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Timothy David Lane

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ESSAYS ON MONETARY CONTROL

by

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Submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
London, Ontario
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ABSTRACT

Since the monetary authorities in a number of countries have adopted targets for the growth rate of the money supply, there has been some debate over how control of the money supply should be implemented. Some issues which have arisen are the appropriate instrument for monetary control (monetary base or interest rate), the time horizon over which this control should be effected, and the correct response to deviations of the money stock from its target path. This thesis is a set of essays addressing these issues.

The first chapter introduces the subject matter of the succeeding essays, and traces the connections among various issues.

The second chapter reviews the literature on the rationale for money-supply targets. Two bodies of literature are traced: the targets-and-instruments literature suggests that the money supply is relevant to policy only as a source of information on the disturbances affecting the level of national income. The literature on monetary rules, in contrast, implies that targets for the money stock should represent a commitment to eschew stabilization policy. These alternative rationales for money-stock targets obviously have quite different implications for the seriousness of departures of the money stock from its target path.

Chapter Three deals with the issue of instrument instability, which pertains to the appropriate time horizon for monetary control. It has
been argued that the lag structure of interest rates in the money-demand function is such that, if the money supply were controlled according to a constant-growth-rate rule, interest rates would behave explosively. This argument is evaluated in the light of two alternative explanations of the apparent influence of interest rates on demand for money: one based on expectations and one based on adjustment costs. It is shown that, if expectations are formed rationally, the danger is not that interest rates would behave explosively if the money supply were controlled, but rather that attempting to stabilize interest rates within the context of money-supply targets would create expectations of patterns of interest-rate movements and thus might be destabilizing. Furthermore, smoothing interest rates may actually generate the empirical evidence which has been adduced to justify such intervention.

Chapter Four considers one aspect of the choice of instrument, namely the argument that the lagged accounting of required reserves makes it difficult to use the monetary base as a monetary instrument, since lagged accounting creates a dependence of current on past deposits and thus gives rise to oscillations in deposits which may be explosive. This argument is evaluated using a model which incorporates optimizing behavior by competitive (deposit-taking) banks. A rational-expectations analogue of the traditional banking-system multiplier mechanism is constructed; it is shown that the deleterious effects of lagged accounting may be mitigated by banks' portfolio adjustment behavior.

The fifth chapter deals with another argument pertaining to lagged reserve accounting: the argument that, under lagged reserve accounting,
the demand for the monetary base is predetermined so that the base cannot be set independently as a policy instrument; this argument has been corroborated with causality tests which show that the money supply causes the monetary base and not vice versa. In this essay, the relevance of the causality-test evidence is challenged: a simple theoretical counter-example is constructed, in which although the authorities can use the base to control the money supply under lagged reserve accounting, causality tests would indicate that money causes base. This is an application of the 'Lucas critique': empirical evidence generated under one regime cannot necessarily be used to judge the consequences of adopting another regime.
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CHAPTER I
INTRODUCTION

Controlling the money supply with reference to some pre-specified target path or rule has widely been advocated as a governing principle for monetary policy. The merits of this proposal have been the subject of considerable debate. Indeed, for some time this has been perhaps the central issue in monetary policy, an issue on which much important work has been focussed.

In recent years, the central banks of a number of Western countries have acknowledged the desirability of such monetary control; they have established targets for rates of monetary expansion, and ostensibly direct their actions towards attaining those targets. The debate over the advisability of a monetary rule has continued despite this apparent change in policy regime; however, there has also been some shift in the emphasis of discussions of the appropriate conduct of policy (Sumner 1979). The practicability, as distinct from the desirability, of exercising close control over the money supply has been at issue; this has increasingly been the case because of the considerable variability of the monetary aggregates, which has continued, in most cases, since the targets were introduced. Central banks have generally either been notably unsuccessful in attaining their declared objectives for the money stock (Pierce 1978) or have defined these objectives in a rather unambi-
tious way (Sumner 1979); in either case, growth rates of the money supply have been allowed to fluctuate considerably in the short run.

Cynics have interpreted this erratic behavior of the money supply as evidence that the authorities have not been making a serious attempt to control it (Kane 1980): the monetary targets, they have claimed, are really just a public relations device superimposed on a policy regime which has not changed. Others, however, have seen it in a more charitable light: some have argued that it indicates flaws in the technique of monetary control which has been employed, and some that the money supply is just not susceptible of very close control by the monetary authorities (see Courchene et al. 1979, Meltzer et al. 1982).

Central banks have frequently attempted to control the money supply by manipulating an interest rate in order, in effect, to pick a point on a money-demand curve. The authorities forecast national income and the price level, and assume these to be predetermined; they then substitute these forecasts into an estimated money-demand function; an interest rate is then selected in order to give a conditional forecast of money demand which is within the authorities' target range for the money stock. Open-market operations and other policy actions are then conducted in such a way as to move the interest rate to this level (Par-kin 1978).

\[1\] In fact, this appears to be by far the most common technique of monetary control. The recent U.S. policy experience may be an exception. Whether or not it is is unclear: the Federal Reserve does set targets for unborrowed reserves, but it appears that the authorities see their actions as affecting the money supply via the traditional channels of free reserves and the Federal Funds rate, rather than through a money-supply function (see Lang 1980).
This technique of monetary control—known as "interest-rate control"—has been criticized for a number of reasons. For one thing, there has been some uneasiness about the assumption that income and prices are fixed while the interest rate is being manipulated: this seems to assume away the danger of Wicksellian instability (Wicksell 1906) which may be present if the interest rate is pegged for a period of time. In addition, there is the notion that interest-rate control is needlessly indirect: since central banks generally influence interest rates by manipulating bank reserves, why not use bank reserves to influence deposits in a more direct way—via the money multiplier relationship, for example? Interest-rate control seems needlessly to use the interest rate as an intermediate target in this context; the use of intermediate targets has been shown in general to waste or misuse information (Kaesken, Muench and Wallace 1973, Benjamin Friedman 1975, 1977). It is not very satisfactory to answer this question by arguing that central bankers have developed rules-of-thumb which enable them to control interest rates quite effectively without enabling them to put the same implicit knowledge to work in controlling the money supply more directly: accepting such an argument would imply endorsing the principle that no policy should ever be changed. Another objection to interest-rate control is that it means that the monetary aggregate which is controlled must be one with an interest-sensitive demand-function (White in Courchene et al. 1976); this may lead to a choice of monetary aggregate

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2 as described, for example, by Davis in the Federal Reserve Bank of New York 1975 volume.
which is inappropriate in other respects (Courchene 1976, 1979). There are also, of course, political considerations such as the fact that if the banking-system multiplier does not fluctuate too much, the money supply could conceivably be controlled tolerably well with a constant growth rate of the monetary base, which would obviously make it much easier for the public to scrutinize whether the monetary authorities were adhering to the monetary rule which they had announced.

The usual starting point for a comparison of alternative techniques of monetary control is a simple stochastic money-supply and demand model, associated with the "targets and instruments" framework (see Benjamin Friedman 1975). Under interest-rate control, the authorities are presumed to supply enough reserves to the system in order to keep the interest rate at the chosen level; any variation in the money stock around its target level thus results from errors in forecasting national income and the price level and from errors in predicting money demand conditional on those variables. A linear combination of these three errors constitutes the disturbance to money demand conditional on the interest rate. The money supply function is also stochastic, for a given interest rate, when the authorities set the stock of high-power money. According to the traditional banking-system multiplier analysis, the money supply depends on the behavior of the public and the commercial banks as well as on the actions of the monetary authorities (see Cagan 1965). The public chooses the form in which it holds money—it chooses the proportions in which to hold currency, demand deposits and other types of deposits which may bear different reserve requirements.
The commercial banks allocate their portfolios between reserves and earning assets, subject to the restrictions which reserve requirements place upon them; they may also influence the monetary base via their borrowings from the central bank. Because the behavior underlying the money-supply function is interest-sensitive, the money supply is believed to be an increasing function of the rate of interest. In addition, because these behavioral relationships are not perfectly predictable, the money supply is stochastic for a given interest rate and monetary base. Thus, the choice of instrument for controlling the money supply appears to involve a rather straightforward comparison of two sources of error: if the interest rate is controlled, the money supply will deviate from its target by the disturbance to the money demand given the interest rate. If the authorities set the base as a policy instrument, the money stock and interest rate will be those which equate money supply and demand; the money stock which emerges will differ from the target money stock by a linear combination of the disturbance to money demand given the interest rate and the disturbance to money supply given the interest rate and the monetary base (cf. Courchene 1976: p. 40). The interest rate will deviate from its expected value by another linear combination of the same disturbances. Thus one could compare the effects of base- or interest rate-control by comparing the resulting variance of the money stock around its target. Alternatively, one could consider the variance of nominal income which would result from the use of one policy instrument or the other; this will be another linear combination of the variances of the money-market disturbances and of dis-
turbances affecting the real side of the economy (see Pierce and Thompson 1972, Parkin 1978, Sparks in Courchene et al. 1979); the precise results will be different, but the general nature of the analysis is the same. In either case, the analysis implies that the issue is, in principle, very simple. The main question is an empirical one: how large are the variances of the various sources of error which have been specified? Answering this question could surely be left to the econometricians in the central banks, who are well equipped for this kind of task.

It would be misleading, however, to suppose that this targets-and-instruments framework reduces the choice of monetary control technique to a straightforward empirical question in this manner. This becomes apparent, first of all, if one considers what variances are being compared. If, on the one hand, one considers the case in which one instrument or the other is being adjusted continually in response to each new piece of information, in order to minimize the variance of the money stock around its target level, it becomes important to recognize that both the monetary base and the interest rate are really intermediate targets, which are controlled through the ultimate actions of policy (such as open market operations). Random disturbances which affect the relationship between the monetary base and the interest rate are thus initially reflected in fluctuations in both variables; if the authorities were trying to keep the interest rate constant over a period of time, they would then have to carry out appropriate adjustments to the base, so that over that period the base could be regarded as endogenous.

If, however, they were attempting to adjust the interest rate using all
available information in order to minimize the variance of the money supply, they would instead incorporate any new information provided by a change in the interest rate associated with a given monetary base into their view of the interest rate appropriate for this task. Similarly, if they were carrying out a policy of adjusting the monetary base using all available information to minimize the variance of the money supply, their setting of the base should change in response to information provided by the interest rate. In short, if base-control and interest-rate-control are carried out using all pertinent information in order to minimize the variance of the money supply around its target path at each point in time, the two control techniques are identical.

At the other extreme, one could consider a comparison of the two control techniques wherein no information is available to the authorities in setting their instruments. In this case, interest-rate control would mean pegging the interest rate at some level. According to Wicksell's classic analysis (Wicksell 1906), this would lead to cumulative movements of the price level, unless by coincidence the level chosen for the interest rate were the natural rate. Under rational expectations, pegging the interest rate does not even give rise to a determinate price level (Sargent and Wallace 1975). Setting the monetary base, on the other hand, is generally supposed to result in some determinate money stock and price level. This points out a difference between base- and interest-rate-control as they are generally conceived: base control, according to the traditional analysis of the money supply, works automatically and should be stable; interest-rate control, on the other
hand, requires careful action since the interest rate must be adjusted promptly and in a particular way in order to avoid cumulative movements of the money supply and the price level (see Howitt and Laidler 1979).

Thus, if the chosen policy instrument is adjusted continually in response to all information that is useful in controlling the money supply, base- and interest rate-control converge; if the instrument is never adjusted in response to any new information, on the other hand, only base control is viable. For there to be a non-trivial choice of control technique, then; it is necessary that the instruments of policy be adjusted in response to some, but not all, pertinent information. Barring fixed costs of adjusting instruments, why ever should this be the case? why would the authorities fail to use all available information in setting their policy instruments?

In considering this question, one loses the sharp distinction between "strategy and tactics" which was implicit in the foregoing discussion (see Guttentag 1966, Friedman 1971). It is conventional to treat some things--such as the time path of the money supply--as important to the overall thrust of policy; others--such as the appropriate adjustments of the monetary base or the interest rate--are regarded as tactical details in the implementation of this strategy. This nice hierarchical distinction breaks down to the extent that the details of a monetary regime may reflect, in no small way, the purposes which it is supposed to serve.

Some economists see the use of targets for the money supply as no more than a public declaration that circumspection is being practiced in
the use of interest rates as instruments of policy. Monetary and credit aggregates provide information on whether interest rates are at their appropriate level, information which can be useful in moving towards slower rates of inflation. If monetary targets are no more than an announcement that the authorities are taking account of the information contained in monetary aggregates, it would certainly not be desirable for them to adhere to these targets rigidly, in other words, to minimize the variance of the money stock around its target path. For one thing, to the extent that money-supply data are being used as a source of information in stabilizing nominal income, it would be necessary for the authorities to estimate the extent to which a particular departure of the money stock from its target path resulted from a shock to nominal income which should be counteracted by an adjustment of the interest rate (Duguay and Jenkins 1978). In addition, this view of monetary targets would admit empirical evidence according to which any influence of money on nominal income operates with a long distributed lag (Pierce and Thompson 1972, Duguay and Jenkins 1978; Poole 1976); such evidence has been used to argue that month-to-month fluctuations in the money supply are essentially irrelevant. From this standpoint, it has been argued that continually attempting to minimize the variance of the money stock around its target path would be pointless. Instead, only the average growth rate of the money stock over a longish interval should be of any concern to the authorities; given that average growth rate, they are free to act on their traditional preferences for orderly financial and foreign-exchange markets (cf. Poole 1976). This would imply, of course,
that the authorities would not use all the available information in order to minimize the variance of the money supply; it would not, however, mean that it would be appropriate to choose the instrument of monetary control on the basis of whether it yields a higher or a lower variance of the money stock over some particular time horizon. Instead, the fundamental operations of policy should be conducted in such a way as to maximize an objective-function in which all the various criteria of policy are contained. In that case, the monetary targets are simply an announcement about the authorities’ preferences and beliefs, an announcement that these are such that the authorities’ objective-function can appropriately be maximized with the aid of some information provided by the growth rate of the money stock.

From a different perspective, though, the establishment of money stock targets would mean much more that announcing that the money stock is being used to provide information useful for stabilizing national income and achieving other objectives. Instead, it would amount to a redefinition of the monetary standard (see Yeager 1962). The existence of a clearly-defined monetary standard implies that agents can base their decisions on reliable information about the value of the monetary unit.³ The monetary unit need not be constant in value; it might be argued, however, that it should not be subject to deliberate manipulation designed to induce agents to alter their behavior. Specifying a rule for the growth rate of the money stock has frequently been proposed

³ The implications of a changing monetary standard are discussed in Klein 1975.
on the grounds that it would eliminate the scope for such manipulation (see Friedman in Yeager 1962). Discretionary monetary policy has been opposed since it is an important form of government intervention in a market economy, which places considerable power in the hands of the monetary authorities. It has also been argued that such policy—be it designed to stabilize national income or to smooth interest rates—is nugatory (Friedman 1968). Such policy is subject to serious information problems (Friedman 1954, *inter alia*) which makes it difficult for the authorities to act in such a way as to offset shocks impinging on the economy. This information problem does not vanish as the authorities gain knowledge about the structure of the economy, since private agents also learn and this enables them to respond to these shocks in an appropriate manner themselves; it is difficult for the authorities to improve on these private adjustments (Barro 1976). Also, private agents learn about the behavior of the authorities, and alter their own behavior as the policy regime changes; this implies that the structure of the system changes when the policy environment changes, so that it is impossible to make a reliable assessment of the effects of a hypothetical change in policy; this makes it difficult to design appropriate policy feedback to counter shocks to the economy (Lucas 1976). In short, according to this view, the authorities cannot treat the economy as a mechanism which they can manipulate by suitable adjustments of their policy instruments. On this basis, it has been argued that they should eschew all attempts to stabilize the economy: these are as likely as not to be counterproduc-
tive. Instead, they should find a viable rule for monetary policy, and adhere strictly to that rule. A rule specifying the growth rate of the money supply has generally been proposed, since it pins down a nominal variable and thus makes the price level determinate; a rule which pegged interest rates would not have this property (supra p. 7). Given a rule for the growth rate of the money supply, it has been argued, any stabilization of interest- or exchange-rates which was desirable could be left to private speculators (Poole 1976); to the extent that shocks affecting national income could be foreseen, they would be also be taken care of by private adjustment behavior.

This rationale for the control of the money supply—as part and parcel of a monetary rule—suggests that monetary policy should at all times be guided toward the target rate of growth of the money supply. Short-run fluctuations of the money supply are undesirable for two reasons: if the public believes that a monetary rule is in effect, short-run fluctuations of the money stock around the target path are "unanticipated money", which is believed to lead to fluctuations in real income (Barro 1977); in addition, if the public is not convinced that policy is being governed by a constant-growth-rate rule, fluctuations of the money supply would provide the public with confusing information about the way in which policy is being conducted, undermining the credibility of the alleged rule. Perhaps most important is the fact that accepting the

This, of course, glosses over the distinction between a rule in the sense of preannouncement of policy and a rule in the sense of absence of feedback from economic events; this distinction is elaborated below (pp. 55-57).
idea that the rule can with impunity be abandoned in the "short run" in pursuit of other goals would defeat one of the supposed purposes of a monetary rule, which is to keep the authorities from trying to do what the market can do equally well and to confine them to the role of providing a stable policy environment within which the market can work.

There is, however, a difficulty with policy designed to minimize the variance of the money supply around a target path. Such policy might be acceptable if the money supply were endogenous only in a trivial sense—if, that is, it depended only on the actions of the authorities via some predictable and mainly institutionally-determined relationships. If, however, the money supply can meaningfully be regarded as the unintended consequence of private optimizing behavior, "fine-tuning" the money supply in accordance with a monetary rule poses problems quite similar to those involved in fine-tuning any other economic variable. The literature on the microeconomic underpinnings of the money-supply process frequently analyzes the behavior which determines the money stock—that of the commercial banks and the general public—as optimal behavior under uncertainty (e.g. Morrison 1966, Modigliani, Rasche and Cooper 1970, Parkin, Gray and Barrett 1969, Orr and Mellon 1961). To the extent that this literature is of any relevance, it implies that the money-supply function will be different when there is any change in the probability distribution of deposits, which affects the commercial banks' decisions. Thus, if there is a change in the policy environment which gives rise to such a change in the stochastic behavior of the money supply, this will in turn alter the money supply
function itself. This problem, which is an example of the "Lucas problem" (Lucas 1976), may make it difficult to fine-tune the money supply, just as in another context it makes it difficult to fine-tune the level of national income. The control of the money supply may, in short, be an economic problem rather than just a logistical one.

An additional difficulty which it has been argued might arise if the authorities attempted to maintain strict control of the growth rate of the money supply is that of instrument instability. Instrument instability is a problem which may occur if lagged values of a policy instrument affect a policy target; if this is the case, the appropriate value of the instrument at any time depends on its own past values. The difference equation which accordingly describes the time path of the instrument may or may not be stable. If it is unstable, then the associated policy (that is, the chosen time path for the target variable) is clearly inadmissible5 (Holbrook 1972). In this case, instead of trying to minimize the variance of the target variable around its desired level at all times, the authorities must minimize a loss function which also places a small weight on the instrument variable's variance, in order to prevent that variance from becoming infinite (Sims 1974). It has been argued that the instrument instability problem is likely to arise if the

5 It is interesting to note that the instrument instability argument was originally used as part of the case for a constant-growth-rate rule for the money supply: it was argued that money affects national income via a distributed lag which is such that, if the authorities attempted to manipulate the money supply in order to minimize the variance of national income, the money supply itself would follow an explosive path (Cagan and Schwarz 1973); accordingly, it was argued that stabilization policy is not viable, and thus that the authorities should adhere to a monetary rule.
authorities attempt to control the money supply closely in accordance with a monetary rule: according to empirical evidence, lagged interest rates affect the demand for money and as a result the interest rate necessary to equate money demand with a constant or steadily-growing target money stock would be a function of previous interest rates; empirical evidence has been adduced to show that the time path of the money stock so defined would be explosive (Pierce and Thompson 1972, Ciccolo 1974, White 1976). This would imply that some smoothing of interest rates would actually be necessary even in principle in order to keep the variance of interest rates from becoming infinite. It would be desirable to examine the basis of this argument in more detail: in particular, it is not clear why lagged interest rates should affect the demand for money in the first place; if their role reflects either expectations or adjustment costs, they might well turn out to have a different influence under a different policy regime. In particular, the very smoothing of interest rates which is sometimes rationalized in terms of avoiding the danger of instrument instability may shape the influence of lagged interest rates on the demand for money and thus provide a kind of spurious justification for itself. Thus, the instrument instability problem should be examined in the light of the economic interpretation of the lag structure from which the fear of this problem arises.

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*This is not a problem associated solely with the use of the interest rate to control the money supply: if it is a problem, it would arise regardless of the technique of monetary control used.*
The discussion so far has indicated that if the authorities attempt to follow a rule for the growth rate of the money supply strictly, there is no distinction between control mechanisms which operate via the monetary base and those which operate via the interest rate: both use the same information and lead to the same movement of both variables. However, it has also been suggested that it may not be particularly desirable, or may actually be impossible, to direct monetary policy solely toward keeping the money stock near to its target path. For one thing, there is the view according to which money supply targets only prescribe circumspection in the adjustment of interest rates, rather than implying that controlling the money stock becomes the cardinal goal of monetary policy. In addition, there is the view that, even if strict adherence to a monetary rule were desirable, it would be, for all practical purposes, impossible because of the Lucas problem and/or the instrument-instability problem. While both of these lines of argument suggest that keeping the money supply close to its target path may not be desirable—in which case the equivalence of base- and interest rate-control may not be applicable—they do not lend support to the alternative which is usually considered in the literature on the techniques of monetary control—that is, fixing one instrument or the other for a particular unit of time and then adjusting it. Thus, there still do not seem to be any grounds for choosing the appropriate technique of monetary control on the basis of a comparison of the variance of the money supply when one or the other of the instruments is fixed. The choice of monetary instrument cannot properly be separated from that of the feedback rule governing that instrument.
This discussion suggests, therefore, that the appropriate course of action for the monetary authority is not to be found by comparing reduced-form variances. The obvious alternative is to derive optimal policy. This, however, poses problems of its own. For one thing, to derive optimal policy under quite general conditions would involve solving a rather complicated stochastic control problem for a system whose structure itself changes when the policy regime changes: finding optimal policy under any but the most severe simplifying assumptions seems to be quite an intractable problem. In addition, some of the recent rational expectations literature has cast doubt on the whole idea of "optimal policy": it has indicated that when agents' current behavior depends on their expectations of future policy, optimal policy is likely to be time-inconsistent, in the sense that the policy actions planned for dates in the future will not turn out to be optimal when those dates arrive (Kydland and Prescott 1977). In this case, an attempt to derive optimal policy will be fruitless.

Does this mean, then, that nothing much can be said about policy? Perhaps not: it is still of considerable interest to examine whether certain widely-advocated policy rules are admissible and sustainable: it is this which this thesis sets out to do. The thesis is structured as a set of four essays--a literature review and three theoretical essays, each of which deals with some particular problem in monetary control. Each chapter of the thesis is a distinct essay, but the thesis as a whole has certain unifying threads. Not only is the essays' subject matter related; their approach is also similar. In all of the
theoretical essays, the implications of rational expectations are traced in contexts in which they had not previously been considered; an attempt is made to go beyond estimated ratios and behavioral relationships and to consider how these would change when expectations are formed rationally under a different policy regime. The essays also share, to a certain extent, a focus on the implications of alternative policy rules for the dynamics of the money supply, the interest rate and the price level.

The second chapter of the thesis is a review of the literature on the rationale for selecting the money stock as the object of monetary policy. The literature pertaining directly or indirectly to this subject is, of course, vast. An extended (but necessarily selective) survey of this literature is included in the thesis partly in order to provide some perspective on the importance of the money stock. It is also included partly in order to suggest that, to some extent, disagreements over how the money stock should be controlled—although ostensibly arising within the framework of money-supply targets on which there is general agreement—may simply mirror disagreements over this prior question of why it should be controlled. Accordingly, two main strands in the literature are traced: one rationale for controlling the money stock emerges from the rules-vs.-discretion literature, while quite a different rationale emerges from the targets-and-instruments literature. The idea of following a monetary rule—whether it is motivated by the policy ineffectiveness proposition, the time-inconsistency problem and the pitfalls of econometric policy evaluation which the rational expectations literature has thrown in the way of activist policy, or by the
uncertainty about the system's structure and the scope for the abuse of discretion to which the earlier literature referred—has led to a focus on the money stock as the object of such a rule. It has long been recognized that a rule would have to refer to the money stock rather than the interest rate, because of the danger that a Wicksellian cumulative process would ensue should the interest rate turn out to be set above or below its natural rate. The rational expectations literature has reinforced this position: it has been shown (Sargent and Wallace 1975) that a rule for the interest rate does not lead to a determinate price level under rational expectations, regardless of the level at which the interest rate is set: it is necessary that a rule specify the value of some nominal variable such as the money stock.

Thus, from one perspective, targets for the money stock might be regarded as incipient monetary rules: these would constitute genuine constraints on the actions of policy-makers and, as such, create new information about policy. In this light, avoidable deviations of the money stock are undesirable, since they indicate to the public that the stated monetary rule is not really being followed. In the light of the targets-and-instruments literature, on the other hand, setting money-supply targets at most communicates information about the preferences

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7 Bennett McCallum (1981) has recently shown that a rule of adjusting the interest rate in response to deviations of the money stock from a target level will also pin down the expected price level and thus yield a determinate solution for the current price level.

9 This characterization of what it means to follow a rule rather than to attempt to carry out optimal policy—that a rule creates new information rather than just communicating existing information—is due to Nicholas Rowe (1980). Following a rule implies that one binds himself
and beliefs of policy-makers, announcing (believably?) that these have changed in such a way that the optimal combination policy now places more weight on the path of the money stock: from this perspective, there is no special cause for concern if the money stock departs temporarily, or even permanently, from its target path. Thus, disputes about the seriousness of departures of the money stock from its target path do not seem to be readily resolved. The survey of the literature in Chapter Two thus provides a background to the succeeding essays, which deal with issues pertaining to whether and how such departures may be minimized.

The third chapter of the thesis considers the issue of instrument instability—the notion that the lag structure of interest rates in the demand-for-money function is such that interest rates would follow an explosive path if the money supply were controlled rigidly according to a constant-growth-rate rule (supra, pp. 14-15). This argument has been used as part of the justification of the smoothing of interest rates: it has been argued that it is necessary even in principle for the authorities to smooth interest rates to some extent, in order to prevent them from exploding. In this essay, the alleged danger of instrument instability is examined in the light of two alternative explanations of why to keep past commitments rather than simply formulating plans which are optimal for the present and future. Rowe characterizes this as forming a plan of action using the deontological, rather than the teleological method.

i.e. this argument is supposed to remove the case for smoothing interest rates from the plane on which it is necessary for the authorities to specify the costs of the interest-rate fluctuations which they are attempting to reduce, and necessary to demonstrate that these costs are such as to justify compromising the money-stock targets in order to avoid incurring them.
lagged interest rates should affect the demand for money in the first place. The first explanation considered is that this results from the effect of past interest rates on the expected rate of interest; the expected rate of interest may influence money demand if decisions on cash balances are made at intervals shorter than the term to maturity of the asset which is treated as the alternative to money. In the essay, current and expected interest rates are incorporated in the demand for money on this basis, and the implications for the time path of the money stock are traced on the assumption that interest rate expectations are formed rationally; this is done first in a Keynesian very-short-run case in which prices and output are predetermined (the case in which the instrument-instability argument is framed in the existing literature); it is then extended to a simple descriptive rational-expectations model. It is shown that in neither case does controlling the money supply according to a constant-growth-rate rule lead to a problem of instrument instability if expectations are formed rationally. On the other hand, smoothing interest rates—the practice for which instrument instability was presented as a justification—may well lead to unstable behavior of the interest rate, unless the steady-state interest rate is chosen as the initial rate.\footnote{Of course, if the interest rate is allowed to start at the equilibrium rate the interest rate will not follow an explosive path in any model (provided that an equilibrium interest rate exists). For the instrument-instability question to be meaningful, one must allow the possibility that the initial rate be the wrong one.} The reason for this is that if the authorities intervene to restrict interest-rate movements, this intervention introduces a systematic element into interest-rate movements; such systematic
movements, of course, will be reflected in rationally-formed interest-rate expectations, and this can cause responses in asset-holding behavior which turn out to be destabilizing. In addition, since such a policy of smoothing alters the lag structure of interest rates in the demand-for-money function—as this lag structure incorporates the systematic interest-rate movements generated by policy—it will lead misleading conclusions to be drawn about its own effects. If one interpreted the lag structure as "structural", and examined it in the way in which the existing instrument-instability literature has done, one would quite likely conclude that the interest rate would follow an explosive path if the money stock were controlled according to a constant-growth-rate rule; the circumstances under which this conclusion would be drawn are the same as those under which, within the same framework, smoothing interest rates itself turns out to be destabilizing. Thus, smoothing interest rates may be in some sense a self-justifying policy rule: empirical evidence generated under such a rule, if interpreted as reflecting an invariant structure, would lead one to conclude that smoothing is necessary in order to keep interest rates from following an explosive path.

