1987

A General Equilibrium Model Of Person Specific Information In The Labour Market

M Mobinul Huq

CANADIAN THESES

THÈSES CANADIENNES

NOTICE

The quality of this microfiche is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Previously copyrighted materials (journal articles, published tests, etc.) are not filmed.

AVIS

La qualité de cette microfiche dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

Les documents qui font déjà l'objet d'un droit d'auteur (articles de revue, examens publiés, etc.) ne sont pas microfilmés.

THIS DISSERTATION HAS BEEN MICROFILMED EXACTLY AS RECEIVED

LA THÈSE A ÉTÉ MICROFILMÉE TELLE QUE NOUS L'AVONS REÇUE

A GENERAL EQUILIBRIUM MODEL OF PERSON SPECIFIC INFORMATION IN THE LABOUR MARKET

By

M. Mobinul Huq

Department of Economics

Submitted in partial fulfilment
of the requirements for the degree of
Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
London, Ontario
June 1987

ISBN 0-315-36588-9

ABSTRACT

In recent years there has a greater emphasis given to the matching/sorting models of human capital investment. In these models, education generates better person specific information which leads to a return from improved firm-worker matching. The first purpose of this thesis is to explain the factors that determine the real return for education through better firm-worker matching. The second purpose is to examine how this return is affected by changes in the economy. Effects of two changes, one in the labour market and the other a government policy, are examined in detail: firstly the change in the age composition of the labour force as the baby boom generation is passing through the labour force, and secondly the introduction of a social pension plan. Unlike the previous studies, the effects of these changes are examined in terms of a change in the allocative benefit from schooling.

The thesis shows that the real return from better firm-worker matching is determined by the structure of demand for final output. The greater the demand for output produced by the 'older worker intensive' industry, the

larger will be the gain from specialization through investment in person specific information. It is also shown that the individually chosen investment in information will be in general smaller than the 'golden rule' steady state level. For the economy as a whole a higher level of productivity gains from schooling will lead to a lower allocative gain.

Any change in the age structure of the labour force leads to a change in the real gains from better firm-worker matching and the level of investment in information. When the number of younger workers increases, through changes in the demand conditions the allocative gains from education increase, and the level of investment in information is also increased. An increase in the number of older workers has the opposite effects.

In the presence of a social pension plan if the growth rate of the pension contribution and the rate of interest are not equal, the return from, as well as level of, investment in information will be affected. In the case of proportional or regressive tax rates, if the pension fund grows at a faster (slower) rate, the existence of such a plan will increase (decrease) the return from information but decrease (increase) the actual level of investment in information.

DEDICATION

This work is dedicated to my mother, Jobeda Huq.

ACKNOWLEDGEMENTS

I express sincere appreciation to the three members of my Thesis Committee, James B. Davies, Peter Kuhn and James Melvin, for their constructive help in the completion of this research. Especially James B. Davies, my chairman, has been most helpful in providing generous amount of time, insightful suggestion and encouragement. I also benefited immensely from discussions with Glenn M. T. MacDonald in the initial stages of the research.

The greatest portion of appreciation, however, goes to my wife, Ruma, for her quiet patience and cheerful long suffering for the last few years. She also deserve a special thanks for the hard work she put in typing the manuscript.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

I. THE MODEL

Economic theories of human capital investment focus on a variety of questions, such as: What determines the return from schooling? What are the determinants of individual's demand for education? What is the optimum level of social provision of education? These questions have been widely examined in the last three decades. Broadly speaking we can distinguish the following three approaches in the resulting literature:

1. Productivity theory: The idea is that education increases the amount of resources embodied in an individual. So productivity of an individual is positively related with the level of education and the wage difference between a more and a less educated individual reflects the contribution of education. (Becker 1964, Schultz 1960, Mincer 1974, Ben-Porath 1967).

2. Information Transfer Theory: The labour market is

characterized by heterogeneity and diversity in the supply side, where different workers have different types/levels of ability. It is assumed that the worker knows his/her own ability but it is not known by others. Human capital investment is the process through which information regarding his/her ability is transferred to the employer. (Arrow 1973, Spence 1974, 1976, Riley 1975, 1979, Stiglitz 1975, Willis and Rosen 1979, Guasch and Weiss 1981, Willis 1986).

One such model is developed in Willis (1986) by extending the Willis-Rosen model (1979). It is assumed that individuals have different occupation specific ability and face a given occupational wage structure. By choosing different levels and types of education they reveal their ability and accordingly enter different occupations.

The behaviour of a firm under such asymmetric information is examined in Guasch and Weiss (1981) in a partial equilibrium framework. They showed that if workers are risk averse or risk neutral, there exists a wage structure which can be used to sort the workers.

3. Information Production Theory: In this type of model also it is assumed that workers have different types of ability, but neither the worker nor the employer are assumed to have information regarding the ability of a

worker. When information is produced through human capital investment it becomes known to everyone (MacDonald 1980, Jovanovic 1979, Johnson 1978, Flinn 1986, Miller 1984).

MacDonald (1980) showed that there are both private and social returns from information-generating human capital investment. His emphasis is on the individual's optimization decision, where the return from better information is given exogenously.

Most other models of this type studied human capital in the form of on the job training (OJT) and examined its role in determining job turnover. Jovanovic (1979) studied a production process which generates information. His model is extended to a continuous time case in Flinn (1986). Johnson (1978) and Miller (1984) also looked at OJT as an information generating process. In all these studies it is assumed that the value of information generated through human capital investment is exogenously given and the emphasis is on the problem of individual's optimization.

Rosen (1978) examined the use of person-specific information from the point of view of a firm when the final demand for the firm's output is given. In his model workers take no action to alter abilities or information.

Thus in the context of 'information production' models the question 'What determines the return from human capital investment?' remains unexplained. The first purpose of my thesis is to provide one answer to this question. This issue is examined in chapters 2 and 3.

The gain from schooling in the information production model comes from an increased productivity as a result of an improved match of a worker and a firm. It can be argued that the real return from improved firm-worker matching is affected by the structure of demand for final output. In an extreme case, as an economy specializes in the production of a single commodity, all the factors of production are used in one industry and the gain from person specific information will be at its minimum. Thus in order to study what determines the return from better job matching we will examine the following question: How do the demand conditions affect the return from better job matching?

Since we are going to examine the structure of demand for the economy the most appropriate method is to use a general equilibrium framework. Using the labour market structure of MacDonald's model (1980) a simple general equilibrium model of imperfect person specific information in the labour market is developed in chapter 2. We will call it 'the basic information model', since it is assumed that human capital investment only generates information.

The thesis shows that the real return from better firm-worker matching is determined by the structure of demand for final output. The greater the demand for output produced by the 'older worker intensive' industry, the larger will be the gain from specialization through investment in person specific information.

But schooling also leads to an increase in the quantity of resources embodied in individuals, which is the productivity augmenting effect (Becker 1964, Schultz 1960, Mincer 1974, Ben-Porath 1967). In chapter 3 of this thesis I have developed a model where schooling increases productivity of a worker as well as generating information. It is found that for the economy as a whole a higher level of productivity gains from schooling will lead to a lower allocative gain.

II. APPLICATIONS

Basic data on earnings by education show that the return from schooling is not constant over time. Thus the question arises: What causes a change in the return from education? In my thesis I have examined the role of two different factors which might have some effect on the returns from education through better firm-worker matching.

Age-Structure of the Labour Force

Beginning in the early 1970's with the labour market entry of the post world war II 'baby boom' generation the number of younger workers in the labour force rose significantly compared to the number of older workers. This motivated a number of studies to examine the process through which cohort size affects the return from, and the optimum choice of human capital investment (Freeman 1975, 1979, Easterlin 1978, Welch 1979, Berger 1985, Dooley 1986, Dooley and Gottschalk 1984).

Welch (1979) argued that the change in the age structure of the labour force affects the relative earnings of the different cohorts of workers, since workers with different levels of experience are imperfect substitutes for one another. So a relative increase in the number of younger workers will decrease their relative earnings. Welch examined the wage depressing effect over time in terms of a career phase model, where each worker passes through different phases of his/her career. He predicted faster income growth for a larger cohort. In his analysis the relative earnings of different cohorts are affected by relative supply changes only.

On the other hand Freeman (1975, 1979) argued that the depressed wage of the larger cohort in the 70's was a result of changes in both supply and demand conditions. He argued that in the 1970's the U.S. economy experienced a shift in labour demand against the group with higher schooling due to a change in the industrial structure. He also argued that without a favorable demand condition the supply effect will dominate and the members of the larger cohort will not experience a faster income growth rate. In his study the change in the industrial structure and the demand condition is considered to be exogenously given.

Berger (1985) in an empirical paper estimated a modified version of the Welch model and concluded that the members of the larger cohort in fact experienced a slower income growth rate.

Dooley (1986) examined the cohort size effect in terms of relative supply change for the Canadian economy. His findings are generally consistent with those of Welch, that is the members of the larger cohort experienced a depressed wage rate, with the downward effect decreasing with level of experience. But his study cannot explain the fact that the earnings gap is narrowing between workers with different schooling levels for both young and old workers. Dooley, like Freeman, also concluded that the change in the structure of demand should be studied in order to fully explain the cohort size effect.

Dooley and Gottschalk (1984) examined the effect of a change in age composition of the labour force on relative earnings and income inequality. They studied the effects from supply side point of view, and argued that the most appropriate approach to examine the problem is to use a general equilibrium model where the rate of change in the return from schooling and optimum level of schooling are determined simultaneously.

All these studies attempt to explain the effect of a change in the age structure of the labour force in terms of relative supply change. As shown in Conneily (1986) the predictions in these studies differ due to different types of production function assumed in different models. Also all the studies so far considered that the cohort size has nothing to do with the demand condition for labour and demand conditions are given exogenously. But with a change in the size and the age structure of the labour force the production possibility frontier for the economy will shift. This will result in a change in both the relative prices and the amount of production of different commodities. In the first part of my thesis it is shown that the market demand condition is directly related with the allocative benefit of schooling. So for a complete picture of how cohort size affects human capital investment, one should not ignore the effects on the return from, and the amount of schooling, through changing demand conditions.

An appropriate theoretical framework for studying the demand effects is to examine the allocative benefits from schooling in a general equilibrium model where the rate of change in return from schooling and optimum schooling level are determined simultaneously. Thus the general equilibrium model of the labour market developed in chapters 2 and 3 is applied to examine the problem addressed here. This is done in chapter 4 of this thesis.

It has been shown that any change in the age structure of the labour force leads to a change in the relative output price. This changes the allocative benefits from, and equilibrium level of schooling. As a baby boom generation passes through the labour force, due to changes in the demand conditions, they will face a depressed market wage and will choose a higher level of schooling.

In addition to the baby-boom effects two other changes in the age composition of the labour force are also examined. One is a selective immigration policy, where a large number of skilled older workers are allowed to immigrate which results in a change in the age-composition of the labour force. The other is increased labour market participation of the secondary workers who are mostly unskilled and have relatively shorter working life.

An increase in the number of secondary workers, increases both the allocative gain and the optimum level of schooling. This results in an increased earnings inequality between skilled and unskilled. An increased immigration of skilled workers has opposite effects.

Social Pension Plan

Human capital theory looks at the schooling level as an individual's decision, where he/she determines the level of investment on the basis of costs and benefits from the investment. Since the cost and benefit structure from schooling is affected by the existence of a social pension plan, it will affect individuals' optimum choice of schooling.

Drazen (1978) and Black (1987) examined the effects of a social pension plan on human capital investment. The study by Black examined the incentives created by intragenerational transfers through a social pension plan. He showed that such a program increases human capital investment made by lower income groups while that made by higher income groups will decrease. On the other hand Drazen examined the incentives created through intergenerational transfers due to such a program. Drazen looked at a model where individuals don't choose their own human capital investment, but rather determine that of the next generation. Both of these studies examined only the productivity augmenting effects of schooling in a partial equilibrium model. The return from schooling is assumed to be given exogenously and fixed.

In chapter 2 of this thesis it is shown that in a general equilibrium framework individuals' optimization through changes in the demand structure might affect the returns (through better firm-worker matching) from schooling. In chapter 5, I have examined the effects of a social pension plan on the return from schooling in terms of the general equilibrium model. It is found that a social pension plan will affect both the return from, as well as level of, investment in information, if the growth rate of the pension contribution and the rate of interest are not equal.

Finally a summary of the findings and some concluding remarks are given in Chapter 6.

CHAPTER 2

THE BASIC INFORMATION MODEL

The purpose of this chapter is to examine the role of the structure of final demand in determining the return from as well as the equilibrium level of investment in information. Using the labour market structure of MacDonald's model (1980) a simple general equilibrium model of imperfect person-specific information in the labour market is developed. Unlike the partial equilibrium models, the return from investment in information is endogenous in this model. In the next section the structure and notation are given. Section III discusses the derivation of the production possibility curves. The preferences are given in section IV. The steady state golden rule production plan and an individual worker's optimization problem is examined in section V and VI respectively. The actual equilibrium for the economy is studied in section VII and the nature of the capital market is discussed in section VIII.

II. STRUCTURE AND NOTATION

There are two goods α and β, each of which is produced using two inputs (tasks) T_1 and T_2. The production functions are assumed to be Leontief type fixed coefficient function[1],

(2.1.a) $Y^{\alpha} = \text{Min}\,(T^{\alpha}{}_1/a_1,\ T^{\alpha}{}_2/a_2)$

(2.1.b) $Y^{\beta} = \text{Min}\,(T^{\beta}{}_1/b_1,\ T^{\beta}{}_2/b_2)$

where Y^j represent output of industry j, $T^j{}_i$ denotes the amount of task i used in industry j, a_i and b_i are the task-output ratio for task i in industry α and β respectively. Industry α is assumed to be task 1 (T_1) intensive while β is T_2 intensive, that is

$$a_1/a_2 > b_1/b_2$$

There are two types of workers in the workforce, A and B. Every one in each type is identical. The only difference between type A and B is in terms of the amount of tasks they can perform, which are,

$$A : (t^A{}_1,\ t^A{}_2)$$

$$B : (t^B{}_1,\ t^B{}_2)$$

where $t^j{}_i$ stands for maximum amount of task i that can be performed by one individual of type j. Neither A nor B type workers in this model are assumed to have absolute advantage in all tasks. Thus

$$t^A{}_1 > t^B{}_1 \text{ and } t^A{}_2 < t^B{}_2$$

i.e. type A is efficient in task 1 and type B in task 2. The production functions, equation (2.1.a) and (2.1.b) are assumed to be such that type A workers are perfectly matched with industry α and B with industry β[2]. So $Y^j{}_i$, the output of one individual of type i in industry j will be,

$$(2.2.a) \qquad Y^{\alpha}{}_A = t^A{}_1/a_1 = t^A{}_2/a_2 \ , \quad Y^{\beta}{}_A = t^A{}_2/b_2 < t^A{}_1/b_1$$

$$(2.2.b) \qquad Y^{\beta}{}_B = t^B{}_1/b_1 = t^B{}_2/b_2 \ , \quad Y^{\alpha}{}_B = t^B{}_1/a_1 < t^B{}_2/a_2$$

In this chapter it is assumed that each worker lives for two periods and that at any particular period of time there are two groups of workers of equal size, one group at their initial period of life (young workers) and the other in the final period of their life (older workers).

Let the total number of A and B type workers in each group be $\bar{A}$ and $\bar{B}$ respectively. Then the total availability of task from each group of workers is:

$$(2.3.a) \qquad T_1 = t^A{}_1\bar{A} + t^B{}_1\bar{B}$$

$$(2.3.b) \qquad T_2 = t^A{}_2\bar{A} + t^B{}_2\bar{B}$$

The total use of a task in production from one group cannot exceed these availabilities. The cohstraints are,

$$(2.4.a) \qquad T^{\alpha}{}_1+T^{\beta}{}_1 = T_1$$

$$(2.4.b) \qquad T^{\alpha}{}_2+T^{\beta}{}_2 = T_2$$

<u>Information</u>

It is assumed that one half of the workers in any group (young or old) is type A and the other half is type B (i.e.

$\bar{A} = \bar{B}$) and this information is known by everyone in this model. But nobody knows which type of worker one particular individual is. For young workers no other information is available. However a test is available to determine which type of worker one is. This test will assign them a value `a' or `b' and the probability of a randomly selected individual getting `a' or `b' is ½. $P(j/k)$ is defined as the probability that given label k one is type j worker. This probability varies from ½ to 1 and increases with the intensity of the test. It is assumed that,

$$P(A/a) = P(B/b) = P > \tfrac{1}{2}$$

$$P(B/a) = P(A/b) = (1-P) < \tfrac{1}{2}$$

It is assumed that the only cost of the test is the fraction of time required for the test, say $S(P)$, and it increases at an increasing rate with the test intensity P. i.e. $S'(P) > 0$, $S''(P) > 0$, further $S(\tfrac{1}{2}) = 0$ and $S(1) = 1$[3]. In any period for perfect information all the time is required for the test. Information generated through this test cannot be used in that period. So in any period only the younger workers will go for the test.

<u>Expected output</u>

Young worker: If any young worker is chosen at random, the probability that he/she is of type A is ½ and the probability that he/she is of type B is also ½, since $\bar{A}$ =

B. So if any young worker (in period 0 of his/her life) is employed in industry α, the expected output Y^{α}_{O} will be,

$$
\begin{aligned}
Y^{\alpha}_{O} &= \tfrac{1}{2}Y^{\alpha}_{A} + \tfrac{1}{2}Y^{\alpha}_{B} \\
&= Y^{\alpha}_{B} + \tfrac{1}{2}(Y^{\alpha}_{A}-Y^{\alpha}_{B}) \qquad \text{let } Y^{\alpha}_{B}= \alpha_{O},\ (Y^{\alpha}_{A}-Y^{\alpha}_{B})= \alpha_{1} \\
&= \alpha_{O} + \tfrac{1}{2}\alpha_{1}
\end{aligned}
$$

If one young worker chosen at random is employed in industry β his/her expected output will be,

$$
\begin{aligned}
Y^{\beta}_{O} &= \tfrac{1}{2}Y^{\beta}_{A} + \tfrac{1}{2}Y^{\beta}_{B} \\
&= Y^{\beta}_{A} + \tfrac{1}{2}(Y^{\beta}_{B}-Y^{\beta}_{A}) \qquad \text{let } Y^{\beta}_{A}= \beta_{O},\ (Y^{\beta}_{B}-Y^{\beta}_{A})= \beta_{1} \\
&= \beta_{O} + \tfrac{1}{2}\beta_{1}
\end{aligned}
$$

Payment to the workers (wage rate) is assumed to be equal to their expected level of output. In a competitive equilibrium the wage rate of a young worker (W^{O} in units of good $\bar{\alpha}$) in industry α or β should be equal. Thus the following equality should hold,

$$W^{O} = \alpha_{O}+ \tfrac{1}{2}\alpha_{1} = (\beta_{O}+ \tfrac{1}{2}\beta_{1})\pi$$

where π is the price of β goods in terms of α. This implies,

$$\pi = (\alpha_{O}+ \tfrac{1}{2}\alpha_{1})/(\beta_{O}+ \tfrac{1}{2}\beta_{1})$$

Older workers : There will be two types of workers in the older group, one with label `a' and the other with label `b' and the probability $P(A/a) = P(B/b) = P$. Denote expected output of one older worker (in period 1 of their life) with label k working in industry j as Y^{j}_{k}, then we have the following four cases,

i) Worker with label `a' working in industry α,

$$Y^{\alpha}{}_{a} = Y^{\alpha}{}_{A}P + Y^{\alpha}{}_{B}(1-P)$$
$$= \alpha_0 + \alpha_1 P$$

ii) Worker with label `a' working in industry β,

$$Y^{\beta}{}_{a} = Y^{\beta}{}_{A}P + Y^{\beta}{}_{B}\ (1-P)$$
$$= \beta_0 + \beta_1(1-P)$$

iii) Worker with label `b' working in industry β,

$$Y^{\beta}{}_{b} = Y^{\beta}{}_{B}P + Y^{\beta}{}_{A}(1-P)$$
$$= \beta_0 + \beta_1 P$$

iv) Worker with label `b' working in industry α,

$$Y^{\alpha}{}_{b} = Y^{\alpha}{}_{B}P + Y^{\alpha}{}_{A}(1-P)$$
$$= \alpha_0 + \alpha_1(1-P)$$

Thus the expected output of an older worker (wage rate) will depend on what label he/she gets in the younger life and in which industry he/she is working. The wage rate of those workers employed in industry i with label j, $W^{i}{}_{j}$, will be,

$$W^{\alpha}{}_{a} = \alpha_0 + \alpha_1 P = W^{o} + \alpha_1(P-\tfrac{1}{2})$$
$$W^{\alpha}{}_{b} = W^{o} - \alpha_1(P-\tfrac{1}{2})$$
$$W^{\beta}{}_{a} = W^{o} - \beta_1(P-\tfrac{1}{2})$$
$$W^{\beta}{}_{b} = W^{o} + \beta_1(P-\tfrac{1}{2})$$

III. THE PRODUCTION POSSIBILITIES

Young and old workers separately

Young workers: Given the total availability of tasks $\bar{T}_1$ and $\bar{T}_2$ from equation (2.3.a) and (2.3.b) the factor box diagram is drawn in Figure 2.1.a. Taking 0^α and 0^β as the origins the isoquant maps of industries α $(\alpha_1, \alpha_2, \ldots, \bar{\alpha})$ and β $(\beta_1, \beta_2, \ldots, \bar{\beta})$ are drawn respectively.

For the younger workers the only information available is that the numbers of A and B type workers are equal. So any worker chosen will be expected to perform tasks 1 and 2 in the proportion $(\bar{T}_1/\bar{T}_2)$. Thus the only feasible production points are those on the diagonal $0^\alpha 0^\beta$ in Figure 2.1.a. Corresponding to this the production possibility curve (PPC) will be a downward sloping straight line like $\bar{\alpha}\bar{\beta}$ in Figure 2.1.b. If all the tasks are used in industry α, $\bar{\alpha}$ amount can be produced and using all tasks in β, $\bar{\beta}$ can be produced.

If the young workers spend some time generating information regarding their abilities, then the amount of tasks available for production will decrease and the constraints (2.4.a) and (2.4.b) become,

$$T^\alpha{}_1+T^\beta{}_1 = (1-S(P))\bar{T}_1$$
$$T^\alpha{}_2+T^\beta{}_2 = (1-S(P))\bar{T}_2$$

When information of quality P_1 is generated then $S(P_1)$

fraction of time will be taken away from production. In terms of Figure 2.1.a the factor box will be reduced by the amount $\bar{T}_2S(P_1)$ horizontally and by $\bar{T}_1S(P_1)$ vertically and the new factor box will be $0^{\alpha}0^{\beta\prime}$. For the same reason as before production will take place along the diagonal $0^{\alpha}0^{\beta\prime}$. The corresponding PPC will be $\alpha_4\beta_4$. With an increase in P the factor box will be reduced further and the PPC will shift towards the origin. In the extreme case when P=1, we will have S(P)=1 and no task will be left for producing α or β.

The slope of the PPC will be = $0\bar{\alpha}/0\bar{\beta}$

$$\text{Since } 0\bar{\alpha} = \bar{T}_1/a_1 \text{ and } 0\bar{\beta} = \bar{T}_2/b_2$$

$$\Longrightarrow \quad 0\bar{\alpha}/0\bar{\beta} = (b_2/a_1)(\bar{T}_1/\bar{T}_2) = (b_2/a_1)(t^e{}_1/t^e{}_2)$$

where $t^e{}_i$ is the expected amount of task i performed by one worker.

So, the slope of the PPC = $(t^e{}_1/a_1)/(t^e{}_2/b_2)$

$$=(\alpha_o + \tfrac{1}{2}\alpha_1)/(\beta_o + \tfrac{1}{2}\beta_1)$$

Older workers: In Figure 2.2.a and 2.2.b the factor box and the PPC for the older workers are drawn. For the older workers the shape and the position of the PPC will depend on the intensity of information generated in their younger period. If no information is generated then the level of P will be $\frac{1}{2}$ for the old workers also and the PPC will be the same as that of young workers, which is shown by $\bar{\alpha}D\bar{\beta}$ in

Figure 2.2.b. If perfect information was generated (i.e. P=1) then workers with label 'a' will supply tasks in the proportion (t^{A}_{1}/t^{A}_{2}) and 'b' in the proportion (t^{B}_{1}/t^{B}_{2}). From equation (2.2.a) and (2.2.b) we know $(t^{A}_{1}/t^{A}_{2})=(a_1/a_2)$ and $(t^{B}_{1}/t^{B}_{2})= (b_1/b_2)$. So in this case production at any point in the region $0^{\alpha}A0^{\beta}$ is feasible, but production on the lines $0^{\alpha}A$ and $A0^{\beta}$ will be efficient. Corresponding to this the PPC will be $\bar{\alpha}A\bar{\beta}$ in Figure 2.2.b. If all the factors are used in industry β, a total amount of $\bar{\beta}$ can be produced. Now to produce some α it would be optimal to release some 'a' label workers from β and employ them in α. Production of β will decrease at a rate $Y^{\beta}_{A} = \beta_o$ and production of α will increase by $Y^{\alpha}_{A} = \alpha_o+\alpha_1$ and production will move along the line $\bar{\beta}A$. When point A is reached all the 'a' labeled worker are taken out of β. In order to increase the production of α further some 'b' label workers will have to be taken out of β and be employed in α. So $Y^{\beta}_{B} = \beta_o+ \beta_1$ will be sacrificed for $Y^{\alpha}_{B} = \alpha_o$ and the production point will move along the curve $A\bar{\alpha}$. At $\bar{\alpha}$ all the factors are used in industry α.

When information quality is between ½ and 1 the PPC will be in between the above two . When P level of information is generated one 'a' label worker will be expected to perform the tasks in the ratio

$$(t_1/t_2)^P{}_a = (a_1/a_2)P + (b_1/b_2)(1-P)$$

We know $(\bar{T}_1/\bar{T}_2) = \frac{1}{2}(a_1/a_2) + \frac{1}{2}(b_1/b_2)$

$$\Longrightarrow \quad (t_1/t_2)^P{}_a = \lambda(a_1/a_2) + (1-\lambda)(\bar{T}_1/\bar{T}_2)$$

where $\lambda = 2P-1$

Similarly one `b' label worker will be expected to perform the tasks in the ratio.

$$(t_1/t_2)^P{}_b = \lambda(b_1/b_2)+(1-\lambda)(\bar{T}_1/\bar{T}_2)$$

Let these ratios for P_1 be $(t_1/t_2)^{P1}{}_a$ and $(t_1/t_2)^{P1}{}_b$. They are shown by the lines $0^{\alpha}B$ and $0^{\beta}B$ respectively in Figure 2.2.a. In this case the efficient production points are shown by the line $0^{\alpha}B$ and $0^{\beta}B$ and the PPC will be $\bar{\alpha}B\bar{\beta}$ in Figure 2.2.b. In this way we can derive different PPC's for different levels of P. As P approaches $\frac{1}{2}$, the PPC will approach $\bar{\alpha}D\bar{\beta}$ and as P approaches 1 the PPC will approach $\bar{\alpha}A\bar{\beta}$.

In this case we have two straight line portions of the PPC. The slope of these will differ depending on the value of P. The slope of the part $\bar{\alpha}B$ will be $\bar{\alpha}\alpha_2/\alpha_2B$. Further $\bar{\alpha}\alpha_2 = T^b{}_1/a_1$ and $\alpha_2B = T^b{}_2/b_2$ where T^b_i is the total amount of task i supplied by `b' label workers. $T^b{}_i = t^b{}_iN^b$ where $t^b{}_i$ is the amount of task i performed by one `b' label worker and N^b is the total number of `b' label worker.

$$\text{Thus,} \quad \bar{\alpha}\alpha_2/\alpha_2B = (T^b{}_1/a_1)/(T^b{}_2/b_2)$$
$$= (t^b{}_1/a_1)/(t^b{}_2/b_2)$$

(2.5) $= (\alpha_o+\alpha_1(1-P))/(\beta_o+\beta_1 P)$

Similarly, the slope of the $B\bar{\beta}$ part will be,

$$B\beta_1/\beta_1\bar{\beta} = (T^a{}_1/a_1)/(T^a{}_2/b_2)$$

$$= (\alpha_o+\alpha_1 P)/(\beta_o+\beta_1(1-P))$$

With higher level of P, $(t_1/t_2)^b$ decreases and the $\bar{\alpha}B$ part will become flatter, while $(t_1/t_2)^a$ increases, and the $B\bar{\beta}$ part will becomes steeper.

In Figure 2.2.b. only at the `kink' points like A and B, all the `a' label workers will be employed in industry α and `b' label in $\beta$. On the flatter part $A\bar{\alpha}$ or $B\bar{\alpha}$ some `b' label workers will be employed in α and on $A\bar{\beta}$ or $B\bar{\beta}$ some `a' label workers will be working in β.

The distance between the PPC with no information ($\bar{\alpha}D\bar{\beta}$) and the PPC with perfect information ($\bar{\alpha}A\bar{\beta}$) will depend on the difference in the task intensities in industry α and β, that is the difference between (a_1/a_2) and (b_1/b_2). The more these differ the further away will be $\bar{\alpha}A\bar{\beta}$ from $\bar{\alpha}D\bar{\beta}$. The reason is that when (a_1/a_2) and (b_1/b_2) differ more it implies factors (tasks) are more industry specific and who works where becomes more important, so that the benefit from better information and worker-industry matching is greater.

<u>Young and old workers together</u>

PPC for the Economy: In this model at any particular period of time there are two equal size groups of workers, one group at their initial period of life (young workers) and other at their second period of life (older workers). So the total production possibilities of the economy will be the sum of the PPC's of these two groups.

If no test is available in the economy to determine the productive capacities of the workers, the PPC of both young and old workers will be the identical, shown by $\bar{\alpha}\bar{\beta}$ and $\bar{\alpha}D\bar{\beta}$ in Figures 2.1.b and 2.2.b respectively. The PPC for the whole economy can be derived by adding these two as shown by $2\bar{\alpha}2\bar{\beta}$ in Figure 2.3.

Now if a test is introduced of the nature described in Section II, only the younger workers will go for the test and their PPC will shift inward , while in that period the PPC of the older workers will remain the same. So the PPC of the economy will approach $\bar{\alpha}\bar{\beta}$ as P tends to unity. Say for P_1 it would be the straight line joining C and F, while for P_2 it would be the straight line joining G and J.

In the next period the PPC for the older workers will shift upward say to $\bar{\alpha}A\bar{\beta}$ for P_1, $\bar{\alpha}B\bar{\beta}$ for P_2 in Figure 2.3. The PPC for young workers in this period will also depend on P. In a steady state case the value of P chosen by the workers in different time periods will be the same. So to derive the steady state PPC of the economy we add the PPC's of the younger and the older workers which correspond to the same level of P. The result is shown in Figure 2.3 by CDEF for P_1, GHIJ for P_2.

From the PPC's of the economy for different levels of P we can derive the steady state grand PPC which shows the maximum attainable steady state output combinations under different P's. When P can take only two values, P_1 and P_2, then the grand PPC will be $2\bar{\alpha}$KLHIMN$2\bar{\beta}$ in Figure 2.3. When P can take any value then the grand PPC will look like the curve ABCD in Figure 2.4. It will have two convex parts in the corners like AB and CD and in between there will be a straight line segment BC.

The PPC for the whole economy will move upward if the increase in output of the older workers due to better information is more than the decrease in output of the younger workers due to reduced working time. The grand PPC will be reached when the increase in output of the older workers and decrease in that of the young workers are equal.

We can also derive the condition required for reaching the grand PPC. From the construction of the grand PPC we can see that the level of schooling (P) corresponding to the straight line segment and the kinked point will be the same. Since this level of P allows us to produce on the grand PPC we will call this the 'golden rule' level of schooling (P^G).

