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A Dynamic Agency Model With Borrowing

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by

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ABSTRACT

Recent research on agency models has emphasized multiperiod contracts. However, most research has assumed the principal could control agents' consumption, choosing to deny them access to a capital market. The few papers that have examined the role of capital markets in an agency model (Braverman and Stiglitz (1982), Rogerson (1985a)) do not allow agents a joint choice of effort and borrowing. This thesis extends these models by allowing agents this joint choice of borrowing and effort, showing how previous results change.

Agents are allowed access to two types of imperfect capital markets. In one model of the capital market, agents can borrow a maximum amount, while in the other model agents can borrow more, risking default and payment of default costs. In addition, compensation contracts are restricted to dynamic rank-order tournaments. Given this specific structure, the agent's and the principal's problems are solved, the optimal forms of contracts are derived, and specific testable predictions are generated about observable variables.

The major prediction, contrary to Fellingham and Newman (1985), is that with risk-neutrality and borrowing, memory contracts dominate nonmemory contracts. This generalizes the results found under risk-aversion by Rogerson (1985a) and Lambert (1983). Second, within these multiperiod memory contracts, the spread and mean of consumption will be rising over the length of the contest, as will the mean value of wages. The third prediction is that agents who do not receive promotions will be observed working extra hours, and those who have missed two promotions in a row will work a higher number of extra hours. These are new, testable predictions, not found in other agency models.

This thesis extends agency models by introducing capital markets in a fuller manner then previously done, and by concentrating on a dynamic rank-order tournament, also in a fuller manner then previously done. The importance of these extensions is shown by the fact that the introduction of a capital has made a crucial difference to the results. Results from models without capital markets are generalized or changed, and new testable predictions are generated.

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Chapter 1

Introduction and Literature Survey

In recent years considerable research has centered on how models of uncertainty and asymmetrical information may explain economic behaviour that seems puzzling in terms of the standard certainty models of microeconomics. This thesis will examine one of these areas of research, the principal-agent problem. There is a broad spectrum of examples of this problem, including insurer and insured, doctor and patient, employer and employee, bank and borrower, society and polluter, stockholders and management, and landlord and sharecropper.

The underlying feature that these models share is that output is produced as a result of the interaction of the agents' effort (or actions) and a stochastic shock, both of which are unobservable to the principal. (Alternatively, output is produced jointly via team production (with or without unobservable shocks), and individual marginal products are unobservable.) Thus, the insurance company cannot tell if you locked up your bicycle before it was stolen and the firm cannot tell if its salesman was slacking, or whether he/she merely met a lot of people who had no desire to buy encyclopedias (although the relevant agent knows his or her effort). This information asymmetry and the incentive problem it creates is known as moral hazard.[1]

In these situations it is obvious that standard textbook contingent contracts are not feasible, and that new methods must be found to deal with these asymmetries. There are three standard responses (MacDonald (1984), Stiglitz (1975)): rearranging production patterns to reduce the impact of the asymmetry (optimal assignment problems); expending money to gather extra information (monitoring effort); or designing optimal incentive schemes that elicit and use information as a byproduct of exchange. As with the agency literature, this thesis will emphasize the latter two responses. In addition, although much of the literature deals with insurance problems, the emphasis will be on the employer-employee case. Here one must design a compensation scheme to elicit the desired productivity response.

In order for a meaningful problem to exist, three things must occur. There must be some delegation of decision-making (the agent chooses effort), the effort choice must be unobservable to the principal (and not perfectly inferrable from the observables), and effort must enter the firm and the agent's utility functions in a conflicting manner (effort is costly to the agent, but more effort increases the firm's profits). In this situation, compensation must be based on something observable - usually the output produced. If the agent is risk-neutral, often first best, efficient contracts can be designed by giving him the claim to output net of a constant - he has the correct marginal incentives. However, with risk-averse agents, there is an inherent conflict between optimal risk-sharing and incentives. Thus a risk-averse agent desires a fixed wage, but this leaves him with no incentive to work. The resulting contract is a compromise between the

[1] The emphasis is on the hidden action problem, as opposed to the hidden information or adverse selection problem (although a few cases will be mentioned below where there is a joint problem). An example of the latter would be life insurance companies attempting to discern the health of their clients (e.g., congenital heart conditions).

two goals. For example, insurance contracts carry deductibles and salesmen's wages have a guaranteed floor plus commission.

This chapter will survey the basic results of the principal-agent literature. From this survey some of the areas that require further research will become clear, and an explanation of how this thesis shall expand these areas will be given. Of necessity this survey cannot cover every area, and must emphasize those important to the thesis (primarily dynamic models and rank-order tournaments). For more complete surveys the reader is advised to read MacDonald (1984), Hart and Holmström (1987), or Parsons (1986).[2] In addition, many models will only be sketched or mentioned, and the reader is referred to the original for explicit details.[3]

The first part of this chapter will examine the general principal-agent problem, for a single agent and for team production. Here some of the basic results on contracts will be derived – including that too little effort is expended, and that monitoring is optimal. In the second section, the recent research on extending the model to a dynamic multiperiod setting will be examined, and results on how contracts evolve over time will be derived. The third section will examine the brief research on capital markets in an agency setting, while the fourth will examine in some detail the research on a specific type of contract, the rank-order tournament. (Both of these sections help point in the direction of the new research of this thesis.) The fifth section will examine the small amount of empirical research that has been done on the agency problem. The final section will summarize the research examined here, emphasizing what areas need further exploration, and discuss how this thesis will explore some of these areas.

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[2] Arrow (1985) has an interesting, casual (but idiosyncratic) survey.

[3] For example, there will not be an examination of the research done under efficiency wages (see Akerlof and Yellen(1986)). This area emphasizes the use of dismissal as a penalty strategy, and hence is only peripheral to the research this thesis emphasizes.

1.1 The General Principal-Agent Problem

1.1.1 The Single-Agent Setting

Early research on agency-type of problems had included work by Simon (1951) and Liebenstein (1963), but the first important research was in the papers by Spence and Zeckhauser (1971), Alchian and Demsetz (1972) and Ross (1973).[4] The single agent setting will be examined first of all, based on the work of Holmström (1979).[5] As mentioned earlier, the problem is fairly simple with risk-neutral agents, so only the case with risk-averse agents will be examined here.

There are many questions about the format of the problem. However, in this survey the 'flavour' of the principal-agent problem will be presented, without getting bogged down in details. Hence a basic version of the problem will be presented, making certain assumptions about the agent's utility function and also using the so-called first order approach to the principal's problem. Altering this methodology will be discussed below.

After Holmström (1979), let:

- a = the agent's choice of effort (action),
- θ = the random state of nature, unknown to the principal or agent,
- $x = x(a,\theta)$ be output, with $\partial x/\partial a > 0$,
- $S(x)$ = the sharing function,[6] the agent's salary.

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[4]Other related papers include Stiglitz (1974, 1975) and Mirrlees (1975)

[5]See the article, Harris and Raviv (1979), or Shavell (1979) for details

[6]$S(x)$ need not be continuous, it can be random, but penalty contracts are precluded. See MacDonald (1984) for details

- $G(x - S(x))$ = the principal's utility function. $G' > 0$, $G'' \leq 0$.
- $U(S(x)) - V(a)$ = the agent's utility function. $V' > 0$, $U' > 0$, and $U'' < 0$

A Stackleberg game exists between the principal and the agent. The latter is the follower, and selects effort to maximize expected utility given the sharing function and the distribution of θ. The principal is the leader, and moves first by selecting $S(x)$ to maximize expected utility subject to two constraints. One constraint is a participation constraint that the agent's expected utility be at least as high as he could receive elsewhere. The second constraint is the moral hazard constraint, which takes account of the fact the agent will be selecting effort to maximize utility for himself.

Following Holmström (1979) and others θ will be suppressed, and instead x is viewed as a random variable from the principal's viewpoint, with a distribution $F(x,a)$ and with a density function $f(x,a)$, with $\partial f/\partial a$ and $\partial^2 f/(\partial a)^2$ well defined.[7] Then the principal's problem is to select $S(x)$ and a to:

$$\max \int G(x - S(x))f(x,a)dx, \tag{1.1}$$

$$\text{s.t.} \int [U(S(x)) - V(a)]f(x,a)dx \geq H, \tag{1.2}$$

$$\int U(S(x))\frac{\partial f(x,a)}{\partial a}dx = V'(a), \tag{1.3}$$

where (1.2) is the participation constraint and (1.3) is the moral hazard constraint. Letting λ and μ be the multipliers for (1.2) and (1.3), the maximum principle yields:

$$\frac{G'(x - S(x))}{U'(S(x))} = \lambda + \mu\frac{f_a(x,a)}{f(x,a)}. \tag{1.4}$$

From this and other first order equations the optimal sharing rule can be recovered (see Holmström). Unfortunately, not a lot can be said about this rule

[7]Thus, instead of a distribution over θ, the relevant distribution is over x. See Holmström for a further discussion.

without further assumptions. However, a few points can be noted. First, optimal risk-sharing would require the R.H.S. of (1.4) to be a constant. Given the assumptions about $F(x,a)$ this requires $\mu = 0$ (no moral hazard problem). However, under the above assumptions Holmström is able to show μ must be positive ((1.3) binds). This implies a second-best solution in risk-sharing. Second, with the moral hazard constraint binding, there is a second-best solution in incentives – the principal would like to see the agent increase effort at the constrained optimum. In addition, Holmström notes that f_a/f can be interpreted as a benefit-cost ratio of the deviation from optimal risk-sharing – a signal of how far off the optimum the actual a is. Therefore, the sharing function should depend on this ratio.

Further results on the exact form of $S(x)$ depend on the specific example – one cannot even say if it is concave or convex. However, if $S(x)$ is continuous, one result is $0 < S' < 1$ (Harris and Raviv (1979)) – there is 'coinsurance' between principal and agent. Thus a compromise exists between risk-sharing and incentives. Examples could include insurance contracts carrying deductibles and salesmen's wages having a guaranteed floor plus a commission.

There are several further results in the general single-agent problem, which will be briefly summarized here. The reader is referred to the survey articles by MacDonald (1984) or Hart and Holmström (1987), or the original articles for exact details.

The first important extension to the basic results of the agency model was the investigation of the use of monitors (imperfect estimates of a) to mitigate the moral hazard problem. Holmström (1979) showed that any information, no matter how imperfect (as long as it was not completely useless), can be pareto-improving. Examples of such monitoring include accounting systems with audits; examining whether an insured party has taken 'due care' (Arrow (1985)); and observing the punctuality of employees or the frequency of salesmen's calls.[8]

Other authors have relaxed two of the other assumptions earlier research has made. The first deals with the use of the agent's first order condition as the moral hazard constraint. This constraint may not yield a unique, global optimum, as Grossman and Hart (1983) first noted. Rogerson (1985b) presents the extra. sufficient conditions on $f(x,a)$ such that using the first order condition is valid.[9] The second assumption relaxed is the one that the agent has no idea of the value of θ when he selects a. Both Holmström (1979) and Sappington (1983) have examined this case. The latter has an interesting model where the agent can declare bankruptcy after observing the shock. He shows in general, due to 'externality' effects, the second-best contracts will have these situations of zero effort sometimes occuring.

This discussion of the general single-agent problem indicates that the most important result seems to be the paucity of results. Other than results on the desirability of monitoring and the presence of coinsurance, there are really no strong predictions – certainly none on the form of $S(x)$.

1.1.2 Team Production – Many Agents

The above results are all derived for a principal and a single agent, but a more normal assumption for a labour setting would be team production, where agents have some common link either through a joint production function or a common error structure. In these cases, compensation will in general depend not only on

[8] Gjesdal (1982) expanded these results, showing monitoring had the ability to improve both the incentive problem and the risk-sharing problem between the principal and the agent. In addition, he showed that if the agent's utility function is no longer assumed seperable in effort and income, then a purely random, noninformative signal can be pareto-improving.

[9] See Rogerson (1985b) or MacDonald (1984) for a discussion of these conditions.

one's own output, but on other agents' output too. The case where agents' output is separable ($\partial x_i/\partial a_j = 0$ for all $j \neq i$) but they are linked by a common shock will be explored below in the section on rank-order tournaments. Tournaments are a specific form of the relative-worth compensation contracts that turn out to be optimal with common shocks.

This subsection will concentrate on situations with joint production, where agents' effort choices have impacts on other agents' outputs ($\partial x_i/\partial a_j \neq 0$ for some (all) $j \neq i$). This is a standard externality or property rights problem, and can be solved either by assigning these rights to a monitor or manager (Alchian and Demsetz (1972)), or by designing incentive schemes that attempt to 'internalize' these externalities by making agents' compensation depend on aggregate output (Groves (1973), Holmström (1982a)).

Once again, the reader is directed to MacDonald (1984) or the original articles for exact models and their details – just the basic results will be discussed here. Assume total output of the firm (X) is such that:

$$X = \sum_{i=1}^{n} x_i(a_1, a_2, \ldots, a_n) \tag{1.5}$$

where there exists an i, j such that

$$\partial^2 x_i/\partial a_i \partial a_j \neq 0. \tag{1.6}$$

These extra indirect effects are the externality effect. If agents are paid based on individual marginal productivities only ($\partial x_i/\partial a_i$), effort will be less than it should be given their true marginal productivity ($\partial X/\partial a_i$), and there is a classic moral hazard result of too little effort.

As Alchian and Demsetz (1972) note, this is a property rights problem. Their suggested solution is for the owner to hire a 'monitor', a manager with a comparative advantage in monitoring the agents' effort levels. Giving him the claim to the residual on output motivates him to profit-maximize, and allowing him to alter team membership allows him to keep members from shirking. However, monitoring costs do mean some inefficiency remains.

Holmström (1982a) examines the above problem, and emphasizes that the problem arises equally from the budget-balancing constraint as from the free-riding problem.[10] He derives a sharing scheme such that if overall output (X) is too low, all agents receive zero output. Thus, if any agent deviates from optimal effort, the firm does not profit-maximize, and all agents get fired, bearing the full brunt of the shirking – no agent will shirk in equilibrium. Holmström notes examples of this do exist, the most extreme example being the firing of a board of directors.[11] Note that self-enforcement will not work here, as some output is wasted. Thus the principal is needed to enforce the contract and claim the residual. Holmström states that the principal's primary role is to administer and police, not to monitor as in Alchian and Demsetz. In addition, Holmström extends the analysis to the case where there is joint output and a random shock, showing a similar sharing scheme works.[12]

In summary, team production implies strong externalities which lead to a free-rider problem. Alchian and Demsetz (1972) argued that a firm's owner/manager is hired as a monitor, while Holmström (1982a) argued instead that owners/man-

[10] A related paper is that of Groves (1973), which is discussed by Holmström.

[11] Another example of this collective punishment can be found in Stanley Kubrick's movie *Full Metal Jacket*. The Marine drill instructor attempts to force a slow recruit to learn faster by forcing all the recruits to bear his punishment. This is successful in speeding up his learning, but has the side-effect of turning him into a homicidal psychotic. It is not clear such side-effects exist more generally.

[12] It should be noted there are some problems with this scheme. As Holmström himself shows, if agents are risk-averse and their endowments are limited, his penalty scheme may not work. In addition, the principal has an incentive to lie and claim the residual, something not analyzed here. Arrow (1985) also notes that several Nash equilibriums may exist, including ones where agents cover up for cheaters. Finally, MacLeod (1986) argues that cooperatives may dominate capitalistic firms in this setting.

agers were hired to break the budget balance; and to create credible sharing rules that can achieve the first best solution. Given this, basic predictions of this research are, (a) that in the presence of joint production, one should observe firms instead of individualistic workers, and (b) that the compensation contracts in these firms should depend on aggregate output, with bonus or penalty schemes built in.

1.2 Dynamic Agency Models

Recent research has considered multiperiod extensions of the agency model, built on the stylized fact that many contracts are long term. Important papers include Lazear (1979), Radner (1981), Lambert (1983) and Rogerson (1985a). The dynamic setting is richer, allowing broader predictions. The strongest of these is the presence of memory effects – compensation in the current period depends not only on current output, but past output. Examples of this would include a rise in future deductibles and insurance rates after an accident, or promotion to president being contingent on having already been promoted to vice-president for past accomplishments.

Early dynamic work included research by Becker and Stigler (1974) and Lazear (1979, 1981). In these models agents were randomly monitored and if found to be shirking, were fired. Agents are induced to not shirk by posting a bond they lose if fired. This is achieved by paying an agent a wage less than the value of his marginal product early in his career, and one higher than VMP later in his career. (Examples include seniority-based wage contracts and non-vested pensions.) Important predictions of this research include the result that wage profiles are steeper than productivity profiles, and that since wage is greater than the marginal product later in life, mandatory retirement is needed (Lazear).[13]

Radner (1981, 1985) examined a repeated agency game. In an infinitely repeated version of the one-period game with no discounting (and hence a different objective function then the earlier models), he shows the first best solution can be achieved. With a long enough time period, the principal has enough observations to tell whether the agent has set effort below the first best. In this case, he punishes the agent by lowering wages. The lack of discounting guarantees the punishment hurts enough to prevent shirking. The result is a first best solution with fixed wages and optimal effort. However, with discounting or a finite time-period, then the solution is at best within 'epsilon' of the first best solution.[14]

Both the Radner and the Lazear dynamic agency models imply that as the length of the contract rises, incentive problems will fall, due to the fact the players interact more than once (a point emphasized by MacDonald (1984) and Hart and Holmström (1987)). Indeed, Fama (1980) has taken this argument even further and argued that incentive problems disappear in a dynamic setting. He argues managers' concern over the capital value of their reputation will internalize the costs of deviating from the first best. However, as Holmström (1982b) shows, this argument does not hold up under risk-aversion and discounting.

The research of Lambert (1983) and Rogerson (1985a) (which has a less artificial structure then the Radner model), can be seen as extending Holmström's (1979) methodology to a multiperiod setting. These papers have discounting and finite periods, and show multiperiod contracts pareto-dominate single pe-

[13]The major problem with these models is that the principal has an incentive to fire agents even when they have not shirked, once wages exceed marginal product. Kuhn (1986) deals with this problem in some detail, noting it can alter the above wage profile results.

[14]This epsilon-equilibrium does suffer from an endgame effect – in the last period it is always optimal to shirk, and therefore in the second last period it is also optimal to shirk, etc. The optimization strategy chosen essentially assumes away this problem.

riod ones within this context. Following Lambert,[15] consider a two-period model where output is $x_t = x(a_t, \theta_t)$, for $t = 1, 2$, which is independent across time. The agent faces a sharing rule $S_1(x_1)$, $S_2(x_1, x_2)$ where memory effects can exist ($\partial S_2/\partial x_1 \neq 0$). Via a dynamic programming approach, the agent's problem can be solved (see Lambert for details). Due to the fact S_2 may depend on x_1, the agent's choice of a_2 is a function of x_1. In turn, when the agent selects a_1 he must account for its potential effect on his second period choices. This in turn makes his problem somewhat more complex.

Lambert solves the principal's problem in a similar way to equations (1.1) - (1.3) above, although of course the moral hazard constraints are more complex. His f.o.c.'s are natural extensions of Holmström's, and reveal the principal would like the agent to work harder in both periods. In addition he is able to show memory effects do exist ($\partial S_2/\partial x_1 \neq 0$).[16] It is these additional effects that give dynamic agency contracts their 'edge' over single-period contracts. Indeed, Lambert is able to show the longer the agency relationship lasts, the more the moral hazard problem is reduced. In addition, Rogerson is able to show that sufficient conditions for these contracts to exhibit rising or falling wages over time is that $1/U'$ is concave or convex.

These results demonstrate that there is a potential gain to long-term contracts. Fellingham and Newman (1985) have identified conditions under which optimal multiperiod contracts will have no gain (one will choose to play the repeated agency game). The reader is referred to the article for explicit results, but basically in an agency game with moral hazard, domain additivity and pref-

[15] Rogerson's paper is more general as it uses a global incentive condition. However, Lambert's paper does get the same result, and the notation and methodology is a straightforward extension of Holmström (1979).

[16] In addition to Rogerson, Stiglitz and Weiss (1983) find similar results in a specific model of a borrowing agency problem (see the next section for details). Townshend (1982) also demonstrates gains to multiperiod contracts in an income insurance scheme

erence separability imply that contracts will have no memory. For the additively separable utility functions the literature has usually considered, this implies risk-neutrality for t e agent.

Fudenberg et al. (1987) have also explored conditions when long-term contracts are not needed. They find that unless today's effort affects future periods' products, short-term contracts are sufficient. This result seems dependent on their assumption that agents have access to a perfect capital market (which they do not discuss in any great detail), as well as their requirement that contracts are sequentially efficient. This latter point requires that contracts meet a minimum profit constraint over the remaining periods of the contract, something not required in the models discussed earlier. This result is similar to one discussed by Lambert (1983), where a model is explored in which neither the agent nor the principal can commit to a long-term contract.

An interesting dynamic model related to the agency literature is that of MacLeod and Malcomson (1988). In this model output is not subject to random shocks, but agents are of an ability level unknown to the firm (an adverse selection problem). They derive sequential wage contracts where workers are promoted or demoted based on the level of their previous periods' output relative to the level expected of them given their rank. Eventually workers settle in at a rank reflecting the firm's Bayesian forecast of the worker's ability. Although in a different context then the research emphasized here, the contracts do show memory effects. Workers' ranks (and hence wages) are a function of past performance levels, not current levels.[17]

Long-term contracts are superior as players can interact more than once, al-

[17] A different result is found in two other papers by the same authors. Macleod and Malcomson (1986,1987) examine multiperiod versions of efficiency wage models (with no adverse selection) (see Akerlof and Yellen (1986)). However, efficiency wage models typically get the result that wages are independent of performance levels, and the two MacLeod and Malcomson papers find this result in the multiperiod models. Thus their wages structures reveal no memory effects.

lowing broader strategy and more efficient contracts. Examining these contracts results in more general predictions about observables than the results from examining single period contracts. These include the prediction that multiperiod contracts will be selected over single-period contracts, and that these contracts will show memory effects (and potentially Rogerson's wage paths). However, it is possible to extend the research on dynamic models. One of these areas of expansion is considered in the next section.

1.3 Capital Markets in an Agency Setting

One of the natural areas of expansion of dynamic agency models would be to examine a case where agents simultaneously select effort and borrowing or saving. Most earlier research assumed firms were able to control the consumption flow of agents (and chose to set it equal to the income flow). This thesis will relax this assumption, and explore how the problem changes. Since the agency problem is partially due to imperfect risk-diversification, allowing agents access to a capital market should alter the problem and the results. Only a small amount of research has explored capital markets in agency models. As will be discussed below, this research has been somewhat imperfect. No research has been done where agents fully and freely jointly choose effort and borrowing or saving. No research has been done on the impact of the borrowing agency problem on the employee agency problem, or vice-versa. This thesis will begin to address these issues.

Two papers by Stiglitz and Weiss (1981, 1983) have examined the lender-borrower relationship as an agency problem. In their first paper, they note that banks bear some of the downside risk of the borrower because his maximum loss is his collateral. This means it is sometimes optimal to keep the rate of interest below the market-clearing rate, and ration loans. Raising the interest rate encourages riskier projects by borrowers, and lower profits for lenders.[18] Their second paper extends this model to a multiperiod setting. Here they show that it is often optimal in the second-period to refuse to loan to first period defaulters. These multi-period incentive effects can be optimal, despite the apparent failure to capitalize on gains from trade.

Only two papers so far have examined the role of capital markets in an agency model (Braverman and Stiglitz (1982) and Rogerson (1985a)),[19] but in neither model is a simultaneous choice of effort and borrowing or saving examined. Braverman and Stiglitz examine a one-period, partial equilibrium situation. They examine the impact of exogenously given debt on the agent's effort choice. They show in general more debt means more effort, a result strengthened by bonded labour for defaulters, weakened by bankruptcy constraints. They show that the principal may wish to 'control' the agent's access to borrowing, as it will affect effort and therefore the principal's profits. Although Braverman and Stiglitz show the crucial result that the returns to the principal are affected by the borrowing of the agent, their model is not truly multiperiod, and in addition assumes the principal completely controls the agent's debts.

In Rogerson (1985a), after period one's outcome is announced, agents are suddenly and unexpectedly allowed to borrow (i.e., when they selected first period effort they did not know this choice would exist). He shows in general agents will wish to save, and that this action will reduce both sides' utility levels.[20] As in Braverman and Stiglitz it would seem optimal for the principal to control the agents' access to the capital market.[21] However, Rogerson's results would seem

[18]This is clearly related to the research on efficiency wages summarized in Akerlof and Yellen (1986).

[19]Fudenberg et al. (1987) allow agents access to a perfect capital market, without examining the viability of such a scheme, or its impact on the agent's choices.

[20]It should be noted that Rogerson's proof contains two (offsetting) errors, although the end result is still correct.

[21]A similar result is found in Diamond and Mirrlees (1978), where agents save early on in

to be dependent on the timing story, and particularly the myopic nature of the first period effort choice.

It is clear that introducing capital markets into agency models is fruitful. Even the small amount of research done so far shows that results can change quite drastically with this introduction. Yet agency models with capital markets are only imperfectly explored. Stiglitz and Weiss have begun some of the exploration of the agency problems in a capital market. The other papers have explored a bit of the impact of saving and borrowing on the agency contract. None of these papers have explored models where agents can jointly choose effort and borrowing, or explored the impact of the borrowing and employee agency problems on each other. This thesis will develop a model where agents have access to a capital market, and address these issues.

1.4 The Rank-Order Tournament Model

In section one above the moral hazard problem in teams was explored, in which agents' output was linked through joint production. In this section the case in which output is separable ($\partial x_i/\partial a_j = 0$ for all $j \neq i$), but where agents' may potentially face a common shock, is considered. As Holmström (1982a) has shown, if agents' shocks are correlated, then using compensation schemes that depend only on the agent's individual output is inferior. Valuable information on the common shock, gained by comparing agent's outputs, is useful in designing the compensation scheme. For example, comparing two adjacent sharecroppers' yields lets the landlord derive information on the impact on final output of inputs

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their lifetime in order to be able to afford to falsely claim disability insurance at a later date. Restricting savings reduces this moral hazard problem.

such as individual effort versus common shocks such as weather conditions.[22]

Holmström shows that more general, comparative compensation schemes will dominate specific restrictive ones such as rank-order tournaments. However, it is extremely difficult to characterize what the optimal scheme will look like. Introducing rank-order tournament contracts will severely restrict the class of optimal schemes, but doing so yields much more explicit results. As MacDonald (1984) has stressed, this is a desirable direction for agency models to take. This, plus the observation tournament-like schemes seem prevalent contract forms, suggests studying them is worthwhile.

1.4.1 Single-Period Tournaments

The case to be explored involves separable output, and agents who face a common shock. Examples of a common shock may include a randomly inaccurate counting machine in a widget factory, a sales force in a given city facing fluctuations in average city income (and therefore sales), or golfers in a tournament facing variable weather conditions. As the research to be discussed shows, paying workers based on their output relative to other agents' output exploits extra information on the value of the common shock.

The original article by Lazear and Rosen (1981) examined only the two-player rank-order tournament, but research by Green and Stokey (1983), Nalebuff and Stiglitz (1983) and Malcomson (1986) showed the results generalize to the n-player case. In a tournament, the agent with the higher output receives a fixed higher payment, regardless of the level of output. As Lazear and Rosen show, the tournament always achieves the first best solution with risk-neutrality.

[22] Clearly in this compensation setting there is a role for collusion between agents, as well as destructive behaviour towards competitors (for example, burning your neighbour's crops). These questions have been only briefly explored in the literature – see MacDonald (1984).

Agents are motivated by the gamble, by the difference between the winner's and the loser's wages. An interesting fact is that although average wages equal expected product, in general actual wages will not equal actual product. Lazear and Rosen argue that they can thus explain why an executive's salary may rise dramatically upon promotion, although it seems unlikely his productivity has jumped as dramatically. The promotion is a prize in a contest that motivates those seeking it.

Lazear and Rosen also examine tournaments under risk-aversion, finding one no longer obtains the first best solution. They compare tournaments to linear piece-rates here, and find that even in the case without a common shock, it is possible the tournament dominates. (As Holmström (1982a) notes, this is not surprising given the restriction to linear piece-rates.) It seems possible this domination is due to the nonseparable utility function ($U = u(W - C(E))$). As Gjesdal (1982) has noted, with this form of a utility function, randomization (inherent in a tournament) can be beneficial.

Green and Stokey (1983) get somewhat different results in comparing tournaments to optimal individual contracts. With separable utility functions ($U = u(W) - C(E)$), they find with no common shock, the tournament is always inferior – it only adds extra uncertainty. However, if the common shock is large enough, they confirm Lazear and Rosen's intuitive result that tournaments will dominate. Here, the tournament's insurance against the common shock makes up for the extra randomness of the tournament gamble.[23] In addition to this result, Green and Stokey show that the tournament will start to dominate as the number of players rises. Essentially, with more players it is possible to have more prizes, and therefore a smoother, more flexible (and more optimal) compensation rule.[24]

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[23] Crudely speaking, it is a question of comparing the variance of the common shock to the variance of the individual shock.

[24] There have been several papers dealing with extensions of this analysis. Malcomson (1986)

As Holmström (1982a) has shown, relative-worth contracts can be optimal and should be studied. Rank-order tournaments are a somewhat restricted version of these relative-worth contracts. However, adopting this explicit structure makes it possible to generate more specific predictions, something the agency literature lacks. In addition, tournaments do seem empirically relevant, and there are several major reasons they would be preferred theoretically (Green and Stokey (1983)). The most important reason is that tournaments automatically insure agents against common uncertainty (thus golfers are insured against bad weather or a tough course). Malcomson (1984, 1986) has emphasized that tournaments solve certain problems with respect to output being unobservable to third parties (such as a court). With tournaments, the firm is committed to pay someone the prize, and has no ability or incentive to renege on the contract. O'Keeffe et al. (1984) have emphasized several other (mostly) unmodelled reasons. One is the utility of the contest itself (sports). A second is one might be dealing with fixed indivisible rewards (only one assistant professor can get tenure). A third is that a contest may be cheaper to administer (only rank has to be observed). Finally, tournaments promote the best agent on average (see Rosen (1986) as well).

has examined a model with a continuum of agents He shows in this setting that the tournaments are at least as good as (nonlinear) piece-rates under fairly general conditions. Nalebuff and Stiglitz (1983) emphasized the flexibility of tournaments in the face of shocks to the (slightly different) technology. O'Keeffe et al. (1984) examine what happens when there are agents of unknown heterogeneity in a tournament, while Carmichael (1983) shows tournaments can dominate in models where the principal has a role in output (in essence, he is the common shock). Dixit (1987) examines the impact on agents' choices of the ability to precommit to effort levels. Berkowitz and Kotowitz (1988) find that tournaments can be optimal as they reduce bankruptcy problems for agents. Additional papers include Mookherjee (1984).

1.4.2 Dynamic Tournaments

There has been only a small amount of research extending specific models such as tournaments to a dynamic setting.[25] Malcolmsen's (1984) model has several special features. The most important of these is that, first, there is a minimum level of observable effort, and second, output can only be observed with a one-period lag. Within this setting he shows that a two-period tournament model clearly dominates single-period payment schemes. The logic is straightforward. Given his one-period lag in observation, the two-period contract is needed to construct an enforceable incentive contract. Malcomsen's model is quite different from the normal tournament model, and does not deal with such dynamic issues as timepath of wages, etc., except in a simple manner.

Rosen (1986) constructs a model more like a standard tournament, as it is explicitly modelled after sequential elimination sports tournaments, such as those found in tennis. This too creates a special structure, although a very insightful one. Thus, agents are eliminated if they lose, and play no more contests (the optimality of this is not examined). In addition, Rosen has no established value of output, and therefore no profit side. Compensation is set with the goal of maintaining nondecreasing effort levels through the steps of the tournament.

Much of Rosen's work involves developing the agents' reaction functions in this setting, a nontrivial and interesting task. He establishes conditions under which symmetric (even chance) and asymmetric contests arise, and examines the situations with both homogeneous talents and heterogeneous talents (known and unknown). The major result is with respect to the pattern of wages over successive stages. These must rise steadily to induce agents to want to continue to play (expend effort). In addition, there must be a discrete jump in the final period to reflect the 'end game' effect (there are no further contests, no further

[25] The research of Cooper and Ross (1988) on intertemporal warranties can be interpreted as a complicated version of a dynamic piece-rate

potential prizes). His second result is that if agents are of unknown, heterogenous talent, then in all likelihood a contest winner is of higher ability. However, agents have an incentive to signal they are of lower ability (reducing future opponents effort levels), and this tends to depress effort levels in earlier stages. This socially useless activity can be reduced by concentrating less of the prize money at the top.

The above research has shown that dynamic tournament models show promising potential insights – for example, a sequential promotional structure within a firm (division heads promoted to vice-president based on ability, and the best of these promoted to president) seems to resemble a dynamic tournament. However, the limited work done so far reveals the modelling difficulty, reflected in the simplifying assumptions used to get the few (but interesting) results generated so far. These include the basic result that multiperiod contracts can dominate single-period contracts (given risk-aversion), as incentive problems can be reduced by repeated contracts. Second, these contracts will show memory effects. Third, the more general tournament results show that the larger the common shock or the larger the number of players, the more likely the tournament is to dominate other forms of contracts.

Clearly there is a great deal of room for a fuller, dynamic tournament model – for example, one where the full problem (including the relation of output to profits) is dealt with, and where a capital market is integrated into the dynamic structure. This thesis will begin this research.

1.5 Empirical Research

It has been emphasized that the literature on agency models has generated few predictions on observable variables. This in turn has meant that there is very little empirical research available to confirm or falsify the theories. Most of the empirical work is still casual observations of the prevalence of deductibles, piece-rates, etc. However, Lazear's (1979) model of a steep wage profile plus mandatory retirement has generated some empirical research. This work tests his agency model versus competing models of retirement. He finds that the probability of mandatory retirement is higher for longer-tenured and more able workers, backing up his agency model versus theories that argue retirement is used to remove less able workers in favour of more able. In a related work, Lazear and Moore (1984) compare the wage-earning profiles of salaried workers to self-employed workers. The fact the salaried workers' profiles are much steeper backs up Lazear's arguments that a steep wage profile is used for incentive purposes, as opposed to reflecting only on the job training. The empirical work in Hutchens (1986) also supports this model.

Murphy (1986) similarly compares an agency model to a more standard model. He derives a simplified dynamic agency model, and compares it to a version of Harris and Holmström's (1982) wage dynamics model. Both models argue wage profiles are upward-sloping over time. However, the Harris and Holmström model argues that the variance of earnings should fall with tenure as more information is accumulated on the workers' true productivity. With the agency model, the variance of income rises with tenure due to the presence of memory effects, and the end period effect. Murphy's data on the salary of chief executive officers seems to somewhat support the learning model. The weakness of the results suggests that both effects are present (Hart and Holmström (1987))

Several of the conference papers summarized in Raviv (1985) find evidence that executive compensation depends on the performance of the firm, supporting the arguments of Holmström (1979, 1982a) and others that agency models would be prevalent and that under likely restrictions $\partial S/\partial X > 0$. Finally, Wolfson (1985) finds an empirical return to establishing a reputation in the oil drilling market – the general partners drill more wells then would seem optimal for them personally in the short-run. However, along the lines of Fama's (1980) arguments, this helps them establish a reputation that attracts future limited partners (who desire these extra drillings). There is a return to reputation, there are memory effects present.[26]

The final piece of major research is the experimental work of Bull et al. (1987), examining how easy it is to implement tournaments versus piece-rates. The experiments are set up such that one can argue for risk-neutrality, in which case both the tournament and the piece-rate should yield the optimal effort level (observable in this setting). Players do seem to solve the tournament correctly, and converge to the optimal effort level. In addition, they react correctly to changes in the variance of the random shock. However, players seem to have trouble with the strategic behaviour and the more difficult calculations of the tournament – the range of effort responses is much wider for the tournaments than for a piece-rate system. Subject to one's reservations about experimental structure, this research is a strike against tournaments.

The empirical research done so far is too limited, and the results too vague, to really deliver any judgements on the validity of agency models. (This is partially because the models examined so far suggest no clear hypotheses to be tested.) Certainly there is scope for more empirical work. An interesting point which Hart and Holmström (1987) emphasize is that most of this empirical research is in a dynamic setting. This backs up the arguments presented above that dynamic models generate more testable predictions on observables than do single-period models.

[26]See Hart and Holmström (1987) or Wolfson for further details.

1.6 Summary of the Literature and Overview of the Thesis

There are several important research results shown in this survey. The most important result is that with risk-averse agents imperfect contracts will end up trading off risk-sharing and incentives – contracts will have deductibles and coinsurance. The second important prediction is that the principal will use all available information. Examples include the use of imperfect monitors of effort, or in the face of common shocks, the use of relative worth contracts.

The extension to dynamic models yielded extra predictions. First, multi-period contracts are superior, they will be used where possible. In addition, these contracts will have memory effects, compensation today will depend on current and past output, and under some restrictions the variance in income will rise over time.

Finally, restricting the format contracts can take, it was found rank-order tournaments tend to be preferred to individualistic schemes the larger the variance of the common shock and/or the larger the number of workers in the firm. In addition, if the principal has a role in output, relative worth contracts would be used.

This literature survey has revealed several areas in which agency models could be improved.[27] Most certainly dynamic models would seem to be an important area of expansion. Especially lacking in this setting is a good integration of capital markets. None of the research in the literature has adequately explored a

[27] In addition to those discussed in the body of the paper, two of the major areas would deal with how principals and agents get selected, or how agents get sorted into types of jobs depending on their degree of risk-aversion. This research would clearly have to be imbedded in a dynamic model. The reader's attention is also drawn to the ideas suggested in MacDonald (1984) and Hart and Holmström (1987)

joint choice of borrowing/saving and effort. Observation reveals agents can borrow and save, and restricting their ability to do this is an unreasonably strong assumption. As other research has hinted, and as this research shall show, introducing capital markets does make a significant difference – this research will generalize the study of dynamic agency models.