In the second part of the essay, an alternative explanation of the effect of lagged interest rates on money demand is considered: this is the explanation based on costly adjustment. If individuals have to adjust their cash balances towards the desired level in ways that are costly, then if these costs are convex they will adjust their cash balances gradually. If this is the case, then clearly an individual's
Currently chosen cash balances will depend on previous as well as on current interest rates and income. Can the resulting lag structure of interest rates in the demand-for-money function give rise to interest-rate instability if the money supply is controlled? This question is addressed using a model which explicitly incorporates optimizing behavior with respect to the adjustment of cash balances: It is shown that, if the money stock is given from the standpoint of the economy as a whole, costly adjustment at the individual level does not in general have observable effects on the time path of the interest rate or the price level. The reason for this result is that if the price level (or even the interest rate) can be adjusted costlessly, then it will adjust until cash balances are at the level that would be desired in the absence of costs of adjusting cash balances at the individual level: this is the only price level that is consistent with equilibrium, in the sense that at any other price level agents will be actively attempting to exchange some of their excess cash balances for other goods or assets (or conversely), and not everyone can do this simultaneously (see also Laidler 1981). Thus, if one supposes—as the proponents of the instrument instability argument do—\(^{11}\)—that interest rates rather than the money stock have been controlled in the past, the effects of lagged interest rates on money demand resulting from adjustment costs would not give rise to any predictable pattern of interest-rate movements if the money stock were controlled.

\(^{11}\) As is obvious, since the argument implies that if the money supply had been controlled in the past, the interest rate would already have exploded.
Thus, this third chapter of the thesis casts considerable doubt on the validity of the instrument-instability argument. In both of the cases considered, both that in which lagged interest rates affect money demand because of expectations and that in which they affect it because of adjustment costs, parallel results emerge. If the money supply were controlled, the lag structure would be different, so that no explosive path of interest rates would result.

The fourth chapter deals with some problems associated with controlling the money stock by means of controlling the monetary base. In particular, it has been argued that a certain feature of the institutional framework, namely the lagged accounting of required reserves, makes it difficult to use base control. It has also been argued that, because of lagged reserve accounting, if the monetary base itself were subjected to a constant-growth-rate rule, the money stock would likely oscillate explosively (e.g., Judd 1977, Freedman 1980). Under lagged reserve accounting, the greater part of the demand for the monetary base depends not on the current level of deposits but on that in the previous accounting period; thus, for a given monetary base, deposits this period depend negatively on deposits last period, and the result is a path of oscillation of deposits which would quite likely be explosive. This conclusion results from analysis in which there is a fixed ratio of excess reserves and currency to deposits and a fixed ratio of required reserves to lagged deposits. In this essay, the assumption that these behavioral ratios are given is replaced with a model of optimizing behavior by banks. It is presumably not optimal for banks to experi-
ence violent oscillations in their deposits, and the notion that deposits may oscillate in a predictable way thus seems to imply some kind of myopia on the part of banks. The complication is that, in the usual characterization of a competitive banking system, the individual bank does not choose its deposits, but rather acts as a deposit-taker and adjusts its loans in order to attain its desired portfolio balance. Thus, in the fourth chapter of the thesis a model is constructed in order to examine the dynamic implications of optimal individual asset-adjustment behavior under rational expectations. It is assumed that it is not costless for a bank to adjust its loans—the costs of adjustment may be visualized as the paperwork of issuing or calling loans in large numbers at short notice. In addition, there are assumed to be costs of departing from the optimal portfolio mix. For the sake of tractability, it is assumed that both these costs are quadratic. Using this framework, it is shown that the money supply process is generally stable—in the sense that deposits converge to a particular equilibrium level, as in the banking-system-multiplier story—if required reserves are based on current-period deposits.

Under lagged reserve accounting, on the other hand, any of three results is possible: if it is relatively costly to adjust loans rapidly, then even with lagged reserve accounting deposits converge towards their steady-state level without oscillations; if these costs are somewhat smaller, there will be oscillations but these will be damped; finally, if banks can adjust their loans at very little cost there may be no rational expectations equilibrium. This suggests that lagged reserve
accounting may under some circumstances have deleterious effects on monetary control, but that these effects may under other conditions be offset by the optimal portfolio-adjustment behavior of banks.

The analysis has some other important implications. For one thing, the costly adjustment of the type specified in the model would, if the interest rate is being controlled, give rise to the same kind of partial adjustment of bank portfolios which is implied by the ad hoc stock adjustment models frequently found in the empirical work on banks' short-run portfolio choice (e.g. Fraser and Rose 1973); however, explicit use of the optimization model under the assumption that expectations are formed rationally gives rise to dynamic behavior of the money supply which is quite different from that which is implied by treating the adjustment parameter in the partial adjustment framework as a structural parameter. Secondly, it is frequently assumed that fluctuations of excess reserves around their average level are to be regarded as "noise" vis a vis the use of the monetary base to control the money supply; the analysis in this essay emphasizes the opposite idea—the idea that since excess reserves, if they are held, will act to some extent as a buffer stock, optimizing banks may well allow them to fluctuate in such a way as to mitigate the effects of lagged reserve accounting, as well as the effects of anything else which would otherwise cause the money stock to fluctuate in a predictable way.

The fifth chapter of the thesis deals with another aspect of the lagged reserve accounting issue. It has been argued (Feige and McGee 1977, Clinton and Lynch 1978) that under lagged reserve accounting
the demand for the monetary base is essentially predetermined, and that base control is thus inherently impossible since if the base were determined independently by policy an equilibrium could not result except by coincidence. This contention has been corroborated by empirical evidence, in particular, the results of causality tests: such tests have shown that lagged values of the money stock are correlated with the monetary base, but lagged values of the monetary base are uncorrelated with the money stock—in other words that money causes base and not vice versa. It has thus been argued that one cannot use the base to control the money stock. In the final essay of the thesis, this argument is confronted with a simple theoretical counter-example: a simple model of optimal commercial-bank portfolio behavior is presented; in this model it is shown that, under certain assumptions about the informational structure, a regime of base control can be quite effective in controlling the money supply notwithstanding lagged reserve accounting. Under the same assumptions, though, it is shown that under a regime that involves controlling the interest rate and accommodating the demand for reserves that occur under lagged-reserve accounting at that interest rate, causality tests would give results which are similar to those which have actually been obtained—i.e. the money stock would cause the monetary base. This suggests that the use of causality tests to rule out the possibility of base control is inappropriate. The results of such causality tests would, it is shown, depend on the policy regime that is in effect.
The fourth and fifth chapters show that there is no convincing case against the use of the monetary base as an instrument of monetary policy, even under lagged reserve accounting. This does not mean, of course, that monetary control would not be more straightforward and reliable if lagged reserve accounting were replaced with contemporaneous accounting; it does, however, weaken this institutional fact as an insuperable barrier to base control.

It obviously cannot be claimed that this set of essays is the last word on how to control the money supply. The essays seem, at least, to make some contribution, though: they apply models of optimizing behavior under rational expectations to areas from which they have previously been conspicuously absent; they show that in these areas, too, the conclusions drawn in a rational expectations framework are often quite different from those which emerge from other analysis. These conclusions are important both with respect to analyzing the processes which determine the dynamic behavior of the money supply and with respect to suggesting the appropriate way of implementing monetary policy. In addition, the analysis, although it is entirely theoretical, reinterprets some existing empirical evidence and also suggests some empirical work which it would be desirable to do in the future. It may, therefore, be hoped that this thesis is a useful step towards a fuller understanding of monetary policy issues.
CHAPTER II

THE RATIONALE FOR MONEY-SUPPLY TARGETS: A SURVEY

The idea of controlling the money supply has widely been conceded an important part in the formulation of monetary policy. Money supply targets have been adopted in some form in an increasing number of countries during the nineteen seventies. As a result, debates over monetary policy have tended to focus on more technical issues--on issues related to the implementation of monetary control. These issues have included the choice of the aggregate whose growth rate is to be controlled, the choice of instrument (monetary base or interest rate) to be used in effecting this control, the importance of deviations of the money stock from its planned path and the appropriate policy response to these deviations.

The literature on the implementation of monetary control is generally separated as much as possible from that on the reasons for controlling the money supply. An attempt is made to separate monetary policy issues into "strategy and tactics": debate over the "technical" issues is framed in terms of how to attain monetary targets on which

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12 A survey of the issues which have risen in connection with the introduction of these targets is contained in Sumner 1980.

13 These issues are surveyed in Sumner (1980); they are also the subject of two symposia, Courchene et al. (1979) and Meltzer et al. (1982). See also papers contained in Federal Reserve Bank of New York (1974), and Federal Reserve Bank of Boston (1972).
there is general agreement. Another aspect of this separation of different levels of policy is that very different considerations are sometimes held to apply to policy in the short run than in the long run.

It appears, however, that this neat compartmentalization of monetary policy conceals some important connections between the different levels of policy-making. Among economists who agree on the advisability of controlling the rate of growth of the money supply, there remain real differences of opinion on the nature and role of money-supply targets, differences which can hardly fail to have implications at the "tactical" level. Depending on how one perceives the role of monetary targets, the implications of deviations of the money stock from its target level, as well as of the specific form of targets that is chosen, will be different.

Accordingly, it seems to be worthwhile to examine the analytical literature which has been used to provide a rationale for money-stock targets. This may provide some indication of the extent to which disagreement over how the money stock should be controlled is really a reflection of disagreements over this prior question of why it should be controlled.

The most commonly-used framework for discussing stabilization policy is one in which the authorities attempt to solve a stochastic optimization problem, either for one period or for the entire future. They attempt to maximize some objective-function subject to the constraints implied by the structure or "laws of motion" of the economy. Their objective-function is generally supposed to be in some way related
to the deviations of national income from some target level or path. This kind of framework is meant to provide common ground on which the views of different economists can be compared: it is considered that any disagreements about policy can be reduced to disagreements about either the structure of the economy or the appropriate objective-function; disagreements of the latter type, however, are generally assumed to be minor, since economists generally agree that "stability"—defined in terms of some measure of the size of deviations of income from its target path—is desirable.

Most of the literature on the use of the money stock as the object of policy has (at least implicitly) been set in the context of such an optimization problem. Some, however, has been set in opposition to the use of such a framework (see Prescott 1978): it has been argued that it is inappropriate to analyze policy as a "game against nature" and to prescribe the optimal way of manipulating a system whose structure is given. The latter view is associated with the idea of following a rule in policy; the money supply is generally proposed as the object of a monetary rule. In this essay, these different strands of the literature will be traced.
The literature on the targets, instruments and indicators of monetary policy might motivate the recent preoccupation with the money stock. That literature has attempted to demarcate the variables on which the authorities should focus in formulating policy. The usual starting point is the schema presented by Tinbergen (1955) and, much later, restored by Benjamin Friedman (1975): this schema draws a distinction between the targets of policy—the variables about which the authorities are ultimately concerned—and the instruments—which they directly manipulate in order to influence the targets. This is meant to establish a neat separation of means and ends: the authorities are presumed not to care about the instruments apart from their influence on the targets. This clarity is soon lost, however: to some extent the reason is that different authors have failed to agree on terminology; a more important problem, though, is that it turns out to be very difficult—and often inappropriate—to compartmentalize different aspects of policy as the targets—-instruments distinction attempts to do. If the distinction between targets and instruments is to be drawn as strictly as possible, only the atomic actions of policy—for instance, the particular open-market-operations which are carried out—can really be regarded as instruments, and only the most all-inclusive goals—social welfare—can be regarded as targets. All other variables are, in Benjamin Friedman's (1975) terminology, "irrelevant variables" which may provide information on the relationship between the targets and the instruments, information which may aid in finding the appropriate set-
ting for the instruments, but in and of themselves should play no role in policy-making. Understandably, few authors on targets and instruments have been anxious to carry terminological and analytical rectitude to this extreme. Even though instruments are supposed to be directly manipulable by the authorities, many authors have wanted to treat as instruments variables that can be influenced quite strongly by the authorities even though they cannot be controlled perfectly. Secondly, many authors have held that the authorities should focus short-run policy on variables other than their ultimate targets: these intermediate variables were often also described as targets, although in another sense they were seen as instruments; Benjamin Friedman calls these variables "intermediate targets" (1975), and argues that their use is suboptimal. Finally, there are some variables which are generally regarded as instruments of policy—the interest-rate, for instance—but about which the authorities seem to care for their own sake: in some sense, the target-instrument distinction takes on a normative role, and becomes a prescription that the authorities should not pay too much attention to certain variables. In all, it is difficult to carry the neatness of the classification scheme with any rigor without giving up the idea of saying something about the issues with which observers of policy have been concerned.

\[^{14}\] see, for instance, the interchange between Courchene and White (in Courchene et al. 1979): White expresses concern over "instrument variability"—over interest-rate fluctuations, that is; Courchene chides the Bank of Canada for being too heavily concerned about instrument variability and not enough about target variability.
The targets-and-instruments literature generally replaces the notion that the authorities are trying to maximize some social welfare function with the assumption that policy has one goal: to reduce fluctuations in real national income. Some models of the choice of monetary instrument have assumed that the price level is fixed; others have assumed that the price level may vary and that the price level and output are determined by the interaction of aggregate supply and aggregate demand relationships. Selecting the variability of output as the minimand of policy poses some problems, of course: for one thing, if there are shocks to aggregate supply which arise from changes in tastes or technology, the resulting variations in output may well be economically efficient; the same would apply to some of the variations in output which are brought about by fluctuations in aggregate demand—to those which result, for instance, from changes in consumers’ preferred time-profile of consumption or from changes in the marginal productivity of capital. In addition, variability of the price level may alter the steady-state level of national income independent of any effects on the magnitude of fluctuations of income around that level; it may also alter the level of social welfare which is associated with a given level of output. There may, of course, be other objectives of policy which are not captured in the national accounts. These issues are generally swept aside in the targets-and-instruments literature, and the alternative policy instruments are compared on the basis of their implications for the variability of income.
The practical issue which stimulated debate over the targets and instruments of monetary policy was the U.S. Federal Reserve's preoccupation, in its policy formulation, with money market conditions and with free reserves. For some time, the Federal Reserve concentrated on influencing the level of free reserves\textsuperscript{15}. It was pointed out—notably by Meigs (1962)—that a given level of free reserves is consistent with more than one configuration of the money stock and the interest rate, especially if one allows the possibility that banks may at any point be in disequilibrium with respect to their reserve positions (i.e., that adjustment of reserves to their desired levels takes time). Meigs used a stock adjustment model to analyze banks' behavior. He emphasized the fact that the level of free reserves is not unambiguous evidence of the 'ease' or 'tightness' of policy; what is important, he argued, is the gap between banks' actual and desired free reserves—a gap which brings about adjustments by the banking system. Thus policy is misdirected if it focuses on the level of free reserves alone. Similar criticisms were later made of other, similar, variables used to guide central bank policy. Measures of money market disequilibrium—"tone and feel of the market"\textsuperscript{16}, were used by many central banks, and are vulnerable to the

\textsuperscript{15} i.e. total bank reserves less required reserves and borrowed reserves.

\textsuperscript{16} Actually, while "tone and feel" can be interpreted in terms of excess supply and demand pressure, it may be just a vaguely defined measure of some composite of interest rates, their rates of change and other financial-market variables; this would perhaps be more palatable than the notion that financial markets could long be in disequilibrium. See, for instance, the papers by Anderson and Atkinson in Brunner (ed.).1969.
same criticism: there is no direct relationship between "tone and feel" and the monetary aggregates and interest rates that are of more direct economic importance; after money markets have adjusted to a disturbance, they can presumably run smoothly at the new equilibrium configuration of prices and quantities, so that the feel of the market tells very little.\(^{17}\)

The debate over specific problems with free reserves broadened into one over targets and indicators in general. In part, the discussion reflected divisions between neo-Quantity Theorists—who considered money to play, by its very nature, a strategic role in the economy—and the "New View"—according to which money is a collection of liabilities which are very much like other liabilities, and which play any distinctive role only because of the government's regulation of the banking system (Brainard and Tobin 1963, Tobin 1967). According to the neo-Quantity Theorists, money exerts an important influence on nominal income via a transmission mechanism whose workings are largely left unspecified but which is presumed to encompass a complex set of portfolio adjustments, expectations and price changes (Laidler 1978). The money stock is thus an important variable in its own right, a variable to which the authorities should pay close attention. According to the

\(^{17}\) A more recent example is the Bank of Canada's use of the Canadian Liquid Asset Ratio as a guide to monetary policy: Courchene has argued (1976, ch. 4) that the CLA ratio was misinterpreted as a measure of the ease or tightness of monetary policy in the first half of the 1970's, and that this led the Bank to adopt too expansionary a monetary policy stance; this excessively expansionary policy has been blamed for much of the inflation which Canada experienced in the 1970s.
"New View", on the other hand, the transmission mechanism consists almost entirely of changes in the rates of return on alternative financial assets in response to changes in the relative supplies of different assets; open market operations affecting the money stock are more effective than those involving other assets only because government regulations fix the rate of return on money at zero so that larger adjustments in rates of return on alternative assets are brought about in order to restore equilibrium to financial markets (Brainard and Tobin, 1963). The existence of time deposits further complicates both demand and supply of money: it leads both velocity and the money multiplier to be highly and unpredictably sensitive to changes in rates of interest (Gramley and Chase in Brunner (ed.) 1969, pp. 219-249). Since money demand depends on the whole array of rates of return on alternative assets, there is no very close relationship between money and nominal income; thus the money stock is an irrelevant variable, a by-product of the relationship between open-market operations and the configuration of rates of return, asset prices and credit availabilities that is of prime importance. Thus, according to the New View, monetary policy must be conducted with a view to its effects on interest rates as well as possibly the stock market (Tobin in Brunner (ed.) 1969, pp. 165-174); concentrating on the money supply is a misguided exercise.

This basic disagreement over the role of money in the economy conditioned the discussions over the merits of alternative intermediate targets of monetary policy. In the course of this debate, some attempt was made to provide an analytical framework within which the question
could be examined. An early effort in this direction is that of Brunner and Meltzer (in Brunner (ed.) 1969, pp. 1-26). They discussed the choice of monetary target, where policy is subject to two kinds of uncertainty: uncertainty over the validity of hypotheses concerning the structure of the economy, and uncertainty over the exogenous shocks which affect the economy. Uncertainty on both these accounts affects the ability of the authorities to achieve their ultimate goals given that they achieve their intermediate targets. Accordingly, Brunner and Meltzer proposed a criterion for choice among alternative intermediate targets: they considered, for each value of the exogenous shocks, the maximum (over all possible hypotheses governing the workings of the economy) of the deviation of the goal-variable from its desired level; they took the maximum of this maximum deviation over all possible exogenous shocks that may occur; they then compared this worst conceivable "miss" for different targets used, choosing the target that minimizes the maximum possible deviation of income from its desired level. This minimaximax criterion is an unusual one: it implies extreme risk aversion on the part of the monetary authorities.

Brunner and Meltzer used the minimaximax criterion in order to argue for the use of the money stock as the intermediate target of central bank policy. They argued that while the money stock and the interest rate both have procyclical biases, that in interest rates is more

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18 in the sense that a policy instrument which is generally unsuitable but cannot conceivably lead to a huge error in policy would, according to this criterion, be preferred to one which is generally perfect but has a tiny chance of leading to a larger error in policy.
serious, since using the interest rate as a policy target may mean moving policy in the wrong direction, while targeting the money stock will only mean moving it too far in the right direction. This argument appears to be invalid for two reasons: firstly, as Friedman showed (1953) it is entirely possible for policy to be more damaging by being too aggressive than by being "perverse." Furthermore, it begs the question of how a "neutral" policy is to be defined: Brunner and Meltzer assumed that a neutral policy is one that leaves the monetary base unchanged, but this is by no means the obvious criterion, since some changes in the economy are offset, as a matter of course, and these offsetting movements should not always be taken to indicate changes in the tenor of policy. Thus the authors' argument for focusing on the money stock carries little weight.

William Poole (1970) developed a more widely-accepted framework for comparing alternative monetary instruments. Poole's analysis is cast in a simple static linear IS-LM model; in his model, there are additive disturbance to both IS and LM curves, resulting from uncertainty about autonomous expenditure and the intercept of the money-demand function, respectively; the structure of the system, on the other hand, is assumed to be known. Since the system is linear and the authorities' objective-function is quadratic, the authorities set their policy instrument at

19 this being, of course, the rationale for focusing on the monetary base rather than simply the central bank's open-market portfolio, for instance.

20 A quadratic loss-function is clearly not as eccentric as the minimax criterion used by Brunner and Meltzer (1969): although it is desirable mainly for its analytical tractability, it can be justified
a level such that the expected value of income equals the target level. Since the authorities cannot set both the money supply and the interest rate independently, they must choose between the two as the instrument of policy; the choice affects the variance of income around its expected value. Setting the interest rate means that only the IS disturbance affects the level of income, since a shifting IS curve is then sliding along an LM curve made horizontal by the policy followed. Setting the money supply means that the effect of the disturbance on the IS side is moderated by a change in interest rates, as the IS curve shifts along an upward-sloping LM curve; this disturbance is, however, then supplemented by the disturbance to the demand for money. Thus, instrument choice involves a comparison of the variance of the expenditure disturbance with a linear combination of the variances of expenditure and money-demand disturbances.

One use of Poole's analysis is that, in principle, it provides some basis for bringing empirical evidence to bear on the choice of instrument. Poole's simple static IS-LM model does not lend itself very well to direct estimation, but it does provide a rationale for an empirically-based comparison of alternative instruments: within many models, one can compare the reduced-form variance of income when the interest-rate is given to that when the money supply is given.

in terms of its similarity to a consumer-surplus measure (this, however, begs the question of whether macroeconomic fluctuations can be considered to represent excess supplies and demands analogous to those which result from market distortions).
Another purpose to which Poole's model has been put is to place the debate over monetary instruments into perspective. In Poole's model, the choice of instrument depends on the relative stability of the real (IS) and monetary (LM) sides of the economy, where "stability" is defined in terms of the relative variances of the disturbances to aggregate-expenditure and money-demand relationships. Keynesians' advocacy of the use of interest rates vs. monetarists' advocacy of the use of the money supply as the monetary instrument has accordingly been traced to Keynesian vs. monetarist debates over issues such as the relative stability of the expenditure multiplier and the velocity of money. This link, drawn in Poole's model, is suggestive; it does not, however, summarize all the major points of contention over monetary instruments.

For one thing, the uncertainty in Poole's model is limited to additive disturbances, whereas uncertainty over the multiplier and velocity relationships constitutes uncertainty over how such disturbances affect the economy. Secondly, the usual monetarist contention that there is a stable demand for money does not imply that most monetarists attribute economic fluctuations to aggregate-expenditure disturbances, as this interpretation of Poole's model would suggest; rather, business cycles are frequently attributed to fluctuations in the money supply. In addition, some of the differences over choice of monetary instrument can be traced to different views of the transmission mechanism: many monetarists are not prepared to accept the interest rate-investment connection as the way in which money affects the economy. Some monetarists have suggested that there may be a variety of different channels of monetary
influence on economic activity—including real-balance effects, effects on exchange rates and effects on expectations (see Laidler 1978);

Poole's static IS-LM model confines the transmission mechanism to the simple Keynesian one, and thereby ignores part of the case for controlling money rather than interest rates.

Another argument which appears in Poole's paper is that it is generally suboptimal to use either the money supply or the interest rate as the instrument of policy. Rather, the optimal policy is a "combination policy"—a policy of controlling a linear combination of the money supply and the interest rate. Such a policy—which is equivalent to making the money supply a function of the interest rate—can be carried out with weights placed on the two variables which minimize the variance of income.

A related issue has been emphasized by Benjamin Friedman (1975, 1977), who has been forceful in stressing the suboptimality of intermediate targets. Friedman focuses in particular on the use of the money stock as the intermediate target, when the authorities are using the interest rate as the instrument. Friedman supposes that the authorities can control the interest rate deterministically, and that they are considering whether to use the interest rate directly in order to set the expected level of income at the desired level, or whether to use it to attain a certain level of the money stock which, in turn, yields an

\[ a \text{ linear combination only because Poole's model is linear and his objective-function quadratic; in other models, a more general function of the money supply, the interest rate and other variables would be controlled. } \]
expected level of national income equal to the desired level. Friedman builds on analysis presented by Kareken, Muench and Wallace (1973); these authors treated the true policy instruments as the variables manipulated directly by the authorities, such as either the central bank's open-market portfolio or the price it offers on the bills it buys and sells. From this standpoint, potential intermediate targets—such as either the interest rate or the money supply—are properly to be regarded as "information variables": they are variables that may or may not provide guidance in the appropriate adjustment of the instruments.

In the time-frame set out by Kareken et al., the instruments are observed continuously, while the intermediate targets are observed less frequently (daily, weekly or monthly); the ultimate target, national income, is only observed at the end of every quarter. Thus the variables such as the interest rate or the money stock can provide information about the level of national income, information which may be useful in adjusting the instruments in the mean time. Friedman (1977) shows, however, that the appropriate adjustment of the instrument in response to movements of the information variables will depend on the lags in receiving information. The intermediate target procedure—the procedure, that is, of picking either the money supply or the interest rate as a surrogate objective of policy, and keeping it at a pre-determined level until real output can next be observed—can be shown to be suboptimal: not only does this procedure misuse the information provided by the variable used as an intermediate target; it also throws away information embodied in other potential information variables. In the case-
where the authorities use the money stock as an intermediate target in determining how to adjust the interest rate in order to achieve the desired level of national income, this procedure is equivalent to assuming that all deviations of the money stock from its desired level are due to fluctuations in national income; the interest rate is accordingly adjusted to offset this imputed disturbance to aggregate expenditure and not in response to any other information. Presumably, one could also carry the same argument to a lower level of policy-making, and argue that if the interest rate cannot be controlled perfectly, it is inappropriate to use it as an intermediate target vis a vis open-market operations or other primary actions of policy.

An important limitation of Poole's simple model is that it considers only one form of uncertainty, namely uncertainty about the position of the IS and LM curves, not about their slopes: thus the structure of the system is assumed to be known with certainty, while the exogenous variables and the money-demand intercept are not. This assumption was considered more carefully in a paper contemporaneous to Poole's, a note by John Kareken (1970). Kareken considered the case in which both exogenous variables and slope parameters are uncertain. His paper presents expressions for the appropriate policy settings, which in this case differ from those which set the expected value of income equal to its desired level. The mean square deviations of income from its target are then calculated, depending on whether it is the money supply or the interest rate that is used as the instrument of policy. The expressions for these mean square deviations are too complicated to be analyzed in
the general case; unlike Poole's results, they do not lend themselves to any straightforward interpretation in terms of the relative stability of monetary and real sectors.

Another limitation of Poole's basic analysis is that it is framed in a static setting. Accordingly, later in the same paper (1970) Poole presents a dynamic model in which the main conclusions of his static analysis are supported. In his model, he posits that aggregate expenditure depends on lagged income, so that income follows a second-order difference equation; this is the result which would emerge from an accelerator-multiplier interaction model. In this model, the authorities can reduce the variance of income by offsetting the effect of past income, which is known to them: as a result, the criteria for instrument choice are not greatly altered by making the model dynamic in this respect.

Instrument choice in dynamic models has also been analyzed in papers by Sargent (1971) and Turnovsky (1975). These papers present models in which money demand depends on lagged income, and in which there is also multiplier-accelerator interaction and a Robertsonian (one-period) consumption lag. Sargent's model also allows the price level to vary: his model includes an expectations-augmented Phillips curve with static inflationary expectations; Turnovsky treats the price level as given. Neither Sargent's nor Turnovsky's analysis considers

Turnovsky points out that allowing the price level to vary makes little difference to his analysis if the price level and output are determined by the interaction of a linear aggregate supply schedule with an aggregate demand function derived from a model like his: in that case, price-level variability can easily be represented in terms of a
inflation to be a concern of the policy maker: it is assumed that only the variability of income is important. In such dynamic models, income follows a difference equation whose stability can be influenced by means of a feedback rule on previously-observed levels of income. Conditions can be found for the stability of the system under alternative choices of instrument. Also, given stability, the asymptotic variance of income can be found, minimized for each instrument, and compared for alternative instruments. It turns out that in the general case, in which the structure of the system is not known with certainty by the policy-maker, no straightforward results can be obtained: which instrument is superior depends on a complicated expression depending on the means, variances and covariances of all the parameters and variables in the model. A few other mildly interesting conclusions emerge from these papers, but one of the main messages is that Poole's simple policy conclusions are not repeated in any straightforward way in a model that incorporates both dynamic interrelations and parameter uncertainty.

A paper by Basil Moore (1972) also deals with the choice of monetary instrument in a dynamic context. Moore's model is quite different from those just discussed: it is a variant of David Laidler's (1968)

money-supply function function which depends on the level of income. Needless to say, neither the assumption of a fixed aggregate-supply schedule nor Sargent's assumption of static inflationary expectations is particularly appealing.