In the straight line segment the young workers are employed in both α and β. In a competitive equilibrium the value of their output in terms of α must be identical,

$$(\alpha_o+\tfrac{1}{2}\alpha_1) = \pi(\beta_o+\tfrac{1}{2}\beta_1)$$

For someone spending the fraction S(P) of time at school this will be,

$$(\alpha_o+\tfrac{1}{2}\alpha_1)(1-S(P)) = \pi(\beta_o+\tfrac{1}{2}\beta_1)(1-S(P))$$

In the second period there is ½ probability that one will have label 'a' (b) and will be employed in α (β). Thus the expected level of output in the older period is

$$\tfrac{1}{2}[(\alpha_o+\alpha_1 P)+\pi(\beta_o+\beta_1 P)]$$

When the number of workers in each cohort is ($\bar{A}+\bar{B}$) then the total output in the economy (say Q) will be,

$$Q = (\alpha_o+\tfrac{1}{2}\alpha_1)(1-S(P))(\bar{A}+\bar{B}) + \tfrac{1}{2}[(\alpha_o+\alpha_1 P)+\pi(\beta_o+\beta_1 P)](\bar{A}+\bar{B})$$

In order to reach the grand PPC, P should be chosen in such a way that Q is maximized, and the first order condition for this is;

$$\delta Q/\delta P = -(\alpha_o+\tfrac{1}{2}\alpha_1)(\bar{A}+\bar{B})S'(P)+\tfrac{1}{2}(\alpha_1+\pi\beta_1)(\bar{A}+\bar{B}) = 0$$

$$\Longrightarrow \quad (\alpha_o+\tfrac{1}{2}\alpha_1)S'(P) = \tfrac{1}{2}(\alpha_1+\pi\beta_1)$$

Thus the grand PPC will be reached when P is such that the increase in the expected output of one older worker $\tfrac{1}{2}(\alpha_1+\pi\beta_1)$ and the decrease in the output of one younger worker $[(\alpha_0 + \frac{1}{2}\alpha_1)\, S'(P)]$ are equal. These are the marginal benefit and marginal costs of schooling respectively.

Alternatively, the above condition can be written as,

$$S'(P) = \tfrac{1}{2}(\alpha_1 + \pi\beta_1)/(\alpha_0 + \tfrac{1}{2}\alpha_1)$$

$$S'(P) = \tfrac{1}{2}(\alpha_1/(\alpha_0+\tfrac{1}{2}\alpha_1)) + \tfrac{1}{2}(\beta_1/(\beta_0+\tfrac{1}{2}\beta_1))$$

From this we can say that P^G depends on α_0, α_1, β_0 and β_1. If α_0 or β_0, the level of output of a worker in 'wrong' industry, goes up optimum P^G will go down. This is so because higher α_0 or β_0 implies smaller loss from wrong matching of industry and worker, which leads to a higher cost of schooling. On the other hand an increase in α_1 or β_1 will imply a higher level of gain from better worker-industry matching and also an increase in the cost of schooling. But the first effect will dominate and there will be an increase in the optimum level of information generated.

The second order condition for maximization is also satisfied, as

$$\delta^2 Q/\delta P^2 = -(\alpha_0 + \tfrac{1}{2}\alpha_1)(\bar{A}+\bar{B})S''(P) < 0$$

Since $(\alpha_0 + \frac{1}{2}\alpha_1) > 0$, $(\bar{A}+\bar{B}) > 0$ and $S''(P) > 0$

In this analysis so far we assumed that there is no growth in the labour force. We can easily incorporate a constant growth rate in the labour force, say n% per period. In that case the factor box for the younger workers will be (1+n) times larger than that of the older workers, but there will be no change in the construction and shape of the grand PPC. However, the condition for reaching the grand PPC will change. At any period when the number of older workers are $(\bar{A}+\bar{B})$, the number of younger worker in that period will be $(\bar{A}+\bar{B})(1+n)$ and total output in the economy in terms of α will be,

$$Q=(\alpha_o+\tfrac{1}{2}\alpha_1)(1-S(P))(\bar{A}+\bar{B})(1+n)+\tfrac{1}{2}[(\alpha_o+\alpha_1 P)+\pi(\beta_o+\beta_1 P)](\bar{A}+\bar{B})$$

The first order condition for maximization is,

$$\delta Q/\delta P = -(\alpha_o+\tfrac{1}{2}\alpha_1)(\bar{A}+\bar{B})(1+n)S'(P)+\tfrac{1}{2}(\alpha_1+\pi\beta_1)(\bar{A}+\bar{B}) = 0$$

(2.6) $$\Longrightarrow \quad (\alpha_o+\tfrac{1}{2}\alpha_1)S'(P)(1+n) =\tfrac{1}{2}(\alpha_1+\pi\beta_1)$$

In this case the grand PPC will be reached when P is such that the increase in the second period output is (1+n) times the decrease in initial period output. This condition can also be written as

(2.7) $$S'(P) =[\tfrac{1}{2}(\alpha_1/(\alpha_o+\tfrac{1}{2}\alpha_1))+\tfrac{1}{2}(\beta_1/(\beta_o+\tfrac{1}{2}\beta_1))]/(1+n)$$

The effect of α_o, β_o, α_1 and β_1 on the golden rule level of P^G in this case is same as before. But now there is one additional factor n, which will also affect P^G. The relationship is inverse, the higher is the labour force growth rate, the lower will be the optimum level of P, as a larger number of younger workers implies larger output loss due to schooling.

IV. **PREFERENCES**

An individual's ordinal utility index depends upon his consumption of α and β in period 0 and 1,

$$U = U(C^{\alpha}_{o},\ C^{\beta}_{o},\ C^{\alpha}_{1},\ C^{\beta}_{1})$$

In order to obtain interesting results it is necessary to put considerable structure on this utility function. Accordingly it is assumed to be intertemporally and additively separable, so that it can be written as,

(2.8) $U = u(C^{\alpha}_{o},C^{\beta}_{o})+(1/(1+\rho)u(C^{\alpha}_{1},C^{\beta}_{1})\quad u_{\alpha},u_{\beta}>0,\ u_{\alpha\alpha},u_{\beta\beta}<0$

where ρ is the rate of time preference.

The function $u(C^{\alpha}_{i},C^{\beta}_{i})$; $i = 0,1$, is assumed to be continuous, differentiable and homothetic. In the steady state equilibrium, the relative price of β in terms of α will be constant, so that in each period α and β will be consumed in a fixed proportion. In that case the utility function can be written as a function of the amount of consumption in terms of α,

$$C_{i} = C^{\alpha}_{i}+C^{\beta}_{i}\pi\ ,\quad i=0,1$$

and the utility function can be written as

(2.9) $U = V(C_{o}) + (1/(1+\rho))V(C_{1})$

This utility function is shown diagrammatically in Figure 2.15 by indifference curve (IC). The slope of the indifference curve is,

$$\delta C_{1}/\delta C_{o} = -(1+\rho)V'(C_{o})/V'(C_{1})$$

where $V'(C_1) = \delta V(C_1)/\delta C_1$. Along a 45^{o} line, where $C_0 = C_1$, the slope of the IC will be $-(1 + \rho)$.

All the individuals in the economy are assumed to have identical preferences. Given all these assumptions, for a particular point of time, we can draw the group IC for the younger and older group of workers and also the community indifference curve (CIC's) as shown in Figure 2.5, by C_I, C_{II} and CIC_I respectively. C_I is drawn taking point E as the origin. All these three sets of indifference curves are assumed to be continuous, differentiable and homothetic.

When we look at the utility function from one individual's point of view there may be uncertainty regarding second period income, because their earnings will depend on what label they get in the initial period and in which industry they are employed in the second period.

By assuming that there exist a costless insurance in the economy, the individuals may be insured in such a way that his or her earnings in the second period are independent of the label he or she gets in the initial period. In that case the second period income will be certain at its expected level. Throughout this chapter we will assume that such insurance exists in the economy.[4]

In this model all the individuals in the younger period are identical, and their income will be insured at an amount which depends on the schooling level. The schooling level as well as the label one gets in the older period is known to everyone in this model. Given these, there will be no moral hazard or adverse selection problem in this model.

V. THE STEADY STATE "GOLDEN RULE" OUTPUT PLAN

Diagrammatically the steady state 'golden rule' output plan is shown in Figure 2.4. The grand production possibility curve (PPC) is shown by the curve ABCD. The point of tangency between the grand PPC and the community indifference curve (CIC) will determine the 'golden rule' outputs of α and β (α^G, β^G). Depending on the shape of the CIC's there are three possible kinds of (α^G, β^G) as given below:

i) in the straight line segment (BC) of the grand PPC (when CIC_I show the preferences).

ii) in the convex part (AB or CD segment) of the grand PPC (when CIC_2 show the preferences).

iii) at the kinked point (B or C) of the grand PPC (when CIC_3 show the preferences).

The nature and some properties of these production plans are discussed next.

i) Output plan in the straight line (BC) segment: One such possible (α^G, β^G) is shown in Figure 2.5. The preferences of the workers are given by CIC_1 and the steady state output plan is determined at point E. Production by the two groups of workers will take place at point P, where all `a' level workers will be employed in $\alpha$ and all `b' in β, while young workers will be employed in α and β industries in the proportion GP/PH.

The steady state price ratio (π^*_1) is given by the slope of the CIC (or the PPC) at (α^G, β^G), which is equal to the slope of the line BC i.e. the slope of the young worker's PPC. Thus,

$$\pi^*_1 = (b_2/a_1)/(T_1/T_2) = (\alpha_0+\tfrac{1}{2}\alpha_1)/(\beta_0+\tfrac{1}{2}\beta_1)$$

Given this price ratio and the production point P, the steady state consumption point will be at J, with the young workers on C_1 and older workers on C_{II}. So long as, at this price ratio, the income consumption curve (OJE in Figure 2.5) is in the range OB and OC, the golden rule output plan will be in the straight line part of the grand PPC. From the construction of the PPC we know that the part BC is associated with a certain level of schooling (P^G) and that P will be the steady state `golden rule' level of test intensity.

Next we will examine the steady state `golden rule' level of real income of a representative worker in his/her initial and older period of life. Given the golden rule level of relative output price (π^*_1) and the real income we will also look at his/her feasible consumption set in different periods.

The wage rate of a representative younger worker employed in α will be equal to his/her expected output, which is

$$(\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P))$$

and someone employed in β will earn,

$$(\beta_0+\tfrac{1}{2}\beta_1)(1-S(P))$$

Given the equilibrium relative price (π^*_1) the latter in terms of α will be

$$\pi^*_1(\beta_0+\tfrac{1}{2}\beta_1)(1-S(P)) = (\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P))$$

Thus the maximum amount of α that can be consumed by one representative younger worker from his/her current income is

$$(\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P))$$

The feasible consumption frontier for one young worker (without borrowing and lending) can be expressed as,

$$\alpha = (\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P)) - \pi^*_1\beta$$

where $\pi^*_1 = (\alpha_0+\tfrac{1}{2}\alpha_1)/(\beta_0+\tfrac{1}{2}\beta_1)$

This is shown in Figure 2.6, by the straight line AB.

When an older worker is chosen at random, the probability is ($\tfrac{1}{2}$) that he/she has a label 'a' and in that case he/she will be employed in α and will earn (equal to expected output),

$$(\alpha_0+\alpha_1P)$$

Also the probability is ($\frac{1}{2}$) that he/she is a worker with label 'b' and will be working in β, and earn

$$(\beta_0+\beta_1P)$$

We are going to assume that individual's earning in the second period is insured at their expected level, in terms of α which will be

$$W_1 = \tfrac{1}{2}[(\alpha_0+\alpha_1P) + (\beta_0+\beta_1P)\pi^*_1] > (\alpha_0+\tfrac{1}{2}\alpha_1)$$

since $(\beta_0+\beta_1P)\pi^*_1>(\alpha_0+\frac{1}{2}\alpha_1)$.

The feasible consumption frontier is given by,

$$\alpha = \tfrac{1}{2}[(\alpha_0+\alpha_1P)+(\beta_0+\beta_1P)\pi^*_1] - \pi^*_1\beta$$

This set is shown in Figure 2.6 by the straight line CD.

In this case with higher level of schooling and specialization there is no change in the relative price. So younger workers have no gain from specialization at the new output plan and all the gains goes to the older workers.

ii) Output plan in the convex (AB) part: Golden rule output plan in the convex part is shown in Figure 2.7[5]. The CIC and the PPC are tangent at point F, which will establish the steady state 'golden rule' output plan. The production and consumption points will be points P and J respectively. All the younger and the 'a' labeled older workers will be employed in α and 'b' labeled older workers will be employed in both α and β. Now the price ratio will be given by the slope of the grand PPC or CIC at F, which is equal to the slope of the less steep part of the older worker's PPC, $\bar{\alpha}P'$. At this price ratio if all 'a' labeled older workers work in α and all 'b' in β, production will take place at P'' and in that case we can see that the desired consumption level of the younger and the older workers will be at points H and J respectively and there is excess demand for α. So some 'b' label workers will be taken out of industry β and will be employed in α. The proportion of 'b' label workers employed in α will depend on the location of (α^G, β^G). The closer is the output plan to the point A the more 'b' labeled older workers will be employed in α.

In terms of Figure 2.7 the golden rule relative price is shown by the slope of the line $\bar{\alpha}P'$, which is from equation (2.5),

$$\pi^*_2 = (b_2/a_1)(t_1/t_2)^b = (\alpha_0+\alpha_1(1-P))/(\beta_0+\beta_1 P)$$

For competitive solution this relative price should be such that the wage rate of one 'b' labeled older worker is the same in industry α and β. The value of P associated with the grand PPC going through the point F (which is GFKLM) will give the golden rule level of test intensity. This level of P will be smaller than that in the previous case

(P^G) and the closer is F to the point A the smaller will be the value of the golden rule amount of P. At point A P=(1/2) is optimal.

Given the golden rule level of relative output price, next we will look at the feasible consumption set of a representative worker by deriving his/her expected income.

All the younger workers will be employed in α and their earnings will be,

$$(\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P))$$

Thus their feasible consumption set can be written as,

$$\alpha = (\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P)) - \pi^*_2\beta$$

where, $\pi^*_2 = (\alpha_0+\alpha_1(1-P))/(\beta_0+\beta_1 P)$

The maximum amount of β that can be consumed (when α =0) will be,

$$\beta^M = (\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P))(\beta_0+\beta_1 P)/(\alpha_0+\alpha_1(1-P))$$

This feasible consumption set is shown by AB in Figure 2.8. With an increase in the level of P the vertical intercept will decrease and at the same time the slope will also decrease. Thus the horizontal intercept can move in any direction. For any given P young workers in this case are better-off compared to case (i) as they are here employed in only α and the relative price is moved in favour of α.

When we look at the situation of one older worker, the probability is $\tfrac{1}{2}$ that he/she is an 'a' labeled worker and in that case he/she will be employed in industry α and will earn,

$$(\alpha_0 + \alpha_1 P)$$

also the probability is ½ that he/she is a `b' labeled worker. But in this equilibrium `b' labeled older workers will be employed in both α and β, and in a competitive solution their income must be the same. If he/she is employed in α, the wage rate will be,

$$(\alpha_0 + \alpha_1 (1-P))$$

and if employed in β, the wage rate in terms of α will be,

$$\pi^*_2(\beta_0 + \beta_1 P) = (\alpha_0 + \alpha_1 (1-P))$$

When their earnings are insured at their expected level, in terms of α that will be,

$$= \tfrac{1}{2}(\alpha_0 + \alpha_1 P) + \tfrac{1}{2}(\alpha_0 + \alpha_1 (1-P))$$

$$= (\alpha_0 + \tfrac{1}{2}\alpha_1)$$

Thus the feasible consumption set (as shown in Figure 2.8 by CD) will be,

$$\alpha = (\alpha_0 + \tfrac{1}{2}\alpha_1) - \pi^*_2 \beta$$

Thus the earnings of an older worker in terms of α are not affected by schooling. But they will change the relative output price which is a source of gain from schooling in this case.

We can also compare the situation of one individual under this case with that under the previous case. The expected level of income of one representative older worker in case (ii) is smaller than that under case (i). But this loss is partially offset by a change in the relative output price.

Thus the young workers are better-off when the steady state (α^G, β^G) is in the convex part of the grand PPC, while the older workers are better-off if the (α^G, β^G) is in the straight line part.

Similarly if the (α^G, β^G) is in the other convex part (CD) of the grand PPC, the steady state level of relative output price will be

$$\pi^* = (\alpha_o+\alpha_1 P)/(\beta_o+\beta_1(1-P))$$

In this case all the younger workers will be employed in β and their wage rate (expected output) will be,

$$W^o = (\beta_o+\tfrac{1}{2}\beta_1)(1-S(P))\pi^*$$
$$= (\beta_o+\tfrac{1}{2}\beta_1)(1-S(P))(\alpha_o+\alpha_1 P)/(\beta_o+\beta_1(1-P))$$

Also all the older workers with label 'b' will be employed in β and will earn

$$W^{\beta}{}_b = (\beta_o+\beta_1 P)(\alpha_o+\alpha_1 P)/(\beta_o+\beta_1(1-P))$$

The older workers with label 'a' may be employed in α or β and they will earn,

$$W^{\alpha}{}_a = (\alpha_o+\alpha_1 P)$$
$$W^{\beta}{}_a = (\beta_o+\beta_1(1-P))\pi^* = \alpha_o+ \alpha_1 P$$

In this output plan all the younger workers are employed in β, and are net buyers of α, while the older group are net buyers of β. Since the price of β is increased compared to case (i), in this case also the younger workers will be better-off, but the older group will be worse-off.

Thus in this case when the older workers specialize in production according to their comparative advantage, the relative price moves against the older worker intensive industry. This makes the older workers worse-off but the younger workers better-off.

(iii) The golden rule output plan at the kinked point B: One such output plan is shown in Figure 2.9. In this case all the younger and 'a' labeled older workers will be employed in α and all the 'b' labeled older workers in β. The steady state output price ratio (π^*_3) will be given by the slope of the CIC at the point (α^G, β^G).

On the one extreme the slope of the CIC may become equal to that of the straight line part of the grand PPC and in that case

$$\pi^*_3 = \pi^*_1 = (\alpha_0+\tfrac{1}{2}\alpha_1)/(\beta_0+\tfrac{1}{2}\beta_1)$$

On the other extreme if the slope of the CIC coincide with that of the convex part of the grand PPC, and

$$\pi^*_3 = \pi^*_2 = (\alpha_0+\alpha_1(1-P))/(\beta_0+\beta_1 P)$$

Thus π^*_3 is determined by the shape of the CIC.

In this case all the younger workers will be employed in

α and their earnings will be,

$$(\alpha_0+\frac{1}{2}\alpha_1)(1-S(P))$$

This is the same amount as in the previous two cases but the feasible consumption set will be different, since the relative price of β in terms of α is not the same, and

$$\pi^*_2 \leq \pi^*_3 \leq \pi^*_1$$

Due to this price change, in this case the younger workers are better-off compared to case (i) but worse-off compared to case (ii).

In this output plan all the `a' labeled older workers are employed in $\alpha$ and `b' in β and the expected earnings in terms of α will be,

$$\frac{1}{2}(\alpha_0+\alpha_1 P) + \frac{1}{2}(\beta_0+\beta_1 P)\pi^*_3$$

as π^*_3 approaches π^*_2, the above term will approach

$$(\alpha_0+\frac{1}{2}\alpha_1)$$

In that case this situation will become identical to that in the convex part of the grand PPC (case ii).

On the other hand as π^*_3 approaches π^*_1 the expected income will be the same as in case (i), and the situation will coincide with that in the straight line part of the grand PPC (case i).

As in case (ii) here also when the older workers specialize in production the relative price moves against the older worker intensive industry. But the relative price change is small compared to the previous case so that the gain from specialization is shared by both groups of workers.

Given the above three possible production plans, we can concentrate on the case (iii) as the other two cases may be considered as a special case of that one. In case (iii) when $\pi^*_3 = \pi^*_1$ the production plan coincides with case (i) and it coincides with the case (ii) if $\pi^*_3 = \pi^*_2$.

Thus in a general equilibrium framework there are two sources of welfare gains. Firstly from the increased production in the economy as shown by an upward movement of the PPC, and this gain always goes to the older group of workers. The second gain comes through specialization in production which allows some group of workers to consume at a point above their group PPC. Distribution of this second gain depends on the relative output price and the gain goes to the older group if the steady state golden rule production plan is in the straight line segment of the grand PPC and it goes to the younger group if it is in the convex part. These two groups of workers will share the gain if (α^G, β^G) is at the kinked point. The distribution of the second source of gain and its effects on the optimization problem are ignored in the partial equilibrium studies.

The steady state feasible combinations of π and P

In this section so far we have looked at the golden rule production plan only. But the actual equilibrium may not be the golden rule one. So in order to determine the actual equilibrium for the economy we will examine the feasible combinations of π and P, for all different levels of P between $\frac{1}{2}$ and 1. In Figure 2.11 the golden rule production plan is shown at the kinked point A" with the level of sohooling at $P''=P^G$, and equilibrium relative price (π^*) is given by the slope of CIC at point A". For different levels of P the PPCs are shown in Figure 2.11. ($\alpha AB\beta$ for $P=\frac{1}{2}$, $\alpha'A'B'\beta'$ for P', $\alpha''A''B''b''$ for P"). When $P=\frac{1}{2}$ the equilibrium is shown by the point A and the equilibrium relative price (π) will be given by the slope of the younger workers PPC,

$$\pi = (\alpha_0+\tfrac{1}{2}\alpha_1)/(\beta_0+\tfrac{1}{2}\beta_1).$$

With higher level of P the kinked points and the production point will move along AA'A"C... Given homothetic preference we will move towards less steep points of the CICs, and the steady state relative price will decrease. This relationship between P and π is shown in Figure 2.12 by the curve ADC.

At one extreme, the slope of the CIC at the golden rule production plan may become equal to that of the straight line part of the grand PPC and we will have

$$\pi^* = (\alpha_0+\tfrac{1}{2}\alpha_1)/(\beta_0+\tfrac{1}{2}\beta_1)$$

In that case as P increases from $\frac{1}{2}$ toward P" the production point will move along the ray 0A" and there will be no change in π. For higher level of P we will move towards C. Given homothetic preference point C will correspond to a less steep point of CIC and the equilibrium π will decrease. The feasible equilibrium combinations of P and π are shown by the curve ABC in Figure A.5. Up to certain level of P the level of relative price is not affected by the level of schooling. As discussed earlier this kinked point production plan is of the same type as that in the interior of the straight line part, but in the latter case the range of P where π is constant will be even larger. In different partial equilibrium models it is assumed that the relative price is not affected by the level of schooling. Thus we can consider those studies as a special case of our general equilibrium model.

The 'golden rule' level of schooling P (for n=0) as derived in equation (2.7) is given by

$$S'(P) = [\tfrac{1}{2}(\alpha_1/(\alpha_0+\tfrac{1}{2}\alpha_1))+\tfrac{1}{2}(\beta_1/(\beta_0+\tfrac{1}{2}\beta_1))]$$

Given this, point B, D in Figure 2.12 correspond to the golden rule production plan.[6]

VI. INDIVIDUAL WORKER'S OPTIMIZATION

In this section initially we will concentrate on the kinked point production plan. The individuals' optimum choice of schooling intensity will depend on their current opportunity cost of time and expected future benefit. All the young workers in this model are employed in industry α and will earn,

$$W_o = (\alpha_o + \tfrac{1}{2}\alpha_1)$$

An increase in π_o (relative price of β in younger period) will decrease their real earnings and the cost of schooling.

On the benefit side there is ½ probability that one will get label 'a' (or 'b') and will be employed in $\alpha(\beta)$. An increase in π_1 (relative price in the older period) will increase the value of output of a 'b' labeled older worker and as such the expected benefit from schooling will increase. The expected earnings (in the older period) in terms of α can be written as,

$$W_1 = \tfrac{1}{2}[(\alpha_o + \alpha_1 P) + \pi(\beta_o + \beta_1 P)]$$

We are going to assume that the individual's earnings are insured (at their expected level W_1) so that his or her earnings in the second period are independent of the label he/she gets in the initial period. We are also going to assume that there exists a perfect capital market where individuals can borrow or lend any amount at a fixed interest rate (r). Each individual in this model will choose P acting competitively assuming market price as given and constant $(\pi_0=\pi_1)$. Under these assumptions a separation of income generating and consumption decision is possible and the individual's problem is to choose a level of P that maximizes his/her discounted lifetime earnings, given by

$$\bar{W} = (\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P)) + (1/(1+r))\tfrac{1}{2}[(\alpha_0+\alpha_1 P)+\pi(\beta_0+\beta_1 P)]$$

The first order condition for maximization gives,

$$\delta\bar{W}/\delta P= -(\alpha_0+\tfrac{1}{2}\alpha_1)S'(P) + (1/(1+r))\tfrac{1}{2}(\alpha_1+\pi\beta_1) = 0$$

$$(2.9) \qquad (\alpha_0+\tfrac{1}{2}\alpha_1)S'(P) = (1/(1+r))\tfrac{1}{2}(\alpha_1+\pi\beta_1)$$

This equation shows that at equilibrium (P^*) the marginal cost should be equal to the marginal benefit from schooling. For P^* to be the global optimum we also need the condition that the present value of lifetime earnings at P^* should be greater than that at $P=\tfrac{1}{2}$. This condition can be written as,

$$(\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P))+(1/(1+r))\tfrac{1}{2}[(\alpha_0+\alpha_1 P)+\pi(\beta_0+\beta_1 P)] > (\alpha_0+\tfrac{1}{2}\alpha_1) +(1/(1+r))(\alpha_0+\tfrac{1}{2}\alpha_1)$$

$$\Longrightarrow \quad (\alpha_0+\alpha_1 P) + \pi(\beta_0+\beta_1 P) > 2(\alpha_0+\tfrac{1}{2}\alpha_1) + S(P)(\alpha_0+\alpha_1 P)(1+r)$$

$$(2.10) \qquad \Longrightarrow \pi > (\alpha_0+\alpha_1(1-P))/(\beta_0+\beta_1 P)+ S(P)(\alpha_0+\tfrac{1}{2}\alpha_1)(1+r)/(\beta_0+\beta_1 P)$$

The above optimization analysis holds for any case where the actual equilibrium for the economy is at the kinked point of the PPC for the economy. It is possible that the golden rule production point is in the convex part of the grand PPC but the actual equilibrium is at the kinked point of a PPC for the economy.

The case where the actual equilibrium is such that there is some mismatch of firm and worker is studied next. For the sake of exposition imagine that no test is available in the economy and all workers are identical with $P=\frac{1}{2}$, and the relative price is $\pi = (\alpha_0+\frac{1}{2}\alpha_1)/ \ (\beta_0+\frac{1}{2}\beta_1)$. If a test becomes available in period t, all the younger workers will choose a $P>\frac{1}{2}$, given by equation (2.9). Since some 'b' labeled workers are employed in α, the relative price in the next period will be,

$$\pi_{t+1} = (\alpha_0+\alpha_1(1-P))/(\beta_0+\beta_1 P)$$

With myopic expectations corresponding to π_{t+1} the younger workers of that period will choose $P=\frac{1}{2}$, since for any $P>\frac{1}{2}$ condition (2.10) will not be satisfied. This will make the situation in period t+2 identical to that in period t, and individuals will choose $P=\frac{1}{2}$. Thus under myopic expectations there will be no equilibrium with mismatch of older workers. This argument also holds for the kinked point of the grand PPC, if condition (2.10) is not satisfied.[7]

The above conclusion does not depend on our simple myopic dynamics. Given any anticipated π it will pay individual young worker to diverge from the P which would support that π. Suppose we are at a steady state equilibrium with schooling level P^s and relative price π^s. In that case individual workers' utility maximizing level of schooling will be $P=\frac{1}{2}$. Thus any $P>\frac{1}{2}$ is not a feasible steady state equilibrium.

If at the kinked point production plan the relative price is such that condition (2.10) is satisfied or the golden rule production plan is reached at a point on the straight line segment, individuals will choose a $P>\frac{1}{2}$. This level of P can be determined from equation (2.9). Alternatively the condition given in equation (2.9) can be written as,

$$(2.11) \qquad S'(P) = \tfrac{1}{2}(\alpha_1+\pi\beta_1)/[(1+r)(\alpha_0+\tfrac{1}{2}\alpha_1)]$$

Thus the individual's optimum choice of schooling level (P^*) is a function of,

$$P^* = P(\alpha_0,\ \alpha_1,\ \beta_1,\ \pi,\ r)$$

Since $S''(P)>0$, β_1 and π will have a positive effect on P^* but an increase in α_0 or r will decrease P^*. The effect of α_1 is indeterminate[8]. The interpretation of these effects is as follows:

α_0: a higher level of α_0 implies higher income for the younger workers. Thus it will lead to a higher cost of schooling and a lower level of P chosen.

r: higher r implies a lower present value of benefit from schooling and hence a lower level of P.

β_1: this is directly related with the benefit of schooling.

π: a higher value of β in terms of α will increase the benefit from schooling if someone end up with label `b' and employed in β.

α_1: the cost of schooling as well as the benefit of schooling is directly related with α_1. Thus its effect on P is indeterminate. However it will have a positive effect on P if the following condition is satisfied

$$2\alpha_o > \beta_1\pi$$

This relationship between π and the individual's choice of schooling can also be shown in terms of a diagram. From equation (2.10) we can derive,

$$\delta\pi/\delta P = \{[-\alpha_1 + S'(P)(\alpha_o + \tfrac{1}{2}\alpha_1)(1+r)](\beta_o + \beta_1 P) - [(\alpha_o + \alpha_1(1-P)) + S(P)(\alpha_o + \tfrac{1}{2}\alpha_1)(1+r)]\beta_1\}/(\beta_o + \beta_1 P)^2$$

$$\Longrightarrow \quad \delta\pi/\delta P < 0 \text{ if } S'(P) < [\alpha_1(\beta_o + \beta_1 P) + (\alpha_o + \alpha_1(1-P))\beta_1] / [(\alpha_o + \tfrac{1}{2}\alpha_1)(1+r)(\beta_o + \beta_1 P)] + S(P)\beta_1/(\beta_o + \beta_1 P)$$

Given S'(P), and S"(P)>0, up to certain level of P this curve will be downward sloping and then it will slope upwards. This relationship is shown by the curve XY in Figure 2.13. Given any combination of π and P on or below the curve, individuals will choose not to spend any time at school, $P=\frac{1}{2}$. Equation (2.9) will give an inverse and convex relationship between π and P, shown by AB. For π below π_2, we will have $P=\frac{1}{2}$. Corresponding to $\pi = (\alpha_0+\frac{1}{2}\alpha_1)/(\beta_0+\frac{1}{2}\beta_1)$, the level of schooling (point A) can be derived from (2.11) as

(2.12) $S'(P) =[\frac{1}{2}(\alpha_1/(\alpha_0+\frac{1}{2}\alpha_1)) + \frac{1}{2}(\beta_1/(\beta_0+\frac{1}{2}\beta_1))]/(1+r)$

An increase in r, α_0 or β_0 (or a decrease in α_1, β_1) will shift it to the north-west direction, say to EF,GD.

VII. EQUILIBRIUM LEVEL OF SCHOOLING

Figure 2.13 shows individual's optimum choice of P for different levels of π. On the other hand given the preferences and the technology Figure 2.12 shows for different levels of P what will be the equilibrium level of π. Combining these two we can determine the equilibrium level of schooling and relative price. This is shown in Figure 2.14. In this model we have the following possibilities:

Firstly, when we have the steady state equilibrium in the straight line part the equilibrium level of schooling will be determined at point B.

