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Dynamic Stochastic Analysis Of A Small Open Economy

Estrada Enrique Mendoza

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DYNAMIC STOCHASTIC ANALYSIS OF A SMALL OPEN ECONOMY

by

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ABSTRACT

This thesis consists of four essays that study the dynamic stochastic behavior of a small open economy. In the framework studied here, rational individuals formulate optimal intertemporal plans in an environment where domestic capital and foreign financial assets are used as alternative vehicles of savings, and where random disturbances affect the production technology or the international terms of trade. As demonstrated in the papers, the separation of savings and investment decisions that characterizes the individual's optimal plans in open economies has important implications for the evolution of macroeconomic aggregates.

The main contribution of this work is that it undertakes a *quantitative* exploration of the equilibrium stochastic process that characterizes the long-run dynamics of the small open economy. Chapters II, III and IV focus on different applications of this computational general-equilibrium modelling strategy, whereas the first chapter concentrates on a purely theoretical analysis. The thesis is ordered as follows. Chapter I presents a dynamic stochastic model and applies dynamic programming techniques to analyze impact and dynamic effects of transitory and persistent disturbances. The potential for this model to explain the kind of comovement and persistence observed in real-world macroeconomic data is formally established. The second essay employs dynamic numerical simulation methods to study the ability of the artificial economy to mimic the stylized facts that typify Canadian business cycles, including the correlation between savings and investment. Since the results of this analysis suggest that the model

allows the domestic capital stock to be adjusted too easily, the third paper investigates how the quantitative performance of the model is improved by introducing capital-adjustment costs. The resulting prototype replicates the majority of the Canadian stylized facts, and thus it constitutes a useful tool for dynamic policy analysis. The last paper utilizes this improved version of the model to quantitatively determine the effects of a policy that stabilizes the balance of trade by introducing tax-enforced capital controls. The results of this experiment indicate that, given the moderate magnitude of typical business cycles in the post-war period, the mentioned policy has minimal effects on economic activity and economic welfare.

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Este trabajo también está dedicado a la memoria de los miles de Mexicanos que murieron en los sismos del 19 y 20 de Septiembre de 1985.

This work is also dedicated to the memory of the thousands of Mexicans killed during the earthquakes of September 19 and 20 of 1985.

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CHAPTER I.

REAL BUSINESS CYCLES AND OPTIMAL FOREIGN-ASSET ACCUMULATION.

1.- Introduction.

The paper presented in this first chapter investigates the impact and dynamic effects that domestic and world-wide technological disturbances have on the evolution of the main macro-aggregates of a small open economy. The novelty of the essay is that it provides an integrated analysis of the dynamic interaction of foreign assets and domestic capital, as alternative vehicles of savings, in an economy subject to productivity or real-exchange-rate shocks.

Formally, the model studied in this paper consolidates real business cycle theory with the recent developments in dynamic open-economy macroeconomics. The basic real business cycle model constructed in the seminal works of Kydland and Prescott (1982) and Long and Plosser (1983) is extended to an environment where the existence of a perfectly competitive, international capital market is allowed. Consequently, the individuals that inhabit this small open economy have the option to borrow or save by trading foreign assets in the world capital market at a fixed real rate of return. Thus, in contrast with closed-economy real business cycle models, savings in the open-economy model can be undertaken in the form of domestic capital as well as in the form of foreign financial assets.

By considering a model where endogenous production and investment choices are made in a stochastic, infinite-horizon environment, this paper also attempts to study a more general formulation of the models typically analyzed in neoclassical dynamic open-economy macroeconomics. Hence, the results obtained in the deterministic infinite-horizon economy studied by Obstfeld (1981a) and the two-period exchange economies analyzed by Sachs (1981) and (1983) and Greenwood (1983) and (1984) are extended to a more complex framework[1].

Following the guidelines of Obstfeld (1981a) and (1981b), the model presented in this paper uses a formulation of lifetime utility where the rate of time preference is an increasing function of past consumption levels. This formulation is utilized to determine a well-defined deterministic stationary equilibrium for the holdings of foreign assets, so that the dynamics of consumption, the current account and the balance of trade can be properly characterized[2]. Furthermore, since the model studied here incorporates uncertainty, the endogenous time-preference framework adopts the specific form of the Stationary Cardinal Utility function formulated by Epstein (1983). This utility function is the analog of the formulation employed by Obstfeld for the case of an economy subject to stochastic disturbances and characterized by the consumption-smoothing effect[3].

The analysis of the impact and dynamic effects caused by transitory and permanent technological shocks illustrates the differences between closed-economy and open-economy real business cycle models. Essentially, the open-economy model is characterized by a separation of the optimal savings and investment decisions. The separation of these two choices allows optimizing agents to adjust the domestic capital stock according to pure portfolio considerations, without having to take into account the intertemporal income and substitution effects generated by the disturbances. The results of the analysis show how consumption-smoothing and portfolio-reallocation effects govern the dynamics of all the main macro-aggregates. In the perfect-foresight version of the model, it is demonstrated that temporary shocks have no persistent effects on capital accumulation, output and employment, and that it is through the process of foreign-asset accumulation that the economy reaches the steady-state equilibrium in savings and consumption. It is also illustrated that persistent disturbances are capable of generating business cycles, in the sense that they can reproduce qualitatively the same kind of comovement and persistence observed in the time paths of the macro-aggregates. Similarly, in the stochastic environment, it is illustrated that serially- uncorrelated shocks affect investment only to the extent that they affect the expected differential in the real rates of return of the two existing vehicles of savings.

The paper proceeds as follows: Section 2 describes the model and studies the optimality conditions that prevail in the stochastic environment. Section 3 analyses the impact and dynamic effects of transitory and permanent technological disturbances in a perfect-foresight environment, considering first an economy where the shocks affect only domestic productivity and then extending the analysis to incorporate world-wide disturbances. The effects caused by serially-uncorrelated, stochastic shocks are investigated in section 4. Section 5 presents the main conclusions and three appendices develop the mathematical analysis needed to support the results presented in the text.

2.- A Dynamic Stochastic Model of a Small Open Economy.

This section of the chapter extends the endogenous-impatience stochastic-growth model constructed by Epstein (1983) to a small open-economy framework. Here, the individuals that inhabit the domestic economy have access to an international, perfectly competitive capital market. The model to be presented is a variation of the prototype originally designed by Brock and Mirman (1972), which in turn served as the starting point for the pioneering work on real business cycles by Kydland and Prescott (1982) and Long and Plosser (1983).

Consider an economy populated by identical, infinitely-lived individuals, with preferences over goods and labor services described by the Stationary Cardinal Utility function[4]:

$$E_0 \left[\sum_{t=0}^{\infty} \left(u(C_t - G(L_t)) \exp\left[-\sum_{\tau=0}^{t-1} v(C_\tau - G(L_\tau)) \right] \right) \right]. \qquad (1)$$

Here, $G(L_t)$ measures the period-by-period disutility of labor L_t and is assumed to satisfy the properties that $G'(L_t)$ and $G''(L_t) > 0$. With this specification, intertemporal preferences can be studied in terms of the composite $C_t - G(L_t)$, where C_t is the individuals' consumption. $v(\cdot)$ is a positive function called the impatience function and $u(\cdot)$ is the instantaneous utility function. As in Obstfeld (1981a) and (1981b), the instantaneous rate of time preference $\exp[v(C_t - G(L_t))]$ is modelled as an increasing function of consumption of the composite good, thus $v'(\cdot)>0$. The rational expectation formulated in (1) is conditional on knowledge of the initial state of the economy and the relevant stochastic processes, all of which are discussed in more detail later.

Since lifetime utility can be studied in terms of consumption of the composite $C_t - G(L_t)$, the conditions of Theorem 5 of Epstein (1983) can be directly introduced here. Epstein's conditions are the following:

$$u(\cdot)<0\ ,\ u'(\cdot)>0\ ,\ u'(0)=\infty,$$
$$v(\cdot)>0,\ v'(\cdot)>0,\ v''(\cdot)<0, \qquad (2)$$
$$\mathrm{Log}[-u(\cdot)]\ \text{convex},$$
$$u'(\cdot)\exp[v(\cdot)]\ \text{non-increasing}.$$

These conditions were employed by Epstein, in the context of a closed-economy model with either a neoclassical or a linear technology, to prove the existence of a stochastic steady state, to show that consumption is always a normal good (see Appendix 2) and to ensure time-consistency, so that dynamic programming methods can be applied[5]. Intuitively, these conditions imply that the model is characterized by an impatience effect, according to which increments in current consumption reduce the subjective weight assigned to future consumption benefits.

The cost of simplifying preferences in this way is that, since the marginal rate of substitution between C_t and L_t depends on L_t only, the wealth effect on the supply of labor is eliminated. This assumption, however, has two important advantages. First, as discussed above and as demonstrated later, it simplifies the formal analysis of the dynamics of the model and helps determine important properties of the value function involved in the solution of the model's equilibrium. Second, it also permits us to focus expressly on the interaction between investment and foreign-asset accumulation, as alternative mechanisms to reallocate resources intertemporally, without having to ignore completely the role of labor services as an input in production.

Domestic production possibilities are characterized by a strictly concave constant-returns-to-scale production function multiplied by a positive stochastic disturbance ϵ

$$Y_t = \epsilon_t F(K_t, L_t). \tag{3}$$

Where $F_K(t)$, $F_L(t)$, $F_{KL}(t) > 0$ and $F_{KK}(t)F_{LL}(t) - (F_{KL}(t))^2 = 0$. Thus, agents in this economy combine the domestic capital stock K_t, which is predetermined one period in advance, with the services of labor to produce a single tradable commodity Y_t. The terms of trade at which this output is exchanged in world markets are taken as given and, without loss of generality, are set equal to unity. Alternatively, as discussed in Greenwood (1983), it is possible to assume that the domestic economy produces an exportable composite good, so that the technological disturbances may take the form of terms-of-trade shocks.

The domestic capital stock is assumed to evolve according to the following condition:

$$K_{t+1} = I_t + K_t(1-\delta). \tag{4}$$

Where I_t is gross investment and δ is a constant depreciation rate.

Individuals also have access to an international, perfectly competitive capital market where they can trade foreign financial assets A_t taking as given their real rate of return $r^*\eta_t$. Where η_t is a stochastic world-wide technological disturbance. In each period, individuals posses a certain initial level of foreign assets, or debt, A_t which is carried over from the previous period. These assets yield a total return equal to $(1+r^*\eta_t)A_t$ which is added to, or substracted from, the total resources available for consumption and savings. Savings are accumulated not only in the form of domestic capital K_{t+1} but also in the form of foreign assets A_{t+1}, with the difference that the latter can be negative, expressing the ability that agents have to borrow from the world capital market against their expected future income. The accumulation of foreign financial assets obeys the following asset evolution equation:

$$A_{t+1} = TB_t + A_t(1+r^*\eta_t), \tag{5}$$

where TB_t is the balance of trade.

The period-by-period resource constraint faced by individuals restricts the sum of domestic absorption, C_t+I_t, plus the current account, $A_{t+1}-A_t$, not to exceed GNP[6]:

$$C_t + I_t + A_{t+1} - A_t \leq Y_t + \eta_t r^* A_t. \tag{6}$$

Given the initial conditions K_0, A_0, ϵ_0 and η_0, the dynamic optimization problem of maximizing (1), subject to the restrictions defined in (3)-(6), the non-negativity constraints on K_t, L_t and C_t and the restriction of intertemporal solvency, determines the optimal intertemporal choices of the individuals[7]. Accordingly, they formulate optimal state-contingent decision rules for consumption, labor supply, capital accumulation and foreign-asset accumulation that characterize the equilibrium of the economy. To solve this problem, individuals must also know the probabilistic structure that governs the evolution of the disturbances, which are assumed to follow Markov processes. Thus, the knowledge of the initial values of K, A, ϵ and η constitutes all the information that agents need in order to rationally formulate their optimal intertemporal plans. The same optimization problem can be identically investigated, in a more tractable manner, by employing dynamic programming techniques.

The functional equation that characterizes the solution of this problem in dynamic programming form can be simplified by merging the state variables that describe the initial situation of the economy in a single aggregate variable Q_t. To define Q_t, start by considering the first-order optimality condition for labor supply obtained from the optimization problem described in the previous paragraph,

$$\epsilon_t F_L(K_t, L_t) = G'(L_t), \qquad \text{for all } t=0,\ldots,\infty. \tag{7}$$

Equation (7) shows that the optimal labor decision is determined by selecting L_t such that the marginal disutility of labor equals its marginal benefit, which is in turn measured by its marginal product. The specific form that this condition adopts is given by the absence of wealth effects on labor supply discussed earlier. This optimality condition requires, therefore, that the optimal labor choice $\hat{L}_t$ be a function of K_t and ϵ_t only:

$$\hat{L}_t = \ell(K_t, \epsilon_t). \tag{8}$$

Furthermore, the assumptions about $F(\cdot,\cdot)$ and $G(\cdot)$ imply that ℓ_K and $\ell_\epsilon > 0$ (see Appendix 2). Thus, from the point of view of an agent for whom the current values of K_t and ϵ_t are predetermined, the optimal labor supply choice is uniquely assigned by the same two state variables.

The aggregate state variable Q_t is defined next. Q_t is a function that describes the maximum units of the composite $C_t - G(L_t)$ that are feasible to consume given the initial values of foreign assets, capital and the shocks:

$$Q_t(K_t,A_t,\epsilon_t,\eta_t) = \epsilon_t F(K_t,\hat{L}_t) + K_t(1-d) + A_t(1+r^*\eta_t) - G(\hat{L}_t). \qquad (9)$$

Thus, Q_t describes completely the current state of the economy for all practical purposes related to the agents' dynamic optimization problem.

Considering equations (8) and (9), the resource constraint (6) can be equally expressed as

$$C_t - G(\hat{L}_t) = Q_t - S_{t+1}. \qquad (10)$$

Where S_{t+1} is the sum of total gross savings allocated to domestic capital and foreign assets

$$S_{t+1} = K_{t+1} + A_{t+1}. \qquad (11)$$

With this simplification in mind, the time-recursive structure imposed with the Stationary Cardinal Utility, the resource constraint (10) and the properties of $u(\cdot)$, $v(\cdot)$ and $F(\cdot,\cdot)$ are all combined to define the following dynamic programming problem:

$$V(Q_t) = \max \{u[C_t - G(\hat{L}_t)] + \exp[-v(C_t - G(\hat{L}_t))] \int V(Q_{t+1}) P(\lambda_{t+1}|\lambda) d\lambda_{t+1}\}$$

$$\text{w.r.t} \quad \{ C_t, K_{t+1}, S_{t+1} \}_{t=0}^{\infty},$$

$$\text{s.t.} \quad C_t = Q_t + G(\hat{L}_t) - S_{t+1}. \qquad (12)$$

Where $Q_{t+1} = \epsilon_{t+1}F(K_{t+1},\hat{L}_{t+1}) + (1+r^*\eta_{t+1})(S_{t+1}-K_{t+1}) + (1-d)K_{t+1} - G(\hat{L}_{t+1})$. The stochastic shocks ϵ_t and η_t are known at the beginning of period t. λ_{t+1} denotes the pair of values that these technological disturbances may take in the next period (i.e. $\lambda_{t+1}=(\epsilon_{t+1},\eta_{t+1})$) and, since λ is assumed to evolve as a Markov process, $P(\lambda_{t+1}|\lambda_t)$ designates the one-step transition probability of facing a particular pair of shocks conditional on today's pair[8]. All possible values of the two shocks are drawn from a compact set Λ. Thus, the initial capital stock, the initial holdings of foreign assets and the currently-observed values of both disturbances contain all relevant information needed to formulate a rational expectation of the value function $V(Q_{t+1})$ -this information is in turn summarized in the initial observed value of Q.

By substituting the constraint in the functional equation, it transpires that the first-order conditions characterizing the optimal savings and capital-accumulation choices are the following[9]:

$$u'(t) - v'(t)\exp[-v(t)]E_t[V(t+1)] = \exp[-v(t)]\ E_t[V'(t+1)(1+r^*\eta_{t+1})], \qquad (13)$$

$$\exp[-v(t)]\ E_t\{\ V'(t+1)\ [\epsilon_{t+1}F_K(t+1)-d - r^*\eta_{t+1}]\ \} = 0, \qquad (14)$$

where $E_t[V(t+1)] = \int V(Q_{t+1})P(\lambda_{t+1}|\lambda)d\lambda_{t+1}$. These optimality conditions illustrate the separation of the optimal savings and investment decisions typical of the open-economy framework. The total amount of savings to accumulate is governed by (13) whereas the optimal allocation of those savings across domestic capital and foreign assets is determined by (14). These two optimality conditions are more easily interpreted by applying the Benveniste-Scheinkman equation to obtain the following expressions:

$$u'(t) - v'(t)\exp[-v(t)]E_tV(t+1) = \qquad (15)$$
$$\exp[-v(t)]\ E_t\{\ (1+r^*\eta_{t+1})\ [u'(t+1)-v'(t+1)\exp[-v(t+1)]E_{t+1}V(t+2)]\ \},$$

$$E_t\{ [\epsilon_{t+1}F_K(t+1)-d]\ U_C(t+1) \} = E_t\{ [r^*\eta_{t+1}]\ U_C(t+1) \}. \quad (16)$$

Where $U_C(t+1) = u'(t+1) - v'(t+1)\exp[-v(t+1)]E_{t+1}[V(t+2)]$ is the lifetime marginal utility of consumption at t+1.

Equation (15) represents the equilibrium condition for savings under uncertainty and illustrates the difference between the constant and variable discount factor cases. The left-hand side denotes the expected marginal lifetime benefit of reducing savings. This consists of two components: (i) the instantaneous marginal utility effect ($u'(t)>0$) and (ii) the impatience effect, by which an increase in the rate of time preference reduces the subjective weight assigned to all consumption benefits expected in the future ($-v'(t)\exp[-v(t)]E_tV(t+1)>0$ since $u(t)<0$ implies $E_tV(t+1)<0$). The right-hand side is the discounted expected marginal cost of reducing savings in utility terms. This is composed of the anticipated return in assets that is given up, converted into next period's lifetime marginal utility, and subjectively discounted to make it comparable with the current period's lifetime marginal utility at the other side of the equation. In equilibrium, lifetime marginal costs and benefits of savings are equalized.

The optimality condition (16) indicates that investment is regulated by a criterion that establishes the equality of expected marginal returns in different vehicles of savings. As can be observed, this equality does not hold exactly period by period, but it holds as a weighted average using as weights the lifetime marginal utilities that can be attained in each state of nature. Again, the form of U_C indicates that changes in any period's consumption have an instantaneous marginal-utility effect and an additional impatience effect. The conditions imposed on u(•) and v(•) ensure that U_C is positive and greater than in the fixed time-preference case.

Equations (15) and (16) can also be used to illustrate how the expected differential in the marginal rates of return $E[DMR_{t+1}]=E[RK_{t+1}-RA_{t+1}]$ is linked to the stochastic structure of the problem. Using (16), it transpires that in equilibrium this differential can be expressed as

$$E[DMR_{t+1}] = \{COV[U_C(t+1),RA_{t+1}]-COV[U_C(t+1),RK_{t+1}]\} / E[U_C(t+1)], \quad (17)$$

where $RK_{t+1} = \epsilon_{t+1}F_K(t+1)+1-d$ is the random return on capital and $RA_{t+1}= (1+r^*\eta_{t+1})$ is the random return on foreign assets. This expression indicates that $E[DMR_{t+1}]$ is determined by the covariance structure between the two marginal rates of return and the lifetime marginal utility of consumption in the same period that these returns are paid. For example, in the case where the disturbances are assumed to affect domestic productivity only, the first covariance term is likely to be negative since a shock with a certain degree of persistence causes $U_C(t+1)$ to fall and RK_{t+1} to increase, and the second covariance term is zero because foreign assets are riskless. Therefore, capital is a risky asset and individuals expect to receive a premium when holding it (i.e. $E[DMR_{t+1}]>0$).

3.- Technological Disturbances in a Perfect-Foresight World.

The theoretical setup outlined in the last section is first studied in the context of a perfect-foresight economy. Here, the impact and dynamic effects of temporary and permanent disturbances are precisely determined and the driving forces of the model's dynamic behavior are clearly isolated. The study of the perfect foresight-economy simplifies the analysis because the absence of uncertainty implies that the rates of return on capital and foreign assets are *exactly* equalized in each period. Consequently, the optimal capital-accumulation choice is independent of the dynamics of consumption and foreign-asset holdings.

The intertemporal optimality conditions in the perfect-foresight case take the following form:

$$u'(t) - v'(t)\exp[-v(t)]V(t+1) = \exp[-v(t)](1+r^{*}\eta_{t+1})V'(t+1), \qquad (18)$$

$$\epsilon_{t+1}F_K(t+1) - d = r^{*}\eta_{t+1}. \qquad (19)$$

This is a block-recursive simultaneous equation system where K_{t+1} is determined by (19) and S_{t+1} is defined by (18). Using the definitions and constraints mentioned in the last section, the solutions of these two variables determine solutions for the rest of the endogenous variables in the model.

Condition (19) shows that, once ϵ_{t+1} and η_{t+1} are both given, the optimal choice for the allocation of savings in the form of capital $\hat{K}_{t+1}$ is determined by equating the next period's net marginal productivity of domestic capital with the real rate of return on foreign assets. Thus, the optimal capital stock is given by

$$\hat{K}_{t+1} = \kappa(\epsilon_{t+1}, \eta_{t+1}), \qquad (20)$$

where $\kappa_\epsilon > 0$ and $\kappa_\eta < 0$ as shown in Appendix 2.

Given (3) and (8), it follows from (20) that the dynamics of labor services and domestic output also depend only on the dynamic behavior of the disturbances. Thus, the time-profiles of investment, capital, labor and output are independent of the dynamics of savings and foreign assets, which govern the evolution of consumption toward the steady-state equilibrium.

The Benveniste-Scheinkman theorem is applied to (18) in order to obtain the next expression:

$$u'(t) - v'(t)\exp[-v(t)]V(t+1) = \exp[-v(t)]\ \{(1+r^{*}\eta_{t+1})[u'(t+1) - v'(t+1)\exp[-v(t+1)]V(t+2)]\}. \tag{21}$$

This equality describes the principle governing optimal savings in this dynamic, deterministic framework. Its interpretation is the same as before, except that both the marginal costs and benefits of postponing consumption are known with certainty.

From (21) it follows that the economy's perfect-foresight stationary equilibrium is characterized as

$$\exp\ \{\ v[\epsilon F(\mathbf{k},\ell(\mathbf{k},\epsilon)) - \mathbf{k}d + r^{*}\eta\mathbf{A} - G(\ell(\mathbf{k},\epsilon))]\ \} = 1 + r^{*}\eta. \tag{22}$$

Boldface letters are used here to refer to steady-state values of variables. Expressions (22) and (19) indicate, therefore, that the stationary equilibrium of the perfect-foresight economy is characterized by a simultaneous equality between the net marginal productivity of capital, the subjective rate of time preference and the world's real interest rate[10].

Although the steady-state equilibrium of some closed-economy real business cycle models is also characterized by an equality of the rate of time preference and the productivity of capital, the manner in which this stationary equilibrium is reached in a small open economy is radically different. Here, whenever the domestic capital stock is below (above) the deterministic steady-state equilibrium, agents are free to borrow (lend) in order to take advantage of the relatively higher return on capital (foreign assets). They do not have to be concerned with having to sacrifice consumption in order to augment investment, and with the consumption-smoothing and consumption-substitution effects involved, since the level of consumption can be sustained by running a deficit in the balance of trade.

3.1 Impact and Dynamic Effects of Domestic Productivity Disturbances.

The study of an economy characterized by purely domestic shocks is the simplest case of open-economy real business cycles. The international real rate of return on assets is assumed to remain constant, either because technology in the rest of the world is not affected by any shocks or because national disturbances cancel out in the global average.

The impact and dynamic effects of temporary and permanent domestic shocks are uncovered by undertaking a series of comparative-statics experiments. Equations (3), (5), (7), (9), (18), (19), the resource constraint in (12) and the definition of savings (11) are differentiated with respect to the endogenous variables S_{t+1}, K_{t+1}, A_{t+1}, L_t, Y_t, C_t and TB_t and the domestic shock, assuming that $\eta=1$ at all dates. The exogenous change is $d\epsilon_t > 0$ for the case of a transitory shock and $d\epsilon_t=d\epsilon_{t+1} > 0$ for a permanent disturbance. The impact effects of an unanticipated temporary shock are found to be the following (see

Appendix 1):

$$dY_t/d\epsilon_t = F(t) + \epsilon_t F_L(t)[dL_t/d\epsilon_t] > 0, \quad (23)$$

$$dC_t/d\epsilon_t = [(1+r^*)b(t)/a(t)+(1+r^*)b(t)]\ F(t) + \epsilon_t F_L(t)\ [dL_t/d\epsilon_t] > 0, \quad (24)$$

$$dL_t/d\epsilon_t = F_L(t)\ /\ [G''(t) - \epsilon_t F_{LL}(t)] > 0, \quad (25)$$

$$dK_{t+1}/d\epsilon_t = 0, \quad (26)$$

$$dS_{t+1}/d\epsilon_t = dA_{t+1}/d\epsilon_t = dTB_t/d\epsilon_t = [a(t)/a(t)+(1+r^*)b(t)]\ F(t) > 0. \quad (27)$$

Where $a(t) = \exp[v(t)][\ u''(t) + u'(t)v'(t)] - V(t+1)v''(t) < 0$ and $b(t) = v'(t)\ V'(t+1) + (1+r^*)V''(t+1) < 0$ as shown in Appendix 2.

The expression in (23) shows the positive impact effect that temporary technological shocks have on current output. The shock has both a direct effect, since it multiplies the production function in the definition of Y_t, and an indirect effect, because it improves the productivity of labor and induces an increase in employment -as shown in (25).

Equation (24) reveals that a once-and-for-all shock increases C_t as the result of two effects. First, a consumption-smoothing effect, denoted by $[(1+r^*)b(t)/a(t)+(1+r^*)b(t)]F(t)$, which directs individuals to use the current-output expansion to increase consumption in all periods. Hence, consumption does not grow as much as output in the same period and the balance of trade moves toward a surplus (as confirmed in (27)). The second effect, represented by the second term in the right-hand side of (24), shows how the increase in employment creates a further enlargement in Y_t that is fully added to C_t.

The result reported in (25), indicating that employment increases unambiguously, follows from the absence of wealth effects on labor supply which are eliminated by adopting the specific forms of v(•) and u(•) defined in (1). This means that only the substitution effect, making current leisure more expensive when a productivity improvement takes place, is affecting the behavior of employment.

Expression (26) illustrates that temporary shocks cannot affect the optimal capital-accumulation choice. Since these kind of disturbances cannot alter the productivity of newly-created capital, the equality of returns established in (19) is not perturbed and hence the optimal K_{t+1} remains unchanged. Furthermore, it follows from the capital-evolution equation (4) that investment in the current period is also completely insensitive to transitory productivity changes.

In contrast, the result in (27) shows that foreign-asset holdings, the balance of trade and total savings react positively and exactly in the same manner to a transitory disturbance[11]. This must be the case because individuals wish to increase consumption permanently and the extra savings needed to achieve this goal are accumulated exclusively in the form of additional foreign-asset holdings. Thus, individuals desire to smooth consumption by expanding their savings and the optimal way to do it, according to the results presented here, is by augmenting their holdings of foreign financial assets without affecting the accumulation of domestic capital.

Since a transitory disturbance leaves unaffected K_{t+1} and I_t, it follows from (3) and (8) that it can neither affect future values of capital, labor and domestic output. The capital stock remains at the level implied by the condition of equality of returns on foreign assets and domestic capital, and output and employment remain at the corresponding levels implied by the production technology (3) and the optimal labor-supply choice (8). Consequently, the dynamics of consumption towards the steady state are governed by the process of foreign-asset accumulation. These dynamic effects of a temporary shock are formally uncovered by solving a comparative-statics experiment using the same equations as before, except that the exogenous change considered now is $dS_t = dA_t > 0$ and the results must be updated one period to obtain the following expressions (see Appendix 2):

$$dY_{t+1}/dS_{t+1} = 0, \tag{28}$$

$$dL_{t+1}/dS_{t+1} = 0, \tag{29}$$

$$dK_{t+2}/dS_{t+1} = 0, \tag{30}$$

$$dA_{t+2}/dS_{t+1} = dS_{t+2}/dS_{t+1}$$
$$= [a(t+1) \;/\; a(t+1)+(1+r^*)b(t+1)]\;(1+r^*) > 0, \tag{31}$$

$$dC_{t+1}/dS_{t+1} = (-dTB_{t+1}/dS_{t+1})$$
$$= [(1+r^*)b(t+1) \;/\; a(t+1)+(1+r^*)b(t+1)](1+r^*) > 0. \tag{32}$$

Equations (28), (29) and (30) illustrate the implications of the fact that the equality of returns on capital and foreign assets is not disturbed by a purely transitory shock -as explained above. The expression in (31) describes the dynamic effects of the shock on savings and foreign-asset holdings, it illustrates how a fraction of the return on savings from the first period is used to increase savings for the future so as to increase consumption permanently. This is in turn implied by the consumption smoothing effect that characterizes the model. The rest of the return from savings is used to increase C_{t+1} as shown in (32). Since Y_{t+1} remains unchanged, this extra consumption is entirely supported by pushing the balance of trade in period t+1 towards a deficit. Thus, C_{t+1} is augmented by increasing imports and the increase in imports is in turn financed with a fraction of the gross return on foreign assets accumulated in the first period as a means of savings.

To summarize, a once-and-for-all, unanticipated shock that hits the economy at date t has the following impact and dynamic effects: (i) it increases C_t, C_{t+1}, L_t, Y_t, S_{t+1}, S_{t+2}, A_{t+1} and A_{t+2}, (ii) it improves TB_t and worsens TB_{t+1} and (iii) it leaves unaffected K_{t+1}, I_t, K_{t+2}, I_{t+1}, L_{t+1} and Y_{t+1}. In sharp contrast with the results that can be obtained by practicing a similar analysis in a closed-economy real business cycle model, these results indicate that transitory productivity shocks in a small open economy cannot reproduce qualitatively *all* of the observed features of the business cycle. The positive comovement between domestic output, employment and consumption is reproduced, as is the persistence in the accumulation of foreign assets and consumption. But there is no comovement nor persistence in investment and capital, the trade balance exhibits negative autocorrelation, employment and output are serially uncorrelated, and foreign-asset holdings and the balance of trade display positive comovement with GDP. Thus, for productivity disturbances to generate the observed properties of actual business cycles in a small open economy, it must be the case that these disturbances affect the productivity of newly-created capital (i.e. that they exhibit a certain degree of positive serial correlation).

The effects of a permanent, fully-anticipated productivity improvement are studied now. The same set of equations as before is analyzed assuming that $d\epsilon_t = d\epsilon_{t+1} > 0$. The impact effects of such a disturbance are the following (see Appendix 1):

$$dY_t/d\epsilon_t = F(t) + \epsilon_t F_L(t)[dL_t/d\epsilon_t] > 0, \tag{33}$$

$$dC_t/d\epsilon_t = [(1+r^*)b(t)/a(t)+(1+r^*)b(t)]\ [F(t) + F(t+1)/(1+r^*)] + \epsilon_t F_L(t)\ [dL_t/d\epsilon_t] > 0, \tag{34}$$

$$dL_t/d\epsilon_t = \epsilon_t F_L(t) / [G''(t) - \epsilon_t F_{LL}(t)] > 0, \tag{35}$$

$$dL_{t+1}/d\epsilon_t = \{ F_{KK}(t+1)F_L(t+1) - F_{LK}(t+1)F_K(t+1) \} / (\cdot) > 0, \tag{36}$$

$$dK_{t+1}/d\epsilon_t = - \{ [G''(t+1) - \epsilon_{t+1}F_{LL}(t+1)]F_K(t+1) + \epsilon_{t+1}^2 F_{KL}(t+1)F_L(t+1) \} / \epsilon_{t+1}(\cdot) > 0, \tag{37}$$

$$dS_{t+1}/d\epsilon_t = [a(t)/(a(t)+(1+r^*)b(t))]\ F(t) - [(1+r^*)b(t)/(a(t)+(1+r^*)b(t))]\ [F(t+1)/(1+r^*)] \gtreqless 0, \tag{38}$$

$$dA_{t+1}/d\epsilon_t = dTB_t/d\epsilon_t = dS_{t+1}/d\epsilon_t - dK_{t+1}/d\epsilon_t \gtreqless 0. \tag{39}$$

Where $(\cdot) = G''(t+1)F_{KK}(t+1) - \epsilon_{t+1}[F_{LL}(t+1)F_{KK}(t+1) - F_{KL}(t+1)^2] < 0$ and $a(t)$ and $b(t)$ are as before.

From (33) and (35) it is clear that the impact effects of a permanent shock on Y_t and L_t are exactly the same as in the case of a transitory shock. This is another result that follows from the elimination of the wealth effect on labor supply.

Equation (34) illustrates that the consumption-smoothing effect pushes up current consumption in an amount proportional to the present value of the permanent output increase. Thus, having more output in each period causes a larger increase in wealth to be allocated across consumption in all periods.

Expression (36) shows that in this case L_{t+1} is also affected. The expression contains two effects that affect positively the productivity of labor and force individuals to substitute leisure for additional hours of employment. First, the increase in ϵ_{t+1} augments directly the marginal product of labor. Second, as explained below, there is a further increase in this marginal product caused by the enlarged holdings of domestic capital in combination with the assumptions of strict concavity of $F(\cdot,\cdot)$ and constant returns to scale.