22 For instance, in both papers it is shown that an optimal interest-rate feedback policy may require that the money supply fluctuate pro-cyclically; this counters the intuition that pro-cyclical movements of the money-stock indicate a destabilizing monetary policy (cf. Brunner and Meltzer 1969).
model, a dynamic IS-LM framework in which the dynamic behavior results from the inclusion of permanent income in both consumption and money-demand functions. Moore discusses feedback policy of two types: proportional and derivative. In Moore’s model, although the parameters are assumed to be non-stochastic the covariances among the various additive disturbances are considered. Here, unless the covariances between money-demand and consumption disturbances are negative the variance of income is generally larger when the money stock is used as the policy instrument. Moore also introduces a term-structure equation, embodying the expectations hypothesis: long-term rates—which are the rates affecting aggregate expenditure—are supposed to equal a geometric average of expected short-term rates. Moore shows that considering the term structure strengthens the case for interest-rate targeting.

All the papers just discussed attempt to analyze the issue of instrument—or intermediate-target—choice within models simple enough to permit the derivation of explicit analytical criteria for a decision. As shown in several of these papers, however, a model does not have to be very complicated before it becomes totally intractable for this pur-

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24 terms devised by A.W. Phillips (1954–1957): proportional feedback is feedback in response to the level of output, whereas derivative feedback responds to the rate of change of output.

25 It is not unlikely that this condition should hold, actually: an individual facing a budget constraint will normally increase his expenditure on goods when he reduces his demand for money.

26 Here again, the result is conditioned by the fact that Moore does not consider direct real-balance effects of money on consumption, which would continue to operate despite the weakened effect of the money supply on the long-term interest rate which is implied by this term-structure formulation.
pose. Accordingly, some authors have taken an alternative route: that of simulating the use of alternative policy instruments. An example of this kind of approach is a paper by Roger Craine and Arthur Havenner (1977). These authors simulate the use of alternative policy instruments using the MPS model for the U.S.; they apply stochastic control methods to a deterministic version of the model, in order to find the optimal path of each instrument. They then compare the loss incurred—based on a loss function including real income and the price level—under alternative instruments. According to their empirical results, using the interest-rate as the policy instrument would result in somewhat smaller loss, although it appeared that neither instrument would have been very successful in stabilizing the economy in the face of the massive shocks affecting the economy in the mid-1970s.

Such results are, it would appear, of little value in themselves. This is not just because of the particular method used in one particular study; more generally, specification of the structure of the model used has often already decided some crucial empirical issues, before any estimation or simulation has been carried out. For instance, the transmission mechanism assumed and the effect of inflation on interest rates are crucial features on which alternative models differ. Thus with larger as with small models, any policy conclusions must be carefully scrutinized.

\footnote{\textit{i.e.} treating the parameter estimates as known with certainty, and treating the regression residuals from the model as the disturbances that actually occurred.}
Another problem which plagues all this kind of analysis is one identified by Robert Lucas (1976): the structure of the system is liable to change depending on the policy regime. One aspect of the policy regime is the choice of instrument of monetary policy; to the extent that this choice of policy instrument affects the variance of income, it may be that the lag-structures of consumption and money-demand functions, the slope of the Phillips curve, the interest sensitivity of investment, and other behavioral relationships may well alter when a different policy instrument is used: all these relationships depend on agents' reading of the information provided by price or income fluctuations. This problem affects both the derivation of theoretical criteria for choice of policy instrument and the econometric evaluation of policy; it further limits the relevance of simple criteria for instrument choice.

At the end of this discussion, what can one conclude about the use of the targets-and-instruments literature as a basis for money-stock targets? Evidently, this literature does not provide a very strong rationale for controlling the money supply. The solution to the instrument problem, according to this literature, is not to control the money supply, nor is it to control the interest rate. Rather, the optimal policy is a combination policy: it is a policy of controlling an appro-

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28 This occurs since all of these behavioral relationships can be seen as reflecting optimal choice under uncertainty. If expectations are formed rationally, agents will extract different information from their observations of economic data under a different policy regime. See below, pp. 76-77.
priate combination of the money supply and the interest-rate--as well as any other variables which embody any information about the level of national income implied by current policy actions.

If this is the appropriate role of the money supply--as one of the variables included in the optimal combination policy--then how can one interpret central banks' adoption of monetary targets? Three alternative explanations carry some weight. Perhaps the targets are a mistake: they would be a mistake, in the light of the targets-and-instruments literature, if the authorities saw them as real goals of policy and directed their policy actions towards minimizing the deviations of the money supply from its target path (see Benjamin Friedman 1977). A second possibility is that they are a sham: announcing targets for the money stock could be merely a public relations device, designed to give the impression that the central bank adopting them is trying out a fresh new approach to policy, when in actual fact it continues to mould its policies in response to political pressure from free-spending governments, financial institutions and the construction industry (see Kane 1980). A third possibility, though, is that the targets are designed to convey some (not necessarily incorrect) information about policy. In the context of the targets-and-instruments literature, the only thing which adopting money-supply targets would likely announce is that the weight of the money supply in the optimal combination policy had

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29 a linear combination in the standard case in which the system is linear and the authorities' objective-function quadratic.

30 by "policy actions" is meant, here again, the atomic operations of policy--particular open market operations, for example.
increased. Why, though, should the weight of the money supply in the optimal combination policy change? Presumably, this would result from a change in the weights which policy-makers attach to the various criteria of policy, or from a change in what they suppose to be the structure of the economy. Adopting monetary targets would then constitute, not a surrogate goal of policy but an announcement about the preferences and beliefs of policy-makers.

Of what use would it be for the authorities to make an announcement of a change in the combination policy which they are following? Such an announcement could be seen, in another light, as an exhortation to the public to pay attention to the behavior of the money supply. This possibility is suggested by some central bankers' remarks about the role of money-supply targets in reducing the public's inflationary expectations (see Sumner 1979, Volcker 1978): it is held that if the public is made aware in this way that the monetary authorities are pursuing contractionary policies with a view to reducing rate of inflation, the transition to a lower rate of inflation will be smoother.

This view of the role of money-stock targets seems to be in some way related to the so-called "indicator problem"—the problem of finding a variable indicating the tenor of policy in a manner intelligible to outside observers (see Brunner and Meltzer 1967, various papers in Brunner (ed.) 1969, Saving 1967, papers by Hamburger and Zecher in Holbrook et al. 1970, Hendershott 1968, Starleaf and Stephenson 1969, Carr and Smith 1975). Benjamin Friedman's (1975) assessment of the literature on the indicators of monetary policy seems to be a typical one: he states
that while it may be difficult to find an appropriate indicator of policy, this difficulty is irrelevant for policy-making. The monetary authority knows its own objective-function and information-set; of what additional use might a measure of the thrust of policy be? This argument certainly appears to be correct within the framework of Friedman's model: in that model—a stochastic IS-LM model—expectations, and correspondingly private agents' perceptions of policy do not play any role.

Is the indicator problem important when private agents base their behavior on price-level expectations which are formed using information about the state of the economy and the policy regime? If expectations are fully rational, and if there is no difference between the information available to private agents and that available to the central bank, the indicator problem collapses: the public will anticipate particular movements in the authorities' policy instruments in response to previous disturbances in the economy; they will anticipate, in turn, that these changes in instrument setting will result in particular changes in other economic variables, including the price level; thus the authorities' (anticipated and actual) instrument settings will be part of the information incorporated in private agents' price expectations. No other indicators will be of use: the only relevant indicators of policy will be the instruments used by the authorities.

Thus, in order to attach any relevance to the money supply as an indicator of policy, it would be necessary to posit some asymmetry in the information available to the central bank and to the general public.
Such an asymmetry may well exist: the central bank, by virtue of its role in regulating the banking and financial sectors, has information about these sectors which is not available to the public (some of it being, of course, confidential). Such a difference in information might make it difficult for the public to interpret the effects of changes in the instruments of policy (in the monetary base, for instance); the public may also find it especially costly to process this information. On the other hand, the public may, to some extent, be aware of the link between the rate of monetary expansion and the rate of inflation. If the authorities are intent on reducing the rate of inflation, they might attempt to draw the public's attention to the behavior of the money supply, encouraging the public to use the money supply as an indicator of monetary policy: using the money supply as an indicator may cut down on the public's costs of processing information about monetary policy and incorporating this information in their expectations of the inflation rate. Announcing target ranges for the money stock, and showing that the behavior of the money stock adheres loosely to these targets, might then be designed by central banks to encourage the use of the money supply as an indicator of policy, in order to increase the speed with which inflationary expectations adjust downward as monetary policy is carried out to reduce the rate of inflation. This fits in with central bankers' statements connecting monetary targets with a reduction in inflationary expectations. Thus the adoption of money-supply targets may perhaps be better described as the promotion of money-supply indicators.
Whether this attempt works, of course, is another question. Why should the public start paying attention to the behavior of monetary aggregates just because the central bank tells it to do so? It is quite possible either that the public will see through the behavior of the monetary aggregates to the effects of the specific policies being followed; alternatively, they may fail (because of high costs of information-processing) to build the effects of money on inflation into their expectations (see Feige and Pearce 1976). However, it does seem possible that an important reason for the authorities' preoccupation with monetary aggregates has been to provide the public with information (true or false) about monetary policy.

In conclusion, then, the literature on the targets and instruments of monetary policy does not provide a rationale for controlling the money supply except as one of a number of information variables to be used in an optimal combination policy. In the light of this analysis, targets for the money supply seem to be a fifth wheel vis-à-vis the formulation of optimal policy. To the extent that such targets have any role to play, it is not in influencing the conduct of policy itself, but rather in communicating the nature of policy to the public.
III

Another important body of literature does lead to the conclusion that targets for the money stock should have a real importance in formulating monetary policy: this is the continuing debate over the advisability of conducting monetary policy according to a rule. The case for centering monetary policy around some kind of rule has been developed over time; a monetary rule has generally been associated with the choice of the money supply as the object of such a rule.

Within the question of whether a monetary rule would be desirable, two separate issues are generally distinguished. One question is whether all policy actions should be pre-announced, or whether the authorities should be allowed to carry out whatever policies they deem to be best at the time. The second question is whether monetary policy should be activist--in the sense that an attempt is made to stabilize the economy by responding to economic events--or whether the monetary authorities should confine themselves to keeping monetary policy from being itself a source of instability. This second question is essentially one of whether policy actions should be contingent on the state of the economy, i.e. of whether or not they should incorporate feedback from economic events.

In practice, these two questions appear to be linked: in general, some economists favor a policy regime involving a preannounced noninterventionist policy, while others favor activist policy which need not be preannounced; the other two permutations seem to have few supporters. Most obviously, if a neutral policy were followed there would presumably
be nothing lost by announcing to the public that such a policy was being followed. It is much less obvious whether "rules with feedback"—a policy which involves responding in a pre-specified way to economic events—is an option which may be taken seriously. Whether or not this is a sensible combination depends, of course, on how strictly one defines preannouncement: if it simply means that the authorities should have defensible reasons for their policy actions, and not carry out policy on the basis of whimsy, preannouncement would indeed have few opponents even among the supporters of policy activism. If, on the other hand, rules with feedback implied that the authorities would bind themselves\(^3\) to responding in a particular, pre-specified way to economic disturbances in the future, in order to enable agents to form reliable expectations of the policy environment not only at the present time but on into the future\(^2\), support for such a policy would dissolve: who would argue that the feedback rule which is judged to be appropriate based on the 1982 version of the MPS model (for instance) should be used from now until Kingdom Come? The danger that such a pre-established feedback policy would turn out to be destabilizing would almost certainly be much greater than the dangers of economic instability under a fixed rule. Thus, if one interprets rule-following behavior in the

\(^3\) This strict interpretation seems to make sense: what can "following a rule" seriously mean if the "rule" can be modified whenever the authorities choose?

\(^2\) as implied, for example, in rational expectations models, in which the entire future path of the money supply influences the current price level; see, for instance, Sargent and Wallace 1975.
loose sense that policy not be whimsical or randomized\textsuperscript{33}, it is trivially desirable; if one interprets it to mean that, to the extent to which policy embodies feedback from economic events, the nature of this feedback be pre-specified despite possible changes in the structure of the economy, the authorities' understanding of that structure and other information\textsuperscript{34} available to them, rules with feedback are trivially undesirable. In either case, then, the issue is not an interesting one; the meaningful issue, on which there can be some interesting discussion, is whether or not monetary policy should be guided by a preannounced rule which constrains the authorities from attempting to stabilize real economic variables.

There is a clear connection between the idea of a monetary rule and the choice of the money supply as the appropriate object of such a rule. While it is certainly possible for the authorities to use the money supply as a discretionary stabilization tool, the converse does not hold: since the work of Wicksell (1906), it has been known that attempting to peg the interest rate will lead to cumulative inflation or deflation unless the rate is set equal to the "natural rate" of interest which is determined by the real forces of productivity and thrift.

\textsuperscript{33} which is the way it is generally interpreted in the rational expectations literature—see for instance Lucas 1972, Sargent and Wallace 1976.

\textsuperscript{34} By this is meant changes in the type of information which the authorities can use, changes which perhaps could not be anticipated and thus cannot be specified in the feedback rule; for instance, a feedback rule devised before the national accounts were invented could hardly have specified feedback from national income.
Wicksell's conclusion has been reinforced in the (much-more-recent) literature on rational expectations: in an important paper by Sargent and Wallace (1975), it has been established that, if the public forms its expectations of the price level rationally, the price level will be indeterminate unless monetary policy pins down some nominal variable such as the money supply. Current behavior, and thus the current price level, depends upon the expected price level; thus unless some future price level is determined, the entire time path of the price level will be indeterminate. This will be the case if monetary policy attempts to set an interest rate.

This conclusion drawn by Sargent and Wallace has subsequently been qualified: Parkin (1978) has shown that, if the authorities set an interest rate but associate this interest rate with the attainment of a target for the money supply, this money-supply target will essentially determine the expected price level and thus prevent the actual price level from being indeterminate. This result seems to depend rather heavily on an announcement effect. It requires that the public believe that the authorities will be successful in attaining its announced target for the money stock, even though it the model does not specify how the authorities would respond if the money stock turned out to be at a level other than the target level: the authorities simply set an interest rate, which would in general be consistent with an infinite number of possible money stocks and price levels.

Another model, by Bennett McCallum (1981), has similarly pointed out the possibility that a target for the interest rate may be a viable
monetary rule provided that the interest-rate rule is in some way connected with the behavior of the money supply. McCallum considers a monetary rule which involves adjusting the interest rate in response to deviations of the money supply from a target path. Merely by including the money supply somewhere in the specification of policy, one can avoid the indeterminacy result of Sargent and Wallace (1975); under such an interest-rate feedback rule, the price level is determinate regardless of how small a weight is placed upon the money supply (provided that this weight be nonzero). Thus, the connection between rule-following and controlling the money supply is not as close as had previously been thought—at least, it is confined to fixed rules, to rules without feedback. Nonetheless, the Sargent-Wallace proposition about the optimal monetary instrument has been quite influential in associating a monetary rule with the choice of the money supply as the variable to be controlled.

The case for a monetary rule has been developed in two directions. Some arguments essentially take the conventional theory of economic policy—which portrays the authorities as maximizing some objective function based on economic stability—as given, and attempt to show that the economy would be more stable if a neutral policy were followed than if the authorities attempted to stabilize the economy. Other arguments have rejected this framework, or at least introduced considerations which are not easily accommodated within the framework. The former type which, as has been argued above, is not as restrictive a case as it may seem, in the sense that rules with feedback either are only rules in a rather tenuous sense or are unlikely to draw much support.
of argument is found in the literature on the pitfalls of stabilization policy, the latter in both the neo-conservative political literature and the rational expectations literature. Each line of argument will be sketched in turn.

There has been much discussion of the pitfalls of stabilization policy, which tends toward the conclusion that perhaps a neutral policy would be preferable to activist policy. One important contribution was Milton Friedman's discussion of "The Effects of a Full Employment Policy on Economic Stability" (1953). In that paper, Friedman simply set out the conditions under which stabilization policy reduces the variance of income; he did so in such a way as to draw attention to the possibility that such policy would be destabilizing. He showed that, in order for stabilization policy to be successful, it must not be conducted on too large a scale; timing, expressed in terms of the correlation between policy actions and the swings of income which would occur in the absence of such actions, is also of crucial importance. Using elementary statistical theory—and assuming that policy affects income additively—Friedman emphasized the fact that the beneficial nature of stabilization policy is not a foregone conclusion.

The considerations suggested by this early paper of Friedman's reappeared in much of his subsequent work. For instance, in A Monetary History of the United States he and Anna Schwarz argued that misguided monetary policy had been the major cause of business cycles in the U.S.; their work suggested that the economy would have been much more stable had the money supply been controlled in a neutral manner over the period...
considered. Friedman's work also indicated that money affects nominal income with a "long and variable lag" (e.g. Friedman 1970); the length of the lag suggests that considerable foresight would be required in order to design policies which would turn out to be stabilizing once their effects begin to be felt. The lags' variability further increases the likelihood that ostensibly-sensible countercyclical policies will turn out to have perverse effects.

Some early work by A.W. Phillips (1954, 1957) also pointed out the difficulty of designing stabilization policy which would achieve its desired results; in contrast to Friedman (1953), Phillips was generally attempting to design appropriate countercyclical policies, rather than to show that this cannot be done. Phillips (1954) discussed three general types of countercyclical policy: he considered proportional feedback, which means varying the policy instrument in proportion to the level of national income; he also considered derivative feedback, which responds to income's rate of change; finally, he considered integral feedback, which means adjusting the instruments in response to a cumulative sum of past deviations of income from its steady-state path. He simulated these kinds of policy feedback rules in the context of a multiplier-accelerator model, showing that some combination of the three is generally desirable. He concluded that monetary policy could keep the level of national income quite stable if it were properly designed (1954); however, he also observed that if the feedback rule deviates from the optimal one to a relatively small extent, it could actually

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36 see also Johnson and Winder 1963
make the economy less stable (1957).

Another early paper by William Baumol (1961) showed some of the pitfalls of activist policy, also in the context of a simple multiplier-accelerator model. Baumol's paper was designed to show the thickness of the task of designing policy, and to demonstrate that ostensibly reasonable policy may turn out to be destabilizing. In his model, he considered derivative and proportional feedback policy which may be used to offset the cyclical fluctuations in income that result from multiplier-accelerator interaction. While such feedback policies will increase the stability of the system in that they increase the degree to which income fluctuations will be damped, they also increase the frequency of these fluctuations and so in another sense are destabilizing. More strikingly, if such policies are carried out subject to a one-period information lag, they will not improve the degree of damping while they will still increase the frequency of oscillations; a two period lag implies that feedback policies will destabilize the system in both respects. Thus, even in a very simple dynamic system, "plausible and reasonable contracyclical policies turn out to be capable of increasing the explosiveness and frequency of economic fluctuations" (p. 24). The implication is that, where dynamic influences are at work and where there are time lags in observing the effects of fluctuations, care is required in order to design stabilization policies that do not turn out to be counterproductive.

Uncertainty over the structure of the system was stressed in an important paper by William Brainard: the implication of uncertainty is
that policy must be more cautious than it would be under certainty. Brainard's paper (1967) stands in contradistinction to the usual analysis of policy options in terms of certainty equivalence. Where disturbances enter additively, the system is linear and the authorities' objective-function is quadratic, optimal policy can be formulated without taking account of uncertainty; policy settings can then be such as to set the expected value of the target variable equal to the desired value, while the instruments used will be such as to minimize the variance of the target around that desired level (Theil 1962). However, as Brainard showed, with parameter uncertainty policy will be more cautious than it would be under certainty: instruments will in general be used to a lesser extent, and it may be sensible to move some of the instruments in the "wrong" direction—i.e. in a direction opposite to that required to move the expected value of the target variable towards its desired value. If the utility function is quadratic, "it always pays to do something, but it never pays to aim for y" (p. 417). This means that although some kind of stabilization policy is still desirable, caution is required. Of course, this principle of cautious policy is not usable as a rule of thumb for policy-making unless we specify how a neutral policy is to be defined—otherwise there is no benchmark against which to measure 'caution'. However, it does tend to stress how difficult it is to find an appropriate activist policy when the structure of the system is unknown.

The idea that long lags in the effects of policy necessarily make policy less effective has been addressed in a paper by Philip Howrey
(1969). Using a dynamic IS-LM framework, Howrey showed that an increased speed of policy can be associated with a greater variance of income when this policy is used; it is not in general true that the speed of policy is linked with its effectiveness in stabilizing fluctuations in income. Thus it seems to be the variability of lags which is crucial in the case for rules; the length of the lags does not necessarily vitiate activist policy.

Caution in policy activism can be advised on grounds other than parameter uncertainty: Franklin Shupp (1972) has shown that even if disturbances are all additive, nonlinearity in the model means that optimal policy will not be the same as certainty-equivalence policy. In Shupp’s paper, a simple aggregate supply-aggregate demand model was presented, in which nonlinearity occurs; in this model, optimal policy depends on the degree of uncertainty about autonomous expenditure. In Shupp’s model, nonlinearity combined with additive uncertainty means that policy will be less forceful than it would be under certainty; however, it is conceivable that in some other nonlinear systems optimal policy would become more forceful when uncertainty is present.

In considering the effects of stabilization policy, it is also necessary to consider what is meant by stability. This is quite simple in a static system: one can express stability in terms of the mean square deviation of the target variable from its desired level. However, in a dynamic system the question is not that simple. This has already been seen in Baumol’s (1961) paper, where the frequency of oscillations and the tendency of the system to be explosive are both
part of the usual notion of stability. Philip Howrey showed (1967) that the question is even more complicated when a dynamic system contains random disturbances. In the multiplier-accelerator model considered by Howrey, there are two contending definitions of stability: the speed with which fluctuations are damped and the asymptotic variance of income. Moreover, these concepts do not coincide—in fact, there is a tradeoff between them: "policies which increase the stability of the system in that they increase the rate at which the transient response damps out may actually increase the variance of the time path of the system". (p. 410) This analysis suggests that the problem of designing appropriate stabilization policies is trickier than it might seem at first blush. For one thing, if the notion of stability is ambiguous even in a simple model, it seems rather unrealistic to expect policymakers to "stabilize" the economy through actions taken on a day-to-day basis, when the economy consists of many complex dynamic interrelations. In addition, the fact that stability can mean different things in the same model frustrates the attempt to devise criteria for policy-making which are, in some sense, independent of the preferences of policymakers. In a static model, stabilizing the economy can, with a few reservations, be represented in terms of minimizing the variance of income; in a dynamic model, on the other hand, it is difficult to say unambiguously whether a particular kind of feedback policy makes the economy more or less stable.

Thus there is a considerable body of literature which is pessimistic about the possibility of finding discretionary policy which stabi-
lizes the economy; this is not just a question of finding "optimal" policy, but even of finding policy that does not actually make the economy less stable. Following a monetary rule can thus be advocated as a "neutral" policy which at least prevents money itself from being a destabilizing influence in the economy.

The idea that a monetary rule would be less risky than discretionary policy has been challenged, however. One problem is that alluded to above: what precisely is a "neutral" policy stance: is it one of keeping the interest rate, the money supply, the monetary base or the price level constant? Not all these variables can be set independently of one another; "caution" in altering one of these variables may imply "activism" in altering all the others, so that it is not clear what kind of policy is really appropriate given uncertainty about the structure of the economy. Perhaps "neutral" policy is not an option that exists; perhaps the authorities simply have to do the best they can with the limited information available to them.

Such an argument is set forth in a paper by Michael Lovell and Edward Prescott (1968). These authors analyzed the question using a multiplier-accelerator-interaction model like that contained in Baumol's paper (but where investment is also affected by interest rates). They considered the optimal policy from the standpoint of the stability of the system (degree of damping), and show that there is no presumption of whether this stability will be improved by a "neutral" policy of keeping interest rates constant or by another "neutral" policy of keeping the money supply constant (which implies a particular procyclical
movement of interest rates). Their conclusion was thus to reject monetary rules, suggesting instead that "we continue to rely upon the best judgement of the monetary authorities rather than take the radical step of abandoning discretionary policy in favor of an extreme concept of neutral money" (p. 70).

Much other work has also criticized the introduction of monetary rules. Some of this work has involved applying optimal control techniques to economic policy: such optimal control applications are numerous, and have been surveyed by David Kendrick (1977). Most of these papers simply ignore the problems of parameter uncertainty and non-linearity—many just using certainty equivalence. Usually such analysis finds an optimal policy feedback regime which is not a constant-growth-rate rule for the money supply. Two specific examples are papers by Cooper and Fischer (1974) and by Craine, Havenner and Berry (1978). The former paper, actually, does not explicitly use optimal control, instead simulating a variety of ad hoc feedback rules in a large-scale macroeconomic model; the authors found that some such feedback rules would dominate a k% (i.e. constant-growth-rate) rule for the money supply, although both out-performed the policies actually followed. In the latter study, a variety of policy regimes were simulated; these rules included both k% rules and a variety of feedback rules including one derived by optimal control techniques. In simulations performed for the turbulent 1973-75 period, the k% rule did better than any of the feedback rules\(^\text{37}\), but actual policy involved a still smaller loss. These

\(^{37}\) according to a loss function set arbitrarily for the whole study
conflicting results do not inspire much confidence in the validity of all this kind of simulation analysis; clearly, any empirical assessment of alternative policy regimes must be treated with a great deal of caution.

A problem that seems to be inherent in this literature is that it seems to be attempting to evaluate optimal policy in terms of itself. If "following a rule" is compared to "doing what is expected to be best, given the information available," and the comparison is made on the basis of which is expected to be best, it is hardly surprising that the optimal policy turns out to be optimal. The literature on the pitfalls of stabilization policy develops a case for circumspect policy; it does not make a case for a monetary rule. In order to devise a case for a rule—in the sense that the authorities do not merely behave with caution, but actually commit themselves to behaving in some pre-arranged way—one must introduce some other consideration. One could perhaps argue that there are some fixed costs of running an activist stabilization policy, costs which generally exceed the expected benefits from activism. Alternatively, one could argue that central banks are generally incapable of recognizing their own limitations, and that, if they are given the opportunity, they will attempt to fine-tune the economy to the same extent as they would under certainty. In addition, it would not seem unrealistic to suppose that central banks are subject to political influences which might lead the policies actually chosen to be suboptimal: they may be influenced by governments which are hungry

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31 The resulting policies would only actually be suboptimal if there
for means of financing their deficits, and whose rates of time-preference are warped by the electoral cycle (see Nordhaus 1975); in addition, they may be subject to pressure from special interests which are directly affected by monetary policy (see Kane 1980). Any of these arguments might be used to claim that a monetary rule would lead to policies which are better than those which are actually followed, even though the rule might be dominated by the monetary policy which is socially optimal. Countenancing this kind of argument would mean departing from the conventional approach to policy analysis: within the standard framework, policy analysis is implicitly cast as a problem of prescribing the appropriate actions to be followed by a wise and benevolent despot.

This suggestion—that the objective-function which is being maximized by the authorities cannot be presumed to correspond to economic stability or social welfare—is the basis of the earliest arguments for a monetary rule. Allowing governments to carry out discretionary monetary policy was regarded as an important source of arbitrary power in a market economy, one which should, if possible, be limited by rules. This kind of neo-conservative argument was the basis of Henry Simons' (1936) advocacy of rules rather than discretion in monetary policy: in Simons' classic paper, the effect of a monetary rule on macroeconomic stability is not addressed. The same concern also appears in the papers presented at a conference on designing a monetary constitution (col-

were transactions costs which prevented the "losers" from these policies from launching effective counter-pressure—i.e. from bribing the "gainers" to change the policy.
lected in Yeager 1962): at that conference, various writers dealt with the question of how the government's power over the creation of money can be limited. A monetary rule is one of the proposals which would limit the role of the state in this respect; other proposals for a monetary constitution, such as a return to the gold standard, were also considered. Friedman advocated a monetary rule on the grounds that it would limit the scope for the government to abuse its power to create money, without sacrificing the real benefits of fiat money (primarily that no resource costs are incurred in creating fiat money). In this light, a rule is intended to be, not just a different, more circumspect, way of carrying out monetary policy, but a real constitutional constraint on the way in which policy can be conducted.

However, another, perhaps more vital attack has been made on the conventional theory of economic policy. Whereas the political arguments for rules depart from the orthodox theory of economic policy on the grounds that the authorities' objective-function is not the appropriate one, this second attack concerns the way in which that theory views the structure of the economy. In the orthodox literature on policy analysis, it is supposed that even if the structure of the system is unknown, it is at least given, insofar as the policy-maker is concerned. This supposition has been challenged in the recent literature associated with rational expectations.

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39 in Yeager (1962); for other aspects of his case for a monetary rule, see also A Program for Monetary Stability.
An early step in this direction was Milton Friedman’s presidential address (1968). In that address, Friedman argued that monetary policy affects real output and real interest rates only to the extent that it is unanticipated: a departure of actual from expected prices leads private agents to believe that the relative prices of the goods or services they are selling has changed, so that an increase in output is thereby induced; once expected and actual prices become equal, no real effects are felt. As a result, Friedman argued, monetary policy cannot make the unemployment rate differ from its natural rate except for a transitional period when agents’ expectations of the price level have not yet adjusted to the new policy. Similarly, it is only until the expected rate of inflation becomes equal to the actual rate that monetary policy can affect real interest rates: the real interest rate will eventually equal its natural rate which is determined on the real side of the economy. This argument implies that it is futile for the authorities to attempt to influence the long-run levels of any real variables.