Secondly, if the golden rule production point is at the kinked point of the grand PPC for the economy, and the feasible combinations of P and π locus is such that it intersects the individuals' utility maximizing P-π locus, that intersection point will be the equilibrium for the economy. One equilibrium of this type is show in Figure 2.14, at point E.

As discussed in the previous section, corresponding to the convex part of the grand PPC, or the kinked point when condition (2.10) is not satisfied, there will be no equilibrium situation for the economy.

If the golden rule production point is given at the point A, the corresponding level of schooling will be $P^G=\frac{1}{2}$. If the slope of the CIC at point A is such that condition (2.10) is not satisfied individuals will also choose $P=\frac{1}{2}$ and that will be an equilibrium situation. Otherwise this case will become identical to that with firm worker mismatch and there will be no equilibrium situation for the economy.[9]

From equation (2.7) we can derive the golden rule level of schooling (P^G). When production takes place in the straight line part of the grand PPC the corresponding steady state relative price will be $\pi=(\alpha_0+\frac{1}{2}\alpha_1)/(\beta_0+\frac{1}{2}\beta_1)$. This combination is shown by point B in Figure 2.12. Given this relative price individual's optimum level of P can be determined from equation (2.12). From equation (2.7) and

(2.12) we can see that this level of schooling will be at P^G level if the labour force growth rate (n) is equal to the rate of interest (r). In that case our equilibrium will be reached at point B and the amount of schooling will be at the 'golden rule' level (P^G). If r is greater (smaller) than n the curve FB will shift to the left (right) and, the equilibrium level of P will be lower (higher) than P^G, as the future are discounted at a higher (lower) rate.

When we have equilibrium at the kinked point, corresponding to P^G we will have a lower level of relative price given by point D. With r=n our equilibrium will be at a lower than golden rule level of schooling (point E). If r is greater than n our equilibrium will be further away from the golden rule level. On the other hand if r<n we will move closer to the golden rule level of schooling and there is a possibility that our equilibrium will be at the golden rule level.

Thus only under special cases the equilibrium level of P will be at the golden rule level and in general it will be smaller.

Corresponding to a production plan with mismatch of firm and worker, for the economy as a whole there are gains from investment in information but individuals will choose not in invest in information. The reasons for this Pareto inefficient outcome are as follows: In these situations the source of the gain from investment in information is relative price change as a result of specialization in production, while earnings in terms of α with golden rule level of P are identical to those with $P=\frac{1}{2}$. The relative price movement will make both the younger and the older workers better-off. Thus when one generation invests in information the next generation will be better-off. In terms of Figure 2.7 the younger group of workers consume at a point (J) above their group PPC. Without a mechanism of intergenerational transfers individuals will not take account of this benefit in determining optimum investment in information. The other gain going to the generation making the investment decision, comes through the relative price change. In Figure 2.7 their group consumption point is at J, above the PPC corresponding to $P=\frac{1}{2}$ (straight line joining $\bar{\alpha}\bar{\beta}$). This relative price change has the properties of a 'public good' and the free rider problem arises. This results in an inefficient level of P chosen by individuals.[10]

Comparative statics: An increase in r, α_0 or β_0 will shift the BF curve upward and will lead to a lower equilibrium level of schooling. The reason is that higher α_0 or β_0 implies higher cost of schooling (opportunity cost of young workers time) while a higher r implies lower present value of future benefit from schooling. On the other hand a higher α_1 or β_1 shifts the AB curve to the right and will result in a higher level of schooling, as this means more benefit from better worker-industry matching through schooling.

Thus in a general equilibrium framework the return as well as equilibrium level of schooling is also affected by the demand conditions. A change in the preference towards the output produced by the older worker intensive industry will move ADC curve upward and will result in an increase in the level of schooling. In general the market determined level of human capital investment will not be at the 'golden rule' level.

VIII. THE CAPITAL MARKET

Forms of the capital market

In each generation all the individuals are identical in the initial period of life. Thus in our model either all young workers will be borrowers or will be savers, i.e. it is not possible to have any borrowing or lending within one generation.

In this model it is not possible to have any borrowing or lending between generations also. If the younger generation borrow from the older generation then that loan cannot be repaid because the older generation will die before the loan is repaid. The younger generation cannot be a lender either because in that case the older generation will not have any time left to repay the loan. In this case there are two alternative ways in which we can incorporate capital market in this model.

Firstly, we may assume that the loans are made by the government of the country in the form of real bonds and the rate of interest charged is the market clearing rate.

Alternatively, we can assume the existence of an international bank outside the economy. In that case we will assume that only one commodity (either α or β) is traded so that relative output price is not exogenously given from the international market.

Determination of the rate of interest

First we will look at the determination of the rate of interest when the loan is made by the government. The interest rate will be set in order to clear the market, that is, demand for loanable funds is equal to supply. Initially we will assume that the population growth in the economy is zero and the age distribution is rectangular. So we have an equal number of younger and older workers and this number is $(\bar{A}+\bar{B})$. For different levels of P the earnings (Wo and W1) of a representative individual were derived in section V. From those we can draw an individuals feasible income set as shown by ABCD in Figure 2.15. The preferences given by (2.8) are shown by ICo, IC1. When an individual can borrow or lend any amount at a fixed interest rate, r, he/she will be willing to borrow the amount KH in youth and to repay the amount BK if the interest rate is r. The amount BK is the principal and the interest on the borrowed amount, that is

(2.13) $\quad BK = (1+r)KH$

Now the government faces a total demand for loans of $KH(\bar{A}+\bar{B})$, while it can collect from the older workers (supply of funds) $BK(\bar{A}+\bar{B})$. So r should be such that

(2.14) $\quad KH\ (\bar{A}+\bar{B}) = BK\ (\bar{A}+\bar{B})$

Substituting (2.13) in (2.14) we get

$$r = 0$$

So the market clearing interest rate will be zero.

In general terms if we incorporate a population growth rate of n% per period, the number of young workers will be (1+n) times that of the older workers. Equation (2.28) will be the same but the market clearing condition become,

$$KH\ (\bar{A}+\bar{B})(1+n) = BK\ (\bar{A}+\bar{B})$$

Substituting (2.13) in this equation, we can derive

$$r = n$$

So the market clearing rate of interest should be the population growth rate , which is Samuelson's biological rate of interest (Samuelson 1958).

In case of an international financial institution we will assume that the rate of interest is determined outside the economy and given exogenously in this model.

IX. SUMMARY

In this chapter a simple general equilibrium model of imperfect person specific information in the labour market is developed. Unlike the partial equilibrium models, in our model the real return from investment in information is endogenously determined. It has been shown that the real return from better firm-worker matching is also influenced by the demand structure. The more the demand structure is biased towards the output produced by the older worker intensive industry, the larger will be the gain from specialization through person specific information. In the general equilibrium framework it is found that the individually chosen investment in information will be in general smaller than the 'golden rule' steady state level. One reason for this is that investment made by one generation may make the following generation better-off.

Without any mechanism of intergenerational transfers individuals will not take account of this gain in their investment decision. Also part of the gain from better information may come through changes in the relative price. This has the properties of a 'public good' and the free rider problem arises. In that case the individual's optimum level of investment in information will be smaller than the golden rule level.

NOTES

[1]In appendix 1 it is shown that under certain restrictions our qualitative results will be the same for constant returns to scale production function.

[2]This assumption is relaxed in appendix 2, and it results in no qualitative change in our analysis and findings.

[3]When the assumption of $S(1)=1$ is relaxed the results will be the same as that in case of imperfect firm-worker matching. If $S(1)>1$ our analysis will be similar to that in appendix 2 where

$$(t^{A}_{1}/t^{A}_{2})<(a_1/a_2) \text{ and } (t^{B}_{1}/t^{B}_{2})<(b_1/b_2)$$

that is even with full time schooling in the initial period we cannot produce on the contract curve in the second period. Analysis with $S(1)<1$ will be similar to that with

$$(t^{A}_{1}/t^{A}_{2})>(a_1/a_2) \text{ and } (t^{B}_{1}/t^{B}_{2})>(b_1/b_2).$$

[4]The results with uncertainty is discussed in appendix 3.

[5]In this case a number of other possibilities exists. We may have multiple tangency points between the CIC and PPC or the CIC which is tangent to the convex part of the PPC may also pass through the kinked point. In those cases the golden rule level of P will no longer be unique. Another possibility is that the CIC's are flatter than the convex part of the PPC. In that case the tangency point will be utility minimizing. Later in this chapter it is shown that we will never observe equilibrium in the convex part of the PPC for the economy and as such the above possibilities are ignored.

[6]Two other possible shape of this relationship is discussed in appendix 4.

[7]Technological structure in our model is one of the reasons why there is no equilibrium with mismatch of firm and worker. The technology of information generation in our model is such that all young workers act in the same way. However, if we have a linear schooling cost function without any fixed cost component, it is possible that some individuals invest in information while others don't (both groups will have the same life-time income). In that case the kinked points of the grand PPC will move towards points

A and D. Corresponding to the relative price at the new kinked point there may be an equilibrium for the economy.

[8]In appendix 5 these comparative static results are derived.

[9]If preferences are extremely biased towards α, we may have equilibrium at point A of the grand PPC. As the preferences shift towards β (the golden rule production point may stay at A or move along the convex part or may be at the kinked point) if condition (2.10) is violated there will be no equilibrium situation for the economy. As preferences shift further, we will have equilibrium for the economy corresponding to the golden rule production plan at the kinked point or the straight line segment. The same analysis also holds if preferences are extremely biased towards β and shift towards α.

[10]There are different alternative policies that government could take in order to reach the golden rule level of schooling. If the actual level of schooling is less than that level the policies should be directed towards decreasing the cost and increasing the benefits from schooling. One way of doing this is to impose a tax on

the earnings of the younger workers (this will decrease the cost of schooling by decreasing the opportunity cost of their time) and/or transfer the revenue to the older workers, where the amount of transfer will be positively related with the level of schooling chosen by them in the younger period (this will increase the benefit from schooling).

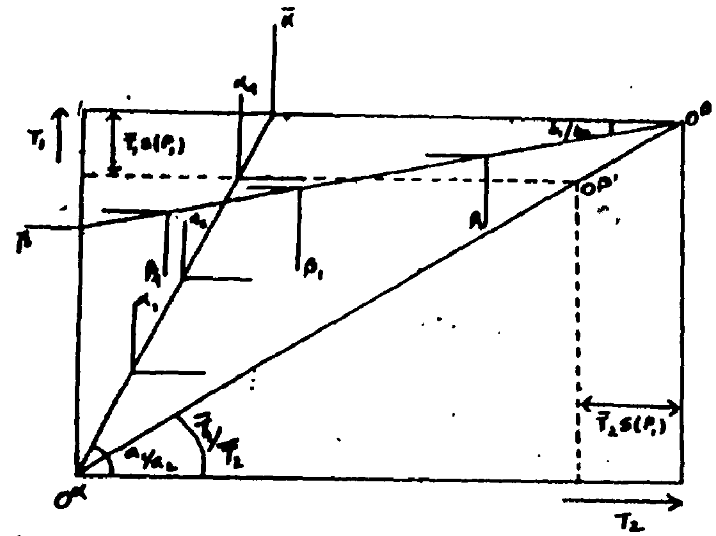


Figure 2.1(a) : Edgeworth Box for Young Workers

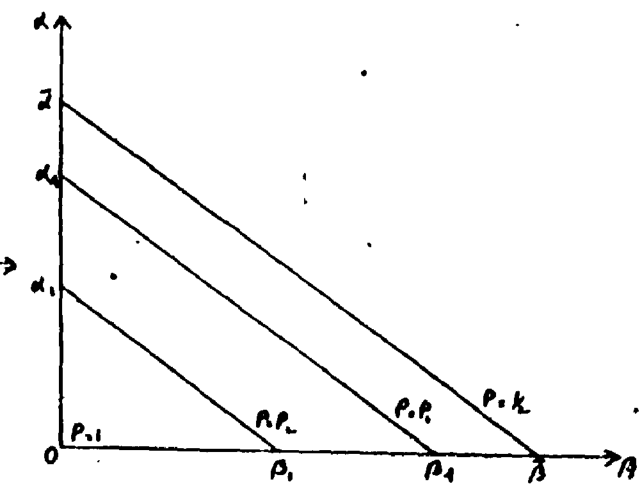


Figure 2.1(b) : PPC for the Young Workers

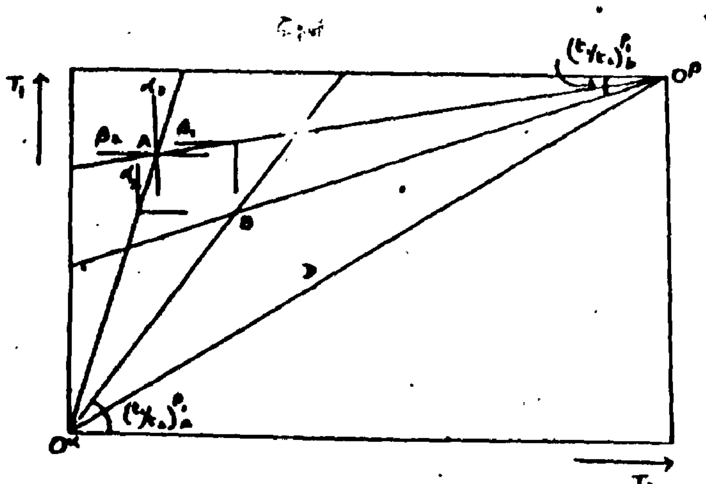


Figure 2.2(a) : Edgeworth Box for Old Workers

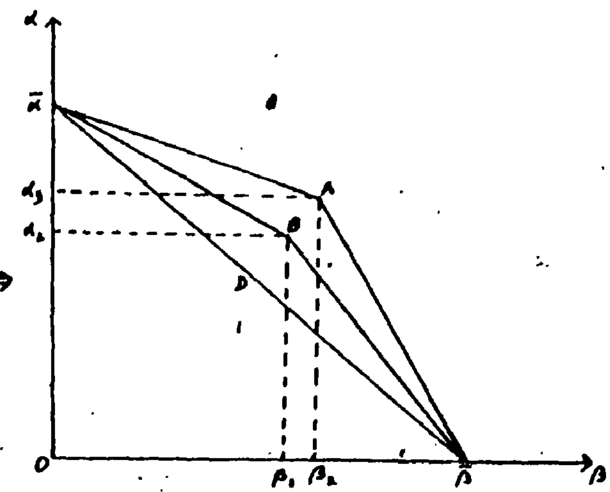


Figure 2.2(b) : PPC for the Old Workers

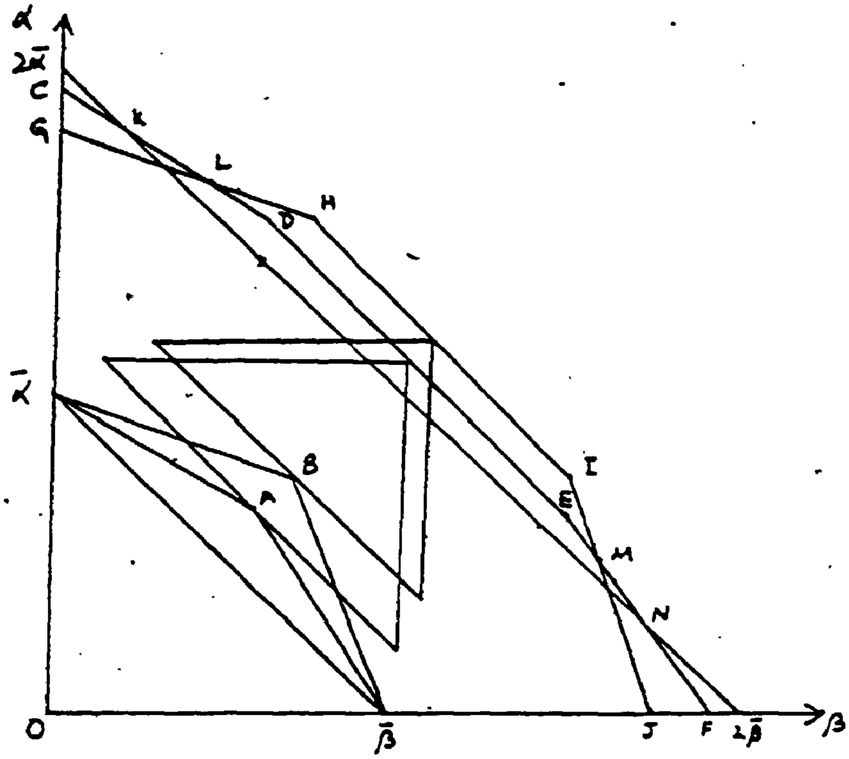


Figure 2.3 : PPC for the Economy

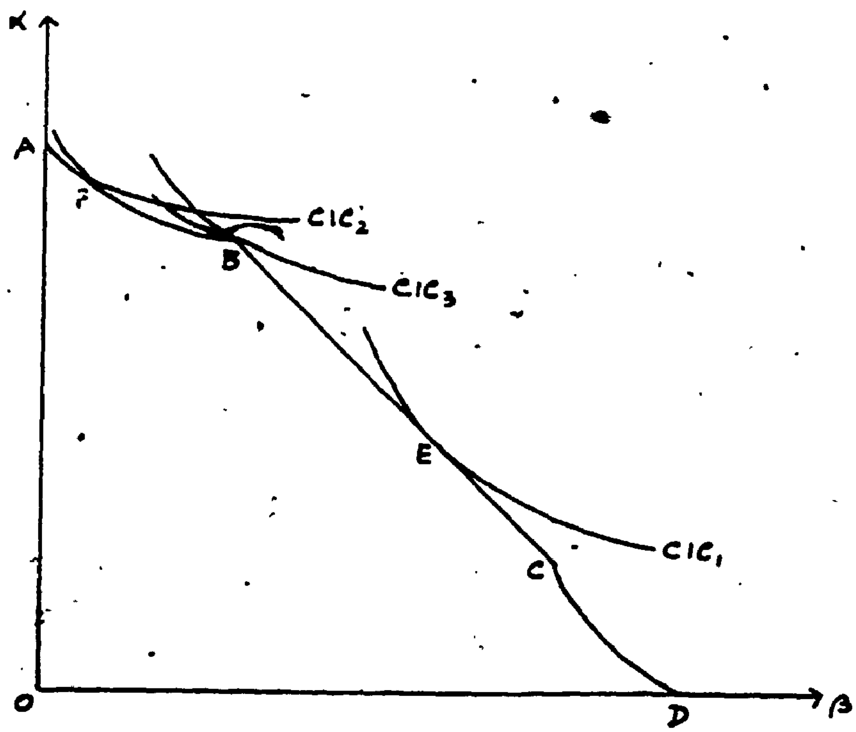


Figure 2.4 : Steady state Grand PPC for the Economy and the 'Golden Rule' Production Plan

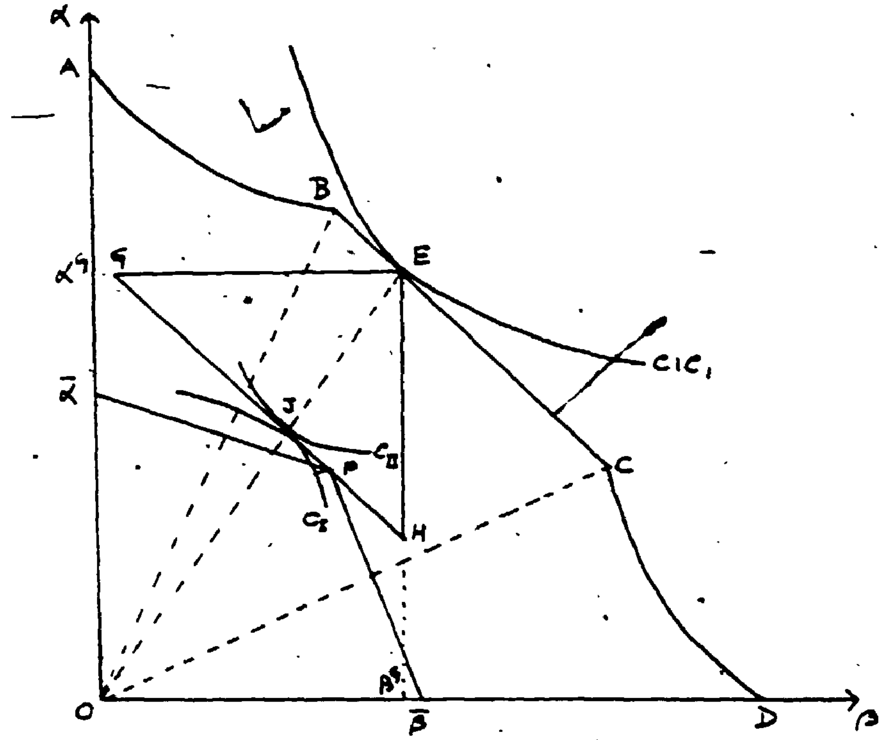


Figure 2.5 : 'Golden Rule' Production Plan in the Straight Line Part of the Grand PPC

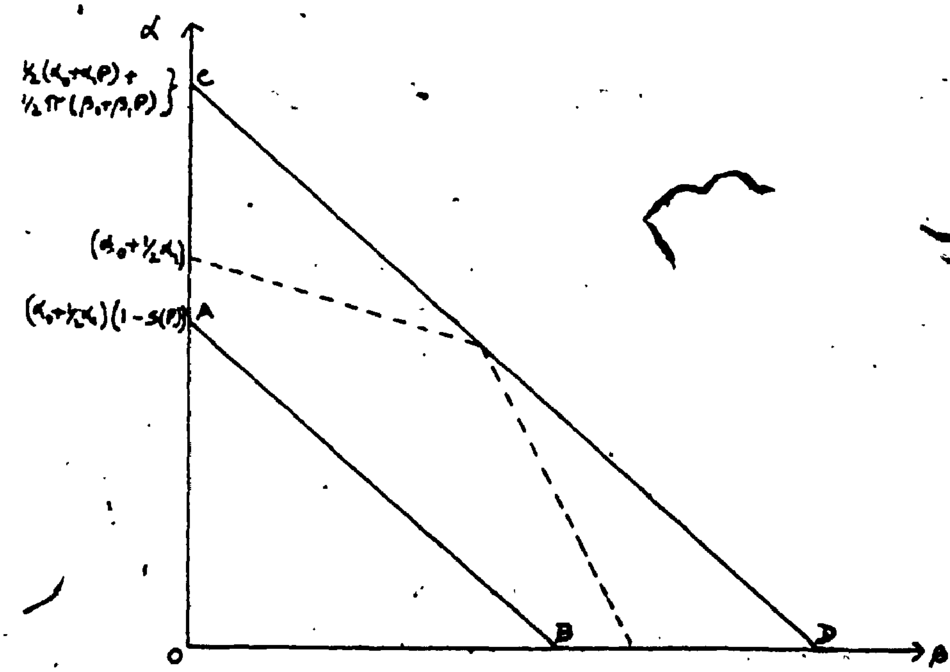


Figure 2.6 : Feasible Consumption Set (Production Plan in the Straight Line Part of the Grand PPC)

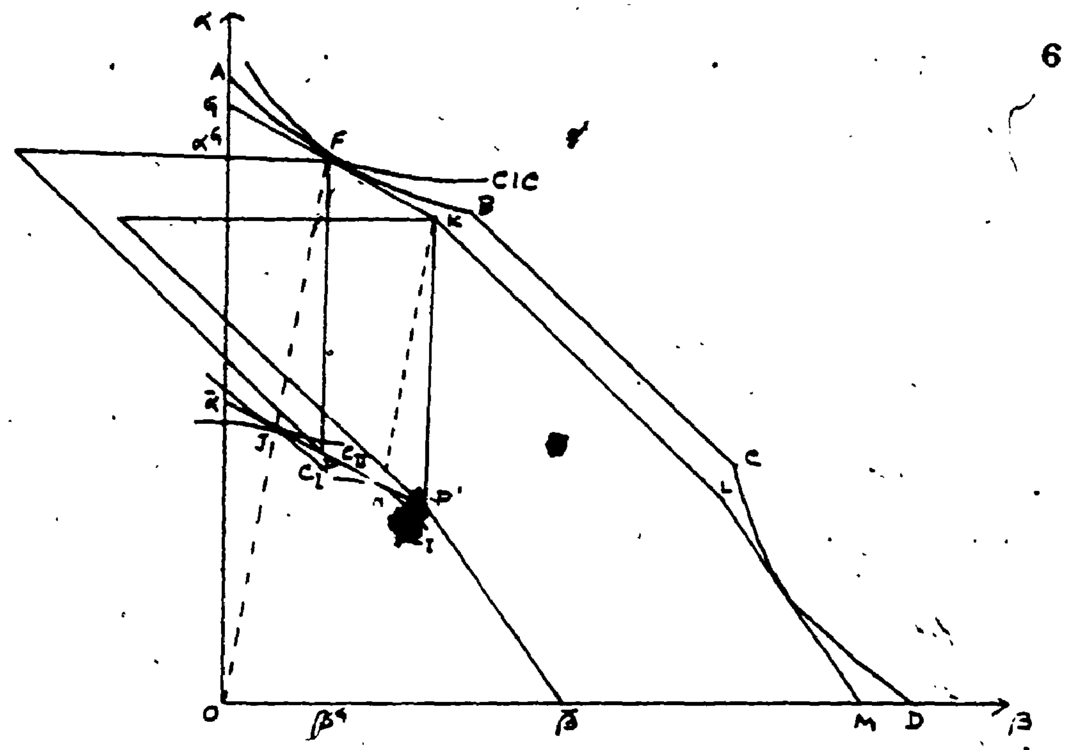


Figure 2.7 : 'Golden Rule' Production Plan in the Convex Part of the Grand PPC

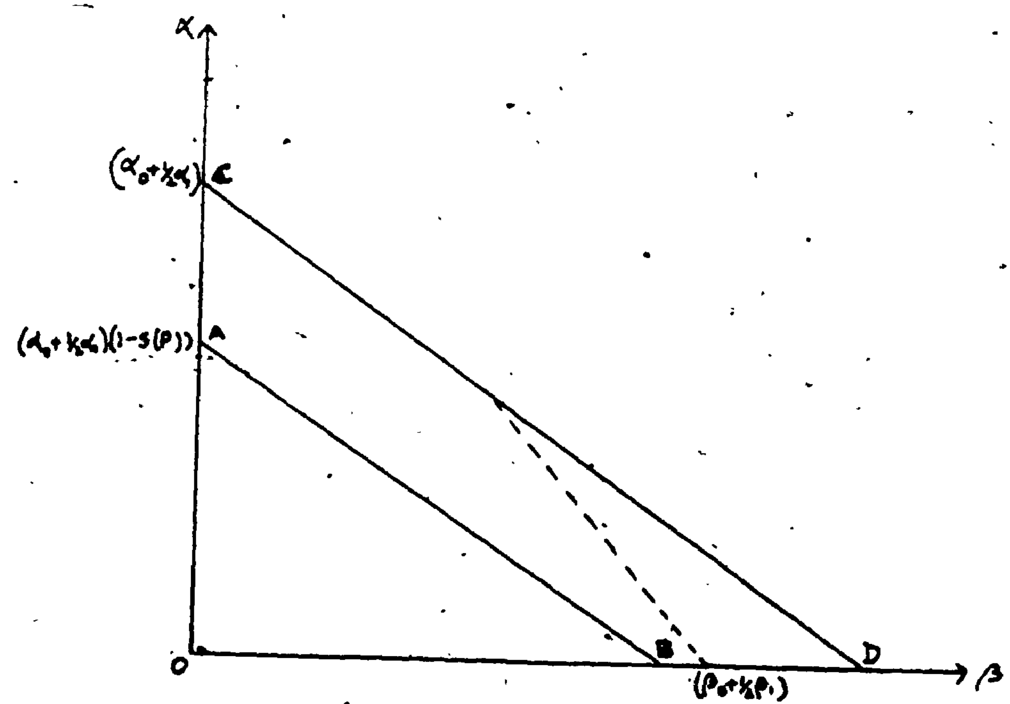


Figure 2.8 : Feasible Consumption Set (Production Plan in the Convex Part of the Grand PPC)

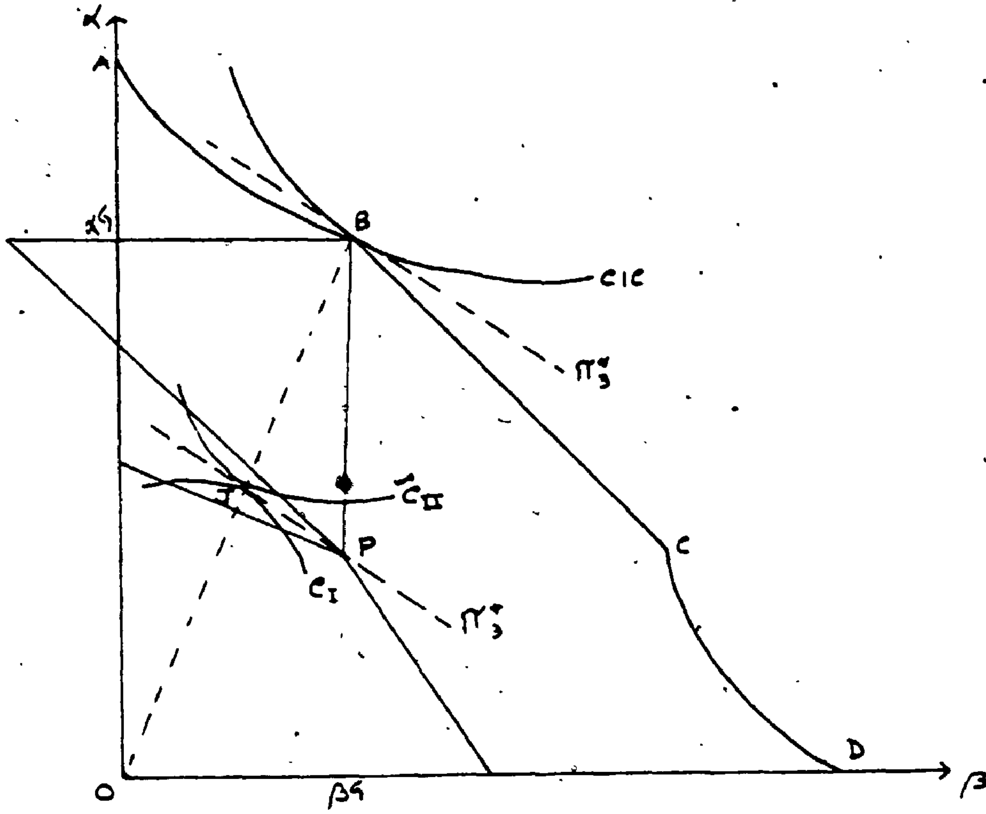


Figure 2.9 : 'Golden Rule' Production Plan at the Kinked Point of the Grand PPC

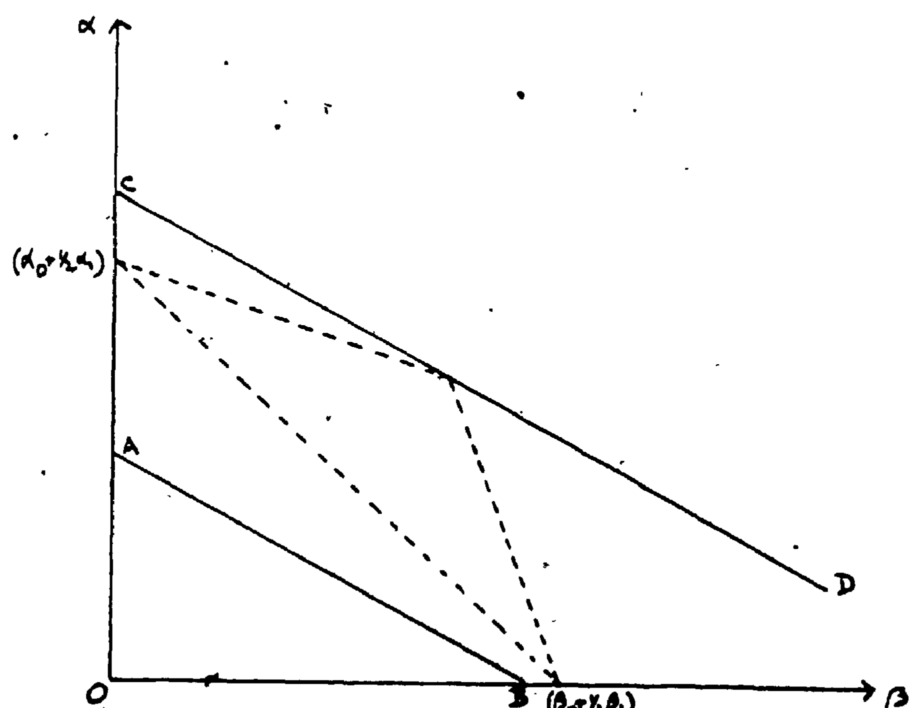


Figure 2.10 : Feasible Consumption Set (Production Plan at the Kinked Point of the Grand PPC)

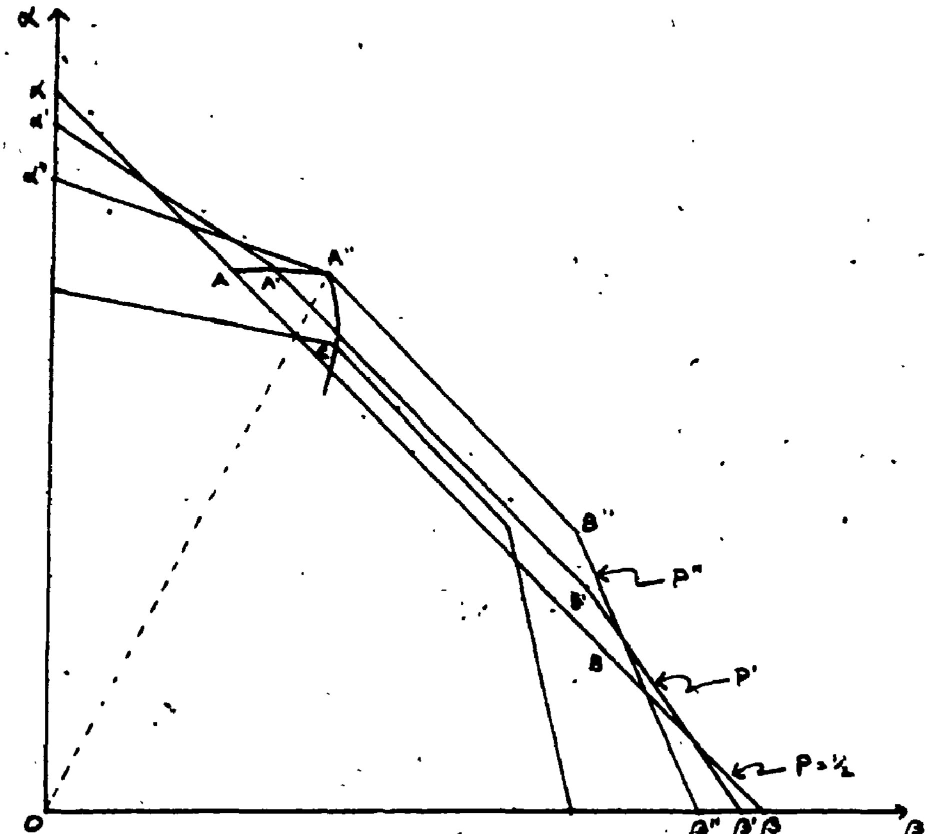


Figure 2.11 : Production Plan for Different Steady State P.