The expression in (37) shows that the permanent shock increases K_{t+1} as well, this is also the result of two complementary effects. First, there is an investment-expansion or portfolio-reallocation effect. This effect operates because the increase in the marginal product of capital in period t+1 motivates agents to increase capital accumulation in order to restore the equality of returns expressed in (19). The second effect is due to the fact that the increase in L_{t+1} enlarges the marginal product of capital even more, as a result of the strict concavity and constant returns to scale that characterize the production technology.

Given the positive impact effects on K_{t+1} and L_{t+1}, it follows that Y_{t+1} is also positively affected. The direct effect of the disturbance that hits the economy at date t+1 is magnified by the increased use of capital and labor, which are the optimal response of individuals expecting the shock to occur with certainty.

Equation (38) illustrates that the impact effect of a permanent shock on first period's savings is ambiguous. A positive effect arises from that fraction of the extra current output that is not allocated for current consumption, but a negative effect also operates due to the increased borrowing induced by the positive impact that the expected expansion in Y_{t+1} has on C_t.

Equation (39) establishes that the resulting impact effects of a permanent shock on TB_t and A_{t+1} are also ambiguous, but always smaller than the impact effects caused by a once-and-for-all shock (i.e. equation (27)). In this case there are three operative forces, one pushing for a trade-balance surplus and two for a deficit. First there is the positive effect from the increase in Y_t that causes a permanent increase in consumption, and therefore makes Y_t grow more than C_t. Second, a negative effect exists because the increase in Y_{t+1} causes C_t to grow by a fraction $(1+r^*)b(t)/[a(t)+(1+r^*)b(t)]$ of the present value of that expected output increase. And third, an additional negative effect exists because the augmented capital holdings imply that savings are increased not only by extra holdings of assets, but also by investing more in the domestic capital stock. Therefore, the pressure for a trade-balance surplus that arises with a transitory disturbance is reduced, and may be dominated, by the negative effects that follow a persistent disturbance.

In summary, a permanent shock exhibits the following impact effects: (i) it increases C_t, L_t and Y_t, (ii) it increases K_{t+1} and I_t, thereby causing L_{t+1} and Y_{t+1} to increase, (iii) and it has an ambiguous, but relatively smaller, effect on A_{t+1} and TB_t. Furthermore, C_{t+1} also increases due to the consumption-smoothing effect and TB_{t+1} may move towards a surplus (due to the increase in Y_{t+1} combined with a less than proportional increase in C_{t+1}), or towards a deficit (due to the pro-borrowing movement that follows from the consumption-smoothing effect caused by the permanent output increase).

These results indicate, therefore, that persistent disturbances are potentially capable of reproducing the features of comovement and persistence that characterize actual business cycles. Specifically, if the disturbances are expected to last long enough, the model may recreate qualitatively all the characteristics of comovement and persistence that the closed-economy aggregates exhibit, as well as the observed negative correlation between the balance of trade and domestic output[12].

It is also important to observe the relationship between the initial state of the economy and the size of the impact effects caused by the disturbances. Equations (38) and (39) show that the larger the initial capital stock is, the stronger the positive effect of the current-output expansion on S_{t+1}, A_{t+1} and TB_t. Thus, the model predicts that, as the economy grows, shocks of the same magnitude cause larger fluctuations in domestic output, consumption, savings and the open-economy aggregates. These findings are consistent with the empirical analysis of Sachs (1983)[13].

The immediate dynamic effects caused by a permanent disturbance (i.e. those that follow changes in S_{t+1}, K_{t+1} and A_{t+1}) can be determined by using a similar strategy as before. But the primary difference between transitory and permanent disturbances is the effect that the latter have in displacing the economy to a new steady-state equilibrium. This displacement will be investigated after the study of the effects caused by international disturbances is completed.

3.2 Impact and Dynamic Effects of World-Wide Shocks.

A more complete model of the world business cycle requires the consideration of both national and international technological fluctuations. In principle, it is possible to model world-wide shocks as affecting all nations at the same time in the same way, or to consider that in addition to the international shocks each country has domestic disturbances of its own. Different assumptions about the persistence and contemporaneous correlation of these disturbances can also be made. In this part of the paper, the impact and dynamic effects of deterministic international shocks are uncovered using a very simple structure that highlights the key factors governing the dynamics of the system. To avoid unnecessary repetition of the mechanisms already exposed in the last section, the analysis focuses on the new features added by fluctuations in the world's interest rate.

First, assume that international disturbances are transmitted to the domestic economy according to the following continuouslydifferentiable function:

$$\epsilon_t = H(\eta_t), \qquad H(\cdot) > 0,\ H'(\cdot) > 0 \ \text{ for all t.} \qquad (40)$$

The condition $H'(\cdot) > 0$ is imposed in accordance with the assumption that world-wide productivity improvements affect positively the real rate of return paid or charged in the world's capital market[14]. $H(\cdot)$ is general enough to allow for national differences in the size of the disturbances, which are associated to such factors as differences in resource endowments, the use of the technology and economies of scale.

Persistence is modelled considering the following dynamic evolution equation for international shocks:

$$\eta_{t+1} = \theta\eta_t, \qquad 0 \leq \theta \leq 1. \qquad (41)$$

Where θ is the coefficient of persistence of the disturbances.

Once (40), (41), θ and the initial value of η are given, agents can determine precisely the dynamic paths of both shocks and possess all information necessary to formulate optimal intertemporal plans. The impact effects of world-wide disturbances on the desired choices of K_{t+1} and S_{t+1} are determined undertaking another comparative-statics experiment. The two main results of such analysis are the following (see Appendix 1):

$$dK_{t+1}/d\eta_t =$$
$$\theta \{r^* - H'(t+1)[F_K(t+1)+H(t+1)F_{KL}(t+1)\ell_\epsilon(t+1)]\} / \{\cdot\} \gtreqless 0, \qquad (42)$$

$$dS_{t+1}/d\eta_t = a(t)\ \{H'(t)F(t) + r^{*}A_t\}\ /\ [a(t) + (1+r^{*}\eta_{t+1})b(t)]$$
$$-\ \theta\ \{b(t)[H'(t+1)F(t+1) + r^{*}A_{t+1}]\}\ /\ [a(t) + (1+r^{*}\eta_{t+1})b(t)] \quad (43)$$
$$-\ \theta r^{*}V'(t+1)\ /\ [a(t) + (1+r^{*}\eta_{t+1})b(t)]\ \gtreqless 0,$$

where a(t), b(t) and $(\cdot) < 0$ are as before.

The expression in (42) illustrates the ambiguous impact effect that a favorable international shock has on domestic capital accumulation. The first term in the numerator represents the effect of the international component of the disturbance. It indicates that the increase in the world's real interest rate motivates agents to substitute domestic capital for foreign assets as a means of savings. The second term in the numerator represents the domestic component of the shock. It is essentially identical to the investment-expansion effect described in (37) and hence it moves domestic capital accumulation in the opposite direction to the first effect. Notice that, since $dK_{t+1}/d\eta_t=0$ when $\theta=0$, the disturbances must have a certain minimum degree of persistence in order to have any effect on capital accumulation. Thus, the result that in a real business cycle model of a small open economy the disturbances must affect the productivity of newly created capital is preserved.

Equation (43) shows the ambiguous impact effect that international shocks have on total savings. The first line illustrates the transitory wealth effects, the first term reproduces the result found in (27) and the second term adds the consumption-smoothing effect that results from the resources gained (lost) when the increase in η raises foreign interest income (payments). Accordingly, the direction of this effect depends on whether the domestic economy is a net borrower or saver in the world's financial markets at the beginning of date t (i.e. whether A_t is positive or negative). The second line represents the persistent wealth effects, the first component comes from the domestic output expansion (the same effect observed in (38)) and the second is caused by the pressure to use the resources gained (lost) by the increase in the interest rate to increase (reduce) consumption permanently. Thus, the direction of this effect depends on the sign of the net holdings of foreign assets initially planned for the following period (i.e. A_{t+1}). The single term in the last line of (43) is the consumption-substitution effect caused by the increment in the intertemporal relative price of consumption $r^*\eta_{t+1}$, it motivates individuals to reduce current consumption and therefore increase savings.

The analysis of equations (42) and (43) also suggests that the additional income and substitution effects introduced by the fluctuations in r^* are likely to be small in practice. According to (42), the size of the effect of the international disturbance on capital accumulation, and hence investment, is given by θr^*. Under normal conditions, where both θ and r^* are less than unity and the latter is less than 10%, this is a very weak effect. Furthermore, (43) indicates that the size of the income and substitution effects affecting S_{t+1} depends on r^*A_t when the shocks are transitory, and also on θr^* and θr^*A_{t+1} when the shocks have some persistence. Thus, as long as the world's real interest rate is low, the disturbances are stationary and foreign-interest income or payments are a relatively small component of GNP, world-wide shocks are not likely to affect significantly the results obtained with purely domestic shocks.

The impact effects that transitory and persistent world-wide disturbances have on the rest of the aggregates are summarized in Table 1. The main differences between domestic and world-wide shocks are illustrated by the additional effects discussed in the previous paragraphs. Mainly, the introduction of additional consumption-smoothing effects that result from changes in foreign interest income, or payments, and the introduction of consumption-substitution effects that result from changes in the intertemporal relative price of consumption. As can be observed in the table, world-wide disturbances must still have a certain degree of positive persistence in order to reproduce qualitatively the comovement among macroeconomic time series that is observed in actual business fluctuations.

Since once-and-for-all world-wide disturbances cannot affect investment, and alter savings only by affecting the accumulation of foreign financial assets, the dynamic effects caused by these shocks are the same as those described in (28)-(32). Thus, transitory shocks can only generate persistent effects on savings, consumption and the open-economy aggregates.

In contrast with the transitory shocks, persistent disturbances do have impact effects on K_{t+1} and I_t and hence they have dynamic effects on investment, output and employment as well. However, long-run dynamic effects can only exist if the coefficient θ is equal to 1.0. Such steady-state adjustments are analyzed next.

3.3 Long-Run Effects of Permanent Technology Shocks.

The study of steady-state comparative statics emphasizes the fact

Table 1.

The Impact Effects of Transitory and Persistent World-Wide Shocks.

Variable	Transitory Shocks $\theta=0$	Persistent Shocks $0<\theta\leq 1$
K_{t+1}	0	+/-
S_{t+1}	$+^{*}$	$+/-^{**}$
A_{t+1}	$+^{*}$	$+/-^{**}$
L_t	+	+
Y_t	+	+
C_t	$+^{*}$	$+^{*}$
I_t	0	+/-
TB_t	$+^{*}$	$+/-^{**}$

Notes: * This result assumes A_t, $A_{t+1} > 0$.
** The impact effect is smaller than when the shock is transitory.
See Appendix 1 for mathematical derivations.

that, in the long run, the allocation of savings across capital and foreign assets is made on the basis of pure portfolio considerations. It also shows how the introduction of the endogenous rate of time preference helps determine the steady-state equilibrium for the holdings of foreign financial assets.

The deterministic stationary equilibrium of the model economy studied here is characterized by equations (19) and (22), considering the steady-state values of the disturbances that result from (40) and (41). The perfect-foresight steady-state of the domestic capital stock K is, therefore, the value that solves

$$\epsilon\, F_K(K, \ell(K, \epsilon)) - d = r^* \eta, \tag{44}$$

given ϵ and η. And the steady-state equilibrium for the holdings of foreign assets A is the solution to

$$\exp\, \{v[\epsilon F(K, \ell(K, \epsilon)) - Kd + r^* \eta A - G(\ell(K, \epsilon))]\} = 1 + r^* \eta. \tag{45}$$

Practicing a comparative-statics exercise for $d\eta$, recalling that $\epsilon = H(\eta)$, it transpires that permanent world-wide disturbances have an ambiguous effect on the long-run equilibrium of the economy (see Appendix 1). The direction of the change in the long-run equilibrium value of the capital stock depends on which of the two shocks, ϵ or η, is stronger. The increase in the world's real rate of return, that results from the world-wide shock, motivates agents to permanently reduce their holdings of domestic capital in order to take advantage of the higher return that their savings can obtain on foreign assets. However, the increase in the productivity of domestic capital, that follows from the domestic shock, induces individuals to expand the steady-state capital stock. Since output and employment are uniquely determined by the capital stock and the shocks, the long-run effects of the disturbances on Y and L also depend on which of the two shocks is stronger. Finally, the direction of the changes in the long-run equilibrium values of consumption, savings, foreign assets and the balance of trade depend not only on the relative strength of the disturbances, but also on consumption-smoothing and consumption-substitution effects. The consumption-smoothing effect is ambiguously signed because a permanent increase in η may have positive or negative income effects depending on the initial sign of A.

4.- **Domestic Shocks in the Stochastic Economy.**

In this section of the paper, the impact and dynamic effects of serially uncorrelated, stochastic disturbances are studied. The impact effects on L_t and Y_t are given by the same conditions as before (i.e. equations (23) and (25)) because the wealth effect on labor supply has been eliminated and K_t is a state variable. To study the effects on the rest of the variables, a comparative-statics experiment is undertaken using equations (5) and (9)-(14). The exogenous change considered for this experiment is $d\epsilon_t>0$, and the assumption that the disturbances are exclusively domestic is adopted for simplicity.

To determine impact and dynamic effects on savings, capital and foreign-asset accumulation that are consistent with observed business fluctuations, it is necessary to impose appropriate conditions on the covariance structure that characterizes the stochastic version of the model. In particular, as Appendix 3 shows, it is necessary to specify adequate correlation properties between the marginal change in valuation V"(t+1) and the differential of rates of return DMR(t+1). However, these properties cannot be arbitrarily imposed, instead they must be identified as characteristics of the value function that follow from the conditions imposed on tastes and technology. But, since closed-form solutions for the kind of functional equation problem defined in (12) do not exist in general, these required characteristics are very hard to establish. Furthermore, as Lucas, Prescott and Stokey (1985) point out, the properties of second and higher order derivatives of the value function are not generally known and, therefore, the conditions required to obtain the desired behavior of V"(t+1) are difficult to produce analytically. For these reasons, the use of numerical simulation techniques is regarded as the best alternative to investigate the ability of models of this kind to recreate actual business cycles. This is the task of the next two chapters.

In spite of the issues mentioned above, it is still worthwhile to perform the comparative-statics experiment in order to observe how the behavior of individuals is altered by the introduction of uncertainty. Assuming that the correct properties are imposed on the covariance structure of the model, the impact effects of a serially-uncorrelated shock can be described as follows (see Appendix 3):

$$dS_{t+1}/d\epsilon_t = E[a(t)F(t)]\ E[c(t+1)] \ / \ |\Delta| \quad > \ 0, \tag{46}$$

$$dK_{t+1}/d\epsilon_t = E[a(t)F(t)]\ E\{(1+r^*)[V''(t+1)DMR(t+1)]\} \ / \ |\Delta| \ \gtreqless 0. \tag{47}$$

where:

$E[c(t+1)] = E[V''(t+1)DMR(t+1)^2+V'(t+1)\epsilon_{t+1}(F_{KK}(t+1)+F_{KL}(t+1)\ell_K(t+1))]$

and $|\Delta|$ is as in Appendix 3.

Equation (46) shows the positive impact effect that serially-uncorrelated disturbances have on total savings. This result follows from the consumption-smoothing effect that characterizes the model, which is in turn implied by the Stationary Cardinal Utility function. Thus, the consumption-smoothing effect is still operative and a movement towards a trade-balance surplus in period t arises.

Expression (47) illustrates the ambiguous impact effect that the shock has on domestic capital accumulation. This result follows from the fact that, given the probability distribution of the disturbances, agents formulate expectations about the future differential of the returns on domestic capital and foreign assets and create state-contingent plans that dictate their actions. Consequently, other things being equal, the size and direction of the impact effect on K_{t+1} depends on how large and persistent are the expected movements of DMR(t+1). As a result, relatively small shocks approximate the perfect-foresight model since K_{t+1} tends to remain relatively immobile. Furthermore, if a risk premium is expected in domestic capital and if the covariance between V"(t+1) and DMR(t+1) is negative, the impact effects of the serially uncorrelated shock on K_{t+1} and I_t are unambiguously positive.

The impact effects on foreign assets and the balance of trade are determined by the difference between (46) and (47). Consequently, they depend on both the strength of the consumption-smoothing effect and the expected permanence and magnitude of the disturbances.

As discussed in Appendix 3, the ambiguity of the impact effect on capital accumulation translates into ambiguities with respect to the direction of all the dynamic effects. The same assumption mentioned above about the sign of COV[V"(t+1),DMR(t+1)], is sufficient to ensure that the total dynamic effects on both K_{t+1} and S_{t+1} are positive. A favorable, serially uncorrelated shock would then cause persistent increments in domestic output, employment, capital and savings. Even then, however, the dynamic effects on investment and the open-economy accounts remain ambiguously signed, depending on the relative size of the increments in K_{t+2}, K_{t+1}, S_{t+2} and S_{t+1}.

In conclusion, the analytical treatment of stochastic disturbances does not yield precise predictions about the behavior of the model economy and can only partially depict its ability to recreate the observed features of the business cycle. The study shows how the choices made by optimizing agents are affected by the presence of uncertainty and the expectations they create. In particular, it clarifies that relatively small shocks produce similar results as those observed in the perfect-foresight case. However, as stated before, the best way to investigate how precisely can this model replicate the qualitative and quantitative features of actual business cycles is to perform an analysis based on numerical simulation.

5.- Concluding Remarks.

In this first chapter, a stochastic dynamic optimizing model that incorporates a perfectly competitive, international capital market into the standard real business cycle model has been constructed and formally analyzed. The optimality conditions that characterize the equilibrium of the artificial economy indicated that small open-economy real business cycle models are typified by a separation of the savings and investment decisions. Savings choices are made so as to equalize the expected lifetime marginal benefits and costs of postponing consumption, whereas the allocation of these savings across domestic capital and foreign assets is chosen so as to equalize their expected marginal rates of return in utility terms.

Impact and dynamic effects of persistent and transitory technological shocks were investigated considering both domestic and world-wide disturbances in a perfect-foresight environment. In sharp contrast with closed-economy real business cycle models, transitory shocks cannot generate persistent effects on investment, employment and output. Furthermore, they cause foreign assets and the balance of trade to follow procyclical time paths. Temporary disturbances still have permanent effects on total savings and consumption, but these operate through the balance of trade instead of investment. On the other hand, recurrent disturbances induce persistence on investment and can recreate all the closed-economy features of the business cycle. The countercyclical nature of the open-economy aggregates can also be reproduced depending on the relative strength of the different consumption-smoothing and consumption-substitution effects involved. Hence, only shocks that affect the productivity of newly created capital are capable of recreating the comovement and persistence features observed in actual business fluctuations. Positive steady-state shocks have ambiguous effects on the stationary equilibrium depending on which of the rates of return of the two existing vehicles of savings is increased more, and also depending on whether the economy started out as a long-run borrower or lender in the world's financial markets.

The formal analysis of the effects caused by stochastic productivity disturbances required the imposition of appropriate conditions on the covariance structure of the economy. Under a certain set of conditions, it is possible to show that serially-uncorrelated shocks cause total savings to increase and have an ambiguous effect on capital accumulation. The adjustment in the dynamic stochastic path of investment depends on how large and persistent are the expected differences on the rates of return of domestic capital and foreign assets. Thus, the relative permanence and magnitude of the disturbances plays a crucial role in explaining the volatility and serial correlation of the endogenous variables in the model. Shocks that are small in size and serially uncorrelated tend to reproduce the results of the deterministic analysis, in the sense that the dynamics of savings tend to rely more on foreign-asset accumulation than in domestic investment.

The impossibility to use comparative-statics methods to determine the potential of the stochastic version of the model to reproduce actual business cycles, indicates that an investigation employing dynamic numerical simulation techniques should be undertaken. This is the goal of the following chapters.

APPENDIX 1.

This appendix describes how impact and dynamic effects of deterministic, transitory and permanent shocks were derived. The case of domestic shocks is treated in detail and the extension to world-wide disturbances is briefly explored. Some important properties of the value function and the optimal labor and capital decision rules are used, these are formally established in Appendix 2.

The impact effects of domestic disturbances are uncovered studying equations (3), (5), (7), (9)-(11), (18) and (19). Start by differentiating equations (7), (18) and (19) with respect to ϵ_t and the endogenous variables L_t, S_{t+1} and K_{t+1}. The resulting system of equations can be described as follows:

(A.1)

$$\begin{bmatrix} [a(t)+b(t)(1+r^*)] & 0 & 0 \\ 0 & [G''(t)-\epsilon F_{LL}(t)] & 0 \\ 0 & 0 & \epsilon_{t+1}[F_{KL}(t+1)\ell_K(t+1)+F_{KK}(t+1)] \end{bmatrix} \begin{bmatrix} dS_{t+1}/d\epsilon_t \\ dL_t/d\epsilon_t \\ dK_{t+1}/d\epsilon_t \end{bmatrix} = \begin{bmatrix} a(t)\ F(t) \\ F_L(t) \\ 0 \end{bmatrix}$$

where: $a(t) = \exp(v(t))[u''(t) + u'(t)v'(t)] - v''(t)V(t+1)$ and

$b(t) = v'(t)V'(t+1) + (1+r^*)V''(t+1)$.

The propositions and lemmas in Appendix 2 show that the first and last elements on the main diagonal of the coefficient matrix are negative and that the middle one is positive. Results (25)-(27) are obtained by solving the system of equations in (A.1). The result reported in equation (23) is then deduced by differentiation of (3), and (24) follows from differentiating (9), (10) and (11).

The dynamic effects reported in equations (28)-(30) are direct implications of the independence arguments to be presented in Propositions 2 and 3. The dynamic effects on savings, foreign assets, consumption and the balance of trade require to differentiate (18) with respect to S_{t+1} and S_t:

$$dS_{t+1}/dS_t = a(t)(1+r^*)/[a(t)+(1+r^*)b(t)] > 0 \tag{A.2}$$

The positive sign of this expression follows form property (3) of Proposition 1. Then, (31) is found by updating (A.2), using (30) and differentiating (11) updated. Finally, (32) is determined by differentiation of updated versions of (9)-(10) and using the results obtained in (28)-(31).

The impact effects of a permanent domestic shock are uncovered by analyzing equations (3), (5), (7), (9)-(11), (18), (19) and (7) updated one period. These equations are differentiated with respect to the endogenous variables Y_t, TB_t, L_t, L_{t+1}, S_{t+1}, K_{t+1}, A_{t+1} and C_t and with respect to the exogenous disturbances ϵ_t and ϵ_{t+1} (assuming $d\epsilon_t = d\epsilon_{t+1}$). The total differentials of (7), (18), (19) and (7) updated are condensed in the matrix system (A.3) that appears in the following page.

(A.3)

$$
\begin{bmatrix}
[a(t)+b(t)(1+r^*)] & 0 & 0 & 0 \\
0 & [G''(t)-\epsilon_t F_{LL}(t)] & 0 & 0 \\
0 & 0 & [G''(t+1)-\epsilon_{t+1}F_{LL}(t+1)] & -\epsilon_{t+1}F_{KL}(t+1) \\
0 & 0 & \epsilon_{t+1}F_{KL}(t+1) & \epsilon_{t+1}F_{KK}(t+1)
\end{bmatrix}
\begin{bmatrix}
dS_{t+1}/d\epsilon_t \\ dL_t/d\epsilon_t \\ dL_{t+1}/d\epsilon_t \\ dK_{t+1}/d\epsilon_t
\end{bmatrix}
=
\begin{bmatrix}
a(t)F(t) - (1+r^*)b(t)[F(t+1)/(1+r^*)] \\ F_L(t) \\ F_L(t+1) \\ -F_K(t+1)
\end{bmatrix}
$$

Solving $dS_{t+1}/d\epsilon_t$ and $dL_t/d\epsilon_t$ from (A.3) is straightforward. Propositions 1 and 2 in Appendix 2 determine the signs of the coefficients involved in both solutions, so that expressions (35) and (38) are obtained. The impact effect on Y_t follows from differentiating (3) with respect to ϵ_t using (35). The results for $(dL_{t+1}/d\epsilon_t)$ and $(dK_{t+1}/d\epsilon_t)$ are derived solving the simultaneous system defined by the submatrix located at the lower right corner of (A.3). Propositions 2 and 3 in Appendix 2 show that the determinant of such a system is positive, and the results in (36) and (37) follow from Cramer's rule. Finally, expressions (34) and (39), containing the impact effects $(dA_{t+1}/d\epsilon_t)$ and $(dC_t/d\epsilon_t)$, are obtained differentiating (10) and (11) using results (33) and (35)-(38).

The impact effects of world-wide transitory and persistent disturbances are deduced by adding (40) and (41) to the system of equations (3), (5), (7), (9)-(11), (18), (19) and (7) updated, and by practicing similar comparative-statics experiments as those described in the last paragraphs. The equations are differentiated now with respect to the endogenous variables and the exogenous change $d\eta>0$. As observed in (42) and (43), the properties of the value function and the K_{t+1} and L_t choices that are discussed in the next appendix are still necessary to determine the results.

Finally, to compute the steady-state effects of a permanent disturbance, differentiate (44) using (8) and (25) to replace the dL terms and obtain the following two results:

$$dK/d\eta = \{r^* - H'(\eta)[F_K(K,\hat{L})-H(\eta)F_{KL}(K,\hat{L})\ell_\epsilon(K,\epsilon)]\}/\ x(K,\epsilon) \ \gtreqless\ 0, \qquad (A.4)$$

$$dL/d\eta = \ell_K(K,\epsilon)dK/d\eta + \ell_\epsilon(K,\epsilon)H'(\eta) \ \gtreqless\ 0. \qquad (A.5)$$

where $x(K,\epsilon)=F_{KK}(K,\hat{L})G''(\hat{L}) - \epsilon[F_{KK}(K,\hat{L})F_{LL}(K,\hat{L})-F_{KL}(K,\hat{L})^2] < 0$. The conditions that $H(\eta)>0$, $H'(\eta)>0$, $F_K(K,\hat{L})$ and $F_{KL}(K,\hat{L}) > 0$, $F_{KK}(K,\hat{L}) < 0$ and Proposition 2 determine the signs of both expressions. Finally, differentiation of (45) considering (7), (44), (A.4) and (A.5) determines the effect in the long-run holdings of foreign assets:

$$dA/d\eta = -[H'(\eta)F(K,\hat{L}) + r^*A]/r^*\eta + r^*/v'(\cdot)(1+r^*\eta)r^*\eta - dK/d\eta \ \gtreqless\ 0. \quad (A.6)$$

Thus, a positive permanent shock causes steady-state holdings of foreign assets to be adjusted according to consumption-smoothing, consumption-substitution and portfolio-reallocation effects. These effects are respectively represented by the first, second and third terms in the right-hand side of (A.6).

APPENDIX 2.

Some important properties of the value function, the optimal labor-supply choice and the optimal capital-accumulation decision are established in the next propositions and lemmas. These results assume that the dynamics of the disturbances are known with certainty. An extension of Proposition 1 for an stochastic environment can be found in Epstein (1983).

Proposition 1: *If u(•) and v(•) satisfy the set of properties defined in (2), then the following three properties hold:*

(i) $u''(t) + u'(t)v'(t) \leq 0$,

(ii) $Log[-V(Q)]$ *is convex,*

(iii) $a(t), b(t) < 0$.

Proof:

(i) Since $u'(t)[\exp(v(t))]$ is non increasing, it follows that $u''(t)+u'(t)v'(t) \leq 0$.

(ii) The second property follows from two lemmas proved in Epstein (1983)[15].

Lemma E.1: Define a function H of sequences of the composite good as $\Psi = -\exp[-H]$, where Ψ is the Stationary Cardinal Utility function (1), and assume that u(•) and v(•) satisfy the conditions described in (2), then H is concave.

Lemma E.2: Assume the same properties for u(•) and v(•) as in lemma E.1, then the function $Log[-V(Q)]$, where V is the value function and Q the aggregate state variable defined in (9), is convex[16].

(iii) The fact that a(t) and b(t) are both negative is demonstrated using the two previous results. From (i) it follows straightforwardly that $a(t) < 0$. Next, to show that $b(t) < 0$,

$v'(t)$ is displaced from the expression using optimality condition (18) and (ii) is used to determine the sign of the result.

<u>**Proposition 2**</u>: *If G(L) is a convex function and F(K,L) satisfies strict concavity, with constant returns to scale and positive diminishing marginal returns, the optimal labor-supply function defined in (8) is increasing in both ϵ and K and does not depend on the intertemporal pattern of consumption, foreign-asset accumulation or savings.*

Proof:

The independence of the optimal labor choice from the intertemporal choices of C, A and S in each period follows trivially from observing optimality condition (7), considering that both ϵ_t and K_t are predetermined state variables.

Differentiating (7) with respect to L_t, ϵ_t and K_t allows us to determine the following results:

$$dL_t/d\epsilon_t = \epsilon_t F_L(t) \,/\, [G''(t) - \epsilon_t F_{LL}(t)] > 0, \tag{A.7}$$

$$dL_t/dK_t = \epsilon_t F_{LK}(t) \,/\, [G''(t) - \epsilon_t F_{LL}(t)] > 0. \tag{A.8}$$

The signs of these derivatives is confirmed using the assumptions that $G''(t)$, $F_L(t)$, $F_{LK}(t) > 0$, and $F_{LL}(t) < 0$. The result in (A.8), updated one period, corresponds to $\ell_K(t+1)$ in the last term of the main diagonal of (A.1), the resulting expression for this term is negative as shown in the next proposition.

<u>**Proposition 3**</u>: *If G(L) and F(K,L) are as in Proposition 2, then the*

optimal capital-accumulation choice defined by optimality condition (19) is increasing in ϵ_{t+1} and decreasing in η_{t+1}. Furthermore, it does not depend on the dynamic processes of consumption, foreign assets and savings.

Proof:

The dynamic-independence argument follows trivially from observing (19), and recalling that the optimal labor choice at date "t+1" depends only on K_{t+1} and ϵ_{t+1}. To prove the increasingness result, differentiate (19) with respect to K_{t+1} and ϵ_{t+1} to obtain the following:

$$dK_{t+1}/d\epsilon_{t+1} = -\{ [\epsilon_{t+1}F_{KL}(t+1)\ell_{\epsilon}(t+1)+F_K(t+1)] [G''(t+1)-\epsilon_{t+1}F_{LL}(t+1)] \} \times \{ 1/[\epsilon_{t+1}G''(t+1)F_{KK}(t+1) - \epsilon_{t+1}^2(F_{LL}(t+1)F_{KK}(t+1)-F_{KL}(t+1)^2)] \} > 0. \quad (A.9)$$

K_{t+1} is shown to be decreasing in η_{t+1} by differentiating (19) with respect to each of these two variables:

$$dK_{t+1}/d\eta_{t+1} = r^*\{1/[\epsilon_{t+1}G''(t+1)F_{KK}(t+1)-\epsilon_{t+1}^2(F_{LL}(t+1)F_{KK}(t+1)-F_{KL}(t+1)^2)]\} < 0. \quad (A.10)$$

Expressions (A.9) and (A.10) have the corresponding signs because the assumptions made imply the following: F_K, F_L, F_{KL}, $G'' > 0$, F_{KK}, $F_{LL} < 0$ and $[F_{LL}F_{KK}-F_{KL}^2] = 0$.

APPENDIX 3.

This appendix analyzes the comparative statics of the stochastic version of the model. The impact and dynamic effects of serially uncorrelated disturbances are investigated.

The impact effects on labor supply and output are exactly the same as those obtained in the deterministic economy, due to the independence result of the optimal labor-supply choice (still governed by condition (7)). The impact effects on savings and capital accumulation are deduced by differentiating (13) and (14) with respect to ϵ_t, S_{t+1} and K_{t+1} in order to obtain the following two equation system:

(A.11)

$$\begin{pmatrix} E[a(t) + (1+r^*)b(t)] & (1+r^*)E[V''(t+1)(DMR(t+1)] \\ (1+r^*)E[V''(t+1)(DMR(t+1)] & E[V''(t+1)(DMR(t+1)^2+c(t+1)] \end{pmatrix} \begin{pmatrix} dS_{t+1}/d\epsilon_t \\ dK_{t+1}/d\epsilon_t \end{pmatrix} = \begin{pmatrix} E[a(t)]F(t) \\ 0 \end{pmatrix}$$

where[17]:

$$E[a(t)] = \exp[v(t)] \{u'(t)v'(t)+u''(t)\}-v''(t)E[V(t+1)] < 0,$$

$$E[b(t)] = v'(t)E[V'(t+1)] + (1+r^*)E[V''(t+1)] < 0,$$

$$DMR(t+1) = \epsilon_{t+1}F_K(t+1) - d - r^* \quad \text{and}$$

$$E[c(t+1)] = E[V'(t+1)\epsilon_{t+1}(F_{KK}(t+1)+F_{KL}(t+1)\ell_K(t+1))] < 0.$$

The determinant of the coefficient matrix is given by:

$$|\Delta| = E[a(t)+(1+r^*)b(t)]*E[V''(t+1)(DMR(t+1))^2 + c(t+1)] - (1+r^*)^2\{E[V''(t+1)(DMR(t+1))]\}^2.$$

The stochastic version of Proposition 1, which was proved by

Epstein (1983), the strict concavity with constant returns to scale of the production function and the result in (A.8), show that the first term of the determinant is unambiguously positive. However, it is clear that the second term is negative, so that the sign of $|\Delta|$ cannot be determined unless the appropriate conditions about the stochastic structure of the problem are imposed.

