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The Behavior of a Falling Particle in a Funnel

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A thesis submitted in partial fulfillment of the requirements for the degree in Master of Science

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Abstract

Recent theoretical work has suggested that a frictional, inelastic, spherical particle falling under gravity through a symmetric funnel will display interesting behavior as a function of the angle of the funnel walls. We have studied this system experimentally, using high-speed video to record the particle trajectories. By analyzing the video images, we have analyzed the time the ball spends in the funnel and its energy loss as functions of the angle of the walls with respect to the horizontal. The coefficient of restitution was also varied by using different balls and funnel materials. We found the time spent in the funnel increases for angles greater than 45° as a result of existence a neutrally quasiperiodic orbits. Also, the time spent in the funnel is more independent on the initial location of the particle in the funnel at higher angles than at small angles. These results are similar to the theoretical predictions. On the other side, our experimental results show that the anomalous behavior is more pronounced for high restitution coefficient than small restitution coefficient, which disagrees with the theoretical prediction.

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Now, after two years full of rich science knowledge I can only thank those bright candles that helped to light the way for me to get to the stage where ambition met with reality. The first words of gratitude go to Dr. John deBruyn, who supervised my project and illuminated my path through his guidance. He did not hesitate to provide assistance for the implementation of this project. I appreciate his humility and good scientific guidance which helped my research succeed.

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I give special thanks to my sponsor, the Saudi Arabian Cultural Bureau, and Taibah University, K.S.A, for their financial support over the last three years. I also thank J.Wylie for his cooperation and sending us his computer code. I don't want to forget to thank the Natural Science and Engineering Research Council of Canada for the research funding.

I have more words of thanks and appreciation to my father and mother for their love, support, and compassion that has not been interrupted for one day, and for their prayers for my with success. My parents are symbols of patience and tenderness.

In conclusion, I extend my thanks and gratitude to my husband and children, and all my sisters and brothers for their help and sacrifices for me throughout my study period. I wish them all good health and longevity.

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Chapter 1
Introduction

The research reported in this thesis involved studying the dynamics of a single particle falling through a symmetric funnel under gravity. In this Chapter I introduce granular materials, review some previous work in areas related to my thesis topic, and briefly outline my work and the organization of this thesis.

1.1 Granular materials

Granular materials are made up of large numbers of distinct solid particles. Physicists and engineers are interested in understanding the behavior of granular materials for many reasons. First of all, matter is typically classified as solid, liquid, or gas, but granular materials are different from all of these and have unique behavior and properties. Sometimes they behave like liquids, as dry sand takes the shape of a container it is poured into [1]. Although equations that describe the behavior of traditional solids, liquids and gases are well known, granular materials do not generally obey these equations and there is no general mathematical description of granular materials.

Second, granular materials are of great importance and are widely found in our daily life. They were also important in the ancient world and play an important role in many industries. For example, granular materials are used extensively in farming, mining, the construction of houses and roads, food products, pharmaceuticals, and bulk chemicals [2].

In nearly all applications, contact interactions between particles or between particles and boundaries are important in determining how the granular materials behave. Moreover, granular materials often flow through chutes and hoppers but our understanding of their behavior in these situations is limited [4]. For instance, some factories use devices to store and transport granular materials that depend on the flow of particles through funnels to direct particles to a desired location, or to control their speed and flow rate. Understanding the physics of granular flows will help to increase productivity and maintain the quality and effectiveness of their machines. According to Ref. [3], many factories that transport and sort granular materials waste 60% of their power. This is a result of our poor understanding of the behavior of granular materials. As a result, any improvement in our understanding of how these materials behave could have significant impact in industry.

1.2 Previous work

In recent years there has been a great deal of interest in the behaviour of granular flow through funnels and many groups of scientists have obtained significant results on this and related subjects.

Early work to examine the properties of glass beads flowing in a two dimensional funnel was done by Le Pennec and co-workers [5] who found a direct link between the flow characteristics and the geometry of the funnel. The behavior of a spherical particle bouncing inelastically on a vertically vibrating flat surface has been studied by Mehta and Luck. They found that the particle behaved in a complicated way, and that the dynamics of the system were difficult to predict [6, 7]. McNamara and Young discovered that an infinite number of collisions could occur in a finite time in a system of a finite number of falling particles [8]. Wylie and co-workers studied the behavior of a one–dimensional system of inelastic particles of different masses flowing through a funnel. They observed that the inelastic particles move in one orbit as a result of the breakdown a large number of complicated periodic orbits [9, 10]. Gao and co-workers studied a spherical, inelastic, frictionless particle bouncing in a corner and found that the particle will either leave the system after making a limited number of collisions or undergo an unlimited number of collisions in certain time [11].

More recently, there has been a number of studies on the effect of friction on the behavior of granular materials falling through a funnel, such as that by Fang et al. [9], who showed that the time that the particle spent in the funnel did not decrease monotonically as the angle of the funnel's walls increased. Spheres in a two-dimensional vibrating box were modeled by Luding. He found that the behavior of the system depended on the friction of both particles and walls [12]. Brilliantov et al. introduced a model for collisions in a granular material based on dissipative viscoelastic collisions. They used the impact velocity to calculate the restitution coefficients for normal and tangential motion. They found there is a direct link between the kind of collision and both surface characteristics and velocity of the colliding grains [13]. Foerster et al.

measured the attributes of collisions between small spheres or between small spheres and a flat plate. Their results showed that a collision model that included friction, normal restitution, and tangential restitution explained the observed impact behavior over a large range of incident angles [14].

1.2.1 The work of Wylie and co-workers

Fang et al. [15] performed simulations of a simple system consisting of a single, inelastic, and frictionless particle falling through a symmetric funnel under gravity. They noted that for certain funnel wall angles, the particle spent more time in the funnel and lost more energy than expected. These phenomena were more pronounced when the coefficient of restitution of the system was large than when it was small. Figure 1.1 shows the average time spent by the particle in the funnel as a function of the angle of funnel walls with the horizontal for a restitution coefficient equal to 0.99. These data where calculated using a simulation code provided to us by Dr. J. Wylie. While the time spent by the particle in the funnel generally decreases as the angle increases, there are several peaks in the plot, including a large peak around an angle of 50°. The simulations of Fang et al. indicated that in these peaks, the particle bounces back and forth between the walls in a simple repeating pattern. This leads the particle to spend a long time in the funnel. In contrast, away from these peaks the particle's trajectory is much more random. Fang et al. also observed that the sequence of collisions the ball makes with the funnel walls is highly sensitive to the initial position of the falling ball and to the wall angle. At small angles, the collision sequence is quite random for all initial positions on the funnel walls, both near to the centre of the funnel and far from it. At large angles, however, the particle follows a more uniform pattern, especially for initial positions near to the centre of the funnel. This is illustrated in Figure 1.2, which shows some experimental results that will be discussed in more detail in Chapter 3. Their theoretical analysis showed that these phenomena are a result of the existence of neutrally stable quasiperiodic orbits at certain angles.

Zhang et al. [4] incorporated friction into these simulations. This is physically close to the system that we studied experimentally. In their simulations, the system showed phenomena quite similar to those observed without friction. However, the frictional system showed the anomalous behaviour at all funnel angles steeper than 45° rather than just the small ranges of angles in the frictionless system. Although friction causes the particle to rotate and the dynamics of the particle's collisions become more complicated, the particle trajectory at large angles shows a relatively simple repeating pattern of collisions as in Fig 1.2(a), while at small angles it undergoes a complicated pattern of collisions, as in Fig 1.2(b).

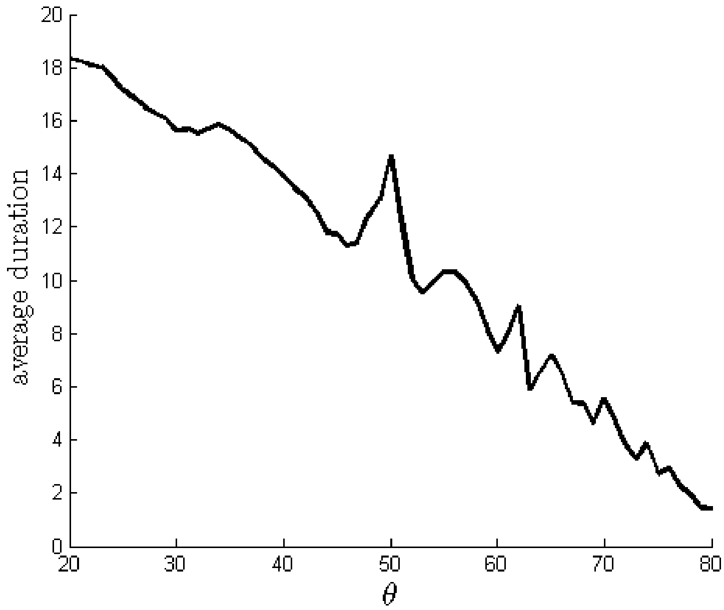


Figure 1.1: Simulation results showing the nondimensionalized average time spent in the funnel as a function of funnel angle for a system with restitution coefficient $e = 0.99$. Simulations were performed using a Matlab program provided by Prof. J. Wylie.

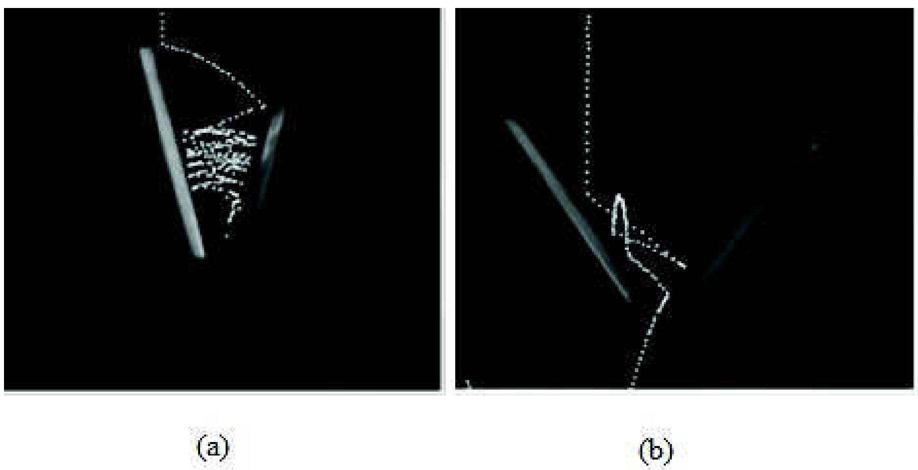


Figure 1.2: Experimentally observed particle trajectories for funnels with (a) $\theta = 60^\circ$ and (b) $\theta = 40^\circ$.

1.3 Outline of this work

For this research, I built a system to study experimentally the behavior of a single particle falling under gravity through a symmetric funnel as in Figure 1.3. In order to choose appropriate materials to use for the funnel walls and the ball, I also carried out an experiment to measure the coefficient of restitution of a variety of different material combinations. Using funnels made from the chosen materials, I recorded the trajectory of the balls as they fell through the funnel using a high speed camera. Matlab programs were written to analyze the data obtained from these experiments. Finally, I compared my results with the theoretical results of Refs. [4, 15]. Our results are in partial agreement with the theoretical results of Zhang et al. that were reported in Ref. [4].

