constant in the source term in stochastic point-source method}

\( D(f) \) \hspace{1cm} \text{Diminution operator}

\( E(\bullet) \) \hspace{1cm} \text{Expectation of the argument}

\( E(M_0, f) \) \hspace{1cm} \text{Source effect term in stochastic point-source method}

\( F \) \hspace{1cm} \text{Effect of the free surface}

\( f \) \hspace{1cm} \text{Frequency}

\( f_0 \) \hspace{1cm} \text{Parameter in the coherency function}

\( G(f) \) \hspace{1cm} \text{Site effect term in stochastic point-source method}

\( I(f) \) \hspace{1cm} \text{An indicator for ground motion type (acceleration, velocity or displacement)}

\( k \) \hspace{1cm} \text{Parameter in the coherency function}

\( M \) \hspace{1cm} \text{Moment magnitude}

\( M_0 \) \hspace{1cm} \text{Earthquake moment}

\( N \) \hspace{1cm} \text{Hamming window parameter}

\( n_R \) \hspace{1cm} \text{Number of stations}

\( P(R, f) \) \hspace{1cm} \text{Path effect term in stochastic point-source method}

\( Q(f) \) \hspace{1cm} \text{Anelastic attenuation term}

\( R \) \hspace{1cm} \text{Source-to-site distance}

\( R_0 \) \hspace{1cm} \text{Reference distance}
<table>
<thead>
<tr>
<th>Notation</th>
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<tr>
<td>$&lt;R_{θθ}&gt;$</td>
<td>Radiation pattern</td>
</tr>
<tr>
<td>$r_{Aj}$</td>
<td>Ratio of the integral of the FAS of the actual records to the integral of $y(M_0, R, f)$</td>
</tr>
<tr>
<td>$r_{li}$</td>
<td>Uncertain multiplication factor to consider interevent correlations</td>
</tr>
<tr>
<td>$S(M_0, f)$</td>
<td>Earthquake source spectrum</td>
</tr>
<tr>
<td>$\overline{S}_{jj}(f)$</td>
<td>Smoothed power spectral density function for a pair of (randomly oriented horizontal) records</td>
</tr>
<tr>
<td>$\overline{S}_{jk}(f)$</td>
<td>Smoothed cross power spectral density function of two records</td>
</tr>
<tr>
<td>$T_n$</td>
<td>Natural vibration period</td>
</tr>
<tr>
<td>$V$</td>
<td>Partition of total shear-wave energy into horizontal components</td>
</tr>
<tr>
<td>$W(n)$</td>
<td>Hamming window</td>
</tr>
<tr>
<td>$Y$</td>
<td>Ground-motion measure, such as PGA and SA</td>
</tr>
<tr>
<td>$Y_C$</td>
<td>Ground-motion measure obtained from the simulated record by considering the coherency structure but ignoring (the uncertainty in) $r_{Aj}$</td>
</tr>
<tr>
<td>$Y_{CU}$</td>
<td>Ground-motion measure obtained from the simulated record by considering both the coherency structure and uncertainty in $r_{Aj}$</td>
</tr>
<tr>
<td>$y(M_0, R, f)$</td>
<td>Fourier amplitude spectrum given in stochastic point-source method</td>
</tr>
<tr>
<td>$\bar{Z}(f)$</td>
<td>Near-surface average seismic impedance</td>
</tr>
<tr>
<td>$Z(R)$</td>
<td>Geometrical spreading term</td>
</tr>
<tr>
<td>$Z_S$</td>
<td>Seismic impedance near the source</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Parameter in the coherency function</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>Shear-wave velocity in the vicinity of the source</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Separation distance between two stations</td>
</tr>
<tr>
<td>$\Delta\sigma$</td>
<td>Stress parameter</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Intraevent residual</td>
</tr>
<tr>
<td>$\gamma(\Delta, f)$</td>
<td>Smoothed coherency between two records</td>
</tr>
</tbody>
</table>
\[ \overline{\gamma}(\Delta, f) \]  
Lagged coherency between two records

\[ \mu_{\ln Y_j} \]  
Mean of \( \ln(Y_j) \)

\[ \rho(\Delta, T_n) \]  
Spatial correlation coefficient of ground-motion measures

\[ \rho(\Delta) \]  
Spatial correlation coefficient of \( \ln(r_{Aj}) \)

\[ \rho_S \]  
Density in the vicinity of the source

\[ \rho_{YC}(\Delta, T_n) \]  
Spatial correlation coefficient between \( \ln(Y_{Cj}) \) and \( \ln(Y_{Ck}) \)

\[ \rho_{YCU}(\Delta, T_n) \]  
Spatial correlation coefficient between \( \ln(Y_{CUj}) \) and \( \ln(Y_{CUk}) \)

\[ \sigma_d(\Delta, T_n) \]  
Standard deviation of the difference of residuals at two sites

\[ \sigma_d(T_n) \]  
Standard deviation of intraevent residuals

\[ \sigma_{\ln Y_j} \]  
Standard deviation of \( \ln(Y_j) \)

\[ \sigma_{\ln r} \]  
Standard deviation of \( \ln(r_{Aj}) \)

\[ \sigma_{\ln YC} \]  
Standard deviation of \( \ln(Y_{Cj}) \)

**CHAPTER 4**

\( A \)  
Parameter in the coherency function

\( A_{jk}(f_i) \)  
Amplitude of cosine wave at frequency \( f_i \)

\( a \)  
Spatial correlation model parameter for \( \ln(r_{Aj}) \)

\( B \)  
Parameter in the coherency function

\( b \)  
Spatial correlation model parameter for \( \ln(r_{Aj}) \)

\( C_s \)  
Design base shear coefficient

\( D_y \)  
Yield displacement capacity of the inelastic SDOF system

\( f \)  
Frequency

\( f_0 \)  
Parameter in the coherency function

\( f_N \)  
Upper cutoff frequency

\( f_\Delta \)  
Frequency interval

\( g \)  
Gravitational acceleration
$h$  Height of the structure

$I_{BT}$  Building index

$K$  Stiffness of the SDOF system

$k$  Parameter in the coherency function

$L$  Aggregate seismic loss

$L(f)$  Lower triangular matrix obtained from Cholesky decomposition of the lagged coherency matrix

$L_{BI}(\delta)$  Business-interruption related loss

$L_{BL}(\delta)$  Building-related loss

$L_{CO}(\delta)$  Contents-related loss

$L_{\text{max}}$  Maximum potential aggregate seismic loss

$l_{jk}(f)$  Element of the lower triangular matrix $L(f)$

$M$  Moment magnitude

$M_0$  Earthquake moment

$m$  Mass of the SDOF system

$N$  Hamming window parameter

$n$  Shape parameter in Bouc-Wen model

$n_G$  Number of simulated records at each site

$n_R$  Number of sites of interest

$Q(f)$  Anelastic attenuation term

$R_N$  Coefficient taking into account that the actual yield strength of a designed structure is greater than $V_d$

$r_{Aj}$  Spatially correlated disturbance of FAS

$r_L$  Seismic loss normalized by its potential maximum

$S_{Aj}(T_n, \zeta)$  Spectral acceleration induced by the earthquake at site $j$

$S_{Dj}(T_n, \zeta)$  Spectral displacement induced by the earthquake at site $j$

$S_0$  Power spectral density function
\( \overline{S}_{jj}(f) \) Smoothed power spectral density function for a pair of (randomly oriented horizontal) records

\( \overline{S}_{jk}(f) \) Smoothed cross power spectral density function of two records

\( T_n \) Natural vibration period

\( u \) Displacement of the SDOF system

\( \ddot{u}_g(t) \) Ground acceleration time history

\( V_d \) Minimum required design base shear force

\( V_{S30} \) Average shear-wave velocity of the top 30 m of earth

\( v_{ap} \) Apparent velocity

\( W \) Total weight of the structure

\( W(n) \) Hamming window

\( W_j(t) \) White noise simulated using spectral representation method

\( Y(T_n) \) Ground-motion measure, such as PGA and SA

\( y_j(M_0, R_j, f) \) Fourier amplitude spectrum at the \( j \)-th site defined by stochastic finite-fault method

\( z \) Hysteretic displacement of the SDOF system

\( \alpha \) Shape parameter in Bouc-Wen model

\( \alpha_0 \) Parameter in the coherency function

\( \beta \) Shape parameter in Bouc-Wen model

\( \beta^{BI}, \beta^{BL} \) and \( \beta^{CO} \) Seismic loss model parameters

\( \gamma \) Shape parameter in Bouc-Wen model

\( \Delta \) Separation distance between two stations

\( \Delta_p \) Projection of separation distance in the direction of wave propagation

\( \Delta \sigma \) Stress parameter

\( \delta_j \) Damage factor for the \( j \)-th building
\( \delta_\eta \)  Stiffness degradation parameter \\
\( \delta_i \)  Strength degradation parameter \\
\( \varepsilon_n \)  Normalized dissipated energy through hysteresis \\
\( \phi_{kl} \)  Random phase angle uniformly distributed between 0 to \( 2\pi \) \\
\( I(f) \)  Lagged coherency matrix \\
\( \mu_{\ln Y_j(T_n)} \)  Mean of \( \ln Y_j(T_n) \) \\
\( \gamma(\Delta, f) \)  Coherency between two records \\
\( \overline{\gamma}(\Delta, f) \)  Smoothed coherency between two records \\
\( \kappa \)  High frequency decay parameter \\
\( \mu_j \)  Ductility demand of the \( j \)-the building \\
\( \mu_R \)  Ductility capacity \\
\( \mu_z \)  Hysteretic displacement normalized by the yield displacement capacity \\
\( \theta \)  Stability factor in Bouc-Wen model \\
\( \rho(\Delta, T_n) \)  (Intraevent) spatial correlation coefficient of ground-motion measures \\
\( \rho_r(\Delta) \)  (Intraevent) spatial correlation coefficient of \( r_{Aj} \) \\
\( \sigma_{\ln Y_j(T_n)} \)  Standard deviation of \( \ln Y_j(T_n) \) \\
\( \tau_{jk} \)  Time lag between site \( j \) and site \( k \) \\
\( \omega_h \)  Natural vibration frequency of the SDOF system \\
\( \xi \)  Damping ratio \\

**CHAPTER 5**

\( A \)  Parameter in the coherency function \\
\( a_1 \)  Empirical model parameter for \( \rho(\Delta) \) \\
\( a_2 \)  Empirical model parameter for \( \rho_{nn,jk}(\Delta) \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_3$</td>
<td>Empirical model parameter for $\rho_{mn,jk}(\Delta)$</td>
</tr>
<tr>
<td>$B$</td>
<td>Parameter in the coherency function</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Empirical model parameter for $\rho(\Delta)$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Empirical model parameter for $\rho_{mn,jk}(\Delta)$</td>
</tr>
<tr>
<td>$b_3$</td>
<td>Empirical model parameter for $\rho_{mn,jk}(\Delta)$</td>
</tr>
<tr>
<td>$c_0, c_1$</td>
<td>Lagged coherency model parameters</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
</tr>
<tr>
<td>$f_0$</td>
<td>Parameter in the coherency function</td>
</tr>
<tr>
<td>$k$</td>
<td>Parameter in the coherency function</td>
</tr>
<tr>
<td>$M$</td>
<td>Moment magnitude</td>
</tr>
<tr>
<td>$M_L$</td>
<td>Local magnitude</td>
</tr>
<tr>
<td>$N$</td>
<td>Hamming window parameter</td>
</tr>
<tr>
<td>$n_R$</td>
<td>Total number of recording sites</td>
</tr>
<tr>
<td>$Q(f)$</td>
<td>Anelastic attenuation term</td>
</tr>
<tr>
<td>$R$</td>
<td>Source-to-site distance</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Empirical model parameter for $\rho_{mn,jk}(\Delta)$</td>
</tr>
<tr>
<td>$r_{Am,j}$</td>
<td>Spatially correlated disturbance of FAS of the $m$-th direction at the $j$-th site</td>
</tr>
<tr>
<td>$\bar{S}_{m,j}(f)$</td>
<td>Smoothed power spectral density functions for the $m$-th component at the $j$-th station</td>
</tr>
<tr>
<td>$\bar{S}_{mn,jk}(f)$</td>
<td>Smoothed cross power spectral density function of the $m$-th component at the $j$-th station and the $n$-th component at the $k$-th station</td>
</tr>
<tr>
<td>$v_{ap}$</td>
<td>Apparent velocity</td>
</tr>
<tr>
<td>$W(n)$</td>
<td>Hamming window</td>
</tr>
<tr>
<td>$y_j(M, R_j, f)$</td>
<td>Fourier amplitude spectrum at the $j$-th site defined by stochastic simulation method</td>
</tr>
</tbody>
</table>
\( \alpha_0 \) Parameter in the coherency function

\( \Delta \) Separation distance between two stations

\( \Delta_p \) Projection of separation distance in the direction of wave propagation

\( \Delta \sigma \) Stress parameter

\( \bar{\gamma}_{mn,jk}(\Delta, f) \) Coherency function between the \( m \)-th component at \( j \)-th station and the \( n \)-th component at \( k \)-th station

\( |\bar{\gamma}_{mn,jk}(\Delta, f)| \) Lagged coherency between the \( m \)-th component at \( j \)-th station and the \( n \)-th component at \( k \)-th station

\( \rho(\Delta) \) (Intraevent) correlation coefficient of the FAS of two randomly oriented components

\( \rho_{mn,jk}(\Delta) \) (Intraevent) correlation coefficient between \( \ln(r_{Am,j}) \) and \( \ln(r_{An,k}) \)
CHAPTER 1. INTRODUCTION

1.1 Background

Strong ground motions induced by earthquakes have uncertain amplitudes, durations and frequency contents. Design and evaluation of structures and infrastructures under seismic hazard require the knowledge of ground motion records if time history analysis is considered. The use of the time history analysis is advantageous because it can deal with material and geometric nonlinearity; its use is recommended in the building codes (NRCC 2010; ASCE 2010). Judiciously selected and scaled historical or simulated ground motion records for scenario events characterized by earthquake magnitude and source-to-site distance (Iervolino and Cornell, 2005; Baker and Cornell, 2006; Hong and Goda 2006) can be used. However, the available historical records are limited and may not match the desired and identified scenario events; synthetic records may be considered. These records at spatially distributed recording stations are spatially coherent and correlated.

The coherency of two ground motion records at two sites, defined as the ratio between the cross spectral density and the square root of the multiplication of the power spectra, depends on the frequency and separation distance; it is estimated from the power spectral density functions of the records and is not affected by scaling the ground motions (Abrahamson et al., 1991; Zerva 2009). The spatial correlation represents the correlation of the amplitude of ground measures at two sites (Boore et al., 2003; Wang and Takada, 2005; Goda and Hong, 2008a; Hong et al., 2009; Jayaram and Baker, 2009; Goda and Atkinson, 2010; Sokolov et al., 2010).
If the components of records at a single site are of concern, they can be simulated using one of the many available methods, including the stochastic ground motion simulation methods such as the stochastic point-source method (Boore 1983, 2003, 2009), the stochastic finite-fault method (Beresnev and Atkinson, 1997; Motazedian and Atkinson, 2005; Atkinson et al., 2009), and methods using the Kanai-Tajimi power spectral density function or the evolutionary power spectral density functions of the ground motion records (Yeh and Wen 1990; Alamilla et al., 2001; Zerva 2009).

The use of the stochastic point-source method and the stochastic finite-fault method is advantageous because they can be directly related to (but not completely defined by) the scenario events characterized by the magnitude and source-to-site distance. The stochastic point-source method is developed by considering that the Fourier amplitude spectrum (FAS) of the far-field accelerations is given by a simple seismological model. The stochastic finite-fault method simulates ground motions by summing the contributions from discretized sub-faults with each sub-fault modeled as a point-source. The updated version of the stochastic point-source method (Boore 2009) can produce similar simulation results as the stochastic finite-fault method even for sites that are close to the source of large earthquakes. The stochastic point-source method and the stochastic finite-fault method are used in various engineering applications, including the development of the ground motion prediction equation (GMPE) for regions without sufficient historical records (Atkinson and Boore, 2006; Atkinson and Macias, 2009).

The advances in computational power and the availability of efficient simulation packages (e.g., SMSIM and EXSIM, http://daveboore.com/software_online.html) facilitate the applications of these stochastic methods in engineering practices.
Some of these methods have been extended to simulate ground motion records by including the spatial coherency (Hao et al., 1989; Abrahamson, 1992) for uni-directional ground motions at different sites. However, none of these methods considered the potential spatial correlation of ground-motion measures such as the FAS, even though the spatial correlation of ground-motion measures can significantly affect the estimated risk of spatially distributed building stocks (Goda and Hong 2008b). Moreover, structures and infrastructure systems such as irregular building with different dynamic characteristics in two horizontal directions and bridges with multiple supports can be sensitive to bi-directional and/or multiple-support excitations (Clough and Penzien 2003; Zerva 2009). The orientation of the records can also affect the characteristics of records and responses of the structures (Arias 1970, 1996; Penzien and Watabe 1975; Hong and Goda, 2010). Algorithms that incorporate both the spatial coherency and spatial correlation for simulating records with multiple components at multiple stations by considering scenario events are lacking.

1.2 Objectives and thesis organization

This study focuses on the development of approaches to simulate single- and multiple-component multiple-station ground motion records with target spatial coherency and spatial correlation structures for scenario events. To develop such approaches, spatial correlation of the FAS for a random orientation and for two orthogonal directions is assessed, and empirical equations are suggested. Moreover, spatial coherency of ground motion records for two orthogonal horizontal directions is estimated using actual records from seven seismic events. Empirical coherency function is suggested for the components of records in two orthogonal horizontal directions at single and multiple
recording stations. The developed spatial correlation and spatial coherency models are incorporated in the stochastic point-source model and in the stochastic finite-fault model to generate synthetic records with single randomly oriented component at each of the multiple stations and records with two orthogonal horizontal components at multiple stations. An application of the simulated single component records for seismic risk assessment of a group of hypothetical buildings located in Vancouver is also given.

The thesis contains six chapters and is organized according to the integrated manuscript format specified by the School of graduate and post-graduate studies. The subsequent five chapters are summarized in the following.

Chapter 2 investigates the differences between spatial correlation of simulated records obtained by the stochastic finite-fault method and the spatial correlation calculated from the 1999 Chi-Chi Taiwan earthquake records. It is shown that although on average the simulations match response spectral characteristics of records, they do not reproduce the observed spatial correlation. This is expected as the method was developed to simulate records at individual site and does not have the built-in spatial correlation features - a common problem for several available algorithms for generating synthetic records.

Chapter 3 proposes an extension to the stochastic point-source method for generating records with single randomly oriented component at each of the multiple stations. The extension incorporates target spatial coherency structure and spatially correlated FAS. The proposed extension facilitates the application of the stochastic point-source method for seismic analysis of structures with multiple supports and for seismic risk assessment of portfolios of structures distributed in a region. This proposed approach that
incorporates the spatial coherency and correlation can also be adopted by other available algorithms for simulating records that match the target spatial coherency and correlation.

Chapter 4 focuses on the extension of the stochastic finite-fault simulation method that incorporates the spatial coherency and the spatially correlated disturbance in the FAS. The extended model is used to generate synthetic records for a scenario event to investigate the sensitivity of the statistics of aggregate seismic loss of a portfolio of hypothetical buildings distributed in downtown Vancouver to the spatially correlated excitations. The use of the spatially correlated and coherent synthetic records for scenario seismic event facilitates the consideration of different nonlinear inelastic behaviours for the group of buildings and avoids the need to develop and use the GMPE and ductility demand rules that are compatible with the scenario seismic event.

Chapter 5 estimates the spatial coherency of ground motion records in two orthogonal horizontal directions using records from SMART-1 array for seven seismic events. Empirical spatial coherency function is suggested for the components of records in two orthogonal horizontal directions. The spatial coherency and correlation of the records in the two orthogonal horizontal directions are used to establish a framework for simulating bi-directional horizontal ground motions at multiple stations. For the simulation it is considered that the reference FAS for scenario events can be defined using the stochastic point-source method or the stochastic finite-fault method. Samples of synthetic records are illustrated.

Finally, Chapter 6 summarizes the concluding remarks of the thesis and provides suggestions for future research.
1.3 Format of the thesis

This thesis is prepared in a manuscript format as specified by the School of Graduate and Postdoctoral Studies at the University of Western Ontario. Each chapter, except Chapters 1 and 6, is presented in a manuscript format with its own list of notations and references.