Of course, Friedman’s argument does not imply that rules are superior to discretionary policy in the short run, but only that the long-run average rate of unemployment cannot be altered through monetary policy. In Friedman’s presidential address, he indicated that the transitional period, during which monetary policy does have some effect on real variables, may last several years: there is nothing in his argument to rule out the possibility of stabilization policy in that time-frame.

—a rate reflecting all non-monetary sources of unemployment, including measured unemployment which does not reflect willingness to work at the market wage, and also including search unemployment.
Thus, although Friedman's argument is used to build a case for a monetary rule, it cannot rule out the possibility that stabilization policy may play a role in insulating the economy from aggregate-demand and aggregate-supply disturbances; such shocks will otherwise have real effects for as long as expected and actual prices continue to diverge.

Once one dismisses the possibility that monetary policy can alter the trend rate of growth of output, and limits the role of policy to stabilizing the path of income around that trend, a further problem arises: if the authorities wish to stabilize income in this way, they must first know what national income's long-run trend is.1 However, for this to make stabilization policy fruitless requires a strong assumption about the degree of ignorance of the monetary authorities; for one thing, if accelerating or decelerating trend rates of monetary growth are needed to keep rates of unemployment and interest above or below their normal rates, this would itself provide some information about what these rates are. Thus Friedman's address indicates that the policy problem cannot be analyzed using some kind of hydraulic approach, which treats the level of output and employment as being susceptible of almost unlimited manipulation by the authorities. However, it does not demonstrate the superiority of rules over some kind of discretionary policy if there are unpredictable and uncontrollable events which have significant temporary effects on the level of real national income, and if it takes some time for the economy to adjust to an equilibrium at which natural

1 A criticism made, more recently, of the Bank of Canada's approach to money-supply targets: see, for instance, Courchene in Courchene et al. 1979, p. 613.
rates of unemployment and interest prevail.

The rational expectations literature has carried through a more striking critique of macroeconomic stabilization policy. A major theme of this literature is that in a market economy information soon becomes available to those for whom it is useful; thus, if any model remains the economy well, private agents will behave as though they had access to the predictions of that model. Thus it is argued that if the authorities did have information about the economy which could enable them to offset the supply and demand shocks which impinge upon the economy, private agents could equally well act in such a way as to offset those shocks; purely as a consequence of behaving optimally given the information available to them (e.g. Lucas 1972, Sargent and Wallace 1975, Barro 1976, Sargent and Wallace 1976); if the authorities did have more information about the shocks affecting the economy than did private agents, the authorities could just as well make use of this information by disseminating it to the public as by adjusting the money supply to offset the shocks (Barro 1976). If the authorities have the same information as the public, "discretionary" policy can be portrayed as random policy (Lucas 1972) since it means departing from any well-defined policy rule incorporating feedback from observable economic variables; put in this way, it can easily be shown to be unambiguously detrimental. The next

As mentioned earlier, this does not seem to capture what is usually meant by discretion—as reflected, for example, in Samuelson's "look at everything, react to everything" dictum. In this light, discretion may alternatively be taken as meaning that policy is formulated taking account of a varying body of information depending on what information becomes available, and that policy reacts to this information in a way that cannot be pre-specified, since it will vary over time.
step, after having rejected discretionary policy, is then to consider whether or not pre-specified policy rules should incorporate feedback from past economic disturbances. It can be shown that, since past disturbances can easily be incorporated into private decision-making, any policy action designed to offset the effects of these disturbances will be nugatory (e.g. Sargent and Wallace 1976, Barro 1976). Thus, deviating from a fixed rule is, at best, futile; worse, it may actually destabilize the economy if it incorporates random discretionary elements; it may also be wasteful if there are administrative costs associated with rules with feedback.

This argument, known as the "policy-ineffectiveness proposition", would, if accepted, lead to a striking change in how the effects of policy are viewed. It implies that only random policy has any effect on the economy, either for good or ill. No systematic policy can either cause the unemployment rate to differ from its natural rate or speed its return to the natural rate; random or unanticipated policy, on the other hand, while it is held to affect the level of output (e.g. Barro 1977), has effects which are unambiguously detrimental to the stability of the economy.

As sweeping as these policy conclusions are, it is hardly surprising that they have not been without critics. Criticisms have focused on either of the two features of the models from which the policy-ineffectiveness proposition is derived. Some criticism has focused on the

as more knowledge about the structure of the economy becomes available.
informational assumptions implied by the assumption of rational expectations: it has been argued that it is costly for private agents to process the required information for their own purposes, and that it may be the case that information on macroeconomic shocks can best be offset by the actions of the monetary authorities—especially if the shock under consideration is a money-demand disturbance, whose effects can easily be dealt with by altering money supply but whose effects on individual markets for goods are uncertain (Howitt 1978). It has been pointed out that it may be rational for individual agents to form their expectations without incorporating the information that is embodied in the money stock, since incorporating the latter provides little incremental improvement in forecasting (Feige and Pearce 1976): if agents ignore the behavior of the money supply in forming their expectations of the price level, as it may be rational for them to do if information-gathering and -processing is costly, it will take time for a change in the time path of the money supply to be incorporated in price-level expectations, and this change in monetary policy will have some real effects for a time. Thus, there could be scope for monetary policy to have real effects during the period of "transition to rational expectations", during which time the authorities and the public have different views of the policy regime that is in effect (see also Taylor 1977). It has been argued, then, that the assumption that expectations are rational is not necessarily an implication of economic rationality if information is costly, or if the authorities have access to more information about their own policies than does the public.
The case in which the authorities have more information about the policy regime than the public does, however, implies only that policy can have some real effects, not that it can have beneficial ones. In fact, if the level of real output which would prevail under full information is optimal⁴³, it is clearly undesirable for the authorities to carry out policies which depend on the public's being unaware of these policies; such policies can only increase the variance of output around its full-information level (Barro 1976). Nonetheless, it may be argued that in general, agents are learning about the structure of the economy as well as about the nature of policy, so that it may not be fruitful to model their behavior as though they could form expectations in such a way that any errors in these expectations is orthogonal to all information available to them. However, there simply does not seem to be any better way of modelling expectations: a model incorporating rational expectations will be in error to the extent that agents do not know the structure of the economy, but so will one which incorporates an ad hoc expectations-formation mechanism; the latter type of model, however, has the drawback that it suggests a scope for influencing real variables by exploiting the expectations-formation mechanism, scope which can surely only exist as long as it is not actually exploited.

⁴³ This is not an empty restriction: if there are distortions in the economy, there is nothing particularly desirable about keeping unemployment at its natural rate. However, it might well be argued that it would be better to deal with such distortions directly, rather than by carrying out macroeconomic policies designed to keep unemployment above or below its (distorted) natural rate.
Other criticisms and modifications of the policy-ineffectiveness proposition have focused on the natural rate hypothesis. It is possible that anticipated changes in the money supply may have real effects on the economy; in that case the policy-ineffectiveness proposition would not hold, even if expectations are formed rationally. Attention has been drawn to barriers to the immediate adjustment of prices, in particular the existence of contracts: it can easily be shown (Fischer 1977; Taylor 1979) that if there are contracts which set wages or prices in advance, there is some scope for activist monetary policy to offset the effects of economic shocks which become known only after some contracts have been made. Similarly, if it is costly for private agents to adjust prices, anticipated changes in the money supply will have some real effects in the short run, so systematic monetary policy will not be irrelevant (Parkin 1980). In a sense, contracts only restore some respectability to the assumption that the authorities have more knowledge than the public: with contracts, the authorities can carry out stabilization policy based on information which is not available to private agents at the time they set their wages or prices. Their existence thus suggests that there may be some scope for activist policy in the short run.

Despite any reservations about the policy-ineffectiveness proposition, one may still attach considerable weight to some criticisms of the orthodox theory of economic policy which have arisen from the rational expectations literature. Perhaps the most important contribution in this area is Robert Lucas' critique of econometric policy evaluation
(1976). Lucas criticizes the tradition of policy prescription which sees the authorities as optimizing some welfare-function subject to the structure of the economy, where this structure is given. Lucas points out that the structure of the economy as measured by an econometrician is actually the result of the decision-rules being followed by private agents; these rules of thumb enable agents to make intertemporal choices reasonably well given the type of uncertainty to which they are subject; a change in the pattern of the things affecting private agents, such as would result from a change in the policy regime being implemented by the authorities, will lead these decision-rules to alter. Thus the structure of the system is not invariant with respect to a change in policy regime; as a result, either theoretical or quantitative analysis which attempts to simulate the effects of a change in policy will not be valid; attempts to find the optimal policy given the structure of the system are also invalid. Furthermore, in the short run the impact of policy is unpredictable, since one cannot model agents' behavior while they are learning about the patterns of the random variables that affect them. Thus, the best that the authorities can do is to follow simple, pre-announced policy rules, which at least give private agents a firm basis on which to form their expectations.

Another critique of orthodox policy analysis is that made by Kydland and Prescott (1977). These authors argue that optimal policy is time-inconsistent, and thus reject the idea of using optimal control to find the appropriate policy. When agents' behavior depends on their anticipations of future policy, the optimal policy for the current per-
iod is generally different from the one which was previously planned: this is because agents' current behavior depends on their anticipations of future policy but it is only their past behavior--now beyond alteration--which depended on current policy in this way. This time inconsistency problem arises in the case of monetary policy as well as in other cases: it may well be optimal for the monetary authorities to expand the money supply at a rapid rate in the first period while expanding it at a much slower rate in subsequent periods: the reason for this is that it is only the future rates of monetary expansion which affect the expected rate of inflation**. This will be the authorities' optimal plan from this period onward, and thus it would be optimal (from this period onward) for them actually to follow that plan; however, next period they will be faced with the same control problem, and the solution will be similar; it will imply that in that period, once again, the optimal policy is one which ignores "bygones"--i.e. which ignores the effect of that period's policy on the expectations which were formed previously. Thus, where agents' expectations are influenced by future actions, the principle of optimal control--the principle that an optimal plan formed at any time is expected** to be, at any point in the future,

** This argument can be cast in the context of a model in which the authorities are attempting to influence the level of output by creating unanticipated increases in the money supply (Kydland and Prescott 1977); alternatively, it can be cast in a model in which they are attempting to achieve the optimum quantity of money (i.e. to induce the public to hold real cash balances which are such that the marginal benefit that an individual receives from holding cash balances is equal to the marginal social cost of creating cash balances--zero, that is), while at the same time they care about the revenues from money creation (Calvo 1976).
the same as the plan which is optimal from that point onward—is violated (Prescott 1978). Needless to say, a regime under which the authorities always plan and announce a policy which they do not subsequently follow is one under which their plans and announcements would not be heeded. The alternatives are two: one is that in which the public expects the authorities to behave as they actually do, and in which the authorities plan their policies without consideration of the effects of their future policies on agents' current expectations. The other alternative is for the authorities to attempt to follow the time-inconsistent policy: they could achieve a higher payoff if they were somehow able to commit themselves to adhering to the policy which they had planned, regardless of the fact that it was not optimal in subsequent periods. This is how Kydland and Prescott characterize the idea of following a rule: the authorities attempt to bind themselves to following the plan which is optimal overall even though it is not optimal from any particular period onward: this requires that new information be created, in the sense that taking account of one's past commitments in choosing one's current actions is not deducible directly from dynamic optimization subject to one's preferences and constraints (Rowe 1980). Whether it is possible for the monetary authorities to make this kind of commitment, of course, is another question. However, the argument

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5 It is important to note that the time-inconsistency problem has nothing to do with the possibility that new information may become available: the "plans" to which the literature refers should be understood to be state-contingent plans.

6 It is clear that the possibility of pre-commitment to a time-inconsistent plan is not an irrelevant one: there are many time-inconsistency
certainly does cast doubt on the traditional view of macroeconomic policy as a dynamic-stochastic optimization problem: it suggests that it may better be seen in game-theoretic terms.

To some extent, this rational-expectations approach to policy implies that rules should be followed in conducting monetary policy. In another sense, though, it implies that rules generally are followed—that policy actions are a function of the state of the economy, a function of which private agents are to some extent aware and which they incorporate in forming their expectations of the variables that affect them. This implies a different approach to describing policy: as one of the relationships in the economic system rather than as a deus ex machina operating on the system from outside. Thus, in effect, the rational expectations treatment of rules is more than a prescription for a particular set of policy measures: it is a wholesale assault on the theory of economic policy that sees the authorities as maximizing an objective-function subject to the "laws of motion" of the economic system. In this context, an important characteristic of any rule is the information it creates and conveys to private economic agents: simple rules, such as increasing the money supply at a constant rate, have the problems, both in individual and in public affairs; these time-inconsistency problems at the individual level (e.g. whether to perform one's half of a bargain once the other party has performed his half) are generally dealt with either by involving a third party (invoking legal sanctions) or by establishing and valuing a reputation; at the public level, of course, there is no "third party", so that valuing one's reputation is the most important way in which time-inconsistent plans (e.g. punishing crimes which have already been committed, or responding to the seizure of one's territory by entering a war which is costlier than the loss of territory) can be followed (see Rowe 1980).
advantage of providing agents with simple and clear information on which to base their expectations.

Thus there is more than one way of rationalizing the adoption of a rule for a steady rate of monetary expansion. The argument for rules in terms of rational expectations is quite a strong one, if one accepts both the natural rate hypothesis and the proposition that private agents are not likely to be consistently mistaken in forming expectations about variables that are important to them. Even if one does not accept both rational expectations and the natural rate hypothesis, the Lucas critique and the time inconsistency problem carry some weight, provided that one is not prepared to assume that expectations are myopic or that they are formed by some unchanging mechanism: both of these arguments suggest that there is something fundamentally unsound about the conventional view of policy as an optimization problem.

The older rationale for monetary rules is based on the difficulty of carrying out stabilization policy under uncertainty; this line of argument seems to be weaker as an argument for rules, since it is really mainly an argument for circumspection—carefully formulated and consistent policy as opposed to impulsive or ad hoc policy—rather than for rules. It only becomes an argument for rules if one supplements it either with an argument that there are large fixed costs of operating an adjustable monetary policy, or with a political argument that it is unlikely that central bankers can be relied upon to show the necessary degree of self-restraint in conducting stabilization policy under uncertainty. Of these two arguments, the political argument seems to carry
the more weight; however, as has been mentioned, the political argument can stand on its own (provided that one endorses the associated political principles), and this argument was in fact made before the literature on the pitfalls of stabilization policy under uncertainty was developed.

IV

Thus, there appear to be two quite different lines of argument, each of which might provide a rationale for the adoption of targets for the money stock. Depending on which of these arguments one considers, the nature of the targets will be quite different. One argument runs through the literature on the targets and instruments of monetary policy: it suggests that the money supply is of concern to policy-makers as an information variable—as a variable which provides information on the level of national income which is implied by current policy actions. However, the conclusion which emerges from that literature is that it is suboptimal for the authorities to use the money supply as an intermediate target (that is, as a surrogate objective of policy): attempting to adhere strictly to a preannounced path of the money supply would mean not only making inappropriate use of the information which is incorporated in the money supply, but also discarding the information which could be provided by other variables. The optimal way to carry out policy, then, is not to control the money supply, but rather to control a combination of the money supply and other information variables. In this case, it is hard to make sense of the announcement of money-supply targets, except as an exhortation to the public to pay attention to
the behavior of the money stock in forming their expectations and as an announcement that the preferences and beliefs of the authorities have changed in such a way that the optimal combination policy now places a greater weight on the money supply than it previously did. In this case, there is clearly no reason that the authorities should be concerned if the money stock departs from its target path in the short run, or even permanently: this may well occur in the course of carrying out the optimal combination policy.

The other explanation for focusing monetary policy-making on the money supply is that it is connected with the adoption of a monetary rule. Two cases for a monetary rule have been developed: one case essentially argues that it is very difficult for activist policy to exert a stabilizing influence on the economy, and that, unless it is carried out very cautiously, activist policy may well actually exacerbate economic fluctuations. However, this line of argument, as has been shown, does not really build a case for a monetary rule so much as prescribe caution and circumspection in the conduct of policy. Money-supply targets could presumably be seen as indicating circumspection in policy, in the sense that the authorities refrain from bringing about excessively large movements of the money stock from a stable growth path under most circumstances. However, in this case, the money-supply targets would once again be a "fifth wheel" in the formulation of policy: one could define policy completely in terms of the authorities' preferences and their beliefs about the structure of the economy; the money-supply targets would then have no additional role in shaping policy.
Here again, the only significance of the targets would be in communicating the nature of policy to the (credulous?) public:

No other arguments for monetary rules suggest that targets for the money supply should play an important role in themselves—that they should constitute a real constraint on the actions of the authorities. One of these arguments is a political one: according to this argument, discretionary power of any kind is liable to be abused, the power to control the money supply being no exception; this power should therefore be limited by a monetary constitution, an important part of which would be a rule for the growth rate of the money supply. The second argument emerges from the rational-expectations literature: it has been argued that monetary policy affects real output only to the extent that it is unsystematic and thus unanticipated. Furthermore, unanticipated policy, while it does affect output, will, on balance, affect it in a detrimental way: it will increase the variance of output. In addition, the Lucas critique and the time-inconsistency problem suggest that finding optimal policy may not just be very difficult, it may not be a well-defined optimization problem. Thus, it has been argued, it is preferable for the authorities to bind themselves to a monetary rule rather than attempting to carry out an optimal policy which is ill-defined.

Either of these latter two approaches to rationalizing a money-supply rule suggest that the orthodox theory of policy analysis, which views policy as the solution to a dynamic stochastic optimization problem, is fundamentally unsound. Accordingly, both view the introduction of a monetary rule as a substitute for discretion, rather than merely as
an announcement of the particular discretionary policies which are being implemented. Thus, from either of these two perspectives, money-stock targets should constitute a real constraint on the monetary authorities; the authorities should not have the right to abandon these targets, either temporarily or permanently, in pursuit of other objectives.

According to the political argument, allowing such scope for departing from the targets would defeat the purpose which the targets are supposed to serve, namely to keep the government from tampering with the money supply. From the rational expectations perspective, too, departures from a rule are supposed to be serious: first, such departures constitute "unanticipated money", which generally has a destabilizing influence (Barro 1977). Second, they can be taken as an indication that the authorities are not really following a rule, and will generally be interpreted as such by the public; they therefore obviate the possibility that, in following a monetary rule, the authorities can achieve a higher value of their criterion-function by following a time-inconsistent policy.

Thus, money-supply targets can be rationalized on different grounds, and depending on which line of reasoning one follows, short-run deviation of the money stock from its target path will have very different implications. This may cast some light on the recent debates over the techniques of implementing monetary control: those who favor controlling the money-supply on rational-expectations or on political grounds would obviously be concerned about short-run departures of the money stock from its target range; those who view the targets as mainly
an announcement that more weight is being placed on the money stock, or that more cautious and circumspect policy is being followed, would not.

The next question is, "What do the monetary authorities think they are doing?" There certainly seems to be no indication that, in setting targets for the money supply, they see themselves as adopting a monetary rule. It seems unlikely that the strong arguments for a monetary rule have been endorsed by central bankers; as for the political arguments, it seems unlikely that the monetary authorities should see the need for constitutional constraints on their own actions; it is also noteworthy that in several cases the money-supply targets have been self-imposed. As for the rational-expectations arguments, there is far from whole-hearted acceptance of the rational-expectations literature in central banks; certainly, the rational-expectations critique of activist policy is not easy for central bankers to accept, since it implies that their policies have no real effects except for detrimental ones.

As one might expect, there seems to be no notion, in the adoption of monetary targets, that the authorities consider themselves bound to a course of action which they would not otherwise choose to follow. In many cases, targets are set for only a short period in advance; when they are set for longer periods (over a year) the targets are considered to be subject to revision as is deemed appropriate. The frequency with which targets can be altered is such that it has been suspected in some

\[\text{In some cases, the requirement that money-supply targets be announced has been imposed externally (e.g. in the U.S. case, where it was imposed by Congress: see Weintraub 1978), but the targets themselves can be chosen by the monetary authorities.}\]
cases that the targets are really just forecasts of the money stock. In some cases, too, the time horizon over which the targets are to be met has been deliberately left vague; in addition, in several countries the practice followed is that, when targets are not met, the level of the money stock that actually occurs is used as the base from which subsequent growth rates are calculated—so that ‘misses’ mean permanent shifts in the planned time path of the money stock (Sumner 1980). Also, needless to say, the authorities incur no penalties when the targets are not met.

There is not really an attempt to control the money stock very precisely: central bank economists argue that deviations of the money stock from its target path are not a problem provided that these deviations are corrected within a few quarters (Duguay and Jenkins 1978, Pierce and Thompson 1972, Anderson and Karnosky 1972); more concern is expressed over the supposed instability of interest rates that would result if the money stock were kept on target in the short run (White 1976, p. 99). Thus, despite the eagerness of some monetarists to identify the central banks’ adoption of money stock targets with monetarism, the targets do not seem to correspond to rules as they have usually been envisaged.

Those who would like to see the adoption of money-supply targets as an incipient monetary rule may interpret departures of the money stock from its target path as evidence that faulty techniques are being used to control the money supply. If, however, the purpose of targets is to announce policy rather than to mould it, the resulting criticism—unless it is actually an attempt to alter the role which the targets play—will
be ineffective. If the goal of the authorities is, not to minimize the probability that the money supply will depart from the target range, but to influence the level of economic activity and the inflation rate, their actions will naturally be different from those which are desirable under a monetary rule. The authorities' actions may still be inappropriate for attaining their goals; their goals may likewise be misguided or unattainable. However, criticism which assumes that their goals are other than they actually are is likely to be misdirected.
CHAPTER III
INTEREST-RATE INSTABILITY AND SHORT-TERM CONTROL OF THE MONEY SUPPLY

How would interest rates behave if the money supply were controlled rigidly in accordance with a constant-growth-rate rule? Static analysis indicates that—by definition—interest rates would fluctuate more if the money supply were controlled than if the interest rate were controlled; whether output would fluctuate more or less, though, is left as an empirical question. Some simple dynamic analysis suggests a concern of a different order of importance: it has been argued that the lag structure of interest rates in the demand-for-money function is such that, if the money supply were controlled continuously according to a constant-growth-rate rule, interest rates would follow an explosive path—that is, they would have an infinite asymptotic variance. If this were true, rigid adherence to such a rule would clearly be inadmissible: it would be necessary, even in principle, for the authorities to smooth interest rates,** to some extent, in the short run.

**This proposition is worked out formally in Sims 1974, for the general case in which using the instrument to minimize the variance of the target would lead the instrument to follow an unstable path. Sims' solution is for the authorities to minimize a loss function in which variances of both the target and the instrument are given some weight.
The analysis leading to this conclusion is based on the idea of "instrument instability". In general, instrument instability is a problem which arises when a policy instrument is manipulated to control a target variable whose value depends on both current and lagged values of the instrument. In this case, the appropriate setting of the instrument at any point in time depends on that instrument's own previous values; the instrument thus follows a difference (or differential) equation which may or may not be stable (Holbrook 1972).

A commonly-used method of controlling the money stock is viewed as consisting of "picking a point on a money-demand curve"—i.e. using an interest rate as a policy instrument, and manipulating it in such a way that predicted money demand equals the target level of the money stock (Parkin 1978). The instrument instability problem may then arise if both current and lagged interest rates affect the demand for money.

Suppose, for instance, that the money-demand equation is

\[ m_d^t = \alpha + \gamma y^p_t + \delta r_t + \sum_{k=1}^{k} w_k r_{t-k} + p_t + \varepsilon_t \]  \hspace{1cm} (3.1)

where \( m_d^t \), \( p_t \) and \( y^p_t \) are the logarithms of money demand, the price level and permanent income respectively, where \( r_t \) is the interest rate, where \( \alpha, \gamma, \delta \) and \( w_k \) are coefficients and \( \varepsilon_t \) is a random disturbance term.

In order to achieve a target money stock \( m^* \), the authorities then set
\[
\begin{align*}
\tau_t - \frac{1}{\delta_t} \sum_{k=1}^{\infty} \omega_k \tau_{t-k} &= \frac{1}{\xi_t} \left[ \alpha + \gamma y_t^P + p_t + \varepsilon_t - \mu_t \right] \quad (3.2)
\end{align*}
\]

The assumption which is then implicitly made is that the monetary control problem is set in the Keynesian "very short run", in which the money market is in equilibrium but the goods' market is not yet affected by the interest rate: \( y^P \) and \( p \) are thus treated as predetermined, and equation (3.2) is examined as a difference equation in \( \tau_t \). Empirical evidence on the demand for money, generally based on monthly data, has been adduced to show that the lag structure \( \omega_1, \omega_2, \ldots \) is such that the roots of equation (3.2) do not all lie in the unit circle: on this basis, it has been argued that controlling the money supply too rigidly would set interest rates on an explosive path (see Pierce and Thompson 1972, Ciccolo 1974, White 1976).

In this essay, this contention will be examined in the light of alternative explanations of the lag structure\(^{\text{a9}}\) of interest rates in the demand-for-money function. In many empirical estimates of the demand for money, lagged interest rates play some role. This role may take the form of a lagged dependent variable, which in a simple money-demand equation is equivalent to including lagged income and interest rates in the function (subject to the restriction that the coefficients be pro-

\(^{\text{a9}}\) Another consideration is that econometric estimates of the lag structure may give misleading impressions of the possibility of instrument instability; this may occur especially if the empirical evidence consists of a discrete-time approximation of continuous lag distribution and thus (if the lag structure is one-sided) underestimates the weight on the current period (Sims 1974a).
portional to those on current income and interest-rates). Alternatively, distributed lags on income and interest rates may be estimated separately (Laidler 1977). In either case, the reason that lagged interest rates should appear to affect money demand is something of a puzzle. Explanations generally run along either of two lines: it is sometimes held that past interest rates influence expected rates, and that these affect the demand for money (e.g. White 1976). Alternatively, it is sometimes claimed that what is being observed is some kind of partial adjustment behavior, which leads cash balances to depend on their own past levels (e.g. Darby 1972, Santomero and Seater 1981).

In this essay, two such explanations are considered: in the second part of the essay, the explanation based on interest-rate expectations will be considered, and its implications traced. In the third part, an explanation based on the existence of costs of adjusting cash balances is presented. In each case, the implications for the instrument instability argument are traced.

II

One explanation for the appearance of lagged interest rates in the demand-for-money function is that they influence expected interest rates. Why should the expected rate of interest influence the demand for money? The simplest explanation is that the interest rate which is used as a measure of the opportunity cost of holding money is an asset whose term to maturity is longer than the holding period over which individuals' decisions to hold cash are made. If this is the case, the expected holding period yield on the alternative asset—i.e. the oppor-
tunity cost of holding money--differs from the yield to maturity by some element of capital gain or loss. For example, if cash balances are adjusted continuously, the opportunity cost of holding money is the instantaneous yield on the alternative asset; if this asset is a bill whose term to maturity is $T$ months, this opportunity cost is

$$R^h_t = r_{T,t} - T \frac{dr_{T,t}}{dt}$$

(3.3)

where $r_{T,t}$ is the $T$-month rate of interest. Using the mean-value theorem, this can be written as

$$R^h_t = r_t + T(r_{t+1} - r_t) + T \left( \frac{dr(t)}{dt} - \frac{dr(t)}{dt} \right)$$

$$= r_{T,t} - T(r_{t+1} - r_t)$$

(3.4)

for some $\psi$, where $0 < \psi < 1$. In short, if the yield on the asset alternative to money is expected to change, the opportunity cost of holding money depends not only on the current yield, but also on the yield expected in the future. 50

Accordingly, one formulation of the demand for money function might be

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50 The expression derived is an approximation rather than an exact relationship in discrete time because (3.3) compounds a movement along a yield curve with a movement through time, while the expected change in the interest rate consists only of the latter.
\[ m_t = a + \gamma y_t - \delta r_t + \kappa r_{t+1} + p_t + \epsilon_t, \]  

(3.5)

where \( r_{t+1} \) is the interest rate now expected to prevail next period. When the expected interest rate is given as a distributed-lag on current and past interest rates

\[ r_{t+1} = \sum_{k=0}^{\infty} \omega_k r_{t-k} \]  

(3.6)

this would give a formulation of the demand for money equivalent to equation (3.1), where \( \delta' = \delta - \kappa \omega_0 \) and \( \omega_k = \kappa \omega_k, k = 1, \ldots \). The approximation given in equation (3.4) suggests that

\[ \kappa = (T+1/T)\delta \]  

(3.4')

so that in general \( \kappa < \delta \).

One might then ask what determines the lag structure given by equation (3.6). Such a lag structure is often explained in terms of some ad hoc regressive-cum-extrapolative expectations mechanism (e.g. White 1976). Such a mechanism can, of course, be criticized from a rational-expectations perspective: under rational expectations, expectations would only be formed as in (3.6) if (3.6) also described the actual behavior of the interest rate (up to a white-noise disturbance term). This, of course, puts restrictions on the lag structure, and also implies that the lag structure would depend on the policy regime that is
in effect—and in particular on whether the authorities are attempting to smooth interest rates.