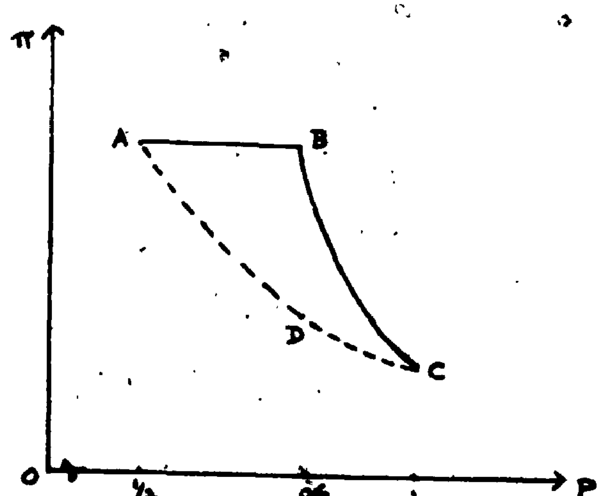


Figure 2.12 : Steady state Feasible Combinations of P and π

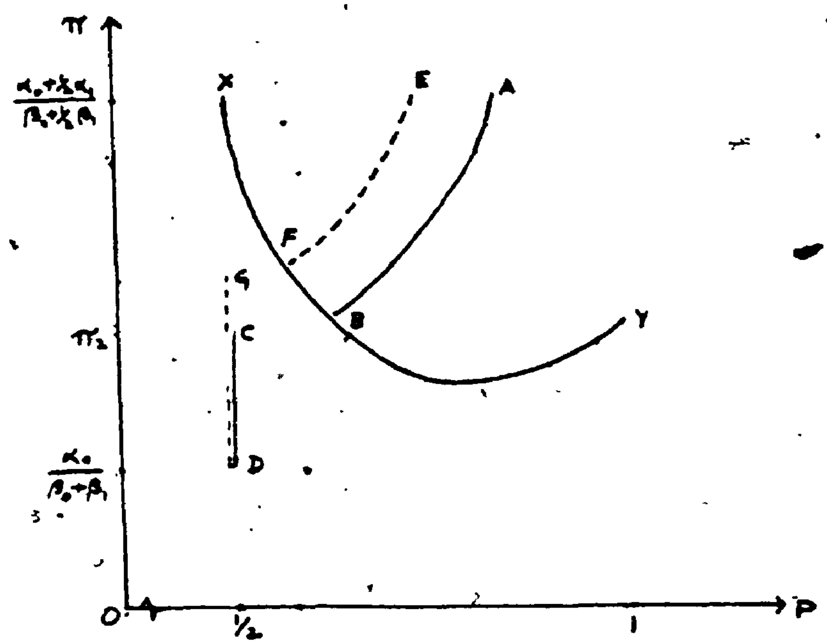


Figure 2.13 : Individual's Optimum Choice of P Given π

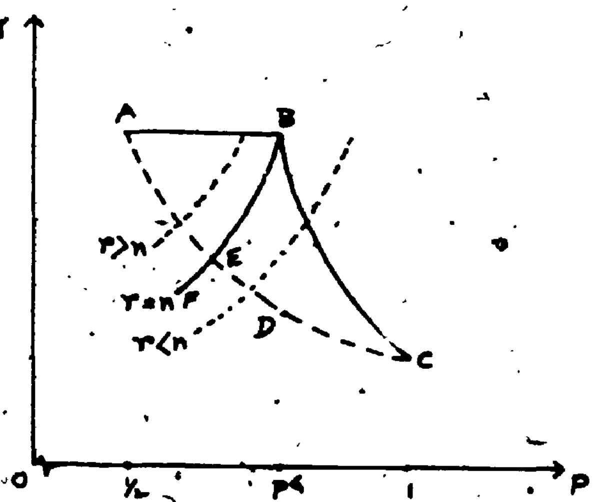


Figure 2.14 : Equilibrium Level of Information P.

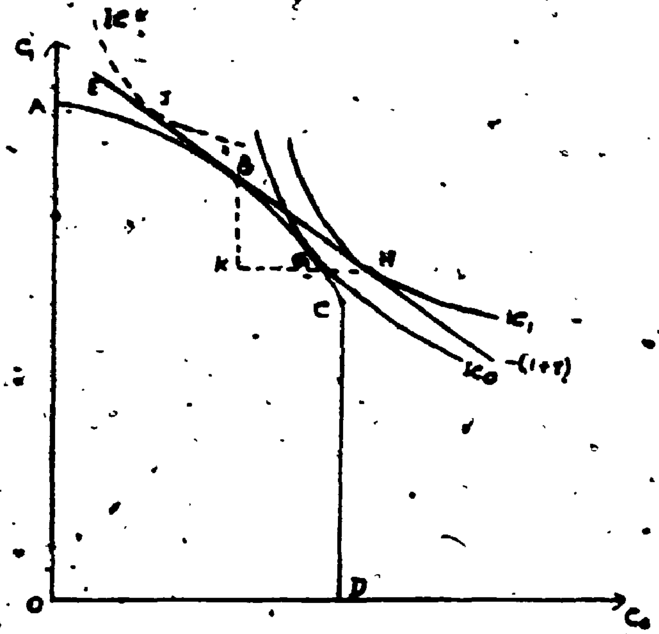


Figure 2.15 : Interest Rate Determination in Domestic Capital Market

CHAPTER 3

SCHOOLING WITH ALLOCATIVE AND PRODUCTIVITY AUGMENTING EFFECTS

In the basic information model developed in the previous chapter, it was assumed that schooling provides better information about workers' productive capacities while the absolute level of productivity is unaffected by schooling. So the only source of gain from schooling is the increased output and wages from better matching between worker and industry. But schooling also lead to an increase in the quantity of resources embodied in individuals, which is known as the productivity augmenting effect. The purpose of this chapter is to extend the basic information model to include both the allocative and productivity augmenting effects of schooling.

The structure of the model and the notation used in this chapter are the same as those in the basic information model and are briefly given in the next section. In section III we have developed the basic productivity model of schooling, where schooling only increases the productivity of the workers, while no information is generated. In section IV the two basic models are combined in a single model of schooling with both the productivity and the allocative effects of schooling.

II. STRUCTURE AND NOTATION

As in chapter 2 there are two goods α and β, each of which is produced using two inputs (tasks) T_1 and T_2. The production functions are assumed to be Leontief type fixed coefficient functions,

$$Y^{\alpha} = \text{Min}\,(T^{\alpha}{}_1/a_1,\ T^{\alpha}{}_2/a_2)$$

$$Y^{\beta} = \text{Min}\,(T^{\beta}{}_1/b_1,\ T^{\beta}{}_2/b_2)$$

where Y^j represent output of industry j, $T^j{}_i$ denotes the amount of task i used in industry j, a_i and b_i are the task-output ratios for task i in industries α and β respectively. Industry α is assumed to be task 1 (T_1) intensive while β is T_2 intensive.

There are two types of workers, A and B. Everyone in each type is identical. The only difference between type A and B is in terms of the amount of tasks they can perform. Type A workers are assumed to be more efficient in task 1 and type B in task 2. The production functions are assumed to be such that type A workers are perfectly matched with industry α and B with industry β. It is also assumed that each worker lives for two periods and that at any particular period of time there are two groups of workers of equal size, one group at their initial period of life (young workers) and the other in the final period of their life (older workers). We assume that one half of the workers in each group (young and old) are type A and the other half is type B (i,e. $\bar{A} = \bar{B}$) and this information is known by everyone. But nobody knows which type of worker one particular individual worker is.

III. THE BASIC PRODUCTIVITY MODEL

Structure of schooling

It is assumed that schooling increases the maximum amount of all tasks that can be performed by a worker equi-proportionally. The only input required for schooling is the fraction of worker's time spent at school (S). For someone who has never attended school the efficiency level (e) is assumed to be equal to 1. e increases with S, and the relationship is given by

(3.1) $e = E(S)$

where $E(0)=1$, $E(1)=e^M$

Further,

$$E'(S)>0 \text{ for } 0<S<1$$

$$E'(0)=\infty \text{ and } E''(S) < 0$$

This relationship is shown diagrammatically in Figure 3.1.

It is further assumed that if someone goes to school his/her productivity increases in the next period. This implies that only the younger workers will go to school.

The maximum amount of tasks that can be performed by one worker with efficiency level $E(S)$ is,

For type A worker : ($t^A{}_1E(S)$, $t^A{}_2E(S)$)

For type B worker : ($t^B{}_1E(S)$, $t^B{}_2E(S)$)

<u>Expected output of one worker</u>

The production functions are assumed to be such that type A workers are perfectly matched with industry α and B with β. So $Y^j{}_i(S)$, the output in industry j produced by one i type worker with efficiency level given by S, will be:

$$Y^\alpha{}_A(S) = (t^A{}_1E(S))/a_1 = (t^A{}_2E(S))/a_2$$

$$= Y^\alpha{}_AE(S) \qquad \text{where } t^A{}_1/a_1=t^A{}_2/a_2=Y^\alpha{}_A$$

(3.2) $$= (\alpha_0+\alpha_1)E(S) \qquad \text{where } Y^\alpha{}_A = \alpha_0+\alpha_1$$

Similarly,

$$Y^\beta{}_A(S) = (t^A{}_2E(S))/b_2 < (t^A{}_1E(S))/b_1$$

$$= Y^\beta{}_AE(S)$$

(3.3) $$= \beta_0E(S) \qquad \text{where } Y^\beta{}_A = \beta_0$$

$$Y^\alpha{}_B(S) = (t^B{}_1E(S))/a_1 < (t^B{}_2E(S)/a_2$$

$$= Y^\alpha{}_BE(S)$$

(3.4) $$= \alpha_0E(S) \qquad \text{where } Y^\alpha{}_B = \alpha_0$$

$$Y^\beta{}_B(S) = (t^B{}_1E(S))/b_1 = (t^B{}_2E(S))/b_2$$

$$= Y^{\beta}{}_{B}E(S)$$

(3.5) $$= (\beta_o+\beta_1)E(S) \qquad \text{where } Y^{\beta}{}_{B} = \beta_o+\beta_1$$

In this basic productivity model schooling does not generate any information. So any individual worker (younger or older) cannot be identified as type A or B, rather all of them are average workers within the group. Since $\bar{A}=\bar{B}$, the probability that any worker is type A is ½ and the probability that he/she is type B is also ½.

The expected level of output of one young worker chosen at random and working in industry α will be ,

$$Y^{\alpha}{}_{o} = \tfrac{1}{2}Y^{\alpha}{}_{A} + \tfrac{1}{2}Y^{\alpha}{}_{B}, \quad \text{as } E(S)=1 \text{ for young workers.}$$

(3.6) $$\Longrightarrow \quad Y^{\alpha}{}_{o} = \alpha_o+\tfrac{1}{2}\alpha_1$$

Similarly,

(3.7) $$Y^{\beta}{}_{o} = \beta_o+\tfrac{1}{2}\beta_1$$

These will be the levels of output if all the available time of the young workers is spent in production. When S amount of time is spent at school the expected output will be

(3.8) $$Y^{\alpha}{}_{o} = (\alpha_o+\tfrac{1}{2}\alpha_1)(1-S)$$

(3.9) $$Y^{\beta}{}_{o} = (\beta_o+\tfrac{1}{2}\beta_1)(1-S)$$

For one older worker (in period 1 of life) the expected output will depend on the level of schooling and is given by,

$$Y^{\alpha}{}_{I}(S) = \tfrac{1}{2}Y^{\alpha}{}_{A}E(S) + \tfrac{1}{2}Y^{\alpha}{}_{B}E(S)$$

(3.10) $= (\alpha_0+\frac{1}{2}\alpha_1)E(S)$

(3.11) and $Y^{\beta}_1(S) = (\beta_0+\frac{1}{2}\beta_1)E(S)$

As in chapter 2 the wage rate is assumed to be equal to the value of the expected output of any worker. Since all the workers in a group are identical, in a perfectly competitive model, one young worker (or older worker) should have the same wage in both the α and β industries. Thus the relative price (π) of β in terms of α should be,

$$\pi = (\alpha_0+\tfrac{1}{2}\alpha_1)/(\beta_0+\tfrac{1}{2}\beta_1)$$

Availability of tasks

Without any information all the workers are of equal expected productivity and the amount of task i expected from one worker is

$$\tfrac{1}{2}(t^A{}_i+t^B{}_i)E(S) \qquad i=1,2$$

There are $(\bar{A}+\bar{B})$ young workers, each spending (1-S) fraction of time working and having E(S)=1. Thus the total amount of tasks available from this group of workers are

$$T_1 = (\tfrac{1}{2})(t^A{}_1+t^B{}_1)(1-S)(\bar{A}+\bar{B})$$

$$T_2 = (\tfrac{1}{2})(t^A{}_2+t^B{}_2)(1-S)(\bar{A}+\bar{B})$$

This is shown in Figure 3.2(a). Note that the task box decreases proportionately with an increase in S.

All the older workers will be working full time and E(S) will be greater than 1. The total amount of tasks available from this group will be,

$$T_1 = (\tfrac{1}{2})(t^{A}{}_1+t^{B}{}_1)E(S)(\bar{A}+\bar{B})$$

$$T_2 = (\tfrac{1}{2})(t^{A}{}_2+t^{B}{}_2)E(S)(\bar{A}+\bar{B})$$

This is shown by the task box in Figure 3.3(a), which will expand proportionately with an increase in S.

<u>Production possibilities</u>

Given the total amount of tasks available, we can derive the production possibility curves. For both the younger and the older workers the only feasible production points are the points on the diagonal in the task boxes, as no information is available or generated in this model. Corresponding to this the PPC will be a downward sloping straight line.

When the younger workers spend all the available time working (S=0), the corresponding task box is shown by $0^{\alpha}0^{\beta}$, and the PPC is shown by the straight line AB in Figure 3.2(b). If S_1 amount of time is spent at school the corresponding factor box will be $0^{\alpha}0^{\beta\prime}$ and the PPC will be CD in Figure 3.2(b). If all the young workers are working in industry α the production point will be A, C or E depending on the level of S. If one worker is shifted from industry α to β, production of α will decrease by $(\alpha_0+\tfrac{1}{2}\alpha_1)(1-S)$ amount and that of β will increase by $(\beta_0+\tfrac{1}{2}\beta_1)(1-S)$ amount. Thus the slope of the younger worker's PPC will be

$$-((\alpha_0+\tfrac{1}{2}\alpha_1)(1-S))/((\beta_0+\tfrac{1}{2}\beta_1)(1-S))$$
$$= -(\alpha_0+\tfrac{1}{2}\alpha_1)/(\beta_0+\tfrac{1}{2}\beta_1)$$

In the same way we can also derive the PPC for the older workers from the task box in Figure 3.3(a). In this case with an increase in S the task box will expand and the corresponding PPC's are GH for S=0, IJ for $S=S_1$ and KL for $S=S_2$ in Figure 3.3(b). The slope of these PPC's will be

$$-((\alpha_0+\tfrac{1}{2}\alpha_1)E(S))/((\beta_0+\tfrac{1}{2}\beta_1)E(S))$$
$$= -(\alpha_0+\tfrac{1}{2}\alpha_1)/(\beta_0+\tfrac{1}{2}\beta_1)$$

For the economy as a whole at any particular period there are two (equally sized) groups of workers, one at its initial period of life (young workers) and the other at its second period of life (older workers). The steady state PPC for the economy will be the sum of the two group PPC's corresponding to the same level of S. Thus for different levels of S we will have different steady state PPC's for the economy. The latter will shift upward if the increase in S shifts the PPC of the older workers by a greater amount than the downward shift in young worker's PPC. By examining the alternative steady state PPC's we can find out the grand steady state PPC for the economy. In Figure 3.4 the grand steady state PPC is derived for $S=S_2$ by adding EF of Figure 3.2(b) and KL in Figure 3.3(b).

Given the relative price of β in terms of α

$$\pi = (\alpha_0+\frac{1}{2}\alpha_1)/(\beta_0+\frac{1}{2}\beta_1)$$

The expected output of one young worker (working in α or β) will be

$$(\alpha_0+\frac{1}{2}\alpha_1)(1-S)$$

and that of an older worker,

$$(\alpha_0+\frac{1}{2}\alpha_1)E(S)$$

If in the economy the labour force is growing at a constant % rate in each period, then the number of younger workers in any period will be $(1+n)$ times that of the older workers. Assuming the number of older workers is $(\bar{A}+\bar{B})$, the total output in the economy in terms of α (say Q) will be

$$Q=(\alpha_0+\frac{1}{2}\alpha_1)(1-S)(\bar{A}+\bar{B})(1+n)+(\alpha_0+\frac{1}{2}\alpha_1)E(S)(\bar{A}+\bar{B})$$

In order to reach the grand PPC (golden rule), S should be chosen in such a way that Q is maximized. The first order condition is,

$$(3.12) \qquad \delta Q/\delta S = -(\alpha_0+\frac{1}{2}\alpha_1)(\bar{A}+\bar{B})(1+n) + (\alpha_0+\frac{1}{2}\alpha_1)E'(S)(\bar{A}+\bar{B}) = 0$$

$$\Longrightarrow \qquad E'(S) = 1+n$$

i.e. the golden rule level of S is determined where the slope of E(S) is one plus the labour force growth rate. Given $E'(0)=\infty$, this optimum level of S will be always positive. Since $E''(S)<0$, a higher level of n will give a smaller optimum level of S. When n=0 this condition become $E'(S)=1$. Determination of this golden-rule level of S is shown in Figure 3.5.

The steady state equilibrium

In order to determine the equilibrium production, consumption and price level, we will have to incorporate preferences (the demand side) in this model. Preferences are assumed to be the same as in the basic information model, and are shown by the community indifference curves in Figure 3.4. The tangency point between the PPC and the CIC will give the equilibrium.

The equilibrium production point is shown by A in Figure 3.4. α^G amount of α and β^G amount of β is produced. The equilibrium relative price of β in terms of α (π) will be equal to the slope of the PPC.

$$\pi^* = (\alpha_o+\tfrac{1}{2}\alpha_1)/(\beta_o+\tfrac{1}{2}\beta_1) \tag{3.13}$$

Given homothetic preferences, at this relative price each worker will consume α and β in the same proportion $0\alpha^G/0\beta^G$ and the actual level of consumption will depend on their income.

Income (wages) of a worker in the i th period of his/her life working in industry j, denoted by $W^j{}_i$ will be ,

$$W^{\alpha}{}_o = (\alpha_o+\tfrac{1}{2}\alpha_1)(1-S)$$

$$W^{\beta}{}_o = (\beta_o+\tfrac{1}{2}\beta_1)(1-S)\pi^* = (\alpha_o+\tfrac{1}{2}\alpha_1)(1-S)$$

$$W^{\alpha}{}_1 = (\alpha_o+\tfrac{1}{2}\beta_1)E(S)$$

$$W^{\beta}{}_1 = (\beta_o+\tfrac{1}{2}\beta_1)E(S)\pi^* = (\alpha_o+\tfrac{1}{2}\alpha_1)E(S)$$

Individual workers' optimization

If we assume that the capital market is perfect then the problem of an individual worker is to maximize his/her discounted life-time income. This is given by

$$\bar{W} = (\alpha_0+\tfrac{1}{2}\alpha_1)(1-S)+(1/(1+r))(\alpha_0+\tfrac{1}{2}\alpha_1)E(S)$$

where r is the rate of interest.

The first order condition for maximization is,

$$(3.14) \quad \delta\bar{W}/\delta S = -(\alpha_0+\tfrac{1}{2}\alpha_1)+(1/(1+r))(\alpha_0+\tfrac{1}{2}\alpha_1)E'(S)=0$$

$$\Longrightarrow \quad E'(S) = (1+r)$$

Thus the private optimum will coincide with the golden-rule level (condition for reaching the grand PPC, derived in equation 3.12) if and only if the rate of interest (r) is equal to the population growth rate (n).

IV. SCHOOLING WITH BOTH ALLOCATIVE AND PRODUCTIVITY EFFECTS

In this part the information and productivity aspects of schooling are incorporated in a single model. The structure of schooling is the starting point for combining the previous two basic models.

Structure of schooling

Schooling is now assumed to both increase the efficiency level/productivity of the workers and generate information regarding the comparative advantage of workers in different tasks (i.e. whether workers are of type A or type B). The former is the productivity augmenting effect of schooling, while the latter is the allocative effect.

In the information model we have assumed that the workers choose information quality P, which in turn determines the amount of time to be spent at school (S), given by

$$(3.15) \qquad S=S(P)$$

$$S'(P)>0, \quad S''(P)>0$$

$$\text{and } S(\tfrac{1}{2})=0, \quad S(1)=1$$

In the basic productivity model we have assumed that individuals choose the fraction of time to be spent at school (S), which then will determine the increase in productivity, given by

$$(3.16) \qquad e = E(S)$$

$$E'(S)>0, \quad E''(S)<0$$

further $E(0)=1$, $E(1)=e^M$, $E'(0)=\infty$

In the integrated model given P, by equation (3.15), S is determined which in turn increases the productivity given by equation (3.16). Substituting (3.15) in (3.16) we can write e as a function of P.

$$e = E(S) = E(S(P))$$

$$(3.17) \qquad e = e(P)$$

$$P=\tfrac{1}{2} \implies S=0 \implies e=1, \quad E'(S(\tfrac{1}{2})) = \infty, \quad e'(\tfrac{1}{2}) = \infty$$

$$P=1 \implies S=1 \implies e=e^M$$

$$e'(P) = E'(S(P))S'(P)$$

$$E'(S)>0,\ S'(P)>0 \Longrightarrow e'(P)>0$$

$$e''(P) = E''(S(P))S'(P)S'(P) + E'(S(P))S''(P)$$

Since $E''(S)$ is negative and $S''(P)$ is positive, the sign of $e''(P)$ is indeterminate. For diagrammatic simplicity it is assumed that $e''(P)<0$. This assumption will not affect any of our results.

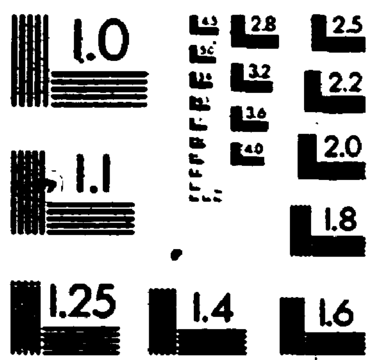
This relationship is shown diagrammatically in Figure 3.6. When an individual worker chooses P_1 level of schooling he/she will have to spend S_1 amount of time in school (from quadrant III) which will increase the efficiency level to e_1 (from quadrant II). And this relationship is shown in the first quadrant by point A. Similarly points B and C represent no schooling at all and full time schooling respectively.

As in the previous case in this model we are assuming that the information generated through schooling is useful in the future period only and that schooling increases productivity in the future only. This implies that only the younger workers will undertake schooling.

Expected output of one worker

Young workers : As in the basic model the expected output of one young worker employed in industry j, (Y^j_0), who is spending $S(P)$ amount of time at school will be:

2



(3.18) $$Y^{\alpha}_{o} = (\alpha_o+\tfrac{1}{2}\alpha_1)(1-S(P))$$

(3.19) $$Y^{\beta}_{o} = (\beta_o+\tfrac{1}{2}\beta_1)(1-S(P))$$

Older workers : Schooling will affect the expected output of an older worker in two ways. Firstly, better information generated through schooling will lead to a better matching between workers and industries. Secondly, schooling increases the amount of tasks that can be performed by one worker.

From the basic productivity model we can derive the expected output of one i type worker with S(P) amount of time spent at school (in younger period), working in industry j (denoted by $Y^{j}_{A}(P)$) as

(3.20) $$Y^{\alpha}_{A}(P) = (\alpha_o+\alpha_1)e(P)$$

(3.21) $$Y^{\beta}_{A}(P) = \beta_o e(P)$$

(3.22) $$Y^{\alpha}_{B}(P) = \alpha_o e(P)$$

(3.23) $$Y^{\beta}_{B}(P) = (\beta_o+\beta_1)e(P)$$

Since schooling now has information content there will be two types of older workers, one with label \`a', the other with label \`b'. The probability that given label \`a' (\`b') a worker is an \`A'(\`B') type worker is P. Thus the expected output level of one \`a' label older worker working in industry α will be

$$Y^{\alpha}_{a}(P) = (\alpha_o+\alpha_1)e(P)P + \alpha_o e(P)(1-P)$$

(3.24) $$= (\alpha_o+\alpha_1 P)e(P)$$

Similarly we can derive,

$$Y^{\beta}{}_{a}(P) = \beta_o e(P)P + (\beta_o+\beta_1)e(P)(1-P)$$

$$\text{~~(3.25)~~} \qquad = (\beta_o+\beta_1(1-P))e(P)$$

$$(3.26) \qquad Y^{\alpha}{}_{b}(P) = (\alpha_o+\alpha_1(1-P))e(P)$$

$$(3.27) \qquad Y^{\beta}{}_{b}(P) = (\beta_o+\beta_1 P)e(P)$$

Figure 3.7 shows these in terms of isoquants. For one young worker the expected amount of tasks performed is shown by point A. For the older worker the expected amount of tasks performed will move along path (1) or (2) depending on what label the worker has. For one 'a' label worker the expected amount of tasks will move along path (1) and with P=1 it is given by point B. For 'b' movement is along path (2) with point C being reached when P=1.

Production possibilities

In this model the total availability of tasks will be the same as in the basic productivity model, since information generated through schooling does not affect the availability of tasks. The task box will be same as before as shown in Figure 3.8(a) and 3.9(a) for the younger and the older workers respectively.

Younger workers' PPC : The derivation and shape of the young workers' PPC will be the same as in the basic models. With a higher level of P the PPC shifts towards the origin, initially by a smaller amount and then by larger amount as

$S''(P)>0$. The slope of the PPC will also be the same as before,

$$(\alpha_0+\tfrac{1}{2}\alpha_1)/(\beta_0+\tfrac{1}{2}\beta_1)$$

These PPC's are shown in Figure 3.8 (b).

Older worker's PPC : With a higher level of schooling the task box of older workers will expand. The feasible production set in the task box also increases. The former effect is the result of the productivity increase while the latter is the result of better information. In Figure 3.9(a) with the P_1 level of schooling the factor box expands from $0^{\alpha}0^{\beta}$ to $0^{\alpha}0^{\beta'}$. Now due to better information it will be possible to produce at any point in the triangular area $0^{\alpha}B0^{\beta'}$ and on the straight lines $0^{\alpha}B$ and $B0^{\beta'}$ production will be efficient. Corresponding to this the PPC is ABC in Figure 3.9(b). In this way the PPC for P_2 is derived which is DEF. If $P=\tfrac{1}{2}$ the PPC will be the same as that of the younger workers. The older workers' PPC will have two straight line segments. As we move from A to B (or D to E) 'b' labeled workers are taken from α and employed in β, so the slope of these parts will be,

$$\begin{aligned} d\alpha/d\beta &= -(\alpha_0+\alpha_1(1-P))e(P)/(\beta_0+\beta_1 P)e(P) \\ &= -(\alpha_0+\alpha_1(1-P))/(\beta_0+\beta_1 P) \end{aligned}$$

and when we move from B to C (or E to F) 'a' labeled workers will be taken from α and will be employed in β and the slope will be

$$d\alpha/d\beta = -(\alpha_0+\alpha_1 P)e(P)/(\beta_0+\beta_1(1-P))e(P)$$
$$= -(\alpha_0+\alpha_1 P)/(\beta_0+\beta_1(1-P))$$

These are the same as in the basic information model. The only difference is that the PPC is moved outward in this model by schooling.

PPC for the economy : By adding the PPC of the young workers and that of the older workers corresponding to the same level of P one can derive the steady state PPC for the economy corresponding to that level of P. In this way for different levels of P we will have different PPC's for the economy. Figure 3.10 show the PPC's AB (for P=½), CDEF (for P_1), GHIJ(for P_2). The grand PPC is shown by the red curve in Figure 3.10, which shows the maximum attainable output combinations, when P can take any value. The shape and slope of the grand PPC in this model is same as in the basic information model[1]. The only difference is that the PPC here is expanded outward compared to that of the former model.

