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# A Three Phase Temperature-density Model To Simulate And Compare Potential Snowmelt Runoff

K.H. Rahman

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**LA THÈSE A ÉTÉ  
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A THREE PHASE TEMPERATURE - DENSITY  
MODEL TO SIMULATE AND COMPARE  
POTENTIAL SNOWMELT RUNOFF

by

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Department of Geography

Submitted in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy

Faculty of Graduate Studies  
The University of Western Ontario  
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## ABSTRACT

A three phase temperature - density model was developed to simulate the temperature patterns of a snowpack and generate snowmelt. This one dimensional model is based on the heat flow equation with air and ground temperature as the two boundary conditions. With four measured input variables the model permits the computation of the temperature patterns, density, heat flux and sublimation of snow over specified time intervals.

The model was applied to the Medway drainage basin near London, Ontario. Six snow courses were operated for the winter of 1977-78 to collect information on snow temperature, density and water equivalent at various depths in the snowpack. The model was tested on fifty five sampling points of which six were randomly selected, one from each snow course to present the results. The results indicate that:

1. The model can simulate the temperature - density patterns in a snowpack with a high degree of accuracy.
2. It is possible to identify different kinds of metamorphism in the snowpack.
3. That the computed heat flux is a good indication of the thermal budget of a snowpack over time.
4. A negative heat flux increases density in the top layers of a snowpack. The reverse is true for a positive heat flux except for cases where melt is produced at the top.

layers. Under such circumstances, especially during the spring melt season, phase transition of water is more important and active than transference of heat by conduction.

5. Sublimation is a function of the temperature gradients in the snowpack and can be estimated quantitatively.
6. There is a high correlation between measured and simulated runoff volumes.
7. Over shorter time intervals, as for the spring runoff period, March 16 - April 15, the melt predicting capability of the model is dependent on the characteristic temperature.
8. The model is not unique in terms of geographical location and can be easily tested in any drainage basin.
9. To improve the efficiency and accuracy of the model it is suggested that the role of other variables be considered.

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## CHAPTER I

### INTRODUCTION

Snowmelt and snowmelt runoff are usually the most significant hydrological effect in watersheds in Southern Ontario. However, there have been few studies which have examined or simulated snowmelt studies in this area. In general, snowmelt studies tend to explain the development and application of hydrologic models. The principal aim of most of these models is to predict and estimate the amount of runoff which can be expected from different basins. This is accomplished by the use of energy balance and aerodynamic principles which take into account the factors that control the physical properties of the snowpack.

Two general types of models have evolved, those based on simple empirical relations and those based upon energy flux theory. The empirical models suggest index techniques and research in this general area can be linked back to studies in the early part of this century when the turn in physics and meteorology was becoming more dynamic (Baker, 1917; Rolf, 1915). These models were based on the physical approach to snowmelt studies using equations which express the physical relation of wind, air temperature and water vapor pressure as they interact to make energy available for snowmelt and evaporation (Garstka, et al., 1958). Later studies by the U.S. Corps of Engineers (1956) have shown that such relationships for calculating snowmelt can be utilized according to the characteristics of the basin and the

available meteorological data. In most cases emphasis was placed on the mechanics of meltwater flow and the thermal characteristics of the snowpack were considered to be of secondary importance.

The energy flux models calculate snowmelt upon concepts well based in the theories of physics, mathematics and thermodynamics. Jumikis (1966, 1977) in his engineering approach suggested two general theories which can be applied to problems of heat flow. These theories are based on the natural laws that 1) heat flows from regions of higher temperature to regions of lower temperature, 2) the amount of heat in a differential element of snow or any other material is proportional to its mass and its temperature and 3) the rate of heat flow across an area is proportional to its size and temperature gradient. Fourier's (1878) theory of the steady state conduction of heat was that the rate of heat flow per unit of time in a homogeneous, isotropic medium is proportional to the thermal conductivity, cross sectional area through which the heat flows, and the rate of change in temperature with respect to the thickness of the medium through which the heat is flowing. When the flow of heat in the medium has reached a steady state, the temperature at each point in the medium remains constant and is a function of position within the medium and not of time. This theory can be used to calculate the rate of heat flow in a snowpack but because of its limited assumptions it does not hold true. This is particularly so when the temperature at the surface of the snowpack is assumed not to fluctuate over time.

The process of heat flow in unsteady conditions involves the variations in temperature as a function of both time and position. The assumption here is that the temperature at a point in space is not only a function of its location but also of the elapsed time since a change in the rate of heat flow took place. The solution to the differential equation (see below) for one directional heat flow considers that a) the snowlayer is an infinite medium; b) the heat flows in only one direction, c) the snowlayer has a given initial temperature distribution and d) the boundary conditions of the snowlayer are governed by the temperature conditions at the top and bottom of the layer. The general form of the equation for unsteady state conditions is,

$$\frac{\delta^2 \theta}{\delta x^2} = \frac{1}{\alpha^2} \frac{\delta \theta}{\delta t} \quad \dots\dots 1.1$$

where the rate of change of temperature  $\frac{\delta \theta}{\delta t}$ ; varies with time as well as with the position of the point concerned  $\frac{\delta^2 \theta}{\delta x^2}$ ;  $\alpha^2$  is the thermal diffusivity of the medium. Traditionally the users of such models (Kuzmin, 1972; Kondrat'eva, 1954; Yoshida, 1963; Colbeck, 1978) have shown that such techniques are more useful to establish relationships between thermal conductivity, specific heat, thermal diffusivity, density and temperature. The application of such a model to compute snowpack temperatures is constrained by assumptions necessary for their operation and the lack of data.

This research reports on the development and testing of an analytical model for calculating snowpack temperatures, densities

and snowmelt. It incorporates the unsteady state heat equation in the modelling procedure. In comparison to existing models, this model is of value because it includes not only the evaluation and quantification of the thermal characteristics of the snowpack but also a comparison of the snowmelt generated by the model and the actual discharge. The specific objectives of this study were twofold:

1. Examination of the thermal characteristics of the snowpack with
  - a) an estimation of quantities and the direction of movement of snow in the sublimation process,
  - b) the metamorphism of snow by processes such as sublimation, melting and refreezing and by pressure effects, and
  - c) the estimation of heat flux
2. Calculation and comparison of potential snowmelt runoff through
  - a) an analysis of hydrographs and
  - b) regression analysis.

The three-phase model was applied to the Medway drainage basin located north of London, Ontario where data was collected for the winter of 1977-78 in six snow courses. The related meteorological data were obtained from London Airport located close to the basin and snowmelt was compared to the discharge measured from the basin by the recorder operated by the Water Survey of Canada.



The results indicate that the model can simulate snowpack thermal conditions on an hourly basis and compute snowmelt amounts which are strongly correlated with the measured discharge. The accuracy of the model is dependent on the strictness of the assumptions.

The thesis is divided into six chapters. Chapter two provides a background to research related to snow studies and a brief review of the state of the art in snowmelt modelling with emphasis on heat flux theories. In chapter three an outline is given of the modelling procedure along with the formulated empirical relationships. The field component of the research is discussed in chapter four with a detailed description of the snow course surveys. Chapter five presents the results of the model on a phase by phase procedure and shows the comparison between snowmelt and runoff. In the last chapter some general conclusions are derived along with the strengths and weaknesses for future research in this field.

## CHAPTER II

### RESEARCH ON SNOW

#### 2.1 Introduction

All research on snow can be categorized into three main classes: 1) Formation of snow 2) Characteristics of the snowpack, i.e. temperature, density, water equivalent, depth, as they relate to changes over time 3) Snowmelt. Each of these are interrelated and when studied together can provide an overall picture of snow research from the viewpoint of thermodynamics and hydrology. The first section of this chapter provides a descriptive review of snow and its formation. This is followed by a discussion of the nature of the snowpack, its thermodynamic properties plus the terminology used. Section three examines the snowmelt-runoff relationships and the present state of the art in snowmelt simulation through deterministic component models. Finally, section four outlines the theory behind heat flow models and the way in which they will be used in this thesis.

#### 2.2 Snow and its formation

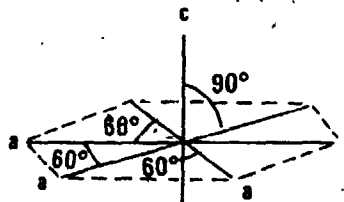
Snow is a form of precipitation which begins in the atmosphere as a water vapor and changes its form as it falls through the atmosphere. By the time it reaches the earth's surface it is an aggregate of ice crystals, the composition and characteristics of which depend on the atmospheric conditions present at that time.

7

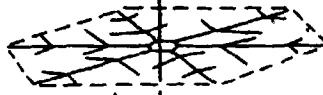
Nakaya (1954) studied the nature of snow crystals and has shown the conditions of formation. LaChapelle (1970) notes that the structural characteristics of a frozen molecule of water takes a "crystal lattice" shape. It is because of the arrangement of the lattice in ice which generates solid forms with hexagonal symmetry in one plane. This plane is called the crystallographic a-axis plane and is fixed with respect to the lattice. In this plane there are three such a-axes, each separated by  $60^\circ$  from the next. The principal axis of the crystal which is called the c-axis is at right angles to the plane of hexagonal symmetry. Figure 2.1 shows the arrangement of these axes. Each crystal is a different shape primarily because of the relative growth along each individual axis. This relative growth is dependent upon the degree of supersaturation under different temperatures. For example, Nakaya (1954) has demonstrated that needle shaped crystals form only in a narrow range of temperatures around  $-5$  to  $-8^\circ\text{C}$ , but over a wide range of supersaturation. In 1966 Magono and Lee published a classification for solid precipitation which embraces about ninety nine percent of the snow crystals observed in nature. Their classification makes the distinction between different combination types and recognizes the various degrees of riming.

It is seen from the above discussion that the mechanism of snow formation and the crystal growth patterns is rather complex and depends entirely upon the environmental temperatures. The relation between the snowfall rate and the quantity deposited

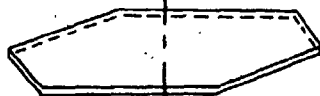
Crystal Axes



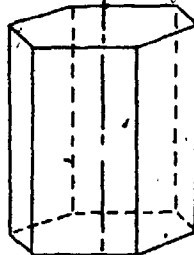
Star



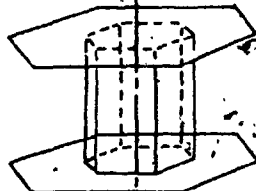
Plate



Column



Capped Column



Scroll (Cup)



**FIGURE 2.1 STRUCTURAL ARRANGEMENT OF THE PRINCIPAL TYPES OF SNOW CRYSTALS IN RELATION TO THE CRYSTAL AXES OF ICE**

Source: La Chapelle, E.R. (1970). Field Guide to Snow Crystals, p. 6.

depends on various factors (UNESCO/IASH/ WMO, 1970) but the areal variability of snowfall within a storm is usually small. McKay (1972) has suggested that most of the snowfall produced in South Western Ontario is of cyclonic origin except for areas on the lee of the lakes where convective precipitation increases snowfall in the well known snowbelt areas. For the Medway drainage basin the spatial variation in snow cover for the 1977-78 winter season is shown in Figure 2.2. The slope of the surface in these three dimensional maps is dominantly from the north to the south indicating the fact, that the northern fringes of the basin may be under the effect of the snowbelt. Table 2.1 shows that the average snowdepth was maximum during early March resulting in a snowpack volume of  $95.18 \times 10^6$  cubic metres over the 180 square kilometer drainage basin.

2.3 The nature of the snowpack

Once the snow is on the ground, its characteristics change, notably in terms of its density which is related to the water holding capacity of the snowpack. These changes which are variable in time and space are also involved in the metamorphism of the snow cover. Simply stated it is the transformation of snow by processes such as internal sublimation, melting and refreezing and by pressure effects. Changes in temperature are singled out to be the most important control of these transformations.

To understand the stratified nature of the snowpack an

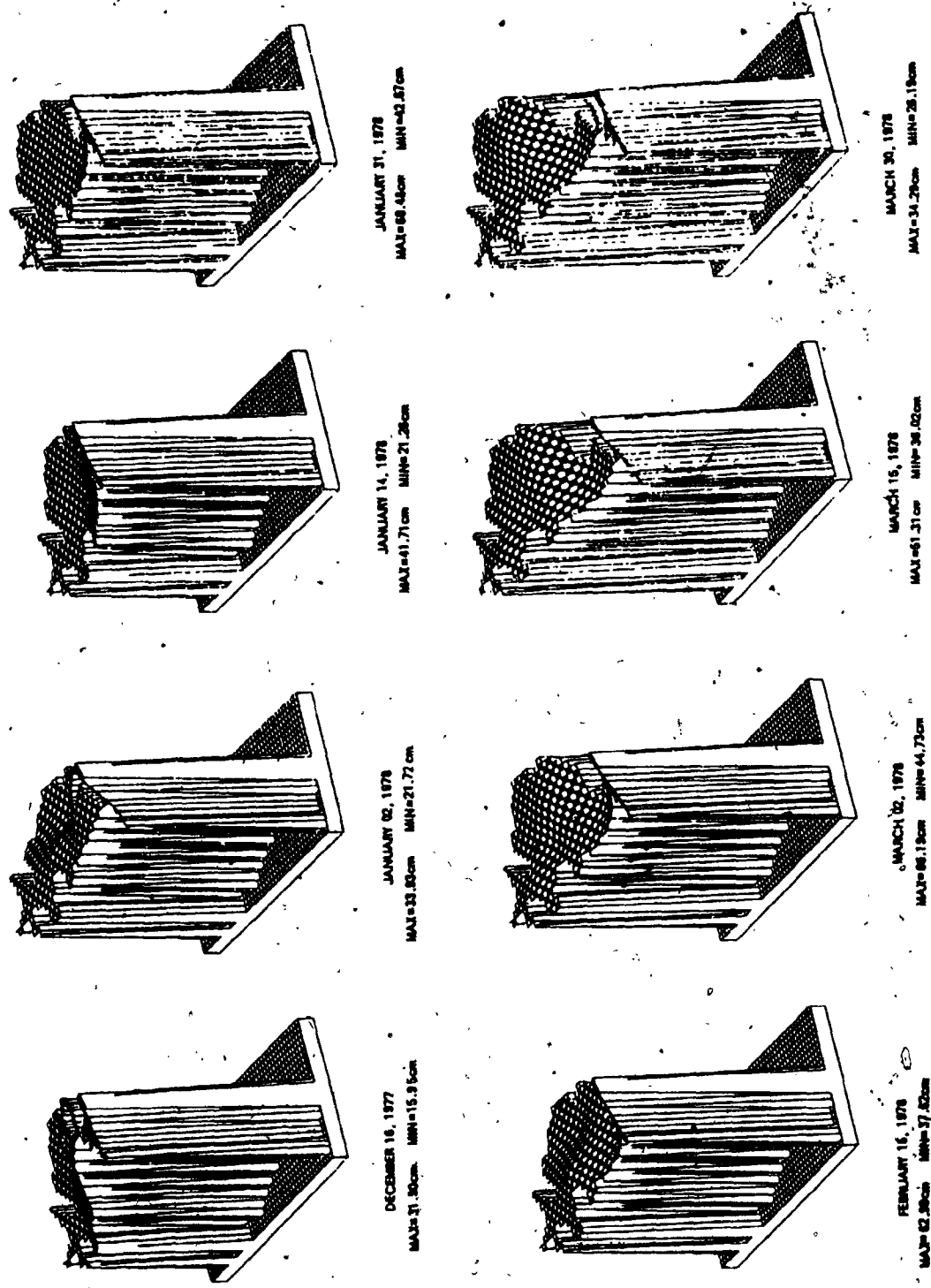


FIGURE 2.2 SNOW DEPTH OVER MEDWAY BASIN

Table 2.1

Spatial and Temporal Characteristics of Snow  
over Medway Basin

	Average W.E.* in cm	Snowpack Volume in cubic metres ( $\times 10^6$ )	Average Snowdepth in cm
Dec. 16, 1977	7.05	39.13	21.73
Jan. 02, 1978	5.39	45.41	25.23
Jan. 14, 1978	7.92	46.21	25.67
Jan. 31, 1978	13.59	86.19	47.88
Feb. 15, 1978	13.09	80.62	44.79
March 02, 1978	14.78	95.18	52.88
March 15, 1978	15.96	80.43	44.69
March 30, 1978	12.28	55.78	30.99

\* W.E. = water equivalent

extensive amount of literature can be cited regarding the theory of densification of snow. Bader (1962), Colbeck (1973, 1979), Perla (1978a), deQuervain (1963), Sommerfeld, et al. (1970), Wakahama (1968, 1975), Yoshida (1963), Muller (1971) have all shown various procedures of numerical computations and relationships in depth-density profiles. In the absence of further precipitation average snowpack density increases with time. Short reversals of this trend may occur due to lowering the mean density of the snowpack. Average densities of snow in its various states have been given by Seligman (1962) and are shown in Table 2.2.

The relationship between density and temperature has not yet been clearly established. Keeler (1967) states that densification is not affected by temperatures in the range  $-1$  to  $-10^{\circ}\text{C}$  but is inversely proportional to grain size. Marbouty (1979) relates crystal growth as an increasing function of temperature. According to him, crystal size is a decreasing function of density with a lower limit of  $0.15 \text{ gm cm}^{-3}$  below which destructive metamorphism occurs. The degree to which the density of new snow is below that of ice is controlled by the air space between the crystals (Colbeck, 1978). The major factors contributing to the size of this space are i) crystal shape, ii) crystal size and iii) compaction.

The U.S. Corps of Engineers (1956) have suggested several physical processes which contribute to the diagenesis of snow.



Table 2.2  
Snow Densities

<u>Snow Type</u>	<u>Density (gm cm<sup>-3</sup>)</u>
Wild snow	0.01 - 0.03
Ordinary new snow immediately after falling in still air	0.05 - 0.065
Settling snow	0.07 - 0.19
Settled snow	0.20 - 0.30
Snow very tightly toughened by wind immediately after falling	0.063 - 0.08
Average wind toughened snow	0.28
Hard Wind slab.	0.35
New firn* snow	0.40 - 0.55
Advanced firn snow	0.55 - 0.65
Thawing firn snow	0.60 - 0.70

\* snow consolidated partly into ice

The most important factors in relation to this research are:

- i) temperature variation within the snowpack
- ii) heat exchange at the surface of the snowpack and
- iii) heat exchange at the ground surface

These processes produce changes in the density, water holding capacity and temperature of the snowpack which are of considerable hydrologic importance. When snow is deposited in layers it is stratified with distinctly different densities. The upper surface is subjected to weathering effects of radiation, rain and wind, the under surface to ground heat and the interior to fluctuating conditions at the two boundaries plus the percolation of water and vapor. When the ground is unfrozen in early winter a granular layer is formed at the ground surface. Bader, et al. (1954) and de Quervain (1963) state that the change in form of the snow crystals is a result of sublimation and the movement of water. The temperature differences in the snowpack create a flow of heat and water vapor which result in rounding and growth of snow crystals. This flow tends to equalize the temperature and vapor pressure within the snowpack making it more and more homogeneous in nature as the winter progresses.

#### 2.31 Kinds of metamorphism

Muller (1971) distinguishes five major types of metamorphism due to different processes:

- (a) mechanical damage during deposition (deposition metamorphism)
- (b) change of surface free energy under homogeneous temperature conditions (destructive metamorphism)
- (c) transport of vapor under varied temperature conditions (constructive metamorphism)
- (d) melt-freeze metamorphism and
- (e) pressure densification.

#### Deposition metamorphism:

Modification of the snowflake may begin from the moment of formation and continue throughout its descent and deposition. It is presumed that the weather conditions during and immediately following deposition have a direct link to this process. Wind driven or packed snow may have densities ranging up to  $0.30 \text{ gm cm}^{-3}$  whereas they are less than  $0.10 \text{ gm cm}^{-3}$  with no wind.

#### Destructive metamorphism:

In this process dry snow crystals decompose from their well developed form into single fragments of small uniform shapes. Yen (1969) states that the end product of this kind of metamorphism usually consists of fine grained snow of density varying between  $0.15$  and  $0.25 \text{ gm cm}^{-3}$ . The system produces equ-dimensional spherical grains less than 1 mm in size. Müller (1971) and de Quervain (1963) also suggest that 'sintering' may take place

during destructive metamorphism. During this process the ice spheres initially in point contact with each other, adhere and bond.

#### Constructive metamorphism:

In the case of temperature gradient metamorphism, evaporation and condensation produce growth of the crystals and, as such the number of crystals per unit volume decreases. The rate of transformation is slow if the density is high (Yen, 1969). If it is lower than  $0.30 \text{ gm cm}^{-3}$  the process can become rapid because of accelerated air flow. A very distinctive type of snow known as depth hoar with a density of about  $0.40 \text{ gm cm}^{-3}$  is the end product of this process.

#### Melt-freeze metamorphism:

In most 'warm' snowpacks, freeze-thaw activity takes place. Under such conditions water which is melted in the snowpack refreezes giving the crystals a coating of ice with a strong bonded structure. The crystals grow to a maximum size of 3 mm and composite grains to about 15 mm. This kind of metamorphism is most active during the spring melt season and produces the well known 'ripe' snow of the season.

#### Pressure metamorphism:

Pressure densification occurs primarily in the lower layers of the snow where the weight of the snow along with the compression, and compaction by wind at the surface leads to increased densities. Density values range between 0.55 and 0.58 gm cm<sup>-3</sup> for snow formed under such conditions. In some cases further densification may create viscous-plastic deformation of the crystals and their bonds leading to densities around 0.80 to 0.83 gm cm<sup>-3</sup>. At the upper limit of this density range, transference of water and vapor ceases and as such the material is classed as ice. If further packing and expulsion of air bubbles take place in this kind of material, the result is pure ice with a density of 0.917 gm cm<sup>-3</sup>.

#### 2.32 Heat transfer in the snowpack

The main features of the snowpack are depth, density, temperature and water equivalent. Besides density the other variables can be measured directly. The density of snow is a function of water equivalent and snow depth.

Knowledge of the metamorphism processes in snow is needed to understand the heat transfer process in the snowpack. Generally, during the winter the temperatures in the snowpack are below freezing and the direction of heat flow is dependent on the boundary conditions of the snowpack. During the spring runoff season the snowpack becomes isothermal at 0°C and the incoming heat is used to melt the snow. The U.S. Corps of Engineers (1956)

and others have shown that the transfer of heat may be accomplished by various processes such as conduction, convection, sublimation or percolation and refreezing of liquid water from melt in the top layers of the snowpack.

It has been demonstrated earlier that metamorphism is a stage-time procedure and as such the determination of the thermal properties of snow is complex. The important thermal properties are:

- a) Specific Heat ( $c$ ) - which is the amount of heat required to raise the temperature of one gram of snow one degree Celsius.
- b) Thermal Conductivity ( $\lambda$ ) - which is a measure of the rate of heat transfer. It is expressed as calories transmitted through one cubic centimeter of snow in one second when the temperature difference between two opposite faces is one degree Celsius.
- c) Thermal Diffusivity ( $\alpha^2$ ) - which is the temperature conductivity of snow and is expressed as,

$$\alpha^2 = \lambda/\rho c$$

where  $\rho$  is the density of snow. It can also be stated as the temperature change in degrees Celsius that occurs in one second when the temperature gradient is one degree Celsius per centimeter for each centimeter depth.\*

The factors which affect the thermal conductivity and diffusivity of snow are: a) the structural and crystalline character of the snowpack, b) the degree of compaction, c) the

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\* Since these definitions are very general they have been directly quoted from U.S.C.E. (United States Corps of Engineers; 1956).

extent of ice planes, d) the water content and e) the temperature of the snow (U.S.C.E., 1956). Experimental work has established that density is a satisfactory index of the thermal properties of snow. Table 2.3 shows some of the values according to different authors. The values for thermal conductivity in the 0.4 to 0.5 gm cm<sup>-3</sup> range indicate some differences of opinion among the three researchers.

Kondrat'eva (1954) determined more precisely the dependence of the coefficient of thermal conductivity upon density in the zone of higher densities. His approach which is used in this research is discussed below. The main objectives were to determine the vaporization - sublimation process and the coefficient of thermal conductivity. His experiment consisted of packing snow of known density in wooden boxes. The boxes were equipped with metal bottoms and were installed in such a manner so that it was in contact with a solution of NaCl which was kept at a constant temperature. Thermocouples at different heights in the snow were used to monitor the temperature. The sides of the box were insulated with sawdust. The temperature distribution and thus the velocity of heat flow was recorded over time. It is depicted in Figure 2.3. According to him, if there is a sublimation process directed from the bottom to the top layers, the temperature gradient curve will be different from that in the absence of sublimation. It is expected that the thermal conductivity at the bottom will be smaller, owing to the lower density and the differences in temperature will be greater. In the upper layers

Table 2.3

## Thermal Properties of Snow, Ice and Water

Density ( $\rho$ ) gm/cm <sup>3</sup>	Specific Heat (c)		Thermal Conductivity ( $\lambda$ )			Diffusivity ( $\alpha^2$ ) °C/cm <sup>2</sup> /sec
	By Weight cal/gm/°C	By Volume cal/cm <sup>3</sup> /°C	Kondrat'eva cal/cm <sup>2</sup> /°C/cm/sec	Abel cal/cm <sup>2</sup> /°C/cm/sec	Jansson	
1.000	1.0	1.0000	0.00130			0.00130
0.900	0.5	0.4500	0.00535			0.0119
0.540	0.5	0.2700	0.00246		0.00162	0.00911
0.500	0.5	0.2500	0.00205	0.00170	0.00095	0.00820
0.440	0.5	0.2200	0.00167	0.00132	0.00089	0.00760
0.365	0.5	0.1825	0.00110	0.00091	0.00075	0.00603
0.351	0.5	0.1755	0.00087	0.00084	0.00072	0.00494
0.340	0.5	0.1700	0.00075	0.00079	0.00070	0.00441
0.330	0.5	0.1650	0.00070	0.00074	0.00068	0.00422
0.250	0.5	0.1250	0.00042		0.00053	0.00336
0.130	0.5	0.0650	0.00011		0.00029	0.00169
0.050	0.5	0.0250	0.00002		0.00010	0.00080
0.001	0.24				0.00006	

Source: U.S.C.E. (1956).



the temperature difference will be much less. When densities of the various layers of the snow were determined at the end of the experiment the functional relation between thermal conductivity and density was used to calculate the changes in density. According to the conditions of the experiment, since layer thicknesses remain the same the relationship is,

$$\lambda_1 (\Delta\theta)_1 = \lambda_2 (\Delta\theta)_2$$

where the subscripts indicate the values of thermal conductivity ( $\lambda$ ) and temperature of layer ( $\theta$ ) at the beginning and end of the experiment and  $\Delta\theta$  indicates the difference in temperature between the upper and lower surfaces of a given layer. The temperature differences  $(\Delta\theta)_1$  and  $(\Delta\theta)_2$  are read from Figure 2.3 and  $\lambda_2$  is known, the relationship can be solved for  $\lambda_1$ , and consequently for snow density. Comparison with the original density showed the sublimation process quantitatively. A simplified example with reference to Figure 2.3 is stated below.

Top layer ( $L_1$ ) - Surface to 11 cm depth

$$\rho = 0.37 \text{ gm cm}^{-3}, \lambda_2 = 0.0085 (\rho)^2 = 0.00116 \text{ cal cm}^{-1} \text{ } ^\circ\text{C sec}^{-1}$$

$$\Delta\theta_1 = 0.55^\circ\text{C}, \Delta\theta_2 = 0.85^\circ\text{C}$$

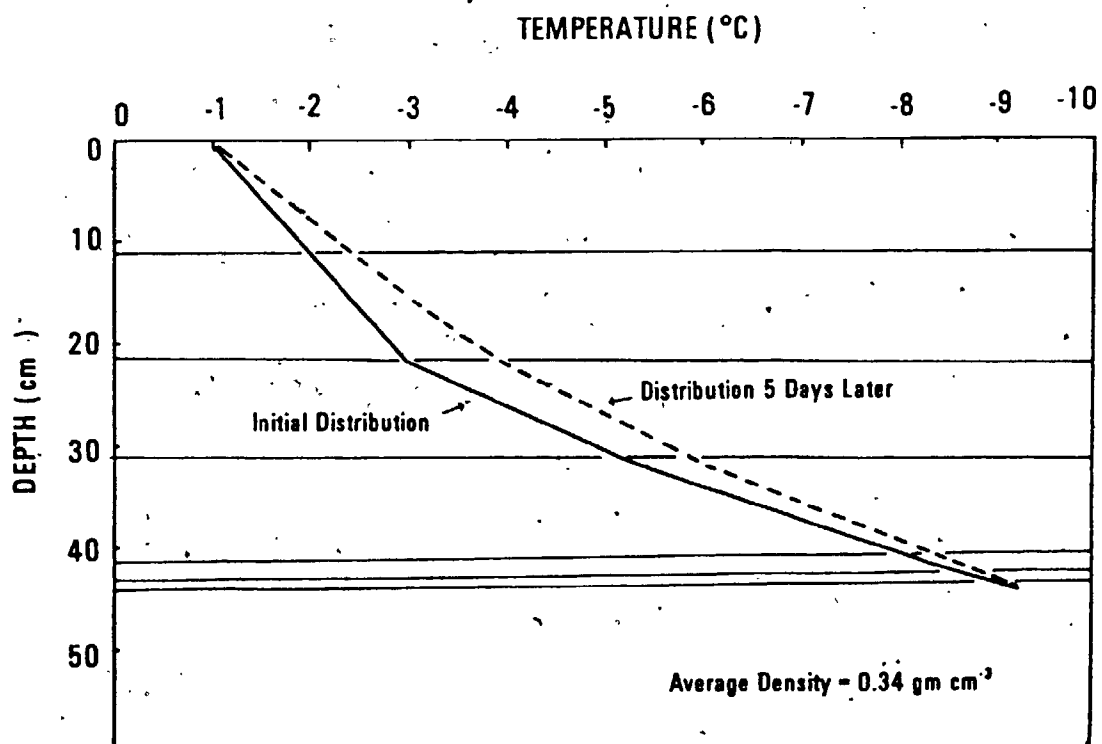
$$\lambda_1 = \lambda_2 (\Delta\theta)_2 / (\Delta\theta)_1$$

$$\rho_n = \sqrt{\lambda_1 / 0.0085} = 0.45 \text{ gm cm}^{-3}$$

$$\Delta\rho = 0.08 \text{ gm cm}^{-3}$$

where  $\rho_n$  is the new density and  $\Delta\rho$  is the change in density.

( $L_2$ ) - 11 cm to 21 cm



**FIGURE 2.3 CHANGE IN SNOW TEMPERATURE AT DIFFERENT LEVELS OF A SNOW SAMPLE AT THE BEGINNING AND AT THE END OF THE EXPERIMENT**

Source: Kondrat'eva, A.S. (1954), Thermal Conductivity of the Snow Cover and Physical Processes Caused by the Temperature Gradient, p. 11.

$$\rho = 0.35 \text{ gm cm}^{-3},$$

$$\rho_n = 0.395 \text{ gm cm}^{-3},$$

(L<sub>3</sub>) - 21 cm to 31 cm

$$\rho = 0.35 \text{ gm cm}^{-3},$$

$$\rho_n = 0.36 \text{ gm cm}^{-3},$$

(L<sub>4</sub>) - 31 cm to 41 cm

$$\rho = 0.34 \text{ gm cm}^{-3},$$

$$\rho_n = 0.32 \text{ gm cm}^{-3},$$

$$\lambda_2 = 0.00104 \text{ cal cm}^{-1} \text{ } ^\circ\text{C sec}^{-1}$$

$$\Delta\rho = 0.045 \text{ gm cm}^{-3}$$

$$\lambda_2 = 0.00104 \text{ cal cm}^{-1} \text{ } ^\circ\text{C sec}^{-1}$$

$$\Delta\rho = 0.01 \text{ gm cm}^{-3}$$

$$\lambda_2 = 0.00079 \text{ cal cm}^{-1} \text{ } ^\circ\text{C sec}^{-1}$$

$$\Delta\rho = 0.02 \text{ gm cm}^{-3}$$

After five days  $0.155 \text{ gm cm}^{-3}$  of snow was transferred by sublimation from layer 1,2,3,4 to lower layers. From his experiments and with reference to other work done, Kondrat'eva has suggested the use of the relation between thermal conductivity and density as:

$$\alpha^2 = 0.0133\rho, \quad \lambda = 0.0068\rho^2$$

for densities less than  $0.35 \text{ gm cm}^{-3}$  and

$$\alpha^2 = 0.0165\rho, \quad \lambda = 0.0085\rho^2$$

for densities above  $0.35 \text{ gm cm}^{-3}$

These relationships have been confirmed by other authors. Kondrat'eva's procedure for calculating the thermal conductivity was through the use of a Fourier equation. Wilson (1941) has also demonstrated that the diffusion of heat through snow is a very slow process. Assuming a deep snow cover of initial temperature  $0^\circ\text{C}$  and then a sudden cooling by the air that maintains the temperature at the snow surface at  $-10^\circ\text{C}$ , he has computed the rate

of heat diffusion (with  $\lambda = 0.003 \text{ cal cm}^{-1} \text{ }^\circ\text{C sec}^{-1}$ ,  $c = 0.50 \text{ cal gm}^{-1} \text{ }^\circ\text{C}$ ,  $\rho = 0.20 \text{ gm cm}^{-3}$ ) as shown in Table 2.4. Figure 2.4 depicts the temperature profiles under the above conditions using a standard Fourier formula as suggested by Wilson.

#### 2.4 Snowmelt models based on heat flow

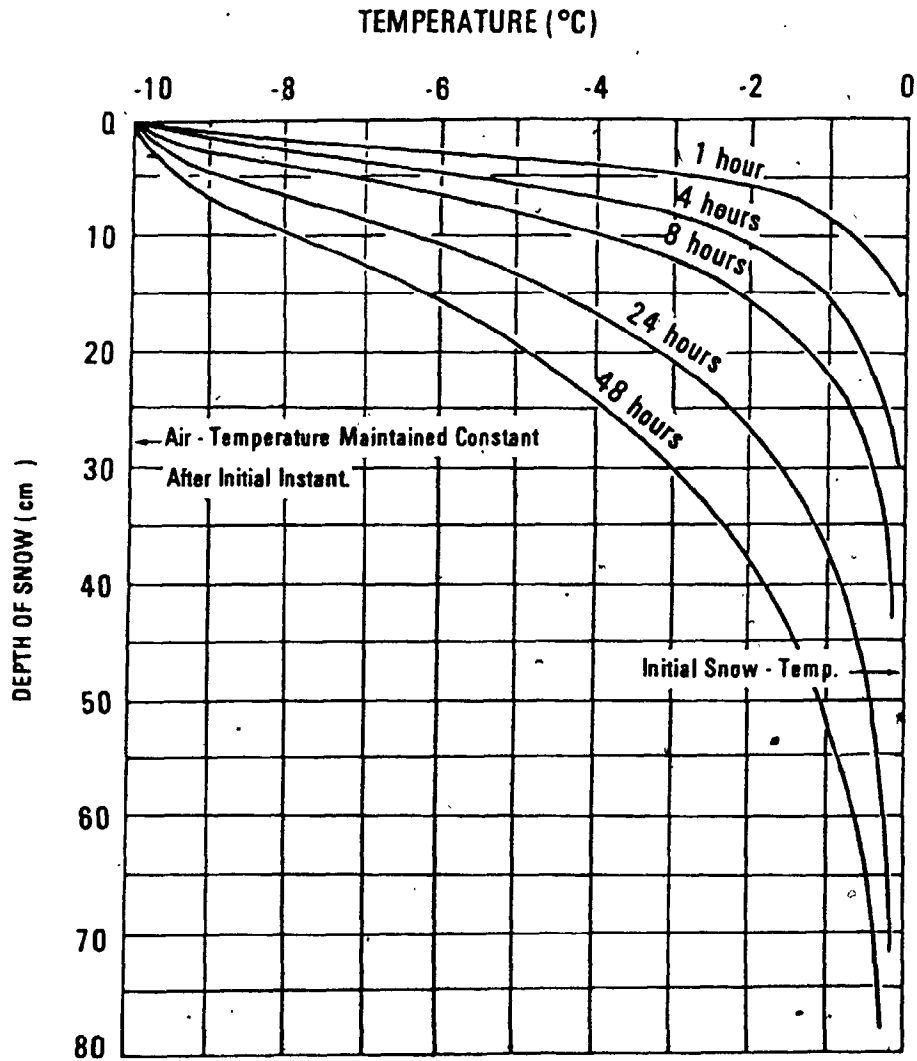
Most of the published work on snowmelt hydrology deals with the empirical energy budget approach to calculate runoff. Indices and empirical relationships have been used in a wide variety of drainage basins. Most of these relationships predict with a certain degree of accuracy but they do not take into account the actual thermodynamic properties of the snowpack. This section will attempt to show the current analytical methods employed in snowmelt hydrology to predict runoff and a general description of how the heat flow equation can be used to calculate temperatures in a medium under given conditions.

##### 2.41 Heat Flux theories

Energy balance methods have been outlined in most texts and are frequently used for design purposes under different climatic conditions. Price (1975) defined the physical processes involved in daily snowmelt in the Subarctic using an energy balance model. The U.S. Corps of Engineers (1956) have produced a general set of equations which are used for calculating snowmelt at a point or basinwide under different conditions. Gray (1973) has noted that

Table 2.4  
Heat Diffusion through Snow

Time, Hours	Heat transfer, Total	cal cm <sup>-2</sup> Per Hour, during last hour	through snow
1	1.9	1.9	
4	3.7	0.5	
8	5.3	0.3	
16	7.4	0.2	
24	9.1	0.2	
48	12.9	3.8	during second day
96	18.2	2.4	during fourth day



**FIGURE 2.4 TEMPERATURE - PROFILES THROUGH SNOW WITH INITIAL SNOW TEMPERATURE OF 0°C, WITH SURFACE SUDDENLY COOLED TO -10°C AND MAINTAINED AT -10°C CONTINUOUSLY**

Source: Wilson, W.T. (1941). An Outline of the Thermodynamics of Snow-melt. Transactions of the American Geophysical Union, Sacramento, p. 184.

the methods that are used for prediction depend on a wide range of factors. These include a) the purpose of prediction, b) the degree of accuracy necessary, c) forecast time, d) the hydrological nature of the basin and e) the availability of tools for forecasting, such as computers. The following methods are currently in use:

1. Degree day analysis
2. Degree day plus recession analysis
3. Generalized snowmelt equations
4. Index plots and regression analysis
5. Hydrograph synthesis and streamflow routing

Logan (1976) formulated a computerized mathematical model to represent the physical processes in a snowpack on an energy flux and mass transfer basis. His model was calibrated and tested with real data in a IHD Representative Basin. The model calculates the heat equivalent of melt for given time increments based on i) heat capacity of the pack, ii) change in heat stored in pack, iii) net radiative energy flux, iv) net conductive and convective energy flux, v) heat of rain and vi) ground heat. The potential melt is then computed in water depth equivalent for that time period. The simulation algorithms allow the model to account for snow accumulation, evaporation and sublimation. The model also considers simulations relative to basin segments of elevation zones and options for computing interpolated or weighted meteorological variables. Estimates are also made for rain and no-rain situations. Although the model has four different options

of data input it does not allow for density, water equivalent and snowpack temperature measurements for various layers over time. Only initial average temperatures are used. Logan's model is similar in approach to the one proposed by Amorocho and Espildora (1966) except that it is adjusted for local snowpack conditions.

In 1973 Leaf and Brink designed a model (MELTMOD) to simulate the melting of snowpacks. This was later modified by Solomon, et al. (1976) to estimate daily and weekly snowmelt rates for snowpack conditions found in the South Western United States. The program called SNOWMELT is applicable to simulating both continuous and intermittent snowpack conditions. However, it has been tested only under intermittent snowpacks. This model is of value for comparison because of its thermal diffusion algorithm which uses a central differencing technique and a snowpack radiating temperature. In general SNOWMELT simulates: 1) winter snow accumulation, 2) energy balance, 3) snowpack conditions and 4) snowmelt. It also has the added component of separating rain and snow events. Four daily input variables are required for SNOWMELT. They are; maximum temperature, minimum temperature, precipitation and short wave radiation. The program also requires some initial values for a) snowpack temperature, b) solar radiation transmissivity coefficient, c) forest cover density, d) water equivalent, e) a threshold value for calculating snowpack reflectivity, f) slope and aspect of watershed, g) latitude, h) atmospheric absorption coefficient and i) a time interval. The general flowchart is shown in Figure 2.5 and a summary of the



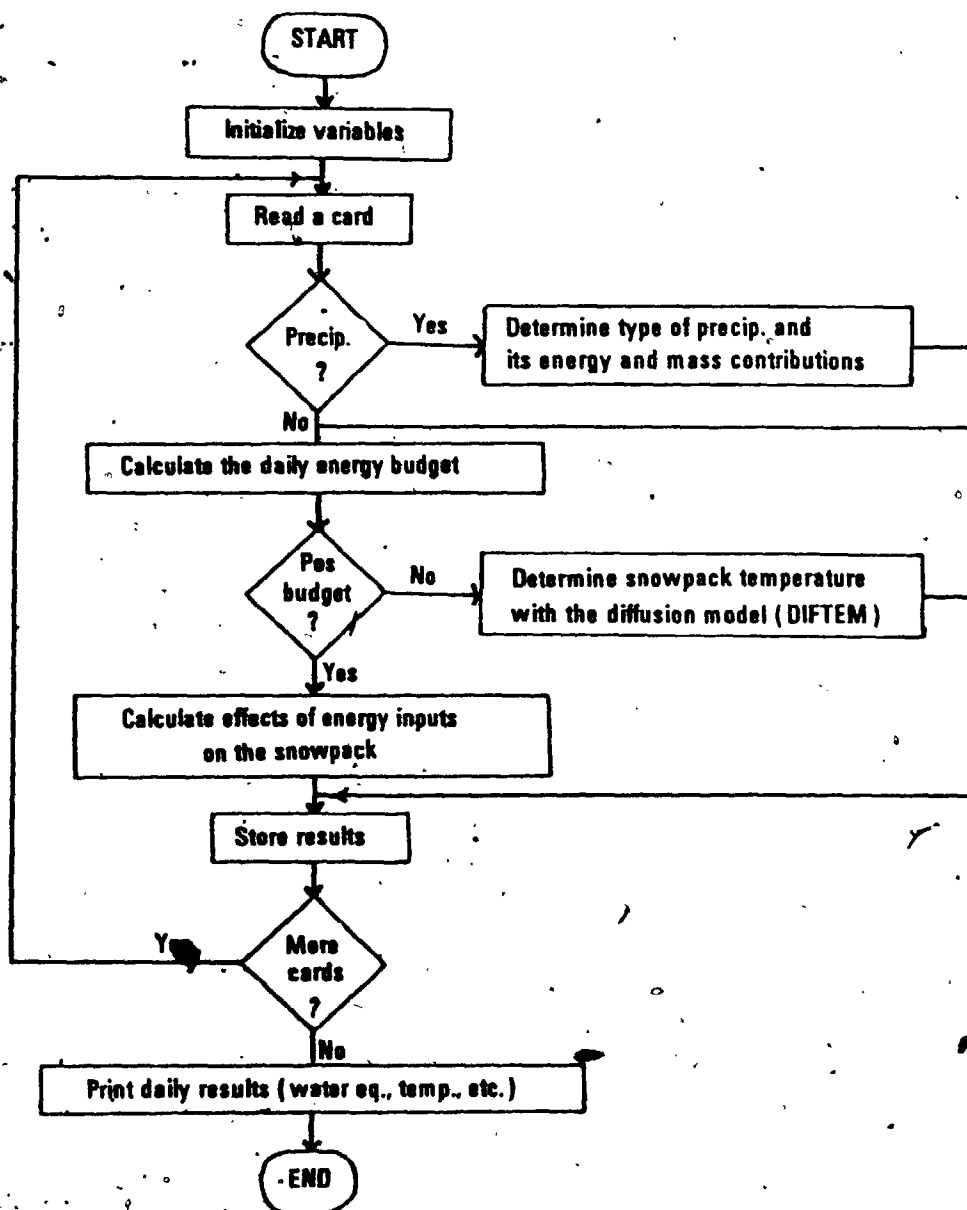


FIGURE 2.5 GENERAL FLOW CHART FOR PROGRAM SNOWMELT

Source: Solomon, et al. (1976). Computer Simulation of Snowmelt. USDA Forest Service Research Paper RM - 174, October 1976, p. 4.

simulation follows.

Step 1: Separation of rain and snow events: The following guidelines are used to separate the events;

SNOW: Maximum temperature drops below  $4.4^{\circ}\text{C}$  and minimum temperature does not exceed  $1.6^{\circ}\text{C}$

RAIN: Minimum temperature exceeds  $1.6^{\circ}\text{C}$

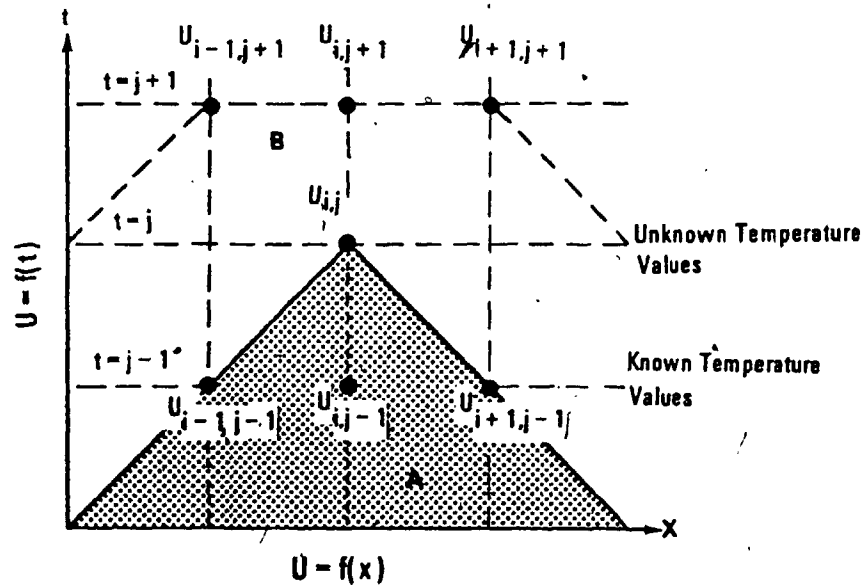
SNOW and RAIN mixed: Minimum temperature drops below  $1.6^{\circ}\text{C}$  and maximum temperature exceeds  $4.4^{\circ}\text{C}$

Step 2: Synthesis of solar radiation data.

In this set of subroutines (SOLAR and CLOUD), total insolation for a day is computed. It takes into account some of the initializing variables mentioned in Section 2.3. Cloud coverage was calculated by using an equation presented by Gates (1962). This setup computes the daily energy budget.

Step 3: Determining snowpack temperature and calculating effects of energy inputs into the snowpack.

This thermal diffusion subroutine was designed to control the temperature of the snowpack to below  $0^{\circ}\text{C}$ . A graphical representation of the implicit central differencing technique is shown in Figure 2.6. Leaf and Brink (1973a) used a forward differencing procedure where they have shown that only those values of  $U$  ( $U_{i-1,j-1}$ ,  $U_{i,j-1}$ ,  $U_{i+1,j-1}$ ) in the pyramidal (shaded) area A have influence on the value of  $U_{i,j}$ . Whereas it is known that points in time earlier than  $t_j$  in area B also influence the



**FIGURE 2.6 IMPLICIT CENTRAL DIFFERENCING METHOD USED IN SNOWMELT**

Source: Solomon, et al. (1976), Computer Simulation of Snowmelt  
 USDA Forest Service Research Paper RM - 174,  
 October 1976, p. 4.

value of  $U_{i,j}$ . In SNOWMELT, values of  $U_{i,j}$  are evaluated at the advanced point in time  $t_{j+1}$  instead of  $t_j$  providing a wider base from which  $U_{i,j}$  can be estimated.

...

The model has been tested in different Arizona watersheds over 13 seasons. The results have been presented statistically as seasons of 'good' fit, 'average' fit and 'poor' fit. It has been suggested that refinements are necessary in the model particularly in relation to snow density.

Kuz'min (1972) reports that the amount of snow melting depends on the amount of heat energy received and that the heat balance method is the only correct method for investigating the course of snowmelt and determining the thermal characteristics of the snowpack. He has provided the analytical expressions to calculate the heat balance of individual layers of snow and the determination of snow temperature by the heat balance. In the former case the value of heat balance according to layers is found as a function of depth of the layer in question. The snowpack is considered as having the largest temperature fluctuations on the uppermost layers because of radiant and turbulent heat exchange. Most of the heat is transmitted from the top to the bottom by conduction. When the mean temperature of the layer in consideration changes, a phase transition of water takes place. His expressions to calculate the amount of heat absorbed or emitted in the particular layer take into account 1) the heat of crystallization, 2) the amount of melting or freezing water, 3)

the overall density of snow and 4) the moisture content as a function of total water equivalent. His equation of heat balance of a snow layer at a depth  $z_1 > \delta$  (below the surface of the snow) is of the form,

$$(I' + i)(1 - r)(e^{-\alpha z_1} - e^{-\alpha z_{i+1}}) - \lambda_{z_1} \frac{\delta \theta}{\delta z_{z=z_1}} + \lambda_{z_{i+1}} \frac{\delta \theta}{\delta z_{z=z_{i+1}}} - c\rho \frac{\delta \bar{\theta}}{\delta t} (z_{i+1} - z_1) - L \frac{\delta \omega_1}{\delta t} (z_{i+1} - z_1) = 0 \quad \dots 2.41$$

where the component  $(I' + i)(1 - r)(e^{-\alpha z_1} - e^{-\alpha z_{i+1}})$  determines the amount of solar energy absorbed by the snow layer,

$$-\lambda_{z_1} \left| \frac{\delta \theta}{\delta z_{z=z_1}} \right| \quad \text{and} \quad + \lambda_{z_{i+1}} \left| \frac{\delta \theta}{\delta z_{z=z_{i+1}}} \right|$$

are the heat fluxes due to the vertical temperature gradient across the upper ( $z_1$ ) and lower ( $z_{i+1}$ ) boundaries of a snow layer with a thermal conductivity,  $\lambda$ . The heat required to produce a temperature change is expressed as the product of the volume heat capacity of the snow ( $c\rho$ ), the change in the mean temperature and the thickness of the layer ( $z_{i+1} - z_1$ ). The last component in equation 2.41 determines the amount of heat absorbed or emitted in the snow layer during melting and freezing as a function of the latent heat of crystallization ( $L$ ), the amount of melting snow or freezing water ( $\omega_1$ ) in  $\text{gm cm}^{-3}$  and the overall density of moist snow ( $\rho$ ).

In equation 2.41, the gradients  $\frac{\delta \theta}{\delta z}$  are considered positive when the snow temperature increases with increasing depth, the values of  $\frac{\delta \theta}{\delta t}$  are positive when during the period in question the

mean temperature of the snow layer  $\bar{\theta}_c$  increases, and the values of  $\frac{\delta \omega_1}{\delta t}$  are considered positive when the snow melts.

Besides the heat balance method of analyzing the temperature regime of dry snow there are other ways of computing the temperature based on the solution of differential equation of heat conduction. The snowpack is heated with the onset of thaw according to the relation (Kuz'min, 1972),

$$\theta = \frac{z}{l} \theta_0 \sum_{n=1}^{\infty} \frac{1}{n} e^{(-\alpha^2 \pi^2 n^2 t / l^2)} \sin \frac{n\pi z}{l} + \theta_1 \frac{z}{l} \quad \dots \dots 2.42$$

where,

$\theta$  = temperature at depth  $z$

$\theta_0$  = initial temperature of the snow surface

$\theta_1$  = temperature at the lower boundary of the snowpack

$t$  = time in seconds after the instant when thaw sets in

$\alpha^2$  = coefficient of thermal diffusivity of snow

$l$  = thickness of snow layer

$z$  = depth of layer

Calculations based on the above equation and with some initial values of  $\theta_0 = -10^\circ\text{C}$ ,  $\theta_1 = 0^\circ\text{C}$ ,  $\rho = 0.30$ ,  $\alpha^2 = 0.0041^\circ\text{C cm}^{-2} \text{sec}^{-1}$ , show that an entire 100 cm of snow is heated to  $0^\circ\text{C}$  only after 15 days. Once the snow begins to melt, percolating water increases the density in such a way that a same thickness of snow layer is heated by phase transitions of water within a matter

of 35-40 minutes.

The discussion so far has concentrated on the methods of analyzing the heat balance and temperature of the snowpack by different authors. Since this research is based on the heat flux theory knowledge of the heat flow equation is necessary to understand the way in which the temperature at a depth at any time can be derived for any homogeneous medium. Applications of this technique are cited in most engineering and mathematics texts. A simplified version is presented below.

#### 2.42 General heat flow theory

According to equation (1.1) under steady state conditions, when  $\delta\theta/\delta t = 0$ ,

$\theta$  is a linear function of the distance  $x$  (Hildebrand, 1976). So if steady state conditions exist initially,  $\theta(x,0)$  must be a linear function of the form,

$$f(x) = \theta_1^0 + (\theta_2^0 - \theta_1^0) \frac{x}{l} \quad \dots\dots 2.43$$

where  $\theta_1^0$  and  $\theta_2^0$  are initial temperatures at ends, and  $l$  is the length.

At instant  $t = 0$  if temperature at end  $x=0$  is changed to a new value,  $\theta_1$ , and temperature at end  $x=l$  is changed to  $\theta_2$  and maintained at those constant values thereafter, then

$$\theta(0,t) = \theta_1, \quad \theta(l,t) = \theta_2 \quad (t>0) \quad \dots\dots 2.44$$

The temperature distribution is then expressed as a function of  $x$  and  $t$ , as the sum of two distributions, one of which is to

represent the limiting steady state distribution (independent of  $t$ ) after transient effects have become negligible, and the other of which is to represent the transient distribution (which must then approach zero as  $t$  increases indefinitely). As such it becomes,

$$\Theta(x,t) = \Theta_s(x) + \Theta_t(x,t) \quad \dots\dots 2.45$$

The function  $\Theta_s(x)$  must be a linear function of  $x$  satisfying equation (2.44), and hence is of the form,

$$\Theta_s(x) = \Theta_1 + (\Theta_2 - \Theta_1) \frac{x}{l} \quad \dots\dots 2.46$$

and  $\Theta_t(x,t)$  is a particular solution of equation (1.1). The function  $\Theta_t$  must be determined in such a way that it vanishes when  $t \rightarrow \infty$ ,

$$\Theta_t(x,\infty) = 0 \quad \dots\dots 2.47$$

Therefore the sum of  $\Theta_s + \Theta_t$  satisfies the initial condition,

$$\Theta(x,0) = f(x) \quad \dots\dots 2.48$$

Since  $\Theta_s(x)$  satisfies equation (2.44), it follows that  $\Theta_t$  must vanish at the ends  $x=0$  and  $x=l$  for all positive values of  $t$ ,

$$\Theta_t(0,t) = \Theta_t(l,t) = 0 \quad \dots\dots 2.49$$

Thus the transient distribution satisfies homogeneous end conditions. Solutions of equation (1.1) satisfying equations (2.47) and (2.49) can be obtained in the form,

$$\Theta(x,t) = B_n \sin \frac{n\pi x}{l} e^{(-n^2 \pi^2 \alpha^2 t / l^2)} \quad (n=1,2,3\dots) \quad \dots\dots 2.50$$

So by combining equation (2.46) and a superposition of solutions of this type, the required function  $\Theta(x,t)$  is expressed as,

$$\Theta(x,t) = \sum_{n=1}^{\infty} B_n e^{(-n^2 \pi^2 \alpha^2 t / l^2)} \sin \frac{n\pi x}{l} + \Theta_1 + (\Theta_2 - \Theta_1) \frac{x}{l} \quad \dots\dots 2.51$$

where the Fourier coefficient  $B_n$  is determined the following way,



$$B_n = \frac{2}{1} \int_0^1 f(x) \sin \frac{n\pi x}{1} dx + \frac{2}{n\pi} (\Theta_2 \cos n\pi - \Theta_1) \quad \dots\dots 2.52$$

Since  $e^{(-n^2\pi^2\alpha^2t/l^2)}$  decreases rapidly with increasing  $n$ , only two terms of the series are required. Thus the final equation for the distribution of temperature in a medium can be expressed as (Kondrat'eva, 1954),

$$\Theta(x,t) = \frac{2\Theta_3 - \Theta_2 - \Theta_1}{\pi} e^{(-\pi^2\alpha^2t/l^2)} \sin \frac{\pi x}{1} + \frac{\Theta_2 - \Theta_1}{2\pi} e^{(-4\pi^2\alpha^2t/l^2)} \sin \frac{2\pi x}{1} + \Theta_1 + (\Theta_2 - \Theta_1) \frac{x}{1} \quad \dots\dots 2.53$$

where:  $\Theta_1$  and  $\Theta_2$  are boundary conditions and

$\Theta_3$  is the initial condition.

The models mentioned in this chapter are some examples of a variety of models in the literature. Essentially, the models differ from each other in their use of physically based calculation procedures for the particular thermodynamic process that was of most interest to the developer of the model.

The present study reports on the development and testing of a three phase model which is designed to simulate snowpack conditions and the consequent runoff produced by changes which take place in the snowpack.

## CHAPTER III

### A THREE PHASE TEMPERATURE - DENSITY MODEL

This chapter describes the three phase temperature - density model hypothesized to incorporate the relationship between temperature, density and snowmelt.

#### 3.1 Objectives of the model

Specifically the model is designed to,

1. Simulate temperature conditions in a snowpack
2. Simulate density changes and heat fluxes
3. Simulate the sublimation processes which result from density changes
4. Resimulate temperature conditions in the snowpack with adjusted density values.
5. Compute melt generated from the snowpack.

Further, to reduce the preparation required in order to apply the model in a basin, a major objective during model development was to limit the input parameters and variables to those readily available in published format or obtainable with a minimum of field investigation.

#### 3.2 Theories and relationships

This section is divided into three stages. Stage 1 involves the mathematical theory for calculating snowpack temperatures. Stage 2 explains the empirical relationships used in calculating

heat fluxes and density changes. Finally stage 3 describes the process of latent heat transfer as related to the melting or freezing of the snowpack.

### 3.21 The temperature model

Consider a one-dimensional problem of heat flow in a homogeneous snow layer of depth  $h$ . The temperature ( $\Theta$ ) depends only on the distance  $x$  from the surface of the layer and the time  $t$  (Figure 3.1). The heat flow equation can then be described as,

$$\frac{\delta^2 \Theta}{\delta x^2} = \frac{1}{\alpha^2} \frac{\delta \Theta}{\delta t} \quad \dots 3.1$$

where  $\alpha^2$  is the thermal diffusivity.

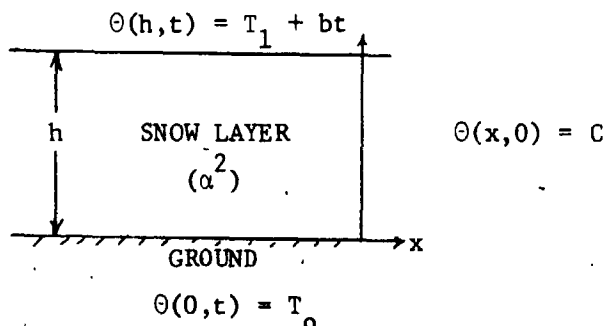


FIGURE 3.1

If the  $x$  - origin is taken at the bottom of the layer the problem can be set up with boundary conditions,

i)  $\Theta(h,t) = T_1 + bt \quad \dots 3.2$

representing temperature as linear in time at the upper boundary ( $T_1$  and  $b$  are constants)

ii)  $\Theta(0,t) = T_0 \quad \dots 3.3$

representing a constant temperature at the lower boundary.

These are based on the assumptions that atmospheric temperature fluctuates linearly over time,  $\Delta t$  (simulation time) and the ground is frozen at a constant temperature. If multiple layers of snow were present then each layer of snow was taken to have the same temperature throughout, in accordance to the above boundary conditions. Snow densities were assumed to be uniform for each layer.

Take

$$\Theta(x,0) = C \quad \dots\dots 3.4$$

to be the initial temperature for the layer, constant for all depths  $0 < x < h$ .

To determine a solution  $\Theta(x,t)$  of equation (3.1) satisfying the initial and boundary conditions, let

$$\Theta(x,t) = w(x,t) + \frac{bx^3}{6\alpha^2 h} - \frac{bhx}{6\alpha^2} + T_0 + \frac{x}{h}(T_1 - T_0 + bt) \quad \dots\dots 3.5$$

The term  $w(x,t)$  will be the solution for homogeneous boundary conditions, namely,

$$i) \quad \Theta(0,t) = T_0 = w(0,t) + T_0 \quad \dots\dots 3.6$$

$$\text{Therefore } w(0,t) = 0 \quad \dots\dots 3.7$$

$$ii) \quad \Theta(h,t) = T_1 + bt = w(h,t) + \frac{bh^2}{6\alpha^2} - \frac{bh^2}{6\alpha^2} + T_0 + (T_1 - T_0 + bt) \quad \dots\dots 3.8$$

$$= w(h,t) + T_1 + bt \quad \dots\dots 3.9$$

$$\text{Therefore } w(h,t) = 0 \quad \dots\dots 3.10$$

Taking the derivative of equation (3.5) with respect to  $x$ , we obtain

$$\frac{\partial \Theta}{\partial x} = w_x + \frac{bx^2}{2\alpha^2 h} - \frac{bh}{6\alpha^2} + \frac{1}{h}(T_1 - T_0 + bt) \quad \dots\dots 3.11$$

Taking the derivative of equation (3.11) with respect to  $x$ , we get

$$\theta_{xx} = w_{xx} + \frac{bx}{\alpha^2 h} \quad \dots\dots 3.12$$

Taking the derivative of equation (3.5) with respect to  $t$ , we get

$$\theta_t = w_t + \frac{bx}{h} \quad \dots\dots 3.13$$

Substituting equations (3.12) and (3.13) into (3.1), eliminating

$\theta_{xx}$  and  $\theta_t$ , we have

$$w_{xx} + \frac{bx}{\alpha^2 h} = \frac{1}{\alpha^2} (w_t + \frac{bx}{h}) \quad \dots\dots 3.14$$

Therefore

$$w_{xx} = \frac{1}{\alpha^2} w_t \quad \dots\dots 3.15$$

Hence  $w$  satisfies the heat equation with homogeneous boundary conditions,

$$w(0,t) = w(h,t) = 0 \quad \dots\dots 3.16$$

Since the heat equation is a linear partial differential equation, it can be solved by the method of separation of variables (Kreyszig, 1972). Thus we assume that we can write  $w(x,t)$  as the product of a function of  $x$  and a function of  $t$  as follows

$$w(x,t) = X(x)T(t) \quad \dots\dots 3.17$$

Substituting this expression and its derivatives into equation (3.15) we obtain

$$X''T = \frac{1}{\alpha^2} T' \quad \dots\dots 3.18$$

where the primes denote differentiation with respect to the argument. We divide both sides of this equation (3.18) by  $XT$ , finding

$$\frac{X''}{X} = \frac{T'}{\alpha^2 T} \quad \dots\dots 3.19$$

Since the expression on the left hand side of equation (3.19) is a function of  $x$  alone, it cannot vary with time. The expression on the right hand side depends only on  $t$  so that it cannot vary with

x. Hence both members must be equal to a common constant value, say  $-k$  (Churchill, 1969). Thus

$$\frac{X''}{X} = \frac{T'}{\alpha^2 T} = -k \quad \dots\dots 3.20$$

or equivalently,

$$X'' + kX = 0, \quad T' + k\alpha^2 T = 0 \quad \dots\dots 3.21$$

If  $w$  is to satisfy the first of conditions (3.16), then  $X(0)T(t)$  must vanish for all  $t$  ( $t > 0$ ). The case  $T(t) = 0$  for all  $t$  is trivial since the function  $w = 0$  always satisfies linear homogenous equations; hence  $X(0) = 0$ . Likewise, the last condition of (3.16) is satisfied by  $w$  if  $X(h) = 0$ . Thus  $w$  satisfies conditions (3.16) when  $X$  and  $T$  satisfy the following two equations:

$$X''(x) + kX(x) = 0; \quad X(0) = 0, \quad X(h) = 0 \quad \dots\dots 3.22$$

$$T'(t) + k\alpha^2 T(t) = 0 \quad \dots\dots 3.23$$

Now, if  $k < 0$ , it can be shown that equation (3.22) has just the trivial solution  $X(x) = 0$  (Churchill, 1969). Only for the following discrete set of values of the parameter  $k$ ,

$$k = n^2 \pi^2 / h^2, \quad n = 1, 2, \dots \quad \dots\dots 3.24$$

does equation (3.22) possess non trivial solutions except for a constant factor. That is, when  $k = n^2 \pi^2 / h^2$ , equation (3.22) is a distinct problem for each different positive integer  $n$ . For a fixed integer  $n$ , it has the solution (3.23) and equation (3.22) becomes,

$$T'(t) + \frac{n^2 \pi^2 \alpha^2}{h^2} T(t) = 0. \quad \dots\dots 3.25$$

The general solution, except for a constant factor is

$$T(t) = e^{-\lambda_n^2 t} \quad \dots\dots 3.26$$

where  $\lambda_n = n\pi a/h$

Hence the functions

$$w_n(x,t) = \sin \frac{n\pi x}{h} e^{-\lambda_n^2 t} \quad n=1,2,\dots \quad \dots 3.27$$

are solutions of equation (3.15) satisfying (3.16). Hence, by the principle of superposition, the general solution is

$$w(x,t) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{h} e^{-\lambda_n^2 t} \quad \dots 3.28$$

where the constant  $B_n$  is to be determined. To this end, we now have that

$$\begin{aligned} \theta(x,t) = & \frac{b}{6\alpha^2 h} (x^3 - h^2 x) + T_0 + \frac{x}{h} (T_1 - T_0 + bt) \\ & + \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{h} e^{-\lambda_n^2 t} \quad \dots 3.29 \end{aligned}$$

So, to determine the constant  $B_n$ , we consider the initial condition (3.4) and find that

$$\begin{aligned} \theta(x,0) = C = & \frac{b}{6\alpha^2 h} (x^3 - h^2 x) + T_0 + \frac{x}{h} (T_1 - T_0) + \\ & \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{h} \quad \dots 3.30 \end{aligned}$$

Rearranging, we have that

$$\sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{h} = C - \frac{bx^3}{6\alpha^2 h} + \frac{bxh}{6\alpha^2} - T_0 - \frac{x}{h} (T_1 - T_0) \quad \dots 3.31$$

Using the orthogonal relation for  $\sin \frac{n\pi x}{h}$  (Appendix I), it can be shown that

$$\begin{aligned} B_n = & \frac{2}{h} [(C - T_0)(h/n\pi) [(1 - (-1)^n)]] + \left[ \frac{T_0 - T_1}{h} + \frac{bh^2}{6\alpha^2} \right] [(-1)^{n+1} \frac{h^2}{n\pi}] \\ & + \left[ \frac{-b}{6\alpha^2 h} \left( \frac{(-1)^n h^4}{n\pi} \right) \left( \frac{6}{2n^2} - 1 \right) \right] \quad \dots 3.32 \end{aligned}$$

Therefore,  $\theta(x,t) = \frac{b}{6\alpha^2 h} (x^3 - h^2 x) + T_0 + \frac{x}{h} (T_1 - T_0 + bt) +$

$$\sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{h} e^{-\lambda_n^2 t} \dots\dots 3.33$$

where  $B_n$  is given by (3.32).

Before actual data was tested in the model a theoretical simulation was done using the mathematical functions cited in this chapter. The results are presented diagrammatically in the following pages according to input data on Table 3.1.

As indicated in the table the difference between the two graphs in each figure is in terms of its density which is represented by the diffusivity ( $\alpha^2$ ). In each case the upper graphs are assumed to have low densities (less than .05 gm cm<sup>-3</sup>) while the ones below have densities around .330 gm cm<sup>-3</sup>. In Figure 3.2, since the snow surface is warmer than the ground and the snow beneath, the depth of the temperature wave extends to 50 cm and 100 cm over 100 hours before it becomes isothermal. In Figure 3.3, the snow surface is colder and the bottom warmer than the snow beneath. Under such conditions a cooling and a warming wave extends from both directions. In the upper graph (Figure 3.3), because of the low density there is an isothermal layer of about 20 cm whereas with higher densities the layer diminishes to a point, approximately 55 cm below the surface. Figure 3.4 represents a condition somewhat the reverse of what is shown in Figure 3.2, i.e. the temperature wave is directed upwards from the bottom of the snow. These graphs indicate the time rate of heat transfer in snowpacks with varied depths. (Fortran program to compute values, given in Appendix II).