In this section of the essay, the implications of rationally-formed expectations for the possibility of instrument instability will be considered in a "descriptive" rational-expectations framework. Using descriptive models—rather than attempting to set up a complete model of optimizing behavior under rational expectations—clearly limits the purposes to which the results can be used. Since the purpose of this analysis is essentially critical—to assess the validity of the instrument-instability argument when the lag structure is endogenous rather than, for example, to compare alternative policies—the use of descriptive models seems to be justified. Because we are here considering an explanation of the lag structure which is based on the effect of expected interest rates on money demand, it is a straightforward step to make the formation of expectations endogenous and rational.

For the sake of argument, the implications of rational expectations can first be considered in the context in which instrument instability is usually discussed: the very short run in which both prices and output are treated as predetermined. Of course, this case is broadly inconsistent with rational expectations; however, it is useful to examine it initially as a "strong case", in order to show that any rejection of the instrument instability argument does not hinge on the assumption of perfect price flexibility and instantaneous market clearing. Furthermore, it might be argued that this case is not a bad caricature of much neo-Keynesian analysis, according to which financial markets use information
efficiently but behavior in other markets is myopic and/or constrained by contracts and other institutions.

Even in this case, there does not seem to be much justification for policy intervention to prevent explosive interest-rate movements: on the contrary, such intervention may actually give rise to such movements.

If the authorities controlled the money supply rigidly at \( m^* \), and if the money demand function were given by (3.5), the path of the interest rate would be given by

\[
\frac{r_t}{\delta} = \frac{\kappa}{\delta} r_{t+1} = \frac{1}{\delta} (a + y_{t+1}^P + p_t + \varepsilon_t - m_{t+1}^*) \quad (3.7)
\]

Since \( \kappa/\delta < 1 \), the solution for the interest rate would be

\[
r_t = \sum_{i=0}^{\infty} \left( \frac{\kappa}{\delta} \right)^i (a + y_{t+i}^P + p_{t+i} - m_{t+i}^*) + \frac{1}{\delta} \varepsilon_t \quad (3.8)
\]

Thus, in this case there seems to be no reason to suppose that the interest rate would follow an unstable path if the money stock were controlled: equation (3.8) implies that if income, prices and the target money stock were constant, any fluctuations of the interest rate would be white noise.

What would happen, in this case, if the authorities attempted to smooth interest rates? One case that could be considered—and which corresponds to what the authorities frequently claim they are doing (e.g. White 1978)—is that in which, instead of adhering rigidly to a
monetary rule, the authorities manipulate the interest rate according to an interest-rate rule with feedback from deviations of the money stock from its target level. A simple representation of such a rule would be

$$r_t = r_{t-1} + \theta(m^*_t - m_t^*)$$  \hspace{1cm} (3.9)

If this rule were followed, and if the money demand function were as described by (3.5), then the interest rate would follow

$$r_t = \frac{(1 - \theta \delta)}{(1 - \theta \kappa)} r_{t-1} = \frac{\theta}{(1 - \theta \kappa)} \left( m_t^* - 2 \gamma_{t-1}^P - p_{t-1} \right)$$  \hspace{1cm} (3.10)

Four possible results emerge. If the authorities smooth the interest rate very strongly (if, that is, $\theta < 1/\delta$), the interest rate follows an explosive path from a given initial level, unless the initial level is that given in (3.8). If the interest rate is adjusted rather quickly ($\theta > 1/\kappa$), it converges to the level given by (3.8). In the intermediate case ($1/\kappa > \lambda > 1/\delta$) there are oscillations which may be either convergent or explosive (see Appendix). Of course, such instability would never be observed if the authorities started smoothing interest rates with the appropriate initial interest rate; if, however, they decided at some moment to begin following the policy rule given by (3.9) or to change the parameter $\theta$, taking the previous period's interest rate as given, this might lead to ever-widening interest-rate movements. The case in which the authorities start their smoothing with the long-run equili-
brium interest-rate has no bearing on the instrument-instability argument, since that argument pertains to the consequences of starting with an initial interest rate which may not be the long-run equilibrium rate.

How would following such a rule affect the time path of the money stock? In a simplified case in which permanent income, the price level and the target money stock are constant, the money stock follows a path given by

$$m_t = m^* + \frac{1}{\theta} (c + \gamma_p + p - r_0 \lambda (1 - \lambda) - m^*) \lambda^t + \sum_{i=0}^{t-1} \lambda^i \epsilon_{t-i}$$  \hspace{2cm} (3.11)

Thus, although a feedback rule such as (3.9) is an intuitively plausible interpretation of what the policy-makers often claim that they are doing—that is, resisting interest-rate fluctuations in the short run while gradually adjusting the interest rate to offset departures of the money stock from its target path, with a view to adhering to monetary targets in the long run—such a policy may, in this model, give rise to explosive movements of both the interest rate and the money stock. This occurs even in a model chosen as the most favorable to the case for interest-rate smoothing, a model in which departures of the money stock from its target do not cause income and the price level to fluctuate.

Thus, attempting to smooth interest rates may have effects which are the opposite of those intended, under precisely the circumstances under which it has been argued that such smoothing would be desirable: those in which expected interest rates affect the demand for money.
What is the reason for this result? If the authorities adjust the interest rate gradually in response to money-stock deviations, starting from a historically-given initial rate, this means that future movements of interest rates can be anticipated; expectations of interest-rate movements in turn influence the demand for money, and the resulting process is unstable.

The next step is to drop the (rather contrived) assumption about the fixity of the price level and of output. This could be done in a simple ad hoc rational expectations model like the following:

\[ m_t^d = \alpha + \gamma y_t^p - \delta r_t + \kappa r_{t+1} + p_t + \xi_t \]  

(3.5)

\[ y_t^d = y_t + \xi(p_t^c - p_t^c - \rho, p_t - r_t) + \nu_t \]  

(3.12)

\[ y_t^s = y_t + \mu(p_t^c - p_t^c - 1) + \eta_t \]  

(3.13)

\[ y_t^d = y_t^d \]  

(3.14)

Here (3.5) is the money demand function reproduced from page 95, (3.12) is an aggregate demand function expressing aggregate demand in terms of the deviation of the real interest rate from its natural rate, (3.13) is a Lucas "surprise" aggregate supply function (Lucas 1973) and (3.14) is the condition that aggregate supply and demand be equal. The trend level of output is \( \bar{y} = \bar{y}^p \). For simplicity, it is assumed that trend
output is constant, that monetary targets specify a constant level of the money stock $m^*$, and that the disturbance terms $\epsilon_t, \nu_t$, and $\eta_t$ are mutually and serially uncorrelated.

If the authorities do not attempt to smooth interest rates, but simply attempt to set $m_t = m^*$ for all $t$, the result is a first-order system, which can be written

$$\kappa P_{t+2}^* = (\kappa+\delta)p_{t+1}^* + (1+\delta)p_t^* + (\delta \mu / \xi + \delta \mu)(P_t^* - P_t^{*-1})$$

$$= (m^* - \alpha - \delta y_t^* + (\delta - \kappa)\rho) - \delta \left( \eta_t^* + \nu_t^* \right) - \epsilon_t^*$$  \hspace{1cm} (3.15)

The solution to this system is

$$p_t^* = m^* - \alpha - \delta y^* + (\delta - \kappa)\rho$$

This implies that in the absence of interest-rate smoothing, the price level fluctuates randomly around its steady-state level; the interest-rate will accordingly fluctuate randomly around the natural rate $\rho$: 

$$p_t = m^* - \alpha - \delta y + (\delta - \kappa)\rho$$

$$- \frac{1}{1 + \delta + \delta \mu / \xi + \delta \mu} \left[ \frac{\delta}{\xi} \left( \eta_t + \nu_t \right) - \epsilon_t \right]$$
\[ r_t = \rho \]

\[
+ \left( 1 + \frac{1}{\xi} \right) \frac{1}{1 + \delta + \delta \mu \xi + \mu \xi} \left( \frac{\delta}{\xi} \right) \left[ (\eta_t + \nu_t) - \varepsilon_t \right] + \frac{1}{\xi} (\eta_t + \nu_t)
\]

Thus, in this simple rational expectations model, there is no indication that the interest rate would follow an unstable path if the money stock were controlled according to a constant-growth-rate rule. Under this regime, output is given by

\[ y_t = y - \frac{\mu}{1 + \delta + \delta \mu \xi + \mu \xi} \left[ \frac{\delta}{\xi} \left( \eta_t + \nu_t \right) - \varepsilon_t \right] + \eta_t \]

that is, it will deviate from the natural level as a result of shocks to aggregate supply, and demand and to money demand.

What would happen in this model if the authorities attempted to smooth interest rates? In this case, the system would consist of equations (3.5), (3.12), (3.13), (3.14) and (3.9). Solving these simultaneously gives a difference equation for the price level which can be written
(1 - θκ)p_{t+1}^{t-1} - [2 - θ(κ + δ)]p_t^{t-1} + (1 - θδ - θ)p_{t-1}^{t-1}

+ (p_t^t - p_{t+1}^{t-1}) - (1 + \eta_\xi)(p_t - p_t^{t-1}) + \frac{1 - θδ}{\xi}(p_t - p_t^{t-1})

= θ(α + θy - (δ - κ)p - m*) + θε_{t-1}

+ \frac{1}{\xi}(\eta_t - v_t) - \frac{1 - θδ}{\xi}(\eta_{t-1} - v_{t-1})

Taking expectations as of time t-2, we have

(1 - θκ)p_{t+1}^{t-2} - [2 - θ(κ + δ)]p_t^{t-2} + (1 - θδ - θ)p_{t-1}^{t-2}

= θ(α + θy - (δ - κ)p - m*)^θ

(3.20)

a second-order equation for the time path of the expected interest rate. Its solution can be written as

\[ p_{t+k}^t = m^* - a - \frac{\bar{y}_t}{\eta_t} + (δ - κ)p + A_1\lambda_1^{t+k} + A_2\lambda_2^{t+k} \]  

(3.21)

where \( \lambda_1 \) and \( \lambda_2 \) are characteristic roots and \( A_1 \) and \( A_2 \) are constants.

Subtracting equation (3.20) from equation (3.19) gives
\( (1 - \theta \kappa)(p_{t+1} - p_{t+1}) - [2 - \theta(\kappa + \delta)](p_{t} - p_{t-1}) \)

\[ + [1 - \theta \delta - \theta + \frac{(1 - \theta \delta \mu)}{\xi}](p_{t-1} - p_{t-1}) \]

\[ + (p_{t+1} - p_{t+1}) - (1 + \frac{\mu}{\xi})(p_{t} - p_{t-1}) \]

\[ = \theta \varepsilon_{t-1} + \frac{1}{\xi} (\eta_{t} - \nu_{t}) - \frac{(1 - \theta \delta)}{\xi} (\eta_{t-1} - \nu_{t-1}) \]

Taking expectations as of time \( t-1 \) in equation (3.22) yields

\( (1 - \theta \kappa)(p_{t+1} - p_{t+1}) - [2 - \theta(\kappa + \delta)](p_{t} - p_{t-2}) \)

\[ + [1 - \theta \delta - \theta + \frac{(1 - \theta \delta \mu)}{\xi}](p_{t-1} - p_{t-2}) = \theta \varepsilon_{t-1} + \frac{1}{\xi} (\eta_{t} - \nu_{t}) \]

which can be subtracted from (3.22) to yield

\[ (p_{t+1} - p_{t+1}) - (1 + \frac{\mu}{\xi})(p_{t} - p_{t-1}) = \frac{1}{\xi} (\eta_{t} - \nu_{t}) \] (3.24)

\[ ^{51} \text{Equation (3.25) describes the time path of the price level if } \lambda_1 < 1; \]

if \( \lambda_1 > 1 \), then \( A_1 = 0 \) and \( \phi_1 = 0 \); in addition, the expression for \( \phi_2 \) will be different from the one given here.
Finally, using equations (3.21) to (3.24), the following equation for the time path of the price level can be found:

\[ p_t = m^* + \gamma y + (\delta - \kappa) \rho + \lambda_1 \lambda_{1t} + \lambda_2 \lambda_{2t} \]

\[ + \frac{t-1}{\xi} (\phi^1 \theta_t - 1) u_j + \frac{\xi}{\xi + \mu} (\lambda_1 \phi^1 + \lambda_2 \phi^2) u_t - \frac{1}{\xi + \mu} (\eta_t - \nu_t) \]

\[ u_t = \frac{1}{1 - \theta \kappa} \left[ \theta \varepsilon_t + \left( \frac{1}{\xi + \mu} - \frac{1 - \theta \delta}{\xi} \right) (\eta_t - \nu_t) \right] \]

\[ \phi = \left( \lambda_m \left( 1 + \frac{\theta \xi}{(1 - \theta \kappa)(\xi + \mu)} \right) \right)^{-1} \]

\[ - \lambda_n \left( 1 + \frac{\theta \xi}{(1 - \theta \kappa)(\xi + \mu)} \right) \frac{[(\xi + \mu)(1 - \theta \delta) - \theta \xi] \lambda_m - (1 - \theta \xi - \theta \xi) \lambda_n - (1 - \theta \delta - \theta \xi) \lambda_n}{(1 - \theta \kappa)(\xi + \mu)} \]

\[ \gamma (1 - \frac{(\xi + \mu)(1 - \theta \delta) - \theta \xi) \lambda_n - (1 - \theta \delta - \theta \xi)(1 - \theta \kappa)(\xi + \mu)}{(\xi + \mu)(1 - \theta \delta) - \theta \xi) \lambda_n - (1 - \theta \delta - \theta \xi)(1 - \theta \kappa)(\xi + \mu)} \]

As can be seen, the smoothing of interest rates implies that the price level follows an autoregressive process. Will the time path of the price level be stable? This depends on the values of the characteristic roots and on the initial conditions. If interest-rate smoothing is introduced starting with a historically-given interest rate, the constants \( A_1 \) and \( A_2 \) cannot both take values which ensure stability: only one
of them can be chosen independently, since even though the price level is free to take any initial level, the relationship between two initial expected price levels is pinned down by an initial interest rate through equation (3.12). Thus if both the characteristic roots \( \lambda_1 \) and \( \lambda_2 \) lie outside the unit circle, the price level will be subject to explosive oscillations. Whether this will occur depends on the values of all the parameters of the model: the possible cases are given in the Appendix. An examination of the characteristic roots indicates that although the system will not necessarily be unstable because of the smoothing policy (3.9), it is quite possible that such a policy will be destabilizing in such a rational expectations framework.*

If the time path of the price level is given by (3.25), then the time path of the interest rate can be found by substituting from (3.25) into (3.12) and using (3.13):

\[
 r_t = \rho + (\lambda_1 - 1) \sum_{j=1}^{t-1} \phi_1 \lambda_1^{-j}u_j + (\lambda_2 - 1) \sum_{j=1}^{t-1} \phi_2 \lambda_2^{-j}u_j + A_1(\lambda_1 - 1)\lambda_1^t + A_2(\lambda_2 - 1)\lambda_2^t
\]

(3.26)

It is clear from the similarity between the form of (3.25) and (3.26) that if conditions are such that the price level's path is unstable, that of the interest rate will also be unstable. Thus, once again,

* The most plausible cases are those in which \( \theta > 1/\kappa \): if this is so, at least one and possibly both of the roots lie outside the unit circle, so that it is quite possible that the system will be unstable.
attempting to carry out a policy of resisting movements in interest rates may, by causing predictable patterns in interest-rate movements, cause both the interest-rate and the price level to explode.

What will such a policy imply about the behavior of the money stock? The time path of the money stock is given by

\[ m_t = m^* + \frac{1}{\delta} \left[ (\lambda_1 - 1)\phi_1 \sum_{j=1}^{t-1} \lambda_1^{t-j} u_j + (\lambda_2^* - 1)\phi_2 \sum_{j=1}^{t-1} \lambda_2^{t-j} u_j \right] \]

\[ + \frac{1}{\delta} \left[ A_1 (1 - \lambda_1)^2 \lambda_1^{t-1} + A_2 (1 - \lambda_2)^2 \lambda_2^{t-1} \right] \]

(3.27)

Thus, if interest rates are smoothed, the money stock also follows an autoregressive process; this process will be unstable if the process generating the price level is unstable.

It is also interesting to consider the time path of output. Because of the "surprise" aggregate supply function given by (3.13), the fluctuations in output will be white noise:

\[ y_t = \tilde{y} + \mu \left( \frac{\xi}{\xi + \mu} (\lambda_1 \phi_1 + \lambda_2 \phi_2) u_t - \frac{1}{\xi + \mu} (\xi \eta_t - v_t) \right) \]

(3.28)

Unfortunately, because of the complexity of the expressions for \( \phi_1 \), \( \phi_2 \), \( \lambda_1 \) and \( \lambda_2 \), it is not possible to make any interesting comparisons between the variance of output with smoothing as given in (3.28) with that without smoothing as given in equation (3.19).
A further question can also be considered: suppose that the system consisted of equations (3.5), (3.12), (3.13) and (3.14), and that the policy regime consisted of manipulating interest rates according to (3.9). If expectations are formed rationally, expected future interest rates are related to past and current rates in a way implied by taking the expected value in equation (3.26). Suppose that this relationship is interpreted as "structural", as it is reflected in the weights of current and lagged interest rates in the demand-for-money function. Suppose that these weights are then used to infer the behavior of interest rates which would occur under an alternative policy regime involving rigid control of the money supply. What misleading conclusions would then be drawn about the danger of interest-rate instability under a monetary rule?

This question essentially considers the conclusions which would be drawn from the data generated by the present model if they were analyzed as they have been in the existing literature on instrument instability (as discussed supra. p. 2). The question can be answered by using (3.26) to find the next period's interest rate in terms of current and past rates:

\[ r_t = \frac{2 - \theta (\kappa + \delta)}{1 - \theta \kappa} r_{t-1} + \frac{1 - \theta \delta - \theta}{1 - \theta \kappa} r_{t-2} + \frac{\theta}{1 - \theta \kappa} \rho \]  

\[ + (\lambda_1 - 1) \phi_1 (\lambda_1 - \frac{2 - \lambda (\kappa + \delta)}{1 - \theta \kappa}) u_{t-2} + (\lambda_2 - 1) \phi_2 u_{t-1} \]
Taking expectations, substituting into the money demand function (3.5) and solving for \( r_t \) as in (3.2) gives the following difference equation describing the time path which the interest rate would supposedly follow if the money supply were controlled:

\[
\frac{r_t - \kappa(1 - \delta - \theta)}{\delta - 2\kappa + \kappa^2 \theta} r_{t-1} = a + \bar{\gamma}y + p \frac{1}{1 - \theta \kappa} \rho - m
\]  

(3.30)

This difference equation may or may not be stable. It is interesting to note that in the case in which, according to the analysis presented earlier, smoothing interest rates is most likely to cause explosive movements of prices and interest rates (specifically when \( \theta > 1/\kappa \)), equation (3.30) will imply instrument instability. Ironically, in the very case in which smoothing interest rates is most likely to be destabilizing, empirical evidence generated under that regime would—if interpreted as it has been in the literature—lead one to infer that abandoning such a regime would cause the interest rate to explode. This application of the "Lucas critique" (Lucas 1976) indicates that intervention to limit interest-rate movements may be, in some sense, a self-justifying policy: under such a regime, empirical evidence is generated which suggests that abandoning the regime would be catastrophic; this occurs in a model in which the system is quite stable without such intervention, and under conditions under which the intervention is itself likely to be destabilizing.
Because of the ad hoc nature of the model used in this section, the results obtained therefrom are obviously not conclusive. They are, however, suggestive. They suggest that intervention to stabilize interest rates within a framework of long-run control of the money supply, far from being necessary to prevent interest-rate instability, may well be itself a source of instability. It may lead to movements of interest rates; moreover, it may cause widening divergences of the money supply from its target. Moreover, such intervention may be self-justifying. Interest-rate smoothing lends some predictability to movements in interest rates; such policy-induced interest-rate trends will be incorporated in the lag structure of interest rates in the demand-for-money function; if this lag structure is interpreted as "structural", it would lead one to infer that interest rates would follow an unstable path if the money supply were controlled rigidly in the short run.

\[ m_t = p_t u_t (y_t^p, r_t) \quad (3.31) \]

Another explanation of the apparent effect of lagged interest rates on the demand for money is that it is the result of costly adjustment. This explanation is often embodied in an ad hoc stock adjustment framework: individuals have desired cash balances and adjust their money balances gradually towards this desired level according to
\[ m_t - m_{t-1} = \theta (m_t - m_{t-1}) \]  

This, of course, implies a short-run demand-for-money function which depends on exponentially-weighted distributed lags on past permanent income and interest rates:

\[ m_t = \sum_{i=0}^{\infty} \theta (1-\theta)^i P_{t-i} u_i Y_{t-i}^P r_{t-i} \]  

(3.33)

if permanent income is itself given by a distributed lag on income then obviously \( Y \) and \( r \) will influence money demand via different lag structures. Such a stock adjustment model does not give rise to instrument instability: if we suppose that the money-demand function is log-linear, equation (3.31) gives rise to a (supposed) time path of interest rates

\[ r_t + \sum_{i=1}^{\infty} (1-\theta)^i r_{t-i} = \frac{\theta}{\delta} \left( \alpha + \sum_{j=0}^{\infty} (1-\theta)^j (\gamma y_{t-j}^P + p_{t-j}) \right) - m^* \]  

(3.34)

The exponentially-declining weights on \( r_{t-i} \) imply that if, for example, the right-hand side were constant, then if \( r_t \) satisfied (3.34) a value of \( r_{t+1} \) equal to the right-hand side of (3.34) would also satisfy (3.34); thus, while a disturbance to income, prices or the target money
stock would cause the interest rate to overshoot its steady-state level, no sustained movement of interest rates will result. One might suppose that instrument instability would occur, however, if adjustment costs were specified in a way that did not impose the same restrictions on the lag structure.

Laidler (1981) has criticized this stock adjustment framework as an explanation of the empirical evidence on the demand for money. Laidler points out that if, for the economy as a whole, the money stock is given, all individuals cannot simultaneously adjust their cash balances toward the desired level by exchanging money for other goods or assets—the type of adjustment which may be costly. In the aggregate, the only way in which cash balances can be adjusted toward their desired level is through changes in the general price level, and/or through changes in the arguments of the money-demand function. In that case, if prices are flexible such adjustment will take place instantaneously: one will never observe points at which real balances are not at their desired level as given by the long-run demand-for-money function (3.31), since such points would imply disequilibrium—disequilibrium, that is, in the sense that there would be excess supply or demand for money even when agents' demands were formulated while taking account of adjustment costs.

The argument can perhaps be expressed more clearly in a model that takes explicit account of adjustment costs: this may pinpoint the "fallacy of composition" which Laidler argues is committed by applying the ad hoc stock adjustment mechanism at the aggregate level. An attempt
will be made to take due account of what is exogenous and what endogenous from the standpoint of the individuals whose adjustment behavior is being modelled and from the standpoint of the economy as a whole.

One can treat the adjustment of cash balances using a quadratic stock adjustment framework, in which there are assumed to be costs of departing from one's desired cash balances and costs of adjusting one's cash balances toward the desired level. Adjustment costs may consist of costs of searching for alternative assets such as consumer durables (Santomero and Seater 1981) or they may be seen as 'costs' of altering one's consumption from the steady-state level where there is diminishing marginal utility of consumption (Lane 1982). However one interprets these costs, the crucial assumption is that both being "out of equilibrium" and adjusting one's cash balances are subject to increasing marginal costs.\(^\text{53}\)

The quadratic stock-adjustment framework involves minimizing a quadratic loss function for all future periods:

\[^{53}\text{This treatment of costs may be objectionable to some, on the grounds that the "out-of-equilibrium" costs smack of "money in the utility function" (i.e. there is nothing in the model to explain why money is held and why it is costly to have more or less than one's desired cash balance). This problem is addressed in Lane 1982, in which something equivalent to this quadratic-adjustment-cost framework is shown to emerge from a model in which money's role as a consumption-smoother is modelled explicitly. It was felt, however, that the argument of this chapter could be made more succinctly using the standard adjustment-cost framework.}\]
\[
\min \sum_{t=0}^{T} \beta^t (\frac{1}{2} \left( \frac{M_t}{P_t} - \mu_t \right)^2 + \frac{1}{2} \left( \frac{X_t}{P_t} \right)^2 - \frac{M_t - M_{t-1} - X_t}{P_t})^2
\]  
(3.35)

The first-order conditions for minimizing this loss function are

\[
\left( \frac{M_t}{P_t} - \mu_t \right)(1/P_t) + \gamma \left( \frac{M_t - M_{t-1} - X_t}{P_t} \right)(1/P_t)
\]

\[- \beta \gamma \left( \frac{M_{t+1} - M_t - X_{t+1}}{P_{t+1}} \right)(1/P_{t+1}) = 0
\]

(3.36)

\[t = 0, 1, \ldots\]

This set of Euler equations must be satisfied by the adjustment path of an individual's cash balances; in an economy of identical individuals, the aggregate behavior of money balances and prices must also satisfy (3.36). Such a formulation of adjustment costs is, of course, the basis for a stock-adjustment model like (3.32): if there were no transfers, and if the price level were given, cash balances would follow

\[
M_{t+1} \left( 1 + \frac{1}{\beta} + \frac{\beta \gamma}{1 - \beta} \right) M_t + \frac{1}{\beta} M_{t-1} = -\frac{\mu}{\beta \gamma}
\]

(3.37)

whose solution can be written

\[
M_t = M_{t-1} + \theta (M - M_{t-1})
\]

(3.38)
where
\[ \theta = \frac{1}{2}(1 - \beta - 1/\gamma) + \frac{1}{2}(1 + \gamma + \beta\gamma)^2 - 1/\beta)^{1/2} \]

and
\[ \underline{M} = \frac{\mu_t}{\beta^2(1 - \gamma) + \gamma} \]

When, however, the money stock rather than the price level is given, the behavior of the price level cannot be found simply by inverting this stock adjustment mechanism. If the money stock is altered by transfers to the public—even if these transfers are unequally distributed among different individuals—the second and third terms of (3.36) vanish, and the equilibrium price level is simply

\[ p_t = \frac{M_t}{\mu_t} \]  
(3.39)

Thus, regardless of adjustment costs, the gradual adjustment of cash balances toward their desired level will never be observed: adjustment costs have no influence on the time path of the price level.\(^\text{54}\)

\(\text{54}\) Note that this result does not in any way depend on whether the transfer whereby the money supply was altered was anticipated or unanticipated. Darby and Carr (1981) suggest that this is a crucial distinction, but they give no convincing argument that this should be so.
Does this result hinge on the assumption that all prices adjust instantaneously to clear all markets? Evidently not: if the price level were fixed, the interest rate could still adjust, altering desired cash balances \( \mu_t \) in (3.36) until long-run demand for money was equal to the fixed real money stock. In this case, any other interest rate would imply that supply and demand for money would not be equal, in the sense that agents would be actively attempting to exchange money for other tradable objects or vice versa. Thus, this analysis is a blow to the orthodox explanation of partial adjustment in the demand for money, both in the flexible-price case and in the Keynesian fixed-price case; the crucial assumption is that there is some variable which adjusts to equate money supply and demand: this assumption is made in both types of model.

Does the analysis depend on the assumption that the cost of adjustment and the costs of being out of equilibrium are both quadratic? If so, this analysis would not be a blow to the instrument instability argument, since instrument instability would not be anticipated, in any event, from the linear stock-adjustment framework. As it turns out, however, the result can quite easily be generalized to convex cost functions with continuous time, provided that one assumes away distributional effects \(^{55}\).

The individual's problem is

\(^{55}\) by assuming that the transfers whereby the money supply is altered are distributed equally to all (identical) individuals.
\[ \min_{\mathcal{P}} \int_{0}^{t} e^{-pt} \left[ f\left( \frac{M}{P} - \mu \right) + g(y_t) \right] dt \]  

(3.40)

where

\[ y \equiv \frac{1}{p} \frac{dM}{dt} - x_t \]

where \( x_t \) is the transfer received from the monetary authority, and where

\[ f(\cdot) \geq 0, \ f(0) = 0, \ f'(0) = 0, \ f''(\cdot) > 0 \]
\[ g(y) \geq 0, \ g(0) = 0, \ g'(0) = 0, \ g''(y) > 0 \]

The Hamiltonian for this problem is

\[ H = f\left( \frac{M}{P} - \mu \right) + g(y) - \lambda \left( y - \frac{1}{p} \frac{dM}{dt} - x \right) \]  

(3.41)

which, using the Maximum Principle, yields the first-order conditions

\[ \frac{\partial H}{\partial y} = g'(y) - \lambda = 0 \]  

(3.42)

\[ \frac{\partial \lambda}{\partial t} + \rho \lambda = \frac{\partial H}{\partial M} \frac{1}{P} f'\left( \frac{M}{P} - \mu \right) - \alpha \frac{d^2 M}{dt^2} \]

subject to the transversality conditions
\[
\lim_{t \to \infty} e^{-\rho t} \lambda(t) \geq 0 \quad \text{and} \quad \lim_{t \to \infty} e^{-\rho t} \lambda(t)M(t) = 0
\]

Here again, if the time path of prices were given, the adjustment path of each individual's cash balances would obey

\[
[g'(y) - g''(y)] \frac{dy}{dt} + pg'(y) + \frac{1}{P} \frac{dP}{dt} y = \frac{1}{P} \epsilon' \left( \frac{M}{P} - \mu \right) \tag{3.43}
\]

However, if the money supply is given for the economy as a whole, and altered only through transfers \(x(t)\), the entire left-hand side of equation (3.43) vanishes, and once again we have

\[
P_t = \frac{M_t}{\mu_t} \tag{3.39}
\]

Thus, once again, if prices are perfectly flexible, costly adjustment of cash balances at the individual level will have no influence on the time-path of the price-level, nor on that of the interest rate or of national income. One could easily show that interest-rate flexibility will also guarantee this result.