References


CHAPTER 2. INTRAEVENT SPATIAL CORRELATION
CHARACTERISTICS OF STOCHASTIC FINITE-FAULT SIMULATIONS

2.1 Introduction

Spatially correlated strong ground motions can cause severe accumulation and concentration of seismic damage and loss to spatially distributed buildings and infrastructure. The spatial correlation of peak ground motions and response spectra for a given event (i.e., the intraevent spatial correlation) has recently been studied by several researchers (Boore et al., 2003; Kawakami and Mogi, 2003; Wang and Takada, 2005; Goda and Hong, 2008; Jayaram and Baker, 2009; Goda and Atkinson, 2009; Goda and Atkinson, 2010). Common findings are that, for a given earthquake, ground-motion measures at different sites are spatially correlated; the correlation decreases as the separation distance increases. Goda and Hong (2008) developed an empirical spatial correlation model for peak ground acceleration (PGA) and spectral acceleration (SA) at various natural vibration periods, using ground-motion records from the Pacific Earthquake Engineering Research (PEER) Next Generation Attenuation (NGA) database. Goda and Atkinson (2010) compared the spatial correlation of strong ground-motion data from SK-net, a regional network in Japan having dense station coverage, with the Goda-Hong model developed from the NGA data (Goda and Hong, 2008); they concluded that the extended dataset, which includes the Japanese data, is also consistent with this model.

The stochastic finite-fault method is a widely used ground motion simulation technique that has been shown to be simple and effective in the generation of synthetic
ground-motion records (Beresnev and Atkinson, 1997; Motazedian and Atkinson, 2005; Atkinson et al., 2009; Boore, 2009). Stochastic simulations do a good job in matching response spectra and peak motions observed in earthquakes over a broad frequency range, but are particularly applicable at high frequencies (>1 Hz). More detailed broadband simulation methods use deterministic waveform modeling at low frequencies, but also revert to stochastic methods at high frequencies (e.g. Hartzell et al., 1999; Frankel, 2009; Halldorsson et al., 2011). Thus the performance of stochastic simulation methods is an important issue.

In this chapter, whether the spatial correlation structure of ground motions simulated by the stochastic finite-fault method match that of real records is assessed, using the 1999 M 7.6 Chi-Chi earthquake as a case study. To evaluate the intraevent spatial correlation characteristics of stochastic finite-fault simulations, simulations for a test event, the Chi-Chi earthquake, are performed and the obtained spatial correlation is compared to that calculated from the records. For the simulations, the EXSIM stochastic finite-fault program of Motazedian and Atkinson (2005) is used. This is a freely-available and widely-used program that has been validated under a range of conditions (Motazedian and Atkinson, 2005; Atkinson et al., 2009; Boore, 2009). The Chi-Chi earthquake is a well-recorded event, with more than 400 records from the main shock available, and its spatial correlation characteristics are already available (Wang and Takada, 2005; Goda and Hong, 2008). In this chapter, a simulation model for the Chi-Chi earthquake is developed using EXSIM, and the intraevent spatial correlation coefficient is assessed using the simulated records. The obtained spatial correlation is compared to that of the actual records. The results indicate that the former does not match the latter
satisfactorily. To overcome this deficiency, it is believed a post-processing fitting approach aimed at improving this aspect of the synthetic records generated using the stochastic finite-fault method could be valuable.

2.2 Intraevent spatial correlation of ground-motion measures

Let \( Y_i(T) \) denote a ground-motion measure (i.e., such as PGA or SA) at the \( i \)-th recording site for a specific seismic event:

\[
\ln Y_i(T) = f(R_i, \lambda_i, T) + \varepsilon_i(T)
\]  

where \( f(R_i, \lambda_i, T) \) represents the median of the (logarithmic of) ground-motion measure; \( R_i \) denotes the distance to the \( i \)-th recording site; \( T \) represents the natural vibration period (if it is required); \( \lambda_i \) denotes an explanatory parameter such as \( V_{S30} \) (the average shear-wave velocity of the top 30 m of earth); and \( \varepsilon(T) \) is the intraevent residual, which is normally distributed with zero mean and standard deviation \( \sigma(\varepsilon(T)) \). Note that the interevent residual is ignored in Equation (2.1) as a single event is considered. Throughout this chapter, unless otherwise indicated, \( Y_i(T) \) is used to represent the geometric mean of two orthogonal horizontal components of the ground-motion measure.

The intraevent spatial correlation coefficient (i.e., spatial correlation within a single event) can be expressed as (Goda and Hong, 2008):

\[
\rho(\Delta, T) = \frac{1 - \left[ \frac{\sigma_\varepsilon(\Delta, T)}{2\left[ \sigma(\varepsilon(T)) \right]} \right]^2}{2\left[ \sigma(\varepsilon(T)) \right]^2},
\]  

(2.2)
where $\Delta$ is the separation distance between the $i$-th and $j$-th recording stations (or observation points) and $[\sigma_{\Delta}(\Delta, T)]^2$ is the variance of $\varepsilon_i(T) - \varepsilon_j(T)$.

### 2.3 Modeling the Chi-Chi Ground motion

#### 2.3.1 Ground motion prediction equations and residuals

The ground-motion records of the Chi-Chi earthquake were obtained from the PEER-NGA database (PEER Center, 2010). Following Wang and Takada (2005), poor-quality data, recorded with an older-type instrument (A800), are discarded, resulting in a total of 389 selected records.

To estimate the intraevent spatial correlations, a set of ground motion prediction equations (GMPEs) for the Chi-Chi earthquake is first developed by carrying out regression analysis for the observed ground-motion measures PGA and SA. The following functional form is adopted:

$$
\ln A = c_0 + c_1 \ln R + c_2 R + c_3 \ln \left( \frac{V_{S30}}{V_{\text{ref}}} \right) + \varepsilon
$$

(2.3)

where $A$ is the ground-motion measure; $R = \sqrt{R_{\text{rup}}^2 + h^2}$, where $R_{\text{rup}}$ is the closest distance from the recording site to the fault rupture plane and $h$ is an “added depth” term that builds in near-source saturations due to finite-fault effects. The value of $h$ is determined by searching within the range from 1 to 10 km to find the value that minimizes the standard deviation of the residuals. $V_{\text{ref}}$ is set to 760 m/s, $c_0$, $c_1$, $c_2$ and $c_3$ are regression coefficients to be determined, and $\varepsilon$ is the intraevent residual. The obtained regression coefficients are summarized in Table 2.1. The intraevent residuals $\varepsilon$ for SAs at 0.3 s and
1.0 s with respect to $R_{rup}$ are plotted in Figure 2.1. It can be seen that the intraevent residuals are generally unbiased in terms of $R_{rup}$.

Table 2.1 Regression coefficients for GMPE and intraevent variability of Chi-Chi records

<table>
<thead>
<tr>
<th>Ground-motion measure</th>
<th>$h$</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$\sigma_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
<td>1.8</td>
<td>-0.558</td>
<td>-0.427</td>
<td>-0.007</td>
<td>-0.145</td>
<td>0.5137</td>
</tr>
<tr>
<td>SA at 0.1 s</td>
<td>1.7</td>
<td>-0.227</td>
<td>-0.371</td>
<td>-0.011</td>
<td>-0.035</td>
<td>0.5505</td>
</tr>
<tr>
<td>SA at 0.2 s</td>
<td>1.7</td>
<td>-0.155</td>
<td>-0.242</td>
<td>-0.015</td>
<td>-0.021</td>
<td>0.5366</td>
</tr>
<tr>
<td>SA at 0.3 s</td>
<td>1.2</td>
<td>-0.174</td>
<td>-0.200</td>
<td>-0.015</td>
<td>-0.091</td>
<td>0.5528</td>
</tr>
<tr>
<td>SA at 0.5 s</td>
<td>2.8</td>
<td>0.1113</td>
<td>-0.373</td>
<td>-0.009</td>
<td>-0.207</td>
<td>0.5344</td>
</tr>
<tr>
<td>SA at 1.0 s</td>
<td>3.3</td>
<td>0.2996</td>
<td>-0.721</td>
<td>0.0025</td>
<td>-0.517</td>
<td>0.5636</td>
</tr>
<tr>
<td>SA at 2.0 s</td>
<td>5.1</td>
<td>0.1259</td>
<td>-0.942</td>
<td>0.0065</td>
<td>-0.747</td>
<td>0.5648</td>
</tr>
<tr>
<td>SA at 3.0 s</td>
<td>9.9</td>
<td>1.0928</td>
<td>-1.383</td>
<td>0.0116</td>
<td>-0.841</td>
<td>0.5775</td>
</tr>
<tr>
<td>SA at 5.0 s</td>
<td>9.6</td>
<td>0.1656</td>
<td>-1.157</td>
<td>0.0053</td>
<td>-0.760</td>
<td>0.6542</td>
</tr>
</tbody>
</table>

Figure 2.1 Regression residuals vs. $R_{rup}$ (closest distance from the recording site to the fault rupture plane) for spectral accelerations (SAs) at 0.3 and 1.0 s.

2.3.2 Simulation model using EXSIM

Roumelioti and Beresnev (2003) simulated the Chi-Chi earthquake using FINSIM, an earlier version of EXSIM. Following their study, a rectangular fault with dimensions of
110 km by 40 km (Chi et al., 2001; Ma et al., 2001), a strike of 5°, and an easterly dip of 34° (Chang et al., 2000) is used in the EXSIM simulations. The fault plane is discretized into 176 sub-faults, each with dimension of 5 km by 5 km. The fault is assigned a homogenous slip; its surface projection is shown in Figure 1 in Roumelioti and Beresnev (2003) by the dashed line. The hypocenter is located at 23.853°N 120.816°E with a focal depth of 8 km. In the implementation of the stochastic finite-fault method, the path effects are modeled by an empirical geometric spreading and \( Q \) function. The upper-crustal amplification is taken into account by multiplying the simulated spectra by the factors proposed for generic rock sites in Western North America (Boore and Joyner, 1997). The spectra were additionally attenuated by the \( \kappa \) operator (Anderson and Hough, 1984) with \( \kappa = 0.06 \) s. The modeling parameters are summarized in Table 2.2.

No additional site amplification (beyond that of crustal amplifications) is considered for the simulation at 200 rock stations (NEHRP site class A, B and C). For the 189 soil sites (NEHRP site class D), the transfer function representing site amplification that was calculated by Roumelioti and Beresnev (2003) based on mainshock and aftershocks records is adopted. Only the mean value of the transfer function, shown in Figure 7(b) in Roumelioti and Beresnev (2003), is used in this chapter. For consistency, the response spectra for the simulated records as well as the actual Chi-Chi records are evaluated using a geometric mean based smoothing technique. The average (based on 389 stations) of the model bias defined as \( \log_{10}(\frac{Y_i(T) \text{ from actual record}}{Y_i(T) \text{ from the simulated record}}) \) is calculated and shown in Figure 2.2. It is noted that the stochastic simulation model matches the observations well on average. The results presented in the figure are based on a single trial (or simulation run); however, results using the average of multiple
(i.e., 30) simulation runs follow the same trend as shown in the figure. Note that no special effort was made to achieve the best fit when the modeling parameters were selected, beyond choosing the best-fit stress parameter (100 bars).

Table 2.2 Modeling parameters in EXSIM for Chi-Chi earthquake

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fault orientation (strike/dip)</td>
<td>5°/34°</td>
</tr>
<tr>
<td>Fault dimensions along strike and dip (km)</td>
<td>110 by 40</td>
</tr>
<tr>
<td>Depth of the upper edge of the fault (km)</td>
<td>0</td>
</tr>
<tr>
<td>Mainshock moment (dyn·cm)</td>
<td>$2.8 \times 10^{27}$</td>
</tr>
<tr>
<td>Sub-fault dimensions (km)</td>
<td>$5 \times 5$</td>
</tr>
<tr>
<td>Stress parameter $\Delta \sigma$ (bar)</td>
<td>100</td>
</tr>
<tr>
<td>Radiation-strength factor</td>
<td>1.0</td>
</tr>
<tr>
<td>Number of subsources summed</td>
<td>176</td>
</tr>
<tr>
<td>$Q(f)$</td>
<td>$117 f^{0.77}$</td>
</tr>
<tr>
<td>Geometric spreading</td>
<td>$\frac{1}{R}$ for $R &lt; 50$ km</td>
</tr>
<tr>
<td></td>
<td>$\frac{R^0}{R}$ for $50$ km $\leq R &lt; 150$ km</td>
</tr>
<tr>
<td></td>
<td>$\frac{R^{0.5}}{R}$ for $R \geq 150$ km</td>
</tr>
<tr>
<td>Windowing function</td>
<td>Cosine-tapered boxcar</td>
</tr>
<tr>
<td>Kappa (s)</td>
<td>0.06</td>
</tr>
<tr>
<td>Crustal amplification</td>
<td>Boore and Joyner (1997) western North America generic rock site</td>
</tr>
<tr>
<td>Crustal shear-wave velocity (km/s)</td>
<td>3.2</td>
</tr>
<tr>
<td>Rupture velocity (km/s)</td>
<td>$0.8 \times$ (shear-wave velocity)</td>
</tr>
<tr>
<td>Crustal density (g/cm$^3$)</td>
<td>2.7</td>
</tr>
</tbody>
</table>

The attenuation of SA at 1.0 s for the simulated records (based on one trial) is compared to that of the Chi-Chi records in Figure 2.3 as an example. Note that there are two trends in the simulated amplitudes can be seen, representing rock and soil sites. This is because there is no any added variability is included in the stochastic simulations to mimic site and attenuation variability. Such variability is much more pronounced in the actual observations. Appropriate variability can be introduced in the simulation by applying random variability in the input parameters (e.g., Atkinson and Boore, 2006); this point will be revisited later.
Figure 2.2 Averaged model bias at 389 stations. 95% confidence interval of the mean is shown in dashed lines.

Figure 2.3 Ground motions of Chi-Chi records and simulation (one trial) versus $R_{jb}$ (closest horizontal distance from site to surface projection of the rupture) in units of g for spectral acceleration (SA) at 1.0 s.
2.4 Comparison of intraevent correlation coefficients for real and simulated records

The intraevent correlation coefficients of both the observed and simulated (one EXSIM trial) Chi-Chi records are estimated using Equation (2.2), and plotted against the separation distance $\Delta$ in Figure 2.4 for SAs at 0.3, 1.0 and 3.0 s. By adopting the following functional form for $\rho_\varepsilon(\Delta, T)$,

$$\rho_\varepsilon(\Delta, T) = \exp\left(-\alpha \Delta^\beta\right)$$  \hspace{1cm} (2.4)

where $\alpha$ and $\beta$ are the model parameters, the fitted empirical correlation curves are also illustrated in Figure 2.4. Note that a bin size of 3 km is used for the analysis throughout to ensure that there are adequate residual samples within each bin.

As expected, the results presented in Figure 2.4a are similar to those given in Goda and Hong (2008). Comparison of Figure 2.4a and Figure 2.4b indicates that the records from the EXSIM simulation are generally less correlated than the real records for SAs at 0.3 and 1.0 s (Note: observations for periods less than 0.3 s, and for PGA, are similar to those shown for 0.3 s.). However, the intraevent spatial correlation coefficient of SA at 3.0 s for the simulated records is significantly greater than that for the Chi-Chi records. Furthermore, note that the intraevent spatial correlation coefficients calculated using the Chi-Chi records are similar at different natural vibration periods, while that of the simulated records show an increasing trend with increasing natural vibration period. In effect, the EXSIM records show too little spatial correlation at short periods and too much correlation at long periods. To assess if the results shown in Figure 2.4b are biased by use of a single simulation trial, the simulation analysis is repeated using 30 trials. The
estimated spatial correlation coefficients from multiple (i.e., 30) simulation runs are almost identical to those shown in Figure 2.4b. It is concluded that the stochastic model records, while matching the average response spectral characteristics, do not match their inherent spatial correlation structure.

![Figure 2.4 Estimated intraevent spatial correlation coefficient $\rho_\varepsilon(\Delta,T)$ samples and their fitted curves of spectral accelerations (SAs) for $T$ equal to 0.3, 1.0 and 3.0 s: (a) using the Chi-Chi records; (b) using simulation records (one trial).](image)

It could be valuable if the stochastic finite-fault method can be modified such that the spatial correlations of the simulated records are more closely matching those of real records. For a possible simple modification, it is noted that the correlation, $\rho_\varepsilon(\Delta,T)$, is affected by both $\sigma_d(\Delta,T)$ and $\sigma_\varepsilon(T)$ (see Equation (2.2)). It was observed that the standard deviations of the residuals $\varepsilon_i(T)$ shown in Equation (2.1) for the simulated records, denoted by $\sigma_{\text{sim}}(T)$, are systematically smaller than those for the real records. For
example, on average (based on 30 simulation trials), $\sigma_{\text{sim}}(T)$ equals 0.338 for PGA, 0.367 for SA at 0.3 s, 0.369 for SA at 1.0 s, and 0.390 for SA at 3.0 s. These values are significantly less than those shown in Table 2.1. This can be remedied in practice by introducing more variability into the simulations, such as by adding a randomly-drawn increment to the average site or attenuation terms (e.g., Atkinson and Boore, 2006). Comparison of $\sigma_d(\Delta,T)$ obtained by using the Chi-Chi records and the simulated records is shown in Figure 2.5. The figure shows that $\sigma_d(\Delta,T)$ for the simulated records becomes larger as the separation distance increases, which follows the observed trend from actual records. However, the values of $\sigma_d(\Delta,T)$ for the simulated records are generally smaller than in the real records. Again, the simulation analysis is repeated 30 times; the observations from such an analysis are the same as for a single trial.

![Figure 2.5](attachment:figure25.png)

Figure 2.5 Intraevent spatial variability (i.e., the standard deviation of difference of regression residuals at two sites), $\sigma_d(\Delta,T)$, of SAs at 0.3, 1.0 and 3.0 s: (a) using the Chi-Chi records; (b) using simulation records (one trial).
To see whether a pragmatic simple approach can lead to an improved match of the record characteristics, an additional intraevent variability (i.e., an error term $\varepsilon_E(T)$) is introduced to the ground-motion measure from the EXSIM simulations, $\varepsilon_{\text{sim}}(T)$, whose standard deviation is $\sigma_{\text{sim}}(T)$, such that the new error term from the simulations after this post-processing, $\varepsilon'_{\text{sim}}(T)$, is given by,

$$\varepsilon'_{\text{sim}}(T) = \varepsilon_{\text{sim}}(T) + \varepsilon_E(T),$$  \hspace{1cm} (2.5a)

where $\varepsilon_E(T)$ is a normal variate with zero mean and standard deviation of $\sigma_E(T)$ such that the variance of $\varepsilon'_{\text{sim}}(T)$, $[\sigma'_{\text{sim}}(T)]^2$,

$$[\sigma'_{\text{sim}}(T)]^2 = \sigma_{\text{sim}}^2(T) + \sigma_E^2(T),$$  \hspace{1cm} (2.5b)

equals the variance obtained from the actual records.

With the additional error term, the ground-motion measures from the EXSIM simulations for a single trial are illustrated in Figure 2.6 and compared to those of the Chi-Chi records. As expected, the dispersion of the simulated amplitudes is now similar to that of the real records; the clear distinction between rock and soil sites is diminished. The corresponding $\sigma_d(\Delta,T)$ and intraevent spatial correlation coefficient $\rho_d(\Delta,T)$ for the simulated records are shown in Figure 2.7. In this case, although the agreement between the estimated $\sigma_d(\Delta,T)$ from the simulated records to those from the Chi-Chi records is improved significantly, the desired monotonic increasing trend of $\sigma_d(\Delta,T)$ versus the separation distance was not maintained or achieved by the use of the additional error term $\varepsilon_E(T)$. The intraevent spatial correlation coefficients of the simulated records with and without the additional error term $\varepsilon_E(T)$ are compared with that observed from the Chi-Chi
records for SAs at 0.3 and 3.0 s in Figure 2.8. The comparison shows that the intraevent spatial correlation coefficient for SA at a long period (i.e., 3.0 s) becomes closer to the observation after applying the post-processing approach, while the SA becomes even less spatially correlated at short periods (i.e., 0.3 s). These observations were confirmed by repeating the simulation 30 times.

Figure 2.6 Comparison of ground-motion measure of Chi-Chi records and EXISM simulation (one trial, with additional random error term) in the unit of g for spectral acceleration (SA) at 1.0 s.

The results indicate that the use of $\varepsilon_{E}(T)$ effectively corrects the deficiency of the simulation method in matching the residuals at a randomly selected recording station, but it is deficient in reproducing the observed $\sigma_{d}(\Delta,T)$ and the spatial correlation. In
particular, stochastic simulations can be easily manipulated to lessen their intraevent correlations ($\rho_\varepsilon(\Delta,T)$) by increasing the random variability input to the simulations, and this is moving in the correct direction at long periods. But at short periods greater intraevent correlation is needed, so one would essentially need to make the simulations less random. This implies that high-frequency motions have a deterministic component, likely due to details of crustal structure. Moreover, the degree of determinism of the high-frequency motions varies with the scale length, and perhaps regionally (e.g. Sato and Fehler, 1998; Fehler and Sato, 2003). Thus it is needed to introduce more distance dependency to $\sigma_d(\Delta,T)$ in the simulations to more closely model the observations (e.g. see Figure 2.5). To solve this problem, and to reproduce both the observed characteristics of the ground-motion measures and their spatial correlation, one must assign a joint normal probability distribution to the zero-mean intraevent residuals at all recording stations, where the second statistical moments of the intraevent residuals match those from the actual records.