Other authors have argued (MacDonald (1984), Hart and Holmström (1987)) that agency models are short on testable predictions, and that dynamic models are better at generating estimable predictions, as the timepaths of variables come into play. Further, it is possible to get specific predictions by making specific assumptions. Therefore, this thesis will examine a dynamic model, but the feasible choices of the principal will be restricted by concentrating on rank-order tournaments, with a small digression comparing them to linear piece-rates.

In Chapter Two a basic risk-neutral model of a sequential rank-order tournament is developed, based on the structure of Lazear and Rosen (1981). It is first shown that allowing agents access to a perfect capital market where they can borrow[28] up to mean income results in a serious existence problem. The remainder of Chapter Two develops an intermediate, imperfect capital market based on restricted borrowing. Repeated one-period, nonmemory and memory two-period contracts are then considered. It is proved that memory contracts are optimal, and hence some of the results of the dynamic literature are generalized.

Chapter Three develops a simple dynamic linear piece-rate model with restricted borrowing. This allows for the generalization of the optimality of intertemporal models under borrowing. In addition, piece-rates and tournaments are compared in this setting. It is shown neither completely dominates the other, with the tournament tending to dominate as the value of output rises or the variance of the individual shock falls.

Chapter Four develops a more sophisticated capital market. Here agents

[28] In this structure the savings case turns out to be trivial.

are able to 'default' on their loans, but must pay a form of default cost. The modelling is more analytically complex, but yields much richer predictions. (For example, it is shown that when the principal cannot use memory contracts, in the second period the first period winner will 'coast' (choose less effort compared to the first period loser).) It is also shown that the memory contract predictions hold under this form of capital market as well.

Chapter Five pulls together and extends many of the results shown in the earlier chapters. Among the strong predictions of the model are that memory contracts will be observed, they are pareto-superior. In addition, predictions about the timepaths of consumption and wages in these models are generated. As an example, it is predicted mean wages will rise over the length of the contest. The default cost model also generates several predictions about observable labour supply curves and the extra work hours of agents. For example, it is predicted agents who lose contests will be forced to work extra hours (compared to winners) to pay off their debts.

Chapter 2

Rank-Order Tournaments With Restricted Borrowing

The literature survey in Chapter One revealed that one important extension of agency models would be to examine the case of dynamic models with agents able to freely and jointly choose borrowing and effort. This chapter begins this research by examining borrowing in a sequential rank-order tournament. It is first shown that a perfect capital market leads to an equilibrium with zero output. The remainder of the chapter examines an imperfect capital market (based on restricted borrowing), and examines its impacts on the agency problem. Several types of contracts are developed, and it is shown that in this setting memory contracts will dominate nonmemory contracts. Details of these contracts are also developed, and predictions about observables are made.

2.1 The Basic Model

A model in which agents consume in periods 0, 1, 2 and work in periods 1 and 2 will be developed. In each period of work, output is produced via the following simple production function, for worker j:

$$q_{jt} = E_{jt} + \epsilon_{jt}. \tag{2.1}$$

Output is q, effort E, and ϵ is a random shock with mean zero and variance σ^2. It is assumed ϵ is independent across time and agents, and the density function is such that $f(\epsilon|E) = f(\epsilon)$ for all E. Effort is selected before the shock is observed by the agent, and the principal can only observe the quantity of output.[1]

Agents are assumed identical, and have the following utility functions:

$$V = Z_0 + \beta(Z_1 - C(E_1)) + \beta^2(Z_2 - C(E_2)) \tag{2.2}$$

where $\beta = (1+\rho)^{-1}$ is the discount factor, Z consumption and $C(E)$ reflects the disutility of effort ($C(0) = 0$, $C'(0) = 0$, $C' \geq 0$, $C'' \geq 0$). Agents select effort and the amount they wish to save or borrow at the interest rate r to maximize expected utility subject to whatever borrowing constraints are in place, the form of the compensation contracts, and their conjectures of other agents' effort. It is assumed $0 < r < \rho < 1$, so that agents always want to borrow some amount.[2]

Solving the agent's problem yields the reaction functions for borrowing and effort. The principal takes this into account when solving his problem. He selects the wage structure to maximize the agents' expected utility (with the optimal reaction functions substituted in) subject to a constraint that expected profits be nonnegative. This problem replicates the outcome of competition, where firms and workers are maximizing expected profits and expected utility respectively. Thus the synthetic problem is to select the wage structure to solve:

$$\max L = V(w, E(w), B(w), \ldots) + \lambda(\pi(w, E(w), B(w), \ldots)),$$

[1] It is assumed collusion between agents, and destructive behaviour towards one's opponents is precluded (see MacDonald (1984) for a discussion). A common shock term has been left out of the production function, as it turns out to have no role in these models - not even in the piece rate versus rank-order tournament comparisons of Chapter 3.

[2] It is possible that agents could have endowments each period, which would be fully borrowed and consumed in period 0. These would in no way alter any results, and hence are omitted.

where w represents the respective compensation structure analyzed below, and where $V(w, E(w), B(w), \ldots)$ is essentially an indirect utility function.[3] The lagrange is as written to reflect the indirect effects of the wage structure on the agent's choices. In this setup all the rents go to the agent, but this in no way alters any crucial results about effort levels, etc.

Firms hire four workers, who are paired off in sequential rank-order tournament contests. In each contest the worker with the highest level of output receives the wage of a winner (denoted as W_w), and the other the wage of a loser (W_l). The probability that an agent i wins a particular contest is:

$$P = \text{prob}(q_i > q_j) = \text{prob}(E_i - E_j > \epsilon_j - \epsilon_i) = G(E_i - E_j), \tag{2.3}$$

where $G(\cdot)$ is the cumulative density function of $\epsilon_j - \epsilon_i$, and $\partial P / \partial E_i = g(E_i - E_j)$. It will be assumed throughout the resulting equilibrium is Nash.

There are three types of contests examined – repeated contests (workers replay each other, identical wage structures in each period), nonmemory contests (workers replay each other, different wage structures) and memory contests (workers are sorted so that winners play winners, etc. and all wage structures can differ). The sequence of events follows Table 1.

This structure can be thought of as a sequential promotion structure, where division heads compete for the prize of being selected vice-president (with the resulting jump in salary), and then in turn compete as vice-presidents to be selected president. A similar story could be salespeople competing to be salesperson of the year, with a free trip to Hawaii, etc.

[3] The question of potential mobility constraints on the principal's problem are discussed below

TABLE ONE Sequence of Events		
Period 0		
P announces wages.	A selects B_1 and consumes.	
Period 1		
A selects E_1.	q_1 observed, wages paid out.	Winner(loser) selects $B_w(B_\ell)$ and consumes(works off debt)
Period 2		
Winner(loser) selects $E_w(E_\ell)$.	$q_w(q_\ell)$ observed, wages paid out.	Second period winner(loser) consumes(works off debt).

2.2 Introducing the Capital Market

In this section a standard capital market will be introduced, and it will be shown that in this setting only a trivial solution to the principal's problem exists. Consider a one-period tournament. As discussed above, the probability of winning W_w is $P(E_i, E_j)$, etc. It is assumed a Nash equilibrium exists, computable by all parties, and with identical agents $E_i = E_j$, yielding $P(E^*) = .5$.

Suppose a bank offers the following 'fair bet' loan at the rate $r < \rho$.[4] Agents borrow B for immediate consumption in period zero, where

$$B = \frac{x(P(E^*)W_w + (1 - P(E^*))W_\ell)}{1 + r},$$
$$x \leq \mathrm{F}.$$

Winners repay $x \cdot W_w$, losers $x \cdot W_\ell$. Note that banks earn nonnegative profits (assuming the cost of funds is $\leq r$), and agents can borrow up to their mean

[4]Other borrowing schemes can be constructed, but yield under-effort and/or negative profits for the bank. Examples include a scheme where borrowing B is subject to $W_w \geq B \geq W_\ell$

income. Note if the agent selects $x = 1$, consumption in period one is zero. It is assumed the bank can calculate $P(E^*)$, straightforward in the current case where agents are identical, and make identical choices of effort, so that $P(E^*) = 5$

Proposition 1 *In the above described fair-bet borrowing arrangement, only a trivial Nash equilibrium exists in the tournament firm, with zero effort and zero output on average for each agent.*

<u>Proof</u>: Each agent's problem is to select x (and hence B) and E, subject to $x \leq 1$, to solve the following:

$$\max_x V = \frac{xP(E^*)W_w + x(1 - P(E^*))W_\ell}{1 + r} + \max_E \beta(P(E)W_w(1 - x) + (1 - P(E))W_\ell(1 - x) - C(E)).$$

Solving this backwards from the final period, each agent selects E to solve

$$\frac{\partial P}{\partial E} \cdot (W_w - W_\ell)(1 - x) - C'(E) = 0$$

Assuming a symmetric Nash equilibrium, each identical agent will make the same selection of effort, E^*, given the already chosen optimal value of x.

Next, in the first period, given he will be selecting E optimally in the final period, each agent selects x, subject to $x \leq 1$, to maximize

$$V = \frac{xP(E^*)W_w + x(1 - P(E^*))W_\ell}{1 + r} + \beta(P(E^*)(W_w(1 - x) + (1 - P(E^*))W_\ell(1 - x) - C(E^*)).$$

With λ the multiplier on the borrowing constraint, the necessary condition is:

$$\frac{P(E^*)W_w + (1 - P(E^*))W_\ell}{1 + r} - \frac{P(E^*)W_w + (1 - P(E^*))W_\ell}{1 + \rho} - \lambda \leq 0.$$

Examination given $r < \rho$ reveals $\lambda > 0$, and $x = 1$. Agents take the full amount of the fair bet. Substitution into the effort equation yields:

$$\frac{\partial P}{\partial E} \ (0) = C'(E).$$

which under the assumptions on $C(E)$ yields E = expected output = 0. □

The intuition is that the agent selects and eats B in period zero. At the beginning of period one, he selects effort. At this stage, he is committed to repay the outstanding debt, and whether he wins or loses, he consumes zero. Unless he is somehow precommitted to select $E > 0$, clearly the maximizing choice is zero effort. The entrance of a third economic entity into the game destroys workers' incentives, and hence the tournament firm's equilibrium collapses.

In response to this problem, two types of imperfect capital markets are developed in this thesis, both of which avoid this problem. Neither of these types is necessarily optimal. Developing an optimal setup is a complex task left for future research. There it would be necessary to describe maximizing behavior for the bank, and set out the exact game theory for this new triangular situation.

Two different models of capital markets will be developed that approximate those actually observed.[5] These models will be internally consistent in that banks will make zero profits on average, and in the resulting equilibriums effort and output will be positive. In the first model, developed in this chapter and the next, agents are allowed to borrow up to an amount they can commit to repay (the wage they earn in the worst case scenario). In Chapter Four, agents will be able to borrow past this amount (risking bankruptcy). However, they face default costs, in that they are committed to work off outstanding debts at an outside job. These models are only approximations of more complicated capital markets, but their simple structure yields specific, testable predictions.

The idea of an imperfect capital market providing useful insights is not unique. One interesting example is Haltiwanger and Waldman (1986) who examine adding a capital market to Harris and Holmström's (1982) contract model.

[5]Both models are developed not only for comparison purposes, but because each yields unique insights into the impact of borrowing on agency models.

They show that an imperfect capital market generates predictions about wage patterns that are closer to the stylized facts than the predictions of models with either no capital market or a perfect capital market (like the one used above).

A second model with an imperfect capital market is developed in Farrow (1986). He considers a model where firms have superior information on the value of marginal product, and are negogiating with workers over wages. If these firms face bankruptcy constraints and default costs (somewhat resembling those developed in Chapter Four), then this 'illiquidity phenomena can make a risk neutral firm act 'as if ' it were risk averse.'[6] As shall be shown, the imperfect capital markets developed in this thesis will make risk-neutral workers act as if they were risk-averse. Clearly, exploring imperfect capital markets is fruitful.

2.3 Repeated One-Period Contracts

In this section contracts will be restricted to a particular form. Firms will be constrained to offer identical zero-profit contracts in each period, although agents are free to borrow across two periods. This is a case where neither side can commit for more then one period, and where workers are not identifiable as previous winners or losers.

A firm hires two workers, who compete with each other in two consecutive contests. In each contest the worker with the highest level of output receives W_w, and the other receives $W_\ell (W_w > W_\ell)$. The probability that an agent i wins the first (second) contest is $P_1(P_2)$, where P_1 and P_2 are as defined in equation (2.3). In a Nash equilibrium, assuming symmetry, $E_i = E_j$, and therefore in equilibrium $P_1 = P_2 = .5$.

[6] Farrow (1986), page one

Agents can borrow against their sure income in each period:

$$B_1 \leq \frac{W_\ell}{1+r} + \frac{B_2}{1+r} \; ; \; B_2 \leq \frac{W_\ell}{1+r}. \tag{2.4}$$

B_1 is borrowed (and consumed) in period zero against period one's earnings. Since agents can roll over the outstanding debt of B_2, the above implies:

$$B_1 \leq \frac{W_\ell}{1+r} + \frac{W_\ell}{(1+r)^2} \tag{2.5}$$

Thus agents borrow across two periods. Note that banks always are guaranteed repayment, as an agent earns at least W_ℓ (e.g. $B_2(1+r) = W_\ell$) In addition, it will be demonstrated below that an equilibrium with positive effort levels and positive output exists.

The agent selects effort and borrowing in each period to maximize:

$$\begin{aligned} V &= B_1 + \beta[B_2 - B_1(1+r) + P_1(W_w) + (1-P_1)(W_\ell) - \\ &\quad C(E_1)] + \beta^2[P_2(W_w) + (1-P_2)(W_\ell) - B_2(1+r) - C(E_2)] \end{aligned}$$

subject to the borrowing constraints. Letting λ_1 and λ_2 be the multipliers on the B_1 and B_2 constraints, the first order conditions are:

$$\frac{\partial V}{\partial B_1} = 1 - \beta(1+r) - \lambda_1 = 0, \tag{2.6}$$

$$\frac{\partial V}{\partial B_2} = \beta(1 - (1+r)\beta) - \lambda_2 = 0, \tag{2.7}$$

$$\frac{\partial V}{\partial E_1} = \beta(\frac{\partial P_1}{\partial E_1}(W_w - W_\ell) - C'(E_1)) = 0, \tag{2.8}$$

$$\frac{\partial V}{\partial E_2} = \beta^2(\frac{\partial P_2}{\partial E_2}(W_w - W_\ell) - C'(E_2)) = 0, \tag{2.9}$$

as well as the two constraints. Assume the above equations characterize the global maximum, and a unique solution exists.[7]

[7]There are two important points here. First, this assumption is potentially important, as a global maximum of $E_1 = 0$ is possible - see Lazear and Rosen (1981), O'Keeffe et al (1984) or Rosen (1988) for a further discussion. Second, there is a potential problem if $W_\ell < 0$ - then B_2 or $B_1 < 0$ is possible To avoid this anomaly, it could be assumed agents have an endowment such that the endowment + $W_\ell > 0$ As mentioned above, this endowment can be ignored in subsequent analysis

Given $\rho > r$, the above f.o.c.'s imply $\lambda_1, \lambda_2 > 0$ and thus the borrowing constraints bind. Assume a symmetric Nash equilibrium holds ($E_i = E_j$), and define $\bar{g} = g(0)$. In order to get an explicit solution, it is assumed $C(E) = E^2/2$. Thus, the best response functions are:

$$B_1 = \frac{W_\ell(2+r)}{(1+r)^2}, \tag{2.10}$$

$$B_2 = \frac{W_\ell}{1+r}, \tag{2.11}$$

$$E_1 = E_2 = E = \bar{g} \cdot (W_w - W_\ell). \tag{2.12}$$

The effort choice function reveals a simple marginal cost equals marginal benefit condition. Thus, $E = MC$, while the MB of working equals the gain in wages from winning times the probability density function evaluated at the equilibrium. The last factor represents the randomness due to the shock to output - the lower the variance of this shock, the higher $\bar{g}$ and the higher the return to one's effort (essentially, the lower the chance of an unfortunate draw of ϵ negating your hard work). It can be noted that as long as $W_w > W_\ell$ effort will be positive - agents are motivated by the gamble (consumption of a winner $= W_w - W_\ell$, that of a loser zero). Unlike in Proposition 1, with this capital market a nontrivial equilibrium with positive effort and output exists.

The comparative statics are straightforward. Effort rises as the spread of wages rises, but it falls if the variance of the shock to production rises ($\bar{g}$ falls and chance is more important). With more randomness, the link between effort and compensation is weakened, and effort falls. The comparative statics on B_1, B_2 are clear, too. Finally, note that there are no links between periods, and no interperiod effects - the second period effort is independent of what occured in the first period.

As mentioned earlier, the principal selects the wage structure to maximize the agent's expected utility (evaluated at the Nash equilibrium), subject to a constraint that discounted expected profits equal zero. Defining D as the value of output and r as the principal's discount rate, his problem is to select W_w, W_ℓ to:

$$\max L = \frac{W_\ell(2+r)}{(1+r)^2} + \frac{(2+\rho)}{(1+\rho)^2}\left(\frac{(W_w - W_\ell)}{2} - C(E)\right) + \lambda\frac{(2+r)}{(1+r)^2}\left(DE - \frac{(W_w + W_\ell)}{2}\right)$$

where E comes from the agent's reaction functions, and $P_1 = P_2 = .5$, and the borrowing reaction functions have been substituted in.[8]

Assuming the second-order conditions are met, the first order conditions to the principal's problem are:

$$\frac{\partial L}{\partial W_w} = \frac{2+\rho}{(1+\rho)^2}\left(\frac{1}{2} - C'(E)\frac{\partial E}{\partial W_w}\right) + \lambda\frac{(2+r)}{(1+r)^2}\left(D\frac{\partial E}{\partial W_w} - \frac{1}{2}\right) = 0, \tag{2.13}$$

$$\frac{\partial L}{\partial W_\ell} = \frac{2+r}{(1+r)^2} + \frac{2+\rho}{(1+\rho)^2}\left(-\frac{1}{2} - C'(E)\frac{\partial E}{\partial W_\ell}\right) + \lambda\frac{(2+r)}{(1+r)^2}\left(D\frac{\partial E}{\partial W_\ell} - \frac{1}{2}\right) = 0, \tag{2.14}$$

$$\frac{\partial L}{\partial \lambda} = \frac{(2+r)}{(1+r)^2}\left(DE - \frac{(W_w + W_\ell)}{2}\right) = 0. \tag{2.15}$$

The intuition of these equations is straightforward. For example, as the principal raises W_w this raises the agent's utility via raising consumption, but it lowers his utility by raising effort ($\partial E/\partial W_w = \bar{g} > 0$). The effects on profits are such that raising W_w raises effort and revenue, but also raises costs, with the overall effect ambiguous. The fact $C'' > 0$ guarantees a maximum value for W_w exists. On the other hand, raising W_ℓ always raises utility and lowers expected profits, and once again the convexity of C guarantees an equilibrium.

Proposition 2 *a) The solution to the principal's problem yields:*

$$E = \frac{D(1+\rho)^2(2+r)}{(2+\rho)(1+r)^2} + \frac{(r-\rho)(3+2r+2\rho+\rho r)}{2\bar{g}(1+r)^2(2+\rho)}.$$

[8]The method used here of substituting E into the principal's problem means the 'first order approach' problems discussed in Rogerson (1985b) are not relevant

$$W_w = E(D + \frac{1}{2\bar{g}}) \; ; \; W_\ell = E(D - \frac{1}{2\bar{g}}) \; .$$

b) The principal desires the agent to work harder at the optimum (an observable increase in output is pareto-optimal - both the principal and the agent can be made better off).

Proof: a) Manipulating the principal's first order conditions yields $\lambda = 1$, and the solutions above.

$$\text{b)} \quad \frac{\partial L}{\partial E} = \frac{2+\rho}{(1+\rho)^2}(-C'(E)) + \lambda\frac{(3+r)D}{(1+r)^2}.$$

Evaluating this at the equilibrium values for E and λ:

$$\frac{\partial L}{\partial E} = \frac{(\rho - r)(3 + 2r + 2\rho + \rho r)}{2\bar{g}(1+r)^2(1+\rho)^2} > 0 \; . \; \square$$

This proposition reveals the usual agency result that the principal desires the agent to work harder at the second-best optimum (see Holmström (1979), Nalebuff and Stiglitz (1983)). In addition, the comparative statics are as follows:

$$\frac{\partial E}{\partial D} > 0, \; \frac{\partial E}{\partial \bar{g}} > 0, \; \frac{\partial E}{\partial \rho} \gtrless 0, \; \frac{\partial E}{\partial r} \gtrless 0,$$
$$\frac{\partial (W_w - W_\ell)}{\partial D} > 0, \; \frac{\partial (W_w - W_\ell)}{\partial \bar{g}} \gtrless 0.$$

These are straightforward. For example, if the value of output rises, more effort is induced by raising the spread and mean of wages. A fall in the randomness of output (assuming a symmetric distribution, $\bar{g}$ rises) raises effort.

Finally, solving for the value of the contract to the agent in equilibrium:

$$\text{V(repeated)} = W_\ell\frac{(2+r)}{(1+r)^2} + \frac{(2+\rho)}{(1+\rho)^2}(\frac{W_w - W_\ell}{2} - C(E)).$$

Substituting in the values for W_w and W_ℓ yields:

$$\text{V(repeated)} = \frac{E^2(2+\rho)}{2(1+\rho)^2},$$

where V has been left as a function of E for algebraic simplicity. This value will be useful in comparing repeated contracts to others in Section 2.6 below.

The repeated contracts in this section entailed severe restrictions on the principal's choices - he was restricted to offer identical contracts to all agents in all periods. This means he was unable to differentiate between agents based on their history, and was unable to have interperiod effects in the wage structure. It was shown that in this situation there was an agency problem of undereffort. In the next section, one of these restrictions is released – the principal can offer different wages in different periods.

2.4 Nonmemory Two Period Contracts

In this section the same production structure, utility functions, and capital market are assumed. However, a crucial change is that now firms can offer different wage structures in the two periods. The firm hires two workers who compete with each other in two consecutive contests. The probabilities of winning in the first and second period are P_1 and P_2 respectively,[9] with these probabilities described by equation (2.3). Agents are offered the wage pair (W_w, W_ℓ) in the first period and $(W_{2w}, W_{2\ell})$ in the second period, with the agent with the higher output receiving the (higher) winner's wage (W_w, W_{2w} respectively). A crucial point is that firms are not completely unrestricted in their choice of wage structures. Although they may have wage structures differing between periods, firms are not allowed to differentiate between first period winners and losers in the second period (there are no memory effects). However, despite this lack of wage links between periods, an intertemporal structure will come into play.

With the fact contracts differ between periods, unlike in the repeated case an

[9] In equilibrium, $P_1 = P_2 = .5$

agent may wish to leave the firm after the first period. It will still be assumed the principal signs a legally binding contract over two periods, and initially it will be assumed the agent is legally bound too. It will be shown below that when the agent can leave after one period, the principal will arrange the contract so the agent does not want to.

Agents can borrow against their sure income each period:

$$B_1 \leq \frac{W_\ell}{1+r} + \frac{W_{2\ell}}{(1+r)^2} \; ; \; B_2 \leq \frac{W_{2\ell}}{1+r} . \tag{2.16}$$

Agents select borrowing and effort to solve the following problem:

$$\begin{aligned} \max V \; = \; & B_1 + \beta(P_1(W_w - B_1(1+r)) + (1-P_1)(W_\ell - B_1(1+r)) \\ & + B_2 - C(E_1)) + \beta^2(P_2(W_{2w} - B_2(1+r)) + \\ & (1-P_2)(W_{2\ell} - B_2(1+r)) - C(E_2)) \end{aligned}$$

subject to the borrowing constraints.[10]

The first order conditions to the above problem are similar to those for the repeated problem, and are omitted. In a symmetric Nash equilibrium ($P_1 = P_2 = .5$), the solution to the agent's problem is:

$$B_1 = \frac{W_\ell}{1+r} + \frac{W_{2\ell}}{(1+r)^2} , \tag{2.17}$$

$$B_2 = \frac{W_{2\ell}}{1+r} , \tag{2.18}$$

$$E_1 = \bar{g}(W_w - W_\ell) , \tag{2.19}$$

$$E_2 = \bar{g}(W_{2w} - W_{2\ell}) . \tag{2.20}$$

These results are natural extensions of the previous ones. Note that in general $E_1 \neq E_2$ unless the principal chooses to set wage differentials identical in the two periods. The comparative statics are straightforward, and similar to the earlier

[10] Note that E_w is restricted to equal E_ℓ. It is easy to show that with no leftover debt and facing identical wages in the second period, agents will indeed select $E_w = E_\ell$.

case. Crucial is that there are no interperiod wage effects, so $\partial E_2/\partial W_w = 0$, etc. (These links will be introduced in the next section.)

Given the agent's choices, the principal selects the wage structure to solve the following:

$$\begin{aligned}\max \mathrm{L} &= \frac{W_\ell}{1+r} + \frac{W_{2\ell}}{(1+r)^2} + \beta(\frac{W_w - W_\ell}{2} - C(E_1)) + \\ &\beta^2((\frac{W_{2w} - W_{2\ell}}{2}) - C(E_2)) + \lambda(\frac{DE_1}{1+r} + \frac{DE_2}{(1+r)^2} \\ &- \frac{W_w + W_\ell}{2(1+r)} - \frac{W_{2w} + W_{2\ell}}{2(1+r)^2}) .\end{aligned}$$

(Mobility constraints are considered below.)

Proposition 3 *a) The wage structure is non unique.*

b) The contracts that solve the principal's problem all involve the following values:

$$E_1 = \frac{D(1+\rho)}{1+r} + \frac{r-\rho}{2\bar{g}(1+r)} , \tag{2.21}$$

$$E_2 = \frac{D(1+\rho)^2}{(1+r)^2} + \frac{(r-\rho)(2+r+\rho)}{2\bar{g}(1+r)^2} \tag{2.22}$$

It can be shown that

$$\begin{aligned}E_1 - E_2 &= \frac{(\rho - r)(1+\rho)}{(1+r)^2}(D - \frac{1}{2\bar{g}}) \gtrless 0 . \\ \frac{\partial E_1}{\partial D} &> 0, \ \frac{\partial E_1}{\partial \bar{g}} > 0, \ \frac{\partial E_2}{\partial D} > 0, \ \frac{\partial E_2}{\partial \bar{g}} > 0. \\ Sign\frac{\partial E_1}{\partial \rho} &= -Sign\frac{\partial E_1}{\partial r} = Sign\frac{\partial E_2}{\partial \rho} \\ &= -Sign\frac{\partial E_2}{\partial r} = Sign(2D\bar{g} - 1) \gtrless 0 .\end{aligned}$$

c) At the optimum, the principal desires the agent to work harder.

Proof: a) Consider a set of wages that solves the principal's problem. Carry out the following changes: $dW_w = dW_\ell = x$ and $dW_{2w} = dW_{2\ell} = -x(1+r)$, for some x. From the values of the agent's reaction functions, $dE_1 = dE_2 = 0$. From the principal's problem.

$$dL = \frac{x}{1+r} - \frac{x(1+r)}{(1+r)^2} + \beta(.5(x-x)) + \beta^2(.5(x(1+r)$$
$$-x(1+r))) + \lambda(-\frac{(x+x)}{2(1+r)} + \frac{x(1+r)+x(1+r)}{2(1+r)^2}) = 0 .$$

Thus, any changes in the wage structure as described above does not disturb the optimum.

b) Take the first order conditions for the principal's problem. $\partial L/\partial W_w = 0$ and $\partial L/\partial W_\ell = 0$ can be solved for λ and E_1. These are substituted into either of $\partial L/\partial W_{2w}$ or $\partial L/\partial W_{2\ell}$ and solved for E_2. $E_1 - E_2$ follows simply, as do the comparative statics.

c) Evaluate the following at the equilibrium values of E_1, E_2, and λ:

$$\frac{\partial L}{\partial E_1} = \beta(-C'(E_1)) + \frac{\lambda D}{1+r} = \frac{\rho - r}{2\bar{g}(1+r)(1+\rho)} > 0 .$$
$$\frac{\partial L}{\partial E_2} = \beta^2(-C'(E_2)) + \frac{\lambda D}{(1+r)^2} = \frac{(\rho - r)(2+r+\rho)}{2\bar{g}(1+r)^2(1+\rho)^2} > 0 . \square$$

The nonuniqueness result is common to multiperiod agency problems.[11] There are many different wage patterns that yield the optimum to the problem. What is crucial to the agent is consumption, not wages - in a model with borrowing it is this that motivates an agent. The wage changes described in the proof above are offset by the agent changing his borrowing, leaving his consumption constant. It will be shown below these consumption values are unique.

The effort levels are also unique. There is an intertemporal effect in wages and therefore in effort - in general $E_1 \neq E_2$. When borrowing is present, the firm chooses to create this intertemporal effect, as it maximizes profits. The borrowing itself allows this effect to be created, as it links periods together.

[11] See Lazear (1979) or in an analogous setting Haltiwanger and Waldman (1986).

The equilibrium value of the nonmemory contract will be needed below to compare it to the value of other contracts. This value is:

$$\begin{aligned} V\,(\text{nonmemory}) &= \frac{W_\ell}{1+r} + \frac{W_{2\ell}}{(1+r)^2} + \beta(\frac{W_w - W_\ell}{2} - C(E_1)) \\ &\quad + \beta^2(\frac{W_{2w} - W_\ell}{2} - C(E_2)). \end{aligned}$$

To solve this, start by letting Z index consumption in a given state. Then

$$\begin{aligned} Z_{2w} &= W_{2w} - W_{2\ell} = E_2/\bar{g}, \\ Z_w &= W_w - W_\ell = E_1/\bar{g}, \\ Z_{2\ell} &= Z_\ell = 0, \\ Z_0 &= \frac{W_\ell}{1+r} + \frac{W_{2\ell}}{(1+r)^2} = \frac{\bar{y}}{1+r} - \frac{Z_w}{2(1+r)} - \frac{Z_{2w}}{2(1+r)^2} \end{aligned}$$

where

$$\bar{y} = \frac{W_w + W_\ell}{2} + \frac{W_{2w} + W_{2\ell}}{2(1+r)} = DE_1 + \frac{DE_2}{1+r}.$$

Appropriate substitution of the Z terms into V and simplification yields a result which will be used in Section 2.6 below:

$$V(\text{nonmemory}) = \frac{(E_1)^2}{2(1+\rho)} + \frac{(E_2)^2}{2(1+\rho)^2}. \tag{2.23}$$

Implicit in the above discussion is the assumption that the worker does not wish to leave the firm in the second period, that he is bound to the firm. If this assumption is relaxed, the firm must meet certain second-period mobility constraints. Given there is no specific human capital, the first of these is that second-period profits must be non-positive (so that other firms will not bid away workers):

$$.5(W_{2w} + W_{2\ell}) - DE_2 \geq 0. \tag{2.24}$$

The second constraint is that second-period utility must be higher in the contest than at a reserve job. It is assumed this job entails perfectly observable output.

and pay equals the value of marginal product ($D_r E_r$). Appendix 2.1 examines the agents' choices at such a job, and there it is shown the same constraint will hold for a first period winner or loser:

$$\frac{W_{2\ell}}{1+r} + \beta(.5(W_{2w} - W_{2\ell}) - C(E_2)) \geq \frac{(D_r)^2(1+\rho)}{2(1+r)^2}. \qquad (2.25)$$

Proposition 4 *The mobility constraints do not bind.*

__Proof:__ In the new principal's problem, let λ_1, λ_2, and λ_3 (all nonnegative) be the multipliers for the two-period zero-profit constraint, (2.24) and (2.25) respectively. The first order conditions to this problem then become:

$$\begin{aligned}
\frac{\partial L}{\partial W_w} &= .5\beta - \beta C'(E_1)\frac{\partial E_1}{\partial W_w} + \lambda_1\left(\frac{D}{1+r}\frac{\partial E_1}{\partial W_w} - \frac{1}{2(1+r)}\right) \leq 0, \\
\frac{\partial L}{\partial W_\ell} &= \frac{1}{1+r} - .5\beta - \beta C'(E_1)\frac{\partial E_1}{\partial W_\ell} + \lambda_1\left(\frac{D}{1+r}\frac{\partial E_1}{\partial W_\ell}\right. \\
&\quad \left. - \frac{1}{2(1+r)}\right) \leq 0, \\
\frac{\partial L}{\partial W_{2w}} &= .5\beta^2 - \beta^2 C'(E_2)\frac{\partial E_2}{\partial W_{2w}} + \frac{\lambda_1}{(1+r)^2}\left(D\frac{\partial E_2}{\partial W_{2w}} - .5\right) + \\
&\quad \lambda_2\left(.5 - D\frac{\partial E_2}{\partial W_{2w}}\right) + \lambda_3\left(.5\beta - \beta C'(E_2)\frac{\partial E_2}{\partial W_{2w}}\right) \leq 0, \\
\frac{\partial L}{\partial W_{2\ell}} &= \frac{1}{(1+r)^2} - .5\beta^2 - \beta^2 C'(E_2)\frac{\partial E_2}{\partial W_{2\ell}} + \\
&\quad \frac{\lambda_1}{(1+r)^2}\left(D\frac{\partial E_2}{\partial W_{2\ell}} - .5\right) + \lambda_2\left(.5 - D\frac{\partial E_2}{\partial W_{2\ell}}\right) + \lambda_3\left(\frac{1}{1+r}\right. \\
&\quad \left. - .5\beta - \beta C'(E_2)\frac{\partial E_2}{\partial W_{2\ell}}\right) \leq 0.
\end{aligned}$$

$\partial L/\partial W_w$ and $\partial L/\partial W_\ell$ can be solved for $\lambda_1 = 1$ and

$$E_1 = \frac{D(1+\rho)}{1+r} + \frac{r-\rho}{2\bar{g}(1+r)}.$$

Taking into account the comparative statics from the agent's problem and $\lambda_1 = 1$, adding together $\partial L/\partial W_{2w}$ and $\partial L/\partial W_{2\ell}$ yields $\lambda_2 + \lambda_3/(1+r) \leq 0$, which implies $\lambda_2 = \lambda_3 = 0$. □

The intuition of why the mobility constraints do not bind is as follows. In Proposition 3 it was shown that the firm could make the following changes to wages without disturbing the optimum - lower W_w, W_ℓ by x and raise W_{2w}, $W_{2\ell}$ by $x(1+r)$. Clearly the firm can costlessly meet the mobility constraints by carrying out such actions.

The previous results on wages, consumption and effort go through unchanged. This result of agents choosing not to leave the firm is quite common in the literature on long-term contracts. However, since the principal makes negative profits in the second period, this requires either a legal mechanism or strong reputation effects to prevent the principal from firing the agents after the first period (see Lazear (1979) or Kuhn (1986) for further discussion on this point.)

In this section the principal was allowed to offer two-period contracts, unlike in the previous section. Here, it was shown that there would still be under-effort in the resulting equlibrium. Despite this, firms did choose to offer these two-period contracts as they were superior to repeated one-period contracts. These new contracts generated some new results (for example, the nonuniqueness of wages, and the fact $E_1 \neq E_2$). However, firms were still not allowed to offer agents with different histories different contracts. The next section addresses this issue.

2.5 Memory Contracts

In this model, the technology and preferences are as above, but the firm can sort workers in the second period, based on what they did in the first period. Thus the firm has the possibility of selecting memory contracts - agents' earnings in the second period depend not only on second period effort, but also on whether they won or lost in the first period. A crucial interperiod wage link has been created.

The firm hires four workers, who work in two separate contests in the first period, for wages W_w, W_ℓ as above. In the second period the two winners are now sorted to play each other for wages W_{ww}, $W_{w\ell}$, with the agent with the higher output receiving $W_{ww} > W_{w\ell}$. Similarly, the two losers play each other for $W_{\ell w}$ and $W_{\ell\ell}$, $W_{\ell w} > W_{\ell\ell}$. As a result, there is the possibility of memory effects - what agents do in the first period potentially affects their second-period compensation. The agent with the higher output not only receives a higher wage, but goes on to play in a different contest.[12] An explicit example would be a sequential promotion structure within a firm - division heads with high output get promoted to vice-presidents, from whose ranks a president will be selected.

The probability of winning an individual contest (P_1, P_w, P_ℓ) is formed in a manner analogous to that of the earlier contests. Agents once again face the constraint they can borrow only up to their minimum guaranteed income at each stage:

$$B_1 \leq \frac{W_\ell}{1+r} + \frac{W_{\ell\ell}}{(1+r)^2}, \tag{2.26}$$

$$B_w \leq \frac{W_{w\ell}}{1+r};\ B_\ell \leq \frac{W_{\ell\ell}}{1+r}. \tag{2.27}$$

Note how the sorting means the winner now revises his borrowing.

The agents select borrowing and effort to solve the following problem:[13]

$$\begin{aligned}\max V = \; & B_1 + \beta P_1(W_w - B_1(1+r) + B_w + \beta(P_w(W_{ww} - \\ & B_w(1+r)) + (1-P_w)(W_{w\ell} - B_w(1+r)) - C(E_w))) +\end{aligned}$$

[12] It should be noted if workers differed in ability, this scheme would promote more able workers on average, and sort for the firm. See Rosen (1986) for further discussion.

