1986

Radar Auroral Echo Heights As Seen By A 398 Mhz Phased Array Radar Operated At Homer, Alaska

John Michael Ruohoniemi

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ACKNOWLEDGEMENTS

It is a pleasure to acknowledge the guidance, assistance, and encouragement of Dr. D. R. Moorcroft throughout the course of this study. I am especially grateful for the preparatory work that established the basis for this project and for the many illuminating discussions over the years.

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RADAR AURORAL ECHO HEIGHTS
AS SEEN BY A 398 MHZ PHASED ARRAY RADAR
OPERATED AT HOMER, ALASKA

by

J. Michael Ruohoniemi

Department of Physics

Submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
London, Ontario
October 1985

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ISBN 0-315-29490-6

ABSTRACT

Backscatter data collected with a 398 MHz phased-array radar operated at Homer, Alaska (59.72 deg N, 151.53 deg W) have been analyzed for information on the height of radar auroral echoing. Altitude was resolved through the variation of backscattered power with antenna beam elevation angle. The mean height of backscattering could be determined with an accuracy of 1-2 km over small ($\sim 20 \times 20$ km²) areas and short (~ 1 min) periods. In this thesis, the results are presented in the form of maps of the spatial distribution of echo height.

The data base encompassed approximately 40 hrs of observations carried out in 1973, 1976, and 1978. Echo activity most often spanned a 10-20 km range of height centred on 100-110 km. The echoing region was always sharply bounded from below at 96-98 km. The upper altitude limit of echo activity was 115-120 km. The height range in events of the post-midnight sector was 97-110 km vs. 97-118 km in events of the pre-midnight sector. Echoing was restricted to directions nearly perpendicular to the geomagnetic field, giving rise to systematic spatial and temporal variations of height. Magnetic aspect control of echo height was weaker in events of discrete radar aurora.

The height range of the Homer echo activity is shown to be consistent with the onset of primary two-stream plasma instability within auroral electrojet current. The

modulation of height within the altitude limits of echo activity by the magnetic aspect geometry is attributed to strong directional confinement of plasma wave growth. It is suggested that the spatial and temporal variabilities of radar auroral altitude derive from structure within auroral ionization and variability of auroral electrojet current.

ACKNOWLEDGEMENTS

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(1983-1985) is gratefully acknowledged.

Finally, I thank my parents for their support and understanding, and I acknowledge a special debt to Ann Marie.

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CHAPTER 1

INTRODUCTION

1.1 The nature of radar aurora

Man has long been intrigued by the spectacular displays of lights in the northern and southern skies known as aurorae borealis and aurorae australis, but only in this century has their essential nature come to be understood. The lights are the visual manifestation of relaxation processes associated with the excitation and ionization of atmospheric constituents by beams of energetic particles. The guiding effect of the earth's magnetic field restricts particle bombardment to auroral latitudes (65-75 deg) except during periods of extreme disturbance. Auroral light is emitted primarily in the E (90-150 km) and F (150-600 km) layers of the ionosphere.

Reflections of radio waves from the auroral ionosphere were first studied by Harang and Stoffregen [1938]. The name radar aurora (also radio aurora) applies to scatter that is generated within auroral ionization by small-scale non-thermal electron density irregularities. Radar aurora is observed in the HF (3-30 MHz), VHF (30-300 MHz), and UHF (300-3000 MHz) bands. Since the maximum frequencies associated with critical reflection are low by comparison (~ 3 MHz in the E region, ~ 10 MHz in the F region) radar auroral scatterers are 'underdense' and most of the transmitted power is lost into space. Although

irregularities exist at every level of the ionosphere, observational constraints limit studies of radar aurora on VHF and UHF to the E region,

The occurrence of radar aurora depends on the presence of current within auroral plasma. The most important sources of current are large-scale electric fields that originate at the boundary of the magnetosphere through the dynamo interaction of solar wind plasma and the earth's magnetic field. In the outer magnetosphere, the motions of charged particles are essentially collisionless and plasma responds to a combination of electric and magnetic forces by undergoing an **E_xB** drift. With decreasing height through the ionosphere, however, the motions - first of ions, then of electrons - become dominated by collisions with neutrals. The 90 to 120 km altitude interval is part of a transition region in which the predominant electron motion is an **E_xB** drift while ion motion is severely limited by collisions. The difference in drifts, when combined with significant levels of ionization at these heights, causes a strong Hall current to flow. The patterns of plasma convection and particle precipitation in the magnetosphere are such that the largest Hall current flows at auroral latitudes. The principal component of the auroral current (as determined by ground-based sensing devices) reverses direction around magnetic midnight. The evening sector is characterized by westward flow of electrons (eastward electrojet) and the morning sector by eastward flow

(westward electrojet). Radar aurora of the evening sector is known to be closely associated with the intensified current of the eastward electrojet (Greenwald et al., 1975; Tsunoda et al., 1976b).

A large ionospheric current flows as well in a band a few degrees in width about the geomagnetic equator. This 'equatorial' electrojet is driven by tidal processes in the lower atmosphere, however, not by the magnetospheric dynamo. Radio wave scattering by non-thermal density irregularities is observed in the equatorial electrojet (Bowles et al., 1963) and most theories of the phenomenon were developed first for the equatorial region.

The actions of electric currents and density gradients can render ionospheric plasma unstable to the growth of perturbations in the ambient ionization density. According to accepted theory, several types of plasma instability are capable of generating spatially periodic structures within the ionization, i.e. plasma density waves. Through constructive interference, plasma density waves in the auroral ionosphere scatter electromagnetic radiation at very high frequencies to produce radar aurora.

At first, it was thought that radar aurora arose within the structures of ionization responsible for visual aurora. Although the regions of maximum occurrence of visual and radar aurora roughly coincide in an 'auroral oval' about the geomagnetic pole, efforts to establish a one-to-one correspondence in specific events have yielded

ambiguous results (Tsunoda et al., 1974). The complexity appears to derive from the two-parameter dependence (significant ionization plus agent of plasma destabilization) of radar aurora. There are even instances of anti-correlation resulting from reduction of electric field within the density enhancements of visual forms (Tsunoda et al., 1976b). Visual and radar aurora have a common origin in the complex electrodynamics of the auroral regions but are sufficiently differentiated to warrant separate disciplines.

Despite continuing improvements in observational and analytical techniques, there remain serious shortcomings in our knowledge of the altitude characteristics of radar aurora. The difficulty in making direct measurements of the height of echo activity at high latitudes can be traced to a severe observational constraint imposed by the nature of the scattering phenomenon. For scattering to occur, the radar wave vector must be nearly perpendicular to the magnetic field in the scattering volume. The angle off-perpendicular is known as the magnetic aspect angle. As shown in Fig. 1.1, fulfillment of the condition of zero aspect angle means that the auroral radar must be located equatorward of the auroral zone and look at small angles to the horizontal. The radar acceptance beam is typically several degrees in width, corresponding to vertical scale sizes comparable to that of the entire E region (~ 60 km) at typical ranges (500-1000 km). The configuration is not

well suited to resolution of altitude structure through backscattered power variations with either radar slant range or elevation angle. The insensitivity to altitude is usually so large that effectively no height resolution is available within the 95-120 km vertical extent of the echo region.

Fig. 1.1 Geometry of E region auroral radar experiments
(after Balsley and Ecklund, 1972, as adapted
by Fejer and Kelley, 1980.)

The situation is different at the equatorial electrojet where the magnetic field is horizontal and observations can be made with the radar beam intersecting the echo region at vertical incidence. There, selection of

short radar pulse lengths in a system with high sensitivity affords excellent (~ 1 km) discrimination in altitude (Fejer et al., 1975b). Results on the height of echoing in equatorial studies cannot simply be carried over to radar aurora, however - an essential difference between the two regions is the degree of electrical coupling between ionospheric layers, which depends on the inclination of geomagnetic field lines to the horizontal. Other important differences include considerably larger electric fields in the auroral zone (50 mV/m vs. 10-15 mV/m) and a much more dynamic plasma density structure at high latitudes because of auroral particle precipitation.

Despite the difficulties, a number of studies of the height of radar auroral echoing have been made. These have almost all sought to characterize echo activity in terms of a mean height of backscattering, usually on quite large horizontal and temporal scales. We present an abbreviated summary of the findings to date in the next section. More attention is given to the work of Unwin [1957, 1959, 1981] who has been a prominent figure in this field.

1.2 Previous studies of the altitude characteristics of radar aurora

Currie, Forsyth and Vawter [1953] were among the first to estimate the height of auroral echoing. Working at Saskatoon with an antenna consisting of a highly directional aerial array, they studied auroral returns on

56 and 106.5 MHz. Average heights over large quantities of data were estimated by comparing the elevation angles of various features in the aerial radiation pattern to their positions in plots of echo frequency occurrence as a function of range, a technique that was widely employed in early studies. The average echo heights were found to lie in the 105-118 km interval.

Unwin and Gadsen [1957], using a Lloyd's mirror interferometer, carried out the first high resolution measurements of echoing height. Their 55 MHz radar located at Bluff, New Zealand overlooks the sea at an altitude of 270 m. With this device, a thin sheetlike layer of radar aurora results in a well-defined aerial lobe pattern being formed on range-time records through interference of direct and sea-reflected rays. The height of the layer is determined by measuring the slant range location of each of the multiple narrow-beam elevation lobes. The accuracy of the height determination reaches 0.5 km for very thin (<2.5 km) layers. These workers cite an average echo height for one period of 110 km. Unwin [1959] carried out an extensive survey of auroral echo characteristics on 55 MHz as seen by the Bluff radar. He identified two main classes of echo layers - those extended in range (diffuse) and those limited in range to the resolution of the radar (discrete). The four sub-classes and their characteristics are listed in table 1.1.

TABLE 1.1Echo layer types according to Unwin [1959]

<u>Type</u>	<u>Occurrence</u>	<u>Duration</u>	<u>Height</u> (km)	<u>Thickness</u> (km)
Diffuse	afternoon/ evening	> 10 min	103 - 117	< 5
Diffuse w/ structure	evening/early morning	< 5 min	115	5 - 14
Short discrete	evening/early morning	< 1 min	120	10 - 26
Long discrete	early morning	> 10 min	100	15

Leadabrand et al. [1959] carried out radar auroral observations on 398 MHz with a narrow-beam (3 deg) antenna located at College, Alaska. By sweeping the beam in elevation they were able to measure the heights of more localized echoing regions. The altitude limits of echo activity were reported to be 75 and 135 km, with the majority of echoes originating in the 100-110 km height interval. Also at College, Presnell et al. [1959] carried out the first comparative study of the height of echoes on VHF and UHF. They estimated the altitude ranges for the different frequencies to be: 70-160 km (216 MHz), 75-135 km (398 MHz), 100-120 km (780 MHz).

Watkins [1960] used a 37 MHz rotating aerial operated at Jodrell Bank to study the height and geometry of auroral echoes. From theoretical range-number curves calculated for the elevation polar diagram of the aerial he estimated

the height of the centre of the echo region to be 110 km. Bullough [1961] monitored echo occurrence on 7.3 MHz with a rotating antenna installed at Terre Adelie, Antarctica for observations during the IGY. Using the vertical polar diagram technique, he obtained an estimate of 106 km for the mean height of echo occurrence and found a slight diurnal variation (107 km in the evening, 104 km in the morning). Lockwood [1961], using the 448 MHz Prince Albert radar, compared experimental contours of auroral echo occurrence with the distribution of visual aurora to determine that most echoes arise in the 100-105 km height interval.

Barber et al. [1962] extended the Jodrell bank observations into the UHF band with narrow-beam observations on 300 and 500 MHz. The height range of echo occurrence was reported to be 104-124 km, with a mean height for strong afternoon echoes of 110 km. The aspect sensitivity (i.e. the decrease in echo power with angle between the radar wave vector and the perpendicular to the magnetic field in the scattering volume) was estimated to be 10 dB/deg. Leadabrand et al. [1965] made simultaneous multifrequency (30, 401, 800 MHz) observations of radar aurora from a site in Scotland. On 400 MHz they found that most echoes arise from a layerlike form of auroral scatterers whose thickness is about 10-20 km. By comparing the spatial occurrence of radar aurora with the magnetic aspect geometry they estimated the height range of 400-MHz

echoes to be 100-120 km. The aspect sensitivity on the higher frequencies was estimated to be 10 dB/deg.

Abel and Newell [1969] studied the radar aurora on 1298 MHz with an exceptionally small-beamwidth (0.6 deg) antenna operated at Millstone Hill. The echoes were found to arise from a layerlike region having a thickness of 5-20 km located at an average height near 110 km. They detected variability in the height of echoing; in one three minute period the height of the echo peak was observed to change from 108 to 118 km.

Unwin and Johnston [1981] reported a double-layer altitude structure in the afternoon diffuse radar aurora. Using the 55 MHz Bluff radar they found cases where the power spectrum of the backscattered signal consisted of two distinct spectral distributions. They attributed these distributions to layers of irregularities at altitudes of 106 and 112 km. Timofeev and Miroshnikov [1982] reported similar instances of layering within the echo region. Their 90 MHz radar located at Karmaselga, Karelia is a modified Lloyd's mirror interferometer. They also reported extreme altitude values of 100 and 123 km, an east-west altitude asymmetry, and distinctly smaller echo layer thicknesses in the westward electrojet.

The Lloyd's mirror interferometer technique, as employed by the New Zealand and Soviet workers, produces the finest resolution of vertical structure when certain conditions on the echo activity are satisfied. However,

whenever the echoing is extensive in altitude the antenna lobe structure in the backscatter returns is smeared out and sensitivity to height drastically reduced. In addition, these systems have very limited ability to resolve horizontal spatial structure; an echo layer slant range extent of more than 100 km is required for definition of the antenna lobe pattern and the horizontal beamwidths of the antennas are quite large (>9 deg). The narrow-beam studies afforded greater sensitivity to horizontal spatial variations but their ability to resolve vertical structure, even at the level of identifying a mean height of backscattering, was very limited.

Radar observations of coherent scattering provide indirect evidence of the presence of enhanced plasma density fluctuations in the ionosphere. With rocket probes, it is possible to sample the density fluctuations and their attendant fields directly. The rocket findings on the height range of plasma instability have important consequences for our understanding of radar aurora.

Kelley and Mozer [1973] observed oscillating electric fields and plasma density fluctuations in a strongly driven auroral electrojet current. They demonstrated that the waves were electrostatic and included composition at wavelengths associated with radar auroral scattering. The height range of broadband density fluctuation was reported to be 90 to 120 km.

Pfaff et al. [1984] detected a layer of electrostatic

plasma turbulence in the unstable auroral E layer covering heights from 96 to 121 km. The layer was sharply defined at both the upper and lower edges. The waves covered a broad spectrum from hundreds of metres to a few centimetres wavelength. Observations from two simultaneous payloads indicated that the electrojet turbulence was uniform over distances of kilometres.

1.3 Present work

Accounting for altitude variation is an important requirement of a successful theory of radar aurora. In addition, the derivation of ionospheric electric fields from radar auroral backscatter in STARE-type experiments may be subject to altitude factors (Nielsen and Schlegel, 1983). If radar aurora is to be used as a diagnostic tool for the study of E region electrodynamics, accurate knowledge of its altitude characteristics is essential. For these reasons, we have undertaken an analysis of auroral backscatter data from a sophisticated experiment for information on echoing height.

The observations were made with a 398 MHz phased-array radar located at Homer, Alaska. A large (~ 40 hr) pool of data from campaigns in 1973, 1976, and 1978 constitutes the subject matter of our analysis.

As in previous studies, resolution of vertical structure is limited to the characterization of echo activity in terms of a mean height of backscattering.

However, with the added dimension in spatial coverage afforded by the unique phased-array capability of the Homer radar, the backscatter data are amenable to higher forms of analysis than are possible with fixed-beam and mechanically-steered systems. In particular, we are able to measure mean echoing height with high accuracy (1-2 km) over large areas ($\sim 500 \times 500 \text{ km}^2$) with good spatial ($\sim 20 \text{ km}$) and temporal ($\sim 1 \text{ min}$) resolutions. Because of the rapid antenna scan rate, small-scale spatial and temporal variations are unambiguously resolved in the Homer data.

1.4 Outline of the thesis

Chapter 2 begins with a review of non-relativistic scattering theory. An expression for the radar scattering cross-section is derived and applied to backscattering from plasma density waves. The solution of the dispersion relation for electrostatic plasma waves in the ionosphere is discussed. The instability conditions for wave growth in the long-wavelength (fluid) regime are obtained. The generation of secondary waves within the disturbance wavefields of primary waves is described, and the nonlinear saturation of wave growth is briefly discussed.

Chapter 3 consists of an outline of the experiment. Brief descriptions are given of the capabilities of the radar and the process of data collection. A summary of the observational periods is presented in tabular form.

Chapter 4 describes the analysis used to extract

altitude information from the Homer data. The radar equation is derived. The assumptions involved in simplifying its dependence on altitude are described and justified. The physical significance of the echo height measurement is clarified and errors are discussed.

Chapter 5 presents the results of the analysis of altitude. Maps of the spatial distribution of echo height are examined and interpreted in terms of the vertical distribution of small-scale electron density irregularity and the dependence of echoing on aspect angle. Specific events are analyzed in detail; aspect control of echo height is seen to vary from total to negligible. Temporal variability of echo height is examined and a diurnal variation is reported. A simple model of scattering is developed that accounts in a semi-quantitative manner for the most pronounced spatial variation of echo height.

Chapter 6 presents the results of simplified numerical analyses of plasma instability in the auroral E region. The predictions of linear theory are shown to be in reasonable agreement with the vertical extent of the echo activity observed by the Homer radar. Plausible explanations are offered for the properties of echo height reported in Chapter 5. The limitations of linear theory are described and some relevant nonlinear considerations are outlined.

Chapter 7 summarizes the results, makes comparisons with previous studies, and offers directions for future

research.

Appendix A deals with the derivation of the antenna sensitivity function for a thin-layer reflector. Appendix B consists of an examination of secondary effects (e.g. atmospheric refraction) on the determination of echo height. Appendix C describes in some detail an analysis of fading applied to the Homer data.

C H A P T E R 2

THEORY

2.1 Introduction

An extensive body of theory has been developed to account for underdense radio wave scattering in the ionosphere. The central problem is to devise mechanisms that will give rise to suitable density structures in a magnetized, weakly ionized (~ 1 part in 10^8) plasma. The subject is introduced here with a brief review of the non-relativistic scattering of electromagnetic radiation.

An electron 'scatters' an electromagnetic wave by re-radiating wave energy out of the direction of wave propagation. The energy scattered per unit solid angle per unit incident power density is given by $(r_e \sin \chi)^2$ where r_e ($= 2.82 \cdot 10^{-15}$ m) is the classical electron radius and χ is the angle between the direction of the incident electric field and the direction to the receiving apparatus. The radar cross-section of a target is conventionally expressed as the cross-sectional area of an isotropic scatterer which, if placed at the position of the real target, would scatter the same amount of energy to the observer. It follows that the radar cross-section of an electron, σ_e , is given by $\sigma_e = 4\pi(r_e \sin \chi)^2$. For a plasma it makes sense to speak of a scattering cross-section per unit volume, σ_v . The behaviour of σ_v depends critically on whether the

exploring radar wavelength is small or large compared to the Debye length of the plasma ($\sim .3$ cm in the auroral E region). In the small-wavelength regime, the phases of the signals scattered by the electrons vary independently with time and the effective volume scattering cross-section is simply $\sigma_v = N_e \sigma_e$, where N_e is the electron density. For radiation wavelengths large compared to the Debye length, the influence of the ions causes the electrons to exhibit collective behaviour. The scattering can then be thought of as arising from density fluctuations brought about by longitudinal oscillations in the plasma (Evans, 1969). If the density fluctuations are due to thermal turbulence, the scattering is said to be incoherent and the returned signal has properties related to such basic parameters of the plasma as density and temperature. The incoherent scatter technique has been used for ionospheric studies on a routine basis for a number of years (e.g. Farley, 1971).

Radar aurora is an example of coherent scatter, which is stronger by several orders of magnitude. It is accessible to radar systems of much lower power (~ 50 kW vs. ~ 4000 kW). The electron density fluctuations responsible for coherent scatter have been enhanced over thermal levels by plasma instability processes. For radar aurora, the most important sources of energy for plasma instability are relative electron-ion drifts and ionization density gradients. These are associated with the two-stream and gradient-drift instabilities respectively

(e.g. Fejer and Kelly, 1980). The presence of enhanced order in auroral plasma at scale lengths (15-.1 m) characteristic of HF, VHF, and UHF radar systems is the physical basis for radar aurora.

In this chapter, the theory of radio wave scattering from a diffuse medium is reviewed. A dispersion relation describing the conditions of plasma wave growth at the threshold of instability is developed. Finally, a nonlinear theory of the saturation of plasma wave growth is outlined. Some of these aspects of plasma instability in the auroral ionosphere are related to experimental findings in Chapter 6.

2.2 Scattering from a diffuse medium

Experimental studies have shown that the irregularities responsible for radar auroral scatter are uncorrelated in amplitude and phase in volumes that are large compared to the scale size of the irregularities (Hultqvist and Egeland, 1964). Since this condition on size is satisfied by radar sampling volumes, the amplitude of returned signal obeys fluctuation statistics. Consequently, it is convenient to approach the analysis of auroral scattering from a statistical point of view.

Let the presence of electron density irregularities be described in terms of fluctuations of dielectric permittivity, i.e.

$$\epsilon(r,t) = \langle \epsilon \rangle + \Delta\epsilon(r,t) , \quad [2.1]$$

where $\langle \epsilon \rangle$ = mean permittivity
 $\Delta\epsilon(r,t)$ = fluctuation from mean permittivity.