The remaining chapters of the thesis are as follows: In Chapter 2 I describe the apparatus and the experiments performed. In Chapter 3 I present our results. In addition, I will compare our observations and results with the theoretical results. Finally, in Chapter 4 I discuss our results, summarize, and conclude our work.

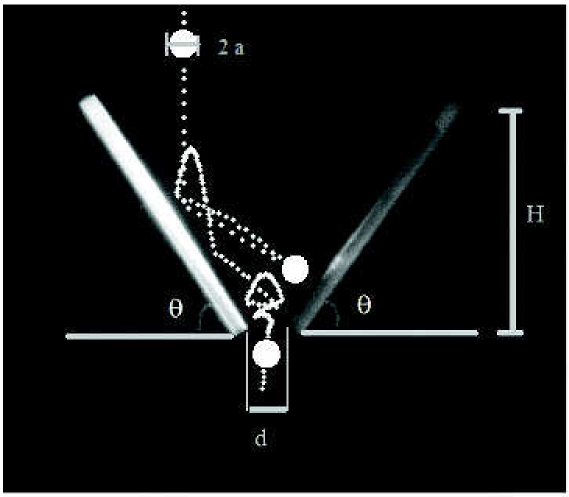


Figure 1.3: An image from my experiment showing the system that is used in this thesis for our study of a particle falling through a funnel. H is the height from the bottom of the funnel, a is the radius of the ball, d the gap at the bottom of the funnel, and θ is the angle of funnel's walls with the horizontal.

Chapter 2
Experimental Apparatus and Procedure

As part of the work carried out for this thesis an experimental apparatus for studying the behavior of a particle falling through a symmetric funnel was designed. In this chapter, we describe our apparatus and discuss the experimental procedures.

2.1 Restitution experiment

An experiment was carried out to measure the restitution coefficient e for various combinations of balls and plates. The restitution coefficient is the ratio of the speed after a collision to the speed before the collision. Its value is between 0 and 1. $e = 0$ means the collision is perfectly inelastic and $e = 1$ means it is perfectly elastic. $1 - e^2$ is the fraction of kinetic energy that remains after a collision between two objects. The restitution coefficient is an important parameter in our funnel experiments because the particle loses energy as it falls through the funnel as a result of inelastic collisions. If e is small, the anomalous behavior reported in Ref. [4] is less pronounced, while it is more pronounced for large values.

The apparatus used to measure the restitution coefficient is shown in Figure 2.1; the schematic diagram in Fig 2.2 illustrates the setup of the device. The ball is dropped from a holder onto a metal plate. The holder has a mechanism to control the ball's release. Two plates were used for this experiment, one made from aluminium and the other from steel. The size of the plates was 12.1 cm x 7.6 cm. The plates are fixed on an optical table by four screws at their corners as seen in Figure 2.3. A high speed camera operating at 250 frames per second was used to record the position of the particle as it bounces on the plate. A MiDAS Motion Trigger device is used to trigger the camera when the ball bounces on the plate. The camera is connected to a computer which is used to save and analyze the data, and to calculate the restitution coefficient. Two 500 watt halogen lights are used to illuminate the funnel and the ball to make them clearly visible in the recorded images.

At the beginning of the experiment, the ball was dropped from a height H onto the flat plate. The ball is released from the holder with zero initial velocity and without applying any force other than gravity by moving a small metal tab that holds the ball in place. The ball bounces several times on the plate before stopping. The high speed camera records the position of the bouncing ball. The highest height recorded by the ball after the first collision is h. A matlab program was used to determine h and to calculate the restitution coefficient as follows. The restitution coefficient is

$$e = \frac{v_f}{v_i}, \tag{2.1}$$

where v_i and v_f are the velocity of the ball before and after it hits the plate. Since the ball is released from rest, its velocity when it hits the plate is given by conservation of energy as

$$v_i = \sqrt{2gH}, \tag{2.2}$$

where g is the acceleration due to gravity. The ball bounces off the plate with velocity

$$v_f = ev_i \tag{2.3}$$

and rises to a height h. Applying conservation of energy after the collision gives

$$v_i = \sqrt{2gh} \tag{2.4}$$

so

$$e = \frac{v_f}{v_i} = \sqrt{\frac{h}{H}}. \tag{2.5}$$

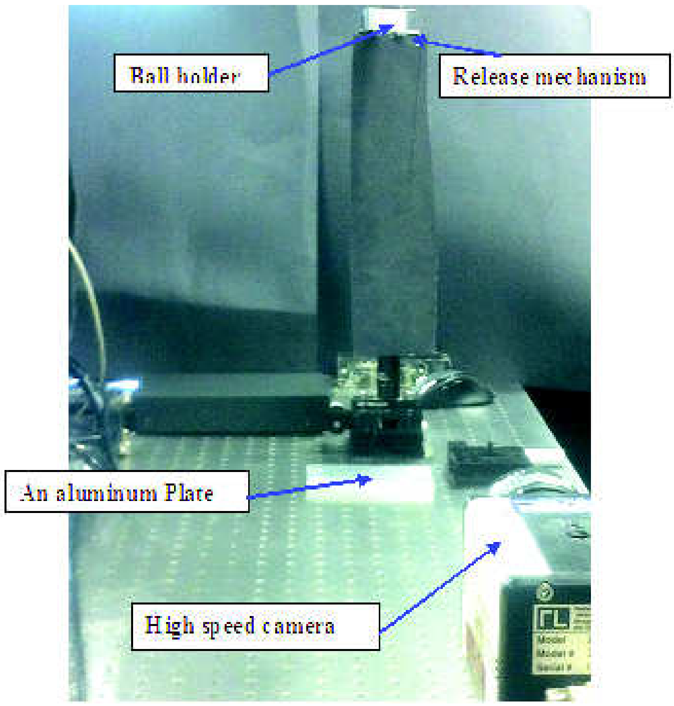


Figure 2.1: Photograph of the experiment used to measure the coefficient of restitution.

2.2 Funnel system

The system used for the funnel experiment reported here consists of a ball holder, the funnel itself, a high speed camera, and a computer. The ball was released with zero initial velocity from a height H above the bottom of the funnel and at a horizontal location x_0 measured from the central axis of the funnel (see Figure 1.3). The components of the apparatus are shown in Figure 2.4, which is a photograph of the actual experimental system. The apparatus mainly consisted of a ball holder which has at its top end a mechanism that controls the ball's release. The release mechanism was operated by pulling a thread that moved a metal key, allowing the ball to fall out of the ball holder. The holder was mounted on a post 40 cm tall. It was also fixed on a plate that could be moved by adjusting a micrometer screw to adjust the particle's starting position. Two different funnels were used for this experiment, one made from aluminium and the other from steel. The size of the funnel plates was 15.1 cm x 5.3 cm. They are attached by hinges to a pair of horizontal plates as shown in Figure 2.4. The angle of the funnel walls could be adjusted, and was measured with a protractor with 1.0° accuracy. One of funnel walls was fixed on a plate that could be moved by micrometer to adjust the gap d at the bottom of the funnel. The gap d was measured using Vernier calipers with 0.02 mm accuracy. A high speed camera was used to record the position of the particle from the time it enters to the time it leaves the funnel. The camera is triggered by a MiDAS Motion Trigger when the ball enters the funnel. The recorded images and the time at which they were taken were sent to the computer for analysis using a Matlab program which is included in Appendix A. The Matlab program tracked the position of the ball and then calculated its velocity in each video frame. It then calculated the energy loss at each collision with the funnel walls, the total energy lost in the funnel, and the time spent in the funnel. Two 500 watt halogen lights illuminate the funnel and the ball to make them clearly visible in the recorded images.

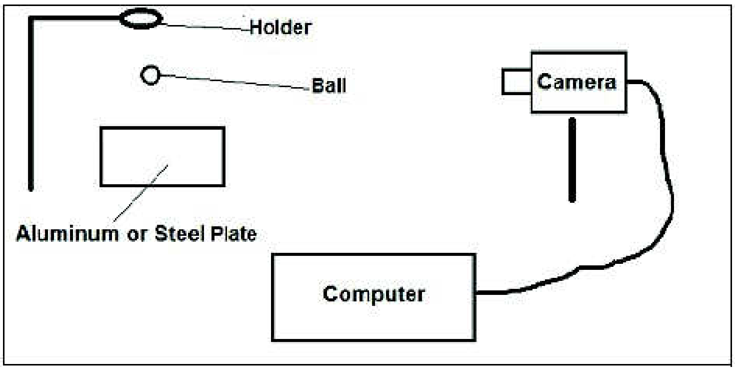


Figure 2.2: Schematic of the device used in the coefficient of restitution experiment.

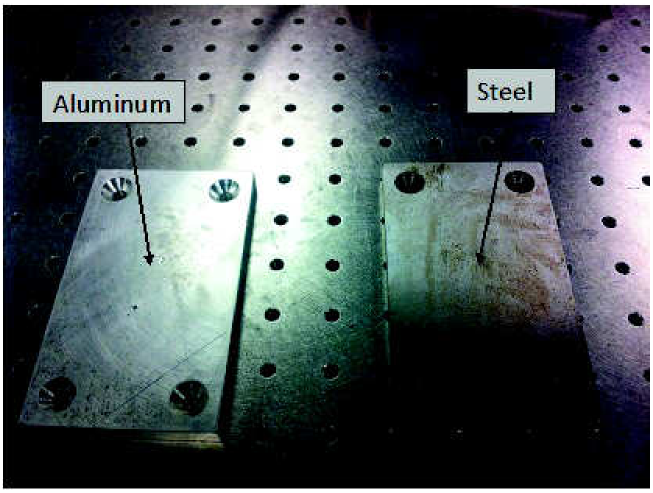


Figure 2.3: Materials used in restitution coefficient experiment.

2.3 Data analysis

In order to calculate the total energy that is lost by the sphere, the Matlab code determined the total energy

$$E = P + K, \tag{2.6}$$

$$E = mgh + \frac{1}{2}mv^2, \tag{2.7}$$

where P is the potential energy, K is the kinetic energy, m is the mass of the particle in kg, g is gravitational acceleration in $m.s^{-2}$, h is the particle height above the bottom of the funnel, and v is the particle's velocity in m/s. E, h, v vary with time while the particle is in the funnel. We calculated the total energy of the ball when it enters and leaves the funnel using Eq. (2.7) and subtracted these quantities to get the energy loss. The trajectory of the sphere is determined by the sequence of collisions it has against the walls, with free-fall motion under gravity between the collisions. We studied the dependence of these quantities on the wall angle θ, e, and on the starting position x_0. In the next chapter, we present our results.