[13] It is assumed that

$$W_w - W_\ell + \frac{W_{w\ell} - W_{\ell\ell}}{1+r} > 0.$$

This is needed to guarantee that a first period winner has positive consumption, a constraint similar to the earlier one that losers have nonnegative consumption.

$$\beta(1-P_1)(W_\ell - B_1(1+r) + B_\ell + \beta(P_\ell(W_{\ell w} - B_\ell(1+r)) + (1-P_\ell)(W_{\ell\ell} - B_\ell(1+r)) - C(E_\ell)))$$
$$-\beta C(E_1) + \lambda_1(\frac{W_\ell}{1+r} + \frac{W_{\ell\ell}}{(1+r)^2} - B_1) + \lambda_2(\frac{W_{w\ell}}{1+r} - B_w)$$
$$+\lambda_3(\frac{W_{\ell\ell}}{1+r} - B_\ell),$$

where E_w is the effort in the winners contest, E_ℓ in the losers contest. Assuming once again a unique symmetric Nash equilibrium (so that $P_w = P_\ell = P_1 = .5$), the solution is:

$$B_1 = \frac{W_\ell}{1+r} + \frac{W_{\ell\ell}}{(1+r)^2}, \quad (2.28)$$

$$B_w = \frac{W_{w\ell}}{1+r}, \quad (2.29)$$

$$B_\ell = \frac{W_{\ell\ell}}{1+r}, \quad (2.30)$$

$$E_1 = \bar{g}(W_w - W_\ell + \frac{W_{w\ell} - W_{\ell\ell}}{1+r} + (.5\beta(W_{ww} - W_{w\ell}) - \beta C(E_w)) - (.5\beta(W_{\ell w} - W_{\ell\ell}) - \beta C(E_\ell))), \quad (2.31)$$

$$E_w = \bar{g}(W_{ww} - W_{w\ell}), \quad (2.32)$$

$$E_\ell = \bar{g}(W_{\ell w} - W_{\ell\ell}). \quad (2.33)$$

Clearly, first period effort depends on wages in the second period (for example, $\partial E_1/\partial W_{\ell\ell} \neq 0$). Sorting creates intertemporal links that the principal will be able to exploit to get a more profitable solution, compared to the nonsorting model. Thus (2.31) reveals that the return to winning the first period is not only the wage differential, but includes a sort of capital gain comprised of the difference (expected value (winner's contest) - expected value (loser's contest)).

Comparative statics for the above values can be easily derived. The values for the borrowing choices are straight forward, while those for equations (2.31) - (2.33) are more complex and require solving a 3x3 system (see Appendix 2.2 for details). The results are as follows (where $R = (1+r)^{-1}$).

$$\frac{\partial E_1}{\partial W_w} = \frac{\partial E_w}{\partial W_{ww}} = \frac{\partial E_\ell}{\partial W_{\ell w}} = g > 0,$$

$$
\begin{aligned}
\frac{\partial E_1}{\partial W_\ell} &= \frac{\partial E_w}{\partial W_{w\ell}} = \frac{\partial E_\ell}{\partial W_{\ell\ell}} = -\bar{g} < 0, \\
\frac{\partial E_1}{\partial W_{ww}} &= .5\beta\bar{g} - \beta\bar{g}^2 E_w \gtrless 0, \\
\frac{\partial E_1}{\partial W_{w\ell}} &= \bar{g}(R - .5\beta) + \beta\bar{g}^2 E_w > 0, \\
\frac{\partial E_1}{\partial W_{\ell w}} &= -.5\bar{g}\beta + \beta\bar{g}^2 E_\ell \gtrless 0, \\
\frac{\partial E_1}{\partial W_{\ell\ell}} &= \bar{g}(.5\beta - R) - \beta\bar{g}^2 E_\ell < 0, \\
\frac{\partial E_w}{\partial W_w} &= \frac{\partial E_w}{\partial W_\ell} = \frac{\partial E_w}{\partial W_{\ell w}} = \frac{\partial E_w}{\partial W_{\ell\ell}} = 0, \\
\frac{\partial E_\ell}{\partial W_w} &= \frac{\partial E_\ell}{\partial W_\ell} = \frac{\partial E_\ell}{\partial W_{w\ell}} = \frac{\partial E_\ell}{\partial W_{ww}} = 0.
\end{aligned}
$$

The primary differences from the earlier models are the intertemporal results, the fact comparative statics such as $\partial E_1/\partial W_{ww}$ are nonzero. These create the intertemporal links the principal can exploit in addition to the borrowing links. The above comparative statics are all logical. For example, as W_{ww} rises this tends to raise E_1 (more wages if you win), but also tends to lower E_1 (once you win, in the winners' contest effort rises ($\partial E_w/\partial W_{ww} > 0$), lowering the value of winning). The result is ambiguous. One does not always want to be vice-president if one has to work excessively hard.

Once again, the principal selects the wage structure to maximize the agents' utility, given the optimal choices of borrowing and effort, and subject to a zero-profit constraint:

$$
\begin{aligned}
\max \mathrm{L} \;=\; & \frac{W_\ell}{1+r} + \frac{W_{\ell\ell}}{(1+r)^2} + \beta(.5(W_w - W_\ell + \frac{W_{w\ell} - W_{\ell\ell}}{1+r}) - \\
& C(E_1)) + .5\beta^2(.5(W_{ww} - W_{w\ell}) - C(E_w) + .5(W_{\ell w} - W_{\ell\ell}) \\
& -C(E_\ell)) + \lambda(\frac{DE_1}{1+r} + \frac{D(E_w + E_\ell)}{2(1+r)^2} - \frac{W_w + W_\ell}{2(1+r)} \\
& -\frac{W_{ww} + W_{w\ell} + W_{\ell w} + W_{\ell\ell}}{4(1+r)^2}).
\end{aligned}
$$

Proposition 5 *a) The wage structure is nonunique.*

b) The contracts that do solve the above problem all involve the following values.

$$E_1 = \frac{D(1+\rho)}{1+r} + \frac{r-\rho}{2\bar{g}(1+r)}. \tag{2.34}$$

$$E_w = \frac{D(1+\rho)}{1+r} + \frac{r-\rho}{2\bar{g}(1+r)}. \tag{2.35}$$

$$E_\ell = \frac{D(1+\rho)^2}{(1+r)(1+2r-\rho)} + \frac{(r-\rho)(3+2r+\rho)}{2\bar{g}(1+r)(1+2r-\rho)}. \tag{2.36}$$

where

$$E_\ell - E_w = \frac{(\rho-r)(1+\rho)}{(1+r)(1+2r-\rho)}(2D-\frac{1}{g}) \gtrless 0.$$

$$\frac{\partial E_1}{\partial D} = \frac{\partial E_w}{\partial D} > 0, \frac{\partial E_1}{\partial \bar{g}} = \frac{\partial E_w}{\partial \bar{g}} > 0.$$

$$\frac{\partial E_1}{\partial \rho} = \frac{\partial E_w}{\partial \rho} \propto (2D\bar{g}-1) \gtrless 0, \frac{\partial E_1}{\partial r} = \frac{\partial E_w}{\partial r} \propto -(2D\bar{g}-1) \gtrless 0.$$

$$\frac{\partial E_\ell}{\partial D} > 0, \frac{\partial E_\ell}{\partial \bar{g}} > 0, \frac{\partial E_\ell}{\partial \rho} \propto (2D\bar{g}-1) \gtrless 0.$$

$$\frac{\partial E_\ell}{\partial r} \propto -(2D\bar{g}-1) \gtrless 0.$$

c) At the optimum, the principal desires the agent to work harder in the first period and in the second period winners' contest.

Proof: Analogous to Proposition 3.

As in the nonmemory model, the wage structure is non-unique. However, effort and the consumption values are unique, and can be solved for. It can be seen there are memory effects present - in general $E_\ell \neq E_w$ and hence wages will differ in the two second period contests, given equations (2.32) and (2.33). The principal chooses to set these wages at different levels, helping to create the capital gain mentioned above (expected value (winner) - expected value (loser)). The presence of these memory effects creates links between periods, an agent's compensation in the second period is a function of effort in both the second and the first period. These extra tools lead to a more profitable, pareto-superior outcome. Indeed, both the memory and the nonmemory contracts can be explicitly solved for their expected values to the agent. It can in turn be shown that EV(memory)$\geq$ EV(nonmemory).[14] Memory contracts pareto-dominate nonmemory contracts. (See Section 2.6.) The multiperiod, sorting framework helps to eradicate the moral hazard costs. However, these costs are not totally eradicated, as seen by the fact the principal would still like the agent to work harder at the optimum (see Lambert (1983) and Fama (1980) for a further discussion).

In these contracts, an examination of the consumption values in each state allows some insight into what motivates agents. Once again, define Z as consumption in a given state, such that:

$$\begin{aligned}
Z_w &= W_w - W_\ell + \frac{W_{w\ell} - W_{\ell\ell}}{1+r}, \\
Z_{ww} &= W_{ww} - W_{w\ell}, \\
Z_{\ell w} &= W_{\ell w} - W_{\ell\ell}, \\
Z_\ell &= Z_{w\ell} = Z_{\ell\ell} = 0, \\
Z_0 &= \frac{W_\ell}{1+r} + \frac{W_{\ell\ell}}{1+r}.
\end{aligned}$$

Recall equations (2.31) - (2.33) plus the zero-profit conditio

$$\begin{aligned}
\bar{y} &= .5(W_w + W_\ell) + \frac{(W_{ww} + W_{w\ell} + W_{\ell w} + W_{\ell\ell})}{4(1+r)} \\
&= DE_1 + \frac{D(E_w + E_\ell)}{2(1+r)}. \qquad (2.37)
\end{aligned}$$

It is possible to solve Z_0, Z_w, Z_{ww} and $Z_{\ell w}$ as functions of the known values E_1, E_w and E_ℓ by manipulation of (2.31) - (2.33) and (2.37) (see Appendix 2.3). This yields:

$$\begin{aligned}
Z_{ww} &= E_w/\bar{g};\ Z_{\ell w} = E_\ell/\bar{g}; \qquad (2.38) \\
Z_w &= \frac{E_1}{\bar{g}} - .5\beta\left(\frac{E_w}{\bar{g}} - \frac{E_\ell}{\bar{g}} + E_\ell^2 - E_w^2\right), \qquad (2.39)
\end{aligned}$$

[14]The inequalities are all strict except in the unlikely case where $D = 1/(2\bar{g})$.

$$Z_0 = \frac{DE_1}{1+r} + \frac{D(E_w + E_\ell)}{2(1+r)^2} - \frac{1}{2(1+r)}[\frac{E_1}{\bar{g}} - .5\beta(\frac{E_w - E_\ell}{\bar{g}} + E_\ell^2 - E_w^2)] - \frac{E_w + E_\ell}{4\bar{g}(1+r)^2}. \quad (2.40)$$

With this information, one can first of all solve for the equilibrium value of the memory contract to the agent (useful in Section 2.6):

$$\begin{aligned} V(\text{memory}) &= \frac{W_\ell}{1+r} + \frac{W_{\ell\ell}}{(1+r)^2} + \beta(.5(W_w - W_\ell + \frac{W_{w\ell} - W_{\ell\ell}}{1+r}) \\ &-C(E_1)) + .5\beta^2(.5(W_{ww} - W_{w\ell}) - C(E_w) + \\ &.5(W_{\ell w} - W_{\ell\ell}) - C(E_\ell)), \end{aligned}$$

which by substitution of the above Z values and simplification yields

$$V(\text{memory}) = \frac{(E_1)^2(3+2r)}{4(1+r)(1+\rho)} + \frac{(E_\ell)^2(1+2r-\rho)}{4(1+r)(1+\rho)^2}. \quad (2.41)$$

Interesting comparative statics include:[15]

$$\begin{aligned} \frac{\partial V}{\partial \bar{g}} &= \frac{\partial V}{\partial E_1}\frac{\partial E_1}{\partial \bar{g}} + \frac{\partial V}{\partial E_\ell}\frac{\partial E_\ell}{\partial \bar{g}} > 0, \\ \frac{\partial V}{\partial D} &= \frac{\partial V}{\partial E_1}\frac{\partial E_1}{\partial D} + \frac{\partial V}{\partial E_\ell}\frac{\partial E_\ell}{\partial D} > 0. \end{aligned}$$

The fact equilibrium utility at the optimum is increasing in the agents' effort levels is hardly surprising given Proposition 5, and therefore neither are the other results. Note that as 'chance' falls (g rises), equilibrium expected utility rises.

Second, comparisons of consumption effects across periods are possible. Define Z as mean consumption and Δ as the spread of consumption in a contest, such that:

$$\begin{aligned} Z_1 &= .5(Z_w + Z_\ell), \; Z_w = .5(Z_{ww} + Z_{\ell\ell}), \\ \Delta_1 &= Z_w - Z_\ell, \; \Delta_w = Z_{ww} - Z_{w\ell} \end{aligned}$$

[15] It should be noted that V(memory) has been written in the form $V(E(\quad)\quad)$ for simplicity of presentation and manipulation - the values of E are the optimal values from equations (2.31)-(2.33). Thus, the comparitive statics as written do not violate the envelope theorem.

Proposition 6 *It can be shown that:*

$$\Delta_1 \leq \Delta_w \text{ and } \tilde{Z}_1 \leq \tilde{Z}_w,$$

where the inequalities are strict unless $2D\bar{g} = 1$.

Proof: See Appendix 2.3.

Although other comparisons are ambiguous (see Appendix 2.3), the ones above clearly show memory effects in consumption values, effects that are testable. These are analogous to wage patterns in multiperiod models without borrowing (see Rogerson (1985a), Rosen(1986)) – with borrowing, it is consumption that motivates agents. These are endperiod effects, as in the last period there are no further periods to spread incentives across, so a higher spread is required to motivate effort. This in turn requires a higher mean to compensate the agent.[16]

Finally, consider again the introduction of mobility constraints into the principal's problem. There are two constraints on each of the winners and losers' contests. First, the constraints that second-period profits are nonpositive:

$$.5(W_{ww} + W_{w\ell}) \;\geq\; DE_w, \tag{2.42}$$
$$.5(W_{\ell w} + W_{\ell\ell}) \;\geq\; DE_\ell. \tag{2.43}$$

The other constraints are that second-period utility must be higher at the contests than at the reserve job described above:

$$\frac{W_{w\ell}}{1+r} + \beta(.5(W_{ww} - W_{w\ell}) - C(E_w)) \;\geq\; \frac{(D_r)^2(1+\rho)}{2(1+r)^2}, \tag{2.44}$$
$$\frac{W_{\ell\ell}}{1+r} + \beta(.5(W_{\ell w} - W_{\ell\ell}) - C(E_\ell)) \;\geq\; \frac{(D_r)^2(1+\rho)}{2(1+r)^2}. \tag{2.45}$$

(See Appendix 2.1 for a formal derivation of these constraints).

[16]See Rosen(1986) for a precise discussion of similar wage effects in a stylized sequential tournament model without borrowing.

Proposition 7 *a) The mobility constraints do not bind.*
b) $.5(W_{ww} + W_{w\ell}) > .5(W_w + W_\ell)$.

Proof: a) Analogous to the proof of Proposition 4. See Appendix 2.1 for details. b) Given (2.42) and (2.43) are strict inequalities, in combination with the two-period zero-profit constraint, this means $DE_1 > .5(W_w + W_\ell)$. Since $E_1 = E_w$, this in turn implies:

$$.5(W_{ww} + W_{w\ell}) > DE_w = DE_1 > .5(W_w + W_\ell).\square$$

The memory model thus predicts that mean wages rise over the length of the contest, with the principal making positive profits in the first period, and negative profits in the second. This is a familiar agency result (see Lazear (1979)), and hence the results of this model have generalized the settings in which these effects hold.

2.6 Comparisons of Restricted Contracts

In the above section, three types of models were developed, with the principal's choices constrained in different manners. One of the purposes for doing this was to be able to compare the optimality to the principal of the three types of contracts . It was possible to solve explicitly the equilibrium values of the repeated, the nonmemory and the memory contracts. It is thus possible to show the following:

Proposition 8 *The memory contracts pareto-dominate the nonmemory contracts, which in turn pareto-dominate the repeated contracts. Given firms always earn zero profits in equilibrium, the following holds:*

$$V(memory) \geq V(nonmemory) \geq V(repeated), \tag{2.46}$$

where strict inequality holds unless $2D\bar{g} = 1$.

Proof: See Appendix 2.4.

The intuition is straightforward. Moving from right to left in equation (2.46) gives the principal more 'tools' with which to fine-tune the compensation structure, and better motivate agents. Thus, in comparing the nonmemory contracts to the repeated contracts, the principal can use intertemporal effects, setting $W_{2w} \neq W_w$ and $W_{2\ell} \neq W_\ell$. The agents' borrowing links the periods together, and by setting wage values to differ across periods, the principal gets an extra intertemporal effect that boosts expected utility and/or expected profits.

In comparing memory to nonmemory models, a similar effect holds. When the principal carries out sorting, he creates the extra return to winning of being promoted to the winning contest. This provides extra incentive effects (memory effects) – one's compensation in the second period is a function of one's output in the first period. The principal takes advantage of these effects, in that losers' contests do differ then winners' ($W_{ww} \neq W_{w\ell}$, $E_w \neq E_\ell$, etc.). These memory contracts are pareto-superior, given agents are borrowers. This pareto-dominance does not occur if agents are not borrowers – see Appendix 2.5 for this result in a tournament setting, or Fellingham and Newman (1985) for the result in a general setting.

2.7 Summary

This chapter explored rank-order tournaments with restricted borrowing. This borrowing scheme led to an equilibrium in a tournament firm with positive effort and output, unlike in the model with fair-bet (perfect) capital markets. It was further shown that borrowing does make a crucial difference. Contrary to Fellingham and Newman's (1985) result, memory contracts dominate nonmemory contracts under risk-neutrality and borrowing. In conjunction with the results of Rogerson (1985a) and Lambert (1983) for risk-aversion and no capital market, this result leads to a very robust prediction – unless otherwise constrained, the principal will choose to offer memory contracts, where compensation in the second period depends on output in both the first and second periods.

In examining the specifics of the nonmemory and memory contracts, it was found that the wage structure will be nonunique, although both effort levels and consumption levels are unique. Thus, nonmemory contracts exhibited intertemporal wage and effort differences, while the memory contracts revealed in addition to these results other memory effects. For example, the spread and mean of consumption and the mean of wages are both higher in the winner's contest than in the first period contest, and first period winners and losers will face differing wage structures in the second period. These effects occur only with borrowing.

APPENDICES TO CHAPTER TWO

2.1 The Second Period Reserve Job

If an agent were to leave the tournament job and go to the reserve job, he would have some leftover debt (described below). Assume he can borrow enough at the reserve job such that his debt + $B_r \geq 0$, where B_r is borrowing at the reserve job. It is necessary only that this condition is met, and that effort and borrowing at the reserve job are independent of outstanding debt, previous effort levels, etc. Given this, the mobility condition amounts to the following: Debt + value of continuing $\geq$ Debt + value of moving, or value of continuing $\geq$ value of moving.

To construct a typical value of moving that meets these independence conditions, consider a reserve job with perfectly observable output, and a piece-rate D_r. Then the agents' choice in the second period is to select (B_r, E_r) to:

$$\begin{aligned}
\max \text{V(move)} &= B_r + \beta(D_r E_r - B_r(1+r) - C(E_r)) \\
\text{s.t.} &\quad D_r E_r \geq B_r(1+r). \\
\frac{\partial V_r}{\partial B_r} &= 1 - \frac{1+r}{1+\rho} - (1+r)\lambda = 0, \\
\frac{\partial V_r}{\partial E_r} &= \beta(D_r - C'(E_r)) + \lambda D_r = 0, \\
\frac{\partial V_r}{\partial \lambda} &= D_r E_r - B_r(1+r) = 0.
\end{aligned}$$

These first-order conditions can easily be solved for the borrowing and effort choices, and substitution of these values back into the agent's expected utility function and simplification yields:

$$\text{V(move)} = \frac{(D_r)^2(1+\rho)}{2(1+r)^2}.$$

In the nonmemory contract, both winners and losers face identical contracts and values if they stay, so substitution reveals the mobility constraint:

$$\frac{W_{2\ell}}{1+r} + \beta(.5(W_{2w} - W_{2\ell}) - C(E_2)) \geq \frac{(D_r)^2(1+\rho)}{2(1+r)^2}.$$

In the memory contract, winners and losers face different values of continuing, ($W_{2\ell} = W_{w\ell}$ and $W_{\ell\ell}$ respectively, etc.), so that substitution of these values yields the appropriate constraints.

2.2 The Memory Contract's Agent's Comparative Statics

In order to perform the comparative statics on the equilibrium values of E_1, E_w and E_ℓ, first completely differentiate the agent's effort choices:

$$\begin{aligned}
dE_1 &= \bar{g}(dW_w - dW_\ell) + .5\beta\bar{g}dW_{ww} + \bar{g}(R - .5\beta)dW_{w\ell} - .5\beta\bar{g}dW_{\ell w} \\
&\quad + \bar{g}(.5\beta - R)dW_{\ell\ell} - \beta\bar{g}E_w dE_w + \beta\bar{g}E_\ell dE_\ell, \\
dE_w &= \bar{g}(dW_{ww} - dW_{w\ell}), \\
dE_\ell &= \bar{g}(dW_{\ell w} - dW_{\ell\ell}),
\end{aligned}$$

where $d\bar{g}$, dR and $d\beta$ have been left out for simplicity. These equations can be arranged in matrix form:

$$\begin{pmatrix} 1 & \beta\bar{g}E_w & -\beta\bar{g}E_\ell \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dE_1 \\ dE_w \\ dE_\ell \end{pmatrix} =$$

$$\begin{pmatrix} \bar{g} & -\bar{g} & .5\beta\bar{g} & \bar{g}(R - .5\beta) & -.5\beta\bar{g} & \bar{g}(.5\beta - R) \\ 0 & 0 & \bar{g} & -\bar{g} & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{g} & -\bar{g} \end{pmatrix} \begin{pmatrix} dW_w \\ dW_\ell \\ dW_{ww} \\ dW_{w\ell} \\ dW_{\ell w} \\ dW_{\ell\ell} \end{pmatrix}$$

or $AE = BW$. Note that A is the negative of the Hessian (presented in this manner due to the way the problem has been formulated), and that the determinant of A is positive, the correct sign. Using Cramer's Rule, substituting the appropriate column of B into the appropriate column of A yields:

$$\frac{\partial E_1}{\partial W_w} = \frac{|A_1|}{|A|} = |A_1|$$

$$= \begin{vmatrix} \bar{g} & \bar{g}\beta E_w & -\beta\bar{g}E_\ell \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \bar{g},$$

and by similar manipulations all the other comparative statics in the text are obtained. Note that these are the comparative statics at the Nash equilibrium, so that $P_1 = P_w = P_\ell = .5$, and $\bar{g} = g(0)$ is assumed.

2.3 The Consumption Values in the Memory Contract

For the proof of proposition 6 of Chapter Two, recall the consumption values for each time-state as defined in the text:

$$\begin{aligned} Z_0 &= \frac{W_\ell}{1+r} + \frac{W_{\ell\ell}}{(1+r)^2}, \\ Z_w &= W_w - W_\ell + \frac{W_{w\ell} - W_{\ell\ell}}{1+r}, \\ Z_{ww} &= W_{ww} - W_{w\ell}, \\ Z_{\ell w} &= W_{\ell w} - W_{\ell\ell}, \\ Z_\ell &= Z_{w\ell} = Z_{\ell\ell} = 0. \end{aligned}$$

Recall the following equations:

$$\begin{aligned} E_1 = {} & \bar{g}(W_w - W_\ell + \frac{W_{w\ell} - W_{\ell\ell}}{1+r} + \beta(.5(W_{ww} - W_{w\ell}) \\ & -.5(W_{\ell w} - W_{\ell\ell}) + C(E_\ell) - C(E_w)), \end{aligned}$$

$$
\begin{aligned}
E_w &= \bar{g}(W_{ww} - W_{w\ell}).\\
E_\ell &= \bar{g}(W_{\ell w} - W_{\ell\ell}).\\
\bar{y} &= .5(W_w + W_\ell) + .25R(W_{ww} + W_{w\ell} + W_{\ell w} + W_{\ell\ell})\\
&= DE_1 + \frac{D}{2(1+r)}(E_w + E_\ell).
\end{aligned}
$$

The values for E_1; E_w and E_ℓ (and hence for $\bar{y}$) can be written as functions of the exogenous parameters:

$$
\begin{aligned}
E_1 &= E_w = \frac{D(1+\rho)}{1+r} + \frac{r-\rho}{2\bar{g}(1+r)}.\\
E_\ell &= \frac{D(1+\rho)^2}{(1+r)(1+2r-\rho)} + \frac{(r-\rho)(3+2r+\rho)}{2\bar{g}(1+r)(1+2r-\rho)}
\end{aligned}
$$

The next step is to manipulate the equations for E_1 - $\bar{y}$ to yield the equations for Z_0 - $Z_{\ell w}$ as a function of the exogenous parameters. (The resulting Z terms will be left as functions of the effort levels for ease of exposition).

First, inverting E_w and E_ℓ yields:

$$Z_{ww} = E_w/\bar{g};\ Z_{\ell w} = E_\ell/\bar{g}.$$

Next, inverting E_1 yields:

$$
\begin{aligned}
Z_w &+ \beta(.5(W_{ww} - W_{w\ell}) - .5(W_{\ell w} - W_{\ell\ell}) + C(E_\ell) - C(E_w))\\
&= E_1/\bar{g}.
\end{aligned}
$$

Substituting in Z_{ww} and $Z_{\ell w}$ and collecting terms yields:

$$Z_w = \frac{E_1}{\bar{g}} - .5\beta(\frac{E_w - E_\ell}{\bar{g}} + E_\ell^2 - E_w^2).$$

Finally, consider the term :

$$\frac{\bar{y}}{1+r} - \frac{Z_w}{2(1+r)} - \frac{(Z_{ww} + Z_{\ell w})}{4(1+r)^2}.$$

Substitution of $\bar{y}$, Z_w, Z_{ww}, and $Z_{\ell w}$ yields:

$$
\begin{aligned}
&\frac{W_w + W_\ell}{2(1+r)} + \frac{(W_{ww} + W_{w\ell} + W_{\ell w} + W_{\ell\ell})}{4(1+r)^2} - \frac{(W_w - W_\ell)}{2(1+r)}\\
&\quad - \frac{(W_{w\ell} - W_{\ell\ell})}{2(1+r)^2} - \frac{(W_{ww} - W_{w\ell} + W_{\ell w} - W_{\ell\ell})}{4(1+r)^2}.
\end{aligned}
$$

Collection of terms yields:

$$\frac{W_\ell}{1+r} + \frac{W_{\ell\ell}}{(1+r)^2} = Z_0.$$

Thus, it can be seen:

$$\begin{aligned} Z_0 &= \frac{\bar{y}}{1+r} - \frac{Z_w}{2(1+r)} - \frac{(Z_{ww} + Z_{\ell w})}{4(1+r)^2} \\ &= \frac{DE_1}{1+r} + \frac{D(E_w + E_\ell)}{2(1+r)^2} - \frac{.5}{1+r}[\frac{E_1}{\bar{g}} - .5\beta(\frac{E_w - E_\ell}{\bar{g}} \\ &\quad + E_\ell^2 - E_w^2)] - \frac{(E_w + E_\ell)}{4\bar{g}(1+r)^2}, \end{aligned}$$

upon substitution of the values for $\bar{y}$ and the Zs. Thus the consumption values can be solved for as functions of effort and hence of the exogenous parameters using the values for E_1, E_w and E_ℓ.

Proceeding to the complete set of comparisons of means and spreads of income in the different contests, recall the definitions of the spreads Δ_1 and Δ_w from the text, and define Δ_ℓ as $Z_{\ell w} - Z_{\ell\ell}$. Then, since $Z_\ell = Z_{\ell\ell} = Z_{w\ell} = 0$, therefore $\Delta_1 = Z_w$, etc. Recalling $E_1 = E_w$, substituting Z_{ww} and $Z_{\ell w}$ into Z_w yields:

$$\begin{aligned} \Delta_1 &= \Delta_w - .5\beta(E_w - E_\ell)(\frac{1}{\bar{g}} - (E_\ell + E_w)), \text{ or} \\ \Delta_1 &- \Delta_w = .5\beta(E_\ell - E_w)(\frac{1}{\bar{g}} - (E_\ell + E_w)). \end{aligned}$$

Substitution yields:

$$\begin{aligned} &\Delta_1 - \Delta_w \\ &= .5\beta(E_\ell - E_w)(\frac{1}{\bar{g}} - \frac{D(1+\rho)}{1+r} + \frac{\rho - r}{2\bar{g}(1+r)} - \frac{D(1+\rho)^2}{(1+r)(1+2r-\rho)} \\ &\quad + \frac{(\rho - r)(3 + 2r + \rho)}{2\bar{g}(1+r)(1+2r-\rho)}), \\ &= .5\beta(\frac{(\rho - r)(1+\rho)}{(1+r)(1+2r+\rho)}[2D - \frac{1}{\bar{g}}])(\frac{(2+2r)(1+2r-\rho)}{2\bar{g}(1+r)(1+2r-\rho)} \\ &\quad + \frac{(\rho - r)(4+4r)}{2\bar{g}(1+r)(1+2r-\rho)} - \frac{D(1+\rho)(1+2r-\rho+1+\rho)}{(1+r)(1+2r-\rho)}). \end{aligned}$$

$$= \frac{(\rho - r)}{2(1+r)(1+2r-\rho)}[2D - \frac{1}{\bar{g}}](\frac{1+2r-\rho+2\rho-2r}{\bar{g}(1+2r-\rho)} - \frac{2D(1+\rho)}{1+2r-\rho})$$

$$= \frac{(\rho - r)(1+\rho)}{2(1+r)(1+2r-\rho)^2}[2D - \frac{1}{\bar{g}}][\frac{1}{\bar{g}} - 2D], \text{ or}$$

$$\Delta_1 - \Delta_w = \frac{(r-\rho)(1+\rho)}{2(1+r)(1+2r-\rho)^2}(2D - \frac{1}{\bar{g}})^2 \leq 0.$$

Since $\bar{Z}_1 = .5\Delta_1$, $\bar{Z}_{ww} = .5\Delta_w$, this provides the proof for Proposition 6 of Chapter Two. □

Other contest comparisons yield ambiguous results. Recall that:

$$E_w = E_\ell + \frac{(\rho - r)(1+\rho)}{(1+r)(1+2r-\rho)}(\frac{1}{\bar{g}} - 2D).$$

Substitute this plus E_1 into Z_w:

$$\Delta_1 = \Delta_\ell + \frac{(\rho - r)(1+\rho)}{\bar{g}(1+r)(1+2r-\rho)}(\frac{1}{\bar{g}} - 2D) + (\Delta_1 - \Delta_w).$$

Substitution of $\Delta_1 - \Delta_w$ plus manipulation yields:

$$\Delta_1 - \Delta_\ell = \frac{(\rho - r)(1+\rho)}{(1+r)(1+2r-\rho)}(\frac{1}{\bar{g}} - 2D)(\frac{1}{\bar{g}} + (2D - \frac{1}{\bar{g}})(\frac{1}{2(1+2r-\rho)}))$$

$$= \frac{(\rho - r)(1+\rho)}{2\bar{g}(1+r)(1+2r-\rho)^2}(\frac{1}{\bar{g}} - 2D)(2D\bar{g} + 1 + 4r - 2\rho).$$

If :

a) $2D\bar{g} \geq 1$, then $\Delta_1 \leq \Delta_\ell$ given $0 < r < \rho < 1$, or if

b) $2D\bar{g} \leq 1$, and if $\rho \geq .5 + 2r + D\bar{g}$, then $\Delta_1 \leq \Delta_\ell$, or if

c) $2D\bar{g} \leq 1$, and if $\rho \leq .5 + 2r + D\bar{g}$, then $\Delta_1 \geq \Delta_\ell$.

Finally, it is simple to see that:

$$\Delta_w - \Delta_\ell = \frac{2(E_w - E_\ell)}{\bar{g}}$$

$$= \frac{(\rho - r)(1+\rho)}{\bar{g}(1+r)(1+2r-\rho)}(\frac{1}{\bar{g}} - 2D).$$

$$\Delta_w \begin{cases} \geq \Delta_\ell & \text{if } 2D\bar{g} \leq 1. \\ < \Delta_\ell & \text{if } 2D\bar{g} > 1. \end{cases}$$

Since $\bar{Z}_1 = .5\Delta_1$, etc., the mean comparisons follow trivially.