The results reported in (46) and (47) are correct as long as the determinant is positive, as can be seen by solving the system using Cramer's rule. A sufficiency condition for $|\Delta| > 0$ is the following:

$$E[b(t)]E[DMR(t+1)^2V''(t+1)] + (1+r^*)var[V''(t+1)DMR(t+1)] > (1+r^*)E[V''(t+1)^2DMR(t+1)^2]. \quad (A.12)$$

The dynamic effects are found by updating the results of another comparative-statics exercise. The exogenous changes are now assumed to be dS_t and dK_t instead of $d\epsilon_t$. The Chain Rule is then applied in order to compute the total dynamic effects as the results of the following two equations:

$$dS_{t+2}/d\epsilon_t = [dS_{t+2}/dS_{t+1}][dS_{t+1}/d\epsilon_t] + [dS_{t+2}/dK_{t+1}][dK_{t+1}/d\epsilon_t], \quad (A.13)$$

$$dK_{t+2}/d\epsilon_t = [dK_{t+2}/dK_{t+1}][dK_{t+1}/d\epsilon_t] + [dK_{t+2}/dS_{t+1}][dS_{t+1}/d\epsilon_t]. \quad (A.14)$$

FOOTNOTES

1. More recently, Kimbrough (1987) presented a general review of the existing work in dynamic optimizing models of the open economy and Frenkel and Razin (1987) carried out a thorough analysis of an overlapping generations model with endogenous production in the presence of uncertainty.

2. As discussed in Helpman and Razin (1982) and Frenkel and Razin (1987), foreign-asset accumulation reaches the stationary equilibrium when the rate of time preference and the world's real interest rate are equalized. As long as the interest rate is greater (smaller) than the rate of time preference, individuals will accumulate (deplete) foreign assets in order to finance an increasing (decreasing) consumption stream. Clearly, the constant-time-preference formulation of preferences cannot explain the process by which this long-run equilibrium is reached.

3. Epstein (1983) and Epstein and Hynes (1983) showed that this type of utility function constitutes a less restrictive representation of the individuals' preferences, which allows to address a series of issues that cannot be studied within the constant time-preference framework. Epstein (1983) also showed that the assumptions required by the Stationary Cardinal Utility function imply that the discount factor cannot vary too much, in a way that is made clear later in the paper. Thus, the use of the Stationary Cardinal Utility function should not be seen as a radical departure from the standard framework.

4. Epstein (1983) derived the axioms necessary to obtain Stationary Cardinal Utility (SCU) as a representation of the individual's preference order. These axioms are related to the risk independence of the consumption bundles over time. SCU requires that preferences over uncertain future consumption be independent of the consumption levels from the current and previous (i.e. certain) periods. This is a weaker condition than the one required by the traditional time-separable utility function with a fixed discount factor, because it requires that preferences over all period's consumption bundles must be independent from each other. Thus it requires not only that the future must be risk independent of the past, but also that the past must be risk independent of the future.

5. These conditions restrict the degree of variability of the rate of time preference, defined as $\exp[v(\cdot)]$. Specifically, the condition that $u'(\cdot)\exp[v(\cdot)]$ be non-increasing implies that

$$0 < \delta \exp[v(\cdot)]/\delta(\cdot) \leq -u''(\cdot)\exp[v(\cdot)]/u'(\cdot) \quad .$$

6. The role of international trade in complete contingent claims is ignored for simplicity. Individuals are still allowed to seek insurance against the risk of domestic shocks by purchasing foreign assets. The degree of protection that these assets can give is

determined by the covariance structure of ϵ and η. Furthermore, recent developments by Cole and Obstfeld (1988) suggest that, for some configurations of tastes and technology, the competitive allocations are independent of the completeness of international financial markets.

7. If the intertemporal solvency requirement is satisfied, the condition that $\text{Lim}_{t\to\infty} A_{t+1}/\Pi_0{}^t(1+r^*\eta_t)=0$ must hold for every A that has non-zero probability of being reached in the stochastic steady state of the economy. Chamberlain and Wilson (1984) have shown that, in the constant-discount case, the restrictions that enforce such solvency requirement may take complicated forms. In the endogenous time-preference framework, however, Theorem 3 of Epstein (1983) proved that, if the conditions defined in (2) hold and a linear technology like $(1+r^*\eta)$ is considered, the restrictions that $\exp[v(0)]<1+r^*\underline{\eta}$ and $\text{Lim}_{C-G(L)\to\infty}\exp[v(C-G(L))]>1+r^*\overline{\eta}$ are sufficient to guarantee the existence of a stationary limiting distribution of the state variables. Here, $\underline{\eta}$ and $\overline{\eta}$ are the lowest and highest possible realizations of the world-wide shock. In this case, there exists a well-defined boundary for asset holdings below which they are not depleted in the long run, and the solvency condition holds for every A in the stochastic stationary equilibrium.

8. These probability transitions are defined as follows:

$$P(\lambda_{t+1}=\lambda^r|\lambda_t=\lambda^s) = dG(\lambda_{t+1}=\lambda^r|\lambda_t=\lambda^s) \;/\; d\lambda^r$$

where $G(\lambda_{t+1}=\lambda^r|\lambda_t=\lambda^s) = \text{Prob}\;\{\;\lambda_{t+1} \le \lambda^r \mid \lambda_t=\lambda^s\;\}$ and $G(\cdot)$ is a continuous transition-distribution function.

9. The index t denotes the time period at which the corresponding function must be evaluated.

10. Epstein (1983) used the properties of $v(\cdot)$, $u(\cdot)$ and $F(\cdot,\cdot)$ to demonstrate the existence, uniqueness and stability that characterize the stochastic steady state of a closed economy version of the model discussed here.

11. Notice that these results, predicting positive comovement between output and the balance of trade in response to a transitory current-income expansion, are infinite-horizon extensions of conclusions obtained in the two-period exchange economies studied by Sachs (1981) and Greenwood (1983) and the overlapping generations model analyzed by Frenkel and Razin (1987).

12. The model can be equivalently used to study the current account instead of the trade balance. Since initial assets are a predetermined state variable, once the behavior of next period's assets is determined it is possible to explain either the total accumulation of these assets (i.e. the current account) or the international flow of commodities (i.e. the balance of trade).

13. Sachs (1983) identified that larger current-account improvements, among small open economies, tend to occur in the countries with the lowest rate of domestic investment.

14. The response of r^* to international disturbances can be studied by constructing a world-wide real business cycle model. This model would look the same as any standard closed-economy model, so that the manner in which the shocks affect the interest rate depends on their degree of persistence. For example, using a simplified version of Long and Plosser (1983), it is possible to obtain the following closed-form solution for r^*:

$$r^*{}_t = (\epsilon_{t+1}/\epsilon_t)\alpha^\alpha(L/\beta Y_t)^{1-\alpha} - 1$$

Where Y_t is output defined by a Cobb-Douglas technology, α is the capital's share on output, β is the constant subjective discount factor and L is the fixed supply of labor that characterizes the Long-Plosser model. If the disturbances were deterministic and stationary, evolving according to a serially autocorrelated process $\epsilon_{t+1}=\rho\epsilon_t$ with $0<\rho<1$, the above expression could be rewritten as

$$r^*{}_t = (\rho\epsilon^\alpha{}_t)\alpha^\alpha(L/\beta Y_t)^{1-\alpha} - 1$$

Hence, under these conditions, world-wide productivity improvements will always induce an increase in the world's real rate of return on assets.

15. The proofs constructed by Epstein (1983) apply specifically to a stochastic, closed-economy version of the model. They are easily extended to the deterministic, open-economy framework studied in Section 3 of this paper.

16. Since $Log[-V(Q)]$ is convex, it follows that $V'(Q)/V(Q)$ is increasing and hence $V''(Q)V(Q)-(V'(Q))^2 > 0$. Notice that since $V(Q)<0$, the latter implies that the value function is concave.

17. The negative signs of E[a(t)] and E[b(t)] follow from stochastic versions of lemmas E.1 and E.2 demonstrated by Epstein (1983).

CHAPTER II

REAL BUSINESS CYCLES IN A SMALL OPEN ECONOMY: THE CANADIAN CASE.

1.- Introduction.

Real models of economic fluctuations, such as Kydland and Prescott (1982), Prescott (1986) and Hansen (1985), have been very successful in reproducing some of the observed features of postwar U.S. business cycles. In these models, random productivity disturbances generate intertemporal income and substitution effects that induce optimizing agents to adjust investment and smooth consumption. These effects, in conjunction with the stochastic properties of the disturbances, cause the models to exhibit a pattern of dynamic behavior similar to that observed in actual business cycles. In all these models, however, the domestic capital stock is the only vehicle that can be used to reallocate consumption intertemporally when a technological disturbance occurs. Therefore, it seems interesting to wonder whether an extension of this theory to an open-economy environment, where international trade also serves as a means to support intertemporal consumption planning, can be as successful.

The empirical evidence suggests that foreign-asset accumulation may be just as important as investment for the understanding of the dynamics of savings. Trade-balance and current-account fluctuations are large in size and negatively correlated with domestic economic activity. In the case of Canada, for example, the volatility of net foreign interest payments reaches 15.25% whereas the variability of domestic private investment is 9.82%. Hence, the integration of the foreign sector appears to be an interesting additional way of evaluating the empirical performance of real business cycle theory.

Another motivation for the study of real business cycles in a small open economy is the desire to empirically analyze a stochastic version of a dynamic model widely used in theoretical open-economy macroeconomics. Although the pure analysis of issues in international finance from the perspective of dynamic optimizing models has been the subject of numerous studies, as reviewed in Frenkel and Razin (1987) and Kimbrough (1987), the empirical analysis of these models is rarely pursued[1]. In fact, as Hercowitz (1986b) noted, some of the observed features of a small open economy do not seem to fit well with the predictions of a simple dynamic optimizing framework[2].

Quantitative studies of dynamic stochastic models of the open economy are also important in the context of the ongoing debate on the meaning of the high degree of correlation between savings and investment observed in industrialized economies. The controversial work of Feldstein and Horioka (1980) documented high cross-sectional correlations between savings and investment and attributed them to a low degree of international capital mobility. Since then, the high correlation of savings and investment has been established in various time-series and cross-sectional samples of data. There is, however, disagreement as to what this empirical evidence indicates. In contrast with the Feldstein-Horioka argument, the findings of Obstfeld (1985) and Finn (1988) have determined that the correlation between savings and investment does not provide information about the degree of capital mobility. These authors performed numerical simulations in stochastic overlapping-generations models and found that, by varying the degree of persistence of the disturbances considered, almost any intensity of correlation between savings and investment can be produced. An interesting experiment that could help settle this matter is one in which the productivity disturbances are set so as to replicate the main features of the business cycle. Then one could observe whether the disturbances that allow the model to replicate actual economic fluctuations can also explain the comovement between savings and investment.

In the light of the previous arguments, this paper endeavors to study real business cycles in the context of a small open economy. The dynamic stochastic model constructed in the last chapter is numerically solved and calibrated in order to evaluate its ability to mimic the stylized facts, and the correlation between savings and investment, that characterize Canadian post-war business cycles. As explained before, in this model agents are allowed to transfer resources for future consumption either in the form of foreign financial assets or in the form of domestic physical capital. Thus, in contrast with real business cycle models developed to date, the role played by international asset trading in the intertemporal allocation of consumption is stressed in this essay.

In the spirit of the work by Obstfeld (1981a) and (1981b), the model studied here features an endogenous rate of time preference. This is utilized to determine a well-behaved deterministic stationary equilibrium for the holdings of international assets. Such a deterministic stationary state is obtained when the world's real interest rate and the rate of time preference are equalized. As Helpman and Razin (1982) and Frenkel and Razin (1987) note, when the interest rate is greater (smaller) than the rate of time preference, individuals will rationally choose to accumulate (deplete) foreign assets in order to finance an increasing (decreasing) consumption stream. When the market and the subjective discount factors are equal, individuals attain the steady state in consumption and the accumulation of foreign assets stops[3]. Clearly, the constant-time-preference representation of preferences cannot be employed to describe the process by which this long-run equilibrium is reached; in this framework, either (a) there is no stationary equilibrium, because the interest rate and the rate of time preference are not set to be equal, or (b), when the two *are* equal, the economy begins at a steady-state equilibrium, which depends on the initial level of foreign-asset holdings, and never deviates from it. In the uncertain environment studied in this paper, the endogenous rate of time preference is utilized to determine a stable *stochastic* steady state. In order to consistently introduce stochastic disturbances in the endogenous time-preference framework, the Stationary Cardinal Utility function (SCU) formulated by Epstein (1983) is adopted as the time-recursive expression of preferences. A set of restrictions is imposed on this utility function to allow the use of dynamic programming techniques, to guarantee the normality of consumption in all periods, and to ensure the existence of a stationary distribution of the state variables in the stochastic steady-state equilibrium. These restrictions also imply that the use of SCU does not constitute a major deviation from the standard time-separable setup[4].

The equilibrium stochastic process of the artificial economy is numerically computed following a procedure suggested in Bertsekas (1976). This methodology was utilized by Sargent (1980) and was introduced to the real business cycle literature by Greenwood, Hercowitz and Huffman (1988). The procedure is based on value-function iteration defined on a discretized version of the state space. Using the optimal state-contingent decision rules obtained from the solution of the value function, the exact joint limiting distribution of the state variables is computed and used to calculate moments of variability, comovement and persi tence of all variables in the model. The model mimics fairly well the cyclical behavior of GNP, consumption, foreign interest payments, savings and the trade-balance-output ratio. In contrast, the volatility, persistence, output-correlation and savings-correlation of private investment are poorly approximated. As will be explained later, the model's lack of accuracy in duplicating the behavior of investment is connected to the frictionless manner in which it can be undertaken. Thus, it appears that the neoclassical view of capital accumulation, as a process free of adjustment costs or other frictions, may be unsatisfactory for the study of real business cycles in a small open economy.

The rest of the chapter proceeds as follows: The next section describes the structure of the model, the functional forms employed and the manner in which the stochastic disturbances are treated. Section 3 discusses the solution procedure and the specification of the set of parameters used to calibrate the model. Section 4 presents the results of various simulation exercises, comparing the performance of the open-economy model with the results obtained in some closed-economy prototypes. Some concluding remarks are presented in section 5.

2.- Structure of the Artificial Economy.

Production Technology and Financial Structure: The representative-agent economy studied here produces an internationally tradable composite commodity. The production technology is described in standard Cobb-Douglas form

$$\epsilon_t F(K_t, L_t) = \exp(e_t)\, K_t^{\alpha}\, L_t^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{1}$$

where L_t are labor services, K_t is the capital stock, α is the capital's share in output and ϵ_t is a technology disturbance that follows a stochastic process to be described later. Following Greenwood (1983), it is also possible to assume that GDP is only an exportable commodity and that the disturbances represent terms-of-trade shocks.

Agents in this model economy also have access to a perfectly competitive, international capital market. In this market, foreign financial assets A_t paying the non-random real rate of return r^* are traded with the rest of the world[5]. There are no restrictions on the international flow of goods and services, and the domestic economy is assumed to be small relative to the size of the world's capital market. Thus, r^* is taken as given by the agents when formulating their optimal intertemporal plans.

The laws of motion for international assets and domestic capital are given by the following two equations:

$$K_{t+1} = (1-\delta)K_t + I_t, \qquad 0 \leq \delta \leq 1, \tag{2}$$

$$A_{t+1} = TB_t + A_t(1+r^*). \tag{3}$$

Where I_t is gross investment, δ is a constant rate of depreciation and

TB_t is the balance of trade.

In accordance with the aggregate resource constraint, the sum of consumption C_t, investment, and the balance of trade, cannot exceed gross domestic product[6]:

$$C_t + I_t + TB_t \leq \exp(e_t)\, K_t^{\alpha}\, L_t^{1-\alpha}. \qquad (4)$$

Preferences: The identical and infinitely-lived agents allocate C_t and L_t intertemporally so as to maximize the expected value of their lifetime utility as given by the Stationary Cardinal Utility function:

$$E_0\left[\sum_{t=0}^{\infty}\{\, u(C_t - G(L_t))\exp[-\sum_{\tau=0}^{t-1} v(C_\tau - G(L_\tau))]\ \}\right]. \qquad (5)$$

Instantaneous utility is given the isoelastic form

$$u[C_t - G(L_t)] = (1-\gamma)^{-1}\,[(C_t - L_t^{\omega}/\omega)^{1-\gamma} - 1], \qquad \omega > 1, \quad \gamma > 1, \qquad (6)$$

where γ is the coefficient of relative risk aversion and ω is equal to 1 plus the inverse of the intertemporal elasticity in labor supply. The impatience function is specified in the logarithmic form

$$v[C_t - G(L_t)] = \beta \,\mathrm{Ln}\,[1 + C_t - L_t^{\omega}/\omega], \qquad \beta > 0, \qquad (7)$$

where β is related to the consumption elasticity of the rate of time preference.

In order to focus the analysis expressly on the dynamic interaction of foreign assets and domestic capital as alternative vehicles of savings, lifetime utility has been formulated in such a way that it can be studied in terms of the composite defined by consumption minus the disutility of labor. The functions u(•) and v(•) were constructed so as to make the marginal rate of substitution between C and L dependent on the latter only, and hence the wealth effect on labor supply has been eliminated. This simplification has the additional advantage that it allows the conditions established in Theorem 5 of Epstein (1983) to be easily introduced. As explained in the previous chapter, Epstein determined in this theorem a set of sufficiency conditions for the instantaneous-utility and impatience functions to guarantee the existence of a stationary limiting distribution of the state variables and to ensure that consumption is always a normal good. These conditions are the following:

$$u(\cdot)<0 \ , \ u'(\cdot)>0 \ , \ u'(0) = \infty, \tag{8.1}$$

$$v(\cdot)>0, \ v'(\cdot)>0, \ v''(\cdot)<0, \tag{8.2}$$

$$u'(\cdot)\exp[v(\cdot)] \text{ non-increasing}, \tag{8.3}$$

$$\text{Log}[-u(\cdot)] \text{ convex}. \tag{8.4}$$

The isoelastic form of (6) and the logarithmic form of (7) ensure that (8.1)-(8.4) are all satisfied as long as $\beta \leq \gamma$.

Stochastic Equilibrium: The stochastic structure of the model is simplified following the same strategy utilized by Greenwood, Hercowitz and Huffman (1988). Accordingly, the disturbances are assumed to follow a two-point Markov process, so that in any given period the productivity or terms-of-trade shocks take one of two values

$$e_t \in E = \{ e^1 , e^2 \}. \tag{9}$$

The transition probabilities of the disturbance starting in state s and moving to state r in one period are denoted as π_{sr}, for $s,r=1,2$. The usual properties that $0\leq\pi_{sr}\leq1$ and $\pi_{s1}+\pi_{s2}=1$ for $s,r=1,2$ must be satisfied. Then the assumptions that $\pi_{11}=\pi_{22}=\pi$ and $e^1=-e^2=e$ are adopted. These symmetry conditions imply that the asymptotic standard deviation, σ_e, and the first-order autocorrelation coefficient, ρ_e, that characterize the stochastic shocks are given by $\sigma_e=e$ and $\rho_e=2\pi-1$ respectively.

Given the initial values of K, A and e, and the knowledge of the stochastic process that governs the disturbances, rational agents formulate optimal state-contingent decision rules for consumption, labor supply, capital accumulation and foreign-asset accumulation. These decision rules are obtained by maximizing (5) subject to (1)-(4), the usual non-negativity restrictions on C, K and L, and the requirement of intertemporal solvency[7]. The same problem can be more easily analyzed and solved by applying dynamic programming techniques[8].

The state of the economy at any given date is described by the observed value of the disturbance, the initial capital stock and the initial holdings of international assets (i.e. the triple $(e^s{}_t,K_t,A_t)$). Given these, agents decide how much to consume, how much labor services to supply and how much to save in the form of both domestic capital and foreign assets (i.e. they must choose C_t, L_t, K_{t+1}, and A_{t+1}). The time-recursive nature of the structure of preferences employed here implies that such decisions can be characterized by the following functional equation problem:

$$V(K_t, A_t, e^{s}_t) = \max_{[C_t, K_{t+1}, A_{t+1}]} \{ (1-\gamma)^{-1} [(C_t - \hat{L}_t^{\omega}/\omega)^{1-\gamma} - 1] + \exp[-\beta \, Ln(1+C_t - \hat{L}_t^{\omega}/\omega)][\sum_{r=1}^{2} \pi_{sr} V(K_{t+1}, A_{t+1}, e^{r}_{t+1})] \}, \quad (10)$$

s.t.

$$C_t = \exp(et) K_t^{\alpha} \hat{L}_t^{1-\alpha} - K_{t+1} + K_t(1-\delta) + (1+r^*)A_t - A_{t+1},$$

with:

$$\hat{L}_t = \underset{L_t}{\operatorname{argmax}} \{ \exp(et) K_t^{\alpha} L_t^{1-\alpha} - L_t^{\omega}/\omega \}.$$

As shown in the first chapter, the concavity of the value function in this case is guaranteed by the properties of $u(\cdot)$, $v(\cdot)$ and $F(\cdot,\cdot)$ specified before[9].

3.- Numerical Solution of the Model.

Since this is a case where the value function cannot be solved for analytically, a numerical procedure based on a discretization of the state space is utilized. As mentioned before, this methodology follows the works of Bertsekas (1976), Sargent (1980) and Greenwood, Hercowitz and Huffman (1988). This technique makes use of the contraction property that mappings of the kind defined by functional equations like (10) exhibit. The functional equation problem is solved by the method of successive approximations starting from an initial guess $V^0(\cdot)$. A computer algorithm performs the iterations on (10), starting with $V^0(\cdot)$ on the right-hand side to obtain a new guess $V^1(\cdot)$ as the result of a maximization routine. The process continues until the state-contingent decision rules for capital and assets converge[10].

Sargent (1980) and Greenwood, Hercowitz and Huffman (1988) discuss how this procedure obtains the exact joint p.d.f. of the state variables in the stochastic stationary equilibrium of closed-economy models. What follows here is a brief review of how the procedure is adapted to the open-economy case.

The first step is to define the discrete state space used to analyze the problem. Capital and assets can take any value contained in the corresponding finite, time-invariant grid: $K=\{K_1, \ldots, K_N\}$ and $A=\{A_1, \ldots, A_M\}$. Hence, the state space for the model is the discrete set defined by the product $K \times A \times E$ of dimensions $N \times M \times 2$. The definition of these grids is an important part of the solution, since they contain the specific set of numbers used to evaluate (10) in search for the maximum. The aim is to define K and A so as to capture the ergodic set for the joint stationary distribution of K, A and e, refining the grids until the covariances among the state variables converge[11]. The two grids are initially centered around the deterministic stationary equilibrium values of domestic capital and foreign assets. In this model, steady-state holdings of A and K are determined by a simultaneous equality between the rate of time preference, the net marginal productivity of capital and the world's real interest rate[12].

The algorithm solves for the value function and determines unique, state-contingent decision rules for both capital and foreign assets. These decision rules have the following form:

$$K_{t+1} = k(A_t, K_t, e^s{}_t) \in K, \qquad (11)$$

$$A_{t+1} = a(A_t, K_t, e^s{}_t) \in A. \qquad (12)$$

These policy rules are used to construct one-step transition probabilities of moving from any initial triple (K^m, A^n, e^s) to any other triple (K^p, A^q, e^r) in one period. Each of these transitions is determined using the following property of the decision rules:

$$\Pr[\ K_{t+1}=K^p,\ A_{t+1}=A^q \mid K_t=K^m,\ A_t=A^n,\ e_t=e^s] = 1.0, \tag{13}$$

only if a) $K^p = k(\ A^n,\ K^m,\ e^s\ )$,

and b) $A^q = a(\ A^n,\ K^m,\ e^s\ )$,

and zero otherwise. The corresponding transition probability $P_{pqr,mns}$ is found by multiplying (13) by its corresponding π_{sr}. Following this procedure, the (2MNx2MN) transition-probability matrix denoted as P is created. This matrix has most of its components set to zero, and the rest are the π_{sr}'s located at the coordinates defined by the decision rules on both grids.

Next, the unique joint stationary distribution function for the state variables is obtained using the fact that the sequence defined by $\rho^1=\rho^0 P$ is also a contraction. Making an initial guess for ρ^0 is possible to iterate repeatedly until the sequence converges to a limiting fixed point ρ^*. This 1x2MN vector contains the set of stationary probabilities for each triple of foreign assets, domestic capital and the stochastic disturbances. Such limiting distribution is then used to compute population moments of all variables in the system, since they are all functions of the decision rules (11) and (12). Consequently, expected values, variances, autocorrelations and correlations of C, I, GDP, GNP, L, K, $-r^*A$ (foreign interest payments), S (savings) and TB/Y (the trade- balance-output ratio) can be computed.

In order to evaluate the empirical performance of the artificial economy, the probabilistic structure of the model is calibrated so as to mimic a particular subset of the actual moments that characterize Canadian business cycles, and then the rest of the moments from the artificial economy are compared with those actually observed. Once the structural parameters are fixed, the calibration exercise is performed by setting the magnitude of the shocks e (i.e. the standard deviation σ_e) and their transition probability π (i.e. the first-order autocorrelation coefficient ρ_e) so as to mimic the percentage standard deviation and first-order autocorrelation of annual, postwar, detrended Canadian GDP. The particular values of all the other parameters are selected using long-run actual averages and the restrictions imposed by the deterministic stationary equilibrium of the model, and also by considering the estimates obtained in some of the relevant empirical literature. The structural parameters for the Canadian economy have been assigned the following values:

$$\alpha=0.32, \quad \beta=0.11, \quad \gamma=1.001 \text{ or } 2.0, \quad \delta = 0.1, \quad \omega=1.455, \quad r^*=0.04. \qquad (14)$$

The parameter α was determined from the long-run average of the ratio of labor income to net national income at factor prices. The rate of depreciation $\delta=0.1$ corresponds to the one commonly used in the real business cycle literature. With this value, the average investment-output ratio is 23.0%, which is very close to the 21.5% observed in the data[13]. The value of ω is in the range of the estimates of the intertemporal elasticity of substitution in labor supply $(1/(\omega-1))$ obtained by the studies of MaCurdy (1981) and Heckman and MaCurdy (1980, 1982). MaCurdy (1981) estimated that, for adult males not younger than 25 years, the intertemporal elasticity of substitution in labor supply at the intensive margin is around 0.3. Heckman and MaCurdy (1980,1982) obtained corresponding estimates for females at both the intensive and extensive margins of about 2.2. The value of this parameter in the present model is 2.19, which although very close to the upper bound of the range (0.3,2.2) it allows the model to mimic the percentage variability of hours worked very closely[14]. A sensitivity analysis of the model's statistical moments to changes in the value of this parameter showed that the behavior of all the statistics, except the volatility of labor supply, is generally independent of the particular number assigned to it. The value of r^* corresponds to what Prescott (1986) and Kydland and Prescott (1982) consider to be the long-run annual real interest rate in the U.S. economy. The parameter β is then determined combining the deterministic steady-state equation for assets, the specified values for the other parameters and the actual long-run ratio of GNP to GDP over the sample period. Hence, this parameter is set so as to equalize the rate of time preference with the world's interest rate in the non-random, stationary equilibrium. Following Greenwood, Hercowitz and Huffman (1988) -from here on GHH-, two alternative sizes of the risk aversion parameter were used. The value of 1.001 is consistent with the findings of Hansen and Singleton (1983) and the value of 2.0 is taken from the results obtained by Friend and Blume (1975), although the models analyzed by these authors are structurally different from the one studied here. A sensitivity analysis of the model's behavior to changes in this parameter is performed later.

4.- Results of the Simulations and Analysis.

A series of numerical simulations were performed in order to calibrate the model and evaluate its sensitivity to changes in the various parameters specified before. The results of these experiments are reported in 4.1 and an economic interpretation of the model's behavior is provided in 4.2.

4.1 Results of the Numerical Simulations.

The first exercise was designed to achieve a close comparison with the existing work on closed-economy real business cycles. The risk aversion parameter was set to the value $\gamma=2.0$, which corresponds to what GHH consider best to fit a closed economy with variable utilization and investment shocks. Two evenly-spaced grids containing 22 points each were chosen to contain the admissible values of capital and foreign assets, the **K** grid spans the interval [3.25,3.56] and the **A** grid the interval [-1.42,0.08]. Hence, there are a total of 484 different options for the allocation of savings. The actual statistics used as references to evaluate and calibrate the model are obtained from per-capita, postwar, annual data in terms of the 15+ population, logged and detrended with a quadratic time trend (see notes in Table 1).

Given the parameter and grid specifications mentioned before, the model mimics the percentage standard deviation and first-order serial autocorrelation of GDP with a technological disturbance that exhibits 1.18% standard deviation and 0.35 first-order autocorrelation. The complete results obtained from the artificial economy and the statistics from the actual data are reported in Table 1. The corresponding marginal probability density of capital and foreign assets is depicted in Figure 1.

The inspection of Table 1 illustrates that the model performs well in reproducing the volatility and persistence of some of the aggregates. The model is consistent with the facts in generating a consumption process which is less volatile than either domestic or national output, gross savings, investment and foreign interest payments. It is also compatible with actual observations in that S, I and $-r^*A$ are the most volatile variables. As column A of the model's results shows, the percentage standard deviation of the majority of the aggregates is quite well approximated, except those of savings, investment and foreign interest payments. The volatility of savings in the artificial economy is only 5.8%, which is below the 7.3% observed in the data. In contrast, the model exaggerates the variability of both investment and foreign interest payments (21.0% instead of 9.8% for the former and 19.6% instead of 15.3% for the latter). The first-order serial correlations reported in column B also mimic the majority of the actual statistics, except in the cases of investment, capital and the trade-balance-output ratio. The results reproduce the fact that C and $-r^*A$ possess the highest serial autocorrelation coefficients, whereas S, I and K have the lowest. The artificial economy also matches the facts in generating foreign interest payments as the most serially correlated variable, and is very close in duplicating the persistence of savings and consumption.

Table 1

Statistical Moments: Canadian Data and Artificial Economy[a].

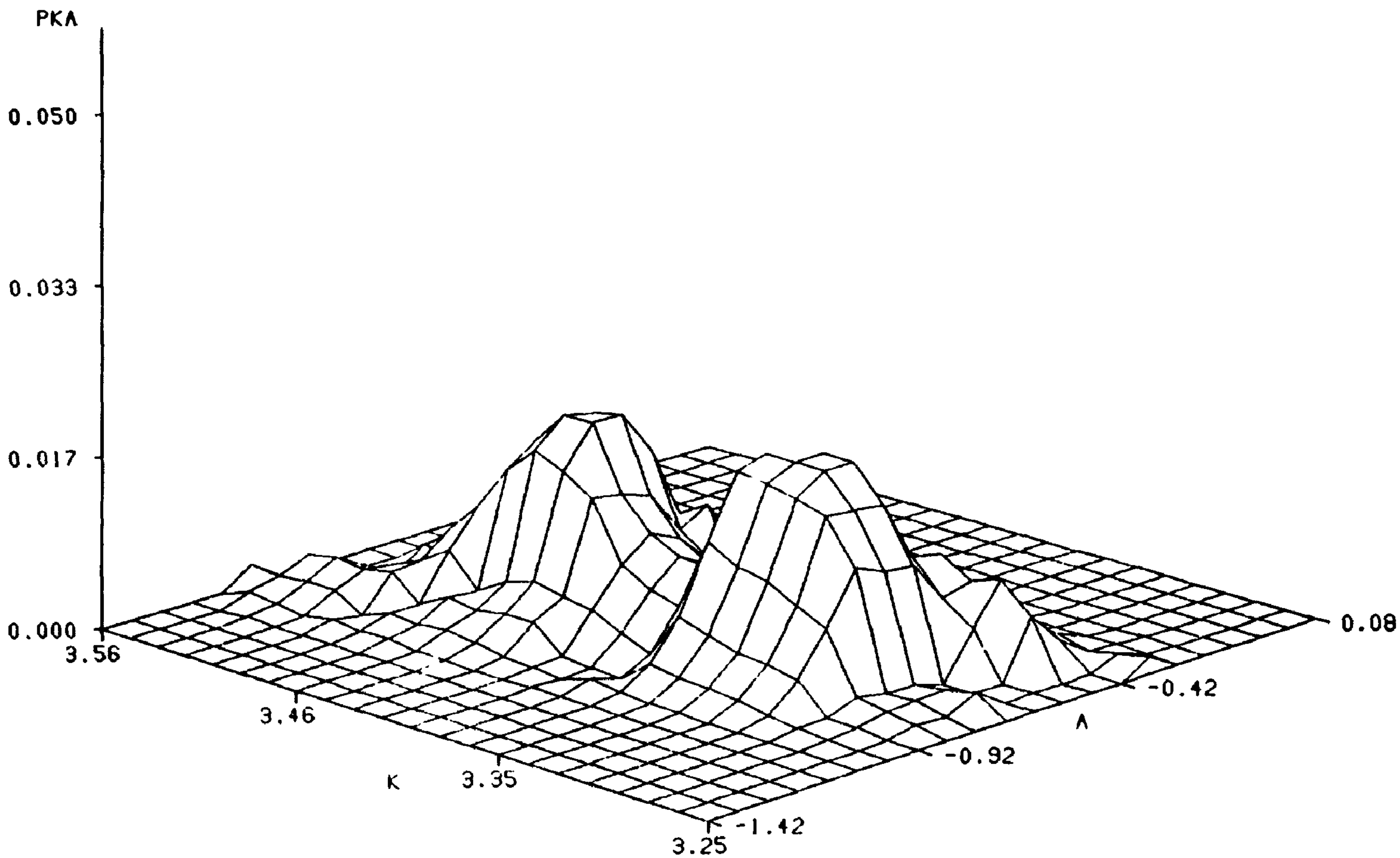
Variables	Canadian Data 1946-1985 (A)[b]	(B)[c]	(C)[d]	Artificial Economy $\gamma=2.0$, $\sigma_\epsilon=1.18\%$, $\rho_\epsilon=0.356$ (A)[b]	(B)[c]	(C)[d]
1) GDP	2.810	0.615	1.000	2.810	0.615	1.000
2) GNP	2.950	0.643	0.995	2.821	0.619	0.990
3) C	2.460	0.701	0.586	2.086	0.693	0.944
4) S	7.306	0.542	0.662	5.772	0.599	0.932
5) I	9.820	0.314	0.639	21.056	-0.319	0.235
6) K	1.380	0.649	-0.384	1.980	0.377	0.669
7) L	2.020	0.541	0.799	1.936	0.615	1.000
8) $-r^*A$	15.250	0.727	-0.175	19.566	0.886	-0.198
9) TB/Y	0.019	0.623	-0.129	0.046	-0.312	0.032
	CORR(S,I) = 0.434			CORR(S,I) = 0.251		

[a]The data are measured in per-capita terms of the 15+ population, logged and detrended with a linear quadratic time trend. 1)-3), 5) and 8) are the totals from the national income accounts in 1981 dollars. 6) was obtained from the end-of-period net stocks of fixed non-residential capital in manufacturing and non-manufacturing industries, in 1981 prices. The labor data is an index of man hours worked by paid workers with 1981=100.0. Savings in 4) is generated as investment plus the trade-balance surplus. The source of all is the CANSIM data retrieval.