Here a method is proposed to control both $\sigma_e(T)$ and $\sigma_d(\Delta,T)$ based on the azimuth and source distance of the recording sites. A schematic diagram is shown in Figure 2.9 for a hypothetical event. The idea is to divide the overall recording region into small “blocks” (such as the grey one shown in Figure 2.9). The error terms for each and every block may be specified, such that their first two statistical moments, including the cross moments, match that from the observations. By doing this, not only is the overall intraevent variability matched, but also the residuals in each block will fit statistically to the observations. $\sigma_d(\Delta,T)$, should then be appropriately reproduced at small distances. It is noted that the size of the block is crucial, because there might not be enough samples in
each block if the blocks are too small; alternatively the standard deviations and $\sigma_d(\Delta, T)$ may not be able to differentiate the difference between blocks if the blocks are too big. This idea is presented in concept only at this time. To properly test it, an appropriate dataset with a dense distribution of recording stations is needed; this will be the subject of future studies. It should be emphasized that the proposed scheme is aimed at reproducing the spatial correlation of the ground-motion measures, rather than the spatial coherence structure of the records as a whole. There are other more deterministic methods of simulation that would be better suited to matching the overall coherence structure of the time series, provided that sufficiently detailed information on crustal properties is available to resolve motions over the frequency range of interest.

Figure 2.7 Results after applying the random error term of the peak ground acceleration (PGA) and SAs at 0.3, 1.0 and 3.0 s (one trial): (a) intraevent spatial correlation coefficient, $\rho_e(\Delta, T)$; (b) spatial variability, $\sigma_d(\Delta, T)$. 
Figure 2.8 Comparison of the intraevent spatial correlation coefficient of the Chi-Chi records, EXSIM simulation and modified simulation with added variability for spectral acceleration (SA) at 0.3 and 3.0 s (only fitted curves are shown).

Figure 2.9 Schematic diagram of a hypothetical earthquake source area and the blocks that could be used to subdivide the region by both distance and azimuth (the recording stations are shown only in the highlighted block).
2.5 Conclusions

It is concluded that the stochastic finite-fault method, while reproducing average response spectral characteristics of records, does not reproduce observed spatial correlation, at least for the Chi-Chi records. Although only the program EXSIM is used in this chapter, it is expected that this is an inherent common problem in other currently-available stochastic simulation packages, and with the high-frequency part of broadband simulations. It is also expected the problem to be common to other events, although only the Chi-Chi event was studied here. This is a fundamental limitation of stochastic simulations, which may be important if considering spatially-distributed infrastructure. To remedy this deficiency would require controlling the variability of the simulations spatially. Alternatively, more deterministic modeling techniques could be used, but these would require detailed knowledge of crustal structure at a fine scale, and the extension of deterministic modeling techniques to higher frequencies (>3 Hz), which is not presently feasible. Due to the difficulties of extending deterministic modeling to high frequencies, the introduction of improved models of coherence into stochastic simulation methods is of significant practical importance, and worthy of more detailed investigations.

2.6 Data and Resources

The ground-motion records of the Chi-Chi earthquake were obtained from the PEER-NGA database at [http://peer.berkeley.edu/nga/](http://peer.berkeley.edu/nga/) (last accessed August 2010). EXSIM program was obtained from David M. Boore’s personal website at [http://daveboore.com/software_online.html](http://daveboore.com/software_online.html) (last accessed March 2011).
References


CHAPTER 3. SIMULATION OF MULTIPLE-STATION GROUND MOTIONS USING STOCHASTIC POINT-SOURCE METHOD WITH SPATIAL COHERENCY AND CORRELATION CHARACTERISTICS

3.1 Introduction

As the ground motion excitations at multiple sites for a seismic event are originated from the waves generated at the same source and propagated through the random medium, the records have a spatial coherency structure, and the ground-motion measures such as the peak ground acceleration (PGA) and the spectral accelerations (SAs) are spatially correlated. The coherency function of two ground motion records at two stations for a seismic event is a function of frequency and the distance between the stations (i.e., inter-station distance). Evaluation and modeling of spatial coherency have been carried out based on the recordings from dense arrays such as the El Centro differential array (Bycroft, 1980), the SMART-1 array (Bolt et al., 1982) and the LSST array (Abrahamson et al., 1991). The spatial coherency structure for the ground motion records has been proposed by Harichandran and VanMarcke (1986), Hao et al. (1989), and Der Kiureghian (1996) based on the analysis results of actual records or random vibration theory. These models are considered to be applicable for an inter-station distance or separation distance less than a few kilometers as this is the inter-station distance of the records used to develop and calibrate the models. The coherency decreases as the inter-station distance and frequency increases; it approaches to negligible value for the inter-station distance greater than about 5 km. Empirical spatial correlation models of the PGA and SAs have been proposed in the literature for seismic events in
California, Taiwan and Japan (Boore et al., 2003; Wang and Takada, 2005; Goda and Hong, 2008a; Hong et al. 2009, Jayaram and Baker, 2009; Goda and Atkinson, 2010; Sokolov et al. 2010). In contrast to the coherency function, they showed that in some cases the spatial correlation can still have values greater than about 0.4 for a separation distance more than 10 km.

The spatial correlation can affect the probabilistic characteristics of the seismic risk of a group of buildings distributed in a region (Goda and Hong, 2008b; 2009). In their seismic risk analysis, the seismic hazard is characterized in terms of SAs that are predicted using the ground motion prediction equations (GMPEs) with spatial correlation. An enhanced seismic risk assessment can be carried out for spatially distributed buildings or critical infrastructures through nonlinear dynamic analysis if the spatially correlated records are available for a range of scenario seismic events that cover the considered earthquake magnitude and site to seismic source distance combinations. The records could be selected from a database, matching the seismic scenario (and spatial distribution of actual records). As the available historical ground motion records are insufficient for such a purpose, the use of synthetic records for scenario events could be an alternative.

Stochastic ground motion simulation methods such as the stochastic point-source method (Boore, 2003 and 2009) and the stochastic finite-fault method (Beresnev and Atkinson, 1997; Motazedian and Atkinson, 2005; Atkinson et al., 2009; Boore, 2009) have been widely used to generate synthetic ground motion records. The simulated records using these methods match historical records over a broad frequency range and the level of ground-motion measures. Broadband hybrid simulation methods use deterministic waveform modeling at low frequencies, but also revert to stochastic
methods at high frequencies (e.g. Hartzell et al., 1999; Frankel, 2009). Other simulation methods include the empirical methods that consider the evolutionary properties of the ground motion records (Alamilla et al., 2001).

An advantage of the stochastic point-source and finite-fault methods is that they can be directly related to (but not completely defined by) seismic scenario events (e.g., earthquake magnitude and source-to-site distance). The stochastic finite-fault method does not reproduce the observed intraevent spatial correlation characteristics of peak ground-motion measures (i.e., PGA and SAs) (Chpter 2; Liu et al., 2012). As will be seen in this chapter, this is also the case for the stochastic point-source method. Moreover, the coherency among the records for a given event is not part of the built-in feature of the method. Aimed at generating ground motion records that mimic the observed features of coherency and spatial correlation of the ground-motion measures, an extension is suggested to the stochastic-point source method by introducing the target spatial coherency structure and the spatially correlated uncertainties in the Fourier amplitude spectrum for each site (recording station). The use of the extended model to simulate the multiple-station records is illustrated; the spatial correlations of the PGA and SAs for the simulated records are compared with those obtained from the records of the 1999 Chi-Chi Taiwan earthquake.

3.2 Stochastic point-source method

The stochastic point-source method (Boore, 2003 and 2009) is a simple-to-use method to generate records that match specified Fourier amplitude spectrum $y(M_0, R, f)$ of shear wave at a (hypocentral) distance $R$ from a fault with seismic moment $M_0$. $y(M_0, R, f)$ is given by,
\[ y(M_0, R, f) = E(M_0, f)P(R, f)G(f)I(f), \quad (3.1) \]

where \( f \) is the frequency in Hz; \( E(M_0, f) \), \( P(R, f) \) and \( G(f) \) represent the effects from source, path and site, respectively; \( M_0 \) is the seismic moment that can be converted to moment magnitude; \( I(f) \) is an indicator for ground motion type (acceleration, velocity or displacement) (see Table 3.1 for details). The steps to sample a record (Boore 2003) are to: a) Sample Gaussian noise with zero mean and unit variance; b) Modulate the signal in time; c) Calculate the Fourier amplitude spectrum (FAS) of the modulated signal; d) Normalize the FAS by the square-root of the mean squared amplitude spectrum; e) Multiply the normalized spectrum by the point source spectrum defined by Equation (3.1); and f) Calculate the synthetic ground motion record by applying the inverse Fourier transform to the spectrum obtained in Step e).

The method can closely reproduce the specified target Fourier amplitude spectrum. As the method does not contain built-in coherency or spatial correlation features (Boore, 2003), it is unlikely to reproduce any coherency structure or spatial correlation of the PGA or SAs observed from the historical records. To illustrate this, consider the 389 recording stations for the Chi-Chi earthquake shown in Figure 3.1, which was considered in Chapter 2 and by Liu et al. (2012), and the simulation parameters shown in Table 3.2.

The parameters are selected based on Roumelioti and Beresnev (2003) and Liu et al. (2012) for the stochastic finite-fault method, and adjusted such that the median of the ratio of the integral of the Fourier amplitude spectrum (0.1 to 20 Hz) of the actual records to the integral of \( y(M_0, R, f) \) (0.1 to 20 Hz) equals one. This ratio, denoted as \( r_{Aj} \) where \( j \) denotes the \( j \)-th recording station, is shown in Figure 3.2a; its natural logarithm is plotted
on normal probability paper in Figure 3.2b. The plots indicate that \( \ln(r_{ij}) \) could be assumed to be a normal variate and independent of the source-to-site distance. Note that each randomly oriented horizontal component of the records is used as a sample shown in Figure 3.2a, and only 328 of 389 original Chi-Chi records are processed with a high-pass filter corner frequency \( \leq 0.1 \text{ Hz} \) and a low-pass filter corner frequency \( \geq 20 \text{ Hz} \). The investigation of \( \ln(r_{ij}) \) as a function of frequency and its inclusion in the stochastic point-source model, although is important, is beyond the scope of this chapter.

Table 3.1 Summary of stochastic point-source model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source effect</td>
<td>( C = \frac{\langle R_{oo} \rangle VF}{4\pi\rho_s \beta_s^2 R_0} ) is constant, where ( \langle R_{oo} \rangle ) is the radiation pattern; ( V = 1/\sqrt{2} ) is the partition of total shear-wave energy into horizontal components; ( F = 2 ) represents the effect of the free surface; ( \rho_s ) and ( \beta_s ) are the density and shear-wave velocity in the vicinity of the source, and ( R_0 = 1 \text{ km} ) is a reference distance. ( M_0 ) is given by ( \log M_0 = 1.5M + 16.1 ), where ( M ) is the moment magnitude. ( S(M_0, f) = 1/\left[1 + \left(f/f_0\right)^2\right] ) is the ( \omega )-square displacement source spectrum.</td>
</tr>
<tr>
<td>Path effect</td>
<td>( P(R, f) = Z(R) \exp\left(-\frac{\pi f R}{Q(f) c_Q}\right) ) is the geometrical spreading and ( Q(f) ) is the attenuation term.</td>
</tr>
<tr>
<td>Site effect</td>
<td>( A(f(z)) = \sqrt{Z_S / \tilde{Z}(f)} ) is the amplification term where ( Z_S ) and ( \tilde{Z}(f) ) are the seismic impedance near the source and the near-surface average seismic impedance, respectively. ( D(f) ) is the diminution operator, can be the ( f_{\text{max}} ) filter or the ( \kappa_0 ) filter.</td>
</tr>
</tbody>
</table>

For the adopted model parameters given in Table 3.2 and the recording stations shown in Figure 3.1, records are sampled using the stochastic point-source simulation program that is publicly available at Boore’s personal website (see Data and Resources section).
In particular, two records are presented in Figures 3.3a and 3.3b for two selected recording stations: Station HWA007 and Station HWA009 with the inter-station distance, $\Delta$, equal to 0.534 km. For the simulation, the sampling frequency of 200 Hz is used.

![Figure 3.1 Considered recording stations for the Chi-Chi earthquake: stations at rock sites (NEHRP site class A, B and C) and soil sites (NEHRP site class D) are shown in black and grey triangles, respectively.](image)

Table 3.2 Modeling parameters in stochastic point-source model (selected based on Roumelioti and Beresnev, 2003 and Liu et al., 2012).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress parameter $\Delta\sigma$</td>
<td>103 (bar)</td>
</tr>
<tr>
<td>$Q(f)$</td>
<td>$117 \cdot f^{0.77}$</td>
</tr>
<tr>
<td>Geometrical spreading</td>
<td>$1/R$ for $R &lt; 50$ km, $1/R^0$ for $50$ km $\leq R &lt; 150$ km and, $1/R^{0.5}$ for $R \geq 150$ km</td>
</tr>
<tr>
<td>Windowing function</td>
<td>Exponential (Saragoni and Hart, 1974)</td>
</tr>
<tr>
<td>Kappa</td>
<td>0.08 (s)</td>
</tr>
<tr>
<td>Crustal amplification</td>
<td>Generic rock site (Boore and Joyner, 1997)</td>
</tr>
<tr>
<td>Crustal shear-wave velocity</td>
<td>3.2 (km/s)</td>
</tr>
<tr>
<td>Crustal density</td>
<td>2.7 (g/cm$^3$)</td>
</tr>
</tbody>
</table>
Figure 3.2 Ratio $r_{Aj}$ for the considered records and the adopted model parameters: (a) variation of $r_{Aj}$ versus hypocentral distance, (b) normal probability plot of $\ln(r_{Aj})$.

It is noted that the unsmoothed lagged coherency (absolute of coherency) estimate for a pair of records would be identically equal to unity, and smoothing procedure is often carried out using Hamming window, $W(n)$, given by (Zerva 2009)

$$W(n) = 0.54 - 0.46\cos\left(\frac{\pi(n+N)}{N}\right) \quad n = -N, \ldots, N$$

(2.2)

where $N$ is a parameter and the window width is given by $2N + 1$. The use of $N = 5$ for time series less than approximately 2000 points has been suggested (Abrahamson et al. 1991). Since the number of the points for the simulated records is 9740, $N$ is varied from 10 to 200 as a sensitivity analysis for estimating lagged coherency. As expected, an increased $N$ leads to smoother coherency estimation. Figure 3.3c illustrates the calculated lagged coherency for $N = 100$ between the two simulated records presented in Figures 3.3a and 3.3b.
Figure 3.3 Illustration of two sampled records \((\Delta = 0.534 \text{ km})\), estimated lagged coherency, and the spatial correlation coefficient for PGA by using the stochastic point source model.

This lagged coherency which is low is a consequence of that the stochastic point-source model does not contain built-in coherency (Boore, 2003). The low lagged coherency is also typical for any pair of simulated records and is independent of \(\Delta\), which is not in agreement with the observed coherency structure from actual records (e.g.,
Harichandran and VanMarcke, 1986). Moreover, by repeating the simulation 100 cycles and collecting the PGA from each station shown in Figure 3.1, 100 sets of the PGA, each set containing the PGA for one simulation cycle, are obtained. The spatial correlation coefficient \( \rho(\Delta, T_n) \) for the sets of samples is calculated using,

\[
\rho(\Delta, T_n) = E\left(\left(\ln\left(Y_j\right) - \mu_{\ln Y_j}\right)\left(\ln\left(Y_k\right) - \mu_{\ln Y_k}\right)\right)/\left(\sigma_{\ln Y_j}\sigma_{\ln Y_k}\right),
\]

where \( T_n \) is the natural vibration period; \( E(\cdot) \) represents expectation; \( Y_j \) represent the ground-motion measures such as PGA and SAs at the \( j \)-th station with mean and standard deviation of \( \ln(Y_j) \) denoted by \( \mu_{\ln Y_j} \) and \( \sigma_{\ln Y_j} \), respectively; and \( Y_k, \mu_{\ln Y_k} \) and \( \sigma_{\ln Y_k} \), are defined similarly but for the \( k \)-th station. Use of Equation (3.3a) to estimate \( \rho(\Delta, T_n) \) is advantageous for simulated samples with many replications since it directly uses the residuals (i.e., \( \left(\ln(Y_j) - \mu_{\ln Y_j}\right) \)) at any single station, and there is no need to develop the GMPEs and the statistics of the intraevent residual \( \varepsilon_j \) for the \( j \)-th station. Equation (3.3a) differs from,

\[
\rho(\Delta, T_n) = 1 - \frac{\left(\sigma_d(\Delta, T_n)\right)^2}{2\left(\sigma_e(T_n)\right)^2},
\]

that is used by others (Boore et al. 2003; Goda and Hong 2008a; Liu et al. 2012) to deal with actual records, where \( \sigma_d(\Delta, T_n) \) is the standard deviation of the difference of \( \varepsilon_j \) (or variability) at two sites separated by \( \Delta \), and \( \sigma_e(T_n) \) is the standard deviation of \( \varepsilon_j \). Unlike the application of Equation (3.3a), the use of Equation (3.3b) requires the knowledge of the GMPE and statistics of \( \varepsilon_j \) from regression analysis. An implicit assumption in Equation (3.3b) is that the mean of \( \varepsilon_j \) is zero at each station. This assumption cannot be
verified for a given historical event because there is no replication available. For practical applications, the mean of $\varepsilon_j$ for all stations equal to zero is often used as a proxy to imply that the mean of $\varepsilon_j$ at each station equals zero.

The calculated $\rho(\Delta, T_n)$ by using Equation (3.3a) for each pair of stations with separation distance less than or equal to 50 km is shown in Figure 3.3d, indicating that the trend for the spatial correlation coefficient obtained from simulation results does not agree with that observed from the Chi-Chi earthquake (Goda and Hong, 2008a; Liu et al., 2012). Similar analysis is carried out for the SAs at different $T_n$ values and the observation made to $\rho(\Delta, T_n)$ for the PGA is equally applicable to those for the SAs.

It is noteworthy that by repeating this analysis using the stochastic finite-fault method, instead of the stochastic point-source method, the same observation for $\rho(\Delta, T_n)$ again can be made. This observation differs from the one given in Chapter 2 and Liu et al. (2012), showing that $\rho(\Delta, T_n)$ for the stochastic finite-fault method decrease with increased distance. To explain this discrepancy, it is noted that in Chapter 2 and Liu et al. (2012) $\rho(\Delta, T_n)$ was calculated using Equation (3.3b). A re-analysis and subsequent scrutiny indicate that although the mean of the residuals for all stations equals zero, the residuals (from the regression analysis) decrease with increasing average shear-wave velocity in the upper most 30 m. This heterogeneity and the application of Equation (3.3b) instead of Equation (3.3a) are the source of the observed discrepancy between Liu et al. (2012) and this chapter. This emphasizes that the stochastic point-source method as well as the stochastic finite-fault method do not reproduce any spatial coherency structure of the records or the spatial correlation of the ground-motion measures.
3.3 Extension of the stochastic point-source method for simulation of multiple-station records

The proposed extension of the stochastic point-source method includes two parts: one is the integration of coherency function; the other is the inclusion of spatially correlated disturbance in the Fourier amplitude spectrum. The extended method includes the use of coherent Gaussian white noises as the input and an uncertain spatially correlated scaling factor for the Fourier amplitude spectrum. The application of the extended method should simulate ground motion records with predefined coherency and result in $\rho(\Delta, T_n)$ values that mimic those observed from historical seismic events.

3.3.1 Integration of coherency

It is proposed to integrate the coherency into the stochastic point-source method to reproduce realistic target coherency function and to use the modified algorithm to simulate multiple-station ground motion records. The integration can be done by replacing the white noise shown in Step a) (of the simulation procedure) with a vector of coherent white noises, and by considering that $y(M_0, R, f)$ in Step e) is replaced by $y_j(M_0, R_j, f)$, where $R_j$ represents the distance of the source to the $j$-th recording station. The dimension of the vector of white noises is $n_R$ if the generation of records for $n_R$ stations due to a single event is of interest. For each time series in the vector, the Steps b) to f) are repeated resulting in $n_R$ ground motion records with predefined coherency.