2.4 The Rareto-Dominance of Memory Contracts

To prove Proposition 8 of Chapter Two, recall from the text it was shown.

$$\begin{aligned} V(\text{repeated}) &= V(R) = \frac{(E_R)^2(2+\rho)}{2(1+\rho)^2}, \\ V(\text{nonmemory}) &= V(N) = \frac{(E_{1N})^2}{2(1+\rho)} + \frac{(E_{2N})^2}{2(1+\rho)^2}, \\ V(\text{memory}) &= V(M) = \frac{(E_{1M})^2(3+2r)}{4(1+r)(1+\rho)} + \frac{(E_{\ell M})^2(1+2r-\rho)}{4(1+r)(1+\rho)^2}. \end{aligned}$$

$$\begin{aligned} E_R &= \frac{D(1+\rho)^2(2+r)}{(2+\rho)(1+r)^2} + \frac{(r-\rho)(3+2r+2\rho+\rho r)}{2\bar{g}(1+r)^2(2+\rho)}, \\ E_{1N} &= E_{1M} = \frac{D(1+\rho)}{1+r} + \frac{r-\rho}{2\bar{g}(1+r)}, \\ E_{2N} &= \frac{D(1+\rho)^2}{(1+r)^2} + \frac{(r-\rho)(2+r+\rho)}{2\bar{g}(1+r)^2}, \\ E_{\ell M} &= \frac{D(1+\rho)^2}{(1+r)(1+2r-\rho)} + \frac{(r-\rho)(3+2r+\rho)}{2\bar{g}(1+r)(1+2r-\rho)}, \end{aligned}$$

where

$$\begin{aligned} E_R - E_{1M} &= \frac{(1+\rho)(\rho-r)}{(1+r)^2(2+\rho)}[D - \frac{1}{2\bar{g}}] \gtrless 0, \\ E_R - E_{\ell M} &= \frac{(1+\rho)^2(3+2r)(\rho-r)}{(1+r)^2(2+\rho)(1+2r-\rho)}[\frac{1}{2\bar{g}} - D] \gtrless 0, \\ E_R - E_{2N} &= \frac{(\rho-r)(1+\rho)^2}{(1+r)^2(2+\rho)}[\frac{1}{2\bar{g}} - D] \gtrless 0, \\ E_{2N} - E_{\ell M} &= \frac{(\rho-r)(1+\rho)^2}{(1+r)^2(1+2r-\rho)}[\frac{1}{2\bar{g}} - D] \gtrless 0. \end{aligned}$$

To prove pareto-dominance start with:

$$V(M) - V(R)$$

$$= \frac{(E_{1M})^2(3+2r)}{4(1+r)(1+\rho)} + \frac{(E_{\ell M})^2(1+2r-\rho)}{4(1+r)(1+\rho)^2} - \frac{(E_R)^2(2+\rho)}{2(1+\rho)^2}.$$

$$= \frac{(3+2r)}{4(1+r)(1+\rho)}\left(\frac{D(1+\rho)}{1+r} + \frac{r-\rho}{2\bar{g}(1+r)}\right)^2 + \frac{(1+2r-\rho)}{4(1+r)(1+\rho)^2}\times$$
$$\left(\frac{D(1+\rho)^2}{(1+r)(1+2r-\rho)} + \frac{(r-\rho)(3+2r+\rho)}{2\bar{g}(1+r)(1+2r-\rho)}\right)^2 - \frac{(2+\rho)}{2(1+\rho)^2}\times$$
$$\left(\frac{D(1+\rho)^2(2+r)}{(2+\rho)(1+r)^2} + \frac{(r-\rho)(3+2r+2\rho+\rho r)}{2\bar{g}(1+r)^2(2+\rho)}\right)^2.$$

$$= \frac{3+2r}{4(1+r)(1+\rho)}\left(\frac{D^2(1+\rho)^2}{(1+r)^2} + \frac{(r-\rho)^2}{4\bar{g}^2(1+r)^2} + \frac{D(1+\rho)(r-\rho)}{\bar{g}(1+r)^2}\right)$$
$$+\frac{(1+2r-\rho)}{4(1+r)(1+\rho)^2}\left(\frac{D^2(1+\rho)^4}{(1+r)^2(1+2r-\rho)^2} + \frac{(r-\rho)^2(3+2r+\rho)^2}{4\bar{g}^2(1+r)^2(1+2r-\rho)^2}\right.$$
$$\left.+\frac{D(1+\rho)^2(r-\rho)(3+2r+\rho)}{\bar{g}(1+r)^2(1+2r-\rho)^2}\right) - \frac{(2+\rho)}{2(1+\rho)^2}\left(\frac{D^2(1+\rho)^4(2+r)^2}{(2+\rho)^2(1+r)^4}\right.$$
$$+\frac{(r-\rho)^2(3+2r+2\rho+\rho r)^2}{4\bar{g}(1+r)^4(2+\rho)^2}$$
$$\left.+\frac{D(1+\rho)^2(2+r)(r-\rho)(3+2r+2\rho+\rho r)}{\bar{g}(2+\rho)^2(1+r)^4}\right).$$

Now collect terms for three variables: D^2, $(r-\rho)^2/\bar{g}^2$ and $D(r-\rho)/\bar{g}$, to get:

$$\frac{D^2(1+\rho)}{4(1+r)^4(2+\rho)(1+2r-\rho)}[(3+2r)(2+\rho)(1+2r-\rho)(1+r)$$
$$+ \quad (1+\rho)(1+r)(2+\rho) - (2+r)^2(1+2r-\rho)(2+2\rho)]$$
$$+ \quad \frac{(r-\rho)^2}{16\bar{g}^2(1+r)^4(1+\rho)^2(1+2r-\rho)(2+\rho)}[(3+2r)(1+\rho)(1+r)(1+$$
$$2r-\rho)(2+\rho) + (3+2r+\rho)^2(1+r)(2+\rho) - (3+$$
$$2r+2\rho+\rho r)^2(2+4r-2\rho)] + \frac{D(r-\rho)}{4\bar{g}(1+r)^4(2+\rho)(1+2r-\rho)}[(3+$$
$$2r)(1+r)(2+\rho)(1+2r-\rho) + (3+2r+\rho)(1+r)(2+\rho)$$
$$-(4+2r)(3+2r+2\rho+\rho r)(1+2r-\rho)].$$

First, consider the D^2 term. The term in the square brackets can be rewritten as:

$$(2+r)(1+2r-\rho)((2+\rho)(1+r) - 2(2+r)(1+\rho)) +$$

$$
\begin{aligned}
&(1+r)(2+\rho)((1+2r-\rho)(1+r)+1+\rho)\\
=\;& (2+r)(1+2r-\rho)(-2-3\rho-\rho r)+(1+r)(2+\rho)(2+3r+2r^2-\rho r)\\
=\;& -4-6\rho-2\rho r-8r-12\rho r-4\rho r^2+4\rho+6\rho^2+2\rho^2 r-2r-3\rho r\\
& -\rho r^2-4r^2-6r^2\rho-2\rho r^3+2\rho r+3\rho^2 r+\rho^2 r^2+4+6r\\
& +4r^2-2\rho r+2\rho+3\rho r+2r^2\rho-\rho^2 r+4r+6r^2+4r^3\\
& -2\rho r^2+2\rho r+3\rho r^2+2r^3\rho-\rho^2 r^2\\
=\;& -12\rho r+6\rho^2+6r^2+4r^3+4\rho^2 r-8r^2\rho\\
=\;& 6\rho(\rho-r)+6r(r-\rho)+4r^2(r-\rho)+4\rho r(\rho-r)\\
=\;& (\rho-r)^2(6+4r).
\end{aligned}
$$

Thus the first term is

$$\frac{D^2(1+\rho)(\rho-r)^2(6+4r)}{4(1+r)^4(2+\rho)(1+2r-\rho)}.$$

Second, consider the $(r-\rho)^2/\bar{g}^2$ term. The term in square brackets can be written as:

$$
\begin{aligned}
&(3+2r)(1+\rho)(1+r)(1+2r-\rho)(2+\rho)+(3+2r+\rho)^2(1+r)(2+\rho)\\
&-(2+4r-2\rho)((3+2r+\rho)^2+(\rho+\rho r)^2+2(\rho+\rho r)(3+2r+\rho))\\
=\;& (3+2r)(1+\rho)(1+r)(1+2r-\rho)(2+\rho)-(2+4r-2\rho)((\rho+\rho r)^2\\
& +(2\rho+2\rho r)(3+2r+\rho))+(3+2r+\rho)^2(3\rho-2r+\rho r)\\
=\;& (3+2r)(1+\rho)(1+r)(1+2r-\rho)(2+\rho)-2\rho^2(1+r)^2(1+2r-\rho)\\
& +(3+2r+\rho)((3+2r+\rho)(3\rho-2r+\rho r)-(2\rho+2\rho r)(2+4r-2\rho))\\
=\;& (1+2r-\rho)(1+r)((3+2r)(2+3\rho+\rho^2)-2\rho^2-2\rho^2 r)\\
& +(3+2r+\rho)(5\rho-6r-5\rho r+7\rho^2-4r^2-6\rho r^2+5\rho^2 r)\\
=\;& (1+3r-\rho+2r^2-\rho r)(6+9\rho+\rho^2+4r+6\rho r)+\\
& (3+2r+\rho)(5\rho-6r-5\rho r+7\rho^2-4r^2-6\rho r^2+5\rho^2 r)\\
=\;& 6+9\rho+\rho^2+4r+6\rho r+18r+27\rho r+3\rho^2 r+12r^2+18\rho r^2-6\rho
\end{aligned}
$$

$$-9\rho^2 - \rho^3 - 4\rho r - 6\rho^2 r + 12r^2 + 18r^2\rho + 2r^2\rho^2 + 8r^3 + 12\rho r^3$$
$$-6\rho r - 9\rho^2 r - \rho^3 r - 4r^2\rho - 6\rho^2 r^2 + 15\rho - 18r - 15\rho r + 21\rho^2$$
$$-12r^2 - 18\rho r^2 + 15\rho^2 r + 10\rho r - 12r^2 - 10\rho r^2 + 14\rho^2 r - 8r^3$$
$$-12\rho r^3 + 10\rho^2 r^2 + 5\rho^2 - 6\rho r - 5\rho^2 r + 7\rho^3 - 4r^2\rho - 6\rho^2 r^2$$
$$+5\rho^3 r$$
$$= 6 + 18\rho + 4r + 12\rho r + 18\rho^2 + 12\rho^2 r + 6\rho^3 + 4\rho^3 r$$
$$= 6(1 + 3\rho + 3\rho^2 + \rho^3) + 4r(1 + 3\rho + 3\rho^2 + \rho^3)$$
$$= (6 + 4r)(1 + \rho)^3.$$

Thus the second term is

$$\frac{(r-\rho)^2(6+4r)(1+\rho)}{16\bar{g}^2(1+r)^4(1+2r-\rho)(2+\rho)}.$$

Third, examine the $D(r-\rho)/\bar{g}$ term. The term in square brackets can be written as:

$$(3+2r)(1+r)(2+\rho)(1+2r-\rho) + (3+2r+\rho)(1+r)(2+\rho)$$
$$-(3+2r+2\rho+\rho r)(2+4r-\rho)(1+1+r)$$
$$= (3+2r)(1+r)(2+\rho)(1+2r-\rho+1) + \rho(1+r)(2+\rho)$$
$$-(3+2r)(1+r)(2+4r-2\rho) - (3+2r)(2+4r-2\rho) - (2\rho$$
$$+\rho r)(2+4r-2\rho)(2+r)$$
$$= (3+2r)(1+r)((2+\rho)(2+2r-\rho) - 2 - 4r + 2\rho) + \rho(1+r)(2+\rho)$$
$$-(3+2r)(2+4r-2\rho) - (2\rho+4\rho r-2\rho^2)(4+4r+r^2)$$
$$= (3+2r)(1+r)(4+4r-2\rho+2\rho+2\rho r-\rho^2-2.-4r+2\rho)$$
$$+\rho(2+\rho+2r+\rho r) - (3+2r)(2+4r-2\rho) - (2\rho+4\rho r$$
$$-2\rho^2)(4+4r+r^2)$$
$$= (3+5r+2r^2)(2+2\rho+2\rho r-\rho^2) + (2\rho+\rho^2+2\rho r+\rho^2 r)$$
$$-(3+2r)(2+4r-2\rho) - (2\rho+4\rho r-2\rho^2)(4+4r+r^2)$$

$$
\begin{aligned}
&= 6 + 6\rho + 6\rho r - 3\rho^2 + 10r + 10\rho r + 10\rho r^2 - 5r\rho^2 + 4r^2 + 4r^2\rho \\
&\quad +4\rho r^3 - 2\rho^2 r^2 + 2\rho + \rho^2 + 2\rho r + \rho^2 r - 6 - 12r + 6\rho - 4r - 8r^2 \\
&\quad +4\rho r - 8\rho - 16\rho r + 8\rho^2 - 2\rho r^2 - 4\rho r^3 + 2\rho^2 r^2 - 8\rho r - 16r^2\rho \\
&\quad +8\rho^2 r \\
&= 6\rho + 6\rho^2 - 2\rho r - 4r^2\rho + 4\rho^2 r - 6r - 4r^2 \\
&= 2(\rho(3 - r - 2r^2) + \rho^2(3 + 2r) - r(3 + 2r)) \\
&= (6 + 4r)(\rho - \rho r + \rho^2 - r) \\
&= (6 + 4r)(1 + \rho)(\rho - r).
\end{aligned}
$$

Thus the third term is

$$
\frac{D(r-\rho)(6+4r)(1+\rho)(\rho-r)}{4\bar{g}(1+r)^4(2+\rho)(1+2r-\rho)}.
$$

Thus, collecting the three terms and taking in account common factors:

$$
\begin{aligned}
V(M) - V(R) &= \frac{(6+4r)(1+\rho)(\rho-r)^2}{4(1+r)^4(2+\rho)(1+2r-\rho)}[D^2 + \frac{1}{4\bar{g}^2} - \frac{D}{\bar{g}}] \\
&= \frac{(6+4r)(1+\rho)(\rho-r)^2}{4(1+r)^4(2+\rho)(1+2r-\rho)}[D - \frac{1}{2\bar{g}}]^2 \geq 0.
\end{aligned}
$$

The next step is to calculate:

$$
\begin{aligned}
&V(N) - V(R) \\
&= \frac{(E_{1N})^2}{2(1+\rho)} + \frac{(E_{2N})^2}{2(1+\rho)^2} - \frac{(E_R)^2(2+\rho)}{2(1+\rho)^2} \\
&= \frac{(E_{1N}+E_R)}{2(1+\rho)}(E_{1N} - E_R) + \frac{(E_{2N}+E_R)}{2(1+\rho)^2}(E_{2N} - E_R).
\end{aligned}
$$

Recall $E_{1N} = E_{1M}$, and $E_R - E_{1N}$ and $E_R - E_{2N}$ have been calculated above.

Now calculate :

$$
\begin{aligned}
&E_{1N} + E_R \\
&= \frac{D(1+\rho)}{1+r} + \frac{r-\rho}{2\bar{g}(1+r)} + \frac{D(1+\rho)^2(3+r)}{(2+\rho)(1+r)^2} + \frac{(r-\rho)(3+2r+2\rho+\rho r)}{2\bar{g}(1+r)^2(2+\rho)}
\end{aligned}
$$

$$= \frac{D(1+\rho)((2+\rho)(1+r)+(1+\rho)(2+r))}{(1+r)^2(2+\rho)}$$
$$+\frac{(r-\rho)((1+r)(2+\rho)+3+2r+2\rho+\rho r)}{2\bar{g}(1+r)^2(2+\rho)}$$
$$= \frac{D(1+\rho)(4+3r+3\rho+2\rho r)}{(2+\rho)(1+r)^2}+\frac{(r-\rho)(5+4r+3\rho+2\rho r)}{2\bar{g}(1+r)^2(2+\rho)}.$$

$$E_{2N}+E_R$$
$$= \frac{D(1+\rho)^2}{(1+r)^2}+\frac{(r-\rho)(2+r+\rho)}{2\bar{g}(1+r)^2}+\frac{D(1+\rho)^2(2+r)}{(2+\rho)(1+r)^2}$$
$$+\frac{(r-\rho)(3+2r+2\rho+\rho r)}{2\bar{g}(1+r)^2(2+\rho)}$$
$$= \frac{D(1+\rho)^2(4+\rho+r)}{(2+\rho)(1+r)^2}+\frac{(r-\rho)((2+r+\rho)(2+\rho)+3+2r+2\rho+\rho r)}{2\bar{g}(1+r)^2(2+\rho)}$$
$$= \frac{D(1+\rho)^2(4+\rho+r)}{(2+\rho)(1+r)^2}+\frac{(r-\rho)(7+4r+6\rho+2\rho r+\rho^2)}{2\bar{g}(1+r)^2(2+\rho)}.$$

Thus, substituting in various terms yields:

$$V(N)-V(R)$$
$$= \left(\frac{D(1+\rho)(4+3r+3\rho+2\rho r)}{(2+\rho)(1+r)^2}+\frac{(r-\rho)(5+4r+3\rho+2\rho r)}{2\bar{g}(1+r)^2(2+\rho)}\right)\times$$
$$\frac{r-\rho}{2(1+r)^2(2+\rho)}(D-\frac{1}{2\bar{g}})+$$
$$\left(\frac{D(1+\rho)^2(4+\rho+r)}{(2+\rho)(1+r)^2}+\frac{(r-\rho)(7+4r+6\rho+2\rho r+\rho^2)}{2\bar{g}(1+r)^2(2+\rho)}\right)\times$$
$$\frac{(\rho-r)}{2(1+r)^2(2+\rho)}(D-\frac{1}{2\bar{g}})$$
$$= (D-\frac{1}{2\bar{g}})\frac{(\rho-r)}{2(1+r)^4(2+\rho)^2}[D(1+\rho)((1+\rho)(4+\rho+r)-$$
$$4-3r-3\rho-2\rho r)+\frac{(r-\rho)}{2\bar{g}}(7+4r+6\rho+2\rho r$$
$$+\rho^2-5-4r-3\rho-2\rho r)]$$
$$= (D-\frac{1}{2\bar{g}})\frac{(\rho-r)}{2(1+r)^4(2+\rho)^2}[D(1+\rho)(2\rho-2r+\rho^2-\rho r)$$
$$+\frac{(r-\rho)(2+3\rho+\rho^2)}{2\bar{g}}]$$
$$= (D-\frac{1}{2\bar{g}})\frac{(\rho-r)^2}{2(1+r)^4(2+\rho)^2}[D(1+\rho)(2+\rho)-\frac{(1+\rho)(2+\rho)}{2\bar{g}}]$$

$$= (D - \frac{1}{2\bar{g}})^2 \frac{(\rho - r)^2(1+\rho)}{2(1+r)^4(2+\rho)} \geq 0.$$

Finally, the last step is to solve:

$$\begin{aligned}
& V(M) - V(N) \\
= \; & V(M) - V(R) - [V(N) - V(R)] \\
= \; & \frac{(6+4r)(1+\rho)(\rho - r)^2}{4(1+r)^4(2+\rho)(1+2r-\rho)}[D - \frac{1}{2\bar{g}}]^2 \\
& - \frac{(1+\rho)(\rho - r)^2}{2(1+r)^4(2+\rho)}[D - \frac{1}{2\bar{g}}]^2 \\
= \; & [D - \frac{1}{2\bar{g}}]^2 \frac{(1+\rho)(\rho - r)^2[6 + 4r - 2(1+2r-\rho)]}{4(1+r)^4(2+\rho)(1+2r-\rho)}. \\
= \; & [D - \frac{1}{2\bar{g}}]^2 \frac{(1+\rho)(\rho - r)^2}{2(1+r)^4(1+2r-\rho)} \geq 0.
\end{aligned}$$

Thus $V(M) \geq V(N) \geq V(R)$, where the inequalities are strict except for the unlikely case of $2D\bar{g} = 1$. □

2.5 No Borrowing Rank-Order Tournaments

Consider a model with risk-neutral agents and principals, $r = \rho$ and the ability to offer two-period memory contracts. This is essentially an extension of Lazear and Rosen (1981) to two periods. Note that if $r = \rho$, it is easy to see agents will be indifferent to borrowing, so it is assumed $B_1 = B_w = B_\ell = 0$ for simplicity. Hence the agent's problem is to select E_1, E_w and E_ℓ to solve the following:

$$\begin{aligned}
\max V \; = \; & P_1(W_w + \beta(P_w W_{ww} + (1 - P_w)W_{w\ell} - C(E_w))) + \\
& (1 - P_1)(W_\ell + \beta(P_\ell W_{\ell w} + (1 - P_\ell)W_{\ell\ell} - C(E_\ell))) - C(E_1) \\
\frac{\partial V}{\partial E_1} \; \rightsquigarrow \; & E_1 = \bar{g}(W_w - W_\ell + \beta(.5(W_{ww} + W_{w\ell}) - C(E_w) - \\
& .5(W_{\ell w} + W_{\ell\ell}) + C(E_\ell))),
\end{aligned}$$

$$\frac{\partial V}{\partial E_w} \rightsquigarrow E_w = \bar{g}(W_{ww} - W_{w\ell}),$$
$$\frac{\partial V}{\partial E_\ell} \rightsquigarrow E_\ell = \bar{g}(W_{\ell w} - W_{\ell\ell}).$$

The comparative statics to this can be solved in a manner similar to those of the memory contract (see Appendix 2.2), and doing so yields:

$$\begin{aligned}
\frac{\partial E_1}{\partial W_w} &= \frac{\partial E_w}{\partial W_{ww}} = \frac{\partial E_\ell}{\partial W_{\ell w}} = \bar{g} > 0, \\
\frac{\partial E_1}{\partial W_\ell} &= \frac{\partial E_w}{\partial W_{w\ell}} = \frac{\partial E_\ell}{\partial W_{\ell\ell}} = -\bar{g} > 0, \\
\frac{\partial E_1}{\partial W_{ww}} &= .5\beta\bar{g} - \beta\bar{g}^2 E_w \gtrless 0, \\
\frac{\partial E_1}{\partial W_{w\ell}} &= .5\beta\bar{g} + \beta\bar{g}^2 E_w > 0, \\
\frac{\partial E_1}{\partial W_{\ell w}} &= -.5\beta\bar{g} + \beta\bar{g}^2 E_\ell \gtrless 0, \\
\frac{\partial E_1}{\partial W_{\ell\ell}} &= -.5\beta\bar{g} - \beta\bar{g}^2 E_\ell < 0,
\end{aligned}$$

with other wage comparative statics equal to zero.

Thus, the principal's problem is to select wages to solve:

$$\begin{aligned}
\max \mathrm{L} &= .5(W_w + W_\ell) + .25\beta(W_{ww} + W_{w\ell} + W_{\ell\ell} + W_{\ell w}) - C(E_1) \\
&\quad -.5\beta C(E_w) - .5\beta C(E_\ell) + \lambda(D[E_1 + \frac{E_w + E_\ell}{2(1+\rho)}] \\
&\quad -.5(W_w + W_\ell) - \frac{(W_{ww} + W_{w\ell} + W_{\ell w} + W_{\ell\ell})}{4(1+\rho)}).
\end{aligned}$$

The first-order conditions are:

$$\begin{aligned}
\frac{\partial L}{\partial W_w} &= .5 - C'(E_1)\frac{\partial E_1}{\partial W_w} + \lambda(D\frac{\partial E_1}{\partial W_w} - .5) \leq 0, \\
\frac{\partial L}{\partial W_\ell} &= .5 - C'(E_1)\frac{\partial E_1}{\partial W_\ell} + \lambda(D\frac{\partial E_1}{\partial W_\ell} - .5) \leq 0, \\
\frac{\partial L}{\partial W_{ww}} &= .25\beta - C'(E_1)\frac{\partial E_1}{\partial W_{ww}} - .5\beta C'(E_w)\frac{\partial E_w}{\partial W_{ww}} + \lambda(D\frac{\partial E_1}{\partial W_{ww}} + \\
&\quad \frac{D}{2(1+\rho)}\frac{\partial E_w}{\partial W_{ww}} - .25\beta) \leq 0,
\end{aligned}$$

$$\frac{\partial L}{\partial W_{w\ell}} = .25\beta - C'(E_1)\frac{\partial E_1}{\partial W_{w\ell}} - .5\beta C'(E_w)\frac{\partial E_w}{\partial W_{w\ell}} + \lambda(D\frac{\partial E_1}{\partial W_{w\ell}} + \frac{D}{2(1+\rho)}\frac{\partial E_w}{\partial W_{w\ell}} - .25\beta) \leq 0,$$

$$\frac{\partial L}{\partial W_{\ell w}} = .25\beta - C'(E_1)\frac{\partial E_1}{\partial W_{\ell w}} - .5\beta C'(E_\ell)\frac{\partial E_\ell}{\partial W_{\ell w}} + \lambda(D\frac{\partial E_1}{\partial W_{\ell w}} + \frac{D}{2(1+\rho)}\frac{\partial E_\ell}{\partial W_{\ell w}} - .25\beta) \leq 0,$$

$$\frac{\partial L}{\partial W_{\ell\ell}} = .25\beta - C'(E_1)\frac{\partial E_1}{\partial W_{\ell\ell}} - .5\beta C'(E_\ell)\frac{\partial E_\ell}{\partial W_{\ell\ell}} + \lambda(D\frac{\partial E_1}{\partial W_{\ell\ell}} + \frac{D}{2(1+\rho)}\frac{\partial E_\ell}{\partial W_{\ell\ell}} - .25\beta) \leq 0.$$

In a manner analogous to that used in Chapter Two, it can be shown that the wage structure is nonunique, but the effort levels are unique. Thus, solving $\partial L/\partial W_w$ and $\partial L/\partial W_\ell$ yields $E_1 = D$ and $\lambda = 1$. Substitution of these values into $\partial L/\partial W_{ww}$ and $\partial L/\partial W_{\ell w}$ yields $E_w = E_\ell = D$ also. Thus, there are no memory effects, the two-period model is merely a repeated one-period model (e.g. see Lazear and Rosen (1981), where $E = D$ in the one-period setting). Note that the principal does not want the agent to work harder at the optimum. For example, evaluate the following at the optimal values for E and λ:

$$\frac{\partial L}{\partial E_1} = -C'(E_1) + \lambda D = -D + D = 0.$$

As in Fellingham and Newman (1985), risk-neutrality and no borrowing imply no memory or intertemporal effects.

Chapter 3

Piece-Rates With Restricted Borrowing

Chapter Two explored a rank-order tournament with restricted borrowing. In that setting, it was shown that intertemporal, memory contracts would be chosen by the principal, a result driven by the presence of borrowing. This chapter develops a simple model of piece-rate contracts, also under restricted borrowing. This is done for two reasons. First, it allows for a test of the generality of the results developed under tournament contracts. Second, it allows for a comparison of tournaments and piece-rates under restricted borrowing, generating some empirical predictions on the relative prevalence of tournaments versus piece-rates.

The relative optimality of these types of contracts has been much analyzed in nondynamic, nonborrowing models (see Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), or MacDonald (1984)). The basic result is that rank-order tournaments can dominate only if the common portion of the random shock is large enough (as the contest automatically insures against the common shock), or as the number of workers becomes large enough (as rank tends to become a virtually perfect estimate of effort).[1] In a somewhat different model (with a nonlinear output function), Nalebuff and Stiglitz have emphasized the flexibility of the contest in the face of changes in the environmental variables. In this chapter, an extension of this comparison to a dynamic setting where agents have access to a capital market is undertaken.

The production function and the utility functions will be the same as those described in Chapter Two. However, now the principal is restricted to use a linear piece-rate of the form $I + Sq$, where I is a fixed payment independent of output, S is the piece-rate and q is individual output. In this setting, once again the introduction of fair-bet borrowing would create severe incentive problems. Therefore, as in Chapter Two, it is assumed agents can only borrow up to their sure income in the next period ($B \leq I/(1+r)$). While this is not necessarily the optimal form of borrowing, as in the tournament the resulting equilibrium does have positive output, unlike under fair-bet borrowing.

Piece-rate systems have a simpler structure than tournaments due to the fact they are individualistic – there is no strategic interaction between agents, reducing the complexity of the results. However, even in this simple structure intertemporal effects arise, confirming some of the results of Chapter Two (for example, two-period contracts pareto-dominate repeated one-period contracts). In addition, the relative optimality of piece-rate contracts compared to tournament contracts is considered.

3.1 Repeated One-Period Contracts

In this section, contracts will be restricted to be identical (zero-profit) contracts in each period, although once again agents will be free to borrow across two

[1] See Green and Stokey (1983) for details.

periods. Thus, each individual agent will receive an identical wage of $I + Sq_t$, where I is fixed income and S the piece rate. Agents can borrow fully against their sure income each period:

$$B_1 \leq \frac{I}{1+r} + \frac{I}{(1+r)^2}; \; B_2 \leq \frac{I}{1+r};$$

The agent selects (E_1, E_2, B_1, B_2) to maximize expected utility subject to the borrowing constraints:

$$\begin{aligned} L &= B_1 + \beta(I + SE_1 - B_1(1+r) + B_2 - C(E_1)) + \\ &\quad \beta^2(I + SE_2 - B_2(1+r) - C(E_2)) + \lambda_1(\frac{I(2+r)}{(1+r)^2} - B_1) \\ &\quad + \lambda_2(\frac{I}{1+r} - B_2). \end{aligned}$$

The f.o.c.'s are:

$$\frac{\partial L}{\partial B_1} = 1 - \frac{(1+r)}{1+\rho} - \lambda_1 = 0, \quad (3.1)$$

$$\frac{\partial L}{\partial B_2} = \beta(1 - \frac{(1+r)}{1+\rho}) - \lambda_2 = 0, \quad (3.2)$$

$$\frac{\partial L}{\partial E_1} = \beta(S - C'(E_1)) = 0, \quad (3.3)$$

$$\frac{\partial L}{\partial E_2} = \beta^2(S - C'(E_2)) = 0. \quad (3.4)$$

Assuming a unique solution holds, these can be solved for the optimal values of borrowing and effort:[2]

$$B_1 = \frac{I(2+r)}{(1+r)^2}; \; B_2 = \frac{I}{1+r}, \quad (3.5)$$

$$E_1 = E_2 = E = S. \quad (3.6)$$

Once again, as in the repeated tournament contracts, there are no links between periods, and the comparative statics are simple. However, note that unlike in the tournament, effort is not a function of the variance of output.

[2]Note in the individualistic set-up there is no problem with the global incentive condition of the tournament. Also, $E > 0$ as long as $S > 0$.

The principal selects I and S to solve:

$$\max L = \frac{I(2+r)}{(1+r)^2} + \frac{SE(2+\rho)}{(1+\rho)^2} - \frac{E^2(2+\rho)}{2(1+\rho)^2} + \lambda\frac{(2+r)}{(1+r)^2}(DE - I - SE),$$

where $E = S$ and (3.5) has been substituted in.[3] The first order conditions are:

$$\frac{\partial L}{\partial I} = \frac{2+r}{(1+r)^2} - \lambda\frac{(2+r)}{(1+r)^2} = 0, \quad (3.7)$$

$$\frac{\partial L}{\partial S} = \frac{2+\rho}{(1+\rho)^2}(E + S\frac{\partial E}{\partial S} - E\frac{\partial E}{\partial S}) + \lambda\frac{(2+r)}{(1+r)^2}(D\frac{\partial E}{\partial S} \quad (3.8)$$

$$-E - S\frac{\partial E}{\partial S}) = 0. \quad (3.9)$$

$$\frac{\partial L}{\partial \lambda} = \frac{(2+r)}{(1+r)^2}(DE - I - SE) = 0. \quad (3.10)$$

Proposition 1 *a) The solution to the principal's problem yields:*

$$S = E = \frac{D(2+r)(1+\rho)^2}{2(2+r)(1+\rho)^2 - (2+\rho)(1+r)^2},$$

$$I = \frac{D^2(2+r)(1+\rho)^2(3+2\rho+2r+\rho r)(\rho - r)}{[2(2+r)(1+\rho)^2 - (2+\rho)(1+r)^2]^2}.$$

Some interesting comparative statics include:

$$\frac{\partial E}{\partial D} > 0,\ \frac{\partial E}{\partial \bar{g}} = 0,\ \frac{\partial E}{\partial \rho} < 0,\ \frac{\partial E}{\partial r} > 0;$$

$$\frac{\partial I}{\partial D} > 0,\ \frac{\partial I}{\partial \bar{g}} = 0,\ \frac{\partial I}{\partial \rho} > 0,\ \frac{\partial I}{\partial r} < 0.$$

b) The principal desires the agent to work harder at the optimum.

<u>**Proof:**</u> a) Solve (3.7) - (3.10).

b) Evaluate the following at the optimal values for S and λ:

$$\frac{\partial L}{\partial E} = \frac{S(2+\rho)}{(1+\rho)^2} - \frac{E(2+\rho)}{(1+\rho)^2} + \lambda\frac{(2+r)}{(1+r)^2}(D-S)$$

$$\propto \frac{D(3+2\rho+2r+\rho r)(\rho - r)}{2(2+r)(1+\rho)^2 - (2+\rho)(1+r)^2} > 0.\square$$

[3]Once again, the one-period contract implies that mobility constraints are not relevant, and the methodology used precludes any worries about the 'first-order approach'.

Several aspects of this solution are worth noting. As in Lazear and Rosen's (1981) risk-averse case, the principal provides positive insurance ($I > 0$), even though in this case the agent is risk-neutral. This is due to the role of the borrowing, which acts much like risk-aversion in creating a consumption distortion. So, although normally risk-neutrality implies $I = 0$ (see Lazear and Rosen (1981)), here the fact borrowing is contingent on I leads to $I > 0$. However, since it is positive, the zero-profit condition requires the marginal return to the agent (S) to be less than his marginal productivity (D). There is a tradeoff between the return to borrowing (lowering S) and the return to incentives (raising S). As a result the correct marginal incentives are not set, and E is too low at the second-best optimum.

The comparative statics with respect to D are straightforward. An interesting result is that the probability density function of the random shocks ($\bar{g}$) has no impact on any variable. This is identical to the result for the risk-neutral case in Lazear and Rosen (1981) (where $I = 0$), but unlike their risk-averse case (where $I > 0$). In contrast, recall that the rank-order tournament model of Chapter Two found effort a function of $\bar{g}$ – this difference will be crucial below.

The other set of interesting results are those coming from changes in ρ or r. As ρ rises, so does the return on borrowing (reflected in its shadow price in equations (3.1) and (3.2)). This means that on the margin the principal raises I and hence the amount of borrowing. However, the zero-profit condition then requires the value of S to fall, and with it the level of E. On the other hand, if r rises the cost of borrowing rises, and an analogous, but reverse version of the above line of reasoning works.

Finally, from the values of S, I, and E the equilibrium value of the contest to the agent is:

$$\text{V(repeated)} = \frac{I(2+r)}{(1+r)^2} + \frac{SE(2+\rho)}{(1+\rho)^2} - \frac{E^2(2+\rho)}{2(1+\rho)^2} = \frac{DE(2+r)}{2(1+r)^2}.$$

This will be useful in Proposition 4 below.

In this section, the principal was restricted to offer identical contracts in all periods. He was therefore unable to create interperiod effects in wages. In Chapter Two it was shown with rank-order tournaments that the principal would desire to create these interperiod effects. To test the generality of these results, the next section will examine allowing the principal to offer different wages in different periods, within a piece-rate structure.

3.2 Two-Period Contracts

In this section the same production structure, utility functions and capital markets are assumed. However, in period one agents now receive the payment $I_1 + S_1 q_1$, and in period two they receive $I_2 + S_2 q_2$, where I is fixed income and S the piece rate.[4] Agents can borrow fully against sure income at each stage, such that:

$$B_1 \leq \frac{I_1}{1+r} + \frac{I_2}{(1+r)^2};\ B_2 \leq \frac{I_2}{1+r}.$$

The agent selects effort and borrowing to solve the following problem:

$$\begin{aligned}\max L &= B_1 + \beta(I_1 + S_1 E_1 - B_1(1+r) + B_2 - C(E_1)) \\ &\quad + \beta^2(I_2 + S_2 E_2 - B_2(1+r) - C(E_2)) \\ &\quad + \lambda_1\left(\frac{I_1}{1+r} + \frac{I_2}{(1+r)^2} - B_1\right) + \lambda_2\left(\frac{I_2}{1+r} - B_2\right).\end{aligned}$$

The first-order conditions are similar to those from the repeated case above. Assuming a unique solution, solving the agent's problem yields:

$$B_1 = \frac{I_1}{1+r} + \frac{I_2}{(1+r)^2};\ B_2 = \frac{I_2}{1+r} \quad (3.11)$$

$$E_1 = S_1;\ E_2 = S_2. \quad (3.12)$$

[4]Note this implies the firm is signing legally binding two-period contracts.

These results are quite standard, and require no discussion.

The principal now selects I and S to solve the following (where (3.11) has been substituted in, and E_1, E_2 are determined by (3.12)):

$$\begin{aligned}\max \mathrm{L} &= \frac{I_1}{1+r} + \frac{I_2}{(1+r)^2} + \beta(S_1E_1 - C(E_1)) + \beta^2(S_2E_2 - C(E_2)) \\ &+ \frac{\lambda}{1+r}(DE_1 + \frac{DE_2}{1+r} - I_1 - \frac{I_2}{1+r} - S_1E_1 - \frac{S_2E_2)}{1+r}.\end{aligned}$$

(Mobility constraints will be discussed below.) The first-order conditions are:

$$\frac{\partial L}{\partial I_1} = \frac{1}{1+r} - \frac{\lambda}{1+r} = 0, \tag{3.13}$$

$$\frac{\partial L}{\partial I_2} = \frac{1}{(1+r)^2} - \frac{\lambda}{(1+r)^2} = 0, \tag{3.14}$$

$$\frac{\partial L}{\partial S_1} = \beta(E_1 + S_1\frac{\partial E_1}{\partial S_1} - C'(E_1)\frac{\partial E_1}{\partial S_1}) + \frac{\lambda}{1+r}(D\frac{\partial E_1}{\partial S_1} - E_1 \tag{3.15}$$

$$-S_1\frac{\partial E_1}{\partial S_1}) = 0, \tag{3.16}$$

$$\frac{\partial L}{\partial S_2} = \beta^2(E_2 + S_2\frac{\partial E_2}{\partial S_2} - C'(E_2)\frac{\partial E_2}{\partial S_2}) + \frac{\lambda}{(1+r)^2}(D\frac{\partial E_2}{\partial S_2} \tag{3.17}$$

$$-E_2 - S_2\frac{\partial E_2}{\partial S_2}) = 0, \tag{3.18}$$

$$\frac{\partial L}{\partial \lambda} = DE_1 + \frac{DE_2}{1+r} - I_1 - \frac{I_2}{1+r} - S_1E_1 - \frac{S_2E_2}{1+r} = 0. \tag{3.19}$$

Proposition 2 *a) The values of I_1 and I_2 are nonunique.*

b) The solution to the principal's problem yields:

$$S_1 = E_1 = \frac{D(1+\rho)}{(1+2\rho-r)}, \tag{3.20}$$

$$S_2 = E_2 = \frac{D(1+\rho)^2}{(1+\rho)^2 + (\rho-r)(2+\rho+r)}, \tag{3.21}$$

$$I_1 + \frac{I_2}{1+r} = D^2(1+\rho)(\rho-r)\left[\frac{1}{(1+2\rho-r)^2} + \right. \tag{3.22}$$

$$\left. \frac{(1+\rho)(2+\rho+r)}{(1+r)[(1+\rho)^2+(\rho-r)(2+\rho+r)]^2}\right]. \tag{3.23}$$

The following comparative statics can be demonstrated:

$$\frac{\partial E_1}{\partial D} > 0, \frac{\partial E_1}{\partial \rho} < 0, \frac{\partial E_1}{\partial r} > 0, \frac{\partial E_1}{\partial \bar{g}} = 0;$$

$$\frac{\partial E_2}{\partial D} > 0, \frac{\partial E_2}{\partial \rho} < 0, \frac{\partial E_2}{\partial r} > 0, \frac{\partial E_2}{\partial \bar{g}} = 0;$$

$$\frac{\partial(I_1 + I_2/(1+r))}{\partial D} > 0, \frac{\partial(I_1 + I_2/(1+r))}{\partial \rho} > 0,$$

$$\frac{\partial(I_1 + I_2/(1+r))}{\partial r} \lessgtr 0, \frac{\partial(I_1 + I_2/(1+r))}{\partial \bar{g}} = 0.$$

Further,

$$E_1 - E_2 = \frac{D(1+\rho)(\rho - r)(1+r)}{(1+2\rho - r)[(1+\rho)^2 + (\rho - r)(2+\rho+r)]} > 0.$$

c) At the optimum, the principal desires the agent to work harder.

Proof: a) (3.13) and (3.14) are dependent. Only the joint value $I_1 + I_2/(1+r)$ can be solved for.

b) Solve (3.14) - (3.19), and take derivatives.

c) Evaluate the following at the optimum:

$$\begin{aligned} \frac{\partial L}{\partial E_1} &= \beta(S_1 - C'(E_1)) + R\lambda(D - S_1) \\ &= \frac{D(\rho - r)}{(1+r)(1+2\rho - r)} > 0, \\ \frac{\partial L}{\partial E_2} &= \beta^2(S_2 - C'(E_2)) + R^2\lambda(D - S_2) \\ &= \frac{D(\rho - r)(2+\rho+r)}{(1+r)^2[(1+\rho)^2 + (\rho - r)(2+\rho+r)]} > 0. \square \end{aligned}$$

As in the tournament contracts, in a multiperiod setting there is non-uniqueness - the values of the variables I_1 and I_2 are not independent, one can only solve for the value of $I_1 + I_2/(1+r)$. This is due to the borrowing - the fixed income components are fully borrowed, and thus the individual amounts are irrelevant. For example, if I_1 is lowered by 1 dollar and I_2 raised by $(1+r)$ dollars, then profits are unchanged, and the extra I_2 is just borrowed forward, and hence utility is unchanged. Only the total of $I_1 + I_2/(1+r)$ matters, not the components. However, although the breakdown of $I_1 + I_2/(1+r)$ is irrelevant, the fact the total is positive means that once again the 'insurance effect' discussed in Section 3.1 exists.

The comparative statics on equations (3.20) - (3.23) are analogous to those in Section 3.1. It can be noted that $I_1 + I_2/(1+r)$ is independent of the variance of output. This in turn means the borrowing constraint is independent of the variance, and therefore so are the piece-rates and so are effort levels.

Inspection of equations (3.20) and (3.21) reveal there is an intertemporal effect - $E_2 < E_1$. The model is much like the non-memory tournament developed earlier. The principal selects the intertemporal structure because it is pareto-superior given the presence of borrowing.