[b]Percentage standard deviation, except 9 which is not in percent.

[c]First-order autocorrelation coefficient.

[d]Coefficient of correlation with GDP.



MARGINAL PROBABILITY DENSITY OF CAPITAL AND FOREIGN ASSETS

Figure 1

In contrast, this model does not prove to be as accurate in recreating the comovement features of Canadian business cycles. The results listed in column C of the two panels of Table 1 show that only the GDP correlations of GNP, $-r^{*}A$ and TB/Y are well approximated. This by itself is an important achievement, since to date the existing work in dynamic international finance has not been very successful in reproducing the countercyclical behavior of the balance of trade and foreign assets[15]. Surprisingly, consumption in the artificial economy is almost perfectly correlated with GDP. This fact is consistent with the predictions of a deterministic model that assumes the existence of exogenous output endowments; in this framework, output and consumption are perfectly correlated because homothetic preferences, as embodied in the isoelastic utility function, imply that agents desire to consume a constant fraction of their wealth each period. In a stochastic framework where the output endowments follow a Markovian process, expected wealth is a fixed proportion of the current-period's endowment, and hence consumption and output would also tend to be highly correlated[16]. A more intuitive interpretation of this result in the context of the present model is discussed later. The fact that labor has perfect positive correlation with domestic output is an implication of the Cobb-Douglas production technology and the utility and time-preference functions that were adopted. The common serial autocorrelation coefficient of both GDP and L also follows from this fact.

From Table 1 it also transpires that the model generates a correlation coefficient between savings and investment of 0.25, which is lower than the 0.43 actually observed. This is implied by the low variability and persistence of the productivity disturbances required to calibrate the model, and is not an indication of the degree of international capital mobility. An analysis of the changes caused by alterations in the degree of persistence of the disturbances indicated that the correlation between savings and investment is positively related to the first-order serial autocorrelation of the shocks. Thus, although the model cannot replicate simultaneously the stylized facts of GDP and the correlation between savings and investment, it does support the argument of Obstfeld (1985) and Finn (1988) claiming that the intensity of the comovement between S and I does not relate to the degree of capital mobility.

Although comparisons with the existing real business cycle literature are complicated by differences in solution methods, filtering procedures and, in the case of the GHH model, the location of the disturbances, this numerical analysis shows that the required exogenous persistence and variability of the productivity shocks appears to be lower than comparable values found in closed-economy models. The shocks in the works of Prescott (1986) and Hansen (1985) follow a stochastic process close to a random walk, the second author estimates their quarterly correlation to be 0.95, or 0.81 annually. In the investment shock-endogenous utilization model of GHH, the required annual persistence of the disturbances is 0.51 (when the coefficient of relative risk aversion is given a value of 2). In contrast, the present model necessitates only a yearly autocorrelation of 0.35. Furthermore, following the criterion of GHH for comparing the size of the disturbances using the ratios of their required percentage volatility to that of output, the model of Hansen (1985) produces the values 1.3 and 1.7 and the GHH prototype gives 1.47. The open-economy model generates a ratio equal to only 0.42.

Table 2 reproduces the actual statistics and the results obtained with the artificial economy using a value of γ close to 1.0. A 1.18% shock with 0.34 serial autocorrelation was required to calibrate the model in this case. The results reported in Table 2 are very similar to those obtained with γ=2.0, with the exception of the volatility, comovement and persistence of foreign-asset holdings, which are all reduced[17]. The percentage variability of foreign interest payments is reduced to 11.1%, which is now lower than the 15.3% actually observed. The serial correlation coefficient of $-r^*A$ drops to 0.63 and its GDP correlation falls to -0.35. These changes are explained by the increase in the intertemporal elasticity of substitution in consumption (i.e. the reduction in the risk-aversion coefficient). This enables individuals to attain optimal consumption behavior without having to resort as much to the insurance that foreign assets provide against domestic productivity disturbances. These results also indicate that the appropriate value for γ is between 1.001 and 2. Finally, it is also worth noting that, unlike in closed-economy real business cycle models, the procyclical behavior of consumption does not seem to depend on the value of the risk aversion parameter.

Table 2

Statistical Moments: Canadian Data and Artificial Economy[a].

Variables	Canadian Data 1946-1985 (A)[b]	(B)[c]	(C)[d]	Artificial Economy $\gamma=1.001$, $\sigma_\epsilon=1.18\%$, $\rho_\epsilon=0.34$ (A)[b]	(B)[c]	(C)[d]
1) GDP	2.810	0.615	1.000	2.810	0.615	1.000
2) GNP	2.950	0.643	0.995	2.795	0.611	0.997
3) C	2.460	0.701	0.586	2.105	0.669	0.961
4) S	7.306	0.542	0.662	5.483	0.590	0.947
5) I	9.820	0.314	0.639	21.050	-0.343	0.266
6) K	1.380	0.649	-0.384	1.913	0.322	0.669
7) L	2.020	0.541	0.799	1.931	0.615	1.000
8) $-r^*A$	15.250	0.727	-0.175	11.064	0.632	-0.354
9) TB/Y	0.019	0.623	-0.129	0.046	-0.338	-0.013
	CORR(S,I) = 0.434			CORR(S,I) = 0.268		

[a]See note "a" in Table 1 for sources and calculations.

[b]Percentage standard deviation, except 9 which is not in percent.

[c]first-order autocorrelation coefficient.

[d]GDP correlation coefficient.

4.2 An Economic Interpretation of the Model's Behavior.

A discussion of the economic forces that cause the volatile behavior of investment, the high positive correlation between consumption and GDP and the low persistence of the trade-balance-output ratio in the artificial economy is carried out now. It was formally established in Chapter I that a small open economy differs from a closed economy essentially in that, in the former, there is a separation of savings and investment decisions that follows from the ability that agents have to accumulate savings in the form of foreign financial assets. In this framework, savings are chosen so as to equate the stochastic intertemporal marginal rate of substitution with the intertemporal relative price of consumption $(1+r^{*})$, whereas investment is planned so as to equalize the expected marginal returns of K_{t+1} and A_{t+1} in utility terms. Therefore, it is no longer necessary for individuals to gradually adjust the domestic capital stock in response to the wealth and substitution effects caused by technological disturbances. Specifically, the numerical analysis shows that the wealth effect pushing for the smoothing of consumption is concentrated in the process of foreign-asset accumulation, and that domestic capital is rapidly adjusted in order to ensure the equality of expected marginal returns in utility terms[18].

The manner in which the separation of savings and investment decisions operates can be more precisely illustrated by noting that the volatility of domestic capital is directly related to the serial correlation of the disturbances. A transitory productivity improvement motivates optimizing agents to obtain the desired adjustment in the consumption path by adjusting only their holdings of foreign assets. Investment in the domestic capital stock remains almost unaltered because a once-and-for-all productivity change cannot affect its expected real rate of return relative to the world's interest rate. But, when the productivity improvement has some persistence, capital accumulation fluctuates according to the expected difference in the marginal returns of K and A[19]. For instance, using a 1.18% serially-uncorrelated shock, the percentage standard deviation of K in the artificial economy is only 0.83%, whereas it increases to 4.6% when a shock of the same size exhibits 0.99 first-order autocorrelation.

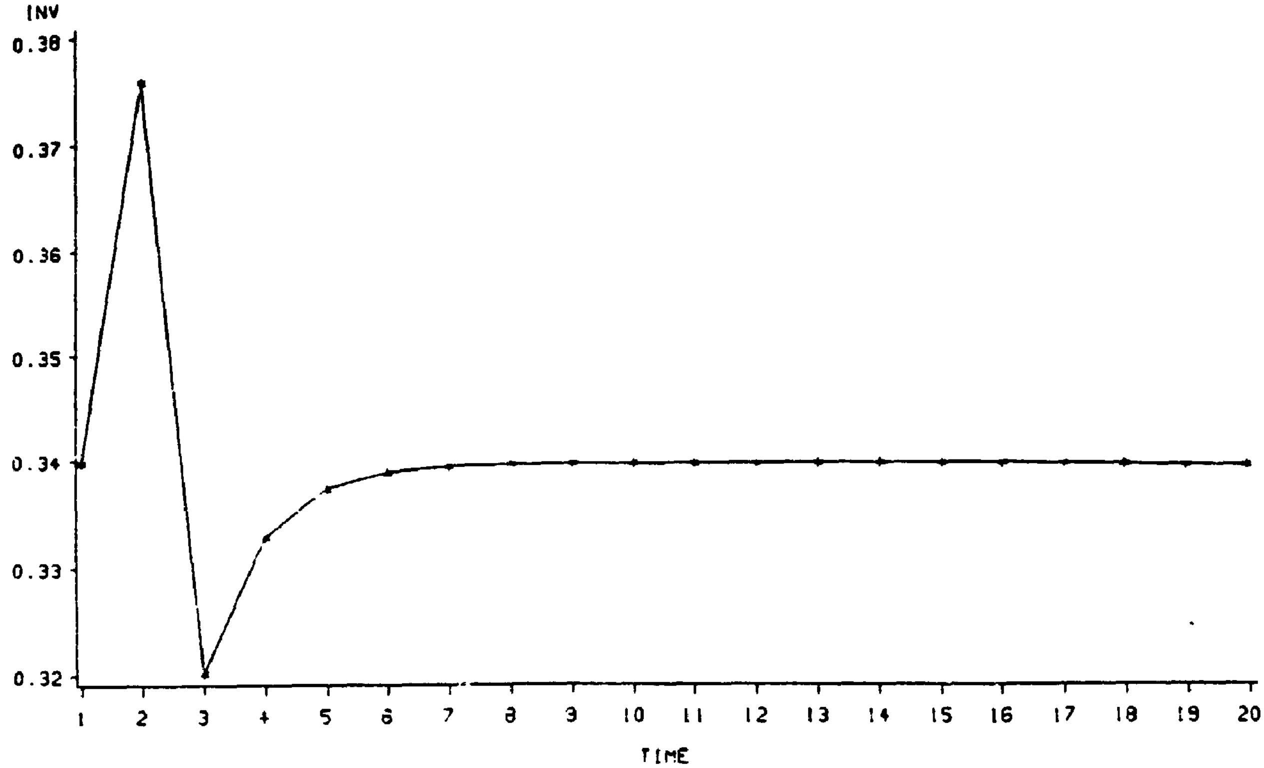
The separation of savings and investment choices justifies the volatile behavior of investment in the artificial economy. Individuals wish to equalize the expected marginal returns paid on domestic capital and foreign assets and, since there are no restrictions on the international flow of commodities nor there are any frictions or costs involved in the process of expanding or contracting the domestic capital stock, they can always borrow or lend from abroad the resources they need to obtain the desired equality of returns. Both the bimodal nature of the marginal limiting distribution of K and A, depicted in Figure 1, and the high percentage variability of investment reflect the fast rate at which K is intertemporally readjusted in response to random productivity changes. Similarly, the negative autocorrelation of I and its weak positive correlation with GDP also follow from the separation feature, in conjunction with the relatively small and short-lived technological shocks required to calibrate the model.

The previous arguments can be illustrated more clearly by considering the perfect-foresight version of the model. Here the evolution of the disturbances is known with certainty and the equality of marginal returns holds in strict sense:

$$\exp(e_{t+1})F'(\hat{K}_{t+1},L_{t+1}) - \delta = r^*. \tag{14}$$

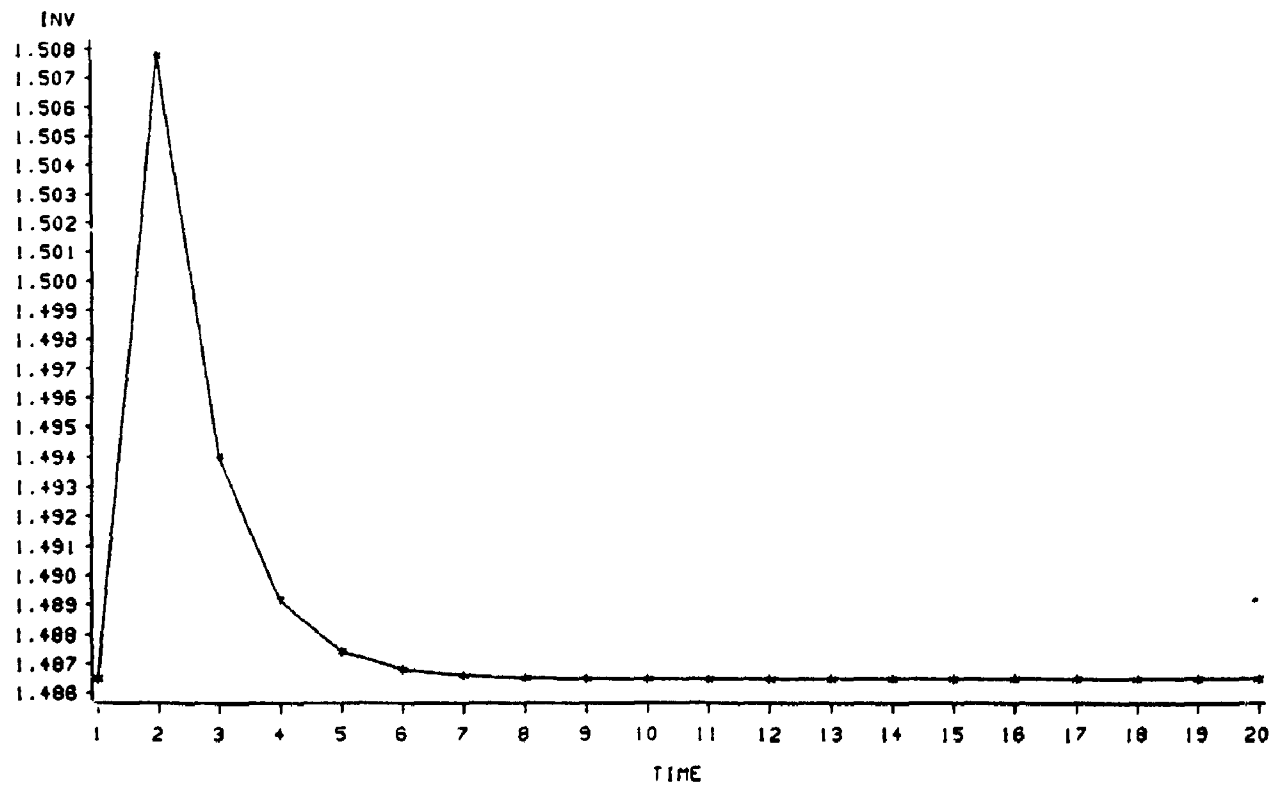
Figures 2 and 3 display the perfect-foresight equilibrium time-profiles of investment and GDP as generated by the following experiment. Consider a 1.18% productivity shock with a serial correlation parameter of 0.35 that hits the economy at date 2. Since $e_{t+1} = 0.35e_t$, this shock affects the productivity of investment (i.e. the marginal product of capital in the following period) by only 0.42%. At date 3 this investment shock declines to 0.15%. Thus, in period 2 investment is enlarged as K_3 is adjusted upward to ensure that the equality in (14) is maintained. In the following period, as the shock starts to vanish, optimizing agents reduce investment so as to accommodate the capital stock to its declining productivity (see Figure 2). In the stochastic version of the model, where positive or negative technological shocks occur randomly, investment will tend to exhibit a pattern of negative first-order serial autocorrelation.

The weak comovement between GDP and investment is also imposed by (14). This condition implies that domestic output increases when the shock hits the economy and then falls monotonically towards its starting value as the effect of the disturbance disappears. Investment, on the contrary, falls below its stationary equilibrium when the economy reaches date 3 (after the expansion in I_2), and from then on increases gradually until it has returned back to it (see figures 2 and 3). If



INVESTMENT IN THE PERFECT-FORESIGHT ECONOMY

Figure 2



OUTPUT IN THE PERFECT-FORESIGHT ECONOMY

Figure 3

this kind of behavior prevails in the stochastic environment, it is reasonable to expect that the correlation between investment and output will be quite weak.

It is important to mention that domestic capital irreversibility constraints, of the type considered by Sargent (1980), are not relevant to generate the type of behavior observed in the investment process of the artificial economy. In all the numerical experiments performed here, total gross investment in every possible state of nature is always positive.

The almost perfect correlation between C and GDP, and hence the high correlation of S and GDP, has the following theoretical justification. In this artificial economy, only the wealth effect caused by the disturbances affects the behavior of consumption. The intertemporal consumption-substitution effect, that operates in a closed economy when a favorable persistent shock causes the interest rate to increase, is no longer at work because the relative price of consumption at different dates is exogenously determined in the world's capital market. This argument also accounts for the observation that private consumption and domestic output remain highly correlated regardless of the degree of risk aversion.

The negative serial autocorrelation of the trade-balance-output ratio is due to the strength of the consumption-smoothing effect present in the model. As formally established in Chapter I, favorable transitory shocks in a small open economy, where savings and investment decisions are separated, motivate optimizing agents to increase foreign-asset accumulation in order to smooth consumption. The balance of trade improves in the same period that the shock occurs, but is negatively affected in all subsequent periods -as agents deplete foreign assets to finance the extra imports required to increase consumption permanently.

Since this model incorporates an endogenous rate of time preference, it is also important to establish whether this factor plays an important role in explaining why the behavior of the aggregates deviates from that observed in closed-economy real business cycle models. Simulating the GHH model using Stationary Cardinal Utility does not dramatically affect the behavior of their results[20]. Furthermore, in the present model the expected value of the discount factor is 0.96 and its variability is less than 0.06%. However, to investigate the issue in more detail, variations of the model for the cases of a closed economy with SCU preferences and an open economy with constant discounting have been simulated. This analysis showed unambiguously that the introduction of the world's capital market is responsible for the volatile behavior of investment. In the case of an open economy with constant discounting, the percentage variability of I is 16.0%, its serial autocorrelation is -0.42 and its comovement with GDP is only 0.22[21]. The correlation of GDP and consumption is still very high, 0.96, and the serial correlation of TB/Y is still negative at -0.47. In contrast, the closed-economy model with variable time preference delivered an investment process with 5.1% volatility, 0.43 autocorrelation and 0.94 output correlation.

5.- Concluding Remarks.

This chapter investigated quantitatively real business cycles in a framework where the existence of an international, perfectly competitive capital market is allowed. The artificial economy studied was characterized by a highly flexible savings mechanism. Both domestic capital and foreign financial assets are used to allocate resources intertemporally in an environment where there are no restrictions on international borrowing and lending, nor there are any frictions or adjustment costs in the process of investment.

The results of various numerical simulations indicated that the process of foreign-asset accumulation may play an important role in real business cycle theory. In comparison with closed-economy prototypes, a relatively less volatile and less persistent technological shock was required to mimic the observed variability and persistence of output. The equilibrium stochastic processes of some of the macro-aggregates were fairly well replicated, as the close match between the observed sample moments and the population moments obtained with the artificial economy suggested. In contrast, the model performed poorly in reproducing the stylized facts of investment, the comovement of consumption and the persistence of the trade-balance-output ratio. These inaccurate results were related to the excessive flexibility of the savings mechanism embodied in the model, particularly the costless manner in which capital accumulation and depletion can be undertaken. Hence, the numerical exploration performed in this chapter indicated that the introduction of frictions in the investment process may be important for the subsequent development of real business cycle models of the open economy.

FOOTNOTES

1. With the exception of works like Ahmed (1986) or Hercowitz (1986b).

2. Hercowitz (1986b) concluded that an exchange-economy, dynamic optimizing model cannot explain the weak countercyclical behavior of the Israeli foreign debt. This is attributed to a strong procyclical behavior of consumption, which is in turn related to either the assumption of exogenous endowments or the existence of capital controls.

3. Alternatively, Frenkel and Razin (1987) explore how a stable steady-state equilibrium can be obtained by assuming that individuals face a positive probability of dying each period.

4. Epstein (1983) showed theoretically that the stochastic growth model with SCU preferences generates similar comovement and persistence features as those obtained with the constant-discount framework. A numerical experiment performed on a variation of Greenwood, Hercowitz and Huffman (1988) illustrated that the variability of the discount factor is negligible. The model's results are almost identical to the ones they obtained with the standard time-separable utility function. The only changes were a 0.2 reduction in the consumption-GNP correlation and a 2.4 point increase in the percentage variability of investment.

5. The assumption that r^* is non-random is not innocuous. Fluctuations in the rate of interest cause consumption-substitution and consumption-smoothing effects, the direction of the latter depending on whether the economy started out as an international net borrower or lender. However, as shown in Chapter I, these effects are likely to be weak as long as the shocks are small and stationary, r^* is small and foreign interest payments are a small fraction of GDP. In the present paper, $r^*=4\%$ and $-r^*A/GDP$ is about 2% in the Canadian post-war average. Consequently, numerical simulations with interest-rate shocks of less than 5% standard deviation did not have large effects on the results to be reported later.

6. The role of international risk sharing arising from the exchange of contingent claims is not explicitly modelled. This is partially replaced with trade in risk-less foreign assets. These assets can be interpreted as a perfectly diversified portfolio consisting of shares of the capital stock in various countries. Furthermore, the numerical analysis of Cole and Obstfeld (1988) suggests that, for some specifications of tastes and technology, the competitive allocations are independent of the completeness of financial markets.

7. Chamberlain and Wilson (1984) showed that solvency restrictions can take complicated forms in stochastic models. However, in the numerical simulations performed here, long-run solvency is obtained

by adopting the conditions from Theorem 4 of Epstein (1983). These are two boundary restrictions on the rate of time preference that ensure the existence of a stable stochastic steady-state in an economy with a linear technology. Intertemporal solvency is then numerically verified by noting that the long-run probability of setting foreign debt below -1.4 in the A grid of the artificial economy is infinitesimal.

8. A detailed analysis of the theoretical properties of the model was presented in Chapter I. Various comparative statics exercises were undertaken to investigate impact and dynamic effects of stochastic and deterministic technological disturbances.

9. Following the analysis of Epstein (1983), a formal proof of the concavity of the value function is presented in Appendix 2 of Chapter I. In fact, not only is V(•) concave but Log[-V(•)] is convex.

10. The use of the SCU function implies that the algorithm suggested by Bertsekas (1976) to speed up the convergence of the decision rules can no longer be used. This, combined with the large memory requirements, increases considerably the time and cost of running the program in a regular mainframe, and seriously restricts the size of the state space than can be analyzed. These technical difficulties were eliminated using an ETA-10P supercomputer with a vector-Fortran compiler.

11. The gradual redefinition of the grids is a complicated process. Since agents are allowed to substitute assets for capital and vice versa, the discretization of the state space restricts not only the divisibility of each savings instrument, but also the degree to which one can be substituted for the other.

12. This steady-state system of equations is studied in more detail in Chapter I.

13. An alternative depreciation rate of 5.2% is determined using the capital evolution equation and the data on net capital stocks and gross investment, computing the long-run average as in Hercowitz (1986a). However, with such a depreciation rate the model generates 43.0% standard deviation in investment and an average I/Y ratio of only 18.0%.

14. Values of ω=4.3 and 4.9 (intertemporal elasticity of labor supply=0.3 and 0.26) were estimated by Hercowitz (1986a) for the Canadian economy. The author points out, however, that these estimates are not very reliable because one of the reduced form coefficients involved is imprecisely estimated. In the present model, a value of 4.3 generates a standard deviation of hours worked of just 0.76%.

15. See note 2.

16. Levhari and Srinivasan (1969) showed that a similar result holds for a stochastic exchange economy where the return on assets is an i.i.d. process that satisfies $ER_t<1/\beta$ (with β as the constant discount factor).

17. The sensitivity analysis for changes in the risk-aversion parameter showed that γ and the volatility, serial correlation and GDP correlation of assets are directly related. Increasing γ to 3.0 rises the volatility of assets to 23.7%, their serial correlation to 0.93 and their GDP correlation to -0.15. The behavior of the rest of the aggregates is not significantly affected by the changes in the value of this parameter.

18. This conclusion is supported by the results of the sensitivity analysis for changes in the coefficient of relative risk aversion, the depreciation rate and the intertemporal elasticity of labor.

19. A more detailed analysis of how the model's optimality conditions deliver these results is presented in Chapter I.

20. See note 5.

21. In this case, the steady-state value of A is not determined and hence there is no well-behaved limiting distribution of foreign assets. The limiting distribution of domestic capital is still stationary and nicely-behaved.

CHAPTER III.

BUSINESS CYCLES, ADJUSTMENT COSTS AND THE THEORY OF INVESTMENT IN A SMALL OPEN ECONOMY.

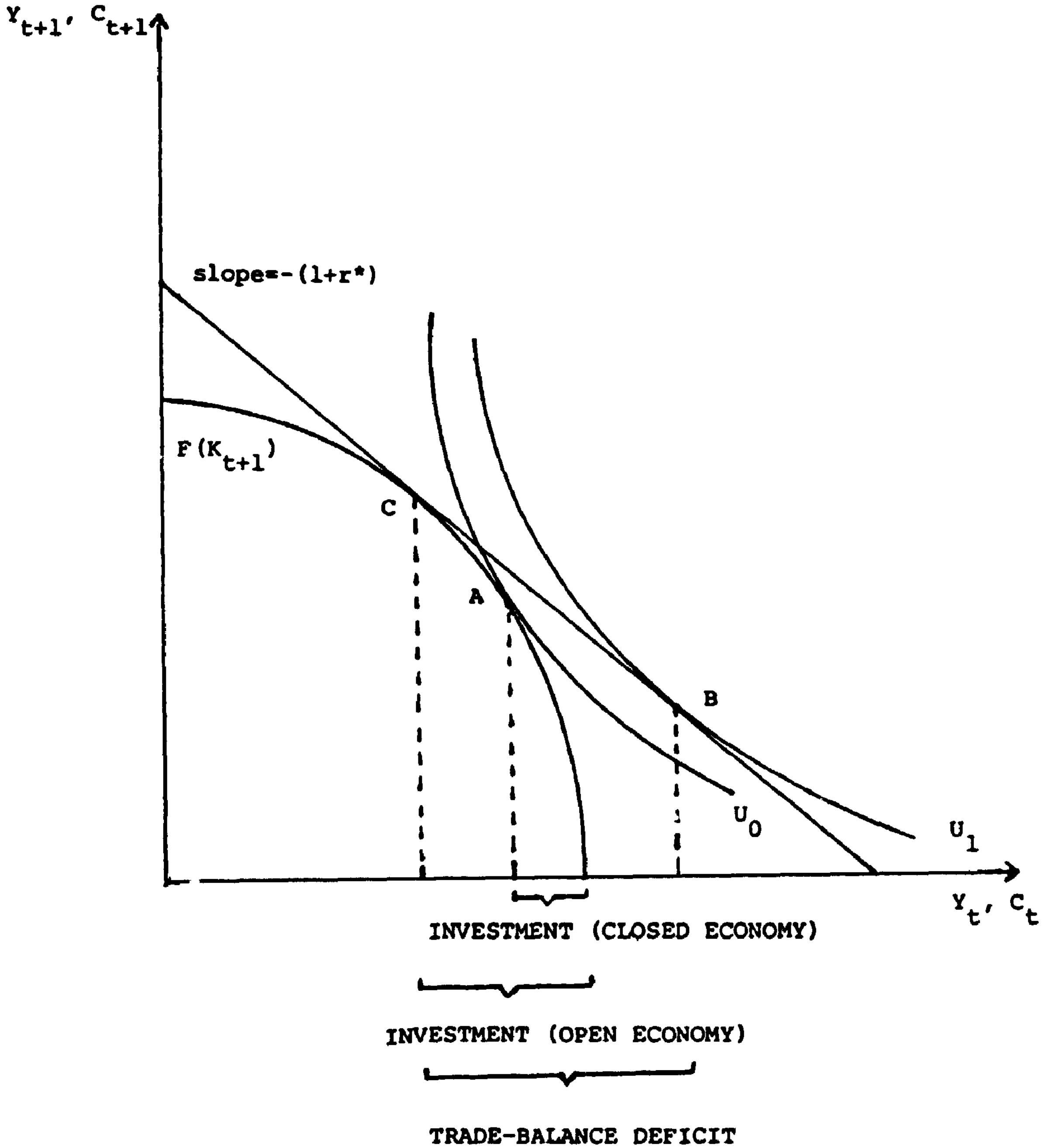
1.- **Introduction.**

This chapter investigates the role that capital-adjustment costs play in the theory of investment of a small open economy. The main reason for undertaking this research is to produce a model that can successfully extend the existing work on closed-economy real business cycles to the international arena. The pioneering work of Kydland and Prescott (1982) and Long and Plosser (1983) preceded the development of a wide variety of closed-economy models capable of replicating some of the observed stylized facts of the U.S. economy[1]. Therefore, an extension of this theory to a small open-economy environment, where foreign financial assets are allowed to play a role in the dynamics of savings and investment, appears to be interesting.

As illustrated in the last two chapters, the development of open-economy real business cycle models is complicated by the fact that they postulate a significantly different theory of investment behavior. With the exception of models like that of Kydland and Prescott (1982), where time-to-build restrictions in the process of investment are considered, most real business cycle prototypes developed to date assume that capital accumulation is not affected by explicit costs or frictions. The only cost associated to additional investment is the marginal utility of the current consumption sacrificed in order to augment the future capital stock. Thus, as King, Plosser and Rebelo (1988) point out, most of the existing real business cycle models are stochastic extensions of the free-adjustment neoclassical model of investment for a closed economy.

In the closed-economy neoclassical model of investment, optimal capital-accumulation decisions are equivalent to optimal savings choices because the capital stock is the only existing vehicle of savings. Optimal investment is determined by equating the intertemporal marginal rate of substitution with the marginal productivity of future capital, which occurs at a tangency point like A in Figure 1. Consequently, the adjustments in investment, and in the intertemporal allocation of consumption, that follow a productivity disturbance are governed by intertemporal consumption-smoothing and consumption-substitution effects. These effects play an important role in determining the ability that real business cycle models possess to reproduce the observed stylized facts. When a favorable, serially correlated productivity disturbance occurs, the desire to smooth consumption causes consumption, investment and output to exhibit positive comovement and persistence. Because investment is equivalent to savings and savings are planned so as to smooth consumption, investment is likely to be more volatile than output and output is likely to be more volatile than consumption. Furthermore, persistent disturbances can also cause the equilibrium interest rate to rise, generating an intertemporal consumption-substitution effect away of current consumption[2]. This consumption-substitution effect dominates the consumption-smoothing effect in certain states of nature where the latter is particularly weak, and hence it generates positive but less-than-perfect correlation between consumption and output -as observed in the actual data.

It has been known for some time that the theory of investment described in the previous paragraph is significantly modified when the neoclassical model is applied to a small open economy. Following the



THE NEOCLASSICAL MODEL OF INVESTMENT

FIGURE 1

writings of Fisher (1907) and (1930), Hirshleifer (1958) illustrated how a different theory of investment is obtained when economic agents have access to financial markets from which they can borrow or lend in order to support consumption planning. The essential difference between the neoclassical investment models for the closed economy and the small open economy is that in the latter savings and investment decisions are separated. Individuals have access to a perfectly competitive world capital market where they can borrow or save by trading foreign assets that pay or charge the real interest rate r^*. Optimal savings are chosen so as to equalize the intertemporal marginal rate of substitution with the world's intertemporal relative price of consumption $1+r^*$, as in point B of Figure 1. Whereas optimal investment is solely decided on the basis of portfolio considerations that equate the marginal rates of return on domestic capital and foreign assets, as in point C of Figure 1[3]. In this environment, productivity shocks affect investment only to the extent that they alter the marginal productivity of domestic capital relative to the world's real interest rate. Thus, investment is not regulated by either consumption-smoothing or consumption-substitution effects. The former are still present, but operating through the accumulation of foreign financial assets, and the latter are eliminated, because the world rate of interest is exogenously determined in the international financial market.