The smoothed coherency function of the ground motions for an earthquake event at two stations $j$ and $k$ is defined as:

$$\tilde{\gamma}(\Delta, f) = \frac{\tilde{S}_{jk}(f)}{\sqrt{\tilde{S}_{jj}(f)\tilde{S}_{kk}(f)}},$$  \hspace{1cm} (3.4)
where $S_{jj}(f)$ and $S_{kk}(f)$ are the smoothed power spectral density functions for the (randomly oriented horizontal) records at the $j$-th and $k$-th recording stations, respectively; and $S_{jk}(f)$ is the smoothed cross power spectral density function of the two records. By using the lagged coherency (or absolute value of coherency), $|\tilde{\gamma}(\Delta, f)|$, which is a measure of the degree to which the ground motion records at the two stations are related by means of a linear transfer function (Zerva, 2009), the coherency can be expressed as the multiplication of $|\tilde{\gamma}(\Delta, f)|$ and the phase spectrum (Zerva 2009). Since the phase spectrum does not affect the PGA and SAs on average, and one of the objectives of this chapter is focused on matching the observed spatial correlation of PGA and of SAs, only $|\tilde{\gamma}(\Delta, f)|$ is considered for the numerical analysis. The consideration of phase spectrum is discussed later.

One of the available functional forms for $|\tilde{\gamma}(\Delta, f)|$ is proposed by Harichandran and VanMarcke (1986):

$$|\tilde{\gamma}(\Delta, f)| = A \exp\left( -\frac{2\Delta}{\alpha \theta(f)} (1 - A + \alpha A) \right) + (1 - A) \exp\left( -\frac{2\Delta}{\theta(f)} (1 - A + \alpha A) \right)$$

(3.5)

where $\Delta$ in this equation is in meters and, $\theta(f) = k \left( 1 + \left( \frac{f}{f_0} \right)^B \right)^{-1/2}$, in which $f$ is the frequency (Hz); $A, \alpha, k, f_0$ and $B$ are model parameters. The coherency function is depicted in Figure 3.4 for $A = 0.636$, $\alpha = 0.0186$, $k = 31200$, $f_0 = 1.51$ and $B = 2.95$, which are suggested by Harichandran (1991) based on the analysis of the records from the SMART-1 Array. The figure shows the change in the coherency function for $\Delta$
greater than 3000 m is small, and the coherency value at \( \Delta = 5000 \) m is about 0.2. Since there is insufficient number of Chi-Chi records with short separation distance, verification of the model parameters cannot be carried out. However, it was observed that the value of \( |\mathcal{H}(\Delta, f)| \) at \( \Delta = 5000 \) m is close to the average that can be estimated from the Chi-Chi records shown in Figure 3.5, which are estimated using Hamming window with \( N = 20 \). The value of \( |\mathcal{H}(\Delta, f)| \) equal to about 0.2 for a large separation distance (up to 50 km) shown in the figure is likely due to noise. Based on these findings, for the numerical analysis, the coherency function shown in Equation (3.5) with the parameters suggested by Harichandran (1991) is used.

![Figure 3.4 Lagged coherency function as suggested by Harichandran (1991).](image)
To illustrate the impact of incorporating the coherency function in the stochastic point source method in the simulated records, consider the 389 stations (i.e., \( n_R = 389 \)) shown in Figure 3.1. The vector of \( n_R \) coherent white noises is sampled using the Auto-regressive Moving-average (ARMA) model (Samaras et al., 1985) for the adopted coherency function. For each time series in the vector, by using the stochastic point-source method and the adopted model parameters shown in Table 3.2, synthetic records...
are generated. Similar to Figures 3.3a to 3.3c, two sampled records (at station HWA007 and station HWA009) are presented in Figures 3.6a and 3.6b. The calculated \( |\gamma(\Delta, f)| \) of the two records for 100 simulation cycles are compared with the target coherency in Figure 3.6c, showing they reproduce the target coherency. The estimated \( |\gamma(\Delta, f)| \) for other three pairs of records with separation distances \( \Delta \) equal to 2.08 km, 4.90 km and 19.61 km are shown in Figure 3.7, indicating that the coherency of the simulated records does not match that of the target if the target coherency is less than about 0.2 – a problem attributed to the noise and post-processing. The estimation of \( |\gamma(\Delta, f)| \) for other pairs of records is also carried out. In all cases, the closeness of the estimated \( |\gamma(\Delta, f)| \) from the sampled records to the target coherency function for a given \( \Delta \) is similar to that shown in Figure 3.6c or 3.7.

To see the spatial correlation coefficient \( \rho(\Delta, T_n) \) for the PGA and SAs from the simulated spatially coherent records, again, the simulation is repeated 100 cycles and \( \rho(\Delta, T_n) \) is calculated using Equation (3.3a). The calculated values of \( \rho(\Delta, T_n) \) are shown in Figure 3.6d for SA at 1.0 s and are compared to the mean values of those obtained from the Chi-Chi records. The comparison indicates that the spatial correlation coefficients produced by the synthetic records are lower than those by the actual records. This indicates that the consideration of coherency alone is deficient in matching the spatial correlation coefficient estimated from the actual ground motion records. Similar analysis shows that the observation made to \( \rho(\Delta, T_n) \) for the SA at 1.0 s is equally applicable to those for the PGA and SAs at different natural vibration periods.
Figure 3.6 Sampled records and estimated lagged coherency by considering target coherency.
3.3.2 Inclusion of spatially correlated disturbance in the Fourier amplitude spectrum

As shown in Figure 3.2, \( r_{Aj} \) representing the ratio of the integral of the Fourier amplitude spectrum of the record to the integral of \( y(M_0, R, f) \) is uncertain. The normal probability paper plot and Kolmogorov-Smirnov test indicate that \( \ln(r_{Aj}) \) could be
assumed to be normally distributed. Moreover, by carrying out statistical analysis for \( \ln(r_{Aj}) \), the estimated spatial correlation coefficient, \( \rho_r(\Delta) \), is shown in Figure 3.8. The results presented in the figure indicate that the spatial correlation for \( \ln(r_{Aj}) \) is similar to that for the (logarithmic of) PGA and SAs of Chi-Chi records (Goda and Hong, 2008a; Liu et al., 2012), and \( \rho_r(\Delta) \) can be approximated by,

\[
\rho_r(\Delta) = \exp(-a\Delta^b),
\]

where \( a \) and \( b \) are the model parameters, and this functional form is the same one used by Goda and Hong (2008a) to represent the empirical spatial correlation coefficient of the (logarithmic of) PGA and SAs. Regression analysis suggests that \( a = 0.17 \) and \( b = 0.49 \) can be adopted for the Chi-Chi records.

![Figure 3.8 Spatial correlation coefficient samples of \( r_{Aj} \) and fitted curve.](image)

A simplistic way to incorporate the uncertainty in \( r_{Aj} \) in the stochastic point-source method is to replace \( y(M_0, R, f) \) by \( y_j(M_0, R_j, f) \) for the \( j \)-th station, where,
\[ y_j(M_0, R_j, f) = r_{Aj}E(M_0, f)P(R_j, f)G(f)I(f). \] (3.7)

In other words, the target Fourier amplitude spectrum in Step e) of the simulation procedure is replaced by \( y_j(M_0, R_j, f). \)

Before carrying out numerical analysis, it is instructive to see the potential influence of the uncertainty in \( r_{Aj} \) on the spatial correlation coefficients of PGA and SAs for the simulated records. Let \( Y_{Cj} \) denote the PGA or SA at the \( j \)-th station obtained from the simulated record by considering the coherency structure but ignoring (the uncertainty in) \( r_{Aj} \), and similarly let \( Y_{CUj} \) denote the same quantity but considering both the coherency structure and uncertainty in \( r_{Aj} \). \( Y_{CUj} \) is related to \( Y_{Cj} \) by \( \ln(Y_{CUj}) = \ln(r_{Aj}) + \ln(Y_{Cj}) \). It is worth noting the similarity on how \( \ln(r_{Aj}) \) and the intraevent residual in the GMPEs (Joyner and Boore, 1993; Hong et al., 2009) affect the spatial correlation and the prediction of ground-motion measures.

Since the logarithmic of the PGA or SAs are commonly considered to be zero mean normally distributed random variable, \( \ln(Y_{Cj}) \) can be considered to be normally distributed. Based on this and that \( \ln(r_{Aj}) \) is also zero mean normally distributed, it can be shown that the spatial correlation coefficient between \( \ln(Y_{CUj}) \) and \( \ln(Y_{CUk}) \), \( \rho_{YCU}(\Delta, T_n) \), is given by,

\[ \rho_{YCU}(\Delta, T_n) = \rho_r(\Delta) \frac{\sigma_{\ln r}^2}{\sigma_{\ln r}^2 + \sigma_{\ln YC}^2} + \rho_{YC}(\Delta, T_n) \frac{\sigma_{\ln YC}^2}{\sigma_{\ln r}^2 + \sigma_{\ln YC}^2}, \] (3.8)

where \( \sigma_{\ln r} \) denotes the standard deviation of \( \ln(r_{Aj}) \) and \( \sigma_{\ln YC} \) represents the standard deviation of \( \ln(Y_{Cj}) \). Equation (3.8) indicates that \( \rho_{YCU}(\Delta, T_n) \) is the weighted value of
\( \rho_r(\Delta) \) and \( \rho_{YC}(\Delta, T_n) \). If the magnitude of the uncertainty in \( \ln(r_{Aj}) \) (i.e., \( \sigma_{inr} \)) is much smaller than that in \( \ln(Y_{Cj}) \) (i.e., \( \sigma_{inYC} \)), \( \rho_{YCU}(\Delta, T_n) \) approaches \( \rho_{YC}(\Delta, T_n) \). However, \( \rho_{YCU}(\Delta, T_n) \) tends to \( \rho_r(\Delta) \) if \( \sigma_{inr} \) is much greater than \( \sigma_{inYC} \). Moreover, if \( \rho_{YC}(\Delta, T_n) \) tends to zero (see Figure 3.6d), \( \rho_{YCU}(\Delta, T_n) \) tends to \( \rho_r(\Delta) \times \left( \frac{\sigma_{inr}^2}{\sigma_{inr}^2 + \sigma_{inYC}^2} \right) \) and is bounded by \( \rho_r(\Delta) \).

To illustrate the behaviour of the simulated records, the analyses that were carried out for the results shown in Figure 3.6 are repeated but including the uncertainty in \( r_{Aj} \) as discussed above. The obtained results are shown in Figure 3.9. The results show that the simulated ground motion records by considering the uncertainty in \( r_{Aj} \) preserve the target coherency structure. Also, \( \rho_{YCU}(\Delta, T_n) \) of simulated records mimics that of the Chi-Chi records, indicating that the inclusion of spatially correlated disturbance in the Fourier amplitude spectrum improves the spatial correlation characteristics of the simulated ground-motion measures significantly.

Furthermore, to see the attenuation of the simulated ground motion records versus that of Chi-Chi records, SA at 1.0 s are compared in Figure 3.10 as an illustration. The figure shows that the dispersion of the simulated amplitudes is similar to that of the actual records; the distinct trends in the simulated ground-motion measures (PGA or SAs), representing rock and soil sites (as shown in Chapter 2 and Liu et al. (2012) are also observed for using the stochastic point-source method), are eliminated.
Figure 3.9 Sampled records, lagged coherency and spatial correlation coefficients by considering target coherency and spatially correlated disturbance in the Fourier amplitude spectrum.
Finally, it is noted that the distance from the source to the recording stations and the earthquake magnitude do not alter $\rho_{YCU}(\Delta, T_n)$ of the simulated records as the extension does not consider that the coherency function and $r_{Aj}$ depend on these parameters.

![Figure 3.10](image.png)

Figure 3.10 Comparison of attenuation of SA of Chi-Chi records and synthetic records.

3.4 Potential further enhancement

The extension described above by considering $|\tilde{g}(\Delta, f)|$ and $r_{Aj}$ can also be easily implemented in other record simulation techniques to generate correlated ground motion records with prescribed spatial correlation and coherency. As shown through numerical analysis, the proposed extension to the stochastic point-source method can achieve the objectives of matching the target coherency structure of the records and the spatial correlation of PGA and of SAs. However, there are still a few limitations that deserve discussion. First, the phase spectrum is not considered in the numerical analysis; second,
the interevent correlation that could influence the reliability of structures during their service periods is not addressed; and third, how to control $p_{YCU} (\Delta, T_n)$ for different $T_n$ values is not entirely clear. At least, the first two limitations can be dealt with rather easily as outlined below.

The phase spectrum incorporates two effects: the wave propagation across the recording stations (wave-passage effect) and random phase variability at each station (site-response effect) (Der Kiureghian 1996). The phase spectrum could be important for structures with a large span such as long span bridges (Nazmy and Abdel-Ghaffar, 1992). A set of coherent band-limited white noises could be simulated for an adopted coherency model, including the phase spectrum (Harichandran and VanMarcke 1986; Hao et al. 1989; Der Kiureghian 1996). The set of white noises is then used as input to generate the ground motion records.

Left out in the extension is the consideration of the interevent correlation (i.e., event to event correlation at the same site). Although numerical investigation is beyond the scope of this chapter, a short discussion is in order. The inclusion of features to reproduce the interevent correlation can be done in a similar fashion as is done for the intraevent correlation if there are sufficient historical records from many earthquakes for a particular seismic source zone. More specifically, an additional uncertain multiplication factor $r_{li}$ could be considered for the Fourier amplitude spectrum such that the Fourier amplitude spectrum for the $i$-th event and at the $j$-th recording station $y_{ij} \left( M_{0i}, R_j, f \right)$ equal to $r_{li} r_{lj} E(M_0, f) P(R_j, f) G(f) I(f)$. $r_{li}$ varies from event to event, $r_{li}$ and $r_{lk}$ are correlated, and their statistical characteristics need to be assigned based on the analysis of historical
records. The statistical assessment of \( \ln(r_{ii}) \) can be carried out based on the logarithmic of the ratio of the Fourier spectrum to \( y_i(M_0, R, f) \) in a similar manner as that for the interevent residuals obtained in the regression analysis for the GMPEs (Joyner and Boore, 1993; Hong et al., 2009). In fact, in cases where there is insufficient available data to characterize \( \ln(r_{ii}) \), the statistical characteristics of the interevent residual for the GMPEs may be considered for the \( \ln(r_{th}) \). This suggestion is based on that \( \ln(r_{th}) \) and the interevent residual for the GMPEs affect the interevent correlation of the ground-motion measures in a similar way.

Other possible modifications to the stochastic point-source method to cope with interevent correlation could be made by considering that the model parameters in \( y(M_0, R, f) \) are interevent correlated. The statistical assessment of these interevent correlated model parameters is likely to be much more involved than that of \( r_{th} \).

### 3.5 Conclusions

An extension to the stochastic point-source method is proposed for generating multiple-station ground motion records. The extension incorporates a target spatial coherency structure and the spatially correlated uncertainties in the Fourier amplitude spectrum. This extension can also be easily implemented in other ground motion simulation techniques to generate correlated ground motion records with prescribed spatial correlation and coherency. The application of the extended method is illustrated by numerical examples focused on the Chi-Chi earthquake, reproducing desired target spatial coherency and spatial correlation of ground-motion measures.
A further enhancement by incorporating the phase spectrum and the interevent correlation is also outlined. The proposed extension facilitates the application of the stochastic point-source method for seismic analysis of structures with multiple supports and for seismic risk assessment of portfolios of structures distributed in a region.

3.6 Data and Resources

The ground-motion records of the Chi-Chi earthquake were obtained from the PEER-NGA database at http://peer.berkeley.edu/nga/ (last accessed August 2010). Stochastic point-source simulation program was obtained from David M. Boore’s personal website at http://daveboore.com/software_online.html (last accessed March 2011).

References


CHAPTER 4. APPLICATION OF SPATIALLY CORRELATED AND COHERENT RECORDS OF SCENARIO EVENT TO ESTIMATE SEISMIC LOSS OF A PORTFOLIO OF BUILDINGS

4.1 Introduction

For seismic loss assessment of a portfolio of buildings, strong ground motion records that are compatible with the study region and the considered earthquake scenarios are needed. These records may not be available in the current ground motion database. To ameliorate this, the estimation can be carried out based on the HAZUS-Earthquake approach (Whitman et al. 1997; FEMA and NIBS 2003), which uses the ground-motion measures, the capacity spectrum method and fragility curve for each building in a scenario earthquake. The aggregate seismic loss of the portfolio of buildings equals the sum of the seismic loss of each individual building; the probability distribution of the annual seismic loss of the buildings can also be estimated by repeating this analysis for a series of scenario events and considering the probabilistic model of the earthquake occurrence. One of the deficiencies of the HAZUS-Earthquake approach is the lack of the consideration of correlated seismic excitations. To overcome this, Goda and Hong (2008b) presented a simulation-based framework using ground motion prediction equation (GMPE) with spatially correlated interevent and intraevent residuals and empirical ductility demand rules. This facilitates the assessment of the statistics of seismic loss of the portfolio of buildings that are affected by multiple scenario events and multiple seismic source zones with different characteristics of the ground-motion measures. Their results showed that the spatial correlation of ground-motion measures
affect the probabilistic characteristics of the seismic loss of a portfolio of buildings distributed in a region.

The framework given in Goda and Hong (2008b) is computationally efficient and incorporates the spatial correlation of the ground-motion measures. However, it cannot directly take into account the time-frequency characteristics of the ground motions of scenario events. This is not important if the use of the adopted GMPEs and the empirical ductility demand rules, which are compatible with the considered scenario event and seismic-tectonic setting, can provide sufficient accurate estimate of the seismic responses of the nonlinear inelastic structural systems. Unfortunately, this cannot be always ensured since some of the buildings could be sensitive to multiple vibration modes and/or P-Δ effects (Esteva 1992; MacRae 1994; Gupta and Krawinkler, 2000). In such a case and for a more refined analysis, it would be ideal that the ground motion records at the sites of spatially distributed buildings can be directly used to estimate the seismic loss for scenario events. These needed ground motion records, that are spatially correlated and coherent for scenario events, are unlikely to be available in the current historical ground motion database. The coherency of two ground motion records at two sites for the same seismic event is a function of the frequency and inter-station distance; it is not affected by scaling the ground motion; it is estimated from the power spectral density functions of the records (Abrahamson et al. 1991; Zerva 2009). The spatial correlation is used to measure the correlation of the peak ground acceleration (PGA) or spectral accelerations (SAs) (i.e., amplitude of ground motion) at two sites. The spatial coherency models were proposed by Harichandran and VanMarcke (1986), Hao et al. (1989) and Der Kiureghian (1996) based on the analysis results of actual records or random vibration theory.
Empirical spatial correlation models of PGA and SAs were proposed for seismic events in California, Taiwan and Japan (Boore et al., 2003; Wang and Takada, 2005; Goda and Hong, 2008a; Hong et al., 2009; Jayaram and Baker, 2009; Goda and Atkinson, 2010; Sokolov et al., 2010).

The spatial correlation of the Fourier amplitude spectrum (FAS) was considered by Chapter 3 and Liu and Hong (2013). Their analysis was focused on Chi-Chi earthquake, and showed that the spatial correlation of the FAS is similar to those obtained for the SAs. The assessment of the spatial correlated disturbance of the FAS was based on the integral of the FAS for selected frequency range. They proposed to incorporate both the coherency of the ground motion records and the spatial correlation of the FAS into the stochastic point-source model (Boore 2003, 2009) to generate spatially coherent and correlated (synthetic) ground motion records. This extended stochastic point-source model was used successfully to generate ground motion records, reproducing the desired target coherency and spatial correlation of ground-motion measures. It was also noted that by incorporating the coherency alone in the stochastic finite-fault model (Beresnev and Atkinson, 1997; Motazedian and Atkinson, 2005) or the stochastic point-source model, the generated synthetic ground motion records cannot reproduce the spatial correlation of ground-motion measures (Liu et al. 2012 and Liu and Hong 2013).

The main objectives of this chapter are to extend the stochastic finite-fault model for generating synthetic ground motion records that match target spatial coherency and correlation, and to investigate the influence of spatially correlated and coherent ground motions on the seismic loss of a portfolio of buildings. Similar to the use of the stochastic-point source model, the use of the stochastic finite-fault model is advantageous
because it can be used to generate synthetic ground motion records that are directly related to seismic scenario events (e.g., earthquake magnitude and source-to-site distance) (Boore, 2003; Beresnev and Atkinson, 1997; Motazedian and Atkinson, 2005). The model takes into account some important parameters from large earthquakes, such as the fault geometry and rupture propagation. Records generated using the model can produce peak ductility demand that are similar to the actual records (Atkinson and Goda, 2010). For the proposed extension, the steps used to extend the stochastic point-source model (Chapter 3; Liu and Hong 2013) are followed, except that in this case the stochastic finite-fault model is first used to define the reference FAS and time modulating function at spatially distributed sites. These are explained in detail in next section. The simulated records are used to investigate the influence of the spatial correlation and coherency on the statistics of the seismic loss of a hypothetical building stock in downtown Vancouver for a scenario event occurring in the Cascadia subduction zone.