Table 3.1

Temperature - Depth Simulation Data Input

	Figure 3.2		Figure 3.3		Figure 3.4		Variable Definition
	Top	Bottom	Top	Bottom	Top	Bottom	
	Diagram		Diagram		Diagram		
T <sub>1</sub>	1°C	1°C	-10°C	-10°C	-10°C	-10°C	Snow Surface Temp.
T <sub>0</sub>	0°C	0°C	-5°C	-5°C	0°C	0°C	Ground Temp.
C	0°C	0°C	-7°C	-7°C	0°C	0°C	Snow Temp.
H	100 cm	100 cm	100 cm	100 cm	80 cm	80 cm	Depth
t	100 hrs.	100 hrs.	100 hrs.	100 hrs.	100 hrs.	100 hrs.	Time
α <sup>2</sup>	.00075	.004	.00075	.004	.00075	.004	Diffusivity in °C cm <sup>-2</sup> sec <sup>-1</sup>
Del	.01	.01	.01	.01	.01	.01	Truncation point of series
b	0	0	0	0	0	0	Constant in equation

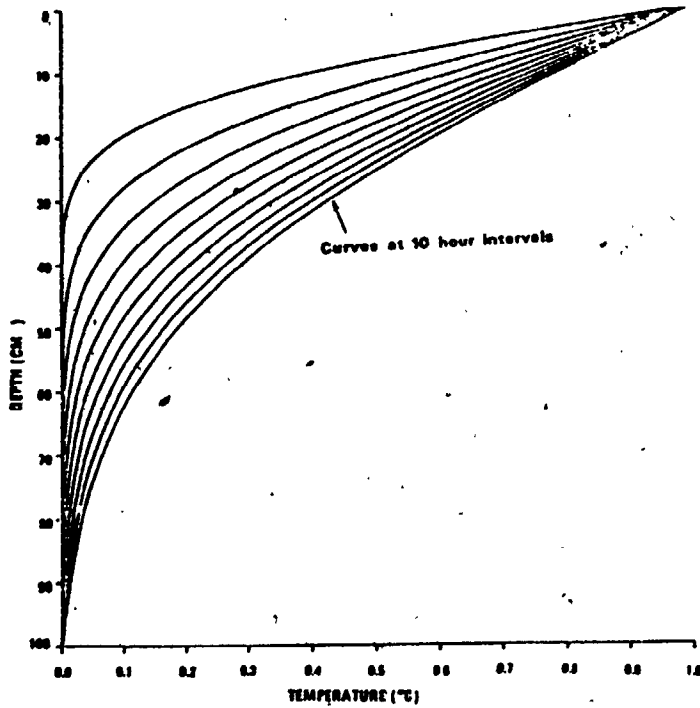
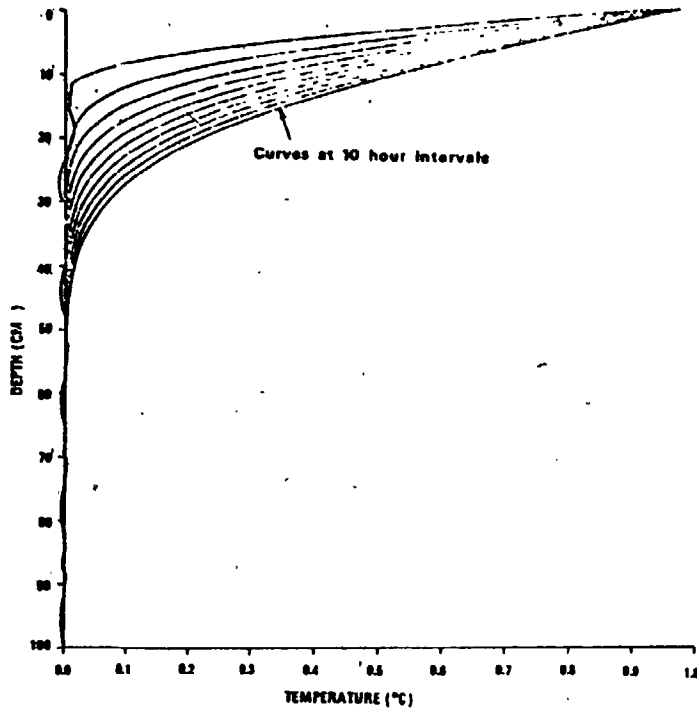


FIGURE 3.2 TEMPERATURE - DEPTH SIMULATION NO. 1

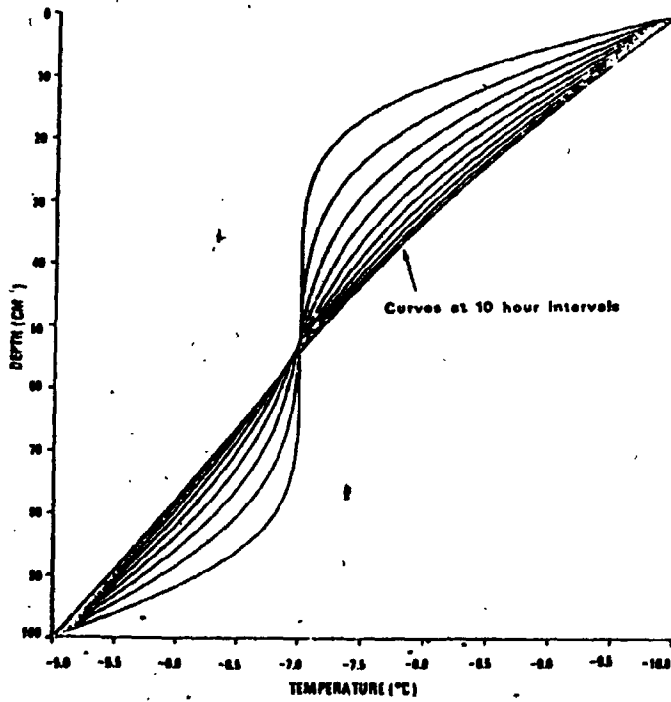
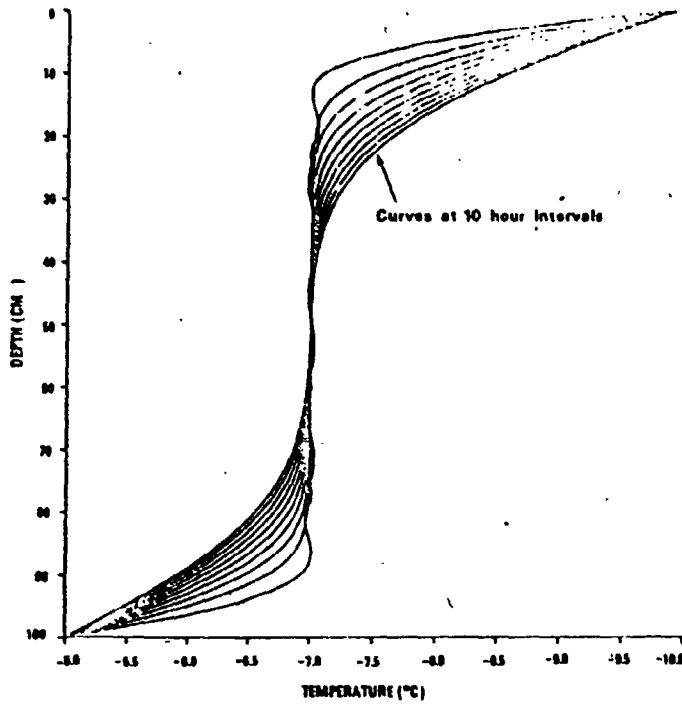


FIGURE 2.3 TEMPERATURE - DEPTH SIMULATION NO. 2

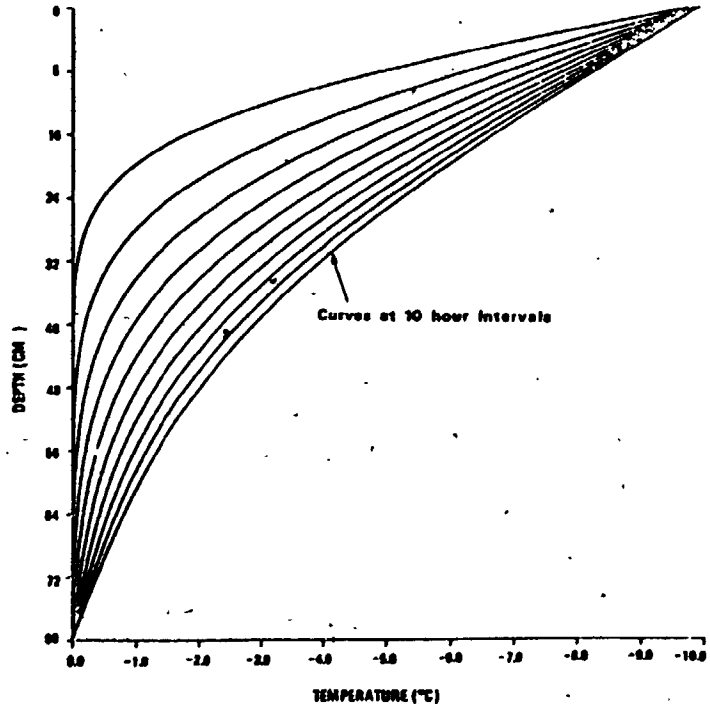
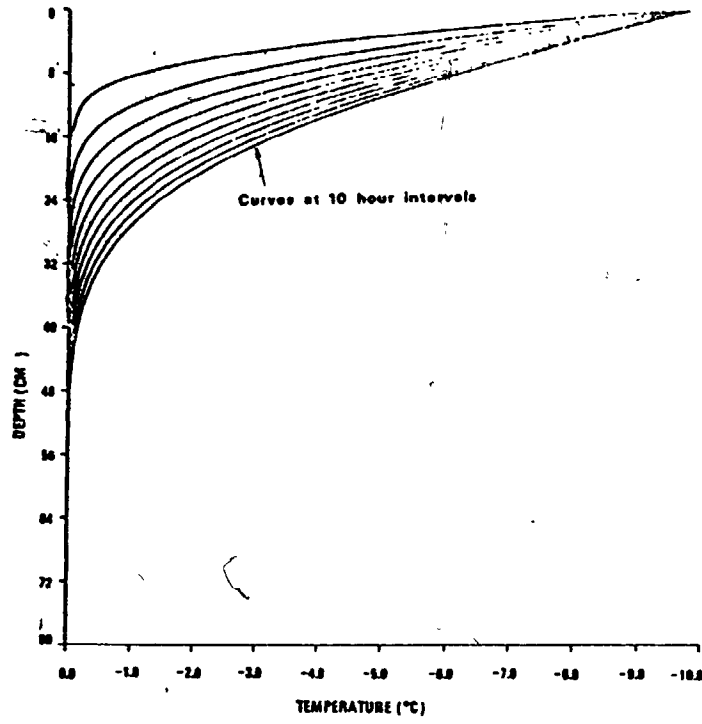


FIGURE 3A TEMPERATURE - DEPTH SIMULATION NO. 3

### 3.22 The density - heat flux model

This particular stage of the model involves the calculation of the heat fluxes over time based on temperature gradients and densities and the calculation of new densities because of temperature changes as computed by the temperature model. This was done to show the direction of heat flow in the snowpack and the sublimation process. The heat flux is calculated by the following relationship.

$$Q = \lambda \theta_0 / \alpha^2 \sqrt{\pi} \times 60 \quad \dots\dots 3.34$$

where,

$Q$  = heat transfer in  $\text{cal cm}^{-2} \text{hr}^{-1}$

$\lambda$  = conductivity in  $\text{cal cm}^{-2} \text{C}^{-1} \text{sec}^{-1}$

$\theta_0$  = the difference in temperature between the midpoint of two layers in  $^{\circ}\text{C}$

$\alpha^2$  = diffusivity =  $\lambda/\rho c$  in  $^{\circ}\text{C cm}^{-2} \text{sec}^{-1}$

$\rho$  = density in  $\text{gm cm}^{-3}$

$c$  = specific heat in  $\text{cal gm}^{-1} \text{C}^{-1}$

In calculating densities the same procedure was followed as outlined in chapter two. The changes in density were also computed for each individual layer.

### 3.23 Melt-freeze model

This particular model calculates the amount of melting or freezing which takes place in the snowpack based on the latent heat transfer between different layers. The equation to calculate

the latent heat transfer as an amount of melt equivalent of liquid water is given as,

$$L = \theta_o c \rho l / 80 \quad \dots\dots 3.35$$

where,

L = amount of melt/freeze in equivalent cm of liquid water

$\theta_o$  = snow temperature in °C

c = specific heat in cal gm<sup>-1</sup> °C<sup>-1</sup>

l = thickness of layer in cm

Positive values of L indicate that the layer is melting while negative values mean that is freezing and at the same time increasing its density. For positive values, the thickness of the layer is adjusted in terms of the amount of melt and the amount of water transferred to the next layer below for calculations of changes in density and L for that layer. A negative value increases the density of that layer by that amount.

Each individual model is explained in terms of how it works. A list of the variables used in the program(s) is stated later in this chapter. Table 2.3 is used to approximate values of specific heat, conductivity and diffusivity for different density values. The general flowchart of the model is shown in Figure 3.5.

### 3.3 Computer simulation using temperature model

The following is a description of how the temperature algorithm works, as designed by the author. For the temperature

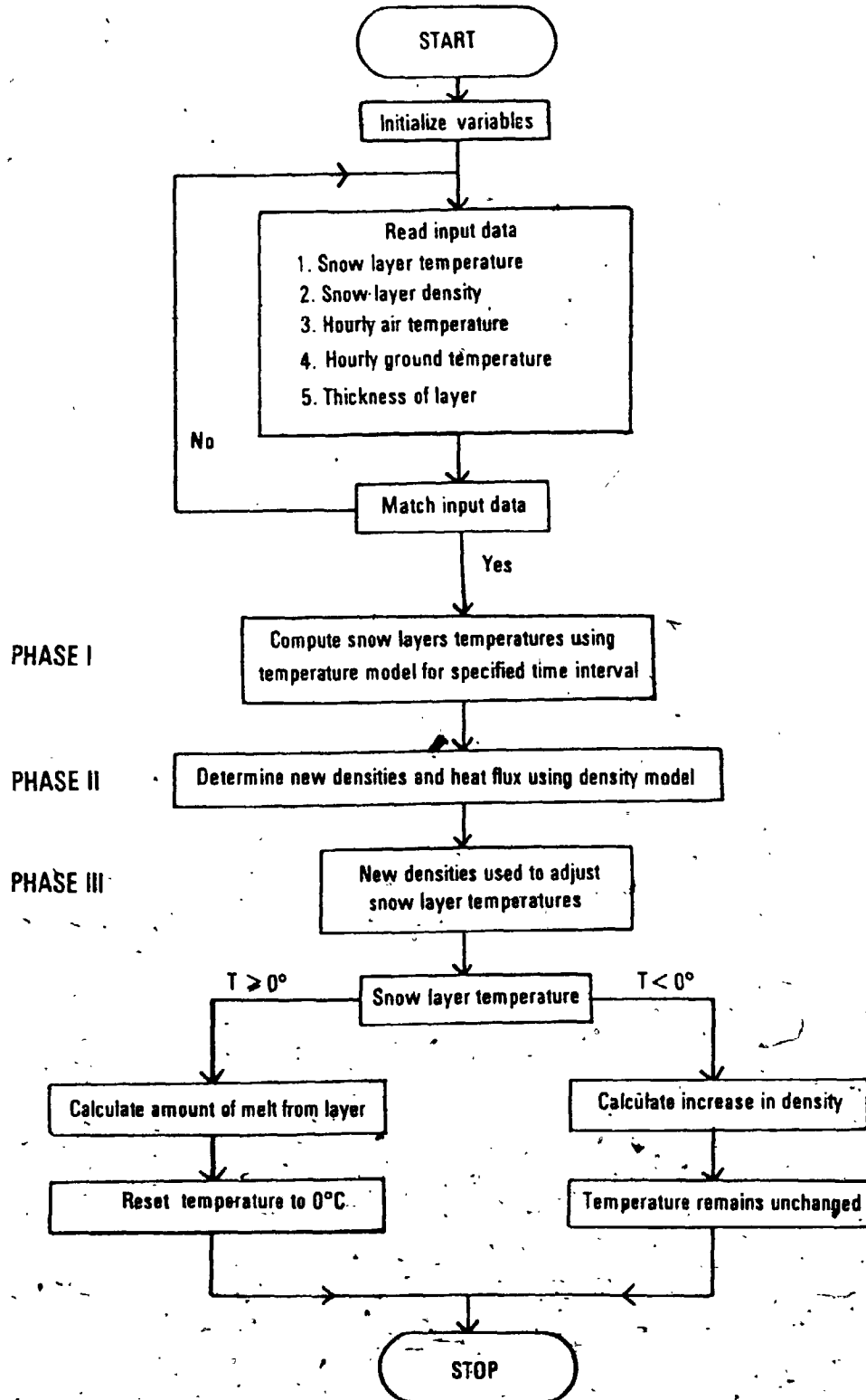


FIGURE 3.5 GENERAL FLOW CHART FOR THE THREE PHASE MODEL

model, consider five layers of thicknesses  $l_1 \dots l_5$  and densities  $\rho_1 \dots \rho_5$  with ground temperature  $GT = -1$ , initial air temperature  $0^\circ\text{C}$  and the air temperature 24 hours later  $-10^\circ\text{C}$ . Initial snow layer temperatures are  $T_1, T_2, T_3, T_4, T_5$  and the time interval selected is 1 hour ( $TS=1$ ). This means the program will solve for the layer's temperature at 1 hour intervals and give the answer for the next hour. The air temperature is assumed linear over the 24 hours, so  $T_a = 0 - bt$ ,  $0 \leq t \leq 1$ . The program sets up a  $9 \times 24$  matrix with the columns representing the time (hour), air temperature, snow layer temperatures, ground temperature and date. Each row represents a time period of one hour. Once the initial conditions are entered a matrix  $TM$  is set up as follows,

TM	Hour	Air Temp.	L <sub>1</sub>	L <sub>2</sub>	L <sub>3</sub>	L <sub>4</sub>	L <sub>5</sub>	GT	Date
	1	0	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	T <sub>5</sub>	-1	780101
	2	-1						-1	780101
	3	-2						-1	780101
	.	.						.	.
	.	.						.	.
	24	-10						-1	780101

All other entries are set to 0. The program solves for each entry starting at  $TM [2;3]$  and running across the rows and down the columns. For example, to find  $TM [2;3]$  it considers,

$$T_a = 0 - bt$$

$\rho = \rho_1$	$\begin{array}{ c } \hline 1 \\ \hline \end{array}$	$\begin{array}{ c } \hline C = T_1 \\ \hline \end{array}$
	$\begin{array}{ c } \hline 1 \\ \hline \end{array}$	$\begin{array}{ c } \hline T_o = T_2 \\ \hline \end{array}$



In this case  $b=-1$ , and the model is used to find  $\Theta(\frac{1}{2},1)$ ; that is the temperature at the midpoint of the first layer after one hour, truncating the series when successive terms drop below a predetermined value. The next step is to find  $TM [2;4]$  using  $T_0=T_3$ ,  $C=T_2$  and finding  $b$  by,

$$b = (T[2;3] - T [2;2] ) \div TS$$

The process continues until  $TM$  is filled, whereby the last row represents the temperature of the snow layer after 24 hours. In solving for the last snow layer  $TM[2;7]$  in every case,  $GT=T_0$ .

The program allows one to limit the possible layer size by assigning the maximum layer depth to  $TF$ . If a layer exceeds the maximum permissible depth, the program lets the user split it into an appropriate number of smaller layers of the same density and initial temperatures as the original layer. This is especially important when dealing with short time intervals as the temperature for each layer is always at the midpoint, so a thick layer will not change temperature over a short time period, yielding inaccurate results. The series is truncated as mentioned before, after 100 terms or whenever the absolute value of the exponential term falls below the user determined variable  $TQ$ . It is recommended  $TQ$  be set at .001 or thereabouts as the marginal improvement in accuracy for lower  $TQ$  values is offset by the increase in run time. The model works from the top of the snowpack down, under the assumption that  $GT$  is not too different from the temperature of the bottom layer, whereas atmospheric

temperature does differ from the first layer.

The Fortran program written to compute temperatures is given in Appendix III. The general flowchart of the program, called SNOW is shown in Figure 3.6. Table 3.2 is a list of the parameters used in the program. The three major functions LL, SEAN, and FOCEF are explained below.

### 3.31 Function LL

In this function layer thicknesses are put in a vector HTT and checked to ensure thickness is  $\leq TF$ , a predetermined value. If a thickness is  $> TF$  it can be split up into equal sizes all less than  $TF$ . Next it divides  $HTT(BI)$  by  $TF$  and rounds it up to the nearest integer. This gives how many layers the  $BI$ th layer must be split into to ensure each is  $\leq TF$ . HT is the final vector of layer thicknesses. To the end of this is added the number of entries determined in each of the thicknesses  $HTT(BI) \div BK$ . Data is the final vector of layer temperature. Added to this are as many entries as determined as above (i.e. BK) with each of the same temperature as the original layer (i.e. DA [BI]). The K values are computed on the basis of the

following relationships as proposed by Kondrat'eva.

$$\alpha^2 = 0.0133\rho \quad \text{for } \rho \leq 0.35 \text{ gm cm}^{-3}$$

$$\alpha^2 = 0.0165\rho \quad \text{for } \rho > 0.35 \text{ gm cm}^{-3}$$

Converting these values to an hourly basis,

$$\alpha^2 = 13.3 \times 3.6 \times \rho \quad \text{for } \rho \leq 0.35 \text{ gm cm}^{-3}$$

$$\alpha^2 = 16.5 \times 3.6 \times \rho \quad \text{for } \rho > 0.35 \text{ gm cm}^{-3}$$

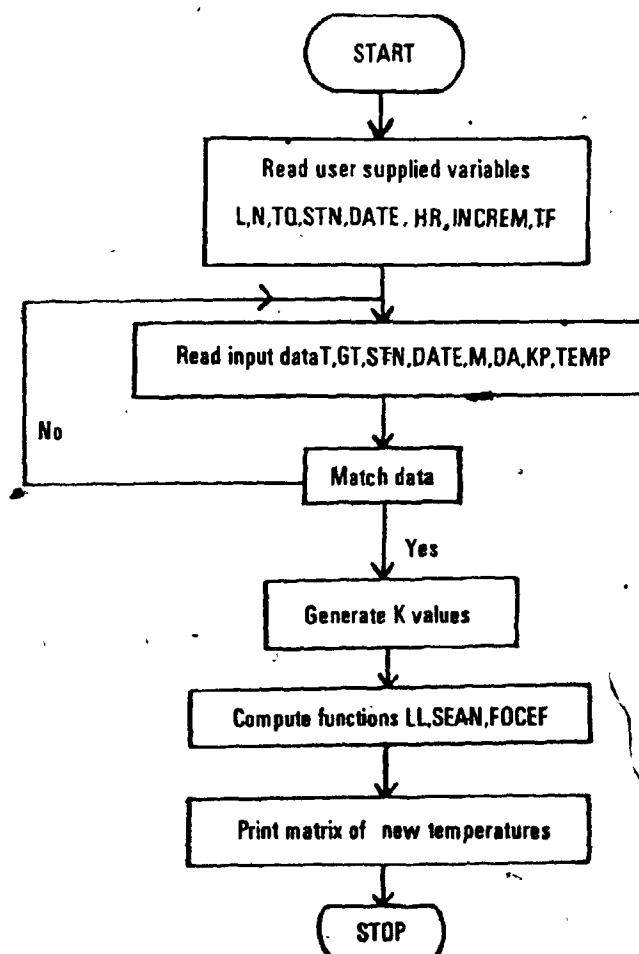


FIGURE 3.6 GENERAL FLOWCHART FOR THE TEMPERATURE MODEL

Table 3.2

List of Parameters for the  
Temperature Model

<u>Parameter</u>	<u>Definition</u>	<u>Remarks</u>	<u>Units</u>
S01,S02,S03 S04,S05,S06	Temperature, Density data files for snow courses	Data is for layers identified in snow-pack at that time	$T = ^\circ\text{C}$ $\rho = \text{gm cm}^{-3}$
L	Time interval over which the model is run, as 1/L hrs.	If L=4, then temperatures are determined every 15 mins. ( $\frac{1}{4}$ ).	Hours
N	Number of hours of simulation	If n = 24, temperatures are computed for 24 hours	Hours
TQ	Variable in model	Set at 0.001	
STN	Station Number	201 means sampling point No. 1 in snow course No. 2	
DATE	Date	Day from which simulation is run. 780115 refers to the 15th day of Jan., 1978	
M	Number of layers	Maximum of 13	
TF	Thickness of layer	Can be set by the user	cm
SDATE	Starting date for simulation	Controls timing for simulation run	
EDATE	Ending date for simulation	Controls timing for simulation run	
SSTART	Starting station number	Controls station number for simulation run	

Table 3.2 (Cont'd.)

<u>Parameter</u>	<u>Definition</u>	<u>Remarks</u>	<u>Units</u>
SEND	Ending station number	Controls station number for simulation run	
INCREM	Increments	Controls increments between printing	
HST	Hour for starting	User supplied	
DA(J)	Vector of snow layer temperatures	Each vector contains 1 sampling point	T = °C
KP(J)	Vector of snow layer densities	Each vector contains 1 sampling point	$\rho = \text{gm cm}^{-3}$
TEMP	Temporary variable stores thicknesses of layer for a certain sampling point	Calculated from total snow depth divided by M	cm
HTT(J)	Vector of snow layer thicknesses	Same as TEMP	
T	Air temperature data on an hourly basis	Matched with date and hour for simulation run	T = °C
GT	Ground temperature data on a hourly basis	Matched with date and hour for simulation run	T = °C

The first entry of each of DATA, HTT and KAY are dropped as each of these vectors was initially set to the single value of 0. M is set equal to the number of entries in HTT as this is the final number of layers. In the next step TM is set as a (M+2) (L+1) matrix of 0's. The columns in the matrix store the hour, air temperature, layer temperature and ground temperature. After setting L, a loop is set to determine the hourly temperatures for N hours. Since the air temperature is assumed to vary linearly the slope is given by the term,

$$T(KI + 1) - T(KI)$$

The following entries in that column are given by,

$$TM(1,n) = T(KI) + [T(KI + 1) - T(KI)] \times \frac{(n-1)}{L}$$

$$\text{i.e. } y = y_0 + b \times t$$

A double iterative process is used to fill in the matrix from results obtained from function SEAN.

### 3.32 Function SEAN

In this function the first step is to read in the data values from function LL. After initializing N, TR and COEF, CO, C1, and C2 are determined. TR is the vector of series terms and is given as,

$$TR(N) = (\text{latest entry in COEF}) \times \text{Maxex} \times \sin \frac{n\pi}{2}$$

where,

$$\text{Maxex} = \exp \left[ -\left(\frac{n\pi}{h}\right)^2 kTS \right]$$

The next step is to calculate the temperature of the

appropriate layer by using function FOCEF. If the absolute value of  $TR(N) < TQ$  this above step is repeated. If not, it increments  $N$  and does it for the next hour. The final temperature is given as,

$$TM[1,j+1] = \frac{-bh^2}{16k} + \frac{T0}{2} + \frac{T1}{2} + \frac{bTS}{2} + \sum_{n=1}^{\infty} TR(N)$$

and rounded off.

### 3.33 Function FOCEF

This function calculates  $I0$ ,  $I1$ , and  $I2$  which is used in COEF.

It is done as follows,

$$I0 = C0 \frac{h}{\pi n} [1 - (-1)^n]$$

$$I1 = (-1)^{n+1} C1 \frac{h^2}{\pi n}$$

$$I2 = (-1)^n C2 \frac{h^4}{\pi n} \left[ \frac{6}{(\pi n)^2} - 1 \right]$$

$$COEF = \frac{2}{h} [I0 + I1 + I2]$$

### 3.4 Computer simulation using density model

This simulation procedure was designed to estimate the effects of the sublimation process which goes on in the snowpack. It is based on the temperatures computed in the temperature model and the empirical relationships cited in chapter two and in the earlier part of this chapter. A general flowchart of the model is presented in Figure 3.7 along with the program in Appendix IV and the related parameters in Table 3.3. To demonstrate how the process works consider Table 3.4 with observed temperatures and densities at hour 1, temperatures simulated for the following two hours, and the air and ground temperature for that particular hour, for a snowpack with seven layers. Computations based on the formulae show the heat flux and new densities for the following hours. The simulated flux (NEWFLX) was calculated using the simulated (new) average densities (NAVDEN), while OAVDEN and OLDFLX represent the observed average density and flux. The results are presented in Table 3.5.

### 3.5 Computer simulation for recalculating temperature and computing melt.

This is an extended version of the temperature model explained previously. In this model two steps are carried out. First, temperatures are recalculated on the basis of new densities and secondly the amount of melting or freezing is computed based on these new temperature values. If the temperature exceeds  $0^{\circ}\text{C}$  they are set to  $0^{\circ}\text{C}$ . The flowchart for this part of the three phase model is shown in the overall flowchart for the entire three phase



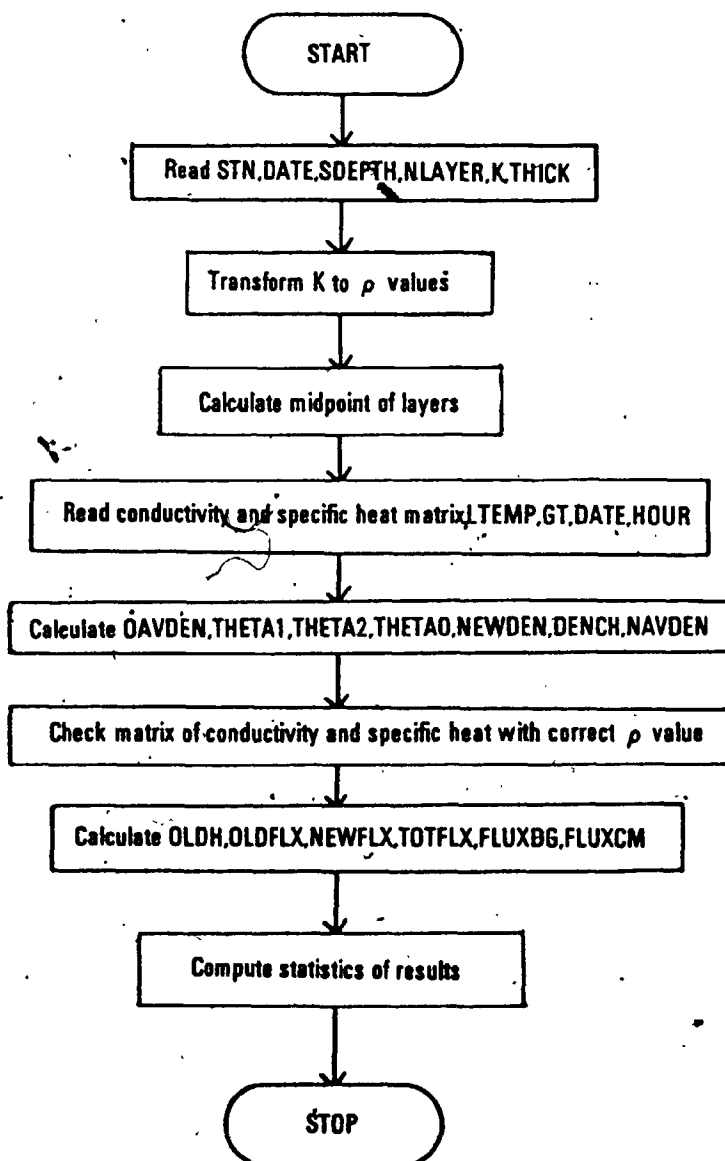


FIGURE 3.7 . GENERAL FLOWCHART FOR THE DENSITY MODEL

Table 3.3

List of Parameters for the  
Density Model

<u>Parameter</u>	<u>Definition</u>	<u>Remarks</u>	<u>Units</u>
STN	Station number	Matched with temperature run	
DATE	Date	Matched with temperature run	
SDEPTH	Snow depth		cm
NLAYER	Number of snow layers	Maximum of 13	
K	Variable computed using density	Calculation performed in temperature model	
THICK	Thickness of layer	Read in from temperature model	cm
DEN(LAYER)	Vector of density values for every layer	Recomputed back from K values	gm cm <sup>-3</sup>
LHDEN	Vector of density values for preceding hour		gm cm <sup>-3</sup>
MIDPT	Midpoint of layer	Calculated for every hour	
MATRIX	Matrix containing values of conductivity and specific heat	Values are matched with densities for every layer	
LTEMP	Layer temperatures	Computed in temperature model	T = °C
THETA1	Temperature difference between midpoint of two consecutive layers	For top layer it is the difference between midpoint of top layer and air temperature. For bottom	°C

Table 3.3 (Cont'd.)

<u>Parameter</u>	<u>Definition</u>	<u>Remarks</u>	<u>Units</u>
THETA1 (cont.d)		layer it is the difference between midpoint of bottom layer and ground temperature	
THETA2	Temperature difference between midpoint of two consecutive layers for the following hour	Calculated in the same way as above for top and bottom layers	$^{\circ}\text{C}$
THETA0	Average density between two successive layers for two consecutive time periods		$^{\circ}\text{C}$
OAVDEN	Original average densities	The average is for two consecutive layers	$\text{gm cm}^{-3}$
LAMDA2	Variable in model	Computed	$\text{cal cm}^{-1}\text{ }^{\circ}\text{C}^{-1}\text{ sec}^{-1}$
LAMDA1	Variable in model	Computed	$\text{cal cm}^{-1}\text{ }^{\circ}\text{C}^{-1}\text{ sec}^{-1}$
NEWDEN	New density of layer	Computed	$\text{gm cm}^{-3}$
DENCH	Density change for individual layers	Change between that layer and original density	$\text{gm cm}^{-3}$
NAVDEN	Average of new densities	The average is for two consecutive layers	$\text{gm cm}^{-3}$
OLDH	Diffusivity between layers	Computed	$^{\circ}\text{C cm}^{-2}\text{ sec}^{-1}$
OLDFLX	Heat flux between layers	Computed with OAVDEN	$\text{cal cm}^{-2}\text{ hr}^{-1}$

Table 3.3 (Cont'd.)

<u>Parameter</u>	<u>Definition</u>	<u>Remarks</u>	<u>Units</u>
NEWFLX	Heat flux between layers	Computed with NAVDEN	$\text{cal cm}^{-2} \text{hr}^{-1}$
TOTFLX	Total flux for that layer	Computed	$\text{cal cm}^{-2} \text{hr}^{-1}$
FLUXDG	Flux per degree	Computed	$\text{cal cm}^{-2} \text{hr}^{-1} \text{ } ^\circ\text{C}^{-1}$
FLUXCM	Flux per centimeter thickness	Computed	$\text{cal cm}^{-2} \text{hr}^{-1} \text{ cm}^{-1}$
GT	Ground temperature		$^\circ\text{C}$

Table 3.4

Temperature - Density Data

	<u>Hour 1</u>	<u>Hour 2</u>	<u>Hour 3</u>
Air Temp. (°C)	-1.3	-1.5	-1.5
Density (gm cm <sup>-3</sup> )	0.04		
Layer 1	-7.21	-4.56	-4.02
Density	0.04		
Layer 2	-7.23	-6.44	-5.67
Density	0.04		
Layer 3	-7.24	-7.00	-6.59
Density	0.04		
Layer 4	-7.25	-7.16	-6.99
Density	0.04		
Layer 5	-7.26	-7.20	-7.14
Density	0.04		
Layer 6	-7.27	-7.22	-4.86
Density	0.04		
Layer 7	-7.22	-2.15	-1.06
Density	0.04		
Ground Temp. (°C)	3.8	3.8	3.8

Table 3.5

Results of density-flux simulation

Density	Hour 1		Hour 2		Hour 3	
	OAVDEN	OLDFLX	NEWDEN	NAVDEN	NEWFLX	NAVDEN
AIR	0.04	.12670		.3878	2.2200	.3218
L1	0.04	.00042	.3878	.3435	0.3650	.3081
L2	0.04	.00021	.2993	.2296	0.0670	.2421
L3	0.04	.00021	.1600	.1200	0.0074	.1609
L4	0.04	.00021	.0800	.0682	0.0007	.5661
L5	0.04	.00021	.0565	.2296	0.0035	1.000 *
L6	0.04	-.00107	.4027	.2160	-0.5845	.1870
L7	0.04	-.23640	.0293	.0293	-0.15556	.0265

\* If density exceeds 1.000 gm cm<sup>-3</sup> it is set to 1.000 gm cm<sup>-3</sup>

model, as shown in Figure 3.5 (Program is given in Appendix V). The empirical relationship used for this melt/freeze calculation has been explained earlier in this chapter.

### 3.6 Summary

The model is designed so that conditions in the snowpack can be simulated for any number of hours. For this particular research, snowpack conditions were simulated for every sampling point where layer information was available. Simulations were performed on every time point on which data was collected. Appendix VI shows the number of hours for which the simulation was done for different snow courses.

S

## CHAPTER IV

### THE FIELD COMPONENT OF THE RESEARCH

#### 4.1 Introduction

Chapter four describes the river basin which was selected to test the model, and the procedures followed to obtain the necessary data for the model.

Since measurements were taken at selected time periods the research did not require instrumenting the basin except for a discharge recorder to determine the runoff from the basin. The other measurements, namely air temperature, snow depth, water equivalent and snow temperature were collected by the researcher and are discussed later in this chapter.

The research area is part of the North Thames River Basin and covers an area about 180 square kilometers. The Medway Creek which flows into the North Thames river is located just north of the city of London, Ontario. It is monitored by the Water Survey of Canada for hydrologic and watershed management purposes. Figure 4.1 shows the location of the Medway Creek drainage basin and the sites where the snow courses were carried out.

The watershed was selected for study for the following reasons:

1. The basin is small enough to enable data to be collected by two individuals in one day.





2. The whole basin is easily accessible from the city of London by roads enabling easier sampling of some areas.
3. The discharge recorder which is maintained by the WSC is located in an optimal position at the basin outlet.
4. The basin is adjacent to a Class One meteorological station (London Airport) which provided part of the meteorological data which could not be collected in the basin on a continuous basis.

#### 4.2 Physiography, Soils, and Landuse

The physiography of the Medway Creek drainage basin is dominated by a gently rolling till plain (Chapman and Putnam, 1973). In places the till plain has a thin lacustrine veneer which consists of stony or clay loam or other waterlaid deposits. As seen in Figure 4.1 the contour height varies from 875 feet in the south of the basin on the outskirts of London to 1025 feet on the northern fringes of the basin. The mean north-south gradient is 3 meters per kilometer. The basin has two major tributary areas which join at Arva and then flow through the narrow neck of the basin to the North Thames.

A typical soil profile of the area ranges from 45 to 55 cm. It consists of a dark coloured A<sub>h</sub> horizon (12-18 cm), a yellowish brown A<sub>e</sub> horizon (10-15 cm), a B horizon (15-20 cm) of loam or clay loam and a greyish brown C horizon of stony till (Chapman and Putnam, 1973). These poorly drained loams often increase the rate

of runoff. Under winter conditions the soil remains frozen to depths of 10 cm till early spring.

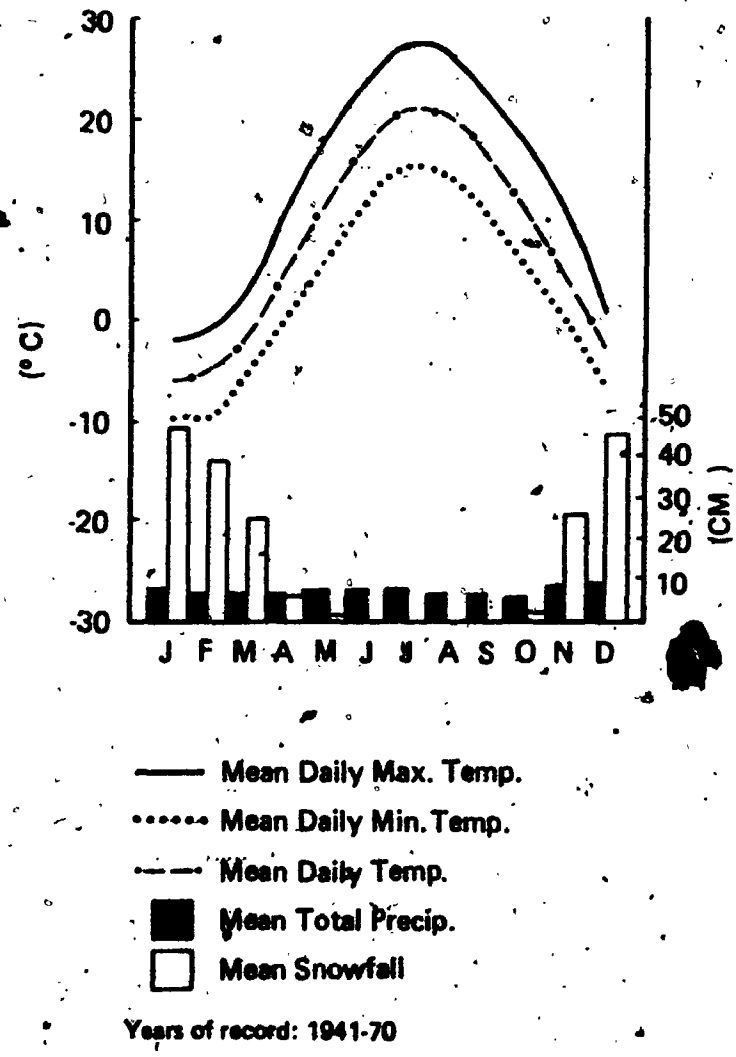
The landuse is dominated by agriculture. Ninety percent of the basin is in agricultural production with only 18 square kilometers under forest cover (Karuka, 1963). The major farming activity is cattle and hog production and therefore the main crops are feed grains and silage corn. There is some urban encroachment at the south end of the basin on the outskirts of London.

#### 4.3 Climate

The climate of Southwestern Ontario is dominated by the westerly circulation. Being located in the mid-latitude belt, the Medway basin has a pronounced variability of weather conditions on a day to day basis. This variability is in response to two factors:

1. The continuous succession of high and low pressure systems formed when northward migrating maritime tropical air masses with a high moisture content converge with dry arctic air masses moving in from northern Canada and the Arctic.
2. The proximity of the Great Lakes, which are a good source of atmospheric moisture and tend to change the moisture balance characteristics of the air masses which traverse them.

A thirty year record of the meteorological data is shown in Figure 4.2. The mean annual values are shown in Table 4.1. The important climatic elements related to this research are discussed



**FIGURE 4.2 METEOROLOGICAL DATA FOR LONDON, ONTARIO**

Source: Harz, F.K. and Thomas, M.K. (1979). Climate Canada.

Table 4.1  
Mean Annual Values of Temperature and  
Precipitation for London,  
Ontario. 1941-70

Mean Daily Maximum Temperature	12.5°C
Mean Daily Minimum Temperature	2.6°C
Mean Daily Temperature	7.5°C
Mean Total Precipitation	92.5 cm
Mean Snowfall	201.1 mm
Mean No. of Precipitation Days	165
Mean No. of Snowfall Days	66

below.

#### 4.31 Air Temperature

Air temperature is one of the primary climatic elements used in this research. The spatial variability in the basin is less than 2 - 3 °C (Webber and Hoffman, 1967). The lowest monthly mean temperature occurs in January but large deviations from the mean may take place both daily and seasonally.

The hourly temperature data necessary for modelling necessitated the use of local temperature data. The cost and time involved in establishing and maintaining temperature data on a continuous basis from each snow course site was prohibitive. It was therefore decided to make use of the air temperature data recorded at London Airport. This meteorological station is located approximately six kilometers east of the nearest snow course and about twenty kilometers south southeast from the farthest snow course. Air temperature data which was measured on days when the snow course was operated was checked with London Airport readings because of anticipated variations. At the sites the temperature was measured at a height of one meter from the snow surface. This comparison showed an overall daily variance of + 0.5 °C and + 1.0 °C. Under this assumption it was felt that the data from the meteorological station would be at least representative of the general magnitude of the air temperature occurring at a particular time in the drainage basin. This

information on daily air temperature is presented in graphical form in the next chapter.

#### 4.32 Precipitation

Precipitation in the region tends to be evenly distributed throughout the year and is primarily cyclonic in nature in response to the mixing of air masses of different characteristics. The annual snowfall for London, Ontario for the period 1940-80 is shown in Figure 4.3. It is apparent from the graph that during the field season for this research (1977-78) the mean annual snowfall was well above the previous 40 year mean. Snowfall and rainfall for the basin for that winter is shown in Figure 4.4. A storm in early December produced about 57 cm of snowfall which was followed by 2.2 cm of rainfall the next week. This apparently depleted a large volume of the snowcover. Another storm in late January produced about 29 cm of snow and about 1 cm of rain.

Snowfall is defined as the rate at which snow falls and is expressed as centimeters of snow depth which fell between two time periods (Potter, 1965). Snow cover refers to the total accumulation of snow on the surface and is measured as total depth in centimeters over the surface. The snow cover season for this region extends from October 18 to April 19 (Hare and Thomas, 1979). During this period snow metamorphism and snowmelt can occur several times. Figure 2.2 shows that the snow cover in this

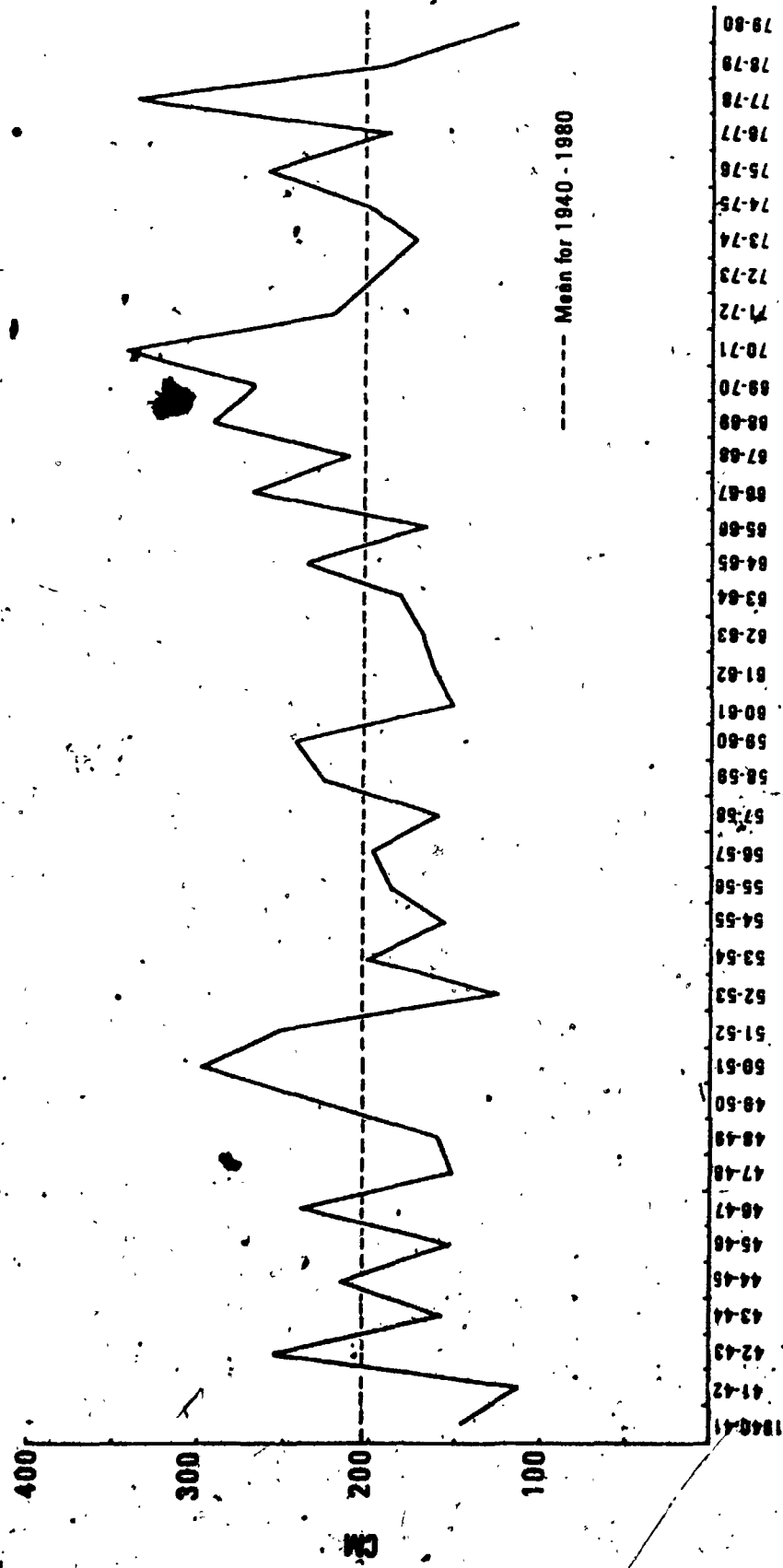


FIGURE 4.3 MEAN ANNUAL SNOWFALL LONDON, ONTARIO 1940 - 1980



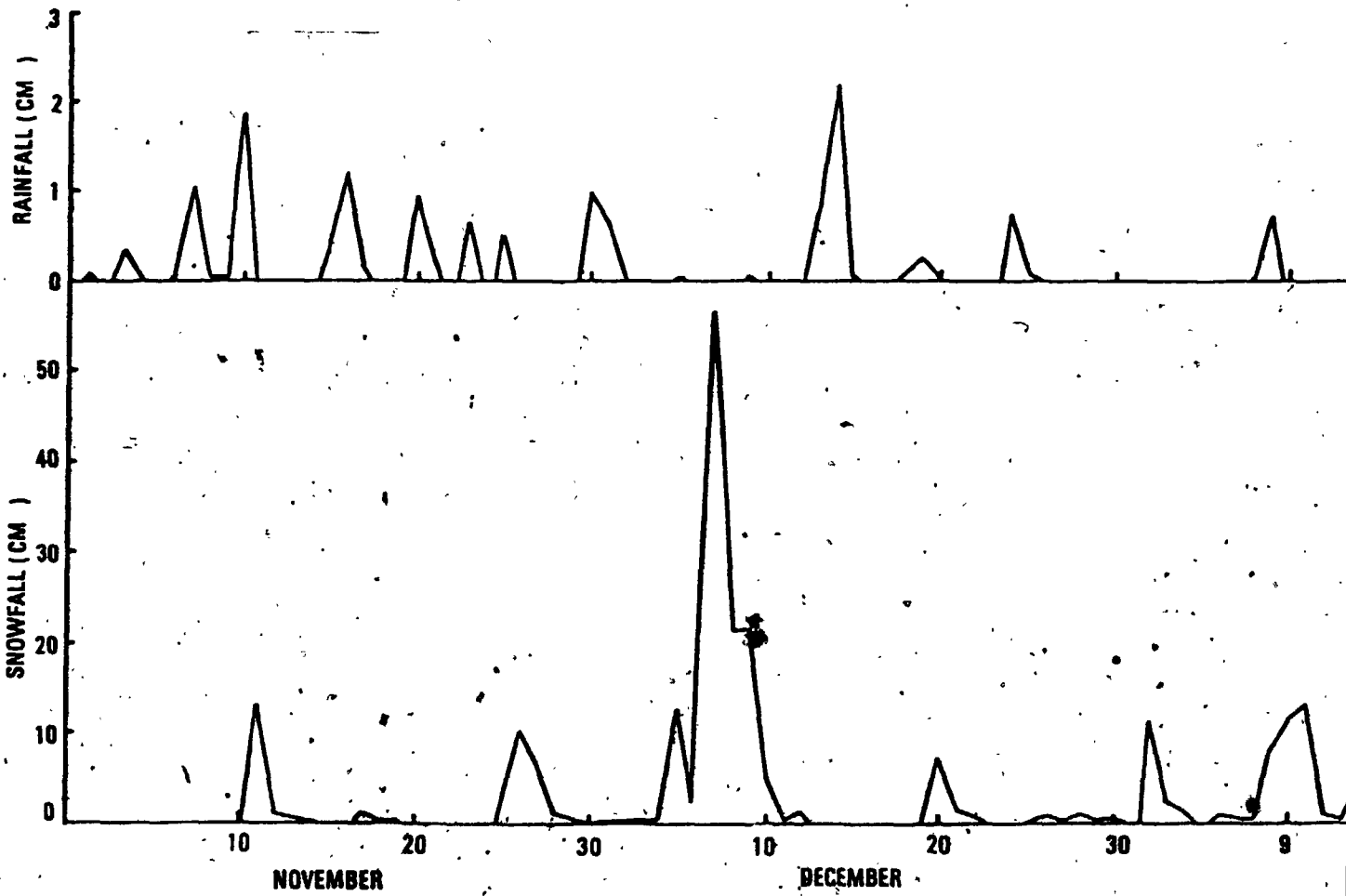
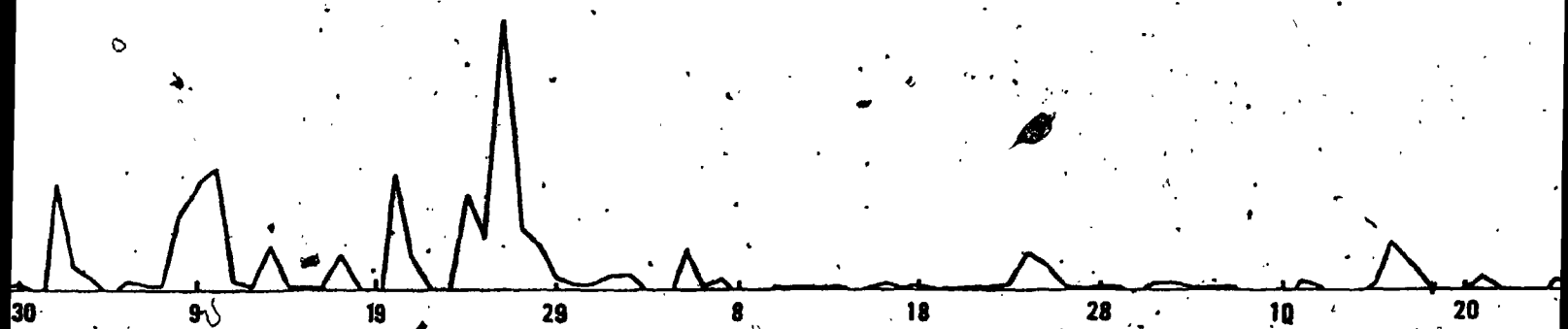


FIGURE 4.4 PRECIPITATION FOR LONDON, ONTARIO

105



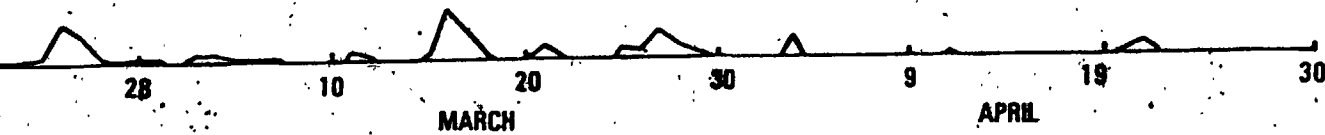
JANUARY

FEBRUARY

MARCH

1977-78

| 20 |



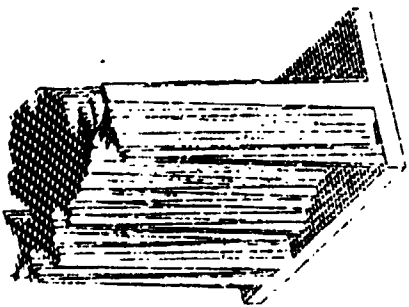
13000

basin in 1977-78 lasted from the middle of December to the end of March and there was a sporadic snow cover in November and April. The maximum depth was reported during the end of January. Snow water equivalent, is shown in Figure 4.5. It is the amount of water which is derived from melting that particular amount of snow. Since the snowpack gets ripened during late winter and early spring it is not unusual to find the maximum values during that time period.

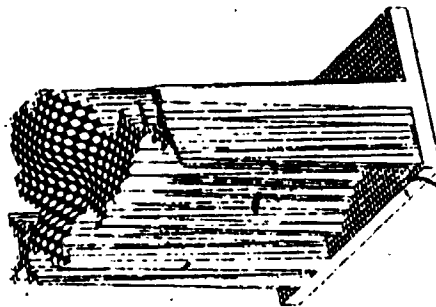
#### 4.33 Other related data

Snow temperature data along with the sampling dates for the snow courses is shown in Appendix VII. A YSI tele-thermometer was used for the measurements. The procedure followed required inserting the probe into the snow profile at various depths and letting it stand for about 60 seconds. When it came to equilibrium a recording was done. It was repeated several times at every depth to check for consistency and accuracy. In some of the sampling points, especially during the melt season a temperature slightly above  $0^{\circ}\text{C}$  was recorded. This is due to the presence of melt water in the snowpack which in some instances have enlarged the pore spaces between the snow grains trapping the outside air.

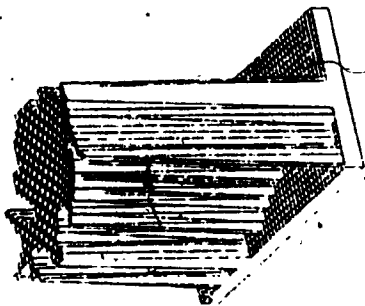
Soil temperatures were also recorded at the sites for those selected time periods when measurements were carried out. The same tele-thermometer was used but with a different probe. However, since continuous data was needed, published data was used



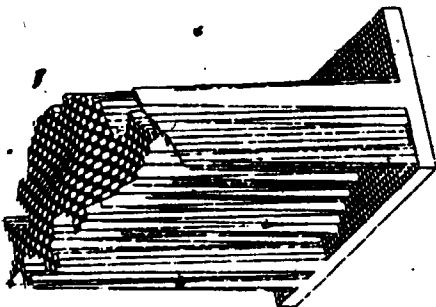
JANUARY 31, 1978  
MAX=17.22cm MIN=12.52cm



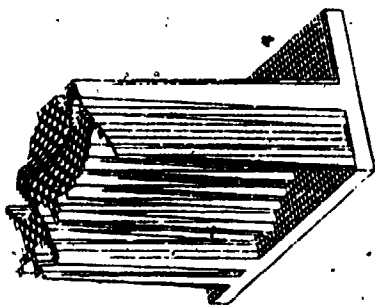
MARCH 30, 1978  
MAX=13.31cm MIN=11.06cm



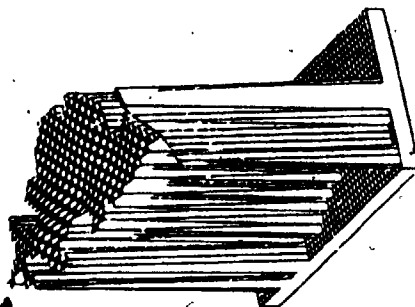
JANUARY 14, 1978  
MAX=11.48cm MIN=6.72cm



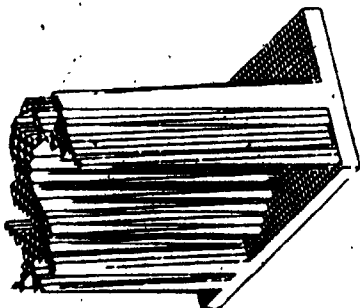
MARCH 15, 1978  
MAX=17.86cm MIN=13.74cm



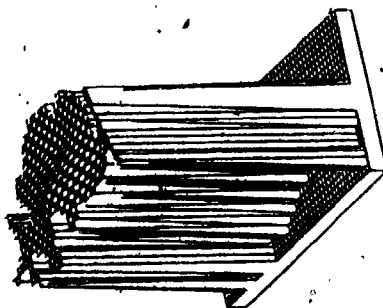
JANUARY 02, 1978  
MAX=7.32cm MIN=4.42cm



MARCH 02, 1978  
MAX=18.24cm MIN=12.60cm



DECEMBER 16, 1977  
MAX=10.19cm MIN=5.13cm



FEBRUARY 16, 1978  
MAX=17.42cm MIN=11.32cm

FIGURE 4.5 WATER EQUIVALENT OVER MEDWAY BASIN

to fill in the dates when data was unavailable. The published soil data is from the Elora Research station, the closest available for this research basin.

#### 4.4 Snow surveying

Normally snow survey methods calculate depth, density and water equivalent for the whole basin. In this study snow surveying in Medway basin included not only total snow depth and water equivalent but also measuring water equivalent at different depths to estimate the densities at those depths. As mentioned earlier temperature measurements at those depths were also made.

##### 4.4.1 Site Selection

The snow cover in a basin varies from point to point over time. This is one of the major considerations which had to be made with respect to data collection. A pilot survey was made prior to the beginning of the snow season to select the most suitable snow courses. Selection was made on the basis of a) the representation of the snow course for that particular area, b) the surface topography of the area, c) protection from wind drifts, d) permanence of the site throughout the entire season and e) accessibility. With the above considerations, six snow courses were selected (see Figure 4.1) at different points on the basin. Five of these were representing 10-point open snow courses and the other a 5-point course located in a wooded area close to the

discharge gauge. It has been suggested that the type of site which yields the most consistent and reliable results is an opening in a wooded area surrounded by hills and sloping to permit runoff beneath the snowpack (Environment Canada, 1973). A number of other factors had to be kept in mind when locating the snow courses at the start of the snow season. These included checking to make sure that snow courses do not get ponded during a rainstorm, absence of shrubs or any kind of vegetation, and away from cross country skiers or snowmobilers.

#### 4.42 Snow survey course

A snow survey course consists of a set of sampling points in a designed area. Two kinds of snow courses were used for this research:

10-point course; consists of ten sampling points at which measurements were taken at the beginning and the middle of the month. It was established along a straight 900 ft. base line with 10 sampling points, 100 ft. apart.

5-point course; consists of five sampling points at which measurements were taken four times a month, preferably on the 1st, 8th, 15th and 23rd. In this case a straight 400 ft. base line is used with each sampling point 100 ft. apart.

These guidelines are used as standard snow course measurements.

#### 4.43 Snow equipment

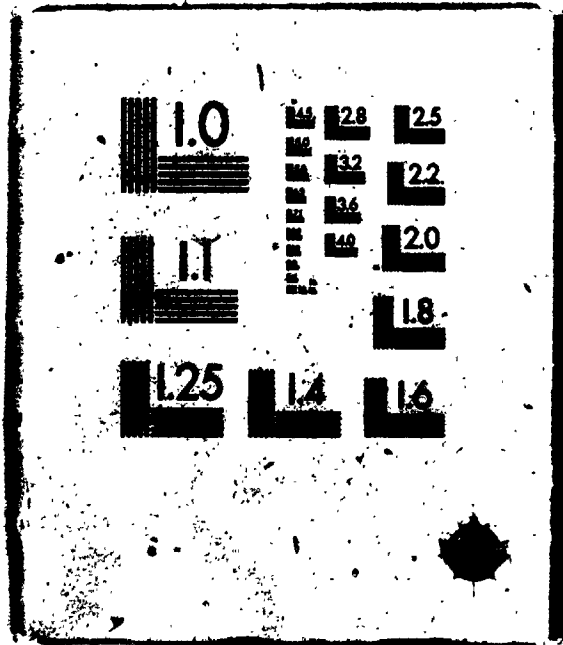
A MSC Type I snow sampler was used to collect the data. The complete set of sampling equipment is shown in Figure 4.6. The equipment consists of the following parts:

1. A single cutter tube, 43 inches in length
2. A spring balance and weighing scale
3. A turning bar handle to facilitate cutting
4. Two snow removal tools
5. A shovel
6. A weighing cradle
7. A set of ten markers

The inside diameter of the cutter tube is 2.776 inches. 3.5 ounces of snow core from this sampler has a water equivalent of 1 inch. In other words if 3.5 ounces of snow from this cutter tube are melted it will produce a 1 inch column of water in the sampling tube. The cutter tube is made of aluminum and is fastened to a plated steel cutter with 16 teeth. The tube is also provided with slots which allow the core to be seen. On the outside of the tube an inch scale is marked to determine the depth of penetration and the length of the snow core. The spring balance has two scales marked on it. For this type of sampler the red scale is used. In this scale each small division represents one-half of water equivalent and each long graduation represents one inch of water equivalent. The spring balance was checked for accuracy ( $\pm 1$  oz.) every month from the beginning of



2



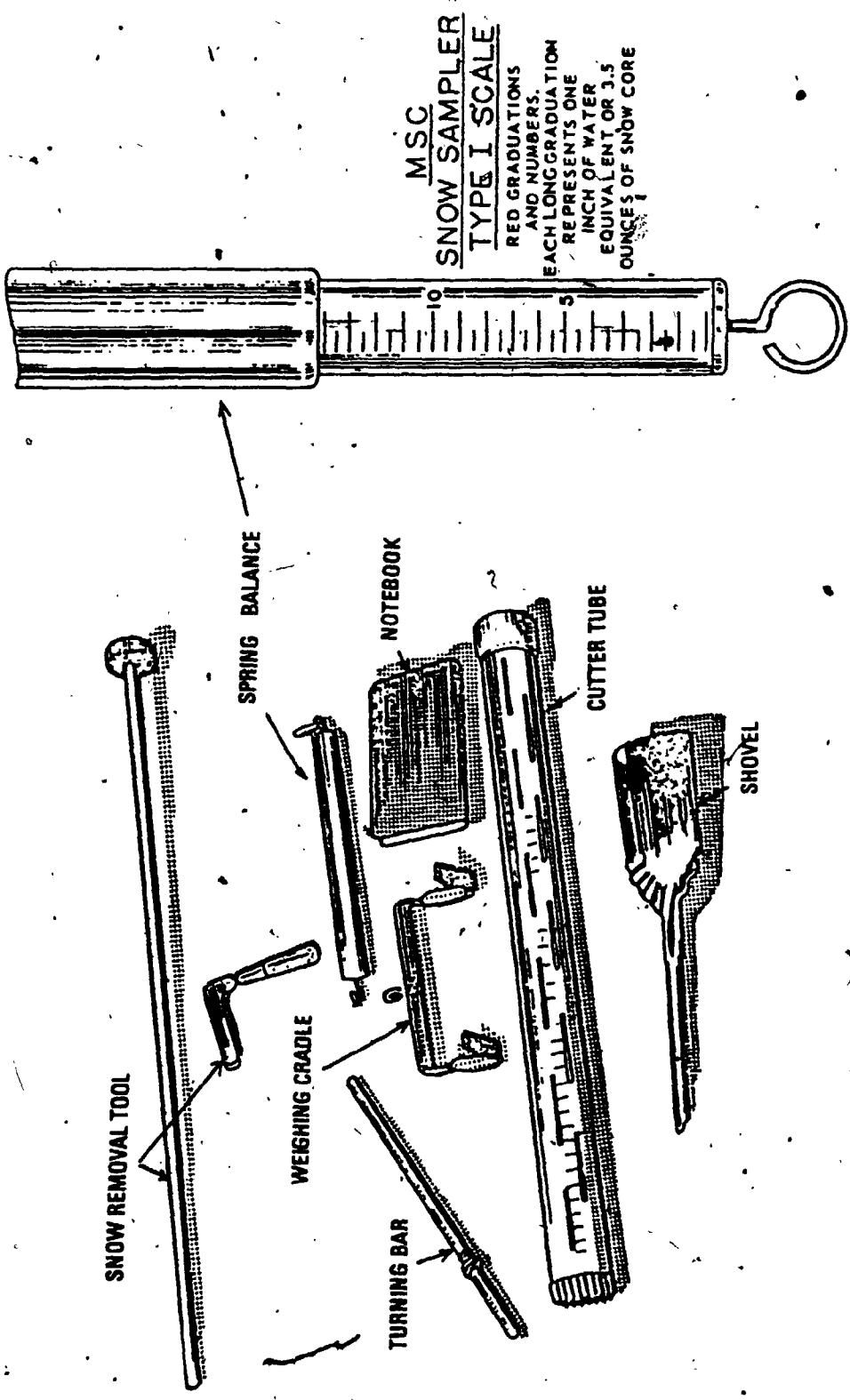


FIGURE 4.6 PARTS OF THE MSC SAMPLER TYPE I

the snow season. This check was performed as follows:

- a) A pail was suspended from the balance, repeating the procedure to give a consistent reading.
- b) One quart of water was added to the pail so that the combined weight of the pail and water was 40 oz. more than the original weight of the pail.

#### 4.44 Snow data collection

Measurements began after the first snow cover of the season reached 5 cm or more in depth and continued till the disappearance of the snow cover. A look at the data set shows that for the five point snow courses measurements began on the 12th of November, 1977 while on the 10-point snow courses it did not begin till the 15th of December. This was because of the sporadic nature of the snow cover in the 10-point courses, which did not allow measurements to be made in at least six or more sampling points. The data for different layers was collected by digging a pit as close to the sampling point as possible. For the 5-point snow course, pits were dug at every sampling point while on the 10-point snow courses every alternate sampling point was selected for pit measurements. It was assumed that there would be little difference between the stratigraphy at such site, an assumption which is borne out by the data. The pits were closed every time with the snow taken out and the new pit for the next time slot was dug as close as possible to the sampling point and the previous pit, making sure it was unaffected by the previous

disturbed pit. The horizontal length of each pit was made about 45 inches to ensure that the core could be inserted horizontally.

Figure 4.7 portrays the way in which the measurements were made. Before recordings were taken, sketches of the profile (pit) were made and pertinent information regarding the number of layers, crust formation, conditions at the bottom of the profile were written down. A special profile data sheet was made to record the data for any one profile (pit) sampling point. The sampling tube was cooled to the ambient air temperature to avoid crust formation on the outside before readings were taken, in the following way:

1. Weigh the empty tube.
2. Insert tube into snowpack vertically, drilling down slowly until the teeth end of the core encountered the ground. If it got stuck, it was turned or rotated slowly in one direction until it was released for further penetration.
3. Record the depth of snow to the nearest tenth of an inch from the scale on the tube.
4. The tube was then extracted vertically making sure the core inside remained intact. Record the length of the core.
5. The tube was then put on the cradle which was then hooked to the spring balance to measure the weight of the tube and the core. Precautions were taken to ensure that there was no spillage from the tube. At times the weighing procedure had to be shielded against the wind for accurate measurements.
6. The water equivalent was calculated by subtracting the weight of the empty tube from the weight of the tube and the core.