What can be concluded? One interpretation is that the money supply was not exogenous over the period during which the empirical evidence on the demand for money was being assembled--either because of fixed exchange rates in open economies or because monetary policy was directed at controlling interest rates. In that case, the apparent partial
adjustment in demand for money could be due to the kind of convex adjustment costs incorporated in the foregoing models. According to this interpretation, the above analysis indicates that if the money supply were controlled no implications of costly adjustment would be observed: in particular, there is no reason to suppose that the interest rate would follow an explosive path. This is clearly an interpretation which applies to the instrument instability argument: that argument assumes that the interest rate has hitherto been controlled, and alleges that disastrous results would follow from controlling the money stock instead.

It would be well to qualify this analysis by discussing some of the assumptions on which it depends. One important assumption pertains to the means whereby money is introduced into the economy: in the model, it is assumed that this is done by transfer. This is an important assumption. The traditional Keynesian analysis (e.g. Kahn 1954) regards the open-market operations whereby the money supply is typically altered in quite a different light: it regards them as working along a money-demand function—in this case, along a short-run money demand function which incorporates adjustment costs. According to that analysis, making an open-market purchase means bidding a price for bonds which is such that individuals are prepared to incur these costs by selling bonds and correspondingly adding to their cash balances; thus adjustment costs would have observable effects on the interest rate. The monetarist view of open-market operations is set in opposition to this: although certainly open-market operations involve exchanging bonds for money, the bond
traders with whom the monetary authorities deal typically do not intend to hold the extra money except for a very short period until they have exchanged the money for other assets—and in any case, the idea that such bond traders face adjustment costs sufficiently large to spread their portfolio adjustments over several months is not very plausible. Similarly, when the money supply expands via credit expansion, borrowers accept the newly created money not because they ultimately intend to hold the money but simply because it is the medium of exchange. Thus, in monetarist models, monetary expansion is generally assumed to take place via a transfer (e.g., the notorious helicopter: Friedman 1969), which can be received costlessly but which may necessitate adjustments in order to restore portfolio balance. Which of these representations of the way in which money is introduced into the economy one accepts is usually regarded as a matter of expository convenience; in the present context, however, it determines whether costly adjustment at the individual level will have observable aggregate implications.

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56 a point which has been clarified in David Laidler's lectures; see also Laidler 1981.

57 It is not a foregone conclusion that it is more appropriate to model monetary expansion as a transfer: one can imagine a case in which open-market operations involved announcing a higher price of bonds, and posting a perfectly-elastic supply of bonds for money at that price, then allowing the public to adjust cash balances in the light of the associated new interest rate; if this occurred, the adjustment costs would reduce the amount by which cash balances would be adjusted in the first period, and thus would imply that a greater change in the interest rate would be required in order to bring about a given change in the money supply. The only problem with this account is that if the authorities merely posted a perfectly-elastic supply of bonds for money at a given price of bonds, the general price level would be indeterminate (as in Sargent and Wallace 1975).
Another thing which may alter the conclusions of the foregoing analysis is the possibility of distributional effects. Distributional effects were ruled out in the quadratic model because of the linearity of the decision-rule (3.42). In the more general version subsequently presented, there may be distributional effects if transfers are not distributed equally to all agents. This can be illustrated in the simple case in which there are two agents, one of whom receives a transfer $x(t)$ and the other a transfer $-x(t)$. In this case, it follows from (3.43) and from the condition that $y_2 = -y_1$ (where subscripts denote individuals) that

$$\left\{ [g'(y_1) + g'(-y_1)] - [g''(y_1) + g''(-y_1)] \right\} \frac{dy_1}{dt}$$

$$+ p[g'(y_1) + g'(-y_1)] = \frac{1}{p} \left\{ f'\left(-\frac{M_1}{p} - \mu\right) + f'\left(-\frac{M_2}{p} - \mu\right) \right\}$$

It is clearly not true in general that this equation is solved for $p = (M_1 + M_2)/\mu$. Thus, distributional effects can cause there to be some aggregate implications of costly adjustment. However, such effects seem to be a weak basis for an explanation of the ostensible partial adjustment phenomenon, simply because in order for one to be able to observe a stable lag structure the money that is added to or subtracted from the money stock would have to be distributed in a similar manner on different occasions.
Another possible explanation for the apparent effect of lagged interest rates and income on the demand for money is that it has something to do with nonconvex adjustment costs. Nonconvex adjustment costs—usually in the nature of set-up costs—are frequently used as the basis of the demand for money (in the inventory-theoretic approaches to transactions demand for money). If such costs are important, the desired average level of cash balances would depend on both current and expected interest rates; in addition, cash balances would not adjust instantly to this desired level after a change in the arguments of the demand-for-money function; also, a change in the money stock would not in general lead the price level, interest rate and income to change instantly to their new equilibrium levels. The implications of nonconvex adjustment costs are very difficult to trace in the case in which the money stock is given; this subject is beyond the scope of this essay. However, a preliminary examination makes it clear that, with such costs, an individual's adjustment pattern will be quite different when the money supply is controlled than when the interest rate is smoothed, simply because the optimal adjustment path will depend on the time paths of an individual's receipts and of interest rates and the price level. Thus, once again it would be invalid to interpret the estimated lag parameters in the money-demand equation as "deep parameters" and to

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The implications for such set-up costs for the lag structure of interest rates and income in the demand for money are explored in Chant 1976. Chant does not, however, work out the implications of these costs when the money supply is exogenous, and some investigation of this question indicates that it is a very difficult one to address.
use them to make inferences about the consequences of moving to a policy of rigid adherence to a money-supply rule.

In Laidler's (1981) discussion of partial adjustment in the demand for money, he draws attention to another assumption on which the irrelevance of costs of adjusting money balances depends: this is the assumption that the price level adjusts instantaneously to clear the money market. Laidler argues that since, for the economy as a whole, real balances are adjusted towards their desired level through changes in the price level, the apparent partial adjustment of cash balances must be the effect of partial adjustment of the price level. However, as mentioned earlier, it is not sufficient that the price level fail to adjust immediately to its full-employment equilibrium level; static Keynesian analysis makes that assumption and yet it does not give rise to a short-run money-demand function which is any different from the long-run function; rather, some combination of income and the interest rate adjusts to equate money supply and demand (along the LM curve). Thus, if the apparent partial adjustment in the demand for money is attributable to the failure of prices to adjust immediately to equilibrium, it must depend on either or both of two other considerations. One is that asset demands may be different in disequilibrium than when all markets clear: in orthodox Keynesian analysis, the only difference that disequilibrium makes to asset demands is reflected in the assumption that money demand depends on income rather than on tastes, technology, endowments and prices alone. A second point is that Keynesian analysis assumes that when prices are not at their equilibrium level, the asset
markets still clear, while goods and labor markets fail to clear in the usual sense: this implies a particular specification of the organization of markets, which is not the only specification possible. A consistent model of disequilibrium behavior, which included a careful specification of the technology governing price setting and the organization of markets in general, might well alter both of these conclusions. An important element in such a model would likely be the holding of inventories of goods and money; such a model would thus have implications for the demand for money in disequilibrium, and might well imply that money balances would differ from their long-run equilibrium levels. Further describing such a model is, of course, well beyond the scope of this essay.

In this essay, it has been shown that the case of convex adjustment costs does not provide any support for the instrument instability argument. If one interprets the money stock as having been exogenous during the period over which empirical evidence on the demand for money was being assembled, convex adjustment costs simply do not provide a satisfactory explanation for the appearance of partial adjustment in the demand for money. In that case, this phenomenon remains something of a puzzle. If one the other hand, one supposes that the interest rate rather than the money supply has been exogenous, the conclusion is different: the conclusion is that if the money supply were rigidly controlled, there is no indication that anything like instrument instability would occur. Since the latter assumption is the one which must underlie the instrument instability argument, the analysis in this sec-
tion of the essay casts further doubt on the possibility that interest rates would follow an explosive path if the money supply were controlled according to a constant-growth-rate rule.

IV

The analysis in this essay does not, of course, absolutely disprove the instrument-instability argument against tight short-term control of the money supply, but it does seem to weaken it considerably. Two standard explanations of the effects of lagged interest rates on the demand for money have been considered: in both cases, the notion that interest rates would explode if the money supply were controlled according to a preannounced target path turns out to be groundless. In addition, it has been shown that the estimated lag structure of interest rates in the money-demand function would be quite different if the money supply were being controlled than if interest rates were being controlled or smoothed: empirical analysis like that which has previously been used to address the issue would thus lead to misleading conclusions. Thus, although the treatment in this essay is not exhaustive, it indicates that barring a stronger case to the contrary, the instrument instability problem should not be taken seriously as a danger of controlling the money supply "too well".
Appendix

In this Appendix, some of the mathematical results in Chapter 3 will, for the sake of completeness, be established.

1. Equation (3.10) is a first-order difference equation in $r_t$. Its characteristic root is

$$\lambda = \frac{1 - \theta\delta}{1 - \theta\kappa}$$  \hspace{1cm} (3A.1)

Thus, as asserted in the text, if $\theta < 1/\delta$, then $\lambda > 0$, but that clearly also implies that $\lambda > 1$. In the intermediate case in which $1/\kappa > \theta > 1/\delta$, we have $\lambda < 0$ with $\lambda < -1$ if $\theta < 2/(\delta + \kappa)$. If $\theta > 1/\kappa$, then $\lambda > 0$, but that obviously means that $0 < \lambda < 1$ since $\delta > \kappa$.

2. Equation (3.15), a first-order system in $P_t$, has characteristic roots

$$\lambda = \frac{\kappa + \delta}{2\kappa} + (\frac{\kappa + \delta}{\kappa} - \frac{4}{\kappa}(1 + \delta + \delta \mu + \delta \eta))^{1/2}$$ \hspace{1cm} (3A.2)

It is not clear whether or not these roots are real: they will be real if and only if

$$(\kappa - \delta)^2 \geq 4\kappa[1 + \mu(\delta + \eta)]$$ \hspace{1cm} (3A.3)

However, both roots can be shown to lie outside the unit circle regardless of whether they are real or complex. Thus, equation (3.15) has the "forward-looking" solution given in equation (3.16).
3. The characteristic roots of the difference equation (3.20) are

\[ \lambda = \frac{2 - \theta(\kappa + \delta)}{2(1 - \theta \kappa)} + \left[ \frac{2 - \theta(\kappa + \delta)}{1 - \theta \kappa} \right]^2 - \frac{4 - 2 \delta - \theta}{1 - \theta \kappa} \cdot \frac{1}{2} \]  

(3A.4)

Unfortunately, little can be said in general terms about these roots. The roots are both real if \( \theta \kappa < 1 \); they are also real if

\[ \theta(\delta - \kappa) > 4(\theta \kappa - 1) \]  

(3A.5)

using the approximation (3.4) this implies that where \( T \) is the term to maturity of the alternative asset, the roots are real unless

\[ 1 < \theta \kappa < \frac{T}{T - 1/4} \]  

(3A.6)

i.e. almost always. Should \( \lambda_1, \lambda_2 \) be nonreal, their modulus would be

\[ R = \left( \frac{1 - \theta \delta - \theta}{1 - \theta \kappa} \right)^{1/2} \]  

(3A.7)

which exceeds unity provided that \( \delta + 1 > \kappa \), i.e. in general. If the roots are real, there are no fewer than ten nonvacuous sub-cases:

a) \( \lambda_1 > \lambda_2 > 1 \)
b) \( \lambda_1 > 1 > \lambda_2 > 0 \)
c) \( \lambda_1 > 1 > 0 > \lambda_2 > -1 \)
d) \( \lambda_1 > 1 > -1 > \lambda_2 \)
e) \( 1 > \lambda_1 > \lambda_2 > 0 \)
f) $1 > \lambda_1 > 0 > \lambda_2 > -1$

    g) $1 > \lambda_1 > 0 > -1 > \lambda_2$

    h) $0 > \lambda_1 > \lambda_2 > -1$

    i) $0 > \lambda_1 > -1 > \lambda_2$

    j) $1 > \lambda_1 > \lambda_2$

Each of these cases has different implications for the behavior of the price level, the interest rate and the money supply. Three of these cases—cases (a), (d) and (j)—give rise to instability if the authorities attempt to introduce the implied regime of smoothing without starting with the appropriate interest rate. These cases occur as follows:

(a) $\lambda_1 > \lambda_2 > 1$

    if $\theta > 1/\kappa$ and $(1 - \theta)(\delta - \kappa) < \theta$

(d) $\lambda_1 > 1 > -1 > \lambda_2$

    if (i) $\theta < 1 - \theta\kappa < (1/4)(\delta - \kappa)$

    and $(\theta^2 - 1)(\delta - \kappa)^2 > -4\theta(1 - \theta\kappa)$

    or if

(ii) $0 < 1 - \theta\kappa < (1/3)[\theta + \delta - \kappa - (\kappa - \theta\delta)]$

    and $(\theta^2 - 1)(\delta - \kappa)^2 > -4\theta(1 - \theta\kappa)$. 
(j) \(-1 > \lambda_1 > \lambda_2\)

\[
\text{if } (1 - \theta \kappa)/(1 - \theta) > (1/4)(\delta - \kappa) > 4(1 - \theta \kappa) > 0
\]

The cases in which \(\theta > 1/\kappa\) appears to be the more plausible ones, since for instance if the alternative interest rate is the three-month bill rate, and the unit of time is one month, a long-run elasticity of money demand of .2 and an average nominal interest rate of 10% imply that \(\kappa = 6\); a value of \(\theta\) as low as .16 implies such a small amount of feedback from the money stock that one would doubt the plausibility of a model whose stability depended on the existence of such feedback.

4. The characteristic root of equation (3.30) is

\[
\omega = \frac{\kappa(1 - \theta \delta - \theta)}{\theta - 2\kappa + \kappa^2 \theta}
\]

(3A.8)

It will be found that \(\omega < -1\) if \(\theta > 1/\kappa\); otherwise, the result is ambiguous, with four different cases:

a) \((1 - \theta \kappa) > 0, (\delta - \kappa) > 1 - \theta \kappa\)

i) if \((1 - \theta \delta) < \theta\), then
   - \(\omega < -1\) iff \((1 - \theta \kappa) < 1/\delta\)
   - \(0 > \omega > -1\) iff \((1 - \theta \kappa) > 1/\delta\)

ii) if \((1 - \theta \delta) > \theta\), then
   - \(\omega > 1\) iff \(2\kappa(1 - \theta) + 1 > \delta(1 + \theta \kappa)\)
   - \(0 < \omega < 1\) iff \(2\kappa(1 - \theta) + 1 < \delta(1 + \theta \kappa)\)

b) \((1 - \theta \kappa) > 0, (\delta - \kappa) < (1 - \theta \kappa)\)
i) if \((1 - \delta) < 0\), then
- \(\omega > 1\) iff \(2\kappa(1 - \theta) + \delta(1 + \theta\kappa) > 1\)
- \(0 < \omega < 1\) iff \(2\kappa(1 - \theta) + \delta(1 + \theta\kappa) < 1\)

ii) if \((1 - \delta) > 0\), then
- \(\omega < -1\) iff \((1 - \theta\kappa) > 1/\delta\)
- \(0 > \omega > -1\) iff \((1 - \theta\kappa) < 1/\delta\).
CHAPTER IV

A SIMPLE PERFECT-FORESIGHT MODEL OF
THE MONEY-SUPPLY PROCESS

It could hardly be complained that the determination of the money supply has been neglected as a subject of economic analysis. Every textbook on money and banking contains a chapter in which the banking-system multiplier process works itself out like some system of hydraulics. Nor has the behavior determining the supply of money been untouched by more sophisticated economic theory: on the contrary, the literature on the microeconomics of banking and its relationship to the money supply process is vast. Early treatments (e.g. Morrison 1966, Brunner and Meltzer 1968), which were designed largely to show the impossibility of a liquidity trap in bank reserves, have been succeeded by ones which have introduced a variety of features of the system including the resource costs of banking (see e.g. Balten spger 1980) with a view to dealing with a number of different issues.

One topic of which there has been no satisfactory treatment, however, is the short-run adjustment behavior of banks and its implications for the money supply process. Bank adjustment is generally relegated to empirical studies, in which ad hoc stock adjustment processes are

"An exception is Carleton and Bryan 1971, in which the interaction of the adjustment processes of individual banks is simulated; the behavior of the individual bank is modelled in terms of an ad hoc stock adjustment model, however."
often posited (e.g. Fraser and Rose 1973). In recent debates over the control of monetary aggregates, this short-run adjustment behavior figures prominently. Central bank economists (e.g. Pierce and Thompson 1972, Clinton and Lynch 1978, Freedman 1980) among others (e.g. Feige and Mcgee 1977, Judd 1977, Laurent 1979, Kaufman and Lombra 1979) have argued that because of some very short-run considerations, the money supply function either does not exist or at least cannot be used to control the money supply without severe financial instability. These authors argue against base control—a technique of controlling the money supply which would otherwise appear to be simple, direct and effective. They have argued that, because of lagged accounting of required reserves, and perhaps also because of the manner in which commercial banks adjust to correct any portfolio imbalances, base control would give rise to oscillations and overshoots in the money supply, and might even make the money supply indeterminate.

The argument that has been made can perhaps best first be presented using a simple "hydraulic" model of the determination of the money supply; in such a model, it is assumed that certain behavioral and institutional ratios are given. If we ignore banks' borrowings of reserves from the central bank and the public's holdings of currency, the demand for the monetary base under current reserve accounting is

66 Of the authors mentioned, Judd and Laurent should be distinguished from the others in that they are arguing against lagged reserve accounting rather than against base control.
\[ R_t = (\rho + x)D_t \]  

(4.1)

where \( \rho \) is the required reserve ratio and \( x \) the ratio of excess reserves to deposits. Under base control, the authorities set the volume of reserves, and the volume of deposits adjusts to satisfy (4.1); the equilibrium is

\[ D_t = \frac{1}{\rho + x} R_t \]  

(4.2)

Under lagged reserve accounting, however, required reserves are based on the previous level of deposits, so that instead of (4.1) we have

\[ R_t = \rho D_{t-1} + xD_t \]  

(4.3)

so that if the base is controlled

\[ D_t = \frac{1}{x} R_t - \frac{\rho}{x} D_{t-1} \]  

(4.4)

a difference equation in deposits which is stable only if \( (\rho/x) < 1 \).

This suggests that there is a significant danger of explosive oscillations in deposits under base control (Judd 1977). Moreover, it is frequently asserted that desired excess reserves are essentially zero: if that is so, the system is overdetermined if the authorities attempt to set total reserves as a policy instrument; they can do nothing but accommodate the banks' demands for reserves; if \( x \neq 0 \), the equilibrium level of deposits given by (4.4) is undefined (Clinton and Lynch 1978).
Will the danger of explosive oscillations in deposits be obviated if banks adjust their portfolios gradually? Apparently, the reverse is true: even under current reserve accounting, gradual adjustment of banks' reserves gives rise to overshooting in the level of deposits. Suppose that desired reserves $R^*_t$ are given by (4.1), while banks adjust to that desired level according to an ad hoc stock-adjustment process

$$R_t = \tilde{R}_{t-1} + (1 - \lambda)(R^*_t - R_{t-1})$$  \hspace{1cm} (4.5)

as suggested by the empirical evidence on the lags in bank portfolio adjustment. This implies that if the central bank adopts base control and tries to set the base at a new level $\bar{R}$, initially we have

$$D_t = \frac{1}{(1 - \lambda)(\rho - x)} R - \frac{\lambda}{(1 - \lambda)(\rho + x)} R_{t-1}$$  \hspace{1cm} (4.6)

while in the next and subsequent periods

$$D_{t+k} = \frac{1}{(1 - \lambda)(\rho - x)} R - \frac{\lambda}{(1 - \lambda)(\rho + x)} R_{t-1} = \frac{1}{\rho + x} R$$  \hspace{1cm} (4.7)

$k = 1, 2, \ldots$

It can easily be seen that deposits first overshoot their equilibrium level as given by (4.7), and then move directly to that level. Under lagged reserve accounting, this overshooting leads into the oscillations in deposits described by (4.4): at first,
\[ D_t = \frac{1}{(1 - \lambda)x} R - \frac{\rho}{x} D_{t-1} - \frac{\lambda}{(1 - \lambda)x} R_{t-1} \]  

(4.8)

and then

\[ D_{t+k} = \frac{1}{(1 - \lambda)x} R - \frac{\rho}{x} D_t - \frac{\lambda}{(1 - \lambda)x} R = \frac{1}{x} R - \frac{\rho}{x} D_t \]  

(4.9)

Thus, under both current and lagged reserve accounting, lags in bank reserve adjustment simply mean that deposits overshoot their steady-state levels for one period after the base is altered, while subsequently behaving much as they would if banks' adjustment were instantaneous.

These results—at which one arrives by assuming desired excess-reserve ratios and stock-adjustment coefficients to be constants—are very puzzling. It would appear that oscillations in deposits, such as those implied by (4.9), would not be in banks' interest, and must thus surely depend on some myopia on their part. This observation, however, does not immediately lead to the conclusion that such cycles cannot occur: in the traditional analysis of the money supply process, individual banks do not choose their own deposits, but act as deposit-takers: the money supply, according to this treatment, is the unintended aggregate consequence of asset adjustment on the part of individual banks. This paper extends this traditional framework to analyzing the implications of optimal asset adjustment for the dynamics of the money supply. An
attempt is made to investigate whether similar cycles and overshooting occur in a model in which agents optimize given their anticipations of the variables which affect them.

No attempt is made to achieve complete generality in this model. Instead, an explicit, perfectly-competitive optimizing model of the behavior underlying the money-supply process is developed in a simple linear-quadratic case. The central feature of the model is that banks' expectations of their deposits are important in influencing their portfolio-adjustment behavior; this behavior, in turn, in the aggregate determines the actual time path of deposits. Perfect foresight is assumed, since that is the most straightforward way of taking account of the crucial nature of banks' anticipations of the future course of their deposits in determining their actual course: in perfect foresight equilibrium, these anticipated and actual time paths are the same.

The analysis starts with the idea that a bank finds it costly to adjust its loans quickly. The existence of some such costs must underlie the kind of stock-adjustment framework which is found in the

\[\text{\footnote{The objection may be made (on the basis of casual empiricism) that while there may be costs of adjusting loans rapidly, there essentially are no costs of adjusting liquid assets, so that banks' gradual adjustment of their loans will not imply that their reserves will be slow to adjust to the desired level. In a model containing more than two bank assets, however, the relative interest rates on loans and on more liquid assets should not be treated as fixed; rather, they must adjust in such a way that the existing aggregate amounts of reserves and other liquid assets will be held; these interest-rate movements would then reflect the costs of adjusting loans. Thus--it is conjectured--results qualitatively similar to those of this paper would emerge in the three-asset case, provided that one of the assets is costly to adjust and neither of the other two is perfectly elastic in supply to the banking system.}}\]
empirical literature. These costs may be visualized as paperwork of expanding or contracting loans (provided that the number and volume of loans are proportional). The important assumption made is that marginal adjustment costs are increasing; for the sake of tractability, it is assumed that these costs are quadratic—that is,

$$C_{1,t} = \eta (\Delta L_t)^2$$

(4.10)

Assuming that banks are faced with a choice of holding cash reserves or loans, the bank's balance-sheet constraint is

$$L_t + R_t + X_t = D_t$$

(4.11)

It follows that under current reserve accounting

$$\Delta L_t = (1 - \rho)(D_t - D_{t-1}) - (X_t - X_{t-1})$$

(4.12)

where $X_t$ is the bank's excess reserves. Holding excess reserves entails an opportunity cost, viz. the interest-rate earned on loans; on the other hand, borrowing from the central bank—represented here as negative excess reserves—has costs, both explicit in terms of the interest rate paid and implicit in terms of the bank's reputation and so forth. Here, it is assumed for simplicity that the costs of holding excess reserves (as compared to loans) are quadratic:
\[ C_{2,t} = 2b(r_t - a)X_t + gX_t^2 \]  

(4.13)

where \( a, b \), and \( g \) are constants and \( r_t \) is the market rate of interest.\(^2\)\(^3\)

The bank must adjust its loans in such a way as to maximize its profits; since it is assumed here that banks are competitive deposit-takers, they can equivalently be seen as adjusting their loans in order to achieve a level of excess reserves that minimizes the present value of sum of the costs of holding reserves and the costs of adjusting loans:

\[ \text{minimize } P \text{ subject to } X_t \leq 0 \]

\(^2\)\(^3\) This coefficient is related to the Bank Rate or discount rate, at which banks can borrow from the central bank; it is not necessary to interpret \( a \) as being equal to this central-bank rate, however. In the model, there have to be some costs of borrowing which are not reflected in a constant central-bank rate: either the rate charged must depend on the amount borrowed, or there must be some form of (pecuniary or non-pecuniary) penalties for "excessive" borrowing; this is necessary in order to yield an increasing marginal cost of borrowing. The assumption that \( a \) is the Bank Rate implies that the other costs of holding negative excess reserves are zero when excess reserves are zero. Note, however, that the value of \( a \) does not affect the dynamic behavior of the money supply; see below.

\(^2\)\(^3\) Such a cost-function is posited mainly for the sake of tractability. This quadratic formulation does, however, have the appeal that when it is used to find the steady-state demand for reserves, it can be viewed as a "canonical form" of a stochastic model of the precautionary demand for reserves, when the cash requirement is uniformly distributed and there is a lump-sum cost of being "caught short" of reserves (Morrison 1966): the latter model yields a cost function which is a quadratic function of reserves held.
\[ \min \left\{ X_t, X_{t+1}, \ldots \right\} \Gamma_t = \sum_{t=1}^{\infty} \beta^{t-1} (C_{1,t} + C_{2,t}) \] (4.14)

where \( \beta \) is a discount factor. Substituting (4.10) and (4.13) into (4.14) using (4.12) and differentiating with respect to \( X_t \) yields a set of Euler equations

\[
0 = \frac{\partial \Gamma_t}{\partial X_t} - \beta \frac{\partial (\Delta L)}{\partial X_t} + 2\beta \frac{\partial (\Delta L_{t+1})}{\partial X_t} + 2b(r - a) + 2aX
\]

for \( t = t, t+1, \ldots \)

\[
= -2\beta [(1 - \rho)(D_t - D_{t-1}) - (X_t - X_{t-1})] + 2\eta \beta [(1 - \rho)(D_{t+1} - D_t) - (X_{t+1} - X_t)] + 2\beta X + 2b(r - a)
\] (4.15)

In order to close the model, one could posit a particular form for the public's demand for deposits; for the sake of tractability, it is assumed that money demand is a linear function of the rate of interest.

Perhaps it should really be assumed that \( \beta = 1/(1+r_t) \). This would, however, make the system nonlinear and thus intractable.

This assumption is common in the money-supply literature. It is not quite as restrictive as it may seem: it can be regarded as a reduced form from an aggregate supply-aggregate demand framework which implies variations in \( P \) and \( Y \); alternatively, it can be rationalized in terms of the assumption that money demand depends on permanent income, and is a demand for real balances denominated in permanent prices (both permanent income and permanent prices can be constants...
\[ D_t = k - \zeta r_t \quad (4.16) \]

If it is assumed that the public holds no base money in the form of currency, and if it is also assumed that all banks are identical so that the banking system behaves like a single competitive bank, the adding-up condition for the unborrowed monetary base is

\[ R_t = \rho D_t + X_t \quad (4.17) \]

If (4.16) and (4.17) are substituted into (4.15), it turns out that if all banks are identical and competitive and anticipate their future deposits, the following time path of aggregate deposits emerges from the optimal asset-adjustment behavior of individual banks:

\[ D_{t+1} = \phi D_t + \omega D_{t-1} + \chi_t \quad (4.18) \]

where

---

if the money supply converges to a constant). This is still not very satisfactory, of course: the price level should be allowed to be expected to vary. The reason for choosing the present formulation (rather than, for example, putting the entire model in logarithms, which would make it possible to include the price-level in a log-linear money-demand schedule) is that one of the main motivations of the paper is to provide a rational-expectations counterpart of the traditional banking-system-multiplier analysis: for that reason, it was considered desirable both to specify the model in such a way that the banks' budget identities hold exactly, and to suppress, as much as possible, the other details of the economic system.
\[ \phi = (1/\beta)[1 + \gamma + g\rho/\eta + b/\eta\zeta] \]

\[ \omega = 1/\beta \]

and

\[ \chi_t = (1/\beta)\{BR_{t+1} - (1 + \beta + g/\eta)R_t + R_{t-1} + (b/\eta)(a - k/\xi)\} \]

Equation (4.18) is a second-order linear difference equation; its characteristic roots are real and positive; in addition, one of the roots is always greater and the other less than unity.\(^6\)

Since this is apparently the first treatment of the "banking-system multiplier" process which does not assume that banks are myopic, some attention should be given to the conditions under which this process will be stable\(^7\). In models of monetary equilibrium, there is generally an infinite number of solutions for the time path of the price level, all but one of which are explosive. The unstable solutions usually yield a lower level of utility than do the stable ones, but all are outcomes of individual optimizing behavior given the current and expected price levels. The unstable branches are generally ruled out using terminal conditions on the price level (for instance, that the discounted value of the future price level approach zero as \(t\to-\infty\), e.g. Sargent and Wallace 1973). Such conditions may be rationalized in terms of some conditions on the primitives of the model, which render the unstable

\(^6\) see Appendix, point 1.