Finally, from (3.20) - (3.23) the equilibrium value of the two-period contract to the agent is:

$$\begin{aligned} \text{V(two-period)} &= RI_1 + R^2 I_2 + \beta(S_1 E_1 - C(E_1)) \\ &\quad + \beta^2(S_2 E_2 - C(E_2)) \\ &= .5RDE_1 + .5R^2 DE_2. \end{aligned}$$

In the above analysis it was implicitly assumed the agent was bound to the firm for the full two periods. To examine the case where he may leave for a reserve job, similar to the one from Chapter Two (see Appendix 2.1), add two extra constraints to the principal's problem:

$$I_2 + S_2 E_2 \geq DE_2, \tag{3.24}$$

$$RI_2 + \beta(S_2 E_2 - C(E_2)) \geq .5R^2 (D_r)^2 (1+\rho). \tag{3.25}$$

The first is that second-period profits must be non-positive (so that other firms will not bid away the workers), while the second is that expected second-period utility must be higher if one stays, as opposed to moving to the reserve job. Condition (3.25) can be derived in a manner analogous to the analysis of Appendix 2.1.

Proposition 3 *The mobility constraints do not bind.*

Proof: Let $\lambda_1, \lambda_2, \lambda_3 \geq 0$ be the constraints on the two-period zero-profit constraint, (3.24) and (3.25). Then the following first-order conditions exist (among others):

$$\frac{\partial L}{\partial I_1} = R - R\lambda_1 \leq 0, \tag{3.26}$$

$$\frac{\partial L}{\partial I_2} = R^2 - R^2\lambda_1 + \lambda_2 + R\lambda_3 \leq 0. \tag{3.27}$$

The first equation yields $\lambda_1 = 1$, and hence the second equation yields $\lambda_2 + \lambda_3/(1+r) \leq 0$, which given $\lambda_2, \lambda_3 \geq 0$ implies $\lambda_2 = \lambda_3 = 0$.□

Thus the previous results go through unchanged. As with the tournament model, the principal's contracts are designed to 'bind' workers to the firm by having second-period profits negative.

In this section the principal was allowed to offer two-period contracts, unlike in the previous section. Here, it was shown that there would still be under-effort in the resulting equilibrium. Despite this, firms did choose to offer these two-period contracts as they were superior to repeated one-period contracts. The following addresses this point in more detail.

From the explicitly solved equilibrium utility values of the contracts, it follows that:

Proposition 4 *The two-period contracts pareto-dominate the repeated contracts, or V(two-period) > V(repeated).*

Proof: See Appendix 3.1.

As in the rank-order tournament case, here the principal prefers the two-period contract over the repeated one-period contracts. The more complex two-period contracts give the principal an extra tool to use to better motivate agents.

Thus the principal selects $S_1 \neq S_2$ which in turn lead to $E_1 \neq E_2$, an intertemporal effect in wages and effort. This extra intertemporal effect boosts expected utility and/or expected profits. Note once again that this intertemporal effect does not occur with risk-neutral agents without borrowing - see Appendix 3.2 for details of this in a piece-rate setting.

The addition of a linear memory effect has also been examined. Here, in period two the agent's payment is $I_2 + S_2 q_2 + S_m q_1$. It turns out this has no impact in equilibrium - given the pure linearity of the piece-rate, there is no advantage to adding this memory variable. Since the value of $S_m q_1$ is known at the end of period one, agents borrow against it, and make its value irrelevant in much the same way they make the breakdown of $I_1 + I_2/(1+r)$ irrelevant. To get memory effects, a more complicated nonlinear piece-rate would be needed.

3.3 A Comparison of Piece-Rates and Tournaments

As mentioned above, there is a history of comparing tournaments to piece-rates, starting with Lazear and Rosen's (1981) seminal article. This section will extend this comparison to a dynamic setting, with borrowing. The results will reveal that some of the earlier results seem quite general, while other new results are generated.

Let starred values represent the values from the rank-order tournament with sorting, and unstarred variables represent the values from the two-period piece-rate system. For the former these values are:

$$\begin{aligned} E_1^* &= E_w^* = RD(1+\rho) + \frac{r-\rho}{2\bar{g}(1+r)}, \\ E_\ell^* &= \frac{D(1+\rho)^2}{(1+r)(1+2r-\rho)} + \frac{(r-\rho)(3+2r+\rho)}{2\bar{g}(1+r)(1+2r-\rho)}, \end{aligned}$$

$$V^* = \frac{(E_w^*)^2(3+2r)}{4(1+r)(1+\rho)} + \frac{(E_\ell^*)^2(1+2r-\rho)}{4(1+r)(1+\rho)^2},$$

while the piece-rate values are:

$$E_1 = \frac{D(1+\rho)}{1+2\rho-r}, \tag{3.28}$$

$$E_2 = \frac{D(1+\rho)^2}{(1+\rho)^2+(\rho-r)(2+\rho+r)}, \tag{3.29}$$

$$V = .5RDE_1 + .5R^2DE_2. \tag{3.30}$$

The values for expected utility have been left as functions of the effort values for notational simplicity.

In addition to the comparative statics on the E^*s and Es derived earlier on, note:

$$\frac{\partial V^*}{\partial \bar{g}} = \frac{\partial V^*}{\partial E_w^*}\frac{\partial E_w^*}{\partial \bar{g}} + \frac{\partial V^*}{\partial E_\ell^*}\frac{\partial E_\ell^*}{\partial \bar{g}} > 0,$$

$$\frac{\partial V^*}{\partial D} = \frac{(3+2r)2(1+\rho)E_w^*}{4(1+r)(1+\rho)(1+r)} + \frac{2(1+2r-\rho)(1+\rho)^2E_\ell^*}{4(1+r)(1+\rho)^2(1+r)(1+2r-\rho)}$$

$$= .5R^2(3+2r)E_w^* + .5R^2E_\ell^* > 0,$$

$$\frac{\partial V}{\partial D} = .5RE_1 + .5R^2E_2 + \frac{D(1+\rho)}{2(1+r)(1+2\rho-r)}$$

$$+\frac{D(1+\rho)^2}{2(1+r)^2[(1+\rho)^2+(\rho-r)(2+\rho+r)]}$$

$$= RE_1 + R^2E_2 > 0,$$

$$\frac{\partial V}{\partial \bar{g}} = \frac{\partial V}{\partial E_1}\frac{\partial E_1}{\partial \bar{g}} + \frac{\partial V}{\partial E_2}\frac{\partial E_2}{\partial \bar{g}} = 0,$$

$$\frac{\partial V}{\partial \rho} = \frac{\partial V}{\partial E_1}\frac{\partial E_1}{\partial \rho} + \frac{\partial V}{\partial E_2}\frac{\partial E_2}{\partial \rho} < 0,$$

$$\frac{\partial V}{\partial r} = .5R^2[(1+r)D(\partial E_1/\partial r) - DE_1]$$

$$+.5R^4[D(1+r)^2(\partial E_2/\partial r) - 2D(1+r)E_2],$$

$$= \frac{D^2(1+\rho)(r-\rho)}{(1+r)^2(1+2\rho-r)^2}$$

$$+\frac{2D^2(1+\rho)^2(r-\rho)(2+r+\rho)}{(1+r)^3[(1+\rho)^2+(\rho-r)(2+r+\rho)]^2} < 0,$$

while $\partial V^*/\partial r$ and $\partial V^*/\partial \rho$ are ambiguous.

Direct comparisons of V^* to V, or of the E^*s to the Es gives results depending on the size of the exogenous variables (D, $\bar{g}$, ρ, r). However, since both V and V^* are increasing in effort, any change that raises the value of the E^*s relative to the value of the Es, raises the value of V^* relative to the value of V.

Some simulation results in this area are presented below, but some of the comparative statics can be discussed without simulation. It is interesting to note that the variance of the individual shock (σ^2) plays a role only in the tournament. If σ^2 rises ($\bar{g}$ falls), to maintain the effort level, the wage spread must rise. However, given the zero-profit condition, this means W_ℓ, $W_{\ell\ell}$ must fall. However, this effort to maintain incentives, and hence expected utility, conflicts with the ability of agents to borrow, which is contingent on W_ℓ and $W_{\ell\ell}$. Thus, as σ^2 rises, the wage spread cannot rise fully (W_ℓ, etc. must fall too much), therefore the E^*s must fall as σ^2 rises, and this in turn means $\partial V^*/\partial\sigma^2 < 0$. However, in the piece-rate, a change in σ^2 has no impact on effort choice because effort and borrowing are independent of σ^2, no conflict is created, and hence $\partial V/\partial\sigma^2 = (\partial V/\partial E)(\partial E/\partial\sigma^2) = 0$. Thus, the lower σ^2, the better the rank-order tournament is relative to the piece-rate system. This is a familiar result from Lazear and Rosen (1981) and Green and Stokey (1983), but under very different circumstances.[5]

[5] The current analysis has left out a common shock. It is easy to show it would be irrelevant to the tournament/piece-rate comparison. Thus, on the one hand, the piece-rate analysis is independent of the distribution of any shock, common or individual. On the other hand, in the tournament, if there was a common shock, the output equation would now look like this:

$$q_j = E_j + \eta_j + \theta,$$

where θ is the common shock. Substitution of this into a typical wage equation still yields

$$W_w > W_\ell \text{ if } E_{j1} + \eta_{j1} > E_{i1} + \eta_{i1},$$

as the common shock is netted out, and hence is irrelevant.

As noted earlier, as the value of output rises (D), so do effort levels and therefore expected utility levels in both types of contracts. However, it is easy to show the following:

$$\begin{aligned}
\frac{\partial E_w^*}{\partial D} &= \frac{1+\rho}{1+r} > \frac{1+\rho}{1+2\rho-r} = \frac{\partial E_1}{\partial D}, \\
\frac{\partial E_\ell^*}{\partial D} &= \frac{(1+\rho)^2}{(1+r)(1+2r-\rho)} > \frac{(1+\rho)^2}{(1+\rho)^2+(\rho-r)(2+\rho+r)} = \frac{\partial E_2}{\partial D}, \\
\frac{\partial V^*}{\partial D} &- \frac{\partial V}{\partial D} = .5R^2(3+2r)E_w^* + .5R^2E_\ell^* - RE_1 - R^2E_2 \\
&= .5R^2(E_w^* - E_2) + R(E_w^* - E_1) + .5R^2(E_\ell^* - E_2) \gtrless 0.
\end{aligned}$$

The value of $\partial V^*/\partial D - \partial V/\partial D$ depends on the levels of the E^*s relative to the Es. Sufficient for it to be positive is $E_w^* \geq E_1$, $E_\ell^* \geq E_2$. Thus, as D rises, since the tournament's effort levels rise faster, it is likely that eventually V^* will surpass V.

In order to get further insight into the comparisons, a set of simulated examples is now presented, with arbitrary values for the different exogenous variables (ρ, r, D, $\bar{g}$), and a normal distribution of σ^2. These examples are presented in the tables at the end of the chapter. These immediately confirm the result that as σ^2 rises, output falls in the tournament, and thus so does its expected utility. Similarly, the results on the impact of changes in D, the value of output, are confirmed. As D rises, effort levels in the tournament rise faster than in the piece-rate system, so that eventually the tournament's expected utility dominates (if it does not already).

The impacts of changes in ρ and r are interesting. As ρ and r approach each other (from either direction), in both systems output converges on the socially optimal level ($E = D$). However, this is not to say a tax on r would create a social optimum, as raising r lowers the gains from borrowing. In the piece-rate system this unambiguously lowers expected utility, while in the tournament it depends on the relative size of the other exogenous variables. Thus, comparing

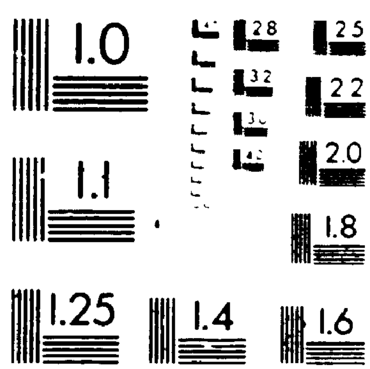
Table Four to Table Two, as r rises, if D is low or σ^2 is high, V^* can actually rise as E_w^* and E_t^* rise. Indeed, it is possible to get a reversal of the ranking of the wage systems. Thus, for $D = 5$, $\sigma^2 = 7.5$, $\rho = .10$, as the rate of interest rises from .05 to .09, the preferred contract form switches from the piece rate to the tournament. A similar reversal can occur if ρ decreases, all else equal (comparing Table Two to Table Three). Once again how V^* changes depends on the values of D and σ^2.

Finally, Table Five examines the impact of lowering r, all else constant. If D is low or σ^2 high, then effort and utility in the tournament falls. For all values of D and σ^2, as r falls effort falls and utility rises in the piece-rate system. Once again, for $D = 5$ and $\sigma^2 = 7.5$, the reversal of the relative ranking of the schemes occurs as r falls. This shows again the relative importance of the levels of D and $\bar{g}$ in determining what goes on in the tournament, although the values of ρ and r also play important but complicated rôles.

To summarize this comparison between piece-rates and tournaments, the most interesting part of the comparison surrounds the role played by the distribution of random shocks. Unlike in previous work comparing these payment schemes (Lazear and Rosen (1981), Green and Stokey (1983), and Nalebuff and Stiglitz (1983)), there is no role played by the common shock. As is well known, it 'washes out' of the tournament. On the other hand, as was demonstrated, due to the risk-neutrality of the agents, the optimal piece-rate scheme is not a function of the distribution of the shocks, common or individual. Thus, in comparing piece-rates to tournaments, the variance of the common shock is irrelevant, unlike in Green and Stokey (1983), where it plays a dominating role.

Green and Stokey, in a model with risk-averse agents, show that if the variance of the common shock is zero, the tournament can never dominate the piece-rate. Here, in a model with risk-neutral agents, the comparision is independent of the value of the common shock. In addition, they show that the smaller the variance

2 of/de 2



of the individual shock relative to the variance of the common shock, the more likely the tournament is to dominate the piece-rate. In the current restricted borrowing model, the lower the variance of the individual shock the more likely the tournament is to dominate. Thus, both the comparative statics and the simulations show that as σ^2 rises, the value of the tournament falls, until at some level (dependent on the values of the other parameters), the piece-rate dominates. Thus, as in other models, the smaller the variance of the individual shock, the more likely it is that the tournament dominates the piece-rate. However, unlike in other models, the relative size of the common shock is completely irrelevant. This is an important new result.

In summary, in comparing tournaments to piece-rates, at the industry level the tournament is more likely to be observed the higher the value of the output produced by the firm, and the lower the variance of output produced by individual workers. These predictions are presumably testable. A less testable prediction is the general, cross-industry prediction that the closer ρ is to r in value, the more likely is the use of tournaments.

3.4 Summary

In this chapter piece-rate contracts were examined with both repeated one-period contracts, and with two-period contracts. The primary purpose of this exercise was to test the generality of the results found for tournaments with restricted borrowing. This generality was indeed confirmed. For example, it was found that the firm would choose to have an intertemporal effect in piece-rate contracts, for reasons similar to those under tournaments. In addition, it was found that piece-rate contracts also had non-uniqueness in their compensation structure, and that once again the resulting level of effort was less than the principal desired. As in the tournament, all these results are driven by the borrowing distortion. As Appendix 3.2 shows, in a piece-rate system with risk-neutrality and no borrowing, none of these results occur.

The secondary purpose of this exercise was to compare tournaments to piece-rates under the restricted borrowing model. Although as in Lazear and Rosen (1981) it was impossible to make a general comparison, some illustrative simulation results were presented. The two forms of contracts are different enough to create different incentive/borrowing interactions, and thus it was found either piece-rates or tournaments could dominate, depending on the values of exogenous variables. However, certain specific predictions were generated – the lower the variance of the individual random shock to output, and/or the higher the value of output, the more likely it is that the tournament dominates. The former prediction is similar to those of Green and Stokey (1983), while the latter is new.

TABLE TWO $\rho = .10,\ r = .05$

	σ^2	E_1^*	E_2^*	V^*	E_1	E_2	V	Best
D=1	.25	1.006	1.017	.882	.9565	.9184	.872	ROT
	.50	.988	.962	.837	.9565	.9184	.872	PRS
	1.00	.964	.882	.776	.9565	.9184	.872	PRS
	6.00	.841	.491	.522	.9565	.9184	.872	PRS
	7.50	.815	.414	.480	.9565	.9184	.872	PRS
	12.00	.754	.214	.390	.9565	.9184	.872	PRS
D=3	.25	3.101	3.321	8.623	2.8700	2.7550	7.849	ROT
	.50	3.083	3.266	8.477	2.8700	2.7550	7.849	ROT
	1.00	3.059	3.187	8.278	2.8700	2.7550	7.849	ROT
	6.00	2.936	2.795	7.321	2.8700	2.7550	7.849	PRS
	7.50	2.911	2.718	7.141	2.8700	2.7550	7.849	PRS
	12.00	2.850	2.518	6.699	2.8700	2.7550	7.849	PRS
D=5	.25	5.196	5.625	24.342	4.7830	4.5920	21.801	ROT
	.50	5.178	5.570	24.095	4.7830	4.5920	21.801	ROT
	1.00	5.154	5.490	23.755	4.7830	4.5920	21.801	ROT
	6.00	5.031	5.099	22.100	4.7830	4.5920	21.801	ROT
	7.50	5.007	5.022	21.790	4.7830	4.5920	21.801	PRS
	12.00	4.946	4.822	20.996	4.7830	4.5920	21.801	PRS

Note: ROT = rank-order tournament, PRS = piece-rate system, all other symbols as defined in the text.

TABLE THREE $\rho = .06,\ r = .05$

	σ^2	E_w^*	E_ℓ^*	V^*	E_1	E_2	V	Best
D=1	.25	1.002	1.004	.921	.991	.982	.917	ROT
	.50	.998	.994	.911	.991	.982	.917	PRS
	1.00	.993	.979	.897	.991	.982	.917	PRS
	6.00	.969	.907	.834	.991	.982	.917	PRS
	7.50	.963	.893	.821	.991	.982	.917	PRS
	12.00	.951	.856	.790	.991	.982	.917	PRS
D=3	.25	3.021	3.062	8.415	2.972	2.946	8.254	ROT
	.50	3.017	3.052	8.384	2.972	2.946	8.254	ROT
	1.00	3.012	3.037	8.343	2.972	2.946	8.254	ROT
	6.00	2.988	2.965	8.148	2.972	2.946	8.254	PRS
	7.50	2.982	2.951	8.105	2.972	2.946	8.254	PRS
	12.00	2.970	2.914	8.007	2.972	2.946	8.254	PRS
D=5	.25	5.040	5.120	23.447	4.953	4.910	22.925	ROT
	.50	5.036	5.110	23.396	4.953	4.910	22.925	ROT
	1.00	5.031	5.095	23.327	4.953	4.910	22.925	ROT
	6.00	5.007	5.023	22.999	4.953	4.910	22.925	ROT
	7.50	5.001	5.009	22.930	4.953	4.910	22.925	ROT
	12.00	4.989	4.972	22.763	4.953	4.910	22.925	PRS

TABLE FOUR $\rho = .10,\ r = .09$

	σ^2	E^*_w	E^*_ℓ	V^*	E_1	E_2	V	Best
D=1	.25	1.001	1.003	.871	.991	.982	.868	ROT
	.50	.998	.993	.862	.991	.982	.868	PRS
	1.00	.993	.978	.850	.991	.982	.868	PRS
	6.00	.969	.906	.791	.991	.982	.868	PRS
	7.50	.964	.892	.779	.991	.982	.868	PRS
	12.00	.952	.855	.751	.991	.982	.868	PRS
D=3	.25	3.020	3.059	7.965	2.973	2.946	7.811	ROT
	.50	3.017	3.049	7.941	2.973	2.946	7.811	ROT
	1.00	3.012	3.034	7.902	2.973	2.946	7.811	ROT
	6.00	2.988	2.962	7.718	2.973	2.946	7.811	PRS
	7.50	2.982	2.948	7.678	2.973	2.946	7.811	PRS
	12.00	2.970	2.911	7.585	2.973	2.946	7.811	PRS
D=5	.25	5.038	5.115	22.191	4.955	4.910	22.697	ROT
	.50	5.035	5.105	22.150	4.955	4.910	22.697	ROT
	1.00	5.030	5.090	22.086	4.955	4.910	22.697	ROT
	6.00	5.006	5.018	21.777	4.955	4.910	22.697	ROT
	7.50	5.000	5.014	21.708	4.955	4.910	22.697	ROT
	12.00	4.988	4.967	21.554	4.955	4.910	22.697	PRS

TABLE FIVE $\rho = .10,\ r = .01$

	σ^2	E_w^*	E_l^*	V^*	E_1	E_2	V	Best
D=1	.25	1.010	1.034	.895	.924	.864	.873	ROT
	.50	.977	.924	.810	.924	.864	.873	PRS
	1.00	.931	.766	.700	.924	.864	.873	PRS
	6.00	.701	—	—	.924	.864	.873	PRS
	7.50	.656	—	—	.924	.864	.873	PRS
	12.00	.539	—	—	.924	.864	.873	PRS
D=3	.25	3.188	3.639	9.401	2.773	2.593	7.858	ROT
	.50	3.156	3.528	9.111	2.773	2.593	7.858	ROT
	1.00	3.109	3.371	8.709	2.773	2.593	7.858	ROT
	6.00	2.880	2.593	6.902	2.773	2.593	7.858	PRS
	7.50	2.834	2.440	6.582	2.773	2.593	7.858	PRS
	12.00	2.717	2.041	5.802	2.773	2.593	7.858	PRS
D=5	.25	5.366	6.243	26.910	4.622	4.322	21.826	ROT
	.50	5.334	6.132	26.415	4.622	4.322	21.826	ROT
	1.00	5.287	5.975	25.722	4.622	4.322	21.826	ROT
	6.00	5.058	5.197	22.473	4.622	4.322	21.826	ROT
	7.50	5.013	5.044	21.868	4.622	4.322	21.826	ROT
	12.00	4.895	4.646	20.350	4.622	4.322	21.826	PRS

Note: For $D = 1$, $\sigma^2 = 6, 7.5, 12$ the values for E_l^* turn out to be negative, so a tournament is not feasible.

Appendices to Chapter Three

3.1 The Pareto-Dominance of Two-Period Contracts

In the text it was shown:

$$
\begin{aligned}
V(\text{Repeated}) &= V(R) = .5R^2DE_R(2+r), \\
E_R &= \frac{D(2+r)(1+\rho)^2}{2(2+r)(1+\rho)^2-(2+\rho)(1+r)^2}, \\
V(\text{Two-period}) &= V(T) = .5RDE_1 + .5R^2DE_2, \\
E_1 &= \frac{D(1+\rho)}{(1+2\rho-r)}, \\
E_2 &= \frac{D(1+\rho)^2}{(1+\rho)^2+(\rho-r)(2+\rho+r)}, \text{ where} \\
E_R - E_1 &= \frac{D(1+\rho)(1+r)(r-\rho)}{(2(2+r)(1+\rho)^2-(2+\rho)(1+r)^2)(1+2\rho-r)} < 0, \\
E_R - E_2 &=
\end{aligned}
$$

$$
\frac{D(1+\rho)^2(1+r)^2(\rho-r)}{(2(2+r)(1+\rho)^2-(2+\rho)(1+r)^2)((1+\rho)^2+(\rho-r)(2+\rho+r))} > 0.
$$

so that $E_1 > E_R > E_2$.

To prove Proposition 4 of Chapter Three, solve:

$$
\begin{aligned}
V(T) - V(R) &= .5RDE_1 + .5R^2DE_2 - .5R^2DE_R(2+r) \\
&= .5RD(E_1-E_R) + .5R^2D(E_2-E_R) \\
&= .5RD\left[\frac{D(1+\rho)(1+r)(\rho-r)}{(2(2+r)(1+\rho)^2-(2+\rho)(1+r)^2)(1+2\rho-r)}\right] +
\end{aligned}
$$

$$
\frac{.5R^2D^2(1+\rho)^2(1+r)^2(r-\rho)}{(2(2+r)(1+\rho)^2-(2+\rho)(1+r)^2)((1+\rho)^2+(\rho-r)(2+\rho+r))}
$$

Define $m = (1+\rho)^2+(\rho-r)(2+\rho+r)$. Then

$$V(T) - V(R) =$$

$$\frac{D^2(1+\rho)(\rho-r)[(1+\rho)^2+(\rho-r)(2+\rho+r)-(1+2\rho-r)(1+\rho)]}{2(2(2+r)(1+\rho)^2-(2+\rho)(1+r)^2)(1+2\rho-r)m}$$

$$\begin{aligned}
&= [+][(1+\rho)(1+\rho-1-2\rho+r)+(\rho-r)(2+\rho+r)] \\
&= [+][(1+\rho)(r-\rho)+(\rho-r)(2+\rho+r)] \\
&= [+][(\rho-r)(1+r)] > 0,
\end{aligned}$$

so that $V(T) > V(R)$.

3.2 Piece-Rates Without Borrowing

Consider a model with risk-neutral agents and principal, $r = \rho$ and two-period contracts. Once again, this is an extension of Lazear and Rosen (1981) to two periods. As in Appendix 2.5, agents will be indifferent to borrowing, so it is assumed $B_1 = B_2 = 0$. Given I_1, I_2, S_1 and S_2, agents select borrowing and effort to solve:

$$\begin{aligned}
\max V &= I_1 + S_1E_1 - C(E_1) + \beta(I_2 + S_2E_2 - C(E_2)), \\
\frac{\partial L}{\partial E_1} &= S_1 - C'(E_1) = 0, \ \leadsto E_1 = S_1, \\
\frac{\partial L}{\partial E_2} &= \beta(S_2 - C'(E_2) = 0, \ \leadsto E_2 = S_2.
\end{aligned}$$

The principal selects I_1, I_2, S_1, and S_2 to:

$$\begin{aligned}
\max L &= I_1 + S_1E_1 - C(E_1) + \beta(I_2 + S_2E_2 - C(E_2)) \\
&\quad + \lambda[DE_1 + \beta DE_2 - (I_1 + S_1E_1) - \beta(I_2 + S_2E_2)]. \\
\frac{\partial L}{\partial I_1} &= 1 - \lambda = 0, \\
\frac{\partial L}{\partial I_2} &= \beta(1 - \lambda) = 0, \\
\frac{\partial L}{\partial S_1} &= E_1 - C'(E_1)\frac{\partial E_1}{\partial S_1} + S_1\frac{\partial E_1}{\partial S_1} + \lambda((D - S_1)\frac{\partial E_1}{\partial S_1} - E_1) = 0, \\
\frac{\partial L}{\partial S_2} &= \beta[E_2 - C'(E_2)\frac{\partial E_2}{\partial S_2} + S_2\frac{\partial E_2}{\partial S_2} +
\end{aligned}$$

$$\lambda((D - S_2)\frac{\partial E_2}{\partial S_2} - E_2)] = 0,$$

$$\frac{\partial L}{\partial \lambda} = DE_1 + \beta DE_2 - (I_1 + S_1E_1) - \beta(I_2 + S_2E_2) = 0.$$

The first two f.o.c.'s yield $\lambda = 1$. Substituting this plus $E_1 = S_1$ into $\partial L/\partial S_1$ yields:

$$S_1 - S_1 + S_1 + D - S_1 - S_1 = 0,$$

or $S_1 = D$. A similar exercise with $\partial L/\partial S_2$ yields:

$$S_2 + S_2 - S_2 + D - S_2 - S_2 = 0,$$

or $S_2 = D$. Substituting these values into $\partial L/\partial \lambda$:

$$D^2 + \beta D^2 - I_1 - D^2 - \beta(I_2 + D^2) = 0,$$

or $I_1 + \beta I_2 = 0$. Therefore there are no memory effects ($E_1 = E_2 = D$). In addition, evaluate the following at the optimum:

$$\frac{\partial L}{\partial E_1} = S_1 - C'(E_1) + \lambda(D - S_1) = 0.$$

Thus, with risk-neutrality and no borrowing, there is no moral hazard distortion, and no intertemporal effects.

Chapter 4

Rank-Order Tournaments With Default Costs

> [G]oing back and forth into court to have my income scrutinized ... is driving me mad! ... [W]hen little Judge Rosenzweig hears I teach only two classes a week, he's ready to send me to Sing Sing! ... They want me to get a paper route, Susan! They wouldn't care if I sold Good Humors![1]

Chapter Two briefly explored a perfect capital market in a rank-order tournament. It was shown in that setting that only a zero output equilibrium would exist in the tournament firm. As a way around this problem, Chapters Two and Three explored a model in which agents could commit to repaying by borrowing only up to their worst-case income. In this chapter a different formulation of the capital market is employed, in which where agents can borrow higher amounts of income. This different formulation will allow for a test of the generality of Chapter Two's results. In addition, the new formulation generates a great many new predictions about observable variables.

[1] Philip Roth, *My Life as a Man*.

In this new formulation of the capital market, should the worst case outcome occur, an agent will have borrowed more then he can repay, and will 'default' on his loans, and must pay a form of default costs. In Jaffee and Russell (1976) these default costs were interpreted as a reduction in future earnings capability, or as 'moral costs'.[2] Here it is assumed these default costs take the form of a legal compulsion to pay off any outstanding debt by working at a second job, or perhaps overtime at the current job. (In essence, you do repay the debt and do not technically default.) This is costly because the defaulter must expend effort equal to (net debt)/A, where A is the value of output and the wage at this second job. It is assumed this job is one with observable effort, and that monitoring costs mean $A < D$, where D is the value of output at the tournament firm.[3] This assumption means the tournament job is preferred to the reserve job, and this helps justify the additional assumption made below that one never borrows more then one's maximum possible wage.

This setup is similar to the 'bonded labour' concept explored by Braverman and Stiglitz (1982), or to earlier institutions such as debtor's prison. Perhaps it may be more usefully viewed as the case of a manager who fails to get a promotion, and must take a night job (such as selling Good Humor ice cream) to pay off the mortgage. The current model is unique in its emphasis on borrowing in a multiperiod setting. The multiperiod setting crucially affects the agent's choices. Consider, for example, if an agent loses the first period contest. He would have net debt outstanding, equal to $B_1(1+r) - W_\ell - B_\ell$. Thus the agent can use B_ℓ to 'rollover' outstanding debts, or he can choose to work off the debt – he has some choice of how to spread the cost of losing across periods. This choice in turn will create a linkage between periods, which the principal will choose to exploit in

[2] The capital markets in Haltiwanger and Waldman (1986) and Farrow (1986) are essentially adaptations of this model.

[3] Alternatively, working overtime is more painful then normal work.

his contract formulation. This linkage will lead to the special, new results of the default cost model.

4.1 A Preliminary Result

The first step is to show the agent will borrow enough to have a positive probability of defaulting. Assume the production function, utility function, and probability function are as defined in Chapter Two. In addition, note the agent is no longer constrained in his borrowing.

Proposition 1 *If the default cost function has a sufficiently small fixed cost, and a sufficiently small marginal cost at zero cost, the agent will always borrow enough such that if he loses any contest, he will have outstanding debt to be paid off.*

Proof: Consider an agent facing repeated contracts for the wages $W_w > W_\ell$ each period (an analogous proof holds for the nonmemory and memory two-period contracts). Suppose the agent borrows so little he does not go to debtor's prison if he loses a contest. Then it must be that B_1, B_2 satisfy:

$$\begin{aligned} B_1(1+r) &\leq W_\ell + RW_\ell, \\ B_2(1+r) &\leq W_\ell, \end{aligned}$$

which in turn means the agent's utility function is:

$$\begin{aligned} V &= B_1 + \beta(P_1(W_w - B_1(1+r)) + (1-P_1)(W_\ell - B_1(1+r)) \\ &\quad + B_2 - C(E_1)) + \beta^2(P_2(W_w - B_2(1+r)) + \\ &\quad (1-P_2)(W_\ell - B_2(1+r)) - C(E_2)). \end{aligned}$$

The first-order conditions on B_1, B_2 are:

$$\frac{\partial V}{\partial B_1} = 1 - \beta(1+r)(P_1 + 1 - P_1) = \beta(\rho - r) > 0,$$

$$\frac{\partial V}{\partial B_2} = \beta[1 - \beta(1+r)(P_2 + 1 - P_2)] = \beta^2(\rho - r) > 0.$$

On the margin, borrowing past the worst-case income (W_ℓ) has a positive value, give $\rho > r$. Therefore, if there is a sufficiently small cost to borrowing just past this margin, the agent will be willing to accept some of the fair bet of borrowing past this margin, with the subsequent risk of incurring the default costs. □

The intuition is that at the margin of W_ℓ, an extra borrowed dollar has a positive marginal utility ($\rho > r$). As long as the cost of borrowing past this margin is less than this benefit (fixed cost + MC(0) < $\beta(\rho - r)$), it always pays to borrow some extra amount, and take the gamble of losing and paying the debt. The assumption made throughout this thesis that $C(E) = .5E^2$ is sufficient to meet the needed cost condition.

Given this result, clearly agents will be 'defaulting' under the correct conditions. This chapter will repeat the analysis of Chapter Two by examining repeated one period contracts, nonmemory two period contracts and memory contracts given default cost borrowing. Examining them in this order follows an order in which the principal is allowed more and more tools in his attempts to use the compensation structure to better motivate agents. It will be shown there are quite different results depending on what type of contracts are used. In addition, many new results will be generated due to the new capital market assumption.

4.2 Repeated One-Period Contracts

Once again, firms are constrained to offer identical zero-profit contracts each period, with the winner receiving W_w and the loser W_ℓ, although agents are free to borrow across both periods. The contests are as in Chapter Two. However, first period losers now incur debt, and hence behave differently than first period winners – there is no longer a symmetric equilibrium in the second contest. This yields the crucial result that first period winners' and losers' effort levels and probabilities of winning will differ in the second period.

In Proposition 1 it was shown agents would borrow enough to risk bankruptcy. This implies the following holds:

$$
\begin{aligned}
B_1(1+r) &> W_\ell + B_\ell, \\
B_\ell(1+r) &> W_\ell, \\
B_w(1+r) &> W_\ell.
\end{aligned}
$$

In addition, in order to make further analysis tractable it is assumed the agent never borrows so much that he must pay the default costs even if he wins a contest.[4] This assumption plus the above result means:

$$
\begin{aligned}
W_w &> B_1(1+r) > W_\ell + B_\ell, \\
W_w &> B_\ell(1+r) > W_\ell, \\
W_w &> B_w(1+r) > W_\ell.
\end{aligned}
$$

Analyzing this as a dynamic programming problem, working backwards from the final period, if one has won the first period contest, there is no leftover debt, and the problem is to choose B_w and E_w ($B_1 > 0$ given at this stage) to maximize:

$$
\begin{aligned}
V_w = {} & W_w - B_1(1+r) + B_w + \beta(P_w(W_w - B_w(1+r)) - \\
& (1-P_w)C[(B_w(1+r) - W_\ell)A^{-1}] - C(E_w)), \qquad (4.1)
\end{aligned}
$$

where $P_w = P_w(E_w, E_\ell)$ is the probability of the first period winner winning the second-period contest, given the loser's effort choice, and where $(B_w(1+r) - W_\ell)/A$ is the necessary effort to pay off one's debt if one loses in the second period, A equalling the wage rate at the second job.

[4] The assumption made earlier that the return to effort at the tournament firm is higher then at the second job ($A < D$) helps to justify this strong but useful assumption.

Alternatively, for the loser of the first contest, there is leftover debt to be paid or rolled over, and the problem is to choose B_ℓ, E_ℓ ($B_1 > 0$ given) to maximize:

$$
\begin{aligned}
V_\ell &= -C[(B_1(1+r) - W_\ell - B_\ell)A^{-1}] + \beta(P_\ell(W_w - B_\ell(1+r)) \\
&\quad -(1-P_\ell)C[(B_\ell(1+r) - W_\ell)A^{-1}] - C(E_\ell)), \qquad (4.2)
\end{aligned}
$$

where $P_\ell = P_\ell(E_w, E_\ell)$ is the probability of the first period loser winning the second period contest given the winner's effort level.[5] Although the winner starts afresh, the loser will have leftover debt. This crucial difference will mean $B_\ell \neq B_w$, and hence $E_\ell \neq E_w$. Thus the second period contest will have an asymmetric equilibrium, unlike in the restricted borrowing case, making the problem simultaneously more difficult but more interesting.