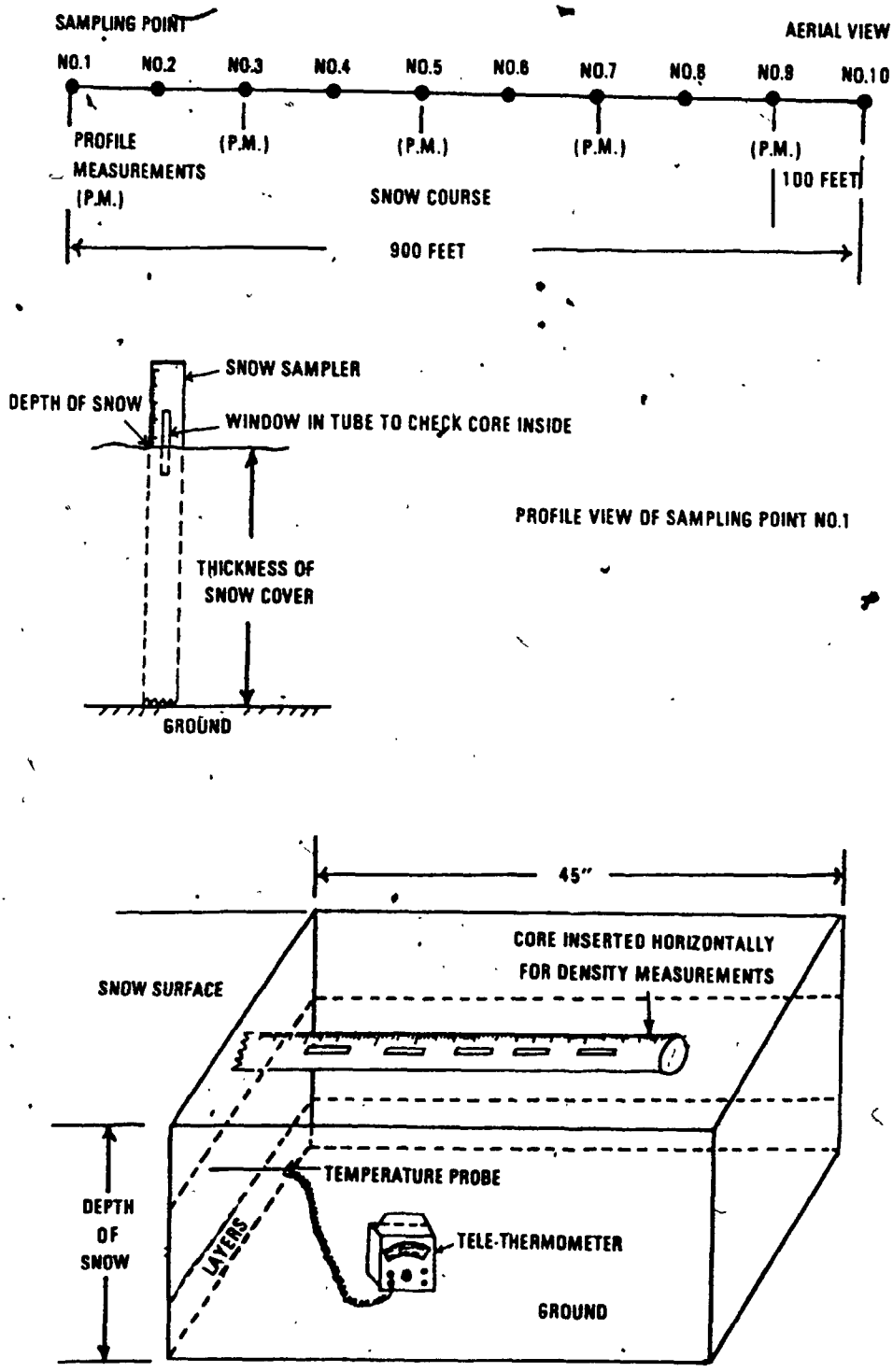


FIGURE 4.7 MEASUREMENT TECHNIQUES

7. The mean density of the snowpack was calculated by dividing the water equivalent by the snow depth.
8. Next, a pit was dug and the number of layers with their thicknesses recorded.
9. The above procedure was then repeated for the layers to calculate their density. In this particular aspect the tube was inserted horizontally to a certain length into the snowpack. It was not always possible to maintain a constant insertion length into the snowpack.
10. The temperature probe was inserted at selected depths for temperature measurements.
11. The probe was also inserted into the soil to measure the soil temperature in the top 5 cm of the soil.
12. After all measurements were taken the pit was filled with snow.

The weight of the empty tube was taken at every alternate sampling point to ensure the accuracy of measurements. If there was any spillage while extracting the core, the measurements were redone. The air temperature was measured on the same spot in the sampling point area, one meter above the snow surface. Each complete set of measurements i.e. for one pit, took about 15-20 minutes on the average.

#### 4.45 Data storage and manipulation

Goodison (1975) has suggested a comprehensive system for data

collection and storage. His procedure of reporting snow course information results in a simple way of coding the data into a computer bank. Based on this notion the data on snow course measurements and the related meteorologic information were compiled into a data base. The format of this data was (see Appendix VIII) selected so that information at various depths in the snowpack can be identified easily. Each profile (pit) measurement consisted of a number of records (cards). Selection can be made on the basis of time or space (sampling point). Since all the information was not used in this research, a filtered data set of similar format was created with the information needed. This set also contained the ground temperature on a daily basis. Since hourly air temperatures were required for the simulation, a separate air temperature file was generated and matched to the exact hour of the data collection when the simulation was done.

## CHAPTER V

### APPLICATION OF THE SIMULATION MODEL

#### 5.1 Introduction

In order to understand the processes which operate in the snowpack and to satisfy the objectives outlined for this research, a simulation of the temperature-density model was carried out with the information collected for the drainage basin. The thermal characteristics and the sublimation process were identified quantitatively and the melt which resulted from the positive energy flux was also computed. The analysis and results are presented in three sections with comparisons to information already available.

In the six snow courses there were a total of 55 sampling points of which pit measurements were done on 30 points. Snow course No. 1 (5-point course) is different from the others as it is located in a wooded area where snow conditions are likely to be more variable. The snow cover in this course extended from the middle of November to the end of April compared to the other snow courses where the snow cover season did not begin till the middle of December. Since the nature of the research implies that conditions be observed under a continuous snow cover, the simulation was carried out till the end of April for each sampling point, although there were periods when there was no snow on the ground.



One sampling point from each snow course was randomly selected to produce the results from the modelling procedure. It is expected that since every sampling point in a snow cover course has similar characteristics, the results generated by the sampling point in question would be representative of that particular site. The overall trends of the snow profiles were also comparable in terms of the temperature-density profiles. Variations occur primarily in relation to snow depth.

The simulation procedure was carried out for the entire snow cover season for Stations 1.04 and 6.09. This was done to show the effect of site on the characteristics of the snowpack. Station 2.03 was simulated from the middle of February to see if the model reacted differently over a short time span. For the other stations, 3.05, 4.01 and 5.03 the simulation was done for the critical spring melt season. The above comparisons over space and time provide indications of the accuracy and usefulness of the model and indicate which portions of the modelling procedure need to be modified.

The results of the simulation for the above stations are provided in Appendix IX. The tables in the appendix indicate the:

1. Heat flux
2. Sublimation
3. Density differences between the top and the bottom of the snowpack

on a daily basis, although the computations were performed on an

hourly basis.

## 5.2 Analysis of thermal characteristics

One of the major components of this research was the examination of the thermal characteristics of the snowpack with special attention to the amount and direction of heat flux. Figures 5.1 through 5.6 have been constructed to provide a graphical view of the thermal simulation procedure. Since it was done in three stages the results are interpreted as Phase I, Phase II and Phase III. Attached within each diagram are air and ground temperatures for that particular time period. This was necessary since they are the boundary conditions and are an integral part of the model. In Phase I the temperatures are computed on an hourly basis given the two boundary conditions, the initial temperature and density of snow at various depths. In Phase II, densities were computed for every hour and for every measurement layer given the hourly simulated temperatures in Phase I. In this phase the midpoints of layers and their thicknesses were also calculated. The simulated density values were then used to adjust the temperatures simulated in Phase I. In Phase II the heat flux was also computed. The temperatures in Phase II are an adjusted version of temperatures seen in Phase I. The major difference between the two phases is that in Phase I, only the initial density values were used to simulate temperatures whereas in Phase II the temperatures are based on simulated densities. The temperature pattern in Phase III is a replica of Phase II for all

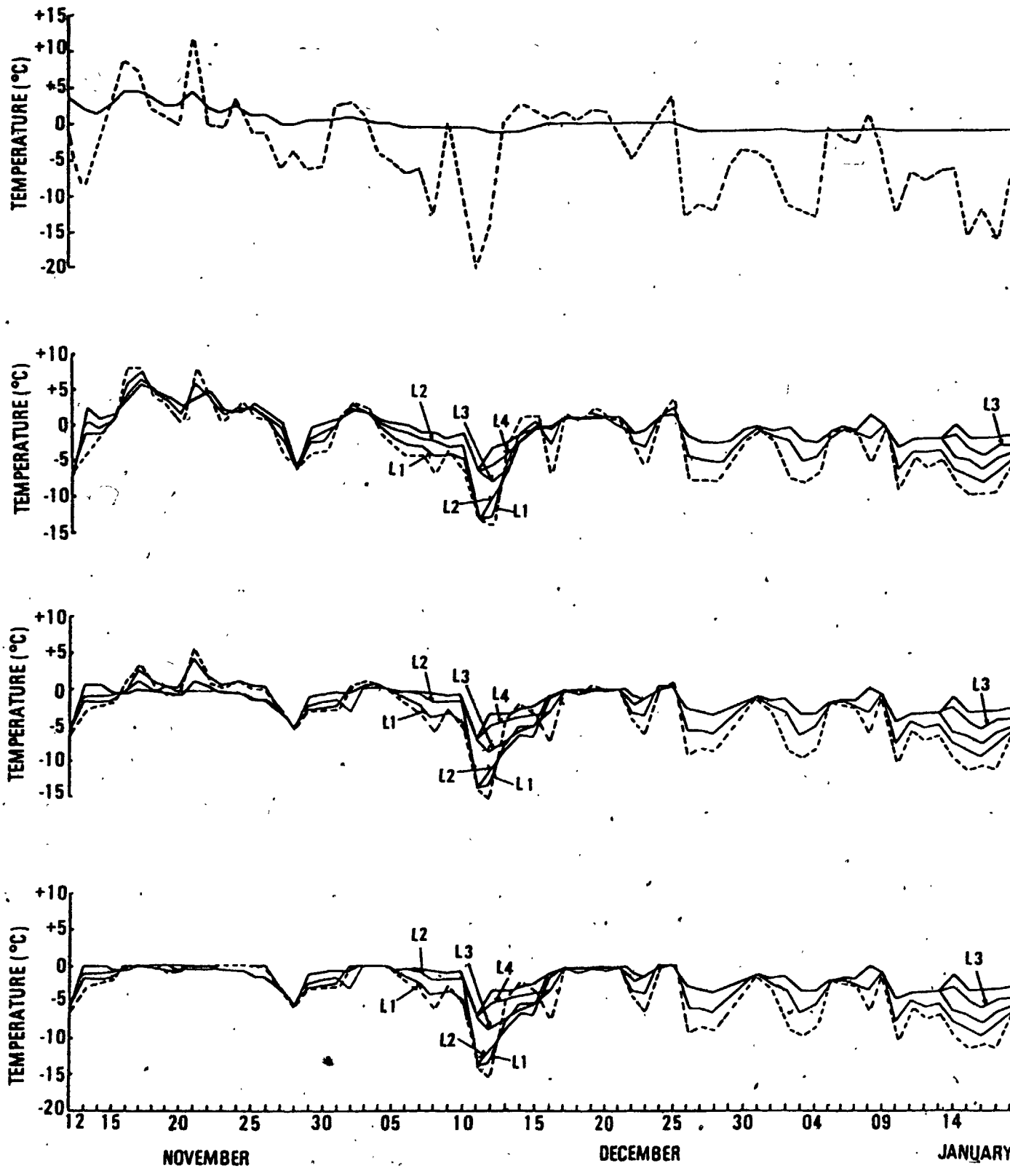
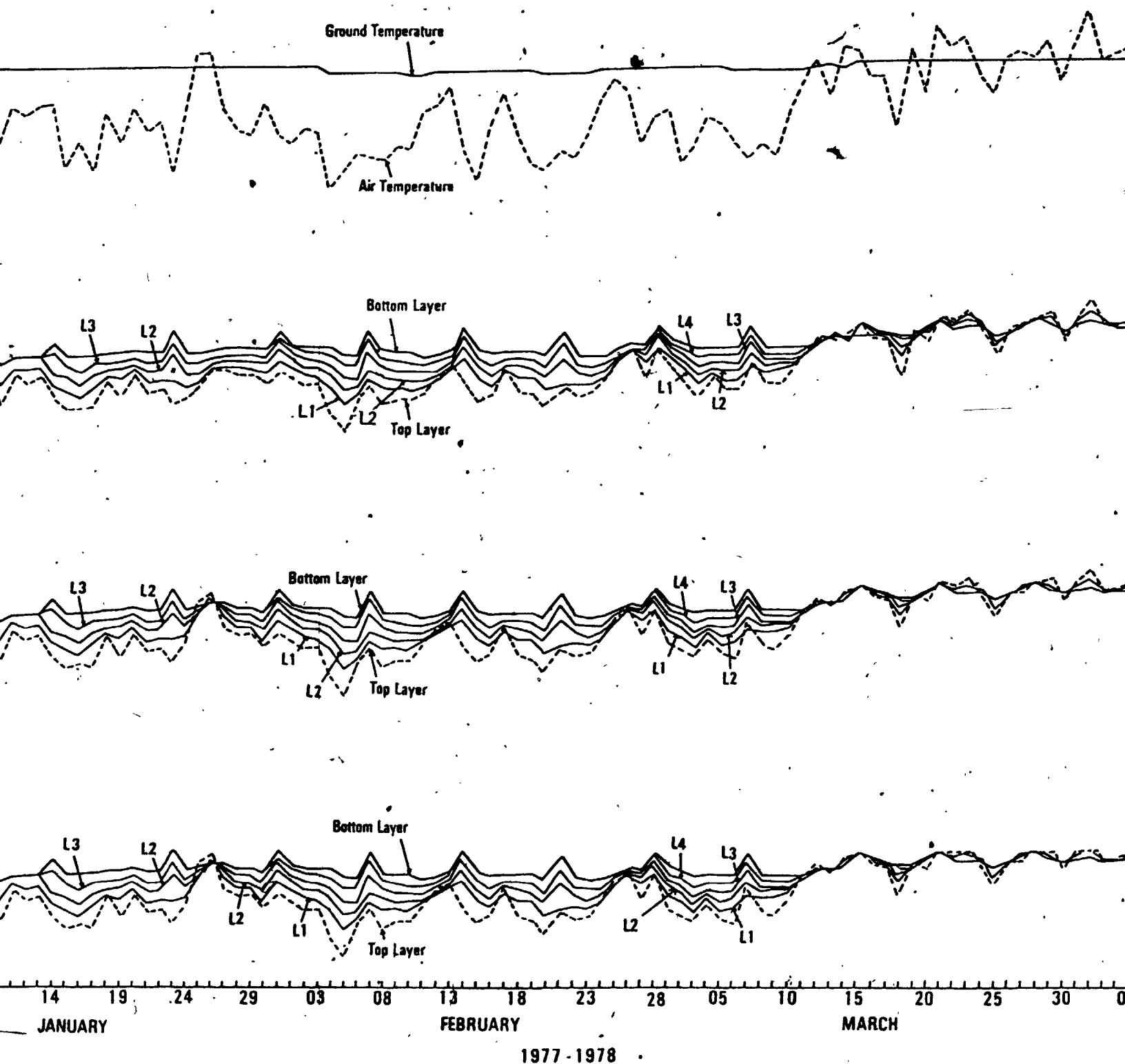
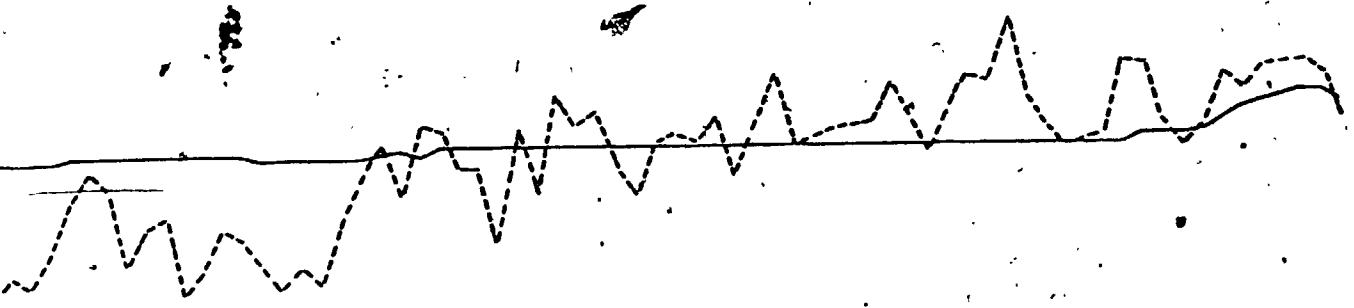
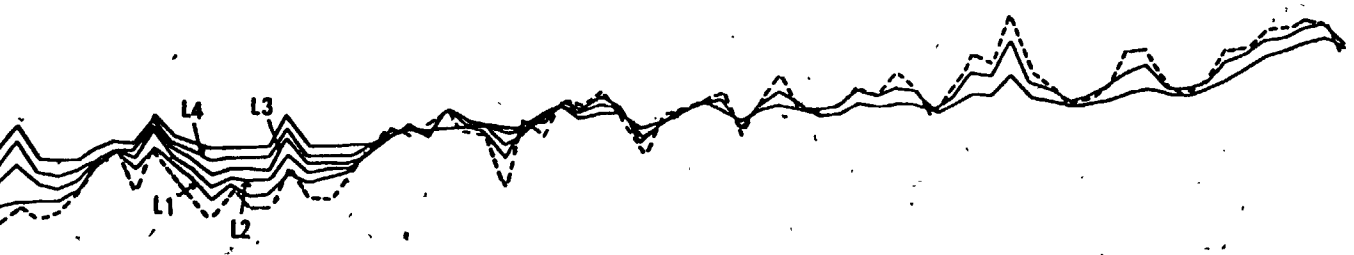


FIGURE 5.1 SNOWPACK TEMPERATURE SIMULATION AT ST. #04

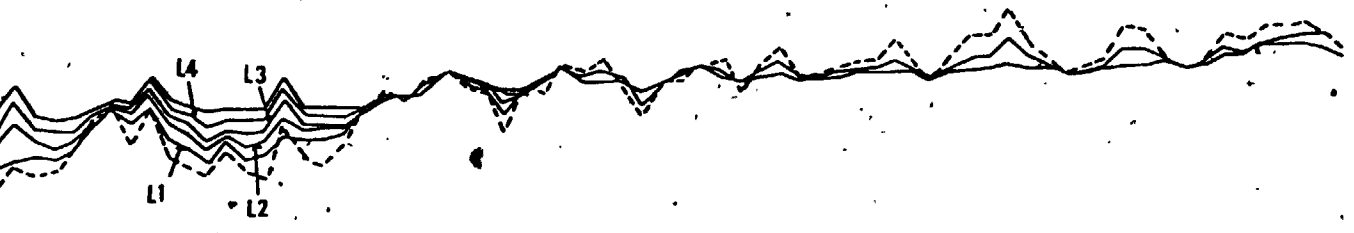




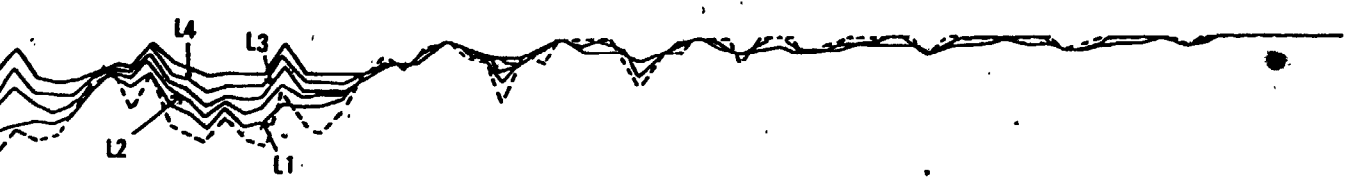
PHASE I



PHASE II



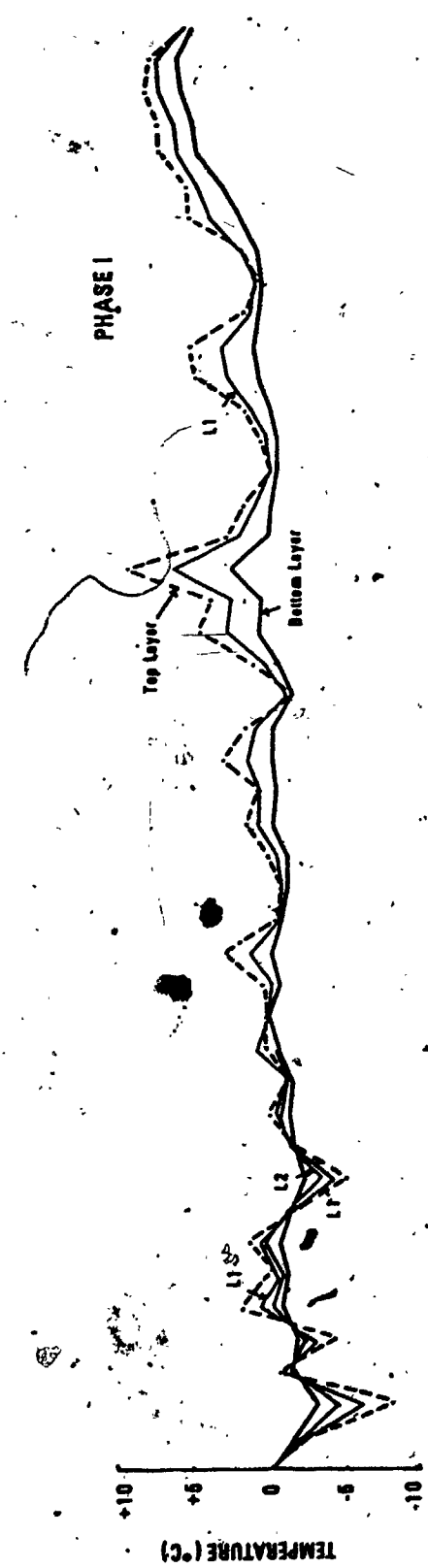
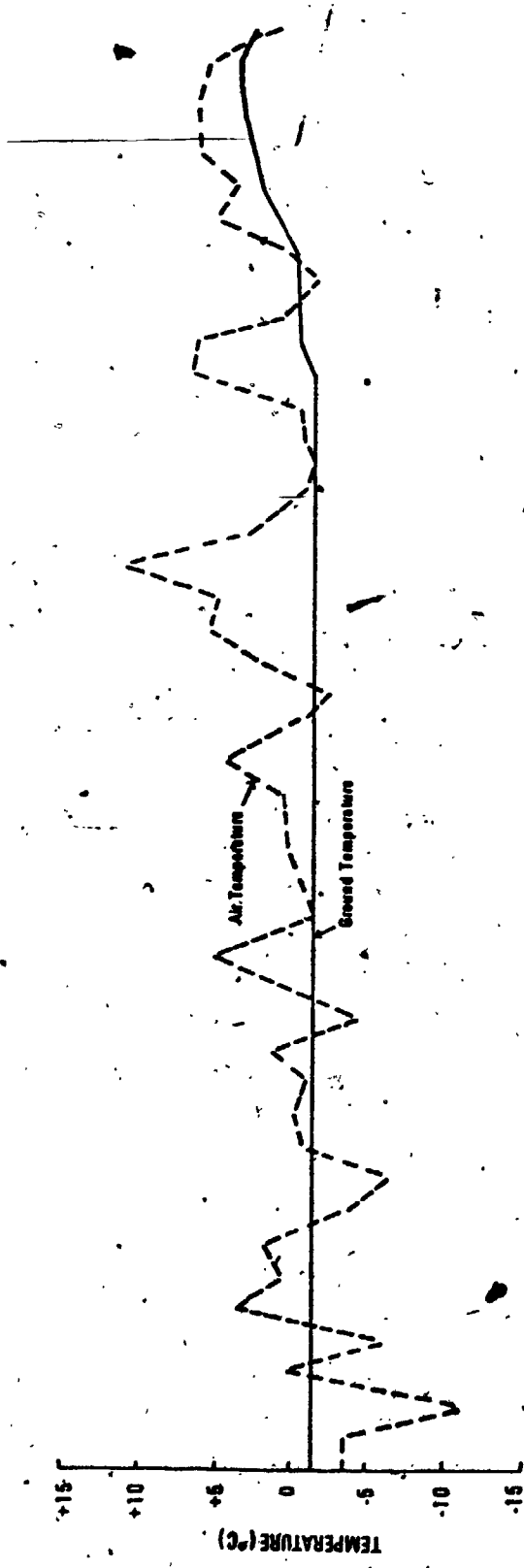
PHASE III



23 28 05 10 15 20 25 30 04 09 14 19 24 30  
MARCH APRIL

7-1978

| 3 of 3 |



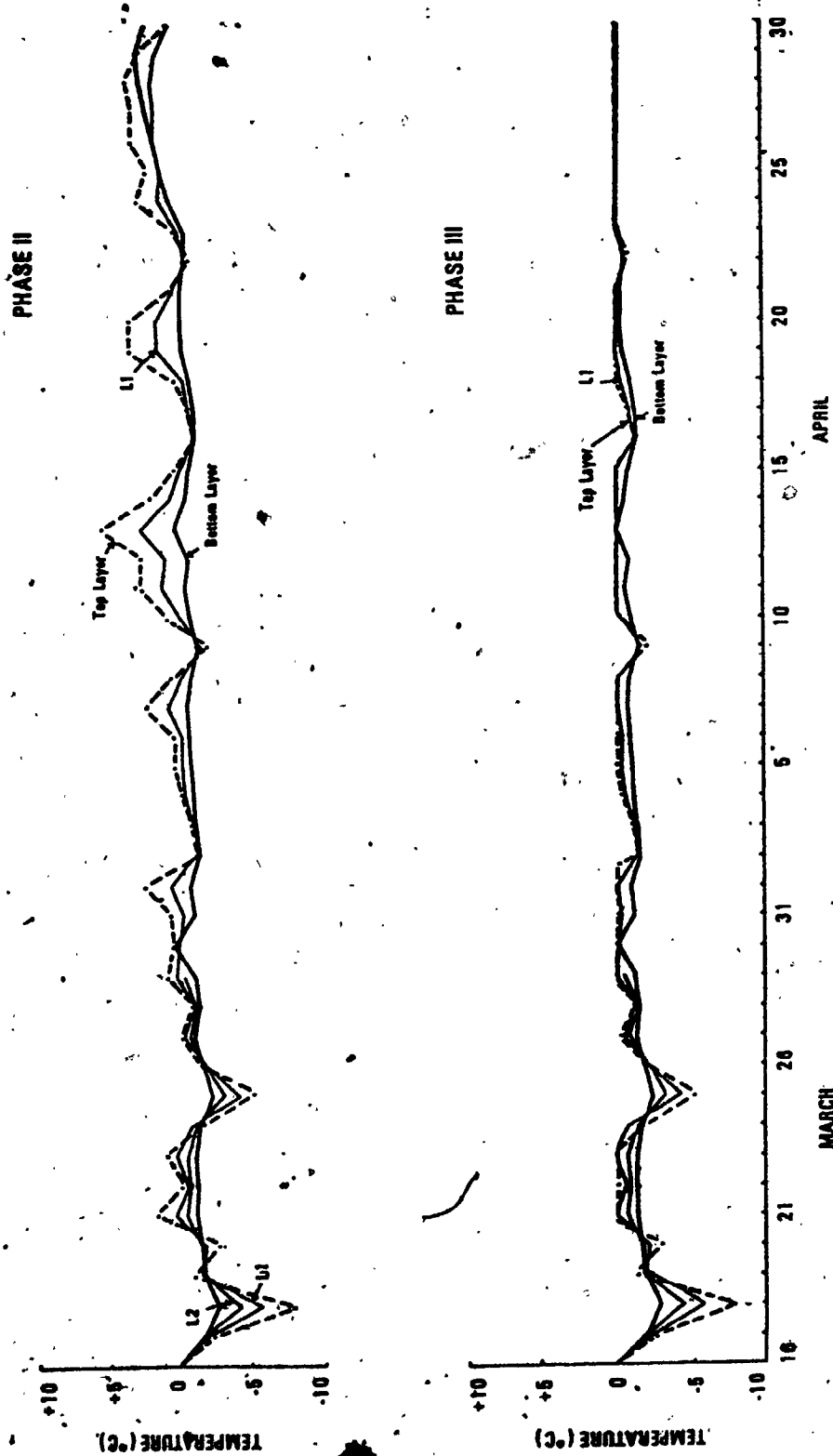
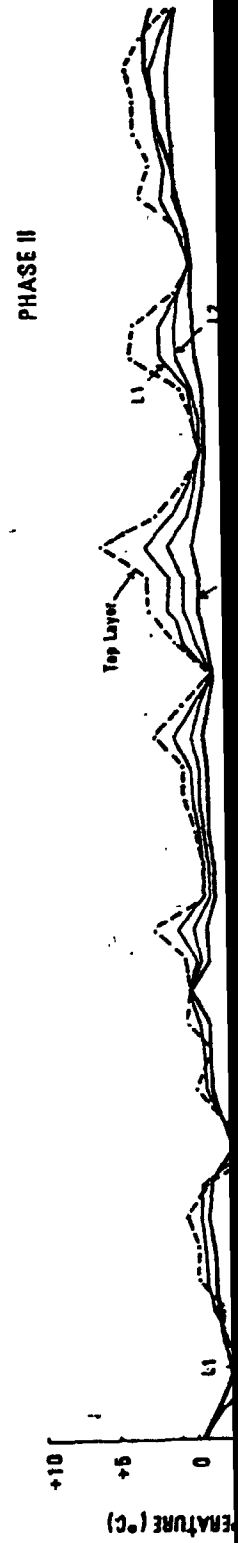
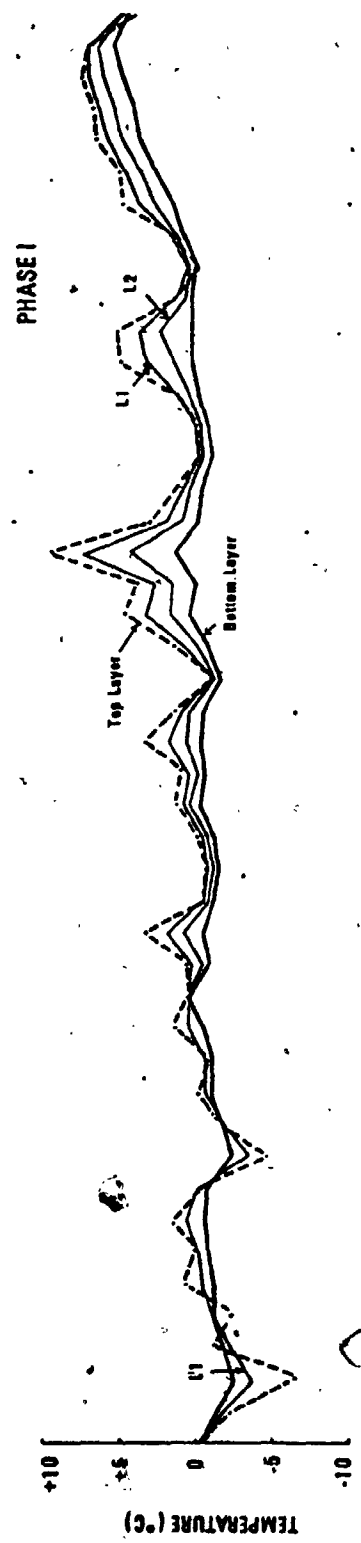
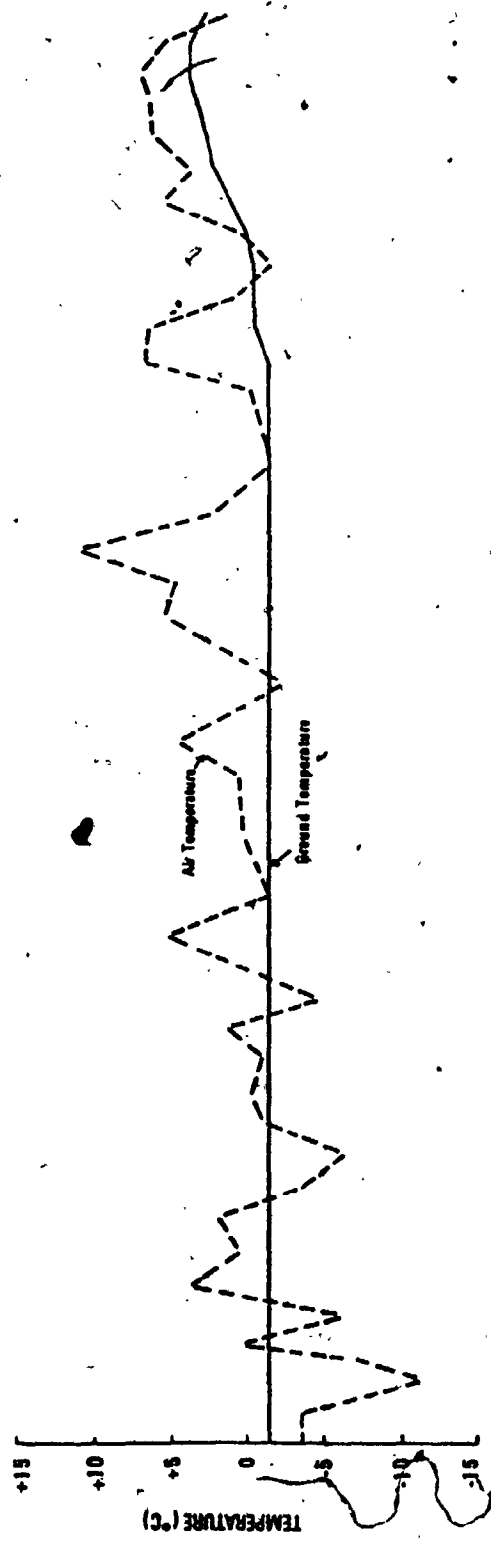


FIGURE 5.2 SNOWPACK TEMPERATURE SIMULATION AT ST. 2.03

2002





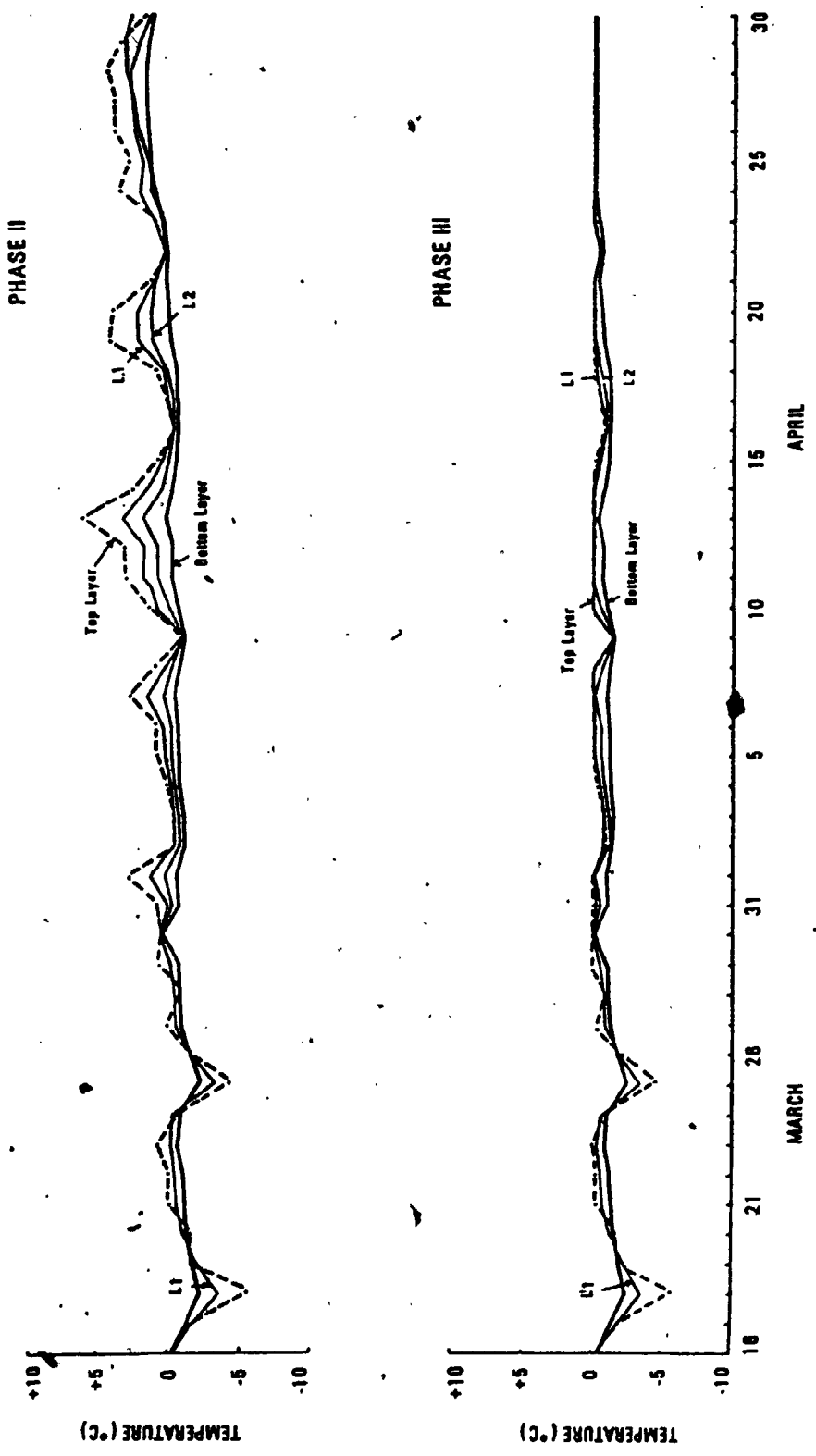
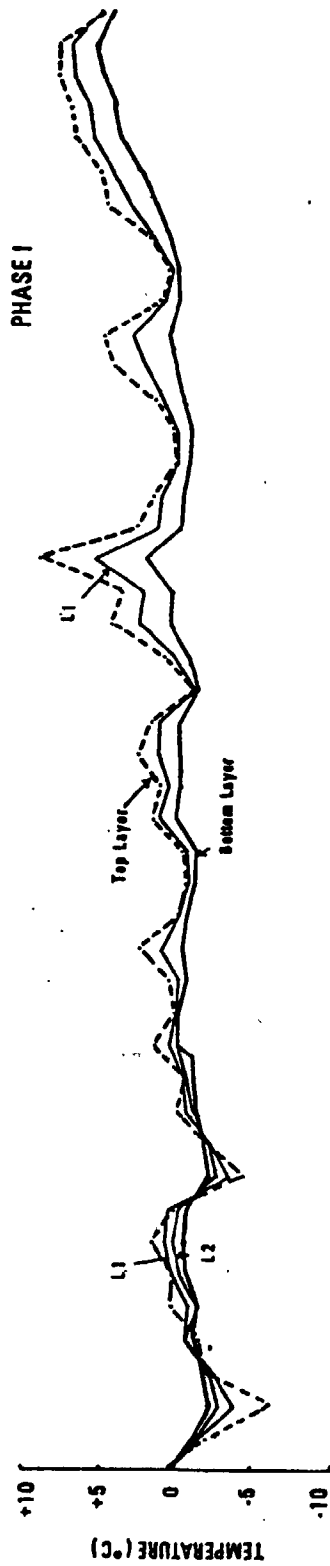
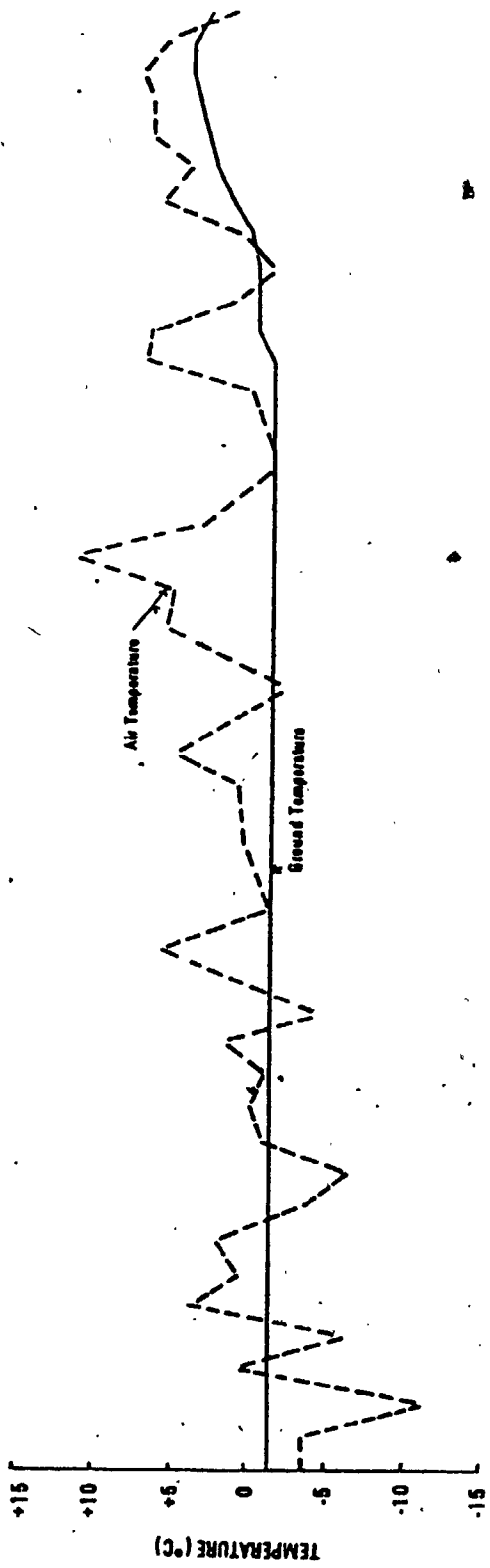


FIGURE 5.3 SNOWPACK TEMPERATURE SIMULATION AT ST. 3.05

2002



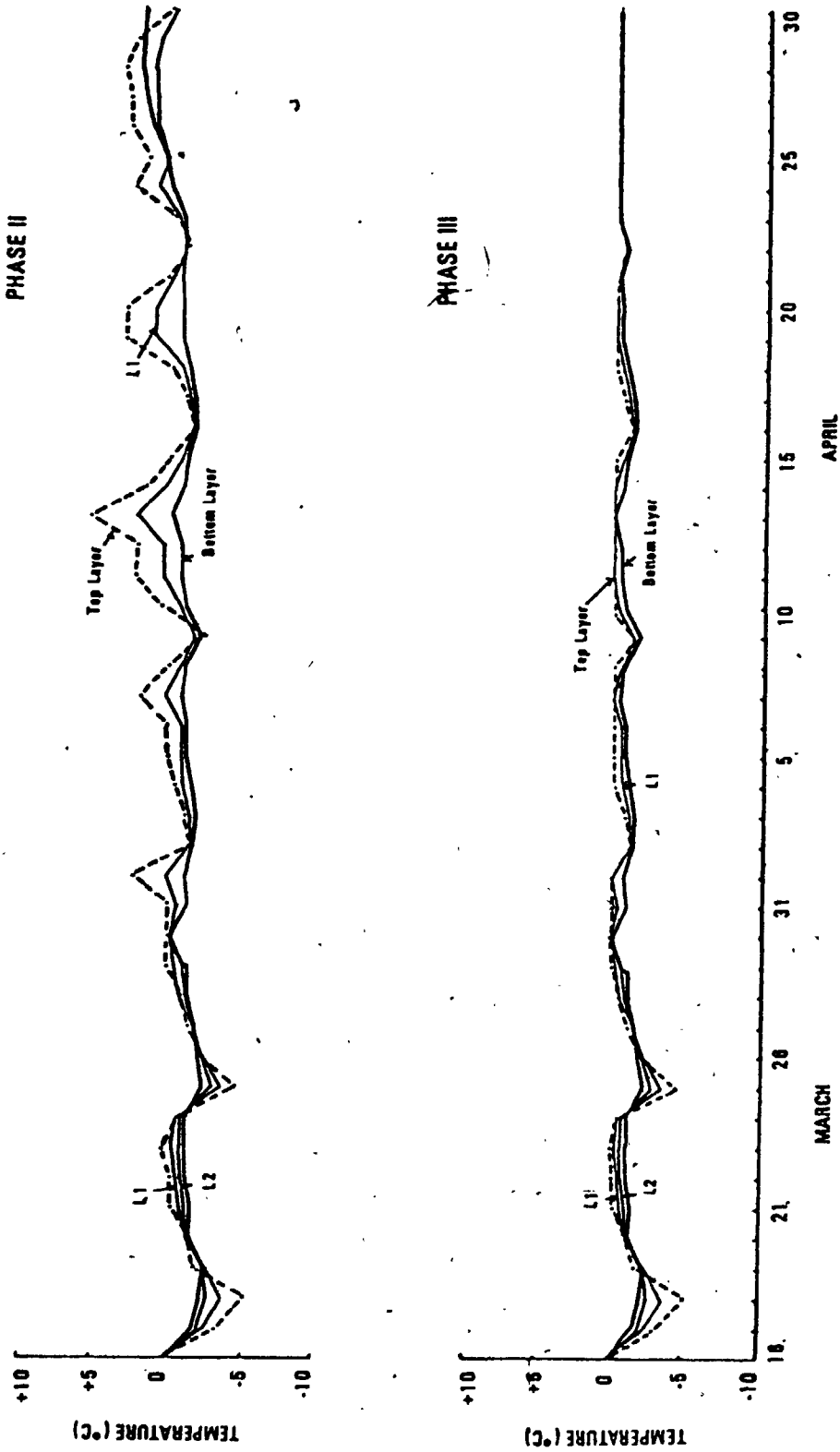
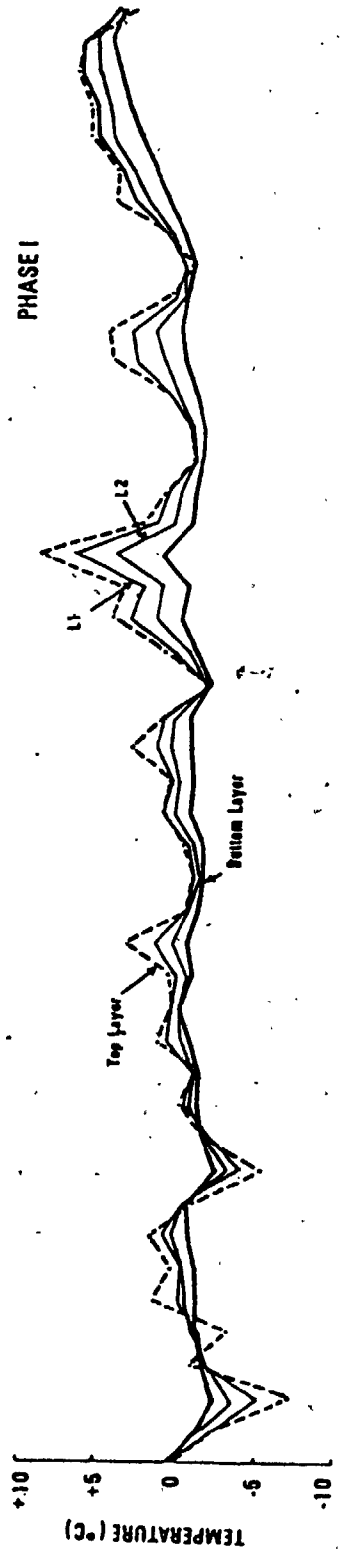
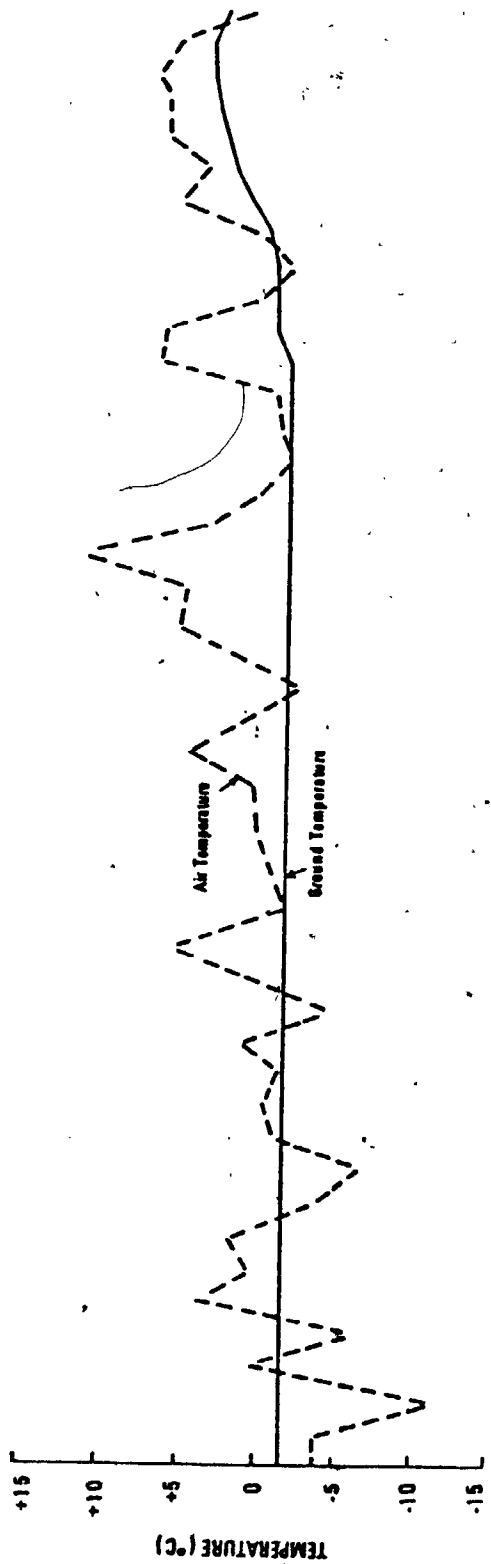


FIGURE 5.4 SNOWPACK TEMPERATURE SIMULATION AT ST. 4.01

120021



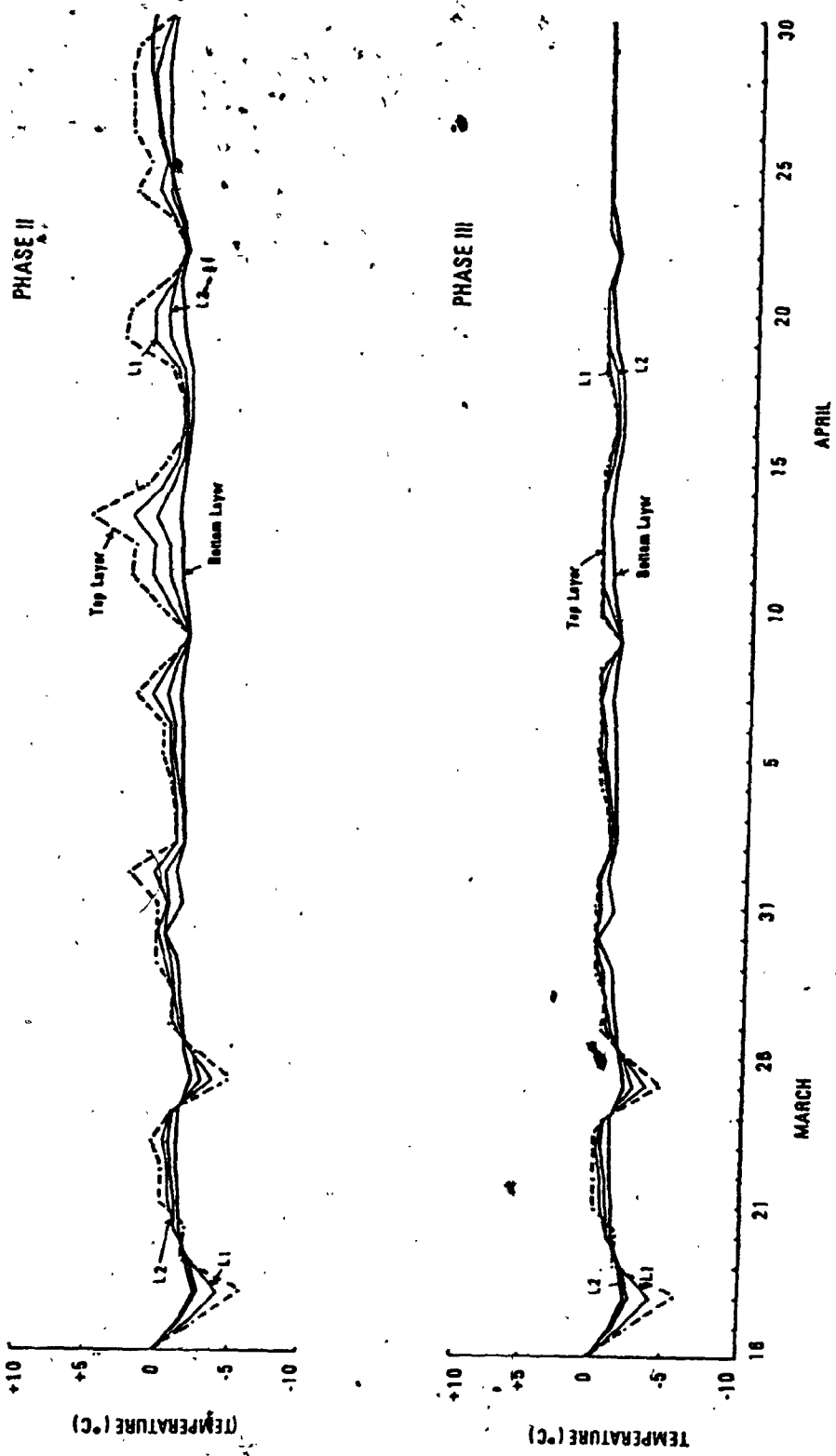


FIGURE 5.5 SNOWPACK TEMPERATURE SIMULATION AT ST. 5.03

2002

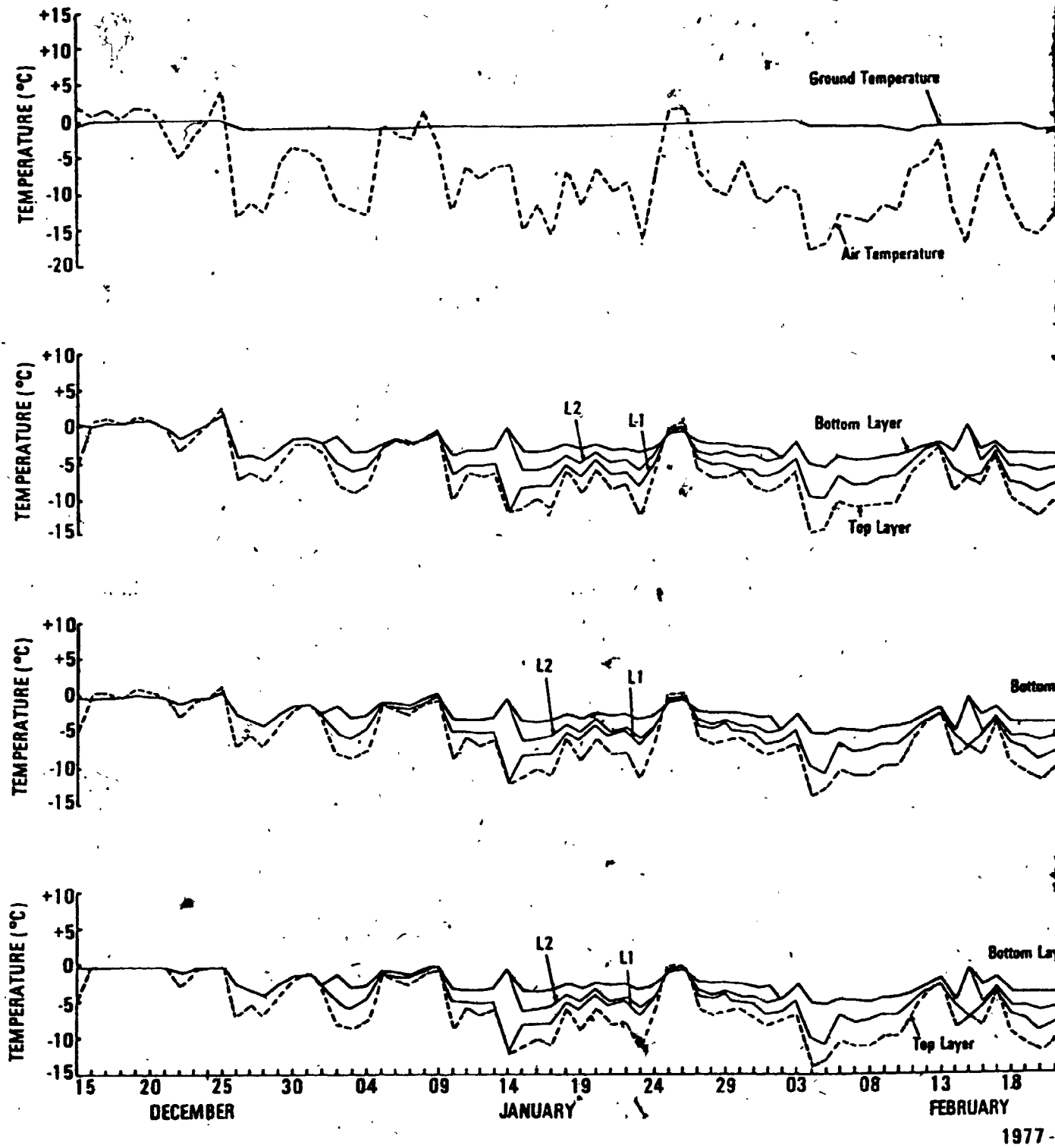
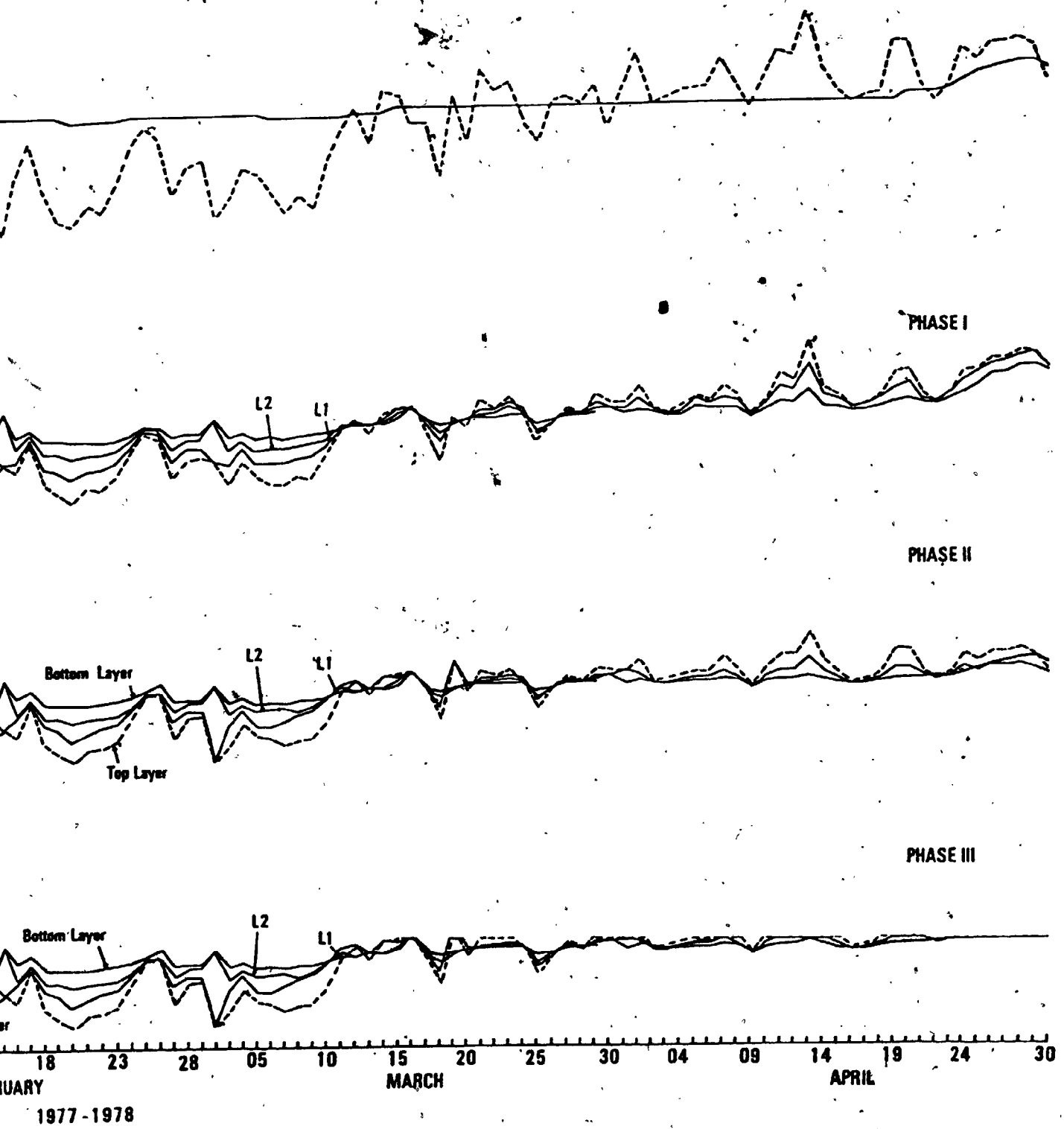


FIGURE 5.8 SNOWPACK TEMPERATURE SIMULATION AT ST. 6.09



20021

values below  $0^{\circ}\text{C}$ . This particular phase considers the effect of melt and this is shown by the flattening of the curves to  $0^{\circ}\text{C}$  whenever the snowpack temperatures were greater than the melting point. Phase III is the final output of the thermal procedure indicating that whenever there is a positive energy flux snow is melted from the top downwards and isothermal conditions dominate. It is also possible that the system can be reversed if the ground surface is much warmer than the upper boundary condition.

A number of general trends can be identified from Figures 5.1 through 5.6.

1. When air temperatures were higher than snowpack temperatures, the top layers were much warmer than the bottom layers. The reverse is true for most of the winter when ground temperatures are higher.
2. During early and late winter, a positive energy flux into the snowpack from either direction melted the snow from the top and from below.
3. During most of the winter the net heat flux is upwards into the atmosphere with temperatures at various depths fluctuating in accordance with the air temperature.
4. The temperature patterns were very similar for every sampling point except for the number of layers which vary over the winter. This variation is due to the depth of snow. For example the depth of snow in Station 1.04 was almost twice that of Station 6.09 in mid-December, resulting in four measurements at Station 1.04



and only two at 6.09.

5. There are days when the snowpack becomes isothermal for a short time period. This results whenever the air temperature curve intersects with the ground temperature inducing uniform temperatures throughout the entire snowpack. Examples are December 3, 24 and March 12 for Station 1.04.

The patterns of temperature shown in Phase I indicate the possibility of temperatures inside the snowpack being greater than  $0^{\circ}\text{C}$ . This is not true and as such those values greater than  $0^{\circ}\text{C}$  are reset to  $0^{\circ}\text{C}$  in Phase III. It is known that the diffusion of heat in dry snow is very slow, but once the energy is increased to produce melt, the rate of heat transfer does not increase considerably. The snowpack is heated by phase transitions of water and may melt within a short time. The time required to accumulate the heat of fusion for a particular layer is therefore a function of the intensity of heat flux and the density of the layer. By similar analogy it can be shown that the propagation of the subzero temperature zone within a snowpack having an initial temperature of  $>0^{\circ}\text{C}$  and a falling air temperature to  $-10^{\circ}\text{C}$  or lower, is relatively rapid in the first few hours and then slows down. This phenomenon was operating on days following December 25 on Station 1.04 and 6.09 when there was a sharp drop in air temperature. It was also seen that when the snow cover was deep and the melting snow was freezing rapidly, only the upper part of the snow cover was totally frozen while the downward seepage of

melt water kept the lower part melting.

The subzero temperature patterns show that during mid-winter the top part of the snowpack can be cooled to  $-10^{\circ}\text{C}$  or lower and remain like that for a considerable length of time. Kuzmin (1972) has shown that the rate of heating of the snow cover depends on the 'reserve of cold' accumulated in the period preceding the thaw and the intensity of heat intake upon heating. The 'reserve of cold' is determined from  $c\rho d\theta$  where,  $c$  = specific heat,  $\rho$  = mean density,  $d$  = depth of snow and  $\theta$  is the mean temperature of the snowpack. He quotes values of  $-68.3 \text{ cal cm}^{-2}$  during the winter of 1915-16 at Sodankyla. Similar computations done for Station 1.04 are shown in Table 5.1. The highest values are recorded during the middle of February when the mean snow temperatures were lowest. When thaw sets in, the snow is heated in the upper layers by energy exchange with the atmosphere and tends to keep the temperatures in that zone close to  $0^{\circ}\text{C}$ . Most of the heat is transmitted downwards by conduction and tends to deplete the 'reserve of cold'. Since the heat is not accumulated in the upper layers the snow does not melt. But as soon as it starts melting, the meltwater seeps downward, compacts these layers and heats them close to  $0^{\circ}\text{C}$  because of the release of latent heat of crystallization. This indicates that heat transmission by percolating water is very rapid compared to heat transmission by conduction. The temperature trends in the Phase diagrams show that as soon as positive temperature values are encountered, melting starts taking place and as such the rate of heat

Table 5.1

Reserve of Cold for Station 1.04

Date	Depth of snow (cm)	Mean Density (gm cm <sup>-3</sup> )	Mean snowpack temperature (0 °C)	Reserve of Cold (cal cm <sup>-2</sup> )
Nov. 13	22.88	.027	-0.7	- .2
Dec. 08	16.80	.014	-4.0	- .5
Dec. 12	69.84	.143	-8.5	-42.4
Dec. 27	34.29	.477	-5.8	-47.4
Jan. 06	38.10	.217	-2.0	- 8.3
Jan. 12	35.04	.397	-5.2	-36.2
Jan. 18 <sup>1</sup>	47.75	.377	-3.5	-31.5
Jan. 26	59.70	.240	-4.2	-30.1
Feb. 02	76.98	.197	-4.5	-34.1
Feb. 09	72.90	.262	-7.5	-71.6
Feb. 20	68.60	.339	-8.5	-98.8
Feb. 26	68.60	.249	-3.2	-27.3
Mar. 03	76.20	.237	-7.9	-71.3
Mar. 09	71.10	.285	-7.3	-73.9
Mar. 18	52.05	.226	-4.5	-26.5
Mar. 25	40.64	.299	-3.2	-19.4

\* The specific heat is 0.5 cal gm<sup>-1</sup>°C<sup>-1</sup> for densities less than or equal to 0.9 gm cm<sup>-3</sup>

transmission is enhanced. Whenever the total daily intake of heat is greater than the 'reserve of cold', i.e. a positive heat balance, the entire snowpack is heated to  $0^{\circ}\text{C}$  on the first day of the thaw. In summary, temperature patterns are significant in assessing the heat transfer process in the snowpack. The conditions vary from dry snow and its properties during most of the winter to wet spring melt 'ripe' snow. Under the latter condition, which is more critical, the lower layers of a snowpack which begins to melt from above can be heated within a short time if thaw is intensive. The seepage of meltwater and the phase transitions which take place with energy being released play an important part in such a process.