The scattering geometry is shown in Fig. 2.1. At upper VHF and UHF frequencies the density irregularities are weak ($|\Delta\epsilon| \ll \epsilon$) and the electric field in the scattering volume is not significantly different from the one that would have existed in the absence of fluctuations (i.e. the Born approximation is valid). The scattering volume is taken to have dimensions that are large compared to the radar wavelength but sufficiently small that the medium can be treated as statistically uniform. The transmitting and receiving antennas are assumed to be great distances from the scattering volume. Each volume element dV can be considered a radiating dipole. The field at the receiver is the sum of the radiation fields of all the elementary dipoles and can be written as (Flood, 1967)

$$E_s = \frac{\mu_0 \omega^2 |E_0| \sin(\chi)}{4\pi R} \int_V \Delta\epsilon(r,t) \exp[-iq \cdot r] dV , \quad [2.2]$$

where

- μ_0 = permeability of free space
- ω = angular frequency of the radar wave
- $|E_0|$ = magnitude of the electric field intensity incident upon the scattering volume
- R = distance from scattering volume to the radar receiver
- $q = k_i - k_s$, the difference of the wave vectors of the incident and scattered fields
- $|q| = (4\pi/\lambda) \sin(\theta/2)$
- λ = operating wavelength of the radar
- θ = angle between incident and scattered waves
- χ = angle between the electric polarization vector of the incident wave and the propagation vector of the scattered wave.

Fig. 2.1 Geometry of scattering.

The receiving radar measures the mean squared field intensity scattered in the direction of $\mathbf{k}_s$. This is given by the ensemble average of the product of E_s and its complex conjugate:

$$\langle |E_s|^2 \rangle = \langle E_s E_s^* \rangle = \left[\frac{\mu_0 \omega^2 |E_0| \sin(\chi)}{4\pi R} \right]^2 \int_V \int_{V'} \langle \Delta \epsilon(\mathbf{r}) \Delta \epsilon^*(\mathbf{r}') \rangle \exp[-i\mathbf{q} \cdot (\mathbf{r} - \mathbf{r}')] dV dV' \quad [2.3]$$

The autocorrelation function of the spatial fluctuation of dielectric permittivity, $\langle \Delta \epsilon(\mathbf{r}) \Delta \epsilon^*(\mathbf{r}') \rangle$, describes statistically the sizes and shapes of irregularities in the medium. Because the fluctuation statistics are uniform throughout V the autocorrelation function depends only on the relative position vector $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ and can be written as $C(\mathbf{R}) = \langle \Delta \epsilon(\mathbf{r}) \Delta \epsilon^*(\mathbf{r}') \rangle$. Carrying out the change in variable of integration in eq. [2.3], we have

$$\langle |E_s|^2 \rangle = \left[\frac{\mu_0 \omega^2 |E_0| \sin(\chi)}{4\pi R} \right]^2 \int_V dV \int_{V_{rel}} C(\mathbf{R}) \exp[-i\mathbf{q} \cdot \mathbf{R}] dV_{rel} \quad [2.4]$$

The correlations that exist between the irregularities are extremely localized. Consequently, the relative volume integral can be extended over all space without incurring significant error:

$$\int C(\mathbf{R}) \exp[-i\mathbf{q} \cdot \mathbf{R}] dV = \langle |\Delta \epsilon(\mathbf{q})|^2 \rangle \quad [2.5]$$

The mean squared field intensity scattered to the receiver from the volume V is

$$\langle |E_s|^2 \rangle = \left[\frac{\mu_0 \omega^2 |E_0| \sin(\chi)}{4\pi R} \right]^2 \langle |\Delta \epsilon(\mathbf{q})|^2 \rangle \times \text{volume} \quad [2.6]$$

and the scattering cross-section per unit volume is

$$\sigma_v = \frac{4\pi R^2 |E_s|^2}{V |E_0|^2} = \frac{(\mu_0 \omega^2)^2 \sin^2(\chi)}{4\pi} \langle |\Delta \epsilon(q)|^2 \rangle . \quad [2.7]$$

At very high frequencies in the ionosphere, collisions between electrons and heavier plasma particles can be neglected and the dielectric permittivity is a pure real quantity. Fluctuations in dielectric permittivity and electron density are then related through the equation

$$\epsilon(r, t) = \epsilon_0 - \frac{N(r, t) e^2}{m_e \omega^2} , \quad [2.8]$$

where ϵ_0 = dielectric permittivity of free space
 e = charge on an electron
 m_e = mass of an electron
 $N(r, t)$ = density of electrons.

In terms of N , σ_v is given by

$$\sigma_v = \frac{(\mu_0 e^2 / m_e)^2 \sin^2(\chi)}{4\pi} \langle |\Delta N(q)|^2 \rangle = \sigma_e \langle |\Delta N(q)|^2 \rangle . \quad [2.9]$$

The radar scattering cross-section is proportional to the power spectrum of the electron density fluctuations,

$\langle |\Delta N(q)|^2 \rangle$, evaluated at $q = k_i - k_s$.

In backscatter operations, for which the same radar is used to transmit and receive, $\chi = 90$ deg, $\theta = 180$ deg, $k_s = -k_i$, and $q = 2k_i$. From eq. [2.9], the volume cross-section for backscattering is

$$\sigma_v = \sigma_e \langle |\Delta N(2k_i)|^2 \rangle , \quad [2.10]$$

i.e. the backscatter power measurement is sensitive to that component of the electron density fluctuations along the radar line of sight with a spatial periodicity equal to one-half the radar wavelength. This is the component with the orientation and spacing necessary for all the very weak partial reflections from each phase front to add

coherently. Any other spacing results in destructive interference. The Homer radar, with an operating wavelength of .75 m, detected reflections from the spatial Fourier component of the electron density fluctuations of wavelength $\lambda/2 = .375 \text{ m}$ ($k = 16.7 \text{ m}^{-1}$).

2.3 Theory of plasma instability

2.3.1 Introduction

The plasma wave model of coherent scattering is now quite widely accepted. The direct evidence includes rocket observations of electric field and density fluctuations such as those produced by plasma instabilities (Kelley and Mozer, 1973; Pfaff et al., 1984). There are several instabilities that can efficiently amplify thermal density perturbations in the lower ionosphere; in this section, the physical mechanisms of the two-stream and gradient-drift instabilities are reviewed.

The theory of the two-stream instability, modified to include ionospheric magnetization and collision effects, was first formulated by Farley [1963] and Buneman [1963]. As its name suggests, the source of free energy for the two-stream instability in a plasma is a differential drift of the ions and electrons. In the ionosphere, the relative electron-ion drift velocity, V_d , maximizes in the 90-120 km altitude interval. If V_d exceeds approximately the ion acoustic velocity ($\sim 350 \text{ m/s}$ in the E region) energy in the streaming particle motion leads to the growth of

longitudinal electrostatic plasma waves.

The existence of the two-stream instability in ionospheric plasma is due entirely to the combined action of collisions and the geomagnetic field; in a collisionless, field-free plasma the threshold for two-stream instability is the electron thermal velocity, which is very much greater than the ion-acoustic velocity and never attained in ionospheric plasma. The magnetic field lowers the threshold for instability by impeding electron (and hence ion) motion in the plane perpendicular to the field, thereby reducing thermal damping.

The gradient-drift instability was studied for laboratory plasmas by Simon [1963] and Hoh [1963], and applied to the theory of irregularity formation in the E region by Maeda et al. [1963] and Knox [1964]. The gradient-drift mechanism requires a gradient in ionization density and a relative electron-ion drift perpendicular to the density gradient. Fig. 2.2 shows the geometry of the instability for the equatorial electrojet. The background electric field is E_0 and the ambient magnetic field is B_0 . The ionization density gradient is directed upwards in the figure. The density is assumed to have been perturbed by a small-amplitude sinusoidal perturbation perpendicular to E_0 . Only the electrons drift, giving rise to secondary polarization fields E' alternating in opposite directions as shown. The plasma drift produced by the interaction of the E' fields with the magnetic field is directed downwards

Fig. 2.2 Geometry of the gradient-drift instability for the daytime equatorial electrojet (after Fejer and Kelley, 1980.)

in the density enhancements and upwards in the density depletions, accentuating the original density perturbations. The basic requirement for a growing instability is that the orientation of the background electric field relative to the magnetic field be such as to move plasma against the background density gradient. If this orientation is reversed, the gradient-drift mechanism damps the density perturbation instead.

The ion-cyclotron instability has also been considered a possible mechanism for forming ionospheric irregularities (D'Angelo, 1973). Unlike the two-stream and gradient-drift instabilities, it derives from flow of current along the geomagnetic field lines. Although this instability has recently been the subject of renewed interest (Fejer et al., 1984a) it is of doubtful importance to UHF radar reflections from the lower E region. Echoing from ion-cyclotron waves is thought to occur only on lower frequencies within regions of intense field-aligned current above electrojet altitudes.

2.3.2 Fluid theory of plasma instability

A considerable simplification of the theory of plasma instability is possible if the various characteristic lengths of the plasma are small compared to the scale length of the wave density irregularity. The wave then interacts with the average response of the individual particles and the plasma can be described by a macroscopic

fluid theory. The fluid theory has been found to correctly predict the qualitative behaviour even for irregularity wavelengths for which the above condition is not strictly satisfied. For these reasons, the fluid theory is developed here in some detail while a discussion of kinetic effects is deferred to Chapter 6.

Important properties of wave growth and propagation in a plasma can be obtained from the dispersion relation for the medium. In the case of E region coherent scatter, the waves are essentially electrostatic since the plasma pressure is much less than the ambient magnetic energy density. The appropriate fluid dispersion relation is derived from the ion and electron continuity equations, the two equations of motion, and Poisson's equation:

$$(\partial N / \partial t) + \nabla \cdot (NV) = Q - \alpha N^2 \quad [2.11]$$

$$m(DV/Dt) = q(-\nabla\phi + \mathbf{V} \times \mathbf{B}) - \nabla P/N - m\nu V \quad [2.12]$$

$$\nabla^2 \phi = e(N_e - N_i)/\epsilon_0, \quad [2.13]$$

where Q and α are the production and recombination rates, D/Dt is the convective derivative, and the collision frequencies ν_e and ν_i represent effective rate coefficients for the transfer of momentum between the charged particle and neutral gases. The equations are linearized and solved by assuming the perturbations in density, velocity, pressure, and potential are proportional to $\exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$, where $\mathbf{k}$ is the wave vector and ω is the complex oscillation frequency. The procedure is outlined in a number of references (e.g. Rogister and D'Angelo, 1970; Sudan et

al., 1973; Fejer et al., 1975b). The dispersion relation for a relative drift velocity

$$\mathbf{V}_d = \mathbf{V}_e - \mathbf{V}_i = \left[1 - \frac{\Omega_i^2}{\Omega_i^2 + \nu_i^2} \right] \frac{\mathbf{E} \times \mathbf{B}}{B^2} - \left[\frac{\nu_e}{\Omega_e} + \frac{\nu_i \Omega_i}{\Omega_i^2 + \nu_i^2} \right] \frac{\mathbf{E}}{B} \quad [2.14]$$

(where Ω_e and Ω_i are the electron and ion gyrofrequencies) has been given by Fejer et al. [1984b]. For simplicity, we also work in the 'electrojet' approximation, for which

$$\mathbf{V}_d = \mathbf{V}_e = \mathbf{E} \times \mathbf{B} / B^2, \quad [2.15]$$

i.e. to a first approximation the only charged particle motion is an electron Hall drift. The ion Hall and Pedersen drifts increase with altitude in the E region and become substantial above 115 km, but the results of the instability analysis are not changed very much below 120 km. In the electrojet approximation, the fluid dispersion equation for electrostatic plasma waves is

$$\omega^* - \mathbf{k} \cdot \mathbf{V}_d + [\omega^* (\nu_i - i\omega) + ik^2 C_s^2] \left[\frac{\psi}{\nu_i} - \frac{i}{k L_N \Omega_i} \right] = 0, \quad [2.16]$$

where $\omega^* = \omega + i2\alpha N_e$,

$$\psi = (\nu_e \nu_i / \Omega_e \Omega_i) (1 + \Omega_e^2 k_{\perp}^2 / \nu_e^2 k^2), \quad [2.17]$$

$C_s = [K(T_e + T_i) / m_i]^{1/2}$ is the ion acoustic velocity, and $L_N = k_{\perp} B N_e / [\mathbf{k} \cdot (\nabla N_e \times \mathbf{B})]$ is the effective scale length of the electron density (Fejer et al., 1984b). The configuration of ambient electric and magnetic fields, electron density gradient, and relative electron-ion drift velocity for the electrojet approximation is shown in Fig. 2.3.

The complex oscillation frequency can be written as the sum of its real and imaginary parts, which represent the wave frequency and growth rate respectively:

Fig. 2.3 Geometry of ambient electric and magnetic fields and electron density gradient. The direction of the relative electron-ion drift velocity is shown for the electrojet approximation.

$$\omega = \omega_r + i\gamma . \quad [2.18]$$

When $\gamma < 0$ the wave is exponentially damped; when $\gamma > 0$ the wave grows exponentially. Assuming $\gamma \ll \omega_r$ (i.e. marginally stable/unstable conditions), the solution of eq. [2.16] is

$$\omega_r = k \cdot V_d / (1 + \psi) \quad [2.19]$$

$$\gamma = \frac{1}{1 + \psi} \left[\frac{\psi}{\nu_i} (\omega_r^2 - k^2 C_s^2) + \frac{\omega_r \nu_i}{k L_N \Omega_i} \right] - 2\alpha N_e . \quad [2.20]$$

The first and third terms on the right hand side of eq. [2.20] are the two-stream and gradient-drift terms, while the second and fourth terms represent diffusive and recombinational damping. Diffusion and recombination oppose wave growth and establish a threshold for instability.

The threshold drift velocity for wave growth follows from eq. [2.20] with $\gamma = 0$. Following, for example, Fejer et al. [1984b]:

$$k \cdot V_d = k C_s (1 + \psi) [(F^2 + 1 + G)^{1/2} - F] , \quad [2.21]$$

where $F = \nu_i^2 / 2\Omega_i k^2 C_s L_N \psi$

$$G = 2\alpha N_e \nu_i (1 + \psi) / k^2 C_s^2 \psi .$$

If the effects of background density gradients and recombination are negligible ($F \ll 1$, $G \ll 1$) for a wave vector k , instability depends solely on the competition between the two-stream effect and diffusive damping. The minimum threshold drift velocity, $V_d(\min)$, is realized for plasma wave propagation in the direction of V_d and

$$V_d(\min) = C_s (1 + \psi_0) , \quad [2.22]$$

where $\psi_0 = \nu_e \nu_i / \Omega_e \Omega_i$ because $k_{\parallel} = 0$. The minimum threshold

drift velocity for ~~pure~~ two-stream instability in the electrojet approximation is independent of k and has the ion-acoustic velocity as a lower bound. The altitude dependence of $V_d(\min)$ is depicted in Fig. 2.4 for the parameters of a model ionosphere. In this theory, two-stream instability sets in over the entire 95 to 120 km height interval for moderate (~ 550 m/sec) relative drifts, given significant levels of ionization. Also shown is the altitude dependence of $V_d(\min)$ for $\lambda = 10$ m calculated from the dispersion relation [Fejer et al., 1984b] appropriate to the more general relation between V_d and E expressed by eq. [2.14]. It can be seen that the electrojet approximation underestimates $V_d(\min)$ for $\lambda = 10$ m by less than 15% below 120 km. Ignoring the effect of finite ion motion, above 100 km $\psi_0 \ll 1$ and $V_d(\min) = C_s$. Rapidly increasing collision frequencies with decreasing height below 100 km account for the steep increase in $V_d(\min)$. It should be noted that the criterion for fluid theory restricts the validity of these results to larger wavelengths with increasing altitude; they are applicable to waves with $\lambda \geq 1$ m at $h = 100$ km and to waves with $\lambda \geq 10$ m at $h = 120$ km.

For a general orientation of the plasma wave vector k relative to the drift velocity V_d , the threshold condition is

$$k \cdot V_d = k C_s (1 + \psi_0) , \quad [2.23]$$

i.e. the wave will be unstable to the two-stream

Fig. 2.4 Variation of threshold drift velocity with altitude for pure two-stream instability (fluid theory): A) in the electrojet approximation B) for the more general relation between E and V_d expressed by eq. [2.14] with $\lambda = 10$ m.

instability and grow in amplitude if the component of the drift velocity in the direction of k exceeds the ion-acoustic velocity by a factor somewhat greater than one. The growth condition is met for waves propagating in a cone of aperture $\theta_c = \cos^{-1}(C_s/V_d)$. The half-angle of the growth cone increases from 0 deg at the threshold of instability ($V_d \sim C_s$) to over 45 deg for moderately strong ($V_d \sim 2C_s$) drift velocities.

The normalized threshold drift velocity $k \cdot V_d / k C_s = (1 + \psi)$ increases rapidly with angle, α , between the plasma wave propagation vector and the plane perpendicular to the magnetic field. Physically, the wave experiences increased Landau damping because of the greater electron mobility along k for $k_{\parallel} > 0$. At an altitude of 105 km the threshold drift velocity is increased by a factor of 2.5 for $\alpha = 1$ deg and by a factor of 7 for $\alpha = 2$ deg. The instability condition is prohibitive for values of α greater than a few degrees.

Strictly speaking, the presence of density gradients in the ionospheric plasma only modifies the instability conditions deriving from a differential flow of electrons and ions. However, the two-stream growth process may be so much less significant that the irregularities may be said to exist in the pure gradient-drift regime. This is the case for large-wavelength irregularities in the presence of sizable density gradients, for which $F \gg 1$ in eq. [2.21]. Fig. 2.5 shows threshold drift velocity for an intermediate

case ($\lambda = 10$ m) for five values of effective electron density scale length. An electron density gradient can stabilize or destabilize the plasma depending on the sign of L_N . A steep electron density gradient ($L_N = 10$ km) changes the threshold electric field from its pure two-stream value by 25%. Fig. 2.6 shows normalized threshold drift velocity as a function of wavelength for five values of L_N at a height of 105 km, assuming a recombination rate $2\alpha N_e = 0.06 \text{ s}^{-1}$. Large-scale (tens of metres) waves can grow easily in the presence of destabilizing density gradients. The growth cones of such waves extend to almost 90 deg in half-angle.

2.3.3 Generation of secondary irregularities

Large-amplitude plasma waves disturb the medium in which they propagate and themselves give rise to instability. Sudan et al. [1973] showed that the disturbance can be sufficiently large in the equatorial electrojet to cause the growth of 'secondary' waves outside the growth cone of 'primary' instability. Greenwald [1974] and Tsunoda [1976] have applied the theory of secondary instability to the auroral ionosphere. Its importance lies in the prediction of short-wavelength plasma instability at large angles to the direction of the primary current.

The sources of secondary instability are the perturbation electric fields and density gradients of the primary plasma waves. In zero-order theory, the

Fig. 2.5 Variation of threshold drift velocity with altitude and electron density scale length (fluid theory) in the electrojet approximation with $\lambda = 10$ m.

Fig. 2.6 Variation of normalized threshold drift velocity with wavelength and electron density scale length (fluid theory) for an altitude of 105 km and a recombination rate $2\alpha N_e = 0.06 \text{ s}^{-1}$ (adapted from Fejer et al., 1984b).

Fig. 2.5

Fig. 2.6

perturbation of plasma parameters that characterize the secondary wave is assumed to be of the form $\exp[i(k_s z - \omega t)]$ with $k_s \gg k_p$, where k_p is the wave number of the primary wave. The primary and secondary wave vectors are assumed to lie in the plane perpendicular to the magnetic field, with k_s perpendicular to k_p . These represent the most favourable conditions for the onset of secondary wave growth. The linearized fluid equations are solved as in primary theory, but with V_d replaced by the first-order perturbation electron velocity v_e and LN replaced by $k_p N/N_e$, where

$$N/N_e = A_p \sin(k_p z - \omega_p t) = A_p \sin(\delta) \quad [2.24]$$

(A_p is the relative amplitude of the primary wave and δ is the normalized displacement in the wave frame of reference). The wave frequency and growth rate are given by (Sudan et al., 1973)

$$\omega_s = -k_s \frac{\Omega_e}{\nu_e} \frac{\psi_0 V_d}{(1+\psi_0)^2} A_p \sin(\delta) \quad [2.25]$$

$$\gamma_s = \frac{\psi_0}{1+\psi_0} \left[-\frac{\Omega_e^2}{\nu_e^2} \frac{k_p V_d}{2} \frac{\psi_0 A_p^2}{(1+\psi_0)^2} \sin(2\delta) + \frac{1}{\nu_i} (\omega_s^2 - k_s^2 C_s^2) \right] \quad [2.26]$$

The first and second terms in square brackets in eq. [2.26] represent the secondary gradient-drift and two-stream instabilities respectively. (Diffusive damping is retained in the third term; in this order of approximation recombinational damping is neglected.)

The instability criterion for the growth of secondary two-stream irregularities derived from eq. [2.26] is

$$V_d > \frac{\Omega_i}{\nu_i} \frac{(1+\psi)^2}{A_p} C_s \quad [2.27]$$

The normalized threshold drift velocity V_d/C_s is a rapidly increasing function of height above 100 km, assuming a constant primary amplitude. The saturation amplitude of the primary waves cannot be deduced from linear theory, but on physical grounds it should not exceed about 10%. Consequently, lower bounds on the normalized threshold drift velocity for secondary two-stream instability are $V_d/C_s = 1$ at 105 km altitude, 2.2 at 110 km, and 4.4 at 115 km. The generation of secondary two-stream waves is heavily favoured at low (< 105 km) altitudes in this theory.