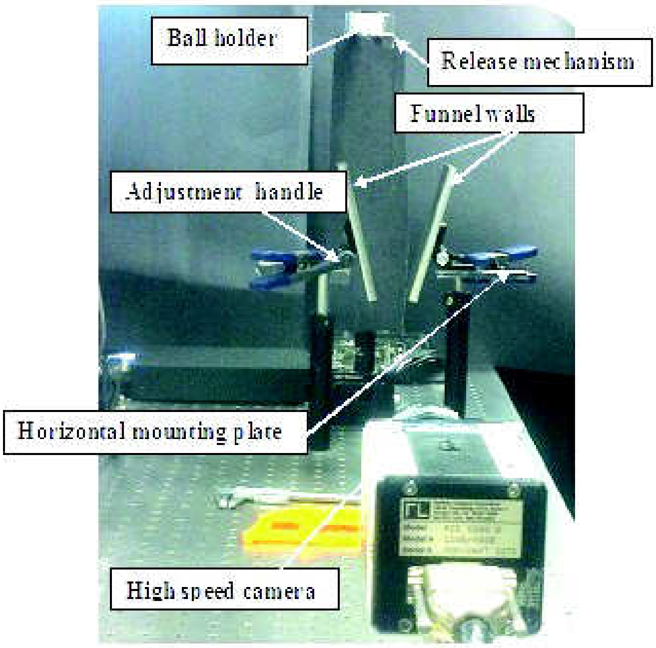


Figure 2.4: Photograph of the funnel experiments.

Chapter 3
Results

3.1 Restitution Coefficient Experiment

The restitution coefficient of several materials was investigated by measuring the height to which a ball bounced after colliding with plates of different metals. The restitution coefficient experiment was performed to aid in selecting materials to use for the funnel experiments. In particular, we wanted to find a system with a high coefficient of restitution in order to make the effects predicted in Ref. [4] more visible.

Figure 3.1 shows an example trajectory followed by an aluminum ball bouncing on a steel plate. These data are used to calculate the restitution coefficient as outlined in section 2.2. The results presented here were obtained using ceramic, aluminium and Plexiglas balls on steel and aluminium plates. Figure 3.2 shows the balls used, and Table 3.1 gives their mass and radius. Table 3.2 shows the measured values of the restitution coefficient e for all six ball and plate combinations. These measurements are the average over 30 measurements for each combination and the uncertainties are found by using the standard deviation of these values. From Table 3.2, we see that the Plexiglas ball has the highest restitution coefficient with both metal plates. Although the combination of Plexiglas ball and aluminium plate had a slightly higher restitution coefficient, we chose to use the combination of Plexiglas ball and steel plate for our funnel experiment because we found the phenomena predicted in Ref. [4] were more pronounced with this combination. We also performed experiments with the aluminum ball and steel funnel and the ceramic ball with an aluminum funnel.

3.2 Funnel Experiment

In our experiment, measurements were done on several ball / plate combinations: a ceramic ball with an aluminum funnel, Plexiglas ball with a steel funnel, and an aluminum ball with a steel funnel. We studied the time the particle spent in the funnel and its energy loss as functions of the angle of the funnel walls and the restitution coefficient of the system.

Figure 3.3 shows the relationship between the angle of the funnel walls and time the particle spends in the funnel for a ceramic ball bouncing in an aluminum funnel. For this system, the restitution coefficient is 0.79 ± 0.10. If we ignore the error bars, the average time spent in the funnel initially decreases as the angle of the funnel walls increases. At an angle of approximately 45° the time starts to increase again. Experiments were performed for a total of nine different initial positions of the ball. The holder was positioned at three different positions along the x–axis using a micrometer screw. At each x–position, the holder was placed at three different y-positions by using another micrometer screw. The height from which the ball was released was the same in all cases. The error bars in Figure 3.3 are standard deviations and show how much variation exists in the experimental values. Some of error bars are quite large, suggesting that the time is very sensitive to small uncontrollable changes of the initial conditions. In order to compare these results with the theoretical predictions, we used our values for the restitution coefficient and ball radius in the simulation software provided to us by Dr. Wylie. The results are shown in Figure 3.4. At small angles, the average time spent in the funnel decreases as the angle increases, but the simulation show no increase at higher angles. In this case, the theoretical and experimental results are similar at small angles but different at large angles. This difference may in part be due to the fact that the simulations were done for a frictionless system, while our experiments do have friction.

Table 3.1: Ball Measurements.

	Mass (g)	Radius (cm)
Ceramic Ball	4.15 ± 0.10	0.90 ± 0.02
Plexiglas Ball	1.26 ± 0.10	0.70 ± 0.02
Aluminum Ball	3.02 ± 0.10	0.60 ± 0.02

Table 3.2: Restitution Coefficients for different combinations of balls and plates.

	Ceramic Ball	Plexiglas Ball	Aluminum Ball
Aluminum Plate	0.79 ± 0.10	0.94 ± 0.04	0.85 ± 0.06
Steel Plate	0.79 ± 0.08	0.93 ± 0.01	0.87 ± 0.05

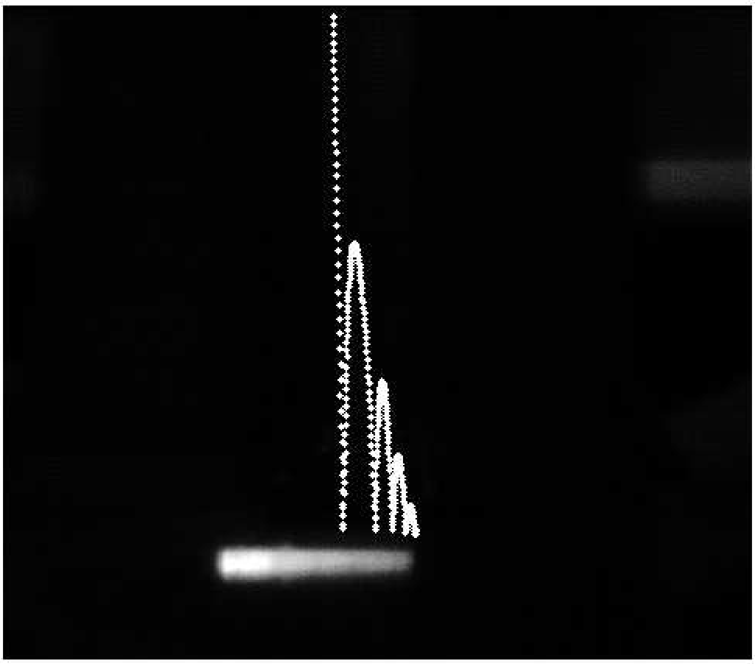


Figure 3.1: Trajectory of an aluminum ball bouncing on the steel plate.

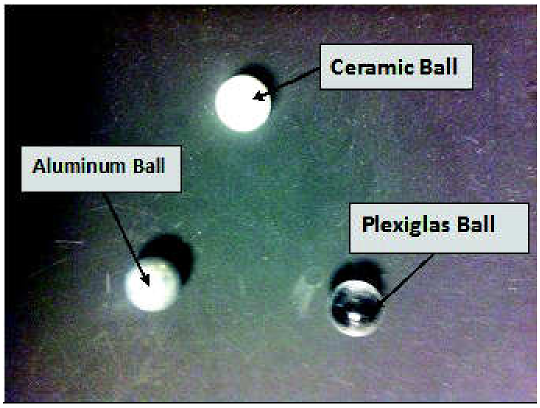


Figure 3.2: Balls used in the restitution coefficient and funnel experiments.

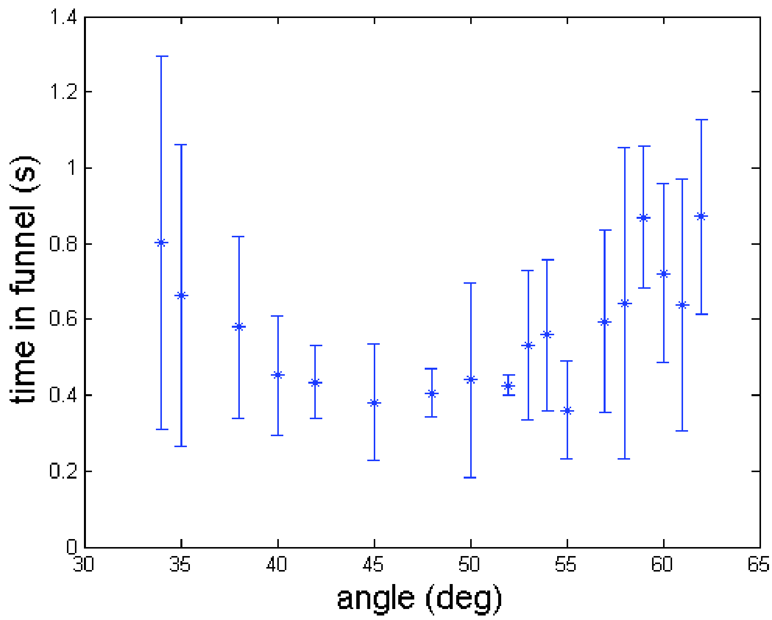


Figure 3.3: Time in funnel vs. funnel angle for the ceramic ball and aluminum funnel, restitution coefficient $= 0.79 \pm 0.10$.

Figure 3.5 shows the average energy lost by the ball falling through funnel for the same materials. The energy lost by the particle in this system is fairly constant at low funnel angles, but starts to increase around 40°. This is similar to the theoretical results of Ref. [4]. Again, error bars in Figure 3.5 are the standard deviations. The error bars reflected the amount of variation in the results at each angle.

Figure 3.6 displays the average time spent in the system for the Plexiglas ball and steel funnel with $e = 0.93 \pm 0.01$. These data are averaged over a total of nine different initial positions of the ball. The holder was positioned at three different positions along the x–axis using a micrometer screw. At each x–position, the holder was placed at three different y–positions by using another micrometer screw. The time is fairly constant as the funnel angle is varied at small angles, but shows a small peak approximately at 45° and an increase starting around 60°. As before, the large error bars reflect a sensitivity to initial conditions. Figure 3.7 shows the theoretical result obtained when we use our experimental parameters in Wylie's simulation program. At small angles the average time spent in the funnel decreases as the angle increases. There is a peak between 45° and 50°, then further decrease; there are smaller peaks at higher angles. In this case, the experimental results are quite different from the theoretical results at small angles. The theoretical results start increasing around 45° while our experimental results have an increase starting around 60°. These difference may again reflect the fact that the simulation was done for a frictionless system while our experiment was done on a system with friction.

In this case, despite the fact that e was large, we observed much less variation in the time then in the previous case, for which e was smaller. This contradicts the theoretical results of Ref. [4] which predicted that the anomalous behavior should be more pronounced for a higher restitution coefficient.

Figure 3.8 shows the energy lost by the Plexiglas ball falling through the steel funnel. The data at each angle are averaged over nine different initial positions of the ball, as above. The energy loss increases as the funnel angle is increased. Similar behavior is seen in the theoretical results for a similar restitution coefficient in Ref. [4]

The results for the aluminum ball and the steel funnel, with restitution coefficient $e = 0.87 \pm 0.05$, are shown in Figures 3.9–3.11. As displayed in Figure 3.9, the average duration of the aluminum ball during the steel funnel is fairly constant with increasing angle of the funnel walls, but shows a sharp increase around 59°. In order to compare this result with the theoretical result, we use our values of the restitution coefficient and ball radius in Wylie's simulation software. The results in Figure 3.10 show that the average time spent in the funnel decreases as the angle increase with a small peak around an angle of 50°. In this case, theoretical and experimental results are different. Again, these differences may be because the simulation were done for a frictionless system while the experiments did have friction.