### 5.3 Heat Transfer and Sublimation

This section is related to the temperature characteristics of the snowpack and involves looking at the time rate of heat distribution in the snowpack. The heat flux has been calculated on an hourly basis and is shown in the upper part of Figures 5.7 through 5.13. The pattern is similar to the temperature patterns with values corresponding well with those established by Wilson (1941). The results are presented along with the values of the amount of sublimation and density differences between the upper and lower part of the snowpack in Appendix IX. Negative values of sublimation indicate that the top layers are increasing in density while the reverse holds true for positive values. Similarly a positive value of density difference indicates that the upper

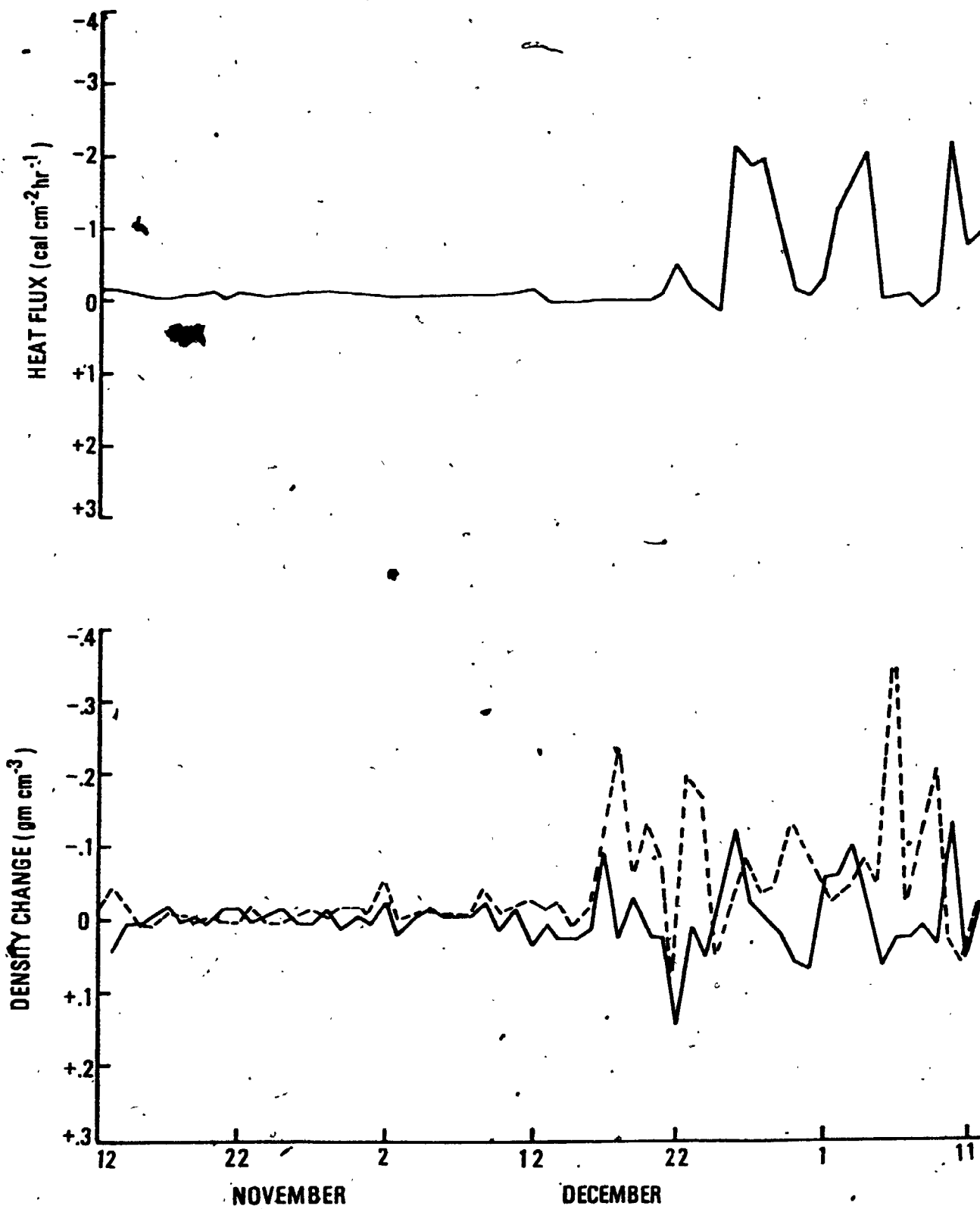
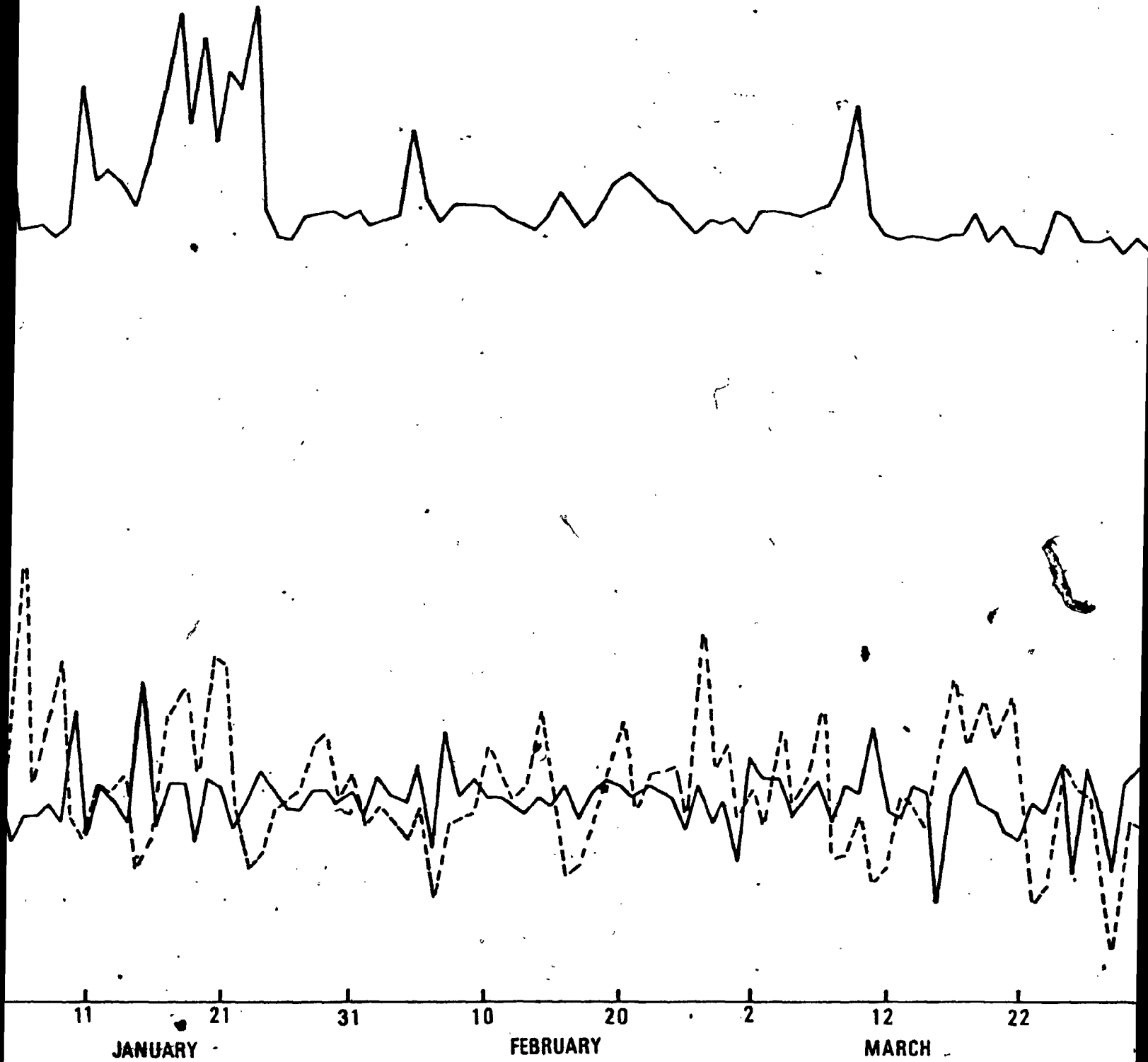


FIGURE 5.7 SUBLIMATION AND DENSITY DIFFERENCES IN RELATION TO HEAT



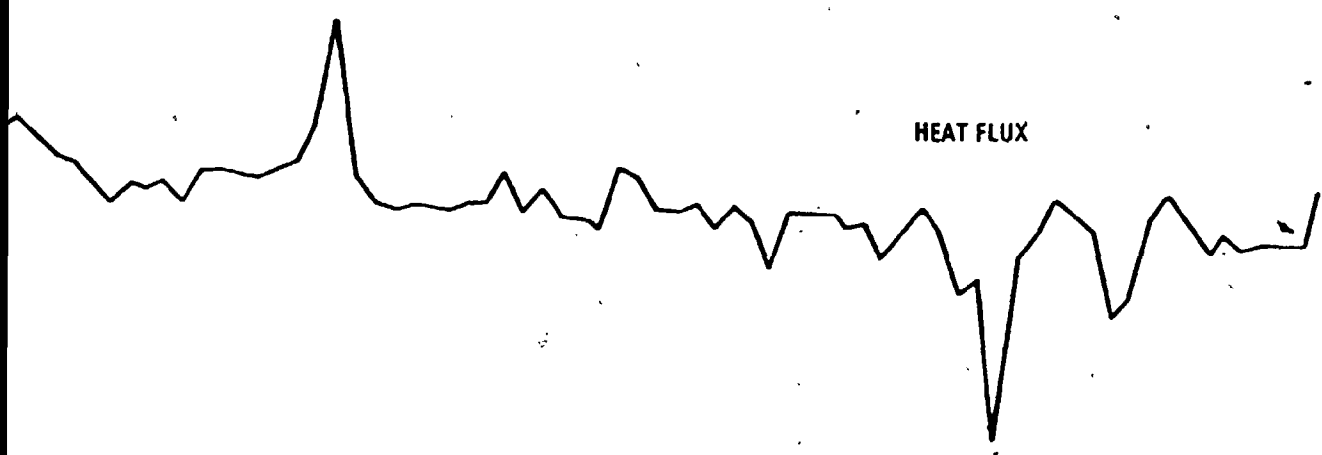
JANUARY

FEBRUARY

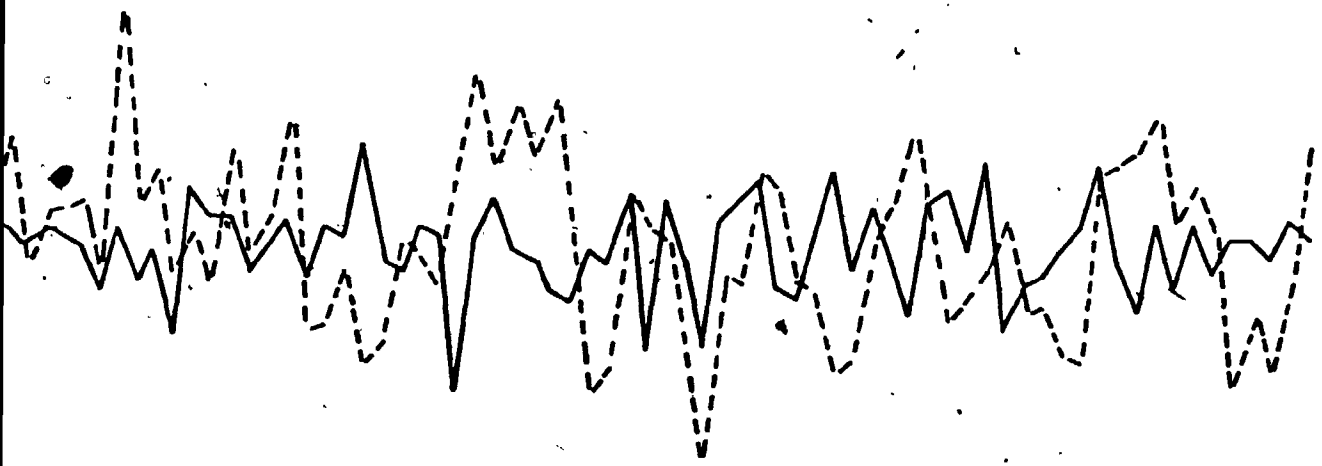
MARCH

TO HEAT FLUX AT ST. 1:04

20F 1



— SUBLIMATION  
- - - DENSITY DIFFERENCE



0 2 12 22 1 11 21 30  
MARCH APRIL

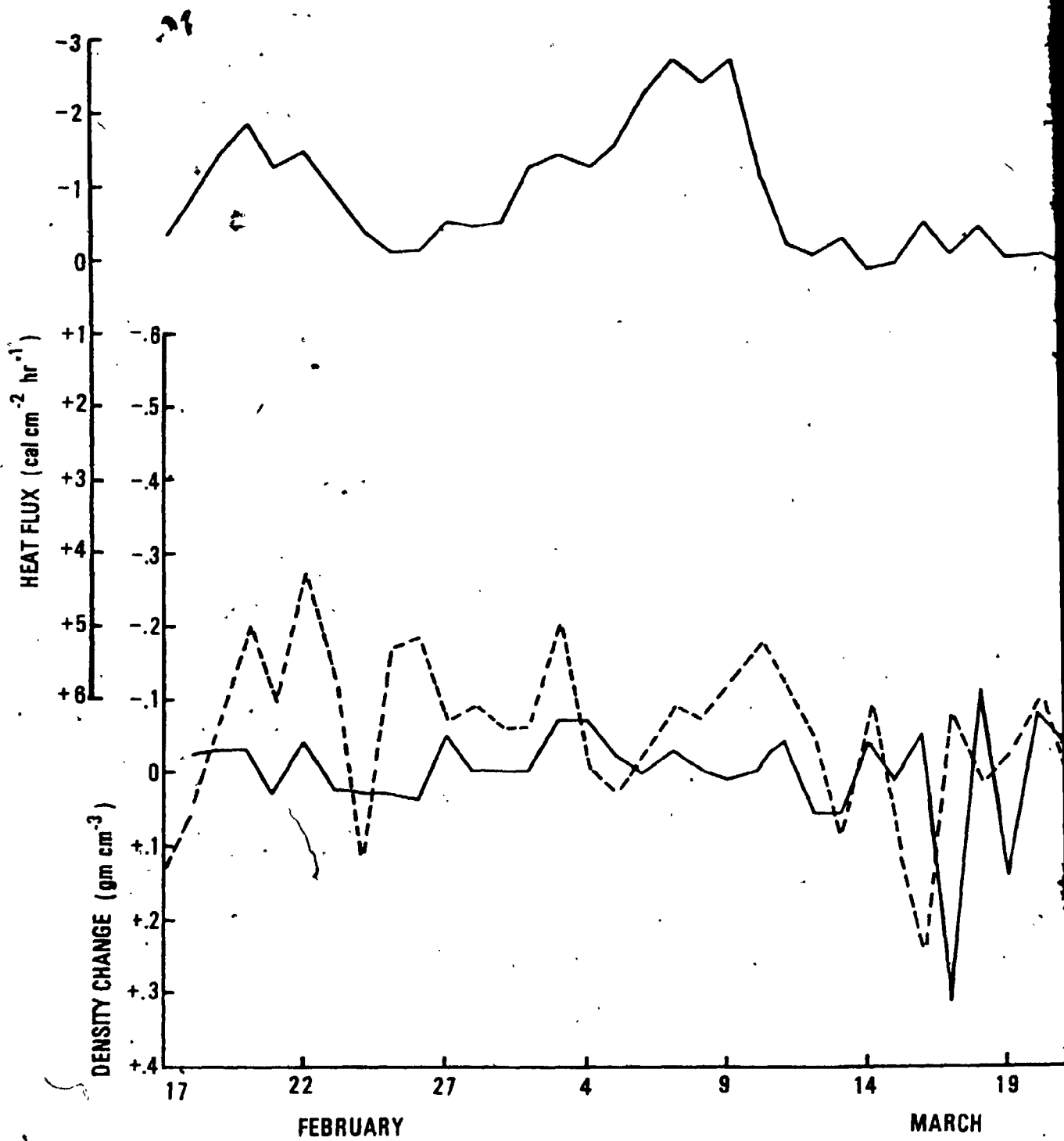
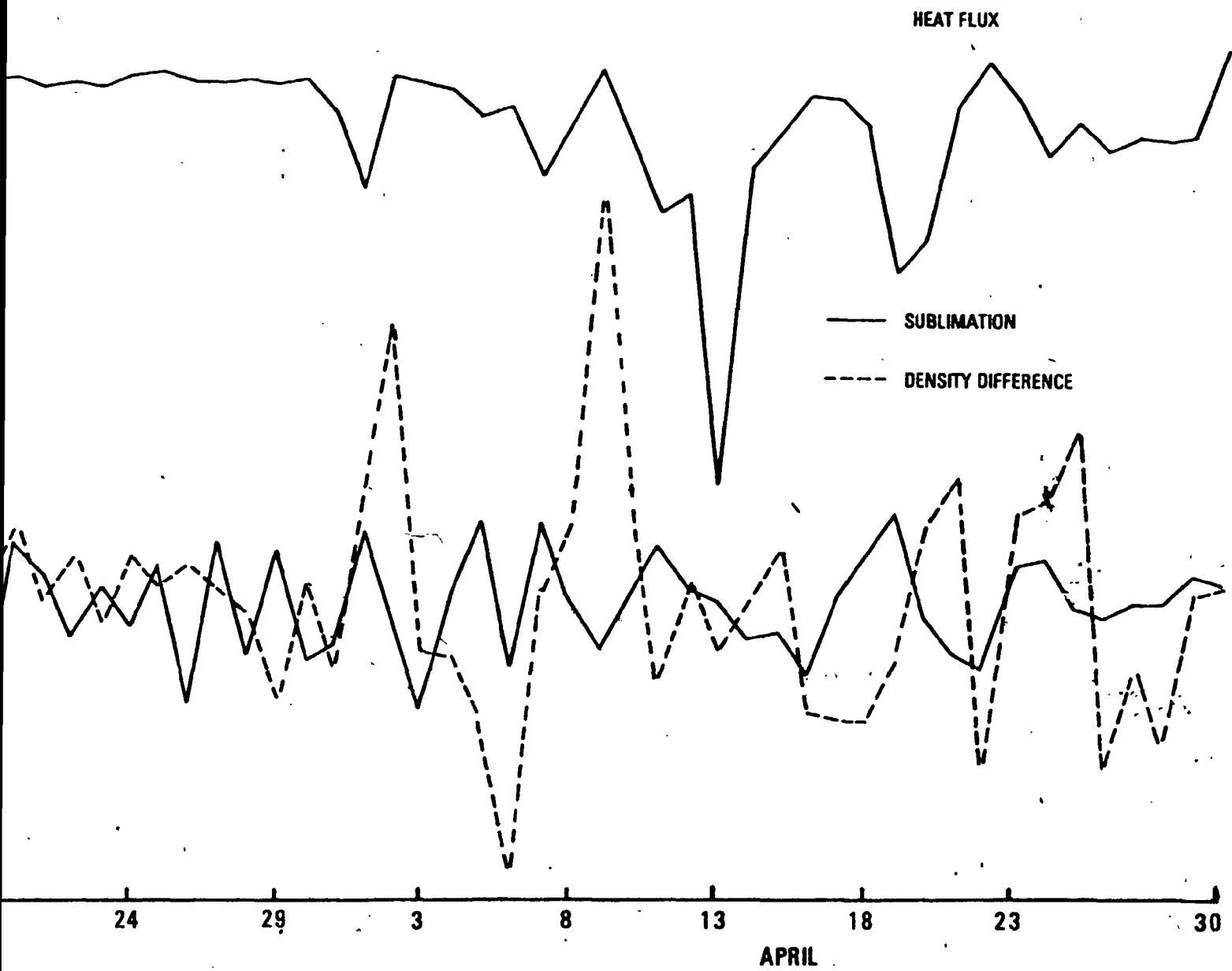


FIGURE 5.8 SUBLIMATION AND DENSITY DIFFERENCES IN RELATION TO HEAT FLUX

1 OF 1





FLUX AT ST. 2.03

| 2002 |

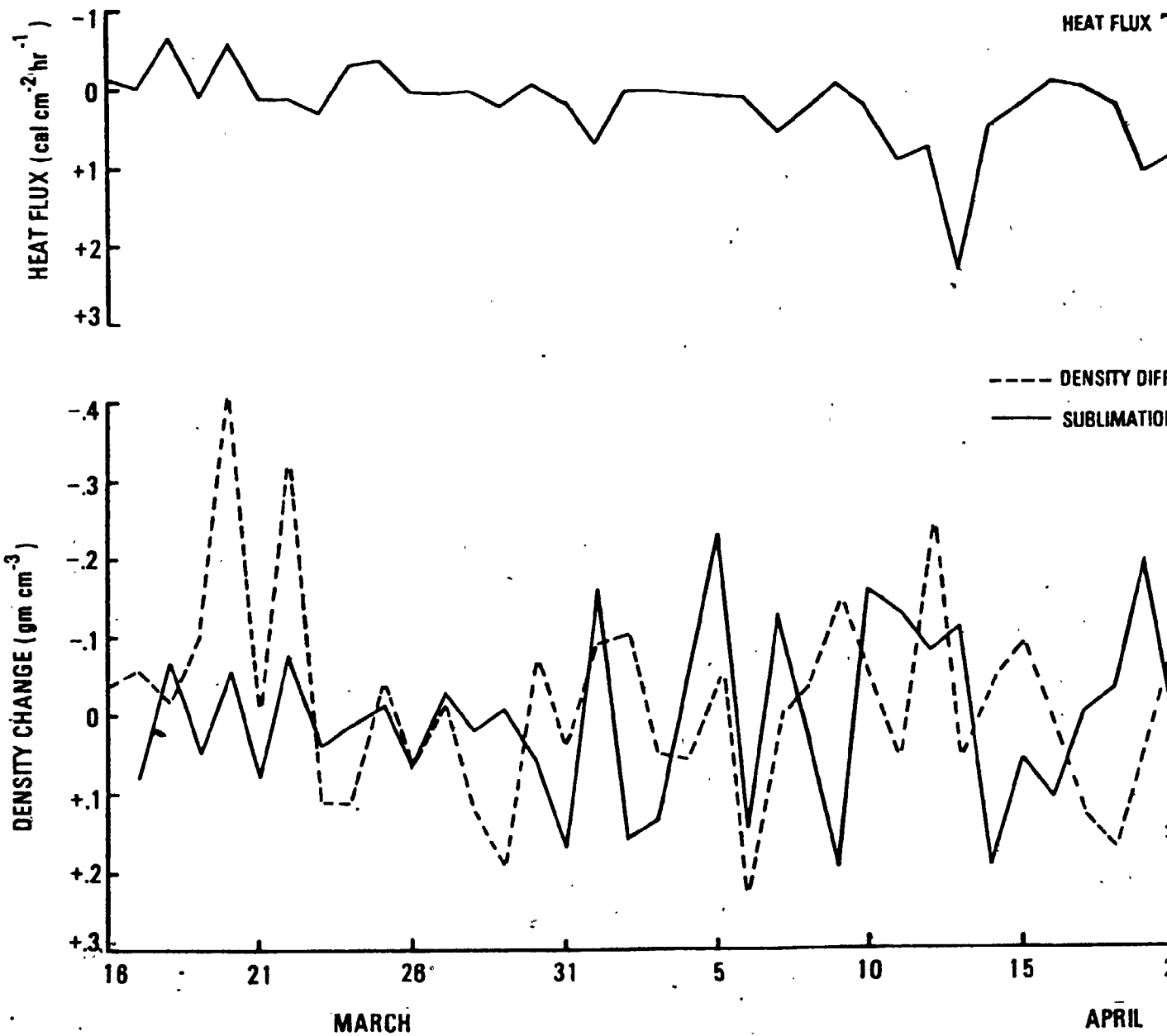
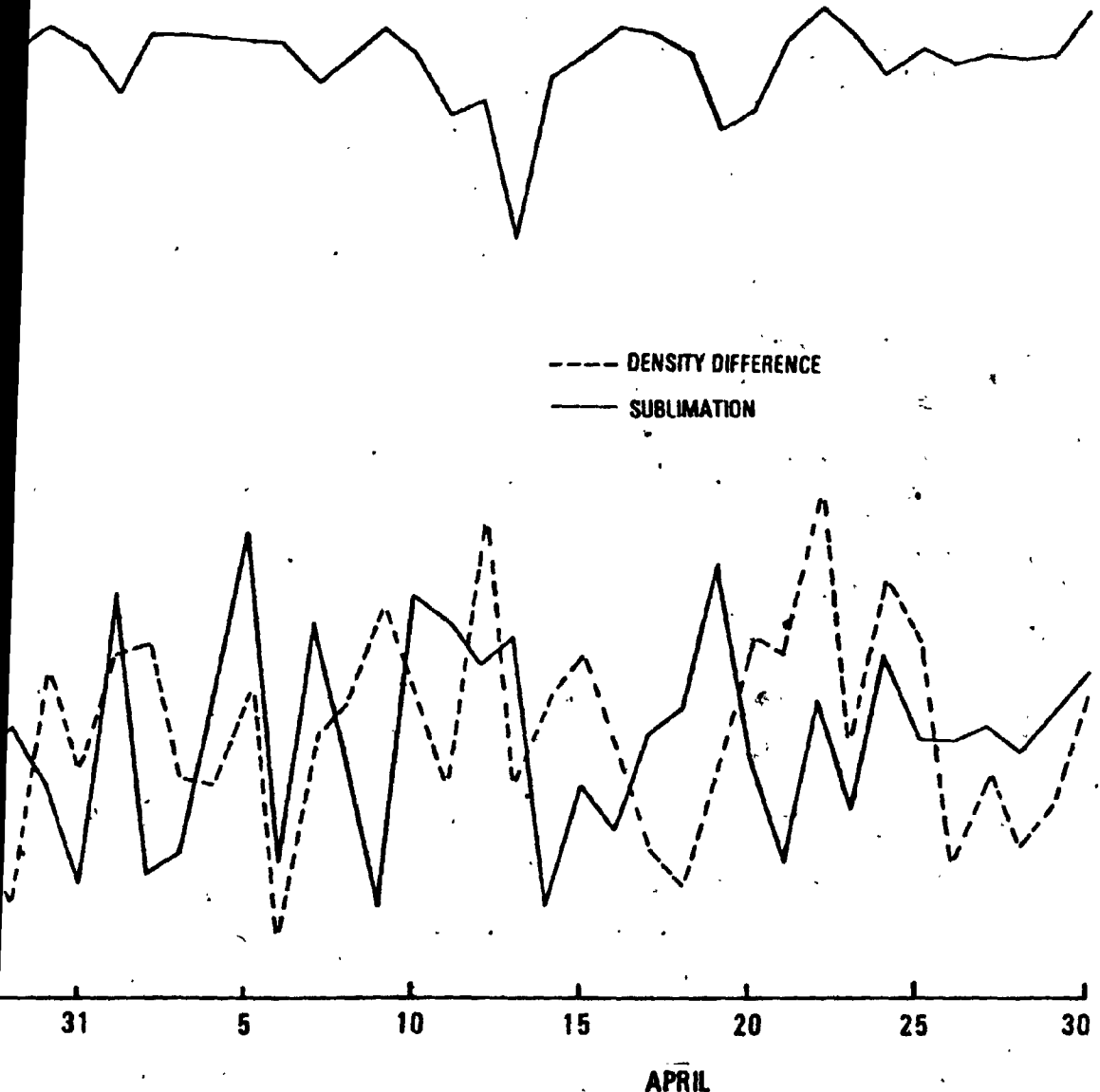


FIGURE 5.9 SUBLIMATION AND DENSITY DIFFERENCES IN RELATION TO HEAT FLUX AT ST

HEAT FLUX

--- DENSITY DIFFERENCE  
— SUBLIMATION



APRIL

DENSITY DIFFERENCES IN RELATION TO HEAT FLUX AT ST. 3.05

2021

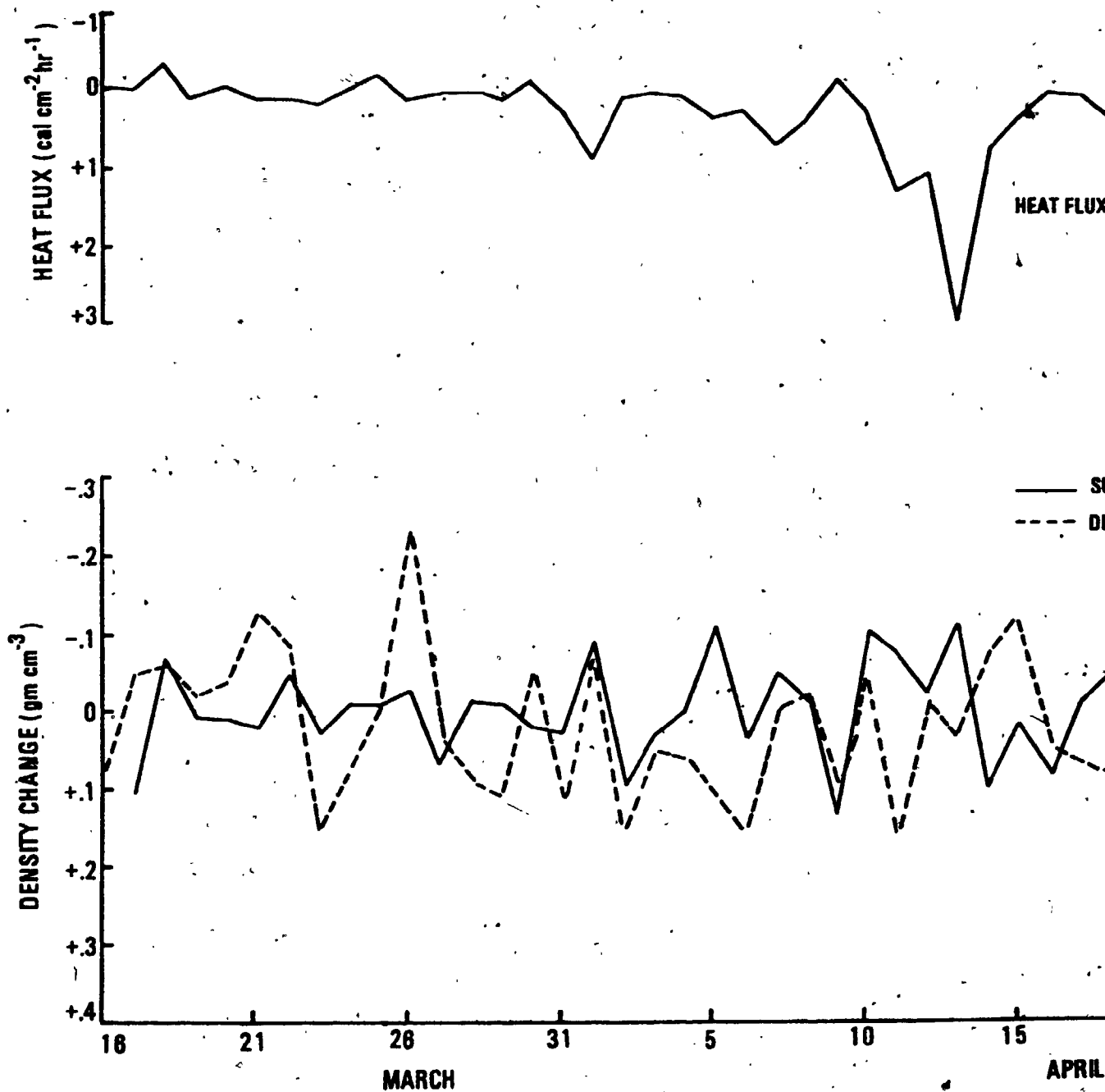
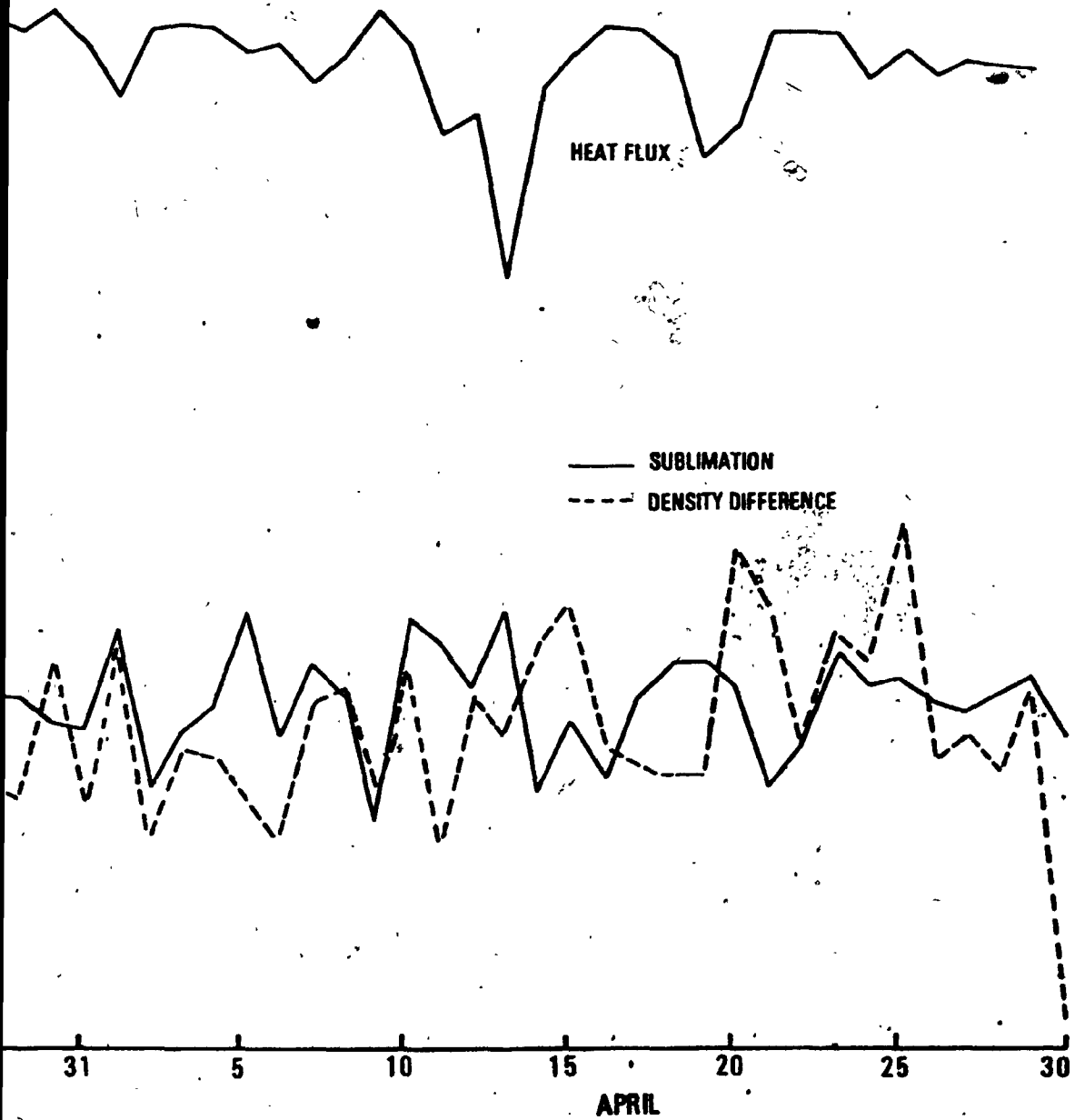


FIGURE 5.10 SUBLIMATION AND DENSITY DIFFERENCES IN RELATION TO HEAT FLUX



D DENSITY DIFFERENCES IN RELATION TO HEAT FLUX AT ST. 4.01

2002

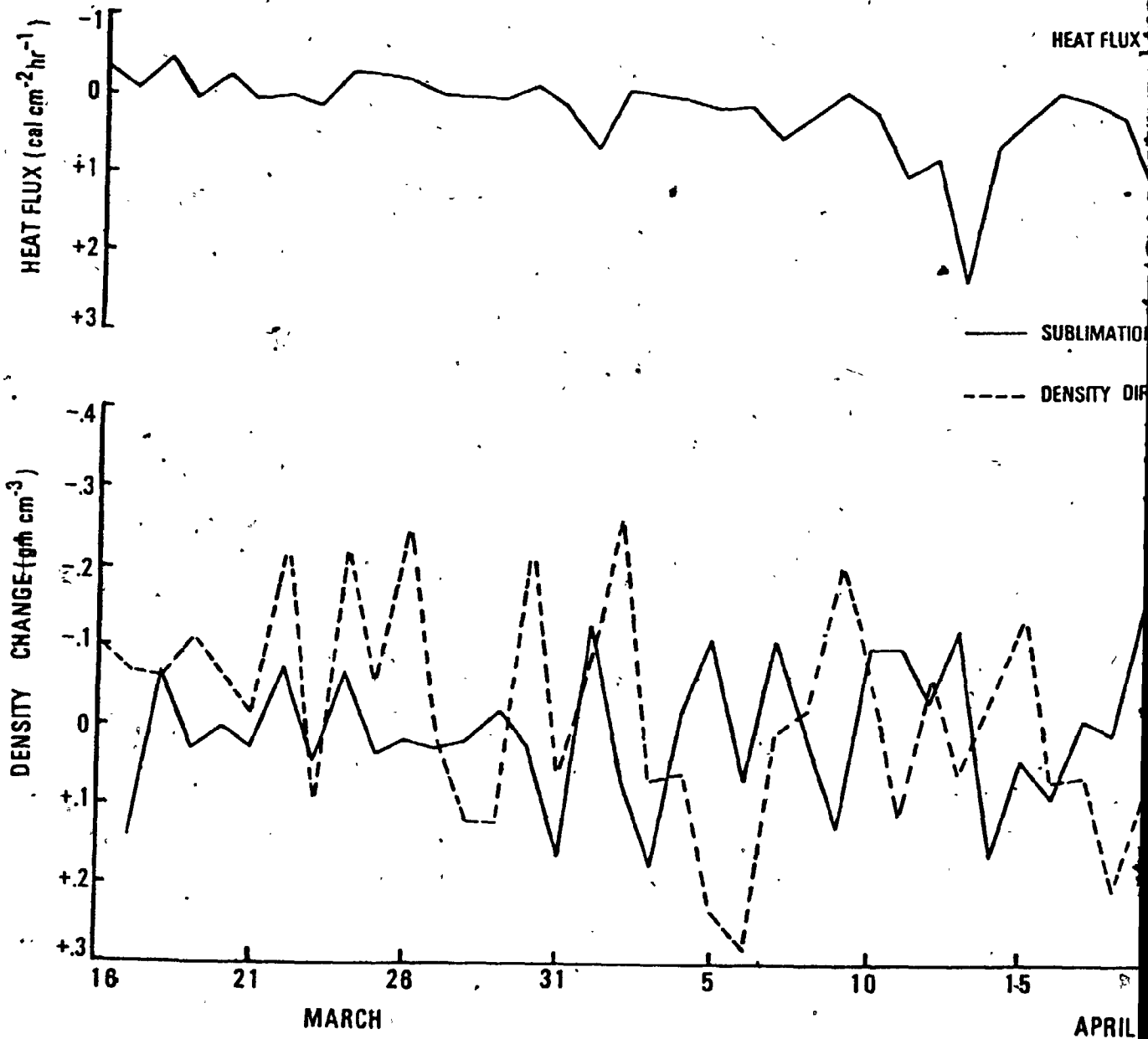
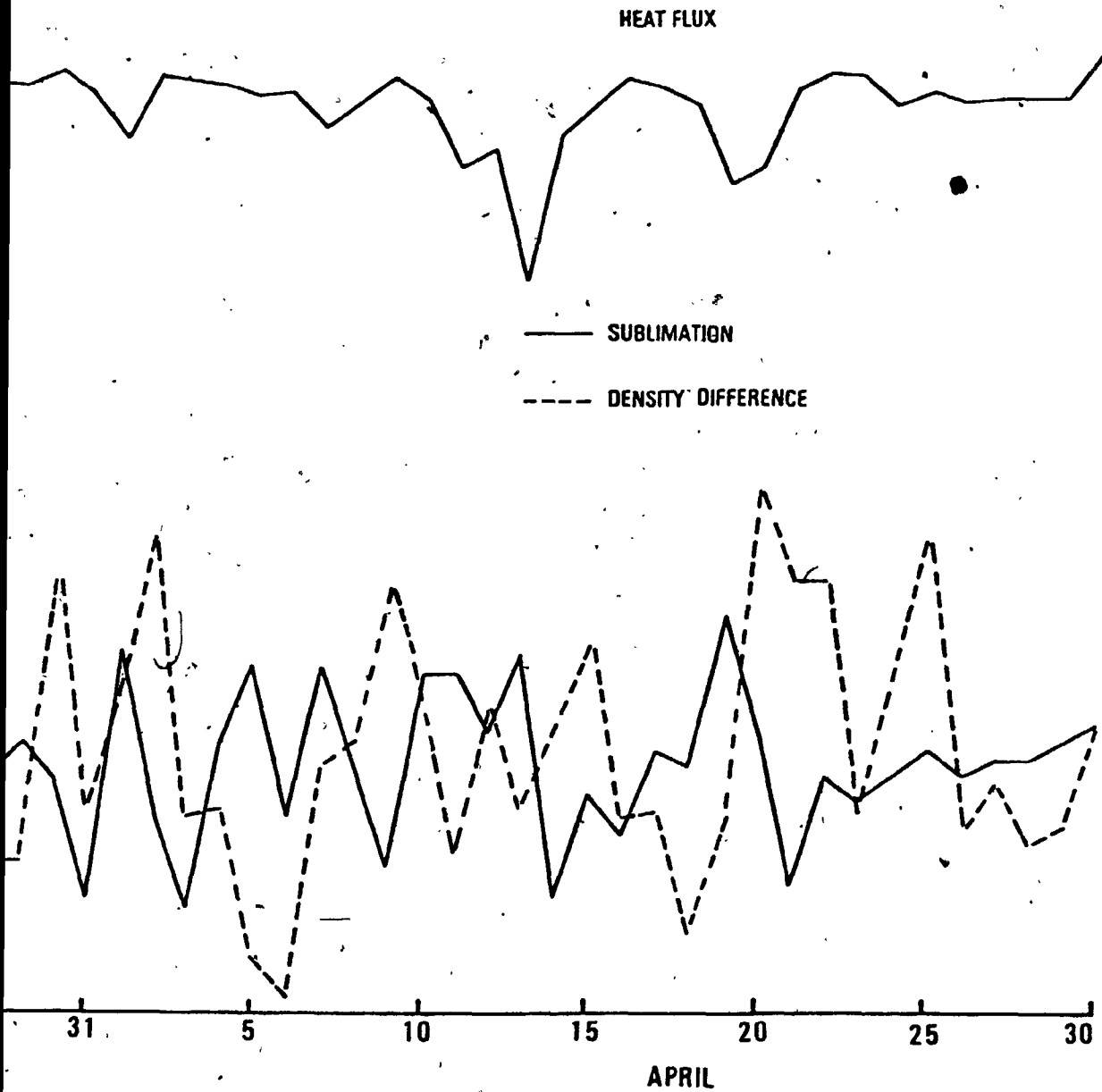


FIGURE 5.11. SUBLIMATION AND DENSITY DIFFERENCES IN RELATION TO HEAT FLUX



DENSITY DIFFERENCES IN RELATION TO HEAT FLUX AT ST. 5.03

2002

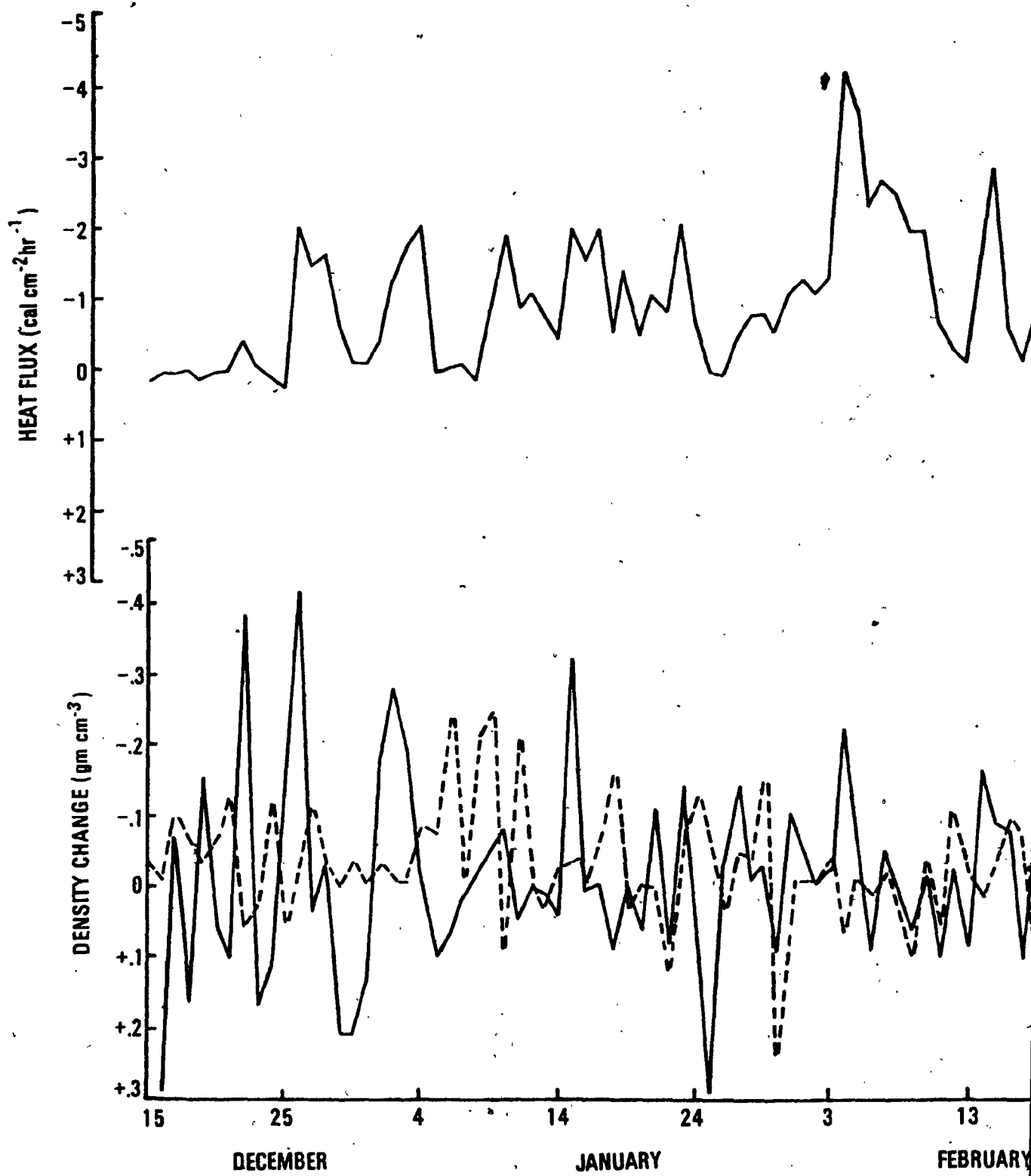
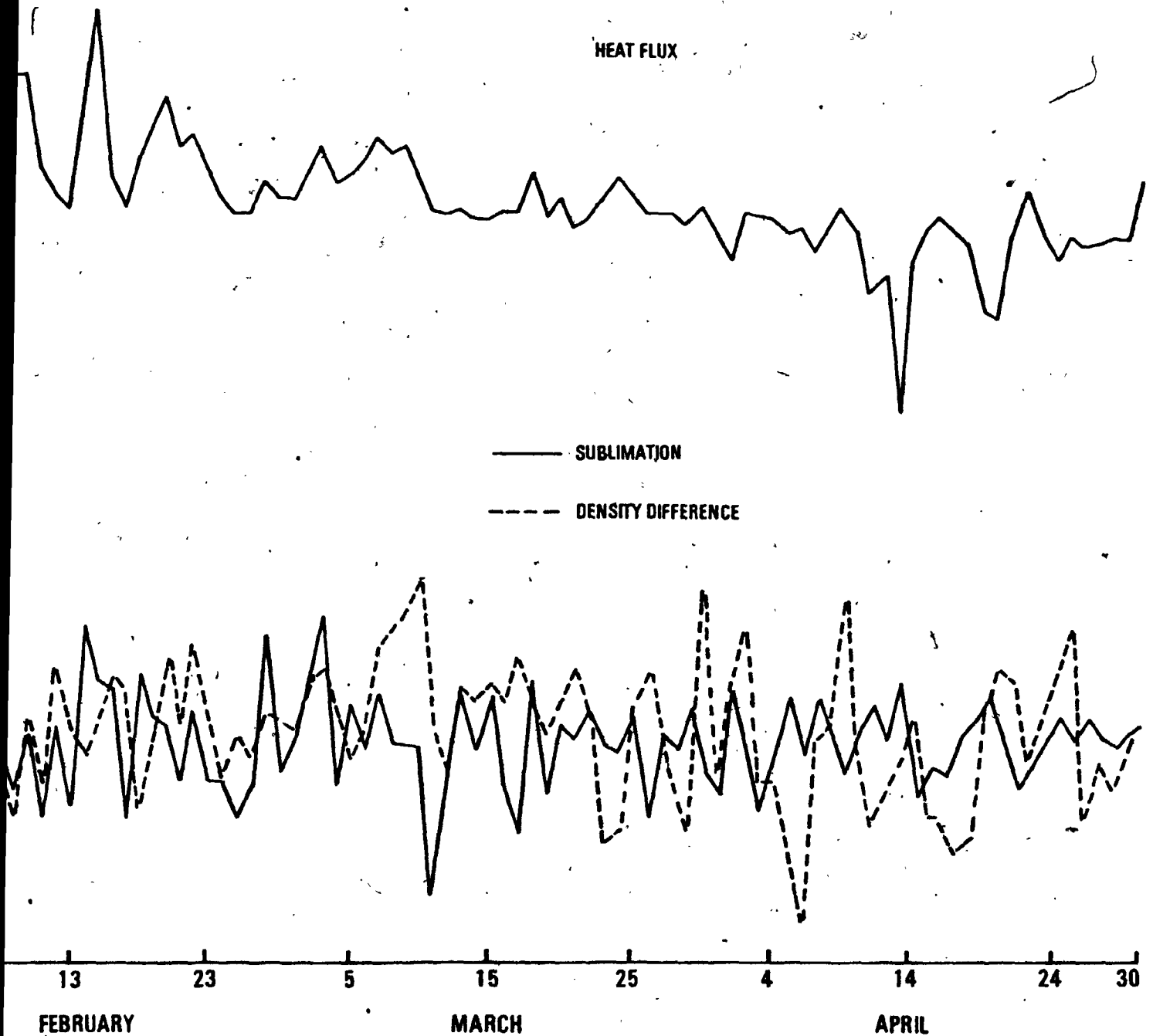


FIGURE 5.12 SUBLIMATION AND DENSITY DIFFERENCES IN RELATION TO HEAT FLUX

1 OF 1





TO HEAT FLUX AT ST. 6.09

2002

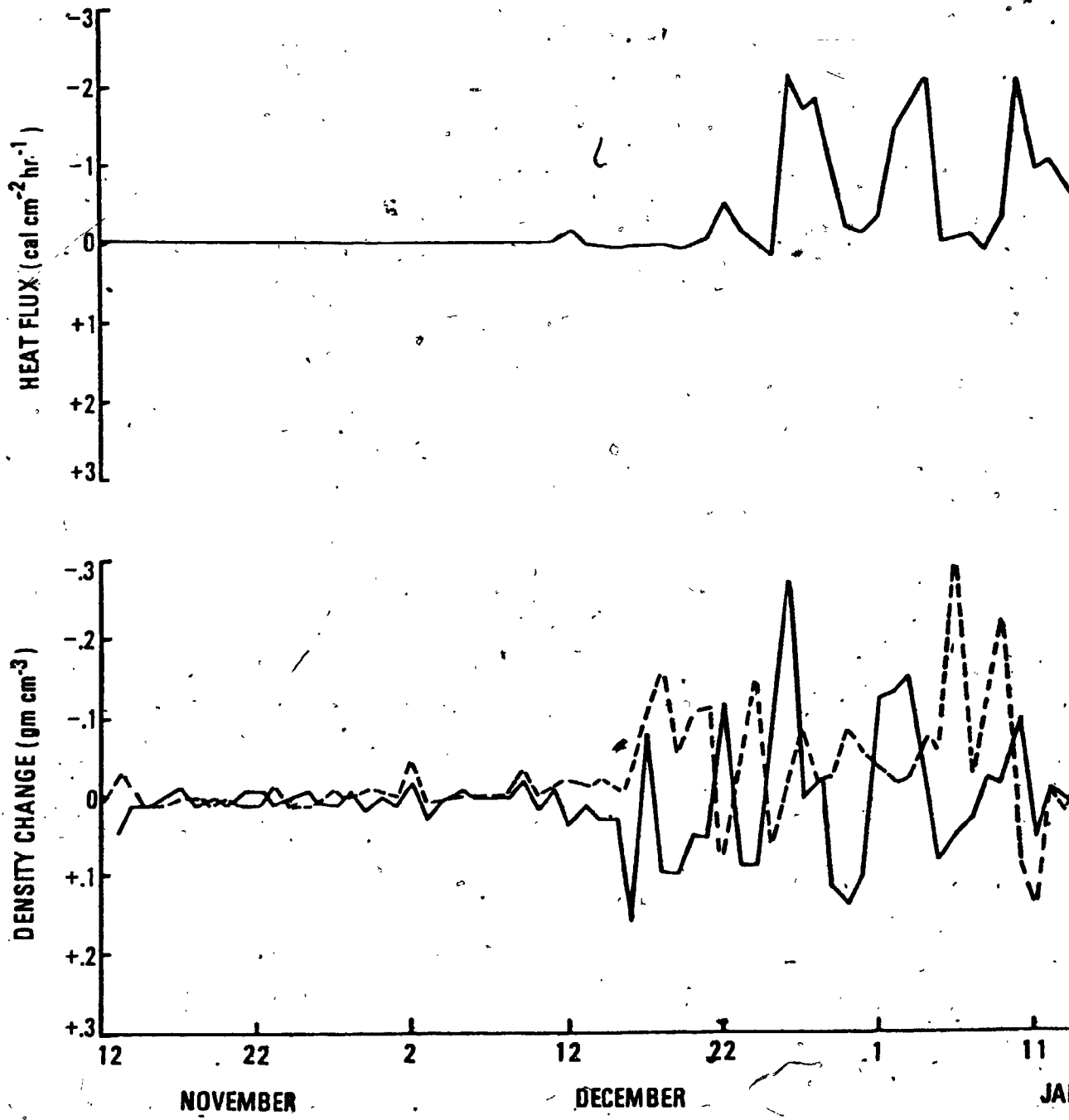
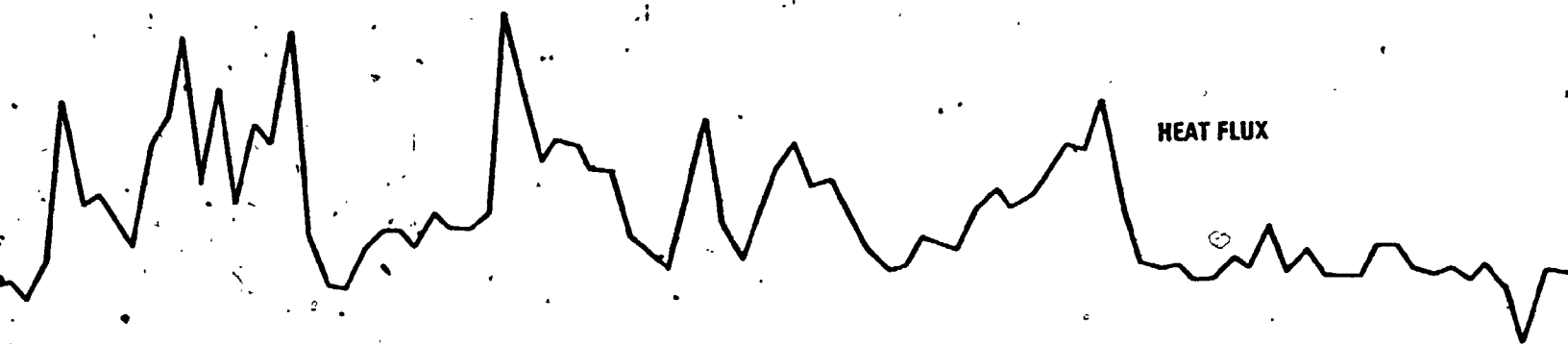
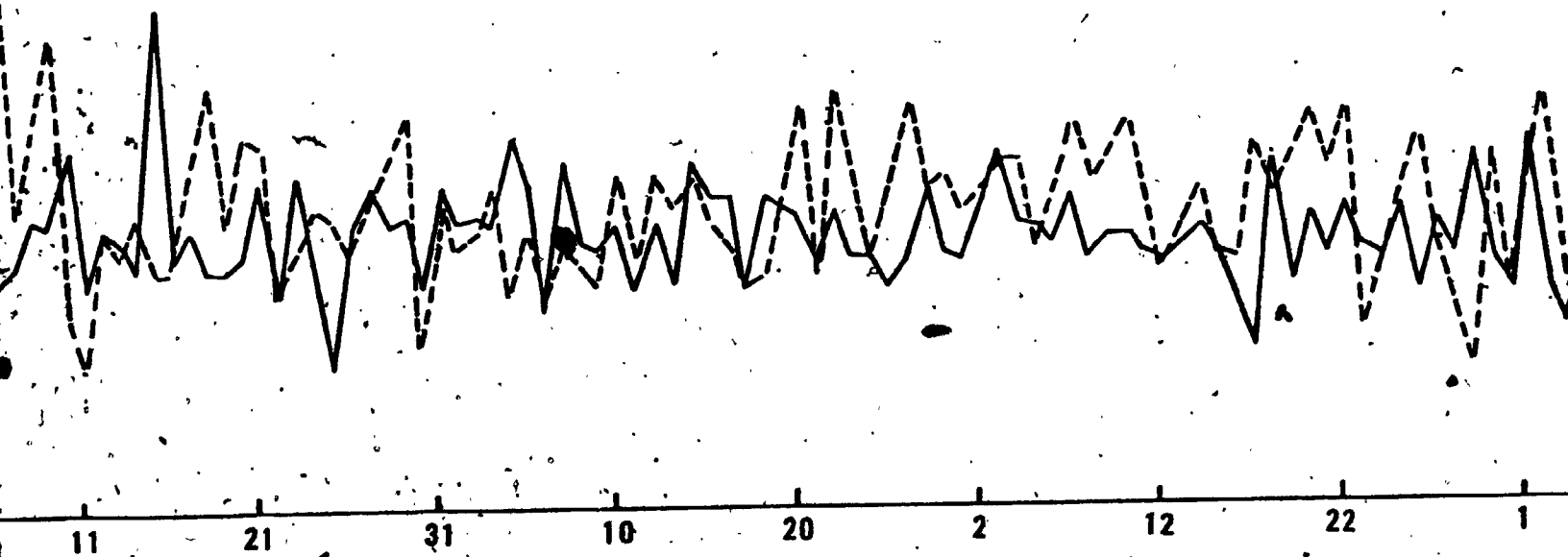


FIGURE 5.13 SUBLIMATION AND DENSITY DIFFERENCES IN RELATION TO HEAT F



— SUBLIMATION  
- - - DENSITY DIFFERENCE



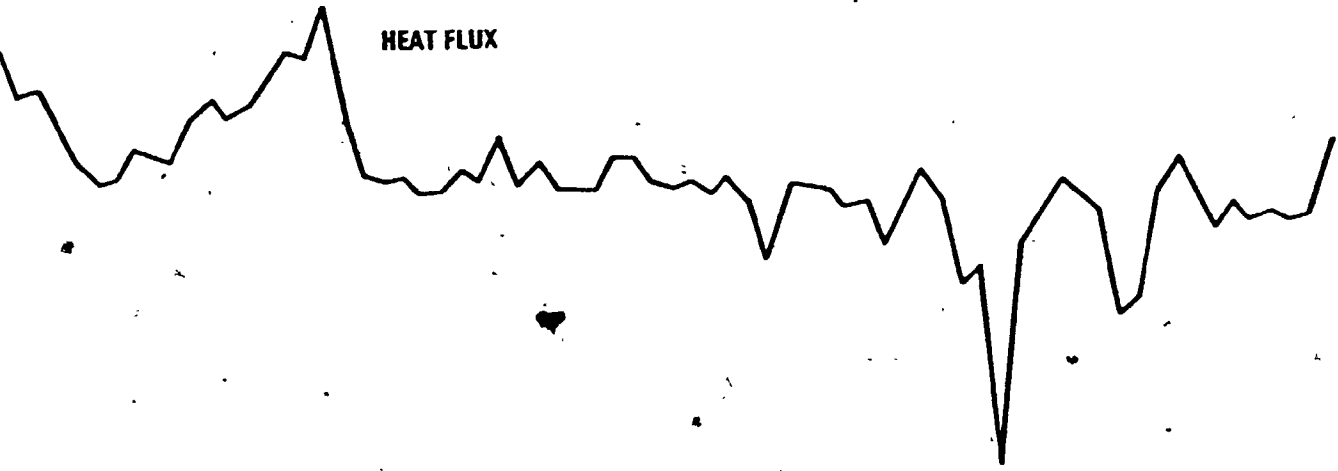
JANUARY

FEBRUARY

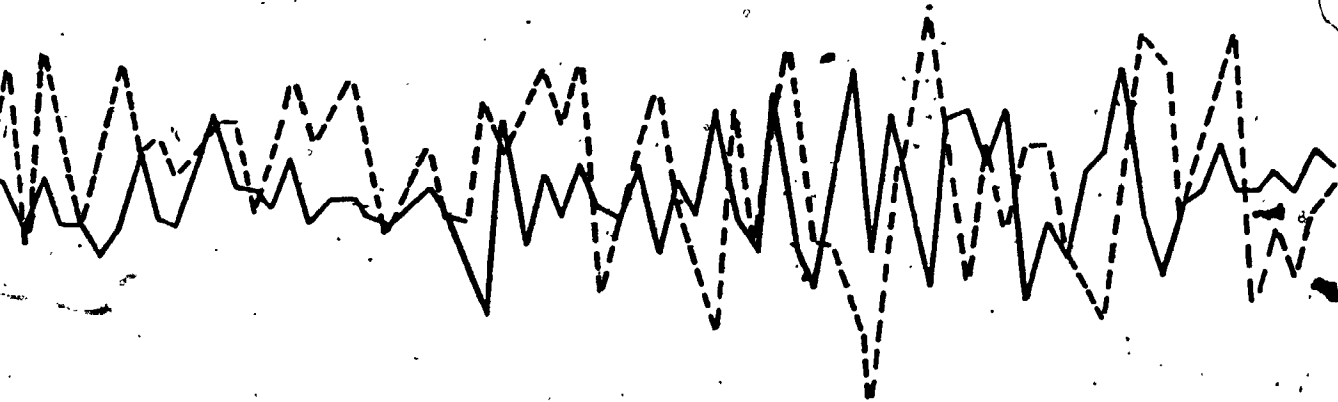
MARCH

TO HEAT FLUX FOR ALL STATIONS

205 /



— SUBLIMATION  
- - - DENSITY DIFFERENCE



20 2 12 22 1 11 21 30  
MARCH APRIL

| 3 of 3 |

layer of the snowpack is much denser than the lower layer. The general pattern in all the graphs for sublimation and density differences show that an increased negative flux increases the density in the upper layers while a decrease in the negative flux induces an increase in the lower layers. The mean values for all stations (Figure 5.13) shows that the maximum rate of heat emission from the snowpack was about  $-2.9 \text{ cal cm}^{-2} \text{ hr}^{-1}$  whereas the maximum input during the melt season was approximately  $+3.1 \text{ cal cm}^{-2} \text{ hr}^{-1}$ . During early winter when the snow cover was shallow and the snow had a low density, there was virtually no heat transfer within the snowpack. The net transference of snow between the upper and lower layers was primarily a result of deposition metamorphism. Since the ground temperature remained positive during that period, part of the snow which was deposited was melted from below creating a tendency of slightly increased densities at the bottom of the snowpack. At the same time, during periods when the air temperature was higher than the ground temperature the sublimation process was directed downwards. If the air temperature was positive and greater than the ground temperature, the net heat flux was positive. Under this condition melt was initiated in the upper layers of the snowpack indicating an increase in density in that zone. An example of the above statement can be illustrated by the flux-sublimation diagram for all stations on the 13th of April when the maximum rate of heat transfer took place. This positive flux caused a melt in the upper layers, increasing the densities although the net

sublimation was positive, indicating movement downwards. This is true under actual conditions because the snowpack is relatively colder in the lower part. In such situations the vapor pressure gradient is directed downwards sublimating snow to lower levels. The increased density at the top is due to melting. When the meltwater seeps down, the heat transmission by conduction is reduced and the lower layers are heated by phase transition. It is during the spring melt season that equi-temperature metamorphism takes place in regions of the snowpack where there is no temperature gradient. These regions are also partly affected by melt-freeze metamorphism. During equi-temperature metamorphism snow of densities around .25 to .35 gm cm<sup>-3</sup> can be expected along with some 'sintering' taking place. The refreezing of the meltwater gives the crystals a coating of ice in the melt-freeze metamorphism and tends to 'ripen' the snowpack. During the mid-winter when the snowpack has a negative heat balance and heat is continuously emitted from the snowpack, the sublimation process reversed (negative values) and increased the density of the upper layers of the snowpack. Under such conditions of temperature gradients, evaporation and condensation produces the growth of grains resulting in temperature-gradient metamorphism. The rate of transformation of snow is dependent upon the initial density and the flow of air through the pores in the snowpack. It was also noticeable that when the negative heat flux was increased and the density was low the process became rapid and may have increased the density values to .40-.45 gm cm<sup>-3</sup>. This often

resulted in depth hoar or an increased number of crust layers in the snowpack.

A comparison of Figures 5.7 and 5.12 indicate that the energy exchange between the snowpack and the atmosphere-ground system was much higher for Station 6.09. This is shown by the increased rates of heat transfer and is also reflected in the temperature patterns in the Phase diagrams. The snowpack is cooled to much lower values in the exposed snow courses and as such creates higher temperature gradients leading to increased flux. The fluctuation in the rates has a tendency to create movement of snow in either direction in much shorter time periods. Quantitatively expressed, about  $.60 \text{ gm cm}^{-3}$  of snow was moved upwards in Station 6.09 during the period December 23-26 whereas in Station 1.04 about  $.30 \text{ gm cm}^{-3}$  was moved downwards and about  $.10 \text{ gm cm}^{-3}$  moved upwards following that, in the same period. During the peak negative flux period (February 3-14) for Station 6.09 the net movement of snow was between  $+.10$  and  $-.20 \text{ gm cm}^{-3}$ , while in Station 1.04 the values were between  $+.15$  and  $-.10 \text{ gm cm}^{-3}$ . The peak emission from Station 1.04 took place between January 17 and 23 resulting in a movement between  $+.10$  and  $-.20 \text{ gm cm}^{-3}$ . During the same period the movement in Station 6.09 was  $+.10$  and  $-.15 \text{ gm cm}^{-3}$ . The slightly increased value during this period for Station 1.04 was partly due to the greater snow depth resulting in more layers and higher temperature gradients. In other words there was a greater volume of snow available for movement. In the middle of February (17-22), a sharp drop in air temperature resulted once

again in increased rates of negative flux. This is seen in Figures 5.7, 5.8 and 5.12. The amount of sublimation was restricted between  $+0.10$  and  $-0.10 \text{ gm cm}^{-3}$  for Station 1.04 and 6.09. For Station 2.03 it was roughly half that amount although there is a considerable change in the density difference between the top and bottom layers, and the flux was greater compared to the other two stations. This leads one to believe that there are other processes which are operating or must be considered to account for such variability. One explanation could be that the surface of the snow was warmer than the air above it or the top layer of the snow resulting in heat being emitted in both directions. Since the temperature below the top layer starts increasing again, an inversion is created whereby a cancelling effect tends to restrict the movement of snow. A similar situation takes place for Station 2.03 for the period March 4-9 and this once again is indicated by the temperature-depth profile (Appendix VII). These conditions are also seen to occur for Stations 1.04 and 6.09. The heat flux by conduction during the spring melt is reduced in every case as seen in Figures 5.7 through 5.13. This is because of two factors: i) the steadily increasing air temperature which results in an increase in the snowpack temperatures and ii) the slow depletion of the snowpack with net decrease in snow depth, i.e. a shallow snowpack. The sublimation process is also slowed down as temperature conditions in the snowpack tend towards equalization.