We can also examine the value of P that will ensure that we are on the grand PPC. A value of P is required such that total output in terms of any one commodity, say α, is maximized. If we look at the straight line segment the price of β in terms of α (π) will be,

$$= (\alpha_0+\tfrac{1}{2}\alpha_1)/(\beta_0+\tfrac{1}{2}\beta_1)$$

This will give the output of one young worker in terms of α as,

(3.28) $$(\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P))=(\beta_0+\tfrac{1}{2}\beta_1)(1-S(P))\pi$$

In the second period there is $\frac{1}{2}$ probability that one will have label 'a'(b) and will be employed in $\alpha(\beta)$. Thus the expected output in the older period will be,

(3.29) $$\tfrac{1}{2}[(\alpha_0+\alpha_1 P) + \pi(\beta_0+\beta_1 P)]e(P)$$

Assuming n% growth rate in the labour force, if in any period the size of the older work force is $(\bar{A}+\bar{B})$ then that of the younger work force will be $(\bar{A}+\bar{B})(1+n)$. Given output of one worker in terms of α in equation (3.28) and (3.29), the total output in the economy (Q) will be,

$$Q=(\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P))(\bar{A}+\bar{B})(1+n)+$$
$$\tfrac{1}{2}[(\alpha_0+\alpha_1 P)+\pi(\beta_0+\beta_1 P)]e(P)(\bar{A}+\bar{B})$$

The first order condition for maximization is,

$$dQ/dP = -S'(P)(\alpha_0+\tfrac{1}{2}\alpha_1)(\bar{A}+\bar{B})(1+n)+\tfrac{1}{2}(\alpha_1+\pi\beta_1)e(P)(\bar{A}+\bar{B})$$
$$+ \tfrac{1}{2}[(\alpha_0+\alpha_1 P)+\pi(\beta_0+\beta_1 P)]e'(P)(\bar{A}+\bar{B}) = 0$$

(3.30) $$\implies S'(P)(\alpha_0+\tfrac{1}{2}\alpha_1)(1+n)$$
$$= \tfrac{1}{2}(\alpha_1+\pi\beta_1)e(P) + \tfrac{1}{2}[(\alpha_0+\alpha_1 P)+\pi(\beta_0+\beta_1 P)]e'(P)$$

From this equation we can derive the level of P which will ensure production on the grand PPC. The left hand side shows the marginal cost of schooling. The first term in the right side shows the marginal benefit from the allocative effect of schooling and the second term is the benefit from the productivity augmenting effect.

Comparing equation (3.30) with (2.6) one can see that the golden-rule level of P in this case will be larger than that in the basic information model.

Preferences

The preferences of the individuals in this model are assumed to be the same as in the basic models. The intertemporal utility function is assumed to be additively separable and can be represented in terms of group indifference curves and community indifference curves (CIC).

The steady state golden rule production plan

As in the basic information model the golden rule production plan (α^G, β^G) discussed in Chapter 2.V, in this case also there are three different kinds of feasible steady state (α^G, β^G). Firstly we will consider the golden rule production plan on the straight line segment of the grand PPC and then we will look at the kinked point. We are omitting the discussion on the production plan in the convex part, since we will never observe an equilibrium here due to the same reasons as in the basic information model.

i) Golden rule production plan in the straight line segment of the grand PPC : In Figure 3.11 this is shown by point E, α^G amount of α and β^G amount of β will be produced. By looking at the group PPC we can say that all 'a' labeled older workers will be working in α and 'b' labeled in β (production point P). Young workers will be allocated between α and β in such a way that their group production will be $\alpha_1\alpha^G$ and $\beta_1\beta^G$.

Steady state relative price (π) of β in terms of α will be given by the slope of the grand PPC at E, which is equal to the slope of the young workers' PPC,

$$\pi^*_1 = (\alpha_0+\tfrac{1}{2}\alpha_1)/(\beta_0+\tfrac{1}{2}\beta_1)$$

Earnings (W) of the workers are assumed to be equal to their expected output. For one younger worker

$$W^o = (\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P)) = \pi^*_1(\beta_0+\tfrac{1}{2}\beta_1)(1-S(P))$$

For one 'a' label older worker it will be

$$W^1_a = (\alpha_0+\alpha_1 P)e(P)$$

and one 'b' label older worker will get

$$W^1_b = \pi^*_1(\beta_0+\beta_1 P)e(P)$$

All individuals (given identical taste) will consume α and β is same proportion $0\alpha^G/0\beta^G$.

ii) Golden rule production plan at point of the kink B (or C). In this case all the young workers and all 'a' label workers will be employed in α. All 'b' label older workers will be working in β. The steady state relative price (π) will be equal to the slope of the CIC at golden rule production plan.

In this case all the younger workers will be employed in α and their earnings will be ,

(3.32) $\qquad W^{o}(P) = (\alpha_0+\frac{1}{2}\alpha_1)(1-S(P))$

When we look at one representative older worker the probability is $\frac{1}{2}$ that he/she will be one with label 'a' and will earn

$$(\alpha_0+\alpha_1 P)e(P)$$

The probability is also $\frac{1}{2}$ that he/she has label 'b' and in that case will be employed in β and his/her earning in terms of α will be ,

$$\pi^*{}_3(\beta_0+\beta_1 P)e(P)$$

So the expected earning of one worker in older period is,

$$\tfrac{1}{2}[(\alpha_0+\alpha_1 P) + \pi^*{}_3(\beta_0+\beta_1 P)]e(P)$$

The nature of the relationship between P and π in this case will be the same as in the basic information model. The only difference in this case is that with higher levels of P, π will decrease at a faster rate. The intuition is as follows: with higher level of schooling the marginal cost (decrease in young workers output α) is the same as before, but the increase in output in the older period will be larger. So the output of the older worker intensive industry β will increase at a faster rate than that of α.

This will lead to a faster decrease in π. Diagrammatically this relationship is shown by the curve AB in Figure 3.12.

Individual worker's optimization

In the presence of a perfect capital market with fixed interest rate (r) and given relative output price, the optimization problem of an individual worker is to maximize his/her discounted life-time income $(\bar{W})^2$. When we have the steady state equilibrium on the straight line segment or at the kinked point of the grand PPC the present value of the life-time income of an individual worker in terms of α is given by,

$$\bar{W}=(\alpha_o+\tfrac{1}{2}\alpha_1)(1-S(P))+(1/(1+r))\tfrac{1}{2}[(\alpha_o+\alpha_1 P)+\pi(\beta_o+\beta_1 P)]e(P)$$

The first order condition for maximization is,

$$d\bar{W}/dP=-S'(P)(\alpha_o+\tfrac{1}{2}\alpha_1)+(1/(1+r))(e(P)(\alpha_1+\pi\beta_1) + \tfrac{1}{2}[(\alpha_o+\alpha_1 P) + \pi(\beta_o+\beta_1 P)]e'(P)=0$$

$$\Longrightarrow \quad P=P(\alpha_o,\ \alpha_1,\ \beta_o,\ \beta_1,\ r,\ \pi)$$

As in the basic information model in this case also an increase in α_o, β_o or r will decrease the optimum level of P, while an increase in α_1, β_1 or π will increase it.

Comparing this with the condition for reaching the grand PPC (equation 3.30), both will give same level of P if and only if r=n. Thus in this case also individual's optimization will coincide with the optimization for the society if the rate of interest is equal to the population growth rate.

From the above first order condition of maximization we can derive the relation between π and individuals optimum choice of P. These two will be directly related it is shown in Figure 3.12 by the XY curve. Comparing this result to that in the basic model (equation (2.9)), in this case for any given level of π individuals will choose a higher level of P as the benefit component now includes some additional terms, as there is an increase in productivity in the older period as a result of schooling in the initial period.

Equilibrium investment in schooling

As in the basic information model the feasible steady state combinations of schooling (P) and relative price (π) and individual's optimum choice of schooling given π will determine the final equilibrium. One such equilibrium is shown by point E in Figure 3.12, with P* level of schooling and π^* relative price. All the comparative static results in this case will be the same as in the basic information model. An increase in α_0, β_0 or r will decrease the equilibrium level of P and will decrease π, while an increase in α_1 or β_1 will have the opposite effects.

Compared to the basic information model (Figure 2.14), the amount of human capital investment may be higher or lower, as the curve XY has shifted to the right while AB shifted to the left and the net result will depend on the relative amount of the shift. The intuition behind this is as follows. For a given level of allocative gain from schooling (determined by π) individuals will choose a higher level of P as there is an additional gain from schooling through productivity increase. But the productivity effect has a negative effect on the marginal allocative gains, as a higher productivity gain results in a faster decrease in π for the economy.

V. SUMMARY

For the economy as a whole a higher level of productivity gains from schooling will lead to an lower allocative gain. The intuition is as follows. Higher productivity gain implies that with more schooling, production of older intensive industry will increase at a faster rate than that of the younger worker intensive industry. This will lead to a relative price movement against the older worker intensive industry which in turn means smaller allocative gain.

NOTES

[1] If the allocative benefit from schooling is very small compared to the productivity gain it is possible that there will be no convex parts in the grand PPC rather in the corners two straight line segments with different slopes. The one towards the axis α will have smaller slope while that touching the axis β will have steeper slope. In that case also there will be two kinked points of the grand PPC and there will be no change in our analysis and results.

[2] Similar to the basic information model in this case also we will have a condition for global optimum. In the following discussion we will study the case where that condition is satisfied and an equilibrium for the economy exists.

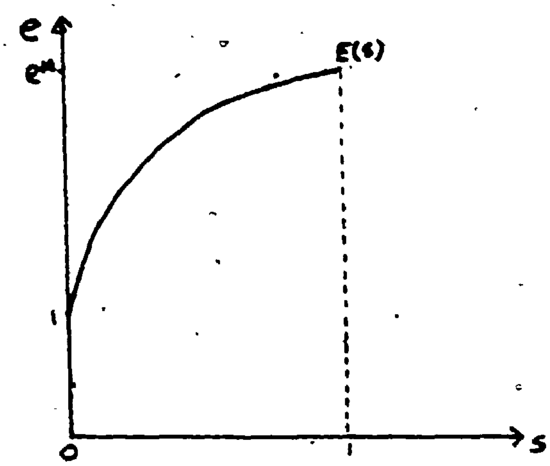


Figure 3.1 : Level of Schooling and Productivity

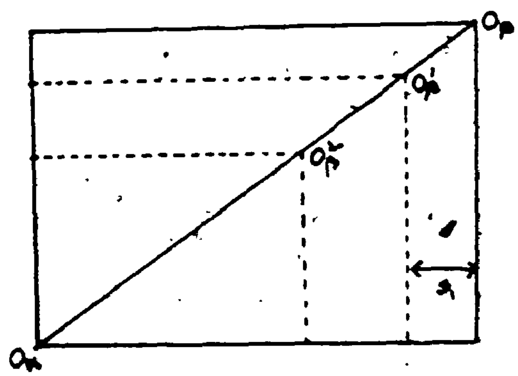


Figure 3.2(a) : Edgeworth Box for Young Workers

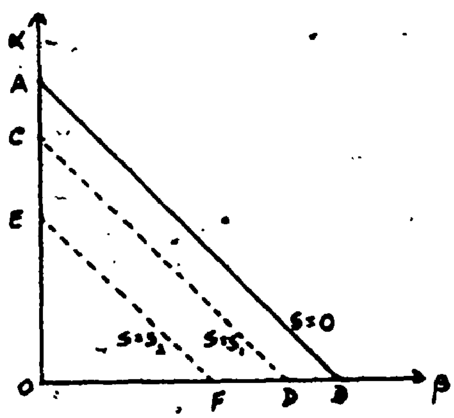


Figure 3.2(b) : PPC for the Young Workers (Productivity Effect only)

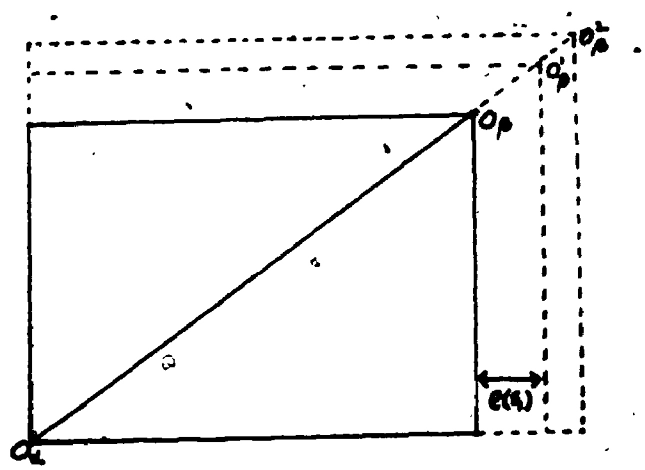


Figure 3.3(a) : Edgeworth Box for Old Workers

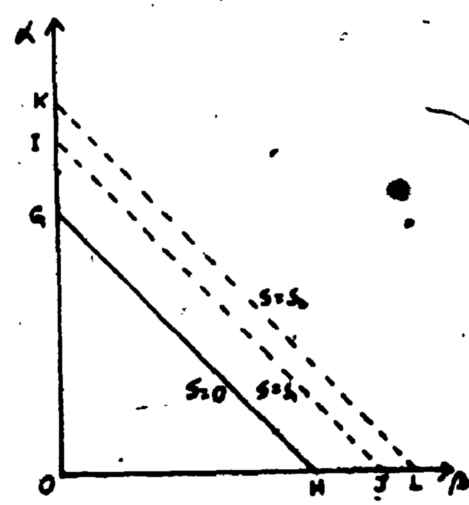


Figure 3.3(b) : PPC for the Old Workers (Productivity Effect only)

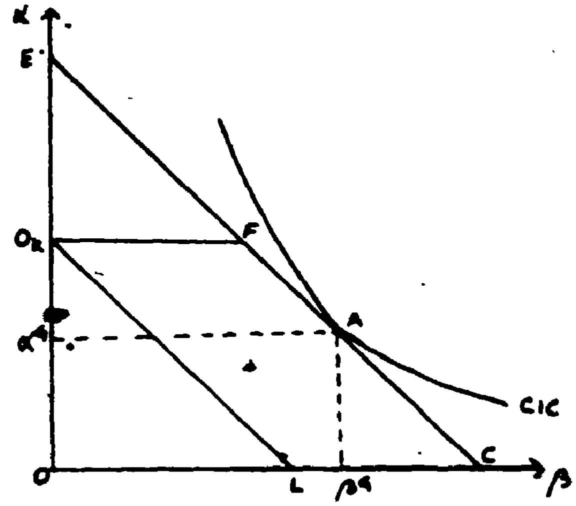


Figure 3.4 : Steady State Grand PPC for the Economy and Equilibrium (with only Productivity Effect)

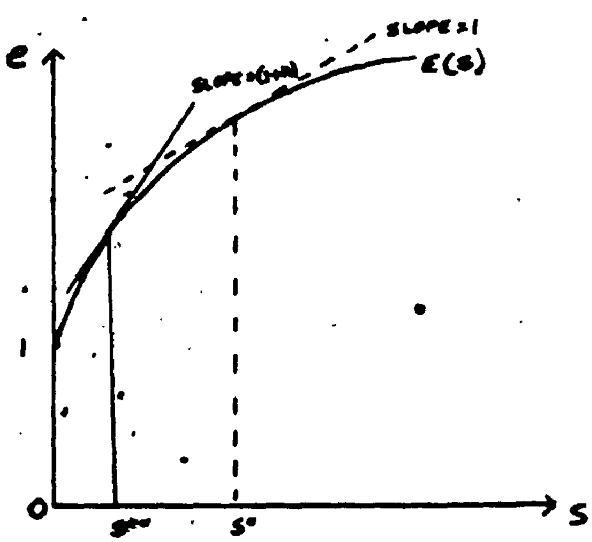


Figure 3.5 : Golden Rule Level of Schooling

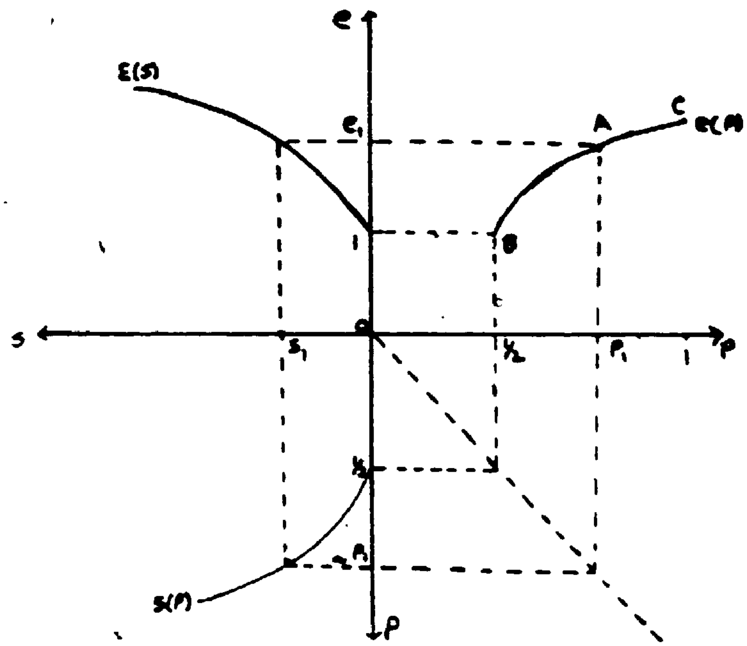


Figure 3.6 : Information Quality and Productivity

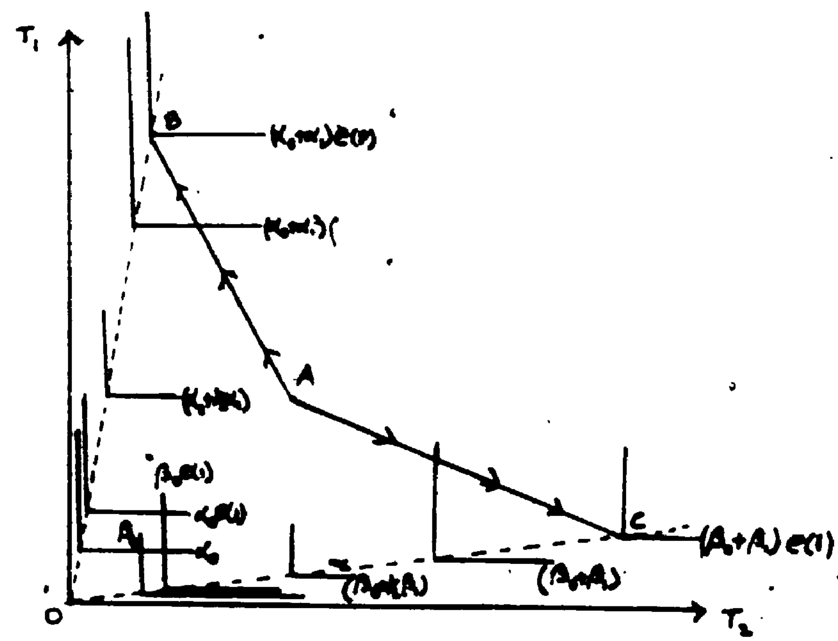


Figure 3.7 : Expected Output of One Worker

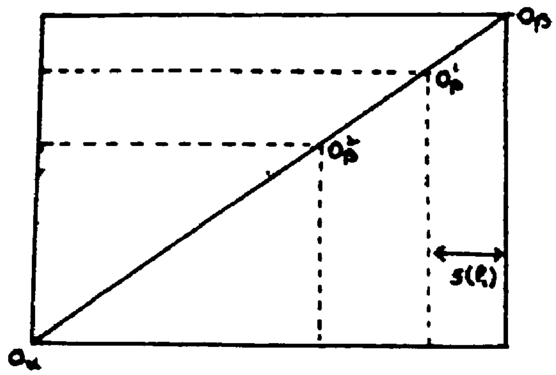


Figure 3.8(a) : Edgeworth Box for Young Workers

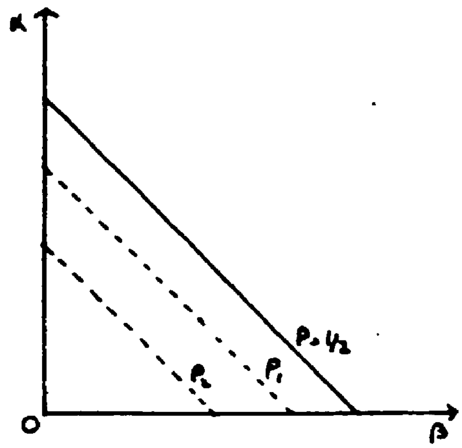


Figure 3.8(b) : PPC for the Young Workers (both Productivity and Allocative Effects)

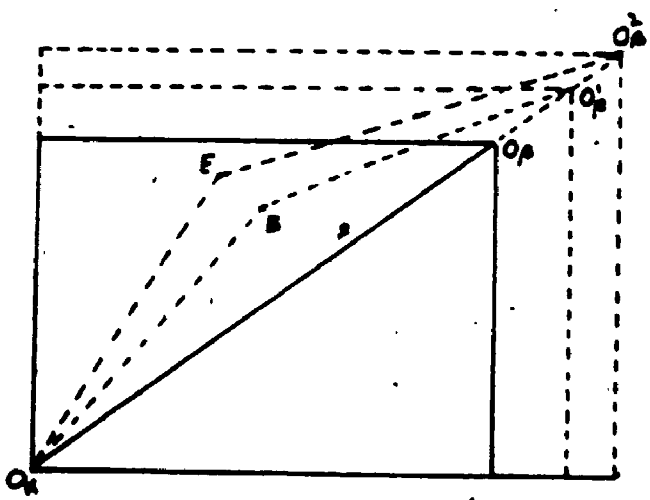


Figure 3.9(a) : Edgeworth Box for Old Workers

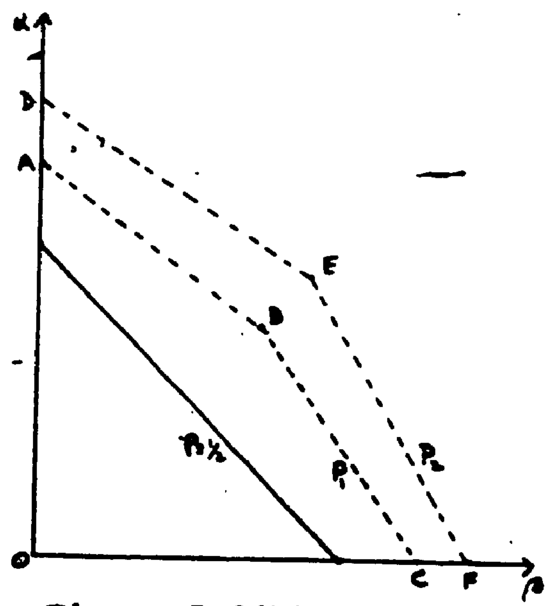


Figure 3.9(b) : PPC for the Old Workers (both Productivity and Allocative Effects)

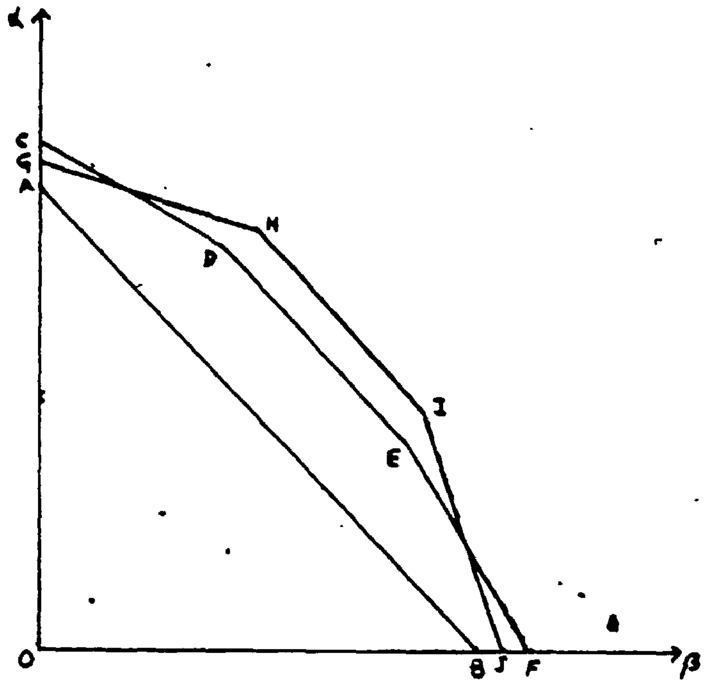


Figure 3.10 : Steady State Grand PPC for the Economy (with both Productivity and Allocative Effects)

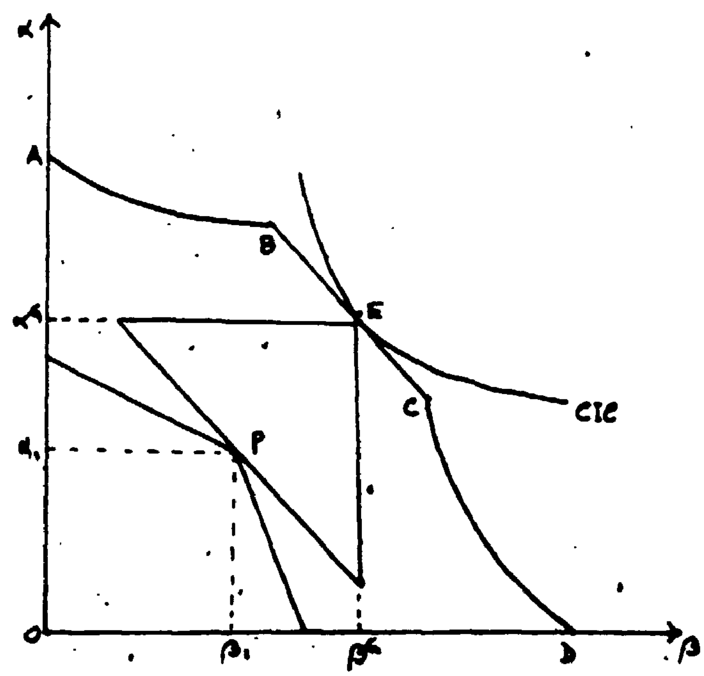


Figure 3.11 : Steady State Golden Rule Production Plan (with both Productivity and Allocative Effects)

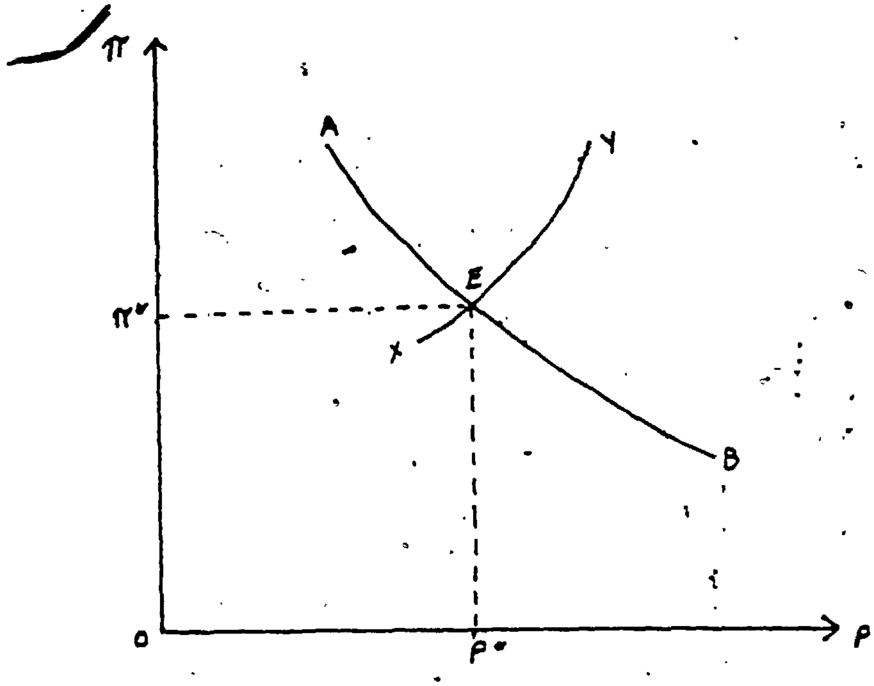


Figure 3.12 : Equilibrium Level of P and π

CHAPTER 4

EFFECTS OF A CHANGE IN THE AGE STRUCTURE OF THE LABOUR FORCE

The objective of this chapter is to examine the effect of a change in the age structure of the labour force on the real return from, as well as on the optimum amount of schooling. The first section provides the motivation for the study through an examination of the relevant literature. The second section discusses the effects of a change in the age structure of the labour force on the feasible equilibrium combinations of schooling and relative price. The third section examines the individual's optimization problem in and out of a steady state situation. Three different types of changes in the age composition of the labour force are examined in the three subsequent sections. Some concluding remarks are given in the final section.

I. RELATIONSHIP TO PREVIOUS WORK

Beginning in the early 1970's, the Canadian and the U.S. economies experienced a dramatic change in the age composition of the labour force (see appendix 6). With the labour market entry of the post world war II 'baby boom' generation the number of younger workers in the labour force were significantly increased compared to the number of older workers. A number of studies were performed in order to determine the labour market effects of this change (Freeman 1975, 1979, Easterlin 1978, Welch 1979, Berger 1985, Dooley 1986, Dooley and Gottschalk 1984).

One of the general findings of these studies is that the members of the larger cohort faced a depressed market wage in their younger life, and this wage depressing effect was more pronounced for the workers with higher levels of schooling. At the same time there was a significant increase in the average level of schooling chosen by individuals. Also there is an increase in the earnings inequality both between the members of different cohorts and within cohorts, observed during the last 20 years.

From these observations it has been argued that the returns from schooling, as well as the level of schooling chosen by individuals, are affected by the size of the cohort one belongs to. This resulted in a number of studies to examine the process through which cohort size affects the return and the optimum choice of human capital investment.

Welch (1979) argued that the change in the age structure of the labour force affects the relative earnings of the different cohort of workers, since workers with different levels of experience are imperfect substitutes for one another. So a relative increase in the number of younger workers will decrease their relative earning. The smaller is the elasticity of substitution the greater will be the wage depressant effect. Welch further argued that workers with different levels of schooling are imperfect substitutes for one another and that the elasticity of substitution between different experience levels decreases with the level of schooling. From this he concluded that the wage depressing effect will be greater the larger the cohort and the higher its investment in human capital. Welch examined the wage depressing effect over time in terms of a career phase model, where different phases are identified by the activities in which one devotes his/her time. Each worker is assumed to pass through different phases of his/her career (the simplest case consists of two phases, young and old). It is assumed that the time required for transition from one phase to the next is exogenously given and is independent of cohort size. Also, the wage rate earned by one representative worker in the period of transition is assumed independent of the cohort size. In Welch's model the members of the larger cohort start with a lower income level but reach the same level of income at the transition point. This implies a faster earnings growth for the larger cohort. Thus in Welch's analysis the relative earning of different cohorts are affected by relative supply change only and for the baby boom generation he predicted a faster earnings growth than for other cohorts.

On the other hand Freeman (1975,1979) argued that the depressed wage of the larger cohort in the '70's was a result of both changes in supply and demand conditions. In his studies the supply side effect is also examined in terms of imperfect substitution between workers with different levels of experience. But Freeman (1975) argued that in the 1970's the U.S. economy experienced a shift in labour demand against the group with a higher level of schooling due to a change in the industrial structure. He argued that the shift in labour demand is coming to an end. Without a favorable demand condition the supply effect will dominate and the members of the larger cohort will not experience a faster income growth rate. In his studies the demand condition is considered to be exogenously given. In his later study Freeman (1979) argued that the wage depressing effect on the members of the larger cohort is a permanent one, at least for the following three reasons. Firstly they started at a lower starting wage and as they move up in the job ladder they will always have lower income. Secondly due to competition some of them may have chosen a job with a lower earning growth path and thirdly they will face competition from a larger pool of workers for promotion or raise. And he concluded that to understand the relation between earnings and the age composition of the labour force more emphasis should be put on demand conditions.