Figure 3.11 displays the average energy loss of the aluminum ball in the steel funnel. The energy loss increases as the angle increases. This combination, which consists of an aluminum ball and steel funnel, and the previous combination, which consists of a Plexiglas ball and steel funnel, have similar experimental results for time spent in the funnel and energy lost as a function of funnel walls angle. Although these two systems have higher restitution coefficients than the ceramic ball and aluminum funnel, we found they showed less pronounced anomalous behavior than the system with the smallest restitution coefficient. This disagrees with the theoretical predictions of Refs. [4, 15].

Figure 3.12 illustrates the experimentally observed trajectories of the balls as they fall through the funnels. One observes that at large angles for all three systems the balls bounce back and forth across the funnel many times in a regular pattern. On the other hand, at small angles, the balls bounce much more randomly through the funnels. The former case leads to an increase in the time spent in the funnel, and because the number of collisions increases, to a corresponding increase in the energy loss. This change in the particle dynamics is in agreement with the theoretical work.

Figure 3.13 and Figure 3.14 show the results of experiments performed to show how the time spent in funnel depended on the starting locations x_0 of the ball, where x_0 is measured from the centre of the funnel. Figure 3.13 shows results for the Plexiglas ball and the steel funnel at angles 40°, 50°, and 60°, and Figure 3.14 shows results for the aluminum ball and the steel funnel at similar angles. These figures illustrate how much the duration is sensitive to the starting location. Figure 3.13(a) and Figure 3.14(a) show that in both systems the duration at an angle of 40° is quite scattered, indicating a very high sensitivity to initial location. At this angle, the ball's trajectory in quite random, as seen in Fig 3.12(d) and (e). On the other hand, Figure 3.13(c) and Figure 3.14(c) show that at high angles the duration varies smoothly with initial position. At this angle, the ball's trajectories follow a more organized pattern that is independent of the initial location. These results are in strong agreement with the theoretical results.

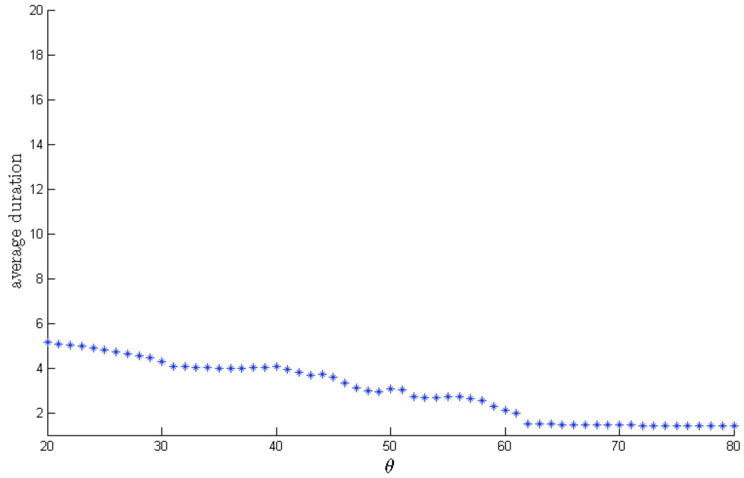


Figure 3.4: Simulation results for nondimensionalized time in funnel vs. funnel angle for the ceramic ball and aluminum funnel, restitution coefficient $= 0.79 \pm 0.10$.

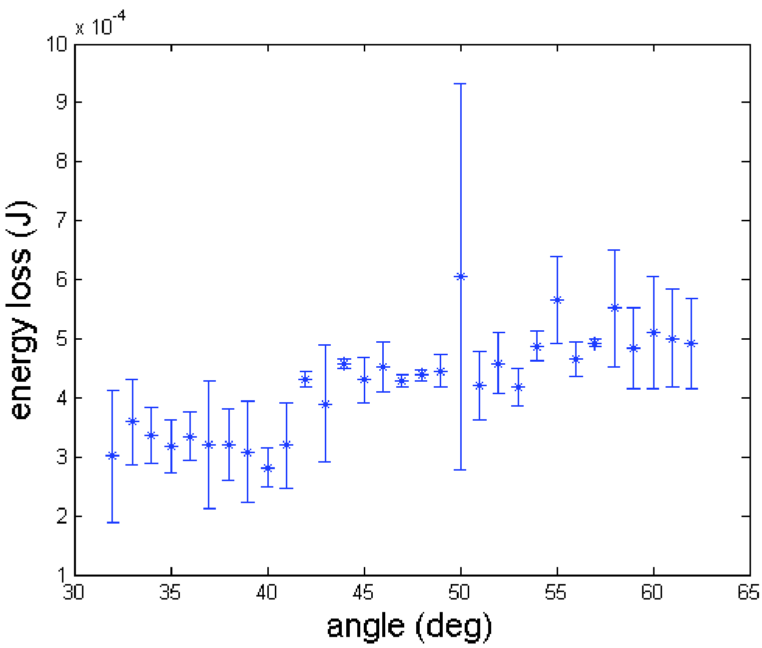


Figure 3.5: Energy loss vs. funnel angle for the ceramic ball and aluminum funnel, restitution coefficient $= 0.79 \pm 0.1$

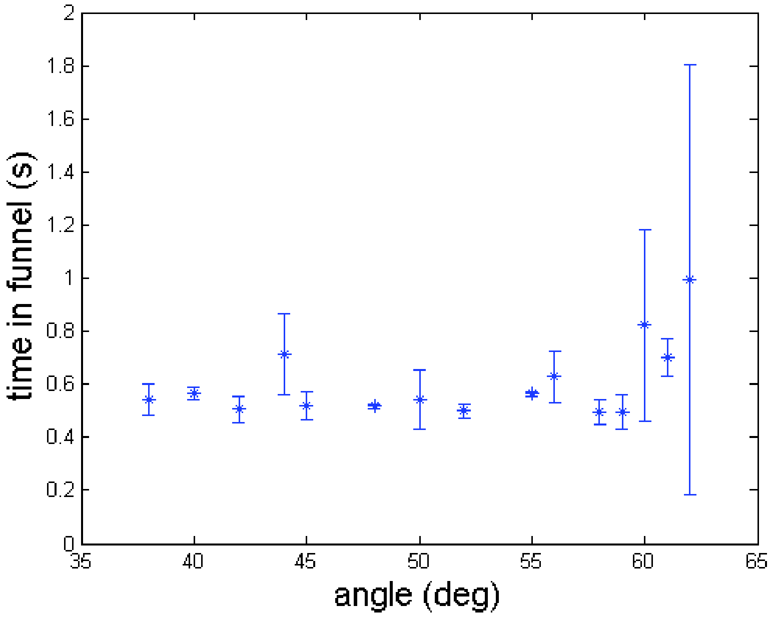


Figure 3.6: Time in funnel vs. funnel angle for the Plexiglas ball and steel funnel, restitution coefficient $= 0.93 \pm 0.01$.

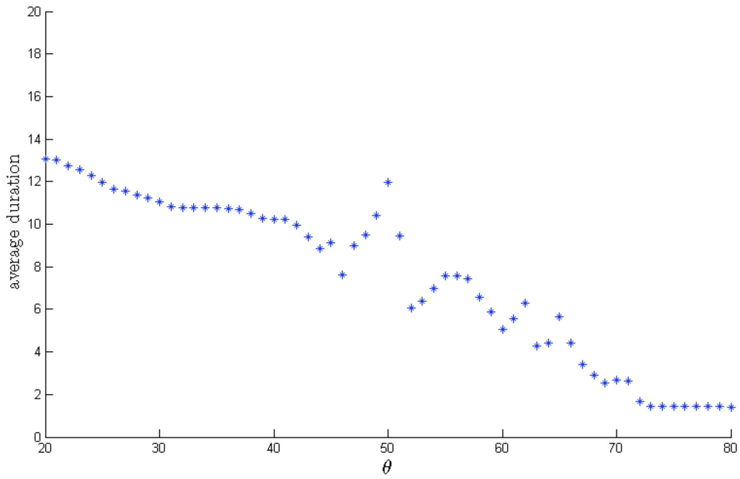


Figure 3.7: Simulation results for nondimensionalized time in funnel vs. funnel angle for the Plexiglas ball and steel funnel, restitution coefficient $= 0.93 \pm 0.01$.

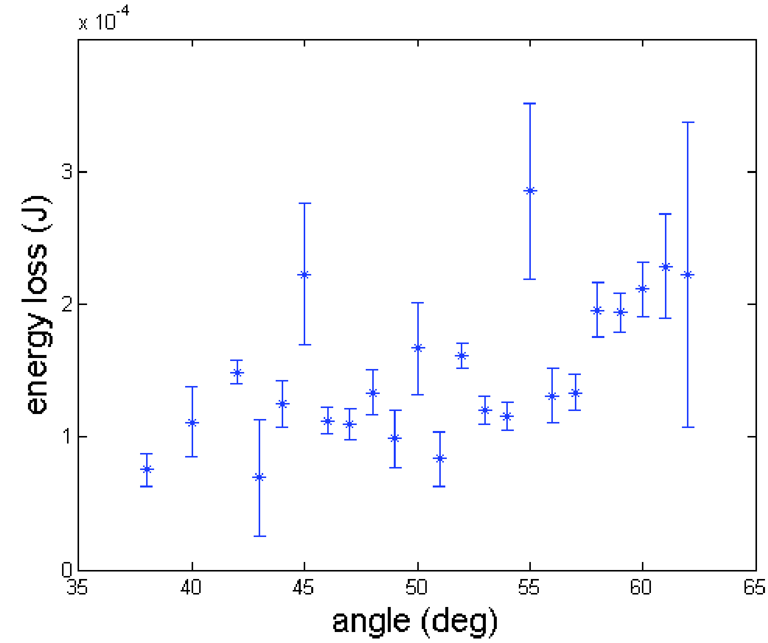


Figure 3.8: Energy loss vs. funnel angle for the Plexiglas ball and steel funnel, restitution coefficient $= 0.93 \pm 0.01$.

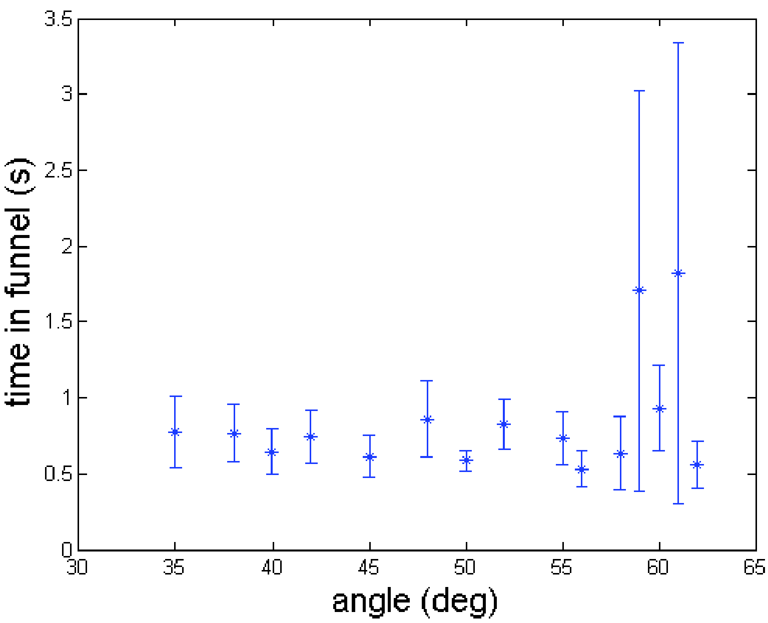


Figure 3.9: Time in funnel vs. funnel angle for the aluminum ball and steel funnel, restitution coefficient $= 0.87 \pm 0.05$.