The conditions on April 1 and 2 for Station 6.09 are depicted in Figure 5.14 on an hourly basis. These two days have been selected on the basis of the discharge hydrograph which shows that the peak discharge took place on April 2. The graph shows the relation between air temperature and density changes. With an increase in temperature there was an increase in density downwards because the snowpack was getting colder below. At the same time since a positive flux generates melt at the top, densities at the top of the snowpack were close to conditions observed at the bottom. So, on an hourly basis the density difference was small until April 2 when a drop in air temperature increased densities at the top. This was followed by another reversal when the air temperature started rising to values above 0°C. Two points are significant from here. Firstly, density changes are sharply related to air temperature and secondly the sublimation process is tied to fluctuating temperature conditions. Warming in the region above the snow surface enhances negative sublimation and if temperatures remain positive it also increases the density at the top. When temperatures fall below zero, there is positive sublimation. Between hours 10 and 13 on April 2 when the air temperature started increasing again and remained below zero, the bottom of the snowpack was much denser than the top. In the next few hours although the temperature kept increasing, this time to values above zero, the density differences were smaller. This indicates that the model dampens the sublimation effect as it should when temperatures are above freezing. Phase transitions of

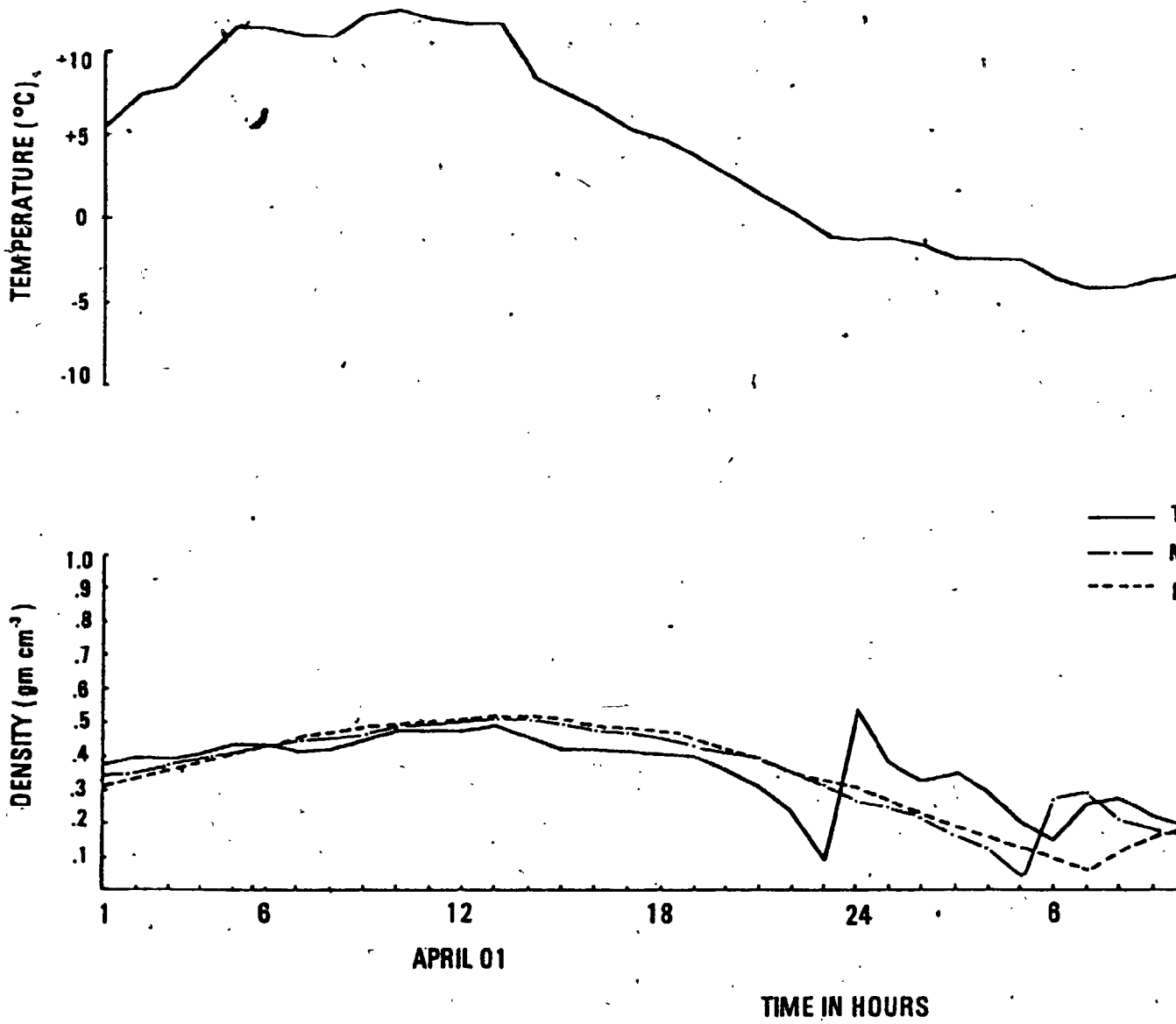
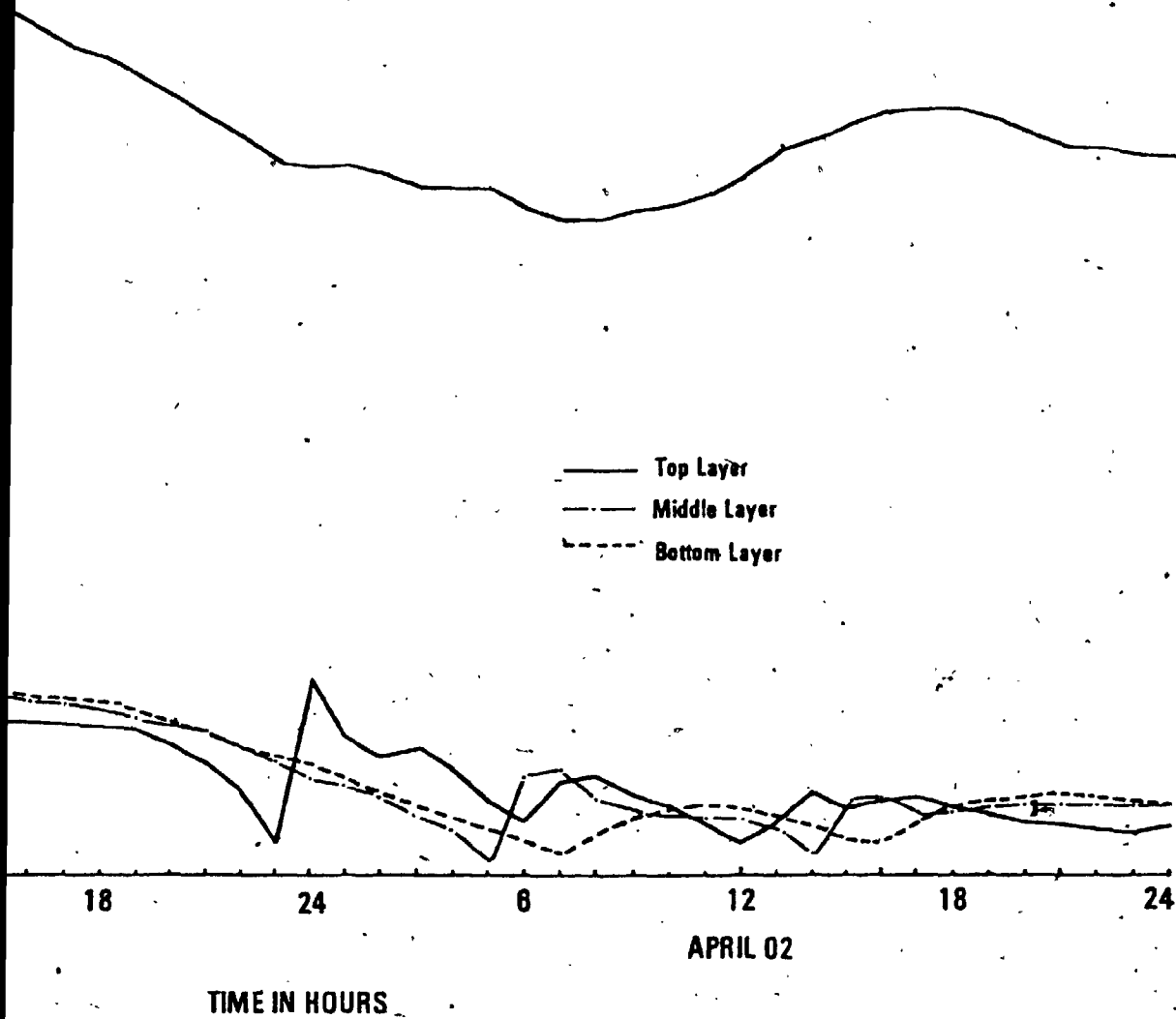


FIGURE 5.14 DENSITY CHANGES AT STATION 6.09 IN RELATION TO AIR TEMPERATURE



STATION 6.09 IN RELATION TO AIR TEMPERATURE

| 2002 |

water and melt-freeze mechanisms operate more freely under those conditions, rather than conductive heat transfer.

In summary, during most of the winter there is a negative flux of heat from the snowpack upwards resulting in negative sublimation or increased density at the top. During the spring melt season, heat flux is positive and into the snowpack enhancing melt from the top downwards. This meltwater moves under the influence of gravity towards the lower layers, often refreezing and releasing heat. The heat transfer by conduction is reduced and the warming in the lower layers is primarily a function of phase transition of water. Sublimation is also reduced and depending on the boundary conditions (air and ground temperatures) snow will be moved either up or down. Depositional metamorphism is characteristic at the beginning of winter, followed by temperature gradient metamorphism when snowpack remains subzero creating increased rates of sublimation. Over the spring melt season isothermal conditions give rise to equi-temperature metamorphism with small amounts of sublimation.

#### 5.4 Runoff and regression analysis

This section deals with the analysis of melt simulated from the model. The objective was to check the usefulness and accuracy of the model in relation to the observed discharge in the basin. To make such comparisons a correlation-regression analysis was employed.

Since there was very little melt during the early and mid-winter period an assumption was made to compare the melt against the observed runoff for certain time intervals over the entire winter. These time intervals were selected on the basis of days between data collection. For instance information for Station 1.04 was collected 18 times over the winter, as such it has been broken down into 18 time slots. Time slot No. 1 consists of 16 days (Nov. 12 to Nov. 27), whereas slot No. 2 consists of 13 days (Nov. 28 to Dec. 11) and so on. For the other stations, the days have been worked out in the same way. Table 5.2 shows the starting date for each snow course for the simulation. These are dates when actual measurements were made. To make comparisons simpler and in the same units, the discharge values (after baseflow separation) were converted to centimeters depth over the basin. It is apparent from the observed hydrograph that most of the runoff takes place during the spring melt season and so it was decided to examine the time period between March 16 and April 15 on a daily basis. Figures 5.15 through 5.20 shows the cumulative snowmelt and runoff in centimeters depth over the basin for different stations. The data is provided in Tables 5.3 through 5.6. It can be seen from the tables that there is some variation in the total values when compared to the observed runoff, not only in terms of snow courses but also from sampling point to point. The best results are obtained from snow course No. 1. Station 4.07, 4.09 and 6.01 tend to overestimate while stations 1.05, 2.07, 2.09, 3.07, 5.07, 5.09, 6.05 and 6.07 are reasonably below



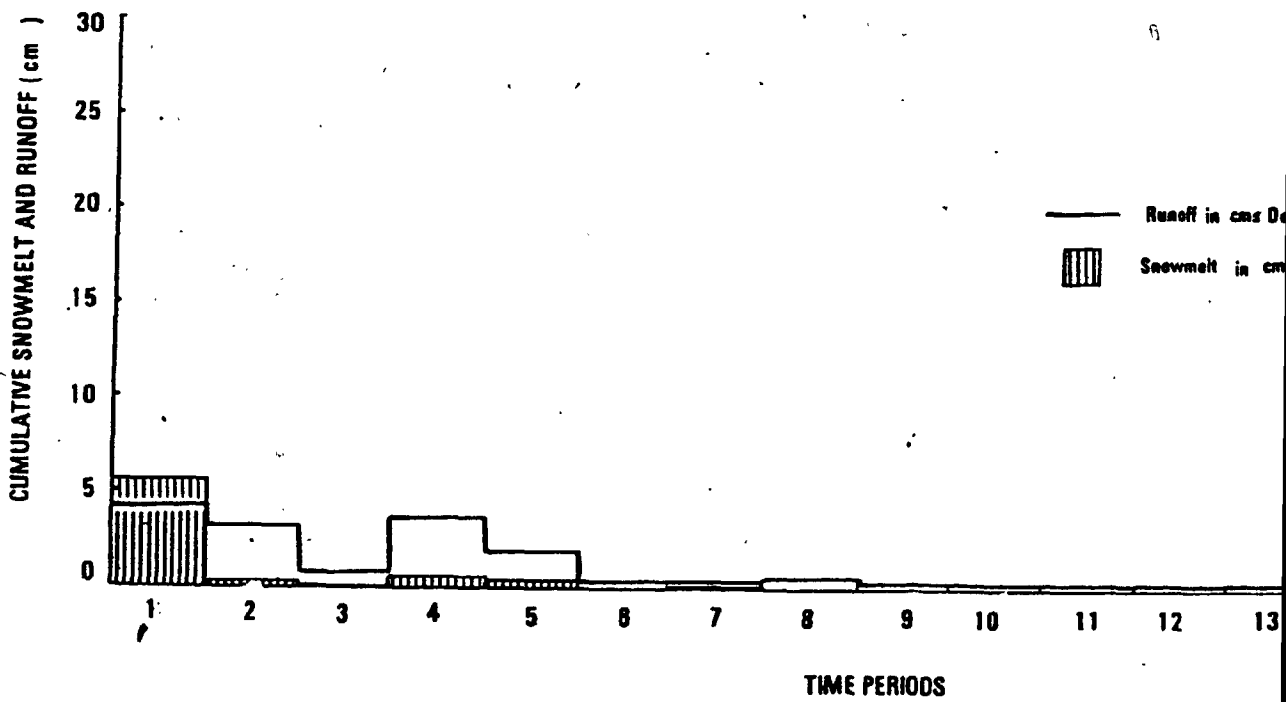
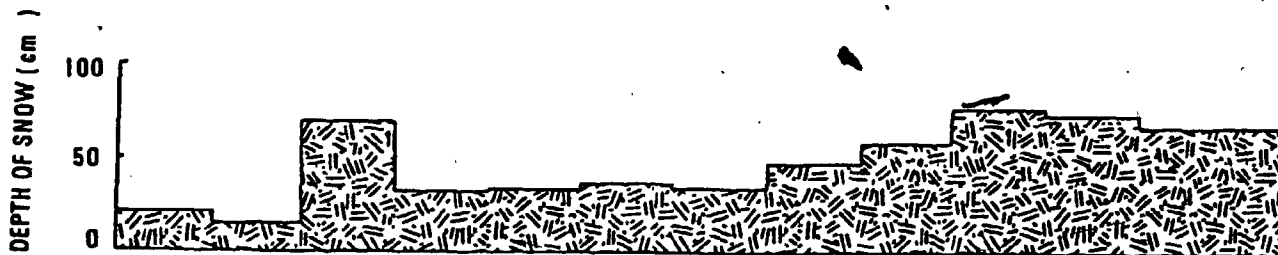
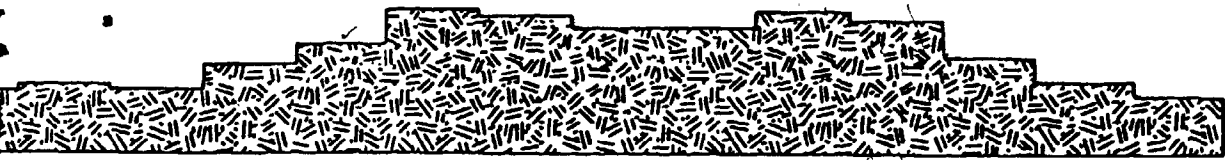


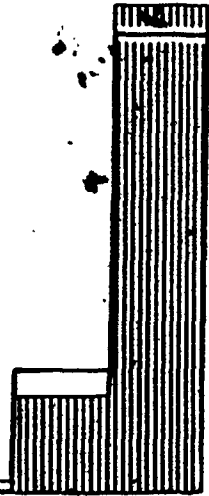
FIGURE 5.15 COMPARISON OF CUMULATIVE SNOWMELT AND RUNOFF AT ST. 1.04



— Runoff in cms Depth Over Basin  
▨ Snowmelt in cms Depth Over Basin

6 7 8 9 10 11 12 13 14 15 16 17 18

TIME PERIODS



RELATIVE SNOWMELT AND RUNOFF AT ST. 1.04

2002



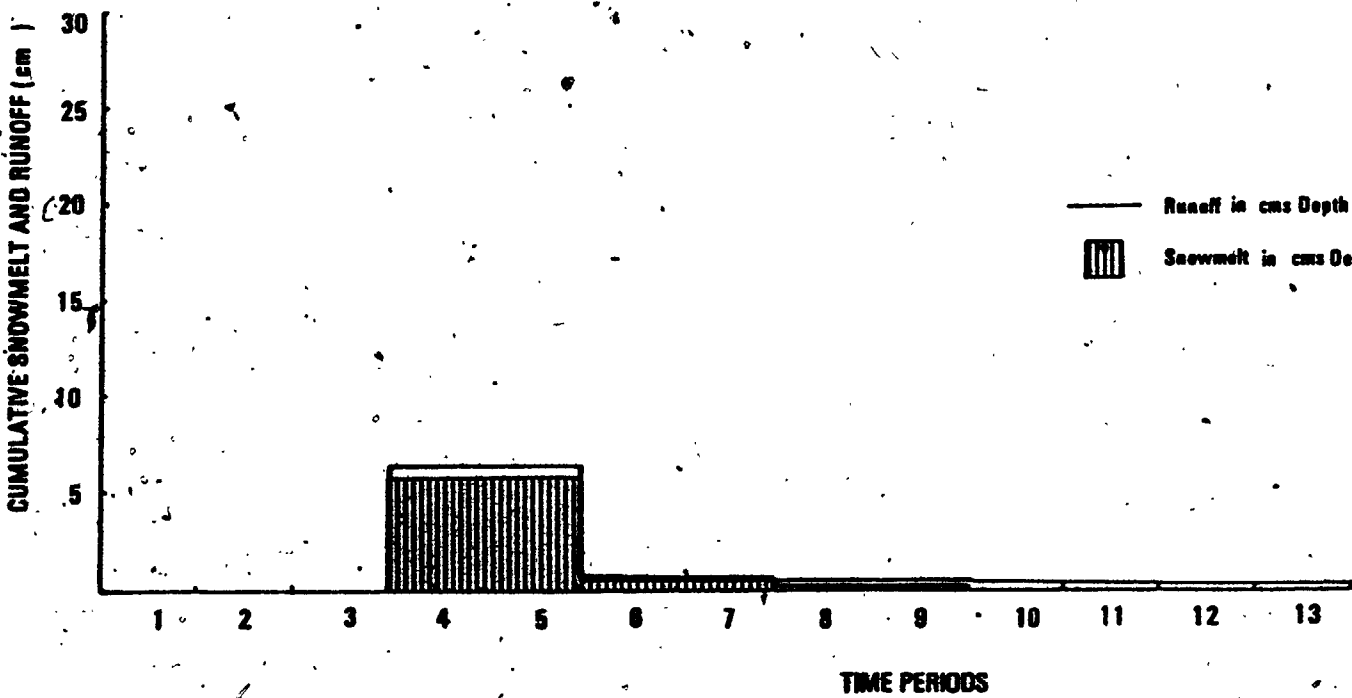
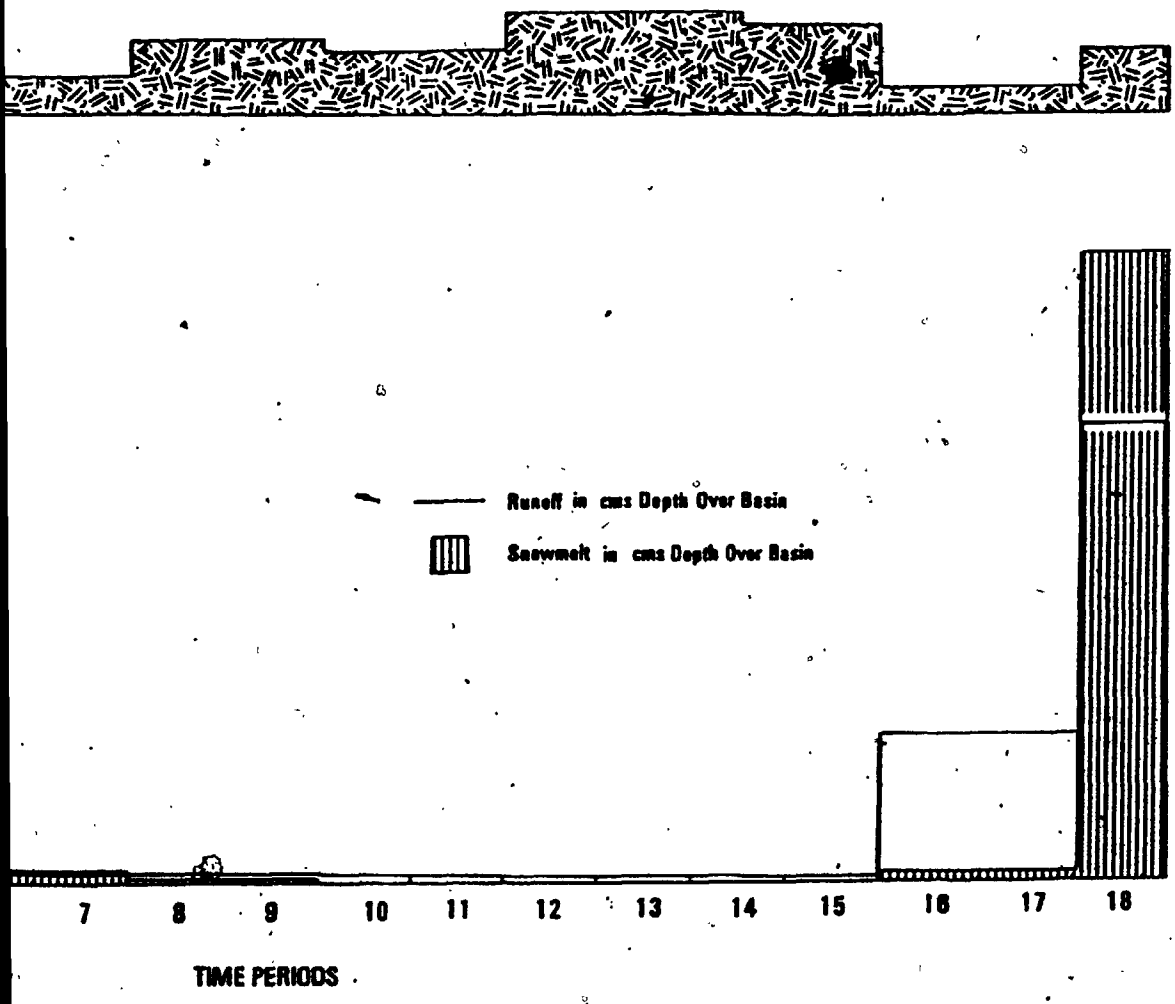


FIGURE 5.16 COMPARISON OF CUMULATIVE SNOWMELT AND RUNOFF AT ST. 2.03

1 OF 1



VE SNOWMELT AND RUNOFF AT ST. 2.03

2002

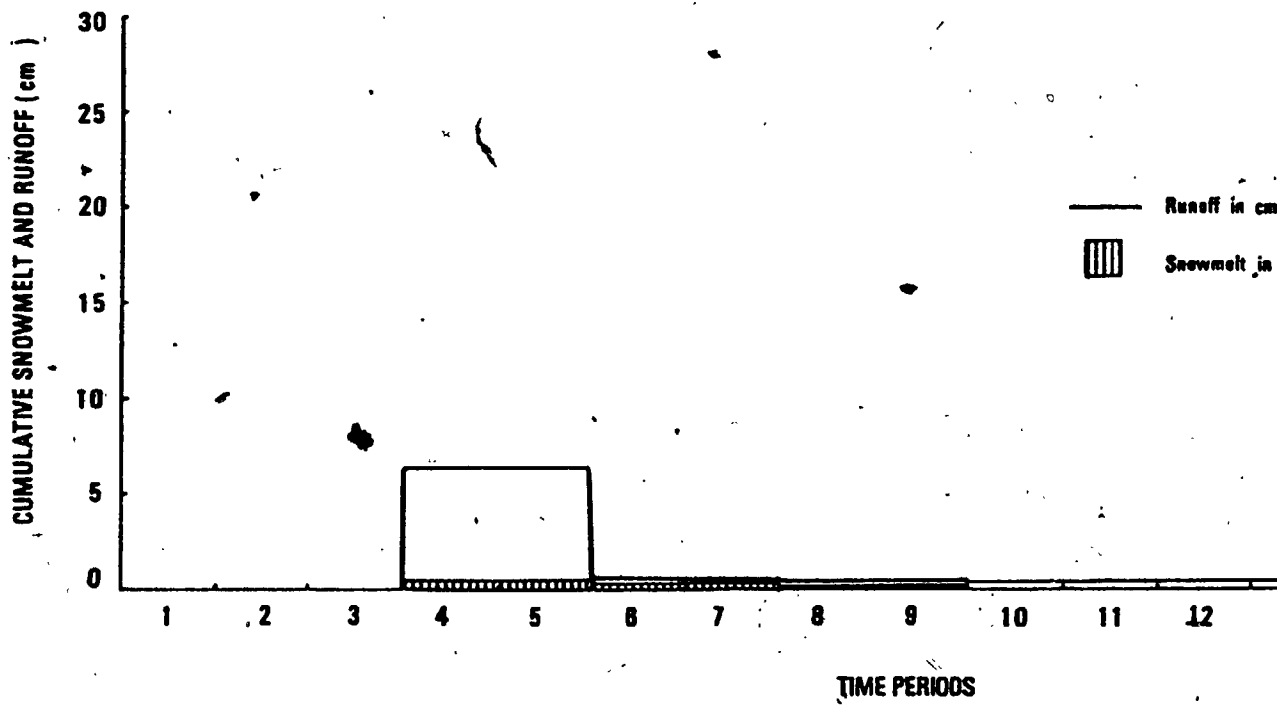
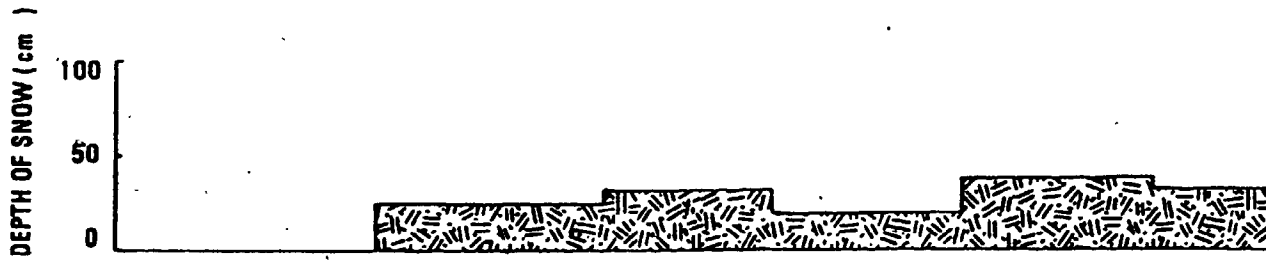
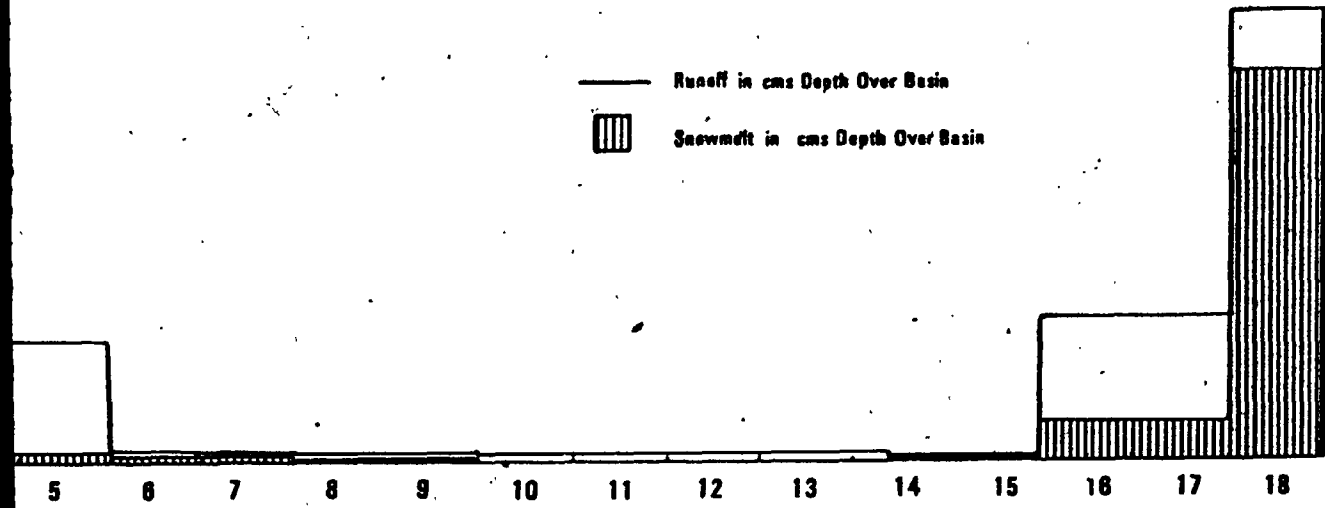


FIGURE 5.17 COMPARISON OF CUMULATIVE SNOWMELT AND RUNOFF AT ST. 3:05

1 OF 1



— Runoff in cms Depth Over Basin  
▨ Snowmelt in cms Depth Over Basin



TIME PERIODS

OF CUMULATIVE SNOWMELT AND RUNOFF AT ST. 3.05

2024

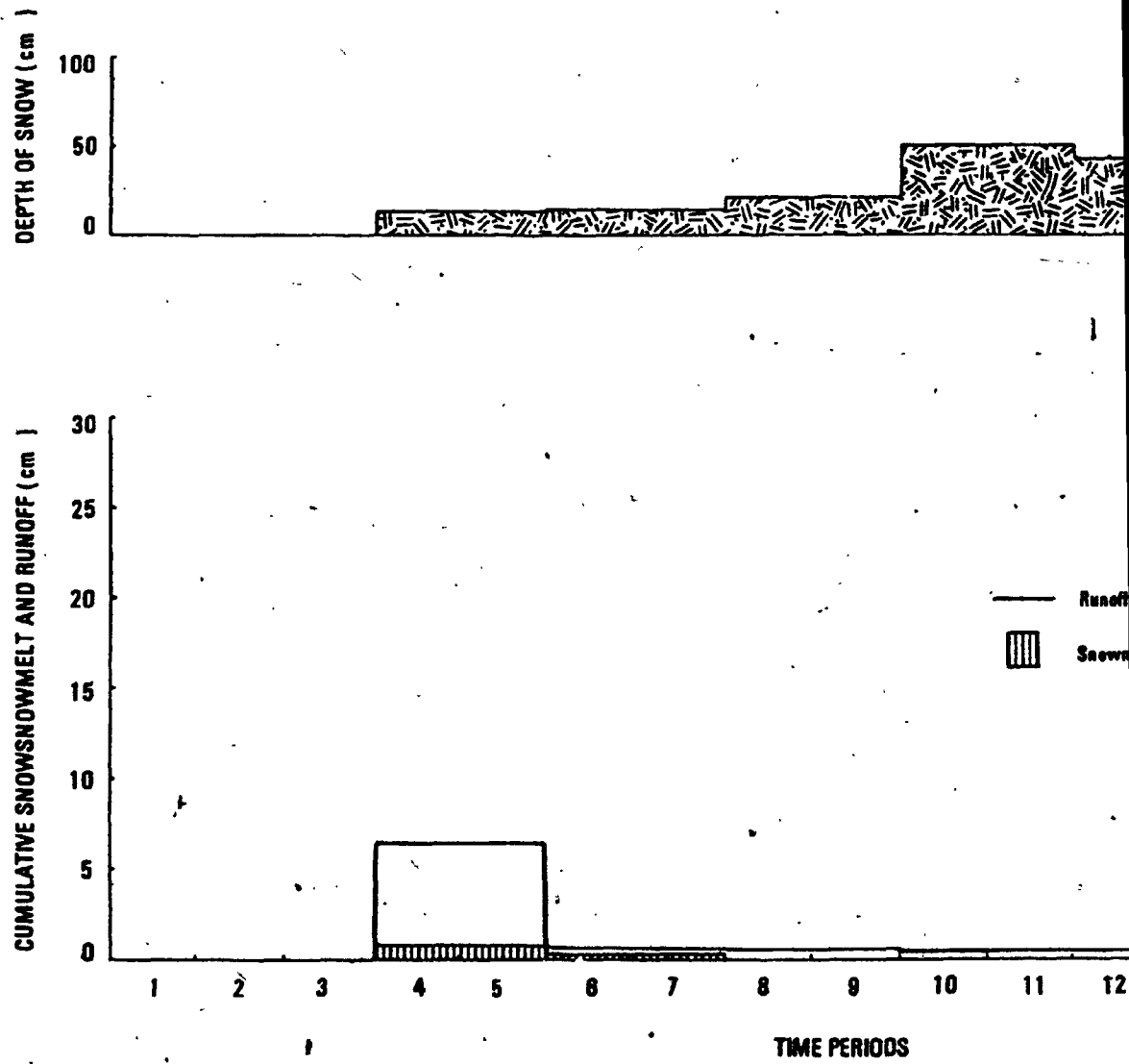
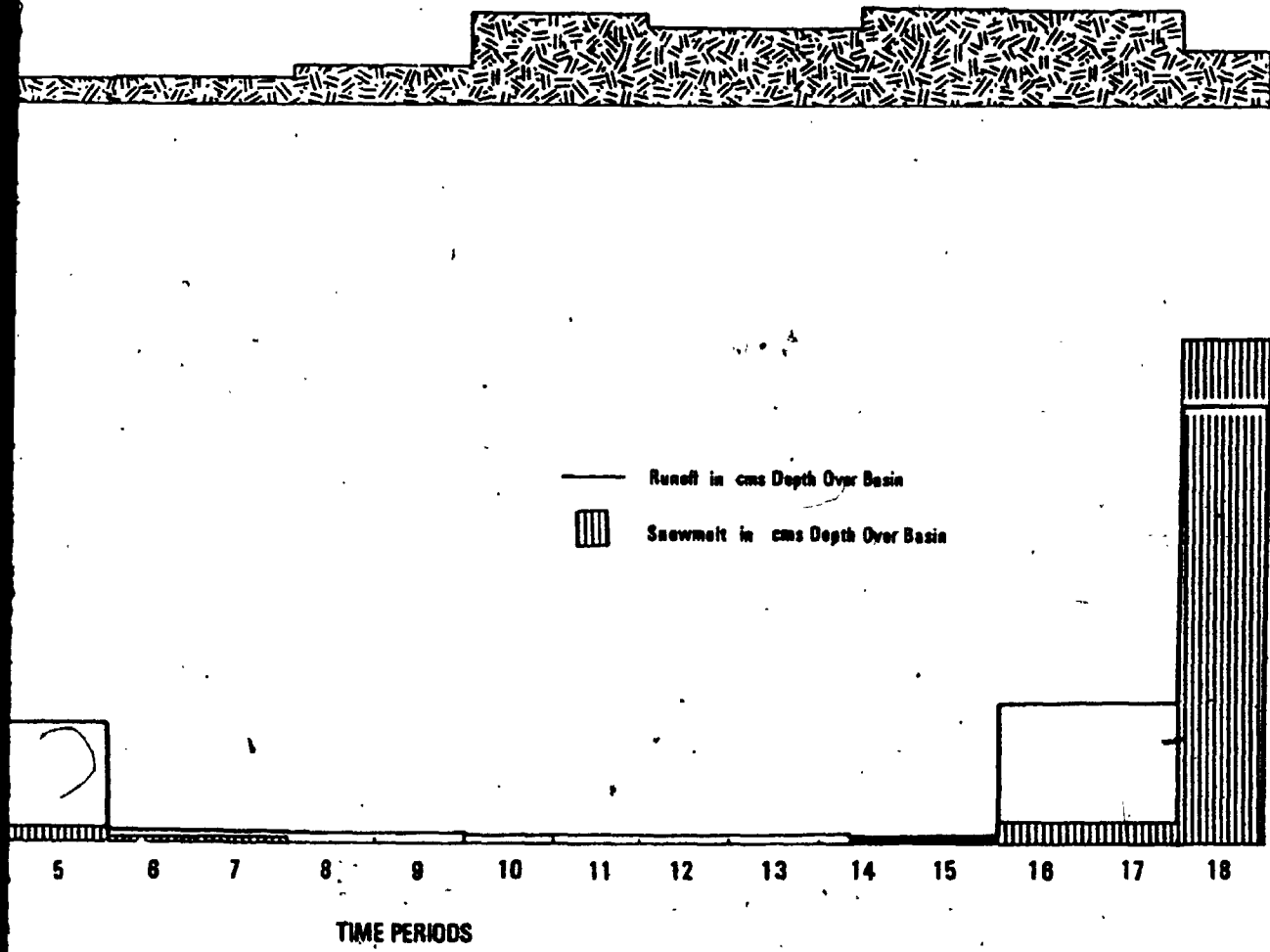


FIGURE 5.18 COMPARISON OF CUMULATIVE SNOWMELT AND RUNOFF AT ST.4.01

1 OF 1



OF CUMULATIVE SNOWMELT AND RUNOFF AT ST.4.01

2002

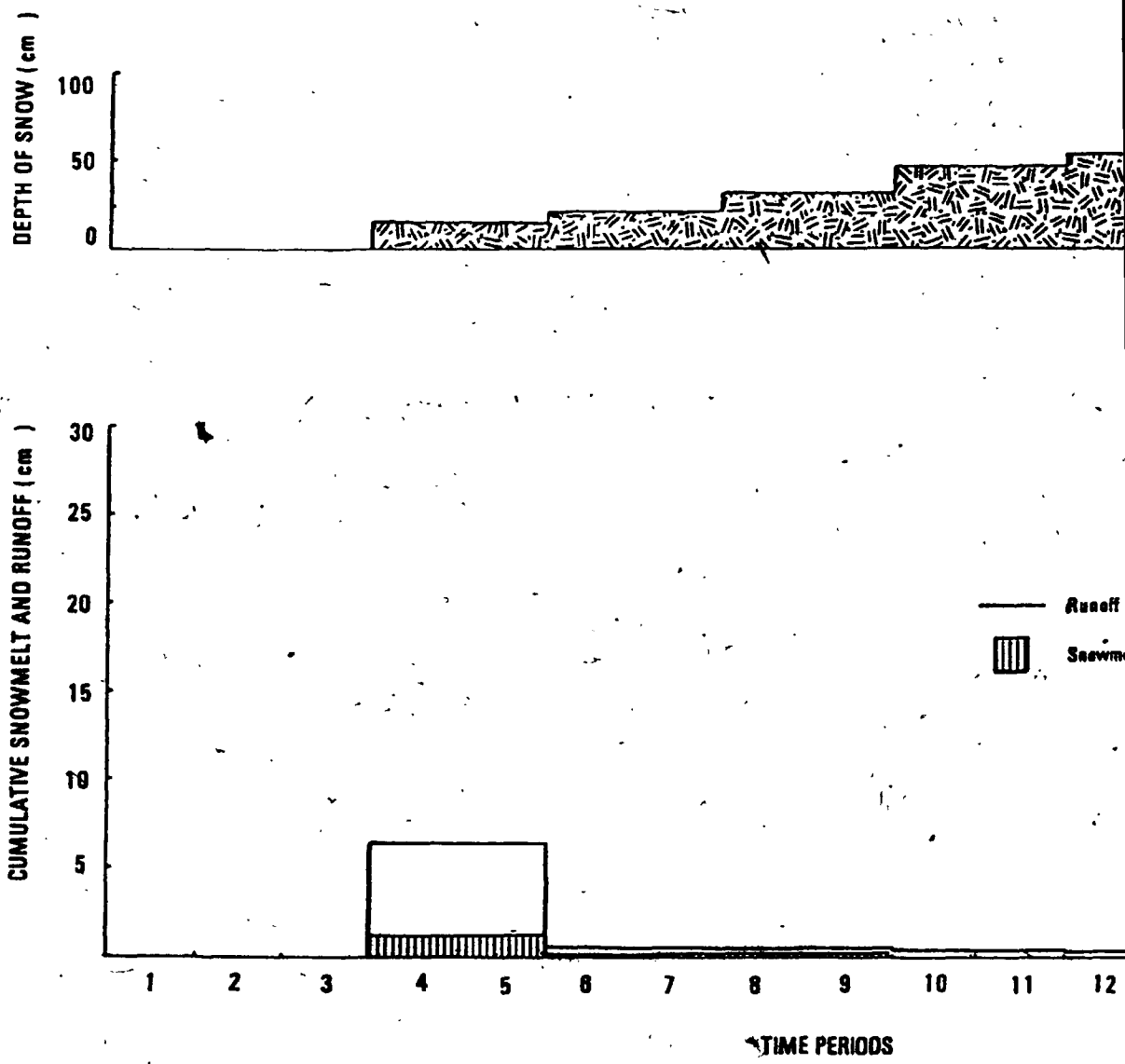
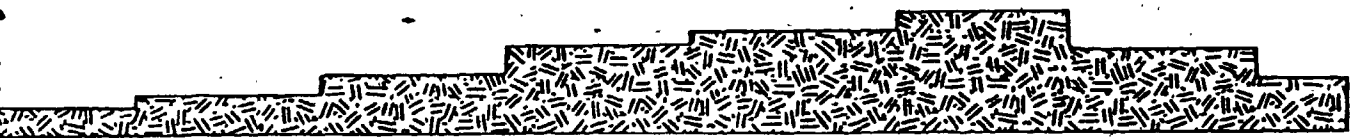
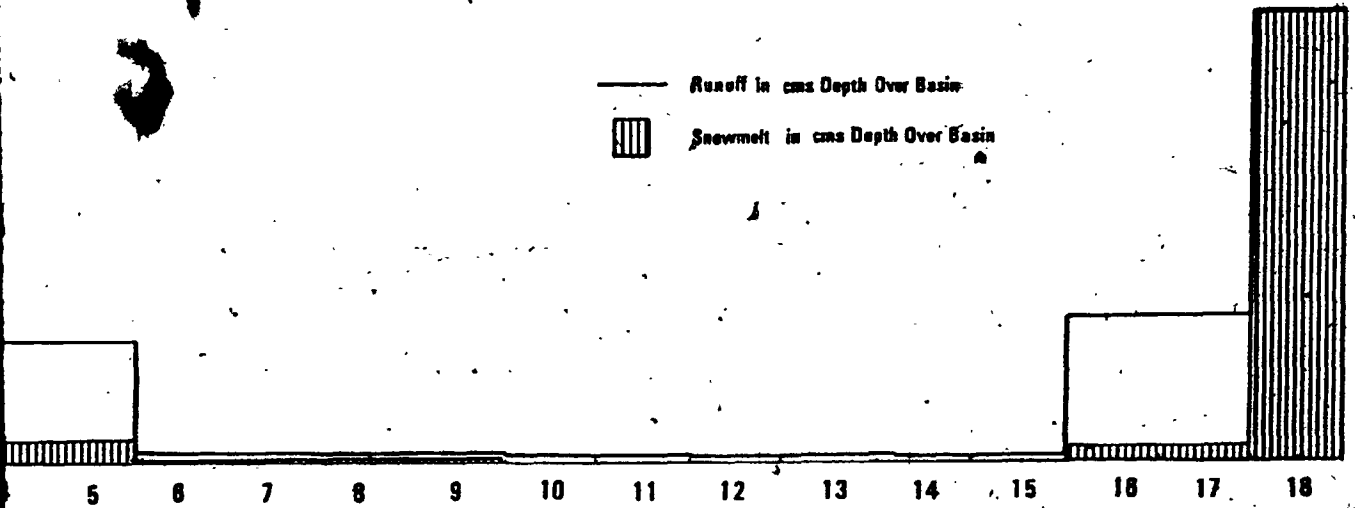


FIGURE 5.19 COMPARISON OF CUMULATIVE SNOWMELT AND RUNOFF AT ST. 5.03



— Runoff in cms Depth Over Basin  
▨ Snowmelt in cms Depth Over Basin



TIME PERIODS

OF CUMULATIVE SNOWMELT AND RUNOFF AT ST. 5.03

2002 1



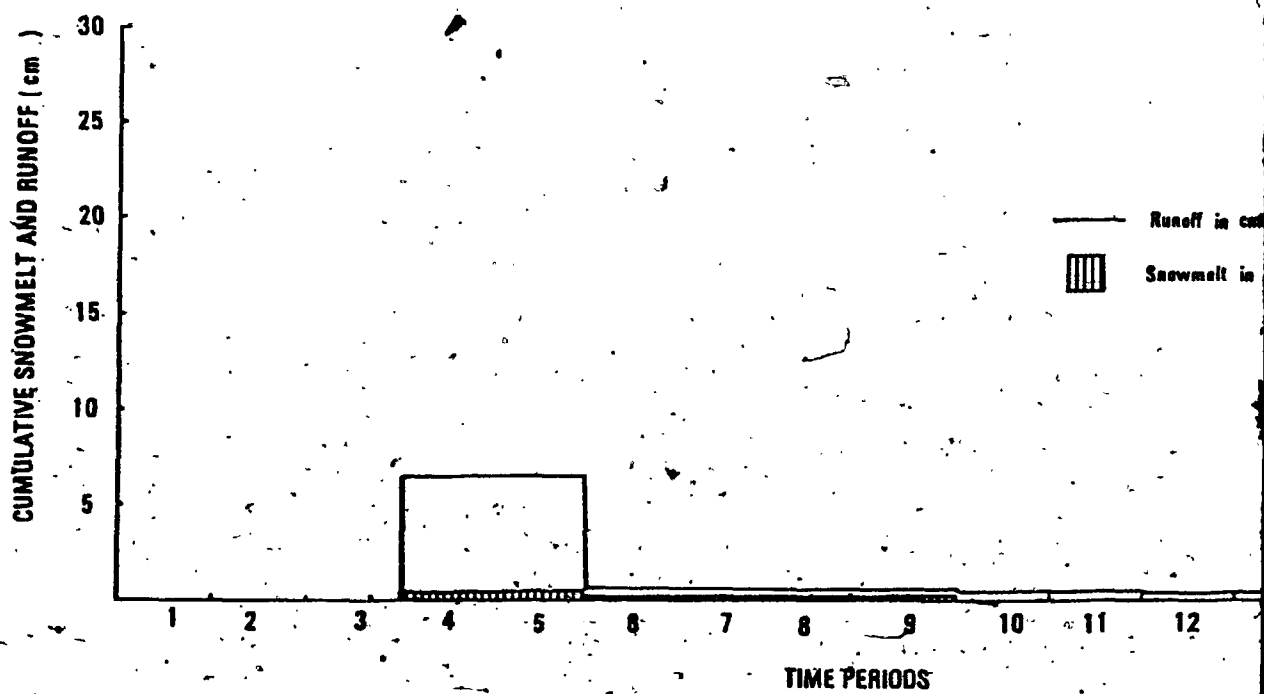
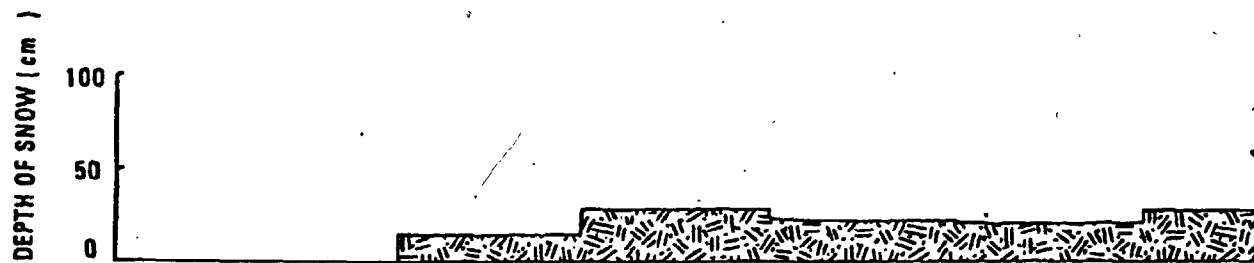
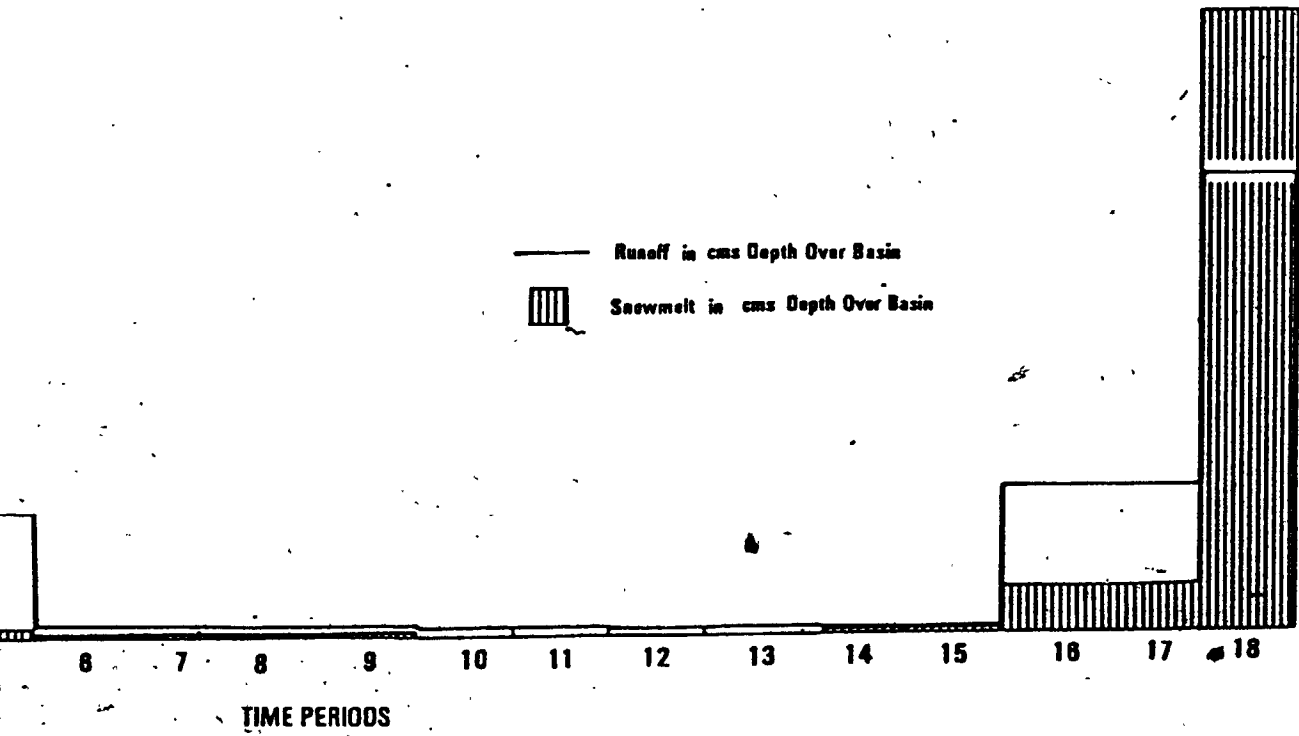


FIGURE 5.20 COMPARISON OF CUMULATIVE SNOWMELT AND RUNOFF AT ST. 6.09

1 OF 1



— Runoff in cms Depth Over Basin  
▨ Snowmelt in cms Depth Over Basin



CUMULATIVE SNOWMELT AND RUNOFF AT ST. 6.09

2002

Table 5.3

Simulated Melt vs. Observed Runoff  
for Snow Course No. 1

S T A T I O N N U M B E R

M E L T ( c m )

Time Slot	Observed Runoff (cms.)	S T A T I O N N U M B E R				
		Stn. 1.01	Stn. 1.02	Stn. 1.03	Stn. 1.04	Stn. 1.05
1	4.15	6.01	6.70	0.04	5.45	0.02
2	3.23	0.01	0.00	0.00	0.28	0.00
3	0.88	0.00	0.00	0.00	0.01	0.00
4	3.83	0.07	0.32	0.22	0.69	0.75
5	1.98	0.39	0.40	0.62	0.35	0.80
6	0.35	0.00	0.00	0.00	0.00	0.00
7	0.38	0.77	2.47	0.65	0.02	0.29
8	0.45	0.00	0.00	0.00	0.00	0.00
9	0.36	2.32	0.00	0.00	0.00	0.00
10	0.31	0.00	0.00	0.00	0.00	0.00
11	0.26	0.00	0.00	0.00	0.00	0.00
12	0.23	0.00	0.00	0.00	0.00	0.00
13	0.21	0.00	0.00	0.00	0.00	0.00
14	0.20	0.00	0.00	0.00	0.00	0.00
15	0.22	0.00	0.08	0.03	0.00	0.07
16	0.58	0.01	0.01	0.02	0.04	0.17
17	6.45	3.27	3.04	4.91	5.09	5.13
18	24.11	27.31	29.69	23.76	25.83	19.39
Tot.	48.18	40.01	42.71	30.25	37.76	26.62

Table 5.4

Simulated Melt vs. Observed Runoff  
for Snow Course No. 2 and 3

S T A T I O N N U M B E R

M E L T ( c m )

Time Slot	Observed Runoff (cms.)	S T A T I O N N U M B E R				
		2.01	2.03	2.05	2.07	2.09
4	6.57	0.55	5.99	1.32	2.59	0.94
6	0.79	0.15	0.76	0.05	0.04	0.33
8	0.76	0.00	0.15	0.06	0.16	0.03
10	0.67	0.00	0.00	0.00	0.00	0.00
12	0.42	0.00	0.00	0.00	0.00	0.00
14	0.38	0.00	0.00	0.00	0.02	0.98
16	7.83	5.99	0.44	2.83	4.95	2.74
18	23.27	31.75	32.88	26.54	11.56	12.60
Tot.	40.69	38.44	40.22	30.80	19.42	17.62
		S T A T I O N N U M B E R				
		3.01	3.03	3.05	3.07	3.09
4	6.57	8.01	2.97	0.45	0.92	0.66
6	0.79	0.08	0.10	0.19	0.02	0.20
8	0.90	0.03	0.01	0.01	0.01	0.01
10	0.53	0.00	0.00	0.00	0.00	0.00
12	0.42	0.00	0.00	0.00	0.00	0.00
14	0.38	0.72	0.30	0.03	0.60	0.51
16	7.83	2.69	5.24	2.16	2.59	1.66
18	23.27	17.84	32.22	20.47	14.81	30.01
Tot.	40.69	29.37	40.84	23.31	18.95	33.06

Table 5.5

Simulated Melt vs. Observed Runoff  
for Snow Course No. 4 & 5

S T A T I O N N U M B E R

MELT (cm )

Time Slot	Observed Runoff (cms.)	S T A T I O N N U M B E R				
		4.01	4.03	4.05	4.07	4.09
4	6.57	0.86	0.59	0.52	1.33	0.56
6	0.79	0.04	0.30	0.40	0.15	0.17
8	0.85	0.00	0.04	0.11	0.00	0.06
10	0.51	0.00	0.00	0.00	0.00	0.00
12	0.49	0.00	0.10	0.00	0.00	0.49
14	0.38	0.01	0.08	0.02	0.00	3.88
16	7.83	1.19	4.15	1.82	1.25	0.40
18	23.27	27.57	25.99	26.44	51.92	45.91
Tot.	40.69	29.67	31.25	29.31	54.65	50.98
		5.01	5.03	5.05	5.07	5.09
4	6.57	0.36	1.27	2.57	1.90	1.69
6	0.79	0.11	0.10	0.51	0.12	1.04
8	0.85	0.03	0.10	0.10	0.06	0.04
10	0.51	0.00	0.00	0.00	0.00	0.00
12	0.49	0.00	0.00	0.00	0.00	0.00
14	0.38	0.83	0.00	0.88	0.00	0.07
16	7.83	0.32	0.91	2.19	2.79	2.52
18	23.27	36.07	23.45	33.24	12.52	10.63
Tot.	40.69	37.72	25.83	39.49	17.39	15.99

Table 5.6

Simulated Melt vs. Observed Runoff  
for Snow Course No. 6

S T A T I O N N U M B E R

M E L T ( c m )

Time Slot	Observed Runoff (cms.)	Stn. 6.01	Stn. 6.03	Stn. 6.05	Stn. 6.07	Stn. 6.09
4	6.57	2.98	1.53	1.14	5.21	0.52
6	0.74	0.72	0.04	0.51	0.05	0.03
8	0.95	0.10	0.11	0.08	0.09	0.02
10	0.46	0.00	0.00	0.00	0.00	0.00
12	0.46	0.00	0.00	0.00	0.00	0.00
14	0.41	0.15	0.51	0.00	0.66	0.19
16	7.83	2.81	0.97	3.90	1.08	2.53
18	23.27	24.21	23.16	11.64	10.53	32.58
Tot.	40.69	50.97	26.32	17.27	17.62	35.57

the recorded values. If only the total values are considered station 2.03 and 3.03 predict with a high degree of accuracy. Table 5.7 presents the correlation - regression results for this part of the analysis. Although each individual station has a significant correlation value, the standard error of estimate gives an indication of the accuracy of the model. To understand some of the discrepancies in the model, examination by time slots was necessary. One important factor which has to be borne in mind is that the melt is produced only under positive energy flux (i.e. temperature  $> 0^{\circ}\text{C}$ ). On days when the temperature was below  $0^{\circ}\text{C}$  no melt was produced by the model although small amounts of discharge under ice were reported on the gauge in Medway creek. This can be observed by looking at time slots 8 through 14 in most snow courses. During time periods 1, 2, 4, 16, 17 and 18 the amount of melt produced was a function of the amount of energy available and the depth of snow. For example the reported depth of snow was considerably higher in stations 4.07 and 4.09 as compared to station 5.07 or 6.07. Similar values can be extrapolated for other sampling points. A graphical comparison can also be made by examining Figures 5.15 through 5.20, although in these specific cases the variations are not too great. In order to account for these and other differences the simulation model needs to be modified with other factors which were not included in this research. These would include variables like precipitation inputs, effects of site and vegetation and a forcing function which would generate melt up to a certain critical negative

Table 5.7

Correlation-regression analysis of simulated  
melt versus observed runoff

<u>Station</u>	<u>N</u>	<u>r</u>	<u>S.E.E.</u>	<u>Equation</u>
1.01	18	0.97	1.41	RO = 0.85M + 0.79
1.02	18	0.96	1.43	RO = 0.77M + 0.84
1.03	18	0.97	1.26	RO = 0.97M + 1.03
1.04	18	0.98	0.99	RO = 0.90M + 0.78
1.05	18	0.98	1.17	RO = 1.19M + 0.91
2.01	8	0.96	1.95	RO = 0.69M + 1.76
2.03	8	0.97	1.62	RO = 0.85M + 1.24
2.05	8	0.96	2.08	RO = 0.82M + 1.91
2.07	8	0.99	0.87	RO = 1.90M + 0.47
2.09	8	0.97	1.79	RO = 1.79M + 1.14
3.01	8	0.96	2.00	RO = 1.20M + 0.67
3.03	8	0.97	1.64	RO = 0.69M + 1.53
3.05	8	0.95	2.23	RO = 1.06M + 1.99
3.07	8	0.97	1.83	RO = 1.51M + 1.52
3.09	8	0.94	2.50	RO = 0.71M + 2.14
4.01	8	0.94	2.47	RO = 0.77M + 2.21
4.03	8	0.96	2.04	RO = 0.84M + 1.78
4.05	8	0.94	2.43	RO = 0.81M + 2.11
4.07	8	0.94	2.59	RO = 0.41M + 2.30
4.09	8	0.92	2.94	RO = 0.45M + 2.19
5.01	8	0.93	2.78	RO = 0.58M + 2.35
5.03	8	0.94	2.42	RO = 0.92M + 2.13
5.05	8	0.95	2.26	RO = 0.66M + 1.83
5.07	8	0.99	1.04	RO = 1.82M + 1.13
5.09	8	0.98	1.26	RO = 2.16M + 0.76
6.01	8	0.95	2.26	RO = 0.49M + 1.94
6.03	8	0.94	2.40	RO = 0.93M + 2.01
6.05	8	0.98	2.51	RO = 1.92M + 0.93
6.07	8	0.94	2.51	RO = 1.97M + 0.75
6.09	8	0.95	2.38	RO = 0.66M + 2.12

RO = Cumulative Runoff in cm depth over basin

M = Simulated Melt in cm depth over basin



temperature. The overestimation could probably be calibrated by accounting for a snow depletion factor over time. Some of these modifications would necessitate monitoring the snowpack on a continuous basis. The best predictor equation from Table 5.7 is the one for station 2.07 where with no melt being simulated only 0.47 cm of runoff is predicted. This is comparable to values cited for station 2.07 in Table 5.4 for time slots 12 or 14.

The results for observed versus simulated discharge on a daily basis are produced in Figure 5.21. The observed values are derived after baseflow separation. The trend of the peaks indicates the lag between snowmelt and runoff. Since the simulated melt is dependent on the temperature conditions at the two boundaries, the values on a daily basis are higher than the observed discharge. On an hourly basis the correlation is also higher. For instance, the maximum instantaneous discharge which took place on April 1, hour 2307, was  $94.5 \text{ m}^3 \text{ sec}^{-1}$  and the maximum daily discharge was  $63.7 \text{ m}^3 \text{ sec}^{-1}$  (April 2). Air temperature being the important variable producing melt, a correlation - regression analysis was performed. These results are shown in Tables 5.8 through 5.11. As expected the highest correlation was with maximum air temperature, indicating that the more energy was available the higher the melt. This leads one to believe that threshold values are needed to modify the model. The best results are obtained from stations 2.09, 5.09 and 6.07. Significant results are also obtained from the mean temperature values, but when minimum temperatures are considered, some of the

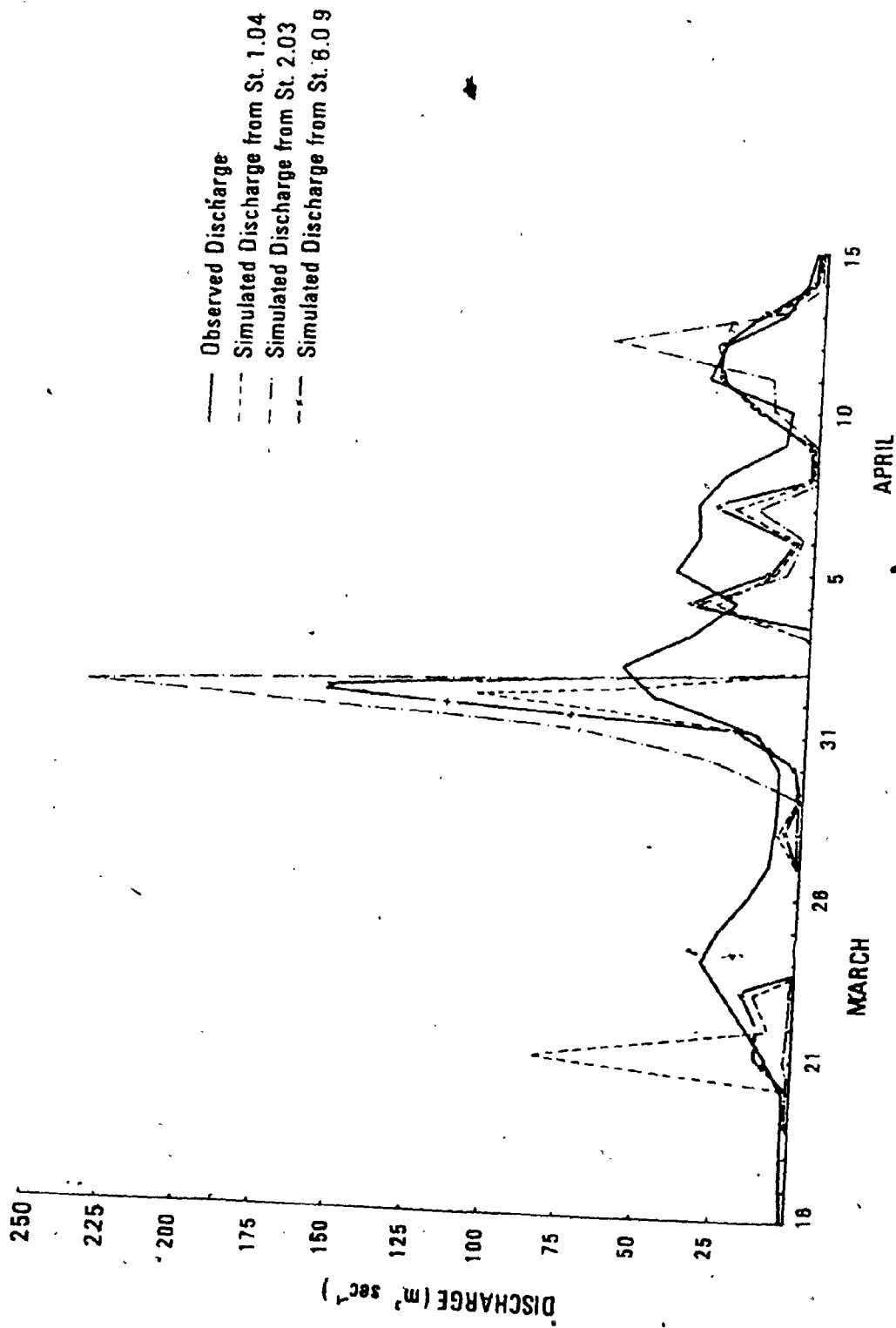


FIGURE 5.2.1 OBSERVED vs SIMULATED DISCHARGE

Table 5.8

Correlation-regression analysis of simulated  
melt versus maximum air temperature

March 16 - April 15

<u>Station</u>	<u>r</u>	<u>S.E.E.</u>	<u>Equation</u>
1.01	0.69	.03	M = .14T - .09
1.02	0.61	.03	M = .14T - .06
1.03	0.52	.04	M = .12T - .01
1.04	0.57	.03	M = .13T - .01
1.05	0.51	.03	M = .10T + .04
2.01	0.60	.04	M = .16T - .01
2.03	0.50	.07	M = .20T - .25
2.05	0.57	.04	M = .15T - .10
2.07	0.57	.02	M = .06T - .09
2.09	0.66	.02	M = .06T - .01
3.01	0.65	.02	M = .09T - .04
3.03	0.61	.04	M = .17T - .04
3.05	0.63	.02	M = .11T - .07
3.07	0.64	.02	M = .08T - .01
3.09	0.53	.05	M = .16T - .15
4.01	0.55	.04	M = .14T - .15
4.03	0.60	.03	M = .13T - .05
4.05	0.56	.04	M = .14T - .12
4.07	0.69	.04	M = .23T - .26
4.09	0.54	.07	M = .25T - .31
5.01	0.68	.04	M = .19T - .25
5.03	0.61	.04	M = .13T - .15
5.05	0.53	.05	M = .18T - .15
5.07	0.65	.01	M = .06T + .11
5.09	0.68	.01	M = .05T + .00
6.01	0.74	.03	M = .16T - .14
6.03	0.69	.02	M = .10T - .11
6.05	0.66	.02	M = .06T + .44
6.07	0.77	.01	M = .05T - .04
6.09	0.59	.04	M = .16T - .14

All r values are significant at the .05 level.

M = Simulated Melt in cm

T = Maximum Air Temperature in °C

N = 31

Table 5.9

Correlation-regression analysis of simulated  
melt versus mean air temperature

March 16 - April 15

<u>Station</u>	<u>r</u>	<u>S.E.E.</u>	<u>Equation</u>
1.01	0.64	.03	M = .15T + .50
1.02	0.56	.04	M = .15T + .52
1.03	0.51	.04	M = .14T + .51
1.04	0.54	.04	M = .15T + .54
1.05	0.50	.04	M = .12T + .46
2.01	0.54	.05	M = .17T + .67
2.03	0.41	.08	M = .20T + .64
2.05	0.48	.05	M = .15T + .55
2.07	0.55	.02	M = .64T + .33
2.09	0.61	.02	M = .07T + .28
3.01	0.58	.03	M = .10T + .36
3.03	0.55	.05	M = .18T + .66
3.05	0.56	.03	M = .11T + .39
3.07	0.57	.02	M = .08T + .31
3.09	0.45	.06	M = .16T + .53
4.01	0.46	.05	M = .15T + .48
4.03	0.54	.04	M = .14T + .52
4.05	0.48	.05	M = .14T + .47
4.07	0.63	.06	M = .24T + .70
4.09	0.47	.09	M = .25T + .74
5.01	0.60	.05	M = .19T + .53
5.03	0.53	.04	M = .14T + .43
5.05	0.45	.07	M = .18T + .64
5.07	0.60	.02	M = .07T + .28
5.09	0.62	.02	M = .06T + .24
6.01	0.68	.04	M = .18T + .56
6.03	0.61	.03	M = .10T + .31
6.05	0.63	.02	M = .07T + .31
6.07	0.69	.01	M = .06T + .19
6.09	0.51	.05	M = .16T + .54

All r values are significant at the .05 level.

M = Simulated Melt in cm

T = Mean Air Temperature in °C

N = 31 Days

Table 5.10

Correlation-regression analysis of simulated  
melt versus minimum air temperatures

March 16 - April 15

<u>Station</u>	<u>r</u>	<u>S.E.E.</u>	<u>Equation</u>
1.01	0.46*	.04	M = .11T + 1.01
1.02	0.39*	.05	M = .11T + 1.00
1.03	0.39*	.05	M = .11T + 1.00
1.04	0.41*	.05	M = .11T + 1.04
1.05	0.39*	.05	M = .09T + 0.87
2.01	0.36*	.06	M = .12T + 1.22
2.03	0.21	.10	M = .11T + 1.18
2.05	0.29	.06	M = .10T + 1.00
2.07	0.41*	.02	M = .05T + 0.55
2.09	0.43*	.02	M = .05T + 0.51
3.01	0.39*	.03	M = .07T + 0.68
3.03	0.37*	.06	M = .12T + 1.23
3.05	0.36*	.04	M = .07T + 0.74
3.07	0.38*	.03	M = .05T + 0.57
3.09	0.26	.07	M = .10T + 1.00
4.01	0.27	.06	M = .08T + 0.91
4.03	0.36*	.05	M = .10T + 0.98
4.05	0.28	.05	M = .08T + 0.89
4.07	0.43*	.07	M = .17T + 1.49
4.09	0.29	.10	M = .16T + 1.50
5.01	0.39*	.06	M = .13T + 1.15
5.03	0.33	.05	M = .09T + 0.85
5.05	0.26	.08	M = .10T + 1.16
5.07	0.41*	.02	M = .05T + 0.51
5.09	0.43*	.02	M = .04T + 0.45
6.01	0.47*	.04	M = .12T + 1.15
6.03	0.40*	.03	M = .07T + 0.64
6.05	0.47*	.02	M = .06T + 0.56
6.07	0.47*	.01	M = .04T + 0.37
6.09	0.32	.10	M = .10T + 1.04

\* Significant at the .05 level.

M = Simulated Melt in cm

T = Minimum Air Temperature in °C.

N = 31.

Table 5.11

Correlation-regression analysis of simulated  
melt versus observed runoff

March 16 - April 15

<u>Station</u>	<u>r</u>	<u>S.E.E.</u>	<u>Equation</u>
1.01	0.36*	.13	RO = .27M + .79
1.02	0.35	.11	RO = .23M + .82
1.03	0.23	.12	RO = .15M + .87
1.04	0.27	.12	RO = .18M + .85
1.05	0.20	.14	RO = .16M + .88
2.01	0.37*	.10	RO = .21M + .79
2.03	0.35*	.06	RO = .13M + .86
2.05	0.35*	.10	RO = .21M + .83
2.07	0.21	.27	RO = .32M + .84
2.09	0.32*	.27	RO = .50M + .79
3.01	0.37*	.18	RO = .39M + .79
3.03	0.38*	.10	RO = .21M + .79
3.05	0.40*	.15	RO = .35M + .79
3.07	0.36*	.21	RO = .45M + .79
3.09	0.39*	.08	RO = .19M + .83
4.01	0.41*	.10	RO = .23M + .83
4.03	0.37*	.12	RO = .25M + .81
4.05	0.40*	.10	RO = .24M + .82
4.07	0.50*	.07	RO = .23M = .76
4.09	0.41*	.06	RO = .14M + .84
5.01	0.46*	.10	RO = .25M + .78
5.03	0.41*	.11	RO = .28M + .81
5.05	0.38*	.07	RO = .17M + .83
5.07	0.32	.27	RO = .49M + .80
5.09	0.32	.31	RO = .57M + .80
6.01	0.47*	.11	RO = .32M + .73
6.03	0.42*	.17	RO = .45M + .79
6.05	0.30	.27	RO = .46M + .79
6.07	0.40*	.36	RO = .86M + .76
6.09	0.41*	.10	RO = .23M + .81

\* Significant at the .05 level.