The threshold condition for the secondary gradient-drift instability is met when

$$k_p V_d A_p^2 > \frac{2\Omega_i \nu_e (1+\psi)^2}{\Omega_e \nu_i^2} k_s^2 C_s^2 \quad [2.28]$$

From this expression, the minimum primary wave number k_p that will lead to secondary gradient-drift wave growth can be determined for given values of V_d and A_p . Greenwald [1974] has concluded that 3 m secondary gradient-drift plasma waves are readily generated in the auroral E region for moderate values of drift velocity and primary irregularity amplitude.

It should be pointed out that eqs. [2.25] - [2.28] are derived on the basis of the size criteria $\lambda_p \gg \lambda_s \gg \lambda_m$, where λ_p and λ_s are the wavelengths of the primary and secondary waves and λ_m is the ion mean free path. The

two-step mechanism of Sudan et al. [1973] is probably operative over a much larger range of relative scale sizes, but no more general version of the theory exists at the present time.

2.3.4 Nonlinear theory of plasma instability

The linear theory of plasma instability can account for a number of features of coherent scatter, such as the existence of threshold electric fields (e.g. Tsunoda and Presnell, 1976; Moorcroft, 1979). However, the unstable plasma waves presumably do not remain for long in the marginal state to which linear theory applies (e-fold growth times are of the order of milliseconds). There are thought to be primarily two means for limiting plasma wave growth: (i) energy cascade from unstable modes to regions of k space that are linearly damped (ii) wave-enhanced diffusion of electrons across field lines. In the generally accepted picture, irregularities are first generated at small angles to the direction of the electrojet current and then couple into wave growth in other directions [Farley et al., 1978]. The process may lead to a two-dimensional cascade of energy in the plane perpendicular to the magnetic field. The medium is then 'turbulent', with waves of all scale sizes interacting with one another in varying degrees. The problem, as considered by Sudan [1983], is outlined here.

The starting point is the set of fluid equations

[2.11]-[2.13] with the addition of the largest-order nonlinear term, which appears in the electron continuity equation. This term leads to mode-coupling of irregularities. In the presence of a suitable density gradient, energy enters the system at large wavelengths through the gradient-drift mechanism and cascades to the damped short wavelength region. The medium becomes turbulent through the nonlinear interaction of triads of waves k , k' , and k'' such that $k = k' + k''$ and $k \sim k' \sim k''$. In the pure gradient-drift regime ($V_d < C_s [1 + \psi]$) this process is sufficiently rapid that full two-dimensional turbulence develops in the plane perpendicular to the magnetic field. In the two-stream regime ($V_d > C_s [1 + \psi]$) there is a direct mechanism for the growth of waves and the cascade process is not sufficiently rapid to be effective in establishing equilibrium over all of k space. Sudan [1983] states that the nonlinear cascade of energy to higher modes establishes steady state only in the region $k < k_*$, where k_* is a cut-off wavelength that depends on V_d , L_N , and k_0 (k_0 is the wavenumber corresponding to the largest scale of irregularity in the medium). For $k > k_*$ the dominant saturation mechanism is wave-enhanced diffusion of electrons across field lines, which is equivalent to an increase in the effective electron-neutral collision frequency. The enhanced collision frequency shuts off the instability and establishes its saturation level. On the basis of this theory, the density perturbation associated

with saturated two-stream waves propagating in the direction of primary current above 100 km in the auroral E region for V_d somewhat greater than C_s can be shown to be

$$A_I = (\Omega_i / \nu_i) (1 - C_s / V_d) \quad [2.29]$$

The perturbation decreases monotonically to zero at the boundary of the growth cone. However, it is still possible to generate waves at large angles to the direction of the primary current by the mechanism of Sudan et al. [1973].

This does not necessarily lead to isotropic turbulence.

The nature of the interaction between primary waves in the two-stream regime and secondary waves remains largely unexplored.

CHAPTER 3

THE EXPERIMENT

3.1 Introduction

The data used in this study were collected with the DNA Project 609 398-MHz radar system. The site of the radar at Homer, Alaska (59.72 deg N, 151.53 deg W) afforded broad visibility of the auroral zone over central Alaska. The same radar was used for transmitting and receiving so that the whole system was monostatic. A campaign at the Homer site was typically several days in duration, with actual data collection limited to occasional (~ once a day) periods of pronounced activity. These periods could last from a few minutes to several hours.

In this chapter, the capabilities of the radar are discussed. The operation of the radar is examined in detail with the aid of a representative data record.

3.2 The radar

The phased-array antenna was installed at the Homer auroral radar facility in the winter of 1972-1973. Originally conceived as a diagnostic tool for studying changes in the upper atmosphere brought about by nuclear explosions, it was readily adapted to the task of tracking naturally occurring structures in auroral ionization. The antenna ceased operation in 1978.

The most valuable features of the Homer radar were its

electronic beam forming and beam steering capabilities. A photograph of the antenna at the Homer site is shown in Fig. 3.1. It had a radiating face consisting of a planar array of 780 short-backfire (dipole-excited, open-ended cavity) radiators, each phase-controlled through a 4-bit phase shifter by a PDP-11/20 general-purpose computer. In a phased-array antenna, the beam is formed by adjusting the phases at the individual radiating elements to form a cone of maximum interference in an assigned direction. On reception, those signals arriving from the direction in which the phased array is directed undergo constructive interference and exit the receiver at an increased power level. The beam is steered electronically by generating phase gradients across the face of the array. In the Homer system the electronic beam steering could be performed through 20 deg in both azimuth and elevation about the central boresight setting of the planar array. Within this limited solid angle the antenna was capable of a very rapid scan rate compared to conventional systems.

The radar was operated at a frequency of 398 MHz and had a total antenna gain of 36 dB. The polarization was vertical and the beamwidth was 2.3 deg. The peak power for transmit was 50 kW. The central boresight of the array could be steered mechanically over the full hemisphere. The feed system was a centre-fed-delay-line compensated coaxial distribution system using directional couplers and hybrid dividers/combiners. The more relevant

Fig. 3.1 The Homer 398 MHz phased-array antenna.

characteristics of the system for the current study are summarized in Table 3.1. Detailed descriptions have been given by Presnell et al. [1972] and Presnell [1974].

Table 3.1

Operating characteristics of the Homer phased-array radar

Frequency	398 MHz
Peak power	50 kW
Pulse length	300 microseconds
Pulse repetition frequency	75 pulses/s
Antenna diameter	20.7 metres
Antenna beamwidth	2.3 deg
Antenna gain	36 dB
Antenna element	Short backfire radiator
Polarization	Vertical
Electronic scan in azimuth and elevation	+/- 20 deg
Mechanical steerability	Full hemisphere
Receiver bandwidth	36 kHz
Receiver dynamic range	80 dB approximately

3.3 Description of the experiment

The fundamental measurement made by the radar was of backscattered power as a function of time, beam position, and echo range. The position of the beam was designated by the azimuth and elevation settings of the centre of the beam, with azimuth referenced to geographic true north and elevation to the plane of the local horizon. Range information was obtained by pulsing the transmission and alternating the radar between transmit and receive modes, i.e. a pulse of finite length (300 microseconds) was transmitted, then the power of the signal backscattered from the pulse recorded as a function of time delay.

Because velocity retardation is negligible in the ionosphere at UHF frequencies, the delay represents a direct measure of the slant range to the scattering centres. The actual detection of signal was performed as follows. For each beam position three pulses were transmitted in succession. The auroral echoes from each pulse were received and heterodyned down to an IF centre frequency. The detected signal was passed through a logarithmic amplifier and sampled by an A/D converter to provide readings of power level at regularly-spaced time intervals. The readings for the three pulses were summed and averaged by the computer and the result stored on magnetic tape. Thus, a complete data record consists of diagnostic information, such as the azimuth and elevation of the beam position, and a one column array of power indexed by time delay. Most of the entries in the power array correspond to background noise, but sometimes the power level was greatly enhanced, indicating the occurrence of radar aurora.

An example of a data record from an active period is shown in Table 3.2. The upper line lists the time of pulse transmission (Day 91, 08 hr 57 m 40.364 sec UT), the azimuth and elevation settings of the beam (22.4 and 10.1 deg respectively), and the dwell time, i.e. the number of pulses emitted per beam position (3). The 51-element data block constitutes the power array. The first entry lists the average logarithmic power backscattered over the three

pulses from 300 km slant range. The time delay to each successive entry was 67 microseconds, corresponding to a range gate length of 20 km. The pulse length, $\tau = 300$ microseconds, corresponds to a range resolution of $c\tau/2 = 45$ km. The 51 entries thus represent the power backscattered from 45-km long sampling volumes centred on slant ranges from 300 to 1300 km with a step in slant range of 20 km. In this sample significant backscattered power was received over the 420 - 500 km slant range interval. The pulses were separated by 13.3 milliseconds and the collection period for each data record was .04 seconds. A complete data record includes entries (not shown) representing switching noise and calibration data.

Table 3.2 - A representative data record

91085740.364					22.4	10.1	3		
24	14	14	22	27	38	114	146	109	206
151	10	27	15	33	29	29	25	15	20
26	13	21	20	31	12	26	15	18	23
15	20	33	22	11	19	40	20	18	17
13	21	09	29	19	40	17	27	24	22
11									

The aim of the experiment was to monitor radar auroral activity over an extended area. To achieve this, the antenna beam was steered electronically over a two-dimensional grid of beam positions. At each position, a data record analogous to that of Table 3.2 was compiled. The scan sequence was standardized. The beam steering was always begun at the westernmost azimuth and the highest

elevation. The scan was performed in elevation before stepping in azimuth. The angular steps in both azimuth and elevation were 1 deg. The scan extended over 41 azimuth points (40 deg azimuthal sector) and 13 elevation points (12 deg elevation sector). A data set from a complete scan consists of $41 \times 13 = 533$ data records. With a dwell time of 3, a scan required about 17 seconds; with time for writing data onto tape, a scan could be repeated in about 22 seconds.

The geographical coverage provided by the electronic scan is indicated by the fan-shaped region over central Alaska shown in Fig. 3.2. Because of magnetic aspect conditions, the azimuthal sector was usually centred on or near the magnetic meridian at 24.5 deg E true azimuth. The boresight was invariably set at an elevation of 7 deg. The electronic scan in elevation allowed coverage of the E region from 62 to 70 deg magnetic latitude.

In usual operation, backscatter data was recorded continuously as the radar was repeatedly swept over a fixed region, leading to a time series of auroral activity. Often, because of equipment requirements or other considerations, the scanning was interrupted, causing gaps to appear in the scan record. Moreover, in some campaigns (notably those in 1978) it was decided to monitor echo activity at extremes of azimuth (>20 deg from the magnetic meridian). Since the electronic beam steering was limited to an azimuthal sector of 40 deg centred on the array

Fig. 3.2 Geographical coverage of the Homer phased-array radar.

boresight, this necessitated mechanical steering between boresight settings, with requisite time losses.

3.4 Periods of data collection

Table 3.3 lists the periods of echo activity considered in this study: There are gaps in some of these records, which, for clarity, have not been indicated.

Table 3.3 - Periods of data collection

<u>Year</u>	<u>Date</u>	<u>Day number</u> UT	<u>Begin time</u> hr/min/sec UT	<u>End time</u> hr/min/sec UT
1973	March 20	79	083517.565	105250.364
1973	March 20	79	114748.252	141501.765
1973	March 30	89	072552.297	083447.535
1973	March 30	89	113317.255	125705.452
1973	April 1	91	085727.001	093944.877
1973	April 3	93	093743.071	095659.521
1976	February 27	58	155813.024	174739.156
1976	February 27	58	175458.398	193210.462
1978	February 27	58	044526.834	062215.038
1978	February 28	59	063843.267	085851.718
1978	February 28	59	090753.435	111954.435
1978	February 28	59	112938.081	133659.895
1978	March 1	60	065249.527	092346.242
1978	March 1	60	093312.191	185914.744
1978	March 1	60	190215.766	200303.015
1978	March 3	62	081958.930	094537.956
1978	March 3	62	095111.567	114859.674
1978	March 3	62	120025.754	133919.510
1978	March 4	63	090229.127	095327.182
1978	March 4	63	104743.466	111000.456
1978	March 5	64	083659.537	104358.672
1978	March 5	64	111523.502	133113.093

CHAPTER 4

ANALYSIS

4.1 Introduction

The data from the Homer experiment can be applied to the study of many features of radar auroral echoing. Contributions have been made in interpreting radar auroral echo signatures in terms of auroral substorm concepts (Tsunoda et al., 1974; Tsunoda and Fremouw, 1976a,b), relating radar aurora to visual aurora and ionospheric current systems (Tsunoda et al., 1976a; Tsunoda et al., 1976b), investigating the Doppler spectra of radar aurora (Tsunoda, 1976; Moorcroft and Tsunoda, 1978), and characterizing the properties of radar aurora in terms of plasma instability processes (Tsunoda, 1975; Moorcroft, 1979; Moorcroft, 1980). The current work represents the first concerted effort to extend the study to altitude characteristics.

In this chapter, the analysis applied to the Homer data to extract information on altitude is described. The complete radar equation for the phased-array antenna is developed. Echoing 'height' is defined and its physical significance clarified. The sources of error in the determination of height are discussed. The importance of short-term amplitude variation to the accuracy of the results is noted.

4.2 The radar equation

The problem is to relate the power transferred from a transmitting antenna to a receiving antenna, assuming that the antennas are in free space and are separated from the scattering volume by large distances. The reflectivity of the medium is characterized by the effective volume cross-section for scattering, σ_v (sec. 2.2). The increment to the power density at the receiving antenna, dP , due to scattering from a volume element dV is given by (e.g.

Antenna Engineering Handbook, 2nd edition)

$$dP = \frac{G_t P_t}{4\pi R_t^2} \frac{\sigma_v}{4\pi R_r^2} dV, \quad [4.1]$$

where P_t is the power emitted by the transmitting antenna, R_t and R_r are the distances separating the transmitting and receiving antennas from the scattering volume element, and G_t is the gain of the transmitting antenna in the direction of dV . The increment to the received power, dP_r , is equal to the product of dP and the effective aperture, A_e , of the receiving antenna, i.e.

$$dP_r = dP A_e = \frac{G_t P_t}{4\pi R_t^2} \frac{\sigma_v}{4\pi R_r^2} \frac{G_r \lambda^2}{4\pi} dV, \quad [4.2]$$

where G_r is the gain of the receiving antenna in the direction of the scattering volume element and λ is the sampling radar wavelength.

In a monostatic radar system $G_t = G_r = G'$ and $R_t = R_r = R$. For a phased-array antenna the gain and beam shape depend on the off-boresight angle. When the antenna looks

at an angle ρ away from the boresight direction the gain is $G' = G_0 \cos(\rho)$ (where G_0 is the gain when the beam is directed along boresight) and the beam is broadened in elevation by a factor of $1/\cos(\rho)$. With the appropriate substitutions and inclusion of the beam broadening factor, eq. [4.2] becomes

$$dP_r = \frac{G_0^2 \lambda^2 \sigma_v}{(4\pi)^3 R^4} P_t \cos^3(\rho) dV \quad [4.3]$$

Within the antenna beam, the power received from a given direction is proportional to the two-way or 'effective' gain $G(\theta, \phi) = G_0^2(\theta, \phi)$, where θ is the angular distance measured from the beam axis and ϕ is the azimuthal angle measured with respect to a vertical plane containing the beam axis. In terms of the effective gain, we have finally, for the increment to the returned power due to scattering from a volume element dV , the expression

$$dP_r = \frac{G(\theta, \phi) \lambda^2}{(4\pi)^3 R^4} \sigma_v P_t \cos^3(\rho) dV \quad [4.4]$$

A backscatter radar detects returns over the entire volume defined by the radar pulse in the scattering region. There may be considerable variation in the value of σ_v within this volume. Introducing loss terms, the most general expression for the power received at the radar is

$$P_r = \int_V dV dP_r = \frac{\lambda^2 P_t \cos^3(\rho)}{(4\pi)^3} L_T L_R L_p \int_V dV \int d\Omega \frac{G(\theta, \phi) \sigma_v(\theta, \phi, R, t)}{R^2} \quad [4.5]$$

where L_T , L_R , and L_p are the losses between the transmitter output and free space, between the antenna face and the signal detector, and along the propagation path. In eq.

[4.5] the dependence of σ_v on position within the sampling volume and on time has been made explicit.

The basic measurement available for analysis is P_r . Our aim is to extract information on the altitude variation of σ_v from the variations of P_r .

4.3 Analysis of altitude

For the purpose of resolving the primary features (i.e. mean height, vertical extent) of the altitude distribution of the radar auroral scatterers, it is not necessary to solve eq. [4.5] in complete detail. In this section, the analytical procedure for determining the mean height of echoing in the Homer data is explained and justified. The basic method is to isolate the altitude variation of σ_v by making comparative measurements of power, P_j/P_i , that have no zero-order dependence on the horizontal and temporal variations of σ_v .

As the Homer radar stepped in azimuth and elevation, backscattered power was recorded as a function of slant range. The scan sequence could be taken to define a range-azimuth grid composed of cells with dimensions set by the sampling intervals of 20 km in slant range and 1 deg in azimuth angle. With temporal variability minimized by the rapid antenna scan rate, the backscattered power at each cell was a function only of elevation. Echo height is determined by converting the elevation angle of maximum backscattering into altitude. The end product of the

analysis for each scan is a map expressing echo altitude as a function of position over the range-azimuth grid.

The steps in the height algorithm are most simply illustrated for the case of backscattering from a thin, stable, sheet-like layer. This situation is depicted in Fig. 4.1 for a planar geometry. The antenna beam is shown intersecting the echo layer at two settings of elevation. We are specifically interested in estimating the height of the portion of the echo layer within the slant range interval shown bracketed by the two curved lines. The appearance of the layer in beam cross-section at this range, for two beam positions separated by 1 deg in elevation, is illustrated in the lower diagram. The layer uniformly fills the beam in the horizontal plane but is of infinitesimal extent in the vertical dimension. (Because of the inclination of the radar beam to the horizontal and the 45 km length of the Homer sampling volume, the image of the layer in beam cross-section would usually be spread over a finite range of elevation angle. For the present discussion, the spreading is taken to be negligible.) The relative strengths of the backscatter returns at the two beam positions can be used to estimate the elevation of the echo layer at the stated slant range, and hence the height of echoing.

The important consideration for the determination of height is the sensitivity of the antenna to the angular position of the layer in the beam. This can be found from

Fig. 4.1 (Top) Observing geometry at Homer for a thin, sheet-like scattering layer (not to scale). For clarity, the beam is shown at only two settings of elevation angle. (Bottom) The 'views' of the thin layer in cross-section in the beam at the upper (left) and lower (right) beam positions, for a beam separation of 1 deg.

the gain function of the antenna for a point reflector, $G(\theta, \phi)$, by integrating the contributions to the backscattered power from all points at the same height, as described in Appendix A. The antenna gain function for a thin layer is shown in Fig. A.2. It is azimuthally symmetric with a half-power width of 1.6 deg.

Of course, the volume backscattering cross-section, σ_v , is in general a function of all three spatial coordinates and of time. The analysis of its altitude dependence is simplified in the following way. We assume that the relative strengths of backscatter returns collected within a scan sampling period from volumes separated only in elevation are dominated by the altitude variation of σ_v . That is, at each cell in the range-azimuth grid σ may be expressed as a function only of height, z , for each scan. Justification of this assumption follows from consideration of the scale lengths (>50 km in the horizontal dimension, 3-20 km in the vertical dimension) and times (~ 1 min) of the auroral scattering phenomenon and the consistency of the results of the analysis.

The essential dependence of the returned power on elevation at a range-azimuth cell can be expressed as

$$P[z_i] \sim \int W(e[z]) \sigma[z] dz, \quad [4.6]$$

i.e. the power, $P[z_i]$, backscattered from a Homer pulse centred on height z_i is proportional to the convolution of the effective gain function, $W(e[z])$, with the effective

backscattering cross-section, $\sigma[z]$, over the height interval sampled by the radar pulse. ($e[z] = (z-z_i)/R$ is the angle to the height z measured off the beam axis centred on height z_i and slant range R .) The integration nominally extends from $z = 0$ to $z = \infty$ but the integrand is non-zero only at E region heights. In principle, it is possible to invert this equation for $\sigma[z]$ by deconvolving the antenna weight function from $P[z_i]$, but direct inversion cannot be carried out because of practical considerations (e.g. usually only two or three $P[z_i]$ values at a range-azimuth cell represent signal).

For the situation depicted in Fig. 4.1, the altitude profile of the effective backscattering cross-section approaches a delta function (i.e. $\sigma[z] = \sigma \delta[z-z_0]$) and

$$P[z_i] \sim W(e[z_0]) . \quad [4.7]$$

The height, z_0 , of the scattering layer can be deduced from two (or more) measurements of signal, $P[z_i]$. The procedure is outlined graphically in Fig. 4.2. The backscattering profile for a thin layer is shown as a narrow, shaded rectangle on a background of intersecting antenna sensitivity curves. The curves correspond to overlapping beam patterns separated by 1 deg in elevation angle. Since the effective beam half-width is .8 deg, the echo layer is illuminated at both beam positions and comparable levels of backscattered power are recorded. For a given position of the layer, the relative echo strengths expected for the two beam positions can be found from the intercepts with the

Fig. 4.2 Backscattering profiles for thin (narrow, shaded rectangle) and thick (wide, unshaded rectangle) layers on a background of intersecting antenna sensitivity curves, corresponding to echo returns from an interval of elevation angle between the settings of the two beam positions.

sensitivity curves. Conversely, two suitable observations of echo strength can be compared with the effective antenna gain function and the elevation of the echo layer, $e[z_0]$, determined. The altitude of the echo layer then follows from the slant range of the returns. Complications due to refraction in the atmosphere and other effects are discussed in Appendix B.