4.2 Extension of the stochastic finite-fault model to generate spatially correlated and coherent ground motions

The stochastic finite-fault model (Beresnev and Atkinson, 1997; Motazedian and Atkinson, 2005; Atkinson et al., 2009) is used to generate ground motions by summing the contributions from discretized sub-faults with each sub-fault modeled as a point-source. The model, similar to the stochastic-point source model, does not have built-in features to reproduce the spatial correlation characteristics observed from actual records (Liu et al., 2012; Chapter 3; Liu and Hong, 2013). To overcome this, the stochastic finite-fault model is extended by incorporating the target coherency and spatial correlation of the FAS. This is done by following the steps used to extend the stochastic
point-source model (Chapter 3; Liu and Hong 2013) but with two important distinctions: the stochastic finite-fault model is first used to define the reference FAS and time modulating function at spatially distributed $n_R$ sites of interest. There are two stages in using this extended model to generate the synthetic records. In Stage I, $n_G$ records at each of the sites are generated using the stochastic finite-fault model for a considered scenario event with seismic moment $M_0$. Using the simulated $n_G$ records at the $j$-th site, the average time window profile (i.e., time modulating function) is estimated using Hilbert transform (Hao, 1989) and the average FAS (i.e., reference FAS) is calculated using Fourier transform, $y_j(M_0, R_j, f)$, where $f$ is the frequency in Hz and $R_j$ is the distance from the $j$-th site to earthquake source (e.g., horizontal distance to the surface projection of the fault plane, Joyner-Boore distance). Note that in case of the stochastic point-source model, both the reference FAS and the time modulating function are explicitly given.

In Stage II, the steps illustrated in Figure 4.1 are carried out and described below:

(a) Generate the white noises with zero mean for each considered site but incorporating the target spatial coherency which is discussed in more detail below;

(b) Apply the corresponding time modulating function to each white noise at each site;

(c) Calculate the FAS of each time modulated record;

(d) Normalize the estimated FAS by its square-root of the mean squared amplitude spectrum;
Figure 4.1 Illustration of simulating spatially correlated and coherent record using the extended finite-fault model (in Stage II).
(e) Multiply the normalized spectrum by its corresponding reference FAS (determined in Stage I) and by the spatial correlated scaling factor (which will be explained below as well); and,

(f) Apply the inverse Fourier transform to the spectra obtained in step (e) to compute the synthetic ground motion records.

For Step (a), a target coherency function needs to be considered. According to Harichandran and VanMarcke (1986), the coherency function for two sites with separation distance \( \Delta \) (in kilometers), \( \gamma(\Delta, f) \), can be expressed as,

\[
\gamma(\Delta, f) = |\gamma(\Delta, f)| \exp\left(-i2\pi f \frac{\Delta_p}{v_{ap}}\right),
\]  

(4.1)

where \( \Delta_p \) is the projection of \( \Delta \) in the direction of wave propagation; \( v_{ap} \) represents the apparent velocity; the term \(-2\pi f \Delta_p / v_{ap}\) is known as phase angle of the wave passage effect (Der Kiureghian 1996); the smoothed lagged coherency \( |\bar{\gamma}(\Delta, f)| \) is given by (Harichandran and Vanmarcke, 1986),

\[
|\bar{\gamma}(\Delta, f)| = A \exp\left(-\frac{2000\Delta}{f_0 \theta(f)}(1-A + \alpha_0 A)\right) + (1-A) \exp\left(-\frac{2000\Delta}{\theta(f)}(1-A + \alpha_0 A)\right),
\]  

(4.2)

in which \( \theta(f) = k \left(1 + \left(\frac{f}{f_0}\right)^B\right)^{-1/2} \), and \( A, \alpha_0, k, f_0 \) and \( B \) are model parameters. The model parameter selection was given in Harichandran (1991).
Given the coherency function, spatially coherent band-limited white noises can be simulated using spectral representation method (Shinozuka and Jan, 1972; Hao et al., 1989). This leads to the white noises at the \( j \)-th site given by,

\[
W_j(t) = \sum_{k=1}^{j} A_{jk}(f_l) \cos\left(2\pi f_l \left(t - \tau_{jk}\right) + \phi_{kl}\right), j = 1, \ldots, n_R
\]  

(4.3)

where \( \Delta_{jk} \) is the separation distance between \( j \)-th and \( k \)-th sites, \( j, k = 1, \ldots, n_R, f_l = l f_\Delta \), \( f_\Delta = f_N/N \), \( f_N \) represents an upper cutoff frequency, the random phase angle \( \phi_{kl} \) is independent uniformly distributed between 0 to \( 2\pi \), \( \tau_{jk} = \Delta_{jk} v_{ap} \) is the time lag between site \( j \) and site \( k \), and the amplitudes \( A_{jk}(f_l) \) is given by

\[
A_{jk}(f_l) = \sqrt{8\pi S_0 f_\Delta} \left| l_{jk}(f_l) \right|.
\]  

(4.4)

For the simulation, \( S_0 \) in this equation is taken equal to one because a normalized of the FAS for the simulated noises is carried out in Step d), and the simulated records in Step f) are independent of the value of \( S_0 \). \( l_{jk}(f) \) is the element of a lower triangular matrix \( L(f) \) that is obtained from the Cholesky decomposition of the lagged coherency matrix, \( \Gamma(f) \), whose \((j, k)\) element equals \( \tilde{\gamma}(\Delta_{jk}, f) \).

To include the spatial correlation in the ground motions (see Step (e)), the reference FAS at the \( j \)-th site is considered to be \( r_{Aj} \chi_f (M_0, R_j, f) \), where \( \ln(r_{Aj}), j = 1, \ldots, n_R, \) can be modeled as joint normally distributed random variables, with the zero mean, and standard deviation of 0.694 and, the (intraevent) correlation coefficient, \( \rho_r(\Delta) \), defined by (Chapter 3; Liu and Hong 2013),
\[ \rho_r(\Delta) = \exp(-a\Delta^b), \tag{4.5} \]

where \( a \) and \( b \) are model parameters that are estimated empirically. Note that the selection of standard deviation of \( \ln(r_{Aj}) \) is based on the similarity of \( \ln(r_{Aj}) \) to residuals of GMPE (Chapter 3; Liu and Hong, 2013; Hong and Goda, 2007).

Records simulated based on the above procedure as well as their use for aggregate seismic loss estimation for a portfolio of hypothetical buildings located in Vancouver are presented in the following sections. For the simulation, the parameters that are summarized in Table 4.1 are adopted. The parameters for the coherency model are selected based on Harichandran (1991). Estimated \( a \) and \( b \) based on SA for California earthquakes and on SA and the integral of the FAS for Chi-Chi earthquake (Goda and Hong, 2008a; Liu and Hong, 2013) are used as a guide to select the model parameters shown in Equation (4.5) for the sensitivity analysis.

### 4.2.1 Earthquake scenario and simulation parameters

A scenario earthquake described in Atkinson and Macias (2009) is considered. The event has a moment magnitude \( M_{8.5} \) and a rupture plane of \( 380 \times 90 \text{ km}^2 \), placed symmetrically about a perpendicular line from the Juan de Fuca trench to the city. The top corner of the fault plane is located at \([47.1^\circ \text{N}, 124.5^\circ \text{W}]\), 10 km deep from the ground surface. The strike and dip angle are equal to \( 310^\circ \) and \( 10^\circ \), respectively. The parameters of the stochastic finite-fault model for the scenario event are shown in Tables 4.1 and 4.2. The consideration of this scenario event for seismic risk assessment is justified since such an event can have a relatively significant impact on the seismic risk even though the
Cascadia subduction events do not affect the Uniform Hazard Spectra (UHS) values significantly (Hong and Goda 2006).

Table 4.1 Selected modeling parameters for generating records.

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Parameter value and notes</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged coherency model</td>
<td>([A, \alpha_0, k, f_0, B] = [0.636, \ 0.0186, \ 31200, \ 1.51, \ 2.95])[v_{ap} = 2000 \text{ m/s}]</td>
<td>Harichandran (1991)</td>
</tr>
</tbody>
</table>
| Spatial correlation model              | \([a, b] = [0.66, 0.43]\)

\(\ln(r_{Al})\) is normally distributed with zero mean and standard deviation equal to 0.694 | Goda and Hong (2008a); Liu and Hong (2013) |
| Moment magnitude                       | \(M_{8.5}\)                                                      | Atkinson and Macias (2009)             |
| Orientation (º) [strike, dip]          | \([310, 10]\)                                                    |                                       |
| Dimensions (km) [strike×dip]           | \([380×90]\)                                                    |                                       |
| Depth (km) [top of the plane]          | 10                                                               |                                       |
| Location (top; º) [latitude, longitude]| \([47.1, -124.5]\)                                               |                                       |
| Number of sub-faults [strike, dip]     | \([38, 9]\)                                                    |                                       |
| Pulsing area (%)                       | 50                                                              |                                       |
| Slip distribution and hypocenter location\(^1\) | Random                                                        |                                       |
| Stress parameter \(\Delta \sigma\) (bar)\(^2\) | 230                                                             |                                       |
| Anelastic attenuation \(Q = Q_0 f^n\) | \(180^{0.45}\)                                                  |                                       |
| Geometrical spreading                  | \(1/R\) for \(R \leq 40\) km; \(1/R^{0.5}\) for \(R > 40\) km |                                       |
| Duration term (sec/km) [\(d\)]        | 0.10                                                            |                                       |
| \(\kappa\) factor (sec)\(^3\)         | 0.005                                                           |                                       |
| Crustal shear wave velocity (km/sec)   | 3.8                                                             |                                       |
| Crustal density (g/cm\(^3\))           | 2.8                                                             |                                       |

Note: 1) The hypocenter is randomly placed on the fault plane and the slip distribution is assigned in EXSIM; 2) \(\Delta \sigma = 90\) (bar), and \(\kappa = 0.03\) (sec) together with older version of the stochastic finite-fault model were by Atkinson and Macias (2009).

The average response spectrum at \((49.25ºN, 123.13ºW)\) was estimated by Atkinson and Macias (2009) for the scenario event and the model parameters shown in Tables 4.1 and 4.2. This average response spectrum is depicted in Figure 4.2. The version of the stochastic finite-fault program used by Atkinson and Macias (2009) differs from the
newer version of the stochastic finite-fault program (EXSIM) (see Boore’s personal website http://daveboore.com/software_online.html, last accessed March 2011), that includes several changes (Boore, 2009; Atkinson et al., 2009). Since the use of the model parameters shown in Tables 4.1 and 4.2 in the EXSIM does not lead to the same average response spectrum, a higher value of the stress parameter Δσ and a lower value of nonlinear dispersion factor κ (see Table 4.1) are needed for the response spectrum to be comparable to that given in Figure 4.2. The estimated average response spectrum with 10 replications by using the EXSIM and parameters shown in Tables 4.1 and 4.2 is compared with that given by Atkinson and Macias (2009) in Figure 4.2, indicating that there is adequate match.

Table 4.2 Soil amplification factor for Vancouver (from Atkinson and Macias 2009).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>1.6482</td>
<td>0.40</td>
<td>2.8427</td>
<td>1.58</td>
<td>2.9521</td>
<td>6.31</td>
<td>1.9631</td>
</tr>
<tr>
<td>0.13</td>
<td>1.8377</td>
<td>0.50</td>
<td>2.9649</td>
<td>2.00</td>
<td>2.8665</td>
<td>7.94</td>
<td>1.6872</td>
</tr>
<tr>
<td>0.16</td>
<td>2.0672</td>
<td>0.63</td>
<td>3.0152</td>
<td>2.51</td>
<td>2.745</td>
<td>10.00</td>
<td>1.4313</td>
</tr>
<tr>
<td>0.20</td>
<td>2.2882</td>
<td>0.79</td>
<td>3.0493</td>
<td>3.16</td>
<td>2.5946</td>
<td>12.59</td>
<td>1.1291</td>
</tr>
<tr>
<td>0.25</td>
<td>2.4976</td>
<td>1.00</td>
<td>3.0411</td>
<td>3.98</td>
<td>2.4135</td>
<td>15.85</td>
<td>0.8976</td>
</tr>
<tr>
<td>0.32</td>
<td>2.6893</td>
<td>1.26</td>
<td>3.0157</td>
<td>5.01</td>
<td>2.1958</td>
<td>19.95</td>
<td>0.6219</td>
</tr>
</tbody>
</table>

Note: These values are calculated using the quarter-wavelength approach based on the shear-wave velocity profiles for generic-site condition (top of the Pleistocene deposits) in Vancouver region.

It is noted that the soil amplification factors shown in Table 4.2 are calculated using the quarter-wavelength approach considering the shear-wave velocity profiles for generic-site condition (top of the Pleistocene deposits) in Vancouver region, defined by the average shear-wave velocity in the uppermost 30 m, \( V_{S30} \), equal to 414 m/sec. The local site condition in downtown Vancouver is considered to be NEHRP (National Earthquake Hazards Reduction Program) site class C (Cassidy and Rogers, 2004), where
\( V_{S30} \) ranges between 360 and 760 m/s (NRCC 2005). Therefore, it is assumed that the use of \( V_{S30} = 414 \) m/sec for the portfolio of hypothetical buildings located in downtown Vancouver is adequate.

Figure 4.2 Comparison of the average response spectrum given by Atkinson and Macias (2009) and EXSIM with parameters shown in Tables 4.1 and 4.2.

4.2.2 Spatial coherency and correlation of simulated ground motion records

Consider the 100 sites of the portfolio of hypothetical buildings distributed in downtown Vancouver shown in Figure 4.3. The simulation of the sites and characteristics of the buildings is to be discussed shortly. Using the procedure described in the previous sections, and the model parameters shown in Tables 4.1 and 4.2, ground motion records for the sites of the portfolio of hypothetical buildings shown in Figure 4.3 are simulated for the described scenario event. The simulation is carried out for 100 replications. An illustration of the simulated ground motion records for the scenario
earthquake is presented in Figure 4.4 for eight selected sites within the area depicted in Figure 4.3.

For the simulated records, the smoothed coherency between a pair of records is estimated based on definition,

$$
\tilde{\gamma}(\Delta, f) = \frac{\tilde{S}_{jk}(f)}{\sqrt{\tilde{S}_{jj}(f)\tilde{S}_{kk}(f)}},
$$

(4.6)

where $\tilde{S}_{jj}(f)$ and $\tilde{S}_{kk}(f)$ are the smoothed power spectral density functions for the records at the $j$-th and $k$-th sites, respectively; and $\tilde{S}_{jk}(f)$ is the smoothed cross power spectral density function of the two records. The adopted smoothing window is Hamming window, $W(n)$, given by (Zerva 2009),

$$
W(n) = 0.54 - 0.46\cos\left(\frac{\pi(n + N)}{N}\right) \quad n = -N, \ldots, N,
$$

(4.7)
where $N$ is a parameter which is taken equal to 80 and the window width is given by $2N + 1$. The obtained lagged coherency and its mean and +/- standard deviation for the white noises and the synthetic ground motion records are illustrated in Figure 4.5 for a pair of sites ($\Delta = 0.48$ km) of 100 replications.

Figure 4.4 Illustration of simulated spatially correlated and coherent records for eight selected sites ((x, y) in each plot refers to the coordinate system shown in Figure 4.3).

Figure 4.5 shows that the lagged coherency of the simulated white noise and of the corresponding synthetic records are similar, and that the lagged coherency from the simulated records follows the trend of the target lagged coherency. Figure 4.6 shows the
unwrapped phase spectra of the estimated coherency for the corresponding records and sites. They are in good agreement with the target phase spectrum. Further analysis results indicate that these observations are equally applicable to other pairs of the sites with different separation distances.

Figure 4.5 Comparison of lagged coherency to the target: (a) estimated using simulated ground motion records; (b) estimated using simulated white noises.

Figure 4.6 Comparison of unwrapped phase spectra to the target: (a) estimated using simulated ground motion records; (b) estimated using simulated white noises.
Also, the PGA or SAs for each simulated record and each replica are calculated, these values are used to estimate the spatial correlation coefficients of the ground-motion measures \( \rho(\Delta, T_n) \) using,

\[
\rho(\Delta, T_n) = E\left(\ln(Y_j(T_n)) - \mu_{\ln Y_j(T_n)}\right)\ln(Y_k(T_n)) - \mu_{\ln Y_k(T_n)}\right)\right) / \sigma_{\ln Y_j(T_n)} \sigma_{\ln Y_k(T_n)},
\]

where \( Y_j(T_n) \) is the PGA or SA at \( j \)-th station, \( T_n \) is the natural vibration period of single-degree-of-freedom (SDOF) system, and \( \mu_{\ln Y_j(T_n)} \) and \( \sigma_{\ln Y_j(T_n)} \) are the mean and standard deviation of \( \ln Y_j(T_n) \). The calculated \( \rho(\Delta, T_n) \) based on 100 simulation cycles is depicted in Figure 4.7. The average of the spatial correlation coefficient estimated from the simulated records is close to the target value. This observation is consistent with that reported for the extended stochastic point-source model (Chapter 3; Liu and Hong 2013).

Figure 4.7 Spatial correlation coefficient of the ground-motion measures of the simulated records: (a) PGA; (b) SA at 0.3 s.
4.3 Assessment of seismic loss of a group of buildings

4.3.1 Building inventory

For the seismic loss assessment, the set of 100 hypothetical buildings, consists of 18 building types with different structural systems and occupancies (40 buildings for residential use and 60 buildings for commercial use), is sampled based on the statistical information describing the existing building stocks in downtown Vancouver (Munich Re. 1992, Onur 2001). The detailed information of each building type is shown in Table 4.3, which was used by Goda and Hong (2008) for their study but considering 2000 buildings randomly distributed over the same area. The set of 100 hypothetical buildings is considered to be randomly distributed over a square area of 2.5 km by 2.5 km centered at (49.2° N, 123.2° W), which contains 4000 property lots, each with an area of 25×50 m². The sites for the 100 hypothetical buildings are simulated and illustrated in Figure 4.3.
4.3.2 Seismic loss estimation

For the estimation of the building damage and seismic loss caused by the strong ground motion induced by the scenario earthquake, first, it is assumed that each building can be modeled as a (generalized) SDOF system without considering the P-Δ effect. For the building inventory shown in Figure 3, consider the j-th building with yield displacement capacity \( D_{yj} \) and displacement ductility capacity \( \mu_{Rj} \). If the SA induced by the earthquake is \( S_{Aj}(T_n, \zeta) \), where \( \zeta \) is the damping ratio, and the corresponding displacement demand \( S_{Dj}(T_n, \zeta) = S_{Aj}(T_n, \zeta)/(2\pi/T_n)^2 \) is larger than \( D_{yj} \), the structural damage is measured using the damage factor, \( \delta_j \), defined as

\[
\delta_j = \max\left( \min\left( \frac{\mu_j - 1}{\mu_{Rj} - 1} \right), 1 \right) 0, \quad (4.9)
\]

where \( \mu_j \) is the displacement ductility demand (maximum nonlinear inelastic displacement normalized by yield displacement) of the j-th structure due to the scenario earthquake. If seismic demand is less than the yield displacement capacity, \( \delta_j \) equals zero (no damage); while if seismic demand exceeds the ultimate capacity, \( \delta_j \) equals one (complete damage). \( \delta_j \) ranges between zero and one for partial damage.
Table 4.3 Hypothetical building inventory for downtown Vancouver (Goda and Hong, 2008, Munich Re. (1992), Onur (2001), and FEMA and NIBS (2003).