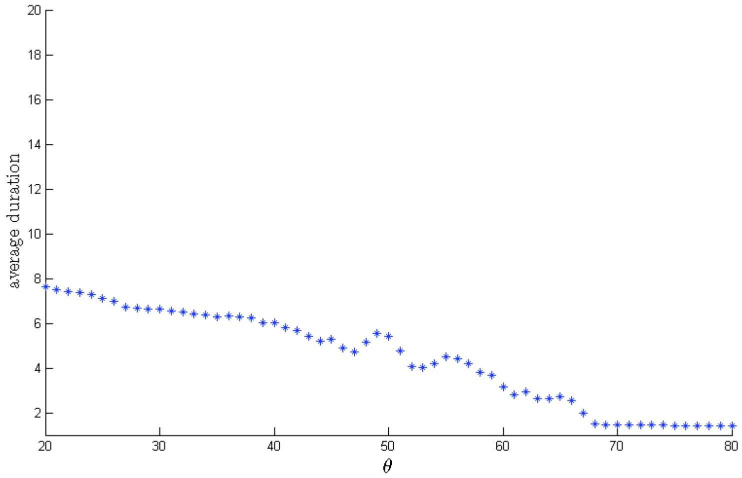


Figure 3.10: Simulation results for nondimensionalized time in funnel vs. funnel angle for the aluminum ball and steel funnel, restitution coefficient = 0.87 ± 0.05.

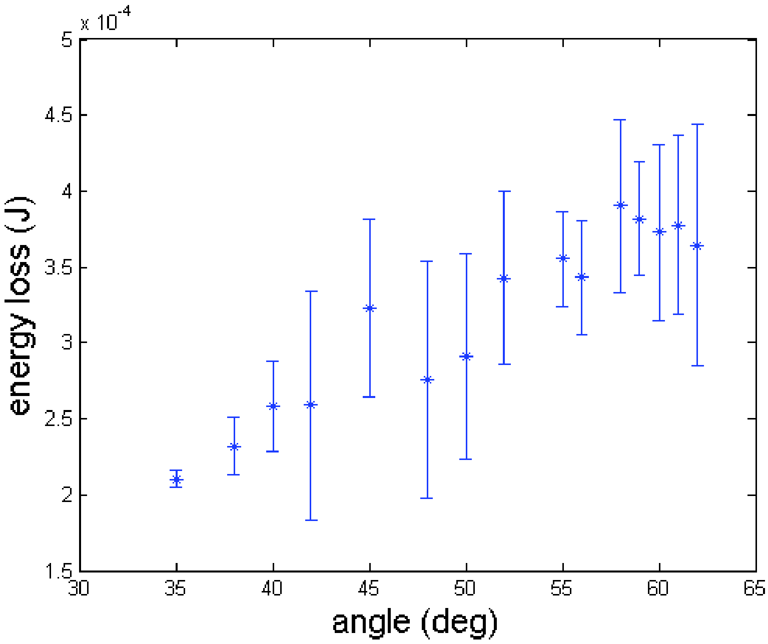


Figure 3.11: Energy loss vs. funnel angle for the aluminum ball and steel funnel, restitution coefficient = 0.87 ± 0.05.

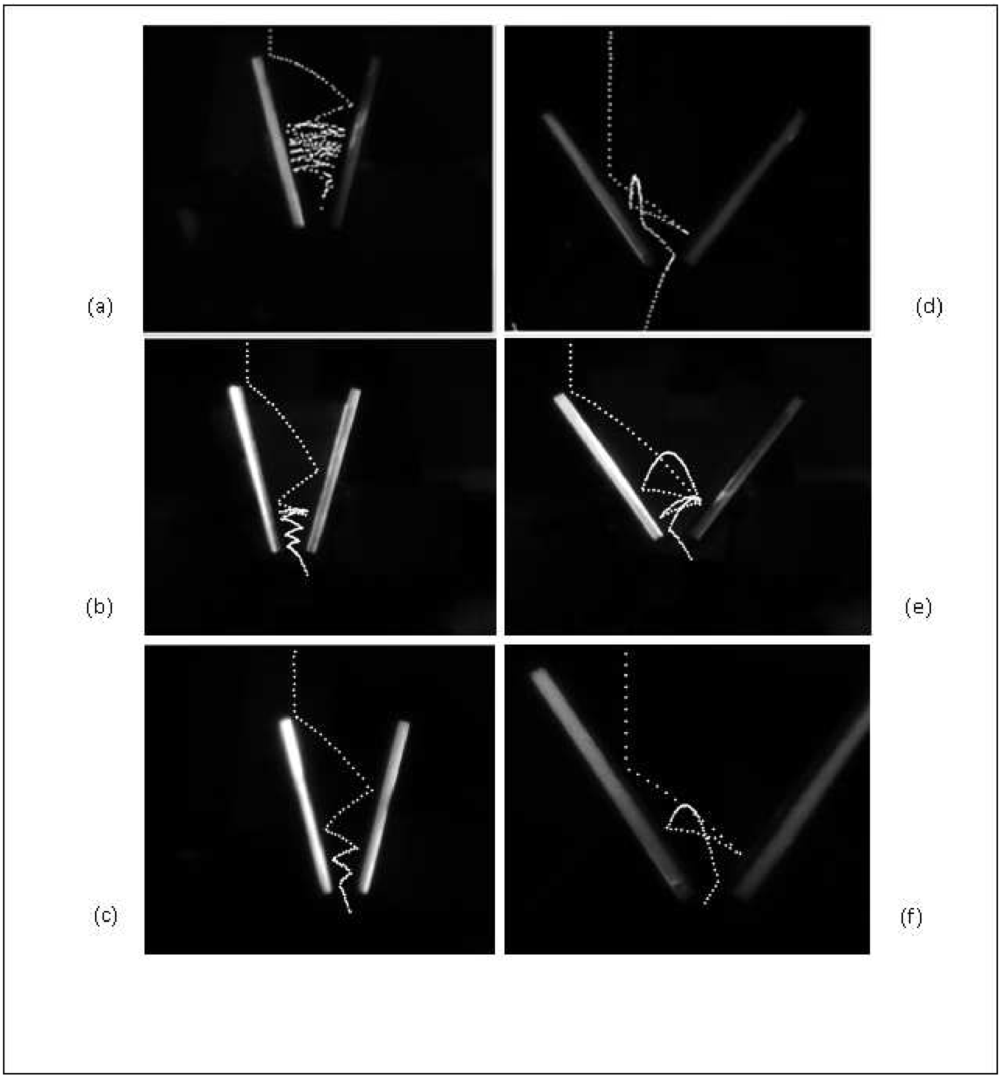
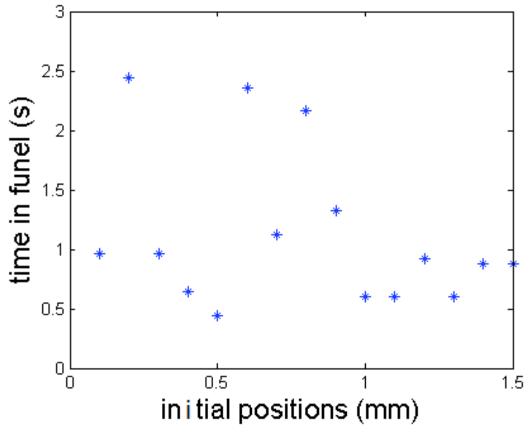
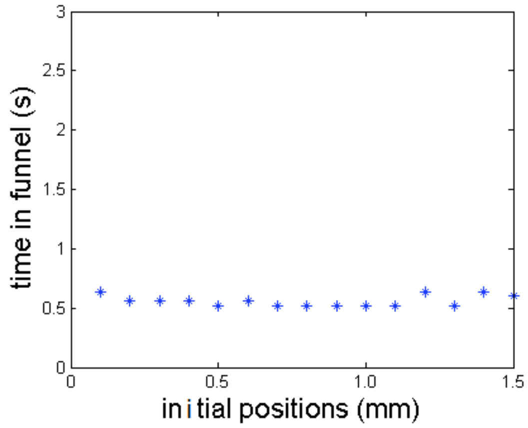


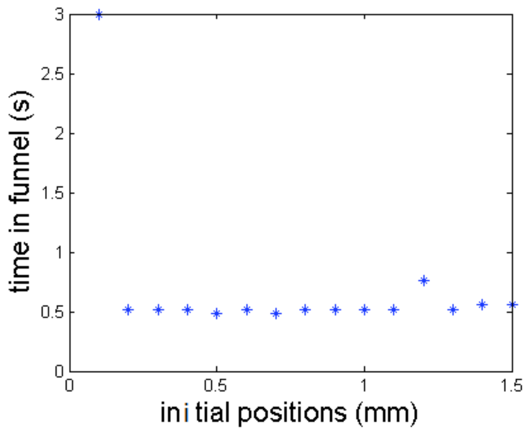
Figure 3.12: The trajectory of a ball falling through the funnel measured in six experiments. (a) Plexiglas ball and steel funnel, $e = 0.93 \pm 0.01$, $\theta = 60°$. (b) aluminum ball and steel funnel, $e = 0.87 \pm 0.05$, $\theta = 60°$. (c) ceramic ball and aluminum funnel, $e = 0.79 \pm 0.1$, $\theta = 60°$. (d) Plexiglas ball and steel funnel, $e = 0.93 \pm 0.01$,$\theta = 40°$.(e) aluminum ball and steel funnel, $e = 0.87 \pm 0.05$, $\theta = 40°$.(f) ceramic ball and aluminum funnel, $e = 0.790.1$, $\theta = 40°$.