Given the above, in the first period the agent's problem is to select E_1, B_1 to maximize

$$V = B_1 + \beta(P_1(V_w) + (1-P_1)V_\ell - C(E_1)),$$

where V_w and V_ℓ are the maximized values from equations (4.1) and (4.2). For ease of exposition V_w and V_ℓ will be substituted into this equation. Thus, the agent selects borrowing and effort to solve:

$$
\begin{aligned}
\max V &= B_1 + \beta P_1[W_w - B_1(1+r) + B_w + \beta(P_w(W_w - \\
&\quad B_w(1+r)) - (1-P_w)C[(B_w(1+r) - W_\ell)A^{-1}] \\
&\quad -C(E_w))] + \beta(1-P_1)[-C[(B_1(1+r) - W_\ell - B_\ell)A^{-1}] + \\
&\quad \beta(P_\ell(W_w - B_\ell(1+r)) - (1-P_\ell)C[(B_\ell(1+r) - W_\ell)A^{-1}] \\
&\quad -C(E_\ell))] - \beta C(E_1).
\end{aligned}
$$

The first-order conditions are:

$$\frac{\partial V}{\partial B_1} = 1 - \beta(1+r)[P_1 + A^{-1}(1-P_1)C'[(B_1(1+r) - W_\ell$$

[5] Note that $P_\ell = 1 - P_w(E_w, E_\ell)$, but from the viewpoint of the agent in the second period, this is irrelevant.

$$-B_\ell)A^{-1}] = 0. \tag{4.3}$$

$$\frac{\partial V}{\partial B_w} = \beta P_1(1-\beta(1+r)[P_w + A^{-1}(1-P_w)C'[(B_w(1+r) -W_\ell)A^{-1}]) = 0. \tag{4.4}$$

$$\frac{\partial V}{\partial B_\ell} = \beta(1-P_1)(A^{-1}C'[(B_1(1+r)-W_\ell-B_\ell)A^{-1}] - \beta(1 +r)(P_\ell + A^{-1}(1-P_\ell)C'[(B_1(1+r)-W_\ell)A^{-1}])) = 0. \tag{4.5}$$

$$\frac{\partial V}{\partial E_1} = \beta[\frac{\partial P_1}{\partial E_1}[W_w - B_1(1+r) + B_w + \beta(P_w(W_w - B_w(1+r)) - (1-P_w)C[(B_w(1+r)-W_\ell)A^{-1}] - C(E_w)) + C[(B_1(1 +r)-W_\ell-B_\ell)A^{-1}] - \beta(P_\ell(W_w - B_\ell(1+r)) - (1- P_\ell)C[(B_\ell(1+r)-W_\ell)A^{-1}] - C(E_\ell))] - C'(E_1)] = 0. \tag{4.6}$$

$$\frac{\partial V}{\partial E_w} = \beta^2 P_1[\frac{\partial P_w}{\partial E_w}(W_w - B_w(1+r) + C[(B_w(1+r)-W_\ell)A^{-1}]) -C'(E_w)] = 0. \tag{4.7}$$

$$\frac{\partial V}{\partial E_\ell} = \beta^2(1-P_1)[\frac{\partial P_\ell}{\partial E_\ell}(W_w - B_\ell(1+r) + C[(B_\ell(1+r) -W_\ell)A^{-1}]) - C'(E_\ell)] = 0. \tag{4.8}$$

Now define the probability of the first period winner winning the second period, assuming a Nash equilibrium, as P_2 $(= P_w = 1 - P_\ell)$, and define $\tilde{g} = g(E_\ell - E_w) = \partial P_w/\partial E_w = \partial P_\ell/\partial E_\ell$, assuming a normal distribution on $\epsilon_j - \epsilon_i$.[6] Given this, the first-order conditions can be solved for:

$$B_1 = R^2 W_\ell(2+r) + h, \tag{4.9}$$

$$B_w = RW_\ell + y, \tag{4.10}$$

$$B_\ell = RW_\ell + x, \tag{4.11}$$

$$E_1 = \tilde{g}[W_w - W_\ell + y - h(1+r) + C[(h(1+r)-x)A^{-1}] + \beta(P_2(W_w - W_\ell - (1+r)y + C[x(1+r)A^{-1}]) - (1 - P_2)(W_w - W_\ell - (1+r)x + C[y(1+r)A^{-1}]) + C(E_\ell)$$

[6]The assumption of a normal distribution is made to obtain explicit values for $g(E_\ell, E_w)$ later on.

$$-C(E_w))],\tag{4.12}$$

$$E_w = \tilde{g}(W_w - W_\ell - (1+r)y + C[y(1+r)A^{-1}]),\tag{4.13}$$

$$E_\ell = \tilde{g}(W_w - W_\ell - (1+r)x + C[x(1+r)A^{-1}]),\tag{4.14}$$

where

$$h = \frac{A^2[(1+2\rho-r)(1+\rho+(1+r)^2P_2)-(1+r)^2(1-P_2)]}{(1+r)^4P_2} > 0,\tag{4.15}$$

$$y = \frac{A^2[1+\rho-P_2(1+r)]}{(1+r)^2(1-P_2)} > 0,\tag{4.16}$$

$$x = \frac{A^2[(1+2\rho-r)(1+\rho)-(1+r)^2(1-P_2)]}{(1+r)^3P_2} > 0.\tag{4.17}$$

Given $B_1(1+r) > W_\ell + B_\ell$ from Proposition 1, B_ℓ is never consumed, being used only to change the default costs. Thus the agent is able to spread the monetary and effort costs of his debt between periods, giving himself some flexibility. This means B_1 and B_ℓ are determined jointly, while B_w is determined alone. This in turn means $B_w \neq B_\ell$, with the value of the difference depending on the value of P_2 and hence on the value of E_ℓ versus E_w.

Proposition 2 *a) Assuming $\epsilon_j - \epsilon_i$ is distributed normally, the Nash equilibrium solution to the second-period contest is asymmetrical ($E_w \neq E_\ell$).*
b) Assuming a unique solution exists, it includes $E_\ell > E_w$, $P_2 < .5$ and $B_\ell > B_w$

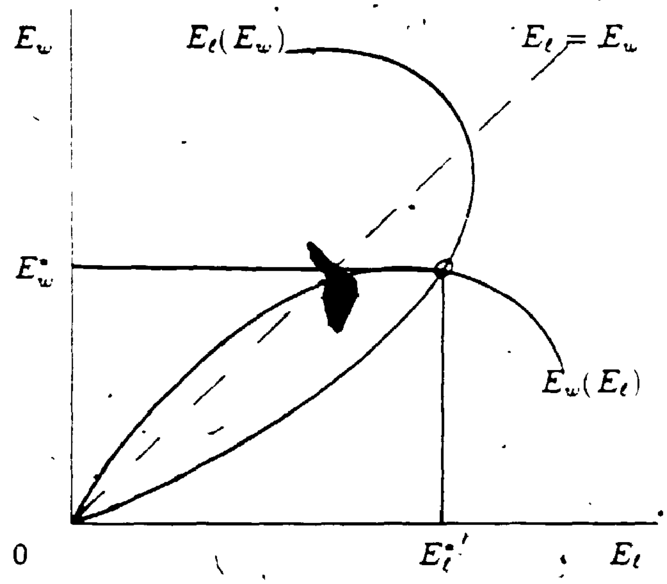
<u>**Proof:**</u> See Appendix 4.1.

The proof is algebraically arduous, but intuitively straightforward. Essentially, equations (4.13) and (4.14) are best-response functions, $E_w(E_\ell)$ and $E_\ell(E_w)$ respectively. The appendix demonstrates they are single-turning and shaped as in Figure 1, so that in equilibrium $E_\ell > E_w$, hence $P_2 < .5$ and $B_\ell > B_w$.[7]

The logic of this is straightforward. Unlike in the restricted borrowing case, here a loser has a debt burden after the contest. He can choose to roll over

[7] I would like to thank Sam Bucovetsky for a useful discussion on this proof

FIGURE ONE



some of this debt by borrowing, and indeed will decide to do so ($B_\ell > W_\ell$). However, this means that compared to a winner, he has borrowed more going into the second-period, and the cost of losing is higher ($B_\ell > B_w$). With this higher incentive, he works harder (∂ effort / ∂ debt > 0), and hence $E_\ell > E_w$ and $P_2 < .5$. Losers work harder, winners coast.

This is a powerful new result not found in the restricted borrowing case. The model of a risk-neutral agent with default costs approximates the illiquidity of a risk-averse agent. The resulting prediction that losers' effort levels will be higher is strong and testable. Although effort is unobservable, output is observable. With enough observations, mean effort can be approximated by mean output. Therefore, this new borrowing model has yielded a refutable prediction, something in general lacking in agency models.

The comparative statics for the agent's problem are as follows (see Appendix

4.1 for the derivations):

$$\begin{aligned}
\frac{\partial B_1}{\partial W_\ell} &> 0, \frac{\partial B_w}{\partial W_\ell} > 0, \frac{\partial B_\ell}{\partial W_\ell} > 0, \\
\frac{\partial B_1}{\partial r} &< 0, \frac{\partial B_w}{\partial r} < 0, \frac{\partial B_\ell}{\partial r} < 0, \\
\frac{\partial B_1}{\partial \rho} &> 0, \frac{\partial B_w}{\partial \rho} > 0, \frac{\partial B_\ell}{\partial \rho} > 0, \\
\frac{\partial E_1}{\partial W_w} &= \bar{g}(1 + \beta(2P_2 - 1)) + \beta g\bar{g}(E_\ell - E_w) > 0, \\
\frac{\partial E_1}{\partial W_\ell} &= -\frac{\partial E_1}{\partial W_w} < 0, \\
\frac{\partial E_w}{\partial W_w} &= \frac{\partial E_\ell}{\partial W_w} = \bar{g} > 0, \\
\frac{\partial E_w}{\partial W_\ell} &= \frac{\partial E_\ell}{\partial W_\ell} = -\bar{g} > 0.
\end{aligned}$$

The intuition of these comparitive statics is familiar and straightforward. Note that compared to the restricted borrowing, repeated contract case, the default cost has created intertemporal linkages ($\partial E_1/\partial W_w$ depends on the selections of second period effort). Thus, in the restricted case $\partial E_1/\partial W_w = \bar{g}$, as compared to the complications created here by the asymmetric solution ($E_\ell > E_w$, $P_2 < .5$). Now, despite identical wages in each contest, the asymmetry in effort levels creates a capital gain to winning (reflected by the extra terms in $\partial E_1/\partial W_w$) consisting of avoiding the default costs of losing, a capital gain not present in the restricted borrowing, repeated contract case. Further comparisons of the default cost model to the restricted model will be made later, along with comparisons between the comparative statics in the default cost, repeated contract model and the nonmemory and memory models.

The next step is to solve the principal's problem. With (4.9) - (4.11) substituted in, and given (4.12) - (4.14), the principal selects the wage structure to solve:[8]

$$\begin{aligned}\max L &= R^2W_\ell(2+r)+h+.5\beta[W_w-W_\ell+y-h(1+r)-\\&C[((h(1+r)-x)A^{-1}]]+.5\beta^2P_2(W_w-W_\ell-y(1+r)-\\&C[x(1+r)A^{-1}])+.5\beta^2(1-P_2)(W_w-W_\ell-x(1+r)\\&-C[y(1+r)A^{-1}])-.5\beta^2C(E_w)-.5\beta^2C(E_\ell)-\beta C(E_1)\\&+R\lambda(DE_1+.5RD(E_w+E_\ell)-.5R(W_w+W_\ell)(2+r)).\end{aligned}$$

The first-order conditions are:

$$\begin{aligned}\frac{\partial L}{\partial W_w} &= .5\beta+.5\beta^2P_2+.5\beta^2(1-P_2)-.5\beta^2C'(E_w)\frac{\partial E_w}{\partial W_w}-\\&.5\beta^2C'(E_\ell)\frac{\partial E_\ell}{\partial W_w}-\beta C'(E_1)\frac{\partial E_1}{\partial W_w}+R\lambda[D\frac{\partial E_1}{\partial W_w}+\\&.5RD(\frac{\partial E_w}{\partial W_w}+\frac{\partial E_\ell}{\partial W_w})-.5R(2+r)]=0, \quad (4.18)\\\frac{\partial L}{\partial W_\ell} &= R^2(2+r)-.5\beta-.5\beta^2P_2+-.5\beta^2(1-P_2)-\\&.5\beta^2C'(E_w)\frac{\partial E_w}{\partial W_\ell}-.5\beta^2C'(E_\ell)\frac{\partial E_\ell}{\partial W_\ell}-\beta C'(E_1)\frac{\partial E_1}{\partial W_\ell}\\&+R\lambda[D\frac{\partial E_1}{\partial W_\ell}+.5RD(\frac{\partial E_w}{\partial W_\ell}+\frac{\partial E_\ell}{\partial W_\ell})-.5R(2+r)]=0, \quad (4.19)\end{aligned}$$

[8] Note the zero-profit condition is quite similar to the restricted borrowing case, despite the fact $P_2 \neq .5$, etc. The reason is simple – it is a per person two-period discounted zero-profit constraint. Thus, viewed from the initial period, there is a 50% chance an agent will win in the first period, and hence select E_w (with expected second-period income $P_2W_w+(1-P_2)W_\ell$), and a 50% chance he/she will lose and select E_ℓ (with expected second-period income of $(1-P_2)W_w+P_2W_\ell$) Thus, per person expected, discounted profits are:

$$R(DE_1-.5(W_w+W_\ell))+.5R^2(DE_w-P_2W_w-(1-P_2)W_\ell)+R^2(DE_\ell -(1-P_2)W_w-P_2W_\ell),$$

which upon rearrangement yields:

$$RDE_1+.5R^2D(E_w+E_\ell)-.5R(W_w+W_\ell)-.5R^2(W_w+W_\ell).$$

$$\frac{\partial L}{\partial \lambda} = DE_1 + .5RD(E_w + E_\ell) - .5R(W_w + W_\ell)(2 + r) = 0. \quad (4.20)$$

Adding together equations (4.18) and (4.19) yields $\lambda = 1$. Substitution of this plus the comparative statics of the agent's problem into (4.18) leaves an equation linear in $W_w - W_\ell$. This equation plus the zero-profit constraint can be solved exactly for W_w and W_ℓ, but the resulting answer is horribly complex, and intuitively unappealing, as the asymmetry in this case creates great complications in the algebra. However, an exact answer is unnecessary for obtaining the crucial results from this section, as they were derived in Proposition 2. First, the contract structure forces an asymmetric solution in the second period game. This in turn leads to the coasting result, that first period losers work harder than first-period winners. Secondly, although contracts are identical in each period, agent's effort levels are not. There are 'memory' effects in output, created by the asymmetry, which in turn is created by the default cost borrowing structure. These memory effects did not occur in the repeated contracts with restricted borrowing.

This section's repeated contracts restricted the principal's choices. He was unable to differentiate between agents based on their history, and he was unable to create interperiod effects in the wage structure. In the next section the principal is allowed to create these interperiod effects.

4.3 Nonmemory Two-Period Contracts

In this section, the principal offers two-period wage contracts identical to those in Chapter Two, Section 4 – wages are $W_w > W_\ell$ in the first period and $W_{2w} > W_{2\ell}$ in the second period. Thus, although there are intertemporal effects in wages, there is no sorting (first period winners and losers play each other in the second period) and hence no wage memory effects. In all contract aspects, the wage structure is identical to the Chapter Two model, although the new borrowing structure means equilibrium wage values will be different.

As per the discussion above, the following conditions on wages hold:

$$
\begin{aligned}
W_w &> B_1(1+r) > W_\ell + B_\ell, \\
W_{2w} &> B_\ell(1+r) > W_{2\ell}, \\
W_{2w} &> B_w(1+r) > W_{2\ell}.
\end{aligned}
$$

Once again, in each condition the first set of inequalities is for analytical tractibility (although quite logical give $A < D$), while the second set is an implication of Proposition 1.

These conditions mean that the agent's problem if he or she has won the first period contest (and has no leftover debt), is to choose B_w and E_w (B_1 given at this stage) to maximize:

$$
\begin{aligned}
V_w &= W_w - B_1(1+r) + B_w + \beta(P_w(W_{2w} - B_w(1+r)) \\
&\quad +(1-P_w)[-C[(B_w(1+r) - W_{2\ell})A^{-1}]] - C(E_w))
\end{aligned}
$$

where $P_w = P_w(E_w, E_\ell)$ is the probability of the (first period) winner winning the second period contest, given the losers' effort choice. If one has lost the first contest, there is leftover debt to be rolled over, and with B_1 given at this stage, the problem is to choose B_ℓ, E_ℓ to maximize:

$$
\begin{aligned}
V_\ell &= -C[(B_1(1+r) - W_\ell - B_\ell)A^{-1}] + \beta(P_\ell(W_{2w} - B_\ell(1+r)) \\
&\quad +(1-P_\ell)(-C[(B_\ell(1+r) - W_{2\ell})A^{-1}]) - C(E_\ell)),
\end{aligned}
$$

where $P_\ell = P_\ell(E_w, E_\ell)$ is the probability of the loser winning the second period contest given the winner's effort level.

Note that except for the W_{2w}, $W_{2\ell}$ terms, this problem is identical to the repeated contract case. Indeed, this section's problem shares many of the characteristics of the previous section's problem, including most crucially the asymmetric equilibrium in the second period ($B_\ell > B_w$, $E_\ell > E_w$). However, the principal's problem is much less complex to solve.

Substituting in the values of V_w and V_ℓ into the agent's first period problem gives the agent's complete, overall problem. He selects borrowing and effort to:

$$\begin{aligned}\max V \; = \; & B_1 + \beta P_1[W_w - B_1(1+r) + B_w + \beta(P_w(W_{2w} \\ & -B_w(1+r)) - (1-P_w)C[(B_w(1+r) - W_{2\ell})A^{-1}] \\ & -C(E_w))] + \beta(1-P_1)[-C[(B_1(1+r) - W_\ell - B_\ell)A^{-1}] \\ & +\beta(P_\ell(W_{2w} - B_\ell(1+r)) - (1-P_\ell)C[(B_\ell(1+r) - \\ & W_{2\ell})A^{-1}] - C(E_\ell))] - \beta C(E_1)\end{aligned}$$

This problem and its solution are virtually identical to that of the repeated case. Once again, given the Nash equilibrium values of E_ℓ and E_w, define $P_2(= P_w = 1 - P_\ell)$ as the probability of the first period winner winning the second period contest, and define $\bar{g} = g(E_\ell - E_w) = \partial P_w/\partial E_w = \partial P_\ell/\partial E_\ell$ (assuming a normal distribution). Given this the first-order conditions to the above problem can be solved to yield:

$$B_1 = RW_\ell + R^2 W_{2\ell} + h, \tag{4.21}$$

$$B_w = RW_{2\ell} + y, \tag{4.22}$$

$$B_\ell = RW_{2\ell} + x, \tag{4.23}$$

$$\begin{aligned}E_1 = \; & \bar{g}[W_w - W_\ell + y - h(1+r) + C[(h(1+r) - x)A^{-1}] + \\ & \beta(P_2(W_{2w} - W_{2\ell} - (1+r)y + C[x(1+r)A^{-1}]) - \\ & (1-P_2)(W_{2w} - W_{2\ell} - (1+r)x + C[y(1+r)A^{-1}]) \\ & +C(E_\ell) - C(E_w))],\end{aligned} \tag{4.24}$$

$$E_w = \bar{g}(W_{2w} - W_{2\ell} - (1+r)y + C[y(1+r)A^{-1}]), \tag{4.25}$$

$$E_\ell = \bar{g}(W_{2w} - W_{2\ell} - (1+r)x + C[x(1+r)A^{-1}]), \tag{4.26}$$

where x, y and h are as in equations (4.15) - (4.17).

As noted, these best-response functions are virtually identical to those for the repeated contracts, except for the W_{2w}, $W_{2\ell}$ terms. This similarity is not surprising, as these contracts share the condition of no sorting in the second period, forcing agents to play in an asymmetric contest.

Proposition 3 *a) Assuming $\epsilon_j - \epsilon_i$ normally distributed, the Nash equilibrium for the second-period contest is asymmetric ($E_w \neq E_\ell$).*

b) Assuming a unique solution exists, it involves $E_\ell > E_w$, $P_2 < .5$ and $B_\ell > B_w$.

<u>**Proof:**</u> Analogous to that of Proposition 2.

As noted, the results of the current contract setup strongly resemble those of the earlier repeated contracts. Indeed, B_w, B_ℓ, E_w and E_ℓ differ only in substituting $W_w = W_{2w}$ and $W_\ell = W_{2\ell}$ into them, and thus the proof of Proposition 2 applies here.

Appendix 4.2 shows the formal derivation of the following comparitive statics for the agent's supply functions:

$$\begin{aligned}
\frac{\partial B_1}{\partial W_\ell} &> 0,\ \frac{\partial B_w}{\partial W_\ell} = 0,\ \frac{\partial B_\ell}{\partial W_\ell} = 0,\\
\frac{\partial B_1}{\partial W_{2\ell}} &> 0,\ \frac{\partial B_w}{\partial W_{2\ell}} > 0,\ \frac{\partial B_\ell}{\partial W_{2\ell}} > 0,\\
\frac{\partial B_1}{\partial r} &< 0,\ \frac{\partial B_w}{\partial r} < 0,\ \frac{\partial B_\ell}{\partial r} < 0,\\
\frac{\partial B_1}{\partial \rho} &> 0,\ \frac{\partial B_w}{\partial \rho} > 0,\ \frac{\partial B_\ell}{\partial \rho} > 0,\\
\frac{\partial E_1}{\partial W_w} &= \bar{g} > 0,\ \frac{\partial E_1}{\partial W_\ell} = -\bar{g} > 0,\\
\frac{\partial E_1}{\partial W_{2w}} &= \bar{g}\beta(2P_2 - 1) + \beta\bar{g}\bar{g}(E_\ell - E_w) \gtrless 0,\\
\frac{\partial E_1}{\partial W_{2\ell}} &= -\frac{\partial E_1}{\partial W_w} \gtrless 0,\\
\frac{\partial E_w}{\partial W_{2w}} &= \frac{\partial E_\ell}{\partial W_{2w}} = \bar{g} > 0,\\
\frac{\partial E_w}{\partial W_{2\ell}} &= \frac{\partial E_\ell}{\partial W_{2\ell}} = -\bar{g} < 0.
\end{aligned}$$

Note that the comparative statics for E_1 are such that:

$$\frac{\partial E_1}{\partial W_w}(\text{nonmemory}) + \frac{\partial E_1}{\partial W_{2w}}(\text{nonmemory}) = \frac{\partial E_1}{\partial W_w}(\text{repeated}).$$

This equality is only logical considering in the repeated case W_w is 'playing' a 'double' role. Also, comparing these comparative statics to those for the restricted borrowing case, here the default cost borrowing has created special linkages between the periods – $\partial E_1/\partial W_{2w}$, $\partial E_1/\partial W_{2\ell}$ are nonzero.

The principal's problem, given (4.21) - (4.26), is to solve:[9]

$$\begin{aligned}
\max \text{ L} \;=\;& RW_\ell + R^2W_{2\ell} + h + .5\beta[W_w - W_\ell + y - h(1+r) \\
& -C[(h(1+r) - x)A^{-1}]] + .5\beta^2 P_2(W_{2w} - W_{2\ell} - y(1+r) \\
& -C[x(1+r)A^{-1}]) + .5\beta^2(1-P_2)(W_{2w} - W_{2\ell} - x(1+r) \\
& -C[y(1+r)A^{-1}]) - .5\beta^2 C(E_w) - .5\beta^2 C(E_\ell) - \beta C(E_1) \\
& +R\lambda(DE_1 + .5RD(E_w + E_\ell) - .5(W_w + W_\ell) \\
& -.5R(W_{2w} + W_{2\ell}))
\end{aligned}$$

Proposition 4 *a) The wage structure is non-unique.*

b) The contracts that solve the above problem all involve the following values:

$$E_1 = RD(1+\rho) + \frac{r-\rho}{2\bar{g}(1+r)}, \tag{4.27}$$

$$E_\ell = \tau + R^2D(1+\rho)^2 + .5R(1+\rho)(k), \tag{4.28}$$

$$E_w = \tau + R^2D(1+\rho)^2 - .5R(1+2r-\rho)(k), \tag{4.29}$$

where

$$\begin{aligned}
\tau &= \frac{(r-\rho)(3-2P_2+\rho+r(2-2P_2))}{2(1+r)^2\bar{g}} < 0, \\
k &= \bar{g}[(1+r)(y-x) + C[x(1+r)A^{-1}] - C[y(1+r)A^{-1}]] > 0,
\end{aligned}$$

and

$$\begin{aligned}
E_\ell - E_w &= k > 0, \\
\frac{\partial E_1}{\partial D} > 0, \; \frac{\partial E_1}{\partial \bar{g}} > 0, \; & \frac{\partial E_1}{\partial r}, \frac{\partial E_1}{\partial \rho} \gtrless 0.
\end{aligned}$$

[9] An exercise similar to that done in the repeated model yields the zero-profit condition-

$$\frac{\partial E_\ell}{\partial D} > 0, \ \frac{\partial E_\ell}{\partial \bar{g}} > 0, \ \frac{\partial E_\ell}{\partial r}, \frac{\partial E_\ell}{\partial \rho} \gtrless 0.$$
$$\frac{\partial E_w}{\partial D} > 0, \ \frac{\partial E_w}{\partial \bar{g}}, \frac{\partial E_w}{\partial r}, \frac{\partial E_w}{\partial \rho} \gtrless 0.$$

c) At the optimum the principal desires the agent to work harder in the first period, he desires a winning agent to work harder in the second period, while for a losing agent in the second period he desires the agent to work more or less then he does work.

Proof: a) An exercise similar to that of Proposition 3 of Chapter Two yields the result.

b) See Appendix 4.2 for the solving of the first-order conditions, etc.

c) Evaluate the following at the equilibrium values for the effort levels and for λ:

$$\frac{\partial L}{\partial E_1} = -\beta C'(E_1) + R\lambda D = \frac{\rho - r}{2\bar{g}(1+r)(1+\rho)} > 0,$$
$$\frac{\partial L}{\partial E_w} = -.5\beta^2 E_w + .5R^2\lambda D = -.5\beta^2 r + .25R\beta^2(1+2r-\rho)k > 0.$$
$$\frac{\partial L}{\partial E_\ell} = -.5\beta^2 E_\ell + .5R^2\lambda D = -.5\beta^2 r - .25R\beta k \gtrless 0.\square$$

It is easier to get a solution in this problem then in the repeated contract case. This ease is due to the two-period nature of the solution, which makes comparative statics such as $\partial E_1/\partial W_w$ much less complex. However, as in the earlier case a toasting result holds – first period winners work less in the second-period than first period losers. As well, there are again memory effects in output. These crucial results from the repeated contracts generalize to the two-period contracts.

Finally, consider the introduction of second-period mobility constraints on the principal's problem. Since first period winners and losers will act differently in the second period, separate constraints are needed for each. The second-period profit constraints are:

$$P_2W_{2w} + (1-P_2)W_{2\ell} - DE_w \geq 0, \quad (4.30)$$

$$(1-P_2)W_{2w} + P_2W_{2\ell} - DE_\ell \geq 0. \quad (4.31)$$

Second, the constraint that utility is higher if a worker stays with the firm than if he moves to a reserve job, can be constructed as in Chapter Two and Three. These constraints are formally constructed in Appendix 4.3, and for a winner and a loser are respectively:

$$RW_{2\ell} + y + \beta(P_2(W_{2w} - W_{2\ell} - y(1+r)) -$$

$$(1-P_2)C[y(1+r)A^{-1}] - C(E_w)) - .5R^2(D_r)^2(1+\rho) \geq 0, \quad (4.32)$$

$$-C[(h(1+r)-x)A^{-1}] + \beta((1-P_2)(W_{2w} - W_{2\ell}$$

$$-x(1+r)) - P_2C[x(1+r)A^{-1}] - C(E_\ell)) - \bar{V}_\ell \geq 0, \quad (4.33)$$

where

$$\frac{\partial \bar{V}_\ell}{\partial W_{2\ell}} < 0, \ \frac{\partial \bar{V}_\ell}{\partial W_\ell} = 0, \ \frac{\partial \bar{V}_\ell}{\partial W_w} = 0, \ \frac{\partial \bar{V}_\ell}{\partial W_{2w}} = 0.$$

Proposition 5 *The mobility constraints do not bind.*

Proof: Let λ_1 be the multiplier on the zero-profit constraint and $\lambda_2 - \lambda_5$ be the multipliers on the new constraints (4.30) - (4.33). The first-order conditions for the new principal's problem for $\partial L/\partial W_w$ and $\partial L/\partial W_\ell$ are identical to the case without mobility constraints, and hence still yield $\lambda_1 = 1$ and equation (4.27). The first-order conditions for $\partial L/\partial W_{2w}$ and $\partial L/\partial W_{2\ell}$ are:

$$\begin{aligned}\frac{\partial L}{\partial W_{2w}} := \ & .5\beta^2(P_2 + 1 - P_2) - .5\beta^2(C'(E_w)\frac{\partial E_w}{\partial W_{2w}} + C'(E_\ell)\frac{\partial E_\ell}{\partial W_{2w}}) \\ & -\beta C'(E_1)\frac{\partial E_1}{\partial W_{2w}} + R\lambda_1(D\frac{\partial E_1}{\partial W_{2w}} + .5DR(\frac{\partial E_w}{\partial W_{2w}} \\ & + \frac{\partial E_\ell}{\partial W_{2w}}) - .5R) + \lambda_2(P_2 - D\frac{\partial E_w}{\partial W_{2w}}) + \lambda_3(1 - \\ & P_2 - D\frac{\partial E_\ell}{\partial W_{2w}}) + \lambda_4(\beta P_2 - \beta C'(E_w)\frac{\partial E_w}{\partial W_{2w}})\end{aligned}$$

$$+\lambda_5(\beta(1 - P_2) - \beta C'(E_\ell)\frac{\partial E_\ell}{\partial W_{2w}}) = 0. \quad (4.34)$$

$$\begin{aligned}\frac{\partial L}{\partial W_{2\ell}} = {} & R^2 - .5\beta^2(P_2 + 1 - P_2) - .5\beta^2(C'(E_w)\frac{\partial E_w}{\partial W_{2\ell}} + \\ & C'(E_\ell)\frac{\partial E_\ell}{\partial W_{2\ell}}) - \beta C'(E_1)\frac{\partial E_1}{\partial W_{2\ell}} + R\lambda_1(D\frac{\partial E_1}{\partial W_{2\ell}} \\ & + .5DR(\frac{\partial E_w}{\partial W_{2\ell}} + \frac{\partial E_\ell}{\partial W_{2\ell}}) - .5R) + \lambda_2(1 - P_2 - D\frac{\partial E_w}{\partial W_{2\ell}}) \\ & + \lambda_3(P_2 - D\frac{\partial E_\ell}{\partial W_{2\ell}}) + \lambda_4(R - \beta P_2 - \beta C'(E_w)\frac{\partial E_w}{\partial W_{2\ell}}) \\ & + \lambda_5(-\beta(1 - P_2) - \beta C'(E_\ell)\frac{\partial E_\ell}{\partial W_{2\ell}} - \frac{\partial \bar{V}_\ell}{\partial W_{2\ell}}) = 0. \quad (4.35)\end{aligned}$$

Add together (4.34) and (4.35), recalling:

$$\lambda_1 = 1, \text{ and } \frac{\partial E_1}{\partial W_{2w}} = -\frac{\partial E_1}{\partial W_{2\ell}}, \frac{\partial E_w}{\partial W_{2w}} = -\frac{\partial E_w}{\partial W_{2\ell}}, \frac{\partial E_\ell}{\partial W_{2w}} = -\frac{\partial E_\ell}{\partial W_{2\ell}}.$$

This yields

$$\lambda_2 + \lambda_3 + \lambda_4(R) + \lambda_5(-\frac{\partial \bar{V}_\ell}{\partial W_{2\ell}}) = 0.$$

Given $\lambda_2, \lambda_3, \lambda_4, \lambda_5 \geq 0$ by construction and $\partial \bar{V}_\ell / \partial W_{2\ell} < 0$, this implies $\lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0$. None of the constraints bind. □

The fact that the mobility constraints do not bind means that all previous results on the values of effort levels, and most especially the result $E_\ell > E_w$, go through unchanged. The intuition of why this result holds follows the intuition of the similar results of Chapter Two, section 4.

In this section the principal was allowed to choose two-period contracts over repeated one-period contracts, and did indeed choose to do so. However, firms were still not allowed to offer agents with different histories different contracts. The next section addresses this issue.

4.4 Memory Contracts

In this section, the principal now offers memory wage contracts identical to those of Section 5 of Chapter Two. Thus, once again in the second period, winners are sorted to play each other for W_{ww}, $W_{w\ell}$, and losers play each other for $W_{\ell w}$, $W_{\ell\ell}$. (First period wages are still W_w, W_ℓ.) The sorting allows the principal to use memory effects in compensation. With sorting an asymmetric equilibrium no longer results in the second period (although winners and losers contests are still different). This in turn makes the problem much simpler, despite the more complex wage structure.

The probability of winning an individual contest (P_1, P_w and P_ℓ) is formed analogously to the earlier contests. The following conditions hold on wages:[10]

$$\begin{aligned} W_w &> B_1(1+r) > W_\ell + B_\ell, \\ W_{ww} &> B_w(1+r) > W_{w\ell}, \\ W_{\ell w} &> B_\ell(1+r) > W_{\ell\ell}. \end{aligned}$$

Therefore, the agent's problem is to select borrowing and effort to maximize the following (with the appropriate V_w, V_ℓ substituted in):

$$\begin{aligned} V = {} & B_1 + \beta P_1[W_w - B_1(1+r) + B_w + \beta(P_w(W_{ww} \\ & -B_w(1+r)) - (1-P_w)C[(B_w(1+r) - W_{w\ell})A^{-1}] \\ & -C(E_w))] + \beta(1-P_1)[-C[(B_1(1+r) - W_\ell - B_\ell)A^{-1}] \\ & +\beta(P_\ell(W_{\ell w} - B_\ell(1+r)) - (1-P_\ell)C[(B_\ell(1+r) - W_{\ell\ell})A^{-1}] \\ & -C(E_\ell))] - \beta C(E_1). \end{aligned}$$

In a symmetric Nash equilibrium, $P_w = P_\ell = P_1 = .5$. Solving the above for the agent's actions yields:

$$B_1 = RW_\ell + R^2W_{\ell\ell} + h. \tag{4.36}$$

[10] As above, in each condition the first set of inequalities is assumed and the second set comes from Proposition 1

$$B_w = RW_{w\ell} + y, \tag{4.37}$$

$$B_\ell = RW_{\ell\ell} + x, \tag{4.38}$$

$$\begin{aligned} E_1 = {} & \bar{g}[W_w - W_\ell + y + R(W_{w\ell} - W_{\ell\ell}) - h(1+r) + \\ & C[(h(1+r) - x)A^{-1}] + \beta(.5(W_{ww} - W_{w\ell} - (1+r)y \\ & -C[y(1+r)A^{-1}]) - .5(W_{\ell w} - W_{\ell\ell} - (1+r)x - \\ & C[x(1+r)A^{-1}]) + C(E_\ell) - C(E_w))], \end{aligned} \tag{4.39}$$

$$E_w = \bar{g}(W_{ww} - W_{w\ell} - (1+r)y + C[y(1+r)A^{-1}]), \tag{4.40}$$

$$E_\ell = \bar{g}(W_{\ell w} - W_{\ell\ell} - (1+r)x + C[x(1+r)A^{-1}]). \tag{4.41}$$

where x, y and h are as before, with $P_2 = P_w = P_\ell = .5$.

The above functions reveal complex intertemporal links, as in the earlier case with restricted borrowing. Clearly, first period effort depends on wages in the second period (for example, $\partial E_1/\partial W_{\ell\ell} \neq 0$). Sorting creates intertemporal links that the principal will be able to exploit to achieve a more profitable solution, compared to a nonsorting (nonmemory) model. Examining E_1 reveals that the return to winning the first period contest consists of an immediate wage differential, the avoidance of debt, and the capital gain of the winners' contest over the losers' contest.

The comparitive statics are derived in Appendix 4.4, and are as follows:

$$\frac{\partial B_1}{\partial W_r} > 0,\ \frac{\partial B_w}{\partial W_\ell} = 0,\ \frac{\partial B_\ell}{\partial W_\ell} = 0,$$

$$\frac{\partial B_1}{\partial W_{\ell\ell}} > 0,\ \frac{\partial B_w}{\partial W_{w\ell}} > 0,\ \frac{\partial B_\ell}{\partial W_{\ell\ell}} > 0,$$

$$\frac{\partial B_1}{\partial r} < 0,\ \frac{\partial B_w}{\partial r} < 0,\ \frac{\partial B_\ell}{\partial r} < 0,$$

$$\frac{\partial B_1}{\partial \rho} > 0,\ \frac{\partial B_w}{\partial \rho} > 0,\ \frac{\partial B_\ell}{\partial \rho} \gtrless 0,$$

$$\frac{\partial E_1}{\partial W_w} = \frac{\partial E_w}{\partial W_{ww}} = \frac{\partial E_\ell}{\partial W_{\ell w}} = \bar{g} > 0,$$

$$\frac{\partial E_1}{\partial W_\ell} = \frac{\partial E_w}{\partial W_{w\ell}} = \frac{\partial E_\ell}{\partial W_{\ell\ell}} = -\bar{g} < 0,$$

$$\frac{\partial E_1}{\partial W_{ww}} = .5\beta\bar{g} - \beta\bar{g}^2 E_w \gtrless 0,$$
$$\frac{\partial E_1}{\partial W_{w\ell}} = \bar{g}(R - .5\beta) + \beta\bar{g}^2 E_w > 0,$$
$$\frac{\partial E_1}{\partial W_{\ell w}} = -.5\bar{g}\beta + \beta\bar{g}^2 E_\ell \gtrless 0,$$
$$\frac{\partial E_1}{\partial W_{\ell\ell}} = \bar{g}(.5\beta - R) - \beta\bar{g}^2 E_\ell < 0,$$
$$\frac{\partial E_w}{\partial W_w} = \frac{\partial E_w}{\partial W_\ell} = \frac{\partial E_w}{\partial W_{\ell w}} = \frac{\partial E_w}{\partial W_{\ell\ell}} = 0,$$
$$\frac{\partial E_\ell}{\partial W_w} = \frac{\partial E_\ell}{\partial W_\ell} = \frac{\partial E_\ell}{\partial W_{w\ell}} = \frac{\partial E_\ell}{\partial W_{ww}} = 0.$$

Note once again, there is a relationship between the current comparative statics and those from the repeated model (where W_w plays a 'triple' role)

$$\frac{\partial E_1}{\partial W_w}(\text{memory}) + \frac{\partial E_1}{\partial W_{ww}}(\text{memory}) + \frac{\partial E_1}{\partial W_{\ell w}}(\text{memory}) = \frac{\partial E_1}{\partial W_w}(\text{repeated}),$$

allowing for $P_2 = .5$. Second, comparing these comparative statics to those of the restricted borrowing memory case, note that the comparative statics are identical for the wage terms (the comparative statics for r or ρ would not be the same, although in both cases they are quite complex and of ambiguous sign). This is not surprising, as the extra terms in the default cost model are not functions of the wages, and hence the comparative statics are the same.