Quantitative research in open-economy real business cycles has revealed that the neoclassical model of investment in its original form is not capable of replicating the observed stylized facts. Using a two-country model, Backus, Kehoe and Kydland (1989) noted that, unless time-

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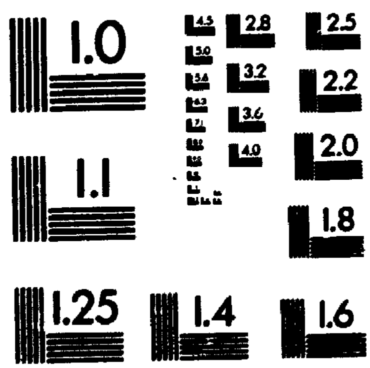


to-build restrictions and international spill-overs of technological disturbances are considered, the variability of investment is largely exaggerated by the model. Furthermore, in Chapter II it was observed that a model of a small open economy exaggerates the variability of private investment and underestimates its first-order serial autocorrelation, its correlation with domestic output and its correlation with aggregate savings. Consequently, in order to postulate a solution for this problem, this chapter proposes to modify the neoclassical model of investment for a small open economy by adopting a different view of the technology. An explicit capital-adjustment cost is introduced here to force optimizing agents to undertake changes in capital accumulation in a gradual manner, so that random productivity disturbances may generate a relatively less volatile pattern of investment behavior[4].

As in the previous chapter, the present work has also two empirical motivations. First, since the intertemporal reallocation of consumption in a real-world economy is achieved not only by changes in investment, but also by accumulating or depleting foreign financial assets to finance a trade deficit or surplus, it is important for real business cycle theory to explain the behavior of both investment and the balance of trade. In fact, the empirical evidence indicates that trade-balance movements are large and countercyclical, suggesting that they may play as much as an important role as changes in private investment do. The second empirical motivation follows from the debate on the meaning of the high degree of correlation between savings and investment observed in industrialized economies. This debate started with the empirical work of Feldstein and Horioka (1980), whom considered these high correlations as evidence indicating that the degree of international capital mobility is very low. In contrast, numerical experiments performed with overlapping-generations models by Obstfeld (1985) and Finn (1988) indicate that, considering productivity disturbances with the right intensity of persistence, a high degree of correlation between savings and investment can be theoretically consistent with the assumption of perfect capital mobility. In this context, the numerical investigation undertaken in this paper studies the ability of a dynamic stochastic model to match the correlation between savings and investment when restricted to employ the kind of disturbances required to replicate actual business cycles.

Technically, the artificial economy studied here is an extension of theoretical developments made by Obstfeld (1981a) and (1981b) and Epstein (1983) to an environment where investment is costly to adjust. The work of Obstfeld utilizes an endogenous rate of time preference to obtain a well-behaved deterministic stationary equilibrium for the holdings of international assets. As discussed in the last two chapters, such a deterministic stationary equilibrium is produced when the world's real interest rate and the rate of time preference are equalized. It is a well-established fact, documented among others by Helpman and Razin (1982) and Frenkel and Razin (1987), that as long as the world's interest rate is greater (smaller) than the rate of time preference, individuals will rationally choose to accumulate (deplete) foreign assets so as to finance an increasing (decreasing) consumption stream[5]. Thus, the usual constant-discount formulation of preferences cannot explain the process by which this long-run equilibrium is reached; in this framework either *(a)* there is no steady state, because the rate of time preference is not preset at the level of the world interest rate, or *(b)*, when the two are equal, the economy starts out in a stationary equilibrium, which is *contingent* on the agents' initial level of asset holdings, and will never be displaced from it. In an economy with uncertainty, the endogenous rate of time preference is utilized in a similar manner to determine a stable *stochastic* stationary state. Epstein (1983) extended the endogenous time-preference framework to an uncertain environment and determined sufficient conditions to obtain a stationary joint limiting distribution of the state variables in the stochastic steady-state. His analysis also showed that this formulation of preferences does not imply a radical departure from the standard time-separable setup. The costs of adjusting investment are introduced in accordance with the principles summarized in Brechling (1975), adopting the convex, quadratic specification employed by Gould (1968), Craine (1975) and Eichenbaum (1984).

The model is numerically analyzed using the same solution method employed in the last chapter. This procedure computes the exact joint limiting distribution of the state variables in the stochastic stationary equilibrium of the economy, making use of an algorithm that solves numerically the functional equation problem for a discretized version of the state space. The statistical moments that characterize the random processes of the model's endogenous variables are calculated using this limiting distribution, and are then compared with the actual sample moments from detrended, Canadian data.

The rest of the chapter is ordered as follows: Section 2 describes the structure of the artificial economy to be studied here and comments on the strategy used to implement the model empirically. Section 3 presents the results of the numerical experiments, stressing the differences between the model with adjustment costs and the free-adjustment neoclassical model and comparing the results with those obtained for some closed-economy prototypes. Some concluding remarks are presented in the last section.

2.- The Structure of the Model and the Solution Technique.

The analysis starts with the description of the artificial economy and an explanation of the numerical solution method employed to analyze it. This section presents first a dynamic stochastic model of a small open economy that incorporates explicit costs of adjustment in the capital stock and an endogenous rate of time preference. The analysis of the model is then simplified by applying dynamic programming techniques that are used later to calculate numerically the equilibrium stochastic process of the economy.

2.1 Structure of the Model.

Preferences: All agents are identical and infinitely-lived, with preferences described by the Stationary Cardinal Utility function formulated by Epstein (1983):

$$E_0 \left[\sum_{t=0}^{\infty} \{ u(C_t - G(L_t)) \exp[-\sum_{\tau=0}^{t-1} v(C_\tau - G(L_\tau))] \} \right]. \tag{1}$$

Here, C_t is private consumption and L_t are labor services. The Stationary Cardinal Utility is assumed to embody the following instantaneous utility and time-preference functions:

$$u[C_t - G(L_t)] = (1-\gamma)^{-1} [(C_t - L_t^{\omega}/\omega)^{1-\gamma} - 1], \qquad \omega > 1, \quad \gamma > 1, \qquad (2)$$

$$v[C_t - G(L_t)] = \beta \text{ Ln } [1 + C_t - L_t^{\omega}/\omega], \qquad \beta > 0. \qquad (3)$$

Which satisfy the following conditions:

$$u(\cdot)<0 \text{ , } u'(\cdot)>0 \text{ , } u'(0) = \infty, \qquad (4.1)$$

$$v(\cdot)>0, \ v'(\cdot)>0, \ v''(\cdot)<0, \qquad (4.2)$$

$$u'(\cdot)\exp[v(\cdot)] \text{ non-increasing}, \qquad (4.3)$$

$$\text{Log}[-u(\cdot)] \text{ convex}. \qquad (4.4)$$

The formulation of preferences presented in (1) is che same as in the previous chapters. It ensures the existence of a well-behaved stationary equilibrium for the holdings of international assets by assuming that the rate of time preference is an increasing function of past consumption levels. As in the work by Obstfeld (1981a) and (1981b), the deterministic long-run equilibrium of foreign-asset holdings is determined by equating the rate of time preference with the world's real interest rate. According to the specific functional forms adopted in (2) and (3), lifetime utility is studied in terms of the composite commodity defined by consumption minus the disutility of labor. This allows the model to focus expressly on the dynamic interaction of domestic capital and foreign financial assets as alternative vehicles of savings. The cost of this simplification, however, is that the intratemporal marginal rate of substitution between consumption and labor is forced to depend on the latter only, and hence the wealth effect affecting the supply of labor is eliminated. Finally, conditions (4.1)-(4.4) are sufficiency conditions identified by Epstein (1983) to satisfy the requirements of dynamic programming, to ensure that consumption in any given period is a normal good and to prove the existence of a stationary probability distribution that characterizes the stochastic steady-state of the economy[6].

Technology and Financial Structure: The mechanism by which resources for consumption or investment are made available has two components. First, the domestic production technology is described as follows:

$$G(K_t, L_t, K_{t+1}) = \exp(e_t)\, K_t^{\alpha} L_t^{1-\alpha} - (\Phi/2)(K_{t+1}-K_t)^2, \qquad (5)$$
$$0 < \alpha < 1, \qquad \Phi > 0,$$

where e_t is a random technological shock to be discussed in more detail later, $K_t^{\alpha}L_t^{1-\alpha}$ is a neoclassical constant-returns-to-scale production function, K_t is the domestic capital stock currently productive and $(\Phi/2)(K_{t+1}-K_t)^2$ is the cost of adjusting the capital stock as a function of net investment $In_t=K_{t+1}-K_t$. The capital evolution equation is given by

$$K_{t+1} = (1-\delta)K_t + Ig_t, \qquad 0 \leq \delta \leq 1, \qquad (6)$$

where δ is a constant rate of depreciation and Ig_t is gross investment.

Individuals also have access to a perfectly competitive, international capital market where foreign financial assets A_t paying, or charging, the real rate of return r^* are exchanged with the rest of the world. The holdings of foreign financial assets evolve according to

$$A_{t+1} = TB_t + A_t(1+r^*), \tag{7}$$

where TB_t is the balance of trade.

By combining domestic production with foreign borrowing or savings, the aggregate resource constraint dictates that domestic absorption plus the balance of trade cannot exceed GDP net of adjustment costs[7]:

$$C_t + Ig_t + TB_t \leq \exp(e_t)\, K_t^{\alpha}\, L_t^{1-\alpha} - (\Phi/2)(K_{t+1}-K_t)^2. \tag{8}$$

Domestic absorption is defined here as the sum of private consumption and gross investment[8].

The convex quadratic formulation of the adjustment cost included in (5) ensures that the total cost of changing the capital stock by a fixed amount is larger the faster the adjustment. Hence, optimizing agents are motivated to adjust investment in a gradual manner in order to avoid large adjustment costs[9]. Also, the domestic economy is assumed to be small relative to the size of the international capital market, so that individuals take r^* as given when formulating their optimal intertemporal plans. For simplicity, world-wide productivity disturbances are assumed to cancel each other on the average and thus r^* is considered to be non-random[10].

2.2 The Dynamic Programming Problem and the Solution Technique.

The intertemporal equilibrium of the artificial economy described above is characterized by a set of state-contingent decision rules for consumption, domestic capital, labor supply and foreign financial assets that maximize (1), given K_0, A_0, e_0 and the stochastic process governing the disturbances, subject to (5)-(8), the non-negativity restrictions

$K_t \geq 0$, $L_t \geq 0$ and $C_t \geq 0$, and the condition of intertemporal solvency[11]. The same problem can be studied and numerically solved in a more tractable manner by applying Bellman's optimality principle.

At any given date, the optimal intertemporal choices of rational agents involve selecting K_{t+1}, A_{t+1}, C_t and L_t given the state of the economy as described by K_t, A_t and e_t. By taking advantage of the time-recursive structure of the Stationary Cardinal Utility, and by simplifying the stochastic process of the disturbances in the manner to be discussed below, the optimal decision rules that characterize the equilibrium stochastic process of the economy can be obtained as the solutions of the following dynamic programming problem:

$$V(K_t, A_t, e^s{}_t) = \max_{[K_{t+1}, A_{t+1}, C_t]} \{ (1-\gamma)^{-1} [(C_t - \hat{L}_t{}^{\omega}/\omega)^{1-\gamma} - 1] + \exp[-\beta \, Ln(1+C_t - \hat{L}_t{}^{\omega}/\omega)] [\sum_{r=1}^{2} \pi_{sr} V(K_{t+1}, A_{t+1}, e^r{}_{t+1})] \}, \tag{9}$$

s.t.

$$C_t = \exp(e^s{}_t) K_t{}^{\alpha} \hat{L}_t{}^{1-\alpha} - K_{t+1} + K_t(1-\delta) - (\Phi/2)(K_{t+1} - K_t)^2 + (1+r^*)A_t - A_{t+1},$$

$$\hat{L}_t = \underset{L_t}{\operatorname{argmax}} \{ \exp(e^s{}_t) K_t{}^{\alpha} L_t{}^{1-\alpha} - L_t{}^{\omega}/\omega \}.$$

Here, π_{sr} for $s,r=1,2$ denotes the transition probability of the next-period's technological disturbance conditional on its observed value in the current period. Accordingly, the usual properties that $0 \leq \pi_{sr} \leq 1$ and $\pi_{s1} + \pi_{s2} = 1$ for $s,r=1,2$ must be satisfied. Note that the stochastic structure of the problem has been simplified by assuming that the disturbances follow a two-point Markov process, so that in any given period the productivity shock takes one of two values

$$e_t \in E = \{ e^1 , e^2 \}. \quad (10)$$

Then the assumptions that $\pi_{11}=\pi_{22}=\pi$ and $e^1=-e^2=e$ are adopted. These symmetry conditions imply that the asymptotic standard deviation, σ_e, and the first-order autocorrelation coefficient, ρ_e, that characterize the stochastic shocks are given by $\sigma_e=e$ and $\rho_e=2\pi-1$ respectively.

As in the previous chapter, the particular values of the parameters γ (coefficient of relative risk aversion), ω (1 plus the inverse of the intertemporal elasticity of substitution in labor supply), α (capital's share in output), δ (depreciation rate) and β (the consumption elasticity of the rate of time preference), are selected using long-run averages of actual data and the restrictions imposed by the theoretical structure of the model, and also by considering some estimates obtained in the relevant empirical literature. Accordingly, these structural parameters are assigned the following values[12]:

$$\alpha=0.32, \quad \beta=0.11, \quad \gamma=1.001 \text{ or } 2.0, \quad \delta=0.1, \quad \omega=1.455 \quad \text{and} \quad r^*=0.04. \quad (11)$$

The model is calibrated by adjusting the parameters Φ (the rate of change of the marginal adjustment cost), e and π. The first parameter is selected so as to mimic the observed volatility of private investment, and the second and third are set so as to replicate the variability and first-order serial autocorrelation of Canadian post-war, detrended GDP[13].

The dynamic programming problem is solved following a procedure suggested in Bertsekas (1976) and employed by Sargent (1980) and Greenwood, Hercowitz and Huffman (1988). This methodology was dicussed in detail in Chapter II. In short, the technique starts by specifying a discrete grid of points to approximate the state space. In this case, two evenly-spaced grids containing the admissible values of domestic capital $K=(K_1,...,K_M)$ and foreign assets $A=(A_1,...,A_N)$ need to be defined. Thus, the state space of this artificial economy is given by the set KxAxE that contains 2MN elements. The next step is to construct an algorithm that performs successive iterations in the functional equation (9). The algorithm iterates on (9) using the set of numbers included in KxAxE until the sequences of optimal, state-contingent decision rules for domestic capital and foreign assets converge. The decision rules obtained in this way are combined with the conditional probabilities π_{sr}, for s,r=1,2, to define the one-step transition probabilities of moving from any initial triple of domestic capital, foreign assets and the technological disturbances to any other such triple in one period. These transition probabilities are condensed in a matrix P of dimensions (2MNx2MN), which is used to calculate the stationary probabilities of each triple of K, A and e. The long-run probabilities are calculated by iterating on the sequence $\rho^1=\rho^0 P$, where ρ^0 is an initial-guess vector of dimensions (1x2MN) and ρ^1 is a vector of identical dimensions that is used as the new guess in the following iteration. These iterations eventually converge to a unique fixed point ρ^*, which is the joint limiting probability distribution of K, A and e that characterizes the stochastic steady-state of the economy. This probability distribution is used to compute population moments of variability, comovement and persistence of all endogenous variables in the model.

3.- Calibration and Analysis of the Results.

In this section, the quantitative performance of the model with adjustment costs is evaluated by comparing its own statistical moments with those obtained from the original neoclassical model, which were studied in the previous chapter, and with the actual moments from the Canadian economy. This is done for two sets of calibration exercises, one for each value of the risk aversion parameter. The sample moments calculated from the Canadian data are listed in panel A of tables 1 and 2[14].

3.1 The Neoclassical Model without Adjustment Costs.

The statistical moments calculated for the neoclassical model are listed in panel B of tables 1 and 2. Panel B of Table 1 presents the results for the case where $\gamma=2.0$ and the same panel in Table 2 lists the moments for $\gamma=1.001$, the marginal probability distribution of domestic capital and foreign assets that corresponds to the first case is plotted in Figure 2. These two artificial economies were calibrated to replicate the observed percentage variability and first-order serial autocorrelation of GDP.

In the case where $\gamma=2.0$, the technological disturbances are set to exhibit 1.18% volatility and 0.36 first-order autocorrelation. As panels A and B of Table 1 indicate, the neoclassical model exaggerates the volatility of private investment (21.0% instead of 9.8%) and underestimates its first-order autocorrelation (-0.32 v. 0.31), its correlation with GDP (0.24 v. 0.64) and its correlation with aggregate savings (0.25 v. 0.43)[15]. Furthermore, it also exhibits too much

Table 1

Statistical Moments: Canadian Data and Artificial Economies[a].
(γ=2.0)

Variables	A Canadian Data 1946-1985			B Free-Adjustment Economy σ_e=1.18% ρ_e=0.36 Φ=0.0			C Costly Adjustment Economy σ_e=1.29% ρ_e=0.42 Φ=0.028		
	(I)[b]	(II)[c]	(III)[d]	(I)[b]	(II)[c]	(III)[d]	(I)[b]	(II)[c]	(III)[d]
1) GDP	2.810	0.615	1.000	2.810	0.615	1.000	2.810	0.615	1.000
2) GNP	2.950	0.643	0.995	2.821	0.619	0.990	2.891	0.622	0.987
3) C	2.460	0.701	0.586	2.086	0.693	0.944	2.250	0.689	0.932
4) S	7.306	0.542	0.662	5.772	0.599	0.932	5.582	0.629	0.896
5) I	9.820	0.314	0.639	21.056	-0.319	0.235	9.888	-0.017	0.505
6) K	1.380	0.649	-0.384	1.980	0.377	0.669	1.464	0.752	0.575
7) L	2.020	0.541	0.799	1.936	0.615	1.000	1.937	0.615	1.000
8) $-r^*A$	15.250	0.727	-0.175	19.566	0.886	-0.198	23.131	0.986	-0.045
9) TB/Y	0.019	0.623	-0.129	0.046	-0.312	0.032	0.019	0.032	0.023
	CORR(S,I) = 0.434			CORR(S,I) = 0.251			CORR(S,I) = 0.501		

[a]The data are measured in per-capita terms of the 15+ population, logged and detrended with a linear quadratic time trend. 1)-3), 5) and 8) are the totals from the national income accounts in 1981 dollars. 6) was obtained from the end-of-period net stocks of fixed non-residential capital in manufacturing and non-manufacturing industries, in 1981 prices. The labor data is an index of man hours worked by paid workers with 1981=100.0. Savings in 4) is generated as investment plus the trade balance surplus. The source of all is the CANSIM data retrieval.

[b]Percentage standard deviation, except 9 which is not in percent.

[c]First-order autocorrelation coefficient.

[d]Coefficient of correlation with GDP.

Table 2

Statistical Moments: Canadian Data and Artificial Economies[a].
(γ=1.001)

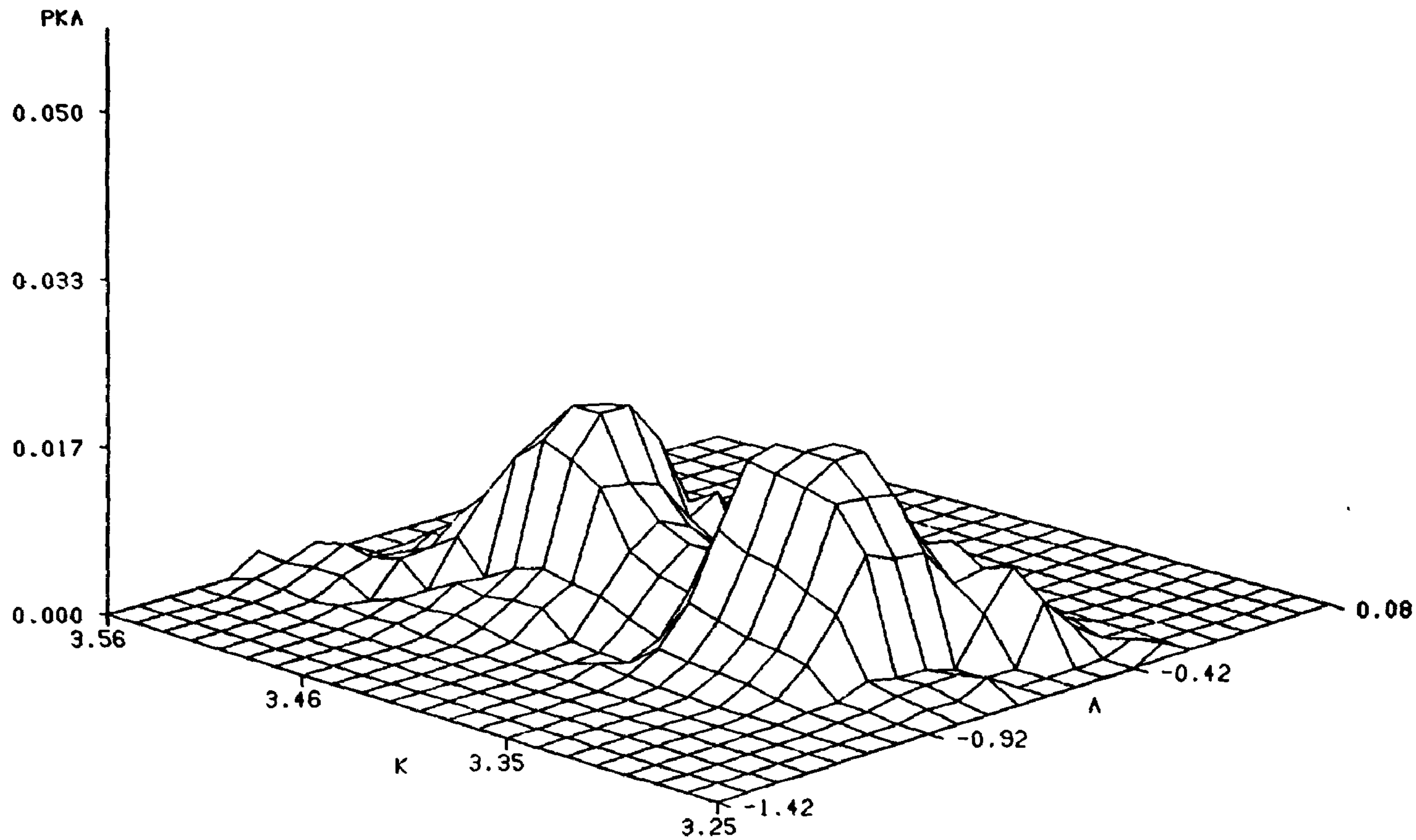
Variables	A Canadian Data 1946-1985			B Free-Adjustment Economy σ_e=1.18% ρ_e=0.34 Φ=0.0			C Costly Adjustment Economy σ_e=1.29% ρ_e=0.41 Φ=0.023		
	(I)[b]	(II)[c]	(III)[d]	(I)[b]	(II)[c]	(III)[d]	(I)[b]	(II)[c]	(III)[d]
1) GDP	2.810	0.615	1.000	2.810	0.615	1.000	2.810	0.615	1.000
2) GNP	2.950	0.643	0.995	2.795	0.611	0.997	2.849	0.613	0.998
3) C	2.460	0.701	0.586	2.105	0.669	0.961	2.234	0.689	0.957
4) S	7.306	0.542	0.662	5.483	0.590	0.947	5.209	0.568	0.926
5) I	9.820	0.314	0.639	21.050	-0.343	0.266	9.837	-0.052	0.571
6) K	1.380	0.649	-0.384	1.913	0.322	0.669	1.347	0.792	0.595
7) L	2.020	0.541	0.799	1.931	0.615	1.000	1.927	0.615	1.000
8)-r*A	15.250	0.727	-0.175	11.064	0.632	-0.354	9.551	0.929	0.085
9) TB/Y	0.019	0.623	-0.129	0.046	-0.338	-0.013	0.017	-0.045	-0.080
	CORR(S,I) = 0.434			CORR(S,I) = 0.268			CORR(S,I) = 0.616		

[a]See note "a" in Table 1 for sources and calculations.

[b]Percentage standard deviation, except 9 which is not in percent.

[c]first-order autocorrelation coefficient.

[d]GDP correlation coefficient.



MARGINAL PROBABILITY DENSITY OF CAPITAL AND FOREIGN ASSETS
IN THE NEOCLASSICAL MODEL
Figure 2

positive comovement between consumption and GDP (0.94 instead of 0.59) and results in negative persistence in the trade-balance-output ratio (-0.31 instead of 0.62). The neoclassical model with $\gamma=1.001$ is calibrated using a 1.18% shock with 0.34 first-order serial autocorrelation. Comparing the results in panel B of tables 1 and 2, it transpires that changes in the degree of risk aversion only have sizeable effects on the stochastic process of foreign-asset holdings. The model still misrepresents the moments that characterize investment in the actual data in approximately the same manner as before.

As discussed in Chapter II, the process of capital accumulation is badly reproduced by the free-adjustment neoclassical model of investment because of the frictionless manner in which it can be undertaken. Since this is a small open economy, individuals choose separately the optimal amount of savings and the optimal portfolio allocation of those savings across domestic capital and foreign assets. The portfolio decision is made so as to equalize the expected marginal returns, in utility terms, of both vehicles of savings. Optimal savings, on the other hand, are determined by equating the stochastic intertemporal marginal rate of substitution with the real rate of return on foreign assets. When a productivity shock hits this economy, the domestic capital stock is rapidly and freely adjusted to maintain the equality of expected returns (see Figure 2), and the optimal alterations in the dynamic path of savings are mainly achieved through changes in foreign-asset accumulation.

As discussed in the introduction, investment is not as volatile in closed-economy real business cycle models, which are also based on the neoclassical framework, because there the optimal savings and investment decisions are the same. Hence, in closed-economy models the accumulation of the capital stock responds to both consumption-smoothing and consumption-substitution effects and faces an increasing supply price. In contrast, in the small open-economy framework the consumption-smoothing effect operates mainly through the current account, the intertemporal consumption-substitution effect is eliminated, and investment faces a constant supply price of the capital stock given by r^*.

Given that a real business cycle prototype based on the neoclassical model of investment for a small open economy is not capable of mimicking the stylized facts of Canadian private investment, it is reasonable to deduce that the actual savings mechanism operating in the economy is not as flexible as this framework indicates. The ability that agents in this model have to equalize returns across alternative vehicles of savings can be limited in various ways. It is possible to introduce capital market imperfections or to impose controls on international capital flows. However, the Canadian economy has been historically characterized by the absence of capital controls and a high degree of integration with the U.S. financial markets. Thus, it is more likely that the frictions affecting optimal portfolio allocation are located in the process of domestic investment instead of the current account.

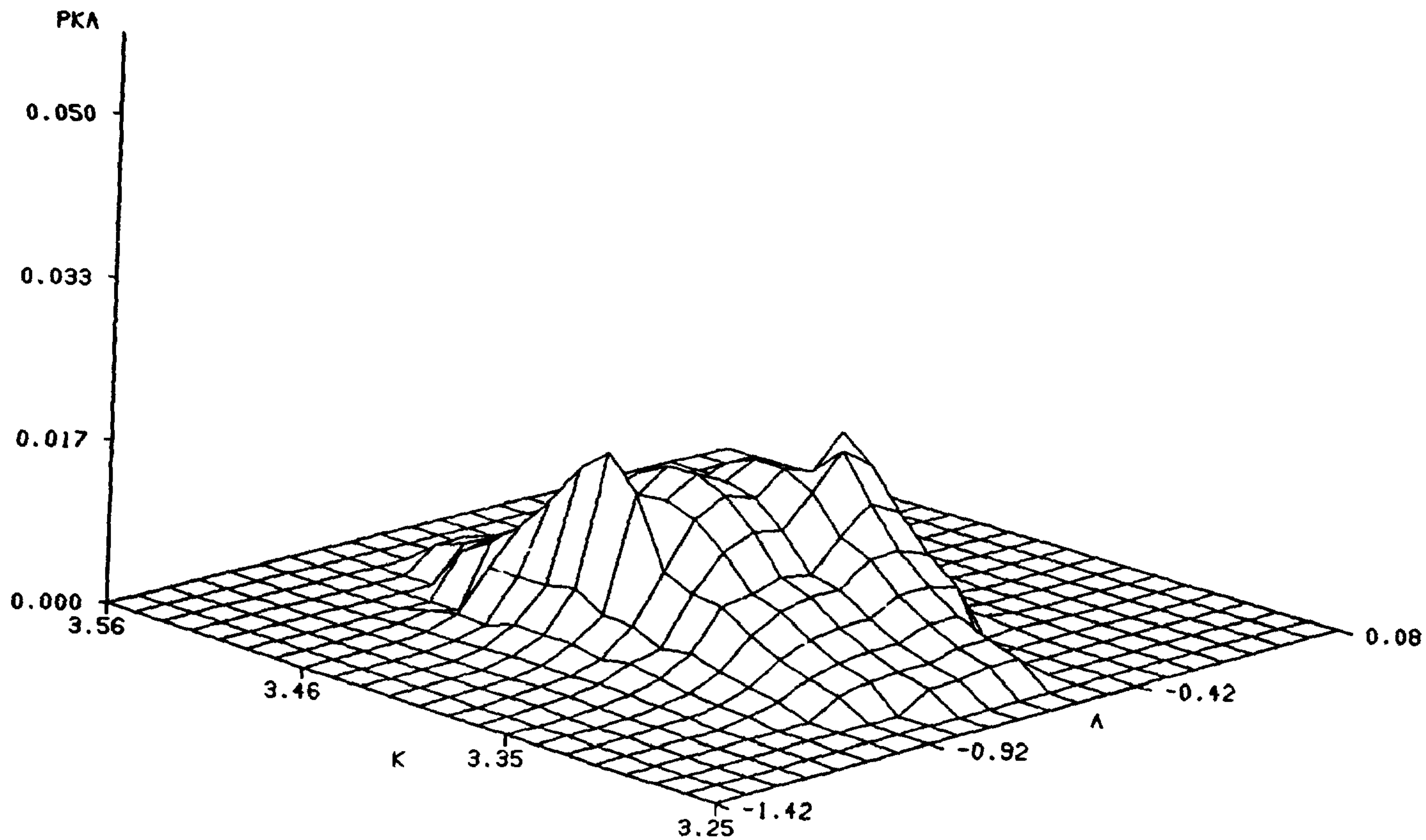
One possible friction affecting the accumulation of domestic capital is the irreversibility of investment studied in Sargent (1980). However, gross and net investment in the simulations of the free-adjustment model resulted always positive in every state of nature. Hence, investment irreversibility is not a binding constraint for this particular model. Another friction that complicates investment, which has been largely studied in the closed-economy context, are adjustment costs in the process of accumulating capital. The role of such adjustment costs is thoroughly analyzed next.

3.2 The Model with Adjustment Costs.

When the costs of adjusting the capital stock take the form introduced in (5), the marginal cost of altering investment is increasing with the absolute size of the adjustment. Firms will then have to consider both current and future marginal adjustment costs when formulating their optimal intertemporal plans. As a result, the reaction of the capital stock to a given change in domestic productivity will be more gradual and the overall behavior of the investment process will be less volatile.

The statistical moments that characterize the equilibrium stochastic process of the artificial economy with investment-adjustment costs are listed in panel C of tables 1 and 2. Panel C of Table 1 presents the results for the case where γ=2.0, and the corresponding marginal probability density of K and A is depicted in Figure 3. Panel C of Table 2 lists the results for γ=1.001.

In the case where the risk-aversion parameter is given a value of 2.0, the artificial economy with adjustment costs is calibrated setting Φ=0.028, σ_e=1.29% and ρ_e=0.42[16]. The value of Φ was not predetermined as the other parameter values because of the partial-equilibrium, non-



MARGINAL PROBABILITY DENSITY OF CAPITAL AND FOREIGN ASSETS
IN THE COST-OF-ADJUSTMENT MODEL
Figure 3

structural nature of most of the existing empirical work on adjustment-cost models of investment. However, the value Φ=0.028 is consistent with the findings of Craine (1975) for the U.S. economy. Using a ratio of price indices to approximate the relative price of capital goods in terms of output, Craine (1975) estimated that the coefficient of the quadratic adjustment cost function is 0.025. Furthermore, the fact that the average adjustment cost in the artificial economy is only about 0.1% of GDP is also consistent with the findings of Brechling (1975) and Eichenbaum (1984), in the sense that investment-adjustment costs are statistically significant but relatively small in size. Thus, the numerical investigation performed here indicates that modest adjustment costs are sufficient to induce a less volatile pattern of investment behavior.

The analysis of panels A and C of Table 1 indicates that the model with capital-adjustment costs is capable of replicating the majority of the stylized facts that characterize Canadian business cycles. First, it generates procyclical behavior in consumption, investment, employment and savings, and produces stochastic processes for the ratio of the balance of trade to output and foreign interest payments that are almost serially uncorrelated. Second, it replicates the same ranking of percentage variability in which the actual aggregates are ordered. In fact, with the exception of $-r^*A$, it mimics the actual percentage standard deviations very closely. Third, the model replicates some of the first-order autocorrelation coefficients calculated with the actual data.

Table 1 also illustrates that the adjustment-cost model is capable of replicating the correlation between savings and investment. In fact, the coefficient of comovement between S and I is already higher than what is actually observed in the Canadian data (0.50 v. 0.43). This result shows that the model can replicate the observed correlation between savings and investment and at the same time explain actual business cycles, all without affecting the assumption of perfect international mobility of financial capital. Thus, the findings of the theoretical analysis, based on numerical simulations of overlapping-generations models, by Obstfeld (1985) and Finn (1988) are successfully extended to a calibration exercise in the context of an infinite-horizon economy, and the argument of Feldstein and Horioka (1980) appears to be invalidated.