<table>
<thead>
<tr>
<th>(I_{BT})</th>
<th># of bldgs.</th>
<th># of stories</th>
<th>Size (m)</th>
<th>Structural &amp; occupancy types</th>
<th>(L_{BL}(1), L_{CO}(1), L_{BI}(1)) (CAD/ft²)</th>
<th>(\beta_{BL}, \beta_{CO}, \beta_{BI})</th>
<th>(T_n) (s)</th>
<th>Mean of (R_N)</th>
<th>Mean of (\mu_R)</th>
<th>Target (C_S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
<td>10×12</td>
<td>W1-RES1</td>
<td>87.6, 21.9, 19.9</td>
<td>0.75, 0.68, 0.57</td>
<td>0.4</td>
<td>2</td>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1</td>
<td>8×12</td>
<td>W1-RES1</td>
<td>87.6, 21.9, 19.9</td>
<td>0.75, 0.68, 0.57</td>
<td>0.4</td>
<td>2</td>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>2</td>
<td>15×30</td>
<td>W2-RES3</td>
<td>111.4, 27.9, 26.3</td>
<td>0.81, 0.68, 0.62</td>
<td>0.4</td>
<td>2</td>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>2</td>
<td>15×30</td>
<td>W2-COM1</td>
<td>47.8, 26.5, 23.9</td>
<td>0.81, 0.68, 0.43</td>
<td>0.4</td>
<td>2</td>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>5</td>
<td>18×36</td>
<td>S4M-RES3</td>
<td>111.4, 27.9, 26.3</td>
<td>0.69, 0.58, 0.53</td>
<td>0.7</td>
<td>2.25</td>
<td>4</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>5</td>
<td>18×36</td>
<td>S4M-COM4</td>
<td>103.5, 51.7, 163.9</td>
<td>0.70, 0.58, 0.57</td>
<td>0.7</td>
<td>2.25</td>
<td>4</td>
<td>0.1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>13</td>
<td>18×36</td>
<td>S4H-RES3</td>
<td>111.4, 27.9, 26.3</td>
<td>0.69, 0.59, 0.53</td>
<td>1.4</td>
<td>2.25</td>
<td>3</td>
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</tr>
<tr>
<td>8</td>
<td>1</td>
<td>13</td>
<td>18×36</td>
<td>S4H-COM4</td>
<td>103.5, 51.7, 163.9</td>
<td>0.70, 0.59, 0.57</td>
<td>1.4</td>
<td>2.25</td>
<td>3</td>
<td>0.075</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>2</td>
<td>15×30</td>
<td>C2L-RES3</td>
<td>111.4, 27.9, 26.3</td>
<td>0.76, 0.64, 0.58</td>
<td>0.4</td>
<td>2.5</td>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>2</td>
<td>15×30</td>
<td>C2L-COM1</td>
<td>47.8, 26.5, 23.9</td>
<td>0.75, 0.64, 0.41</td>
<td>0.4</td>
<td>2.5</td>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td>5</td>
<td>18×36</td>
<td>C2M-RES3</td>
<td>111.4, 27.9, 26.3</td>
<td>0.75, 0.64, 0.58</td>
<td>0.6</td>
<td>2.5</td>
<td>5</td>
<td>0.12</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>5</td>
<td>18×36</td>
<td>C2M-COM4</td>
<td>103.5, 51.7, 163.9</td>
<td>0.77, 0.64, 0.62</td>
<td>0.6</td>
<td>2.5</td>
<td>5</td>
<td>0.12</td>
</tr>
<tr>
<td>13</td>
<td>6</td>
<td>15</td>
<td>18×36</td>
<td>C2H-RES3</td>
<td>111.4, 27.9, 26.3</td>
<td>0.76, 0.64, 0.58</td>
<td>1.65</td>
<td>3</td>
<td>3</td>
<td>0.05</td>
</tr>
<tr>
<td>14</td>
<td>13</td>
<td>15</td>
<td>18×36</td>
<td>C2H-COM4</td>
<td>103.5, 51.7, 163.9</td>
<td>0.77, 0.64, 0.62</td>
<td>1.65</td>
<td>3</td>
<td>3</td>
<td>0.05</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>2</td>
<td>15×30</td>
<td>URMLR-RES3</td>
<td>111.4, 27.9, 26.3</td>
<td>0.81, 0.69, 0.62</td>
<td>0.35</td>
<td>2</td>
<td>5</td>
<td>0.08</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td>2</td>
<td>15×30</td>
<td>URMLR-COM1</td>
<td>47.8, 26.5, 23.9</td>
<td>0.81, 0.69, 0.43</td>
<td>0.35</td>
<td>2</td>
<td>5</td>
<td>0.08</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>3</td>
<td>20×40</td>
<td>URMMR-RES3</td>
<td>111.4, 27.9, 26.3</td>
<td>0.81, 0.69, 0.63</td>
<td>0.5</td>
<td>2</td>
<td>3.3</td>
<td>0.08</td>
</tr>
<tr>
<td>18</td>
<td>4</td>
<td>3</td>
<td>20×40</td>
<td>URMMR-COM2</td>
<td>61.0, 33.4, 19.5</td>
<td>0.80, 0.69, 0.49</td>
<td>0.5</td>
<td>2</td>
<td>3.3</td>
<td>0.08</td>
</tr>
</tbody>
</table>

\(a\) \(I_{BT}\) is the building index.

\(b\) The structural and occupancy types are related to the ones defined in HAZUS-Earthquake (FEMA and NIBS 2003).

\(c\) The target \(C_S\) is used to represent the seismic design level for existing buildings.
For the estimation of \( \mu_j \), nonlinear dynamic structural analysis is carried out where each building is modeled as a nonlinear inelastic SDOF system, where the Bouc-Wen model (Foliente, 1995; Ma et al, 2004; Goda et al, 2009) is adopted to mimic the hysteretic behaviour of the system. The model is able to simulate various hysteretic behaviours including degradation and pinching effects. The equation of motion of the SDOF system can be expressed in the following using the normalized displacement:

\[
\ddot{\mu} + 2\zeta_0 \omega_n \mu + \alpha \omega_n^2 \mu + (1 - \alpha) \omega_n^2 \mu = -\ddot{u}_g(t) / D_y \\
\dot{\mu}_z = \frac{1}{1 + \delta_n \varepsilon_n} \left[ \ddot{\mu} - (1 + \delta_n \varepsilon_n) \left( \beta \| \dot{\mu}_z \|^{\alpha - 1} \mu_z + \gamma \| \dot{\mu}_z \|^{\gamma} \right) \right], \tag{4.10}
\]

\[
\varepsilon_n = (1 - \alpha) \int_0^T \dot{\mu}_z \, dt
\]

where \( \mu \) and \( \mu_z \) are the displacement and hysteretic displacement normalized by the yield displacement capacity of the inelastic SDOF system, \( D_y \) (i.e., \( \mu = u / D_y \) and \( \mu_z = z / D_y \), in which \( u \) and \( z \) are the displacement and hysteretic displacement, respectively); \( \omega_n = (K / m)^{0.5} \) is the natural vibration frequency, in which \( K \) and \( m \) are the stiffness and mass of the system; \( \ddot{u}_g(t) \) is the ground acceleration time history; \( \varepsilon_n \) is the normalized dissipated energy through hysteresis; \( \alpha, \beta, \gamma \) and \( n \) are shape parameters; \( \delta_n \) and \( \delta_\varepsilon \) are stiffness and strength degradation parameters, respectively. The hysteretic behaviours of the SDOF system (i.e., normalized restoring force \( \alpha \mu + (1 - \alpha) \mu_z \) versus normalized displacement \( \mu \)) for different sets of parameters are illustrated in Figure 4.8.
Figure 4.8 Normalized force-deformation curve of the Bouc-Wen models subjected to harmonic excitations with an increasing amplitude.

The yield displacement $D_y$ of the nonlinear inelastic model could be approximately related to the 2005 NBCC seismic design code (NRCC, 2005), where the minimum required design base shear force $V_d$ is given by $V_d = C_s W$, $W$ is the total weight of the structure and $C_s$ is the design base shear coefficient given in Table 4.3 for different building types. It can be shown that $D_y$ is

$$D_y = \frac{R_N C_s W}{K},$$  \hspace{1cm} (4.11)

where $R_N$ is the coefficient taking into account that the actual yield strength of a designed structure is greater than $V_d$. $\mu_R$ and $R_N$ are considered to be lognormally distributed with
mean values shown in Table 4.3 and coefficient of variation (cov) of 0.3 and 0.15 (Ellingwood et al., 1980; Ibarra, 2003), respectively. $\mu_R$ and $R_N$ are assumed to be independent for each building.

Furthermore, seismic losses associated with building operation are categorized into three types: building-related loss $L_{BL}(\delta)$, contents-related loss $L_{CO}(\delta)$, and business-interruption related loss $L_{BI}(\delta)$. These damage-loss functions can be expressed as,

$$L_{BL}(\delta) = \delta^{\beta_{BL}} L_{BL}(1), \quad L_{CO}(\delta) = \delta^{\beta_{CO}} L_{CO}(1), \quad \text{and} \quad L_{BI}(\delta) = \delta^{\beta_{BI}} L_{BL}(1),$$

(4.12)

where the values of losses for the complete damage $L_{BL}(1), L_{CO}(1), \text{and } L_{BI}(1)$, as well as the model parameters $\beta_{BL}, \beta_{CO}, \text{and } \beta_{BI}$ are shown in Table 4.3 for each building type. By using the damage-loss functions, the aggregate seismic loss $L$ for $m$ buildings subjected to the scenario earthquake is calculated as:

$$L = \sum_{j=1}^{m} \left( L_{BL} \left( \delta_j \right) + L_{CO} \left( \delta_j \right) + L_{BI} \left( \delta_j \right) \right).$$

(4.13)

where $\delta_j$ denotes the damage factor for the $j$-th building. The maximum possible aggregate loss, $L_{\max}$, is given by Equation (4.13) with $\delta_j = 1$.

As the reference case, it is considered that each building can be modeled as (quasi-)bilinear SDOF system with $\xi = 0.05$, and model parameters shown in Table 4.4 is subjected to the simulated records based on the information given in Tables 4.1 and 4.2.
Table 4.4 Parameters defining the cases, and the estimated statistics of aggregate loss.

| Case   | Parameters for
|        | Extended
|        | Stochastic
<table>
<thead>
<tr>
<th></th>
<th>finite-fault model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. Case</td>
<td>Tables 4.1 and 4.2 for the scenario event</td>
</tr>
<tr>
<td>Case A</td>
<td>[\alpha, \beta, \gamma, n, \delta_\eta, \delta_\varsigma] = [0.05, 0.5, 0.5, 25, 0, 0]; (\rho_r(\Delta) = 1, |\gamma(\Delta)| = 0, \rho_{aj} = 1)</td>
</tr>
<tr>
<td>Case B</td>
<td>(\rho_r(\Delta) = 1, |\gamma(\Delta)| = 1)</td>
</tr>
<tr>
<td>Case C</td>
<td>(\rho_r(\Delta) = 0, |\gamma(\Delta)| = 0)</td>
</tr>
<tr>
<td>Case D</td>
<td>Spatial coherency alone, (\rho_r(\Delta) = 0)</td>
</tr>
<tr>
<td>Case E</td>
<td>Spatial correlation alone, (|\gamma(\Delta)| = 0)</td>
</tr>
<tr>
<td>Case F</td>
<td>Smoothed hysteretic behaviour, (n = 1)</td>
</tr>
<tr>
<td>Case G</td>
<td>Small damping ratio, 2%</td>
</tr>
<tr>
<td>Case H</td>
<td>Strength and stiffness degradation considered, ([\delta_\eta, \delta_\varsigma] = [0.3, 0.05]) for building types 13 and 14</td>
</tr>
<tr>
<td>Case I</td>
<td>(P-\Delta) effect considered, (\theta = 0.05) for building types 7, 8, 13 and 14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>Nonlinear inelastic system parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
</tr>
<tr>
<td>Ref. Case</td>
<td>0.21</td>
</tr>
<tr>
<td>Case A</td>
<td>0.15</td>
</tr>
<tr>
<td>Case B</td>
<td>0.26</td>
</tr>
<tr>
<td>Case C</td>
<td>0.23</td>
</tr>
<tr>
<td>Case D</td>
<td>0.23</td>
</tr>
<tr>
<td>Case E</td>
<td>0.20</td>
</tr>
<tr>
<td>Case F</td>
<td>0.17</td>
</tr>
<tr>
<td>Case G</td>
<td>0.29</td>
</tr>
<tr>
<td>Case H</td>
<td>0.21</td>
</tr>
<tr>
<td>Case I</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Note: Case A is identical to EXSIM as downloaded. Parameters for Case A-I are the same as those shown for the Reference Case except those described in the table. \(r_{L-10\%}\) and \(r_{L-5\%}\) represent the (1-10\%)- and (1-5\%)-quantile of \(r_L\), respectively.

By using the simulated (coherent and correlated) records for the scenario event, as discussed previously, and carrying out nonlinear inelastic time history analysis for each building defined for the reference case, the ductility demand for each building and the damage loss for the portfolio of buildings were estimated using Equation (4.13). The estimation was done using 100 replications. The samples of the aggregate loss
normalized with respect to $L_{\text{max}}$ (i.e., $r_L$), are presented on Gumbel probability paper in Figure 4.9. The mean of $r_L$ equals 0.21 and the cov of $r_L$ equals 0.81. This indicates that the variability of the damage loss for the considered scenario event is very significant. Since the samples in the upper tail follow almost a straight line, the aggregated seismic loss in the upper tail region may be fitted by the Gumbel distribution. The estimated (1-10%) and (1-5%)-quantiles of $r_L$ are listed in Table 4.4. These values indicated that if the portfolio of buildings are all insured against the scenario event with zero deductibles, a reserve of $0.45L_{\text{max}}$ ($0.51L_{\text{max}}$) corresponding to the ruin probability (i.e., the probability of the insurer to reach insolvency) equal to 10% (5%) is needed.

![Figure 4.9 Distribution of $r_L$ plotted on Gumbel probability paper for different spatial coherency and correlation parameters.](image-url)
4.3.3 Influence of uncertainty in FAS, and spatial correlation and coherency

To investigate the sensitivity of the aggregate loss to the inclusion or exclusion of the uncertainty in the FAS and the spatial coherency and correlation, the above analysis is repeated for several cases defined in Table 4. Case A, which can be considered as an “extreme case”, neglects the uncertainty in the FAS. It represents a direct application of the EXSIM as downloaded; it effectively assumes that all buildings are subjected to the same ground acceleration time history.

The obtained empirical distribution of the cost loss ratio is also shown in Figure 4.9, and the statistics of \( r_L \) are shown in Table 4.4. Table 4.4 shows that the mean, cov, (1-10%)-quantile and (1-5%)-quantile are smaller than those obtained for the Reference
Case. The (1-10%)- and (1-5%)-quantile of $r_L$ are about 0.20 and 0.21, respectively. This could have very serious implications for insurance industry. For example, if the required reserve to pay the “catastrophic loss” is determined based on (1-5%)-quantile of $r_L$ for Case A but Reference Case reflects better the reality, the ruin probability is equal to 45\% for the considered scenario event.

Case B considers that ground motions at different sites are fully coherent and $r_{Ai}$ is uncertain but identical for all sites (i.e., the reference FAS at different sites are scaled by the same $r_{Ai}$). Note that an alternative interpretation of Case B is that it implicitly assumed that the variability of the FAS (i.e., uncertainty in $r_{Ai}$) is due to interevent variability rather than intraevent variability. The results obtained for this case, which are included in Figure 4.9 and Table 4.4, indicate that the magnitude of the uncertainty in $r_L$ is largely increased as compared to that for Reference Case. The (1-10%)- and (1-5%)-quantile of $r_L$ are about 0.74 and 0.92, that are much greater than those for Reference Case. The assessment of interevent spatial correlation of the FAS using the actual records, which has important implication and was outlined in Chapter 3 and Liu and Hong (2013), is beyond the scope of this chapter.

Case C neglects the spatial coherency and correlation. Case D considers the spatial coherency alone, while Case E considers the spatial correlation alone. These cases are used to investigate effect of including or excluding the spatial coherency and/or spatial correlation on the estimated aggregate seismic loss. The obtained results for these cases are compared with those for Case A and Case B in Figure 4.9 and Table 4.4 as well. It can be observed that the probability distribution of $r_L$ is not sensitive to the spatial coherency (i.e., comparison between Cases D and C, and between Reference Case and
Case E). However, it is sensitive to spatial correlation (comparison between Reference Case and Case C, and between Cases E and D). The closeness of empirical distributions of \( r_L \) for Reference Case and Case E indicates that the spatial correlation of the FAS has a dominant effect on the statistics of the aggregate seismic loss for a portfolio of buildings.

The distribution of the relative contribution to the mean aggregate seismic loss is presented in Figure 4.10 for Cases A to E and compared to that of Reference Case. The figure shows that the distribution is similar in all cases.

4.3.4 Influence of inelastic behaviour and P-\( \Delta \) effect on the aggregate seismic loss

The results presented in Figure 4.9 are based on the assumption that the buildings can be approximated as bilinear systems - an assumption that is commonly adopted to simplify the analysis in code making and seismic risk assessment. To investigate the influence of different inelastic characteristics on the seismic loss estimates, Cases F, G, H and I tabulated in Table 4.4 are considered. Case F represents a much smoother transition from pre-yielding to post-yielding behaviour (see Figure 4.8); Case G has a reduced damping ratio (from 5% to 2%); and Case H considers the strength and stiffness degradation for some buildings. Case I includes the P-\( \Delta \) effect for some buildings, where the equation of motion presented in Equation (4.10) is replaced by,

\[
\ddot{\mu} + 2\xi \omega_n \dot{\mu} + \omega_n^2 \mu + (1 - \alpha) \omega_n^2 \mu - \theta \omega_n^2 \mu = -\ddot{u}_g (t) / D_y,
\]  

(4.14)

where \( \theta = mg/(Kh) \) is the stability factor (MacRae 1994); \( g \) is the gravitational acceleration; \( h \) is the height of the structure and vertical seismic excitation is neglected.
It must be emphasized that in reality, different buildings and building types have different hysteretic behaviour, damping ratios, strength and stiffness degradations, and P-Δ effects. The consideration of the above hypothetical cases is aimed at investigating the relative impact of these parameters on the estimated aggregate seismic loss, and at illustrating that the presented analysis framework is applicable to buildings with different nonlinear inelastic hysteretic behaviour and second order effects. The obtained results for these cases together with Reference Case are shown in Figure 4.11.

![Figure 4.11 Probability distribution of $r_L$ plotted on Gumbel probability paper for different structural model parameters.](image)

The results presented in Figure 4.11 indicate that there are clear trends for the nonexceedance probability less than 0.9. The increased smooth transition from pre- to post-yield decreases the aggregate seismic loss can be explained by observing that the
ductility demand for hysteretic model with smooth transition is smaller than that for bilinear hysteretic system. The decrease in the damping ratio, the consideration of strength and stiffness degradations, and the inclusion of the P-Δ effect increase structural damage comparing to the Reference Case. These can be explained by noting that the consideration of the strength and stiffness degradations (Case H) leads to decreased energy dissipating capacity; and that a decreased damping ratio (Case G) or the P-Δ effect increases the seismic demand. Note that since the degradation or P-Δ effect is considered for only a subset of buildings (shown in Table 4.4) for Case H and Case I, the increase in ductility demand on these buildings and in the aggregate loss are small. This is because the selected building are already severely damaged without considering degradation or P-Δ effect.

Figure 4.11 also shows that the trends of the empirical probability distributions for the exceedance probability less than 10% are not very clear. This is attributed to the small sample size. In all cases, the magnitude of the increase/decrease in the mean, cov (1-10%)-quantile and (1-5%)-quantile of $r_L$ are tabulated in Table 4.4. Comparison of the results shown in Table 4.4 indicates that the change in the statistics is not trivial, especially if a decision is to be made based on (1-10%)-quantile and (1-5%)-quantile of $r_L$ for selecting the reserve to prevent the insolvency for the scenario event.

To complete the presentation of the analysis results, the distribution of the relative contribution to the mean aggregate seismic loss due to different buildings is presented in Figure 4.12 for Cases F to I. Comparison of the results presented in Figures 4.10 and 4.12 indicates that the distribution for Cases F and I are similar that for Reference Case.
Figure 4.12 Distribution of relative contribution to the mean aggregate seismic loss of different types of buildings.

4.4 Concluding Remarks

In this chapter, an extension of the stochastic finite-fault simulation method, that incorporate the spatial coherency and the spatially correlated disturbance in the FAS, is presented. The extended model is used to generate synthetic records for a scenario event at spatially distributed sites. The synthetic records are used to investigate the sensitivity of the statistics of aggregate seismic loss of a portfolio of hypothetical buildings distributed in downtown Vancouver to the spatially correlated excitations.

The analysis of the synthetic records indicates that the records simulated using the extended stochastic finite-fault model can match the target spatial coherency and correlation. The results also show that the statistics of the aggregate seismic loss of a portfolio of buildings are significantly affected by the spatial correlation; the influence of
the spatial coherency on the aggregate seismic loss for group of buildings is low. These indicate that consideration of the intraevent spatial disturbance on the FAS is essential in the aggregate seismic loss estimation.

As the aggregate seismic loss is based on spatially correlated and coherent (synthetic) records for scenario seismic event, it facilitates the consideration of different nonlinear inelastic behaviours for the group of building and avoids the need to develop and use the GMPEs and ductility demand rules that are compatible with the scenario seismic event. It is conceptually straightforward to include more complex building models and structures with spatially distributed supports within the proposed framework to calculate their aggregate seismic loss and nonlinear inelastic responses under scenario events.