M = Simulated in cm  
 RO = Runoff in cm (observed)  
 N = 31

stations (2.03, 2.05, 3.09, 4.01, 4.05, 4.09, 5.03, 5.05 and 6.09) are not able to simulate melt upto the expected values. From Table 5.11 it is observable that snow courses 2, 3, 4, 5 and 6 generate the best results as compared to observed values for the period March 16 to April 15. The exceptions are sampling points 2.07, 2.09, 5.07, 5.09 and 6.05. The r values for snow course No. 1 (except 1.01) are not significant. It can be understood that this particular site, although it produces good results on a cumulative time basis, does not generate enough melt on shorter time spans to cope up with the actual values. This particular site is located closest to the discharge gauge but in a wooded area. It is presumed that the snow in this particular locality melts at a much slower rate than in the other snow courses.

#### 5.5 Complexities and problems encountered in simulation modelling

Most simulation studies are complex and there is always a potential for discrepancy between observed and synthesized phenomena. The reason is primarily the simplification of the complex processes occurring in the drainage basin. Simplification is necessary because the basin processes are so complex and interdependent that it is difficult to understand them in a quantitative manner. Even if they are well understood, problems of instrumenting the basin to monitor the necessary parameters and variables on a continuous basis would be formidable. Consequently the computer time required to execute the analysis would be prohibitively expensive. It is because of these and other factors that the model discussed is a collection of simplifying

mathematical equations, and empiricisms constructed in such a manner as to provide a relationship between temperature and snowmelt. Therefore one would expect a certain amount of discrepancy between observed and synthesized phenomena. The significant differences which have come out of the analysis suggest that errors exist, either within the model or in the measurement of the phenomena in the field.

In general there are three sources of error which may suggest significant differences between observed and measured hydrologic phenomena. According to McCaskell (1978) these are:

- a) errors in the assumptions employed in the modelling procedure
- b) errors in the data used as input to the model and
- c) errors in field measurement of the observed phenomena.

As mentioned earlier because of the necessity of simplification, for this research, only two boundary conditions were used in the thermal simulation procedure. These were air and ground temperature along with the initial snowpack temperature. One may argue as to how well these conditions are representative of the energy at the two boundaries. It is known that air temperature is a function of the radiant energy available from the sun. As such it would be appropriate to consider air temperature as a major component of energy. In fact, solar radiation could be used as a calibrating factor of energy flux from the top of the snowpack in the model, if one could isolate parameters like albedo, shade, the different wavelengths of radiation and their



extinction coefficients into the snowpack. Some of these parameters can be easily monitored, others are often computed through empirical relations of measured with complex instrumentation. It is expected that if this simulation model is incorporated with such functions as has been done in various other cases (Solomon, et al., 1976; Logan, 1976) it would certainly improve in accuracy and timing. The same analogy holds true for ground heat conditions. Improvements can be made by understanding the conditions apparent in the soil at that particular moment.

Inputs to the model have been minimized to four variables, air, ground, snow temperature and snow density. To understand more precisely the timing of the melt and depletion of the snow cover, one needs to include variables such as snowfall, rainfall and other related meteorological variables. Snowfall would be a good indicator of the rate of ablation or depletion of the snowcover. Rain-on-snow conditions would definitely be a controlling factor not only in the thermal conditions of the snowpack but also in terms of the timing of the melt. This particular parameter which is of major importance has been excluded due to i) the difficulty of monitoring rainfall at the various snow courses and ii) the published information from London is not as representative as it should be for the different snow courses. A careful synthesis of rain-on-snow hydrographs, if data is available, would help in calibrating the model. More accurate baseflow separations and no snow conditions on the ground should be incorporated when such a complexity is dealt with.

The model has been set up as a one-dimensional flow and energy can be simulated in either direction. In actual snowpack conditions the flux is not necessarily unidirectional, though lateral heat flow by conduction or convection is small. This intricacy can only be solved if measurements of energy are available in horizontal direction. This once again would lead to remodelling the simulation procedure into a multi-dimensional system.

The second general source of error is the data input to the model. The data has been carefully processed and filtered, the problem, however, has been the variation of the observed data with the published information and the matching of the data sets. In the former case the air temperatures recorded at the site were slightly different from the published information on some occasions. On these days the measured values were used. Similarly, matching the data sets (published air temperatures and measured variables) to the exact hour was not always precise. The air temperature readings from the London airport were for every hour whereas measurements at every sampling point were not longer than several minutes. In order to account for this discrepancy all sampling points done within that hour were matched with that published hourly value. If the measurement at any sampling point went past the hour it was included as the next hour. Since the model can simulate on small time increments (15 minutes), thermograph tracings should be used to extrapolate values at those intervals, if one needs to do so.

In the simulation procedure it is possible to control the layer thickness by the variable TF. This is especially important when one deals with short time intervals, as the temperature for each layer is always at the midpoint, so a thick layer will not change temperature over a short time period, yielding inaccurate results. It is suggested that the entire snowpack be divided into as many layers as possible. This has been done for quite a few sampling points in this research. The variable TQ controls the truncation point of the Fourier series and it is recommended that it be set at 0.001 or thereabouts.

The last source of error which could arise in the modelling procedure could be due to field measurements. The techniques employed in this research are simple and the accuracy of the data depends on the instruments used. Snow depth and water equivalent measurements for the entire snowpack posed no problems but when estimating density through the use of horizontal measurements, the penetration of the snow sampler was not always to the same length. As such, the water equivalent values derived for each layer were not comparable but gave a good indication of the density for that layer. To make sure that such measurements are consistent, it would be more appropriate that a small device similar to the sampler be constructed which could be fully inserted into the snow and the sample extracted equally and fully in every case.

This kind of device could also make possible measurements when the snowpack is very shallow. In the present case the inside

diameter of the sampler is 2.776 inches, restricting measurement of layers less than that depth. A sampler with a smaller diameter would be more useful in those cases. Other measurement techniques could be used too.

Temperature measurements often posed a problem. As mentioned earlier in chapter four some of the values recorded inside the snowpack were greater than  $0^{\circ}\text{C}$ . This was primarily due to 1) the snowpack temperature being affected by the ambient air temperature and 2) the kind of probe used to determine the value. In the former case the temperatures were taken after the pit was dug and it is presumed that a delay of 10-15 minutes was enough to vary the temperature to some extent, specially if the snow contained a large number of pore spaces. In order to compensate for this a probe long enough to penetrate deeply into the snow could be used.

APPENDIX II

FORTRAN PROGRAM FOR THEORETICAL SIMULATION

published information. The field data was collected according to the technique outlined in chapter four with one snow course being different than the other five. Six sampling points, one from each snow course were randomly selected to present the results.

With boundary conditions, air and ground temperature and with initial values of snow temperature and density as input variables, the model was able to simulate with a good degree of accuracy the thermal patterns in a snowpack. These simulated temperatures, on an hourly basis, were then used to compute densities at various depths in the snowpack. Once the densities were computed, the temperature patterns were reconstructed based on the thermodynamic properties of the snowpack. These properties include thermal conductivity, diffusivity and specific heat. Since the relationship between density and temperature is not fully understood, these thermal properties seem to be the direct link between the two. Experimentation by some authors (Konrat'eva, 1954; Kuzmin, 1972; USCE, 1956; Colbeck, 1979) have shown that the behaviour of snow in relation to temperature and density is very complex. Most of these studies have recommended simple equations which estimate these properties as a function of density and temperature.

It appears from the analysis that the model was able to estimate the heat flux and sublimation process in good agreement with those done by other authors. Since the heat flux is directly related to the energy available at the site the model reacted

differently at snow course No. 1, which is located in a wooded area. During early and late winter, when air and ground temperatures were higher than the snowpack a positive energy flux melted the snowpack from both directions. On the other hand, net heat flux during most of the winter was negative with patterns fluctuating in accordance with the conditions at the two boundaries. Over this period the movement of snow was primarily upwards (negative). Similarly, when conditions change during the spring melt season the direction of movement is also reversed (positive). There are occasions when an inversion of temperature in the snowpack diminishes the sublimation of snow. Computed values of the 'reserve of cold' indicate that it peaks in the middle of February when the mean snowpack temperatures were lowest. This reserve is depleted as soon as thaw sets in.

Although it was not possible to identify precisely the dates over which different kinds of metamorphism take place, it was apparent that four kinds of metamorphism, suggested earlier in chapter two, do take place. In the early stages of formation of the snowpack the compaction was produced by deposition metamorphism. This is followed by temperature-gradient metamorphism during most of the winter where evaporation and condensation produces the growth of snow crystals. Often depth hoar or an increased number of crusty layers was the result. Equi-temperature metamorphism dominates during the melt season when heat transference by conduction is dampened. During this period or at other times, when the snow was melted at the top, the

melt-water seeped down under gravity and often refroze releasing latent heat. The refreezing of this water creates 'sintering' of crystals in the snowpack. It is evident that during melting periods the lower layers are heated by phase transition.

Figures 5.7 through 5.13 are graphical representations of the relationship between heat flux, sublimation and density difference. The 1st variable is the difference in density between the top and the bottom of the snowpack. Positive values, indicating increased densities at the top follow the trend when sublimation is directed upwards. It was not possible to match each individual peak between density difference and sublimation but the general pattern indicates that the heat flux was the major control of the movement of snow. As pointed out in chapter five, during the melt season the conditions were more complex and different and increased densities at the top and bottom were a function of the melting at those regions.

The second part of the analysis dealt with the comparison of hydrographs and the correlation between temperature and snowmelt. The synthesized and measured hydrographs for the six sampling points showed excellent agreement in terms of the cumulative runoff volumes. However, the high  $r$  values should not be interpreted as an indication of the predicting capability of the model. Simplifications in the modelling procedure and the lack of intensive instrumentation in the test basin restricted comparisons between observed and synthesized volumes on a spatial basis.



Those comparisons that were performed however, especially for the spring melt season, suggested that the model predictions were reasonably accurate, given the assumptions employed in the simulation approach. Detailed examination of the model for the period March 16 - April 15 revealed some conclusive results of the relationship between temperature, snowmelt and runoff. As expected, since the model was based on energy as the major component of temperature, the correlation is strongest with maximum air temperature, followed by mean and minimum temperatures. It was evident that threshold values of temperature are needed to calibrate the model for more accuracy. The relationship between snowmelt and runoff for that period suggests that other factors must be included in the model to improve the predicting capability of the model. The overall trend remains the same in most cases with higher  $r$  values for the five open snow courses as compared to snow course No. 1.

In general the simulation results show that the research objectives have been achieved. However, since the model's accuracy cannot be fully assessed with this limited test data, information from an expanded instrumentation network should be tested. Strengths and weaknesses of the model overlooked in this study may be highlighted by using more detailed data. The model could be further refined by incorporating the modelling procedure changes recommended in this study.

The programming and the computational procedure in this

research has been rather extensive. This was done to keep the modelling approach flexible, which resulted in three major programs. The three stages are identified as Phase I, II, and III through which temperature, density and heat flux were computed. It is hoped that in future research the efficiency of the programming will be increased. Addition of new parameters and subroutines, including plotting procedures may also be considered as an investigative approach to simplifying the model. In the present version of the programs the user can vary input variables, for example, sampling point, time, layer thickness and tailor the output in terms of hourly, daily, weekly or monthly results.

Finally, the results of this study suggest that the art of snowpack simulation is limited by the paucity of existing knowledge and by the complexity of the mechanics involved in snowpacks. This model has been successful primarily because of it being grounded in physics. More data is needed to investigate the parameters used to test the simple relationships which govern the thermodynamic properties of snow. Research is already being carried out but practical applications of such are yet to be fully overcome in modelling procedures. In the present model, the complexity of the parameters were simplified to make it useful for general purpose simulation.

APPENDIX I

ORTHOGONAL RELATION FOR  $\sin n\pi x/h$

## Appendix I

- I. Multiplying both sides of equation 3.31 by  $\sin \frac{m\pi x}{h}$  and integrating results from 0 to h,

$$\begin{aligned} & \int_0^h \left( \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{h} \right) \sin \frac{m\pi x}{h} dx \\ &= \int_0^h \left\{ (C - T_0) + x \left[ \frac{T_0 - T_1}{h} + \frac{bh}{6\alpha^2} \right] - \frac{b}{6\alpha^2 h} x^3 \right\} \sin \frac{m\pi x}{h} dx \\ &= \sum_{n=1}^{\infty} B_n \int_0^h \sin \frac{n\pi x}{h} \sin \frac{m\pi x}{h} dx \\ &= \sum_{n=1}^{\infty} B_n \left( \delta_{nm} \frac{h}{2} \right) \\ &= B_n \frac{h}{2} \end{aligned}$$

$\delta_{nm}$  = Kronecker delta  
= 1 iff  $n = m$   
0 otherwise

- II. To find  $(C - T_0) \left( \frac{h}{n\pi} \right) [(1 - (-1)^n)]$

$$\begin{aligned} & \int_0^h (C - T_0) \sin \frac{n\pi x}{h} dx \\ &= (C - T_0) \int_0^h \sin \frac{n\pi x}{h} dx \\ &= (C - T_0) \frac{h}{n\pi} \int_0^{\frac{n\pi}{h} h} \sin \frac{n\pi x}{h} d\left(\frac{n\pi x}{h}\right) \\ &= - (C - T_0) \frac{h}{n\pi} \cos \frac{n\pi x}{h} \Big|_0^h \\ &= - (C - T_0) \frac{h}{n\pi} (\cos n\pi - 1) \\ &= - (C - T_0) \frac{h}{n\pi} [(-1)^n - 1] = (C - T_0) \frac{h}{n\pi} [1 - (-1)^n] \end{aligned}$$

III. To find  $\left[\left(\frac{T_0 - T_1}{h} + \frac{bh}{6\alpha^2}\right) \cdot (-1)^{n+1} \frac{h^2}{n\pi}\right]$

$$\begin{aligned} & \int_0^h x \left[ \frac{T_0 - T_1}{h} + \frac{bh}{6\alpha^2} \right] \sin \frac{n\pi x}{h} dx \\ &= \left[ \frac{T_0 - T_1}{h} + \frac{bh}{6\alpha^2} \right] \int_0^h x \sin \frac{n\pi x}{h} dx \\ &= \left[ \frac{T_0 - T_1}{h} + \frac{bh}{6\alpha^2} \right] \left\{ \frac{h^2}{n^2 \pi^2} \sin \frac{n\pi x}{h} - \frac{h}{n\pi} x \cos \frac{n\pi x}{h} \right\} \Big|_0^h \\ &= \left[ \frac{T_0 - T_1}{h} + \frac{bh}{6\alpha^2} \right] \left\{ -\frac{h}{n\pi} h \cos n\pi \right\} \\ &= -\left[ \frac{T_0 - T_1}{h} + \frac{bh}{6\alpha^2} \right] \left[ \frac{h^2}{n\pi} (-1)^n \right] \\ &= \left[ \frac{T_0 - T_1}{h} + \frac{bh}{6\alpha^2} \right] \left[ \frac{h^2}{n\pi} (-1) (-1)^n \right] \\ &= \left[ \frac{T_0 - T_1}{h} + \frac{bh}{6\alpha^2} \right] \left[ \frac{h^2}{n\pi} (-1)^{n+1} \right] \end{aligned}$$

IV. To find  $\left[\left(\frac{-b}{6\alpha^2 h}\right) \left(\frac{(-1)^n h^4}{n\pi}\right) \left(\frac{6}{n^2 \pi^2} - 1\right)\right]$

$$\begin{aligned} & \int_0^h \left(\frac{-b}{6\alpha^2 h}\right) x^3 \sin \frac{n\pi x}{h} dx \\ &= \left(\frac{-b}{6\alpha^2 h}\right) \int_0^h x^3 \sin \frac{n\pi x}{h} dx \\ &= \left(\frac{-b}{6\alpha^2 h}\right) \left[ \frac{-(h^2 / (n^2 \pi^2)) - 6x}{h^3 / n^3 \pi^3} \cos \frac{n\pi x}{h} \right] \Big|_0^h \end{aligned}$$

IV. (Continued)

$$\begin{aligned}
&= \left(\frac{-b}{6\alpha^2 h}\right) \frac{3(n^2 \pi^2 / h^2) x^2 - 6}{n^4 \pi^4 / h^4} \sin \frac{n\pi x}{h} - \frac{(n^2 \pi^2 / h^2) x^3 - 6x}{(n^3 \pi^3 / h^3)} \cos \frac{n\pi x}{h} \Big|_0^h \\
&= \left(\frac{-b}{6\alpha^2 h}\right) \left\{ \frac{-(n^2 \pi^2 / h^2) h^3 - 6h}{n^3 \pi^3 / h^3} \cos n\pi \right\} \\
&= -\left(\frac{-b}{6\alpha^2 h}\right) \left\{ \frac{(n^2 \pi^2 - 6) h^4}{n^3 \pi^3} (-1)^n \right\} \\
&= \left(\frac{-b}{6\alpha^2 h}\right) \left\{ \frac{(6 - n^2 \pi^2) h^4}{n^3 \pi^3} (-1)^n \right\} \\
&= \left[\left(\frac{-b}{6\alpha^2 h}\right) \left(\frac{6}{n^2 \pi^2} - 1\right) \left(\frac{h^4 (-1)^n}{n\pi}\right)\right]
\end{aligned}$$

APPENDIX II

FORTRAN PROGRAM FOR THEORETICAL SIMULATION

```

CALL ERRSET(0)
PRINT 2
2 FORMAT (' DEL,C,T0,T1,B,H,K,T?')
READ 1,DEL,C,T0,T1,B,H,FK,TT
1 FORMAT (8F)
CALL PLOTS(100,20.0,10.75,2)
DA=(T1-T0)/10.0
CALL AXIS(0.5,0.5,4HTEMP,-4,10.0,0.0,T0,DA)
DA=H/10.0
CALL AXIS(0.5,0.5,5HDEPTH,5,10.0,90.0,0.0,DA)
DO 4 K=1,11
T=TT*(K-1)/10.0
IP=3
DO 4 J=1,100
X=(J-0.5)/100.0*H
U3=(B*X**3-B*H**2*X)/(6.0*FK*H)+T0+(X/H)*(T1-T0+B*T)
U2=0
DO 5 N=1,1000
F10=(C-T0)*(H/3.14159/N)*(1-(-1)**N)
F11=(B*H/(6.0*FK)+(T0-T1)/H)*((-1)**(N+1)*H**2)/(3.14159*N)
F12=-B/(6.0*FK*H)*(-1)**N*H**4/(3.14159*N)*(6.0/(3.14159*N)**2-1)
BN=2.0/H*(F10+F11+F12)
U1=BN*SIN(N*3.14159*X/H)*EXP(-N**2*3.14159**2*FK*T/H**2)
U2=U2+U1
IF (U1.LT.DEL.AND.N.GT.10) GO TO 6
5 CONTINUE
PRINT 7
7 FORMAT (' DEL NOT REACHED IN 1000')
STOP
6 U=U3+U2
XP=0.5+10.0*(U-T0)/(T1-T0)
YP=X/H*10.0+0.5
CALL PLOT(XP,YP,IP)
IP=2
WRITE(3,300) U,U1,U2,U3,F10,F11,F12,XP,YP
300 FORMAT(' ',9F6.2)
4 CONTINUE
CALL ENDPLT
STOP
END

```



APPENDIX III

FORTRAN PROGRAM FOR TEMPERATURE MODEL

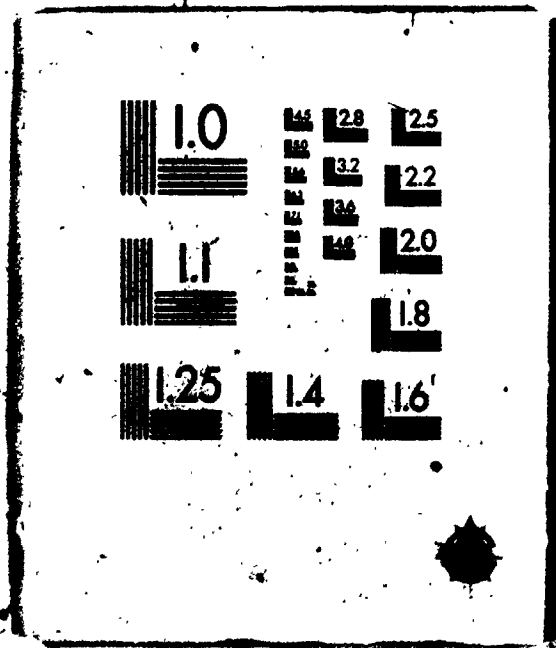
```

C   THIS PROGRAM IS THE AMDAHL VERSION OF 'SNOW'
    DIMENSION DA(13),HTT(13),HT(12),DATA(12),HOLD(12),
    1TM(16,4),COEF(20),TR(20),T(4500),GGT(181),DATEH(181)
    REAL KA(13),KAY(12),KP(13),K,I0,I1,I2,N,N1,MAXEX,IDATE
    INTEGER RHOHTT,BI,BK
C   NOTE CHANGE INTEGER NAME(11) TO INTERGER * 1 NAME(11)
C   FOR MICRO
    REAL NAME(11)
    INTEGER AFF/'Y'/
    INTEGER TERM/'T'/
    INTEGER REPLY
    IG=1
    FLAG=0.0
    IW=0
75  WRITE(3,60)
60  FORMAT(' ',' WELCOME TO PROGRAM: SNOW !! ', 'WHAT ',
1)  'INPUT FILE PLEASE ?'/)
    READ(1,14)NAME
14  FORMAT(11A1)
    CALL OPEN(9,NAME,1)
C   CONVERSATIONALLY DEFINE OUTPUT UNIT 3 FOR TERM 8 FOR D
C   ISK
C   *****NOTE NEXT LINE MUST BE REMOVED FOR MICRO
    GO TO 9153
C   *****
12345 WRITE(3,46)
46  FORMAT(' ','OUTPUT TO TERMINAL OR DISK ? (T OR D)'/
1)
    READ(1,47) REPLY
47  FORMAT(A2)
    IF(REPLY.EQ.TERM)GO TO 50
9153 IOUT1=8
    IOUT2=8
    CALL OPEN (8,'SNOW   DAT';2)
    CALL OPEN(10,'SNOW   DAT',1)
    GO TO 51
50  IOUT1=3
    IOUT2=3
51  WRITE(3,61)
61  FORMAT(' ','ENTER L VALUE AND N OF HOURS AS I1,I4 PLEA
1)SE'/)
    READ(1,4) L,NHOURS
4   FORMAT(I1,I4)
    WRITE(3,62)
62  FORMAT(' ','ENTER TQ VALUE, INCLUDE THE DECIMAL PLEASE
1)'/)
    READ(1,15) TQ
15  FORMAT(F10.3)

```

3 3

OF / DE



```

WRITE(3,63)
63  FORMAT(' ', 'ENTER STARTING AND ENDING STATIONS AS ',
1' I3, 1X, I3  PLEASE'//)
READ(1,64) SSTART, SEND
64  FORMAT(F3.2, 1X, F3.2)
WRITE(3,65)
65  FORMAT(' ', 'ENTER STARTING AND ENDING DATES AS. ',
1' I6, 1X, I6  PLEASE'//)
READ(1,66) SDATE, EDATE
66  FORMAT(F6.0, 1X, F6.0)
WRITE(3,900)
900  FORMAT(' ', 'HOUR FOR START OF PRINTING ? (INCLUDE A ',
1' DECIMAL PLEASE)'//)
READ(1,901)HST
901  FORMAT(F6.0)
IHST=IFIX(HST)
WRITE(3,902)
902  FORMAT(' ', 'HOUR INCREMENT BETWEEN PRINTS ? (INCLUDE ',
1' A DECIMAL PLEASE)'//)
READ(1,903) CREM
903  FORMAT(F7.0)
INCREM=IFIX(CREM)
WRITE(3,9991)
9991  FORMAT(' ', 'OPTIONAL CHANGE OF TF WITH EACH STATION-DA
ITE ??'//)
READ(1,69) REPLY
IF(REPLY.NE.AFF)GO TO 9993
FLAG=1.
9993  WRITE(3,67) NAME, REPLY, L, NHOURS, SSTART, SEND, SDATE, EDAT
1E,
1IHST, INCREM, FLAG
67  FORMAT(' ', 'RUN PARAMETERS ARE: ', /,
1' ', 'FILE NAME: ', I1A1, /,
2' ', 'OUTPUT TO: ', A2, /,
3' ', 'L VALUE : ', I1, /,
4' ', 'N HOURS : ', I4, /,
5' ', 'ST. START: ', F4.2, /,
6' ', 'ST. END : ', F4.2, /,
7' ', 'DA. START: ', F7.0, /,
8' ', 'DA. END : ', F7.0, /,
9' ', 'P. START : ', I4, /,
9' ', 'P. INCR. : ', I4, /,
9' ', 'TF OPT. : ', F2.0, ' 0=NO 1=YES'//)
WRITE(3,68)
68  FORMAT(' ', 'RE-DEFINE PARAMETERS ? (Y OR N)'//)
READ(1,69) REPLY
69  FORMAT(A1)
IF(REPLY.EQ.AFF)GO TO 75

```

```

C   UNIT 7 IS TEMPERATURE AND GT INPUT FILE
    CALL OPEN(7,'TEMPER DAT',1)
    PHI=3.141592654
C   UNIT 1 IS INPUT DATA FILE (I.E. SHAF01 IN APL)
C   READ A DATA CARD
100  READ(9,1,END=1000) STN,DATE,M
    READ(9,16)(DA(J),J=1,M)
    READ(9,16)(KP(J),J=1,M),TEMP
    IG=1
16   FORMAT(9F8.4)
    IF(STN.LT.SSTART.OR.STN.GT.SEND)GO TO 100
    IF(DATE.LT.SDATE.OR.DATE.GT.EDATE)GO TO 100
    DO 110 J=1,M
    HTT(J)=TEMP
110  CONTINUE
1    FORMAT(F4.2,1X,F6.0,1X,I2)
C    GET GT AND T FOR THIS CASE
    REWIND 7
    IS=1
    ISTOP=24
111  READ(7,10,END=115) IDATE,(T(JJ),JJ=1,24),GGT(IG)
    DATEH(IG)=IDATE
10   FORMAT(F6.0,11F5.1/14F5.1)
    IF(IDATE.NE.DATE)GO TO 111
    IF(ISTOP.GE.NHOURS)GO TO 112
    GO TO 113
115  WRITE(IOUT1,11) DATE
    IW=IW+1
11   FORMAT(' ','NO TEMPERATURE DATA FOR: ',F7.0)
    GO TO 100
113  IS=ISTOP+1
    IG=IG+1
    ISTOP=ISTOP + 24
    READ(7,12,END=115) IDATE,(T(JJ),JJ=IS,ISTOP),GGT(IG)
    DATEH(IG)=IDATE
12   FORMAT(F6.0,11F5.1/14F5.1)
    IF(ISTOP.GE.NHOURS)GO TO 112
    GO TO 113
C    CALCULATE AND PRINT TOTAL SNOW DEPTH
112  TEMP=TEMP*FLOAT(M)
    WRITE(IOUT1,3) STN,DATE,TEMP
    IW=IW+1
    IF(IW.GE.1270.AND.IOUT1.NE.3)IOUT1=8
    IF(IW.GE.1270.AND.IOUT2.NE.3)IOUT2=8
3    FORMAT('1','STATION: ',F4.2,5X,'DATE: ',F7.0,5X,
1'TOTAL SNOW DEPTH: ',F8.3 )
    TF=HTT(1)
    IF(FLAG.NE.1.0)GO TO 9982

```

```

WRITE(3,9983) STN,DATE,TF
9983  FORMAT(' ', 'FOR: ',F4.2,2X,F7.0,2X, 'TF IS : ',F10.3,
1CHANGE THIS
1 VALUE ?'/)
READ(1,9984) REPLY
IF(REPLY.NE.AFF)GO TO 9982
9984  FORMAT(A2)
WRITE(3,9985)
9985  FORMAT(' ', 'ENTER NEW TF VALUE INCLUDE A DECIMAL'/)
READ(1,9986) TF
WRITE(3,9987) TF
9987  FORMAT(' ', 'TF CHANGED TO: ',F10.3/)
9986  FORMAT(F10.3)
C     KA GENERATOR
9982  DO 117 KJ=1,M
      IF(KP(KJ).LT.0.35)KA(KJ)=KP(KJ)*3.6 * 13.3
      IF(KP(KJ).GE.0.35)KA(KJ)=KP(KJ)*3.6 * 16.5
117   CONTINUE
      BI=1
      NHT=1
      TS=1.0/FLOAT(L)
C     LINE B1 OF FUNCTION LL
      RHOHTT=M
120   IF(BI.GT.RHOHTT)GO TO 130
      BK=(HTT(BI)/TF) + 0.99
      ISTOP=NHT + BK - 1
      DO 125 KJ=NHT,ISTOP
      HT(KJ)=HTT(BI)/FLOAT(BK)
      DATA(KJ)=DA(BI)
      KAY(KJ)=KA(BI)
125   CONTINUE
      NHT=ISTOP + 1
      BI=BI + 1
      GO TO 120
C     LINE 130 IS FUNCTION LL LINE 7
130   M=BK * M
      WRITE(IOUT1,5) M, (DATA(KJ),KJ=1,M)
      IW=IW+1
5     FORMAT(' ', 'THE PROBLEM IS TAKEN AS ONE OF: ',I3,
1' LAYERS'/' ', 'INITIAL T: ',10F7.3)
      WRITE(IOUT1,6) (KAY(KJ),KJ=1,M)
      IW=IW+1
6     FORMAT(' ', 'K VALUES: ',10F8.4)
      WRITE(IOUT1,7) (HT(KJ),KJ=1,M)
      IW=IW+1
7     FORMAT(' ', 'THICKNESS: ',10F7.3/' ',50(1H-))
      IF(M.EQ.1)WRITE(IOUT2,7001)
      IF(M.EQ.2)WRITE(IOUT2,7002)

```

```

IF(M.EQ.3)WRITE(IOUT2,7003)
IF(M.EQ.4)WRITE(IOUT2,7004)
IF(M.EQ.5)WRITE(IOUT2,7005)
IF(M.EQ.6)WRITE(IOUT2,7006)
IF(M.EQ.7)WRITE(IOUT2,7007)
IF(M.EQ.8)WRITE(IOUT2,7008)
IF(M.EQ.9)WRITE(IOUT2,7009)
IF(M.EQ.10)WRITE(IOUT2,7010)
IF(M.EQ.11)WRITE(IOUT2,7011)
IF(M.EQ.12)WRITE(IOUT2,7012)
IF(M.EQ.13)WRITE(IOUT2,7013)
7001 FORMAT(' /' ',2X,'HOUR',3X,'AIR T.',4X,4X,'L 1',3X,4X
1,'GT',4X,'
1DATE')
7002 FORMAT(' /' ',2X,'HOUR',3X,'AIR T.',4X,4X,'L 1',7X,'
1L 2',7X,'GT
1',4X,'DATE')
7003 FORMAT(' /' ',2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'
1,7X,'L 3',
16X,'GT',4X,'DATE')
7004 FORMAT(' /' ',2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',6X,'GT',4X,'DATE')
7005 FORMAT(' /' ',2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',6X,'GT',4X,'DATE')
7006 FORMAT(' /' ',2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',6X,'GT',4X,'DATE')
7007 FORMAT(' /' ',2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',6X,'GT',4X,'DATE
1')
7008 FORMAT(' /' ',2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'GT'
1,5X,'DATE')
7009 FORMAT(' /' ',2X,'HOUR',2X,'AIR T.',7X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'L 9',
16X,'GT',3X
1,'DATE')
7010 FORMAT(' /' ',2X,'HOUR',2X,'AIR T.',7X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'L 9',
17X,'L 10',
16X,'GT',3X,'DATE')
7011 FORMAT(' /' ',2X,'HOUR',2X,'AIR T.',7X,'L 1',7X,'L 2'

```

```

1,7X,'L-3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'L 9',
17X,'L10',
17X,'L11',6X,'GT',3X,'DATE')
7012 FORMAT(' / ',2X,'HOUR',2X,'AIR T.',7X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'L 9',
17X,'L10',
17X,'L11',7X,'L12',6X,'GT',3X,'DATE')
7013 FORMAT(' / ',2X,'HOUR',2X,'AIR T.',7X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'L 9',
17X,'L10',
17X,'L11',7X,'L12',7X,'L13',6X,'GT',3X,'DATE')
IS=M+2
ISTOP=L+1
DO 135 KJ=1,IS
DO 135 JJ=1,ISTOP
TM(KJ, JJ)=0
135 CONTINUE
KI=1
IG=(KI+23)/24
GT=GGT(IG)
DATEO=DATEH(IG)
INH=IHST
IF(KI.NE.INH)GO TO 140
INH=INH+INCREM
WRITE(IOUT2,8)T(1),(DATA(KJ),KJ=1,M),GT,DATEO
IW=IW+1
8 FORMAT(' ',2X,' 1',2X,F6.2,3X,14(F10.2)/' ',17X,8F10
1.2)
C 140 IS LINE 12 OF LL
140 IF(KI.EQ.NHOURS)GO TO 500
IG=(KI+23)/24
GT=GGT(IG)
DATEO=DATEH(IG)
IS=M+2
ISTOP=L+1
IR=M+2
DO 145 KJ=1,ISTOP
JJ=KJ-1
TM(1,KJ)=T(KI)+((T(KI+1)-T(KI))*FLOAT(JJ/L))
TM(IR,KJ)=GT
145 CONTINUE
C AT PART 2 OF LINE 14 OF LL
TM(1,1)=T(KI)
ISTOP=IS-1
DO 150 KJ=2,ISTOP

```



```

JJ=KJ-1
TM(KJ,1)=DATA(JJ)
150 CONTINUE
TM(IS,1)=GT
J=0
C 155 IS LL1 IN LL
C 400 IS LL4 IN FUNCTION
155 J=J+1
IF(J.EQ.(L+1))GO TO 400
I=2
C 160 IS LL3 IN FUNCTION
C I.E. CALL TO SEAN
C FUNCTION SEAN FOLLOWS
160 C=TM(I,J)
IS=I+1
TO=TM(IS,J)
IS=I-1
ISTOP=J+1
B=(TM(IS,ISTOP) - TM(IS,J))/TS
T1=TM(IS,J)
H=HT(IS)
K=KAY(IS)
C NTR AND NCO ARE NEXT POSITIONS IN VECTORS TR AND COEF
NTR=1
NCO=1
C AT LINE 4 OF SEAN
CO=C-TO
C1=((B*H)/(6*K))+((TO-T1)/H)
N1=1.0-2.0
C2=N1 * (B/(6*K*H))
N=1
C LINE 165 IS LINE START OF SEAN
C LINE 180 IS LINE END OF SEAN
165 IF(N.GT.5)GO TO 180
C START OF FUNCTION FOCEF
IF(N.EQ.2.OR.N.EQ.4)CAT=1
IF(N.EQ.1.OR.N.EQ.3.OR.N.EQ.5)CAT=1.0-2.0
IF(CAT.EQ.1.)DOG=1.0-2.0
IF(CAT.EQ.-1.)DOG=1.0
IO=(CO*(H/(PHI*N)))*(1.-CAT)
I1=(DOG)*(C1*((H**2)/(PHI*N)))
I2=((CAT)*C2*((H**4)/(PHI*N)))*((6/((PHI*N)**2))-1)
COEF(NCO)=(2./H)*(IO+I1+I2)
NCO=NCO+1
C RETURN FROM FOCEF TO LINE 8 OF FUNCTION SEAN
THOLD=((N1*((PHI*(N/H))**2))*K*TS)
MAXEX=EXP(THOLD)
IS=NCO-1

```

```

THOLD=(PHI*N)/2.0
TR(NTR)=(COEF(IS)*MAXEX)*SIN(THOLD)
NTR=NTR+1
IS=N
THOLD=TR(IS)
IF(ABS(THOLD).LT.TQ)GO TO 180
N=N+1.0
GO TO 165
C 180 IS LINE END OF SEAN
180 IS=J+1
ISTOP=NTR-1
C CALCULATE +/TR
SUM=0
DO 185 JJ=1,ISTOP
SUM=SUM + TR(JJ)
185 CONTINUE
TM(I,IS)=((N1*B*(H**2))/(16.0*K))+(T0/2.0)+(T1/2.0)+
1(B*(TS/2.0))+SUM
C END OF FUNCTION SEAN - RETURNING TO LL3 PART 2 IN FUNC
C TION LL
I=I+1
IF(I.EQ.(M+2))GO TO 155
GO TO 160
400 KI=KI+1
IG=(KI+23)/24
GT=GGT(IG)
DATEO=DATEH(IG)
C AT LL4 PART 2 IN FUNCTION LL
C SELECT LAST M+1 ROWS AND LAST COLUMN FROM TM
ICOL=L+1
IROWS1=(M+2)-(M+1)+1
DO 410 JJ=1,M
DATA(JJ)=TM(IROWS1,ICOL)
IROWS1=IROWS1+1
410 CONTINUE
IF(KI.NE.INH)GO TO 935
INH=INH+INCREM
WRITE(IOUT2,37) KI,T(KI),(DATA(KJ),KJ=1,M),GT,DATEO
-IW=IW+1
IF(IW.GE.1270.AND.IOUT1.NE.3)IOUT1=8
IF(IW.GE.1270.AND.IOUT2.NE.3)IOUT2=8
37 FORMAT(' ',2X,I4,2X,F6.2,3X,14(F10.2))/' ',17X,8F10.2)
935 GO TO 140
500 THOLD=TS*60.0
WRITE(IOUT1,9)TS,THOLD
IW=IW+1
9 FORMAT(' ','TS = ',F5.2,' (' ,F6.2,' MIN.))'
GO TO 100

```

```
1000 WRITE(IOUT1,13) NAME
13  FORMAT(' ', 'PROGRAM TERMINATING AT END OF DATA SET: '
1,
111A1)
STOP
END
SUBROUTINE OPEN(IDD,NAME,IDDD)
INTEGER * 2 NAME(11)
C DUMMY SUBROUTINE JCL HANDLED BY CMS EXEC FILE
RETURN
END
```

APPENDIX IV

FORTRAN PROGRAM FOR DENSITY MODEL

```

C
C
C      VARIABLE NAMES IN THIS PROGRAM CORRESPOND TO T
C HOSE      IN THE DESCRIPTION OF THE ALGORITHM
C
C      THE FOLLOWING IS A DESCRIPTION OF ADDITIONAL V
C ARIABLES
C   AT:      AIR TEMPERATURE FOR THE CURRENT HOUR
C   DATEM:   TEMPORAY ARRAY FOR STORAGE OF DATE BEING PR
C OCESSED
C   NLAYER:  NUMBER OF SNOW LAYERS
C   LAYER:   A LAYER WITHIN THE SNOW PROFILE
C   MIDPT:   ARRAY OF MIDPOINTS OF SNOW LAYERS
C   SUMLL:   TEMPORARY SCALAR FOR STORAGE
C   NOFRE:   NUMBER OF READS FROM RAW INPUT FILE
C   LHDEN:   MATRIX OF DENSITY VALUES FOR PRECEEDING HOUR.
C   MATRIX:  COND. SPHEAT MATRIX
C
C
C      COMMON X(20,7),XMEAN(7),XRANGE(7),XVAR(7);XSDEV(7),
1XSKEW(7),XB1(7),XKURT(7),XB2(7),XSE(7)
C      REAL K(20),THICK(20),MIDPT(20),LTEMP(20),THETA1(20),DE
1N(20),
C      1THETA2(20),LAMDA2(20),LAMDA1(20),NEWDEN(20),DENCH(20),
2DATEM(2),THETA0(20),NAVDEN(20),OAVDEN(20),MATRIX(12,3)
C      1,LHDEN(20),
3SPHEAT(20),COND(20),NEWH(20),OLDH(20),OLDFLX(20),NEWFL
1X(20)
C      REAL TTHICK(20),FLUXDG(20),FLUXCM(20),TOTFLX(20)
C
C
C      INTEGER DATE, HOUR(2)
C      INTEGER TOB/'DOWN'/
C
C      UNIT 1 IS THE RAW DATA INPUT UNIT
C      UNIT 2 IS THE COND. SPHEAT MATRIX INPUT UNIT
C
C*****
C      TIME=1.0
C      DO 1098 II=1,20
1098. TOTFLX(II)=0.0
C*****
C      READ N OF HOURS BETWEEN PRINTS
C      READ(1,11) NHOURLS
11  FORMAT(I2)
C      READ FLAG FOR DOWN OR UP FLUX CALCULATION

```

```

      READ(1,10) IFLAG
10     FORMAT(A4)
      IF(IFLAG.EQ.TOB) WRITE(6,12)
12     FORMAT(' ', 'THETAS WILL BE CALCULATED FROM TOP DOWN')
C      READ STATION IDENTIFIER AND ASSOCIATED DATA
100    READ(1,1,END=1000) STN,DATE,SDEPTH,NLAYER
1     FORMAT(T11,F4.2,T26,I6,T56,F12.3/T35,I2)
      NPHOUR=0+NHOURS
      NOFRE=0

C      READ K VALUES
      READ(1,2) (K(LAYER),LAYER=1,NLAYER)
2     FORMAT(/,11X,16F8.4)
C      READ THICKNESSES (THICK)
      READ(1,3) (THICK(LAYER),LAYER=1,NLAYER)
3     FORMAT(12X,16F7.3)
C      CALCULATE DENSITY (DEN) ARRAY. 'LAYER' DENOTES
C      LAYER NUMBER (VALUE OF 1 IS TOP LAYER)
      NSTOP=NLAYER+1
      DO 110 LAYER=2,NSTOP
      IF(K(LAYER-1).LT.16.578)DEN(LAYER)=K(LAYER-1)/47.88
      IF(K(LAYER-1).GE.16.578)DEN(LAYER)=K(LAYER-1)/59.4
      LHDEN(LAYER)=DEN(LAYER)
110    CONTINUE
      DEN(1)=DEN(2)
      LHDEN(1)=DEN(1)

C      CALCULATE MIDPOINT (MIDPT) OF EACH LAYER IN CM
C      S
C      ABOVE THE GROUND
C
C      MIDPOINT OF BOTTOM LAYER IS ONE-HALF OF THICKN
C      ESS
C      OF BOTTOM LAYER
      MIDPT(NLAYER)=THICK(NLAYER)/2.
      TTHICK(NLAYER+1)=MIDPT(NLAYER)
C      CALCULATE MIDPOINTS OF REMAINING LAYERS. MIDP
C      OINT
C      OF LAYER 'LAYER' IS (SUM OF THICKNESSES OF ALL
C      LOWER LAYERS) + (ONE-HALF THICKNESS OF THIS L
C      AYER)
      LAYER=NLAYER
120    LAYER=LAYER-1
      IF(LAYER.EQ.0)GO TO 134
      IL=NLAYER
      SUMLL=0.0
C      SUM THICKNESSES OF ALL LAYERS BELOW THIS ONE
130    SUMLL=SUMLL + THICK(IL)
      IL=IL-1
      IF(IL.GT.LAYER)GO TO 130

```

```

C          CALCULATE MIDPOINT
MIDPT(LAYER)=THICK(LAYER)/2 + SUMLL
TTHICK(LAYER+1)=MIDPT(LAYER)-MIDPT(LAYER+1)
GO TO 120
134  TTHICK(1)=THICK(1)/2.0
GO TO 135
C          MIDPOINTS ARE NOW CALCULATED
C
C          GET COND. AND SPHEAT MATRIX
135  DO 136 IROW=1,12
136  READ(2,7) (MATRIX(IROW,J),J=1,3)
7    FORMAT(F5.3,1X,F3.1,1X,F6.5)
C
C          READ AND PROCESS DATA FOR EACH PRINT HOUR IN I
C          NPUT FILE
C
C          READ HOUR,LAYER TEMPS (LTEMPS.), GT FOR THIS HO
C          UR, AND DATE
IR=1
C          AN HOUR VALUE OF 9999 SIGNALS END OF DATA FOR
C          THIS STN
140  READ(1,4) HOUR(IR),AT,(LTEMP(LAYER),LAYER=1,NLAYER),GT
1,DATEM(IR)
4    FORMAT(3X,I4,F9.2,2X,14F10.2/18X,8F10.2)
NOFRE=NOFRE+1
IF(HOUR(IR).EQ.9999)GO TO 100
IF(IR.EQ.2)GO TO 170
C          CALCULATE THETA1 FOR LOWEST LAYER
145  IF(IFLAG.EQ.TOB)GO TO 155
THETA1(NLAYER+1)=GT-LTEMP(NLAYER)
C          CALCULATE THETA1 FOR REMAINING LAYERS
LAYER=NLAYER
150  LAYER=LAYER-1
IF(LAYER.EQ.0)GO TO 159
THETA1(LAYER+1)=LTEMP(LAYER+1)-LTEMP(LAYER)
GO TO 150
C          CALCULATE THETA1 FOR AIR LAYER
159  THETA1(1)=LTEMP(1)-AT
GO TO 160
C
C          THIS CODE CALCULATES FLUX FROM THE AIR DOWN
C
C          CALCULATE THETA1 FOR AIR LAYER
155  THETA1(1)=AT-LTEMP(1)
NSTOP=NLAYER-1
DO 156 LAYER=1,NSTOP
156  THETA1(LAYER+1)=LTEMP(LAYER)-LTEMP(LAYER+1)
THETA1(NLAYER+1)=LTEMP(NLAYER)-GT

```

```

C
C      READ NEXT HOURS DATA (IF ANY). 'IR' LETS US RE
C      -USE THE
C      READ STATEMENT (LINE 140)
160  IR=2
      GO TO 140
C      CALCULATE THETA2 FOR LOWEST LAYER
170  IF(IFLAG.EQ.TOB)GO TO 185
      THETA2(NLAYER+1)=GT-LTEMP(NLAYER)
C      CALCULATE THETA2 FOR REMAINING LAYERS
      LAYER=NLAYER
180  LAYER=LAYER-1
      IF(LAYER.EQ.0)GO TO 189
      THETA2(LAYER+1)=LTEMP(LAYER+1) - LTEMP(LAYER)
      GO TO 180
C      CALCAULATE THETA2 FOR AIR LAYER
189  THETA2(1)=LTEMP(1)-AT
      GO TO 190
C      THIS CODE CALCULATES THETA2 FOR DOWNWARD FLUX
185  THETA2(1)=AT-LTEMP(1)
      NSTOP=NLAYER-1
      DO 186 LAYER=1,NSTOP
186  THETA2(LAYER+1)=LTEMP(LAYER) - LTEMP(LAYER+1)
      THETA2(NLAYER+1)=LTEMP(NLAYER)-GT
C
C      CALCULATE LAMDA2, LAMDA1, AND NEWDEN FOR EACH L
C      AYER
190  NSTOP=NLAYER
      DO 211 LAYER=1,NSTOP
      IF(NOFRE.EQ.2.AND.DEN(LAYER).GE.0.35)GO TO 200
C      NOTE DECISION USES DEN ONLY FOR FIRST HOUR
C      ALL OTHER HOURS USE OAVDEN
      IF(NOFRE.NE.2.AND.OAVDEN(LAYER).GE.0.35)GO TO 200
C      CALCS WHEN DENSITY < 0.35
      IF(NOFRE.EQ.2)LAMDA2(LAYER)=24.48 * (DEN(LAYER)**2)
      IF(NOFRE.NE.2)LAMDA2(LAYER)=24.48 * (OAVDEN(LAYER)**2)
      IF(THETA1(LAYER).NE.0)GO TO 910
711  IF(NOFRE.EQ.2)NEWDEN(LAYER)=DEN(LAYER)
      IF(NOFRE.NE.2)NEWDEN(LAYER)=OAVDEN(LAYER)
      GO TO 210
910  IF(THETA2(LAYER).EQ.0)GO TO 711
      LAMDA1(LAYER)=(LAMDA2(LAYER)*THETA2(LAYER))/THETA1(LAY
      1ER)
      NEWDEN(LAYER)=(ABS(LAMDA1(LAYER)/24.48)**0.5
      GO TO 210
C      CALCS WHEN DENSITY IS >= 0.35
200  IF(NOFRE.EQ.2)LAMDA2(LAYER)=30.6*(DEN(LAYER)**2)
      IF(NOFRE.NE.2)LAMDA2(LAYER)=30.6*(OAVDEN(LAYER)**2)

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```

IF(THETA1(LAYER).NE.0)GO TO 911
712 IF(NOFRE.EQ.2) NEWDEN(LAYER)=DEN(LAYER)
IF(NOFRE.NE.2) NEWDEN(LAYER)=OAVDEN(LAYER)
GO TO 210
911 IF(THETA2(LAYER).EQ.0)GO TO 712
LAMDA1(LAYER)=(LAMDA2(LAYER)*THETA2(LAYER))/THETA1(LAYER)
NEWDEN(LAYER)=(ABS(LAMDA1(LAYER)/30.6)**0.5
210 IF(NEWDEN(LAYER).GT.1.)NEWDEN(LAYER)=1.
211 CONTINUE
C
C CALCULATE CHANGE IN DENSITY (DENCH) FOR EACH L
C LAYER
C CALCULATE THETA0
DO 220 LAYER=1,NSTOP
C IF HOUR = 1 THETA0 IS SET TO VALUE OF THETA1
THETA0(LAYER)=(THETA1(LAYER)+THETA2(LAYER))/2
IF(NOFRE.EQ.2) THETA0(LAYER)=THETA1(LAYER)
IF(NOFRE.EQ.2)GO TO 771
DENCH(LAYER)=NEWDEN(LAYER)-DEN(LAYER)
GO TO 220
771 DENCH(LAYER)=LHDEN(LAYER)-DEN(LAYER)
220 CONTINUE
C
C CALCULATE HEAT FLUX
C
C NOTE OLDH OLDFLX AND OAVDEN ARE ONLY CALCS FOR
C HOUR = 1
IF(NOFRE.NE.2)GO TO 250
C SET UP OAVDEN VALUES FOR HOUR 1
OAVDEN(1)=DEN(2)
OAVDEN(NLAYER+1)=DEN(NLAYER+1)
LAYER=NLAYER
230 OAVDEN(LAYER)=(DEN(LAYER)+DEN(LAYER+1))/2.0
LAYER=LAYER-1
IF(LAYER.GT.1)GO TO 230
GO TO 400
C CALCULATE NEW AVER. DENSITY FOR PRECEEDING HOU
C R
C NOTE: THIS CAN'T BE DONE FOR THE TOP LAYER
250 IF(NOFRE.NE.3)GO TO 501
LAYER=NLAYER
NAVDEN(1)=(LHDEN(2))
NAVDEN(NLAYER+1)=LHDEN(NLAYER+1)
LAYER=NLAYER
260 NAVDEN(LAYER)=(LHDEN(LAYER)+LHDEN(LAYER+1))/2.0
LAYER=LAYER-1

```

```

IF(LAYER.GT.1)GO TO 260
GO TO 400
501   LAYER=N_LAYER
      NAVDEN(1)=NEWDEN(2)
      NAVDEN(N_LAYER+1)=NEWDEN(N_LAYER+1)
      LAYER=N_LAYER
502   NAVDEN(LAYER)=(NEWDEN(LAYER)+NEWDEN(LAYER+1))/2.
      LAYER=LAYER-1
      IF(LAYER.GT.1)GO TO 502
      GO TO 400
C           STORE THIS NEWDEN AS LAST HOURS DENSITY
C
C
C           CALCULATE DIFFUSIVITY
C
C           LOCATE SPHEAT AND COND FROM MATRIX
400   NSTOP=N_LAYER+1
      DO 300 LAYER=1,NSTOP
      DIFF=1.0E25
      DO 280 MROW=1,12
      IF(NOFRE.EQ.2)CLOSE=((MATRIX(MROW,1)-OAVDEN(LAYER)))
      IF(NOFRE.NE.2)CLOSE=((MATRIX(MROW,1)-NAVDEN(LAYER)))
      CLOSE=ABS(CLOSE)
      IF(CLOSE.GE.DIFF)GO TO 280
      DIFF=CLOSE
      SPHEAT(LAYER)=MATRIX(MROW,2)
      COND(LAYER)=MATRIX(MROW,3)
280   CONTINUE
300   CONTINUE
      NOFRE=NOFRE+1
C
C           CALCULATE NEWH,OLDH,OLDFLX AND NEWFLX
C           NOTE SMALLER LOOP FOR HOUR=1
      IF(NOFRE.NE.3) GO TO 305
C           LOOP FOR HOUR 1
      WRITE(6,51)
51   FORMAT(' ', 'HOUR', 2X, 'LAYER', 3X, 'ORDEN', 5X, 'NEWDEN', 4
1X,
1 'DENCH', 4X, 'FLUX', 6X, 'FLUXDG', 4X, 'FLUXCM', 4X, 'TOTFLUX'
1, 12
2X, 'DATEM')
      DO 303 LAYER=1,NSTOP
      OLDH(LAYER)=(COND(LAYER)/(SPHEAT(LAYER)*OAVDEN(LAYER)
1)**0.5
      OLDFLX(LAYER)=((COND(LAYER)*THETAO(LAYER))/(OLDH(LAYER
1)*1.77))*
160.*(TIME**0.5)

```

```

TOTFLX(LAYER)=TOTFLX(LAYER)+OLDFLX(LAYER)
IF(THETA1(LAYER).EQ.0) GO TO 602
FLUXDG(LAYER)=OLDFLX(LAYER)/THETA1(LAYER)
GO TO 603
602 FLUXDG(LAYER)=OLDFLX(LAYER) ;
603 FLUXCM(LAYER)=OLDFLX(LAYER)/TTHICK(LAYER)
IF(THETA1(LAYER).EQ.0)GO TO 1111
IF(NEWDEN(LAYER).EQ.1) GO TO 866
WRITE(6,50)HOUR(1), LAYER, DEN(LAYER), DEN(LAYER), DENCH(L
1AYE
1R), OLDFLX(LAYER), FLUXDG(LAYER), FLUXCM(LAYER), TOTFLX(LA
1YER),
2DATEM(1)
50 FORMAT(' ', 2I4, 2X, 7F10.5, 11X, F10.2)
WRITE(22, 3880)HOUR(1), LAYER, DEN(LAYER), DATEM(1)
3880 FORMAT(' ', 2I4, 2X, F10.5, 2X, F10.2)
GO TO 1112
866 WRITE(6, 3000)HOUR(1), LAYER, DEN(LAYER), DEN(LAYER), DENCH
1(LAYE
1R), OLDFLX(LAYER), FLUXDG(LAYER), FLUXCM(LAYER), TOTFLX(LA
1YER),
2DATEM(1)
3000 FORMAT(' ', 2I4, 2X, 7F10.5, 2X, ' **', 3X, F10.2)
WRITE(22, 3880)HOUR(1), LAYER, DEN(LAYER), DATEM(1)
GO TO 1112
1111 IF(NEWDEN(LAYER).EQ.1)GO TO 3003
WRITE(6, 2000)HOUR(1), LAYER, DEN(LAYER), DEN(LAYER), DENCH
1(LAYE
1R), OLDFLX(LAYER), FLUXDG(LAYER), FLUXCM(LAYER), TOTFLX(LA
1YER),
2DATEM(1)
2000 FORMAT(' ', 2I4, 2X, 7F10.5, 2X, '*', 8X, F10.2)
WRITE(22, 3880)HOUR(1), LAYER, DEN(LAYER), DATEM(1)
GO TO 1112
3003 WRITE(6, 3009)HOUR(1), LAYER, DEN(LAYER), DEN(LAYER), DENCH
1(LAYE
1R), OLDFLX(LAYER); FLUXDG(LAYER), FLUXCM(LAYER), TOTFLX(LA
1YER),
2DATEM(1)
3009 FORMAT(' ', 2I4, 2X, 7F10.5, 2X, ' * **', 3X, F10.2)
WRITE(22, 3880)HOUR(1), LAYER, DEN(LAYER), DATEM(1)
1112 IF(NPHOUR.EQ.1)NPHOUR=NPHOUR+NHOURS
303 CONTINUE
GO TO 905

C
C LOOP FOR HOURS AFTER 1
305 DO 310 LAYER=1 ,NSTOP
NEWH(LAYER)=(COND(LAYER)/(SPHEAT(LAYER)*NAVDEN(LAYER))

```

```

1)**0.5
NEWFLX(LAYER)=((COND(LAYER)*THETA0(LAYER))/(NEWH(LAYER
1)*1.77))*
160.*(TIME**0.5)
TOTFLX(LAYER)=TOTFLX(LAYER)+NEWFLX(LAYER)
IF(THETA2(LAYER).EQ.0)GO TO 605
FLUXDG(LAYER)=NEWFLX(LAYER)/THETA2(LAYER)
GO TO 606
605 FLUXCM(LAYER)=NEWFLX(LAYER)
606 FLUXCM(LAYER)=NEWFLX(LAYER)/TTHICK(LAYER)
C *****
IF(HOUR(2).NE.NPHOUR)GO TO 310
IF(THETA1(LAYER).EQ.0)GO TO 2002
IF(NEWDEN(LAYER).EQ.1)GO TO 5001
WRITE(6,50)HOUR(2),LAYER,DEN(LAYER),NEWDEN(LAYER),DENC
INCH(LAYER),NEWFLX(LAYER),FLUXDG(LAYER),FLUXCM(LAYER),TOTFLX(LA
YER),
2DATEM(2)
WRITE(22,3880)HOUR(2),LAYER,NEWDEN(LAYER),DATEM(2)
GO TO 310
5001 WRITE(6,3000)HOUR(2),LAYER,DEN(LAYER),NEWDEN(LAYER),DE
INCH(LAYER),NEWFLX(LAYER),FLUXDG(LAYER),FLUXCM(LAYER),TOTFLX(LA
YER),
2DATEM(2)
WRITE(22,3880)HOUR(2),LAYER,NEWDEN(LAYER);DATEM(2)
GO TO 310
2002 IF(NEWDEN(LAYER).EQ.1)GO TO 5002
WRITE(6,2000)HOUR(2),LAYER,DEN(LAYER),NEWDEN(LAYER),DE
INCH(LAYER),NEWFLX(LAYER),FLUXDG(LAYER),FLUXCM(LAYER),TOTFLX(LA
YER),
2DATEM(2)
WRITE(22,3880)HOUR(2),LAYER,NEWDEN(LAYER),DATEM(2)
GO TO 310
5002 WRITE(6,3009)HOUR(2),LAYER,DEN(LAYER),NEWDEN(LAYER),DE
INCH(LAYER),NEWFLX(LAYER),FLUXDG(LAYER),FLUXCM(LAYER),TOTFLX(LA
YER),
2DATEM(2)
WRITE(22,3880)HOUR(2),LAYER,NEWDEN(LAYER),DATEM(2)
310 CONTINUE
IF(HOUR(2).EQ.NPHOUR)NPHOUR=NPHOUR+NHOURS
C
C STORE NAVDEN AS OAVDEN FOR NEXT HOURS CALCS
DO 320 LAYER=1,NSTOP
320 OAVDEN(LAYER)=NAVDEN(LAYER)

```

```

C
905      CONTINUE
        HOUR(1)=HOUR(2)
        DATEM(1)=DATEM(2)
C      STORE THIS NEWDEN AS LAST HOURS NEWDEN
        NSTOP=N_LAYER+1
C *****
C *****
        DO 5555 JJ=1,NSTOP
          X(JJ,1)=NEWDEN(JJ)
          X(JJ,2)=DENCH(JJ)
          IF(NOFRE.EQ.3) X(JJ,3)=OLDFLX(JJ)
          IF(NOFRE.NE.3) X(JJ,3)=NEWFLX(JJ)
          X(JJ,4)=FLUXDG(JJ)
          X(JJ,5)=FLUXCM(JJ)
          X(JJ,6) = TOTFLX(JJ)
5555    CONTINUE
        CALL STATS(NSTOP,6)
        WRITE(8,5556) (XMEAN(JJ),JJ=1,6)
        WRITE(8,5558) (XSDEV(JJ),JJ=1,6)
        WRITE(8,5559) (XRANGE(JJ),JJ=1,6)
        WRITE(8,5560) (XVAR(JJ),JJ=1,6)
        WRITE(8,5561) (XSKEW(JJ),JJ=1,6)
        WRITE(8,5562) (XKURT(JJ),JJ=1,6)
        WRITE(8,5563) (XSE(JJ),JJ=1,6)
5556    FORMAT(' ', 'XMEAN ', 6F13.10)
5558    FORMAT(' ', 'ST.DEV ', 6F13.5)
5559    FORMAT(' ', 'RANGE ', 6F13.5)
5560    FORMAT(' ', 'VAR. ', 6F13.5)
5561    FORMAT(' ', 'SKEW. ', 6F13.5)
5562    FORMAT(' ', 'KURT. ', 6F13.5)
5563    FORMAT(' ', 'S.E. ', 6F13.5)
        DO 270 LAYER=1,NSTOP
270     LHDEN(LAYER)=NEWDEN(LAYER)
          IF(NOFRE.EQ.3)GO TO 190
          DO 8999 LAYER=1,NSTOP
8999    THETA1(LAYER)=THETA2(LAYER)
C
C      CONTROL HERE AT END OF DATA SET ON UNIT 1
C
        GO TO 140
1000   WRITE(6,808)
808    FORMAT(' ', '//, ' * DENOTES NO CHANGE IN TEMPERATURE BETW
1EEN LAYERS')
        WRITE(6,9090)
9090   FORMAT(' ', '//, ' ** DENOTES CALCULATED DENSITY WAS SET
1TO 1.0')
        WRITE(6,6)

```

```

6   FORMAT(' ',//,' PROCESSING TERMINATING AT END OF DATA
1SET')
   STOP
   END
   SUBROUTINE STATS(N,M)
   COMMON X(20,7),XMEAN(7),XRANGE(7),XVAR(7),
1XSDEV(7),XSKEW(7),XB1(7),XKURT(7),XB2(7),XSE(7)
   DIMENSION SUM(7),SXMM(7),SXMM3(7),SXMM4(7)
   REAL MIN(7),MAX(7)

C
C           CALL PARAMETERS
C   X       DATA MATRIX
C   N       NUMBER OF OBS
C   M       NUMBER OF VARIABLES
C   7       MAINLINE ROW DIMENSION OF X
C   4       MAINLINE COLUMN DIMENSION OF X
C
C           INITIALIZE
C   REALN=FLOAT(N)
C   DO 50 IVAR=1,M
C   SUM(IVAR)=0.0
C   SXMM(IVAR)=0.0
C   SXMM3(IVAR)=0.0
C   SXMM4(IVAR)=0.0
C   MIN(IVAR)=1.0E25
C   MAX(IVAR)=-1.0E25
50  CONTINUE
C           SUMS FOR MEAN AND CALC. OF MIN AND MAX.
C   DO 200 IVAR=1,M
C   DO 100 ICASE=1,N
C   SUM(IVAR)=SUM(IVAR)+X(ICASE,IVAR)
C   IF(X(ICASE,IVAR).GT.MAX(IVAR))MAX(IVAR)=X(ICASE,IVAR)
C   IF(X(ICASE,IVAR).LT.MIN(IVAR))MIN(IVAR)=X(ICASE,IVAR)
100 CONTINUE
200 CONTINUE
C           RANGE AND MEAN
C   DO 300 IVAR=1,M
C   XMEAN(IVAR)=SUM(IVAR)/REALN
C   XRANGE(IVAR)=MAX(IVAR)-MIN(IVAR)
300 CONTINUE
C           OTHER SUMS (FOR SKEW. AND KURT.)
C   DO 500 IVAR=1,M
C   DO 400 ICASE=1,N
C   V1=X(ICASE,IVAR)-XMEAN(IVAR)
C   SXMM(IVAR)=SXMM(IVAR)+(V1*V1)
400 CONTINUE
500 CONTINUE

```

```
C      VAR. SDEV: SKEW. BETA1 KURT. BETA2 SE
      DO 600 IVAR=1,M
      XVAR(IVAR)=SXMM(IVAR)/(REALN-1.)
      XSDEV(IVAR)=XVAR(IVAR)**0.5
      XSE(IVAR)=XSDEV(IVAR)/(REALN**0.5)
600    CONTINUE
      DO 700 IVAR=1,M
      DO 650 ICASE=1,N
      IF(XSDEV(IVAR).EQ.0) GO TO 89754
      V1=(X(ICASE,IVAR)-XMEAN(IVAR))/1.0
      GO TO 89755
89754  V1=0.0
89755  SXMM3(IVAR)=SXMM3(IVAR)+(V1*V1*V1)
      SXMM4(IVAR)=SXMM4(IVAR)+(V1*V1*V1*V1)
650    CONTINUE
700    CONTINUE
      DO 800 IVAR=1,M
      XSKEW(IVAR)=SXMM3(IVAR)/REALN
      IF(XVAR(IVAR).NE.0) GO TO 2050
      WRITE(8,2051) IVAR
2051   FORMAT(' ','CAUTION VARIANCE FOR VAR. ',I2,' WAS 0')
      XB1(IVAR)=0.0
      GO TO 2052
2050   XB1(IVAR)=(XSKEW(IVAR)*XSKEW(IVAR))/(XVAR(IVAR)*XVAR(I
      IVAR)
      1*XVAR(IVAR))
2052   XKURT(IVAR)=(SXMM4(IVAR)/REALN)
      IF(XB1(IVAR).NE.0) GO TO 2054
      XB2(IVAR)=0.0
      GO TO 800
2054   XB2(IVAR)=XKURT(IVAR)/(XVAR(IVAR)*XVAR(IVAR))
800    CONTINUE
      RETURN
      END
```

APPENDIX V  
FORTRAN PROGRAM FOR RECALCULATING  
TEMPERATURE AND MELT



```

C
CPM . COMMENTS STARTING WITH: CPM DENOTE ADDITIONS FOR
CPM . VERSION 2
C THIS PROGRAM IS THE AMDAHL VERSION OF 'SNOW'
DIMENSION DA(13),HTT(13),HT(12),DATA(12),HOLD(12),
ITM(16,4),COEF(20),TR(20),T(4500),GGT(181),DATEH(181)
REAL KA(13),KAY(12),KP(13),K,I0,I1,I2,N,N1,MAXEX,IDATE

CPM
DIMENSION SHEAT(12),SMAT(12,2)
REAL LHHEAT(12),LMELT(12),LHCH(12)

C
INTEGER RHOHTT,BI,BK
C NOTE CHANGE INTEGER NAME(11) TO INTERGER * 1, NAME(11)
C FOR MICRO
REAL NAME(11)
INTEGER AFF/'Y'/
INTEGER TERM/'T'/
INTEGER REPLY
CPM READ SP. HEAT MATRIX
DO 4009 I=1,12
4009 READ(23,4008) (SMAT(I,J),J=1,2)
4008 FORMAT(F5.3,2X,F6.4)
IG=1
FLAG=0.0
IW=0
75 WRITE(3,60)
60 FORMAT(' ',' WELCOME TO PROGRAM: SNOW !!'/' ','WHAT ',
1'INPUT FILE PLEASE ?'/)
READ(1,14)NAME
14 FORMAT(11A1)
CALL OPEN(9,NAME,1)
C CONVERSATIONALLY DEFINE OUTPUT UNIT 3 FOR TERM 8 FOR D
C ISK
C *****NOTE NEXT LINE MUST BE REMOVED FOR MICRO
GO TO 9153
C *****
12345 WRITE(3,46)
46 FORMAT(' ','OUTPUT TO TERMINAL OR DISK ? (T OR D)'/
1)
READ(1,47) REPLY
47 FORMAT(A2)
IF(REPLY.EQ.TERM)GO TO 50
9153 IOUT1=8
IOUT2=8
CALL OPEN (8,'SNOW DAT',2)
CALL OPEN(10,'SNOW DAT',1)
GO TO 51
50 IOUT1=3

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```

IOUT2=3
51 WRITE(3,61)
61 FORMAT(' ', 'ENTER L VALUE AND N OF HOURS AS I1,I4 PLEA
1SE '/')
READ(1,4) L,NHOURS
4 FORMAT(I1,I4)
WRITE(3,62)
62 FORMAT(' ', 'ENTER TQ VALUE, INCLUDE THE DECIMAL PLEASE
1'/)
READ(1,15) TQ
15 FORMAT(F10.3)
WRITE(3,63)
63 FORMAT(' ', 'ENTER STARTING AND ENDING STATIONS AS ',
1'I3,1X,I3 PLEASE '/')
READ(1,64) SSTART,SEND
64 FORMAT(F3.2,1X,F3.2)
WRITE(3,65)
65. FORMAT(' ', 'ENTER STARTING AND ENDING DATES AS ',
1'I6,1X,I6 PLEASE '/')
READ(1,66) SDATE,EDATE
66 FORMAT(F6.0,1X,F6.0)
WRITE(3,900)
900 FORMAT(' ', 'HOUR FOR START OF PRINTING ? (INCLUDE A',
1' DECIMAL PLEASE) '/')
READ(1,901)HST
901 FORMAT(F6.0)
IHST=IFIX(HST)
WRITE(3,902)
902 FORMAT(' ', 'HOUR INCREMENT BETWEEN PRINTS ? (INCLUDE',
1' A DECIMAL PLEASE) '/')
READ(1,903) CREM
903 FORMAT(F7.0)
INCREM=IFIX(CREM)
WRITE(3,9991)
9991 FORMAT(' ', 'OPTIONAL CHANGE OF TF WITH EACH STATION-DA
1TE ?? '/')
READ(1,69) REPLY
IF(REPLY.NE.AFF)GO TO 9993
FLAG=1.
9993 WRITE(3,67) NAME,REPLY,L,NHOURS,SSTART,SEND,SDATE,EDAT
1E,
1IHST,INCREM,FLAG
67 FORMAT(' ', 'RUN PARAMETERS ARE: ',/
1' ', 'FILE NAME: ',11A1,/;
2' ', 'OUTPUT TO: ',A2,/;
3' ', 'L VALUE : ',I1,/;
4' ', 'N HOURS : ',I4,/;
5' ', 'ST. START: ',F4.2,/;

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6' ' , 'ST. END : ' , F4.2 , / ,
7' ' , 'DA. START : ' , F7.0 , / ,
8' ' , 'DA. END : ' , F7.0 , / ,
9' ' , 'P. START : ' , I4 , / ,
9' ' , 'P. INCR. : ' , I4 , / ,
9' ' , 'TF OPT. : ' , F2.0 , ' 0=NO 1=YES' / )
WRITE(3,68)
68 FORMAT(' ' , 'RE-DEFINE PARAMETERS ? (Y OR N)' / )
READ(1,69) REPLY
69 FORMAT(A1)
IF(REPLY.EQ.AFF)GO TO 75
C UNIT 7 IS TEMPERATURE AND GT INPUT FILE
CALL OPEN(7, 'TEMPER DAT', 1)
PHI=3.141592654
C UNIT 1 IS INPUT DATA FILE (I.E. SHAF01 IN APL)
C READ A DATA CARD
100 READ(9,1,END=1000) STN,DATE,M
READ(9,16)(DA(J),J=1,M)
READ(9,16)(KP(J),J=1,M),TEMP
IG=1
16 FORMAT(9F8.4)
IF(STN.LT.SSTART.OR.STN.GT.SEND)GO TO 100
IF(DATE.LT.SDATE.OR.DATE.GT.EDATE)GO TO 100
DO 110 J=1,M
HTT(J)=TEMP
110 CONTINUE
1 FORMAT(F4.2,1X,F6.0,1X,I2)
C GET GT AND T FOR THIS CASE
REWIND 7
IS=1
ISTOP=24
111 READ(7,10,END=115) IDATE,(T(JJ),JJ=1,24),GGT(IG)
DATEH(IG)=IDATE
10 FORMAT(F6.0,11F5.1/14F5.1)
IF(IDATE.NE.SDATE)GO TO 111
IF(ISTOP.GE.NHOURS)GO TO 112
GO TO 113
115 WRITE(IOUT1,11) DATE
IW=IW+1
11 FORMAT(' ' , 'NO TEMPERATURE DATA FOR: ' , F7.0)
GO TO 100
113 IS=ISTOP+1
IG=IG+1
ISTOP=ISTOP + 24
READ(7,12,END=115) IDATE,(T(JJ),JJ=IS,ISTOP),GGT(IG)
DATEH(IG)=IDATE
12 FORMAT(F6.0,11F5.1/14F5.1)
IF(ISTOP.GE.NHOURS)GO TO 112