(a)

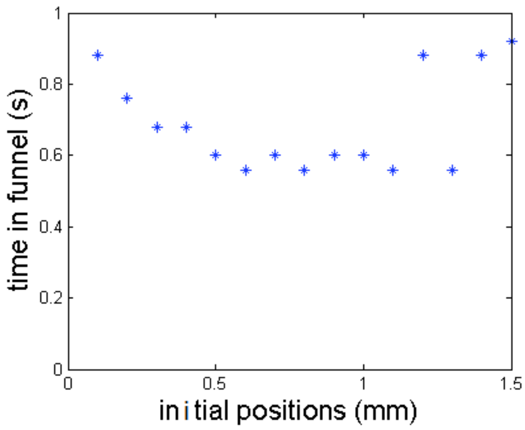


(b)

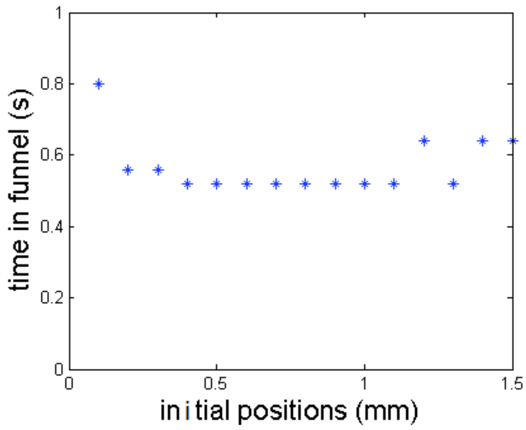


(c)

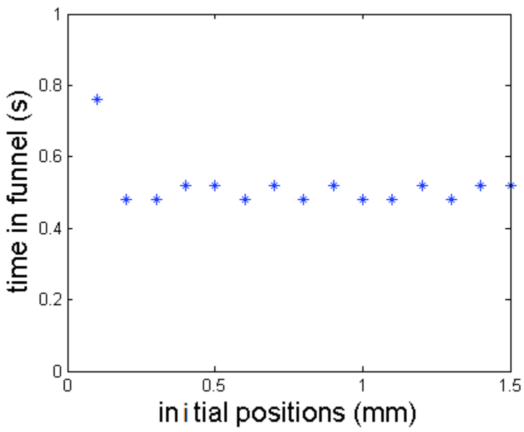
Figure 3.13: Time spent in funnel vs. the initial release location of the ball for the Plexiglas ball and the steel funnel. (a) $\theta = 40°$; (b) $\theta = 50°$; (c) $\theta = 60°$.



(a)



(b)



(c)

Figure 3.14: Time spent in funnel vs. the initial release location of the ball for the aluminum ball and the steel funnel. (a) $\theta = 40°$; (b) $\theta = 50°$; (c) $\theta = 60°$.

Chapter 4
Discussion and Conclusion

4.1 Discussion

The results presented in Chapter 3 will now be discussed and compared with the theoretical work of Refs. [4, 15]

Some of our experimental results agree well with the theoretical prediction for a frictionless particle that falls under gravity and bounces through a 2-dimensional funnel. On the other hand, however, we also found several differences between them. Theoretically, for a system with $e = 1$, peaks in the plot of the time spent in the funnel vs. angle are seen at certain angles at which neutrally stable periodic orbits were shown to exist. For $e < 1$, the peaks observed are caused by the existence of quasi-periodic orbits. Theoretically, peaks are found with simple orbits. For a simple orbit, the locations of the collisions tend to be relatively far from the funnel exit which makes the particle bounce back and forth in a quasi-periodic orbit for a long time as the particle moves down towards the exit of the funnel. However, for more complicated orbits, the locations of the collisions are led to be relatively close to the funnel exit which allows the particle to leave the funnel after only a short time in the funnel. When e is small, the particle loses more energy at each collision, and so leaves the funnel quickly even in simple orbits at high θ. Because of this, the anomalous behavior will become less pronounced as the value of the restitution coefficient decreases [4, 15]. In contrast, our experimental results shown in Figures 3.3, 3.6, and 3.9 show the opposite behavior. The experimental funnel system with the smallest restitution coefficient shows more variation in the time in the funnel than the systems with the larger restitution coefficients. This is a surprising result.

The simulation results show peaks in the time spent in the funnel around funnel angles of 45°, 60° because at these angles, the particle bounces back and forth many times in a coherent sequence of collisions before leaving the funnel. Comparing that with our results we also observed changes around 45°. While we do not see a peak like that in the simulation, we do see a change in the character of the trajectories, as shown in Figure 3.12, that is similar to that seen in the simulations. For angles less than 45°, the particles in our experiments bounced in a complicated and non-repeating pattern, while at angles greater than 45°, they bounced in a coherent repeating pattern.

The simulation results showed that the time spent in funnel by a frictionless particle decreases slightly as the angle of the funnel walls increases, and at angles steeper than 45°, the decrease continues but with small peaks around some angles. Our results also showed a decrease of the time spent in the funnel as the angle increase. The reasons for that differences would be because the simulations were done for a frictionless particle while in reality friction exists between any two contacting surfaces and will have an effect even it is small. Ref. [4] pointed out that friction causes the ball to rotate, leading to a more complicated trajectory, but these complicated orbits become more simple at high angles. Moreover, when the effects of friction are included, the predicted behavior appeared at all angles steeper than 45° rather than around certain angles as for the frictionless particle. Another possible factor that is the degree of smoothness of the funnel surfaces may contribute to the differences between experiment and theory. Surface roughness or small craters in the surface produced by earlier impacts could affect the results. Finally, variations in humidity and temperature of the laboratory where the experiments were performed could also change the condition of the funnel walls.

Both the theoretical and our experimental results showed that, at small angles, the time spent in the funnel by the particle is strongly dependent on the initial position of particle, while at higher angles, the time is much less sensitive to initial position. At low angles, the particle bounced between the left and right funnel walls on complicated and non-repeating trajectories, while at large angles, the particle hopped back and forth in a simple organized pattern.

4.2 Conclusion

The goal of our study was to experimentally investigate behaviour of a single spherical particle falling through a symmetric funnel. From high-speed video recording of the trajectory of the sphere, we were able to measure the time that the particle spends in the funnel, and the energy it loses as a function of the angle of the funnel walls for different restitution coefficients. This behaviour was simulated by Wylie et al. and explained theoretically by them. By comparing our results with the theoretical work, we observed that some of our results were similar to those reported in Refs. [4, 15] while other results were not, and we interpreted our results in terms of the theoretical predictions.

Theoretically, the anomalous behavior is predicted to be more pronounced for higher values of the coefficient of restitution. Surprisingly, in our experimental work these effects are more pronounced for lower values of the coefficient of restitution. Compared with the theoretical work, our experiment shows some phenomena similar to those reported in Refs. [4, 15]. In particular, the increase in time spent in the funnel observed above 45° for the ceramic ball in the aluminum funnel is consistent with the simulations discussed in Refs. [4, 15]. It is likely that this behaviour is due to the fact that the particles bounce back and forth across the funnel many times at steep angles, as seen in Figure 3.11 and in agreement with the theoretical work.

The time spent in the funnel for particles in all systems are highly sensitive to the starting position of the falling particle on the funnel walls. At steep angles of the funnel walls, the trajectory of the particle is more regular than that for small angles, where it is quite random. The trajectories of the falling particle at angles steeper than 45° follow a more coherent, organized pattern than that at angles less 45°, for which their trajectories follow complicated and non repeating patterns.

4.3 Future work

Our experimental results provide support and confirmation for some of the simulation results for the behavior of a single particle falling through a symmetric funnel under gravity. On the other hand, some of our results disagree with the theoretical results. There is a number of questions related to our results that could be addressed in future work.

-We found the results to change when the surface condition of the ball or funnel walls changed. It would be useful to study and understand the behavior of a particle in a symmetric funnel for different controlled surface conditions. .

-It would be useful to investigate several different ball and funnel materials with different restitution coefficients using the same technique that we used.

-We found some of our results agree with the theoretical work while others disagree. These effects are complex and cannot be fully understood from our experiment. It would be interesting to investigate these phenomena further by using a larger funnel in a fully computer-controlled experiment, in which the computer is used to control dropping and imaging the ball automatically.