Returning to the more general expression for received power, eq. [4.6], it may be asked how echo height is to be defined when the backscattering cross-section has a finite vertical extent, possibly with structure. In this regard it should also be noted that the Homer beam was always inclined to the stratified layers of the earth's atmosphere. Consequently, the beam cross-sectional image of even a thin layer in a Homer sampling volume would be spread over a finite range of elevation angle (1.2 deg at 500 km, .7 deg at 750 km, .45 deg at 1000 km slant range). Because of this smearing, the image of a thin layer in a sampling volume simulates the appearance of a thick layer to which eq. [4.6] applies. The smearing distorts the higher order variations of backscattered power with elevation that would otherwise be the signatures of thickness and structure. Because of this complexity we deal here with the altitude properties of an effective layer, one that incorporates the effects of both a finite echo layer thickness and the inclination of the Homer beam to the horizontal. The point is to demonstrate the

validity of the concept of mean echoing height in the most general case.

The consequences of an apparent echo layer thickness on the Homer power measurement can be modelled numerically. The large, unshaded rectangle in Fig. 4.2 represents a backscattering profile for a thick layer, one that is uniform in a limited height interval. The relative strengths of the backscatter at the adjacent beam positions are now found by integrating over the layer profile with weight functions determined by the overlapped portions of the sensitivity curves. Eq. [4.7] becomes

$$P[z_i] \sim \int_{z_c - \Delta z R/2}^{z_c + \Delta z R/2} W(e[z]) dz, \quad [4.8]$$

where z_c is the altitude of the centre of the layer and $\Delta z R$ is its apparent thickness. Because of its two-parameter dependence, the solution of this equation for z_c and $\Delta z R$ requires at least three measurements of signal, $P[z_i]$.

As a defining expression for the mean height of backscattering, z_c , eq. [4.8] suffers the disadvantage of being soluble only in instances of particularly strong echo activity. It may therefore be advantageous to apply the thin-layer algorithm, eq. [4.7], to the Homer data and suitably interpret the resulting 'height' measurement, z_0 . In fact, no significant error is incurred if z_0 is adopted as an approximation to the mean height of backscattering, $\langle z \rangle$, from a layer described by an arbitrary apparent

backscatter profile, $\sigma(z)$, where

$$\langle z \rangle = \int z \sigma[z] dz / \int \sigma[z] dz, \quad [4.9]$$

provided the apparent thickness of the layer, which may be parameterized by Δz , where

$$(\Delta z)^2 = \langle (z - \langle z \rangle)^2 \rangle = \int (z - \langle z \rangle)^2 \sigma[z] dz / \int \sigma[z] dz, \quad [4.10]$$

is not much greater than the beam diameter in the scattering volume. In practice, this is always the case. Consequently, we choose to calculate echo height according to the thin-layer model. It is possible to make estimates of the thickness of the echo layer by removing the smearing effect from Δz but the uncertainties are very large. Direct modelling of echo layer thickness is not discussed further in this thesis.

4.4 Discussion of factors influencing the height of radar auroral echoing

The physical significance of the echo height measurement can be made clearer by considering the factors that control the amplitude of auroral backscatter. The effective volume cross-section for backscattering defined in eq. [2.10] can be written as (e.g. Uspensky et al., 1983; Starkov et al., 1983)

$$\sigma_v \sim N^2 \langle (\Delta N(2k)/N)^2 \rangle F(\alpha) G(\theta), \quad [4.11]$$

where N is the mean electron density, $\langle (\Delta N(2k)/N)^2 \rangle$ is the power spectrum of the electron density fluctuations evaluated at twice the radar wave number, $F(\alpha)$ is the

normalized aspect angle anisotropy function (aspect angle α is measured between the radar wave vector and the normal to the earth's magnetic field), and $G(\theta)$ is the normalized flow angle anisotropy function, expressing the dependence of the cross-section on the angle between the relative drift velocity vector V_d and k . The isolation of the anisotropy functions within this equation is possible if the aspect and flow sensitivities are independent of the power in the spectrum of the electron density fluctuations. The flow angle dependence was studied for 1 m auroral irregularities by Andre [1983]. It is second-order compared to the aspect dependence and should have only a minor effect on echo height. We set $G = 1$ in the following discussion.

For the reasons outlined in the previous section, the normalized backscattering cross-section at each cell in the Homer range-azimuth grid within a scan can be considered a function of altitude alone, i.e.

$$\sigma[z] \sim N[z]^2 \langle (\Delta N[z]/N[z])^2 \rangle F[z] \quad [4.12]$$

In this equation, the two factors in N represent intrinsic properties of the scattering medium, namely the presence of scattering irregularities within a background ionization of density $N[z]$. $F[z]$ is a modulation function expressing the dependence of auroral scattering on the magnetic aspect geometry of the observations.

It is important to note that the parameters $\langle z \rangle$ and Δz of eqs. [4.9] and [4.10] apply to the apparent scattering

layer as seen by the Homer radar. The parameters of the vertical distribution of irregularity intensity represented by $N[z]^2 \langle (\Delta N[z]/N[z])^2 \rangle$ may be quite different. This is because of the effect of $F[z]$ on $\sigma[z]$. In particular, aspect sensitivity may restrict radar visibility to a limited height interval within an irregularity region. The analysis of height in the Homer data pertains to that portion of the irregularity region that actually generated echoes in a set of observations. The analysis cannot be reliably extended to the altitude parameters of the intensity of the irregularities themselves, but the existence of altitude limits to the irregularity region and the influence of the aspect angle anisotropy function can be discerned in the gross properties of the mean height of backscattering.

It is worth remarking that the term $\langle (\Delta N(2k)/N)^2 \rangle$ in eq. [4.12] represents the mean relative intensity of a particular spatial Fourier component of the electron density fluctuations. The altitude variation of this factor depends on complex nonlinear saturation mechanisms. If it may be disregarded as a first approximation, the altitude dependence of σ_v is determined by the electron density term $N^2[z]$ and $F(\alpha)$. Generally, we expect backscattered power from an irregularity region to be weighted according to spatial variations in electron density. There is very little matching electron density data available for analysis with the Homer data and no

attempt has been made to directly model its effect on echo height.

In summary, the term 'height' refers to the altitude of the centre of gravity of backscattered power as determined from direct measurement of echo strength, except where explicitly stated otherwise. The heights of the radar auroral echoes seen by the Homer radar will be interpreted in terms of control by the altitude distribution of small-scale electron density irregularity and the magnetic aspect dependence of the coherent scattering process.

4.5 Discussion of errors

The discussion to this point has indicated that the altitude dependence of the Homer echo activity can be deduced solely from the variation of backscattered power with elevation. If so, echo height can be determined independently at each range-azimuth cell for each scan. However, the results are not satisfactory, owing to rather large uncertainties that have the appearance of random error on the finished heights. To improve the reliability of the height measurement it is necessary to perform a certain amount of additional processing. The purpose of this section is to examine the sources of error on the determination of echo height and to describe the processing applied to the data to obtain the results reported in Chapter 5.

A source of inaccuracy common to all radar systems is the erroneous association of signal from a side lobe of the antenna with the pointing direction of the main beam. Contributions from side lobes would adversely affect the determination of Homer echo height by distorting the variation of backscattered power with main-beam elevation. To evaluate this, we note that the first side lobes of the Homer antenna were located 4 deg away from the axis of the main beam at a level of 24 dB for a main-beam pointing angle of 10 deg to broadside (Presnell et al., 1972). Since the scattering region spanned less than 2 deg of elevation at the radar, sidelobe contamination in this dimension was negligible. Sidelobe contamination in the azimuthal dimension cannot be entirely ruled out, however. When looking at large (>30 deg) angles to the geomagnetic meridian the radar occasionally registered strong returns that were clearly due to intense echo activity in the forward direction sampled by the sidelobes. There is no systematic correction for this effect but the depth of the side lobe levels was such that the bulk of the observations should have been free of any form of side lobe contamination.

An error is incurred on the determination of echo height if non-altitude variations of σ_v significantly influence the variation of backscattered power with elevation. The importance of this effect depends on the relative magnitudes of the spatial gradients of

reflectivity within Homer sampling volumes and on the temporal variability of reflectivity between sampling times. Only the range gradient factor is discussed here, as the azimuthal and temporal factors are not of practical significance.

The effect of a range gradient of reflectivity on the determination of echo height should be most pronounced for discrete radar aurora. In the extreme case, σ_v is non-zero in a range interval that is small compared to the resolution of the Homer radar pulse. The Homer signature of the echoing would be spread over at least 45 km of slant range and there may be measurements of height in several 20-km range gates at a given azimuth. The near edge of the echoing form would appear somewhat low (because of a tendency for sampling volumes at lower elevation to sample the region of finite σ_v more favourably) and the far edge somewhat high. The systematic height variation amounts typically to a rise in height of 5 km over 60 km of slant range. This effect is readily identified in the heights of echoing forms that are limited in slant range extent to the finite length of the Homer pulse.

The heights of more extended forms may still be conditioned to some degree by range variations of reflectivity. In a range interval of positive σ_v -gradient the tendency is for echo height to be underestimated, in a range interval of negative σ_v -gradient the tendency is for height to be overestimated. Of course, in a region of zero

σ_v -gradient no error is incurred. It is difficult to assess the significance of this factor a priori. However, the Homer observations were dominated by events of diffuse radar aurora which are characterized by a high degree of horizontal spatial uniformity (Unwin, 1959; Tsunoda et al., 1974). We also note that echo heights conditioned by range gradients of reflectivity would manifest all the variability of those gradients. Such variability is not generally observed. The predominance of altitude factors over range factors can be firmly established on this basis.

An error is incurred on the determination of mean echoing height using the thin-layer algorithm if the apparent echoing layer is not thin. The amount of error can be estimated on the basis of simple numerical models; it is typically less than a few tenths of a kilometre and should have no serious consequences for the accuracy of the results reported here.

A severe limitation on accuracy is imposed by the random amplitude fluctuation, or fading, of the auroral echoes. It will be immediately apparent that any degree of statistical randomness in the level of backscattered power will degrade the results of the height analysis. The effect of noise fluctuation is minimized by restricting the calculation of height to instances of strong echoing (generally when the signal-to-noise ratio exceeded 5 dB over 2 deg of azimuth for one minute); however, fading is an unavoidable source of signal variability. Fading is

caused by interference at the radar of wavelets scattered by different parts of the auroral ionization. With movement of the scattering centres the relative phases at the radar are altered and the signals changed. In the Homer experiment, fading caused the pulse-to-pulse echo intensity from a given scattering volume to fluctuate about its mean level very rapidly. As discussed in Appendix C, the echo intensity became completely decorrelated in less than the pulse repeat time of 13 milliseconds. The uncertainty introduced on the determination of the mean elevation angle of backscattering by this effect corresponds to a standard error of about .5 deg. Consequently, the standard error on the echo height measurement is

$$\Delta z_p = (\pi/360) \times R, \quad [4.13]$$

i.e., 4.5 km at 500 km range, 9 km at 1000 km range. This is large compared to the 20-km vertical scale length of the echo region and renders the echo height measurement almost meaningless without further processing. Satisfactory results are obtained if height information is averaged first in azimuth over adjacent cells in the range-azimuth grid, then averaged in time by summing at each cell over several scans, and finally averaged in range over adjacent cells. Assuming that the altitude structure of the echo region is invariant over these spatial and temporal scales, the accuracy of a processed echo height, measured in terms of standard error, is

$$\Delta z = (\pi/360) \times (R/3) N^{1/2} \quad [4.14]$$

where N is the number of scans involved in the temporal averaging. Usually N = 3 and the error on processed heights varies from about 1 km at near ranges (~ 500 km) to about 2 km at far ranges (~ 1000 km).

Possible sources of systematic error on the echo heights include inaccuracies in the code for correcting height for tropospheric refraction and oblateness of the earth effects. The error estimates are small (<.5 km), as discussed in Appendix B.

CHAPTER 5

OBSERVATIONS

5.1 Introduction

In most observations of radar aurora, no height information is available and the altitude properties of the echo activity must be assumed for the purposes of analysis. The preferred model appears to be that of scattering from thin (<5 km) layers situated at a mean height of 110 km, following the results of Unwin [1959]. The results presented in this chapter indicate that the situation is more complex and variable. There may be serious consequences for the theory of coherent scatter in the auroral E region.

To investigate the magnetic aspect factor, we have adopted the GSFC/980 model of the earth's magnetic field. This model involves an expansion in spherical harmonics (e.g. Chapman and Bartels, Geomagnetism, 1940) and incorporates the effects of secular variation.

The source regions for radar aurora usually have boundaries that align with lines of force of the earth's magnetic field. This dependence is conventionally described in terms of the L shell coordinate system of McIlwain [1961]. Lines of constant magnetic L locate the intersection with the surface of the earth of field lines whose distance above the equator from the centre of the earth is L earth radii. At a given altitude, contours of

constant L shell define effective parallels of magnetic latitude.

Despite the great diversity of radar auroral events observed by the Homer radar, we have found that the altitude properties of the echo activity can be characterized generally. The results reported in this chapter are felt to be representative of all the data studied.

5.2 Examples of maps of Homer echo height

The first examples of maps of echo height, shown in Fig. 5.1, are taken from the evening of April 1, 1973. The content of these maps will be analyzed after a preliminary discussion of the maps themselves.

The range-azimuth format corresponds to a two dimensional sweep of the radar beam over central Alaska (Fig. 3.2). For this period, the 40 deg azimuth sector was centred on 17 deg N. Although backscatter returns were recorded over a 1000 km slant range interval extending from 300 to 1300 km, echo height could be determined only for slant ranges from 400 to 1060 km owing to the limited scan in elevation. There is no direct information about power in these maps, except insofar as the signal had to exceed a threshold level for height to be calculated. The scan times are shown below each map. Magnetic local time (MLT) at Homer lags Universal Coordinated time (UT) by 11 hr 30 min, so these examples belong to the magnetic pre-midnight

Fig. 5.1 Maps of radar auroral echo height for
consecutive 1 min periods (April 1, 1973, 0909-
0911 UT). For a discussion of these maps, see
text.

Fig. 5.1

sector. Echo height is represented by a simulated grey scale with a 3 km contour interval. The data processing involved averaging height information over 2 deg of azimuth, 40 km of range, and three successive scans, as outlined in sec. 4.5. Height is determined independently at cells separated in slant range by 60 km or in azimuth by 3 deg. The averaging over scans gives the processed height data an integration period of about 1 minute. The rectangular appearance of boundaries and contours is due to the finite (20 km by 1 deg) dimensions of the sampling cell.

This period featured widespread and intense echo activity. The event cannot be characterized by a single value of altitude; over different areas the centre of echo activity varied from 98 to 115 km. Within each map echo height varies systematically with range and azimuth and the overall distribution of height shows marked east-west and north-south consistencies. Changes in detail can be followed from one map to the next. For example, the high altitude feature along the poleward edge of the broad band of echoes shows signs of a westward shift. We conclude that the assumptions underlying the analysis of altitude in the Homer data are sufficiently satisfied to permit reliable measurement of the height of spatially localized radar aurora.

The signatures of radar aurora as seen by the Homer phased-array radar have been categorized by Tsunoda et al.

[1974]. The 'diffuse band' (DB) echo type is geographically widespread with an extent in slant range that exceeds the 45 km resolution of the radar pulse. The broad band of echoes in Fig. 5.1 in the 400-700 km slant range interval is an example of this type. The 'discrete arc' (DA) echo type is characterized by a narrow width that approximately matches the radar range resolution. Portions of the echoing form in Fig. 5.1 in the 900-1000 km slant range interval conform to this definition. The DB and DA echo types are both extended in the magnetic east-west direction in a manner corresponding to alignment with contours of constant magnetic L shell. The orientation of the L shell contours over this viewing area is indicated in Fig. 5.2. A third echo type, the 'discrete patch' (DP), is characterized by limited extents in both the north-south and east-west directions, such as would be obtained if a DA broke up into isolated centres of echo activity. There is no clear example of this echo type in Fig. 5.1. These definitions are ambiguous to the extent that one echo type passes into another as the threshold for detection is varied.

5.3 Characteristics of radar auroral altitude

The height of echoing of radar auroral signal depends first of all on the altitude distribution of electron density irregularities of the appropriate scale size. It has been noted (sec. 4.4) that aspect sensitivity may also

Fig. 5.2 Geometry of L-shell contours for a height of 100 km over the field of view of the Homer radar depicted in the maps of Fig. 5.1.

be important. Maps of the spatial distribution of Homer echo height (e.g. Fig. 5.1) contain information about these factors. In this section, the vertical extent of the irregularity region is established and the question of aspect control of echo height is addressed.

It will be useful to discuss the properties of Homer echo height in several representative periods; the results can be generalized. The first period selected is pre-midnight (0857-0940 UT) from April 1, 1973. Magnetograms recorded at College, Alaska indicated the presence of an intense eastward electrojet in the field of view of the Homer radar. The radar auroral event consisted of strong DB activity at near (<700 km) ranges and weaker DA and DP echoing at intermediate (700-900 km) and far (>900 km) ranges. The cell-by-cell averaging of echo height over the 108 scans conducted during this period results in the map shown in Fig. 5.3. (The number of counts contributing to the calculation of average height at each cell is not indicated but can, to some extent, be inferred from the regularity of the height contours. In this period, the rates of echo occurrence in the 400-600 slant range interval reached 100%.) The 97-114 km range of average echo height indicates an irregularity region almost 20 km thick. There are pronounced systematic variations in the spatial distribution of height that require explanation.

Fig. 5.4 shows a map of height for which the condition

Fig. 5.3 Average echo height over the 108 scans of a pre-midnight period (April 1, 1973, 0857-0940 UT: note error in label). Echo activity was continuous at near ranges (<600 km) and intermittent at far ranges.

Fig. 5.4 Height corresponding to zero aspect angle for the range-azimuth sector of Fig. 5.3.

Fig. 5.3

Fig. 5.4

of zero aspect angle is identically satisfied for the ambient magnetic field. This spatial distribution of height defines the zero aspect surface over the field of view of the Homer radar depicted in Fig. 5.3. In the absence of the aspect effect, there would be no reason why the map of average echo height of Fig. 5.3 should resemble the map of the zero aspect surface of Fig. 5.4. In fact, there are strong similarities, especially over the near and intermediate ranges. We would expect the aspect effect to be more pronounced with decreasing range because the spatial gradients of aspect angle increase toward the near radar horizon (see below). It can be seen that the height contours over the nearer ranges in Fig. 5.3 define a meridian that is displaced towards the east relative to the central azimuth setting of 17 deg; this is consistent with the positioning of the equivalent geomagnetic meridian at 24.9 deg, as indicated by the orientation of the contours in Fig. 5.4.

It has long been known that horizontal variations of aspect angle influence the probability of echo occurrence; Tsunoda et al. [1974], working with the Homer phased-array radar, found a steep decrease in occurrence with aspect angle for angles greater than about 1 deg, assuming a scattering height of 110 km. Representative gradients of aspect angle for the Homer viewing area are listed in Table 5.1. In absolute units the altitude variation of aspect angle always dominates. However, the

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Table 5.1Representative gradients of aspect angle

Slant range (km)	<u>500</u>	<u>760</u>	<u>1020</u>
Altitude: (da/dz)	1.12°/10 km	.72°/10 km	.55°/10 km
S/t range: (da/dR)	.12°/10 km	.0°/10 km	.08°/10 km
Azimuth: (da/ds)	.05°/10 km	.03°/10 km	.02°/10 km

(All gradients have been calculated for a height of 105 km. The altitude and slant range gradients apply to the geomagnetic meridian. The azimuthal gradients apply to azimuths 10 deg east and west of the geomagnetic meridian and are expressed in terms of the equivalent distance gradients.)

unambiguous signature of aspect control is a predictable variation of echo height with position; for this to be observed, there must be sizable horizontal gradients. Fig. 5.4 and Table 5.1 indicate that conditions are most favourable over the nearer ranges. Indeed, allowing for confinement of the echoing region to the 97-114 km height interval, the rise in height of the zero aspect surface with increasing slant range in the 400-700 km interval is closely matched by the experimental data of Fig. 5.3. Identification of aspect control involves a degree of ambiguity in the intermediate (700-900 km) slant range interval because of the flattening of the zero aspect surface in the vicinity of its peak, but the general rise and fall of echo height with slant range in Fig. 5.3 is clearly emulative of the aspect geometry. We conclude that the height of echoing of radar auroral signal in this period was strongly conditioned by altitude variation of aspect angle.

Aspect sensitivity is conventionally quantified in terms of an observed decrease of backscattered power per degree of aspect angle. Such determinations usually involve an added degree of uncertainty because of the lack of accurate height information. With the Homer data, it should be possible to estimate aspect sensitivity at a finer scale of resolution by comparing aspect angles for calculated echo heights to corresponding echo strengths. For the comparison to be valid, the echo strength must be

independent of all other factors. The stable DB activity of the pre-midnight period of April 1, 1973 appears to satisfy this requirement, provided aspect angles and echo strengths are compared only at fixed ranges. (The fixed range condition is necessary to minimize the effect of variations with magnetic latitude and to avoid the normalization problem encountered in making comparisons of backscattered power levels at different ranges when the amount of beam filling is unknown.) The results for one range are shown in Fig. 5.5. The average signal strength, expressed as a signal-to-noise ratio in decibels, is plotted as a function of average aspect angle for one of the ranges occupied by the band. The fit of the data to the solid line indicates that the figure of 10 dB/deg is representative of the bulk aspect sensitivity in this case. It must be noted, however, that the strengths of individual echoes do not always bear a simple relation to aspect angle. More seriously, perhaps, the apparent aspect sensitivity may be unrepresentative of the microscopic or 'true' aspect sensitivity. The difficulty in interpretation is made clear by the following discussion.