The introduction of explicit adjustment costs improves the match between the actual sample moments and the artificial economy's moments. The model with adjustment costs generates statistical moments of volatility, persistence, savings correlation and GDP correlation of private investment that are closer approximations to the actual moments than those obtained with the neoclassical model[17]. In fact, Figure 3 illustrates the less volatile manner in which the accumulation of domestic capital is undertaken in the presence of adjustment costs. Furthermore, the moments of the trade-balance-output ratio and the domestic capital stock are also closer to the observed moments, although the variability, persistence and comovement of foreign assets (and foreign interest payments since r^* is non-random here) are all exaggerated.

It is worth noting that the costly-adjustment model can mimic the variability of both investment and the trade-balance-output ratio very closely. This observation reaffirms the suggestion that the introduction of adjustment costs may play an important role in open-economy real business cycles. Furthermore, following Greenwood (1983), the productivity disturbances can be reinterpreted as terms-of-trade disturbances, and hence these results also indicate that relatively small and transitory shocks to the terms of trade are capable of causing the observed fluctuations in the balance of trade and foreign assets.

Panel C of Table 2 lists the moments calculated for the artificial economy with capital-adjustment costs calibrated under the assumption that γ=1.001. In this case, the model requires a 1.29% shock with 0.41 persistence and an adjustment-cost coefficient Φ=0.023. As in the case of the neoclassical model, it is observed that changes in the degree of risk aversion only affect the stochastic process of foreign-asset holdings. The costly-adjustment model generates a better approximation to the actual moments than the free-adjustment neoclassical model in the same way as before, with the variability of foreign interest payments falling to 9.5%, their first-order autocorrelation increasing to 0.93 and their comovement with GDP changing to -0.085.

An important disadvantage of cost-of-adjustment models in general is that they tend to produce testable vector autoregressions where the effects of highly-persistent shocks cannot be distinguished from the effects of significant adjustment costs[18]. In the context of the present model, a similar problem may arise because the variability of investment in the neoclassical framework can be reduced by increasing the first-order serial autocorrelation of the disturbances (i.e. their one-step transition probability π), instead of introducing the adjustment costs. By increasing the probability of not moving to the opposite state of productivity, the probability of having to accumulate or deplete the capital stock very rapidly is reduced, and hence the variability of investment is also reduced. However, by following this route, the persistence of the shocks is driven to such a high degree that the variability and persistence of all the other aggregates is also increased, and hence the model cannot replicate actual business cycles. For instance, if in the case that γ=2 ρ_e is set to 0.99, the variability of investment is reduced to 5.4%, but the variability of GDP becomes 5.0% and it behaves almost as a random walk. Thus, here it is possible to establish the importance of adjustment costs relative to highly persistence shocks by noting that the latter cannot be employed to mimic real-world economic fluctuations.

3.3 Comparison with Closed-Economy Models and Sensitivity Analysis.

In comparison with the existing work on closed-economy real business cycle models, the two small open-economy prototypes were calibrated using productivity disturbances that exhibit less exogenous volatility and persistence. With respect to the persistence of the disturbances, in the works of Hansen (1985), Prescott (1986) and King, Plosser and Rebelo (1988) the shocks follow a stochastic process close to a random walk, Hansen (1985) estimates their quarterly correlation at 0.95 (0.81 annually). The model with endogenous utilization and investment shocks of Greenwood, Hercowitz and Huffman (1988) requires technological disturbances with 0.51 annual persistence[19]. In contrast, when γ=2.0, the open-economy prototypes require shocks with 0.35 and 0.42 first-order serial autocorrelation for the cases of the neoclassical model and the cost-of-adjustment model respectively.

Considering now the volatility of the disturbances, the ratios of the required volatility of the shocks relative to the observed variability of output are 1.3 and 1.7 in Hansen (1985) and 1.47 in Greenwood, Hercowitz and Huffman (1988). In the open-economy models these ratios are 0.42 for the neoclassical prototype and 0.71 for the model with adjustment costs, when the risk-aversion parameter is assigned a value of 2.0. Thus, although open-economy real business cycle models appear to require smaller and less persistent shocks compared with closed-economy prototypes, the open-economy model with adjustment costs does require relatively larger and more serially correlated disturbances than the neoclassical model.

The introduction of the cost of adjustment in the small open-economy framework has very different consequences than what has been observed in closed-economy real business cycle models. Kydland and Prescott (1982) evaluated the importance of their time-to-build formulation by introducing capital-adjustment costs and studying how this affected the behavior of their closed-economy model. They observed that, setting Φ=1, the statistical moments resulting from the artificial economy were largely inconsistent with U.S. quarterly detrended data. In particular, compared with the model that considers the time-to-build technology instead of adjustment costs, the standard deviation of consumption almost doubled and that of investment expenditures fell by a factor of two. In contrast, in the model studied here adjustment costs are much smaller, since Φ is 0.028 when γ=2 and 0.023 when γ=1.001, and they have the effect of reducing the percentage variability of investment without significantly affecting the behavior of consumption.

It is also worth noting that, relative to the observed sample moments and the results obtained with closed-economy prototypes, both open-economy models exaggerate the comovement between consumption (or savings) and GDP. The introduction of adjustment costs cannot alleviate this problem because such costs do not alter the fact that the intertemporal relative price of consumption is fixed in the world's capital market, and hence the consumption-substitution effects that operate in the closed-economy models are no longer at work.

The comparison of the results listed in Table 2 with the closed-economy models mentioned before shows that the conclusions obtained are robust to changes in the risk-aversion parameter. Small open-economy models require relatively smaller and less persistent shocks than closed-economy models in order to replicate the observed volatility and persistence of domestic output, although the open-economy model with costly adjustment needs disturbances that are relatively larger and more serially correlated than the free-adjustment open economy. Furthermore, the correlation between consumption or savings and output remains very high and seems insensitive to changes in γ. This is due to the mentioned fact that the intertemporal relative price of consumption is not affected by domestic productivity disturbances.

The comparison of panels B and C of tables 1 and 2 has shown that changes in the degree of risk aversion affect only the stochastic process of foreign-asset holdings, independently of whether the domestic capital stock is costly to adjust or not. The study of the overall effects that the reduction in γ from 2.0 to 1.001 causes, indicates that using some intermediate value of this parameter is likely to make the costly-adjustment model replicate the observed sample moments better than any of the two sets of results presented. Specifically, by choosing the appropriate size for the risk-aversion parameter, it may be possible to match the observed moments of foreign interest payments $-r^*A$ without significantly affecting the close match of the other moments. The required value for the coefficient γ would still fall inside the range of the estimates that are commonly regarded as credible.

4.- Concluding Remarks.

This chapter explored the relevance of capital-adjustment costs in the theory of investment of a small open economy, and investigated the ability of a real business cycle model to replicate the stylized facts of Canadian business cycles. The model is capable of replicating the majority of the statistical moments obtained from the actual data, including the correlation between savings and investment, using very small capital-adjustment costs and minimal variability and persistence in the technological disturbances. Specifically, the artificial economy generates similar procyclical and countercyclical patterns as those observed in the macro-aggregates, it replicates the observed ranking and values of the percentage standard deviations, and approximates the ranking and values of some of the first-order autocorrelation coefficients.

In contrast with the costly-adjustment model, the free-adjustment neoclassical model of investment exaggerates the flexibility of the existing savings mechanism, and results in an equilibrium stochastic process where the investment series is extremely volatile. This model exaggerates the percentage variability of investment and underestimates its first-order serial autocorrelation and its correlation with output and savings.

In comparison with some of the closed-economy real business cycle prototypes, the small open-economy model with adjustment costs is capable of replicating the observed variability and persistence of domestic output using smaller and less persistent productivity disturbances. However, this model seems to require relatively larger and more serially correlated shocks than the free-adjustment neoclassical prototype.

Finally, the sensitivity analysis of the model's performance to changes in the degree of risk aversion suggests that setting this parameter to a value inside the rank of credible estimates may result in a very close fit to the actual moments. Such a configuration of the model can be used to evaluate positive and normative effects of a variety of economic policies. This is the task of the following chapter.

FOOTNOTES

1. See for example Long and Plosser (1983), Hansen (1985), Prescott (1986), Greenwood, Hercowitz and Huffman (1988) and King, Plosser and Rebelo (1988).

2. Consider, for example, a simplified version of the model of Long and Plosser (1983) where closed-form solutions can be obtained. The closed-form solution for the equilibrium interest rate is

$$r_t = (\epsilon_{t+1}/\epsilon_t)\alpha^{\alpha}(L/\beta Y_t)^{1-\alpha} - 1,$$

where Y_t is output defined by a Cobb-Douglas technology, α is the capital's share on output, β is the constant subjective discount factor and L is a constant supply of labor. If the disturbances were deterministic and stationary, evolving according to a serially autocorrelated process $\epsilon_{t+1}=\rho\epsilon_t$ with $0<\rho<1$, the above expression could be rewritten as

$$r_t = (\rho\epsilon^{\alpha}{}_t)\alpha^{\alpha}(L/\beta Y_t)^{1-\alpha} - 1.$$

Hence, under these conditions, productivity improvements always cause the equilibrium real interest rate to rise and motivate agents to substitute current consumption for future consumption.

3. In a stochastic model the equality of returns does not hold exactly period by period. Instead, it holds as an expected value where the different realizations of the rates of return are weighted by the corresponding lifetime marginal utility of consumption.

4. Alternatively, as in Backus, Kehoe and Kydland (1989), the time-to-build technology of Kydland and Prescott (1982) could be used to moderate the variability of investment in the open-economy model. The adjustment-cost formulation has the advantage of allowing procyclical fluctuations in the relative price of investment goods, but it forces lagged values of this relative price to be irrelevant for current investment decisions and assumes that investment projects are completed in one period. The time-to-build technology can incorporate different gestation periods, but it assumes that the relative price of investment goods is fixed and independent of the amount of capital being accumulated.

5. A stable steady-state equilibrium for the holdings of foreign assets can also be obtained by assuming that individuals face a positive probability of dying each period. This methodology is explored in detail in Frenkel and Razin (1987).

6. Theorems (3)-(5) of Epstein (1983) show that these conditions, added to a neoclassical production technology satisfying the Inada conditions or a linear technology, are sufficient to guarantee the existence of a stationary joint limiting distribution of the

capital stock and the productivity disturbances in the stochastic steady-state of a closed economy. Given that the instantaneous-utility and time-preference functions (2) and (3) have been defined in terms of the composite C-G(L), it is straightforward to show that Theorem (5) still holds. This was demonstrated in the second appendix of the first chapter.

7. Notice that (8) allows the relative price of investment and consumption goods to vary. The expression can be rewritten as

$$C_t + Ig_t + (\Phi/2)[Ig_t - \delta K_t]^2 + TB_t \leq \exp(e_t)\, K_t^{\alpha}\, L_t^{1-\alpha}.$$

Where the marginal rate of technical substitution between C_t and I_t is given by $q=1+\Phi(Ig_t-\delta K_t)$. In equilibrium, q is the relative price of investment goods in terms of consumption goods.

8. For simplicity, (8) ignores the possibility of international trade in contingent claims and to this extent limits the option of international risk sharing. Note, however, that individuals can still insure themselves against the risk of domestic productivity changes by trading risk-less foreign assets. Furthermore, recent work by Cole and Obstfeld (1988) suggests that, under certain configurations of preferences and technology, the incompleteness of world financial markets does not affect competitive allocations.

9. The formal analysis of the dynamic optimization problem implied by the model illustrates that the evolution of the capital stock around the steady-state can be characterized as in any adjustment-cost model. The steady-state deviations of the capital stock evolve as a stable, first-order difference equation.

10. This simplifying assumption is not innocuous. Interest-rate shocks induce additional consumption-substitution and consumption-smoothing effects, the direction of the latter depending on whether the economy starts out as a net borrower or lender in the world's capital market. Numerical experiments with moderate r^* shocks did not show major changes in the set of results to be discussed in the paper, illustrating that, in the present model, the randomness of the interest rate is probably not as important as the fact that r^* does not depend on the amount of A accumulated. In fact, as shown in Chapter I, interest-rate fluctuations are likely to cause minor changes as long as the shocks are small and stationary, the interest rate is small and foreign interest payments are a small component of GDP.

11. As Chamberlain and Wilson (1984) have shown, solvency restrictions in stochastic models may take complicated forms. Here, however, the boundary restrictions on the rate of time preference for the case of a linear technology, established in Theorem 4 of Epstein (1983), were sufficient to ensure long-run solvency. The solvency requirement was numerically verified by noting that the limiting probability of setting foreign-asset holdings below -1.142 or above

-0.23 is infinitesimal. Thus, the solvency condition that $\lim_{t\to\infty} A_t/(1+r^*)^t = 0$ holds for every A_t that has non-zero probability of being reached in the long run, starting from any triple (K_0, A_0, e_0) in the state space. Additional numerical experiments were performed to confirm that the limiting distribution of the state variables is unique, stationary and stable.

12. The parameter α is determined with the long-run average of the ratio of labor income to net national income at factor prices. The depreciation rate δ has the value commonly used in the real business cycle literature and with it the model generates the same investment-to-output ratio observed in the data. The value of ω is in the range of the estimates of the intertemporal elasticity of substitution in labor supply $(1/(\omega-1))$ obtained by MaCurdy (1981) and Heckman and MaCurdy (1980,1982), it enables the model to mimic closely the percentage variability of hours worked. The world's interest rate r^* is set to the value suggested by Kydland and Prescott (1982) and Prescott (1986) for the real interest rate in the U.S. economy. The parameter γ adopts two different values consistent with the findings of Hansen and Singleton (1983) and Friend and Blume (1975). The value of β is determined using the long-run average of the GNP/GDP ratio and the other parameter values so as to ensure that in the deterministic steady state the rate of time preference equals r^*.

13. An alternative approach to this calibration strategy is to restrict the parameters of variability and persistence of the disturbances according to the results obtained by calculating Solow residuals with actual data. However, the existent data on the Canadian capital stock is not very useful for this purposes, as indicated by the poor results obtained estimating a Cobb-Douglas production function with it and by the countercyclical behavior it exhibits. In any event, a sensitivity analysis of the effects caused by increasing the first-order serial autocorrelation of the shocks is undertaken later in the paper.

14. The actual data from Canada corresponds to annual time series from 1946 to 1985. The data were divided by the 15+ population, logged and detrended with a linear quadratic time trend.

15. The low correlation between savings and investment in the neoclassical model is not related to the degree of international capital mobility, instead it follows from the size and autocorrelation of the shocks used to calibrate the model.

16. As explained in note 7 above, the introduction of adjustment costs allows the relative price of investment and consumption goods q to vary. When the risk-aversion parameter is set to 2, the expected value of q is almost 1 with 1.0% standard deviation, the correlation between q and GDP is about 0.4 and its first-order autocorrelation is -0.001. Thus, as Kydland and Prescott (1982) point out, adjustment-costs formulations generate a fluctuating and

procycilical q, but deliver the counter-factual prediction that lagged q's should not matter.

17. Both artificial economies generate stochastic processes for the supply of labor that exhibit the same first-order serial autocorrelation and output correlation as GDP, this is an implication of the specific functional forms of preferences and technology that have been adopted.

18. The interested reader is directed to the works of Sargent (1978) and Eichenbaum (1984) for a clear illustration of this problem.

19. The results of Greenwood, Hercowitz and Huffman (1988) are easier to compare because the same detrending procedure and solution technique are used here.

CHAPTER IV

A QUANTITATIVE INVESTIGATION OF THE MACROECONOMIC EFFECTS OF CAPITAL CONTROLS AND THE STABILIZATION OF THE BALANCE OF TRADE

1. Introduction.

In the past few years, a series of intertemporal optimizing models that study the macroeconomic effects of economic policy in the context of the open economy have been developed. With the exception of empirical investigations like those undertaken by Ahmed (1986) and Hercowitz (1986b), the majority of these studies have focused on the theoretical analysis of simplified dynamic prototypes. Capital controls, dual exchange rates and other issues of financial policy have been analyzed, using two-period models of perfect-foresight exchange economies, by Adams and Greenwood (1985) and Greenwood and Kimbrough (1987). The analysis of fiscal policy, in a two-period framework that incorporates endogenous production decisions, has been undertaken by Aschauer and Greenwood (1985), Greenwood and Kimbrough (1985) and Kimbrough (1986). In addition to this family of two-period models, a variety of overlapping-generations and infinite-horizon extensions have also been developed. Obstfeld (1981a) and (1981b) investigated exchange-rate dynamics and Obstfeld (1986) and Frenkel and Razin (1986b) analyzed dual exchange rates in deterministic, infinite horizon models. Frenkel and Razin (1986a), (1986b) and (1987) studied fiscal policy and the dual exchange-rate regime using overlapping-generations prototypes.

In spite of the fact that the conclusions reached by the dynamic equilibrium literature mentioned above challenge the traditional wisdom of open-economy policy analysis based on the Mundell-Fleming model, the shortage of empirical work remains an obstacle in the process of translating the theory into specific policy recommendations. More recently, however, a series of developments in dynamic macroeconomic theory have made it possible to undertake quantitative studies of complex extensions of these dynamic models. For instance, it is now possible to numerically calculate the equilibrium stochastic process that characterizes artificial economies where infinitely-lived individuals formulate optimal intertemporal plans. In Chapters II and III of this thesis, following the innovative work on real business cycles by Kydland and Prescott (1982) and Long and Plosser (1983), a model of a small open economy that is capable of mimicking many of the observed stylized facts of the Canadian economy has been constructed. It would be an interesting development to make use of these new techniques to analyze quantitatively the macroeconomic effects of economic policy, which have been so thoroughly studied from a theoretical perspective.

The quantitative analysis of economic policy based on dynamic stochastic models is a very recent advancement even in closed-economy macroeconomics. The novel work by Cooley and Hansen (1987), Greenwood and Huffman (1988) and McGrattan (1988) constitutes the first attempt to undertake such a task. This literature studies the effects of monetary and fiscal policies from the perspective of models where individuals are allowed to adjust their optimal state-contingent plans as the economic environment is altered. By proceeding in this manner, the analysis of economic policy complies with the standards set by Lucas (1976) and (1987). Thus, the agents' rules of behavior are not assumed to be invariant to policy changes and arguments about economic welfare are explicitly linked to lifetime utility-maximizing choices.

In an attempt to extend the research work discussed in the previous paragraphs, the present chapter undertakes a quantitative investigation of the macroeconomic effects of economic policy in the context of a small open economy. This essay studies a policy that stabilizes the balance of trade, at a certain target level, by imposing capital controls. These controls are enforced by introducing a tax or subsidy on foreign-interest income that is rebated to individuals in the form of a lump-sum transfer. The analysis utilizes a dynamic stochastic model, in combination with numerical simulation techniques, to determine the effects of the policy on economic welfare and economic activity, and to calculate the state-contingent schedule of taxes that would allow the government to successfully implement the policy.

The chapter is organized as follows: Section 2 summarizes the dynamic stochastic model of a small open economy studied in the previous chapters. This model considers optimal intertemporal planning in an environment where domestic capital and foreign financial assets can both be utilized as vehicles of savings, and where the rate of time preference is an increasing function of past consumption levels. In Section 3, the potential for the model to be a useful tool for policy analysis is numerically investigated by studying its ability to replicate the observed stylized facts of an actual economy. Section 4 studies the effects of introducing a policy that uses capital controls to stabilize the balance of trade at some desired level. The effect on economic welfare is determined with different measures of compensating variations in terms of constant-consumption paths, and the effect on economic activity is studied by comparing the statistical moments that characterize the equilibrium stochastic process of the policy-restricted version of the model with those of the unrestricted prototype. The same section also analyzes a fiscal strategy that the government could follow to implement the policy. A series of concluding remarks are included in the last section.

2. A Dynamic Stochastic Model of a Small Open Economy: Free Trade and Restricted Trade.

This section presents the artificial economy to be studied in the rest of this chapter. In the first part of the section, the structure of preferences, technology and financial markets that characterizes the economy is described. Since this paper deals with two situations, one where no capital controls or trade-balance targets are in place and one where such a policy is introduced, the financial structure of the artificial economy adopts two forms. In the unrestricted, or free-trade, version of the model, individuals have unlimited access to the world's capital market, whereas in the policy-restricted version of the model they are forced to accumulate a certain predetermined amount of foreign assets. This additional restriction is enforced by a government that levies the appropriate tax, or pays the appropriate subsidy, on foreign interest income. This schedule of taxes and subsidies is studied later in the paper. The second part of the section characterizes the equilibrium of the two versions of the model as the solution of discrete-time dynamic programming problems.

2.1 Preferences, Technology and the Financial Structure.

Preferences: All agents are infinitely-lived and identical, and preferences are described by the Stationary Cardinal Utility function formulated by Epstein (1983)[1]:

$$E_0 \left[\sum_{t=0}^{\infty} \{ u(C_t - G(L_t)) \exp[-\sum_{\tau=0}^{t-1} v(C_\tau - G(L_\tau))] \} \right]. \quad (1)$$

where

$$u[C_t - G(L_t)] = (1-\gamma)^{-1} [(C_t - L_t{}^{\omega}/\omega)^{1-\gamma} - 1], \qquad \omega > 1, \quad \gamma > 1, \quad (2)$$

and

$$v[C_t - G(L_t)] = \beta \text{ Ln } [1 + C_t - L_t{}^{\omega}/\omega], \qquad \beta > 0. \quad (3)$$

Here, C_t denotes private consumption and L_t are the productive services provided by labor.

As explained in chapters I-III, the lifetime utility function formulated in (1) is adopted from Epstein (1983). It constitutes the stochastic analog of the utility function employed by Obstfeld (1981a) and (1981b). In Obstfeld's work, the existence of a well-behaved deterministic stationary equilibrium for the holdings of international assets is ensured by assuming that the rate of time preference is an increasing function of past consumption levels. Such a deterministic steady state is obtained when the world's interest rate and the rate of time preference are equalized. Otherwise, as discussed in Helpman and Razin (1982) and Frenkel and Razin (1987), whenever the rate of interest is greater (smaller) than the rate of time preference, individuals will accumulate (deplete) foreign assets in order to finance an increasing (decreasing) consumption stream. It follows, therefore, that the constant-time-preference representation of preferences cannot explain the process by which this long-run equilibrium is reached and that an alternative utility function, such as the one presented in (1), must be used[2].

The instantaneous-utility and time-preference functions defined in (2) and (3) simplify the analysis by specifying preferences in terms of the composite good $C_t - L_t^{\omega}/\omega$, thus making the intratemporal marginal rate of substitution between consumption and labor depend on the latter only[3]. This formulation of preferences allows the model to focus expressly on the interaction of domestic capital and foreign assets as alternative vehicles of savings. The cost, however, is that the wealth effect on labor supply is eliminated.

Technology: Domestic production is generated as follows:

$$G(K_t, L_t, K_{t+1}) = \exp(e_t)\, K_t^{\alpha} L_t^{1-\alpha} - (\Phi/2)(K_{t+1} - K_t)^2, \qquad (4)$$
$$0 < \alpha < 1, \qquad \Phi > 0.$$

Here, e_t is a random shock to productivity or the terms of trade to be discussed in more detail later, $K_t^{\alpha} L_t^{1-\alpha}$ is a neoclassical constant-returns-to-scale production function, K_t is the domestic capital stock currently productive and $(\Phi/2)(K_{t+1} - K_t)^2$ is the cost of adjusting the capital stock as a function of net investment[4]. Next, the capital evolution equation is given by

$$K_{t+1} = (1-\delta)K_t + Ig_t, \qquad 0 \leq \delta \leq 1, \qquad (5)$$

where δ is a constant rate of depreciation and Ig_t is gross investment.

Financial Structure: The financial structure of the economy adopts one of two forms. In the first case, free trade in foreign financial assets is allowed by the government. Here, agents have unrestricted access to a perfectly competitive, international capital market where foreign assets A_t, paying or charging the non-random real rate of return r^*, are exchanged with the rest of the world[5]. Thus, in the economy without capital controls, the holdings of foreign assets evolve according to

$$A_{t+1} = TB_t + A_t(1+r^*), \tag{6}$$

where TB_t is the balance of trade. In the second case the government permanently restricts foreign-asset accumulation to a predetermined target $\tilde{A}$. Here the evolution of foreign assets is described by

$$A_{t+1} = \tilde{A}. \tag{7}$$

In this case, the balance of trade after the date in which the policy is implemented is given by $TB_t = -r^*\tilde{A}$.

In both versions of the model, the combination of domestic production with foreign borrowing, or savings, results in an aggregate resource constraint according to which domestic absorption plus the balance of trade cannot exceed GDP net of adjustment costs[6]:

$$C_t + Ig_t + TB_t \leq \exp(e_t)\, K_t^{\alpha}\, L_t^{1-\alpha} - (\Phi/2)(K_{t+1}-K_t)^2. \tag{8}$$

The cost of adjustment included in (4) and (8) indicates that the total cost of altering the capital stock increases with the size of the desired adjustment, and hence it implies that investment changes are to be undertaken in a gradual manner. This formulation of the technology also assumes that the domestic economy is a small participant in the world capital market, so that the interest rate r^* is regarded as an exogenous variable.

2.2 Stochastic Equilibrium and the Dynamic Programming Problem.

The Free-Trade Economy: The dynamic equilibrium of the unrestricted model is represented by a set of state-contingent decision rules for consumption, labor supply, capital accumulation and foreign-asset accumulation that maximize (1), given K_0, A_0 and e_0, subject to (4)-(6), (8), the intertemporal solvency restriction and the usual non-negativity restrictions on K, L and C[7]. The time-recursive structure of the Stationary Cardinal Utility function simplifies the analysis and solution of the intertemporal optimization problem because it implies that dynamic programming techniques can be applied.

In each period, the state of the economy can be fully described once K_t, A_t and e_t are observed. Given this information, along with the knowledge of the stochastic process that governs the evolution of the disturbances, individuals choose K_{t+1}, A_{t+1}, C_t and L_t so as to solve the following dynamic programming problem:

$$V(K_t, A_t, e^s{}_t) = \max \{ (1-\gamma)^{-1} [(C_t - \hat{L}_t{}^{\omega}/\omega)^{1-\gamma} - 1] + \exp[-\beta \, Ln(1+C_t - \hat{L}_t{}^{\omega}/\omega)] [\sum_{r=1}^{2} \pi_{sr} V(K_{t+1}, A_{t+1}, e^r{}_{t+1})] \}, \tag{9}$$

subject to

$$C_t = \exp(e^s{}_t) K_t{}^{\alpha} \hat{L}_t{}^{1-\alpha} - K_{t+1} + K_t(1-\delta) - (\Phi/2)(K_{t+1} - K_t)^2 + (1+r^*)A_t - A_{t+1},$$

where

$$\hat{L}_t = \underset{L_t}{\operatorname{argmax}} \{ \exp(e^s{}_t) K_t{}^{\alpha} L_t{}^{1-\alpha} - L_t{}^{\omega}/\omega \}.$$

The symbol π_{sr}, for $s,r=1,2$, denotes the one-step conditional transition probability of the next-period's technological or real-exchange-rate disturbance. These transition probabilities must satisfy the conditions that $0 \leq \pi_{sr} \leq 1$ and $\pi_{s1}+\pi_{s2}=1$ for $s,r=1,2$.

In order to preserve tractability, the stochastic structure of the shocks is simplified by introducing a two-point Markov process. Thus, at any date in time, the shocks can only take one of two values

$$e_t \in E = \{ e^1 , e^2 \}. \tag{10}$$

Furthermore, it is also assumed that the transition probabilities and the shocks themselves are symmetric: $\pi_{11}=\pi_{22}=\pi$ and $e^1=-e^2=e$. Therefore, the asymptotic standard deviation, σ_e, and the first-order autocorrelation coefficient, ρ_e, that characterize the stochastic process of the disturbances are determined by $\sigma_e=e$ and $\rho_e=2\pi-1$ respectively.

As in chapters II and III, the values of the parameters γ (coefficient of relative risk aversion), ω (1 plus the inverse of the intertemporal elasticity of substitution in labor supply), α (capital's share in output), δ (depreciation rate) and β (the consumption elasticity of the rate of time preference), are selected using long-run averages of actual data, the restrictions imposed by the deterministic steady-state equilibrium of the model and also by attempting to approximate some of the estimates obtained in the relevant empirical literature. These structural parameters are assigned the following values[8]:

$$\alpha=0.32, \quad \beta=0.11, \quad \gamma=1.6, \quad \delta=0.1, \quad \omega=1.455 \quad \text{and} \quad r^*=0.04. \tag{11}$$

The model is calibrated by adjusting the value of the free parameters Φ, e and π to replicate a subset of the observed sample moments calculated with detrended post-war Canadian data. The first parameter is selected so as to mimic the volatility of private investment, and the second and third are determined so as to replicate the variability and first-order serial autocorrelation of GDP.

The Restricted-Trade Economy: The equilibrium of the restricted-trade model is characterized in a similar manner as in the free-trade model, except that foreign assets are not a choice variable and (7) replaces (6) because of the capital controls being implemented. The dynamic programming problem defined in (9) is modified to incorporate the restriction that A_{t+1} must equal $\tilde{A}$ in every period. All other elements of the problem remain the same. In fact, this problem is equivalent to the one that characterizes a closed-economy real business cycle model, except that a constant equal to the trade-balance target is added into the resource constraint. Also, since the restricted model is used to evaluate the effects of imposing different trade-balance targets $-r^{*}\tilde{A}$, it adopts the same parameter values used to calibrate the unrestricted model.

3. The Unrestricted Economy: Free Trade.

In order to utilize the unrestricted model as a benchmark for the evaluation of the effects of capital controls and trade-balance stabilization policies, it is important to establish first how useful is it as an accurate description of a real-world small open economy. According to Lucas (1976) and (1987), a useful model for policy-analysis purposes is one that fits historical data while it also separates those elements of economic decision-making that are altered by the policy from those that remain unchanged. Thus, a useful model must replicate the properties of observed time series and must also capture the ability that agents have to modify their behavior as the environment changes. The latter property is obviously satisfied by the model studied here, since it allows individuals to reformulate their optimal intertemporal plans as policy changes are introduced. The task of this section is to enquire whether the first property is also satisfied by undertaking the calibration exercise mentioned earlier and by briefly discussing its results.

The numerical solution of the model follows the methodology suggested in Bertsekas (1976). This techique has been employed by Sargent (1980) and Greenwood, Hercowitz and Huffman (1988), and is also the same procedure employed in Chapters II and III of this thesis. The procedure, which is briefly summarized in the appendix, is based on performing iterations on the value function (9) using a discrete grid of admissible values for domestic capital and foreign assets, along with the two-point Markov structure of the shocks. Since in this case welfare measures are to be considered, the iterative procedure is performed until convergence of the value function is achieved. The state-contingent decision rules for capital and foreign-asset accumulation, associated with the solution of the functional equation (9), are used to calculate the exact stationary joint probability distribution of the state variables of the artificial economy. This probability distribution is in turn utilized to calculate first and second order statistical moments that can be compared with those observed in the actual data, establishing in this manner the ability of the model to mimic observed time series.

The unrestricted model of the small open economy is calibrated setting the parameters σ_e=1.285%, ρ_e=0.41 and Φ=0.023. The statistical moments for both this free-trade artificial economy and the actual Canadian data are listed in Table 1. A graph of the stationary marginal probability distribution of domestic capital and foreign assets is produced in Figure 1, and another graph depicting the marginal limiting distribution of the capital stock is presented in Figure 2.

By comparing the moments of variability, persistence and comovement listed in Table 1, it transpires that the free-trade economy delivers an accurate characterization of the observed stylized facts[9]. First, the model mimics the observed ranking of variability of all the aggregates listed, approximating closely the percentage standard deviation of each one. Second, although the model exaggerates the GDP correlations of consumption, savings and labor, it reproduces a similar pattern of procyclical and countercyclical movements as that observed in the data, as well as the correlation between savings and investment[10]. Third, the model is also capable of partially approximating the ranking of the first-order serial autocorrelation coefficients, except that it underestimates the persistence of investment and the trade-balance-output ratio.

It is particularly important to notice that this benchmark model possesses the ability to replicate some of the moments that characterize foreign interest payments and the ratio of the trade balance to output.

Table 1

Statistical Moments: Canadian Data and Benchmark Economy[a].