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Appendix A
The Matlab Code

A.1: The Matlab code used to calculate the time spent in the funnel and the energy loss for the particle

```
clear all,
clc

cc=hsv(3); % color scheme

degree_list=FoldList; % extract the list of degree folders
angles=degree_list(:,1:2); % extract the angles (usually, 20 to 62)
%fprintf('%d degree folders found\n', size(degree_list,1));
% initialize the matrix to store the averaged values
average_times=zeros(size(degree_list,1),3);
error_times=zeros(size(degree_list,1),3);
average_hits=zeros(size(degree_list,1),3);
error_hits=zeros(size(degree_list,1),3);
average_energies=zeros(size(degree_list,1),3);
error_energies=zeros(size(degree_list,1),3);
figure(3)
for degree_num=1:size(degree_list,1)
    degree_folder=degree_list(degree_num,:);
    cd(degree_folder);
    disp(['current folder: ' degree_folder ', current angle: ',
     angles(degree_num,:)]);
    posit_list=FoldList; % extract the list of position folders
    positions=posit_list(:,9:end); % extract the position numbers
```

```
     (usually, 1 to 3)
    %fprintf('%d position folders found\n', size(posit_list,1));
    subplot(ceil(sqrt(size(degree_list,1))),
    ceil(sqrt(size(degree_list,1))), degree_num);
    hold on;
    for posit_num=1:size(posit_list,1)
        position_folder=posit_list(posit_num,:);
        cd(position_folder);
        disp(['current folder: ' degree_folder '\' position_folder ',
          current position: ', positions(posit_num,:)]);
        experiment_list=FoldList; % extract the list of experiment folders
        experiments=experiment_list(:,2:end); % extract the experiment
         numbers (usually, 1 to 10)
        %fprintf('%d experiment folders found\n', size(experiment_list,1));
        % initialize the vectors used for averaging
        Total_time_in_funnel=zeros(size(experiment_list,1),1);
        Total_number_of_hits=zeros(size(experiment_list,1),1);
        Total_energy_lost=zeros(size(experiment_list,1),1);
        for exp_num=1:size(experiment_list,1)
         experiment_folder=experiment_list(exp_num,:);
         cd(experiment_folder);
         disp(['current folder: ' degree_folder '\' position_folder
        '\' experiment_folder ', current experiment:
         ', experiments(exp_num,:)]);
        bmp_Files = dir('*.bmp');
        [Total_time_in_funnel(exp_num), Total_number_of_hits(exp_num),
        Total_energy_lost(exp_num)] = RunExp(bmp_Files);
        cd ..; % go back to experiment folders
        end
        plot(Total_time_in_funnel,'color',cc(posit_num,:),'LineWidth',2);
        legend(strcat('position',num2str(positions)));
        xlabel('experiment #');
        ylabel('average time, s');
```

```
        %calculate the current set data
        average_times(degree_num,posit_num)=sum(Total_time_in_funnel)
        /size(Total_time_in_funnel,1);
        error_times(degree_num,posit_num)=std(Total_time_in_funnel);
        average_hits(degree_num,posit_num)=sum(Total_number_of_hits)
        /size(Total_number_of_hits,1);
        error_hits(degree_num,posit_num)=std(Total_number_of_hits);
        average_energies(degree_num,posit_num)=sum(Total_energy_lost)
        /size(Total_energy_lost,1);
        error_energies(degree_num,posit_num)=std(Total_energy_lost);
        %fprintf('%d experiment(s):\naverage time in funnel: %d\naverage
         number of hits: %d\naverage energy lost: %d\n', exp_num,
         average_times(degree_num,posit_num),average_hits
         (degree_num,posit_num), average_energies(degree_num,posit_num));
        cd ..; % go back to position folders
    end
    hold off;
    cd ..; % go back to degree folders
end

angles=str2num(angles); % convert before plotting

figure(4)
hold on;
for plot_num=1:size(average_times, 2)
    %plot(angles,average_times(:,plot_num),'color',cc(plot_num,:));
    errorbar(angles,average_times(:,plot_num),error_times(:,plot_num),
    'color', cc(plot_num,:), 'LineWidth',2);
end
hold off;
title('Average times in a funnel');
xlabel('angle, deg');
```

```
ylabel('average time, s');
legend('position1','position2','position3','position4','position5');

figure(5)
hold on;
for plot_num=1:size(average_hits, 2)
    errorbar(angles,average_hits(:,plot_num),error_hits(:,plot_num),
    'color', cc(plot_num,:),'LineWidth',2);
end
hold off;
title('Average hits');
xlabel('angle, deg');
ylabel('average hits');
legend('position1','position2','position3','position4','position5');

figure(6)
hold on;
for plot_num=1:size(average_energies, 2)
 errorbar(angles,average_energies(:,plot_num),error_energies(:,plot_num),
'color',cc(plot_num,:),'LineWidth',2);
end
hold off;
title('Average energy lost');
xlabel('angle, deg');
ylabel('average energy, J');
legend('position1','position2','position3','position4','position5');

%positions=[1, 2, 3, 4, 5]; % change to fit the exact locations
%figure(7)
%title('Average times in a funnel');
%for plot_num=1:size(average_times, 1)
 %subplot(ceil(sqrt(size(average_times, 1))),
 ceil(sqrt(size(average_times, 1))),plot_num),
```

```
 plot(positions,average_times(plot_num,:),
 'color',cc(plot_num,:),'LineWidth',2);
  %  legend(strcat(num2str(angles(plot_num)),'degree'));
   % xlabel('initial location, m');
   % ylabel('average time, s');
%end

fprintf('Done!\n');

\2ndset{language=Matlab , numbers=left , caption=The Program for calculate
the average times spent and the average energies loss ,captionpos=b}
\renewcommand\lstlistingname{Program}
function [Total_time_in_funnel, Total_number_of_hits, Total_energy_lost]
= RunExp(bmp_Files)
Total_time_in_funnel=0;
Total_number_of_hits=0;
Total_energy_lost=0;

if isempty(bmp_Files) % no .bmp files in the folder
    fprintf(2,'No .bmp files in this folder!\n');
    return
end

j=0;
dot_position_i = [];
dot_position_j = [];

filename = bmp_Files(round(length(bmp_Files)/2)).name;%pick the middle shot
Icolor = imread(filename); %Icolor is the middle shot
filename = bmp_Files(1).name; %pick the first shot
I0 = imread(filename); %I0 is the first shot
Icolormap = Icolor - I0; %difference image
dot_max = max(max(Icolormap)); %maximum intensity
```

```
sensitivity = 0.25;
gauge = dot_max - dot_max*sensitivity;
sensitivity_funnel = 0.7;
gauge_funnel = dot_max - dot_max*sensitivity_funnel;
sensitivity_ball = 0.25;
gauge_ball = dot_max - dot_max*sensitivity_ball; %set the brightness which
is considered to be a ball

for t1=1:(size(Icolormap,1))
    for t2=1:(size(Icolormap,2))
        diff = dot_max - Icolormap(t1,t2);
        if diff == 0
         row_pos = t1; %row_pos is the y-coordinate of the brightest spot
         col_pos = t2; %col_pos is the x-coordinate of the brightest spot
       end
    end
end
index_ball = find((Icolormap(row_pos,:))>gauge_ball); %"the ball" points
along the row_pos
radius_ball = index_ball(size(index_ball,2)) - index_ball(1); %the width
 of the ball

mass_ball = 0.003019; %Kg
g = 10; %m/s^2

% position of the funnel ==================================================
kk=0;
for ind01=1:size(I0,1)
    kk=kk+1;

    I0_L=[];
    I0_R=[];
    I0_L_col=[];
```

```
    I0_R_col=[];

    I0_L_col_end(kk) = 0;
    I0_R_col_end(kk) = 0;
    I0_row_end(kk) = ind01;

    % Left half of the funnel ===========
    for ind02=1:((round(size(I0,2)))/2+1)
        if (I0(ind01,ind02))>gauge_funnel
            I0_L=[I0_L I0(ind01,ind02)];
            I0_L_col = [I0_L_col ind02];
            I0_L_col_end(kk) = I0_L_col(size(I0_L_col,2));
        end
    end

    % Right half of the funnel ===========================
    for ind03=((round(size(I0,2)))/2+2):1:(round(size(I0,2)))
        if (I0(ind01,ind03))>gauge_funnel
            I0_R=[I0_R I0(ind01,ind03)];
            I0_R_col = [I0_R_col ind03];
            I0_R_col_end(kk) = I0_R_col(size(I0_R_col,1));
        end
    end

end

[funnel_col_index_top_left,funnel_row_index_top_left] = find
(I0_L_col_end~=0, 1, 'first');
[funnel_col_index_bottom_left,funnel_row_index_bottom_left]
 = max(I0_L_col_end);

index = find(I0_R_col_end~=0);
funnel_row_index_top_right = find(I0_R_col_end~=0, 1, 'first');
```

```
funnel_col_index_top_right = I0_R_col_end(index(1));

[funnel_col_index_bottom_right, II] = min(I0_R_col_end(index));
funnel_row_index_bottom_right = index(II);

I0_row = size(I0,1):(-1):1;

% points describing the funnel -----------------------------------------
y0_L = size(I0,1) - funnel_row_index_bottom_left;
x0_L = funnel_col_index_bottom_left;
y1_L = size(I0,1) - funnel_row_index_top_left;
x1_L = funnel_col_index_top_left;

y0_R = size(I0,1) - funnel_row_index_bottom_right;
x0_R = funnel_col_index_bottom_right;
y1_R = size(I0,1) - funnel_row_index_top_right;
x1_R = funnel_col_index_top_right;

teta_L = atan((abs(y1_L-y0_L))/(abs(x1_L-x0_L)));
teta_R = atan((abs(y1_R-y0_R))/(abs(x1_R-x0_R)));
%=========================================================================
start_frame_num=[];
end_frame_num=[];
%================

collision_Lwing = 0;
collision_Rwing = 0;
%------------------

for k = 2:length(bmp_Files)
filename = bmp_Files(k).name; %start from the second image
I1 = imread(filename);
I = I1 - I0; %calculate the difference between he current image
```

```
and the first one

max_order = [];
for ind=1:size(I,2)
    [m(ind),index_row(ind)]=max(I(:,ind)); %m(ind) stores the
    max value in the "ind" column, index_row(ind) stores its row index
    max_order=[max_order m(ind)]; %max_order stores
    the max values of each column
end

[maxim,index_col]=max(max_order); %maxim stores the
 max value of the whole image, index_col stores its column index;
                   %that's where the peaks start to interchange
                   causing the false detection
%setup an index_col as a MIDDLE POINT of the ball, not the BRIGHTEST ONE
the_ball = find((max_order)>gauge_ball);
if ~isempty(the_ball) % if the ball is still somewhere in the picture
    index_col=round((the_ball(1)+the_ball(end))/2);
end
dot_pos_i(k) = index_row(index_col);
dot_pos_j(k) = index_col;
maximum(k) = maxim;

if maxim > gauge %if the maximum value "maxim" is within the ball
(the brightness is over 155,
                 %the index_col stands for its column in the image
                 %the index_row stands for its row in the image

    % time for the ball to be in the funnel ================================
    if (size(I,1)-dot_pos_i(k))>funnel_row_index_top_left
        start_frame_num = [start_frame_num k];
    end
    if (size(I,1)-dot_pos_i(k))<funnel_row_index_bottom_left
```

```
        end_frame_num = [end_frame_num k];
    end
    %========================================================================
    j = j+1;
    dot_position_i = [dot_position_i size(I,1)-dot_pos_i(k)];
    dot_position_j = [dot_position_j dot_pos_j(k)];
    %----------------------------------------------
end
end
%-----------------------
I0_flipud = flipud(I0);
%figure(1)
%plot(dot_position_j,dot_position_i,'o-')
%title('Particle Trajectory in a Funnel')
%figure(2)
%imshow(I0_flipud)
%hold on;
%plot(dot_position_j,dot_position_i,'w.')
%        for pp = 1:length(dot_position_j)
%            text(dot_position_j(pp),dot_position_i(pp),sprintf('(%d,%d)',
dot_position_j(pp),dot_position_i(pp)),'Color',[.6 .8 .6])
%        end
%set(gca,'ydir','normal')
%axis([0 480 0 420])
%axis([0 size(I,2) 0 size(I,1)])
%hold off;
%------------
NFperS = 250;
num_of_frames = end_frame_num(1) - start_frame_num(1);
Total_time_in_funnel = num_of_frames*(1/NFperS)*10;
%disp(['Total time in the funnel: ' num2str(Total_time_in_funnel)'[sec]'])

dif=[]; %declare the vector of the ball's horizontal velocities
```

```
Hit_L = 0; %reset the counter for left-side hits
Hit_R = 0; %reset the counter for right-side hits
W=[]; %store the energies of each hit
collision_position_col_L = []; %declare the vector storing columns of the
left-side hit elements
collision_position_row_L = []; %declare the vector storing rows of the
left-side hit elements
collision_position_col_R = []; %declare the vector storing columns of
the right-side hit elements
collision_position_row_R = []; %declare the vector storing rows of
the right-side hit elements
Energy_difference_L = []; %declare the left-side energy difference vector
Energy_difference_R = []; %declare the right-side energy difference vector
V_in_R = []; %declare the right-side incoming velocities vector
V_out_R = []; %declare the right-side outcoming velocities vector
V_in_L = []; %declare the left-side incoming velocities vector
V_out_L = []; %declare the left-side outcoming velocities vector

%dot_position_i - vector of the ball's vertical coordinates
%dot_position_j - vector of the ball's horizontal coordinates
%dif - vector of the ball's horizontal velocities

for ii=1:(size(dot_position_i,2)-1)
    dif = [dif dot_position_j(ii+1)-dot_position_j(ii)];
end
%figure(3)
%plot(dif)
%title('Particle Velocity direction in a Funnel')

thresh = 1.5;%define the threshold for the change
                    which is considered a hit
VERT = 1; %start with vertical movement
MOV = []; % -1 - moving to the left,
```

```
            % 0 - moving vertically,
            % +1 - moving to the right.
dif=[dif dif(size(dif,2))]; % increase the length of the dif vector
for jj=2:size(dif,2) %scan every location
     if (abs(dif(jj))<thresh &VERT==1)    % slow or no speed before
      the first hit
         MOV(jj)=0; %the ball moves vertically
 end
 if ((abs(dif(jj))>=thresh &dif(jj)    <0) || (abs(dif(jj))<thresh
&VERT==0 &dif(jj)        <0)) &~(VERT==0 &dif(jj)        <0 &dif(jj    -1)>=0
&dif(jj+1)    >=0) || (VERT==0 &dif(jj)    >=0 &dif(jj    -1)<0 &dif(jj+1)    <0)
% small negative speed after the first hit or fast negative speed
 (excluding the one-time dips):
MOV(jj)=-1; %the ball moves to the left
 VERT=0; %first hit has occured
   end
 if ((abs(dif(jj))>=thresh &dif(jj)    >=0) || (abs(dif(jj))<thresh
&VERT==0 &dif(jj)        >=0)) &~(VERT==0 &dif(jj)        >=0 &dif(jj    -1)<0
&dif(jj+1)    <0) || (VERT==0 &dif(jj)    <0 &dif(jj    -1)>=0 &dif(jj+1)    >=0)
% small positive speed after the first hit or fast positive speed

 (excluding the one-time dips):
 %fprintf('At %d the ball moves to the right: %d.\n',jj,dif(jj));
 MOV(jj)=1; %the ball moves to the right
 VERT=0; %first hit has occured
    end
end

%process the MOV so that there's no dips:
for mm=2:(size(MOV,2)-1)
     if MOV(mm-1)<0 &MOV(mm)    >=0 &MOV(mm+1)    <0 %one-shot
                                                       left wall hit
         MOV(mm)=-1;
```

```
    end
    if MOV(mm-1)>=0 &MOV(mm)    <0 &MOV(mm+1)    >=0 %one-shot
                                                            right wall hit
        MOV(mm)=1;
    end
end

for jj=2:size(dif,2)
    if MOV(jj)<MOV(jj-1) % 0->(-1) or 1->(-1) - right wall hit
        Hit_R = Hit_R +1;
        collision_position_row_R = [collision_position_row_R
        dot_position_i(jj)]; %ok
        collision_position_col_R = [collision_position_col_R
        dot_position_j(jj)]; %ok
        d_in = sqrt((dot_position_j(jj) - dot_position_j(jj-1))^2
        + (dot_position_i(jj) - dot_position_i(jj-1))^2); %ok
        d_out= sqrt((dot_position_j(jj) - dot_position_j(jj+1))^2
         + (dot_position_i(jj) - dot_position_i(jj+1))^2); %ok
        v_in = d_in;
        v_out = d_out;

        %fprintf('Potential: %d --> %d\n', mass_ball*g
        *dot_position_i(jj-1), mass_ball*g*dot_position_i(jj));
        %fprintf('Kinetic: %d --> %d\n', (0.5)*mass_ball*(v_in^2),
        (0.5)*mass_ball*(v_out^2));
        Energy_in = mass_ball*g*dot_position_i(jj-1)
         + (0.5)*mass_ball*(v_in^2);
        Energy_out= mass_ball*g*dot_position_i(jj)
         + (0.5)*mass_ball*(v_out^2);

       W = [W (Energy_in-Energy_out)];
        %Energy_difference_R=sum(C)/length(C);
        V_in_R = [V_in_R v_in];
```

```
        V_out_R = [V_out_R v_out];
    end
    if MOV(jj)>MOV(jj-1) % 0->1 or (-1)->1 - left wall hit
        Hit_L = Hit_L +1;
        collision_position_row_L = [collision_position_row_L
        dot_position_i(jj)]; %ok
        collision_position_col_L = [collision_position_col_L
        dot_position_j(jj)]; %ok
        d_in = sqrt((dot_position_j(jj) - dot_position_j(jj-1))^2
       + (dot_position_i(jj) - dot_position_i(jj-1))^2); %ok
        d_out= sqrt((dot_position_j(jj) - dot_position_j(jj+1))^2
        + (dot_position_i(jj) - dot_position_i(jj+1))^2); %ok
        v_in = d_in;
        v_out = d_out;

       %fprintf('Potential: %d --> %d\n', mass_ball*g
       *dot_position_i(jj-1), mass_ball*g*dot_position_i(jj));
       %fprintf('Kinetic: %d --> %d\n', (0.5)*mass_ball*(v_in^2),
       (0.5)*mass_ball*(v_out^2));
        Energy_in = mass_ball*g*dot_position_i(jj-1)
        + (0.5)*mass_ball*(v_in^2);
        Energy_out= mass_ball*g*dot_position_i(jj)
       + (0.5)*mass_ball*(v_out^2);

       W = [W (Energy_in-Energy_out)];
       %Energy_difference_L=sum(C)/length(C);
        V_in_L = [V_in_L v_in];
        V_out_L = [V_out_L v_out];
    end
end

%hold on;
%plot(MOV*max(abs(dif)),'g');
```

```
%hold off;

Total_energy_lost=sum(W);
%fprintf('Number of collisions on the left side of the funnel:
%d\n',Hit_L) ;
%fprintf('Number of collisions on the right side of the funnel:
%d\n',Hit_R);
Total_number_of_hits=Hit_L+Hit_R;
%fprintf('Total energy lost due to collisions : %d\n', sum(W));

%fileout = sprintf('experiment_%d.mat',exp_num);
%save(fileout, 'V_in_R', 'V_out_R', 'Energy_difference_L',
'Energy_difference_R', 'v_in', 'v_out','Energy_in', 'Energy_out',
'Total_time_in_funnel', 'V_in_L', 'V_out_L','Total_energy_lost');
%save(fileout,'V_in_R','V_out_R','Energy_difference_L',
'v_in', 'v_out', 'Energy_in', 'Energy_out', 'Total_time_in_funnel',
 'Energy_difference_R','V_in_L', 'V_out_L', '-ascii');
clear dif collision_position_row_R collision_position_col_R
Energy_difference_R A V_in_R V_out_R collision_position_row_L
collision_position_col_L Energy_difference_L B V_in_L
V_out_L Hit_R Hit_L;

end

\3rdset{language=Matlab , numbers=left , caption=The Program for
connect between
1stset and 2ndset ,captionpos=b} %frame=single}
function nameFolds = FoldList
d = dir(); % list the files
isub = [d(:).isdir]; % mask the folders
nameFolds = {d(isub).name}'; % list the folders
nameFolds(ismember(nameFolds,{'.','..'})) = []; % exclude the
current/parent folder symbols
```

```
nameFolds=char(nameFolds); % convert from cell to char.
```