What model of the auroral scattering process can account for the spatial distribution of echo height seen in Fig. 5.3? The thin-layer model cannot explain the dependence of echo height on aspect geometry; if echoing were confined to a single height, that height would be recorded over the entire field of view. Detailed

Fig. 5.5 Variation of average backscattered power (in dB above noise level) with average aspect angle over a stable echoing band observed during the period of April 1, 1973 (0857-0940 UT). The data are taken from all azimuths at a fixed slant range. Multiple data points are 'exploded' to show the number of contributing counts. The solid line represents an aspect sensitivity of 10 dB/deg.

inspection of the maps in Figs. 5.3 and 5.4 over the near and intermediate ranges reveals the following features: where the zero aspect surface is below 107 km, average echo height is relatively higher; where the zero aspect surface is above 109 km, average echo height is relatively lower. Between 107 and 109 km average echo height and the height of the zero aspect surface coincide to within a kilometre. The rapid fall of average echo height with decreasing range towards the near radar horizon slows considerably in the 98-100 km height interval. These features are compatible with a model of echoing from a continuous range of heights centred on 107-109 km and extending from somewhat below 100 km to at least the highest individual echo height recorded, which was 117 km. The detail at the high altitude boundary cannot be unambiguously resolved because the greatest height that could be observed at zero aspect angle was 115 km; the lack of echo activity at greater height may be due to unfavourable aspect conditions. The systematic variation of echo height with aspect geometry is evidently due to the weighting of backscattered power from an extended irregularity region towards those heights corresponding to lower aspect angles.

If echo height is variable in this manner, the meaning of a radar measurement of aspect sensitivity where perpendicularity can be achieved in the echoing region (e.g. Fig. 5.5) requires elaboration. This is because

aspect sensitivity at the microscopic level will tend to shift the mean height of echo activity under a change of aspect conditions in such a way as to keep the level of backscattered power unchanged. Of course, shifting cannot be sustained beyond the altitude limits of the irregularity region. Overall there will be a decrease in backscattered power with aspect angle, as indicated in Fig. 5.5, but the precise relation to the microscopic aspect sensitivity will depend on other factors. This point is addressed further in sec. 5.5.

Returning to the analysis of radar auroral altitude in the selected period, it is clear that echo height in the map of Fig. 5.3 does not drop as steeply as the height of the zero aspect surface towards the near radar horizon because a boundary is encountered. The abruptness of the transition at the lower altitude boundary is apparent in the histogram of height for this period shown in Fig. 5.6. This histogram must be interpreted cautiously because of the strong selection effect deriving from spatial variation of both echo activity and aspect geometry. However, the intense DB echoing that dominated this period was ideally situated for accurate height measurement in the 90-100 km height interval under favourable aspect conditions, yet there was almost an order of magnitude drop in echo occurrence between 98 and 96 km. We conclude that the echoing region was sharply bounded from below at a height of about 97 km.

Fig. 5.6 Histogram of Homer echo height for the period of April 1, 1973 (0857-0940 UT). Each measure of echo height as defined in the discussion of Fig. 5.1 contributes a count.

It is interesting to compare the maps of Figs. 5.3 and 5.4 with maps of average echo height for the two periods of March 30, 1973. These are shown in Fig. 5.7. The College magnetogram indicated the usual reversal in electrojet direction from eastward to westward at local magnetic midnight (1130 UT). This was accompanied by a distinct lull in echo activity. The spatial distribution of average echo height for the pre-midnight period (0726-0835 UT) is almost identical to that of the pre-midnight period of Fig. 5.3. However, average echo height for the post-midnight period (1133-1257) was dramatically lower over much of the field of view. The average height data are limited to values of less than 104 km at near ranges (<700 km) and less than about 107 km at far ranges (the extreme north-east and north-west corners excepted). Boundaries are evident in the histograms of height for this period shown in Fig. 5.8. There is again a rapid decline in echo occurrence with decreasing height below 100 km. The steep decrease in echo occurrence between 103 and 105 km shown in the smaller histogram can be associated with an upper altitude boundary over the near ranges. The echoing in this period was confined to a restricted (97-107 km) height interval compared to that (97-117 km) of the pre-midnight periods. Within this restricted interval, aspect conditions at the nearest (<600 km) ranges still endowed echo height with a systematic spatial variation. The striking similarities and differences between these periods

Fig. 5.7 Average echo height for a second pre-midnight
(March 30, 1973, 0726-0835 UT) period (upper) and
for a post-midnight (March 30, 1973,
1133-1257 UT) period (lower).

Fig. 5.7

Fig. 5.8 Histograms of Homer echo height for the post-midnight (1133-1257 UT) period of March 30, 1973. The smaller histogram applies to height data at slant ranges of less than 700 km.

suggest that the height and vertical extent of the region of .4 m radar aurora have a strong diurnal variation.

Two further points will be noted here concerning the maps of Figs. 5.3 and 5.7. First, for the pre-midnight periods there was a distinct tendency for echo height to be greater in the east than in the west at far ranges. The magnitude of the east-west altitude asymmetry over a 20 deg azimuthal sector centred on the geomagnetic meridian is at least 3 km. This asymmetry is observed quite generally for events of the pre-midnight sector (i.e. eastward electrojet). Second, for the post-midnight period echo height was anomalously high towards the extremes of azimuth at intermediate and far ranges. These features cause the greatest deviation of the experimental height contours from the concentric pattern of Fig. 5.4.

5.4 Height of dynamical radar aurora

The Homer data constitute essentially a time series of radar auroral activity. In this respect they are a unique and valuable source of information on radar auroral dynamics. The evolution of particular events is studied in this section, with the aim of characterizing echo height under active conditions.

The first example is taken from the pre-midnight period of March 30, 1973. The four maps of Fig. 5.9 have been selected to show the development of radar aurora over the most dynamic part of this period. The event was

dominated by a pronounced movement of echo activity to lower magnetic latitudes. There is a systematic equatorward movement of the zones of auroral echo occurrence with magnetic local time during the afternoon and evening periods, but the rapid motion seen here is thought to be associated with a global disturbance phenomenon known as an auroral substorm (Tsunoda et al., 1976a). The first map shows a narrow DB at intermediate ranges accompanied by more distant patchy echo activity. Echo height across the DB averages 111 km and increases towards the east; the sense of the azimuthal variation is consistent with the aspect geometry (Fig. 5.4). The period was geomagnetically disturbed and the region of echo activity expanded. As shown in the second map, this expansion was accompanied by a bifurcation of the band, i.e. the emergence of a null interval in echo activity over a range of magnetic latitude. Echo height across the nearer part of the DB is depressed somewhat relative to that of the farther part. The third map shows the situation after 10 minutes. Apart from some DP activity only the nearer, equatorward moving part of the DB survives, with a continuing drop in height. The remaining part of the band reached its point of furthest equatorward expansion after 38 minutes, as shown in the fourth map. Echo height for the DB at this position varies from 98 to 109 km. Fig. 5.10 shows the heights corresponding to zero aspect angle for the same cells in the Homer range-azimuth

Fig. 5.9 Series of four 1-min maps of echo height from the pre-midnight event of March 30, 1973 showing large-scale movement of echo activity.

Fig. 5.9

Fig. 5.9 (cont.)

Fig. 5.9 (cont.) Final map showing the position of the echo activity at its point of furthest equatorward expansion.

Fig. 5.10 Matching heights of zero aspect angle for the diffuse radar auroral band of the final map of Fig. 5.9.

Fig. 5.9
(cont.)

Fig. 5.10

grid. It can be seen that the experimental echo height data follow these heights closely, excepting that they do not drop as steeply with decreasing range towards the near edge.

Fig. 5.11 shows two maps of an extended time series from another pre-midnight period (0840-0850 UT, March 5, 1978). Unlike the previous examples, the azimuthal sector of these maps is centred on the geomagnetic meridian near 25 deg N. The extensive echoing form seen in the first map is of the DB type. It is almost spatially coincident with the highest portion of the zero aspect surface (Fig. 5.4). Consequently, aspect conditioning of the spatial distribution of height is muted. The height variations in the 110-118 km interval over the DB may be evidence of small-scale structure in the irregularity region. Between the times of the first and second maps the near edge of the echo activity was observed to separate and move equatorward. As shown in the second map, the equatorward portion of the echo activity came to constitute a DA with an average altitude of 106 km. This is almost 10 km less than the average altitude of the DB. The DB and DA had a common origin, suggesting that the actual vertical distribution of small-scale density irregularity was the same for both. The implication is that the variable height of echo activity was a function solely of the observing geometry. The features of Figs. 5.10 and 5.11 indicate that the entire 97-118 km height interval was active with

Fig. 5.11 Two 1-min maps of echo height from the pre-midnight event of March 5, 1978. The discrete arc of the second map formed as a result of the detachment and equatorward movement of the near edge of the diffuse band.

Fig. 5.11

irregularities for all the positions of radar aurora in these periods, and that changing echo height was due to spatial variation of the aspect geometry.

Is it ever possible to observe echoing at heights not selected by the aspect effect? In the previous example (Fig. 5.11) there were spatial variations across the DB at intermediate ranges not attributable to the modulating effect of the aspect geometry. To illustrate the point that we must look to further ranges to see the influence of other factors, we note that along the geomagnetic meridian the zero aspect surface has the same height (~ 101 km) at slant ranges of 500 and 1020 km. However, at 500 km slant range an echo height of 110 km corresponds to an aspect angle of 1 deg, which means in effect that echo height at this range never exceeds 110 km. At 1020 km range the corresponding aspect angle is .5 deg. This is sufficiently small that aspect sensitivity does not cut off echo activity above 110 km. Thus, 'anomalous' echo height behaviour can be observed more readily, perhaps exclusively, at greater ranges.

The maps of Fig. 5.12 contain an example of a radar auroral event with unusual height variations. In the first map, there is a well-defined DA at intermediate ranges in the eastern sector with an average height of 111 km. A weaker arc element (DP) is apparent at greater ranges at a considerably lower height, approximately 104 km. (The DB at nearer ranges shows all the usual aspect conditioned height

Fig. 5.12 Series of four consecutive 1-min maps of echo height (April 1, 1973, 0924-0928). Note the differences in height between the arc-like forms in the eastern sector beyond the poleward edge of the broad band of echoes.

Fig. 5.12

Fig. 5.12
(cont.)

variations.) The second map shows the situation one minute later. Both discrete echoing forms have expanded westward. By the third minute, shown in the third map, both features have thickened in range. Despite the smearing of data in the east where their boundaries adjoin the weaker arc element remains measurably lower in height. By the fourth minute the discrete echo activity has expanded over a broad area. The final map shows that the original arcs are still distinguished by their different heights and that a new, high altitude arc has formed at greater ranges besides. The differences in height (as much as 10 km) are contrary to expectations based on the aspect geometry. The weakened aspect control presumably derives from a higher degree of structure in the distribution of the ionization responsible for the discrete echoes.

5.5 Discussion of aspect control of Homer echo height

In the preceding sections spatial variation of echo height was attributed to the modulating effect of aspect sensitivity on the scattering of radio signal within an irregularity layer of finite (10-20 km) vertical extent. With certain assumptions, this spatial variation can be modelled. The particular feature discussed here is the increase in echo height, $\langle z \rangle$, with slant range in the 400-600 km interval seen in the pre-midnight examples. Because of the parallel increase in height, z_0 , of the zero aspect surface, $\langle z \rangle$ can be expressed as a function of z_0 .

over this slant range interval. A plot of the variation of $\langle z \rangle$ with z_0 is shown in Fig. 5.13 for the period of April 1, 1973. The experimental data have been taken from the 11 range gates spanning the 400-600 km interval along the geomagnetic meridian. The increase of $\langle z \rangle$ with z_0 is linear above $z_0 = 98$ km with a dimensionless slope of about .65. Over the $z_0 = 106$ to $z_0 = 110$ km interval the values of $\langle z \rangle$ and z_0 nearly coincide, presumably because this altitude interval corresponds to the centre of the irregularity region. Below $z_0 = 98$ km the increasing divergence between the values of $\langle z \rangle$ and z_0 is attributable to the existence of a lower altitude boundary on the irregularity region.

It is of interest to reproduce the functional dependence of $\langle z \rangle$ on z_0 for assumed values of aspect sensitivity and thickness of the irregularity region. We recall that echo height in the current analysis is defined by (sec. 4.3)

$$\langle z \rangle = \int z \sigma[z] dz / \int \sigma[z] dz, \quad [5.1]$$

where $\sigma[z]$ is the effective altitude cross-section for backscattering. A simplified expression for σ is (sec. 4.4)

$$\sigma[z] \sim \sigma_n[z] F(\alpha[z]), \quad [5.2]$$

where

$$\sigma_n[z] = N[z]^2 \langle (\Delta N[z] / N[z])^2 \rangle, \quad [5.3]$$

i.e. the altitude variations of the mean electron density and mean square amplitude of the electron density

fluctuation have been incorporated into the single parameter σ_n . Substitution of eq. [5.2] into eq. [5.1] gives:

$$\langle z \rangle = \int z \sigma_n[z] F(a[z]) dz / \int \sigma_n[z] F(a[z]) dz . \quad [5.4]$$

To proceed further, it is necessary to adopt a form for the aspect angle anisotropy function. In the following, we assume that

$$F(a) = 10^{-Ba/10} , \quad [5.5]$$

where the decrease in echo intensity over 1 deg of aspect angle is B dB.

Eq. [5.4] still cannot be solved without knowledge of $\sigma_n[z]$. Although direct radar and rocket measurements can be made of the electron density profile $N[z]$, very little is known about the altitude variation of the power spectrum factor $\langle (AN(2k)/N)^2 \rangle$ at $k = 16.7 \text{ m}^{-1}$. We will consider two simple cases: i) $\sigma_n[z]$ constant in a height interval extending from a lower boundary at z_l to an upper boundary at z_u ; ii) $\sigma_n[z]$ Gaussian, centred on height z_c with standard deviation σ_g . In the former case $\langle z \rangle$ is given by

$$\langle z \rangle = z_l \int_{z_l}^{z_u} z \exp[-k|z-z_0|] dz / I_N(z_0) , \quad [5.6]$$

where

$$k = B \ln(10)/10 (da/dz) , \quad [5.7]$$

and $I_N(z_0)$, given by

$$I_N(z_0) = \int_{z_l}^{z_u} \exp[-k|z-z_0|] dz , \quad [5.8]$$

is a measure of the total backscattered power. Eq. [5.6] expresses $\langle z \rangle$ as a function of z_0 for given values of B, da/dz , z_l and z_u . The variation of z_0 with position

over the Homer range-azimuth grid gives rise to a predictable variation of echo height.

Fig. 5.14 shows plots of $\langle z \rangle$ vs. z_0 for $z_1 < z_0 < z_u = z_1 + 20$, four values of aspect sensitivity ($B = 3, 5, 10, 20$), and $da/dz = .11 \text{ deg/km}$ (appropriate to a slant range of 500 km). These are to be compared with the data plot of Fig. 5.13 only over the ranges of z_0 described in the figure caption. All four theoretical curves reproduce the qualitative features of the experimental plot, as expected. With decreasing B , $\langle z \rangle$ is a less and less sensitive function of z_0 . Over the $\langle z \rangle = z_0$ interval the slope of the curve with $B = 10$ is a close match to the slope of the data plot of Fig. 5.13. However, towards the upper and lower boundaries this curve levels off too sharply compared to the experimental data. The lack of a detailed correspondence is not surprising considering the simplicity of the model.

A point worth stressing is that the apparent aspect sensitivity, S (in dB/deg), that would be deduced from the variation of backscattered power with echo height is in general quite different from the aspect sensitivity, B , that entered the model. This is because the backscattered power is proportional to the value of the normalization integral $I_N(z_0)$ rather than to $10^{-Ba/10}$. For the case of $B = 10$, the backscattered power decreases with a shift of z_0 from the centre of the layer ($z_0 = (z_1 + z_u)/2$) to the edge of the layer ($z_0 = z_1, z_u$) by about 3 dB. The range of $\langle z \rangle$

Fig. 5.13 Average echo height as a function of height corresponding to zero aspect angle (i.e. $\langle z \rangle$ vs. z_0) over the 400-600 km slant range interval centred on the geomagnetic meridian. The echo height data are taken from the period of April 1, 1973 (0857-0940 UT) and have been averaged over 2 deg of azimuth. Height is referenced to 100 km. The numbers show the approximate percentage of the 108 scans from the period contributing to each average.

Fig. 5.14 Theoretical $\langle z \rangle$ vs. z_0 for an irregularity layer of uniform backscattering cross-section in a 20-km height interval. Figures refer to values of aspect sensitivity (in dB/deg). The $z_0 = -10$ to $z_0 = 0$ range of these plots matches the $z_0 = 0$ to $z_0 = 10$ range of the experimental data of Fig. 5.13.

Fig. 5.13

Fig. 5.14

over which this drop is measured is 6 km. The aspect sensitivity measured on the basis of power variation with echo height would then be about $S = 5$. Hence, a distinction must be made between aspect sensitivity at the microscopic level and that deduced on the basis of large-scale variations of backscattered power.

The Gaussian model of the altitude dependence of σ_n is probably more realistic. It can be written as

$$\sigma_n \sim \exp[-(z-z_c)^2/2\sigma_g^2] \quad [5.9]$$

The solution of eq. [5.4] with this expression for σ_n is straightforward. Fig. 5.15 shows the variation of $\langle z \rangle$ with z_0 for $\sigma_g = 7$ km, three values of aspect sensitivity ($B = 5, 10, 20$), and $da/dz = .11$ deg/km. A reasonable fit to the data set of Fig. 15.3 is obtained for $B = 10$. Fig. 5.16 shows plots for $B = 10$ and three values of σ_g (5, 7, and 10 km). Comparison of Figs. 5.15 and 5.16 indicates that a family of curves parameterized by (B, σ_g) can reproduce the data set of Fig. 5.13. Using $(B = 10, \sigma_g = 7$ km) as a reference, this family of curves has $(B < 10, \sigma_g > 7$ km), $(B > 10, \sigma_g < 7$ km). On the basis of the zero-order dependence of $\langle z \rangle$ on z_0 it is difficult to distinguish between aspect sensitivity and thickness in this model. However, the variation of backscattered power, I_N , with $\langle z \rangle$ - a first-order effect - allows for some discrimination between the two. The model with $B = 10, \sigma_g = 7$ km gives a drop in backscattered power of 11 dB with a change in aspect angle of 1 deg for an apparent aspect sensitivity

Fig. 5.15 Theoretical $\langle z \rangle$ vs. z_0 for an irregularity layer with a Gaussian height profile of back-scattering cross-section described by $\sigma_g = 7$ km. Figures refer to values of aspect sensitivity. The scales have been adjusted for direct comparison with the experimental data of Fig. 5.13.

Fig. 5.16 Theoretical $\langle z \rangle$ vs. z_0 for an irregularity layer with a Gaussian height profile of back-scattering cross-section and an assumed aspect sensitivity of 10 dB/deg. Figures refer to values of σ_g . The scales have been adjusted for direct comparison with the experimental data of Fig. 5.13.

Fig. 5.15

Fig. 5.16

$S = 11$. For $(B < 10, \sigma_g > 7 \text{ km})$ S is smaller and for $(B > 10, \sigma_g < 7 \text{ km})$ S is larger. The data set of Fig. 5.5 indicated a variation of backscattered power with aspect angle corresponding to $S = 10$. The fit to the model with $B = 10, \sigma_g = 7 \text{ km}$ is preferred on this basis.

In sum, using simple models of radar auroral scattering we can reproduce the dominant feature of the spatial distribution of Homer echo height. The models indicate a convergence of apparent and microscopic aspect sensitivities to a value of about 10 dB/deg.

5.6 Temporal variability

Spatial variation of echo height in the Homer experiment was marked whenever echo activity was widespread. We have seen that large-scale movement of radar aurora lead to a measure of temporal variability in overall height associated with spatial variation of the aspect geometry. Do the Homer echo height data contain evidence of significant short-term temporal variability after removal of the spatial aspect effect? We have approached this question by examining time series of echo height over extended periods for specific cells in the Homer range-azimuth grid. Fig. 5.17 shows an example. The range-azimuth coordinates $R = 500 \text{ km}$, $Az = 24.5 \text{ deg}$ locate the cell in the middle of the strong DB activity that dominated this period. By direct measurement, the mean echo height was 103.5 km with a standard deviation

Fig. 5.17 Time series of Homer echo height at the range-azimuth cell centred on $R = 500$ km, $Az = 24.5$ deg for the period of April 1, 1973 (0857-0940 UT). Time is referenced to the beginning of this period.

$\sigma_h = 1.0$ km. The error analysis of sec. 4.5 applied to this range indicates a standard error on the height measurement of $\Delta z = .9$ km; a value of σ_h significantly greater than this would be unambiguous evidence of temporal variability. It is clear from the smallness of the differential $\Delta\sigma = (\sigma_h^2 - \Delta z^2)^{1/2}$ and the noise-like scatter in the time series that echo height was essentially constant. This was the case for the majority of the cells in this period that recorded significant numbers of counts.

We have applied this analysis to a variety of periods and found that the intrinsic temporal variability of echo height in events of diffuse radar aurora was usually very limited. The temporal invariance of altitude structure is in marked contrast to the usual spatial variation and the dynamism of radar auroral phenomena generally. One notable example of sizeable short-term height variation is presented in sec. 5.8.

5.7 Diurnal variation

The possible existence of a diurnal variation in the height range of .4 m-echo activity was noted in sec. 5.4. In this section, diurnal variation is studied for representative periods from three different years.

To obtain reliable temporal statistics of echo occurrence, we must remove the skewing effect of spatial variation of echo activity. This is accomplished here by making comparisons of height between periods only over a

restricted part of the Homer field of view (620-900 km slant range, 13.5-38.5 deg azimuth). Fig. 5.18 shows histograms of height for a variety of pre- and post-midnight periods. (The shaded histograms apply to the selected range-azimuth cells.) In all cases the pre-midnight range of echo activity was 105-115 km and the post-midnight range was 100-110 km. In two pre-midnight examples the peak in occurrence was above 110 km, while two of the post-midnight examples had their peak below 105 km. Thus, the locus of echo activity was 5-10 km lower in the post-midnight events.