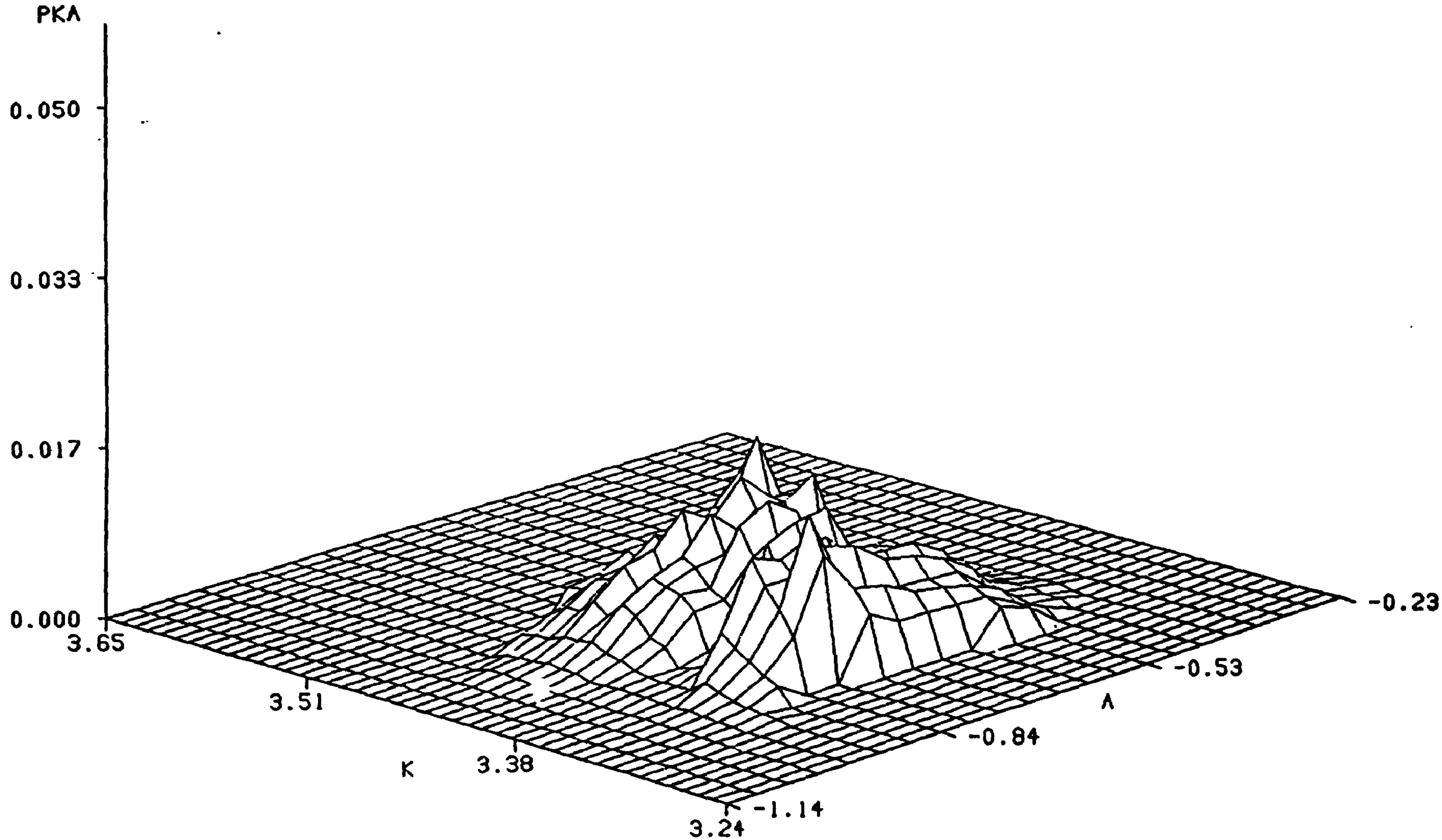
Variables	A Canadian Data 1946-1985 (I)[b]	(II)[c]	(III)[d]	B Benchmark Economy Free Trade (I)[b]	(II)[c]	(III)[d]
1) GDP	2.810	0.615	1.000	2.807	0.614	1.000
2) GNP	2.950	0.643	0.995	2.864	0.616	0.994
3) C	2.460	0.701	0.586	2.140	0.688	0.943
4) S	7.306	0.542	0.662	5.635	0.602	0.923
5) I	9.820	0.314	0.639	10.028	-0.045	0.554
6) K	1.380	0.649	-0.384	1.364	0.705	0.594
7) L	2.020	0.541	0.799	1.929	0.614	1.000
8) $-r^*A$	15.250	0.727	-0.175	15.672	0.971	-0.046
9) TB/Y	0.019	0.623	-0.129	0.019	0.018	-0.019
	CORR(S,I) = 0.434			CORR(S,I) = 0.585		

[a]The data are measured in per-capita terms of the 15+ population, logged and detrended with a linear quadratic time trend. 1)-3), 5) and 8) are the totals from the national income accounts in 1981 dollars. 6) was obtained from the end-of-period net stocks of fixed non-residential capital in manufacturing and non-manufacturing industries, in 1981 prices. The labor data is an index of man hours worked by paid workers with 1981=100.0. Savings in 4) is generated as investment plus the trade balance surplus. The source of all is the CANSIM data retrieval.

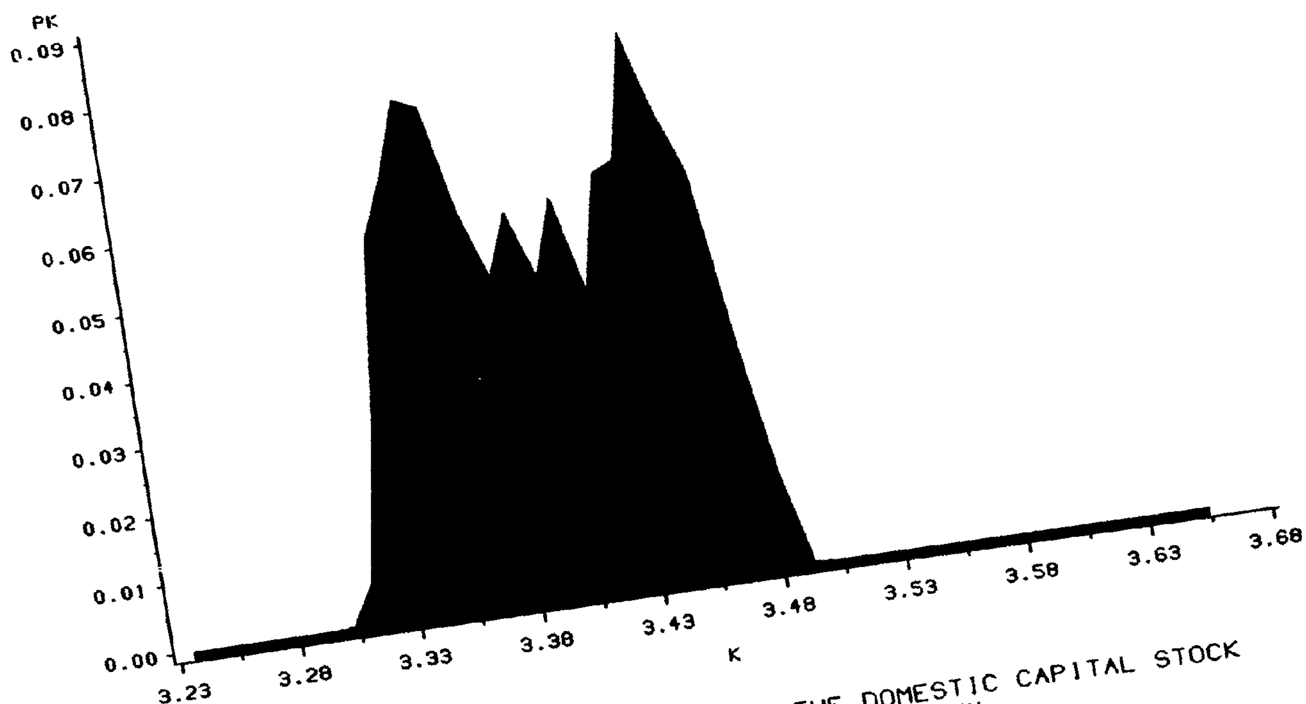
[b]Percentage standard deviation, except 9 which is not in percent.

[c]First-order autocorrelation coefficient.

[d]Coefficient of correlation with GDP.



MARGINAL PROBABILITY DENSITY OF CAPITAL AND FOREIGN ASSETS
IN THE BENCHMARK ECONOMY
FIGURE 1



MARGINAL PROBABILITY DENSITY OF THE DOMESTIC CAPITAL STOCK
IN THE UNRESTRICTED ECONOMY
FIGURE 2

These results indicate that individuals in the artificial economy rationally choose to participate in the world's capital market in a manner such that their holdings of foreign financial assets exhibit 15.7% standard deviation, 0.97 first-order autocorrelation and -0.05 correlation with domestic output. These moments are consistent with, and indeed the result of, lifetime utility-maximizing decisions, and therefore they represent an economic environment in which welfare is maximized.

The rest of the paper studies what happens when the free-trade environment is modified by a government that dislikes the fluctuations in asset holdings and the balance of trade, and imposes capital controls in order to eliminate them or to achieve larger trade surpluses. Since the free-trade benchmark economy is not affected by distortions such as those introduced by labour-income taxes, the theoretical results of Aschauer and Greenwood (1985) follow, and hence capital controls and trade-balance targets will necessarily reduce welfare.

4. Restricted Artificial Economies: Controlled Trade.

The previous section provided evidence illustrating that the unrestricted model is a useful benchmark for the purposes of long-run policy analysis. The goal now is to investigate the effects of introducing a policy that restricts foreign-asset trading in order to stabilize the balance of trade at some target level. This policy is enforced by imposing a tax, or subsidy, on foreign interest income. The revenue from this tax is rebated to agents through lump-sum transfers. This section begins by studying the effects of the policy on economic activity, as expressed by the resulting changes in some of the statistical moments of the main macro-aggregates. The welfare effect of the policy is then analyzed by looking at alternative welfare measures, based on compensating variations of stationary consumption paths that correspond to lifetime utility-maximizing values. This section concludes by considering the feasibility of the policy, which is studied by computing the tax-strategy that the government must follow to successfully achieve its trade-balance target.

4.1 Capital Controls, Trade-Balance Targeting and Economic Activity.

The effects that a policy of imposing capital controls to stabilize the balance of trade can cause on economic activity are uncovered by numerically solving the policy-restricted version of the model. As discussed in section 2, the policy takes the form of a restriction on foreign-asset accumulation according to which $A_{t+1}=\tilde{A}$ for all t. The solution method to be applied is the same, except that the domestic capital stock is the only choice variable and the model does not have to be calibrated. Note that, although the grid of possible initial values for the holdings of foreign assets includes the same elements as before, it collapses to the single point $\tilde{A}$ exactly one period after the policy is implemented. Alternatively, the target value of foreign assets could be allowed to depend upon the state-of-the-world, so that the same methodology could be used to study a somewhat less realistic situation in which the capital controls were state contingent.

In order to obtain a more complete evaluation of the effects of the policy on economic activity, four different strategies have been considered. In the first case the government simply picks $\tilde{A}$ so as to stabilize the balance of trade at its mean value observed in the benchmark economy. In the other three cases, $\tilde{A}$ is adjusted so as to deliver trade-balance surpluses approximately 12%, 30% and 60% higher than the average obtained in the free-trade economy. Recall that, once capital controls are introduced, balance-of-payments equilibrium implies that the value of $\tilde{A}$ is linked to the trade balance by the equality $TB_t = -r^*\tilde{A}$. Thus, a large long-run trade surplus is associated with a high level of foreign debt because sufficient net exports must be generated to meet the debt commitment.

Panels A-E of Table 2 list the means, standard deviations and GDP correlations of the variables in the benchmark economy and in each of the four restricted economies. The long-run effects of the policy on economic activity are analyzed by comparing the moments of the unrestricted economy with the moments of the restricted economies. The general result obtained from this analysis is that capital controls, as a means of stabilizing the balance of trade at some target level, do not have substantial effects on various indicators of economic activity.

Consider first the average values listed in column I of each panel. As can be observed, the policy in question has minimal effects on the mean values of all non-controlled variables. Although the results indicate that the average of each aggregate, except consumption, increases slightly as the trade-balance target is raised, these increments are relatively small. Furthermore, the largest drop in average consumption that the policy can cause is only 0.5%, and that occurs when the trade-balance target is set 60% higher than the mean

Table 2.

Statistical Moments for Alternative Artificial Economies.

	A Benchmark Economy Free Trade			B Restricted Economy 0% Trade-Balance Improvement			C Restricted Economy 12% Trade-Balance Improvement			D Restricted Economy 30% Trade-Balance Improvement			E Restricted Economy 60% Trade-Balance Improvement		
	(I)[a]	(II)[b]	(III)[c]	(I)[a]	(II)[b]	(III)[c]	(I)[a]	(II)[b]	(III)[c]	(I)[a]	(II)[b]	(III)[c]	(I)[a]	(II)[b]	(III)[c]
GDP	1.487	0.042	1.000	1.487	0.042	1.000	1.492	0.042	1.000	1.497	0.042	1.000	1.507	0.042	1.000
GNP	1.459	0.042	0.994	1.458	0.042	1.000	1.460	0.042	1.000	1.460	0.042	1.000	1.461	0.042	1.000
C	1.119	0.024	0.943	1.119	0.027	0.976	1.118	0.027	0.976	1.116	0.027	0.977	1.114	0.027	0.977
S	0.368	0.021	0.923	0.368	0.017	0.939	0.373	0.017	0.939	0.381	0.017	0.940	0.393	0.017	0.941
I	0.340	0.034	0.554	0.340	0.017	0.939	0.342	0.017	0.939	0.343	0.017	0.940	0.347	0.017	0.941
K	3.399	0.046	0.594	3.397	0.064	0.544	3.415	0.064	0.543	3.434	0.064	0.542	3.472	0.064	0.539
L	1.008	0.019	1.000	1.008	0.020	1.000	1.010	0.020	1.000	1.012	0.020	1.000	1.017	0.020	1.000
-A	0.711	0.111	-0.046	0.708	0.000	0.000	0.795	0.000	0.000	0.926	0.000	0.000	1.142	0.000	0.000
TB	0.028	0.028	0.009	0.028	0.000	0.000	0.032	0.000	0.000	0.037	0.000	0.000	0.046	0.000	0.000
	CORR(S,I) - 0.585			CORR(S,I) - 1.000			CORR(S,I) - 1.000			CORR(S,I) - 1.000			CORR(S,I) - 1.000		

[a]Mean.

[b]Standard Deviation.

[c]Coefficient of correlation with GDP.

trade-balance of the benchmark economy. In fact, as column I in panels A and B shows, if the government concentrates only on stabilizing the balance of trade at the level it has in the free-trade economy, the average values of all aggregates are practically the same as in the benchmark economy.

The standard deviations listed in column II of each panel illustrate two findings[11]. First, when capital controls are introduced to stabilize the balance of trade at its mean value in the benchmark economy, the only noticeable changes in the variability of the aggregates affect savings, investment and the domestic capital stock. Since in the restricted economy changes in savings are equivalent to changes in investment, the standard deviation of these two variables is identical. Restricting foreign-asset trading reduces the standard deviation of investment from 0.034 to 0.017. In contrast, the variability of the domestic capital stock increases from 0.046 to 0.064. These changes are consistent with the fact that, by controlling the option of using international trade as a means for the optimal intertemporal allocation of consumption through the cycle, the government forces the dynamics of investment and the capital stock to behave as in a closed economy. Changes in K must now respond to consumption-smoothing and consumption-substitution effects, instead of depending on fluctuations in the relative returns of capital and foreign assets. Therefore, movements in capital accumulation face an increasing supply price of the capital stock, and hence the variability of investment is smaller than what it is in a small open economy -where the supply price of capital is a constant at the level of the world's interest rate r^*.

The second finding that transpires from comparing the standard deviations is that *increasing* the trade-balance target, or *reducing* the value of $\tilde{A}$, does not affect the variability of the aggregates, relative to the situation where trade is stabilized at its average value in the benchmark economy. Thus, the standard deviation of the variables appears to be independent of the *size* of the desired adjustment in the balance of trade.

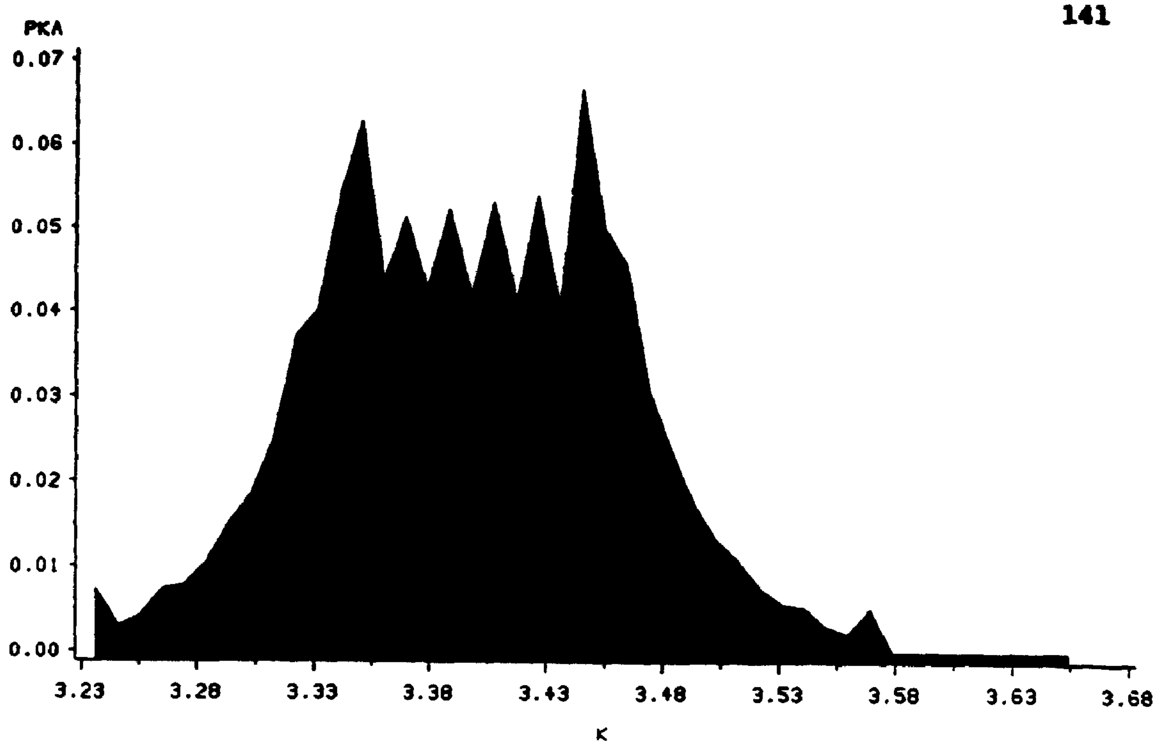
The correlations with domestic output listed in column III of panels A-E illustrate similar observations as the standard deviations. First, when controls are introduced to stabilize the balance of trade, the only noticeable change is that the correlation between investment and output increases substantially from 0.554 to 0.939. This is justified because, as discussed above, changes in savings have to be undertaken by changing investment, and also because the disturbances considered are not large and persistent enough to cause strong consumption-substitution effects. Thus, investment is essentially used as a means to smooth consumption, and hence it exhibits high positive correlation with domestic output. The second observation is that the GDP correlations also appear to be independent of the *size* of the trade-balance surplus target.

Since the only moments that seem to be affected by the *size* of the trade-balance target are the means, it is possible to conclude that the policy under consideration has the effect of shifting the stationary distribution of K and e without affecting its variance. This conclusion is reaffirmed by the graphs of the marginal limiting distribution of the capital stock produced in Figures 3-6. Increasing the trade-balance surplus target displaces the distribution of the domestic capital stock rightwards, without noticeably affecting it in any other way. The direction of this displacement results from the fact that a higher trade-balance surplus reduces consumption in the deterministic stationary equilibrium, and hence it reduces the long-run rate of time preference. This in turn implies that the steady-state capital stock, consistent with an equality between the marginal productivity of capital and the rate of time preference, is rising.

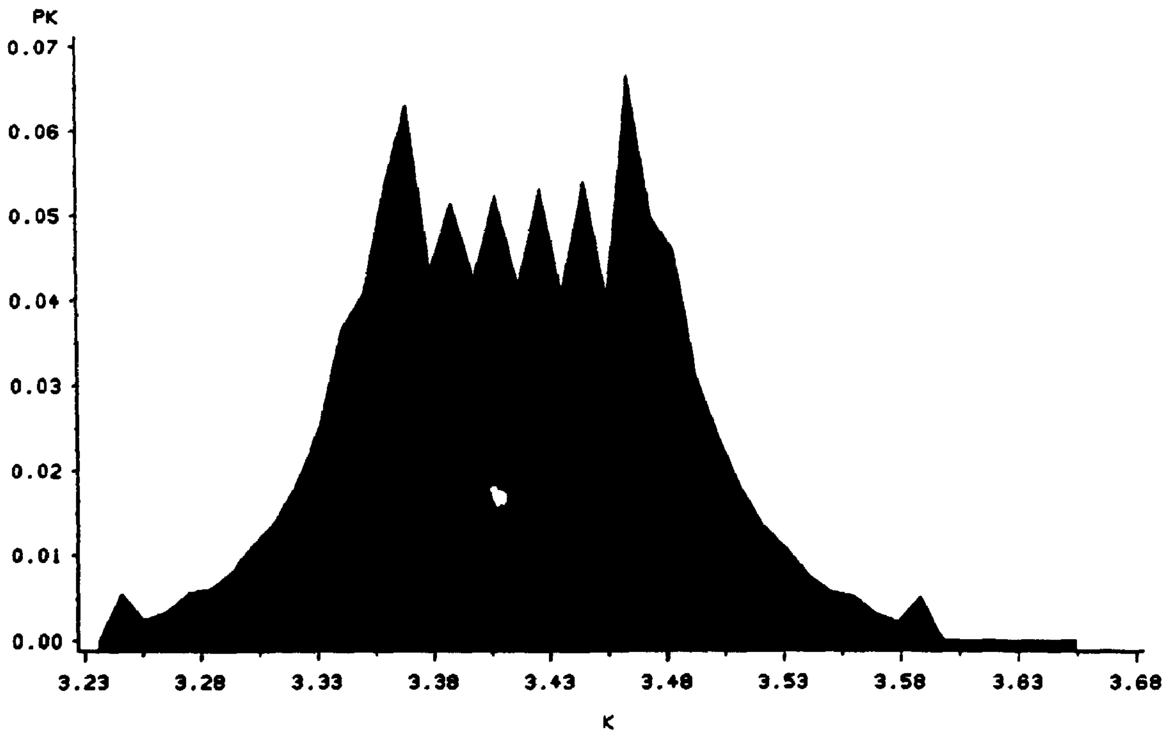
To conclude, the analysis of Table 2 suggests that capital controls, used to stabilize the balance of trade at some target level, do not have substantial effects in the equilibrium stochastic process of the economy. Although is true that the dynamics of the system in the restricted-trade economy work in an entirely different manner, with investment instead of foreign assets being used to smooth consumption, the final result is that output, consumption and the supply of labor behave in almost the same manner. The disturbances that enable the benchmark model to replicate Canadian business cycles are not large and persistent enough to allow capital controls to seriously harm the ability of individuals to smooth consumption through the cycle. The important question, however, is how much welfare is lost by imposing this apparently neutral policy. This issue is analyzed next.

4.2 Capital Controls, Trade-Balance Targeting and Economic Welfare.

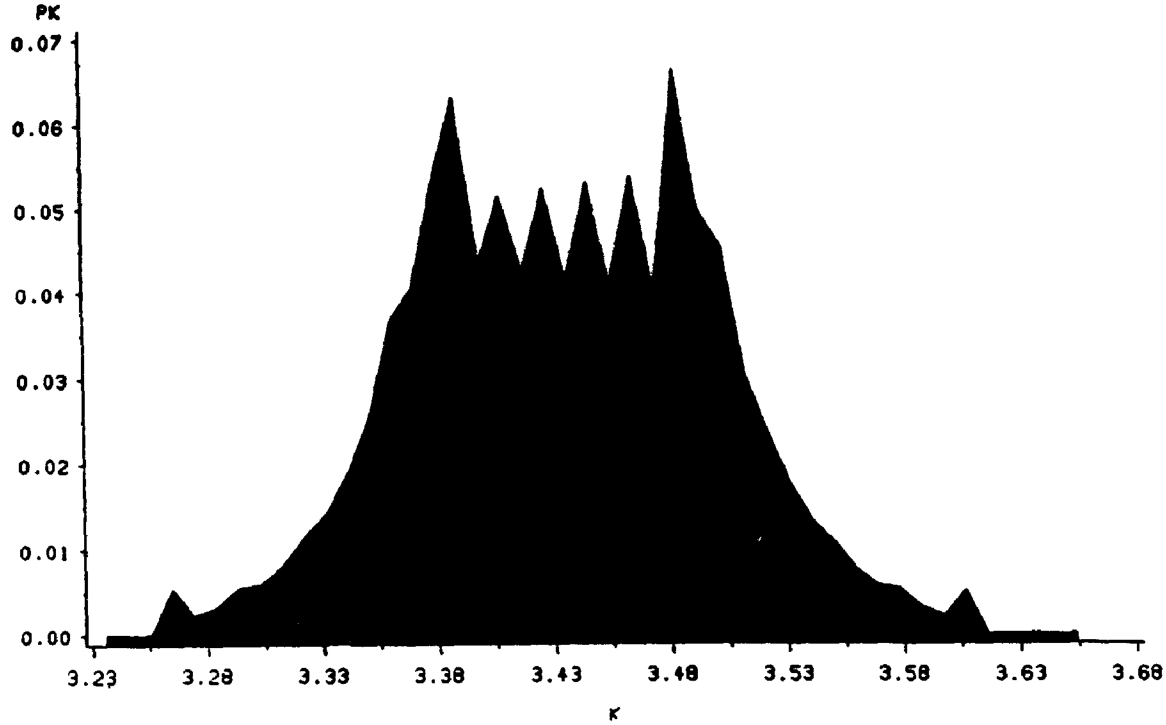
A set of alternative measures have been formulated in an attempt to determine the welfare effect of the policy under study. Following Lucas



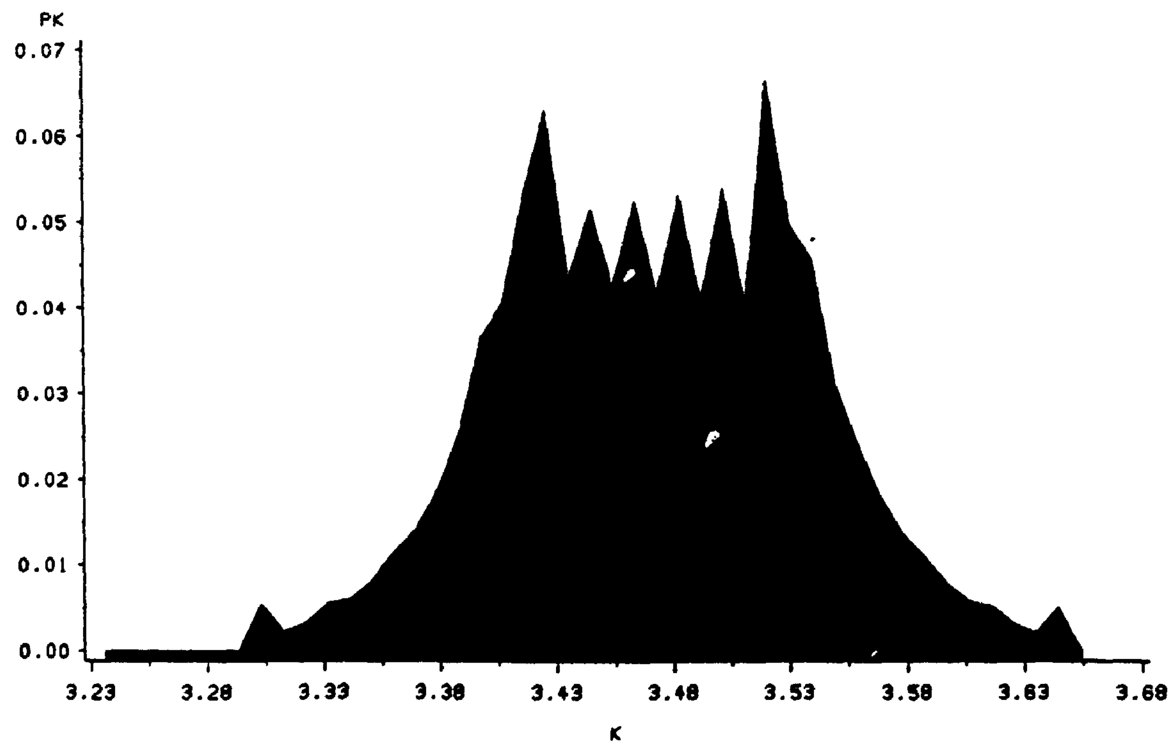
MARGINAL PROBABILITY DENSITY OF THE DOMESTIC CAPITAL STOCK
IN THE RESTRICTED ECONOMY
(0% TRADE-BALANCE IMPROVEMENT)
FIGURE 3



MARGINAL PROBABILITY DENSITY OF THE DOMESTIC CAPITAL STOCK
IN THE RESTRICTED ECONOMY
(12% TRADE-BALANCE IMPROVEMENT)
FIGURE 4



MARGINAL PROBABILITY DENSITY OF THE DOMESTIC CAPITAL STOCK
IN THE RESTRICTED ECONOMY
(30% TRADE-BALANCE IMPROVEMENT)
FIGURE 5



MARGINAL PROBABILITY DENSITY OF THE DOMESTIC CAPITAL STOCK
IN THE RESTRICTED ECONOMY
(60% TRADE-BALANCE IMPROVEMENT)
FIGURE 6

(1987), these welfare measures are based on compensating variations of constant-consumption paths. These compensating variations are calculated as follows. The solution of the dynamic programming problem for both the benchmark and the restricted economies includes a solution for the value function, $V^{u}(K,A,e)$ and $V^{r-\tilde{A}}(K,A,e)$ respectively. Solutions for these two value functions are calculated for each triple (K,A,e) in the state space KxAxE (see appendix). Using the Stationary Cardinal Utility function presented in equation (1), it is possible to write a non-linear equation that spells out a constant-consumption path that yields the same lifetime utility expressed by each $V^{u}(K,A,e)$ and $V^{r-\tilde{A}}(K,A,e)$. Since each restricted economy must imply a reduced level of welfare, given that they constitute more constrained representations of a distortion-free environment, the level of constant consumption associated with each $V^{r-\tilde{A}}(K,A,e)$ is lower than the one that represents each $V^{u}(K,A,e)$. The percentage difference between these two consumption levels, for each triple in the state space and for each of the four values of $\tilde{A}$ considered, is defined here as a compensating variation.

Table 3 presents four alternative measures of the percentage welfare loss, based on the compensating variations, for each one of the restricted economies. First, since given a $\tilde{A}$ there is a V^{u} and a $V^{r-\tilde{A}}$ for each (K,A,e), one can study the compensating variations for each state of nature. From these point-wise comparisons, the maximum and minimum percentage welfare losses are listed in columns I and II of the table. It is difficult, however, to establish an overall judgement of the long-run welfare effect of the policy by looking at these point-wise compensating variations, since they vary so much. In fact, maximum

Table 3.

Long-Run Welfare Effects of Stabilizing the Balance of Trade

% Change in the Balance of Trade[a]	Percentage Welfare Loss I Maximum[b]	II Minimum[b]	III *Ex Ante*[c]	IV *Ex Post*[d]
0	35.00	0.006	0.019	0.008
12	7.00	0.006	0.022	0.009
30	2.15	0.009	0.072	0.015
60	3.29	0.016	0.386	0.038

[a]As a percentage of the mean of the balance of trade in the benchmark economy.

[b]Based on point-wise comparisons of the value function for each triple in the state space.

[c]Expected percentage welfare loss calculated with the stationary probability distribution of the benchmark economy.

[d]Expected percentage welfare loss calculated with the stationary probability distribution of the corresponding restricted economy.

welfare losses are always associated with states of nature that have zero long-run probability of occurrence. For example, in the case where the trade balance is stabilized at its average value in the benchmark economy, a 35% welfare loss occurs if the economy happens to be at the lowest K, the lowest A and the low value of e when the policy is implemented. But, according to the joint limiting distribution of the state variables in the free-trade economy, the long-run probability of starting out from such a situation is infinitesimal (see Figure 1). In general, the largest welfare losses occur when the initial A, at the date the capital controls are introduced, is driven the longest way to reach $\tilde{A}$. But this implies that the initial A must have been located at one of the extremes of the foreign-asset grid, and thus the odds of having to implement the policy when the economy is in that particular situation are null.

More illustrative than maximum and minimum welfare losses are the welfare measures that condense the information provided by the list of all the compensating variations and their associated long-run probability. Two measures of this kind are provided in columns III and IV of Table 3. Column III presents an expected percentage welfare loss calculated using the long-run probability of occurrence of each triple (K,A,e) in the free-trade benchmark economy. This measure is termed the *ex ante* welfare loss because it considers the odds of introducing the policy when the economy is situated at some triple (K,A,e) in the benchmark economy. In contrast, column IV lists an expected welfare loss calculated with the joint stationary probability distribution of the state variables in each restricted economy. This *ex post* welfare loss considers the long-run probability of the restricted economy being at some state $(K,\tilde{A},e)$ and compares the welfare level obtained in this environment with what similar triples would have provided if trade had not been controlled.

The *ex ante* welfare loss appears to be the most accurate to evaluate the policy because it captures the negative welfare effect that, on the average, is caused by moving the economy *from* a free-trade environment *to* a restricted-trade regime. In fact, the *ex post* welfare loss consistently underestimates the *ex ante* loss for each value of $\tilde{A}$ studied. Considering the *ex ante* measure, Table 3 suggests that the welfare losses associated to the policy under study are fairly small. If the government sets $\tilde{A}$ so as to stabilize the trade balance at its mean value in the benchmark economy, the percentage welfare loss is only 0.019%. Even when the policy is designed so as to achieve a 60% trade-balance improvement, the loss in welfare measured in terms of constant consumption is only 0.386%.

The small welfare costs associated with the imposition of capital controls and the stabilization of the balance of trade identified here are consistent with the findings of Lucas (1987) for the welfare cost of business cycles. He encountered that, when the risk-aversion parameter is set to 1 or 5 and the standard deviation of the log of consumption is set to 0.013 or 0.039, the largest cost of consumption instability is about 0.38%[12].