In an empirical paper for the U.S. Berger (1985) estimated a modified version of the Welch model and concluded that the members of the larger cohort in fact experienced a slower earnings growth rate. He argued that the time required for transition from one career phase to the next may not be exogenous and it may be larger for the members of the larger cohort. Berger asserted that the baby boom generation may also have chosen a lower level of human capital investment and on-the-job training, as the expected return from such investment may be lower and this might be one of the reasons for their slower earning growth rate.

Dooley (1986) examined the cohort size effect in terms of relative supply change for the Canadian economy. His findings are generally consistent with those of Welch, that is the members of the larger cohort experienced a depressing wage rate and this wage depressing effect decreases with level of experience. But his study cannot explain the fact that the earning gap is narrowing between workers with different schooling levels for both young and old workers. Dooley, like Freeman, also concluded that the change in the structure of demand should be examined in order to fully examine the cohort size effect.

Dooley and Gottschalk (1984) examined the effect of a change in age composition of the labour force on relative earnings and earnings inequality. They found evidence of an increased income inequality within cohorts as a result of the labour market entry of the baby boom generation. They studied the effects from a supply side point of view and used a two staged approach. The first stage examined the effect of the age structure of the labour force on the return from schooling in a one-sector neoclassical growth model, where workers make no decisions. In a second stage they looked at the effect of a change in the return from schooling on optimal investment in human capital in the Ben-Porath model (1967), where growth rate of return from schooling are given. Thus, in two stages, two different approaches are taken regarding worker's behaviour. They also recognize the fact that a better approach to the problem would be a general equilibrium model where the rate of change in the return from schooling and the optimum level of schooling were determined simultaneously.

All the existing studies attempt to explain the effect of a change in the age structure of the labour force in terms of relative supply change. As shown in Connelly (1986) the predictions of these studies differ partly due to different production functions assumed.

With a change in the size and the age structure of the labour force the production possibility frontier for the economy will shift. This will result in a change in both the relative prices and the amount of production of different commodities. Since skilled (older) and unskilled (younger) workers are employed in different industries in different proportions (Welch 1979, Smith and Welch 1978), any change in the output mix will change the relative demand for skilled or unskilled workers and will lead to a change in their relative earnings. Previous studies have assumed that cohort size has nothing to do with the structure of labour demand and ignored the above general equilibrium effect. For a complete picture of how cohort size affect human capital investment, one cannot ignore this effect. The purpose of this chapter is to examine this general equilibrium effect.

Human capital investment increases individual output not only through productivity increase but also through allocating the workers to their most productive uses (Jovanovic 1979, MacDonald 1980). The increased productivity effect of schooling is determined by the relative supply of different groups of workers, which has been examined in the different studies mentioned above. On the other hand, the structure of market demand is directly related with the allocative benefit from schooling. So one way to study ~~the~~ demand effects is to examine the allocative benefits from schooling in a general equilibrium model where the rate of change in the returns from schooling and the optimum level of schooling are determined simultaneously. Thus the general equilibrium model of the labour market developed in the previous two chapters is appropriate to examine the problem addressed here.

In this chapter in addition to the baby boom effects we are also going to examine two other changes in the age composition of the labour force. A selective immigration policy, where a large number of skilled workers are allowed to immigrate will result in a change in the age composition of the labour force (see appendix 6). In terms of our general equilibrium model this will increase the cohort size of the older workers (skilled) compared to that of the younger (unskilled) workers. In section V we will discuss the effects of such a policy.

Secondary workers in the labour market are mostly unskilled and have relatively shorter working lives. In our model they can be identified as 'young workers'. Any increase in the number of secondary workers (see appendix 6) should have effects similar to an increase in the relative size of the 'younger workers'. The effects of such a change are examined in section VI.

Our analysis here will be on the basis of the equilibrium at the kinked point of the PPC for the economy since the other two situations can be considered special cases of this one. We will study the kinked point where all the young and 'a' labeled older workers are employed in α and all 'b' labeled older workers in β. Thus industry β is older (skilled) worker intensive and α is younger worker intensive.

II. EFFECTS ON THE FEASIBLE EQUILIBRIUM COMBINATIONS OF SCHOOLING AND PRICE

This section examines the effects of two different types of changes in the age structure of the labour force on the grand PPC and on the relationship between the level of schooling (P) and the relative price of β in terms of α (π). In the first case we will look at an increase in the number of young workers while the number of older workers stays at the steady state level. In the second case we will assume that the number of young workers is at the steady state level while the number of older workers is increased compared to the steady state level. For the sake of exposition imagine that we are in a golden rule steady state production plan.

Increase in the Number of Younger Workers

In Figure 4.1 the steady state grand PPC is shown by the curve ABCD and the golden rule production plan is reached at point B with level of schooling P^G. The golden rule relative price corresponding to this is given by the slope of the CIC_1 at point B (shown by ab). When the number of young workers are increased in one period while the number of older workers stays at the steady state level, the group PPC for the younger workers will be enlarged but that of the older workers will remain the same. For the grand PPC this will imply that the length of the straight line part will be increased while that of the convex parts will remain the same. The new grand PPC is shown by EFGH in

Figure 4.1 where point F correspond to the level of schooling P^G. In the case illustrated, the new production plan is shown by point F and the new relative price is given by the slope of the CIC_2 at F (shown by a'b'). From the construction of the grand PPC we can see that point F will be vertically above point B. Given homothetic preference, any point on the CIC's to the left of the OB ray must have a steeper slope than that at point B. So the slope of CIC_2 at F must be greater than that of CIC_1 at B. Thus for P^G, with an increase in the number of younger workers the price ratio for the economy will be increased compared to the steady state level. This is true for any level of P between ½ and 1 as the production point will move along the straight line FJ. In terms of Figure 4.2 the curve showing the feasible equilibrium combinations of P and π for the economy will move upward to ADE from ABC (steady state case). For any given P the relative price moves against the younger worker intensive industry (α).

Increase in the Number of Older Workers

In this case the group PPC for the younger workers will be the same as the steady state one while that of the older workers will be expanded. Thus for the grand PPC the length of the straight line segment will remain the same but that of the convex parts will be increased. In terms of

Figure 4.3 the grand PPC will shift to IJKL from ABCD. The production point corresponding to the steady state golden rule level of P will move from point B to point J and given homothetic preference the relative psice ratio will be decreased compared to the steady state level. By similar reasoning as before for the economy as a whole the feasible equilibrium π-P combination locus will shift downward, say from ABC to AFC in Figure 4.2. For any given P the relative price moves against the older worker intensive industry.

The intuition behind the above changes is as follows. As the size of one group of workers is increased, output of the industry using that group of workers intensively will increase proportionately more. This will result in a relative price movement against that industry.

III. INDIVIDUAL WORKER'S OPTIMIZATION

In this section we will examine an individual's optimization problem, when we are out of the steady state equilibrium and the relative output price is changing. We will denote the relative output price in the younger period by π_0 and in the older period by π_1. From the individual worker's point of view these are given exogenously and fixed.

A representative individual worker's earnings in the two periods of life (W_0 and W_1) will depend on the level of schooling chosen. When earnings are insured at their expected level, (as derived in chapter 2) these are,

(4.1) $$W^0(P) = (\alpha_0+\tfrac{1}{2}\alpha_1)(1-S(P))$$

(4.2) $$W^1(P) = \tfrac{1}{2}[(\alpha_0+\alpha_1 P)+(\beta_0+\beta_1 P)\pi_1]$$

In this case also we are going to assume that there exists a perfect capital market where individuals can borrow any amount (B) at a fixed interest rate (r)[1]. Under such conditions the individual's optimization problem can be written as

$$\text{Maximize } U = u(C^\alpha{}_0,C^\beta{}_0) + (1/(1+\rho))u(C^\alpha{}_1,C^\beta{}_1)$$

Subject to

$$C^\alpha{}_0 + \pi_0 C^\beta{}_0 = W^0(P) + B$$

$$C^\alpha{}_1 + \pi_1 C^\beta{}_1 = W^1(P) - B(1+r)$$

Where B can be either positive or negative. Substituting the constraints in the utility function and using equations (4.1) and (4.2), we have

(4.3) $$U = u(W^0(P)+B-\pi_0 C^\beta{}_0,C^\beta{}_0)+(1/(1+\rho))u(W^1(P)-B(1+r)-\pi_1 C^\beta{}_1,C^\beta{}_1)$$

The first order conditions for maximization are,

(4.4) $$\delta U/\delta C^\beta{}_0 = -u^\alpha{}_0\pi_0 + u^\beta{}_0 = 0$$

(4.5) $$\delta U/\delta C^\beta{}_1 = (1/(1+\rho))(-u^\alpha{}_1\pi_1 + u^\beta{}_1) = 0$$

(4.6) $$\delta U/\delta B = u^\alpha{}_0 -((1+r)/(1+\rho))u^\alpha{}_1 = 0$$

(4.7) $$\delta U/\delta P = u^\alpha{}_0(\delta W^0(P)/\delta P)+(1/(1+\rho))u^\alpha{}_1(\delta W^1(P)/\delta P) = 0$$

Equation (4.4) shows the optimum consumption combination of α and β in period 0 and equation (4.5) shows that in period 1. From equation (4.4), (4.5) and (4.6) we can derive,

(4.8) $$(u^{\beta}{}_{o}/u^{\beta}{}_{1})(\pi_1/\pi_o) = (u^{\alpha}{}_{o}/u^{\alpha}{}_{1}) = (1+r)/(1+\rho)$$

which shows the optimum allocation of consumption between the two periods. Equation (4.7) determines the optimum level of schooling (P). In this equation the first term shows the cost of schooling while the second term is the benefit in terms of increased productivity. Using equation (4.8) the optimum level of P determination given in (4.7) can be written as

(4.9) $$S'(P) = \tfrac{1}{2}(\alpha_1+\pi_1\beta_1)/[(1+r)(\alpha_o+\tfrac{1}{2}\alpha_1)]$$

This is the same as derived in the steady state model developed before except that π_1 replaces π. α_o and r will have a negative effect on P, while β_1, π_1 and under certain conditions α_1 (if $2\alpha_o>\beta_1\pi$) will have a positive effect on P. The current period prices do not affect individual's choice of P. Since future price (π_1) determines the current level of P, individual's expectations regarding π_1 will determine the current choice of schooling level. We will examine two alternative types of expectations formation: firstly a myopic one where workers expect that the current price will prevail in the next period. In that case the relation between the current price and level of schooling will be the same as derived before and shown by the curve RS in Figure 4.4. Secondly, we will examine a 'one period perfect foresight' case where workers can foresee the next period changes in the relative price due to changes in the age composition of the labour force. In this case the optimum choice of P will be independent of the current prices but directly related with the future expected price. This relationship is shown in Figure 4.4 by TU for π_1^1, VW for π_1^2 (where $\pi_1^1 > \pi_1^2$).

IV. ONE LARGER COHORT OF WORKERS PASSING THROUGH THE LABOUR FORCE

We will start with a steady state equilibrium and then increase the number of young workers in one period (period t) while the number of older workers are in the steady state level. In the next period (t+1) we will assume that the number of young workers is at the steady state level while the number of older workers is increased compared to the steady state level. Thus we are looking at the effects when a 'baby boom' generation (generation G_t) passes through the labour force.

Effects on the Actual Level of Schooling and Price

Combining the feasible equilibrium combinations of P and π for the economy (derived in section II) and individual's optimum choice of P given π (derived in the previous section), we can examine the effect on the actual equilibrium level of schooling and relative price for the economy. For the steady state case these are shown by ABC and RS respectively in Figure 4.5. The steady state equilibrium is established at point B with level of schooling P* and relative price π*.

Myopic expectations: In this case there will be no change in the individual's optimum choice of P as the baby boom generation passes through the labour force. But as derived in section II the curve showing the feasible combinations of P-π for the economy will shift upward (to ADE) when the baby boom generation are young and it will shift downward (to AFC) in the next period as they get older. Thus in period t when the baby boom generation enter the labour force as young workers the equilibrium point will be determined at point D which corresponds to a higher than steady state level of both relative price ($\pi_t > \pi^*$) and schooling ($P_t > P^*$). In the next period (t+1) the equilibrium point will move to E with a lower level of both P and π.

Thus the baby boom generation faces an adverse market condition, when they are young the relative output price will move against the young worker intensive industry while in their older period it moves against the older worker intensive industry. Due to this price movement the members of the preceding and the following generations will be better-off. The members of the baby boom generation (G_t) will choose a higher level of schooling while the following generation (G_{t+1}) will choose a lower level. These results are given in columns 2 and 3 of Table 4.1.

One period perfect foresightedness: In this case the equilibrium can be determined by combining the individual's optimization under perfect foresightedness (shown by the vertical curves in Figure 4.4) with feasible equilibrium P-π combinations of the economy (Figure 4.2). This is shown in Figure 4.6. We can see that the equilibrium level of P is determined solely by individuals' optimization, while given that level of P the equilibrium relative price is determined by the feasible P-π locus. With TS and ABC the equilibrium at B shows the steady state case.

In period t-1 the younger generation (G_{t-1}) under perfect foresightedness will expect an increase in the next period relative price (π_1) and the individual's optimization curve will move from TS to VW. Since the size of the labour force is at the steady state level there will be no shift in the ABC locus and the new equilibrium will be at point X. Compared to the steady state level (point

TABLE : 4.1 EFFECTS OF A ONE TIME INCREASE IN THE SIZE OF THE LABOUR FORCE (MYOPIC EXPECTATIONS)*

PERIOD	EFFECT ON PRICE (π)	CHANGE IN SCHOOLING(P)	CHANGE IN EARNINGS[1]	EARNING[2] INEQUALITY
t-1	--	--	--	--
t	↑	↑	Y_Y : ↓ Y_O : ↑	↑
t+1	↓	↓	Y_Y : ↑ Y_O : ?	?
t+2	--	--	Y_Y : - Y_O : ↓	↓
t+3	--	--	--	--

* Changes are relative to the steady state level.
(1) Y_Y: Real earning of young workers,
    Y_O: Real earning of older workers.
(2) Inequality of earning between young and old workers.

TABLE : 4.2 EFFECTS OF A ONE TIME INCREASE IN THE SIZE OF THE LABOUR FORCE (PERFECT FORESIGHTEDNESS)

PERIOD	EFFECT ON PRICE (π)	CHANGE IN SCHOOLING (P)	CHANGE IN EARNINGS	EARNING INEQUALITY
t-2	--	--	--	--
t-1	↓	↑	Y_Y : ↓ Y_O : ↓	?
t	↑	↓	Y_Y : ↑ Y_O : ↑	?
t+1	↓	--	Y_Y : ↑ Y_O : ↓	↓
t+2	--	--	--	--

B) the level of P is increased but π is decreased. In period t the younger generation (G_t) will expect a decrease in π_1 in the next period and their optimization curve will move to QR. In this period as we have a larger cohort of younger workers the feasible equilibrium combinations will move from ABC to AYE and we will have a new equilibrium at Y with higher level of π but a lower level of P. From period t+1 individual's optimization will be back at the steady state case. But in period t+1 the larger cohort of older workers will shift the ABC locus to AZC and we will have equilibrium at Z with a lower level of π but the steady state level of P. Columns 2 and 3 of Table 4.2 show these results.

Effects on Observed Earnings and Inequality in Earnings

The observed level of real earnings of a representative worker in different groups will be affected by the change in relative price as well as the level of schooling. A higher level of P chosen by any generation will imply lower earnings in their younger period but a higher income level in the older period. Since all the younger workers are employed in α a change in π will not affect their earnings. But the real earnings of an older worker are positively related with the level of π. Thus the change in earnings of an older worker with a change in the age structure of the labour force can be derived from equation (4.2) as

(4.10) $dW^1/dN = (\alpha_1+\beta_1\pi)(dP^Y/dN) + (\beta_o+\beta_1 P)(d\pi^o/dN)$

Where P^Y is the level of schooling chosen by them in their younger period, π^o is the relative price in their older period, and N is the cohort size. In any period if dP^Y/dN and $d\pi^o/dN$ are both positive (negative), the older workers of that period will earn more (less) than that in the steady state case. If dP^Y/dN is positive but $d\pi^o/dN$ is negative, earnings of the older workers will be higher than the steady state level if, in absolute value,

$$(\alpha_1+\beta_1\pi)(dP^Y/dN) > (\beta_o+\beta_1 P)(d\pi^o/dN)$$

Alternatively this condition can be written as,

(4.11) $d\pi^o/dP^Y < (\alpha_1+\beta_1\pi)/(\beta_o+\beta_1 P)$

The value of $d\pi^o/dP^Y$ is determined by the shape of the CIC's, and inversely related with the elasticity of substitution between α and β in consumption. $(\alpha_1+\beta_1\pi)$ is the marginal allocative gain from schooling and $(\beta_o+\beta_1 P)$ is the expected output of a 'b' labeled worker in industry β. Thus with the changing age composition of the labour force, the older workers will earn more if : the elasticity of substitution in consumption is large, the marginal allocative gains from schooling is large and/or the productivity of a 'b' labeled worker in industry β is small.

If dP^Y/dN is negative but $d\pi^O/dN$ is positive, the older workers' earnings will increase if

(4.12) $$d\pi^O/dP^Y > (\alpha_1+\beta_1\pi)/(\beta_O+\beta_1 P)$$

Myopic expectations: Up to period t-1 the real earnings of the workers will be at the steady state level. In period t the young workers (the baby boom generation G_t) will earn a lower income as they will spend more time at school. The relative price in this period is changed in favour of the commodity produced by the older worker intensive industry, which will increase the real income of the older workers. Thus we will observe an increase in the income inequality between young and older workers during this period. Alternatively we could say that there will be a steepening of the age profile of observed earnings. I have referred to such steepening as an increase in 'income inequality' throughout this thesis.

In period t+1 the younger workers (G_{t+1}) will spend less time at school which will imply higher earnings for them in that period. The older workers in this period ('baby boom' generation) will face a depressed real wage due to a decrease in π but their higher level of schooling (chosen in the previous period) will have a positive effect on their income. So their observed earnings, and as such income inequality in this period, may move in either direction. If condition (4.11) is not satisfied the real earnings of the older workers will decrease and we will observe a decrease in the earnings inequality compared to both the previous period and the steady state level.

In the next period (t+2), older workers will earn a lower than steady state level of earnings due to a lower level of schooling chosen by them in the previous period but the younger workers' situation will be back at the steady state level. This will result in a lower income inequality.

These effects are summarized in columns 4 and 5 of Table 4.1. The age earnings profile corresponding to these earning patterns are shown in Figure 4.7.

One period perfect foresightedness: Columns 3 and 4 of Table 4.2 summarize the effects under perfect foresight case. Young workers in period t-1 will earn less than the steady state level as they will spend more time at school. As π is decreased in this period real income of the older workers will also be decreased. The inequality of income between younger and older workers may move in either direction.

In period t older workers' earnings will be increased due to the rise in both π and the level of schooling chosen by them when they were young. In this period the young workers' schooling level will be decreased and this will increase their earnings and income inequality may move in either direction.

In period (t+1) the level of schooling chosen by the younger workers and as such their earnings will be at the steady state level. The relative price movement as well as the schooling level will decrease the older worker's earning in this period. Thus the inequality of income in this period will be increased. From periods t+2 we will be back at the steady state equilibrium[2].

Predictions of Our Model and the Stylized Facts

The observed effects of the labour market entry of the baby boom generation are the following. The larger cohort workers faced a depressed market wage when young, and this wage depressing effect was more pronounced for the workers with higher levels of schooling. At the same time there was a significant increase in the average level of schooling chosen by individuals. Also there was an increase in the earnings inequality between age groups observed during that period. Looking just at the most recent studies, for the Canadian economy Dooley (1986) find faster earnings growth for the larger cohort workers while for the U.S. Berger (1985) estimated slower growth.

The predictions of our model depend on expectations formation. Looking at the above facts, we can say that the predictions of our model under myopic expectations are more closely related to what we observe. Using our analysis under myopic expectations, we can give the following explanation for the stylized facts.

When a larger cohort of worker enter the labour force the demand conditions and the relative prices move against the younger worker intensive industry and this leads to decreased real earnings for them. This also results in a windfall increase in the allocative gains from schooling which makes the older workers of that period better-off. These changes result in an increased schooling level for the baby boom generation (as cost is decreased and expected benefit is increased) and increased earnings inequality. So the effects through the changes in the demand conditions reinforce the effects through changes in the supply conditions.

But as the baby boom generation gets older the effects through the demand side and the supply side work in opposite directions. As shown in the Welch model (1979) the supply conditions will result in faster earning growth. In our model on the demand side the relative price will move against the older worker intensive industry and there will be less allocative gain from schooling for the members of the larger cohort. The net effect may go in either direction. This can explain the faster growth rate observed in Canada (Dooley, 1986) but slower growth rate for the U.S. (Berger, 1985).

On the other hand if one period perfect foresightedness is assumed, our predictions are that the baby boom generation will choose a lower level of schooling while that chosen by the preceding generation will be higher. But the observations of the last two decades shows that the level of schooling chosen by the baby boom generation is higher. So if perfect foresightedness is assumed then the changes in the 70's are not a result of labour market entry of the baby boom generation rather result from something else.

V. AN INCREASE IN THE NUMBER OF OLDER WORKERS

In this section we will examine the effects of a one period (say period t) increase in the cohort size of the older/skilled workers. One possible cause for this type of change in the composition of the labour force may be a liberal immigration policy, where a large number of skilled workers are allowed to immigrate.

With such a change the shift in the grand PPC will be the same as that shown in Figure 4.3. Corresponding to this the feasible equilibrium combinations of P and π will shift downward, in terms of Figure 4.2 from ABC to AFC. Under myopic expectations (in this and the next sections we will study only this case) there will be no change in the individuals' optimization problem. In terms of Figure 4.5 the equilibrium point will move from the steady state level B to F. Thus when we have a larger cohort of older workers the relative price will move against the older worker intensive industry and the level of schooling chosen by the younger generation will be decreased. If this is a one time change, in the next period level of P and π will return to the steady state equilibrium.

The net effect on periodic earning of different group of workers will be as follows. Up to period t-1 there will be no change as neither π nor schooling level is changing. In period t young worker's real earning will be greater than the steady state level as they will spend less time in school and more time working. In this period older worker's real income will decrease due to the fall in π. These changes will result in a reduced inequality of earnings between the younger and the older workers in period t. In the next period (t+1) real income of the younger workers will be back at the steady state level, but older worker's

TABLE : 4.3 EFFECTS OF A ONE TIME INCREASE IN THE COHORT SIZE OF THE OLDER WORKERS

PERIOD	EFFECT ON PRICE (π)	CHANGE IN SCHOOLING (P)	CHANGE IN EARNINGS	EARNING INEQUALITY
t-1	--	--	--	--
t	↓	↓	Y_y : ↑ Y_o : ↓	↓
t+1	--	--	Y_y : - Y_o : ↓	↓
t+2	--	--	--	--

TABLE : 4.4 EFFECTS OF A ONE TIME INCREASE IN THE COHORT SIZE OF THE YOUNGER WORKERS

PERIOD	EFFECT ON PRICE (π)	CHANGE IN SCHOOLING(P)	CHANGE IN EARNINGS	EARNING INEQUALITY
t-1	--	--	--	--
t	↑	↑	Y_y : ↓ Y_o : ↑	↑
t+1	--	--	Y_y : - Y_o : ↑	↑
t+2	--	--	--	--

earning will be lower than the steady state level because they had spent less time at school while they were young. So the inequality of income between the young and the older workers will be smaller than the steady state level, but it will be greater than the previous period (t), since in period t decreased π was also contributing towards a lower inequality[3]. All these effects are summarized in Table 4.3.

If we have a permanent increase in the number of older workers in that case we will move from one steady state equilibrium to another. The situation shown in period t, a lower level of P and π and reduced inequality of income will characterize the new equilibrium. Thus we will move to a new steady state equilibrium with lower level of schooling, lower relative price of β and reduced income inequality.

To summarize the effects we can say that the older generation in the period when the cohort size of the skilled workers is increased will be worse-off but younger group in that period will be better-off. The equilibrium level of schooling will be decreased and this will result in a reduced inequality in income.[4]

VI. AN INCREASE IN THE NUMBER OF YOUNGER WORKERS

An increased participation rate by the secondary workers in the economy also changes the age composition of the labour force. This increases the size of the younger work force but the number of older workers remains the same. One example may be increased labour force participation by married women, who on average have much less work experience and OJT, than men of the same age, and have therefore invested much less in information and may be thought of as similar to young workers in the quality of their person-specific information. This change may be of temporary or permanent nature.

If this change takes place only in period t, then the grand PPC will move from ABCD to point EFGH in terms of Figure 4.1. For any given level of P the equilibrium price level (π) will be increased and in terms of Figure 4.5 the feasible equilibrium combinations will move from ABC to ADE. Under myopic expectations there will be no change in the individual's optimization decision and the equilibrium point will move from B to D. The equilibrium level of π and P will be increased. If this is only a one period change then from the next period the equilibrium will be point B.

In period t the young worker's observed earnings will be lower than the steady state level due to their higher level of schooling chosen by them. The increase in π will increase the observed earnings of the older workers of this period. Thus the inequality of income between the two groups will be increased. In the next period younger worker's schooling level and income will move towards the steady state level but older workers' earnings will be higher as they had higher schooling level in their younger period. This will increase the inequality of income. These are shown in Table 4.4.

If this change was a permanent one then we will move from the steady state equilibrium given up to time t-1 to a new one given by that in time t in Table 4.5. In that case at the new steady state equilibrium we will have higher equilibrium level of π and schooling level (P) and also greater inequality in income.

Thus the effects of increased female labour force participation are identical to the cohort size effect when the baby boom generation is in its younger period. The changes observed in the labour market in the last two decades may be a result of both the labour market entry of the baby boom generation and increased female participation. The observations in the late 1980's and 1990's (when the baby boom generation will become older) will show which of these two effects was more important. If no further changes in the return from schooling are observed it could be argued that we are at a new steady state equilibrium and the female labour force participation was more important. On the other hand if we see a decrease in the return from schooling that could be taken to imply that the baby boom generation was mainly responsible for the changes in the 70's.

In summary we can say that when the cohort size of the unskilled workers is increased the members of that group will be worse-off but older group of workers of that period will be better-off due to the relative price change. The equilibrium level of schooling will be increased and this will result in a increased inequality in income.

VII. SUMMARY

In this chapter we have shown that any change in the age structure of the labour force leads to a change in the relative output price. This changes the allocative benefits from, and equilibrium level of schooling. So for a complete picture of how cohort size affects level of schooling we should also add the effects through the structure of labour demand.

When the number of younger workers increases, the output of the younger worker intensive industry will be increased and this will lead to a relative price movement in favour of the older worker intensive industry. Thus the allocative gains from schooling for the current generation of old will be increased and under myopic expectations the young will choose a higher level of investment in information. This will also increase the inequality in earnings between generations. An increase in the number of older workers has the opposite effect.

NOTES

[1]In the presence of a domestic capital market the interest rate will also be affected by the change in the age structure of the labour market. Since our objective is to study human capital investment, we are going to assume that there is an external capital market with a fixed interest rate r.

[2]Throughout this chapter it is assumed that the second period earnings are insured at the expected level. If this assumption is relaxed there will be earnings inequality among older workers also. The level of inequality will be positively related with the level of schooling.

[3]Emigration on the other hand will change the age composition of the labour force towards less skilled (older) workers. In terms of Figure 4.3 the grand PPC will shift from EFGH to ABCD, and this will result in a relative price movement against the younger worker intensive industry. The equilibrium level of P and π will increase.

[4]This may help to explain the fact that the level of schooling chosen by native born Canadians are lower than their counterparts in the U.S. As shown in Table A-2 in appendix 6, immigration to Canada resulted in a change in the composition of the labour force towards more older (skilled) workers. In terms of our model, the result of this is a lower return from schooling. This in turn resulted in a lower schooling level chosen by native born Canadians. But in case of the U.S. the labour force structure was changed in the opposite direction by immigration. This may have resulted in a higher level of schooling chosen by native born Americans.

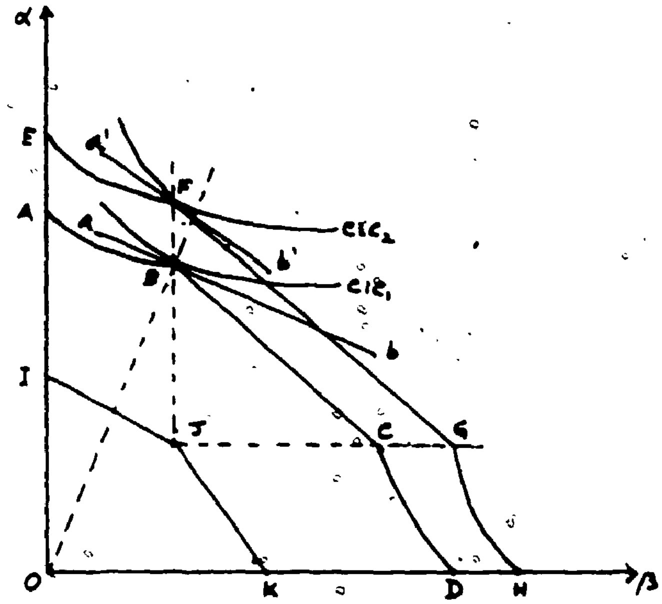


Figure 4.1 : The Grand PPC with Larger Cohort of Younger Workers

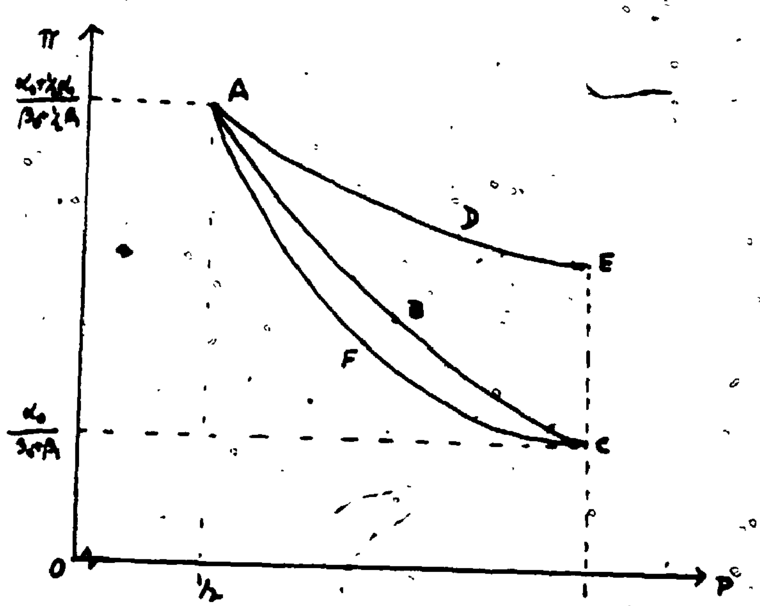


Figure 4.2 : The Feasible Equilibrium Combinations of P and π.