References


CHAPTER 5. ASSESSMENT OF COHERENCY FOR BI-DIRECTIONAL HORIZONTAL GROUND MOTIONS AND ITS APPLICATION FOR SIMULATING RECORDS AT MULTIPLE STATIONS

5.1 Introduction

Structures and infrastructure systems such as irregular building with different dynamic characteristics in two horizontal directions and bridges with multiple supports are sensitive to bi-directional and/or multiple-support excitations (Clough and Penzien 2003; Zerva 2009). Dynamic linear and nonlinear responses of these structures under seismic excitations can be calculated for selected historical ground motion records, and used for design checking or for seismic risk assessment. However, the available historical records are limited and may not match a desired scenario seismic event of interest defined by source-to-site distance and magnitude, which could be identified through seismic hazard and risk deaggregation analysis (Bazzurro and Cornell 1999; Hong and Goda 2006). If only uni-directional horizontal excitations at a site are of concern, there are several methods that can be used to generate synthetic records. For example, they can be simulated using the spectral representation method (Shinozuka and Jan 1972) with the Kanai-Tajimi power spectral density function or evolutionary power spectral density function (Yeh and Wen 1990). They can also be simulated using the stochastic point-source method and the stochastic finite-fault method (Atkinson et al., 2009; Boore, 2009). One of the advantages of using these methods is that the simulated records are directly related to (but not completely defined by) the source-to-site distance and earthquake magnitude.
The stochastic models are extended to simulate uni-directional multiple-station ground motions that match target spatial coherency and correlation (Liu and Hong 2013a, 2013b). The coherency of two ground motion records at two sites depends on the frequency and separation distance; it is estimated from the power spectral density functions of the records and is not affected by scaling the ground motions (Abrahamson et al., 1991; Zerva 2009). The spatial correlation represents the correlation of amplitude of ground-motion measures at two sites such as the peak ground acceleration (PGA) or spectral accelerations (SAs) or the integral of the Fourier amplitude spectrum (FAS).

Empirical spatial correlation models of PGA, SAs and Arias Intensity for the uni-directional excitation with a random orientation were proposed for seismic events in California, Taiwan and Japan (Boore et al., 2003; Wang and Takada, 2005; Goda and Hong, 2008; Hong et al., 2009; Jayaram and Baker, 2009; Goda and Atkinson, 2010; Sokolov et al., 2010; Foulser-Piggott and Stafford, 2012). A model of the spatial correlation for the FAS was presented in Chapter 3 and by Liu and Hong (2013a) for Chi-Chi earthquake. Models for the correlation of SAs in two orthogonal horizontal directions as well as in two horizontal principal axes at a recording site were presented in Baker and Cornell (2006) and Hong and Goda (2010).

The spatial coherency models were proposed by Harichandran and VanMarcke (1986), Hao et al. (1989) and Der Kiureghian (1996) based on the analysis results of actual records or random vibration theory. The coherency of the ground motions at a site in two orthogonal horizontal directions has not been scrutinized or used for simulating bi-directional excitations, although Hao et al. (1989) showed average of the estimated coherency at recording stations for a single seismic event.
Part of this chapter is focused on the estimation of the coherency of ground motion records in two orthogonal horizontal directions at a single and multiple stations. For the estimation, ground motion records from SMART-1 array for seven seismic events in Taiwan are used. A procedure for simulating multiple-station bi-directional horizontal ground motions is presented by incorporating the spatial coherency for two orthogonal horizontal directions and considering that the stochastic point-source method or the stochastic finite-fault method can be used to define the reference FAS and time modulating functions for scenario events. Samples of simulated records as well as their adequacy in matching the target spatial correlation and coherency are presented for a scenario event.

5.2 Assessment of coherency of ground motions in two orthogonal horizontal directions

5.2.1 Ground motion records and data processing

For the analysis, the ground motion records from SMART-1 array for 7 seismic events occurred in Taiwan are considered. These events are described in Table 5.1; these records can be requested from Data Management Center for Strong Motion Seismology of Institute of Earth Sciences (http://www.earth.sinica.edu.tw/~smdmc/). The location and layout of SMART-1 array and the distribution of the separation distance are presented in Figure 5.1. For the assessment, the records from the triggered stations among the 37 stations shown in Figure 5.1 for each considered seismic event are used. The number of triggered stations (i.e., the number of records) and the epicentral distance from the center station for each event are shown in Table 5.1 as well.
Table 5.1 Selected SMART-1 array events.

<table>
<thead>
<tr>
<th>Event No.</th>
<th>Date</th>
<th>Magnitude (M_L)</th>
<th>Epicenter Latitude (°)</th>
<th>Epicenter Longitude (°)</th>
<th>Focal Depth (km)</th>
<th>Azimuth (°)</th>
<th>No. of triggered stations</th>
<th>Epicentral distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1980.11.14</td>
<td>5.5</td>
<td>24.59</td>
<td>121.80</td>
<td>62.1</td>
<td>161.31</td>
<td>16</td>
<td>10.17</td>
</tr>
<tr>
<td>5</td>
<td>1981.01.29</td>
<td>5.9</td>
<td>24.44</td>
<td>121.90</td>
<td>11.1</td>
<td>151.61</td>
<td>27</td>
<td>29.87</td>
</tr>
<tr>
<td>20</td>
<td>1982.12.17</td>
<td>6</td>
<td>24.39</td>
<td>122.88</td>
<td>29.2</td>
<td>105.42</td>
<td>36</td>
<td>117.05</td>
</tr>
<tr>
<td>24</td>
<td>1983.06.24</td>
<td>6.6</td>
<td>23.99</td>
<td>122.62</td>
<td>25</td>
<td>130.96</td>
<td>30</td>
<td>115.36</td>
</tr>
<tr>
<td>37</td>
<td>1985.10.26</td>
<td>4.7</td>
<td>24.42</td>
<td>121.84</td>
<td>1.7</td>
<td>165.71</td>
<td>31</td>
<td>29.29</td>
</tr>
<tr>
<td>40</td>
<td>1986.05.20</td>
<td>6.2</td>
<td>24.09</td>
<td>121.60</td>
<td>15.8</td>
<td>194.74</td>
<td>35</td>
<td>66.93</td>
</tr>
<tr>
<td>45</td>
<td>1986.11.14</td>
<td>6.5</td>
<td>24.00</td>
<td>121.84</td>
<td>15</td>
<td>173.87</td>
<td>34</td>
<td>75.64</td>
</tr>
</tbody>
</table>

a. The event number is corresponding to that in the database of Institute of Earth Sciences.
b. The extended stations are excluded.
c. Epicentral distance of the center station.

Figure 5.1 (a) Location of seismic events and recording stations for SMART-1 array; (b) distribution of separation distance.

To ensure the data quality, the records are processed by applying the zeroth-order correction and the fourth-order low-cut Butterworth filter with corner frequency equal to 0.2 Hz (Boore et al., 2002; Boore and Bommer, 2005).
5.2.2 Estimation of coherency

The coherency function between two ground motion components, $\tilde{\gamma}_{mn,jk}(\Delta, f)$, is defined as:

$$\tilde{\gamma}_{mn,jk}(\Delta, f) = \sqrt{\frac{S_{mn,jk}(f)}{S_{m,j}(f)S_{n,k}(f)}}$$

(5.1)

where $\Delta$ (km) is the separation distance, $f$ (Hz) is frequency; $m, n = 1, 2$ represents the first and second horizontal ground motion component; $j, k = 1, \cdots, n_R$; $n_R$ is the total number of recording sites for a seismic event; $S_{m,j}(f)$ is the smoothed power spectral density functions for the $m$-th (horizontal ground motion) component at the $j$-th station; and $S_{mn,jk}(f)$ is the smoothed cross power spectral density function of the $m$-th component at the $j$-th station and the $n$-th component at the $k$-th station.

It is considered that the orientation of first component of the records is random but parallel to that at the other station. The first and second components at a station are orthogonal. If only two stations are of concern, taking into account the symmetry, there are ten coherency functions that need to be considered. Since $\tilde{\gamma}_{mn,jj}(\Delta, f)$ is equal to one by definition, only six remaining coherency functions need to be considered. The models for the coherency of records along the same direction at two stations $\tilde{\gamma}_{mn,jk}(\Delta, f)$ were proposed in the literature (Sobczyk 1985; Harichandran and VanMarcke 1986; Hao et al. 1989; Der Kiureghian 1996) based on the analysis results of actual records or random vibration theory. $\tilde{\gamma}_{mn,jk}(\Delta, f)$ can be expressed as,

$$\tilde{\gamma}_{mn,jk}(\Delta, f) = |\tilde{\gamma}_{mn,jk}(\Delta, f)| \exp\left(-i2\pi f\Delta_p / v_{ap}\right), \text{ for } j \neq k$$

(5.2)
where $m = 1$ or $2$; $\Delta_{p}$ is the projection of $\Delta$ in the direction of wave propagation; $v_{ap}$ (km/s) represents the apparent velocity; $2\pi f_0 \Delta_{p} / v_{ap}$ represents phase angle of the wave passage effect (Der Kiureghian, 1996); the lagged coherency $|\gamma_{mn,jk}(\Delta, f)|$ is given by (Harichandran and Vanmarcke, 1986),

$$|\gamma_{mn,jk}(\Delta, f)| = A \exp\left(-\frac{2000\Delta}{\alpha_0 \theta(f)}(1 - A + \alpha_0 A)\right) + (1 - A) \exp\left(-\frac{2000\Delta}{\theta(f)}(1 - A + \alpha_0 A)\right), \quad (5.3)$$

in which $\theta(f) = \left(1 + \left(\frac{f}{f_0}\right)^\beta\right)^{-1/2}$, and $A$, $\alpha_0$, $k$, $f_0$ and $B$ are model parameters.

No empirical model has been suggested for the coherency between two orthogonal horizontal directions at a station, $\gamma_{mn,jk}(0, f)$, for $m \neq n$. Furthermore, it is unknown if the model of $\gamma_{mn,jk}(\Delta, f)$ such as the one shown in Equations (5.2) and (5.3) is applicable to the coherency of two components of records in two orthogonal directions, each component at a station, $\gamma_{mn,jk}(\Delta, f)$. This coherency and its corresponding lagged coherency are assessed below using the records listed in Table 5.1.

Although the model for $|\gamma_{mn,jk}(\Delta, f)|$ is already available (Harichandran and VanMarcke, 1986; Hao et al. 1989) such as the one shown in Equation (5.3), the seismic events and records used for their analysis differ from those listed in Table 5.1. For consistency and completeness, an assessment of $|\gamma_{mn,jk}(\Delta, f)|$ by using the records from the events shown in Table 5.1 is carried out. For the analysis, the smoothed power spectral density functions $\tilde{S}_{m,j}(f)$ and the smoothed cross power spectral density
function $\tilde{S}_{mm,jk}(f)$ for the $m$-th components at $j$-th and $k$-th stations for a given seismic event are estimated. For the smoothing, the Hamming window, $W(n)$, given by (Zerva 2009),

$$W(n) = 0.54 - 0.46\cos\left(\pi(n + N)/N\right), \quad n = -N, ..., N,$$  \hspace{1cm} (5.4)

where $N$ is a parameter which is taken equal to 15, is used.

Following Harichandran (1991), the records are first rotated to epicentral direction (radial direction) and its perpendicular direction (transverse direction). The obtained lagged coherency is shown in Figure 5.2 for rotated records from Event 24 shown in Table 5.1. Figures 5.2a to 5.2d show that the lagged coherencies between radial components and between transverse components are similar, and that the averaged value of the lagged coherency is much smoother than that for a single pair of record components, which is expected. Average of the lagged coherency versus separation distance for different frequencies is plotted in Figure 5.2e for the radial direction and in Figure 5.2f for the transverse direction, showing decreasing trends of the magnitude of lagged coherency versus distance. Note that for this event the rate of decrease of $|\tilde{Y}_{mm,jk}(\Delta, f)|$ with respect to distance for the transverse direction is somewhat slower than that for the radial direction.

By adopting the model shown in Equation (5.3) and fitting the model in inverse hyperbolic tangent space, the estimated model parameters are shown in Table 5.2. Fitting to $\tanh^{-1}|\tilde{Y}_{mm,jk}(\Delta, f)|$ rather than $|\tilde{Y}_{mm,jk}(\Delta, f)|$ is because that $\tanh^{-1}|\tilde{Y}_{mm,jk}(\Delta, f)|$ is approximately normally distributed (Jenkins and Watts, 1968; Abrahamson et al., 1991).
Figure 5.2 Samples of lagged coherency and average of lagged coherency for Event 24:
(a) Samples of lagged coherency $\overline{\gamma}_{11,jk}(\Delta, f)$ for a selected pairs of stations; (b) Samples of lagged coherency $\overline{\gamma}_{22,jk}(\Delta, f)$ for a selected pairs of stations; (c) average of $\overline{\gamma}_{11,jk}(\Delta, f)$ for $\Delta = 0.35$ km; (d) average of $\overline{\gamma}_{22,jk}(\Delta, f)$ for $\Delta = 0.35$ km.
Figure 5.2 (cont.) Samples of lagged coherency and average of lagged coherency for Event 24: (e) average of $\overline{\gamma}_{11,jk}(\Delta, f)$ for $f = 0.33$ Hz, 1 Hz and 3.33 Hz; (f) average of $\overline{\gamma}_{22,jk}(\Delta, f)$ for $f = 0.33$ Hz, 1 Hz and 3.33 Hz.

The analysis results for other events listed in Table 5.1 are obtained but not presented because they follow the same trends as those for Event 24. However, the model parameters for $\overline{\gamma}_{mm,jk}(\Delta, f)$ are estimated and presented in Table 5.2. The results indicate that there is no clear trend between the differences in the model parameters for $\overline{\gamma}_{11,jk}(\Delta, f)$ and for $\overline{\gamma}_{22,jk}(\Delta, f)$.

Analysis of $\overline{\gamma}_{mm,jk}(\Delta, f)$ is also carried out by rotating the recording orientations by increment of 5 degrees simultaneously for all recording stations. The conclusions that can be drawn from the results are similar to those from Figure 5.2. This indicates that $\overline{\gamma}_{mm,jk}(\Delta, f)$ is insensitive to the recording orientations.
Table 5.2 Fitted model parameters in Equation 3 for SMART-1 array events.

| Component | Event 2 | | Event 5 | | Event 20 | | Event 24 | | Event 37 | | Event 40 | | Event 45 |
|-----------|---------|-----------------|---------|--------------|---------|------|------|-------|------|-------|------|
|           | Radial  | $A$             | $\alpha_0$ | $k$           | $f_0$  | $B$  |      |       | $A$     | $\alpha_0$ | $k$           | $f_0$  | $B$  |
| Radial    | 0.53    | $2.43 \times 10^{-5}$ | $1.40 \times 10^{-7}$ | 2.59 | 7.38 |
| Transverse| 0.54    | $2.14 \times 10^{-3}$ | $9.66 \times 10^{-4}$ | 8.69 | 6.28 |
| Radial    | 0.39    | $6.28 \times 10^{-5}$ | $1.13 \times 10^{-7}$ | 4.93 | 9.74 |
| Transverse| 0.41    | $2.83 \times 10^{-4}$ | $9.29 \times 10^{-5}$ | 6.40 | 6.55 |
| Radial    | 0.56    | $3.66 \times 10^{-6}$ | $2.49 \times 10^{-8}$ | 0.34 | 4.96 |
| Transverse| 0.55    | $2.00 \times 10^{-4}$ | $7.73 \times 10^{-5}$ | 4.39 | 5.86 |
| Radial    | 0.60    | $1.98 \times 10^{-3}$ | $3.88 \times 10^{-5}$ | 1.91 | 3.81 |
| Transverse| 0.61    | $3.54 \times 10^{-3}$ | $5.25 \times 10^{-5}$ | 0.42 | 2.31 |
| Radial    | 0.33    | $2.67 \times 10^{-4}$ | $9.20 \times 10^{-5}$ | 7.73 | 9.52 |
| Transverse| 0.31    | $4.79 \times 10^{-4}$ | $5.21 \times 10^{-5}$ | 7.55 | 7.84 |
| Radial    | 0.38    | $6.05 \times 10^{-5}$ | $1.75 \times 10^{-7}$ | 1.03 | 5.35 |
| Transverse| 0.37    | $8.01 \times 10^{-5}$ | $8.71 \times 10^{-6}$ | 2.40 | 6.87 |
| Radial    | 0.44    | $7.36 \times 10^{-5}$ | $3.05 \times 10^{-8}$ | 0.026 | 1.80 |
| Transverse| 0.44    | $9.86 \times 10^{-6}$ | $9.27 \times 10^{-7}$ | 0.011 | 2.00 |

The same smoothing window shown in Equation (5.4) is used to assess $|\varphi_{mn,ij}(0,f)|$. The obtained lagged coherency and its mean and mean +/- one standard deviation are illustrated in Figure 5.3 for Event 24 listed in Table 5.1. Figure 5.3 shows the lagged coherency for all the records from Event 24. It indicates that the mean of $|\varphi_{mn,ij}(0,f)|$ can be considered to vary linearly with frequency. The mean value could be considered to be greater than the value attributed to the noise alone (Bendat and Piersol 1971; Abrahamson et al. 1992). The standard deviation of $|\varphi_{mn,ij}(0,f)|$ can be considered to be independent of frequency with a value of 0.18 over the considered frequency range for this event. No assessment of the effect of source-to-site distance on $|\varphi_{mn,ij}(0,f)|$ is carried out because this distance for all recording sites is near 115 km. Based on these observations, the following empirical lagged coherency model could be considered,

$$
|\varphi_{mn,ij}(0,f)| = c_0 - c_1f. 
$$

(5.5)
where $c_0$ and $c_1$ are model parameters. The fitted parameters are shown in Table 5.3.

Figure 5.3 Sample, mean, and mean +/- standard deviation of the estimated lagged coherency $|\tilde{\gamma}_{nn,jj}(0,f)|$ for two orthogonal horizontal directions considering Event 24.

The analysis for $|\tilde{\gamma}_{nn,jj}(0,f)|$ is also carried out for other events listed in Table 5.1 but they are not presented because they follow the same trends as those for Event 24. However, the means and the standard deviations of $|\tilde{\gamma}_{nn,jj}(0,f)|$ for the events are summarized in Figure 5.4, showing the mean varies from event to event, and the standard deviation is very stable. In all cases, the fitted model parameters $c_0$ and $c_1$ for each event are listed in Table 5.3. The value of fitted $|\tilde{\gamma}_{nn,jj}(0,f)|$ at zero frequency (i.e., $c_0$) varies between 0.5 and 0.8, and the rate of decrease with increasing frequency is small.
Table 5.3 Fitted model parameters in Equations 5.5 and 5.6 for SMART-1 array events.

| Event No. | $|\gamma_{mn,ij}(0,f)| = c_0 - c_1f$ | $|\gamma_{mn,jk}(\Delta,f)| = c_0 - c_1f$ |
|-----------|-----------------------------------|-----------------------------------|
| Event 2   | 0.689 $6.6\times 10^{-3}$         | 0.478 $3.7\times 10^{-3}$         |
| Event 5   | 0.721 $4.4\times 10^{-4}$         | 0.613 $3.0\times 10^{-3}$         |
| Event 20  | 0.604 $3.8\times 10^{-3}$         | 0.454 $3.1\times 10^{-3}$         |
| Event 24  | 0.505 $9.5\times 10^{-3}$         | 0.450 $8.6\times 10^{-3}$         |
| Event 37  | 0.781 $2.0\times 10^{-3}$         | 0.678 $2.6\times 10^{-3}$         |
| Event 40  | 0.752 $11.7\times 10^{-3}$        | 0.689 $10.8\times 10^{-3}$        |
| Event 45  | 0.521 $5.4\times 10^{-4}$         | 0.566 $1.0\times 10^{-3}$         |

Figure 5.4 Mean and standard deviation of the estimated of lagged coherency $|\tilde{\gamma}_{mn,ij}(0,f)|$ for different events.

Note that the concept of principal axes has been used to characterize the seismic excitations (Arias 1970, 1996) and to assume that power spectral density function $\tilde{S}_{12,ij}(f)$ of horizontal record components along the principal axes equals zero (Penzien and Watabe 1975; Kubo and Penzien 1979). Also orientation-dependent ground-motion measure was considered for seismic hazard assessment (Hong and Goda 2007).
Therefore, it is instructive to estimate $|\hat{\gamma}_{mm,jj}(0,f)|$ for components of records along the principal axes. The orientations of the principal axes for the horizontal ground motions at each station are calculated, and the records are rotated to the principal directions. $|\hat{\gamma}_{mm,jj}(0,f)|$ is estimated, where $m$ and $n$ in this case represents the major and minor (or minor and major) principal axes in the horizontal plane. The estimated values for records from Event 24 shown in Table 5.1 are presented in Figure 5.5. Also presented in Figure 5.5 are the mean and +/- standard deviation of lagged coherency. The results presented in Figure 5.5 are somewhat surprising because on average $|\hat{\gamma}_{mm,jj}(0,f)|$ for record components oriented along the principal axes is similar to that for randomly oriented orthogonal horizontal components, and is not equal to zero. This implies that the assumption that the cross power spectral density function between the ground motion components along the principal axes at a station $\hat{S}_{12,j}(f)$ equals zero is questionable. Its implication in modal combination rules often used in earthquake engineering for estimating structural responses that are not discussed in this chapter could be important and deserve further investigation.
Figure 5.5 Sample, mean, and mean +/- standard deviation of the estimated lagged coherency $|\tilde{y}_{mn,ij}(0,f)|$ for the components of the records oriented along the two principal axes considering Event 24.