\(^7\) just as the early literature on the money-supply process had to establish the convergence of the process as a geometric series (Ches-ter Phillips 1921).
branches infeasible after a certain point in the future and as a result imply that those paths cannot be rational-expectations equilibria up to that point. For example, in models which contain money in agents' utility functions it is often assumed that the marginal utility of money balances approaches zero as real balances approach infinity and vice versa (Brock 1975); in overlapping-generations models, similar restrictions may be imposed upon the marginal utility of consumption. An alternative is to impose, as a separate restriction, the condition that the price level be based on "market fundamentals", that it be time-consistent (Howitt 1979) or that it be generated by a stationary process (Whiteman 1982). Any of these methods can be used to support the conclusion that the stable branch is followed, without having to invoke some kind of collective rationality.

A similar problem of unstable solutions—self-generating movements of deposits—arises in a simple perfect-foresight model of the money-supply process. Since one of the characteristic roots of (4.18) always exceeds unity, there is an infinite number of unstable solutions which must somehow be excluded. Here again, the unstable branches cannot be ruled out using the transversality conditions from banks' optimization problems, since banks do not choose their own deposits.** In a linear model like this one, the unstable branches could be excluded by invoking the non-negativity constraints on deposits and on nominal interest.

** Likewise, the implausibility of explosive movements of deposits for a given monetary base is no reason to exclude these paths, since they are solutions to the model, and unless some feature of the model can rule them out, their presence suggests that the model itself (or the perfect-foresight assumption) is implausible.
rates. This would be unnecessary, however: the paths which entail explosive increases of deposits imply that excess reserves become increasingly negative; since in this model, excess reserves are treated as borrowings from the central bank, these unstable paths would be infeasible if there were any bound (no matter how high) on borrowings as a fraction of unborrowed reserves. On the other hand, the paths which involve implosive decreases in the money supply for a given monetary base would eventually violate the condition that bank profits be non-negative: if this should occur, and some banks shut down, the remaining banks' deposits would be a larger share of aggregate deposits than is implied by the implosive paths; thus, if banks are allowed to close when their profits become negative, the implosive paths are also infeasible.

Thus equation (4.18) can be solved subject to two side-conditions: an initial level of deposits $D_0$ and a terminal condition that

\[ \lim_{t \to \infty} \beta^t \left( \frac{X_t}{R_t} \right) = 0. \]

This implies that the solution will be a combination of backward- and forward-looking solutions (Blanchard 1979; see also Sargent 1979):
Thus the time path of deposits is described by

\[
P_t = D_0 - \frac{1}{\lambda_1 - \lambda_2} \left( \sum_{i=1}^{\infty} i x_i \right) + \frac{1}{\lambda_1 - \lambda_2} \left( \sum_{j=0}^{\infty} j x_j \right).
\]

The terminal condition implies that \( A_1 = 0 \). The historical level of deposits \( D_0 \) can be used to determine the constant \( A_2 \).

From other rational expectations models, the current level of deposits depends on the entire future course of the monetary base. One aspect of this solution is familiar from the lag operator. Where \( I \) is the lag operator, the current level of deposits is defined as:

\[
A_t = \frac{1}{\lambda_1 - \lambda_2} \left( \frac{1}{\lambda_1 - \lambda_2} \left( \sum_{j=0}^{\infty} j x_j \right) \right) + A_{t+1} + A_{t+2} - \frac{1}{\lambda_1 - \lambda_2} \left( \sum_{j=0}^{\infty} j x_j \right).
\]

(4.19)

\[
D_t = \frac{1}{\lambda_1 - \lambda_2} \left( \sum_{j=0}^{\infty} j x_j \right) + A_{t+1} + A_{t+2} - \frac{1}{\lambda_1 - \lambda_2} \left( \sum_{j=0}^{\infty} j x_j \right).
\]

(4.20)
Suppose, for example, that the monetary base is being held constant from 
$\tau = 0, \ldots$; in this case

$$
\psi = \frac{1}{\beta} \left[ R - (1 + \beta + g/\eta)R + R + \frac{D}{\eta} (a - \frac{k}{\zeta}) \right].
$$

(4.22)

$$
= - \frac{gR + b(k/\zeta - a)}{\beta \eta}
$$

$$
= x
$$

Substituting (4.22) and (4.19) into (4.21) yields

$$
D_t = \frac{1}{\lambda_1 - \lambda_2} \left[ \frac{\lambda_1}{1 - \lambda_1} + \frac{1}{1 - 1/\lambda_2} \right] x
$$

$$
+ \left( D_0 - \frac{1}{\lambda_1 - \lambda_2} \left[ \frac{1}{1 - \lambda_1} + \frac{1/\lambda_2}{1 - 1/\lambda_2} \right] x \right) \lambda_1^t
$$

(4.23)

In other words,

$$
D_t = D^* + (D_0 - D^*) \lambda_1^t
$$

(4.24)
where

\[ D^* = \frac{1}{\lambda_1 - \lambda_2} \left[ \frac{\lambda_1}{1 - \lambda_1} + \frac{1}{1 - 1/\lambda_2} \right] \chi \]

\[ = \frac{\zeta}{b \cdot g \rho \zeta} \left[ gR + b(k/\zeta - a) \right] \]

Thus, the volume of deposits adjusts gradually from its initial level towards a steady-state equilibrium.

What difference is made by lagged reserve accounting in this world of banks endowed with perfect foresight? If a bank's required reserves are a fraction of its deposits in the preceding rather than the current period, the amount by which a bank must adjust its loans is

\[ \Delta L_t = (D_t - D_{t-1}) - \rho(D_{t-1} - D_{t-2}) - (X_t - X_{t-1}) \]  \hspace{1cm} (4.25)

The difference equation which must be obeyed by deposits is then

\[ D_{t+1} - \phi' D_t + \omega' D_{t-1} = \chi' t \]  \hspace{1cm} (4.26)

where

\[ \phi' \equiv (1/\beta)[1 + \beta + b/n\zeta] \]

\[ \omega' \equiv (1/\beta)[1 - 2\rho/n] \]
\[ x'_t = \frac{1}{\beta} [\beta R_{t+1} - (1 + \beta + g/\eta) R_t + R_{t-1} + \frac{b}{\eta} (a - k/\zeta)] \]

This difference equation has characteristic roots\(^{63}\) which are both real; in addition, one of the roots is always greater than unity and the other less than unity. However, under lagged reserve accounting, it is not always the case that both roots are positive. In the limiting case where \(\beta=1\), one of the roots will be negative if \(g\eta > \eta\). This condition is easily interpreted: if the cost of adjusting loans is very small compared to the cost of having a temporary portfolio imbalance, oscillations in deposits will take place. The reason is that if, for instance, deposits were small last period, required reserves will be small this period; unless the bank is content to hold more excess reserves this period than it would otherwise wish, it will expand its loans; if all banks do likewise, deposits will accordingly be large this period, and thus required reserves will be large next period; by the reverse process, this will give rise to a small volume of deposits next period. As this process continues, deposits will oscillate from period to period, around their equilibrium level. Thus, the perfect foresight model can give rise to oscillations in deposits, for much the same reason as does the mechanical model of the money-supply process presented earlier in this chapter (pp. 132-35). However, the model with adjustment costs makes it clear that oscillations will only take place under certain cir-

\(^{63}\) see Appendix, point 2.
cumstances, viz. when banks' costs of having more or less than their long-run desired level of reserves are large compared to their costs of adjusting loans quickly to achieve that level. If banks face large costs of adjusting their loans, deposits will converge towards their equilibrium level without oscillating around it.

A further question is whether, as in the case of current reserve accounting, one of the roots governing the time-path of deposits is greater and the other less than unity in absolute value. If this is so, the equation (4.26) can be solved in much the same way as the corresponding equation under current reserve accounting. If the authorities are following a policy of rigid base control whereby the monetary base is being held constant,

\[ D_t = D^* + (D_0 - D^*) \lambda_1^t \]  \hspace{1cm} (4.27)

where

\[ D^* \equiv \frac{1}{\lambda_1 - \lambda_2} \left[ \frac{1}{1 - \lambda_1} + \frac{1}{1 - \lambda_2} \right] x \]

\[ = \frac{r}{b g p} gR + b(k/t - a) \]

Thus, as can be seen by comparing (4.27) with (4.24), the steady state money stock is the same under current as under lagged reserve accounting.
when the authorities are following a rule of keeping the monetary base constant. However, the adjustment of the money stock towards its equilibrium value is different under current than under lagged reserve accounting. Under lagged reserve accounting, the money supply may oscillate around its steady-state level in the process of converging to that level. However, in the case considered here, instability in this convergence process is ruled out: provided that deposits cannot increase or decrease without bound, there is no tendency for them to follow an explosive path towards their upper or lower bounds, provided that $|\lambda_2| < 1$.

What happens in the alternative case, in which banks' adjustment costs are so low, compared to their out-of-equilibrium costs, that $\lambda_2 < 1$? In this case, there is no rational-expectations equilibrium which is consistent both with the terminal condition and with an initial level of deposits, unless the initial level of deposits is precisely the steady-state level $D^*$ given by (4.27). The latter condition would be presumed to hold if the initial level of deposits were the outcome of the same process after an indefinitely long period. Otherwise, there is no equilibrium, since where there are adjustment costs the historical level of deposits is a constraint on deposits: the "initial" level of deposits cannot "jump" to the level required for stability.

\[7^0\] unlike the price level in rational-expectations models of money and prices (e.g. Sargent and Wallace 1973).
Thus the implications of lagged reserve accounting for the money supply are quite different in a perfect-foresight optimization model than in a model in which various ratios are considered to be given. If banks' costs of adjusting their loans are large, any oscillations in deposits which result from lagged reserve accounting will be moderated; the money supply may then converge to its equilibrium level either with or without oscillations. If, on the other hand, banks can adjust their portfolios very rapidly, it may be the case that no perfect-foresight equilibrium exists. Thus, lagged reserve accounting could possibly be highly destabilizing to the system, but it is quite possible that its effects will be mitigated by banks' holding buffer-stocks of reserves.

In this model, there is a natural criterion for comparing the stability of the system under current and lagged reserve accounting, in the case where banks' cost-functions are such that under lagged reserve accounting the system converges to equilibrium gradually. A suitable criterion is the speed of convergence, which is inversely proportional to the absolute value of the smaller root $\lambda_2$. We can compare the absolute value of this root under current and lagged reserve accounting; the result will be different in different cases. In the case in which the system converges to equilibrium without oscillations in deposits, lagged reserve accounting usually speeds the convergence of deposits to equilibrium—in the limiting case as $\beta + 1$, this generally occurs. Thus, when the banks' technology is such that the system converges to equilibrium without oscillations under lagged reserve accounting, the lagged reserve

\[71\] see Appendix, point 3.
accounting itself makes the system converge more quickly. The intuition behind this result is this: suppose that deposits are at a certain level above equilibrium, and are converging downwards towards equilibrium; required reserves are greater under lagged reserve accounting than they would be under current reserve accounting, since they are based on the previous period's deposits which are larger than this period's deposits; these greater required reserves will then induce banks to contract their loans more quickly, and this faster adjustment of loans is associated with a faster adjustment of deposits.

When λ₂<0, this result does not hold: the effect of lagged reserve accounting on the speed of convergence is, as one might expect, ambiguous. Thus, even when lagged reserve accounting leads to oscillations in the money supply, it is possible that it increases the speed of convergence of deposits towards their equilibrium level; however, it is also possible that it slows convergence to equilibrium, and even more likely that it increases the variance of the money supply in the short run.

Thus, a simple perfect-foresight model leads to somewhat different conclusions about the consequences of lagged reserve accounting for the stability of the money supply under a regime of rigid base control. It might be useful to employ the same model to examine a couple of other policy issues. One such issue is the effect of altering the required reserve ratio. According to the textbooks, the principal purpose of reserve requirements is to facilitate control of the money supply. On the other hand, the literature on lagged reserve accounting suggests

72 see Appendix, point 4.
that reserve requirements may actually be destabilizing, or at best irrelevant. In the perfect-foresight model, the required reserve ratio affects the speed of convergence of deposits to equilibrium. Under current accounting, an increase in reserve requirements leads the system to converge more quickly. Under lagged reserve accounting, on the other hand, an increase in reserve requirements has an ambiguous effect. Increasing the reserve requirement will increase the amplitude of oscillations if there be any (i.e. if \( \lambda_2 < 0 \)) but will otherwise (i.e. if \( \lambda_2 > 0 \)) cause deposits to converge more rapidly to their equilibrium level.

Another issue which may be treated here is the manner in which the central bank sets the rate at which it will lend to the commercial banks. It has been implicitly assumed above that the central bank either holds the discount rate fixed, or varies it with the volume of borrowing. What happens if a "floating bank rate" policy is established, whereby the rate is set as a penalty rate a certain percentage above market interest rates? Supposing that the influence of market interest rates on desired reserves occurs only via the difference between market interest rates and the central bank rate, such a rule

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73 They would be irrelevant if, for example, the authorities were able to take account of the past level of deposits in setting the monetary base.

74 see Appendix, point 5.

75 see Appendix, point 6

76 A typical result from the literature on optimal precautionary reserves is that the demand for reserves depends upon the ratio of the market interest rate to (or its difference from) the central-bank
would eliminate the influence of variations in market interest rates on banks' loan adjustments. Under current reserve accounting, the condition for the bank's optimal holding of excess reserves would be

\[ -[(1 - \rho)(D_t - D_{t-1}) - (X_t - X_{t-1})] \]

\[ + \beta[(1 - \rho)(D_{t+1} - D_t) - (X_{t+1} - X_t)] + (g/\eta)X_t - bg/\eta = 0 \]

where \( \eta \) is a constant. This together with (4.16) and (4.17) can be represented as a difference equation

\[ D_{t+1} - \phi''D_t + \omega''D_{t-1} = \chi''_t \]

(4.29)

where

\[ \phi'' = (1/\beta)[1 + \beta + g\rho] \]

\[ \omega'' = 1/\beta \]

\[ \chi''_t = [R_{t+1} - (1 + \beta + g/\eta)R_t] \]

This equation has characteristic roots which are real and positive; one exceeds and the other falls short of unity\(^77\). Thus the time path of deposits has the same basic form with a penalty rate as with a fixed central-bank rate. The speed of convergence under the alternative rules for setting the discount rate can be compared: the stable root with the floating discount rate exceeds that with the fixed discount rate\(^78\).

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\(^77\) see Appendix, point 7.

\(^78\) see Appendix, point 8.

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See for instance Modigliani Rasche and Cooper 1977.
Thus deposits converge more quickly to equilibrium when the discount rate is fixed than when it is set as a penalty rate above market rates\(^7\). Why should this be so? When the discount rate is fixed, this implies that for instance when deposits are above and thus interest rates below their respective equilibrium levels, desired reserves will be larger than usual, meaning that banks will want to contract their loans more quickly; thus loans and deposits move more quickly towards their steady-state levels. When the cost of borrowing from the central bank moves pari passu with market interest rates, this stabilizing influence of movements in market interest rates is attenuated (or, on the assumptions made here, eliminated).\(^8\)

The foregoing analysis has generally assumed that the regime of base control being followed by the authorities involves keeping the monetary base constant. A more general treatment of rigid base control would also consider the case in which the monetary base is growing (or declining) at a rate \((\gamma-1)\). This implies that the combination of forcing variables \(X\) is now

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\(^7\) The distinction drawn here between a fixed and a floating bank rate should not be confused with two other distinctions: the distinction between a high and a low rate, and that between a tightly and a liberally administered "discount window".

\(^8\) Qualitatively similar results are obtained under lagged reserve accounting. The floating bank-rate slows the speed of adjustment (i.e. increases the value of the stable root) no matter whether the adjustment is direct or includes oscillations.
\[ x_t = \frac{1}{\beta} \left( \delta R_t - \left( 1 + \beta + \frac{\gamma}{\eta} \right) R_{t-1} + \frac{1}{\gamma} R_t^+ + \frac{b}{\eta} (a - k/\zeta) \right) \] 

\[ = (\gamma - 1 + \beta + \frac{\gamma}{\eta} + \frac{1}{\gamma}) R_0^+ x_t + \frac{b}{\eta} (a - k/\zeta), \quad t = 1, \ldots \]

\[ \Omega = \Omega^t + \kappa, \quad t = 1, \ldots \]

In this case, deposits follow the path

\[ D_t = \frac{1}{\lambda_1 - \lambda_2} \left( \frac{\lambda_1}{1 - \lambda_1} + \frac{1}{1 - 1/\lambda_2} \right) x_t \]

\[ + \left( \frac{\lambda_1/\gamma}{1 - \lambda_1/\gamma} + \frac{1}{1 - \gamma/\lambda_2} \right) \Omega^t \]

\[ + \left( D_0 - \frac{1}{\lambda_1 - \lambda_2} \left( \frac{1}{1 - \lambda_1} + \frac{1/\lambda_2}{1 - 1/\lambda_2} \right) x_t + \frac{1}{1 - \lambda_1/\gamma} + \frac{\gamma/\lambda_2}{1 - \gamma/\lambda_2} \right) \Omega^t \}

\[ \times \lambda_1 \]

if \( \lambda_1 < \gamma < \lambda_2 \)

\[ = D_t^* + (D_0 - D_t^*) \lambda_1^t \]

Thus, when the monetary base is growing at a constant rate, there is a moving equilibrium money supply; the actual money supply differs from the equilibrium one by a diminishing proportion of the difference between initial and equilibrium money stocks.
It can be shown that when the monetary base is growing, the steady-state "money-multiplier" will depend on the growth rate. Substituting from (4.19) and (4.30) into (4.31) reveals the ratio of equilibrium deposits to the combination of exogenous variables

\[
\frac{D^*}{x_t} = \frac{1}{\lambda_1 - \lambda_2} \left[ \frac{\lambda_1/x}{1 - \lambda_1/\delta} + \frac{1}{1 - \delta/\lambda_2} \right]
\]  

(4.32)

\[\gamma = \frac{\gamma}{\gamma^2 - (\gamma/\beta)(1 + \beta + gp/n + b/\eta)} + 1/\beta\]

Since this ratio depends on the growth rate of the base, so does the money multiplier. Since

\[
\frac{\partial (D^*/x_t)}{\partial \gamma} = \frac{1/\beta - \gamma}{[\gamma^2 - (\gamma/\beta)(1 + \beta + gp/n + b/\eta) + 1/\beta]^2}
\]

(4.33)

the multiplier increases with the growth rate of the base when \( \gamma^2 < 1/\beta \), while it decreases with the growth rate when \( \gamma^2 > 1/\beta \). In the limiting case in which \( \beta + 1 \), the multiplier is maximized when the base is constant over time. The reason for this is that, if loans are costly to adjust, either growth or contraction of the volume of loans adds to the bank's costs of having a given volume of loans outstanding, and induces it to hold fewer loans and more reserves per dollar of deposits outstanding; this means that the further is the growth rate from zero, in either direction, the smaller is the volume of deposits which will be associated with a given monetary base.
The case of lagged reserve accounting may also be considered. Here, the results are quite similar. However, the ratio of the equilibrium money supply to the base will be different under lagged than under current accounting; it will be

\[
\frac{D^nT}{L^n} = \frac{\gamma}{\gamma^2 - (\gamma/\beta)[1 + \beta + b/\eta\xi] + (1/\beta)[1 - \delta/\xi]} \tag{4.34}
\]

A comparison of (4.34) with (4.32) reveals that although, as under current accounting, the equilibrium money stock for a given monetary base depends on the rate at which the base is growing, this money stock will be different under lagged reserve accounting. The reason for this difference is obvious: when base and deposits are growing, each period's base "backs" the previous period's deposits, which are smaller than this period's ones; thus a larger volume of deposits this period can be sustained by a given base.

The fact that the equilibrium level of deposits given the monetary base is sensitive to the anticipated future growth rate of the monetary base suggests that it is difficult to define the money multiplier in isolation from the rule that is determining the time path of the base. In addition, it suggests that the money multiplier might well be more stable if a regime of rigid base control were being followed than if there were continual swings in the policy governing the behavior of high-power money.
The behavior of the money stock under rigid base control has been examined. What would be observed if the authorities instead attempted to attain a certain level of the money supply and/or the interest rate (equivalent in a non-stochastic model)? At first, the authorities would need to alter the supply of reserves in the system to the point at which the banks are prepared to adjust their loans enough to yield the desired level of deposits (given that they anticipate that this level of deposits will prevail in the future). In succeeding periods, banks would adjust their loans further, while the authorities must alter the supply of reserves in the opposite direction in order to keep the money stock from changing. If the banks anticipate that the target level of deposits \(D^*\) will continue to prevail, their desired holdings of loans and reserves at each stage of the adjustment process can be found; these desired holdings must be accommodated if the money stock is to be kept at the level \(D^*\). Under current reserve accounting, the Euler equations are (cf. equations (4.15)):

\[
-2\bar{n}[(1 - \rho)(D^* - D^*) - (X_t - X_{t-1})] + 2\beta\bar{n}[(1 - \rho)(D^* - D^*) - (X_{t+1} - X_t)] + 2gX_t + 2b(r^* - a) = 0
\]

where \(r^* \equiv 1/\xi (k - D^*)\). Thus

\[
X_{t+1} - \frac{1 + \beta}{\beta} \frac{g/\bar{n}}{X_t + \frac{1}{\beta}X_{t-1}} = \frac{b}{\beta \bar{n}} (r^* - a)
\]
The solution of this equation implies that reserves are adjusted according to a path of the form

\[ R_t = R_{t-1} + (1 - \lambda)[R^* - R_{t-1}] \]  \hspace{1cm} (4.37)

which is the familiar stock-adjustment formulation as presented in (4.5). Thus, when the authorities accommodate banks' demands for reserves, a stock-adjustment process in reserves emerges; as was shown earlier in this paper (pp. 134-35) such a stock-adjustment formulation implies that deposits initially overshoot their equilibrium level under base control; it was also shown that when such a stock-adjustment process is posited, the lagged adjustment does nothing to temper the oscillations in deposits which emerge under base control with lagged reserve accounting. Thus, one policy regime—viz. one in which the authorities accommodate banks' demands for reserves—generates a pattern of stock adjustment in banks' reserves which, when interpreted as part of the structure of the system, indicates that another policy regime—viz. rigid base control—will entail overshooting and cycling of deposits. Overshooting is not predicted when a model of optimizing behavior subject to adjustment costs is applied directly to the question; such a model also shows that adjustment costs would play a considerable part in mitigating or eliminating the cycling which might arise from base control under lagged reserve accounting. This, of course, is another illustration of Lucas' (1976) proposition that, when one is examining

---

\[ \text{see Appendix, point 9.} \]
the results of behavior that depends importantly on agents' anticipations, it is necessary to take tastes and technology, rather than demand functions and adjustment parameters, as given.

The discrepancy between the predictions of the stock adjustment model and those which emerge from an explicit model of optimizing behavior is also related to another point raised in the literature: this is the "fallacy of composition" discussed by Howitt and Laidler (1979 and Laidler 1981). The individual bank may take its deposits as given and adjust its holdings of reserves gradually; on the other hand, when reserves are fixed for the banking system as a whole, the behavior of deposits is not simply the reciprocal of the individual bank's delayed adjustment of reserves; instead, it is loans and deposits that adjust gradually to their equilibrium levels; no overshooting occurs. Clearly, there is a general problem with models which assume stock adjustment processes applying to variables which are exogenous for the system as a whole.

Thus, a simple perfect foresight model of the money supply process points out some of the limitations of analysis which assumes that certain ratios or certain stock adjustment parameters are given. This model also indicates that the holding of excess reserves as buffer stocks may stabilize the system against oscillations in deposits, in a way in which excess reserves which are merely held as a certain fraction of deposits would not: reserves act as a cushion to reduce banks' adjustment costs, and may thereby allow banks to reduce or eliminate the large adjustments associated with oscillations in deposits. Fluctua-
tions in excess reserves, then, far from being a source of variability in deposits, tend to reduce that variability.

The assumption of perfect foresight, therefore, permits the construction of a model of the money supply process which is based on the optimizing behavior of individual banks; this model elucidates some features of the money supply process which may be important in assessing the desirability of a regime of base control.

Appendix

In this Appendix, some of the mathematical results asserted in the text are established.

1. The characteristic roots of equation (4.19) are

\[ \lambda = \frac{1}{2\beta} \left( 1 + \beta + \frac{\delta \rho}{\eta} + \frac{b}{\eta \zeta} \right) \]

\[ + \left[ (1 - \beta)^2 + \left( \frac{\delta \rho}{\eta} + \frac{b}{\eta \zeta} \right)^2 + 2(1 + \beta) \left( \frac{\delta \rho}{\eta} + \frac{b}{\eta \zeta} \right) \right]^{1/2} \]

It is obvious that the roots are real, since the expression of which the square root is taken is positive. Both roots are positive since \((1+\beta)^2 > (1-\beta)^2\); the larger of the roots \(\lambda_1 > 1\) since \((1+\beta)/2\beta > 1\); \(\lambda_2 < 1\) since \((1+\beta) > (1-\beta)\).

2. The roots of equation (4.26) are

\[ \lambda = \frac{1}{2\beta} \left( 1 + \beta + \frac{b}{\eta \zeta} - \left[ (1 - \beta)^2 + \left( \frac{b}{\eta \zeta} \right)^2 + \frac{4\delta \rho}{\eta} + 2(1 + \beta) \beta \frac{\delta \rho}{\eta \zeta} \right]^{1/2} \right) \]

(4A.2)
They are obviously real, and one of the roots is always greater and the other less than unity; the possibility that \((g_p/\eta)\) is large means that one cannot rule out the possibility that both roots lie outside the unit circle.

3. From (4A.1) and (4A.2),

\[
\lambda_1^{\text{CRA}} - |\lambda_1^{\text{LRA}}| = \frac{1}{2\beta} \left( 1 + \beta + \frac{g_p}{\eta} + \frac{b}{\eta \zeta} \right) - \left[ (1 - \beta)^2 + \left( \frac{g_p}{\eta} + \frac{b}{\eta \zeta} \right)^2 + 2(1 + \beta) \left( \frac{g_p}{\eta} + \frac{b}{\eta \zeta} \right) \right]^{1/2}
\]

\[
- \left[ 1 + \beta + \frac{b}{\eta \zeta} - \left( 1 - \beta \right)^2 + \left( \frac{b}{\eta \zeta} \right)^2 + \frac{4g_p}{\eta} + 2(1 + \beta) \frac{b}{\eta \zeta} \right]^{1/2}
\]

When \(\lambda_2 > 0\), this expression becomes

\[
\frac{4g_p}{\eta} + 2(1 + \beta) \frac{b}{\eta \zeta} \left[ \beta - 2(1 - \beta)^2(1 + \beta) \right]
\]

\[
> 3 \left( \frac{b}{\eta \zeta} \right)^2 + 3(1 - \beta)^2 + 4(1 - \beta) \frac{b}{\eta \zeta} \left[ 1 + (1 + \beta) \frac{b}{g_p \zeta} + (1 - \beta)^2 \frac{\eta}{g_p} \right]
\]

This inequality is generally satisfied in the limiting case in which \(\beta = 1\).

4. When \(\lambda_2 < 0\), the right-hand side of (4A.3) becomes
2(1 + \beta + \frac{b}{n\zeta}) + \frac{\epsilon \rho}{\eta} > \{(1 - \beta) + (\frac{\epsilon \rho}{\eta} + \frac{b}{n\zeta})^2 + 2(1 + \beta)(\frac{\epsilon \rho}{\eta} + \frac{b}{n\zeta})\}^{1/2}

+ \{(1 - \beta)^2 + (\frac{b}{n\zeta})^2 + \frac{4\beta\epsilon \rho}{\eta} + 2(1 + \beta)\beta\frac{b}{n\zeta}\} \quad (4A.5)

an expression the sign of which is, predictably, ambiguous.