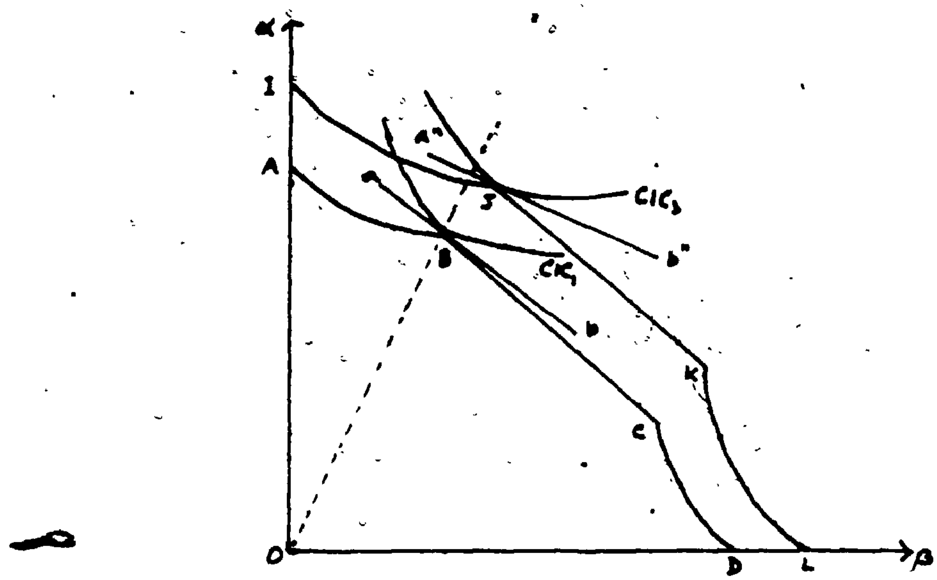


Figure 4.3 : The Grand PPC with Larger Cohort of Older Workers

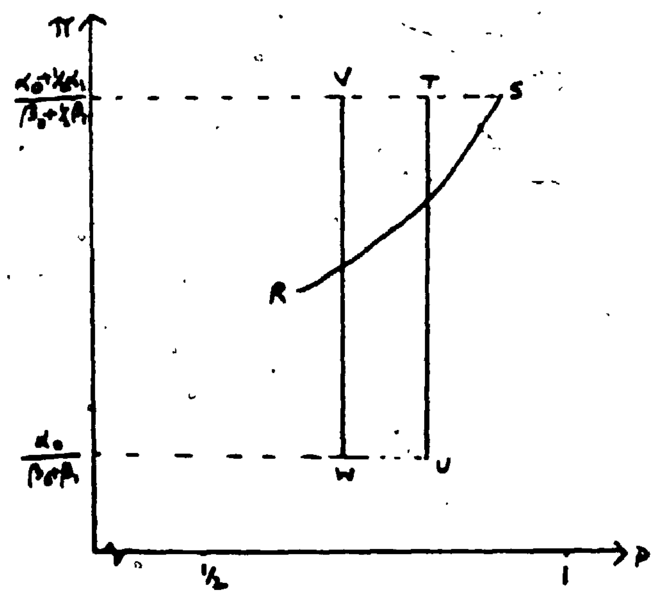


Figure 4.4 : Individual's Optimum Choice of P given π

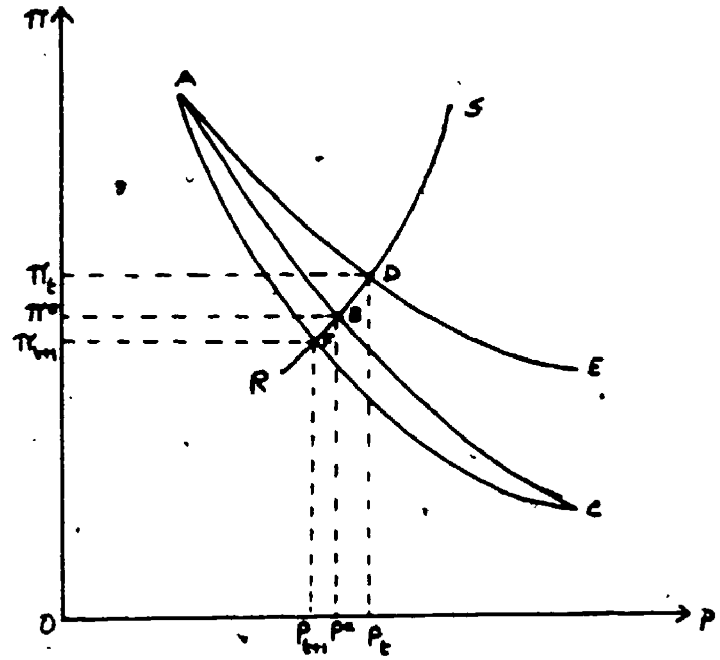


Figure 4.5 : Equilibrium Relative Price and Schooling (Myopic Expectations)

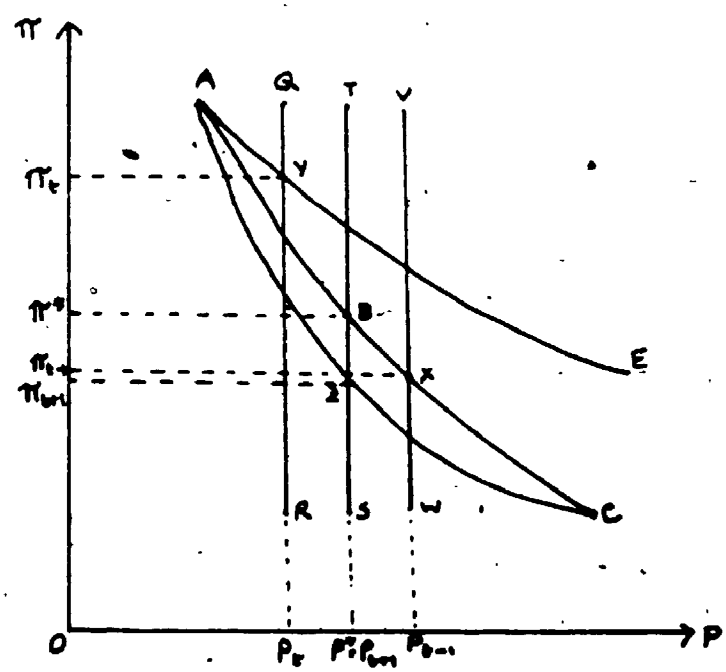


Figure 4.6 : Equilibrium Relative Price and Schooling (Perfect Foresightedness)

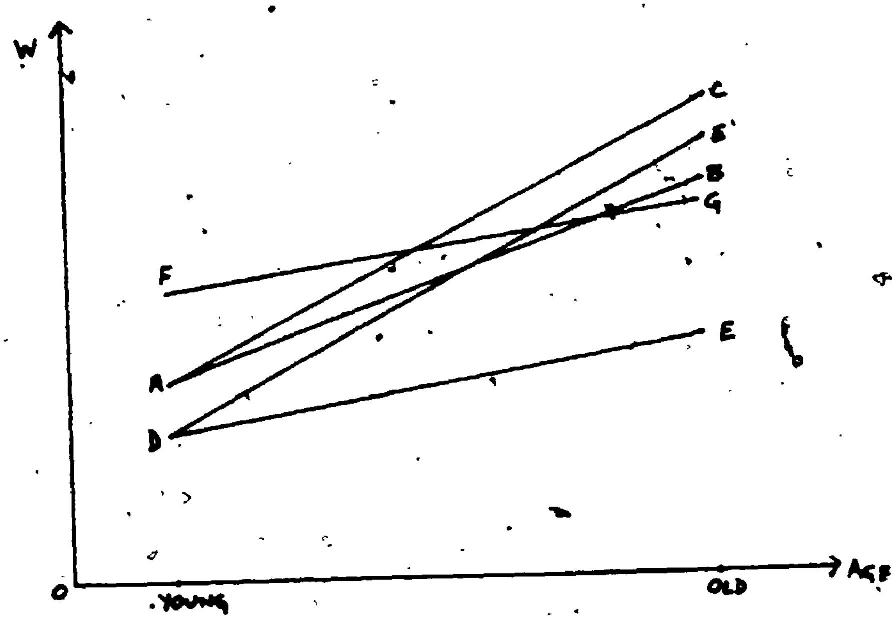


Figure 4.7 : Illustrative Longitudinal Age Earning Profiles: With Baby Boom Effect

AB : The Steady State
DE or DE' : The Baby Boom Generation (G_t)
AC : G_{t-1}
FG : G_{t+1}

CHAPTER 5

EFFECTS OF A SOCIAL PENSION PLAN

As a result of a large projected deficit in the Canada Pension Fund for the early twenty-first century[1], there has been a renewed debate regarding the future of the program (Burbidge 1982, OEC Position Paper 1983, Royal Commission Report 1985, Courchene 1987). Similar debate is also taking place in the U.S. regarding the social security program existing there. In order to evaluate different policy alternatives it is essential to have a clear understanding of the economic effects of such a program.

There is a vast literature on the economic effects of a social pension plan. Much attention has been devoted to the effects on saving, capital formation and labour supply (Samuelson 1958, Aaron 1966, Feldstein 1974, 1977, Barro 1974, Boskin 1977, Quinn 1977, Burkhauser and Turner 1978, Fields and Mitchell 1984). A general survey of the literature is given by Thompson (1983). Barro (1974) has argued that if intergenerational transfers in the form of children's education are operative, a social pension plan would have no effect on human capital investment. Two other papers (Drazen 1978, Black 1987) addressed the effects of the program on the amount of human capital investment. In these studies it is assumed that the return from schooling is not affected by a social pension plan. The one by Black examined the incentives created through intragenerational transfer through a social pension plan. He showed that such a program increases human capital investment by lower income groups while decreasing that of made by higher income groups. On the other hand Drazen examined the incentives created through intergenerational transfers due to such a program. In his study Drazen looked at a model where individuals don't choose their own human capital investment rather that of their children (i.e. the next generation). Utility maximizing division of one's bequest between human capital investment and capital transfer determines the level of schooling for the next generation. Drazen showed that the presence of a social pension plan may affect the level of human capital investment. His model examines only the productivity augmenting effects of schooling. Also it is a partial equilibrium model as the returns from schooling don't depend on the level of schooling chosen in the economy.

One purpose of this chapter is to examine the effects of a social pension plan on the returns from schooling. This effect in turn will affect the equilibrium level of schooling. This implies that one appropriate approach to study the problem is a general equilibrium framework where the return from as well as the equilibrium level of schooling are simultaneously determined. Accordingly, sections III and IV of this chapter incorporate a social pension plan in the general equilibrium model developed in chapter 2 and 3. The structure of such a pension plan is discussed in the next section. Since in our model earnings of all individuals in one generation are identical, there is no intragenerational transfer through the pension plan. Unlike Drazen's model, in our study each generation takes their own schooling decision and our model examines the effects in terms of the allocative benefits of school.

The recent debate over the proposed changes in the social pension plan is a result of the change in the age structure of the labour force due to the labour market entry of the baby boom generation. Thus the effects of the pension plan when the composition of the labour force is changing are also of great interest. These effects are analyzed in section V of this chapter.

In order to examine the problem we are going to assume that each individual lives for a length of three periods. The first two (0 and 1) have the same significance as in previous chapters. Now we will introduce a third period of life (period 2) when the individual is retired and consume past savings and/or pension benefits. This structure is very similar to that considered in the seminal paper by Samuelson (1958).

II. THE NATURE OF THE SOCIAL PENSION PLAN

We will study a social pension plan where the government collects a social pension tax from those who are working and pays a benefit to the retired. The tax and the benefit rate can be determined in a number of different alternative ways. In the usual case the benefit one receives depends on the contribution made while working. In our model it is assumed that storage of any commodity (α or β) is not possible, which implies that consumption equals production within a period. Further, the interest rate is assumed to be at the market clearing level, that is the amount of savings by individuals in any one period is equal to the total amount of dissavings in that period. Under these conditions, assuming there is no administrative cost of the social pension plan, the total amount of tax collected by the government in any period must be equal to the total amount of benefit paid out in that period. Thus the pension plan must be financed on a "current cost" or "pay-as-you-go" basis.

We will examine the relationship between the tax rate and the benefit rate in a steady state situation with an n% growth rate in the labour force. If in any period the number of retired individuals is $(\bar{A}+\bar{B})$, then the number of older workers will be $(\bar{A}+\bar{B})(1+n)$ and that of the younger workers $(\bar{A}+\bar{B})(1+n)^2$. Denote the social pension tax rate by $\tau(W^i)$ and the benefit rate $B(\sigma)$, where W^i is individual earnings in period i and σ is the contribution made to pension fund by one while working. In the case where the plan is financed on a 'pay-as-you-go' basis the following equality must hold

$$\tau(W^o)W^o(\bar{A}+\bar{B})(1+n)^2+\tau(W^1)W^1(\bar{A}+\bar{B})(1+n) = B(\sigma)(\bar{A}+\bar{B})$$

(5.1) $$\tau(W^o)W^o(1+n)^2 + \tau(W^1)W^1(1+n) = B(\sigma)$$

Given this the government will have to either set the tax schedule which will determine the benefit rate or can set the benefit rate which will help to determine the tax schedule. The tax schedule can be made proportional, progressive or regressive. The benefit is dependent on the amount of contribution but the benefit may change at different rates with contribution.

When a social pension scheme is introduced, for the economy as a whole there will be no change in the construction and the shape of the grand PPC. Thus the feasible steady-state combinations of schooling (P) and relative price (π) will be the same with or without the pension plan. This is shown by the curve ABC in Figure 5.1 (same as that in Figure 2.12).

III. INDIVIDUAL'S OPTIMIZATION IN A THREE PERIOD CASE

This section examines the individual worker's optimization problem when the basic information model is extended to a three period case, and there exists a social pension plan of the nature described in section II. The first two periods of life are the same as before; period 0 is when an individual is a younger worker and allocates total available time between working and going to school. In period 1 the individual becomes an older worker and spend all available time working. Now we introduce a third period of life (period 2) when the individual is retired and consume pasts savings and/or pension benefits. As before, in this case we assume that there is no inheritance or bequest and that there exists a perfect capital market where individuals can borrow or lend any amount at a fixed interest rate (r)[2]. Also individuals determine the level of schooling in their younger period of life assuming that the relative price (π_0), pension tax rate (τ) and benefit rate of that period will prevail over their life-time. Again we will examine the individual's optimum choice of schooling when the steady state equilibrium is at the kinked point of the PPC for the economy. Without the pension plan the individual's optimization problem is the same as that in two period case and the utility maximizing combinations of P and π are shown by the curve RS in Figure 5.1 (same as that in Figure 2.13). Next we will examine the individual's optimization in the presence of a pension plan.

One representative individual's expected marginal productivity (μ) in the younger period of life (working full time) is

$$\mu^0 = (\alpha_0 + \tfrac{1}{2}\alpha_1)$$

and for one older worker (in terms of α),

$$\mu^1(P) = \tfrac{1}{2}(\alpha_0 + \alpha_1 P + \beta_0 \pi + \beta_1 P\pi)$$

Of course $\mu^2(P) = 0$, since there is complete retirement in period 2. Earnings (before pension tax) of a representative individual, spending a fraction S(P) of time at school, can be written as,

$$W^0(P) = \mu^0(1-S(P)) = (\alpha_0 + \tfrac{1}{2}\alpha_1)(1-S(P))$$

$$W^1(P) = \mu^1(P) = \tfrac{1}{2}(\alpha_0 + \alpha_1 P + \beta_0 \pi + \beta_1 P\pi)$$

As before the return from schooling is directly related with the level of π.

Taking into account the taxes and benefits at the social pension plan, described in the previous section, an individual's optimization problem is to maximize his/her life-time income given by,

$$\overline{W} = \mu^o(1-S(P))(1-\tau(W^o)) + \mu^1(P)(1-\tau(W^1))/(1+r) + [\mu^o(1-S(P))\tau(W^o)(1+n)^2 + \mu^1\tau(W^1)(1+n)]/(1+r)^2$$

The first order condition for maximization is,

$$\delta\overline{W}/\delta P = -\mu^o S'(P)(1-\tau^o) + \mu^1{}_P(1-\tau^1)/(1+r) - [\mu^o S'(P)\tau^o(1+n)^2 - \mu^1{}_P\tau^1(1+n)]/(1+r)^2 = 0$$

where $\mu^1{}_P=\delta\mu^1/\delta P$, τ^i is the marginal tax rate in period i. The above condition can be rewritten as,

$$(5.2) \qquad S'(P) = \underbrace{\left[\frac{\mu^1{}_P}{\mu^o(1+r)}\right]}_{\Gamma} \underbrace{\left[\frac{1-\tau^1 + \tau^1\left(\frac{1+n}{1+r}\right)}{1-\tau^o + \tau^o\left(\frac{1+n}{1+r}\right)^2}\right]}_{\Sigma}$$

The optimization condition derived in the basic two period information model without social pension plan (equation 2.11) can be written as,

$$(5.3) \qquad S'(P) = [\mu^1{}_P/\mu^o(1+r)] = \Gamma$$

The relationship between P and π derived from this equation is shown by the curve RS in Figure 5.1.

Given $S''(P)>0$, comparing equations (5.2) and (5.3) one can see that in the presence of a social pension plan individual's optimally chosen schooling level will be higher (lower or equal) if Σ is > 1 (<1 or =1).

The value of Σ depends on the following four factors: the rate of return in the capital market (r), the rate of return from pension plan contributions (n), marginal tax rate on income in the two periods (τ^0 and τ^1). Since the current social pension tax rate is regressive (Black 1987), in the following discussions we will concentrate on proportional and regressive tax rate[3]. In order to examine the effects of r and n, initially we will assume that the tax rate is proportional ($\tau^0 = \tau^1$).

If r=n, we will have Σ=1 and there will be no effect on the individual's optimization. Contributions to the pension fund is a form of savings and it generates the same return as that in the capital market. So the only effect will be in terms of reallocation of savings between the pension fund and capital market.

r<n, will give Σ<1 and the level of schooling chosen by individuals will be smaller in the presence of a pension plan. The reason is that pension contributions in this case earn a higher rate of return than is available in the capital market. So individuals have an incentive to contribute more to the pension fund and to make the contribution earlier in life. This will result in a lower optimal level of schooling. Diagrammatically the curve showing individual's choice of P given π, will move to the left, say to QT in Figure 5.1.

$r>n$, will give $\Sigma>1$ and the individually chosen level of schooling will be higher in the presence of a pension plan. Since contributions to the pension fund grow at a slower rate, individuals will try to defer their contribution through higher income in the second period. This is possible if a higher level of schooling is chosen. In terms of Figure 5.1 the curve VW shows the P-π relationship.

If the tax rate is regressive ($\tau^{o}>\tau^{1}$), a higher level of schooling (increased second period earning but a decreased first period one), will decrease the total amount contributed to the pension fund. At the same time a higher level of schooling will defer their contribution and this will further reduce the present value of their contribution. Thus if $r<n$ ($r>n$) individuals have a incentive to decrease (increase) schooling level to increase (decrease) total life-time discounted contribution to the pension fund. This will result in a even greater shift in the curve showing P and π combinations chosen by individuals[4].

For a given level of π, a change in n or r will also affect the level of P chosen by individuals. From equation

(5.2) it can be shown that,

$$\delta\Sigma/\delta n < 0 \quad \text{and} \quad \delta\Sigma/\delta r > 0$$

Given $S''(P)>0$, an increase in n or a decrease in r will shift the locus of P-π combinations (chosen by individuals) to the left.

In Canada, the pension contributions are collected at a constant rate (3.6%) for certain range of income (in 1985 this range was $2,300 ($E_o$) to $23,400 ($E_1$) per year). For income below E_o and above E_1 the marginal contribution rate is zero. In terms of our model, a representative individual faces one of the following situations: Firstly, earnings in the two working periods of life are in the range E_o to E_1, in which case his/her situation will be identical to that under proportional tax rate. Secondly, income in the younger period is in the range E_o and E_1 while that in the older period is higher than E_1. This situation will be the same as the regressive tax rate case. A third theoretical possibility is that in the younger period earnings are below E_o. Since in Canada, E_o is extremely small(only $2,300 per year) we can consider this as a very unlikely case.

IV. EQUILIBRIUM LEVEL OF π AND P

In terms of Figure 5.1, the equilibrium without a pension plan is determined at the point B, with the level of schooling P^* and relative price π^*.

In the presence of a social pension plan (with proportional or regressive tax rate), the position of the steady state equilibrium will depend on the expected rate of growth of the pension fund. If it is expected to grow at a faster rate than r, the equilibrium will be at the intersection point of QT and ABC (point Y), corresponding to a lower P but higher π. Since the allocative benefit from schooling is directly related with π, in this case the introduction of a social pension plan will increase the returns from schooling but decrease the actual level of schooling.

On the other hand, if the pension fund grows at a slower rate, a proportional or regressive tax will have a positive effect on schooling, and the locus of individual's optimum combinations of P and π will move to VW. The equilibrium point will be at X, corresponding to a higher level of schooling but a lower return from schooling.

If the expected growth rate of the pension fund is the same as r, there will be no change in the equilibrium as neither of the two curves in Figure 5.1 will shift. This is similar to the result of Barro (1974), but we have shown here that it will hold in a case without bequest also if individuals take their own schooling decision.

V. ONE LARGER COHORT OF WORKERS PASSING THROUGH THE LABOUR FORCE

In this section we will examine the effects on the return from, and the level of schooling (in the presence of a social pension scheme) when a 'baby boom' generation (generation G_t) passes through the labour force. We will start with a steady state equilibrium and then increase the number of young workers in one period (period t) while the number of older workers and retired individuals are at the steady state level. In the next period (t+1) we will assume that the number of young workers and retired persons are at the steady state level while the number of older workers is increased compared to the steady state level. In period t+2 number of retired individuals is increased.

Change in the Feasible Combinations of P and π

When the age composition of the labour force changes the existence of a social pension plan will not make any difference in the production side for the economy as a whole. So the shift in the feasible steady state combinations of P and π will be the same as that without the pension plan derived in section II of chapter 4. When a baby boom generation enters the labour force as younger workers (period t) it will shift to the right, from ABC (steady-state case) to ADE in Figure 5.2. In their older period (t+1) it shifts to the left (to AFC). In t+2 it will return to the steady state level ABC (same as Figure 4.2).

Change in the Benefit and Tax Structure

With a change in the age composition of the labour force, in order to balance the financing of the social pension plan the government may adjust the tax rate, or adjust the benefit rate or use a combination of the two. During the periods t and t+1 when the baby boom generation is in the labour force (in period 0 and 1 of their life) the number of individuals working in the economy will be larger than the steady state level while the number of individuals collecting pension benefit will be at the steady state level. This will lead to a higher benefit rate, a lower tax rate or a combination of both.

In Chapter 4 we have seen that the baby boom generation will choose a higher than steady state level of schooling, which implies their working time and earning will be lower than the steady state level. Thus the tax rate will have to be adjusted upward while the baby boom generation is young. As discussed in the previous chapter, the return from schooling was observed to have increased in the early 1970's when the members of the baby boom generation were young. This implies that the second effect is smaller than the first one and the overall effect will be a decrease in the tax rate (or an increase in the benefit rate). This means the observed return from pension contribution (n) will be increased in period t.

The higher level of schooling chosen by the baby boom generation will lead to a higher than the steady state level of earnings in their older period (t+1). Also the younger generation of this time (G_{t+1}) will choose a lower level of schooling which will lead to a higher than steady state level of income. This will require a further decrease in the tax rate (or increase in the benefit rate) in period t+1. Thus the observed growth rate of the pension fund will be higher than that in t.

In the next time period when the baby boom generation is ~~retired,~~ the number of individuals receiving benefit will be greater but the number of workers in the labour force will be at the steady state level. This will require a larger amount of benefit payment, collected through taxes from a steady state level of work force. So the tax rate will have to be increased to finance the pension plan. Further the older workers in this period (G_{t+1}) will earn less than the steady state level due to their decreased schooling level. This will require a further increase in the tax rate, which implies an even lower rate of growth of the pension contribution.

Thus when the baby boom generation enter the labour force as young workers (period t) the expected rate of growth of the pension contribution will increase. In the next period (t+1) it will increase further. When they become retired the expected pension fund growth rate will decrease. The magnitude of these changes will depend on the magnitude of the change in the labour force and the change in the level of schooling chosen by G_t and G_{t+1}.

Change in Individual's Optimization

The steady state combinations of P a..d π (chosen by individuals) is shown by RS in Figure 5.2. As discussed in section III for any given level of π, an increase in the rate of return from pension plan contribution will lead to a lower level of schooling if the tax rate is regressive or proportional. Corresponding to the changes in the observed (expected) growth rate of the pension contributions the locus of P-π combinations in different periods are shown in Figure 5.2, QT in period t, UZ in t+1 and VW in period t+2.

Change in the Equilibrium Level of P and π

In terms of Figure 5.2 we can compare the effects of a change in the age structure of the labour force, with and without a pension plan. In both cases the feasible equilibrium combinations of P and π are the same, ADE in period t, AFC in t+1 and ABC (the steady state locus) in t+2. Without a pension plan the locus of P-π combinations chosen by individuals will be the same (RS) in different periods, while with a pension plan it will shift to QT in period t, UZ in t+1 and VW in period t+2. Given these movements the effects of a social pension plan on the return from, and equilibrium level of schooling will be as follows:

Period t: Without a pension plan we will have equilibrium at point D. But with a pension plan it will be at point G, corresponding to a lower level of schooling but a higher level of return from schooling.

Period t+1: Point F and H represents the equilibrium with and without a pension plan respectively. In this period a social pension plan will increase the return from schooling but the equilibrium level of schooling will be decreased.

Period t+2: In this period without a pension plan we will be back at the steady-state equilibrium at B. But with a pension plan our equilibrium will be at point J with a igher level of schooling with a lower return.

The existence of a social pension plan will increase the rate of return from better information but the actual level of schooling will decrease, when a larger cohort of

TABLE 5.1 : EFFECTS OF A SOCIAL PENSION PLAN ON THE RETURN FROM, AND LEVEL OF SCHOOLING **

	Return from Education	Level of Schooling
Pension fund Growth rate (n) greater than the interest rate (r)	↑	↓
n < r	↓	↑
n = r	---	---

** Changes are relative to the steady state level.

TABLE 5.2 : EFFECTS OF A SOCIAL PENSION PLAN WITH CHANGING AGE STRUCTURE OF THE LABOUR FORCE

Period	Expected n	Return from Education	Level of Schooling
t-1	---	---	---
t	↑	↑	↓
t+1	↑	↑	↓
t+2	↓	↓	↑
t+3	---	---	---

workers are employed in the labour force. Expecting a higher growth rate of the pension contributions they will invest more in the pension plan by reducing their schooling level. When we have a larger cohort of retired individuals due to opposite reasoning the level of schooling will decrease but it will generate a higher return.

The above results hold in both the extreme cases of tax, and the benefit schedule adjustments. But these two adjustments will have opposite effects on the welfare of different generations. If the benefit rate adjusts, it will increase in periods t and t+1, while the baby boom generation (G_t) is in the labour force, but will decrease while they are retired in period t+2. This will make the retired individuals in period t and t+1 (generations G_{t-1} and G_{t-2}) better-off but those retired in period t+2 (the baby boom generation) will be worse-off. On the other hand if the tax schedule adjust, the rate will decrease in periods t and t+1 but increase in t+2. This will make the working individuals in periods t and t+1 (generations G_{t-1}, G_t, and G_{t+1}) better-off but those in period t+2 (generations G_{t+1} and G_{t+2}) worse-off. The generation G_{t+1} will be better-off in their younger period but worse off in their older period. The baby boom generation will be better-off if the tax schedule adjusts but will be worse-off if the benefit schedule adjusts. In either case the preceding generation will benefit from the presence of the pension plan.

VI. SUMMARY

In this chapter we have examined the effects of a social pension plan on the return from, and the actual level of schooling. This effect will depend on the relative growth rate of funds in the pension plan and in the capital market. If the contributions to the pension fund grow at a slower rate then individuals will try to contribute less to the pension fund and invest more in the capital market. In the case of proportional or regressive tax rates a higher level of schooling will decrease the present value of the life-time contribution made by an individual. Thus the presence of a social pension plan will increase the level of investment in information if the tax rate is proportional or regressive and the growth rate of the pension fund is smaller than the return in the capital market.

When the members of a baby boom generation (G_t) are working the observed growth rate of the funds in the pension plan will be increased as more workers are contributing to the fund. When the members of that generation retire the observed pension fund growth rate will be decreased as the total benefit will be distributed among more individuals. If the pension tax rate is proportional or regressive, the pension plan will decrease the actual level of schooling chosen by that and the following generation (G_t and G_{t+1}) but will increase that chosen by G_{t+2}. If the tax schedule adjusts with the changing age composition of the labour force, the baby boom generation will be better-off but an adjustment in the benefit schedule will make them worse-off.

Notes

[1] Without any change in the tax schedule the projected deficit in the Canada Pension Plan would have been $5.6 billion in the year 2000 and $10.3 billion in 2003. (Source, "The Canada Pension Plan: Keeping It Financially Healthy",Department of Finance, Ottawa 1985). Begaining in 1987 the Canada Pension Plan tax rate is increased by .2% each year until 1991.

[2] In this three period case it is possible to have a domestic capital market with borrowing and lending between generations (Samuelson 1958). As discussed in chapter 2, government may also provide this service by issuing bond with real interest rate r. With changes in age structure of the population such capital markets would exhibit complex changes in interest rate over time. However in this chapter in order to avoid the coplexity of modeling the effects of interest rate changes, in addition to demographic changes, it is assumed that an external capital market exists.

3 Throughout this thesis the progressivity or regressivity of the pension tax schedule refers to the rate at the margin.

4 Appendix 7 examines the case under progressive tax schedule.

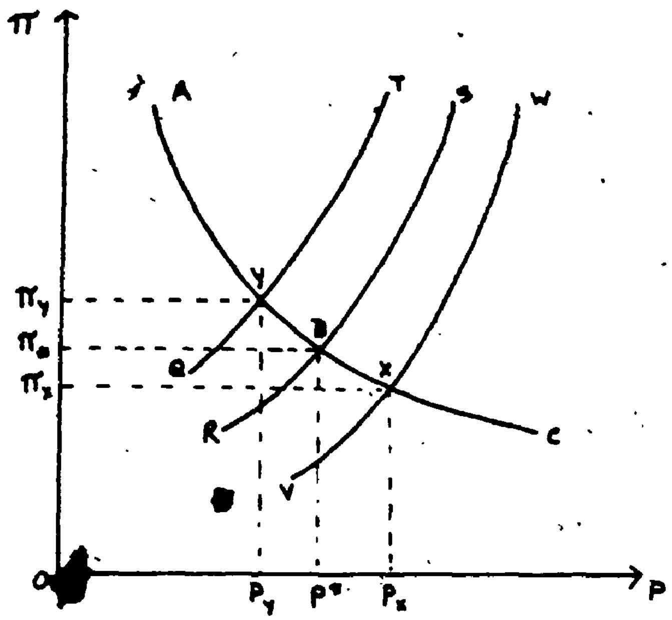


Figure 5.1 : Equilibrium With a Social Pension Plan

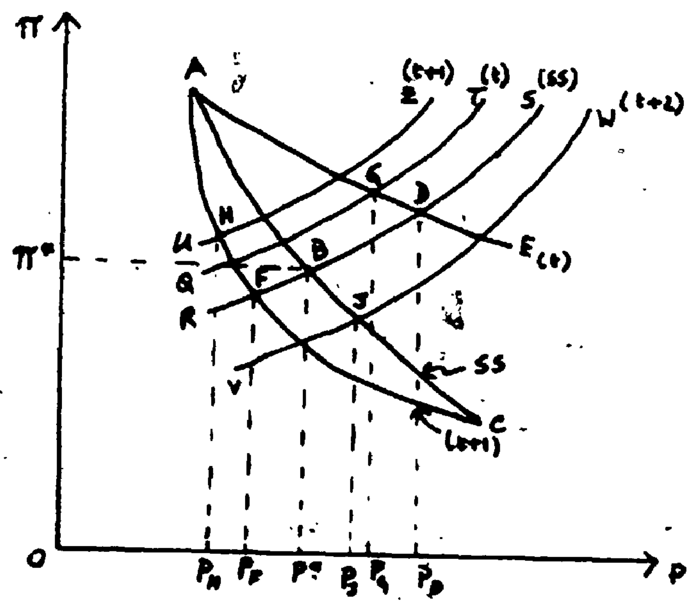


Figure 5.2 : Changing Age-Structure of the Labour Force and the Equilibrium With a Social Pension Plan

CHAPTER 6

SUMMARY AND CONCLUSIONS

In this thesis we have examined a job matching model of human capital investment in a general equilibrium framework. Specifically we have addressed the following two questions:

(a) In the context of a job matching model, what determines the real return from human capital investments?