```

```

GO TO 113
C   CALCULATE AND PRINT TOTAL SNOW DEPTH
112  TEMP=TEMP*FLOAT(M)
      WRITE(IOUT1,3) STN,DATE,TEMP
      IW=IW+1
      IF(IW.GE.1270.AND.IOUT1.NE.3)IOUT1=8
      IF(IW.GE.1270.AND.IOUT2.NE.3)IOUT2=8
3    FORMAT('1','STATION: ',F4.2,5X,'DATE: ',F7.0,5X,
1    'TOTAL SNOW DEPTH: ',F8.3 )
      TF=HTT(1)
      IF(FLAG.NE.1.0)GO TO 9982.
      WRITE(3,9983) STN,DATE,TF
9983  FORMAT(' ','FOR: ',F4.2,2X,F7.0,2X,'TF IS :',F10.3,'
1    CHANGE THIS
1    VALUE ?'/)
      READ(1,9984) REPLY
      IF(REPLY.NE.AFF)GO TO 9982
9984  FORMAT(A2)
      WRITE(3,9985)
9985  FORMAT(' ','ENTER NEW TF VALUE INCLUDE A DECIMAL'/)
      READ(1,9986) TF
      WRITE(3,9987) TF
9987  FORMAT(' ','TF CHANGED TO: ',F10.3/)
9986  FORMAT(F10.3)
C    KA GENERATOR
9982  DO 117 KJ=1,M
      IF(KP(KJ).LT.0.35)KA(KJ)=KP(KJ)*3.6 * 13.3
      IF(KP(KJ).GE.0.35)KA(KJ)=KP(KJ)*3.6 * 16.5
117  CONTINUE
      BI=1
      NHT=1
      TS=1.0/FLOAT(E)
C    LINE B1 OF FUNCTION.LL
      RHOHTT=M
120  IF(BI.GT.RHOHTT)GO TO 130
      BK=(HTT(BI)/TF) + 0.99
      ISTOP=NHT + BK - 1
      DO 125 KJ=NHT,ISTOP
      HT(KJ)=HTT(BI)/FLOAT(BK)
      DATA(KJ)=DA(BI)
      KAY(KJ)=KA(BI)
125  CONTINUE
      NHT=ISTOP + 1
      BI=BI + 1
      GO TO 120
C    LINE 130 IS FUNCTION LL LINE 7
130  M=BK * M
      WRITE(IOUT1,5) M,(DATA(KJ),KJ=1,M)

```

```

C *****THE FOLLOWING WRITES INITIAL DATA ON UNIT 25**
C *****
C *****HOUR ONE IS KY=1*****
  KY=1
  WRITE(25,5060)KY,(DATA(KJ),KJ=1,M)
5060 FORMAT(' ',I4,11F10.4)
  IW=IW+1
5  FORMAT(' ','THE PROBLEM IS TAKEN AS ONE OF: ',I3,
1  ' LAYERS '/' ','INITIAL T: ',10F7.3)
  WRITE(IOUT1,6)(KAY(KJ),KJ=1,M)
  IW=IW+1
6  FORMAT(' ','K VALUES: ',10F8.4)
  WRITE(IOUT1,7)(HT(KJ),KJ=1,M)
C *****THE FOLOWING WRITES INITIAL DATA ON UNIT 25****
C *****
  WRITE(25,5061)KY,(HT(KJ),KJ=1,M)
5061 FORMAT(' ',I4,11F10.4)
  IW=IW+1
7  FORMAT(' ','THICKNESS: ',10F7.3/' ',50(1H-))
  IF(M.EQ.1)WRITE(IOUT2,7001)
  IF(M.EQ.2)WRITE(IOUT2,7002)
  IF(M.EQ.3)WRITE(IOUT2,7003)
  IF(M.EQ.4)WRITE(IOUT2,7004)
  IF(M.EQ.5)WRITE(IOUT2,7005)
  IF(M.EQ.6)WRITE(IOUT2,7006)
  IF(M.EQ.7)WRITE(IOUT2,7007)
  IF(M.EQ.8)WRITE(IOUT2,7008)
  IF(M.EQ.9)WRITE(IOUT2,7009)
  IF(M.EQ.10)WRITE(IOUT2,7010)
  IF(M.EQ.11)WRITE(IOUT2,7011)
  IF(M.EQ.12)WRITE(IOUT2,7012)
  IF(M.EQ.13)WRITE(IOUT2,7013)
7001 FORMAT(' '/' ',2X,'HOUR',3X,'AIR T.',4X,4X,'L 1',3X,4X
1,'GT',4X,'
  IDATE')
7002 FORMAT(' '/' ',2X,'HOUR',3X,'AIR T.',4X,4X,'L 1',7X,'
1L 2',7X,'GT
1',4X,'DATE')
7003 FORMAT(' '/' ',2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'
1,7X,'L 3'
16X,'GT',4X,'DATE')
7004 FORMAT(' '/' ',2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',6X,'GT',4X,'DATE')
7005 FORMAT(' '/' ',2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',6X,'GT',4X,'DATE')
7006 FORMAT(' '/' ',2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'

```

```

1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',6X,'GT','4X,'DATE')
7007 FORMAT(' / ' ,2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',6X,'GT','4X,'DATE
1')
7008 FORMAT(' / ' ,2X,'HOUR',3X,'AIR T.',8X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'GT'
1,5X,'DATE')
7009 FORMAT(' / ' ,2X,'HOUR',2X,'AIR T.',7X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'L 9',
16X,'GT',3X
1,'DATE')
7010 FORMAT(' / ' ,2X,'HOUR',2X,'AIR T.',7X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'L 9',
17X,'L10',
16X,'GT',3X,'DATE')
7011 FORMAT(' / ' ,2X,'HOUR',2X,'AIR T.',7X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'L 9',
17X,'L10',
17X,'L11',6X,'GT',3X,'DATE')
7012 FORMAT(' / ' ,2X,'HOUR',2X,'AIR T.',7X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'L 9',
17X,'L10',
17X,'L11',7X,'L12',6X,'GT',3X,'DATE')
7013 FORMAT(' / ' ,2X,'HOUR',2X,'AIR T.',7X,'L 1',7X,'L 2'
1,7X,'L 3',
17X,'L 4',7X,'L 5',7X,'L 6',7X,'L 7',7X,'L 8',7X,'L 9',
17X,'L10',
17X,'L11',7X,'L12',7X,'L13',6X,'GT',3X,'DATE')
IS=M+2
ISTOP=L+1
DO 135 KJ=1,IS
DO 135 JJ=1,ISTOP
TM(KJ,JJ)=0.
135 CONTINUE
KI=1
IG=(KI+23)/24
GT=GGT(IG)
DATEO=DATEH(IG)
INH=IHST
IF(KI.NE.INH)GO TO 140
INH=INH+INCREM

```

```

WRITE(IOUT2,8)T(1),(DATA(KJ),KJ=1,M),GT,DATEO
IW=IW+1
8   FORMAT(' ',2X,' 1',2X,F6.2,3X,14(F10.2))/' ',17X,8F10
1.2)
C   140 IS LINE 12 OF LL
140 IF(KI,EQ.NHOURS)GO TO 500
CPM
      IF(KI.LE.1) GO TO 4000
CPM *****
CPM SECTION 1 - READ AND USE DENSITIES
CPM
CPM READ NEW DENSITY VALUES
DO 4001 LTVAR=1,5000
READ(22,5000) ITIME
5000 FORMAT(I5,I4,2X,F10.5)
      IF(ITIME.EQ.KI) GO TO 4002
4001 CONTINUE.
      WRITE(3,5001) KI
5001 FORMAT(' ', 'NO DATA IN DENSITY FILE FOR HOUR: ',I4)
      STOP
CPM FIRST LAYER HAS NOW BEEN READ AND DISREGARDED
4002 DO 4003 LTVAR=1,M
4003 READ(22,5002) KP(LTVAR)
5002 FORMAT(11X,F10.5)
CPM RE-CALCULATE KA VALUES
DO 4004 LTVAR=1,M
      IF(KP(LTVAR).LT.0.35)
1   KA(LTVAR)=KP(LTVAR)*3.6 * 13.3
      IF(KP(LTVAR).GE.0.35)
1   KA(LTVAR)=KP(LTVAR)*3.6 * 16.5
4004 CONTINUE
CPM CALCULATE ADDITIONAL KA VALUES FOR SUB-LAYERS
      BI=1
      NHT=1
      RHOHTT=M
4005 IF(BI.GT.RHOHTT) GO TO 4010
      BK=(HTT(BI)/TF)+.99
      IISTOP=NHT+BK-1
      DO 4006 KJI=NHT,IISTOP
4006 KAY(KJI)=KA(BI)
      BI=BI+1
      GO TO 4005
4010 CONTINUE
CPM FIND SPECIFIC HEAT FOR EACH LAYER
DO 4020 LTVAR=1,M
      IWICH=0
      DMIN=1.0E20
      DO 4015 IDUM=1,12

```

```

DIFF1=ABS((SMAT(IDUM,1)-KP(LTVAR)))
IF(DIFF1.GE.DMIN) GO TO 4015
DMIN=DIFF1
IWICH=IDUM
4015 CONTINUE
CPM CLOSEST IS AT POSITION IWICH
SHEAT(LTVAR)=SMAT(IWICH,2)
4020 CONTINUE
CPM END OF SECTION 1
CPM *****
C *****
4000 IG=(KI+23)/24
GT=GGT(IG)
DATEO=DATEH(IG)
IS=M+2
ISTOP=L+1
IR=M+2
DO 145 KJ=1,ISTOP
JJ=KJ-1
TM(1,KJ)=T(KI)+((T(KI+1)-T(KI))*FLOAT(JJ/L))
TM(IR,KJ)=GT
145 CONTINUE
C AT PART 2 OF LINE 14 OF LL
TM(1,1)=T(KI)
ISTOP=IS-1
DO 150 KJ=2,ISTOP
JJ=KJ-1
TM(KJ,1)=DATA(JJ)
150 CONTINUE
TM(IS,1)=GT
J=0
C 155 IS LL1 IN LL
C 400 IS LL4 IN FUNCTION
155 J=J+1
IF(J.EQ.(L+1))GO TO 400
I=2
C 160 IS LL3 IN FUNCTION
C I.E. CALL TO SEAN
C FUNCTION SEAN FOLLOWS
160 C=TM(I,J)
IS=I+1
TO=TM(IS,J)
IS=I-1
ISTOP=J+1
B=(TM(IS,ISTOP) - TM(IS,J))/TS
T1=TM(IS,J)
H=HT(IS)
K=KAY(IS)

```



```

C   NTR AND NCO ARE NEXT POSITIONS IN VECTORS TR AND COEF
    NTR=1
    NCO=1
C   AT LINE 4 OF SEAN
    CO=C-TO
    C1=((B*H)/(6*K))+((TO-T1)/H)
    N1=1.0-2.0
    C2=N1 * (B/(6*K*H))
    N=1
C   LINE 165 IS LINE START OF SEAN
C   LINE 180 IS LINE END OF SEAN
165 IF(N.GT.5)GO TO 180
C   START OF FUNCTION FOCEF
    IF(N.EQ.2.OR.N.EQ.4)CAT=1
    IF(N.EQ.1.OR.N.EQ.3.OR.N.EQ.5)CAT=1.0-2.0
    IF(CAT.EQ.1.)DOG=1.0-2.0
    IF(CAT.EQ.-1.)DOG=1.0
    IO=(CO*(H/(PHI*N)))*(1.-CAT)
    I1=(DOG)*(C1*((H**2)/(PHI*N)))
    I2=((CAT)*C2*((H**4)/(PHI*N)))*((6/((PHI*N)**2))-1)
    COEF(NCO)=(2./H)*(IO+I1+I2)
    NCO=NCO+1
C   RETURN FROM FOCEF TO LINE 8 OF FUNCTION SEAN
    THOLD=((N1*((PHI*(N/H))**2))*K*TS)
    IF(THOLD.LE.-170.0)THOLD=-170.0
    MAXEX=EXP(THOLD)
    WAIT=THOLD
    IS=NCO-1
    THOLD=(PHI*N)/2.0
    IF(COEF(IS).LE..005.AND.WAIT.EQ.-170.0)COEF(IS)=.005
    TR(NTR)=(COEF(IS)*MAXEX)*SIN(THOLD)
    NTR=NTR+1
    IS=N
    THOLD=TR(IS)
    IF(ABS(THOLD).LT.TQ)GO TO 180
    N=N+1.0
    GO TO 165
C   180 IS LINE END OF SEAN
180 IS=J+1
    ISTOP=NTR-1
C   CALCULATE +/TR
    SUM=0.
    DO 185 JJ=1,ISTOP
    SUM=SUM + TR(JJ)
185 CONTINUE
    TM(I,IS)=((N1*B*(H**2))/(16.0*K))+((TO/2.0)+(T1/2.0)+
1(B*(TS/2.0))+SUM
C   END OF FUNCTION SEAN - RETURNING TO LL3 PART 2 IN FUNC

```

```

C      TION LL
      I=I+1
      IF(I.EQ.(M+2))GO TO 155
      GO TO 160
400    KI=KI+1
        IG=(KI+23)/24
        GT=GGT(IG)
        DATEO=DATEH(IG)
C      AT LL4 PART 2 IN FUNCTION LL
C      SELECT LAST M+1 ROWS AND LAST COLUMN FROM TM
      ICOL=L+1
      IROWS1=(M+2)-(M+1)+1
      DO 410 JJ=1,M
      DATA(JJ)=TM(IROWS1,ICOL)
      IROWS1=IROWS1+1
410    CONTINUE
      IF(KI.NE.INH)GO TO 935
      INH=INH+INCREM
      WRITE(IOUT2,37) KI,T(KI),(DATA(KJ),KJ=1,M),GT,DATEO
      IW=IW+1
      IF(IW.GE.1270.AND.IOUT1.NE.3)IOUT1=8
      IF(IW.GE.1270.AND.IOUT2.NE.3)IOUT2=8
37     FORMAT(' ',2X,I4,2X,F6.2,3X,14(F10.2))/' ',17X,8F10.2)
CPM   *****
C     *****
CPM   SECTION 2 - CALCULATE LAYER THICKNESS CHANGES
CPM
935   LTVAR=1
      LHHEAT(LTVAR)=(DATA(LTVAR)*SHEAT(LTVAR)*KP(LTVAR)*H
1T(LTVAR))/80
1     .0
      IF(LHHEAT(LTVAR).LT.0) KP(LTVAR)=KP(LTVAR)-LHHEAT(L
1TVAR)
      IF(LHHEAT(LTVAR).LT.0) GO TO 4030
      HT(LTVAR)=HT(LTVAR)-(LHHEAT(LTVAR)*HT(LTVAR))
      IF(HT(LTVAR).LE.0)HT(LTVAR)=0.0
4030  IF(DATA(LTVAR).GE.0) DATA(LTVAR)=0.0
      DO 4040 LTVAR=2,M
      IF(LHHEAT(LTVAR-1).GE.0)KP(LTVAR)=KP(LTVAR)+LHHEAT(
1LTVAR-1)
      LHHEAT(LTVAR)=(DATA(LTVAR)*SHEAT(LTVAR)*KP(LTVAR)*H
1T(LTVAR))/80
1     .0
      IF(LHHEAT(LTVAR).LT.0)KP(LTVAR)=KP(LTVAR)-LHHEAT(LT
1VAR)
      IF(LHHEAT(LTVAR).LT.0)GO TO 4035
      HT(LTVAR)=HT(LTVAR)-(LHHEAT(LTVAR)*HT(LTVAR))
      IF(HT(LTVAR).LE.0)HT(LTVAR)=0.0

```

```

4035     IF(DATA(LTVAR).GE.0)DATA(LTVAR)=0.0
4040     CONTINUE
5020     FORMAT(' ',I4,11F10.4)
5021     FORMAT(' ',I4,11F10.4)
8008     IF(KI.NE.INH-1)GO TO 8009
        WRITE(25,5020)KI,(DATA(LTVAR),LTVAR=1,M)
        WRITE(25,5021)KI,(HT(LTVAR),LTVAR=1,M)
8009     GO TO 140
CPM     *****
C       *****
500     THOLD=TS*60.0
        WRITE(IOUT1,9)TS,THOLD
        IW=IW+1
9       FORMAT(' ','TS = ',F5.2,' (',F6.2,' MIN.)')
        GO TO 100
1000    WRITE(IOUT1,13) NAME
13     FORMAT(' ','PROGRAM TERMINATING AT END OF DATA SET: '
1,
111A1)
        STOP
        END
        SUBROUTINE OPEN(IDD,NAME,IDDD)
        INTEGER * 2 NAME(11)
C       DUMMY SUBROUTINE JCL HANDLED BY CMS. EXEC FILE
        RETURN
        END

```

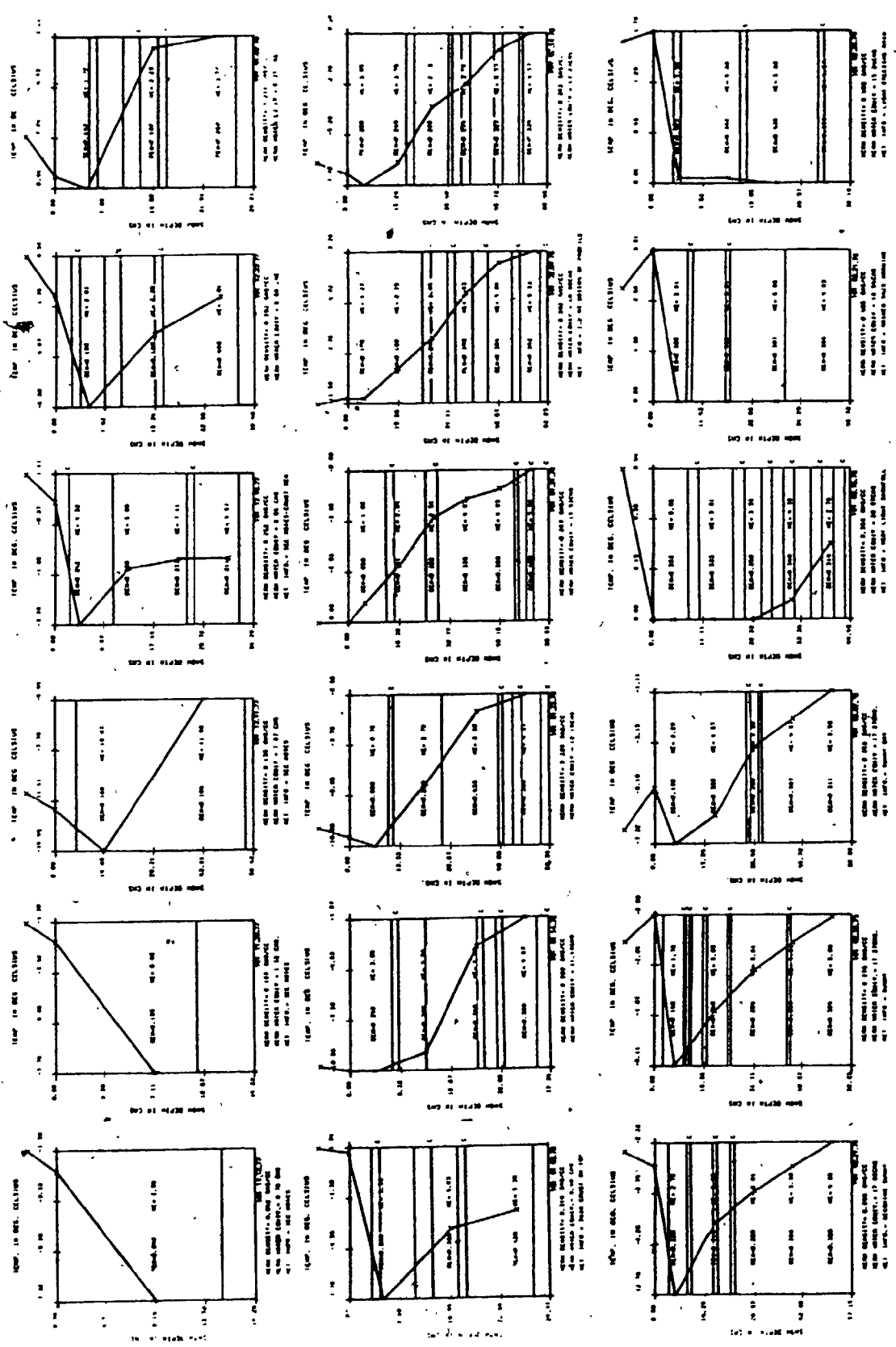
APPENDIX VI  
SIMULATION TIME FOR DIFFERENT  
SNOW COURSES

Appendix VI

Data Collected on	Simulation		NUMBER OF HOURS OF SIMULATION					
	Starts on	Ends on	Station No. 1	Station No. 2	Station No. 3	Station No. 4	Station No. 5	Station No. 6
771112	771112	771127	384	-	-	-	-	-
771128	771128	771210	312	-	-	-	-	-
771211	771211	771215	120	-	-	-	-	-
771215	771215	780101	-	432	432	432	432	432
771216	771216	771222	168	-	-	-	-	-
771223	771223	780101	240	-	-	-	-	-
780102	780102	780107	144	-	-	-	-	-
780102	780102	780113	-	-	-	-	288	288
780102	780102	780114	-	312	312	312	-	-
780108	780108	780113	144	-	-	-	-	-
780114	780114	780122	216	-	-	-	-	-
780114	780114	780201	-	-	-	-	456	480
780114	780114	780202	-	-	-	-	-	-
780115	780115	780130	-	384	-	-	-	-
780115	780115	780201	-	-	-	432	-	-
780115	780115	780202	-	-	456	-	-	-
780123	780123	780130	192	-	-	-	-	-
780131	780131	780206	168	-	-	-	-	-
780131	780131	780216	-	408	-	-	-	-
780202	780202	780214	-	-	-	312	312	288
780203	780203	780214	-	-	-	-	-	-
780203	780203	780216	-	-	336	-	-	-
780207	780207	780213	168	-	-	-	-	-
780214	780214	780220	168	-	-	-	-	-
780215	780215	780301	-	-	-	-	360	360
780215	780215	780302	-	-	-	384	-	-
780217	780217	780302	-	336	336	-	-	-