The logic that explains why the welfare effects of the policy are so small is also consistent with the arguments of Lucas (1987). The disturbances that appear to be responsible for post-war Canadian business cycles are not large and persistent enough to allow capital controls to seriously reduce welfare. Risk-averse individuals wish to insure themselves against the risk of domestic shocks by participating in the world's financial market, but since this risk is very small, not having unrestricted access to the world market does not hurt them very much. In fact, if the productivity or terms-of-trade disturbances are increased from 1.285% to 2.3% so that business cycles of the order of 5.0% standard deviation in GDP are generated, the *ex ante* welfare loss for the case of a 30% trade-balance improvement only rises from 0.072% to 0.166%. Thus, it appears that fairly large shocks and business cycles would be required to make a policy like the one studied here reduce welfare by a large amount. A similar result was also obtained by Cole and Obstfeld (1988). These authors encountered that, under certain specifications of tastes and technology, individuals can attain welfare-maximizing equilibria even when trade in international assets is forbidden.

It is important to mention, however, that this analysis focuses only on the role of international trade as a vehicle to optimally allocate consumption through the business cycle in a representative-agent small open economy. It does not consider other instances of international economic relations, such as the role of the transfer and development of technology in long-run growth and the gains from trade for heterogenous agents or multi-sector economies with traded and non-traded goods, in which depriving individuals or sectors from unrestricted access to world markets could have very harmful effects. Still, the investigation is a useful starting point because it illustrates that, without such considerations, the welfare costs associated to capital controls and the stabilization of the balance of trade are quite modest.

4.3 Capital Controls and the Government's Fiscal Strategy.

The numerical methods applied in this paper can also be utilized to study the fiscal strategy that the government could follow to successfully implement the policy under consideration. By charging the appropriate tax on foreign interest income, and at the same time rebating it in the form of a lump-sum transfer, the government can force individuals to always hold the target level of foreign assets $\tilde{A}$. Under this fiscal regime, the dynamic programming problem that characterizes the free-trade economy would include a period-by-period constraint of the following form:

$$C_t = \exp(e^s{}_t) K_t{}^{\alpha} \hat{L}_t{}^{1-\alpha} - K_{t+1} + K_t(1-\delta) - (\Phi/2)(K_{t+1}-K_t)^2 + [1+r*(1-\tau_\tau)]A_t - A_{t+1} + T_t. \qquad (12)$$

Where τ_t is the percentage tax on foreign interest income and T_t is a lump-sum transfer. The government sets $T_t = r^* \tau_t A_t$, but individuals take both the tax and the transfer as exogenously given.

The procedure utilized to calculate the schedule of taxes and transfers that enables the government to set $A_{t+1} = \tilde{A}$, starting from any initial triple (K_t, A_t, e_t), is the following. The solution of the dynamic programming problem of the policy-restricted version of the model delivers an optimal, state-contingent decision rule for domestic capital accumulation that is used to calculate the consumption-based rate of return of a risk-free asset r_t:

$$U^{r-\tilde{A}}{}_C(t)/\{\exp(-v(t))E[U^{r-\tilde{A}}{}_C(t+1)]\} = 1 + r_t. \quad (13)$$

Here, $U^{r-\tilde{A}}{}_C(t)$ is the lifetime marginal utility of consumption at date t in the restricted economy. This marginal utility includes not only the instantaneous marginal utility but also the marginal change in the rate of time preference and its effect on the valuation of expected future consumption benefits. Next, when interest-income taxes are introduced into the free-trade economy, optimizing individuals will allocate consumption so as to equate the marginal rate of substitution between C_t and C_{t+1} with its effective intertemporal relative price $1+r^*(1-\tau_t)$. Therefore, considering that (13) determines the intertemporal relative price of consumption that must prevail in the free-trade environment if A_{t+1} is to be set at $\tilde{A}$, it follows that when the government sets τ_t so as to make $1+r^*(1-\tau_t)=1+r_t$, it will succeed in implementing the desired policy. This implies that the government's fiscal strategy is to set $\tau_t=1-r_t/r^*$ and $T_t=\tau_t r^* A_t$. By rebating the proceeds of the tax in the form of a lump-sum transfer, this fiscal strategy ensures that no alterations in the value of initial wealth are caused, and hence that no other changes in the behavior of the aggregates than the ones discussed in 4.1 will take place.

Clearly, since r_t depends on the initial state (K_t,A_t,e_t), the tax or subsidy that needs to be charged or paid is also state contingent. Thus, although the individuals take the tax rate as given, this is in fact a random variable. Table 4 presents some of the statistical moments that characterize the stochastic process of the schedule of taxes on foreign interest income for each $\tilde{A}$ target. Graphs illustrating the complete tax schedule for the cases of 30% and 60% trade-balance improvements under favorable and unfavorable disturbances are produced in Figures 7-10.

The mean values listed in the first row of Table 4 suggest that the average tax rate increases with the size of the desired improvement in the trade-balance surplus. If the goal is to stabilize the balance of trade at its average value in the benchmark economy, so that in fact $\tilde{A}$ is at the center of the marginal limiting distribution of foreign assets in the free-trade economy, the average tax rate is close to 0%. As the capital controls are tightened, the mean tax rises gradually to reach about 3% in the case that a 60% trade-balance improvement is desired. Thus, on the average, the taxes or subsidies required to enforce capital controls and stabilize the balance of trade appear to be small, regardless of the desired trade-balance goal.

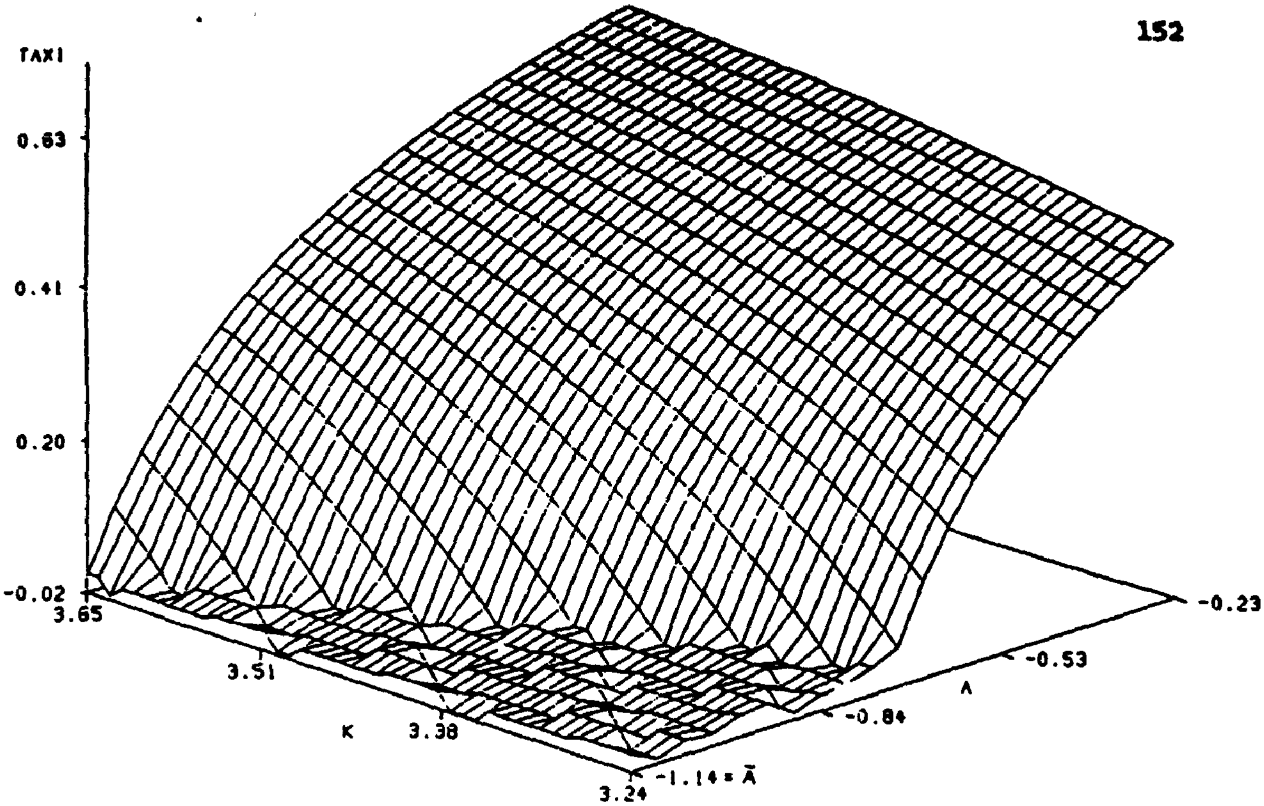
The analysis of the average tax rates contrasts with the tax rates that Figures 7-10 illustrate. The reason is that these graphs depict the required tax, or subsidy, that needs to be imposed to implement the policy starting from every possible initial state of nature contained in the state space. For example, if in the case that a 60% trade-balance improvement is desired the economy starts at $K=3.65$, $A=-0.23$ and experiences a favorable productivity shock, Figure 7 shows that approximately a 63% tax rate must be imposed. However, these large taxrates need only be imposed for one period, because the next date the economy starts at $A=\tilde{A}=-1.14$ and will never deviate from this coordinate in the foreign-asset-holding grid. In the long run, only points where

Table 4.

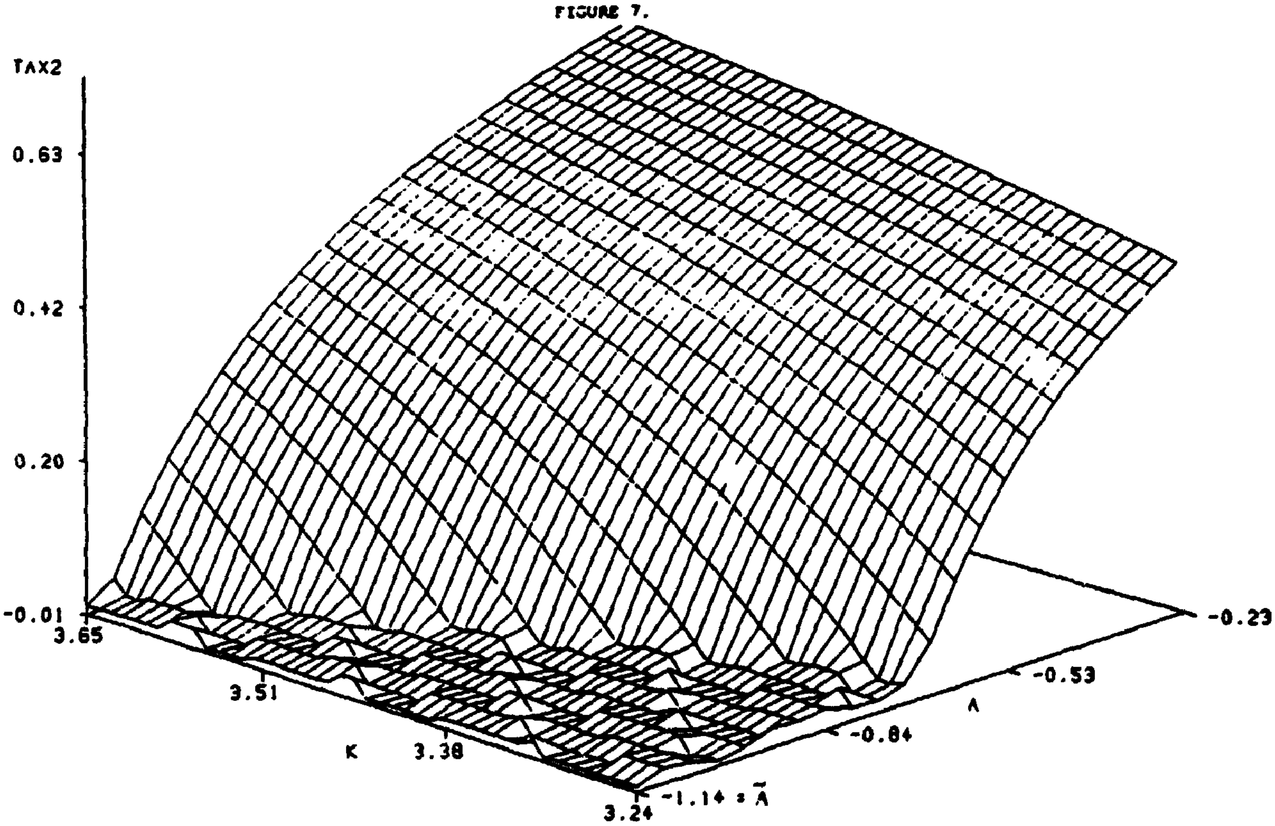
Statistical Moments that Characterize the Stochastic Process of the Foreign Interest Income Tax.

Statistical Moments	Percentage Change in the Balance of Trade[a] I 0	II 12	III 30	IV 60
Mean	-0.0002	0.0054	0.0145	0.0292
Standard Deviation	0.1672	0.1681	0.1690	0.1704
First-Order Autocorrelation	-0.0691	-0.0712	-0.0729	-0.0758
Correlation with GDP	-0.1403	-0.1294	-0.1173	-0.0932

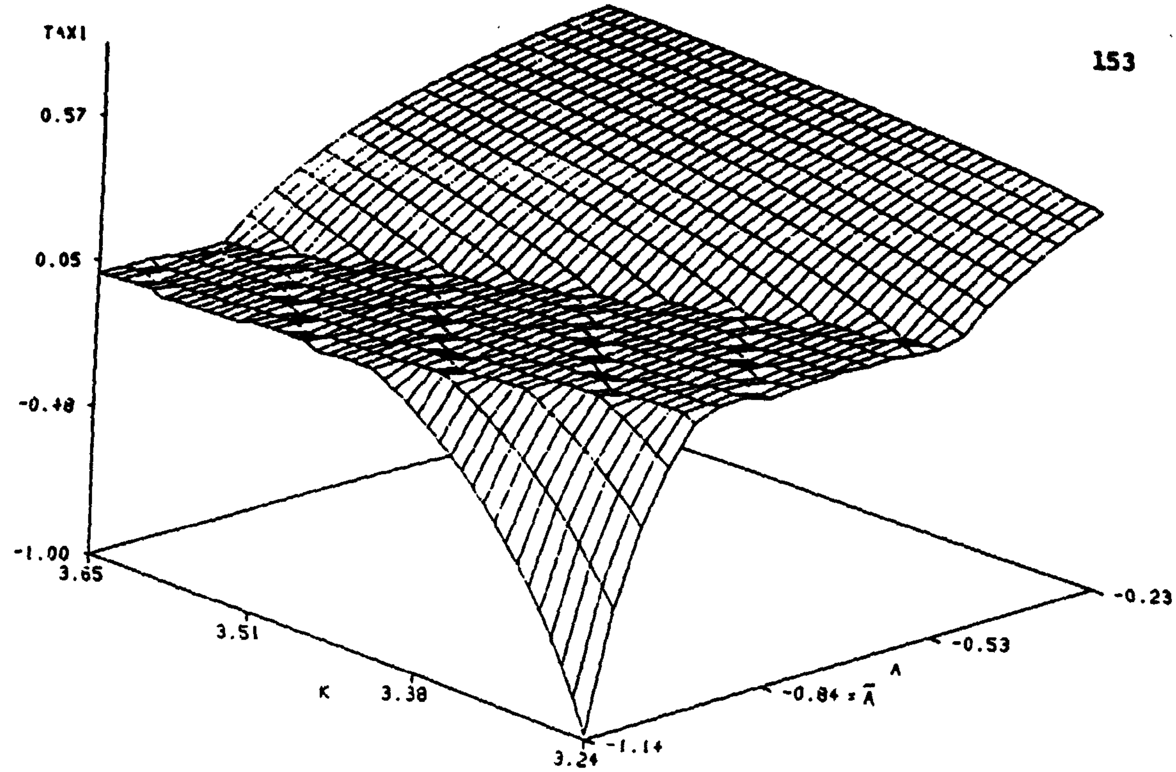
[a]As a percentage of the mean of the balance of trade in the benchmark economy.



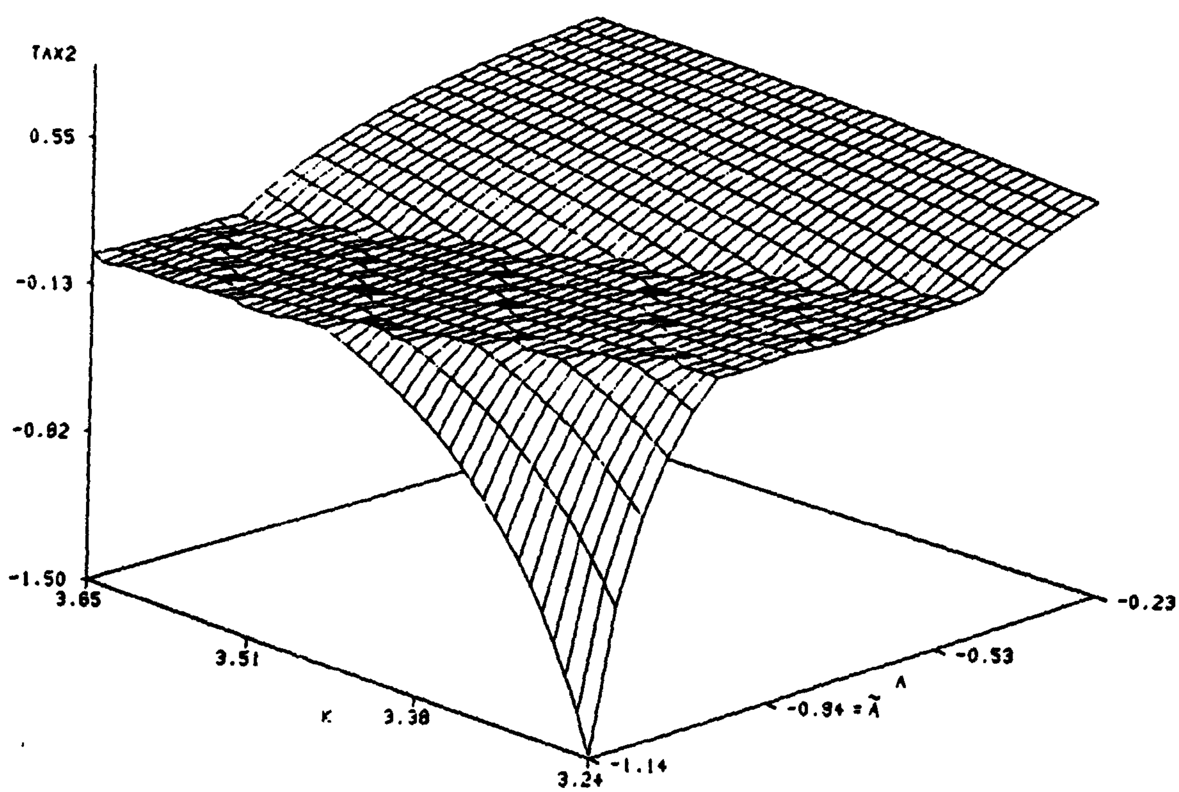
TAX RATES WITH FAVORABLE SHOCK
FOR A 60% INCREMENT IN TRADE SURPLUS
FIGURE 7.



TAX RATES WITH UNFAVORABLE SHOCK
FOR A 60% INCREMENT IN TRADE SURPLUS
FIGURE 8.



TAX RATES WITH FAVORABLE SHOCK
FOR A 30% INCREMENT IN TRADE SURPLUS
FIGURE 9.



TAX RATES WITH UNFAVORABLE SHOCK
FOR A 30% INCREMENT IN TRADE SURPLUS
FIGURE 10.

$A = \tilde{A}$ have non-zero probability of occurrence, so that the one-time large taxes or subsidies have no effect on the statistical moments reported in Table 4. In fact, as the four graphs illustrate, around the area where $A = \tilde{A}$, the schedule of taxes is always relatively flat and close to zero.

The standard deviations and first-order autocorrelation coefficients reported in Table 4 indicate that the limiting distribution of the foreign interest-income tax approximately preserves its variability and persistence as the trade-balance target is raised. There is, however, a tendency for the standard deviation to increase by a very small amount and for the first-order autocorrelation coefficient to fall slightly. In general, the standard deviation appears somewhat large, compared with that of the macro-aggregates listed in Table 2, but the serial autocorrelation is very close to zero in all four cases reported.

The output-correlation coefficients listed in the last row of Table 4 indicate that the tax exhibits weakly countercyclical behavior. This result appears to follow from the weak countercyclical time-path of the trade balance in the benchmark economy (see Table 1). The negative correlation between τ and GDP increases from -0.14, in the case where no trade-balance improvement is planned, to -0.09 in the case where a 60% improvement is the goal.

This analysis of the government's fiscal strategy suggests that, in the long run, the authorities can achieve their goal of restricting foreign-asset trading to stabilize the balance of trade in a relatively easy manner. Although some high taxes or subsidies may be needed initially, in the stochastic steady-state the tax on foreign interest income has a low mean, a constant variance, is almost serially uncorrelated and exhibits weakly countercyclical behavior.

5. Concluding Remarks.

This chapter analyzed, within the context of a dynamic stochastic model of a small open economy, the macroeconomic effects of a policy that utilizes capital controls to stabilize the balance of trade. The effects of this policy on economic activity and economic welfare were determined by employing numerical techniques recently applied in closed-economy dynamic macroeconomic models. The same numerical methods were also used to study a fiscal strategy that would allow the government to successfully implement its policy.

The quantitative results indicated that the policy considered has almost no effect on the equilibrium stochastic processes that describe output, consumption and the supply of labor. The dynamic processes of savings, investment and the capital stock are significantly altered, however, because once foreign-asset trading is restricted, the domestic capital stock is the only vehicle that can be used to reallocate consumption intertemporally. The analysis also determined that the policy has minimal welfare effects, measured in terms of percentage changes in constant-consumption paths. These results suggest that the kind of productivity or terms-of-trade disturbances that explain observed business cycles in post-war Canada, according to the model, are too small and transitory to allow the policy to cause large changes in both the evolution of some of the macro-aggregates and the level of welfare. The analysis of a feasible fiscal strategy that forces individuals to hold the target level of foreign assets suggested that the policy under consideration can be implemented in a relatively easy manner.

These results proved to be robust to changes in the desired target levels of foreign-asset holdings and the surplus in the balance of trade, and they also appeared to remain relatively unchanged for business cycles in which output is almost twice as variable as in the actual data. Thus, it is concluded that much larger disturbances and business cycles would be required for capital controls and trade-balance targeting to cause large effects on economic activity and very harmful effects on economic welfare.

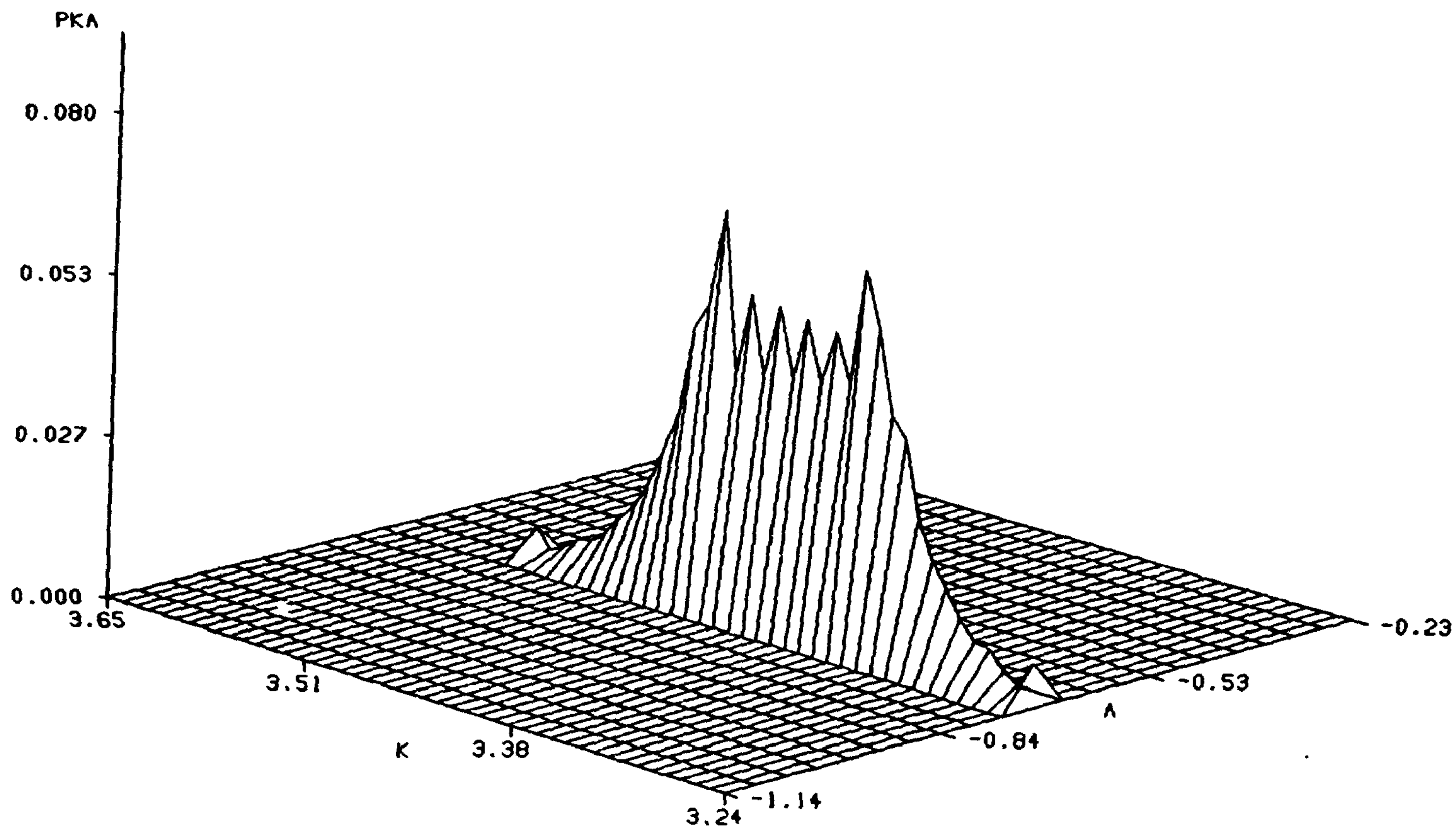
APPENDIX

This appendix describes the method used to solve the dynamic programming problem and calculate the stationary probability distribution of the restricted and unrestricted artificial economies. The dynamic programming problem is solved following a procedure suggested in Bertsekas (1976) and employed by Sargent (1980) and Greenwood, Hercowitz and Huffman (1988). This methodology takes advantage of the contraction property of value-function iteration and uses a discrete grid of points to approximate the state space. In this case, two evenly-spaced grids containing the admissible values of domestic capital $K=\{K_1,...,K_M\}$ and foreign assets $A=\{A_1,...,A_N\}$ need to be defined. Thus, the initial state space of both artificial economies is given by the set $K \times A \times E$ that contains 2MN elements[13].

The next step in the solution process is to construct an algorithm that performs successive iterations in the functional equation (9). The algorithm iterates on (9) starting from the initial guess $V(K_t,A_t,e_t) = 0$, using the set of numbers included in $K \times A \times E$. Since the mentioned functional equation will typically behave as a contraction mapping, the sequence of iterations converges to a function that solves the equation (i.e. the value function). In cases where welfare measures are not needed, the iteration process can be stopped when the sequences of optimal state-contingent decision rules for domestic capital and foreign assets converge.

The decision rules that result from the solution of the functional equation are combined with the conditional probabilities π_{sr}, for $s,r=1,2$, to define the one-step transition probabilities of moving from any initial triple of domestic capital, foreign assets and the technological disturbances to any other such triple in one period. These transition probabilities are condensed in a matrix P of dimensions (2MNx2MN), which is used to calculate the stationary probabilities of each triple of K, A and e. The long-run probabilities are calculated by iterating on the sequence $\rho^1 = \rho^0 P$, where ρ^0 is an initial-guess vector of dimensions (1x2MN) and ρ^1 is a vector of identical dimensions that is used as the new guess in the following iteration. These iterations eventually converge to a unique fixed point ρ^*, which is the joint limiting probability distribution of K, A and e that characterizes the stochastic steady-state of the economy. This probability distribution is used to compute population moments of variability, comovement and persistence of all endogenous variables in both versions of the model.

The solution of the model in the case of the artificial economy where capital controls are in place is simplified because the capital stock is the only choice variable. In the first period, when the controls have just been introduced, the economy may start from any point in KxAxE, but after that date the state space collapses to $K \times \tilde{A} \times E$. Thus, the decision rule with respect to foreign-asset holdings is predetermined to be $\tilde{A}$ at any date. As a result, the long-run probability distribution of capital, foreign assets and the shocks is concentrated in the coordinate where $\tilde{A}$ is found (see Figure A1 for the case of a 0% trade-balance improvement, where $\tilde{A}$ is the 11th point in the A grid).



MARGINAL PROBABILITY DENSITY OF CAPITAL AND FOREIGN ASSETS
IN THE RESTRICTED ECONOMY: 0% TRADE-BALANCE IMPROVEMENT
FIGURE A1

FOOTNOTES

1. The information set needed to formulate the rational expectation defined in (1) is discussed later in the text.

2. Frenkel and Razin (1987) explore in detail how a well-defined steady-state equilibrium can also be obtained by assuming that agents face a positive probability of dying each period, instead of using the endogenous rate of time preference.

3. The functions (2) and (3) were formulated so as to satisfy the conditions from Theorem 5 in Epstein (1983). This theorem proves that the Stationary Cardinal Utility satisfies the requirements of dynamic programming and makes consumption in any period behave as a normal good. Since consumption in the present model can be redefined in terms of the composite C-G(L), it is easy to show that Theorem 5 continues to hold. Also, Theorems 3 and 4 by the same author established that the same conditions, added to either a neoclassical production function or a linear technology, are sufficient to guarantee the existence of a stationary limiting distribution of the state variables.

4. The relevance of capital-adjustment-costs in small open-economy real business cycle models was explored in Chapter III. If capital accumulation does not bear explicit adjustment costs, the desire of individuals to equalize the expected marginal-utility-weighted rates of return paid on domestic capital and foreign assets causes investment in the artificial economy to exhibit too much variability and too little comovement and persistence, relative to actual moments.

5. The assumption that r^* is non-random is introduced for simplicity, but is not a trivial one. A fluctuating interest rate introduces additional consumption-substitution and consumption-smoothing effects. However, interest-rate shocks with less than 5% standard deviation did not have a significant impact in the moments that characterize the equilibrium process of the unrestricted economy. This is because, as formally shown in Chapter I, the income and substitution effects caused by shocks to r^* are very weak as long as these disturbances are small and stationary, the interest rate is low and foreign-interest income is a small component of total output. In the present case, the interest rate is set to 4% and Canadian foreign-interest payments are only 2% of GDP.

6. This specification of the financial structure ignores the role of international trade in contingent claims as a form of risk sharing. However, the introduction of a risk-less international asset still allows individuals to ensure themselves against the risk of fluctuations in domestic productivity. Furthermore, recent findings by Cole and Obstfeld (1988) suggest that, for certain specifications of preferences and technology, the competitive

allocations are independent of the completeness of international financial markets.

7. Although Chamberlain and Wilson (1984) showed that solvency restrictions may take complicated forms in stochastic models, in the numerical investigation performed here the conditions from Theorem 4 in Epstein (1983) were successfully utilized to enforce long-run solvency. In this case, these conditions are:

$$\exp[v(0)] < 1+r^{*} \quad \text{and} \quad \mathrm{Lim}_{(C-G(L))\to\infty}\exp[v(C-G(L))] > 1+r^{*}.$$

Long-run solvency has been verified by noting that the stationary probability of setting foreign-asset holdings below -1.14 is infinitesimal. Thus, the usual solvency requirement that $\mathrm{Lim}_{t\to\infty}[A_t/(1+r^{*})^{t}]=0$ holds for every A_t that has non-zero long-run probability of being reached from any initial triple (K_0,A_0,e_0).

8. The parameter α is determined with the long-run average of the ratio of labor income to net national income at factor prices, δ has the value commonly used in the real business cycle literature, ω is in the range of the estimates of the intertemporal elasticity of substitution in labor supply $(1/(\omega-1))$ obtained by MaCurdy (1981) and Heckman and MaCurdy (1980,1982), r^{*} is set to the value suggested by Kydland and Prescott (1982) and γ is in the rank of the estimates obtained by Hansen and Singleton (1983) and Friend and Blume (1975). The value of β is determined using the long-run average of the GNP/GDP ratio and the other parameter values, so as to ensure that in the deterministic steady-state the rate of time preference equals r^{*}.

9. A detailed analysis of the ability of the free-trade model to mimic and explain Canadian business cycles was carried out in Chapters II and III.

10. The model predicts almost perfect positive correlation between C, S and GDP because the small open-economy assumption, along with the non-random interest rate, eliminates the intertemporal consumption-substitution effect. Also, L exhibits the same persistence and output correlation as GDP because of the Cobb-Douglas structure of the production function and the instantaneous-utility and time-preference functions defined in (2) and (3).

11. Standard deviations, instead of percentage standard deviations, have been considered in order to cancel the effects of changes in the mean of each variable that do not affect their variance.

12. The analysis by Lucas (1987) was performed by imposing a stochastic consumption stream, with trend and cycle components, to an isoelastic, time-separable utility function.

13. The numerical simulations were carried out using an ETA-10P supercomputer with a Vector-Fortran compiler. K contained 45 elements and A 22, so that the state space KxAxE included 1980

combinations and the state-transition probability matrix contained 1980^2 elements.

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