To assess the lagged coherency of two orthogonal components of the records at different stations $|\tilde{y}_{mn,jk}(\Delta,f)|$ for $m \neq n$ and $j \neq k$, the analysis that was done for $|\tilde{y}_{mn,ij}(0,f)|$ is carried out but considering the components of the records from different stations. Samples of $|\tilde{y}_{mn,jk}(\Delta,f)|$ are shown in Figures 5.6a and 5.6b for the components of the records along the radial and transverse directions for the same pair of records in Event 24 used in Figure 5.2. Comparison between results shown in Figures 5.6a and 5.6b to those in Figures 5.2a and 5.2b indicates that the lagged coherency between orthogonal components is significantly lower than that between the components for the same orientation at two stations. The averaged $|\tilde{y}_{mn,jk}(\Delta,f)|$ shown in Figures 5.6c and 5.6d are not sensitive to separation distance and do not show exponential decay with respect to
frequency as \( |\tilde{\gamma}_{mn, jk}(\Delta, f)| \) shown in Figure 5.2, indicating that the application of Equation (5.3) to \( |\tilde{\gamma}_{mn, jk}(\Delta, f)| \) may not be adequate or necessary. By considering \( |\tilde{\gamma}_{mn, jk}(\Delta, f)| \) for all possible pairs of records in Event 24, its mean and standard deviation are shown in Figure 5.6e. It can be seen that \( |\tilde{\gamma}_{mn, jk}(\Delta, f)| \) is similar to \( |\tilde{\gamma}_{mn, ij}(0, f)| \) shown in Figure 5.3. If the simple linear function shown in Equation (5.5) is considered for \( |\tilde{\gamma}_{mn, jk}(\Delta, f)| \) for \( \Delta \neq 0 \) as well, the estimated model coefficients \( c_0 \) and \( c_1 \) are also shown in Table 5.3. Comparison of \( c_0 \) and \( c_1 \) for the case with \( \Delta \neq 0 \) to those for the case with \( \Delta = 0 \) indicates that their differences are small. In general, \( c_0 \) for \( \Delta = 0 \) is about 15% greater than that for \( \Delta \neq 0 \); \( c_1 \) for \( \Delta = 0 \) is smaller than that for \( \Delta \neq 0 \) for three out of seven events. Therefore, it is suggested that the average values of \( c_0 \) and \( c_1 \), which equal 0.61 and \( 4.8 \times 10^{-3} \), could be used for both \( |\tilde{\gamma}_{mn, ij}(0, f)| \) and \( |\tilde{\gamma}_{mn, jk}(\Delta, f)| \) for practical applications. It must be emphasized that this recommendation is for \( \Delta \) less than about 4 km.

To take the wave propagation effect into account, the apparent velocity \( v_{ap} \) needs to be estimated. Previous studies indicate that \( v_{ap} \) varies between 2 km/s to 8 km/s and changes from event to event (Harichandran and VanMarcke, 1986; Hao et al., 1989; Abrahamson et al., 1991; Harichandran, 1991), and that \( v_{ap} \) may be modeled as a function of frequency (Hao et al., 1989; Zerva, 2009). Due to this large variation and the separation distances among the recording sites are small, the assessment of the apparent velocity is not pursued in this chapter.
Figure 5.6 Samples of $|\gamma_{mn,jk}(\Delta, f)|$ for Event 24: (a) and (b), samples of lagged coherency $|\gamma_{mn,jk}(\Delta, f)|$ for a selected pairs of stations ($\Delta = 0.35$ km, the subscript $m$ and $n$ represent radial and transverse component, respectively); (c) and (d), average of $|\gamma_{mn,jk}(\Delta, f)|$ for $\Delta = 0.35$ km.
Figure 5.6 (cont.) Samples of $\hat{p}_{mn, jk} (\Delta, f)$ for Event 24: (e) mean and standard deviation of all possible pairs of records in Event 24.

5.3 Procedure for simulating bi-directional horizontal ground motions

For the simulation of horizontal ground motions, the Fourier amplitude spectrum (FAS), and the time modulating functions need to be defined. For a given scenario event defined by source-to-site distance and moment magnitude $M$, the stochastic point-source method (Boore 2003, 2009) or the stochastic finite-fault method (Beresnev and Atkinson, 1997; Atkinson et al., 2009; Boore, 2009) can be used conveniently to define the FAS, $y_j(M, R_j, f)$, and time modulating functions at spatially distributed locations, where $R_j$ is the distance from the $j$-th site to earthquake source (e.g., closest horizontal distance to the surface projection of the fault plane, Joyner-Boore distance). $y_j(M, R_j, f)$ and the time modulation function are explicitly given for the stochastic point-source method; they can be estimated (Liu and Hong, 2013b) if the stochastic finite-fault method is used. Both $y_j(M, R_j, f)$ and the time modulating function can be considered for the two orthogonal
horizontal directions, which are considered to be random with respect to the source-to-site orientation.

To incorporate the spatial correlation of the FAS for two orthogonal directions, it is considered that the reference FAS of the $m$-th direction at the $j$-th site equals $r_{Am,j} \times y_j(M, R_j, f)$, where $r_{Am,j}$ ($m = 1, 2$) denotes the spatially correlated random (scaling) disturbance of $y_j(M, R_j, f)$. The (intraevent) correlation coefficient between $\ln(r_{Am,j})$ and $\ln(r_{Am,k})$ for $j$ and $k = 1, \ldots, n_R$, $m$ and $n = 1$ or 2, denoted as $\rho_{mn,jk} (\Delta)$ is not available in the literature. However, by focusing on the uni-directional excitations, Chapter 3 and Liu and Hong (2013a) evaluated the correlation coefficient of the FAS of two randomly oriented components separated by distance $\Delta$, $\rho(\Delta)$. Their assessment is based on the integral of the FAS for 389 ground motion records for Chi-Chi earthquake; their results indicate that the standard deviation of $\ln(r_{Am,j})$ equals 0.523 and $\rho(\Delta)$ is given by,

$$\rho(\Delta) = \exp(-a_1 \Delta^b_1), \quad (5.6)$$

where $a_1 = 0.17$ and $b_1 = 0.49$ are empirical model parameters assessed based on the integral of the FAS for 389 ground motion records for Chi-Chi earthquake (Chapter 3; Liu and Hong, 2013a). As their analysis is focused on the randomly oriented uni-directional excitations, for $\Delta > 0$ combinations of record components at different recording stations are used to estimate $\rho(\Delta)$. It should be noted that this estimation requires the development of ground motion prediction equations (GMPEs) using a large number of records from the considered event.
By following the same procedure used in Chapter 3 and Liu and Hong (2013a) but considering the orientations of record components, the estimated average values of the correlation coefficient $\rho_{mn, jk}(\Delta)$ for different combinations of $m$ and $n$ are presented in Figure 5.7a. The figure shows that except for $\Delta = 0$, the differences between $\rho_{mn, jk}(\Delta)$ for different combinations of $m$ and $n$ are not very large. For $\Delta = 0$, by definition $\rho_{mm, ij}(0)$ equals one, and $\rho_{mn, ij}(0)$ differs from unity, as shown in the filled symbols in Figure 5.7a. To increase the sample size, it is assumed that $\rho_{mn, jk}(\Delta)$ equals $\rho_{nm, jk}(\Delta)$ (for $m \neq n$), and $\rho_{11, jk}(\Delta)$ equals $\rho_{22, jk}(\Delta)$. Based on these assumptions, the obtained average values of $\rho_{mn, jk}(\Delta)$ are shown in Figures 5.7b and 5.7c. Also, the fitted empirical correlation coefficient functions

$$\rho_{mn, jk}(\Delta) = \exp\left(-a_2\Delta^{b_2}\right),$$  \hspace{1cm} (5.7a)

and

$$\rho_{mn, jk}(\Delta) = r_0 \exp\left(-a_3\Delta^{b_3}\right),$$  \hspace{1cm} (5.7b)

where $a_2 = 0.17$ and $b_2 = 0.50$; $r_0 = 0.8$, $a_3 = 0.036$ and $b_3 = 0.88$. $r_0$ in Equation (5.7b) represents the correlation coefficient of FAS between two orthogonal horizontal components at same station. It is interesting to note that $r_0$ is close to the intraevent correlation coefficient for the spectral accelerations for different natural vibration periods and for two orthogonal horizontal directions at the same recording station, ranging from about 0.7 to 0.9 (Baker and Cornell 2006; Hong and Goda 2010).
Figure 5.7 Averaged value of estimated $\rho_{mn,jk}(\Delta)$ for Chi-Chi records: a) Estimated $\rho_{mn,jk}(\Delta)$, b) Estimated $\rho_{mn,jk}(\Delta)$ considering $\rho_{11,jk}(\Delta) = \rho_{22,jk}(\Delta)$ for $m \neq n$, and c) Estimated $\rho_{mn,jk}(\Delta)$ considering $\rho_{mn,jk}(\Delta) = \rho_{nm,jk}(\Delta)$ for $m \neq n$.

An attempt was made to use SMART-1 array data to develop spatial correlation relations as those shown in Figure 5.7. However, due to insufficient number of records, the lack of GMPEs and the lack of information on the seismic events to allow for simulation using the stochastic finite-fault method or the stochastic point-source method,
the spatial correlations from SMART-1 array data is not assessed and the model developed based on Chi-Chi records is adopted for the example simulation in the next section.

By integrating the above spatial correlation models and the spatial coherency presented in the previous section, the generation of the synthetic bi-directional ground motions can be carried out:

(a) Generate two band-limited white noises with zero mean for each considered site that incorporate the target spatial coherency;

(b) Apply the corresponding time modulating function to each white noise at each site;

(c) Calculate and normalize the FAS of each time modulated record by its square-root of the mean squared amplitude spectrum;

(d) Multiply the normalized spectrum by its corresponding reference FAS and by the spatial correlated scaling factor; and,

(e) Apply the inverse Fourier transform to the spectra obtained in step (d) to compute the synthetic ground motion records.

For the generation of band-limited white noises with target coherency in Step (a), the method developed by Abrahamson (1992) is adopted. Use of this method rather than the simple spectral representation method (Shinozuka and Jan, 1972) is because the latter
breaks down occasionally in decomposing the lagged coherency matrix if records at multiple stations are to be simulated.

5.4 Samples of bi-directional horizontal ground motions at spatially distributed sites

As an illustration, bi-directional horizontal ground motions are simulated based on the procedure given in the previous section for the locations presented in Figure 5.8a, where site 1 locates at [24.67°N, 121.76°E] that is about 133.2 km from the epicenter for a hypothetical earthquake. For the simulation, the required modeling parameters in the stochastic point-source method are shown in Table 5.4. The parameters in the (bidirectional) coherency model developed for Event 24 shown in Table 5.1 are used as the target coherency to simulate band-limited white noises. As mentioned earlier, the reported apparent velocity based on the records from SMART-1 array by different studies varies widely, $v_{ap}$ equal to 2.5 km/s is considered for the example simulation to illustrate the method. Furthermore, the spatial correlation models given in Equation (5.7) are used as the target spatial correlation.

Table 5.4 Modeling parameters in stochastic point-source model (selected based on Roumelioti and Beresnev, 2003 and Liu et al., 2012).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment magnitude M</td>
<td>7.6</td>
</tr>
<tr>
<td>Epicenter [latitude, longitude]</td>
<td>[23.86°N, 120.80°E]</td>
</tr>
<tr>
<td>Stress parameter $\Delta\sigma$</td>
<td>103 (bar)</td>
</tr>
<tr>
<td>$Q(f)$</td>
<td>$117 f^{0.77}$</td>
</tr>
<tr>
<td>Geometrical spreading</td>
<td>$1/R$ for $R &lt; 50$ km, $1/R^0$ for $50 \leq R &lt; 150$ km and, $1/R^{0.5}$ for $R \geq 150$ km</td>
</tr>
<tr>
<td>Windowing function</td>
<td>Exponential (Saragoni and Hart, 1974)</td>
</tr>
<tr>
<td>Kappa</td>
<td>0.08 (s)</td>
</tr>
<tr>
<td>Crustal amplification</td>
<td>Generic rock site (Boore and Joyner, 1997)</td>
</tr>
<tr>
<td>Crustal shear-wave velocity</td>
<td>3.2 (km/s)</td>
</tr>
<tr>
<td>Crustal density</td>
<td>2.7 (g/cm$^3$)</td>
</tr>
</tbody>
</table>
Samples of the simulated records are shown in Figure 5.8b. The lagged coherency for a few pairs of simulated record components are calculated and illustrated in Figures 5.8c, 5.8d and 5.8e; the corresponding phase spectra are shown in Figures 5.8f, 5.8g, 5.8h. In general, the simulated ground motions match well the target lagged coherency functions, target correlation functions and the target phase spectra, although the agreement shown in Figures 5.8d and 5.8e should be improved.

Figure 5.8 Sites and samples of simulated bi-directional time history: (a) Sites layout, site 1 locates at [24.67°N, 121.76°E]; (b) samples of the simulated records.
Figure 5.8. (cont.) Sites and samples of simulated bi-directional time history: (c), (d) and (e) Lagged coherency for a few pair of simulated record components; (f), (g) and (h) Phase spectrum corresponding to the estimated coherency shown in (c), (d) and (e).
Figure 5.8. (cont.) Sites and samples of simulated bi-directional time history: (c), (d) and (e) Lagged coherency for a few pair of simulated record components; (f), (g) and (h) Phase spectrum corresponding to the estimated coherency shown in (c), (d) and (e).

To test the adequacy of the simulated records in matching the target spatial correlation, 100 sets of bidirectional horizontal records at these five sites are simulated and are used to calculate $\rho_{nm,jk}(\Delta)$. The estimated average values of $\rho_{nm,jk}(\Delta)$ and $\rho_{nm,jk}(\Delta)$ are shown in Figures 5.9a and 5.9b and compared to the target spatial correlation. As expected, the spatial correlation coefficients of the simulated records follow the target values. Moreover, PGA and SAs are calculated using the simulated record components for all records. The estimated spatial correlation coefficients of PGA and SAs at different natural vibration periods are shown in Figures 5.9c and 5.9d. It is interesting to note that the average values of the spatial correlation coefficients of the ground-motion measures are in agreement with those of the FAS. The estimated average values of the correlation coefficient of ground-motion measures between two horizontal
components at the same station are 0.813 for PGA, 0.797 for SA at 0.3 s, 0.791 for SA at 1.0 s and 0.706 for SA at 3.0 s. These values agree well with those reported by Baker and Cornell (2006) and Hong and Goda (2010) for spectral accelerations.

Figure 5.9 Spatial correlation coefficient obtained from simulated ground motion records: (a) and (b), Comparison of the spatial correlation coefficient of FAS versus target; (c) and (d), Comparison of the correlation coefficients of PGA and SAs to the target correlation coefficient of the FAS.
5.5 Conclusions

Estimation of the spatial coherency of ground motion records in two orthogonal horizontal directions is carried out using records from SMART-1 array for seven seismic events. Empirical spatial coherency function is suggested for the components of records in two orthogonal horizontal directions. The analysis results also indicate that the coherency for the components along the major and minor principal axes at a recording station is similar to that for two randomly oriented orthogonal directions.

The spatial coherency and the spatial correlation of the records are used to establish a procedure for simulating bi-directional horizontal ground motions at multiple stations by considering that the reference Fourier amplitude spectrum for scenario events can be defined using the stochastic point-source method or the stochastic finite-fault method. Illustrative simulated records using the proposed procedure are given; it is shown that the spatial lagged coherency and the spatial correlation of the FAS of the simulated records match the targets. Moreover, the correlation coefficient of the ground-motion measures (i.e., PGA and SAs) in two orthogonal directions match those reported in the literature.

References


6.1 Conclusions

In this thesis, the approaches to simulate single- and multiple-component multiple-station ground motion records with target spatial coherency and spatial correlation structures are developed for scenario events. The approaches incorporate both spatial coherency and spatial correlation characteristics observed from actual ground motion records, for both uni-directional and bi-directional excitations. The application of the developed approaches in seismic risk assessment is also illustrated. The conclusions from each chapter are summarized in the following.

Chapter 2 investigates whether the stochastic finite-fault method is able to reproduce the observed spatial correlation characteristics of ground-motion measures. It is concluded that while reproducing average response spectral characteristics of records, the stochastic finite-fault method does not reproduce observed spatial correlation, at least for the Chi-Chi records. And this is expected to be a common problem for the stochastic simulation method.

In Chapter 3, the stochastic point-source method is extended to generate multiple-station ground motion records. The extension incorporates a target spatial coherency structure and the spatially correlated uncertainties in the Fourier amplitude spectrum. Use of the extended method is illustrated by generating ground motion records of the Chi-Chi earthquake, reproducing desired target spatial coherency and spatial correlation of ground-motion measures. This extension can be implemented with other ground motion simulation techniques to generate correlated ground motion records with
prescribed spatial correlation and coherency. A further enhancement by incorporating the phase spectrum and the interevent correlation is also outlined. The proposed extension facilitates the application of the stochastic point-source method for seismic analysis of structures with multiple supports and for seismic risk assessment of portfolios of structures distributed in a region.

In Chapter 4, the stochastic finite-fault simulation method is extended to incorporate the spatial coherency and the spatially correlated disturbance in the FAS. Using the extended method, ground motion records are simulated and are used to investigate the sensitivity of the statistics of aggregate seismic loss of a portfolio of hypothetical buildings distributed in downtown Vancouver to the spatially correlated excitations. The simulated records match the target spatial coherency and correlation adequately. The analyses show that the statistics of the aggregate seismic loss of a portfolio of buildings are significantly affected by the spatial correlation; the influence of the spatial coherency on the aggregate seismic loss for group of buildings is low. These indicate that the consideration of the intraevent spatial disturbance on the FAS is essential in estimating the aggregate seismic loss. As the aggregate seismic loss is estimated based on spatially correlated and coherent (synthetic) records for scenario seismic event, it facilitates the consideration of different nonlinear inelastic behaviours for the group of building and avoids the need to develop and use the GMPEs and ductility demand rules that are compatible with the scenario seismic event.

Chapter 5 estimates the spatial coherency of ground motion records in two orthogonal horizontal directions using records from SMART-1 array for seven seismic events. Empirical spatial coherency function is suggested for the components of records in two
orthogonal horizontal directions. The analysis results also indicate that the coherency for
the records along the major and minor principal axes at a recording station is similar to
that for two randomly oriented orthogonal directions. The spatial coherency and the
spatial correlation of the records are used to develop a procedure for simulating bi-
directional horizontal ground motions at multiple stations by considering that the
reference Fourier amplitude spectrum for scenario events can be defined using the
stochastic point-source method or the stochastic finite-fault method. Illustrative
simulated records using the proposed procedure show that the spatial lagged coherency
and the spatial correlation of the FAS of the simulated records match the targets.
Moreover, the correlation coefficient of the ground-motion measures (i.e., PGA and SAs)
in two orthogonal directions match those reported in the literature.

6.2 Future Work

The work presented in this thesis can potentially be enhanced. Some of the possible
enhancements are listed below:

1. Throughout this study, the Fourier amplitude spectra are integrated to estimate their
spatial correlations. By doing this, the overall spatial correlation characteristics of
the spectra are considered by a simple model; the potential spatial correlation of the
FAS at different frequencies is ignored. The spatial correlation of the FAS at
different frequencies could be important and is missing in the literature. It would
be interesting to see how much this correlation would affect the seismic hazard/risk
assessment results.
2. The simulation approaches are developed for scenario events in this study, although the procedure to include the interevent correlations is outlined in Chapter 3. The interevent variability represents the uncertainties associated with different events. The interevent correlation must be treated appropriately if lifecycle cost analysis needs to be carried out.

3. For estimating the aggregate seismic loss, structures are modeled using single-degree-of-freedom systems. It is conceptually straightforward to include more complex building models and structures with spatially distributed supports within the proposed framework to calculate their aggregate seismic loss and nonlinear inelastic responses under scenario events. However, its implementation is likely to be challenging, an investigation of the impact of the structural modeling on the estimated seismic loss can be of great value.

4. The developed extensions to the stochastic simulation method are implemented for illustration only; they could be integrated to the current version of the simulation packages. The public availability of the extended simulation package user friendly interfaces will facilitate its use for various engineering applications.
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