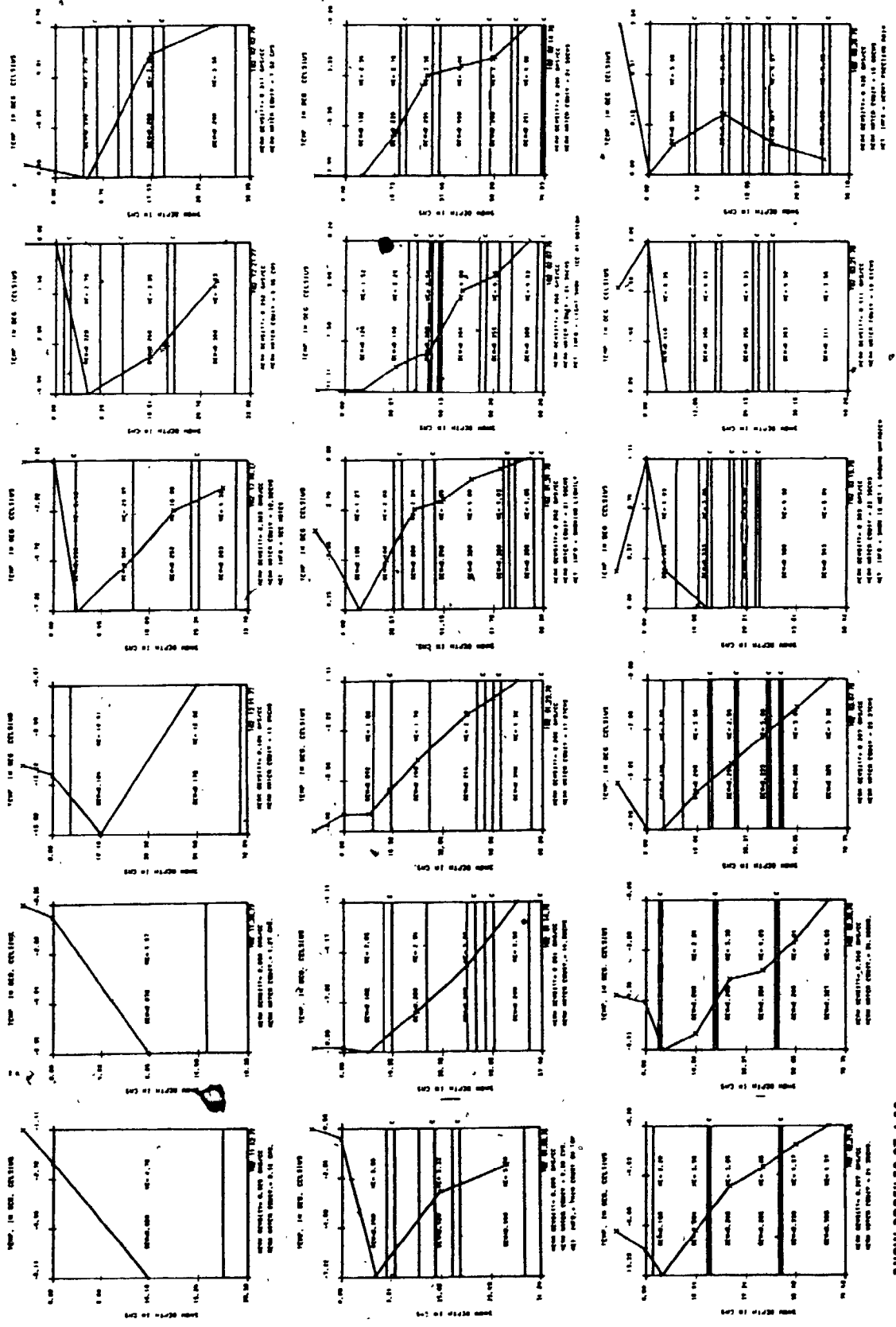


APPENDIX VII  
SNOW PROFILES

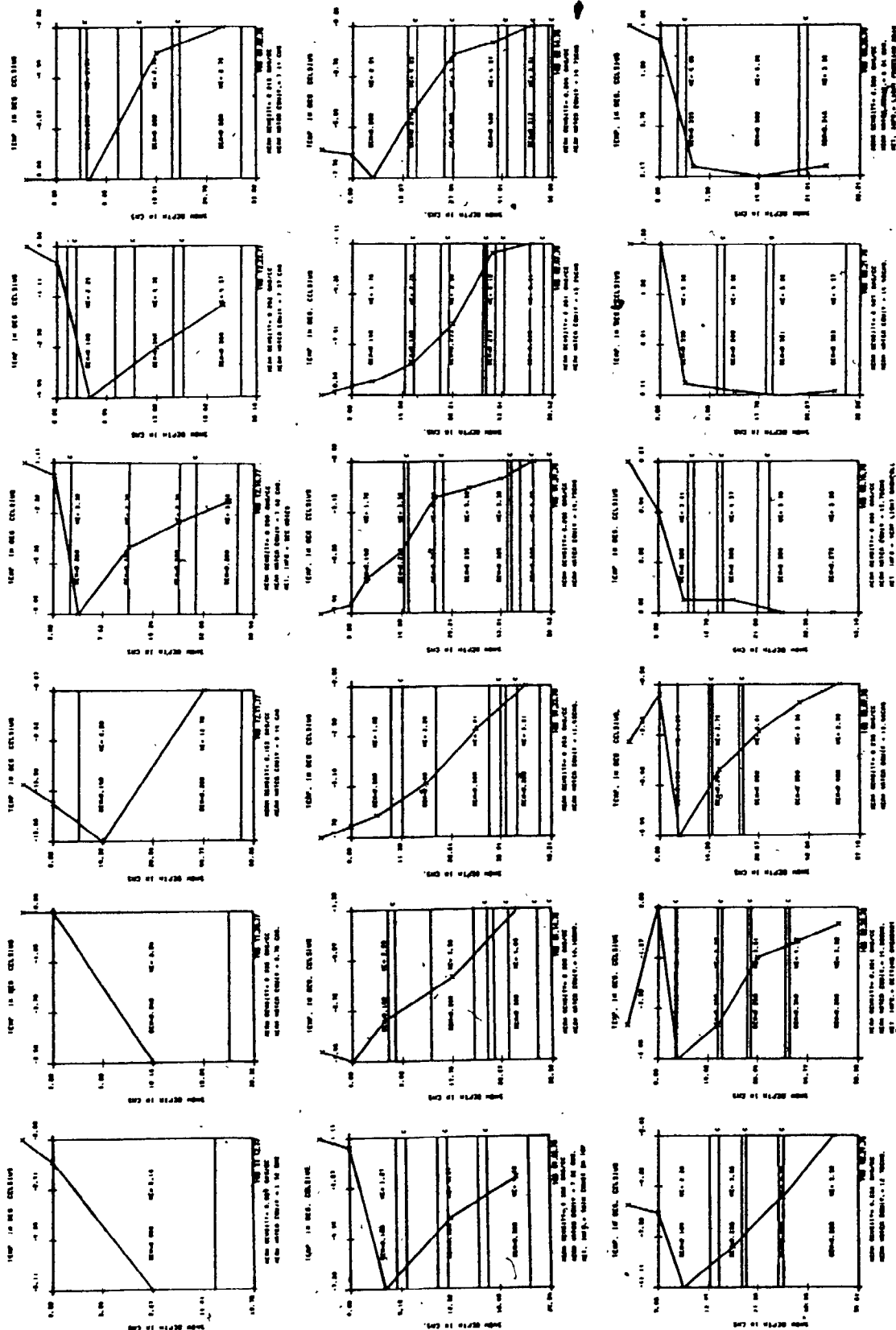


SNOW PROFILES ST. 1.01

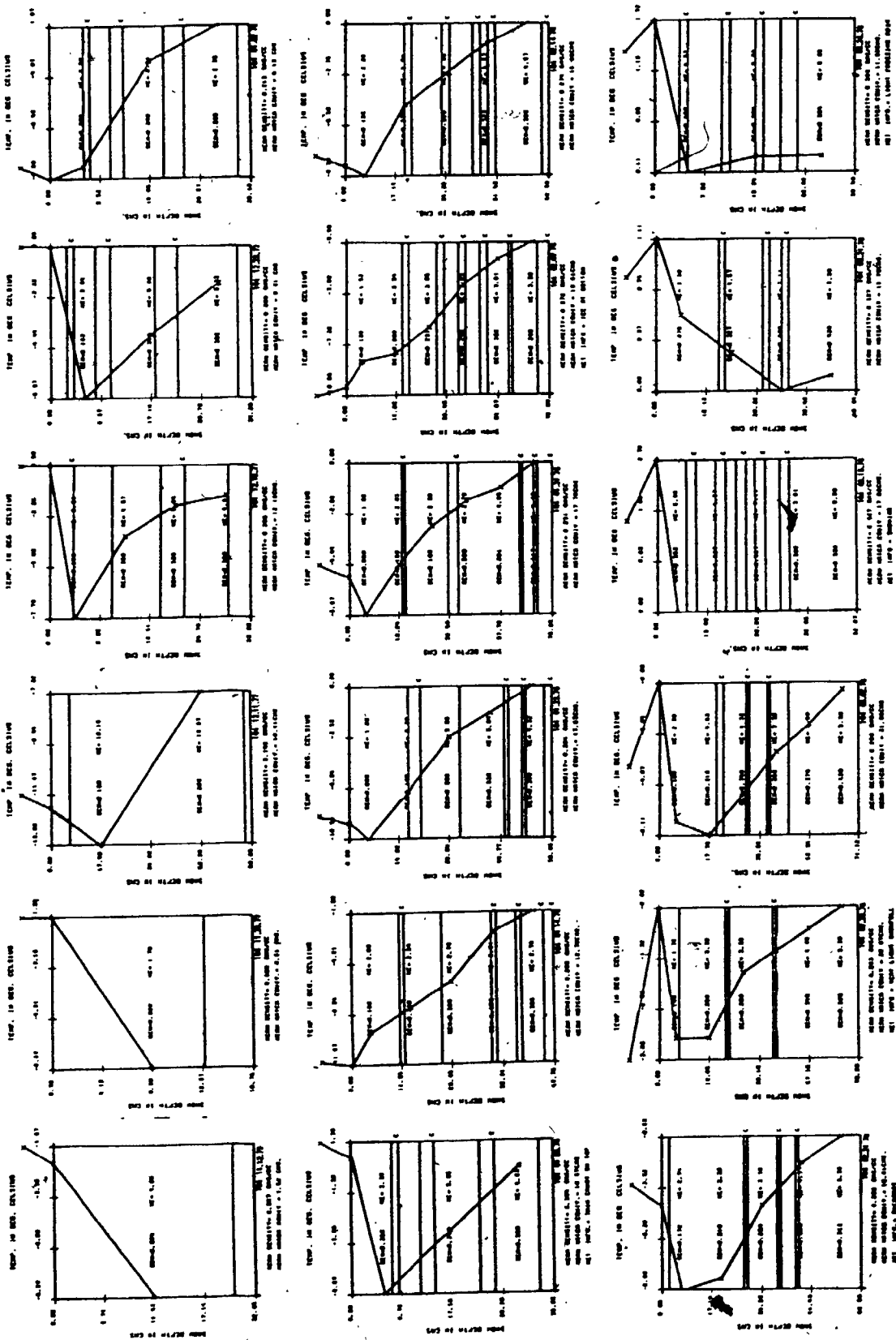




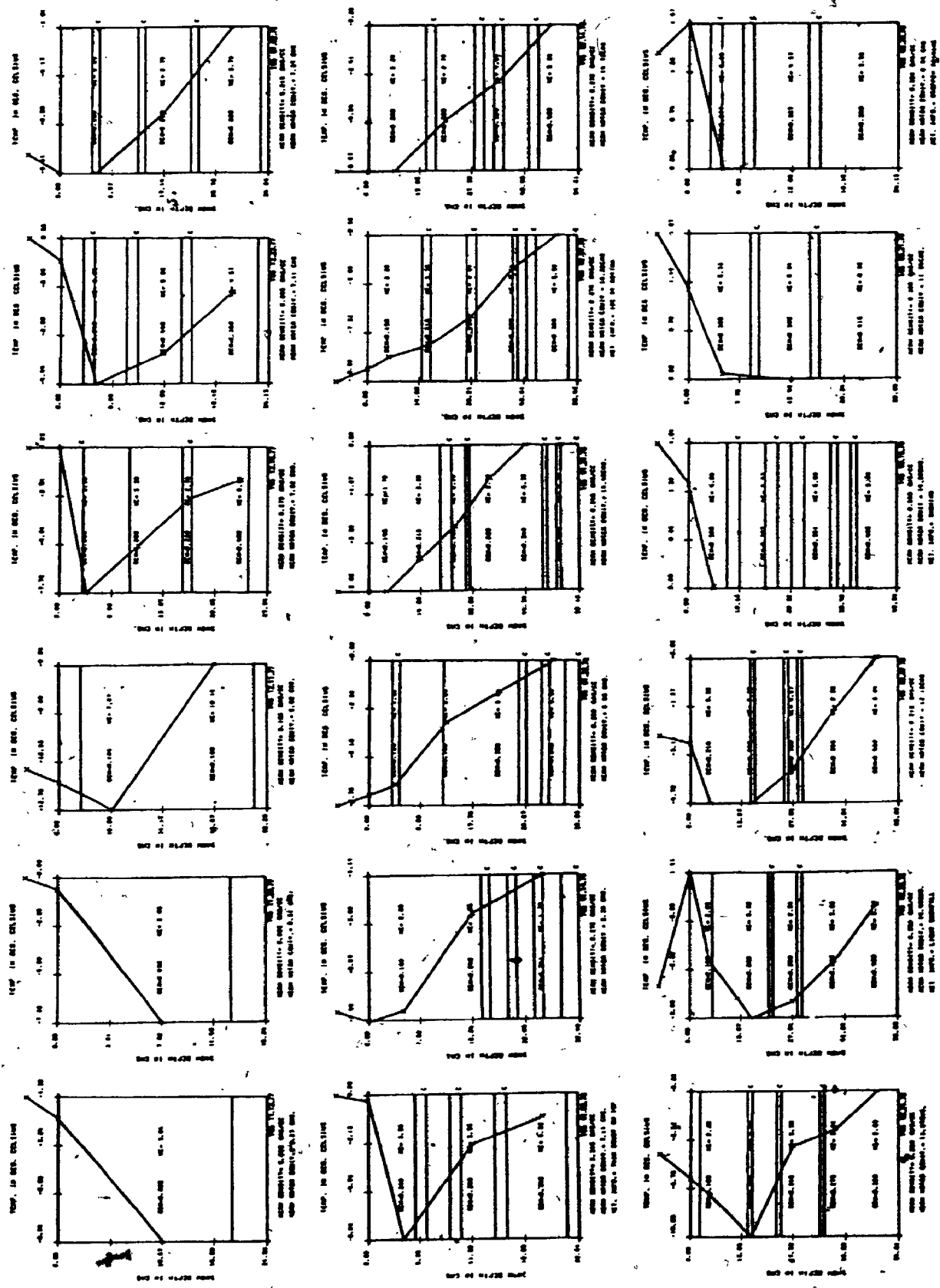
SNOW PROFILES ST. 1.02



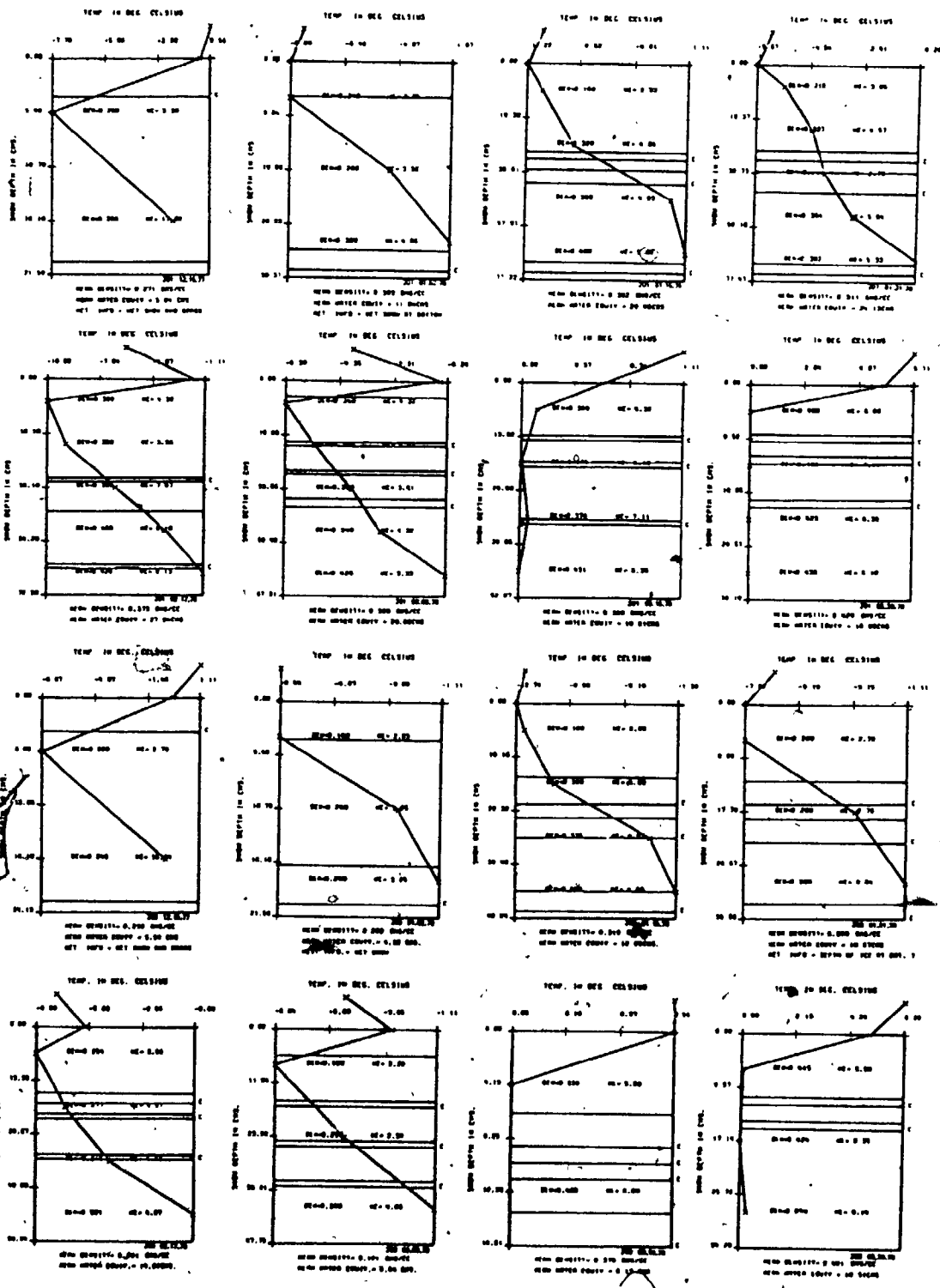
SNOW PROFILES ST. 1.03



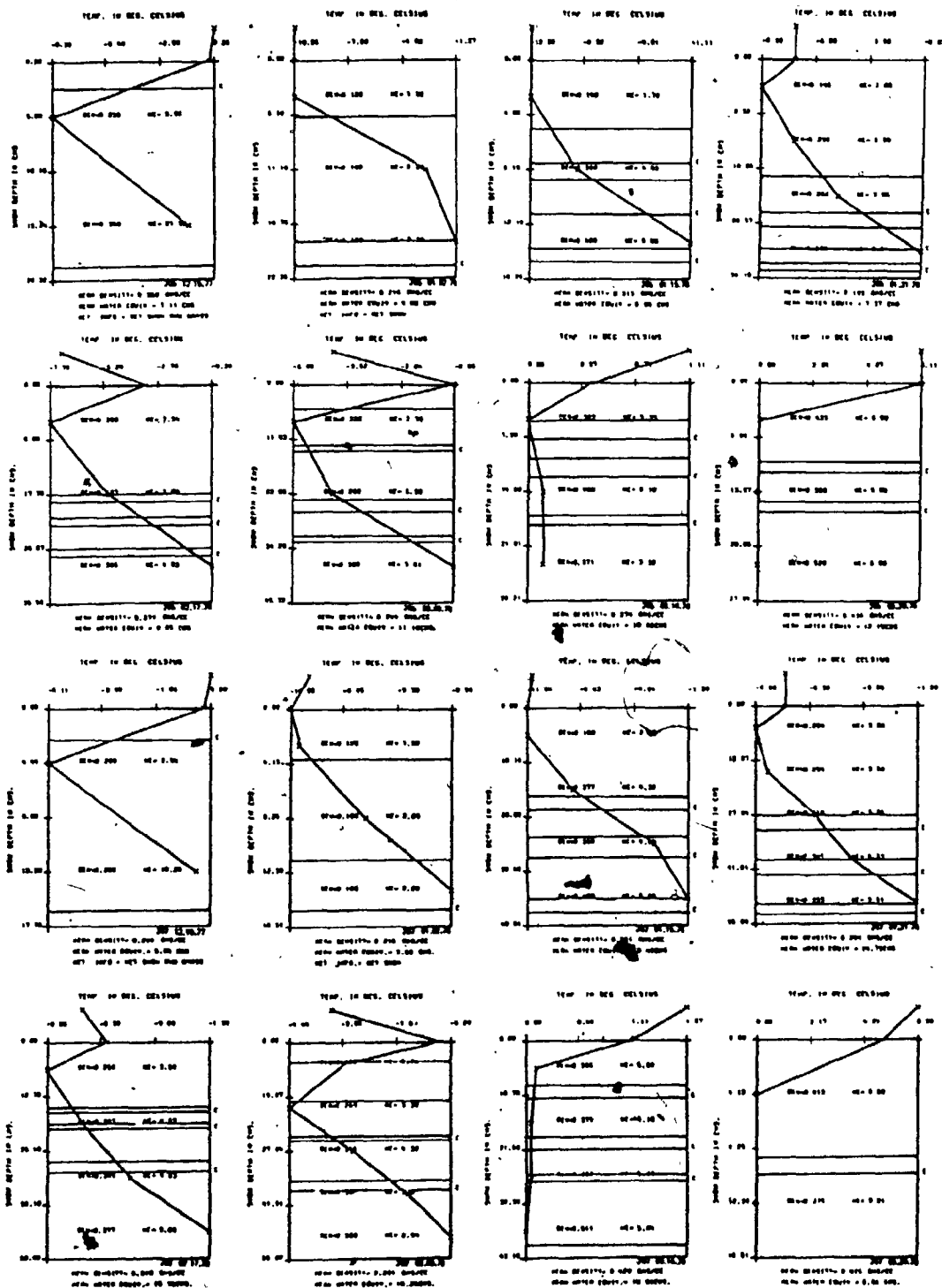
SNOW PROFILES ST. 1.04



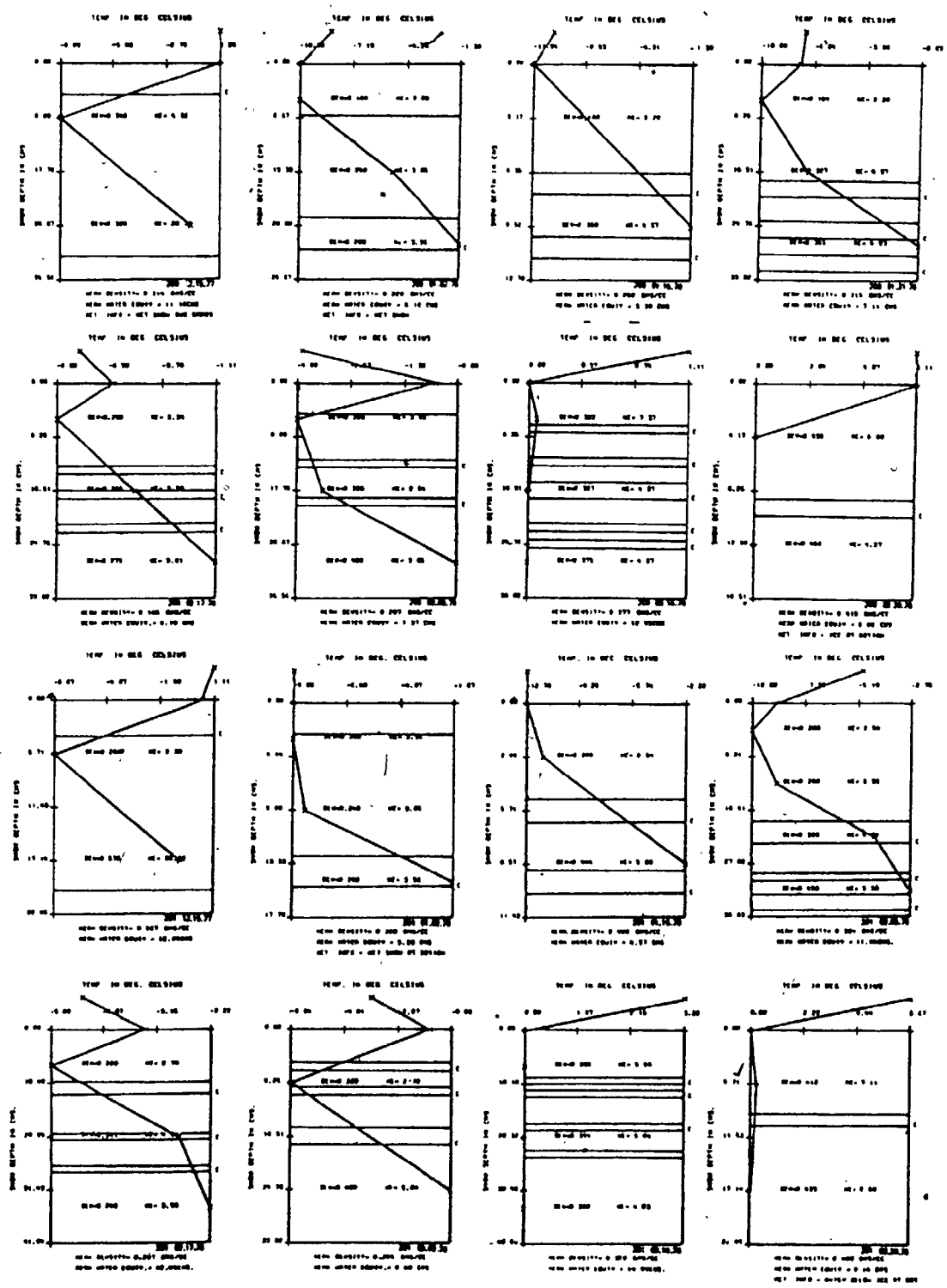
SNOW PROFILES ST. 1.05



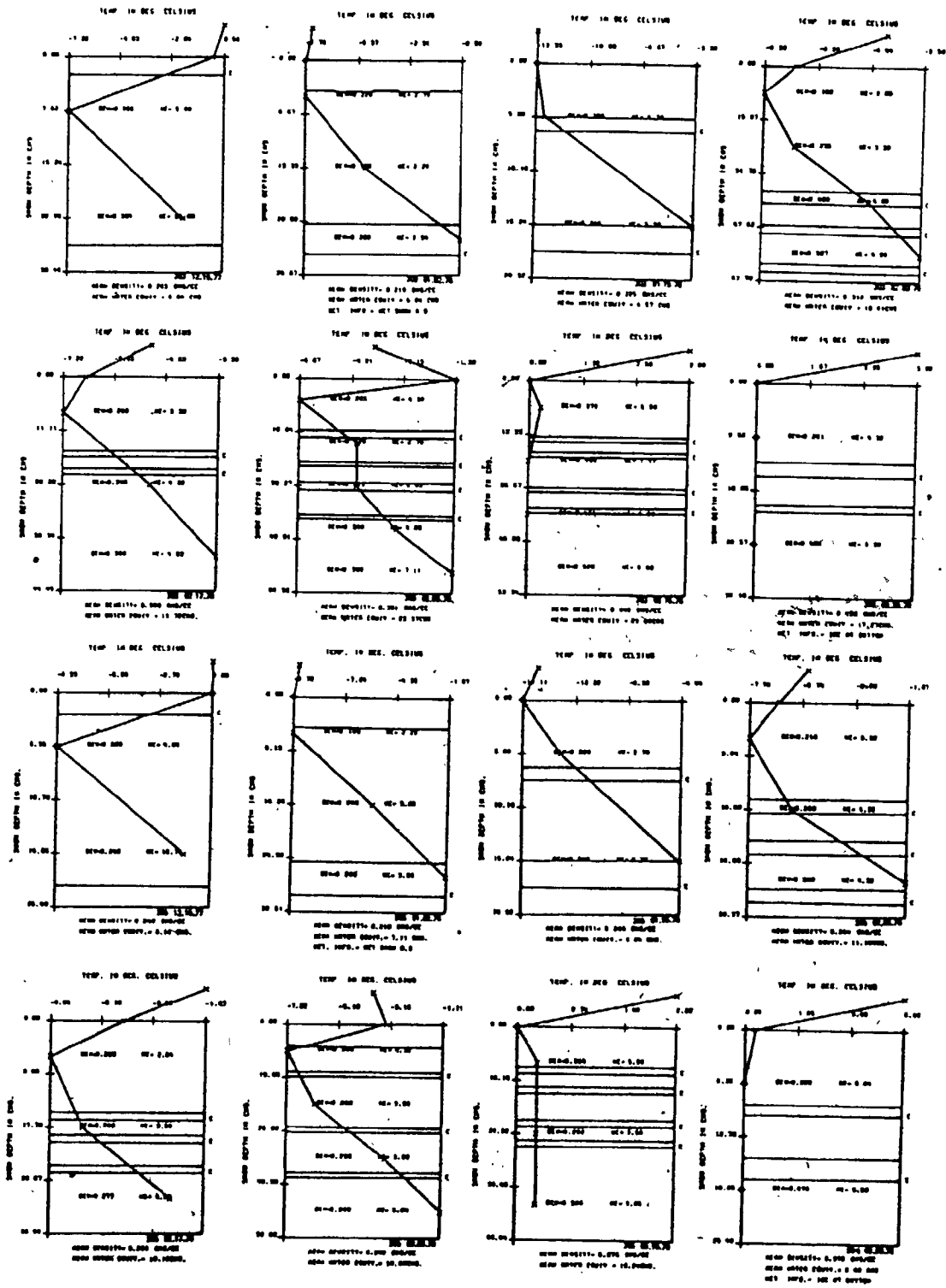
SNOW PROFILES ST. 2.01 and 2.03



SNOW PROFILES ST. 2.05 and 2.07

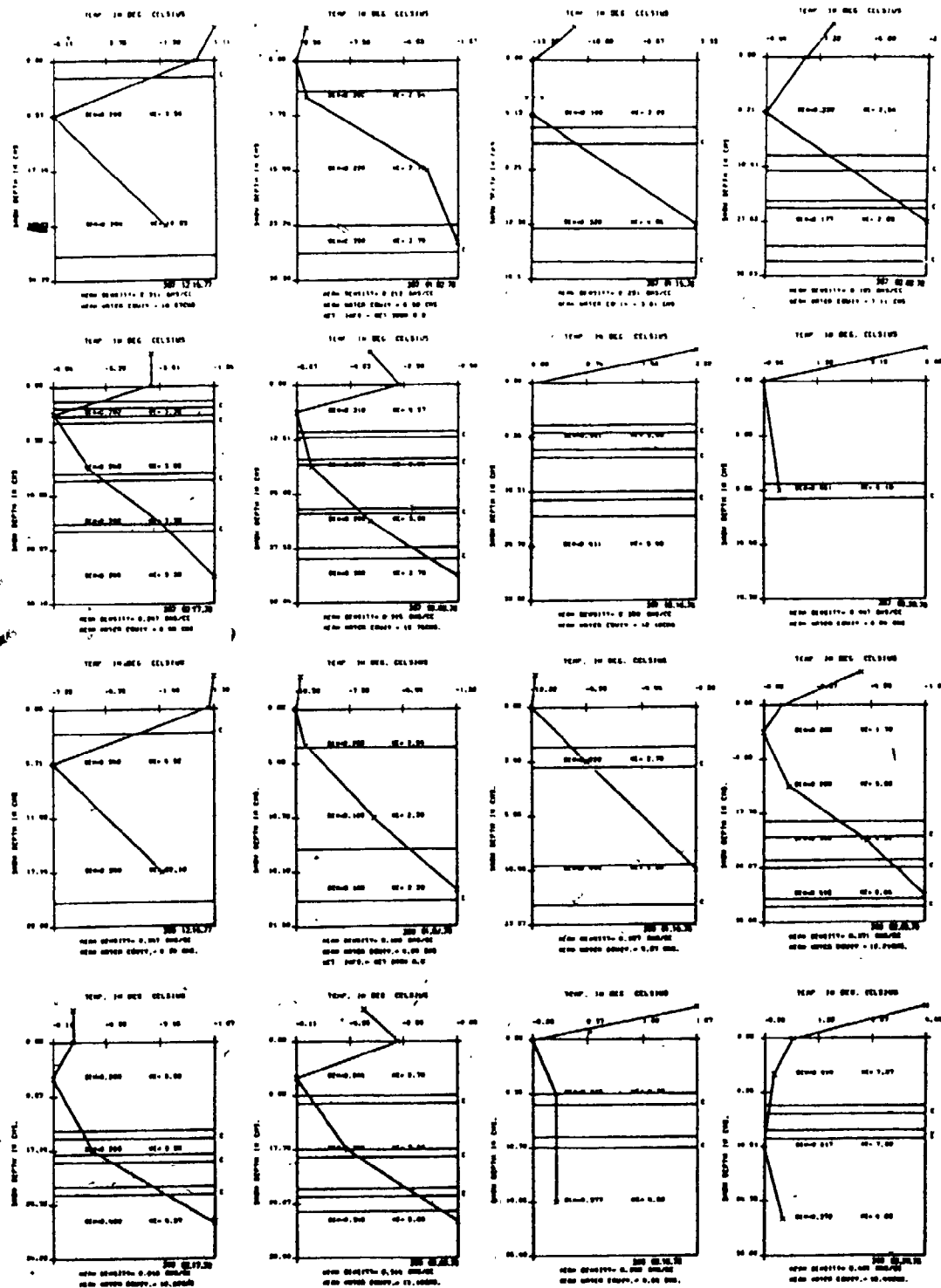


SNOW PROFILES ST. 2.09 and 3.01

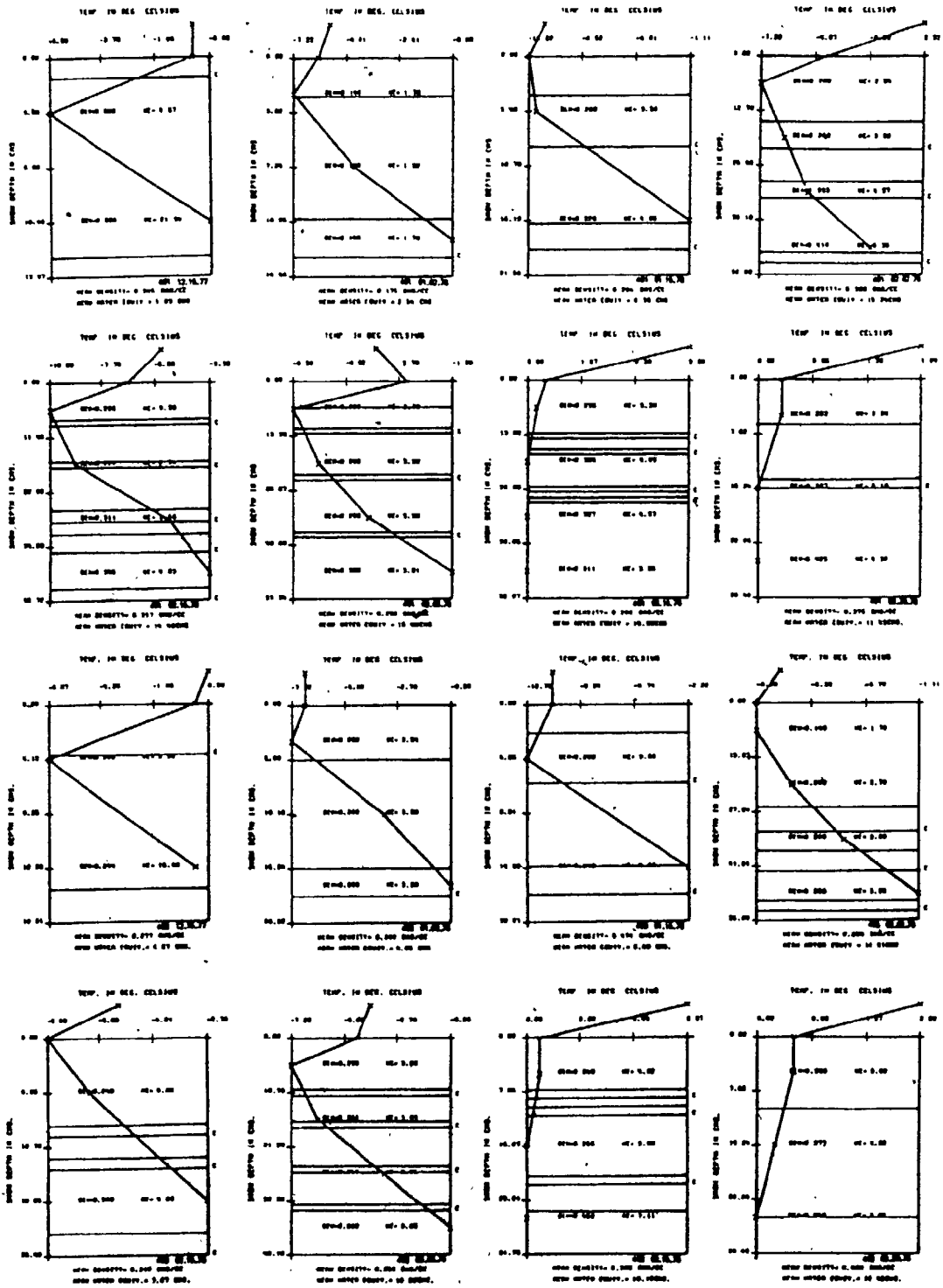


SNOW PROFILES ST. 3.03 and 3.05

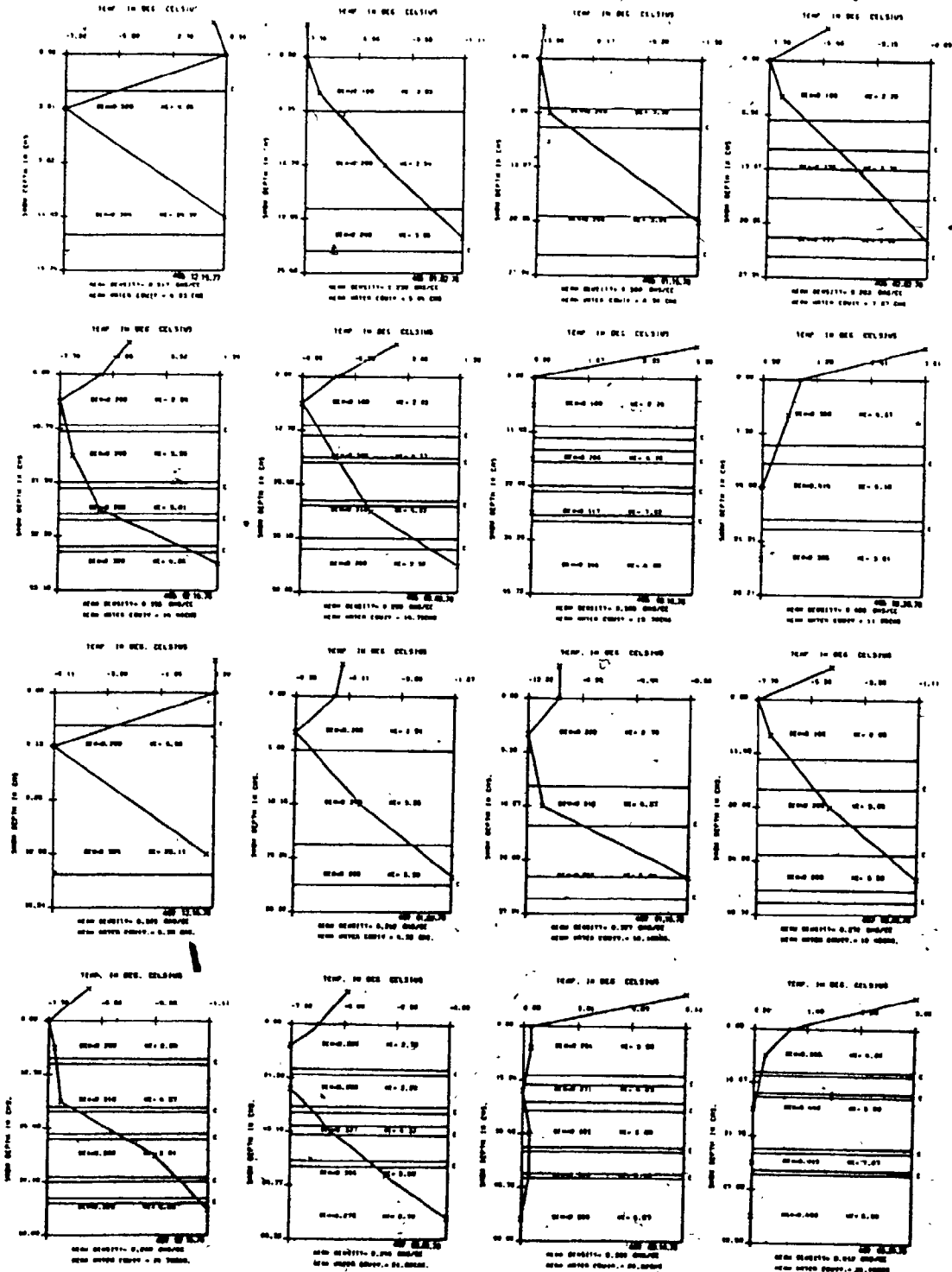




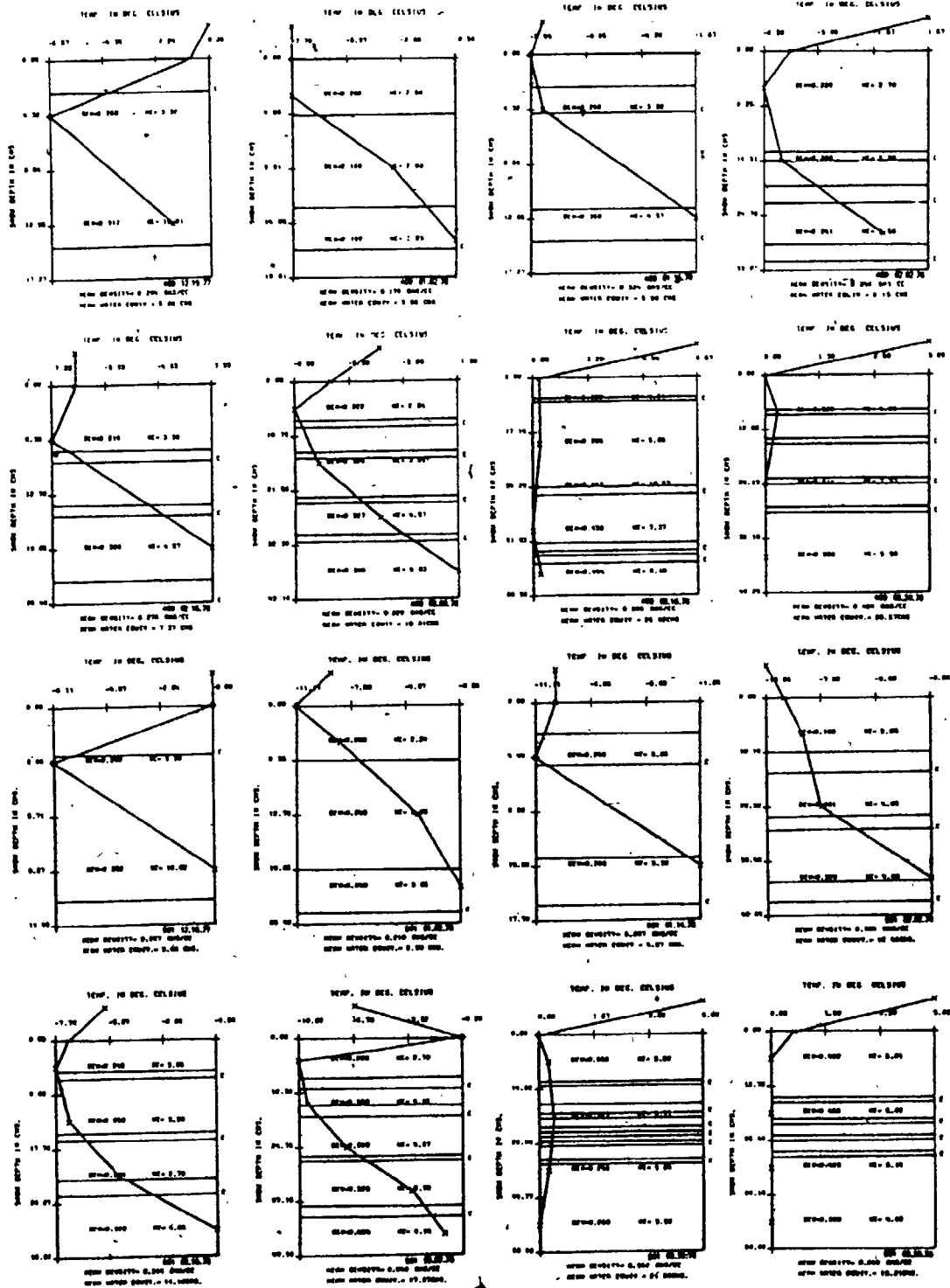
SNOW PROFILES ST. 3.07 and 3.08



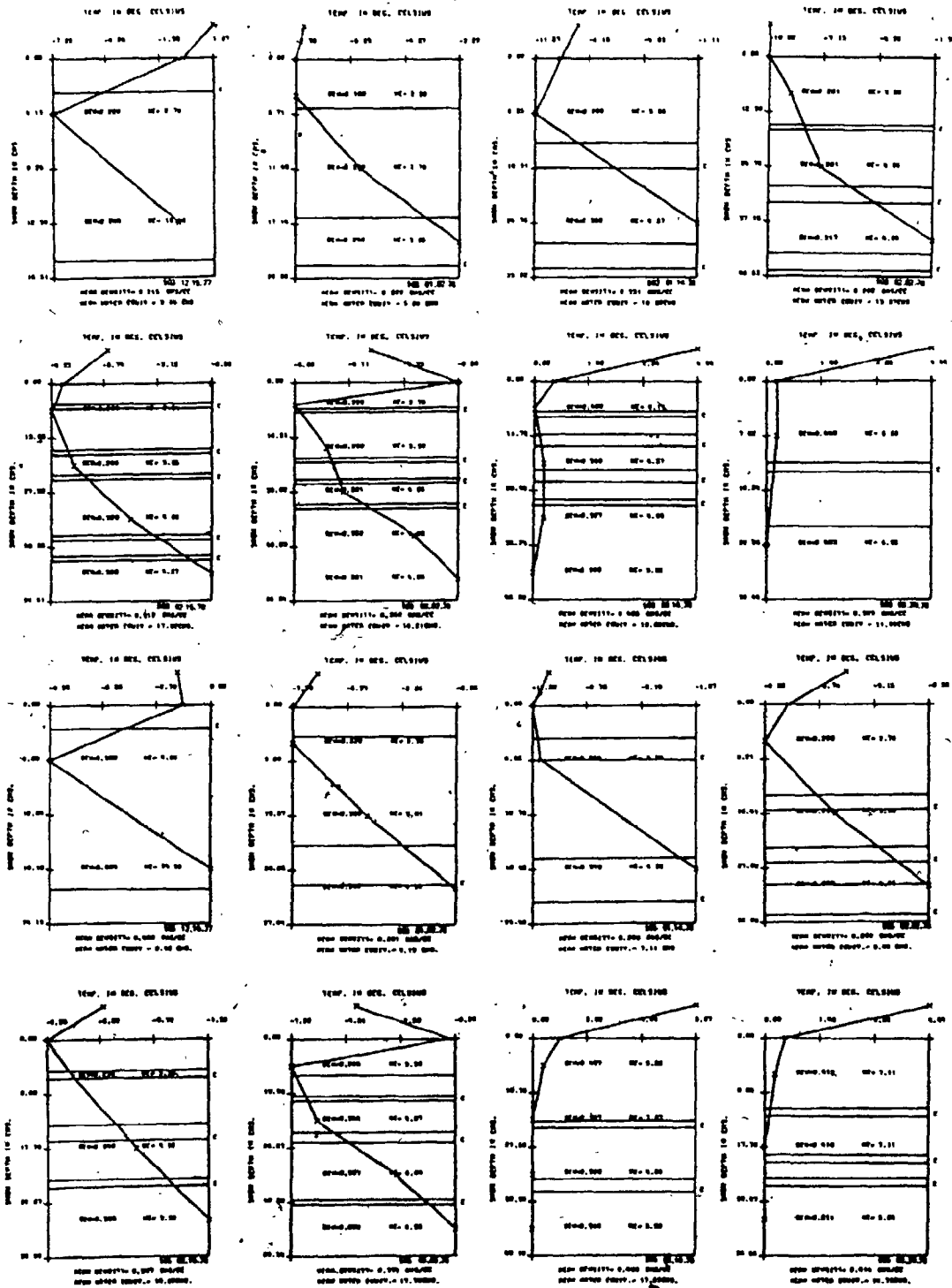
SNOW PROFILES ST. 4.01 and 4.03



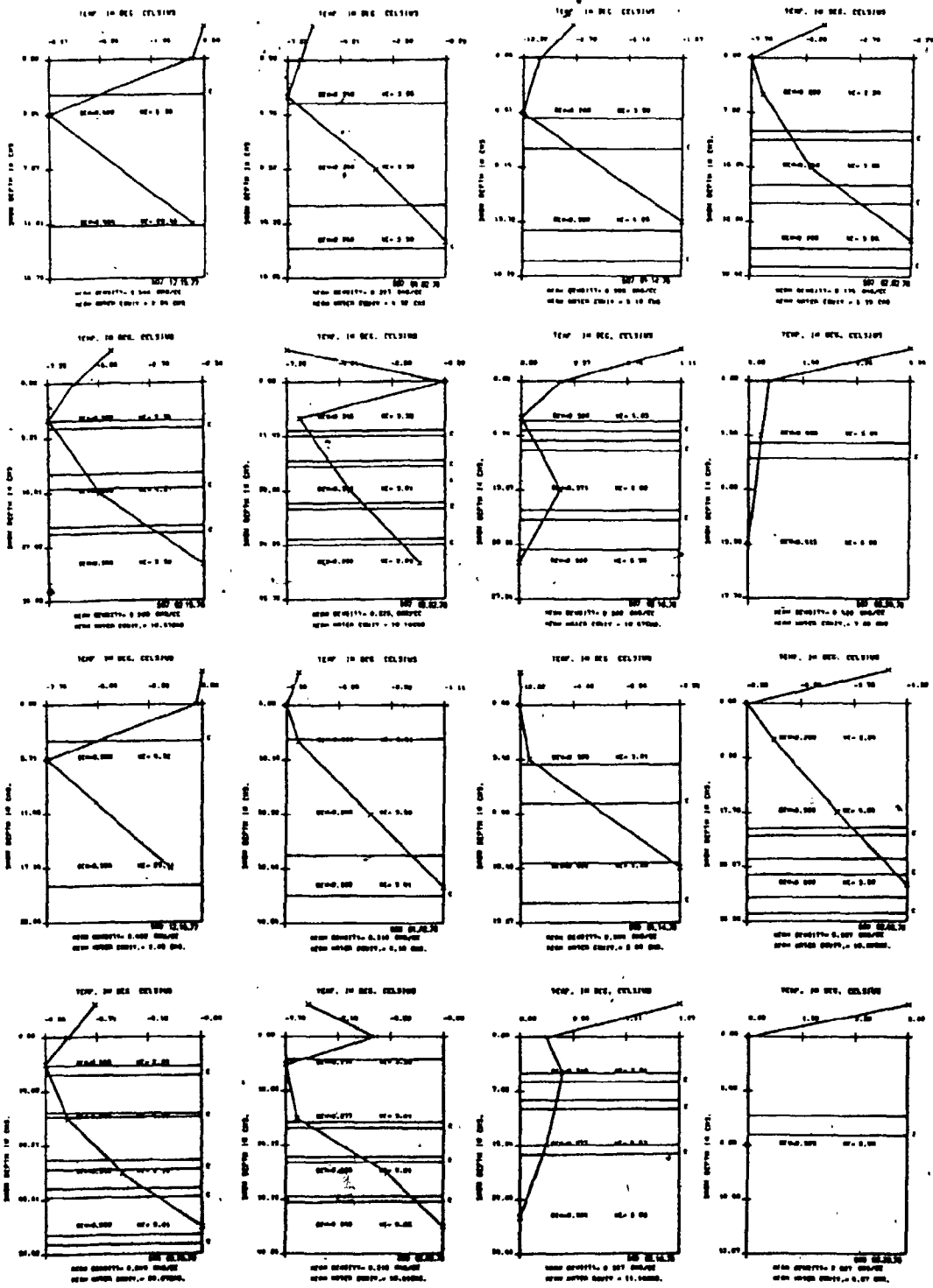
SNOW PROFILES ST. 4.05 and 4.07



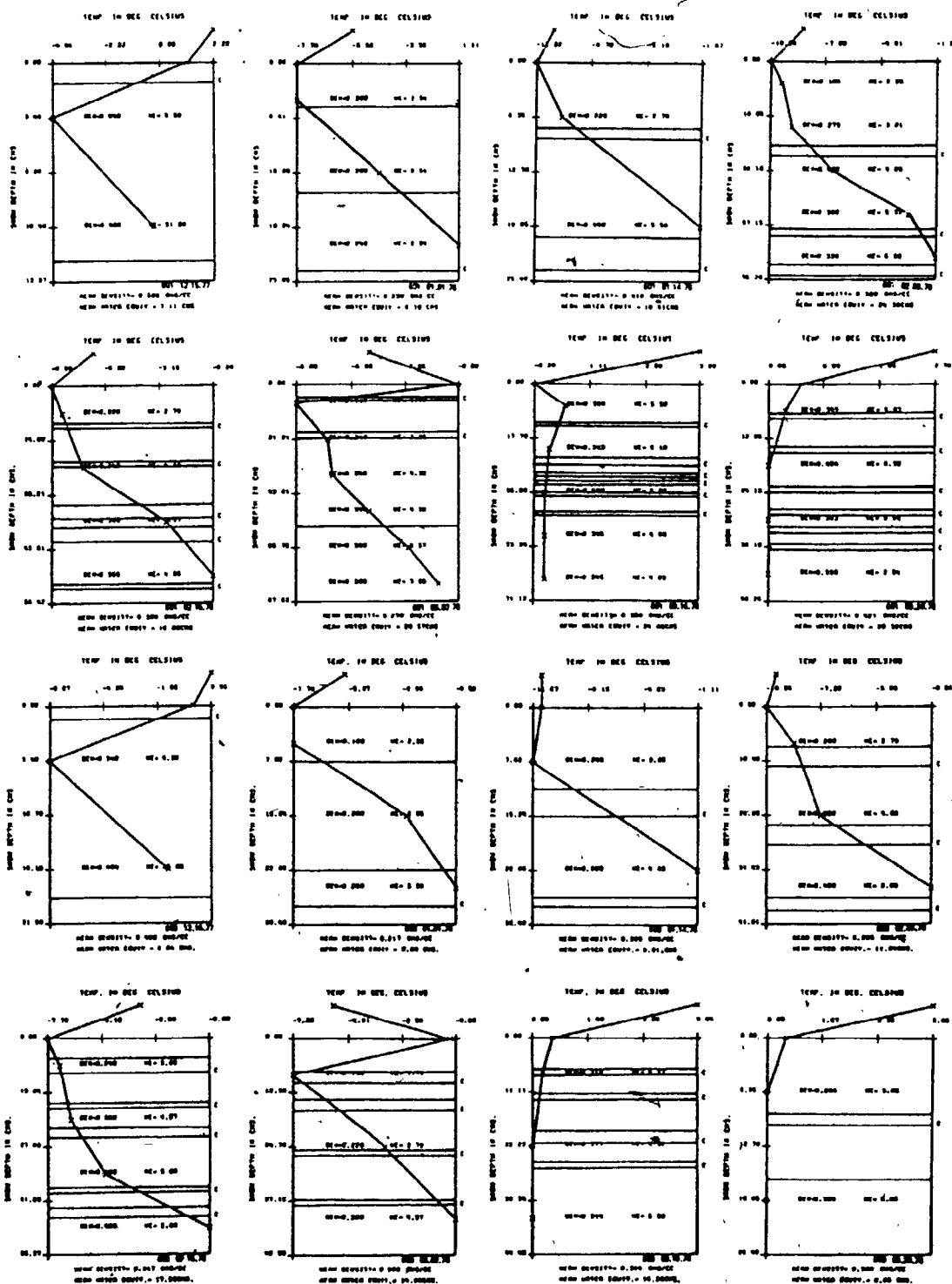
SNOW PROFILES ST. 4.09 and 5.01



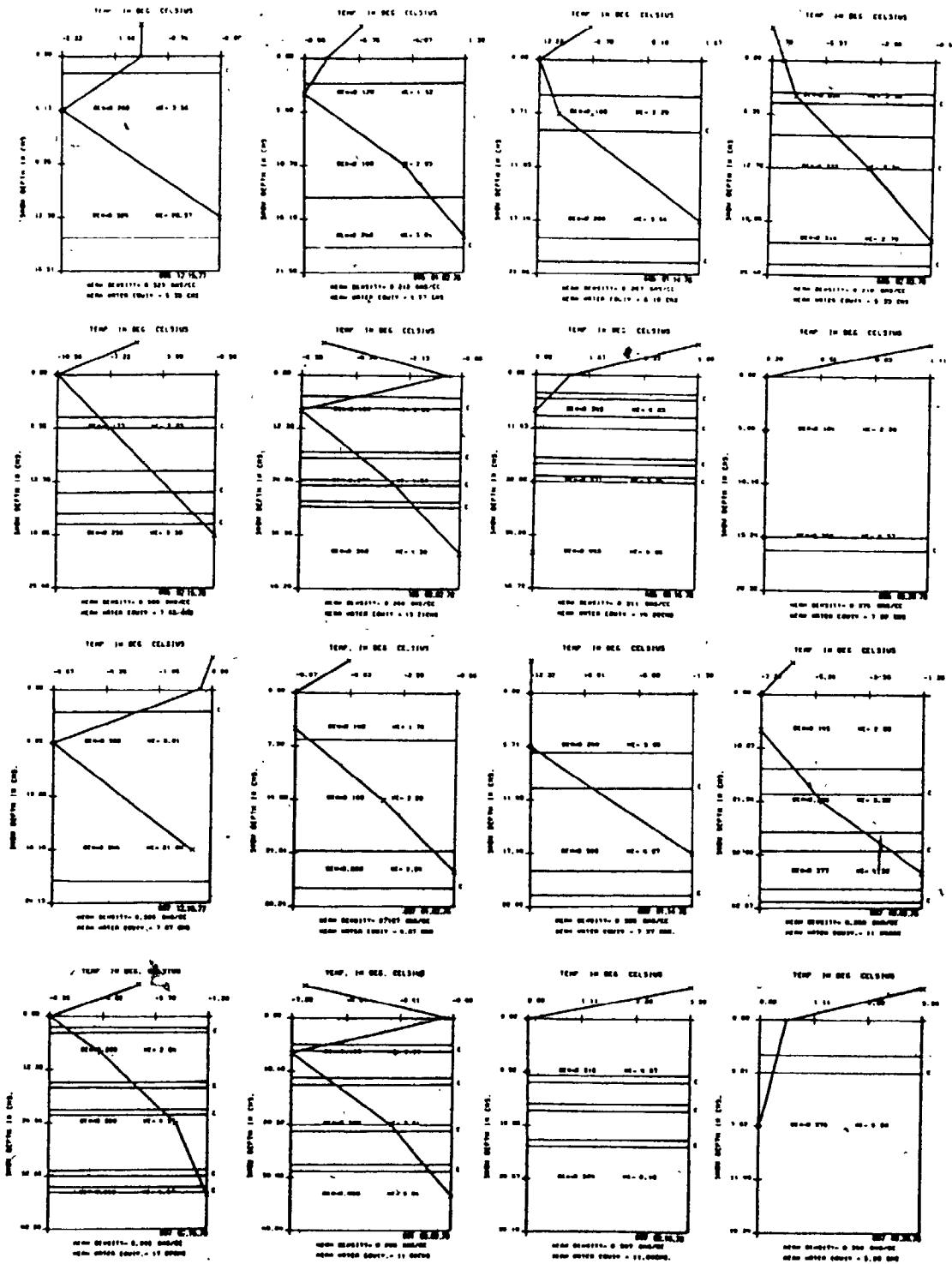
SNOW PROFILES ST. 5.03 and 5.05



SNOW PROFILES ST. 5.07 and 5.08

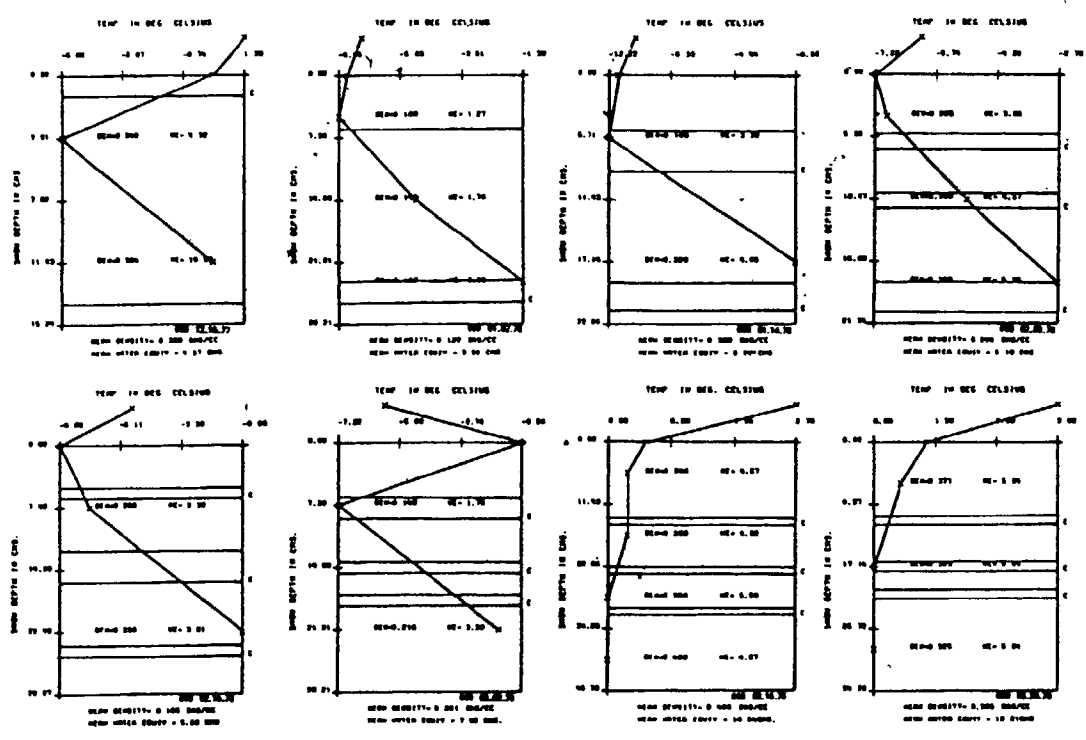


SNOW PROFILES ST. 6.01 and 6.03



SNOW PROFILES ST. 6.05 and 6.07





SNOW PROFILES ST. 6.08

APPENDIX VIII  
SAMPLE OF DATA FILES

1017803281000+350+3521501500841015+350+352000004 LIGHT FREE  
ZING RAIN

1017803280101055055046024022400+322070301505C04505C05505C020

1017803280202055Q55045024021382+322

1017803280303049049043024021428+321

1017803280404050050042024018360+321

1027803280930+330+3201501500870950+330+320000304 HEAVY FREE  
ZING RAIN

1027803280101055055044024020364+322130603005C02005C01005C010  
05C01505020

1027803280202054054047024023426+324

1027803280303055055042024018327+322

1027803280404045045043024019422+321

1037803281030+353+3501151150631045+353+350000003 LIGHT FREE  
ZING RAIN

1037803280101048048040024016333+325050201005C06505C030

1037803280202055055045024021382+323

1037803280303044044038024014318+325

1047803281100+345+3511201100701115+345+351000003 LIGHT FREE  
ZING RAIN

1047803280101045045042024018400+322090401505C02005C01505C010  
10C035

1047803280202068068046024022324+325

1047803280303055055044024020364+325

1057803281130+345+3500950950591145+345+350000003 STOPPED RA  
INING

1057803280101053053044024020377+325070301005C01005C02505C035

1057803280202055055042024018327+325

1057803280303040040038024014350+325

1017803211347+370+3851801680971400+370+385000204 RAINED THI  
S MORNING

1017803210101050050039024015300+325060203005C03005C050060

1017803210202045045039024015333+325

1017803210303068068051024027397+325

1017803210404052052043024019365+325

1027803211325+370+3901901701021342+370+390000005

1027803210101060060049024025416+325090404005C02005C03005C010  
05C070

1027803210202050050043024019380+325

2017803301419+430+4101501500871430+430+410000204  
2017803300101065065050024026400+320070303505C01005C02505C065  
2017803300202060060052024028466+320  
2017803300303078078057024033423+320  
2017803300404055055048024024436+320  
2027803301435+410+4101151130731440+410+410000200 NO PROFILE  
S  
2037803301445+435+4101351300891455+430+410000203  
2037803300101053053046024022415+321050204005C01005C075  
2037803300202059059049024025424+320  
2037803300303055060024036654+325  
2047803301500+430+4101151130741505+430+430000200 NO PROFILE  
S  
2057803301508+430+4301101050721515+430+410000203  
2057803300101060060050024026433+321050204005C01505C045  
2057803300202060060044024020333+320  
2057803300303050050050024026520+320  
2067803301520+430+4301101030601525+430+435000200 NO PROFILE  
S  
2077803301530+435+4100650650511540+435+410000202 WATER AT B  
OTTOM  
2077803300101063063050024026413+321030103505C025  
2077803300202040040039024015375+321  
2087803301545+435+4351201150771550+435+435000200 NO PROFILE  
S  
2097803301600+430+4300650650511610+430+430000202 ICE AT BOT  
TOM  
2097803300101060060050024026433+320030103505C025  
2097803300202040040042024018450+320  
2107803301615+430+4300700700541620+430+430000200 NO PROFILE  
S  
2017803161512+340+3302051801021520+330+330000204  
2017803160101046046041024017369+322070305005C02005C05005C070  
2017803160202064064048024024375+320  
2017803160303074074052024028378+321  
2017803140404058058049024025431+320  
2027803161525+330+3402302001101530+330+340000200 NO PROFILE  
S

3017803301330+440+3200900800601335+440+320000202 WATER BELO  
 W ICE AT BOT  
 3017803300101068068052024028412+325030103505C050  
 3017803300202060060050024026433+320  
 3027803301340+430+3300800750611345+430+330000200 NO PROFILE  
 S  
 3037803301350+410+3201501480921355+410+320000202 ICE AT BOT  
 TOM  
 3037803300101065065041024017261+320050205510C02005C060  
 3037803300202045045045024021466+320  
 3047803301400+410+3401050750721405+410+340000200 NO PROFILE  
 S  
 3057803301410+410+3301000900611415+410+330000202 ICE AT BOT  
 TOM  
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 3057803300202046046046024022478+325  
 3067803301420+400+3201001000721425+400+320000200 NO PROFILE  
 S  
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 3077803300101052052048024024461+320030103605C035  
 3087803301440+410+3250950900591445+410+325000200 NO PROFILE  
 S  
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 3097803300101070070053024029414+320050504005C01005C070  
 3097803300202058058054024030517+315  
 3097803300303043043040024016372+325  
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 S  
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 3017803160101065065046024022338+320090303505C00505C02005C015  
 005065  
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 3017803160303050050043024019380+320  
 3027803161405+390+3251081000671410+392+325000200 NO PROFILE  
 S  
 3037803161415+390+3202102051181420+360+320000204  
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 05C080  
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4037803301100+365+3301201200731110+365+330000203  
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4057803300202058058048024024414+320  
4057803300303038038039024015395+320  
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05C080  
4077803300202050050046024022440+325  
4077803300303070070055024031443+325  
4077803300404046046045024021456+325  
4087803301200+385+3200840840561210+390+320000200 NO PROFILE  
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4097803300101050050040024016320+325090403005C02005C03005C020  
05C070  
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4097803300303040040046024022550+320  
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S  
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5077803300101057057047024023403+326030102005C045  
5077803300202065065051024027415+320  
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05C080

6017803300930+370+3301901801040940+370+330000204  
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6017803300303060060047024023383+320  
6017803300404030030034024010333+320  
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S  
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6037803300202070070051024027386+320  
6047803301010+410+3301501450851015+410+330000200 NO PROFILE  
S  
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6057803300101055055033024009164+325030106005C015  
6057803300202050050042024018360+325  
6067803301030+370+3401701580911035+370+340000200 NO PROFILE  
S  
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6077803300101058058046024022379+320030101005C045  
6087803301050+380+3301701650931055+380+330000200 NO PROFILE  
S  
6097803301100+390+3401351350761110+390+340000203  
6097803300101062062047024023371+330070304005C02005C01005C050  
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6097803300303063063047024023365+320  
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S  
6017803160930+390+3152802501220940+390+315000205  
6017803160101065065046024022338+329130605005C04010C01005C005  
05C01005C020  
6017803160202070070048024024343+322  
6017803160303050050044024020400+320  
6017803160404055055043024019345+320  
6017803160505055055043024019345+320  
6027803160945+390+3202102001080950+390+320000200 NO PROFILE  
S  
6037803160955+400+3301751430881000+400+330000203



1.01 780328	4				
0.1111	0.1111	0.0556	0.0556		
0.4000	0.3820	0.4280	0.3600	9.5250	
1.02 780328	4				
0.1111	0.2222	0.1111	0.0556		
0.3640	0.4260	0.3270	0.4220	9.5250	
1.03 780328	3				
0.2778	0.1667	0.2778			
0.3330	0.3820	0.3180	9.7367		
1.04 780328	3				
0.1111	0.2778	0.2778			
0.4000	0.3240	0.3640	10.1600		
1.05 780328	3				
0.2778	0.2778	0.2778			
0.3770	0.3270	0.3500	8.0433		
1.01 780321	4				
0.2778	0.2778	0.2778	0.2778		
0.3000	0.3330	0.3970	0.3650	11.4300	
1.02 780321	5				
0.2778	0.2778	0.2778	0.2778	0.2778	
0.4160	0.3800	0.3560	0.3030	0.3110	9.6520
1.03 780321	4				
0.2778	0.1667	0.1111	0.1667		
0.3380	0.2800	0.3510	0.3830	8.8900	
1.04 780321	4				
0.5556	0.2778	0.0	0.1111		
0.2760	0.3270	0.4000	0.4200	10.1600	
1.05 780321	3				
0.2778	0.2222	0.2222			
0.3820	0.3960	0.4150	10.3293		

2.01 780330	4				
0.0	0.0	0.0	0.0		
0.4000	0.4660	0.4230	0.4360	9.5250	
2.03 780330	3				
0.0556	0.0	0.2778			
0.4150	0.4240	0.6540	11.4300		
2.05 780330	3				
0.0556	0.0	0.0			
0.4330	0.3330	0.5200	9.3133		
2.07 780330	2				
0.0556	0.0556				
0.4130	0.3750	8.2550			
2.09 780330	2				
0.0	0.0				
0.4330	0.4500	8.2550			
2.01 780316	4				
0.1111	0.0	0.0556	0.0		
0.3690	0.3750	0.3780	0.4310	13.0175	
2.03 780316	2				
0.0	0.0				
0.3380	0.4000	10.7950			
2.05 780316	3				
0.0	0.1111	0.1111			
0.3820	0.4000	0.3710	9.7367		
2.07 780316	4				
0.1111	0.0556	0.0556	0.0		
0.3860	0.3750	0.3820	0.5110	10.7950	
2.09 780316	3				
0.0556	0.0	0.0			
0.3220	0.3270	0.3750	11.0067		

3.01 780330 2  
0.2778 0.0  
0.4120 0.4330 11.4300  
3.03 780330 2  
0.0 0.0  
0.2610 0.4660 19.0500  
3.05 780330 2  
0.2778 0.2778  
0.3960 0.4780 12.7000  
3.07 780330 1  
0.0  
0.4610 19.3040  
3.09 780330 3  
0.0 -0.2778 0.2778  
0.4140 0.5170 0.3720 11.0067  
3.01 780316 3  
0.0 0.0 0.0  
0.3380 0.3540 0.3800 13.5467  
3.03 780316 4  
0.2778 0.0 0.0 0.0  
0.3790 0.4000 0.4330 0.5200 13.3350  
3.05 780316 3  
0.2778 0.2778 0.2778  
0.3380 0.2920 0.3960 13.5467  
3.07 780316 2  
0.0 0.0  
0.3510 0.4110 16.5100  
3.09 780316 2  
0.0 0.0  
0.3150 0.3770 12.7000

4.01	780330	3				
	0.2778	0.0	0.0			
	0.2220	0.3870	0.4250	10.1600		
4.03	780330	3				
	0.5556	0.2778	0.0			
	0.3330	0.3770	0.3430	10.1600		
4.05	780330	3				
	0.5556	0.0	0.0			
	0.3600	0.4140	0.3950	9.7367		
4.07	780330	4				
	0.5556	0.2778	0.2778	0.2778		
	0.3550	0.4400	0.4430	0.4560	15.8750	
4.09	780330	3				
	0.2778	0.0	0.0			
	0.3200	0.4140	0.5500	16.0867		
4.01	780316	4				
	0.2778	0.0	0.0	0.0		
	0.2360	0.3060	0.3270	0.3110	13.0175	
4.03	780316	3				
	0.5556	0.0	0.0			
	0.3400	0.2550	0.4520	10.5833		
4.05	780316	4				
	0.0	0.0	0.0	0.0		
	0.1800	0.2860	0.5170	0.3450	11.4300	
4.07	780316	5				
	0.2778	0.0	0.2778	0.2778	0.0	
	0.2540	0.2710	0.3330	0.3200	0.3600	12.1920
4.09	780316	5				
	0.2778	0.2778	0.1111	0.0	0.2778	
	0.3000	0.2940	0.4420	0.4390	0.4440	13.7160

5.01 780330 4  
0.0 0.0 0.0 0.0  
0.4600 0.4560 0.4230 0.3800 12.7000  
5.03 780330 2  
0.2778 0.0  
0.4400 0.4630 15.2400  
5.05 780330 3  
0.2778 0.0 0.0  
0.4120 0.4180 0.5110 11.8533  
5.07 780330 2  
0.3333 0.0  
0.4030 0.4150 8.8900  
5.09 780330 1  
0.0  
0.3230 13.9700  
5.01 780316 4  
0.2778 0.4444 0.2778 0.0  
0.3660 0.3230 0.2460 0.2600 14.9225  
5.03 780316 4  
0.0 0.2778 0.2778 0.0  
0.4000 0.3600 0.3270 0.3000 11.7475  
5.05 780316 4  
0.4444 0.0 0.0 0.0  
0.4070 0.3970 0.3200 0.3660 10.7950  
5.07 780316 3  
0.0 0.2778 0.0  
0.3960 0.3710 0.3500 9.3133  
5.09 780316 3  
0.4444 0.2778 0.0  
0.3190 0.2770 0.3640 10.1600

6.01 780330 4  
0.2778 0.0 0.0 0.0  
0.3450 0.4540 0.3830 0.3330 12.0650  
6.03 780330 2  
0.0 0.0  
0.2660 0.3860 12.7000  
6.05 780330 2  
0.2778 0.2778  
0.1640 0.3600 10.1600  
6.07 780330 1  
0.0  
0.3790 15.2400  
6.09 780330 3  
0.5556 0.0 0.0  
0.3710 0.3830 0.3650 11.4300  
6.01 780316 5  
0.5000 0.1111 0.0 0.0 0.0  
0.3380 0.3430 0.4000 0.3450 0.3450 14.2240  
6.03 780316 3  
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0.3100 0.3770 0.3140 14.8167  
6.05 780316 3  
0.0 0.0 0.0  
0.3450 0.3710 0.4430 15.2400  
6.07 780316 2  
0.0 0.0  
0.3160 0.3240 19.0500  
6.09 780316 4  
0.2778 0.2778 0.0 0.0  
0.3460 0.3090 0.3660 0.4000 11.4300

APPENDIX IX  
RESULTS OF SIMULATION

Appendix IX

FLUX (cal cm<sup>-2</sup>hr<sup>-1</sup>)

S T A T I O N N U M B E R

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
Nov. 12	-0.03	-	-	-	-	-	-0.03
13	-0.04	-	-	-	-	-	-0.04
14	-0.01	-	-	-	-	-	-0.01
15	0.00	-	-	-	-	-	0.00
16	+0.02	-	-	-	-	-	+0.02
17	+0.01	-	-	-	-	-	+0.02
18	0.00	-	-	-	-	-	0.00
19	0.00	-	-	-	-	-	0.00
20	-0.01	-	-	-	-	-	-0.01
21	+0.02	-	-	-	-	-	+0.02
22	-0.01	-	-	-	-	-	-0.01
23	0.00	-	-	-	-	-	0.00
24	0.00	-	-	-	-	-	0.00
25	-0.01	-	-	-	-	-	-0.01
26	-0.01	-	-	-	-	-	-0.01
27	-0.02	-	-	-	-	-	-0.02
28	-0.01	-	-	-	-	-	-0.01
29	-0.02	-	-	-	-	-	-0.02
30	-0.01	-	-	-	-	-	-0.01
Dec. 01	0.00	-	-	-	-	-	0.00
02	0.00	-	-	-	-	-	0.00
03	0.00	-	-	-	-	-	0.00
04	-0.01	-	-	-	-	-	-0.01
05	-0.01	-	-	-	-	-	-0.01
06	-0.01	-	-	-	-	-	-0.01
07	-0.01	-	-	-	-	-	-0.01
08	-0.03	-	-	-	-	-	-0.03



## Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
09	0.00	-	-	-	-	-	0.00
10	-0.03	-	-	-	-	-	-0.03
11	-0.05	-	-	-	-	-	-0.05
12	-0.19	-	-	-	-	-	-0.19
13	+0.05	-	-	-	-	-	+0.05
14	+0.05	-	-	-	-	-	+0.05
15	+0.01	-	-	-	-	+0.19	+0.10
16	0.00	-	-	-	-	+0.02	+0.01
17	+0.04	-	-	-	-	+0.04	+0.04
18	+0.01	-	-	-	-	+0.01	+0.01
19	+0.08	-	-	-	-	+0.07	+0.08
20	+0.02	-	-	-	-	+0.02	+0.02
21	-0.03	-	-	-	-	-0.02	-0.03
22	-0.48	-	-	-	-	-0.46	-0.47
23	-0.12	-	-	-	-	-0.10	-0.11
24	0.00	-	-	-	-	+0.04	+0.04
25	+0.22	-	-	-	-	+0.25	+0.23
26	-2.10	-	-	-	-	-2.09	-2.10
27	-1.81	-	-	-	-	-1.49	-1.65
28	-1.92	-	-	-	-	-1.70	-1.81
29	-0.92	-	-	-	-	-0.59	-0.76
30	-0.17	-	-	-	-	-0.14	-0.16
31	-0.07	-	-	-	-	-0.11	-0.09
Jan. 01	-0.28	-	-	-	-	-0.30	-0.29
02	-1.25	-	-	-	-	-1.34	-1.29
03	-1.70	-	-	-	-	-1.80	-1.75
04	-2.05	-	-	-	-	-2.03	-2.04
05	+0.03	-	-	-	-	+0.06	+0.04
06	-0.02	-	-	-	-	-0.02	-0.02
07	-0.06	-	-	-	-	-0.08	-0.07

Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
08	+0.10	-	-	-	-	+0.15	+0.12
09	-0.15	-	-	-	-	-0.35	-0.25
10	-2.18	-	-	-	-	-1.92	-2.05
11	-0.71	-	-	-	-	-0.93	-0.82
12	-0.88	-	-	-	-	-1.11	-1.00
13	-0.69	-	-	-	-	-0.82	-0.76
14	-0.39	-	-	-	-	-0.41	-0.40
15	-1.07	-	-	-	-	-2.03	-1.55
16	-2.14	-	-	-	-	-1.54	-1.84
17	-3.32	-	-	-	-	-2.03	-2.67
18	-1.57	-	-	-	-	-0.53	-1.05
19	-2.93	-	-	-	-	-1.40	-2.16
20	-1.28	-	-	-	-	-0.44	-0.86
21	-2.40	-	-	-	-	-1.15	-1.76
22	-2.18	-	-	-	-	-0.83	-1.51
23	-3.44	-	-	-	-	-2.07	-2.75
24	-0.28	-	-	-	-	-0.64	-0.46
25	+0.07	-	-	-	-	+0.04	+0.06
26	+0.08	-	-	-	-	+0.07	+0.08
27	-0.23	-	-	-	-	-0.50	-0.36
28	-0.30	-	-	-	-	-0.82	-0.56
29	-0.32	-	-	-	-	-0.83	-0.57
30	-0.22	-	-	-	-	-0.54	-0.38
31	-0.36	-	-	-	-	-1.10	-0.73
Feb. 01	-0.13	-	-	-	-	-1.25	-0.69
02	-0.20	-	-	-	-	-1.11	-0.66
03	-0.21	-	-	-	-	-1.30	-0.76
04	-1.55	-	-	-	-	-4.25	-2.90
05	-0.56	-	-	-	-	-3.67	-2.11
06	-0.19	-	-	-	-	-2.28	-1.23

## Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
07	-0.41	-	-	-	-	-2.69	-1.55
08	-0.42	-	-	-	-	-2.54	-1.48
09	-0.42	-	-	-	-	-1.95	-1.18
10	-0.40	-	-	-	-	-1.96	-1.18
11	-0.21	-	-	-	-	-0.62	-0.41
12	-0.15	-	-	-	-	-0.32	-0.23
13	-0.06	-	-	-	-	-0.11	-0.08
14	-0.06	-	-	-	-	-1.76	-1.03
15	-0.64	-	-	-	-	-2.91	-1.77
16	-0.46	-	-	-	-	-0.56	-0.51
17	-0.14	-0.27	-	-	-	-0.14	-0.18
18	-0.42	-0.80	-	-	-	-0.82	-0.68
19	-0.78	-1.44	-	-	-	-1.29	-1.17
20	-0.95	-1.85	-	-	-	-1.57	-1.46
21	-0.75	-1.24	-	-	-	-0.95	-0.98
22	-0.50	-1.46	-	-	-	-1.14	-1.03
23	-0.44	-0.93	-	-	-	-0.65	-0.67
24	-0.23	-0.36	-	-	-	-0.25	-0.28
25	-0.06	-0.07	-	-	-	-0.03	-0.05
26	-0.20	-0.12	-	-	-	-0.03	-0.11
27	-0.18	-0.53	-	-	-	-0.48	-0.40
28	-0.22	-0.43	-	-	-	-0.23	-0.29
Mar. 01	-0.07	-0.47	-	-	-	-0.21	-0.25
02	-0.38	-1.27	-	-	-	-0.63	-0.76
03	-0.39	-1.41	-	-	-	-0.97	-0.92
04	-0.32	-1.26	-	-	-	-0.43	-0.67
05	-0.28	-1.57	-	-	-	-0.58	-0.81
06	-0.37	-2.30	-	-	-	-0.76	-1.14
07	-0.43	-2.71	-	-	-	-1.07	-1.40
08	-0.85	-2.36	-	-	-	-0.84	-1.35

## Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
09	-1.97	-2.66	-	-	-	-0.94	-1.85
10	-0.30	-1.17	-	-	-	-0.51	-0.66
11	-0.05	-0.19	-	-	-	-0.03	-0.09
12	+0.01	-0.02	-	-	-	-0.02	-0.01
13	-0.02	-0.22	-	-	-	-0.07	-0.10
14	+0.02	+0.19	-	-	-	+0.04	+0.08
15	+0.03	+0.12	-	-	-	+0.06	+0.07
16	-0.03	-0.49	-0.14	-0.06	-0.27	-0.03	-0.17
17	-0.04	-0.02	-0.04	-0.01	-0.02	-0.03	-0.03
18	-0.35	-0.41	-0.73	-0.38	-0.42	-0.58	-0.48
19	+0.08	+0.01	+0.09	+0.08	+0.08	+0.09	+0.07
20	-0.17	-0.07	-0.63	-0.08	-0.21	-0.21	-0.23
21	+0.10	+0.09	+0.12	+0.09	+0.09	+0.19	+0.11
22	+0.10	+0.03	+0.12	+0.09	+0.05	+0.09	+0.08
23	+0.23	+0.07	+0.27	+0.16	+0.17	-0.21	+0.12
24	-0.42	-0.06	-0.33	-0.05	-0.28	-0.51	-0.27
25	-0.30	-0.10	-0.38	-0.22	-0.23	-0.30	-0.25
26	+0.03	0.00	+0.04	+0.10	-0.17	+0.03	+0.01
27	+0.07	+0.03	+0.07	+0.03	+0.04	+0.04	+0.05
28	-0.02	0.00	+0.01	+0.01	+0.01	+0.01	0.00
29	+0.21	+0.06	+0.21	+0.12	+0.12	+0.17	+0.15
30	-0.06	-0.01	-0.06	-0.11	-0.05	-0.09	-0.06
31	+0.16	+0.45	+0.16	+0.24	+0.17	+0.16	+0.22
Apr. 01	+0.62	+1.48	+0.73	+0.94	+0.74	+0.69	+0.86
02	+0.02	-0.05	+0.01	+0.12	-0.02	-0.04	+0.01
03	+0.02	+0.05	+0.02	+0.05	+0.03	+0.02	+0.03
04	+0.04	+0.16	+0.07	+0.10	+0.08	+0.05	+0.08
05	+0.19	+0.51	+0.14	+0.40	+0.21	+0.29	+0.29
06	+0.16	+0.47	+0.17	+0.30	+0.18	+0.21	+0.25
07	+0.52	+1.32	+0.58	+0.74	+0.59	+0.57	+0.72

## Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
08	+0.24	+0.58	+0.22	+0.42	+0.27	+0.28	+0.33
09	-0.04	-0.19	-0.02	-0.09	-0.03	-0.07	-0.07
10	+0.21	+0.59	+0.24	+0.30	+0.24	+0.25	+0.30
11	+0.88	+1.78	+0.94	+1.37	+1.04	+1.14	+1.19
12	+0.74	+1.47	+0.79	+1.10	+0.81	+0.88	+0.97
13	+2.44	+5.43	+2.38	+3.06	+2.41	+2.80	+3.07
14	+0.47	+1.18	+0.51	+0.82	+0.64	+0.62	+0.71
15	+0.24	+0.62	+0.23	+0.36	+0.27	+0.26	+0.33
16	-0.14	+0.19	-0.03	+0.10	-0.01	+0.18	+0.03
17	+0.06	+0.22	+0.03	+0.12	+0.07	+0.28	+0.13
18	+0.25	+0.58	+0.24	+0.44	+0.29	+0.43	+0.37
19	+1.16	+2.58	+1.16	+1.62	+1.23	+1.42	+1.53
20	+0.93	+2.13	+0.92	+1.26	+1.00	+1.46	+1.28
21	+0.11	+0.27	+0.09	+0.16	+0.09	+0.28	+0.17
22	-0.17	-0.30	-0.14	+0.10	-0.12	-0.32	-0.19
23	+0.10	+0.24	+0.01	+0.16	-0.05	+0.32	+0.13
24	+0.47	+0.99	+0.48	+0.69	+0.28	+0.66	+0.59
25	+0.22	+0.53	+0.20	+0.36	+0.13	+0.34	+0.30
26	+0.37	+0.90	+0.39	+0.65	+0.22	+0.46	+0.50
27	+0.31	+0.72	+0.26	+0.50	+0.18	+0.42	+0.40
28	+0.33	+0.79	+0.34	+0.58	+0.20	+0.35	+0.43
29	+0.36	+0.71	+0.30	+0.59	+0.21	+0.38	+0.42
30	-0.25	-0.47	-0.24	-0.41	-0.34	-0.44	-0.36

SUBLIMATION (gm cm<sup>-3</sup>)

Nov. 13	-0.05	-	-	-	-	-	-0.05
14	-0.01	-	-	-	-	-	-0.01
15	-0.01	-	-	-	-	-	-0.01
16	0.00	-	-	-	-	-	0.00
17	+0.01	-	-	-	-	-	+0.01
18	-0.01	-	-	-	-	-	-0.01

Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
19	0.00	-	-	-	-	-	0.00
20	-0.01	-	-	-	-	-	-0.01
21	+0.01	-	-	-	-	-	+0.01
22	+0.01	-	-	-	-	-	+0.01
23	-0.01	-	-	-	-	-	-0.01
24	0.00	-	-	-	-	-	0.00
25	+0.01	-	-	-	-	-	+0.01
26	-0.01	-	-	-	-	-	-0.01
27	-0.01	-	-	-	-	-	-0.01
28	+0.01	-	-	-	-	-	+0.01
29	-0.02	-	-	-	-	-	-0.02
30	0.00	-	-	-	-	-	0.00
Dec. 01	-0.01	-	-	-	-	-	-0.01
02	+0.02	-	-	-	-	-	+0.02
03	-0.03	-	-	-	-	-	-0.03
04	0.00	-	-	-	-	-	0.00
05	+0.01	-	-	-	-	-	+0.01
06	0.00	-	-	-	-	-	0.00
07	0.00	-	-	-	-	-	0.00
08	0.00	-	-	-	-	-	0.00
09	+0.02	-	-	-	-	-	+0.02
10	-0.02	-	-	-	-	-	-0.02
11	+0.01	-	-	-	-	-	+0.01
12	-0.04	-	-	-	-	-	-0.04
13	-0.01	-	-	-	-	-	-0.01
14	-0.03	-	-	-	-	-	-0.03
15	-0.03	-	-	-	-	-	-0.03
16	-0.02	-	-	-	-	-0.29	-0.16
17	+0.08	-	-	-	-	+0.07	+0.08
18	-0.03	-	-	-	-	-0.17	-0.10
19	+0.03	-	-	-	-	+0.16	-0.10
20	-0.03	-	-	-	-	-0.06	-0.05

## Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
Dec. 21	-0.03	-	-	-	-	-0.10	-0.07
22	-0.15	-	-	-	-	+0.39	+0.12
23	-0.01	-	-	-	-	-0.17	-0.09
24	-0.06	-	-	-	-	-0.11	-0.09
25	+0.02	-	-	-	-	+0.15	+0.09
26	+0.12	-	-	-	-	+0.42	+0.27
27	+0.02	-	-	-	-	-0.04	-0.01
28	0.00	-	-	-	-	+0.03	+0.02
29	-0.02	-	-	-	-	-0.21	-0.12
30	-0.06	-	-	-	-	-0.21	-0.14
31	-0.07	-	-	-	-	-0.13	-0.10
Jan. 01	+0.06	-	-	-	-	+0.18	+0.12
02	-0.03	-	-	-	-	+0.28	+0.13
03	+0.10	-	-	-	-	+0.19	+0.15
04	+0.02	-	-	-	-	+0.02	+0.02
05	-0.07	-	-	-	-	-0.10	-0.09
06	-0.03	-	-	-	-	-0.07	-0.05
07	-0.03	-	-	-	-	-0.02	-0.03
08	+0.01	-	-	-	-	+0.02	+0.02
09	-0.04	-	-	-	-	+0.05	+0.01
10	+0.13	-	-	-	-	+0.08	+0.10
11	-0.06	-	-	-	-	-0.05	-0.06
12	+0.02	-	-	-	-	0.00	+0.01
13	-0.01	-	-	-	-	-0.01	-0.01
14	-0.04	-	-	-	-	-0.04	-0.04
15	+0.17	-	-	-	-	+0.32	+0.25
16	-0.05	-	-	-	-	-0.01	-0.03
17	+0.02	-	-	-	-	0.00	+0.01
18	+0.02	-	-	-	-	-0.09	-0.04
19	-0.07	-	-	-	-	0.00	-0.04
20	+0.03	-	-	-	-	-0.07	-0.02

## Appendix LX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
Jan. 21	+0.03	-	-	-	-	+0.11	+0.06
22	-0.05	-	-	-	-	-0.09	-0.07
23	-0.01	-	-	-	-	+0.14	+0.07
24	+0.04	-	-	-	-	-0.01	-0.03
25	+0.01	-	-	-	-	-0.30	+0.15
26	-0.02	-	-	-	-	+0.03	+0.01
27	-0.02	-	-	-	-	+0.14	+0.06
28	+0.01	-	-	-	-	0.00	+0.01
29	+0.01	-	-	-	-	+0.02	+0.02
30	-0.01	-	-	-	-	-0.10	-0.06
31	+0.01	-	-	-	-	+0.10	+0.06
Feb. 01	-0.03	-	-	-	-	+0.04	+0.01
02	+0.03	-	-	-	-	0.00	+0.02
03	0.00	-	-	-	-	+0.02	+0.01
04	-0.01	-	-	-	-	+0.22	+0.11
05	+0.05	-	-	-	-	+0.05	+0.05
06	-0.08	-	-	-	-	-0.10	-0.09
07	+0.10	-	-	-	-	+0.05	+0.08
08	0.00	-	-	-	-	-0.02	-0.01
09	+0.03	-	-	-	-	-0.07	-0.02
10	0.00	-	-	-	-	+0.01	+0.01
11	0.00	-	-	-	-	-0.11	-0.06
12	-0.01	-	-	-	-	+0.02	+0.01
13	-0.03	-	-	-	-	-0.09	-0.06
14	0.00	-	-	-	-	+0.16	+0.08
15	-0.01	-	-	-	-	+0.08	+0.04
16	+0.02	-	-	-	-	+0.07	+0.04
17	-0.03	-	-	-	-	-0.11	-0.07
18	+0.01	+0.02	-	-	-	+0.09	+0.04
19	+0.03	+0.03	-	-	-	+0.03	+0.03
20	+0.02	+0.03	-	-	-	+0.02	+0.02



## Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
Feb. 21	0.00	-0.03	-	-	-	-0.06	-0.03
22	+0.02	+0.04	-	-	-	+0.04	+0.03
23	+0.01	-0.03	-	-	-	-0.06	-0.03
24	0.00	-0.03	-	-	-	-0.06	-0.03
25	-0.05	-0.03	-	-	-	-0.11	-0.06
26	+0.02	-0.04	-	-	-	-0.06	-0.03
27	-0.04	+0.05	-	-	-	+0.14	+0.05
28	0.00	0.00	-	-	-	-0.05	-0.02
Mar. 01	-0.10	0.00	-	-	-	0.00	-0.03
02	+0.06	0.00	-	-	-	+0.07	+0.04
03	+0.03	+0.07	-	-	-	+0.17	+0.09
04	+0.03	+0.07	-	-	-	-0.07	+0.01
05	-0.03	+0.02	-	-	-	+0.05	+0.01
06	0.00	-0.01	-	-	-	-0.02	-0.01
07	+0.03	+0.03	-	-	-	+0.06	+0.04
08	-0.04	0.00	-	-	-	-0.01	-0.03
09	+0.02	-0.01	-	-	-	-0.01	0.00
10	+0.01	0.00	-	-	-	-0.01	0.00
11	+0.11	+0.04	-	-	-	-0.22	-0.02
12	-0.02	-0.06	-	-	-	0.00	-0.03
13	-0.03	-0.06	-	-	-	+0.06	-0.01
14	+0.02	+0.04	-	-	-	-0.02	+0.01
15	+0.01	-0.01	-	-	-	+0.06	+0.02
16	-0.16	+0.05	-	-	-	-0.07	-0.06
17	+0.01	-0.32	-0.08	-0.11	-0.14	-0.13	-0.13
18	+0.05	+0.11	+0.07	+0.07	+0.07	+0.08	+0.08
19	-0.01	-0.15	-0.05	-0.01	-0.03	+0.08	-0.06
20	-0.02	+0.08	+0.06	-0.01	0.00	+0.02	+0.02
21	-0.05	+0.03	-0.08	-0.02	-0.03	0.00	-0.03
22	-0.06	-0.05	+0.08	+0.05	+0.07	+0.04	+0.03
23	0.00	+0.02	-0.04	-0.03	-0.05	-0.01	-0.02

## Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
Mar. 24	-0.02	-0.04	-0.01	+0.01	+0.07	-0.02	-0.03
25	+0.06	+0.05	+0.01	+0.01	-0.04	+0.04	+0.03
26	-0.12	-0.15	-0.07	+0.03	-0.02	-0.11	-0.07
27	+0.05	+0.08	+0.03	-0.07	-0.03	+0.01	+0.01
28	-0.02	-0.08	-0.02	-0.01	-0.02	-0.02	-0.03
29	-0.11	+0.07	+0.01	+0.01	+0.02	+0.05	+0.08
30	+0.03	-0.08	-0.06	-0.02	-0.03	-0.04	-0.03
31	+0.05	-0.06	-0.17	-0.03	-0.17	-0.07	-0.07
Apr. 01	+0.07	+0.10	+0.16	+0.09	+0.13	+0.07	+0.10
02	-0.05	-0.03	-0.16	-0.10	-0.07	-0.01	-0.07
03	-0.06	-0.15	-0.13	-0.03	-0.18	-0.10	-0.11
04	+0.01	+0.01	+0.02	0.00	+0.02	0.00	+0.01
05	+0.08	+0.11	+0.23	+0.11	+0.11	+0.06	+0.12
06	-0.03	-0.09	-0.15	-0.04	-0.07	-0.03	-0.07
07	+0.04	+0.11	+0.13	+0.05	+0.11	+0.06	+0.08
08	-0.01	-0.01	-0.05	+0.01	-0.02	+0.01	-0.01
09	-0.08	-0.06	-0.20	-0.14	-0.13	-0.05	-0.11
10	+0.05	+0.01	+0.16	+0.10	+0.10	+0.02	+0.07
11	+0.06	+0.08	+0.13	+0.07	+0.10	+0.05	+0.08
12	-0.01	+0.02	+0.08	+0.02	+0.03	-0.01	+0.02
13	+0.09	0.00	+0.11	+0.11	+0.12	+0.08	+0.08
14	-0.09	-0.05	-0.20	-0.11	-0.17	-0.08	-0.12
15	-0.04	-0.04	-0.06	-0.02	-0.04	-0.04	-0.04
16	-0.03	-0.10	-0.11	-0.09	-0.09	-0.05	-0.08
17	0.00	+0.01	0.00	+0.01	+0.01	+0.01	+0.01
18	+0.02	+0.06	+0.03	+0.05	-0.01	+0.03	+0.03
19	+0.09	+0.12	+0.19	+0.05	+0.17	+0.06	+0.12
20	-0.02	-0.02	-0.03	+0.02	+0.03	0.00	-0.03
21	-0.07	-0.07	-0.15	-0.10	-0.15	-0.07	-0.10
22	+0.03	-0.09	+0.04	-0.05	-0.02	-0.04	-0.02
23	-0.04	+0.05	-0.09	+0.06	-0.05	0.00	-0.01

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Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
Apr. 24	+0.03	+0.06	+0.09	-.02	-0.01	+0.03	+0.04
25	-0.03	-0.01	-0.01	+0.03	+0.01	0.00	-0.01
26	+0.01	-0.02	-0.01	0.00	-0.02	+0.03	-0.01
27	+0.01	0.00	+0.01	-0.01	0.00	0.00	+0.01
28	-0.01	0.00	-0.02	+0.01	0.00	-0.01	-0.01
29	+0.03	+0.04	+0.02	-0.03	+0.02	+0.01	+0.03
30	+0.01	+0.03	+0.07	-0.04	+0.04	+0.02	+0.02

DENSITY DIFFERENCE (gm cm<sup>-3</sup>)

Nov. 12	0.00	-	-	-	-	-	0.00
13	+0.04	-	-	-	-	-	+0.04
14	+0.01	-	-	-	-	-	+0.01
15	-0.01	-	-	-	-	-	-0.01
16	-0.01	-	-	-	-	-	-0.01
17	0.00	-	-	-	-	-	0.00
18	0.00	-	-	-	-	-	0.00
19	-0.01	-	-	-	-	-	-0.01
20	0.00	-	-	-	-	-	0.00
21	-0.01	-	-	-	-	-	-0.01
22	-0.01	-	-	-	-	-	-0.01
23	+0.01	-	-	-	-	-	+0.01
24	-0.01	-	-	-	-	-	-0.01
25	-0.01	-	-	-	-	-	-0.01
26	0.00	-	-	-	-	-	0.00
27	+0.01	-	-	-	-	-	+0.01
28	0.00	-	-	-	-	-	0.00
29	+0.01	-	-	-	-	-	+0.01
30	+0.01	-	-	-	-	-	+0.01
Dec. 01	0.00	-	-	-	-	-	0.00
02	+0.05	-	-	-	-	-	+0.05

## Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
Dec. 03	-0.01	-	-	-	-	-	-0.01
04	0.00	-	-	-	-	-	0.00
05	0.00	-	-	-	-	-	0.00
06	0.00	-	-	-	-	-	0.00
07	0.00	-	-	-	-	-	0.00
08	0.00	-	-	-	-	-	0.00
09	+0.03	-	-	-	-	-	+0.03
10	0.00	-	-	-	-	-	0.00
11	+0.01	-	-	-	-	-	+0.01
12	+0.02	-	-	-	-	-	+0.02
13	+0.01	-	-	-	-	-	+0.01
14	+0.02	-	-	-	-	-	+0.02
15	-0.02	-	-	-	-	+0.04	+0.01
16	+0.01	-	-	-	-	+0.01	+0.01
17	+0.10	-	-	-	-	+0.11	+0.11
18	+0.22	-	-	-	-	+0.07	+0.15
19	+0.05	-	-	-	-	+0.04	+0.05
20	+0.12	-	-	-	-	+0.07	+0.10
21	+0.07	-	-	-	-	+0.13	+0.11
22	-0.10	-	-	-	-	-0.06	-0.08
23	+0.18	-	-	-	-	-0.01	+0.09
24	+0.15	-	-	-	-	+0.12	+0.14
25	-0.07	-	-	-	-	-0.06	-0.07
26	+0.01	-	-	-	-	+0.03	+0.02
27	+0.07	-	-	-	-	+0.11	+0.08
28	+0.02	-	-	-	-	+0.01	+0.02
29	+0.03	-	-	-	-	0.00	+0.02
30	+0.12	-	-	-	-	+0.04	+0.08
31	+0.09	-	-	-	-	0.00	+0.05
Jan. 01	+0.04	-	-	-	-	+0.03	+0.04
02	+0.01	-	-	-	-	+0.01	+0.01

## Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
Jan. 03	+0.03	-	-	-	-	+0.01	+0.02
04	+0.07	-	-	-	-	+0.09	+0.08
05	+0.03	-	-	-	-	+0.08	+0.05
06	+0.34	-	-	-	-	+0.24	+0.29
07	0.00	-	-	-	-	+0.01	+0.01
08	+0.10	-	-	-	-	+0.20	+0.15
09	+0.19	-	-	-	-	+0.25	+0.22
10	-0.04	-	-	-	-	-0.09	-0.07
11	-0.07	-	-	-	-	-0.21	-0.14
12	+0.01	-	-	-	-	0.00	+0.01
13	-0.01	-	-	-	-	-0.03	-0.02
14	+0.02	-	-	-	-	+0.03	+0.02
15	-0.12	-	-	-	-	+0.04	-0.04
16	-0.08	-	-	-	-	+0.01	-0.04
17	+0.11	-	-	-	-	+0.06	+0.08
18	+0.17	-	-	-	-	+0.16	+0.17
19	+0.03	-	-	-	-	-0.03	0.00
20	+0.22	-	-	-	-	0.00	+0.11
21	+0.20	-	-	-	-	0.00	+0.10
22	-0.01	-	-	-	-	-0.12	-0.07
23	-0.10	-	-	-	-	+0.06	-0.02
24	-0.08	-	-	-	-	+0.13	+0.03
25	-0.02	-	-	-	-	+0.05	+0.02
26	0.00	-	-	-	-	-0.01	-0.02
27	+0.01	-	-	-	-	+0.05	+0.03
28	+0.08	-	-	-	-	+0.04	+0.06
29	+0.10	-	-	-	-	+0.16	+0.13
30	0.00	-	-	-	-	-0.24	-0.12
31	+0.04	-	-	-	-	+0.01	+0.03
Feb. 01	-0.04	-	-	-	-	+0.01	-0.02
02	-0.01	-	-	-	-	+0.01	0.00

Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
Feb. 03	-0.03	-	-	-	-	+0.04	+0.05
04	-0.06	-	-	-	-	-0.07	-0.07
05	-0.01	-	-	-	-	+0.01	0.00
06	-0.15	-	-	-	-	-0.01	-0.08
07	-0.04	-	-	-	-	+0.02	-0.01
08	-0.03	-	-	-	-	-0.02	-0.03
09	-0.02	-	-	-	-	-0.10	-0.06
10	+0.08	-	-	-	-	+0.04	+0.06
11	+0.03	-	-	-	-	-0.09	-0.03
12	0.00	-	-	-	-	+0.11	+0.06
13	+0.02	-	-	-	-	+0.03	+0.03
14	+0.14	-	-	-	-	-0.03	+0.06
15	-0.01	-	-	-	-	+0.03	+0.01
16	-0.12	-	-	-	-	+0.10	-0.01
17	-0.10	-0.14	-	-	-	+0.07	-0.06
18	-0.03	-0.05	-	-	-	-0.08	-0.05
19	+0.04	+0.07	-	-	-	+0.02	+0.04
20	+0.10	+0.20	-	-	-	+0.12	+0.14
21	-0.14	+0.09	-	-	-	+0.03	-0.06
22	+0.04	+0.27	-	-	-	+0.14	+0.15
23	+0.04	+0.13	-	-	-	+0.05	+0.07
24	+0.05	-0.10	-	-	-	-0.05	-0.03
25	-0.03	+0.17	-	-	-	+0.01	+0.05
26	+0.26	+0.18	-	-	-	-0.02	+0.14
27	+0.04	+0.07	-	-	-	+0.04	+0.05
28	+0.08	+0.09	-	-	-	+0.03	+0.06
Mar. 01	-0.03	+0.06	-	-	-	+0.02	+0.02
02	+0.01	+0.06	-	-	-	+0.08	+0.05
03	-0.04	+0.20	-	-	-	+0.04	+0.06
04	+0.11	+0.01	-	-	-	+0.05	+0.06
05	-0.01	-0.03	-	-	-	-0.02	-0.02

## Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
Mar. 06	+0.03	+0.02	-	-	-	+0.03	+0.03
07	+0.14	+0.09	-	-	-	+0.13	+0.12
08	-0.09	+0.07	-	-	-	+0.16	+0.05
09	-0.08	+0.13	-	-	-	+0.18	+0.08
10	-0.05	+0.17	-	-	-	+0.23	+0.12
11	-0.13	+0.11	-	-	-	+0.03	+0.03
12	-0.10	+0.02	-	-	-	-0.04	-0.04
13	+0.01	-0.09	-	-	-	+0.08	0.00
14	-0.01	+0.09	-	-	-	+0.06	+0.05
15	-0.04	-0.10	-	-	-	+0.08	-0.02
16	+0.11	-0.25	+0.04	-0.08	+0.10	+0.06	-0.03
17	+0.18	+0.08	+0.06	+0.05	+0.07	+0.12	+0.09
18	+0.08	-0.02	+0.02	+0.06	+0.06	+0.06	+0.04
19	+0.15	+0.02	+0.11	+0.01	+0.11	+0.01	+0.07
20	+0.09	+0.10	+0.41	+0.04	+0.07	+0.06	+0.13
21	+0.16	0.00	+0.01	+0.13	+0.01	+0.10	+0.07
22	+0.05	+0.06	+0.32	+0.08	+0.21	+0.04	+0.13
23	-0.16	-0.03	-0.11	-0.15	-0.10	-0.16	-0.12
24	-0.12	+0.06	-0.11	-0.09	+0.22	-0.12	-0.03
25	+0.06	+0.02	+0.04	0.00	+0.05	+0.05	+0.04
26	+0.02	+0.05	-0.06	+0.24	+0.25	+0.10	+0.10
27	+0.01	+0.01	+0.01	-0.03	-0.01	0.00	-0.01
28	-0.10	-0.02	-0.12	-0.09	-0.12	-0.08	-0.08
29	-0.23	-0.14	-0.19	-0.11	-0.12	-0.13	-0.15
30	-0.03	+0.02	+0.07	+0.05	+0.21	+0.21	+0.08
31	-0.04	-0.10	-0.04	-0.12	-0.06	-0.08	-0.07
Apr. 01	+0.08	+0.12	+0.09	+0.08	+0.09	+0.07	+0.08
02	+0.06	+0.37	+0.10	-0.16	+0.26	+0.16	+0.13
03	-0.04	-0.07	-0.05	-0.05	-0.07	-0.05	-0.06
04	-0.05	-0.08	-0.06	-0.06	-0.06	-0.05	-0.06
05	-0.13	-0.18	+0.05	-0.11	-0.23	-0.14	-0.12

Appendix IX (Cont'd.)

<u>Date</u>	<u>1.04</u>	<u>2.03</u>	<u>3.05</u>	<u>4.01</u>	<u>5.03</u>	<u>6.09</u>	<u>Mean</u>
Apr. 06	-0.12	-0.37	-0.23	-0.16	-0.28	-0.24	-0.23
07	0.00	0.00	0.00	0.00	-0.01	0.00	0.00
08	+0.05	+0.11	+0.04	+0.02	+0.02	+0.03	+0.05
09	+0.12	+0.53	+0.14	-0.11	+0.20	+0.20	+0.18
10	+0.05	+0.08	+0.06	+0.03	+0.05	+0.02	+0.05
11	-0.09	-0.11	-0.06	-0.17	-0.12	-0.11	-0.11
12	-0.05	+0.02	+0.24	+0.01	+0.06	-0.07	+0.04
13	-0.03	-0.07	-0.06	-0.04	-0.06	-0.03	-0.05
14	+0.03	+0.01	+0.04	+0.07	+0.04	+0.04	+0.04
15	-0.07	+0.07	+0.09	+0.12	+0.14	-0.10	+0.04
16	-0.06	-0.15	-0.01	-0.04	-0.07	-0.11	-0.07
17	-0.11	-0.16	-0.13	-0.07	-0.06	-0.15	-0.11
18	-0.12	-0.16	-0.17	-0.08	-0.21	-0.13	-0.14
19	+0.08	-0.07	-0.06	-0.08	-0.06	+0.04	-0.04
20	+0.09	+0.10	+0.11	+0.18	+0.32	+0.10	+0.15
21	+0.10	+0.17	+0.09	+0.11	+0.21	+0.08	+0.13
22	+0.14	-0.22	+0.27	-0.05	+0.21	-0.02	-0.05
23	+0.02	+0.12	-0.01	+0.08	-0.07	+0.04	+0.03
24	+0.07	+0.14	+0.18	+0.05	+0.11	+0.09	+0.11
25	+0.01	+0.23	+0.10	+0.21	+0.26	+0.16	+0.16
26	-0.15	-0.23	-0.15	-0.07	-0.09	-0.11	-0.13
27	-0.06	-0.09	-0.04	-0.04	-0.03	-0.03	-0.05
28	-0.13	-0.19	-0.13	-0.08	-0.10	-0.07	-0.11
29	-0.03	+0.01	-0.08	+0.01	-0.08	-0.02	-0.03
30	+0.11	+0.02	+0.04	-0.37	+0.04	+0.03	-0.02



## Glossary of Symbols

The numbers in parentheses refer to the chapters in which the symbol is used in the manner described.

- $\alpha^2$  Thermal diffusivity (1, 2, 3)
- $\theta$  Temperature at depth  $z$  (2)
- $\theta_0$  Initial temperature of snow surface (2)
- $\theta_0$  Difference in temperature between midpoint of two layers (3; Equation 3.34)
- $\theta_0$  Snow temperature (3; Equation 3.35)
- $\theta_1^0$  Initial temperature at one end (2)
- $\theta_2^0$  Initial temperature at other end (2)
- $\theta_1$  New temperature at instant  $t$  at end  $x = 0$  (2)
- $\theta_2$  New temperature at instant  $t$  at end  $x = 1$  (2)
- $\theta_3$  Initial temperature (2)
- $\theta_c$  Snow temperature at the boundary concerned (2)
- $\theta_c$  Mean temperature of snow layer (2)
- $\theta_c$  Temperature at the lower boundary of the snowpack (2)
- $\theta_s$  Temperature with respect to distance (2)
- $\theta_t$  Temperature with respect to time (2)
- $\lambda$  Thermal conductivity (2, 3)
- $\lambda_1$  Calculated thermal conductivity (2)
- $\lambda_2$  Calculated thermal conductivity (2)
- $\rho$  Density of snow layer (2, 3)
- $\rho_n$  New density of snow layer (2)
- $\Delta\rho$  Change in snow density (2)
- $\omega_1$  Amount of melting snow or freezing water (2)
- $b$  Constant in linear equation (3)

$B_n$	Fourier coefficient (3)
$c$	Specific heat (2, 3)
$C$	Initial temperature of layer (3)
$h$	Depth of layer (3)
$i$	Diffuse short wave radiation (2)
$I'$	Direct short wave radiation (2)
$l$	Thickness of layer (2, 3)
$L$	Latent heat of crystallization (2)
$L$	Amount of melting/freezing in equivalent cm of liquid water (3)
$L_1$	Layer number 1 (2, 3)
$L_2$	Layer number 2 (2, 3)
$L_3$	Layer number 3 (2, 3)
$L_4$	Layer number 4 (2, 3)
$Q$	Heat transfer (3)
$r$	Albedo (2)
$t$	Time (2, 3)
$T_0$	Constant temperature at lower boundary (3)
$T_a$	Temperature at upper boundary (3)
$T_1$	Constant in linear equation (3)
$T_2$	Temperature at lower boundary (3)
$Z$	Depth of layer (2)
$Z_1$	Upper boundary of snow layer (2)
$Z_{i+1}$	Lower boundary of snow layer (2)

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