5. The assertion that, under current reserve accounting, an increase in reserve requirements increases the speed of convergence can be established by examining the derivative

\[
\frac{3\lambda_1^{CRA}}{\theta \rho} = \frac{\epsilon \rho}{2\beta n} \{1 - \left[\frac{\epsilon \rho}{\eta} + \frac{b}{n\zeta} + 1 + \beta\right]\} \quad (4A.6)
\]

\[
\theta\{(1 - \beta)^2 + (\frac{\epsilon \rho}{\eta} + \frac{b}{n\zeta})^2 + 2(1 + \beta)(\frac{\epsilon \rho}{\eta} + \frac{b}{n\zeta})^{-1/2}\} < 0
\]

6. The effect of the reserve requirement on the smaller characteristic root under lagged reserve accounting is

\[
\frac{3\lambda_1^{LRA}}{\theta \rho} = -\frac{\epsilon \rho}{\eta} \left\{(1 - \beta)^2 + \frac{b}{n\zeta} + \frac{4\beta\epsilon \rho}{n\zeta} + 2(1 + \beta)\beta\frac{b}{n\zeta}\right\}^{-1/2} < 0 \quad (4A.7)
\]

7. The roots of equation (4.29) are

\[
\lambda = \frac{1}{2\beta} \left\{1 + \beta + \frac{\epsilon \rho}{\eta} - \{(1 - \beta)^2 + (\frac{\epsilon \rho}{\eta})^2 + 2(1 + \beta)\frac{\epsilon \rho}{\eta}\}^{1/2}\right\} \quad (4A.8)
\]
8. The difference between the stable root with a fixed and with a floating bank-rate will be

\[
\frac{1}{2\beta} \left( \left(1 - \beta \right)^2 + \left( \frac{g\rho}{\eta} + \frac{b}{\eta \zeta} \right)^2 + 2(1 + \beta) \left( \frac{g\rho}{\eta} + \frac{b}{\eta \zeta} \right) \right)^{1/2} \quad (4A.9)
\]

\[- \frac{b}{\eta \zeta} \left( \left(1 - \beta \right)^2 + \left( \frac{g\rho}{\eta} \right)^2 + 2(1 + \beta) \frac{g\rho}{\eta} \right)^{1/2} > 0 \]

9. The roots of equation (4.36) are

\[
\lambda = \frac{1}{2\beta} \left( 1 + \beta + \frac{g}{\rho} + \left( \left(1 - \beta \right)^2 + \left( \frac{g}{\eta} \right)^2 + 2 \left(1 + \beta \right) \frac{g}{\eta} \right) \right)^{1/2} \quad (4A.10)
\]

These roots are very similar to those governing the behavior of the money supply under base control. The roots given by equation (4A.10) will clearly be real and positive, with one exceeding and the other falling short of unity.
CHAPTER V

THE CHOICE OF MONETARY CONTROL TECHNIQUE: AN EMPIRICAL MATTER?

After prolonged controversy, central banks in a number of countries have now acknowledged the importance of controlling the growth rate of the money supply. Accordingly, debate over monetary policy has shifted to other ground. One recent point of contention is the choice of the technique of monetary control. The practice of central banks has generally been to control the money supply via some interest rate. In essence, what this technique involves is forecasting national income and the price level, and using an estimated money-demand function to find the interest rate needed to set the expected value of money demand equal to the target money stock: the central bank then supplies commercial banks with enough reserves to keep the interest rate at this level (Par-kin 1978). The interest-rate control technique has been criticized by those who advocate the use of base control. Base control works via the banking-system-multiplier mechanism; it involves setting the monetary base in each period at a level which is predicted to result in the target money supply. Its use therefore requires that the multiplier be forecast, using whatever information is available; errors in attaining the target money stock will arise from errors in these forecasts.
The most common basis for comparing these two control techniques is in terms of a simple stochastic supply-and-demand framework (Pierce and Thompson 1977, Kaminow 1977, Courchene 1977, Parkin 1978, Sparks in Courchene et al. 1979). Money supply and demand both depend on the interest rate. Interest-rate control means bypassing the money-supply function by setting the interest rate; the resulting variations to the money stock about its target level thus arise from errors in forecasting national income and the price level, as well as from disturbances to money demand given those variables. Base control, on the other hand, means setting a money supply function that is presumably an increasing function of the rate of interest; this money supply function, given the monetary base, is stochastic because of uncertainty about the money multiplier. Under base control, then, the money stock deviates from its target level by some combination of the variations in money supply and money demand relations. A comparison of base control and interest-rate control thus means comparing two variances: one is the variance of money demand, while the other is a combination of those of money demand and supply. Which is the appropriate monetary control technique thus appears to be a rather straightforward empirical question.

This representation of the issue has not been universally accepted; however, it has been claimed that base control is essentially impossible under the existing institutional setup. The argument supporting this claim hinges on the lagged accounting of required reserves: in various countries—including the U.S. and Canada—required reserves at any time are a fraction of past, not current deposits. Thus, it is argued,
"In any given statement week, the reserves required to be maintained are pre-determined by the level of deposits existing two weeks earlier...the level of total reserves in any given statement week is also pretty much determined in advance. Since banks have to meet their reserve requirements each week, and they can do nothing within that week to affect required reserves, that total amount of reserves has to be available to the banking system." (Holmes 1969, p. 74)

This argument has been elaborated by others, including Clinton and Lynch (1978) who argue that, because of this situation, "equilibrium requires the mutual coincidence of two predetermined variables and the system is overdetermined" (p. 15) if the central bank attempts to control the monetary base. This sort of argument is not crushing, especially since it is generally held that the authorities control interest-rates by manipulating the monetary base, which seems hard to reconcile with the notion that the base cannot be altered in such a way as to bring about a change in bank assets and liabilities. However, the argument requires careful scrutiny: it implies that, under lagged reserve accounting, the money supply function given the monetary base is not a valid concept.

More interesting, perhaps, is the fact that this theoretical argument against base control has been supported by empirical work. A couple of recent studies have tried to establish that under lagged reserve accounting the monetary base is of necessity endogenous: they have pressed this point by testing causality between money stock and monetary base (Feige and McGee 1977, Clinton and Lynch 1978). Such tests essentially involve correlating the innovation in the money stock with past innovations in the monetary base, and vice versa. Both studies found unidirectional causality from money stock to monetary base: this means that
\[ \text{Var}(B_t | B_{t-1}, B_{t-2}, \ldots, M_{t-1}, M_{t-2}, \ldots) < \text{Var}(B_t | B_{t-1}, B_{t-2}, \ldots) \]

whereas

\[ \text{Var}(M_t | M_{t-1}, M_{t-2}, \ldots, B_{t-1}, B_{t-2}, \ldots) = \text{Var}(M_t | M_{t-1}, M_{t-2}, \ldots) \]

In other words, the money stock provides additional information about future disturbances to the monetary base, while the base does not provide additional information about future disturbances to the money stock (see Clinton and Lynch, 1978, p. 18). The implication is that the proponents of base control are laboring under a misconception about the structure of the economy.

The present paper therefore begins by addressing two questions. First, can the issue of the appropriate money-stock control procedure be settled without recourse to any empirical evidence other than a casual observation of the institutional setup? In other words, does lagged reserve accounting clinch the matter? Second, can the empirical evidence hitherto assembled (or likely to be assembled in response to suggestions made in the literature) decide the issue, if the a priori argument against base control does not? In dealing with these questions, three main points will be established. First, it will be shown that lagged accounting of required reserves does not necessarily make base control of the money stock impossible. Secondly, it will be shown that the type of empirical work done by Feige and McGee and by Clinton and Lynch does not demonstrate the inferiority of base control: a counter-
example will be presented in which, even though the authorities can control the money stock deterministically under base control, causality tests would indicate unidirectional causality from money stock to base. A third, related point is that there is, in the issue of money-stock control techniques, an example of the so-called "Lucas problem". Evaluation of policy alternatives is hampered by the fact that the decision rules followed by private economic agents are not invariant under a change of policy regime (see Lucas 1976). This Lucas problem appears to invalidate most of the empirical work which has been done—and which has been suggested—to decide the base-control vs. interest-rate-control question.

In order to handle these issues as economically as possible, the analysis is stripped down to its bare essentials. The economy being considered is a stationary one, in that permanent income is constant. There are three assets: deposits, high-power money and debt. Demand for these assets depends on permanent income, and agents wish to hold a certain real quantity of each asset, measured in permanent prices. The policy rule being followed is to keep the expected money stock at some target level in the face of random disturbances. Bank portfolio behavior in the presence of such disturbances and of a given money-supply control technique is at the centre of the analysis.

In this world, banks face the effects of three disturbances. The first of these is a disturbance to the demand for money; the second affects the currency ratio; the third is a relative disturbance, which alters the allegiance of the public to particular banks. A bank ini-
tially experiences these disturbances through its own deposits, and cannot fully distinguish one from another. Later, information about the disturbances becomes available both to the authorities and to each individual bank, and policy can be altered accordingly.

The theory of the reserve management of the individual bank is traditionally formulated in a one-period context. A bank plans its reserves given its deposits; it is then faced with a random cash requirement, whose probability-distribution is given (possibly depending on the volume of its deposits). The objective of the bank is to minimize the sum of the expected costs of holding reserves and of being caught short by a cash drain (e.g. Morrison 1966, Modigliani, Rascâ and Cooper 1970, Gray and Parkin 1973). The holding of precautionary reserves results from expected-cost minimization where there is a choice among assets with different rates of return and different unit liquidation costs. The one-period framework involves an asymmetry between an initial portfolio allocation—assumed to be costless—and a subsequent set of liquidations to which costs are attached. In order to adapt such a model to the context in which portfolios are selected and reselected at different points in time, it is necessary to include some feature explaining such an asymmetry: in simple cases, this can be done by positing a lag in the settlement of transactions involving debts—by supposing, for instance, that when a loan is called or an earning asset sold, cash is not received until the week after the transaction, and that it is costly to obtain cash sooner in order to meet reserve requirements.
If reserves have no liquidation costs and bear a zero rate of
turn, the optimal holding of reserves is either zero\textsuperscript{12} or

\[ R^* = G^{-1}(1 - \frac{r}{c}) \]  \hspace{1cm} (5.2)

where \( G(x) \) is the cumulative distribution of the random cash requirement
\( x \), \( r \) is the rate of interest on loans and \( c \) the cost of liquidating
loans.\textsuperscript{13}

The simplest case to consider is a sort of temporary equilibrium in
which banks take their deposits as given and adjust their portfolios
accordingly. This is distinguished from a full equilibrium in that such
changes in the volume of deposits may result in changes in the volume of
deposits, generating a series of portfolio readjustments. Full equili-
brum implies that this process has worked itself out, so that no
further change occurs until there is some further disturbance. The use
of either temporary or full equilibrium for the analysis of the effects
of policy implies a judgement about the relative speeds of the central
bank's receipt of information about the banking system, on the one hand.

\textsuperscript{12} Banks choose the corner solution if \( G(0) \leq 1 - (r/c) \). In this case, some
of the results of the analysis still hold; however, if banks hold
zero excess reserves, the "Lucas problem" does not occur. In addi-
tion, fluctuations in the money supply will be larger if excess
reserves are zero, since then only the currency ratio determines equili-
brum for the system in the face of disturbances, so that larger
movements of deposits are associated with a given disturbance.

\textsuperscript{13} Borrowing from the central bank does not appear explicitly in the
model, but is implicitly represented in terms of the possibility that
excess reserves be negative. The model is best interpreted as one in
which the central bank controls unborrowed reserves, but in which the
banks can borrow from the central bank at a rate \( c \).
one hand, and the completion of the adjustments of bank assets (as in the banking-system-multiplier process) on the other. In the three-asset case, temporary equilibrium can be presented in the following terms: on each Monday, banks open for deposit business with the public, and the interest rate adjusts to clear the markets for money, the monetary base and loans. Given the volume of deposits with which each bank finds itself, it makes a decision to expand or to contract loans, a decision which would result in a different volume of debt outstanding at the beginning of the following week. The authorities also place orders to buy or sell debt, dealing either with the banks or the general public.

For the sake of illustration, specific representations are given for the public's demand for currency and deposits, and for the probability distributions of the disturbances. The public's demand for money (currency plus deposits) is assumed to be

\[ M_t^d = \kappa R_t - \delta \nu_t \]  \hspace{1cm} (5.3)

where \( \kappa \) and \( \delta \) are constants and \( \nu \) is a random disturbance. Their demand for deposits is

\[ D_t = \gamma M_t \]  \hspace{1cm} (5.4)

In the standard banking-system-multiplier analysis, the latter adjustments work themselves out only in infinite time, or in metam.
where \( \xi \) is a random variable. Furthermore, the \( i \)th bank receives a share \( \alpha \) of total deposits. It is assumed that the three random variables are log-normally distributed:

\[
\xi \sim \Lambda(0, \sigma_\xi^2)
\]

\[
\gamma \sim \Lambda(\ln \gamma, \sigma_\gamma^2)
\]

\[
\alpha^i \sim \Lambda(\ln \alpha^i, \sigma_\alpha^2)
\]

If all three of the random variables reflecting tastes are serially uncorrelated, the current period's disturbance need not affect the probability distribution of next period's deposits, unless the money stock is allowed to change. The individual bank's deposits are

\[
D_t^i = \alpha^i_t \gamma_t M_t.
\]

It can be shown that in this framework, if a particular policy rule is introduced, \( M_t = M^* \) can be a rational-expectations equilibrium— in other words that if the banks believe that the authorities are keeping the money stock at its target level, it will be possible for the authorities to do so. In the current period, the total money stock is fixed in the temporary-equilibrium setup, since the public's total wealth is fixed.

**This obviously cannot be strictly true, since \( \sum \alpha^i = 1 \), and since \( \sum i \leq 1 \). The assumption of lognormality is thus only an approximation, assumed (for expository purposes) to hold locally.**
and the volume of debt outstanding is determined by the decisions made in the previous period (determined, that is, by the decisions of the banks and the authorities to buy or sell particular amounts of debt at prevailing prices). Thus, the current period's disturbances only affect the money stock in the next period, as these decisions affect the public's money holdings and its net indebtedness.

What will be the banks' desired reserves under these circumstances? If required reserves are a fraction of current deposits, a bank's cash requirements in the next period will be

\[ x_{t+1}^i = \rho D_t^i - (D_{t+1}^i - D_t^i) - Q_{t+1}^i \]

\[ = D_t^i - (1 - \rho) D_{t+1}^i - Q_{t+1}^i, \]

where \( Q_{t+1}^i \) is the change in next period's reserves resulting from any transactions between the authorities and the bank, and \( \rho \) is the required reserve ratio.

If the authority could control the money stock precisely, the bank's prospective cash requirement would be

\[ x_{t+1}^i = D_t^i - (1 - \rho) x_{t+1}^i. \]

---

**The implicit assumption which is made here is that the authorities' transactions are not subject to the same payments lag as are those made by the commercial banks.**
The first and third terms of (5.8) are known to the bank at time $t$; the second term is lognormally distributed such that

$$E(\ln(1 - \rho)D^{i}_{t+1}) = \ln(1 - \rho) + \ln \alpha^i + \ln \gamma + \ln M^*$$

and

$$\text{Var}(\ln(1 - \rho)D^{i}_{t+1}) = \sigma^2_y + \sigma^2_{\xi}$$

It follows from (5.2), (5.8) and (5.9) that the desired reserves of the $i$'th bank will be

$$R^i_t = D^i_t - Q^i_{t+1} - F^{-1}(\frac{r^*_t}{c})$$

where $F(*)$ is the cumulative lognormal distribution with parameters given in (5.9). If the authorities were to follow a rule of altering the banks' reserves in order to offset the effects of the previous period's interest-rate changes, that is, setting

$$Q^i_{t+1} = F^{-1}(\frac{r^*_t}{c}) - F^{-1}(\frac{r^*_{t+1}}{c})$$

where $r^*$ is the interest-rate that would be consistent with $M^* = M^*$ when $E_t \chi = 0$, reserves would be
\[ R^i_t = D^i_t - F^{-1}(\frac{r^*_t}{c}) \]  

so that

\[ R^i_t - R^i_{t-1} = D^i_t - D^i_{t-1} \]  

(5.13)

i.e. each bank will simply allow its reserves to vary to ride out the in- or outflow of cash which occurs in each week. As a result, the public's net indebtedness will remain undisturbed, and the money stock will remain at the target level as was originally assumed. This implies that the interest-rate must adjust in order to clear the market for money in each period, so that

\[ r_t = \left( \frac{k_t}{H_t^*} \right)^{1/\delta} \]  

(5.14)

Such a policy regime can be characterized as "base control" in the sense that the monetary base is set in each period at the level which is appropriate for minimizing the variance of the money stock around its target level in each period, and in the sense that the interest rate is not stabilized but is allowed to adjust to the level consistent with this monetary target.°

°It is not always clear what is meant by "base control", the characterization here is obviously not the only conceivable one. It is noteworthy that, in this model, adjusting the monetary base in order to minimize the variance of the money supply required feedback from
If, on the other hand, the authorities decided to follow an interest rate-control rule under these circumstances, deterministic control of the money supply would no longer be possible. In this model, the authorities can only peg the interest rate at the desired level by posting a perfectly interest-elastic demand for debt at a fixed interest-rate. The interest rate chosen may be that which sets \( E_{t-1} M_t = M^* \). This interest-rate will be

\[
\begin{align*}
    r^* &= \left( \frac{k}{M^*} \exp(1/2 \sigma^2) \right)^{1/d} \\
    (5.15)
\end{align*}
\]

which implies a money-stock of

\[
M_t = M^* \exp(-1/2 \sigma^2) \epsilon_t \\
(5.16)
\]

As a result, the demand for reserves by the \( i \)th bank is now

\[
D_t^i = D_{t+1}^i - H_{c}^{-1} (r^*) \\
(5.17)
\]

where \( H(*) \) is the cumulative lognormal distribution with parameters

\[
E(\ln(1 - \rho) D_{t+1}^i) = \ln (1 - \rho) + \ln \alpha^i + \ln \gamma + \ln M^* - 1/2 \sigma^2 \\
\]

the interest rate; however, the feedback required is in the opposite direction from that required to smooth the interest rate (this feature, of course, results from the fact that there are only monetary disturbances in the model).
and

\[ \text{Var}(\ln(1 - p)D_t^1) = \sigma_e^2 + \sigma_f^2 + \sigma_\alpha^2 \] (5.18)

A comparison of (5.17) and (5.18) with (5.9) and (5.12) shows that in general, interest-rate-control does not give rise to the same demand for the monetary base as does base control. Thus, even in this extremely simplified model, there is an instance of the "Lucas problem": the monetary base-money supply relationship observed under an interest-rate rule will not be useful either in formulating the feedback rule appropriate for controlling the money supply via the monetary base or in assessing the probable effectiveness of a base-control policy. The sources of the Lucas problem in this context are two: firstly, the knowledge that the money supply is being controlled according to a pre-announced rule is useful to banks in predicting their future deposits. ** Secondly, the particular technique used to control the money supply affects the probability distribution of deposits, as well as allowing a policy regime to create a dependence of future changes in reserves on current deposits and interest rates. Each bank's desired reserves depend on the position and shape of the probability distribution of its future cash requirements. Thus, the demand for high-power money will be conditioned by the knowledge that a constant-growth-rate rule for the money supply is in effect, and that a particular technique is being used to implement it.

What difference would lagged accounting in calculating required reserves make in this model? Under base control, it would change the set of actions to be followed by the central bank, since lagged accounting would otherwise affect the time path of banks' reserve positions. Under lagged accounting, a bank's anticipated cash requirement is

\[ q_t^i = pD_{s,t+1}^i - (D_t^i - D_{t+1}^i) - Q_t^i \]

(5.19)

where \( D_{i,t} \) denotes the statutory deposits on which required reserves are based—i.e. the deposits outstanding in the previous accounting period. The authorities can still control the money supply if these statutory deposits are known to it (as they must be in order to enforce the reserve requirement, in this discrete-time model): they can set

\[ Q_{t+1}^i = D_{s,t+1}^i + J^{-1}(\frac{r^*}{c}) \]

(5.20)

so that

\[ R_t^i = D_t^i - J^{-1}(\frac{r^*}{c}) \]

(5.21)

and once again the money stock can be kept undisturbed at \( M^* \). However, the demand for reserves will be different under lagged accounting: the similarity of (5.21) and (5.12) is misleading, since \( J(\cdot) \) is a different function from \( F(\cdot) \): it is the cumulative distribution of a lognormal variable with parameters
\[ E(\ln D_{t+1}^i) = \ln M^* + \ln \alpha^i + \ln \gamma \]

and

\[ \text{Var}(\ln D_{t+1}^i) = \sigma^2 + \sigma^2 \gamma \]

Thus, one difference which is made by lagged reserve accounting is that it removes the "built-in stability" which current reserve accounting lends to a bank's cash requirement: a change in deposits changes required reserves and actual reserves in the same direction under current reserve accounting, but not under lagged reserve accounting; thus, lagged reserve accounting means that a larger change in a bank's reserve position results from a given change in deposits. A one-dollar outflow of deposits results in a one-dollar deterioration of a bank's reserve position under lagged accounting, while it only causes a \((1 - \rho)\) dollar decline in a bank's free reserves under current accounting. This difference would tend, ceteris paribus, to lead banks to hold larger cushions of excess reserves under lagged reserve accounting. The model of precautionary behavior considered indicates that banks hold enough

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This is somewhat different from the argument made against lagged reserve accounting subsequent to its 1969 introduction in the U.S. Coats (1976), for instance, argued that lagged reserve accounting makes it more difficult for banks to adjust their excess reserves toward the desired level: the reason for this is that only actual reserves, not desired reserves (at least not required reserves), can be changed by the individual bank within the current period. Coats argued that, as a result, lagged reserve accounting may be disruptive for commercial banks.
reserves in order to achieve the same target probability of being "caught short" under each regime; when reserve accounting is lagged, achieving this target probability entails holding more precautionary reserves than will be sufficient for this purpose under current accounting. This larger cushion, in turn, enables banks to ride out fluctuations in deposits while only incurring the same probability of falling short of their requirements in a given period. Thus, it does not appear, a priori, that lagged reserve accounting need make base control of the money supply impossible; base control need not lead to financial chaos, even under the existing institutional setup. Rather, it appears likely that, if base control were adopted, and if policy were carried out in a credible and predictable manner, the decision-rules of banks would alter in a stabilizing way.

If an interest rate-control policy is adopted, the effect of lagged reserve accounting is quite similar. Here, there will be a change in the particular open-market-operations which would be implied by the rule of keeping the interest-rate constant, but the appropriate setting of the interest rate is unaltered by lagged reserve accounting. Lagged accounting will, however, change the observed demand for high-power money: as in the base-control case, precautionary reserves will be increased because a greater change in the cash requirement now corres-

The fact that excess reserves have decreased, rather than increased, since lagged reserve accounting has been introduced must surely be attributable to other circumstances: the development of the Federal Funds Market and increasing sophistication of the banking system in the U.S., and possibly the way in which monetary policy is implemented in Canada.
ponds to a given change in deposits. The reserves demanded will be

\[ B_{S,t+1} = D_{S,t+1}^i + D_t^i + K^{-1}(r^*) \]  \hspace{1cm} (5.23)

Here, \( J(*) \) is the cumulative lognormal distribution with parameters

\[ E(\ln D_{t+1}^i) = \ln M^* - \frac{1}{2} \sigma_e^2 + \ln \alpha_i + \ln \gamma \]

and

\[ \text{Var}(\ln D_{t+1}^i) = \sigma_e^2 + \sigma^2 + \sigma^2 \gamma \]  \hspace{1cm} (5.24)

Thus, with lagged reserve accounting, too, the reserve-demand functions of private banks are different, depending on whether the monetary growth rule is being implemented by means of base- or interest rate-control.

What would be observed, in this model, of the causal links between the money stock and the monetary base? Under interest rate-control and lagged reserve accounting, the following correlations would be found:

\[ \text{Cor}(B_{t-1}, M_t) = 0. \hspace{1cm} (5.25) \]

\[ \text{Cor}(M_{t-1}, B_t) = \frac{\rho}{\ln \gamma} \exp(\sigma_e^2) \exp(\sigma^2 - 1)^{1/2} \]

\[ * [\exp(\sigma_e^2) - 1 + \rho \exp(\sigma_e^2 - \ln \gamma)(\exp(\sigma_e^2 + \sigma^2) - 1)]^{-1/2} > 0 \]
$$\text{Cor}(M_t, B_t) = \left[ \exp(\sigma^2) - 1 \right]^{1/2} \left[ \exp(\sigma^2) - 1 + \rho^2 \exp(\sigma^2 - 2 \ln \gamma) \left( \exp(\sigma^2 + \sigma^2) - 1 \right) \right]^{-1/2} > 0.$$  

By comparing these correlations with the definition of causality (equation (5.1) above) while noting that in this model fluctuations in the money supply and monetary base will be serially uncorrelated, one can infer that under interest rate-control and lagged reserve accounting the money stock causes the monetary base. It is clear, then, that these causal relationships have no bearing on the possibility of base control under the same assumptions under which the pattern of cross-correlations is such that money stock causes monetary base and not vice versa, the authorities can control the money stock deterministically using base control.

Thus, a very simple model has been used to argue two main points. First, the existence of lagged reserve accounting does not, in and of itself, make it impossible to use the monetary base as an instrument for controlling the money supply. Furthermore, base control does not necessarily lead to large fluctuations in deposits and borrowings under lagged reserve accounting: for one thing, the information that reserve accounting is lagged can be used by the authorities in choosing the appropriate time path for the monetary base; secondly, the analysis of precautionary demand for reserves predicts that under lagged reserve accounting banks tend to hold larger precautionary reserves, with a view to keeping the probability of being caught short at some target level, a
level which depends only on the interest-rate and on the cost incurred by a bank when it runs short of reserves.

Secondly, although the invalidity of the a priori argument against base control suggests a need to have recourse to empirical evidence, the evidence which has been assembled to judge the question turns out to be misleading. This is so because the decision rules which private agents follow—and in particular those embodied in banks’ demand for reserves—would not be the same under the alternative control techniques considered. Causality tests indicating that the monetary base is caused by the money supply may cast some light on the question of what policy regime has existed over some period in history; they do not, however, settle the question of whether the monetary base should be used as the instrument of monetary control. In addition, the more traditional method of comparing alternative policy instruments—comparing the reduced-form variances of the money stock in a descriptive macroeconomic model, when alternative policy instruments are taken as exogenous—is also vitiated by the change in the system’s structure to which the above analysis draws attention.

Therefore, the problem of choosing the appropriate technique of controlling the money supply is a real one: it is not rendered trivial by lagged reserve accounting. At the same time, it is not merely an empirical question which can be resolved on the basis of empirical evidence which has already been gathered, or which can be gathered and interpreted without the aid of additional theoretical insights. This suggests that some priority should be given to developing models of the
financial system which take due account of the implications of alternative policy regimes for the decision-rules of private agents. Since much important financial-market activity arises from choice under uncertainty, it is not valid to take demand and supply functions as given when the form of this uncertainty changes. Empirical evidence is essential, but it can be misleading when it is interpreted without an appropriate theoretical scheme.
BIBLIOGRAPHY


______, and Schwarz, Anna J. "How Feasible is a Flexible Monetary Policy?" Columbia University, 1973. (Mimeographed.)


Carr, Jack L., and Smith, Lawrence B. "A Suggestion for a New Monetary Indicator." Journal of Monetary Economics 1, no. 3 (July 1975): 363-68.


—— and ———. "Optimal Monetary Policy with Uncertainty." Board of Governors, Federal Reserve System, 1977. (Mimeographed.)


Friedman, Charles. "Some Theoretical Aspects of Base Control." Queens University, John Deutsch Round Tables on Economic Policy, 1980. (Mimeographed.)


Howitt, Peter W. "Inflation is Not Self-Sustaining in Simple Models of Monetary Equilibrium with Rational Expectations." University of Western Ontario, 1979. (Mimeographed.)


...and Laidler, David E.W. "Recent Canadian Monetary Policy: A Critique." Conference on Issues in Canadian Public Policy, Queens University, 1979. (Mimeographed.)


Lane, T.D. "Uncertain Timing of Receipts and 'Partial Adjustment' in the Demand for Money: A Note." University of Western Ontario, 1982. (Mimeoographed.)

Laurent, Robert D. "Reserve Requirements: Are They Lagged in the Wrong Direction?" *Journal of Money, Credit and Banking* 11, no. 3 (Aug. 1979): 301-10.


"Monetary Policy when Prices are Costly to Adjust." University of Western Ontario, 1980. (Mimeographed.)

; Gray, M.R.; and Barrett, R.J. "The Portfolio Behaviour of Commercial Banks." University of Essex, 1969. (Mimeographed.)


Rowe, P.M.N. "The Rationality of Deontological Methods: A Theory of Property and Communication." University of Western Ontario, 1980. ( Mimeographed.)


__"Policy Choice in Approximate Models." University of Minnesota, 1974a. (Mimeographed.)__


__"Conditions for Unique Solutions in Stochastic Macroeconomic Models with Rational Expectations." Econométrica 45, no. 6 (Sept. 1977): 1377-85.__


__"Controllability of the Money Supply." Bank of Canada, 1978 (Mimeographed.)__


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