In two of the post-midnight histograms, the decrease in echo occurrence with height above its peak is quite steep. Since the 105-115 km height interval was observed under favourable aspect conditions over the selected portion of the Homer field of view, this feature can be interpreted as evidence of a sharp transition at the upper edge of the irregularity region analogous to the transition at the lower edge. We conclude that there is a definite diurnal factor in the altitude distribution of 398 MHz radar aurora and that the irregularity region may be sharply bounded from above as well as from below.

5.8 Further examples of maps of radar auroral altitude

In this section, the altitude features of several events are examined. The intention is to broaden the base of basic results on the height of radar auroral echoing.

Fig. 5.18 Histograms of Homer echo height over the 13.5-38.5 deg N azimuth sector for sample pre-midnight (A) and post-midnight (B) periods. The shaded histograms (discussed in text) apply to the 620-900 km slant range interval.

A1) April 1, 1973, 0857-0940 UT

B1) March 30, 1973, 1133-1257 UT

A2) March 30, 1973, 0726-0835 UT

B2) March 20, 1973, 1148-1415 UT

A3) February 27, 1978, 0445-0622 UT

B3) February 27, 1976, 1558-1748 UT.

Fig. 5.18

Fig. 5.18 (cont.)

Fig. 5.18 (cont.)

Fig. 5.19 shows a one-minute map of echo height from the event of March 5, 1978. The Homer signature of the echo activity is of the DB type. The azimuthal sector of this map is displaced a full 20 deg to the east relative to the geomagnetic meridian. Consequently, the aspect geometry over the eastern part of this field of view is rather extreme, as indicated by the corresponding map of the zero aspect surface shown in Fig. 5.20. The interesting point here is that the magnetic aspect effect simultaneously limits echo occurrence in the horizontal plane and conditions echo height. As to the former, the boundaries of the source regions for radar aurora generally align with contours of constant magnetic L shell. These contours are concave (outward) with respect to the position of the radar (Fig. 5.2). On the other hand, the contours of aspect control implied by the map of Fig. 5.20 are convex at greater ranges. It can be seen in Fig. 5.19 that the poleward boundary of the DB is concave over the 25-35 deg range of azimuth, but becomes convex with increasing azimuth. This is consistent with a transition from control of the poleward boundary by magnetic L shell to control by magnetic aspect angle. By direct calculation, aspect angle over the convex portion of the boundary averages 1 deg. The absence of echo activity towards the eastern edge, where electrojet current undoubtedly flows, shows that backscattered power falls off precipitously over one degree of aspect angle measured in the horizontal plane. It can

Fig. 5.19 1-min map of echo height (March 5, 1978,
1258-1259 UT) over an extreme azimuthal sector.

Fig. 5.20 Map of the zero aspect surface over the range-
azimuth sector shown Fig. 5.20.

Fig. 5.19

Fig. 5.20

also be seen that echo height in the map of Fig. 5.19 has the same large-scale variations as the zero aspect surface towards the eastern edge of the DB. Thus, a strong and consistent aspect dependence is observed in the horizontal and vertical dimensions.

Fig. 5.21 shows a pair of 90 second maps taken from the pre-midnight period of March 3, 1978. This example bears on the altitude dependence of backscattering in highly structured forms. In the first map there is isolated DP echo activity at intermediate and far ranges. The limited slant range extent of these echoes indicates that the echoing region was extremely (<20 km) confined in the slant range dimension. Over a period of 90 sec strong scattering was initiated over almost the entire 40 deg azimuthal extent of the radar scan, leading to the formation of the DA shown in the second map. Height across the DA is uniformly high (i.e. greater than the height of the corresponding portion of the zero aspect surface) although at these ranges the corresponding aspect angles are not excessive, varying from less than .2 deg over the central azimuths to .5 deg at the extremes of azimuth. The uniformity of echo height at 116-118 km (allowing for a systematic rise in height with range because of the finite pulse width) suggests that the altitude profile of the intensity of small-scale density irregularity was sharply peaked in the 115-120 km height interval, weakening the modulating effect of aspect sensitivity. A similar example

Fig. 5.21 Successive 1.5-min maps of echo height (March 3, 1978, 0854-0856 UT) showing the sudden emergence of a high altitude discrete radar auroral arc.

Fig. 5.21

of weakened aspect control of echo height in the DA type of radar aurora was discussed in sec. 5.4. It seems that discrete radar aurora has associated with it a higher degree of structure in the vertical dimension commensurate with its smaller scale of structure in the horizontal plane.

The final example, shown in Fig. 5.22, is taken from the post-midnight period of March 30, 1973. It contains an instance of pronounced short-term temporal variation of echo height. In the first map, a weak DB exhibits a uniformly low height (<107 km) over most of its expanse. The situation 12 minutes later is shown in the second map. The locus of low-altitude echo activity has shifted towards the west. A high altitude (>110 km) feature has emerged in the extreme northeast corner of the field of view. A portion of the nearer-range echo activity over the same azimuthal sub-sector has also experienced an increase in height. A fundamental change in radar auroral altitude has taken place over a period of about 10 minutes and over distances of hundreds of kilometres.

5.9 Summary

The analysis of Homer data described in this chapter has led to the following findings on the height of radar auroral echoing on 398 MHz:

1. The region of echo activity is most often centred on 100-110 km. The extreme altitude limits for echo

Fig. 5.22 1-min maps of echo height from the post-midnight event of March 30, 1973 that contain an example of marked temporal variation of radar auroral altitude.

Fig. 5.22

occurrence are 96-98 km and 115-120 km. In events of diffuse radar aurora the scatter appears to be generated over finite (10-20 km) height intervals. The echoing region is sharply bounded from below and may be sharply bounded from above.

2. Magnetic aspect sensitivity is an important and stable feature of echo activity overall but is especially pronounced as an influence on altitude variation. When the gradient of aspect angle with height is as large as 1 deg/10 km, radar visibility within a region of irregularity is severely restricted. The figure of 10 dB/deg for aspect sensitivity is consistent with certain features of the Homer data.

3. There is a temporal variability in the height of dynamical radar aurora associated with spatial variation of the aspect geometry.

4. The short-term temporal variability of echo height is usually much less than the spatial variability. For diffuse radar aurora echo height is often invariant with time to within the accuracy (1-2 km) of the height measurement.

5. There is a diurnal variation in the height of .4 m echo activity. Events of the post-midnight sector (i.e. westward electrojet) are associated with a lower and narrower range (97-110 km) of height. Only the altitude of the upper boundary of the echoing region is variable.

6. Aspect control of echo height is weaker in events of

discrete radar aurora. Adjacent echoing forms sometimes differ in height by as much as 10 km.

7. There is an east-west altitude asymmetry in events of the pre-midnight sector (i.e. eastward electrojet) with height increasing towards the east.

8. Sizeable short-term variation of radar auroral height is sometimes observed.

CHAPTER 6

INTERPRETATION

6.1 Introduction

An outstanding feature of the theory of coherent scatter is the prediction of a strong altitude dependence (Fejer et al., 1975b). To date, no direct comparison has been made between theory and experiment in this regard for the auroral zone. It is the purpose of this chapter to carry out a detailed analysis of the conditions for plasma instability in the auroral ionosphere in light of the altitude information obtained from the Homer data. To this end, the linear fluid and kinetic dispersion relations are solved for marginally stable and unstable plasma waves. The mechanisms of secondary instability are examined. It is argued that some important properties of radar auroral altitude reported in the previous chapter are consistent with the electrodynamics of the auroral electrojet.

6.2 Models of the E region

Our numerical modelling of plasma instability must be based on assumed height profiles of particle densities and temperatures. In this section, the importance of variation between model profiles is discussed and the working models of the E region are described.

The first uncertainty arises in the choice of neutral atmosphere. However, the exponential variation of neutral

densities with height renders the variations between models of little significance. For example, the densities of the CIRA 72 model differ from those of Banks and Kockarts [1974] by a factor of about 15% throughout the 90-120 km height interval, which corresponds to an altitude offset of less than 1 km. The differences between neutral temperature profiles are similarly negligible. We work with the one model (Banks and Kockarts, 1974) exclusively.

The second uncertainty is in the choice of effective collision frequencies to represent the rates of collisional momentum transfer between charged and neutral particles. We have compared collision frequency profiles for standard references in the field and found differences corresponding to altitude offsets of the order of 1 km. This is sufficiently small to allow the adoption of a single set of collision frequency formulae. We use the expression for the ion-neutral frequency given by Schunk and Walker [1973]

$$\nu_i = 4.34 \cdot 10^{-10} n(N_2) + 4.28 \cdot 10^{-10} n(O_2) + 2.44 \cdot 10^{-10} n(O) \quad [6.1]$$

and the expression for the electron-neutral collision frequency given by Schunk and Nagy [1978]

$$\begin{aligned} \nu_e = & 2.33 \cdot 10^{-11} n(N_2) (1 - 1.21 \cdot 10^{-4} T_e) T_e \\ & + 1.82 \cdot 10^{-10} n(O_2) (1 + 3.6 \cdot 10^{-2} (T_e)^{1/2}) (T_e)^{1/2} \\ & + 8.9 \cdot 10^{-11} n(O) (1 + 5.7 \cdot 10^{-4} T_e) (T_e)^{1/2} \end{aligned}$$

[6.2]

where the densities are in cm^{-3} , T_e is the electron temperature, and the dominant ion is assumed to be NO^+ .

The third and largest uncertainty stems from the possibility that charged particle temperatures in the disturbed auroral ionosphere are elevated above neutral levels. We work with two models of the E region, one assuming uniform neutral and charged particle temperatures at all heights (i.e. $T_e[z] = T_i[z] = T_n[z]$), the other incorporating the heating effect of plasma instability outlined below.

The processes that limit the growth of ionospheric plasma waves and dissipate their energy may serve to heat the ionospheric plasma. Elevated electron temperatures in the auroral ionosphere have been reported by Schlegel and St. Maurice [1981] and discussed in the context of heating through plasma instability by St. Maurice and Schlegel [1981] and St. Maurice and Laher [1985]. Calculations indicate that plasma wave induced heating of the charged particle gases far exceeds that from other sources (Nielsen and Schlegel, 1985). According to the observations, T_e is an unambiguous function of altitude and relative drift velocity. The functional dependence of T_e is depicted in Fig. 6.1, following the example of Schlegel [1983]. Schlegel and St. Maurice [1981] have solved for T_i by using a simplified ion energy balance equation. Their result is shown in graphical form in Fig. 6.2. (The interpolation curves of Figs. 6.1 and 6.2 have been modified for

Fig. 6.1 Dependence of electron temperature on relative electron-ion drift velocity and height (adapted from Fig. 1 of Schlegel, 1983.) The curves represent regression parabolas of the measured data of Schlegel and St. Maurice [1981].

Fig. 6.2 Dependence of ion temperature on drift velocity and height (calculated from relations A8 and A9 of Schlegel and St. Maurice [1981].)

Fig. 6.1

12

Fig. 6.2

consistency with our neutral atmosphere model.) Plasma wave heating reacts on the instability conditions primarily through the temperature dependencies of the electron-neutral collision frequency and ion-acoustic velocity. The heating effect is most pronounced above 110 km. The application of this semi-empirical theory to the study of coherent scatter is not yet firmly established; we adopt it here to demonstrate the possible consequences of charged particle heating on the development of plasma instability.

Finally, the values adopted for the electron and ion gyrofrequencies are $\Omega_e = 9.8 \cdot 10^6 \text{ s}^{-1}$ and $\Omega_i = 1.8 \cdot 10^2 \text{ s}^{-1}$, and the ion mass is taken to be $m_i = 5.1 \cdot 10^{-26} \text{ kg}$.

6.3 Numerical results for primary instability

6.3.1 Introduction

The most fundamental altitude property of the source region for radar aurora is its confinement to a narrow range (95-120 km) of heights. In this section, numerical results are presented that broadly account for these limits. The analysis is based on the susceptibility of auroral plasma to the onset of plasma wave growth as described by the linearized equations of fluid theory (sec. 2.3.2) and kinetic theory. The immediate discussion is limited to the growth conditions for primary waves, i.e. those excited directly by the relative electron-ion drift velocities of background ionospheric currents. While it is

true that the Homer radar observed secondary waves associated with local drifts in other directions, it may be that the altitude distribution of secondaries is not very different from that of the primary waves on which they depend for growth. The two-stream instability is dealt with almost exclusively because of its greater success in explaining the observations.

It is demonstrated here that the drift velocities required for growing .4 m plasma waves are excessive for wave propagation directed by more than a few degrees out of the plane of orthogonality to the magnetic field. This well-known property merits attention in view of the strength of magnetic aspect effects in the Homer backscatter.

6.3.2 Fluid theory

The main limitation on the application of fluid theory to plasma instability in the E region is the assumed smallness of the ion mean free path compared to the irregularity wavelength of interest. The ion mean free path is about .1 m at 100 km, .5 m at 110 km, and 2.1 m at 120 km. Although the assumption is clearly violated at the .4 m irregularity wavelength sampled by the Homer radar, it is nonetheless of interest to determine the fluid conditions for instability. This is because the ionospheric plasma first becomes unstable in the long-wavelength regime described by fluid theory and

because long-wavelength primary instability creates favourable conditions for the generation of short-wavelength secondary irregularities.

A basic result of the application of fluid theory in the pure two-stream regime was given in Fig. 2.4. That figure showed the altitude variation of the threshold drift velocity, V_{th} , required to just excite plasma waves propagating in the direction of electron motion. For other directions of wave propagation the threshold condition on the relative drift velocity is more stringent; for $k \perp B$ it is that $k \cdot V_d / k = V_{th}$, most generally it is that $k \cdot V_d / k = C_s(1+\psi)$, where ψ is a rapidly increasing function of $k_{\parallel}/k$. When dealing with the minimal condition for instability at a given height, the dependence expressed in Fig. 2.4 is sufficient. The unstable layer appears to extend above 120 km for moderate (500-700 m/s) values of V_d . This may, however, be misleading, because it is the electric field - not the relative drift velocity - that is common to all heights. At electrojet altitudes (90-115 km) $E \approx V_d$ and the distinction between threshold electric field, E_{th} , and V_{th} is immaterial, but with increasing height ion magnetization effects become important and the threshold condition in terms of electric field more stringent. Fig. 2.4 is recast in terms of E_{th} in Fig. 6.3. The smallest value of electric field for which instability is possible (in the absence of destabilizing density gradients) is about 20 mV/m. The effect of increasing ion magnetization

Fig. 6.3 Variation of threshold electric field for pure two-stream instability with altitude (fluid theory) A) in the electrojet approximation (but allowing for the effect of finite ion motion on the magnitude of V_d) B) for the more general relation between E and V_d expressed by eq. [2.14] with $\lambda = 10$ m C) same as B but with plasma wave induced heating of the charged particle gases according to the model described in the text.

with height is to make the unstable layer more sharply bounded from above. (Including the effect of recombination enhances this feature to a degree that depends on the value of N_0 and the irregularity wavelength.) Adopting 30 mV/m as a reference value for the electric field, Fig. 6.3 suggests that conditions for the onset of primary two-stream instability are most favourable in the 94-119 km height interval, assuming significant levels of ionization.

Also shown in Fig. 6.3 is the altitude profile of E_{th} for $\lambda = 10$ m under the assumption of wave heating of the charged particle gases. There is a potential self-consistency problem in that heating presupposes the existence of plasma instability. However, the source of heating may lie elsewhere in the k spectrum of the plasma waves or the heating may react sufficiently strongly on the growth of irregularities for the heating effect to represent a real constraint on the development of instability. The profile of E_{th} shown in Fig. 6.3 represents an iterative solution. Wave heating raises the electric field threshold for excitation of the two-stream instability above 110 km by 10-20% and reduces the most-favourable interval to 94-115 km.

A gradient in the background ionization density changes the conditions for plasma instability in the long-wavelength fluid regime. Fig. 2.5 showed the effect of various values of the electron density scale length, L_N , on the drift velocity threshold in the electrojet

approximation for $\lambda = 10$ m. It is unlikely that the electron density gradient ever has a constant value throughout the 90 to 120 km height interval; however, the effect on V_{th} (and hence E_{th}) at any given height is clear. In the extreme case of an ambient electron density gradient corresponding to $L_N = 10$ km the electric field threshold for $\lambda = 10$ m is increased or decreased by about 5 mV/m at any height. The gradient-drift mechanism operates less efficiently with decreasing wavelength and is not capable of generating sub-metre scale irregularities for background density gradients typical of the ionosphere. For reasons discussed in sec. 6.5, the primary gradient-drift instability probably was not responsible for the bulk of the echo activity observed by the Homer radar even allowing for coupling to secondary modes. Hence, we concentrate here on the theory of primary two-stream instability.

Fig. 6.3 indicates that plasma instability, of a kind thought to be responsible for some auroral scatter, should generally set in over an extended (~ 25 km) height interval once E reaches about 30 mV/m. Electric fields this large and larger are often observed in the auroral zone (Tsunoda and Presnell, 1976; Moorcroft, 1979). The analysis of height in the Homer data indicated that echoes are indeed generated from regions extended (10-20 km) in altitude, rather than from thin (~ 5 km) sheets, as is commonly supposed. However, the simple fluid theory predicts an altitude range that is perhaps too large. This difficulty

is ameliorated by extending the analysis to short-wavelength irregularities through the application of kinetic theory.

6.3.3 Kinetic Theory

With decreasing irregularity scale length, the fluid description of plasma instability becomes less tenable. In the 90-120 km interval the first size criterion violated is that $\lambda \gg \lambda_m$, where λ_m is the ion mean free path. As a result, the threshold conditions for primary two-stream instability in the electrojet approximation become explicitly k dependent. Next, the effect of the finite electron gyro-radius (~ 2 cm) becomes important. At still smaller scale lengths, the assumption of quasi-neutrality is not valid because the Debye length ($\sim .3$ cm) is no longer negligible. For the shortest wavelength modes the effect of a finite Debye length is to make the growth conditions dependent on electron density (Lee et al., 1971). The complexity of the calculations in kinetic theory can be handled only with complex numerical codes. The results presented in this section were compiled using the FRLY 03 program provided by D.R. Moorcroft, modified to run on the model atmospheres described in sec. 6.2. The code is based on solution of the dispersion relation, eq. 24, given by Farley [1963]. In the interests of determining the minimal conditions for instability, finite electron gyro-radius and Debye length effects were ignored.

Fig. 6.4 shows threshold electric field as a function of height and radar sampling frequency for pure two-stream instability. With decreasing frequency (i.e. increasing irregularity wavelength) the results approach those of Fig. 6.3. With increasing frequency, the threshold electric field for instability at a given height increases; instability first sets in for an electric field of 27 mV/m at 398 MHz vs. 21 mV/m at 50 MHz. The height interval for instability for an electric field of 30 mV/m is some 4 km smaller at 398 MHz than at lower VHF frequencies. The comparison with the altitude extent of the echoing in the Homer observations is improved somewhat; adopting 35 mV/m as a suitable reference value of electric field, Fig. 6.4 indicates that conditions for plasma instability (and hence echo activity) at the Homer wavelength are most favourable in the 95-117 km height interval.

We seek here to emphasize altitude variation of the instability conditions by examining the height dependence of the linear growth rate, γ . A larger value of γ means a shorter e-fold growth time and presumably a greater wave amplitude at saturation. In a simple extension of linear theory, wave amplitude is taken to vary in the same manner as linear growth rate, i.e. the strongest backscatter (other factors being equal) comes from the altitude interval where the waves are growing fastest. This approach has been adopted by authors interested in other facets of radar auroral observations, such as phase

Fig. 6.4 Variation of threshold electric field for pure two-stream instability with altitude and radar sampling frequency. (The calculation of threshold drift velocity was performed in the electrojet approximation. The effect of finite ion motion was included in the conversion to electric field.)

velocity measurements (e.g. Schlegel, 1983). No sophisticated attempt is made here to establish a causal connection between linear growth rate and wave amplitude; the assumption is valid only within the limited context of linear theory. This said, Fig. 6.5 shows plots of relative linear growth rate for primary two-stream waves as a function of electric field and altitude. The direction of wave propagation has been varied for each electric field-altitude point to find the largest growth rate overall. (The smallest electric field for which the threshold condition $\gamma = 0$ is satisfied is for $k_{\parallel} = 0$. As the electric field is increased, growth rates become positive for small values of $k_{\parallel}$ relative to k - corresponding to angles of tenths of a degree - and maximize away from zero (Wang and Tsunoda, 1975). The selection of growth rate made for Fig. 6.5 is consistent with the assumption that backscatter is dominated by the fastest growing modes.) For moderate (30-50 mV/m) electric fields the growth rate of primary .4 m two-stream waves has a broad maximum in the 100-110 km height interval and a peak between 100 and 105 km. If linear growth rate is indeed representative of wave amplitude in the saturated state, and if the effect of electron density variation in particular is negligible over the heights of interest, the region of .4 m irregularity should generally be centred on 100-105 km, with a somewhat greater range (100-110 km) likely during periods of strong electric field. These

Fig. 6.5 Variation of relative linear growth rate for pure two-stream instability with altitude and electric field for an irregularity wavelength of .4 m. (obtained by scanning over direction of wave propagation for the largest growth rate overall.)

Fig. 6.6 Same as Fig. 6.5 but assuming elevation of electron and ion temperatures on the model of plasma wave induced heating of the charged particle gases described in the text.

Fig. 6.5

Fig. 6.6

predictions are suggestive of the altitude properties of the irregularity region inferred from the Homer data.