The principal selects wages to maximize utility subject to a zero profit condition, given (4.36) - (4.41):

$$\begin{aligned}\max \text{L} = \; & RW_\ell + R^2W_{\ell\ell} + h + .5\beta[W_w - W_\ell + R(W_{w\ell} - W_{\ell\ell}) \\ & y - h(1+r) - C[(h(1+r) - x)A^{-1}]] + .5\beta^2(.5(W_{ww} \\ & -W_{w\ell} - y(1+r) - C[y(1+r)A^{-1}]) + .5(W_{\ell w} - W_{\ell\ell} \\ & -x(1+r) - C[x(1+r)A^{-1}]) - C(E_w) -\end{aligned}$$

$$G(E_\ell) - 3C(E_1) + R\lambda(DE_1 + .5RD(E_w + E_\ell) -$$

$$.5(W_w + W_\ell) - .25R(W_{ww} + W_{w\ell} + W_{\ell w} + W_{\ell\ell})).$$

Proposition 6 *a) The wage structure is nonunique.*

b) The contracts that solve the above all involve the following values:

$$E_1 = \frac{D(1+\rho)}{1+r} + \frac{r-\rho}{2\bar{g}(1+r)}. \quad (4.42)$$

$$E_w = \frac{D(1+\rho)}{1+r} + \frac{r-\rho}{2\bar{g}(1+r)}. \quad (4.43)$$

$$E_\ell = \frac{D(1+\rho)^2}{(1+r)(1+2r-\rho)} + \frac{(r-\rho)(3+2r+\rho)}{2\bar{g}(1+r)(1+2r-\rho)}. \quad (4.44)$$

where

$$E_\ell - E_w = \frac{(\rho-r)(1+\rho)}{(1+r)(1+2r-\rho)}(2D - \frac{1}{\bar{g}}) \gtrless 0,$$

$$\frac{\partial E_1}{\partial D} = \frac{\partial E_w}{\partial D} > 0, \frac{\partial E_1}{\partial \bar{g}} = \frac{\partial E_w}{\partial \bar{g}} > 0,$$

$$\frac{\partial E_1}{\partial \rho} = \frac{\partial E_w}{\partial \rho} \propto (2D\bar{g} - 1) \gtrless 0, \frac{\partial E_1}{\partial r} = \frac{\partial E_w}{\partial r} \propto -(2D\bar{g} - 1) \gtrless 0,$$

$$\frac{\partial E_\ell}{\partial D} > 0, \frac{\partial E_\ell}{\partial \bar{g}} > 0, \frac{\partial E_\ell}{\partial \rho} \propto (2D\bar{g} - 1) \gtrless 0,$$

$$\frac{\partial E_\ell}{\partial r} \propto -(2D\bar{g} - 1) \gtrless 0.$$

c) At the optimum the principal desires the agent to work harder in the first period contest and in the winners' contest.

<u>**Proof:**</u> Analogous to that of Proposition 3 of Chapter Two.

It can be seen here that $E_\ell \neq E_w$ (except in the unlikely case where $2D\bar{g} = 1$). Therefore the contracts exhibit memory effects – the agents' second period effort depends on current and past performance.[11] This result hints at the presence

[11] It is impossible to compare the expected value of the memory to nonmemory contracts in this case, as the algebra is intractable. These results are therefore not as strong as those of Chapter Two.

of memory effects similar to those shown in the restricted borrowing models of Chapter Two. However, since the expected value of the contracts cannot be calculated, it is not possible to state with certainty that memory contracts dominate in the default cost borrowing model.

In comparison to the default cost nonmemory contracts, note that with memory contracts, the agents' equilibrium output in the second period is not a function of debt. In the nonmemory section, the principal was forced to allow winners to play losers. The past of the loser was different than the past of the winner, which affected their behaviour, and in turn affected the wages set and the equilibrium output. In the memory contract losers are sorted to play losers, so that their pasts (debts) are common, and behaviour is identical. Therefore, with symmetry the default costs do not influence the final equilibrium effort levels.

The wage structure will be nonunique, because agents' borrowing 'undoes' any wage changes, leaving consumption unchanged. However, the consumption values for each outcome are unique and can be solved for. Once again, define Z as 'consumption' in a given state:

$$Z_0 = RW_\ell + R^2 W_{\ell\ell} + h, \tag{4.45}$$

$$Z_w = W_w - W_\ell + R(W_{w\ell} - W_{\ell\ell}) + y - h(1+r), \tag{4.46}$$

$$Z_\ell = -C[(h(1+r) - x)A^{-1}], \tag{4.47}$$

$$Z_{ww} = W_{ww} - W_{w\ell} - y(1+r), \tag{4.48}$$

$$Z_{w\ell} = -C[y(1+r)A^{-1}], \tag{4.49}$$

$$Z_{\ell w} = W_{\ell w} - W_{\ell\ell} - x(1+r), \tag{4.50}$$

$$Z_{\ell\ell} = -C[x(1+r)A^{-1}] \tag{4.51}$$

(Note a loser has the disutility of the effort of paying off the default costs, which are defining as 'negative consumption'.) In a manner analogous to the steps in Chapter 2, Section 5, one can solve for the above consumption values as functions of the known values E_1, E_w and E_ℓ (see Appendix 4.4 for details). This allows for a comparison of consumption effects across periods:

Proposition 7 *Define Δ_1 and Δ_w as the spread of consumption in a respective contest, and $\bar{Z}_1$, $\bar{Z}_w$ as the mean consumption, such that:*

$$\Delta_1 = Z_w - Z_\ell, \ \Delta_w = Z_{ww} - Z_{w\ell},$$

$$\bar{Z}_1 = .5(Z_w + Z_\ell), \ \bar{Z}_w = .5(Z_{ww} + Z_{w\ell}).$$

Then, it can be shown that $\Delta_1 < \Delta_w$, $\bar{Z}_1 < \bar{Z}_w$.

Proof: See Appendix 4.4.

This result shows strong, testable memory effects in consumption values. These are analogous to results in wage patterns found in Rogerson (1985a) or Rosen(1986). These effects will be discussed further in Chapter Five.

Finally, consider again the introduction of second-period mobility constraints on the principal's problem. The second-period profit constraints are:

$$.5(W_{ww} + W_{w\ell}) - DE_w \geq 0, \tag{4.52}$$

$$.5(W_{\ell w} + W_{\ell\ell}) - DE_\ell \geq 0. \tag{4.53}$$

In addition, there are the constraints that second-period utility be higher at the contest job, formally constructed in Appendix 4.3:

$$RW_{w\ell} + y + \beta(.5(W_{ww} - W_{w\ell} - y(1+r)) - .5C[y(1+r)A^{-1}]$$
$$-C(E_w)) - .5R^2(D_r)^2(1+\rho) \geq 0, \tag{4.54}$$

$$-C[(h(1+r) - x)A^{-1}] + \beta(.5(W_{\ell w} - W_{\ell\ell} - x(1+r))$$
$$-.5C[x(1+r)A^{-1}] - C(E_\ell)) - \bar{V}_\ell \geq 0, \tag{4.55}$$

where

$$\frac{\partial \bar{V}_\ell}{\partial W_{2\ell}} < 0, \frac{\partial \bar{V}_\ell}{\partial W_\ell} = 0, \frac{\partial \bar{V}_\ell}{\partial W_w} = 0, \frac{\partial \bar{V}_\ell}{\partial W_{2w}} = 0.$$

Proposition 8 *a) The mobility constraints do not bind.*
b) $.5(W_{ww} + W_{wl}) > .5(W_w + W_l)$.

Proof: a) Analogous to the proof of Proposition 5.
b) Analogous to the proof of Proposition 7 of Chapter Two.

Since the mobility constraints do not bind, the earlier results go through unchanged. However, as in the restricted borrowing case, the fact they do not bind implies that mean wages tend to rise over the term of the contract. This is a strong general prediction of both borrowing models.

4.5 Summary

In this chapter the rank-order tournament model has been extended to the situation where the agent has access to a capital market in which he must pay all debts (by working at a 'reserve job'), if necessary. This allowed for generalization of many earlier results, including non-uniqueness in the wage structure of nonmemory and memory contracts (although effort and consumption were unique).

The most powerful predictions of the restricted borrowing model also generalized to the default cost borrowing model. The most important of these predictions is the various memory effects in consumption and effort, effects created by the principal's preference for a contract with memory effects in wages, and by the capital market structure. In addition, in this model it was also shown again that the mean and spread of consumption rise over the length of the memory contract. Finally, the result that mean wages rise over the length of the contract is also confirmed.

The default cost model is a broader model,[12] and hence generates more predictions. Some of these predictions will be dealt with in more detail in Chapter Five, but the most important result is that if a firm is not allowed to sort agents in the second period (perhaps due to union rules), then in the second period, first period losers are predicted to have higher average output than first period winners. That is, losers always work harder ($E_\ell > E_w$), and since on average the error term equals zero, losers would produce higher average output. This is a powerful, testable prediction.

[12]For example, it includes the restricted borrowing model as a special case, where the cost of effort at the reserve job fails to meet the conditions stated in Proposition 1.

APPENDICES TO CHAPTER FOUR

4.1 Details of the Repeated Model

The proof of Proposition 2 of Chapter Four is as follows.

a) The first step is to prove the equilibrium to the second period contest is asymmetric ($E_\ell \neq E_u$). Assume $E_\ell = E_w$. Then $P_2 = .5$. Given this, calculate the following:

$$
\begin{aligned}
E_\ell - E_w &= \bar{g}(C[x(1+r)A^{-1}] - C[y(1+r)A^{-1}] + (1+r)(y-x)) \\
&= \bar{g}(.5A^{-2}x^2(1+r)^2 - .5A^{-2}y^2(1+r)^2 + (1+r)(y-x)) \\
&= .5A^{-2}\bar{g}(1+r)(x-y)[-2A^2 + (1+r)(x+y)] \\
&= .5A^{-2}\bar{g}(1+r)(R^3A^2((1+2\rho-r)(2+2\rho) - (1+r)^2) \\
&\quad -R^2A^2(2+2\rho-(1+r)))[-2A^2 + \\
&\quad (1+r)\left[R^3A^2[(1+2\rho-r)(2+2\rho) - (1+r)^2] + \right. \\
&\quad \left. R^2A^2(2+2\rho-1-r)\right]],
\end{aligned}
$$

where $P_2 = .5$ has been substituted in. Continuing with the simplification.

$$
\begin{aligned}
E_\ell - E_w &= .5\bar{g}A^{-2}(1+r)(R^3A^2[(1+2\rho-r)(2+2\rho-1-r) \\
&\quad -1-2r-r^2])[-2A^2 + (1+r)\left[R^3A^2[(1+2\rho-r)(2+\right. \\
&\quad \left. 2\rho+1+r) - 1 - 2r - r^2]\right]] \\
&= .5R^2g(1+2\rho-r+2\rho+4\rho^2-2\rho r - r - 2\rho r + r^2 \\
&\quad -1-2r-r^2)[-2A^2R^2(1+r)^2 + R^2A^2[(1+2\rho \\
&\quad -r)(3+2\rho+r) - 1 - 2r - r^2]] \\
&= .5R^4\bar{g}A^2(4\rho - 4r + 4\rho^2 - 4\rho r)[-2-4r-2r^2+3 \\
&\quad +2\rho + r + 6\rho + 4\rho^2 + 2\rho r - 3r - 2\rho r - r^2 - 1 \\
&\quad -2r - r^2]
\end{aligned}
$$

$$= .5R^4\bar{g}A^2(4(\rho - r)(1+\rho))[8\rho - 8r + 4\rho^2 - 4r^2]$$
$$= R^4\bar{g}A^2 2(\rho - r)^2(1+\rho)(8 + 4(\rho + r)) > 0.$$

Thus, $E_\ell > E_w$, a contradiction and therefore E_ℓ cannot equal E_w.

b) Given the above result the best response function $E_\ell(E_w)$ crosses the 45° line above $E_w(E_\ell)$. Therefore, in order to prove the second part of the proof ($E_\ell > E_w$), the best response-functions must be examined to see if they are single-turning. To ease the task some simplifying assumptions will be made, similar to those made by Lazear and Rosen (1981). Assume that $G(\cdot)$, the cumulative density function of $\epsilon_i - \epsilon_j$ is a normal distribution with mean zero, and variance $2\sigma^2$. Therefore:

$$g(E_\ell - E_w) = \frac{1}{\sqrt{(4\sigma^2\pi)}} \exp\left(\frac{-.5(E_\ell - E_w)^2}{2\sigma^2}\right) = g(E_w - E_\ell),$$
$$\frac{\partial g(E_\ell - E_w)}{\partial E_\ell} = \frac{(E_w - E_\ell)}{2\sigma^2}[g(E_\ell - E_w)],$$
$$\frac{\partial g(E_\ell - E_w)}{\partial E_w} = \frac{(E_\ell - E_w)}{2\sigma^2}[g(E_\ell - E_w)] = -\frac{\partial g(E_\ell - E_w)}{\partial E_\ell}.$$

Recall also $P_2 = G(E_i - E_j)$, where P_2 = a fixed number only at the Nash equilibrium. Thus, over the range of the BRFs $\partial P_2/\partial E_i \neq 0$. For simplicity, assume the solution is close enough to the Nash equilibrium that the following holds:

$$\frac{\partial P_2}{\partial E_w} = \frac{\partial P_2}{\partial E_\ell} = \bar{g}.$$

Start with the first-order conditions, which define the BRFs:

$$E_w = \bar{g}(W_w - W_\ell - (1+r)y + C[y(1+r)A^{-1}]),$$
$$E_\ell = \bar{g}(W_w - W_\ell - (1+r)x + C[x(1+r)A^{-1}]).$$

Totally differentiate E_w w.r.t. E_w and E_ℓ:

$$dE_w = \frac{E_w}{\bar{g}}\frac{\partial \bar{g}}{\partial E_w}dE_w + \bar{g}(-(1+r) + A^{-2}(1+r)^2 y)\frac{\partial y}{\partial P_2}\frac{\partial P_2}{\partial E_w}dE_w + \frac{E_w}{\bar{g}}\frac{\partial \bar{g}}{\partial E_\ell}dE_\ell + \bar{g}(-(1+r) + A^{-2}(1+r)^2 y)\frac{\partial y}{\partial P_2}\frac{\partial P_2}{\partial E_\ell}dE_\ell.$$

Note the following:

$$\frac{\partial y}{\partial P_2} = \frac{A^2(\rho - r)}{(1+r)^2(1-P_2)^2} > 0.$$

$$-(1+r) + A^{-2}(1+r)^2 y = \frac{\rho - r}{1 - P_2} > 0.$$

Define the following elasticity:

$$\kappa = \frac{\partial \tilde{g}}{\partial E_w}\frac{E_w}{\tilde{g}} = \frac{(E_\ell - E_w)E_w}{2\sigma^2} \gtrless 0 \text{ as } E_\ell \gtrless E_w.$$

Substitution of these values into dE_w yields:

$$dE_w = \kappa dE_w + \frac{\tilde{g}^2 A^2(\rho - r)^2(dE_w + dE_\ell)}{(1+r)^2(1-P_2)^3} - \kappa dE_\ell.$$

or upon rearrangement:

$$\frac{\partial E_w}{\partial E_\ell} = \left(-\kappa + \frac{\tilde{g}^2 A^2(\rho - r)^2}{(1+r)^2(1-P_2)^3}\right) \Big/ \left(1 - \kappa - \frac{\tilde{g}^2 A^2(\rho - r)^2}{(1+r)^2(1-P_2)^3}\right).$$

Note that the denominator of this term is related to the term

$$\frac{\partial^2 V}{\partial E_w^2} = \frac{E_w}{\tilde{g}}\frac{\partial \tilde{g}}{\partial E_w} + \tilde{g}(-(1+r) + A^{-2}(1+r)^2 y)\frac{\partial y}{\partial P_2}\frac{\partial P_2}{\partial E_w} - 1,$$

which is assumed negative (necessary to get a unique solution to the agent's problem). Thus, the denominator is assumed positive as it equals $-\partial^2 V/\partial E_w^2$.

Given that the second term in the numerator is positive, next examine the first term :

$$-\kappa = \frac{(E_w - E_\ell)E_w}{2\sigma^2}.$$

It can be seen that:

$\partial E_w/\partial E_\ell > 0$ if $E_w > E_\ell$.

$\partial E_w/\partial E_\ell > 0$ if $E_w = E_\ell$.

$\partial E_w/\partial E_\ell > 0$ if $E_w < E_\ell$ by a little, so that κ is small.

$\partial E_w/\partial E_\ell < 0$ if $E_w < E_\ell$ by a lot, so that κ is large.

Thus $E_w(E_\ell)$ is single-turning, and has the shape shown in Figure 1 of Chapter Four.

Now totally differentiate $E_\ell(E_w)$:

$$\begin{aligned} dE_\ell &= \frac{E_\ell}{\tilde{g}}\frac{\partial \tilde{g}}{\partial E_\ell}dE_\ell + \tilde{g}(-(1+r) + A^{-2}(1+r)^2 x)\frac{\partial x}{\partial P_2}\frac{\partial P_2}{\partial E_\ell}dE_\ell + \\ &\quad \frac{E_\ell}{\tilde{g}}\frac{\partial \tilde{g}}{\partial E_w}dE_w + \tilde{g}(-(1+r) + A^{-2}(1+r)^2 x)\frac{\partial x}{\partial P_2}\frac{\partial P_2}{\partial E_w}dE_w. \end{aligned}$$

Note that:

$$\begin{aligned} \frac{\partial x}{\partial P_2} &= \frac{A^2(r-\rho)(3+2\rho+r)}{(1+r)^3 P_2^2} < 0, \\ -(1+r) + A^{-2}x(1+r)^2 &= \frac{(\rho-r)(3+2\rho+r)}{(1+r)P_2} > 0. \end{aligned}$$

Define the following elasticity:

$$\psi = \frac{\partial \tilde{g}}{\partial E_\ell}\frac{E_\ell}{\tilde{g}} = \frac{(E_w - E_\ell)E_\ell}{2\sigma^2} \gtrless 0 \text{ as } E_w \gtrless E_\ell.$$

Finally, note that:

$$\frac{\partial^2 V}{\partial E_\ell^2} = \frac{E_\ell}{\tilde{g}}\frac{\partial \tilde{g}}{\partial E_\ell} + \tilde{g}(-(1+r) + A^{-2}(1+r)^2 x)\frac{\partial x}{\partial P_2}\frac{\partial P_2}{\partial E_\ell} - 1,$$

and once again to get a unique solution it is assumed $\partial^2 V/(\partial E_\ell)^2 < 0$. Substitution of these terms into dE_ℓ and rearrangement yields:

$$\frac{\partial E_\ell}{\partial E_w} = \left(-\psi - \frac{\tilde{g}^2 A^2(\rho-r)^2(3+2\rho+r)^2}{(1+r)^4 P_2^3}\right) \Bigg/ \left(-\frac{\partial^2 V}{\partial E_\ell^2}\right).$$

The denominator is positive, and the second term in the numerator is negative. Note that $-\psi = ((E_\ell - E_w)E_\ell)/2\sigma^2$, so that:

$\partial E_\ell/\partial E_w < 0$ if $E_w > E_\ell$,

$\partial E_\ell/\partial E_w < 0$ if $E_w = E_\ell$,

$\partial E_\ell/\partial E_w < 0$ if $E_\ell > E_w$ by a little, so that ψ is small,

$\partial E_\ell/\partial E_w > 0$ if $E_w < E_\ell$ by a lot, so that ψ is large.

Thus $E_\ell(E_w)$ is also single-turning, and shaped as in Figure 1.

With $E_\ell(E_w)$ and $E_w(E_\ell)$ as in Figure 1, clearly the Nash equilibrium is unique with $E_\ell > E_w$, and therefore the probability of a winner in the first-period winning the second period is less than .5 ($P_2 < .5$).[1]

For the final part of the proof of Proposition 2 of Chapter Four, note that,

$$\begin{aligned}
B_\ell - B_w &= x - y \\
&= \frac{A^2[(1+2\rho-r)(1+\rho)-(1+r)^2(1-P_2)]}{(1+r)^3 P_2} \\
&\quad - \frac{A^2[1+\rho-P_2(1+r)]}{(1+r)^2(1-P_2)} \\
&= \frac{A^2}{(1+r)^3(1-P_2)P_2}[(1+2\rho-r)(1+\rho)(1-P_2) - \\
&\quad (1+r)^2(1-P_2)^2-(1+\rho)(1+r)P_2+P_2^2(1+r)^2] \\
&= \frac{A^2}{(1+r)^3(1-P_2)P_2}[(1+2\rho-r)(1+\rho)-P_2(1+\rho)[1+2\rho- \\
&\quad r+1+r]+(1+r)^2[P_2^2-(1-P_2)^2]] \\
&= \frac{A^2}{(1+r)^3(1-P_2)P_2}[(1+2\rho-r)(1+\rho)-2P_2(1+\rho)^2 \\
&\quad +(1+r)^2(2P_2-1)] \\
&= \frac{A^2}{(1+r)^3(1-P_2)P_2}[1+2\rho-r+\rho+2\rho^2-\rho r-1-2r-r^2- \\
&\quad 2P_2((1+\rho)^2-(1+r)^2)] \\
&= \frac{A^2[3\rho-3r+2\rho^2-\rho r-r^2-2P_2(2+\rho+r)(\rho-r)]}{(1+r)^3(1-P_2)P_2} \\
&= \frac{A^2(\rho-r)[3-4P_2+\rho(2-2P_2)+r(1-2P_2)]}{(1+r)^3(1-P_2)P_2} > 0,
\end{aligned}$$

given $P_2 < .5$. Thus $B_\ell > B_w$. □

The next step is to derive the comparative statics for borrowing and effort from the solutions to the agent's problem. As in Lazear and Rosen (1981) these comparative statics are carried out at the Nash equilibrium. Thus, in the first

[1] As Rosen (1986) notes in a similar context, there can be problems in this analysis if the global optimum conditions fails. It is assumed this condition is always met.

period $E_1 = E_2$, $P_1 = .5$ and therefore $dP_1 = 0$ and $\partial\bar{g}/\partial E_1 = 0$. In the second period with $E_\ell > E_w$, as was shown:

$$\frac{\partial\bar{g}}{\partial E_w} = -\frac{\partial\bar{g}}{\partial E_\ell} = \frac{(E_\ell - E_w)\bar{g}}{2\sigma^2} > 0.$$

However, it is assumed the solution is close enough to the Nash equilibrium that the change in $(E_\ell - E_w) \approx 0$ and hence P_2 is constant.

In this situation, the borrowing values are independent of the effort levels, and thus to get the comparative statics on the former one merely takes derivatives. The wage values are trivial, for the others:

$$\begin{aligned}\frac{\partial B_1}{\partial r} &= R^4[(1+r)^2W_\ell - W_\ell(2+r)2(1+r)] + \frac{A^2}{P_2^2(1+r)^8}[(1\\ &\quad +r)^4P_2(-(1+\rho+(1+r)^2P_2) + 2P_2(1+r)(1+\\ &\quad 2\rho - r) - 2(1+r)(1-P_2)) - 4P_2(1+r)^3((1+\\ &\quad 2\rho - r)(1+\rho+(1+r)^2P_2) - (1+r^2)(1-P_2))]\\ &= -R^3(3+r)W_\ell + \frac{A^2}{P_2(1+r)^5}[-P_2(4r^2+2r^3) + P_2 - 1 +\\ &\quad 2\rho(P_2-1) + r^2(P_2-1) + 2r(\rho r P_2 - 1) - (2+2\rho)\\ &\quad +2\rho(2rP_2 - \rho)] < 0,\end{aligned}$$

given $r < \rho < 1$ and $P_2 < .5$.

$$\begin{aligned}\frac{\partial B_w}{\partial r} &= -R^2W_\ell + \frac{A^2}{(1+r)^4(1-P_2)^2}[(1+r)^2(1-P_2)(-P_2)\\ &\quad -2(1+r)(1-P_2)(1+\rho-P_2(1+r))]\\ &= -R^2W_\ell + \frac{A^2}{(1+r)^3(1-P_2)}[P_2(1+r) - 2(1+\rho)] < 0,\\ \frac{\partial B_\ell}{\partial r} &= -R^2W_\ell + \frac{A^2}{(1+r)^6P_2^2}[(1+r)^3P_2(-(1+\rho) - 2(1\\ &\quad +r)(1-P_2)) - ((1+2\rho-r)(1+\rho) - (1+r)^2(1\\ &\quad -P_2))3P_2(1+r)^2]\\ &= -R^2W_\ell + \frac{A^2}{(1+r)^4P_2}[-3 - P_2 - 2P_2r - P_2r^2 + 4r\end{aligned}$$

$$-4\rho + r^2 - \rho^2 + 2\rho r - 2\rho^2 - 3\rho^2 - 6\rho] < 0.$$

$$\frac{\partial B_1}{\partial \rho} = \frac{A^2[2(1+\rho+(1+r)^2P_2)+1+2\rho-r]}{(1+r)^3P_2} > 0.$$

$$\frac{\partial B_w}{\partial \rho} = \frac{A^2}{(1+r)^2(1-P_2)} > 0.$$

$$\frac{\partial B_\ell}{\partial \rho} = \frac{A^2[2(1+\rho)+1+2\rho-r]}{(1+r)^3P_2} > 0.$$

The comparative statics on effort are more complex, as the 3x3 system of E_1, E_w and E_ℓ must be solved. Totally differentiating yields:

$$dE_1 = \bar{g}dW_w - \bar{g}dW_\ell + \bar{g}\beta(P_2 - (1-P_2))dW_w + \bar{g}\beta(-P_2 + (1-P_2))dW_\ell + \beta\bar{g}C'(E_\ell)dE_\ell - \beta gC'(E_w)dE_w.$$

$$dE_w = \bar{g}(dW_w - dW_\ell) + \frac{\partial \bar{g}}{\partial E_w}\frac{E_w}{\bar{g}}dE_w + \frac{\partial \bar{g}}{\partial E_\ell}\frac{E_w}{\bar{g}}dE_\ell.$$

$$dE_\ell = \bar{g}(dW_w - dW_\ell) + \frac{\partial \bar{g}}{\partial E_w}\frac{E_\ell}{\bar{g}}dE_w + \frac{\partial \bar{g}}{\partial E_\ell}\frac{E_\ell}{\bar{g}}dE_\ell.$$

Rearranging terms, and recalling the definitions of κ and ψ, in matrix form:

$$\begin{pmatrix} 1 & \beta\bar{g}E_w & -\beta\bar{g}E_\ell \\ 0 & 1-\kappa & \kappa \\ 0 & \psi & 1-\psi \end{pmatrix}\begin{pmatrix} dE_1 \\ dE_w \\ dE_\ell \end{pmatrix} =$$

$$\begin{pmatrix} \bar{g}(1+\beta(2P_2-1)) & -g(1+\beta(2P_2-1)) \\ \bar{g} & -\bar{g} \\ \bar{g} & -\bar{g} \end{pmatrix}\begin{pmatrix} dW_w \\ dW_\ell \end{pmatrix}$$

or $HE = AW$. First the value and sign of:

$$|H| = (1-\kappa)(1-\psi) - \psi\kappa = 1 - \psi - \kappa$$

$$= 1 - \frac{(E_w - E_\ell)E_\ell}{2\sigma^2} - \frac{(E_\ell - E_w)E_w}{2\sigma^2}$$

$$= 1 + \frac{(E_\ell - E_w)^2}{2\sigma^2} > 0.$$

$$\frac{\partial E_1}{\partial W_w} = \frac{1}{|H|}\begin{vmatrix} \bar{g}(1+\beta(2P_2-1)) & \beta\bar{g}E_w & -\beta\bar{g}E_\ell \\ \bar{g} & 1-\kappa & \kappa \\ \bar{g} & \psi & 1-\psi \end{vmatrix}$$

$$= \frac{1}{|H|}(\bar{g}(1+\beta(2P_2-1))|H|)$$

$$-\frac{\bar{g}}{|H|}\begin{vmatrix} \beta\bar{g}E_w & -\beta\bar{g}E_\ell \\ \psi & 1-\psi \end{vmatrix}$$

$$+\frac{\bar{g}}{|H|}\begin{vmatrix} \beta\bar{g}E_w & -\beta\bar{g}E_\ell \\ 1-\kappa & \kappa \end{vmatrix}$$

$$= \bar{g}(1+\beta(2P_2-1)) + \frac{\bar{g}}{|H|}[\beta\bar{g}E_w\kappa + \beta\bar{g}E_\ell(1-\kappa)$$

$$-\beta\bar{g}E_w(1-\psi) - \beta\bar{g}E_\ell\psi]$$

$$= \bar{g}(1+\beta(2P_2-1)) + \frac{\bar{g}}{|H|}[\beta\bar{g}E_\ell(1-\kappa-\psi)$$

$$-\beta\bar{g}E_w(1-\psi-\kappa)]$$

$$= \bar{g}(1+\beta(2P_2-1)) + \bar{g}\beta\bar{g}(E_\ell - E_w) > 0.$$

$$\frac{\partial E_1}{\partial W_\ell} = -\partial E_1/\partial W_w < 0.$$

$$\frac{\partial E_w}{\partial W_w} = \frac{1}{|H|}\begin{vmatrix} 1 & \bar{g}(1+\beta(2P_2-1)) & -\beta\bar{g}E_\ell \\ 0 & \bar{g} & \kappa \\ 0 & \bar{g} & 1-\psi \end{vmatrix}$$

$$= \frac{1}{|H|}(\bar{g}(1-\psi-\kappa)) = \bar{g} > 0.$$

$$\frac{\partial E_w}{\partial W_\ell} = -\frac{\partial E_w}{\partial W_w} = -\bar{g} < 0.$$

$$\frac{\partial E_\ell}{\partial W_w} = \frac{1}{|H|}\begin{vmatrix} 1 & \beta\bar{g}E_w & \bar{g}(1+\beta(2P_2'-1)) \\ 0 & 1-\kappa & \bar{g} \\ 0 & \psi & \bar{g} \end{vmatrix}$$

$$= \frac{1}{|H|}((1-\kappa)\bar{g} - \psi\bar{g}) = \bar{g} > 0,$$

$$\frac{\partial E_\ell}{\partial W_\ell} = -\frac{\partial E_\ell}{\partial W_w} = -\tilde{g} < 0.$$

4.2 Details of the Nonmemory Two-Period Models

The agent's comparative statics for the borrowing terms in the nonmemory contracts is identical to that of the repeated contracts, except for the substitution of $W_{2\ell}$ for W_ℓ at various stages, and therefore the new comparative statics need not be formally worked out. For the effort terms there is some difference, however.

Begin by totally differentiating E_1, E_w and E_ℓ from the agent's solution, recalling that $dP_1 = dP_2 = 0$:

$$\begin{aligned}
dE_1 &= \tilde{g}dW_w - \tilde{g}dW_\ell + \tilde{g}\beta(2P_2 - 1)dW_{2w} + g\beta(1 - 2P_2)dW_{2\ell} \\
&\quad + \beta\tilde{g}C'(E_\ell)dE_\ell - \beta\tilde{g}C'(E_w)dE_w. \\
dE_w &= \tilde{g}(dW_{2w} - dW_{2\ell}) + \frac{\partial\tilde{g}}{\partial E_w}\frac{E_w}{\tilde{g}}dE_w + \frac{\partial\tilde{g}}{\partial E_\ell}\frac{E_w}{\tilde{g}}dE_\ell. \\
dE_\ell &= \tilde{g}(dW_{2w} - dW_{2\ell}) + \frac{\partial\tilde{g}}{\partial E_w}\frac{E_\ell}{\tilde{g}}dE_w + \frac{\partial\tilde{g}}{\partial E_\ell}\frac{E_\ell}{\tilde{g}}dE_\ell.
\end{aligned}$$

Rearranging terms, and recalling the definitions of κ and ψ, in matrix form

$$\begin{pmatrix} 1 & \beta\tilde{g}E_w & -\beta\tilde{g}E_\ell \\ 0 & 1-\kappa & \kappa \\ 0 & \psi & 1-\psi \end{pmatrix}\begin{pmatrix} dE_1 \\ dE_w \\ dE_\ell \end{pmatrix} =$$

$$\begin{pmatrix} g & -g & \beta g(2P_2 - 1) & -\beta g(2P_2 - 1) \\ 0 & 0 & \tilde{g} & -\tilde{g} \\ 0 & 0 & \tilde{g} & -\tilde{g} \end{pmatrix}\begin{pmatrix} dW_w \\ dW_\ell \\ dW_{2w} \\ dW_{2\ell} \end{pmatrix}$$

or $HE = AW$. Once again,

$$|H| = (1-\kappa)(1-\psi) - \psi\kappa = 1 - \psi - \kappa.$$

$$= 1 + \frac{(E_\ell - E_w)^2}{2\sigma^2} > 0.$$

$$\frac{\partial E_1}{\partial W_w} = \frac{1}{|H|}\begin{vmatrix} \bar{g} & \beta\bar{g}E_w & -\beta\bar{g}E_\ell \\ 0 & 1-\kappa & \kappa \\ 0 & \psi & 1-\psi \end{vmatrix}$$

$$= \frac{1}{|H|}(\bar{g})((1-\kappa)(1-\psi) - \psi\kappa) = \bar{g} > 0.$$

$$\frac{\partial E_1}{\partial W_\ell} = -\partial E_1/\partial W_w = -\bar{g} < 0.$$

$$\frac{\partial E_1}{\partial W_{2w}} = \frac{1}{|H|}\begin{vmatrix} \bar{g}\beta(2P_2 - 1) & \beta\bar{g}E_w & -\beta\bar{g}E_\ell \\ \bar{g} & 1-\kappa & \kappa \\ \bar{g} & \psi & 1-\psi \end{vmatrix}$$

$$= \frac{1}{|H|}(\bar{g}\beta(2P_2 - 1)|H|)$$

$$-\frac{\bar{g}}{|H|}\begin{vmatrix} \beta\bar{g}E_w & -\beta\bar{g}E_\ell \\ \psi & 1-\psi \end{vmatrix}$$

$$+\frac{\bar{g}}{|H|}\begin{vmatrix} \beta\bar{g}E_w & -\beta\bar{g}E_\ell \\ 1-\kappa & \kappa \end{vmatrix}$$

$$= \beta\bar{g}(2P_2 - 1) + \frac{1}{|H|}\beta\bar{g}\bar{g}(E_\ell(1 - \kappa - \psi) + E_w(\kappa + \psi - 1))$$

$$= \beta\bar{g}(2P_2 - 1) + \beta\bar{g}\bar{g}(E_\ell - E_w) \gtrless 0.$$

$$\frac{\partial E_1}{\partial W_{2\ell}} = -\frac{\partial E_1}{\partial W_{2w}} \gtrless 0.$$

$$\frac{\partial E_w}{\partial W_{2w}} = \frac{1}{|H|}\begin{vmatrix} 1 & \bar{g}\beta(2P_2 - 1) & -\beta\bar{g}E_\ell \\ 0 & \bar{g} & \kappa \\ 0 & \bar{g} & 1-\psi \end{vmatrix}$$

$$= \frac{1}{|H|}(\bar{g}(1 - \psi - \kappa)) = \bar{g} > 0,$$

$$\frac{\partial E_w}{\partial W_{2\ell}} = -\frac{\partial E_w}{\partial W_{2w}} = -\bar{g} < 0.$$

$$\frac{\partial E_\ell}{\partial W_{2w}} = \frac{1}{|H|}\begin{vmatrix} 1 & \beta\tilde{g}E_w & \tilde{g}\beta(2P_2-1) \\ 0 & 1-\kappa & \tilde{g} \\ 0 & \psi & \tilde{g} \end{vmatrix}$$

$$= \frac{1}{|H|}((1-\kappa)\tilde{g} - \psi\tilde{g}) = \tilde{g} > 0.$$

$$\frac{\partial E_\ell}{\partial W_{2\ell}} = -\frac{\partial E_\ell}{\partial W_{2w}} = -\tilde{g} < 0.$$

For the proof of Proposition 4 of Chapter Four, the principal's problem must be solved. The first-order conditions are:

$$\frac{\partial L}{\partial W_w} = .5\beta - \beta C'(E_1)\frac{\partial E_1}{\partial W_w} + \lambda R(D\frac{\partial E_1}{\partial W_w} - .5) = 0.$$

$$\frac{\partial L}{\partial W_\ell} = R - .5\beta - \beta C'(E_1)\frac{\partial E_1}{\partial W_\ell} + \lambda R(D\frac{\partial E_1}{\partial W_\ell} - .5) = 0.$$

$$\begin{aligned}\frac{\partial L}{\partial W_{2w}} = {} & .5\beta^2[P_2 + 1 - P_2] - .5\beta^2 C'(E_w)\frac{\partial E_w}{\partial W_{2w}} - \\ & .5\beta^2 C'(E_\ell)\frac{\partial E_\ell}{\partial W_{2w}} - \beta C'(E_1)\frac{\partial E_1}{\partial W_{2w}} + \lambda R(D\frac{\partial E_1}{\partial W_{2w}} + \\ & .5DR(\frac{\partial E_w}{\partial W_{2w}} + \frac{\partial E_\ell}{\partial W_{2w}}) - .5R) = 0.\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial W_{2\ell}} = {} & R^2 - .5\beta^2[P_2 + 1 - P_2] - .5\beta^2 C'(E_w)\frac{\partial E_w}{\partial W_{2\ell}} \\ & -.5\beta^2 C'(E_\ell)\frac{\partial E_\ell}{\partial W_{2\ell}} - \beta C'(E_1)\frac{\partial E_1}{\partial W_{2\ell}} + \lambda R(D\frac{\partial E_1}{\partial W_{2\ell}} \\ & +.5DR(\frac{\partial E_w}{\partial W_{2\ell}} + \frac{\partial E_\ell}{\partial W_{2\ell}}) - .5R) = 0.\end{aligned}$$

plus the zero-profit condition.