Appendix B
The operation of the Matlab programs

In this Appendix I briefly describe the operation of the Matlab programs used to analyze the image data obtained in our experiments. The images are stored on the computers hard drive by the high speed camera. They are later read by the Matlab program. Individual images recorded during the falling the aluminum ball onto the flat steel plate are shown in Figure B.1. The program locates the position of the ball by determines the edges of the shape along x and y axes then use these data to find the center of the ball has been found in each frame, the program displays the trajectory of the bouncing ball as seen in Figure B.2. The program then determines the highest points reached by the ball after each bounce These heights are then used with Eq. (2.5) to determine the value of the restitution coefficient for each bounce. These values are then averaged.

Figure B.1: Individual high-speed video images Some images by Matlab for bouncing aluminum ball on steel flat plate.

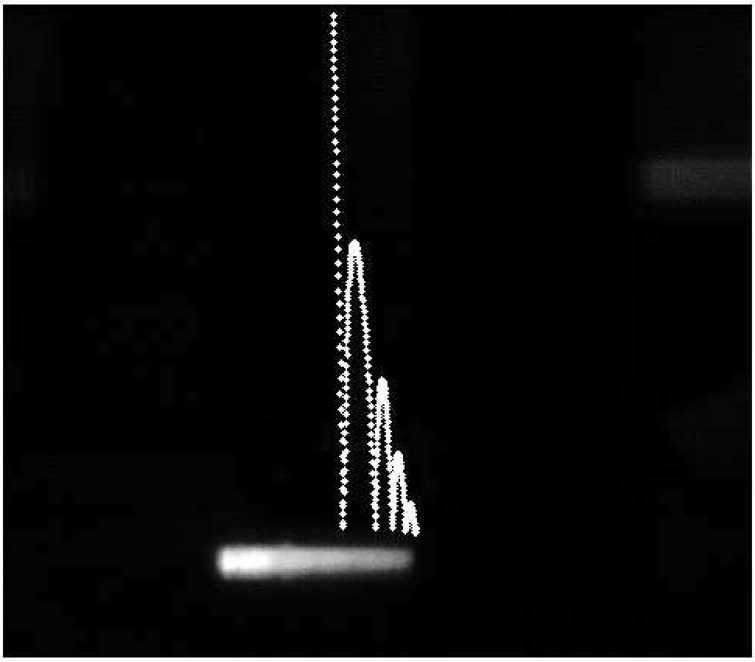

Figure B.2: The trajectory of the bouncing aluminum ball on the steel plate.

The procedure in the funnel experiments in similar. The program reads the images of the falling ball that are recorded by the high speed camera. Some sample images are shown in Figure B.3. Then the program analyzes these images to determine the trajectory of the falling ball as seen in Figure B.4. After that, the program determines time and the x and y coordinates and then calculated the velocities of the ball when it enters and leaves the funnel. By using these values and the law of conservation of energy, the program calculates the total time spent in the funnel and the total energy lost by the falling ball while it is in the funnel. The program then plots these data as a function of the angle of the funnel walls.

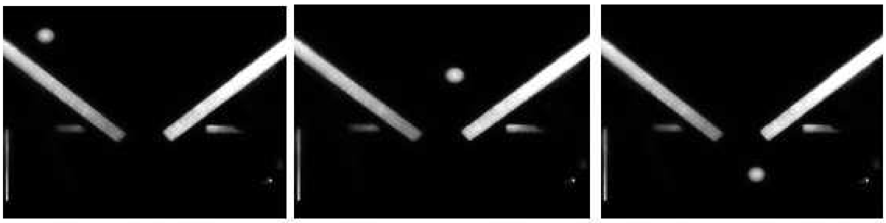

Figure B.3: Some individual images of the ceramic ball falling through the aluminum funnel.

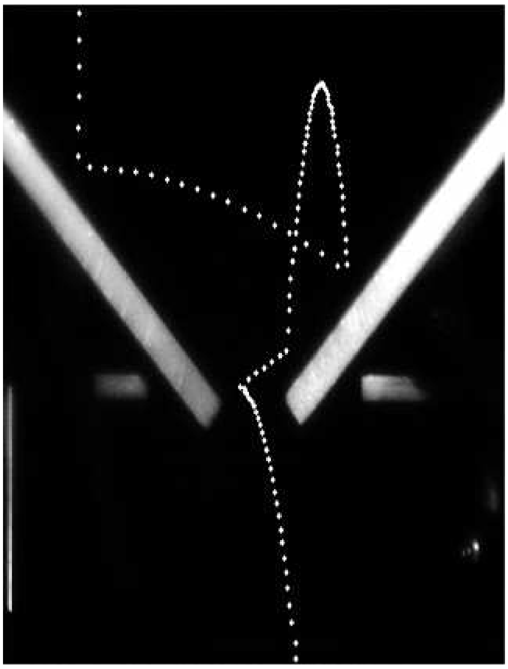

Figure B.4: The trajectory of the ceramic ball falling through the aluminum funnel determined by the Matlab program.

Curriculum Vitae

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