What is the effect of elevated charged particle temperatures on the conditions for growth? The plots of Fig. 6.5 are repeated in Fig. 6.6 for the adopted model of plasma wave-induced heating of the charged particle gases. The effect of heating is severe in the 115-120 km height interval. This suggests a possible cause of the variability observed in the location of the upper altitude boundary. In some periods heating may effectively cut off short-wavelength instability above 115 km.

An important factor in the occurrence of radar aurora is the restriction of growth to plasma waves propagating in or almost in the plane of orthogonality to the magnetic field (Farley, 1963). Fig. 6.7 shows how the threshold value of drift velocity increases with value of $k_{\parallel}/k$ for $\lambda = .4 \text{ m}$ and ionospheric parameters appropriate to a height of 105 km. For drift velocities ($<1000 \text{ m/s}$) more typical of the auroral zone, the propagation vectors of growing waves cannot lie more than about a degree out of the plane of orthogonality. This accounts in a qualitative way for the aspect dependence of the echo activity observed by the Homer radar. However, the theory of coherent scatter has not progressed to the point where a quantitative assessment of aspect sensitivity can be made (Moorcroft, 1985) and there is no possibility of a comparison between theory and observation in this regard.

Fig. 6.7 Variation of threshold drift velocity for pure two-stream instability with angle of wave propagation out of the plane of orthogonality to the magnetic field for a height of 105 km and an irregularity wavelength of .4 m

6.4 Secondary instability

Primary two-stream waves can be generated at sizeable (~ 60 deg) angles to V_d but excessively large drift velocities are required for propagation angles as large as 75 deg. Since the Homer radar detected strong echoes from directions corresponding to orthogonality with the auroral electrojet, the irregularities must have been produced through secondary instability processes. In this section, the altitude dependence of the conditions for secondary instability are discussed.

The criteria for the onset of secondary instability according to the mechanism of Sudan et al. [1973] were given in sec. 2.3.3. It was stated there that the generation of secondary two-stream waves is heavily favoured at lower altitudes. Although this conclusion was reached for irregularities in the long-wavelength fluid regime, extension to .4 m wavelength secondary instability is straightforward. Because of the exponential increase in the drift velocity required for its excitation above 100 km, the secondary two-stream mechanism can only operate below about 105 km for primary amplitudes of a few percent. The linear growth rate for secondary two-stream instability maximizes at about 95 km altitude, somewhat below the region of primary instability. Thus, as presently formulated, the two-step mechanism of Sudan et al. [1973] can account for secondary instability in the two-stream

regime only in the 95-105 km height interval.

The criterion for growth of secondary gradient-drift waves was given in eq. [2.28]. If the condition $k_s \gg k_p$ is not considered restrictive, the mechanism can operate throughout the 90-120 km height interval. However, large values of k_p relative to k_s are required above 110 km and it is not clear that linear theory applies when the scale of the secondary irregularity exceeds that of the primary on which it grows. In any case, for given values of k_s , k_p , and primary amplitude instability is heavily favoured at lower heights. The linear growth rate maximizes at about 95 km. Of course, the growth of secondaries must take place in a region in which primary irregularities can exist; on this basis, the mechanism of Sudan et al.

[1973] predicts the strongest secondary gradient-drift instability at about 100 km altitude, provided the primary instability is of the two-stream variety.

To date, no alternate theory of the generation of secondary two-stream irregularities has been advanced. However, the non-linear theory of the gradient-drift instability has progressed to the point where the mechanism outlined above is subject to modification. Here, a simple synthesis of linear and nonlinear theory is attempted that may have application to the Homer observations.

As discussed in sec. 2.3.4, instability in the pure gradient-drift regime is characterized by the cascade of energy through waves of all scale sizes. One can envision

a modified two-step process whereby the E region plasma becomes unstable to the growth of primary two-stream waves, whose density gradients drive gradient-drift waves qualitatively in the manner described by Sudan et al. [1973], which subsequently develop into full-scale electrostatic plasma turbulence, at least in that portion of the plane of orthogonality that lies outside the growth cone of the primary two-stream waves. The growth of irregularities within the two-stream regime ($V_d \cos(\theta) > C_s[1+\psi]$) is limited by the independent process of orbit diffusion (Sudan 1983). Energy could enter the secondary gradient-drift regime at some optimal scale size and cascade to smaller wavelengths, resulting in uniform fluctuation statistics over a span of propagation angles relative to the direction of the primary current. If the coupling from two-stream primary to gradient-drift secondary is at all efficient, broadly developed secondary instability would generally accompany primary instability at any height. Secondary instability at a given wavelength could be supplied indirectly where the requirements for direct excitation are prohibitive. The crucial question concerns the nature of the two-stream - gradient-drift coupling process. This would appear to require a numerical approach, which apparently has not yet been developed for the large drift velocities associated with primary instability in the two-stream regime (Fejer and Kelly, 1980).

It is not our intention to undertake here an extensive analysis of nonlinear theory. Rather, we seek to interpret our findings on radar auroral altitude in terms of the simplest zero-order effects. The preceding discussion points up the complexity involved in attempting to give a full description of ionospheric plasma in the turbulent state.

We briefly mention an application of the theory of Sudan [1983] to the height of radar aurora. For two-stream instability in the centre of the electrojet (~ 105 km) the theory predicts an integrated density perturbation, A_I , of a few percent. However, the perturbation levels predicted above 110 km are excessive ($>10\%$). This is because of the dependence of A_I on ion collision frequency (eq. 2.29). The Homer data contain many examples of high altitude echoing which, on the basis of linear theory, are more readily attributed to primary instability in the two-stream regime. It seems that auroral echoing at greater altitudes is less well understood in the current theory of coherent scatter than its low altitude counterpart.

6.5 Two-stream vs. gradient-drift instability

We have focused almost exclusively on the two-stream mechanism as the source of primary instability for the Homer observations. There is some basis for this prejudice. Fig. 6.4 showed that the altitude interval for ~ 4 m instability in the pure two-stream regime is 95-117 km

for an electric field of 35 mV/m, in good agreement with the 97-118 km range of Homer echo height. There was undoubtedly variation in the magnitude of the electric field during events, but the theoretical altitude limits for two-stream instability do not vary greatly over the 30-50 mV/m range that was typical of the Homer observations. The gradient-drift instability cannot be entirely excluded on this basis, but we note that growth in the primary gradient-drift regime is possible only where the local electron density gradient is suitably oriented relative to the ambient electric and magnetic fields, and that, for vertical gradients, portions of the 95-120 km height interval are certain to be stabilized against plasma wave growth whenever the density gradient at some level is sufficient to drive instability (Fejer et al., 1975b). For the simplest case of a single peak in electron density, which is considered characteristic of auroral conditions, primary gradient-drift waves could grow only below the peak in the eastward electrojet and only above the peak in the westward electrojet. The electron density profile in the auroral electrojets is variable and generally has its peak in the 100-120 km height interval (Kamide and Brekke, 1977; Moorcroft, 1979). Thus, the primary gradient-drift mechanism is incapable of explaining the generation of echoes below 100 km in the post-midnight sector and the stability of echo height overall. Generally, it does not seem that vertical density gradients are a viable source of

primary instability for the Homer observations.

There is the possibility that horizontal variations of ionization density lead to primary gradient-drift instability. Large horizontal gradients arise in the auroral zone as a result of structured particle precipitation, such as that associated with the formation of visual arcs. Since $\mathbf{E}$ is almost vertical at auroral latitudes, a horizontal gradient is much more effective at driving instability than is the equivalent vertical gradient. We note that steep horizontal density gradients could not have been maintained over the scale distances (~ 100 km) associated with the diffuse type of radar aurora that accounted for the bulk of the Homer observations. The analysis of Homer echo height in terms of the primary two-stream instability is thus deemed to be adequate, although the gradient-drift instability cannot be ruled out for events of discrete radar aurora.

6.6 Discussion of findings on the height of 398 MHz radar auroral echoes

In this section, the findings on radar auroral echo height (sec. 5.9, 1-8) are interpreted. Most of the explanations offered are not definitive, but they are consistent with well-known properties of the auroral ionosphere.

(1) The altitude range of small-scale density irregularity predicted by the linearized theory of the

two-stream instability is in excellent agreement with the results on the heights of echoing obtained from the Homer data. Of course, plasma instability is possible only in the presence of significant ionization and the ionospheric plasma exists in a turbulent state involving saturation processes that cannot be described in linear theory. Nevertheless, the theoretical conditions for plasma wave growth at .4 m wavelength are satisfied in theory above 95 km and below 120 km in the auroral electrojet, and these represent the limits of the echo activity observed by the Homer radar. The most favourable height interval in theory for excitation of growing .4 m plasma waves is 100-110 km, which coincides with the range of the centre of the irregularity region inferred from the Homer data.

The threshold for two-stream instability at 398 MHz increases rapidly from 27 mV/m at 100 km altitude to over 35 mV/m at 94 km. This, combined with the usual decrease in electron density below 100 km, could give rise to the observed abrupt cut-off at the lower edge of the echoing region.

(2) A strong magnetic aspect control is built into the linear theory of plasma instability. A dependence of echo occurrence and intensity on the altitude variation of aspect angle could have been anticipated on this basis.

(3) The variability of echo height in dynamical radar auroral events is consistent with the modulation of auroral scattering within thick (>10 km) irregularity layers by the

aspect geometry.

(4) A somewhat surprising result of the current study is the relative constancy of echo height in fixed scattering volumes. This is in marked contrast to the short-term variability of such parameters and indices of echo activity as electric field, electron density, and echo power. Recalling the discussion of sec. 5.5, a 5 km shift of the altitude profile of the irregularity backscattering cross-section, $\sigma_N[z] = N^2[z] \langle (\Delta N[z]/N[z])^2 \rangle$, would have measurable (2-3 km) consequences for the mean height of backscattered power. The comparative invariance of Homer echo height with time indicates that the irregularity backscattering cross-section at .4 m wavelength must generally have a very stable altitude profile. This is the case even when auroral conditions are so dynamic that short-term (~ minutes) variations in height-integrated backscattered power levels exceed 20 dB. We note that the factors determining the height range of primary two-stream instability are the altitude variations of the electron density, the relative electron-ion drift velocity, and the charged particle collision frequencies. Only the electron density profile possesses variabilities of the kind that could render echo height variable, but this depends on the relation of electron density variations to wave saturation levels. No resolution of this question can be found in linear theory.

(5) The location of the upper altitude boundary of

the irregularity region exhibits a marked (107-117 km) diurnal variation associated with the reversal of electric current between the eastward and westward electrojets. There is no basis for this behaviour in simple linear theory. One possibility is that instability always extends over the 97-118 km height interval but aspect conditions or the background electron density profile heavily weight $\sigma(z)$ towards lower altitudes in the post-midnight sector. For this to be the case, there would have to be a substantial diurnal variation in the aspect geometry or in $N(z)$. We consider these possibilities separately.

The current of the auroral electrojet distorts the ambient geomagnetic field and this distortion possesses a diurnal component. A deflection of field lines through 0.5 deg would shift the zero aspect surface in altitude (as seen by the Homer radar) by about 4 km at 500 km range, 8 km at 1000 km range. In the westward electrojet, this could result in a suppression of echo activity in the 110-115 km height interval provided the current flows predominantly above 115 km. The amount of perturbation of the geomagnetic field due to electrojet current can be estimated. Assuming the current flows in a horizontal layer of thickness τ with a density N_e and a drift velocity V_d , the maximum value of the magnetic field perturbation $\Delta B(N_e, V_d, \tau)$ is $(10^{-21})N_e V_d \tau$ (Gauss) and the corresponding angular deflection is $\Delta \alpha = (10^2)\Delta B$ (deg). For the values $\tau = 6$ km ($\sim$ one neutral density scale height),

$N_e = 5 \cdot 10^{11} \text{ m}^{-3}$, and $V_d = 500 \text{ m/s}$ we have $\Delta\alpha = .15 \text{ deg}$. A change in magnetic field orientation of 0.5 deg requires rather larger values of density and drift velocity than are likely to be maintained over the large areas and extended periods for which low altitude echoing is observed. A further difficulty with this explanation is that the magnetic field perturbation is a minimum in the centre of the current flow where the enhanced level of ionization density would be expected to heighten echo activity (see below). Moreover, no analogous deflection of the zero aspect surface in the pre-midnight sector is implied by the Homer echo height data. For these reasons, the aspect effect can be discounted as the source of the diurnal height variation.

The variation may simply be evidence for a lower height of peak electron density, and hence of current, in the westward electrojet as compared to the eastward electrojet. According to Kamide and Brekke [1977] the current centres for the eastward and westward electrojets are located, on average, at approximately 120 and 100 km altitude respectively during disturbed periods. Thus, the sense of the diurnal variation in the mean electron density profile is in accordance with the variation of echo height reported here.

An overall shifting of the electron density profile to lower height might be expected to be accompanied by a lowering of the lower altitude boundary of the echoing

region. However, the threshold electric field for instability increases very rapidly below 98 km and even under disturbed conditions the decrease in electron density below 100 km is exponential. These two factors control the location of the lower altitude boundary and may effectively deprive it of a diurnal variation.

One difficulty with associating the diurnal variation of Homer echo height with a corresponding variation of $N(z)$ is that the electron density profile in the disturbed auroral E region is highly variable (Moorcroft, 1979) while echo height inferred from the Homer data is not. We consider the possibility of there being a fundamental difference between the plasma environments at the upper and lower boundaries of the irregularity region because of the likelihood of elevated ionization temperatures at greater heights. It was shown in Figs. 6.5 and 6.6 that charged particle heating on the model of Schlegel and St. Maurice [1981] could shut off .4 m wavelength plasma instability above 110 km. This heating acquires a diurnal component if the relevant wave-particle interaction occurs in the long-wavelength gradient-drift regime; in the eastward electrojet, primary gradient-drift waves are unstable in the lower part of the E region (below the height of peak electron density) where charged particle heating is inefficient, while in the westward electrojet the primary gradient-drift instability operates at greater heights, where plasma wave heating is effective at raising charged

particle temperatures. Long-wavelength instability in post-midnight periods could thus shut off short-wavelength instability at greater heights. The critical question concerns the plasma wavelength regime for which charged particle heating is most efficient. St. Maurice and Laher [1985] showed that long-wavelength waves could be responsible for the observed electron temperature enhancements if they have wave vectors departing from perpendicularity to the magnetic field by several (2-5) degrees. They concluded that nonlocal effects could cause gradient-drift waves to propagate at the required aspect angles. Thus, an anti-correlation may exist between long-wavelength gradient-drift instability and short-wavelength two-stream instability above 110 km, of a kind consistent with the diurnal variation of .4 m echo height reported here.

(6) The comparative weakness of aspect control of echo height for discrete radar aurora probably derives from a higher degree of structure in the ionization responsible for such echoes. Highly structured particle precipitation could generate suitable small-scale variations in the height of peak electron density. Considering the insensitivity of echo height for diffuse radar aurora to variability in the electron density profile, the variations in events of discrete radar aurora must be quite extreme to account for the observations.

(7) An east-west altitude asymmetry has been

identified for radar auroral events of the eastward electrojet. The effect is pronounced only at the greater ranges in the Homer field of view, where the vertical variation of aspect angle^{*} is less severe. An east-west altitude asymmetry at 90 MHz was reported by Timofeev and Miroshnikov [1982] but with the opposite inclination to the one reported here. There is a ready explanation for the sense of the asymmetry seen in the Homer data. Because of the changing value of the Hall-to-Pedersen conductivity ratio, the current vector of the auroral electrojet rotates with altitude. Along the geomagnetic meridian the effect of the rotation is to reduce the magnitude of the propagation angle θ , between the radar wave vector, k , and V_d . There is, in addition, a rotation of the current vector relative to k with decreasing/increasing azimuth relative to the geomagnetic meridian. In the eastern sector, in both the eastward and westward electrojets, the rotations in altitude and azimuth are complementary. In the western sector they are opposite. Overall, propagation angle is reduced for increasing eastern azimuth and increasing height. If $\theta < \theta_c$, where θ_c is the half-angle of the growth cone for primary instability, larger amplitude primary irregularities enter the acceptance beam of the radar. Propagation angle is reduced to about 30 deg at an eastern azimuth of 20 deg and a height of 120 km, which is comparable to θ_c for moderate relative drift velocities (~ 500 m/s). Thus, higher altitude primary

irregularities are more favourably illuminated with increasing eastern azimuth and the mean height of backscattered power increases. The effect is derived from the dependence of σ_v on propagation angle (sec. 4.4) and should be the same for both directions of the electrojet. However, it may not be pronounced for events of the westward electrojet because of the generally reduced height of echo activity (current vector rotation with altitude is negligible below 110 km).

(8) Sizeable short-term variation of echo height is less characteristic of .4 m radar aurora but it is sometimes observed. In principle, a variable height of peak electron density could account for the variations. The remarkable point is that radar auroral altitude is not more variable.

CHAPTER 7

CONCLUSIONS

7.1 Summary

We have analyzed a large set of backscatter data collected with a 398 MHz phased-array radar for information on radar auroral echo height. The variation of backscattered power with beam elevation angle has been used to resolve altitude; the measure of echo height so obtained represents an estimate of the first moment of the volume backscattering cross-section in height. Reliable results were anticipated on the basis of the small scale of vertical auroral structure and the rapid scan rate of the antenna. With the processing necessitated by the noise-like character of the auroral signal, mean echoing height has been resolved with an accuracy of 1-2 km over horizontal distances of several tens of kilometres and periods of about 1 minute. The results have been presented in the form of maps of the spatial distribution of echo height. Analysis of these maps has lead to the findings itemized as articles 1 through 8 in sec. 5.9.

Using the linearized equations of fluid and kinetic theory, we have shown that the vertical extent (97-118 km) of 398 MHz radar aurora is consistent with the onset of primary two-stream instability in the auroral electrojet. We have demonstrated that the 100-110 km height interval of

peak echo activity in the Homer observations is matched by the height range of the fastest growing unstable modes in linear theory. It has been suggested that the centre of irregularity formation is shifted downward if the E region plasma is kept nearer threshold by plasma wave-induced heating of the charged particle gases.

Extending the analysis to secondary instability, we have found that the two-step mechanism of Sudan et al. [1973], as presently formulated, can account for the growth of two-stream waves outside the growth cone of primary instability only at lower (95-105 km) heights. The direct excitation of .4 m secondary gradient-drift waves above 105 km by this mechanism is also in doubt, although this may be more of an analytical difficulty related to a breakdown in the assumptions of linear theory than a real physical constraint. We have postulated a synthesis of linear (Sudan et al., 1973) and nonlinear (Sudan, 1983) theories of secondary instability whereby broadly developed gradient-drift turbulence is driven by the perturbation fields and gradients of saturated larger amplitude two-stream primary waves.

Aspect sensitivity has been reported to be an important factor in the occurrence of radar aurora in altitude. Generally, echo activity is most intense in a height interval determined by magnetic aspect modulation of radar visibility within an irregularity region of finite (>10 km) vertical extent. It has been suggested that the

weakening of aspect control of echo height in events of discrete radar aurora is due to sharp spatial variations in electron density.

Two explanations have been advanced for the diurnal variation of the height range of .4 m echo activity, one based on a generally lower altitude of peak electron density in the westward electrojet, the other deriving from wave heating of the charged particle gases of the kind suggested by St. Maurice and Laher [1985]. The east-west asymmetry of echo height in events of the eastward electrojet has been interpreted in terms of well-known properties of current vector rotation with altitude. We have suggested that instances of short-term temporal variability of echo height are due to variability in the electron density profile.

7.2 Comparison with previous studies

The altitude range (97-118 km) of auroral echo occurrence reported here is in general agreement with earlier findings. Because of observational limitations, workers have tended to characterize echoing events by single 'mean' values of height. While the concept has a rough validity, it is an oversimplification if the echo activity is subject to appreciable horizontal variations such as those associated with magnetic aspect conditions. We note that 110 km has often been reported as the mean height of the centre of the echoing region (Unwin and

Gadsen, 1957; Watkins, 1960; Barber et al., 1962; Abel and Newell, 1969). This is consistent with the 97-118 km altitude range of echo activity in the Homer pre-midnight events, but it is somewhat high compared to the 97-110 km range of the post-midnight echoes. Bullough [1961] reported a slight (~ 3 km) diurnal variation in the height of peak echo activity, with reduced height after midnight. More recently, Timofeev and Miroshnikov [1982] reported a contraction of the echo region in altitude between the pre- and post-midnight sectors. Their estimate of vertical extent for pre-midnight events (~ 15 km) compares favourably with our findings, but their estimate for the post-midnight sector (~ 2.5 km) is rather extreme. It does not seem that the characteristics of the dominant echo types in the Homer data can be squared with the classification scheme of Unwin [1959] except in general terms of diffuse vs. discrete; this may be related to the differing scales of irregularity sampled by the two systems (.4 m vs. 2.7 m) or to the sensing peculiarities of the very different radars. In particular, the Homer data invariably indicate a finite (>10 km) vertical extent to the echoing region, while the interferometer radars at times see the region as a very thin (<5 km) sheet. Abel and Newell [1969] obtained thickness estimates of 5-20 km, which are generally consistent with our findings.

The rocket experiments of Kelley and Mozer [1973] and Pfaff et al. [1984] indicated that the unstable layer in

the disturbed auroral E region extends from about 95 km altitude to about 120 km. The irregularity region at .4 m wavelength as determined from the Homer radar data lies within these altitude limits. In addition, Pfaff et al. [1984] reported that the unstable layer is sharply bounded at its edges. The radar data indicate a sharp transition at the lower edge and suggest a similar transition at the upper edge. Thus, the rocket and radar findings on the height range and boundary definition of the unstable layer are in agreement.