Solving $\partial L/\partial W_w$ and $\partial L/\partial W_\ell$, taking into account the comparative statics, yields $\lambda = 1$, and

$$E_1 = RD(1+\rho) + \frac{r-\rho}{2\tilde{g}(1+r)}.$$

Then, substituting these solved values for λ, E_1 and the comparative statics into $\partial L/\partial W_{2w}$ yields:

$$.5\beta^2 - .5\beta^2\tilde{g}^2[W_{2w} - W_{2\ell} - (1+r)y + C[y(1+r)A^{-1}]]$$

$$-.5\beta^2\bar{g}^2[W_{2w} - W_{2\ell} - (1+r)x + C[x(1+r)A^{-1}]]$$

$$-\beta[DR(1+\rho) + \frac{r-\rho}{2\bar{g}(1+r)}]\frac{\partial E_1}{\partial W_{2w}} + DR\frac{\partial E_1}{\partial W_{2w}} + DR^2\bar{g} - .5R^2 = 0.$$

Define the following:

$$\begin{aligned} E_\ell - E_w &= k = \bar{g}[(1+r)(y-x) + C[x(1+r)A^{-1}] \\ &\quad -C[y(1+r)A^{-1}]] > 0. \end{aligned}$$

Substitution of this into the above term plus rearrangement means $W_{2w} - W_{2\ell}$ can be solved for, and therefore E_w, E_ℓ:

$$\begin{aligned} \beta^2\bar{g}^2(W_{2w} &- W_{2\ell}) \\ &= .5\beta^2 - .5\beta^2\bar{g}^2(-(1+r)(y+x) + C[y(1+r)A^{-1}] \\ &\quad +C[x(1+r)A^{-1}]) + \frac{\rho - r}{2\bar{g}(1+r)(1+\rho)}[\bar{g}\beta(2P_2 - 1) \\ &\quad +\beta\bar{g}\bar{g}k] + DR^2\bar{g} - .5R^2 \\ &= .5R^2\beta^2[(1+r)^2 - (1+\rho)^2 + (\rho - r)(2P_2 - 1)(1+r)] \\ &\quad +DR^2\bar{g} + .5R\beta^2(\rho - r)\bar{g}k + .5\beta^2\bar{g}^2((1+r)(x+y) \\ &\quad -C[y(1+r)A^{-1}] - C[x(1+r)A^{-1}]). \end{aligned}$$

Define

$$\tau = \frac{(r-\rho)(3 - 2P_2 + \rho + r(2 - 2P_2))}{2(1+r)^2\bar{g}} < 0.$$

Then:

$$\begin{aligned} W_{2w} - W_{2\ell} &= \frac{\tau}{\bar{g}} + \frac{D(1+\rho)^2}{(1+r)^2\bar{g}} + .5((1+r)(x+y) - C[y(1+r)A^{-1}] \\ &\quad -C[x(1+r)A^{-1}]) + .5R(\rho - r)[(1+r)(y-x) + \\ &\quad C[x(1+r)A^{-1}] - C[y(1+r)A^{-1}]] > 0, \text{ or} \\ W_{2w} - W_{2\ell} &= \frac{\tau}{\bar{g}} + \frac{D(1+\rho)^2}{(1+r)^2\bar{g}} + .5x(1+2r-\rho) + .5y(1+\rho) \\ &\quad -.5RC[x(1+r)A^{-1}](1+2r-\rho) - \\ &\quad .5RC[y(1+r)A^{-1}](1+\rho). \end{aligned}$$

Substitute this into the agent's choice of E_w:

$$\begin{aligned} E_w &= \tilde{g}(\frac{\tau}{\tilde{g}} + \frac{D(1+\rho)^2}{(1+r)^2\tilde{g}} + .5x(1+2r-\rho) + .5y(1+\rho) - \\ &\quad .5RC[x(1+r)A^{-1}](1+2r-\rho) - .5RC[y(1+r)A^{-1}](1 \\ &\quad +\rho) - (1+r)y + C[y(1+r)A^{-1}]), \text{ or} \\ E_w &= \tau + R^2 D(1+\rho)^2 - .5R(1+2r-\rho)k. \end{aligned}$$

Substituting $W_{2w} - W_{2\ell}$ into the agent's choice of E_ℓ and simplifying yields

$$E_\ell = \tau + R^2 D(1+\rho)^2 + .5R(1+\rho)k.$$

The values for $E_\ell - E_w$ and the comparative statics come from simple manipulation: □

4.3 Mobility Constraints

a) Nonmemory Contracts

Recall the reserve job described in Appendix 2.1 – observable work at the rate D_r. For a winner in the first-period of the nonsorting job, the analysis works out much the same under the current default cost case, as under the previous restricted borrowing case. Thus the constraint is leftover debt + value(stay) ≥ leftover debt + value(move), or:

$$\begin{aligned} W_w &- B_1(1+r) + B_w + \beta(P_2(W_{2w} - B_w(1+r)) \\ -(1 &- P_2)C[(B_w(1+r) - W_{2\ell})A^{-1}] - C(E_w)) \geq W_w - B_1(1+r) \\ +B_{rw} &+ \beta(D_r E_{rw} - B_{rw}(1+r) - C(E_{rw})), \end{aligned}$$

where B_{rw}, E_{rw} are selected optimally at the reserve job, B_w and E_w are selected optimally at the tournament, W_w and B_1 are given, and it is assumed $W_w - B_1(1+$

$r) + B_{rw} > 0$.[2] It was shown in Appendix 2.1 what the optimal values of B_{rw}, E_{rw} are, and substitution of these plus the values for B_w, E_w and simplification yields the constraint for the nonsorting winner's case.

For the loser the analysis is more complicated, as he has more leftover debt, and therefore the possibility $W_\ell - B_1(1+r) + B_{r\ell} < 0$. First, however, analyze the case where $W_\ell - B_1(1+r) + B_{r\ell} > 0$. Then the constraint is:

$$-C[(B_1(1+r) - W_\ell - B_\ell)A^{-1}] + \beta((1-P_2)(W_{2w} - B_\ell(1+r)) -$$
$$P_2C[(B_\ell(1+r) - W_\ell)A^{-1}] - C(E_\ell)) \geq W_\ell - B_1(1+r) + B_{r\ell} +$$
$$\beta(D_rE_{r\ell} - B_{r\ell}(1+r) - C(E_{r\ell})).$$

If the loser opts to leave, he selects $B_{r\ell}$, $E_{r\ell}$ to solve:

$$\begin{aligned} \max L &= -RW_{2\ell} - h(1+r) + B_{r\ell} + \beta(D_rE_{r\ell} - B_{r\ell}(1+r) \\ &\quad -C(E_{r\ell})) + \lambda(D_rE_{r\ell} - B_r(1+r)). \\ \frac{\partial L}{\partial B_{r\ell}} &= 1 - \beta(1+r) - \lambda(1+r) = 0, \\ \frac{\partial L}{\partial E_{r\ell}} &= \beta(D_r - C'(E_{r\ell})) + \lambda D_r = 0, \\ \frac{\partial L}{\partial \lambda} &= D_rE_{r\ell} - B_{r\ell}(1+r) = 0. \end{aligned}$$

These can be solved for $E_{r\ell} = RD_r(1+\rho)$; $B_{r\ell} = R^2D_r^2(1+\rho)$, and therefore:

$$\bar{V}_\ell(1) = -RW_{2\ell} - h(1+r) + .5R^2D_r^2(1+\rho),$$

where $\partial\bar{V}_\ell(1)/\partial W_{2\ell} < 0$, and where $\bar{V}_\ell(1)$ is the first version of $\bar{V}_\ell$.

Secondly, suppose the loser does not borrow enough to clear his debt at the reserve job. In this case, $W_\ell - B_1(1+r) + B_{r\ell} < 0$, and he selects $B_{r\ell}$, $E_{r\ell}$ to solve:

$$\begin{aligned} \max V_\ell &= -C[(B_1(1+r) - W_\ell - B_{r\ell})A^{-1}] + \beta(D_rE_{r\ell} \\ &\quad -B_{r\ell}(1+r) - C(E_{r\ell})). \end{aligned}$$

[2] As analysis of the loser's job will show, this assumption is not necessary.

(Clearly, here $B_{r\ell}$ is lower then above, and therefore the constraint $B_{r\ell}(1+r) \leq D_r E_{r\ell}$ no longer binds).

$$\frac{\partial L}{\partial B_{r\ell}} = A^{-2}(B_1(1+r) - W_\ell - B_{r\ell}) - \beta(1+r) = 0,$$
$$\frac{\partial L}{\partial E_{r\ell}} = \beta(D_r - C'(E_{r\ell})) = 0.$$

These imply:

$$B_{r\ell} = -\beta(1+r)A^2 + RW_{2\ell} + h(1+r);\ E_{r\ell} = D_r,$$

which upon substitution yields:

$$\bar{V}_\ell(2) = -C[\beta(1+r)A] + \beta(.5D_r^2 - W_{2\ell} - h(1+r)^2 + \beta(1+r)^2 A^2),$$

where $\partial\bar{V}_\ell(2)/\partial W_{2\ell} < 0$, etc. again.

Thus, with both $\bar{V}_\ell$'s, $\partial\bar{V}_\ell/\partial W_{2\ell} < 0$, $\partial\bar{V}_\ell/\partial$ (other wages) $= 0$.[3] Of course, one $\bar{V}_\ell$ will be higher, but for the current analysis this is irrelevant. Substitution of B_1, B_ℓ, E_ℓ into the left-hand side of the loser's constraint and $\bar{V}_\ell$ into the right-hand side yields the nonsorting loser's constraint used in the text.

b) Memory Contracts

For the construction of the memory constraints, it is easy to see that substituting $W_{ww} = W_{2w}$, $W_{w\ell} = W_{2\ell}$ into the above nonsorting winner's constraint and $W_{\ell w} = W_{2w}$, $W_{\ell\ell} = W_{2\ell}$ into the nonsorting loser's constraint yields the appropriate memory constraints. All the above calculations go through identically with these changes. Also, using these constraints, the proof of Proposition

[3]Note this implies the constraint $W_w - B_1(1+r) + B_{rw} > 0$ is not needed to derive the proof that the mobility constraints do not bind. If this constraint is not met, then $\bar{V}_w = \bar{V}_\ell(2)$. Since $\bar{V}_w$ is then only a function of $W_{2\ell}$, the proof works in an analogous manner to the case where $W_w - B_1(1+r) + B_{rw} > 0$.

8 of Chapter Four, is straightforward, and analogous to the proof used in the restricted borrowing model of Chapter Two.

4.4 Details of the Memory Contracts

As to the comparative statics on the agent's borrowing values for the memory model, note that these are identical to the values for the repeated contracts, except that the wages are slightly different and $P_2 = .5$. These effects are trivial, and the comparative statics in Appendix 4.1 hold for the memory contracts, with slight adjustments (e. g. $P_2 = .5$).

To solve the effort comparative statics, totally differentiate E_1, E_w and E_ℓ from the agent's problem:

$$\begin{aligned} dE_1 &= \bar{g}(dW_w - dW_\ell) + .5\beta\bar{g}dW_{ww} + \bar{g}(R - .5\beta)dW_{w\ell} - .5\beta\bar{g}dW_{\ell w} \\ &\quad +\bar{g}(.5\beta - R)dW_{\ell\ell} - \beta\bar{g}C'(E_w)dE_w + \beta\bar{g}C'(E_\ell)dE_\ell, \\ dE_w &= \bar{g}(dW_{ww} - dW_{w\ell}), \\ dE_\ell &= \bar{g}(dW_{\ell w} - dW_{\ell\ell}). \end{aligned}$$

These are identical to the equivalent equations from Appendix 2.2 for the memory (restricted borrowing) contracts. This is because in the default cost model, the extra default cost terms in the agent's best response functions are not functions of the wage rates. Thus the comparative statics for wages in the default cost case are identical to those of the restricted case, as mentioned in the text.

Next, to prove Proposition 7 of Chapter Four, recall the consumption values for the memory contest:

$$\begin{aligned} Z_0 &= RW_\ell + R^2W_{\ell\ell} + h, \\ Z_w &= W_w - W_\ell + R(W_{w\ell} - W_{\ell\ell}) + y - h(1 + r), \\ Z_\ell &= -C[(h(1 + r) - x)A^{-1}], \end{aligned}$$

$$Z_{ww} = W_{ww} - W_{w\ell} - y(1+r),$$
$$Z_{w\ell} = -C[y(1+r)A^{-1}],$$
$$Z_{\ell w} = W_{\ell w} - W_{\ell\ell} - x(1+r),$$
$$Z_{\ell\ell} = -C[x(1+r)A^{-1}].$$

As before the values for E_1, E_w and $\bar{y} = D(E_1 + .5R(E_w + E_\ell))$ come from the solution to the principal's problem. E_1, E_w, E_ℓ can be rewritten as:

$$E_1 = \bar{g}(Z_w - Z_\ell + 3(.5(Z_{ww} + Z_{w\ell}) - .5E_w^2 - .5(Z_{\ell w} + Z_{\ell\ell}) + .5E_\ell^2)),$$
$$E_w = \bar{g}(Z_{ww} - Z_{w\ell}),$$
$$E_\ell = \bar{g}(Z_{\ell w} - Z_{\ell\ell}).$$

E_w and E_ℓ easily yield:

$$Z_{ww} - Z_{w\ell} = E_w/\bar{g},$$
$$Z_{\ell w} - Z_{\ell\ell} = E_\ell/\bar{g}.$$

Since $Z_{w\ell}$, $Z_{\ell\ell}$ are known from above, Z_{ww} and $Z_{\ell w}$ can be solved for as functions of the exogenous parameters. To solve for $Z_w - Z_\ell$, substitute E_w and E_ℓ into E_1:

$$E_1 = \bar{g}[Z_w - Z_\ell + 3(\frac{E_w}{2g} + Z_{w\ell} - .5E_w^2 - \frac{E_\ell}{2g} - Z_{\ell\ell} + .5E_\ell^2],\ \text{or}$$
$$Z_w - Z_\ell = \frac{E_1}{\bar{g}} - \frac{3E_w}{2\bar{g}} + .53E_w^2 + \frac{3E_\ell}{2\bar{g}} - .53E_\ell^2 + 3(Z_{\ell\ell} - Z_{w\ell})$$

Since Z_ℓ, $Z_{\ell\ell}$, $Z_{w\ell}$ are known from above, as well as E_1, E_w, E_ℓ, therefore Z_w can be solved for as a function of the exogenous parameters.

Finally, to solve for Z_0, postulate that:

$$Z_0 = R\bar{y} - .5RZ_w - .25R^2(Z_{ww} + Z_{\ell w}) + .5h + .25R(y - x)$$

Note that all of these terms are known. Substitute the zero-profit condition and the consumption values from above into this equation:

$$\begin{aligned} Z_0 &= .5R(W_w + W_\ell) + .25R^2(W_{ww} + W_{w\ell} + W_{\ell w} + W_{\ell\ell}) \\ &\quad -.5R(W_w - W_\ell + R(W_{w\ell} - W_{\ell\ell}) + y - h(1+r)) \\ &\quad -.25R^2(W_{ww} - W_{w\ell} - y(1+r) + W_{\ell w} - W_{\ell\ell} - x(1+r)) + \\ &\quad .5h + .25R(y - x). \end{aligned}$$

A collection of terms yields:

$$Z_0 = RW_\ell + R^2W_{\ell\ell} + h,$$

as desired. Thus the postulate is correct, so Z_0 is given by:

$$Z_0 = R\bar{y} - .5RZ_w - .25R^2(Z_{ww} + Z_{\ell w}) + .5h + .25R(y - x).$$

Recalling the definitions of Δ_1 Δ_w, and Δ_ℓ, substituting into $Z_w - Z_\ell$, and rearranging yields:

$$\Delta_1 - \Delta_w = .5\beta(E_\ell - E_w)(\frac{1}{\bar{g}} - (E_\ell + E_w)) + \beta(Z_{\ell\ell} - Z_{w\ell}).$$

In Appendix 2.3 the first term is shown to be nonpositive. For the second term:

$$Z_{\ell\ell} - Z_{w\ell} = C[y(1+r)A^{-1}] - C[x(1+r)A^{-1}],$$

which is proportionate to $y - x$. In Appendix 4.1 it was shown that:

$$x - y = A^2R^34(\rho - r)(1 + \rho) > 0,$$

so that $\Delta_1 < \Delta_w$, proving part of Proposition 7 of Chapter Four.

Recalling

$$E_w = E_\ell + \frac{(\rho - r)(1+\rho)(\frac{1}{\bar{g}} - 2D)}{(1+r)(1+2r-\rho)},$$

and substituting this into $Z_w - Z_\ell$:

$$\Delta_1 - \Delta_\ell = \frac{(\rho - r)(1+\rho)(\frac{1}{\bar{g}} - 2D)}{\bar{g}(1+r)(1+2r-\rho)} + .5\beta(E_\ell - E_w)(\frac{1}{\bar{g}} - (E_\ell + E_w)) + \beta(Z_{\ell\ell} - Z_{w\ell}).$$

The third term is negative, and the first two terms were analyzed in Appendix 2.3, where their sign is shown to be uncertain. The addition of the extra term makes it more likely $\Delta_1 - \Delta_w \leq 0$, but overall the sign is still uncertain.

Finally, it is easy to see:

$$\Delta_w - \Delta_\ell = \frac{(E_w - E_\ell)}{\bar{g}} \gtrless 0,$$

which was analyzed in Appendix 2.3.

For the mean consumption values:

$$\bar{Z}_1 = .5(Z_w + Z_\ell) = .5(\Delta_1 + 2Z_\ell) = .5\Delta_1 + Z_\ell,$$
$$\bar{Z}_w = .5(Z_{ww} + Z_{w\ell}) = .5\Delta_w + Z_{w\ell},$$
$$\bar{Z}_\ell = .5(Z_{\ell w} + Z_{\ell\ell}) = .5\Delta_\ell + Z_{\ell\ell}.$$

Thus, $\bar{Z}_1 - \bar{Z}_w = .5(\Delta_1 - \Delta_w) + Z_\ell - Z_{w\ell}$. The first term is negative, while:

$$Z_\ell - Z_{w\ell} = C[y(1+r)A^{-1}] - C[(h(1+r) - x)A^{-1}].$$

With $P_2 = .5$:

$$\begin{aligned} h(1+r) - x &= R^3A^2[(1+2\rho - r)(2 + 2\rho + (1+r)^2) - (1+r)^2] \\ &\quad - R^3A^2[(1+2\rho - r)(2+2\rho) - (1+r)^2] \\ &= R^3A^2[(1+2\rho - r)(1+r)^2] = A^2R(1+2\rho - r) \\ &= y(1+r), \end{aligned}$$

and thus $Z_{w\ell} = Z_\ell$ and $\bar{Z}_1 < \bar{Z}_w$, proving the final part of Proposition 7 of Chapter Four. □

For the other comparisons:

$$\bar{Z}_1 - \bar{Z}_\ell = .5(\Delta_1 - \Delta_\ell) + C[x(1+r)A^{-1}] - C[(h(1+r) - x)A^{-1}].$$

The first term is of uncertain sign, as discussed above, and the remaining portion is positive (given $x > y$), and therefore, overall the sign is uncertain. Similarly

$$\bar{Z}_u - \bar{Z}_\ell = .5(\Delta_u - \Delta_\ell) + C[x(1+r)A^{-1}] - C[y(1+r)A^{-1}]$$

is of uncertain sign.

Chapter 5

Analysis of the Models and Conclusions

In this chapter results developed in earlier chapters will be expanded on, as well new results presented that are implications of earlier work. Presenting this material in a less technical manner than the earlier chapters makes it possible to emphasize what the introduction of borrowing into a specific model of tournaments and piece-rates has accomplished. These accomplishments fall into two categories. First of all, several results derived in nonborrowing agency models are shown to generalize to a borrowing model. The most important of these is the prediction that memory contracts will be observed. Secondly, the specific models employed in this thesis generate many new results. The most important of these are the predictions on the extra work hours of tournament losers, of those not promoted.

The emphasis in this chapter is on two types of analysis. First, testable predictions about observable variables will be generated. Some of these predictions were originally presented in earlier chapters, while others are new implications of the models. Some of these predictions will be general, while other predictions will be for specific types of models (for example, rank order tournaments with de fault cost borrowing). Second, various comparisons are made – between types of contracts (tournaments versus piece-rates), and between types of capital markets (restricted versus default cost). These comparisons are done as a sensitivity test, in order to emphasize which result are fairly general and which seem dependent on specific assumptions.[1]

5.1 Predictions of the Models

One of the major problems with agency models is the lack of specific testable predictions (see MacDonald (1984) for an extended discussion). The specific structure used in this thesis has resulted in the generation of several predictions. (The emphasis is on tournaments, but it will be noted which results hold for piece-rates.) One of the key problems in agency models is that the most important variable (effort) is unobservable, and thus most comparative statics seems untestable. Here observables such as consumption and wages are focussed on, as well as the variable average output ($\bar{q}$). If one makes enough observations of output over enough time, given $E(\epsilon) = 0$, average output will be a crude proxy for effort. It should be noted that all predictions discussed in this section are for observable variables, and are presumably testable should the relevant data be collected.

5.1.1 Predictions Common to Both Capital Markets

In this section, all of the results discussed hold for both capital markets, and thus are quite robust. The first set of results deals with characteristics of the firm hiring the agents.

[1] Since specific models are used, one cannot say which results are completely general

Prediction 1 *The exogenous variables should have the following observable relationships with average output:*

a) In the tournament:

$$\frac{\partial \bar{q}}{\partial D} > 0, \quad \frac{\partial \bar{q}}{\partial \sigma^2} = \frac{\partial \bar{q}}{\partial \bar{g}}\frac{\partial \bar{g}}{\partial \sigma^2} < 0, \quad \frac{\partial \bar{q}}{\partial r} \gtrless 0;$$

b) In the piece-rate:

$$\frac{\partial \bar{q}}{\partial D} > 0, \quad \frac{\partial \bar{q}}{\partial \sigma^2} = 0, \quad \frac{\partial \bar{q}}{\partial r} < 0$$

None of these predictions are particularly surprising. Indeed, the predictions for the value of output and the piece-rate prediction for the variance are the same as for Lazear and Rosen's (1981) risk-neutral model. The new prediction, and the sharpest, is that in the tournament $\partial \bar{q}/\partial \sigma^2 < 0$. A rise in the variance of the random shock weakens the connection between the agent's effort and eventual compensation, so optimal effort falls.[2] In addition, the results with respect to the rate of interest are also new to the agency literature.

The analysis of Chapters 2, 3 and 4 showed that with multiperiod contracts, the wage structure was nonunique in both tournaments and piece-rates. This is because agents' borrowing actions can undo any wage changes, leaving consumption unchanged. However, the optimal contracts would yield unique consumption and effort values, and these would reveal certain testable effects. The next set of predictions deals with how these values change over the agent's tenure within the tournament firm.[3]

[2] This result is similar to the results of stochastic (but symmetric information) production theory. In stochastic production theory, a risk-averse firm would produce lower output in the face of uncertainty with respect to its demand curve (Lippman and McCall (1982)), or with respect to output (Henderson and Quandt (1980)). The impact on output of an increase in uncertainty depends on the form of the utility function.

[3] It was also shown that agents' effort levels would change over their tenure within the firm. However, the direction of this change was ambiguous, and hence the result is of little predictive power. With respect to the piece-rate system, its simpler structure did not reveal as many

Prediction 2 *Over the typical agent's tenure with the rank-order tournament firm, the following will be observed:*

a) As an agent moves from the first period contest to the second period winners' contest, he will have higher average consumption ($\bar{Z}_1 \leq \bar{Z}_w$). In addition, the spread of consumption will also rise ($\Delta_1 \leq \Delta_w$).

b) As an agent moves from the first period contest to the second period winners' contest, mean wages rise ($.5(W_{ww} + W_{wl}) > .5(W_w + W_l)$).

These are robust predictions of the model, predictions not generated by other agency models. The empirical refutation of these predictions would refute the dynamic borrowing model, as constructed in this thesis. The consumption effects are analogous to wage effects predicted by other models with memory effects (Rogerson (1985a)). The results come from an endperiod effect. Differences in consumption levels motivate agents. In the first period, these consumption differences include immediate effects plus future effects (getting promoted or not). In the last period only immediate consumption incentives are available, so a higher spread of consumption is required to motivate effort. This in turn requires a higher mean to compensate the agent for the reduced borrowing opportunities.

Other models have had some predictions on mean wages over the agent's tenure within the firm. Rosen (1986) finds mean wages rising in a dynamic tournament, but his model does not include a profit-maximizing principal (see Chapter One). Rogerson (1985a) has a more complete model, but does not generate a precise prediction about mean wages, but instead one contingent on the agent's utility function. The current model has generated a precise, testable prediction on mean wages, also contingent on assumptions about the utility function.

testable effects. However, one prediction is that effort will be higher in the first period, then in the second period.

5.1.2 Predictions Specific to Restricted Borrowing

As noted in Chapter Two, the restricted borrowing model is less complex, and it is possible to solve for the equilibrium expected value of a contract to the agent (which here determines its pareto ranking). Thus:

Prediction 3 *a) Wherever possible, in a tournament the principal will elect memory contracts over nonmemory, and nonmemory over repeated.*
$V_T(memory) \geq V_T(nonmemory) \geq V_T(repeated)$,
with strict inequalities unless $2D\bar{g} = 1$.
b) In a piece-rate the principal will select a two-period contract over a repeated.
$V_P(two\text{-}period) > V_P(repeated)$.[4]

Memory contracts pareto-dominate the others examined. The principal structures the first period contract such that a winner receives both a higher wage and a capital gain consisting of promotion to a better contest. This extra incentive effect creates extra linkages or tools that the principal can use to increase utility and/or profits. The principal reduces the conflict between incentives and borrowing by spreading incentives across periods. Therefore, a powerful prediction of this model is that memory contracts should be observed. These contracts will exhibit the memory effects noted in Prediction 2.

Since the equilibrium values of contracts under restricted borrowing could be solved for, it was possible to note the following:

Prediction 4 *In comparing tournaments to piece-rates, the higher the value of output (D) and the lower the variance of the random shock (σ^2), the more likely one is to observe the use of a tournament as a compensation scheme, instead of a piece-rate.*

[4] As noted earlier, a simple linear piece-rate setup precludes memory contracts, and hence they cannot be compared to nonmemory piece-rates.

This prediction states what would be seen in an examination of industry cross-section data.[5] First, in industries with a higher variance of the random shock to output, piece-rate contracts should be more prevalent. This is similar to results found by Green and Stokey (1983) in a model with risk-averse agents, and therefore is a quite robust prediction. Secondly, the higher the value of output, the more prevalent should be the use of tournament contracts. This is a new result not present in Green and Stokey's (1983) analysis.

5.1.3 Predictions Specific to Default Cost Borrowing

The default cost model is more complex, making impossible comparisons of the values of contracts. However, its complexity makes it rich in predictions, especially if the role of the default costs is analyzed. These were assumed to take the form of a legal compulsion to pay off debt by working at a second job (although it could also include overtime at the current job, or other family members being forced to work). Clearly an agent's extra job or overtime is observable, as are the number of hours put in at the extra job.

Prediction 2 discussed how some variables would change over the agent's tenure within the firm. These predictions were generated under both capital market assumptions. In addition, under default cost borrowing the following new predictions about the agent's tenure are generated.

Prediction 5 *a) In any (memory) contest, the loser (the agent not promoted) will work extra hours, either in overtime or at a second job.*

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[5] As Holmström (1982a) and Carmichael (1983) have emphasized, it is likely more subtle, intermediate forms of contracts will dominate the extremities of tournaments and piece-rates. However, actual contracts could be thought of as being mostly contests or mostly piece-rates. Therefore, this prediction tells one which component would dominate in industry cross-section data.

b) The agent who loses two contests sequentially (i.e., has a timepath of 'no promotion, no promotion') will work more extra hours in the second period than an agent works who loses the first period, or who wins the first and loses the second (has a time-path of 'promotion, no promotion').

The extra hours are of course correlated with the outstanding debt of an agent. In terms of the variables developed in Chapter Four:

$$\text{debt(first period loser)} = h(1+r) - x,$$

$$\text{debt(win, lose)} = y(1+r),$$

$$\text{debt(lose, lose)} = x(1+r).$$

It has been shown elsewhere $x > y$, and it has also been shown that $h(1+r) - x = y(1+r)$, so that:

$$\text{debt(lose, lose)} > \text{debt(win, lose)} = \text{debt(first period loser)}.$$

These predictions are new, not generated by any other models. They are a direct result of the borrowing model. These are specific, refutable predictions: In panel data, observations of agents' outstanding debt and/or extra working hours should be correlated with their promotion path if the borrowing model is true. For example, young professors might take out mortgages based on the probability of receiving tenure and pay raises. With this debt hanging over their heads, presumably they work harder to get promotions. Should they fail to be promoted, they could be forced to teach overloads to make ends meet, and would end up working more hours then promoted colleagues.

Prediction 5 came about in situations with principals offering agents memory contracts. It is possible under some situations memory contracts are not allowed, and firms can only offer two-period nonmemory contracts, or even repeated one-period contracts. This leads to:

Prediction 6 *If a firm is not allowed to sort, and must use nonmemory or repeated contracts, then on average the second-period output of first period losers will be higher than that of first period winners ($E_\ell > E_w$ implies $\bar{q}_\ell > \bar{q}_w$), and the first-period losers are more likely to win the second period contest ($P_\ell < .5$).*

A firm may not be allowed to sort due perhaps to union rules, or employee ideas on fairness. For example, one (crude) form of a tournament is to give a bonus to the salesperson with the highest output per year. A memory contract would give a different bonus to someone who had won previously as opposed to someone who had not. It is easy to see this could be perceived as unfair by workers. If firms cannot use memory effects, Prediction 6 is relevant.

For the final result, recall that average output (which proxies effort), consumption, income (wages in the firm plus at the overtime job), and total output (average output at the firm plus average output at the second job) are all observable. Keeping these definitions in mind:

Prediction 7 *a) Observed labour supply curves seem backward-bending. In a given (memory) contest winners and losers (of that contest) put in the same number of average hours at the contest job, but the loser puts in extra outside hours. Hence in a labour force survey such a worker would state he worked more hours for less consumption and less wages – the correlation between consumption and hours and between income and hours is negative.*

b) The correlation between actual income in the tournament firm and actual output is positive (the agent with the highest output receives the higher wages).

c) Define the average propensity to consume as consumption over wages. In a cross-section of the firm one would observe the correlation between the APC and wages as being positive (as wages rise, so does the APC):

$$APC(winner) = \frac{W_w - debt}{W_w} > 0,$$

$$APC(loser) = \frac{W_\ell - debt}{W_\ell} = 0 \text{ (or is negative)}.$$

An inspection of the agent's first-order conditions reveals that a rise in the spread of wages raises effort and average output. However, observing the correlation between hours worked and income earned implies a backward-bending labour supply curve, in line with observed male wage elasticities (Killingsworth (1983)). This is a new result for agency models. In addition, the prediction that the average propensity to consume will be positively correlated with wages is also a new, refutable prediction. Once again the model generates a strong prediction that can be used to test its validity.

5.2 Comparisons

In this thesis several submodels were developed which varied based on assumptions about types of capital markets (restricted or default cost), and about types of contracts (tournament or piece-rate). In this section a recapitulation of these comparisons is undertaken.

A brief examination of a piece-rate system was made primarily to check the robustness of the tournament results, as well as to allow for some comparison of tournaments to piece-rates. First, it was shown several results were robust, including the nonuniqueness of the compensation structure (although once again effort and consumption were unique); the optimality of multiperiod contracts and the resulting presence of intertemporal effects; and the fact mobility constraints would not bind, resulting in negative profits in the second period. These results imply that the analysis of the borrowing distortion does generalize, that it is robust. Second, the comparisons of tournaments to piece-rates emphasized that the optimality of one over another depended on the values of exogenous variables (but not the variance of a common shock, unlike in Green and Stokey (1983)). The higher the value of output and/or the lower the variance of the individualistic random shock, the more likely it is the tournament is preferred to the piece-rate.

The most complicated of comparisons deals with the most important of assumptions, those about the form of the capital market. Many results were not sensitive to the form of the capital market, including the presence of memory effects;[6] the resulting memory effects in consumption and effort; and the result that average wages are higher in the winners' contest as opposed to in the first period. However, several results were sensitive to the form of the capital market. Thus the strong predictions on extra hours for losers, on the backward-bending labour supply curves, and that average output is higher in the second period for first period losers (with no sorting), are all dependent on the assumption of a default cost capital market. The robustness of these results is therefore somewhat suspect.

5.3 Summary and Conclusions

This thesis has extended multiperiod agency models in two crucial ways. First, the realism of the models was extended by introducing borrowing. Borrowing has been only partially developed in the literature (for example, Rogerson (1985a) and Braverman and Stiglitz (1982)). This earlier work was improved upon by examining a simultaneous choice of effort and borrowing, thus yielding a more realistic model and generalizing agency results. Second, additional explicit results were generated by concentrating on specific contract forms, principally rank-order tournaments. Other work on specific multiperiod contract forms (Malcomson (1984), Murphy (1986), Rosen (1986)) dealt with quite different modeling

[6] Even though one cannot explicitly rank the memory versus the nonmemory contracts in the default cost case, the fact the principal selects wages such that $E_\ell \neq E_w$ hints that memory contracts will be optimal here.

structures, and did not have borrowing. So far, this research presents the only complete solution to a dynamic, multiple agent model with borrowing.

It was shown that the introduction of a capital market (principally borrowing) does make a crucial difference. Contrary to Fellingham and Newman's (1985) result, it was found that with risk-neutrality and borrowing, memory contracts dominate nonmemory contracts. In conjunction with the results of Rogerson (1985a) and Lambert (1983) for risk-aversion and no capital market, this result leads to a very robust prediction – where possible, memory contracts will be used. This is certainly the most powerful prediction of the multiperiod literature.[7]

It was also possible to present details of these memory contracts. These details included memory effects in output, and specific comparative statics on output. The strongest predictions were with respect to what consumption and wage timepaths would look like. For the former, the spread and mean of consumption will be higher in the winners' contest than in the first period contest, and for the latter the mean value of wages will be higher in the winners' contest as compared to the first period contest. As discussed earlier, these are endperiod effects. These effects are present only with borrowing, given the risk-neutrality in essence, the borrowing makes a risk-neutral agent act 'as if' he is risk-averse. It should be emphasized these results are general in the sense they occur under both capital market assumptions. In addition, note that these are new, testable predictions not found in other agency models.

The default cost capital market model generated extra results. The most powerful prediction was that under certain circumstances coasting would be observed – in the second period first period winners will produce lower average output than first period losers. This model also generated the prediction that agents who do not get promotions will be forced to take on second jobs, and that the number of hours worked at this second job will be higher for those who have

[7] For a dissenting voice, see Fudenberg et al (1987)

missed two promotions in a row. Finally, the model generated the prediction that observed labour supply curves (for example, in response to labour survey questions on income and hours) would appear to be backward bending. Once again, these are new, testable predictions not found in other agency models.

These new explicit results were generated at a cost, the cost of making simplifying assumptions. Certainly future research should examine more general cases. Four major areas would seem to be open to generalization. Most important would be a more rigorously derived form of an imperfect capital market. This would be difficult (but rewarding), as it would require a three-sided, double moral hazard, game theoretic approach. With such a model, one could better settle the interesting questions about the impact on the principal's optimization problem, of the bank's attempts to solve it's moral hazard problem (a defaulting borrower), and vice versa.[8]

A second generalization would be to examine risk-averse agents and principals. This considerably complicates things as the agent's cross derivatives between effort and savings become very important, and therefore the form of his utility function (Lambert (1983)). Still, it is an important and interesting extension.

A third generalization, examined a great deal in the single period literature but not as much in the multiperiod literature, is to allow agents to be heterogenous, to have differing ability. As Lazear and Rosen (1981) and O'Keeffe et al. (1984) have noted, this considerably complicates the principal's problem. However, it is an obvious extension, as a multiperiod tournament has the potential ability to promote more able agents (on average),[9] and thus helps to reduce the

[8] Some authors have dealt with the bank's problem (for example, Stiglitz and Weiss (1981)), but only when the borrower is a self-employed entrepreneur, not when he is an agent working in a moral hazard firm.

[9] See Rosen (1986) for further discussion.

conflict between sorting and moral hazard.

The final generalization would involve examining other forms of compensation contracts. However, it is to be expected that many of the results of this thesis would survive this generalization, as the examination of both tournaments and piece-rates implies many results are robust.[10]

Most of the results can be expected to survive the above generalizations. Most especially, the optimality of memory contracts, and the presence of memory effects seem to be quite robust. In addition, the coasting result is quite intuitive, and seems likely in many settings.[11]

On the other hand, some of the results derived from the default cost job are less robust. Predictions that the losers of promotions take secondary jobs would seem to be backed up by casual empiricism. However, more specific results such as observed labor supply curves being backward bending would seem to be quite dependent on assumptions about the capital market structure. A generalization of the modeling of the capital market could alter these results.

[10]The most amenable line of extension would seem to be towards either the cardinal relative-worth contracts of Holmström (1982a), of the mixed tournament/piece-rate contracts of Carmicheal (1983).

[11]For example, compare the behaviour of those given and denied tenure, or the behaviour of students receiving an A or a C on a midterm exam. In a different setting, frequently sports teams slow down the pace of a game (effort falls) once they have a lead, in order to reduce risk (MacDonald (1984) p. 435 discusses coasting briefly.)

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