(b) How is this return affected by a change in the age structure of the labour force and by the introduction of a social pension plan?

In order to examine these questions a simple general equilibrium model of imperfect person specific information in the labour market is developed. It has been shown that the real return from better firm-worker matching is determined by the demand structure. The more is the demand structure biased towards the output produced by the older worker intensive industry, the larger will be the gain from specialization through person specific information. It is also shown that the individually chosen investment in information will be in general smaller than the 'golden rule' steady state level. For the economy as a whole higher level of productivity gains from schooling will lead to a lower allocative gain. The intuition is as follows. Higher productivity gain implies that with more schooling, production in the older worker intensive industry will increase at a faster rate than that in the younger worker intensive industry. This will lead to a relative price movement against the older worker intensive industry which in turn means a smaller allocative gain.

This thesis shows that any change in the age structure of the labour force leads to a change in the return from investment in information through changes in the relative output price. When the number of younger workers increases, through changes in the demand conditions the job-worker matching gains from schooling increase, and the level of schooling and earnings inequality is also increased. An increase in the number of older workers has opposite effects. As a baby boom generation passes through the labour force, due to changes in the returns from the information augmenting component of education, they will experience slower earning growth than the previous generation. The effects of an increased female labour force participation are identical to the cohort size effect when the baby boom generation is in its younger period. More immigration of skilled older workers will decrease the equilibrium return from, and the level of schooling.

It is also found that the return from education through better matching may be affected by a social pension plan. This effect depends on the expected growth rate in the pension fund contributions relative to the return from investments in the capital market. In case of proportional and regressive tax rates, if the pension fund grows at a faster (slower) rate, the existence of such a plan will increase (decrease) the return from schooling but decrease (increase) the actual level of schooling. In view of the largely regressive contribution structure of social pension plans in Canada and the U.S., and current views about rates of return on private investment, the 'leading case' seems to be that where a social pension plan increases schooling and decreases the net return from schooling.

When a baby boom generation (G_t) passes through the labour force, the presence of a social pension plan (with proportional or regressive tax rate) will increase the return from schooling in that and the following period. But the actual level of schooling chosen by that and the following generation (G_t and G_{t+1}) will be smaller than the old steady state level. The effects in period t+2 and on Gt+2 will be the opposite. If the tax schedule adjusts with the changing age composition of the labour force, the benefit schedule remaining fixed; the baby boom generation will be better-off. However an adjustment in the benefit schedule, the tax schedule remaining unchanged, will make them worse-off.

As discussed above, the model developed in this thesis has a number of predictions about the labour market effects of different changes in the economy. Thus it is interesting to ask whether our model might help to explain some of the changes observed in the Canadian and the U.S. economies in recent years, and also what future changes may be predicted by the model. The observed changes are as follows: earnings of the young workers have declined compared to the older workers, and the younger workers are choosing a higher level of schooling compared to the previous generations. The findings on the earnings growth rate are not conclusive.

In terms of the general equilibrium model developed in this thesis, these could be the effects of the labour market entry of a larger group of younger workers. Thus one could argue that the recent changes in the labour market are the effect of labour market entry of the post World War II baby boom generation as young workers. At the same time our model also predicts that the effects of an increased female labour force participation or the introduction of a social pension plan (in the leading case) are the same. So the changes observed in the recent years may have been caused by the labour market entry of the baby boom generation, or increased female labour force participation, or social pension plans, or by some combination of all three factors. However the future changes will be different depending on the cause of the past changes. If the baby boom effects were stronger then in the future we should observe decreased relative earnings of the older workers and a lower level of schooling. On the other hand if the female participation and/or social pension plan were mainly responsible and they have stabilized, then the present situation represents a new steady state equilibrium and there will be no further changes in the future.

Our model also shows that the earnings of different cohorts of workers are affected by the level and composition of the immigrant population. Continued immigration of skilled workers in the future will lead to a lower return from schooling and the native born individuals will choose a lower level of schooling, than if there were no immigration.

APPENDICES

APPENDIX 1 : CRS Production Function

In this thesis it is assumed that the production functions are Leontief type fixed coefficient one. If we have a constant returns to scale production function, and certain conditions are satisfied, our analysis and the results will be qualitatively the same. The requirement is that the production function should be such that the test intensity corresponding to the straight line segment of the grand PPC (PG) does not allow us to operate on the contract curve in the factor box.

One situation where the above condition is satisfied is shown in Figure A.1. For the younger workers there will be no change in the construction and shape of the group PPC, since for CRS production functions the PPC corresponding to the diagonal will be a straight line. For different levels of P the efficient production points for the older workers are shown by the straight line segments $0^{\alpha}A$, $A0^{\beta}$ for P_1 and $0^{\alpha}B$, $B0^{\beta}$ for P^G. As the production point moves from 0^{α} to A or B, 'b' labeled workers are reallocated from industry α to β and given CRS production function the marginal rate of transformation will be constant and we will have a straight line segment of the

PPC. Similarly for the points on the straight line AO^{β} or BO^{β} of the factor box there will be another straight line segment of the PPC. Any level of P higher than P^G is inefficient and as such we can ignore those possibilities. Thus for the relevant range of schooling level (between $\frac{1}{2}$ and P^G) our analysis on the production possibilities with a CRS production function is the same as that in case of a fixed coefficient production function.

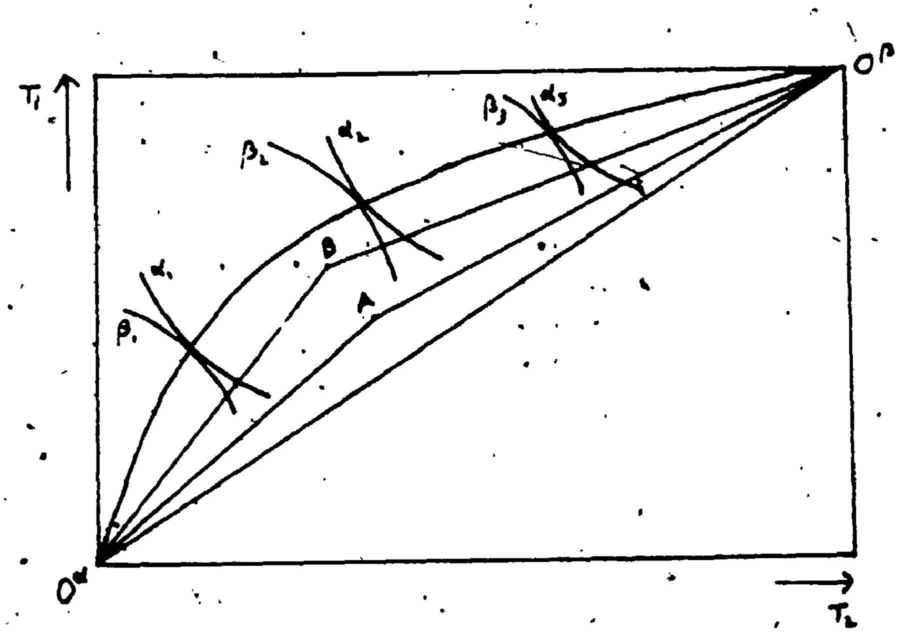


Figure A.1 : Feasible Production Points and CRS Production Function

APPENDIX 2 : Imperfect Firm-Worker Matching

In the model developed in this thesis it is assumed that the production technology and the abilities of the individual workers are such that 'A' type workers are perfectly matched with industry α and 'B' with β, that is,

$$(t^A{}_1/t^A{}_2)=(a_1/a_2) \text{ and } (t^B{}_1/t^B{}_2)=(b_1/b_2)$$

These are shown in Figure A.2 by the ray $0^{\alpha}A$ and $0^{\beta}A$ respectively.

If this assumption of perfect matching is relaxed there will be no qualitative change in any of the results derived in this thesis. One possibility is the following,

$$(t^A{}_1/t^A{}_2)'<(a_1/a_2) \text{ and } (t^B{}_1/t^B{}_2)'<(b_1/b_2)$$

This implies that given the natural abilities of the individuals the production technology is such that some tasks are always wasted. In this case all our analysis will be the same, that is the older workers PPC will shift outward with higher level of P and it will have two straight line segments. The only difference will be that in this case even with perfect information we cannot produce on the contract curve.

On the other hand, if we have

$$(t^A{}_1/t^A{}_2)''>(a_1/a_2) \text{ and } (t^B{}_1/t^B{}_2)''>(b_1/b_2)$$

This implies that, 'a' labeled workers can be perfectly matched with α and 'b' with β, with a level of P less than 1 (say P_1). Thus as P increases from $\frac{1}{2}$ to P_1 the older workers PPC will move upward and all our analysis will be the same, as in the perfect matching case. As P increase further (higher than P_1) there will be no upward shift in the older workers PPC. Hence there will be no benefit from schooling and that possibility can be ignored.

Thus the construction and the shape of the PPC's does not depend on the assumption of perfect firm worker matching.

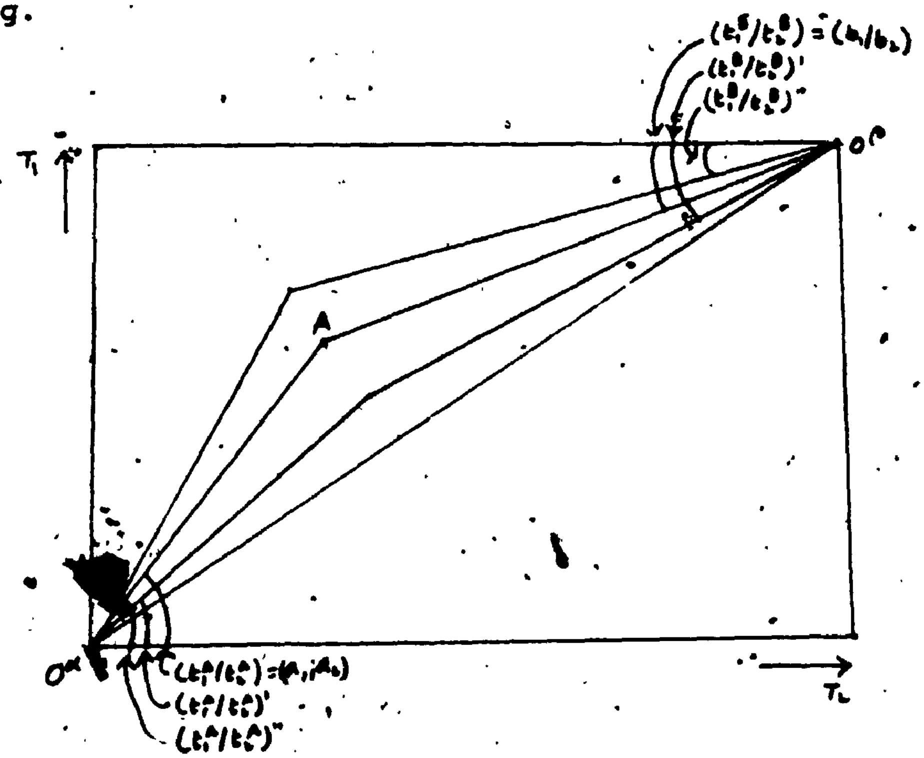


Figure A.2 : Factor Box and Imperfect Firm-Worker Matching

APPENDIX 3 : Uncertain Second Period Earnings

When the assumption that the individuals' second period earnings are assured at its expected level is relaxed, there will be uncertainty regarding second period income. This will not make any difference in the analysis of the production side of the economy but the individuals optimization problem will be different. In this model the level of uncertainty is positively related with the level of schooling. As shown in Levhari and Weiss (1974), if we assume decreasing absolute risk aversion this will result in a lower level of schooling. In terms of our model this will shift the locus of the P and π combinations (chosen by individuals) to the left, from CD to EF in Figure A.3. The equilibrium point will move from point G to point H, and the equilibrium level of π will increase but that of P will decrease. Thus we will have a higher return from schooling and a lower level of uncertainty.

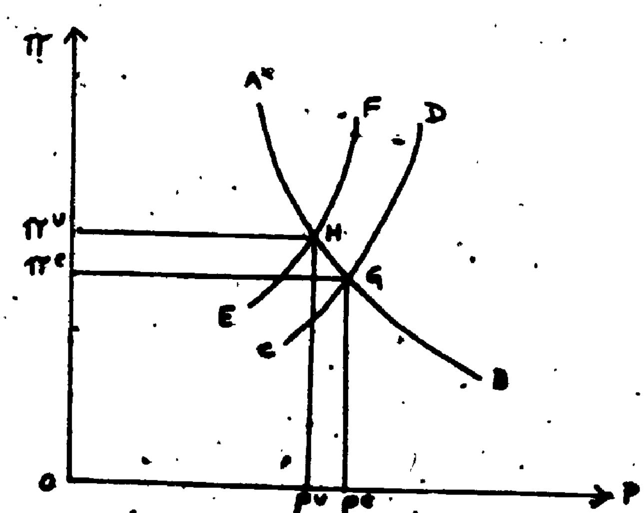


Figure A.3 : Equilibrium with Uncertainty

APPENDIX 4 : Equilibrium P-π Combinations

In this appendix we are going to examine the relationship between P and π when the golden rule production plan is at the kinked point and the steady state price (slope of CIC at A") is equal to the slope of the convex part of the grand PPC. In Figure A.4 it is shown at the kinked point A" with the level of schooling at $P"=P^G$, and equilibrium relative price (π^*) is given by the slope of CIC at point A". For different levels of P the PPCs are shown by αABβ for $P=\frac{1}{2}$, α'A'B'β' for P', α"A"B"b" for P". With higher level of P the kinked point production plan will move along AA'A"C... and we will have

$$\pi = (\alpha_0+\alpha_1(1-P))/(\beta_0+\beta_1 P)$$

$$\delta\pi/\delta P= -(\alpha_0\beta_1+\alpha_1\beta_0+\alpha_1\beta_1)/(\beta_0+\beta_1 P)^2 < 0$$

$$\delta^2\pi/\delta P^2=(\alpha_0\beta_1+\alpha_1\beta_0+\alpha_1\beta_1)\beta_1/(\beta_0+\beta_1 P)^3 > 0$$

ADC curve in Figure A.5 shows this relationship.

Another possibility is that as P increases from $\frac{1}{2}$ up to a certain level the production point stays on the straight line segment and then move along the kinked points (path BA'A"C). Corresponding to this the relationship between P and π is shown by AGC.

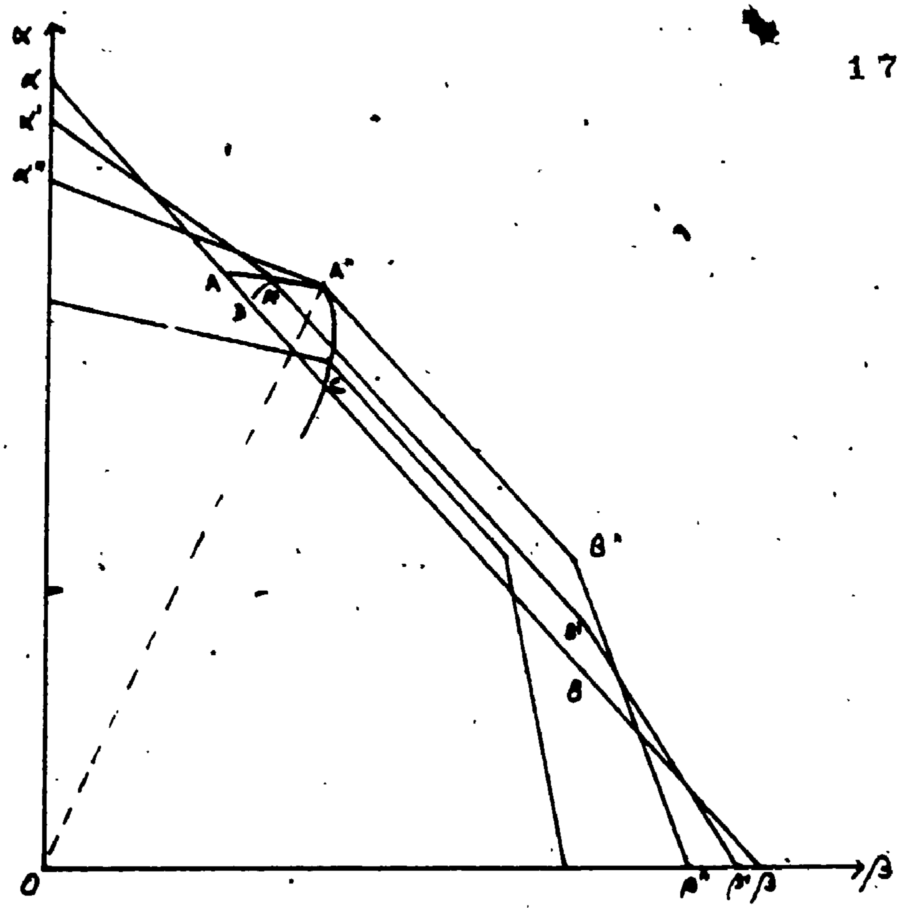


Figure A.4 : Production Point for Different Steady State P

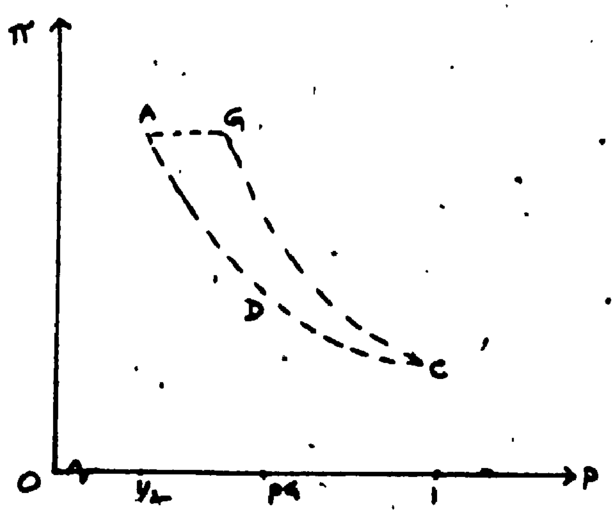


Figure A.5 : Steady-State Equilibrium Combinations of P and π

APPENDIX 5 : Comparative Static Results

Individual's utility maximizing P (derived in chapter 2.VI) is given by,

$$S'(P) - \left[\frac{1}{1+r}\right]\left[\frac{\frac{1}{2}(\alpha_1+\pi\beta_1)}{(\alpha_0+\frac{1}{2}\alpha_1)}\right] = 0$$

$$\Phi(P, \pi, \beta_1, \alpha_0, \alpha_1, r) = 0$$

We can derive the following partial derivative of this implicit function,

$$\frac{\delta\Phi}{\delta P} = S''(P) > 0$$

$$\frac{\delta\Phi}{\delta\pi} = -\left[\frac{1}{1+r}\right]\left[\frac{\frac{1}{2}\beta_1}{(\alpha_0+\frac{1}{2}\alpha_1)}\right] < 0$$

$$\frac{\delta\Phi}{\delta\beta_1} = -\left[\frac{1}{1+r}\right]\left[\frac{\frac{1}{2}\pi}{(\alpha_0+\frac{1}{2}\alpha_1)}\right] < 0$$

$$\frac{\delta\Phi}{\delta\alpha_0} = \left[\frac{1}{1+r}\right]\left[\frac{\frac{1}{2}(\alpha_1+\pi\beta_1)}{(\alpha_0+\frac{1}{2}\alpha_1)^2}\right] > 0$$

$$\frac{\delta\Phi}{\delta\alpha_1} = -\left[\frac{1}{1+r}\right]\left[\frac{\alpha_0+\frac{1}{2}\pi\beta_1}{(\alpha_0+\frac{1}{2}\alpha_1)^2}\right] < 0 \qquad \text{if } 2\alpha_0>\pi\beta_1$$

$$\frac{\delta\Phi}{\delta r} = \left[\frac{1}{(1+r)^2}\right]\left[\frac{\frac{1}{2}(\alpha_1+\pi\beta_1)}{(\alpha_0+\frac{1}{2}\alpha_1)}\right] > 0$$

Using implicit function theorem we can determine the following signs,

$$\frac{dP}{d\pi} = -\left[\frac{\delta\Phi}{\delta\pi}\right]/\left[\frac{\delta\Phi}{\delta P}\right] > 0$$

$$\frac{dP}{d\beta_1} = -\left[\frac{\delta\Phi}{\delta\beta_1}\right]/\left[\frac{\delta\Phi}{\delta P}\right] > 0$$

$$\frac{dP}{d\alpha_0} = -\left[\frac{\delta\Phi}{\delta\alpha_0}\right]/\left[\frac{\delta\Phi}{\delta P}\right] < 0$$

$$\frac{dP}{d\alpha_1} = -\left[\frac{\delta\Phi}{\delta\alpha_1}\right]/\left[\frac{\delta\Phi}{\delta P}\right] > 0 \qquad \text{if } 2\alpha_0>\pi\beta_1$$

$$\frac{dP}{dr} = -\left[\frac{\delta\Phi}{\delta r}\right]/\left[\frac{\delta\Phi}{\delta P}\right] < 0$$

APPENDIX 6: THE AGE AND SEX COMPOSITION OF THE LABOUR FORCE

I. The Baby Boom Effect

Table A-1 shows percentage of the labour force in the age group 25 to 34, since 1971 for Canada. This percentage was dramatically increased in the 1970's and early 1980's. In the recent years it is more or less steady at around 29%. This change is a result of the labour market entry of the post world war II baby boom generation. Beginning in the 1970 as the members of this generation start to reach

TABLE A-1 : PERCENTAGE OF THE LABOUR FORCE IN THE AGE GROUP 25 TO 34, CANADA 1971-1987

1971	22.0%	1980	27.5%
1972	23.3	1981	28.1
1973	23.7	1982	28.2
1974	24.2	1983	28.6
1975	25.0	1984	28.5
1976	26.4	1985	29.0
1977	26.6	1986	29.2
1978	27.4	1987	29.2
1979	27.3		

Source : Statistics Canada, The Labour Force (71-001) Various issues.

age 25 the composition of the labour force started to change towards more younger workers. Beginning in the middle of 1980's the rate of increase diminished as the baby boom generation starts to leave this particular age group. As the members of the baby boom generation get older, the expected future change in the composition of the labour force will be a movement towards a higher percentage of workers in the older age groups.

II. Immigration

The age structure of the immigrant and non immigrant population is shown in Table A-2. For Canada the proportion of non-immigrant population in the age group 25 to 44 (28%) was 58% higher than that of the age group 45 to 64 (17.75%). The corresponding figure for the immigrant population is only 28%. From these observations it can be argued that the average level of work experience of the immigrant population is likely much higher than that of the non-immigrant population. In terms of our model investment in information takes the form of on-the-job training and it will be directly related with the level of experience. Thus we can say that in case of the Canada immigration resulted in a change in the composition of the labour force towards more experienced (older) workers.

TABLE A-2: AGE COMPOSITION OF THE IMMIGRANT AND NON-IMMIGRANT POPULATION (1981), CANADA and U.S.

Age Group	CANADA IMM.	CANADA NON-IMM.	U.S. IMM.	U.S. NON-IMM.
0 - 24	17.9%	46.5%	46.0%	41.6%
25 - 44	36.8	28.0	38.0	28.5
45 - 64	28.8	17.8	12.0	19.3
65+	17.3	7.8	3.0	11.3

Source: 1981 Census of Canada
1981 Statistical Yearbook of the Immigration and Naturalization Service, U.S. Department of Justice.

For the U.S. the proportion of younger workers among the immigrant population is much higher than from that among the non-immigrant population. As such immigration in their case changed the age composition of the labour force towards more younger workers.

III. Female Labour Force Participation

Table A-3 shows the percentage of female workers in the total labour force in Canada. Since 1956 there was a dramatic increase in female labour force participation in Canada which resulted in a larger percentage of female workers in the labour force. In terms of our model we can approximate female workers as 'young workers'. Thus this change will have similar effects as that of a change towards more young workers in the labour force.

TABLE A-3 : PERCENTAGE OF FEMALE WORKERS IN THE LABOUR FORCE .

1946	22.4%	1971	32.8%
1951	22.0	1976	37.6
1956	23.3	1981	40.8
1961	26.7	1986	42.9
1966	30.0		

Source : 1946-1975, Historical Statistics of Canada,(2nd Ed.) Edited by Leacy F.H., Statistics Canada, Series D138, D149.
1976-1986 Historical Labour Force Statistics, 1976, Statistics Canada (71-201), Series D767870, D768001.

APPENDIX 7: Social Pension Plan With a Progressive Tax Rate

The equilibrium level of schooling chosen by individuals is determined by,

$$S'(P) = \underbrace{\frac{\mu^1 P}{\mu^0(1+r)}}_{\Gamma} \underbrace{\left[\frac{1-\tau^1+\tau^1\left(\frac{1+n}{1+r}\right)}{1-\tau^0+\tau^0\left(\frac{1+n^2}{1+r}\right)}\right]}_{\Sigma}$$

Σ will be greater than 1 if,

$$1-\tau^1+\tau^1\left(\frac{1+n}{1+r}\right) > 1-\tau^0+\tau^0\left(\frac{1+n^2}{1+r}\right)$$

(A.1) $\implies$ $\tau^1/\tau^0 < 1+(1+n)/(1+r)$

Under progressive tax rate we will have $\tau^0<\tau^1$, i.e. $\tau^0/\tau^1>0$. If the tax schedule, n and r are such that condition (A.1) is satisfied then even with a progressive tax rate our results will be the same as those with proportional and regressive rates. Otherwise the effects will be the opposite.

References

Aaron, Henry (1966) 'Social Insurance Paradox', Canadian Journal of Economics and Political Science, 32:371-374.

Arrow, Kenneth (1973) 'Higher Education as a Filter' Journal of Public Economics, 2:193-216

Barro, Robert (1974) 'Are Government Bonds Net Wealth?', Journal of Political Economy, 84:1095-1117.

Becker, Gary (1964) 'Human Capital', New York:National Bureau of Economic Research.

Ben-Porath, Yoram (1967) 'The Production of Human Capital and the Life Cycle of Earning', Journal of Political Economy, 75:352-356

Berger, Mark (1985) 'The Effect of Cohort Size on Earnings Growth: a Reconsideration of the Evidence', Journal of Political Economy, 18:561-573.

Black, Dan (1987) 'The Social Security System, the Provision of Human Capital, and the Structure of Compensation', Journal of Labor Economics, 5:242-254.

Boskin, Michael (1977) 'Social Security and Retirement Decisions', Economic Enquiry, 15

Burbidge, John (1982) 'Public Pension Plans in Canada: Alternative Views', in Tax Policy Options in the 1980's, 298

Burkhauser, Richard and John Turner (1978) 'A Time series Analysis on Social Security and Its Effect on the Market Work of Men at Younger Ages', Journal of Political Economy, 86:701-715.

Connelly, Rachel (1986) 'A Framework for Analyzing the Impact of Cohort Size on Education and Labour Earnings', Journal of Human Resources, 21:543-562.

Courchene, Thomas (1987) 'Social Policies in the 1990's : Agenda for Reform', Policy Study No.3, C. D. Howe Institute.

Dooley, Martin (1986) `The Overeducated Canadian? Changes in the Relationship Among Earnings, Education and Age for Canadian Men: 1971-81', Canadian Journal of Economics, 14:142-159.

_______, and Peter Gottschalk (1984), Earnings Inequality Among Males in the United States: Trends and the Effect of Labour Force Growth', Journal of Political Economy, 92:59-89.

Drazen, Allan (1978) `Government Debt, Human Capital, and Bequest in a Life-Cycle Model', Journal of Political Economy, 86:505-516

Easterlin, Richard A. (1978), `What will 1984 be Like? Socioeconomic Implications of Recent Twists in Age Structure', Demography, 15:397-421.

Feldstein, Martin (1974), `Social Security, Induced Retirement and Aggregate Capital Accumulation', Journal of Political Economy, 81:905-926.

---------, (1977) `Facing the Social Security Crisis', Public Interest, 47:88-100.

Fields, Gary and Olivia Mitchell (1984), `Retirement, Pensions, and Social Security', The MIT Press.

Flinn, Christopher (1986) `Wages and Job Mobility of Young Workers', Journal of Political Economy, 94:S88-S110.

Freeman, Richard (1975) `Overinvestment in College Training?' Journal of Human Resources, 10:287-311.

_______, (1979) `The Effect of Demographic Factors on Age-Earnings Profiles', Journal of Human Resources, 14:289-318.

Gausch, Luis and Andrew Weiss (1981), `Self-Selection in the Labour Market', American Economic Review, 71:275-284.

Johnson, William (1978) `A Theory of Job Shopping', Quarterly Journal of Economics, 92:261-277.

Jovanovic, Boyan (1979) `Job Matching and the Theory of Turnover', Journal of Political Economy, 88:1246-1260.

Levhari, David and Yoram Weiss (1974) 'The Effect of Risk on the Investment in Human Capital, American Economic Review, 64:950-963.

MacDonald, Glenn (1980) 'Person Specific Information in the Labour Market', Journal of Political Economy, 89:578-597

Miller, Robert (1984), 'Job Matching and Occupational Choice' Journal of Political Economy, 92:1086-1115.

Mincer, Jacob (1958) 'Investment in Human Capital and Personal Income Distribution', Journal of Political Economy, :281-301.

------, (1974) 'Schooling Experience, and Earnings', New York:National Bureau of Economic Research.

OEC Position Paper (1983), Pensions Today and Tomorrow.

Quinn, J. F. (1977) 'Microeconomic Determinants of Retirement : A Cross Sectional View of White Married Men' Journal of Human Resources, 12:929

Riley, John (1975) 'Competitive Signalling', Journal of Economic Theory, 10:175-186.

-----, (1979) 'Testing the Educational Screening Hypothesis', Journal of Political Economy, 87:S227-S251.

Rosen, Sherwin (1978), 'Substitution in Production', Economica, 45:235-250.

Royal Commission on the Economic Union and Development Prospects for Canada (Macdonald Commission) Report, 1985 Ottawa, Supply and Services Canada.

Samuelson, Paul (1958) 'An Exact Consumption Loan Model of Interest with or Without the Contrivance of Money', Journal of Political Economy, 68:467-482.

Schultz, T. W. (1960) 'Capital Formation by Education', Journal of Political Economy, 68:571-583.

Smith, James and Finis Welch (1978) 'Local Labour Markets and Cyclic Components in Demand for College Trained Manpower', Annales de l'INSEE, 30-31:599-630.

Spance, Michael (1974) 'Market Signalling: Informational Transfer in Hiring and Related Screening Processes', Cambridge, Mass. :Harvard University Press.

------, (1976) 'Competition in Salaries, Credentials and Signalling Prerequisites for Job', Quarterly Journal of Economics, 90:51-75.

Stiglitz, Joseph (1975) 'The Theory of Screening, Education and the distribution of Income', American Economic Review, 65:283-300.

Thompson, Lawrence (1983) 'The Social Security Reform Debate', Journal of Economic Literature, 21:1425-1467.

Welch, Finis (1979) 'Effects of Cohort Size on Earnings: The Baby-boom Babies' Financial Bust', Journal of Political Economy, 87:S65-S97.

Willis, Robert (1986) 'Wage Determination: A Survey and Reinterpretation of Human Capital Earnings Functions', in O. Ashenfelter and R. Layard Edited Handbook of Labor Economics, Vol. 1, Amsterdam:North-Holland.

------, and Sherwin Rosen (1979), 'Education and Self-Selection', Journal of Political Economy, 87:S7-S36.