The resolution of altitude structure available in the Homer data is quite limited; we cannot expect to directly detect layering within the echo region of the kind reported by Unwin and Johnston [1981] and Timofeev and Miroshnikov [1982]. However, we have found that adjacent echoing forms can differ in height by as much as 10 km. In systems with less horizontal resolution such a situation may give rise to the appearance of distinct layers.

7.3 Directions for future work

The height range (97-118 km) of .4 m wavelength echo activity reported here coincides with the expected range for primary two-stream instability in the auroral electrojet. Since the Homer radar observed mostly diffuse radar aurora, the implication is that diffuse echoes are generated mainly by auroral electrojet current, as was concluded by Greenwald et al. [1975] on other grounds.

Discrete radar aurora more often possesses unusual altitude variations, suggesting a different source mechanism.

Recently, Providakes et al. [1985] presented an example of a VHF radar auroral event that was nearly coincident with the poleward edge of a discrete visual arc in the post-midnight sector, for which the orientation of the horizontal density gradient relative to the inferred polarization field of the arc was just that required for growing gradient-drift waves. On this basis, we might expect an association between discrete radar auroral arcs and the gradient-drift instability. The relation of the instability mechanisms to radar aurora is of fundamental importance and warrants further study. Coordinated observations with coherent and incoherent scatter systems of the kind reported by Moorcroft [1979, 1980] offer the best hope of resolving the many outstanding questions.

It remains a formidable experimental task to resolve vertical structure within the source regions for radar aurora. The potential value of such studies can be gauged by the great progress that was made in equatorial studies, when altitude resolution of 1 km became available (Féjer et al., 1975b; Féjer et al., 1976; Farley et al., 1978).

The prospects for procuring a powerful, highly steerable, narrow-beam antenna for operation at an auroral site are uncertain. However, more limited studies involving existing radar systems can be undertaken and are being contemplated (e.g. Nielsen and Schlegel, 1983).

Perhaps the most challenging and potentially rewarding direction for study is the numerical simulation of ionospheric plasma instability. Extensive calculations have been performed for the low drift velocities characteristic of the equatorial electrojet (e.g. Ferch and Sudan 1977; Keskinen et al., 1979) but apparently have not been extended to the high velocities and small wavelengths required to simulate the conditions of radar auroral instability. Newer generations of computers may render the task more feasible. A particularly important question in light of the current study is whether the mechanisms of secondary irregularity formation have a strong altitude dependence, as indicated by linear theory.

The emphasis in experimental radar auroral research has shifted to bistatic systems (STARE, SABRE, BARS) with the ability to stereoscopically map Doppler shift in auroral backscatter. We conclude by mentioning a practical application of our results to such experiments. It has been reported that the phase velocities of plasma waves excited by the auroral two-stream instability are limited to a value near the ion-acoustic velocity (Nielsen and Schlegel, 1985). A similar saturation of phase velocity in events of the equatorial electrojet has long been known (e.g. Cohen 1973). The STARE observations indicate that the limiting value of phase velocity in the westward electrojet is about 100 m/s less than that in the eastward electrojet. It was suggested by the STARE workers that the

difference stems from a height variation of the centre of the scattering region. A variation of the appropriate sense has been reported here; the mean height of the echoing region decreases by 5-10 km between events of the pre- and post- midnight sectors, implying an overall lowering of the ion-acoustic velocity associated with instability in the westward electrojet relative to the eastward. At electrojet altitudes $\bar{C}_s$ changes by about 25 m/s over 5 km of altitude, but this may be increased sufficiently by plasma wave induced heating of the charged particle gases to account for the observations.

APPENDIX A

DERIVATION OF ANTENNA GAIN FUNCTIONS

The array far-field pattern for the Homer radar is given in Fig. 6 of Presnell et al. [1972]. To derive the antenna gain functions, an analytical expression for the pattern is required. The antenna can be treated as a uniformly filled circular aperture with a far-field radiation pattern given by (e.g. Born and Wolf, Principles of Optics, 5th ed.)

$$Y = \left[\frac{2J_1(x)}{x} \right]^2, \quad [A.1]$$

where J_1 is the Bessel function of the first kind of order 1, $x = (2\pi)as\sin(\theta)/\lambda$, a ($= 12.75 \lambda$) is the effective radius of the aperture, and θ is the polar angle of the field point measured from the central axis of the antenna beam. For a point reflector, the level of backscattered power is weighted with respect to angle θ according to

$$W_p(\theta) = Y^2 = \left[\frac{2J_1(\alpha \sin(\theta))}{\alpha \sin(\theta)} \right]^2, \quad [A.2]$$

where α ($= 80.1$) is a composite constant. For a thin-layer reflector, the relevant gain function is found by integrating the contributions to the backscattered power from all points at the same height (Fig. A.1). The level of backscattered power is weighted with respect to angle t according to

$$W(t) = \frac{\int Y^2(s,t) ds}{\int Y^2(s,0) ds}, \quad [A.3]$$

where the limits on the integrations extend from $-\infty$ to $+\infty$.
 The effective antenna gain function for a thin-layer reflector is plotted in Fig. A.2. In the text, the functional dependence of W on angle is expressed as $W(\epsilon)$.

Fig. A.1 Geometry for derivation of the antenna gain function for a thin-layer reflector.

Fig. A.2 Antenna gain function for a thin layer.
 The relative gain is expressed as a function of angle t defined in Fig. A.1.

APPENDIX B

CORRECTIONS TO ECHO HEIGHT AND ASPECT ANGLE

B.1 Introduction

Suppose that backscatter returns are received from a slant range R and a direction denoted by (A, E) , where A and E are the azimuth and elevation pointing angles. If the earth is treated as a perfect sphere of radius R_E and radio wave propagation in the atmosphere is assumed to be rectilinear, echo height, h , is given by

$$h = \left(R_E^2 + R^2 + 2R R_E \sin[E] \right)^{1/2} - R_E \quad [B.1]$$

If the range and directional parameters are referenced to the non-spherical earth with atmospheric refraction (the physically realistic case), the solution for echo height, z , can be approached only numerically. However, satisfactory results are obtained if h (with $R_E = 6371.2$ km) is adopted as a first approximation to z with corrections made subsequently according to $z = h + \Delta h_1$, where the Δh_1 represent small corrections for various effects.

B.2 Antenna geometry

In practice, it makes little difference in which order the corrections are applied. However, the first complication arises because the beam pointing direction reported by the control system did not always coincide with

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the actual pointing direction. For the electronic steering the pointing angles away from boresight (α in azimuth, β in elevation) were referenced to the planar geometry of the array itself. If the central boresight setting was (A_0, E_0) , the control system reported the azimuth and elevation of the pointing direction to be

$$A = A_0 + \alpha, \quad E = E_0 + \beta. \quad [B.2]$$

The azimuth is correct for $E_0 = 0$ and the elevation is correct for $A = A_0$, but for other directions corrections are required due to the fact that surfaces of constant elevation are cones, not planes, and surfaces of constant azimuth are vertical, not necessarily intersecting the radar antenna surface in a line parallel to the edge of the antenna. For the Homer data, the correction on azimuth is always less than .15 deg, which is negligible. The correction on elevation can be expressed as a correction, Δh_a , on the nominal echo height, h . An approximate expression for Δh_a is

$$\Delta h_a = -R_0(\cos(R/R_0) - R_0/(R_0+h))(1 - \cos(\alpha)). \quad [B.3]$$

The correction is always negative and can reach several kilometres in magnitude for $\alpha \sim 20$ deg.

B.3 Oblateness of the earth

Because the earth is not a perfect sphere, the normal to the plane of the horizon at a point on its surface generally departs from the projection through that point of an earth-centred radial vector. The angular difference is

called the deviation of the vertical, δ . At the Homer site, $\delta = .17$ deg. The elevation of a field point, measured from the radar horizon, differs from the elevation reckoned from a fictitious spherical surface by this amount for radar pointing directions characteristic of the Homer observations. In addition, the effective radius of the earth is a decreasing function of geographic latitude. The correction for oblateness on the nominal echo height turns out to be negligible (.17 km at 750 km range, .26 km at 1000 km range). This is because of the offsetting nature of neglecting both the correction to the elevation at the radar and the change in earth radius with latitude to the field point. However, the magnetic field code is earth-centred and an error of the order of δ is incurred on aspect angle if oblateness is ignored. This error is not entirely insignificant. The aspect data (angles and orthogonality contours) reported in the text have been calculated from appropriate global coordinates allowing for the effects of oblateness.

B.4 Atmospheric refraction

Radio waves are refracted in the lower atmosphere because of gradients in refractive index associated with:

- i) the exponential decrease of neutral densities with height in the troposphere
- ii) spatial variation of ionization density in the ionosphere.

The effects of tropospheric refraction occur throughout the radio

frequency spectrum whereas ionospheric refraction is pronounced only at frequencies in the lower part of the VHF band (Unwin, 1966; Moorcroft, 1985). In this section, the correction applied to Homer echo height for tropospheric refraction is described and the effect of refraction on the determination of aspect angle is discussed.

A radio wave traversing the troposphere at a finite angle to the vertical is refracted downward. Most of the deviation from rectilinear propagation occurs in the lowest 15 km. The assumption of straight line propagation implicit in eq. [B.1] leads to an overestimate of the true echo height, i.e. $h > z$. With an assumed tropospheric refractive index profile, the correction, Δh_r , to be applied to the nominal echo height is a single-valued function of slant range. The correction has been given by Unwin [1966] for the profile shown in Fig. B.1. Sample values of Δh_r are presented in Table B.1. The correction can be adapted to other models of the refractive index profile by multiplying Δh_r by the factor $(\mu_0 - 1)10^6 / 320$, where μ_0 is the refractivity at sea level. The differences in refractivity between sites at auroral latitudes and the magnitude of its seasonal variation are sufficiently small ($\sim 10\%$, Segal and Barrington, 1977) that this higher order correction need not be applied in the current analysis. The systematic error possibly incurred on echo height is less than a few tenths of a kilometre generally, as large

as a kilometre only at great (>900 km) ranges for nominal echo heights (<95 km) lower than any actually observed.

TABLE B.1

Correction to echo height for tropospheric refraction (km)

<u>Nominal height (km)</u>	<u>Slant range (km)</u>			
	600	700	800	900
120	-1.0	-1.75	-2.5	-3.5
115	-1.1	-1.8	-2.6	-3.8
110	-1.25	-1.85	-2.8	-4.1
105	-1.4	-2.0	-3.0	-4.4
100	-1.6	-2.2	-3.3	-4.7

The magnetic aspect data reported in the text have been obtained on the assumption that the line-of-sight vector to the field point has the orientation of the radar ray in the sampling volume. Tropospheric refraction actually rotates the radar ray somewhat with respect to the reference vector. The amount of angular deviation, $\Delta\alpha$, represents a possible error on the calculation of aspect angle. The effect is most pronounced at large ranges and small elevation angles. An upper bound on $\Delta\alpha$ can be obtained by assuming that all the tropospheric refraction occurs at a height of 15 km. For $R = 1000$ km, $h = 110$ km, and $\Delta h_r = 6$ km, this gives $\Delta\alpha = .13$ deg. The error on aspect angle incurred by ignoring wave vector rotation in the troposphere is generally less than a tenth of a degree, which is negligible.

On lower VHF frequencies refraction in the ionosphere

has important consequences for the aspect geometry. However, the effect on 398 MHz is quite small. To illustrate this, we estimate the amount of wave vector rotation expected in the ionosphere. The wave propagation obeys Snell's law and an expression for the refractive index in the E region is

$$n = (1 - 80.5 N_e / f^2)^{1/2}, \quad [B.4]$$

where N_e is the electron density in el/m^3 and f is the radar sampling frequency in Hz. The refractive index is set equal to 1 below the E region. For a peak E region electron density $N_e = 5 \cdot 10^{11} \text{ el}/\text{m}^3$ the maximum angular deviation on 398 MHz for a slant range of 1000 km is about .04 deg. The deviation is less for lower densities or shorter range. No adjustment of the aspect data for ionospheric refraction is deemed necessary.

Fig. B.1 Tropospheric refractive index profile
(after Unwin, 1966).

APPENDIX C

FADING OF RADAR AURORAL SIGNAL

C.1 Analysis of fading

In observations of radar aurora, scatter is usually received from sizeable ($\sim \text{km}^3$) sampling volumes. Each scattering centre generates a wavelet that contributes to the returned signal. Relative movement of the centres alters the phases of the wavelets at the receiving site and changes the resultant amplitude (Hultqvist and Egeland, 1964). The amplitude variability is called interference fading. If the scattering centres are numerous the behaviour of the amplitude can be described by fluctuation statistics. For example, if the scatter is caused by many small irregularities uncorrelated in amplitude and phase the probability density function (P.D.F) of the total signal amplitude, A , is of the Rayleigh type (Rice, 1954):

$$P(A) = (A/\sigma^2) \exp(-A^2/2\sigma^2) , \quad [C.1]$$

where by definition $A > 0$, $\int_0^\infty P(A) dA = 1$, and $\sigma^2 = \int_0^\infty W(f) df$, where $W(f)$ is the power spectrum of the scattered radiation. If, in addition to the randomly fading component, the signal contains a steady component (due possibly to specular reflection from a layer), the P.D.F. of the resultant amplitude is (Rice, 1954)

$$P(A) = \frac{A}{\sigma^2} \exp\left[-\frac{A^2+B^2}{2\sigma^2}\right] I_0\left[\frac{AB}{\sigma^2}\right] , \quad [C.2]$$

where I_0 is the Bessel function of zero order and imaginary

argument, B is the amplitude of the specular component, and σ is the same as in eq. [C.1]. If the amplitude of the specular component is considerably greater than that of the random component (i.e. $B/\sigma > 3$), then, over the range of values of A where $P(A)$ is appreciable, eq. [C.3] reduces to (McNicol, 1949)

$$P(A) = \left[\frac{1}{2\pi\sigma^2} \right]^{1/2} \exp \left[-\frac{(A-B_m)^2}{2\sigma^2} \right], \quad [C.3]$$

where $B_m^2 = (B^2 + \sigma^2)$. When specular reflection dominates the received signal the P.D.F. of the resultant amplitude is a displaced Gaussian. The progression in the character of the amplitude density function from Rayleigh to displaced Gaussian with increasing value of B/σ is suggested in Fig. C.1.

Fig. C.1 Probability distribution of amplitude for three values of the Rice parameter $g = f = B/\sigma$ (after Egeland, 1962).

If the amplitude statistics conform to the Rayleigh model described by eq. [C.1] the P.D.F. of the signal intensity, I , can be shown to be

$$P(I) = (1/\langle I \rangle) \exp(-I/\langle I \rangle) , \quad [C.4]$$

where $\langle I \rangle = 2\sigma^2$. In the Homer experiment, the detected signal was logarithmically amplified and sampled by an A/D converter to provide the values s_i (stored locally in the computer) according to $s_i = a \log(I/I_r)$, where a is a calibration constant and I_r is an arbitrary reference intensity level. The s_i values from three pulse transmissions were averaged to produce the value s which appears in the power array. Because of the stochastic variability of the backscattered power, s_i and s obeyed fluctuation statistics. The P.D.F. of the computer value s_i can be expressed in terms of the normalized parameter $t = I/\langle I \rangle$ as

$$P(s_i) = P(a \log(t)) = (1/a \log(e)) t \exp(-t) , \quad [C.5]$$

where e is the base of the natural logarithms and I_r has been set equal to $\langle I \rangle$. The variance of this distribution is

$$\sigma^2_{s_i} = (a \log(e))^2 (\pi^2/6) . \quad [C.6]$$

Observations in the UHF have shown that the pulse-to-pulse echo intensity from a given point in space becomes completely decorrelated in about 10 milliseconds (Jaye et al., 1969). Since the pulse repeat time in the Homer experiment was 13 milliseconds, the s_i values recorded for

successive pulses represented independent measures of the backscattered intensity. The P.D.F. of the array value $s = (s_i[1] + s_i[2] + s_i[3])/3$ can be written parametrically as

$$P(s) = P(\alpha \log(t)/3) = t_0 \int_0^\infty (1/x) \exp(-4t/x^2) K_0(x) dx, \quad [C.7]$$

where K_0 is the modified Bessel function of the second kind of order 0. This is very nearly Gaussian and may be approximated by the formula

$$P(s) = (2\pi\sigma_s^2)^{-1/2} \exp(-s^2/2\sigma_s^2), \quad [C.8]$$

where σ_s^2 is the variance of the density function described by eq. [C.7]. The standard deviations σ_i and σ_s are related in a simple way:

$$\sigma_s = \sigma_i / (3)^{1/2}. \quad [C.9]$$

For Rayleigh fading the solution for σ_s is

$$\sigma_R = (\alpha \log(e)) \pi / 3 (2)^{1/2}. \quad [C.10]$$

The analysis can be repeated for the amplitude distribution, eq. [C.3], appropriate to the case of scatter that is dominated by a specular component. The P.D.F. of the intensity is then

$$P(I) = \frac{1}{(8\pi\sigma^2 I)^{1/2}} \exp\left[-\frac{(I - B_m)^2}{2\sigma^2}\right]. \quad [C.11]$$

The standard deviation, σ_s , of the power array value that derives from this expression is a function of B/σ . For two representative values of this ratio we have

$$\sigma_G = (\alpha \log(e))^{1/2} \cdot 58 / (3) \quad (B/\sigma = 4) \quad [C.12]$$

$$\sigma_G = (\alpha \log(e))^{1/2} \cdot 40 / (3) \quad (B/\sigma = 7)$$

These correspond to variances that are 20% and 10% respectively of the variance expected on the basis of pure Rayleigh fading. The relative fluctuation of the power array value about its expected or mean value is much smaller when the backscattered signal is dominated by a specular component.

C.2 Comparison with Homer data

The stochastic nature of the power measurement is evident immediately in the Homer data. The entries in the power array vary widely even when mean power levels are quite stable. The magnitude of the fading is of some importance to the error analysis in the current work. The nature of the fading (i.e. Rayleigh or non-Rayleigh) is of independent interest.

Studies of radar auroral fading are usually based on analysis of continuous fading traces recorded for fixed antenna pointing directions (e.g. Egeland, 1962; Haldoupis et al., 1980). Because the Homer radar continually scanned a large part of the sky, the variation of signal strength was dominated by geographic variation of echo intensity, not by the statistical properties of the signal itself. It is possible to isolate the stochasticity

of the power measurement by removing, through a form of normalization, the geographical variations of intensity. Consider that for each set of range-azimuth coordinates the Homer radar sampled backscatter returns from thirteen volumes separated successively in elevation by 1 deg. For simplicity, assume the scattering originates at a thin layer. (The beam-layer geometry is depicted in Fig. 4.1.) For a given slant range, the axis of the beam is nearest intersection with the layer at one of the thirteen settings of elevation angle. On average, this is the beam position for which the strongest backscatter is received. The antenna beam at this position is taken to define the primary beam. The secondary beam is defined analogously in terms of the elevation angle that corresponds to next-to-nearest intersection of the beam axis with the layer. The important measures for the fading analysis (as in the analysis of height) are not the absolute levels of the signal intensity recorded in the primary and secondary beams but the relative levels. Every instance of scattering is equivalent in this scheme and the common statistical properties of the mass of data can be considered.

Fading enters the problem in the following way. For a given beam-layer geometry and in the absence of stochastic fluctuation the intensity in the primary beam would exceed that in the secondary beam by a constant ratio. In terms of log power (i.e. the units of the power array) the signal

from the primary would exceed the signal from the secondary by a constant amount. We write the expected or mean difference in power array values as $\langle \Delta s \rangle = s_p - s_s$, where the subscripts refer to the primary and secondary beams, and note that it depends only on the angular separation of the scattering layer from the axis of the primary beam. Because the backscattered signal is noise-like, there is a spread in the value of Δs that could be recorded. The standard deviation of the distribution of Δs can be anticipated from the results of the previous section. It is simply $\sigma_{\Delta s} = \sqrt{2}\sigma_s$, where σ_s for Rayleigh fading is given by eq. [C.10] and for scatter dominated by a specular component by eqs. [C.12].

Nothing can be concluded about the fading from a single value of Δs because the expected value, $\langle \Delta s \rangle$, is not known a priori. However, over a large number of echo events, observed with many different beam-layer geometries, the echo layer will be positioned with equal probability at every angle from 0 to .5 deg to the axis of the primary beam. It is then relatively straightforward to predict the experimental density function, $P(\Delta s)$, given a value for $\sigma_{\Delta s}$. The analytical expression for $P(\Delta s)$ is quite involved owing to contributions from higher order beams and is not given here. Instead, a comparison is made in Fig. C.2 between density functions for two theoretical models of interference fading and an experimental distribution obtained from a one-hour sample of data. Curve A applies

to the case of pure Rayleigh fading with σ_s given by eq. [C.10]. Curve B applies to the case of scatter that is dominated by a specular component with σ_s given by eq. [C.12] and $B/\sigma = 4$. (The value of the coefficient, u , has been taken from a calibration check carried out in 1973.) Curve A is reproduced by the experimental data quite closely. This is found to be the case for all periods for which stable fading statistics can be determined. Density functions for scatter dominated by a specular component are characterized by very limited high- Δ_s tails. Of course, there may be instances of strong specular scattering that cannot be identified against the background of predominant Rayleigh scattering.

Echo height in the current analysis is determined from Δ_s . The stochastic variability of this quantity due to fading imposes a large uncertainty on the result. In fact, the standard deviation σ_{Δ_s} is as large as the largest value of $\langle \Delta_s \rangle$ that could exist (obtained for exact intersection of the axis of the primary beam with the scattering layer.) This corresponds to an uncertainty on the determination of elevation angle of about .5 deg. Considerable processing of the data is required to overcome the effect of fading and recover km-scale resolution of echo height.

Fig. C.2 Probability density functions of Δs as defined in the text. The experimental data are taken from the period of April 1, 1973. Figures refer to the start and stop times (in UT) for data processing and the total number of counts.

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