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Ronald Douglas Hood

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A TWO ASSET MODEL OF
PERSONAL TAXATION

by

Ronald Douglas Hood

Department of Economics

Submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
London, Ontario
March, 1982

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ABSTRACT

In the work presented here, I have examined taxpayer behavior under a simple personal tax system. Taxes in this system are not based on current consumption, expenditure or income. Instead, the taxpayer is taxed on his earnings less whatever part of his savings he invests in a tax deferral portfolio. Any liquidations he makes from this portfolio become part of the tax base in the year they are made. Investment outside the portfolio bears tax free interest. It affords no tax deduction when acquired and can be liquidated tax free.

The essentials of this system can be captured by a two asset model of accumulation. Although the system is degenerate under proportional tax rates, a progressive rate structure ensures that there are unique optimal portfolios of the two savings vehicles at any point in the life cycle.

This system yields ready comparisons with other forms of taxation. If we remove one of the savings vehicles, we have an income tax system. If we remove the other, we have an expenditure tax.

In the thesis I have compared these types of taxation and drawn a variety of conclusions about the static nature.
of the two asset tax and dynamic effects switching to the
two asset system from other systems. In particular, I
have assessed the tax and savings implications of going
from an earnings tax system to the two asset system in a
life cycle context. These are theoretical results and
results from a simulation model.

The main conclusions include the following. Under the
two asset system, people pay a present discounted value of
taxes which is determined by the PDV of their life time
earnings. This horizontal equity feature of the two asset
system derives from the income averaging mechanism inherent
in it. This averaging process is superior in many respects
to the complex averaging proposals previously made for in-
come tax systems.

Movement from almost any tax system to the two asset
system will change individual patterns of lifetime savings
and taxes. In the simulation case studied, we find that
current savings will rise and current taxes will fall in
aggregate. There is a transitional period in which these
changes are larger than in the long run. A "vintage" and
a "catch-up" effect are responsible for this initial over-
shoot.

In transition there will be winners and losers.
Generally, it is those in middle age at the time of reform
who benefit most.

Finally, all the results are dependent on the progressivity of taxes. This is not because the optimal tax-savings pattern depends directly on the degree of progressivity but rather because the costs of being away from the optimum become vanishingly small as we approach proportional tax rates. This has implications for both the magnitude of welfare effects and for the nature and duration of the transition period.

These are just the major points made in the thesis. There are many secondary conclusions at both the theoretical and empirical level. A complete listing of these can be found in the final chapter.
ACKNOWLEDGEMENTS

I would like to thank the members of my committee: Tom Courchene, Jim Davies and Joel Fried, for their encouragement and help. In addition, I would like to thank Michael Parkin and Peter Howitt for their comments and advice.
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CHAPTER I

REVIEW OF LITERATURE AND PLAN OF THESIS

1. INTRODUCTION

In this thesis I will examine the consequences of making certain changes in personal taxation. These changes are motivated by the tax reform proposals made in the late 1970s by the Meadé Committee of the Institute for Fiscal Studies (1978), the U.S. Treasury (1978), Mieszkowski (1978, 1980), Graetz (1980) and Bradford (1980).

While the reform proposals for personal taxation put forward by these writers are not identical, they contain at least one important common element: they provide for much greater deductibility of investment expenditures in the determination of the personal tax base than is currently allowed in either Canada or the U.S. In addition, they generally provide for the inclusion in the tax base of the proceeds of the sale of any asset which previously afforded the holder a tax deduction.

If one wishes to analyse a system of personal taxation which has been amended in such a way, one must know two things. First, what was the nature of the tax system before the amendment? That is, will one be dealing with an amended income tax, an amended wage tax or some other type of tax? Second, to what class or classes
of assets do the new tax treatments apply? Is it all assets? Is it just certain assets? Can the taxpayer choose how much he wishes to be subject to the new treatment?

In this thesis I analyse the consequences of reform in the case where the pre-existing system is a progressive wage tax and the new tax treatment of assets is an option which the taxpayer can apply to as much or as little of his asset holdings as he pleases.

The analysis is done in a life cycle context where the wage tax system is represented by a one asset model of accumulation and the reformed system is represented by a two asset model. The effects of reform can then easily be identified with the repercussions of adding the second asset. But before we delve into the details of this process it will be useful to give some background information on alternative forms of personal taxation so that we can put the reform proposals in perspective.

2. BASIC TYPES OF PERSONAL TAXATION

Four types of personal taxation have been widely discussed in the literature (see Pechman 1977, 1980 for references). These are income taxes, consumption taxes, expenditure taxes and wage taxes. There are advantages and disadvantages to each. Generally, comparisons are made on the
basis of equity, efficiency and administrative practicality.

The Haig (1921) and Simons (1938) income tax uses as a base a comprehensive notion of income including all income from property. Arguments in favour of this tax base derive from the idea that the objective of equity in taxation will best be served by a system which measures the ability to pay taxes by adding up the money value of the net accretions to economic power between two points in time (Haig 1921, p. 7).

This definition, however, presents a number of administrative and conceptual difficulties. It is difficult, for instance, to measure the component of the increase in economic power which is due to capital gains on assets unless those gains are realized in the accounting period. Should one include increases in human capital through education or experience when one is measuring "economic power"? How is one to deal with changes in "money value" which are due solely to changes in the value of money? How can one accurately measure that component of income which comes in the form of service flows from durable goods or owner occupied housing? Should the promise of future economic power through pensions and insurance be taxed when the entitlements accrue or later, when the payments are actually made? When interest rates change there will be changes in the current value of securities which yield a fixed stream of returns. Such capital
gains and losses will lead to changes in taxation even though the stream of future returns is unchanged. This might seem to subvert the ability to pay notion. These are problems that centre on the notion of equity in the Haig-Simons income tax. There are further difficulties that arise in connection with the efficiency of income taxation.

As Fisher (1937, p. 47) pointed out, under income taxation there is a double taxation of savings. Not only is the income out of which savings is done taxed, but so is the return to savings itself. This puts an inefficient distortion in the savings market. The real rate of return to capital will differ from the after tax rate of return to savings.

If interest and capital gains are excluded from the tax base, as is the case under expenditure and consumption taxation, the savings distortion disappears. The principle of equity underlying these two taxes is different from that of the income tax. While the income tax depends on a notion of equity based on the ability to pay, the expenditure and consumption principle states that people should be taxed according to the amount they withdraw from the system (goods and services) rather than the amount they contribute to it (labour services and capital goods) (Kaldor 1955, p. 5).

The simplest form of expenditure tax is the general
sales tax. This is a practical and widely used form of taxation but it is limited in that it is difficult to apply a progressive rate structure to it. Progressive taxation of expenditures can be achieved by subtracting savings (including reinvested interest and capital gains) from comprehensive income. Using this as the tax base may be simpler in many cases than using comprehensive income itself. Any unrealized capital gain will cause problems under income taxation. But under the expenditure tax, capital gains and reinvested interest count both as income and as saving. They net out in the calculation of the tax base (income less saving). It is not necessary to know either to calculate the base. The situation is exactly symmetric for depreciation and capital consumption allowances.

One of the difficulties with the expenditure tax is that purchases of durable goods and especially housing will cause the taxpayer to have unusually high taxes in one year. This will cause problems under a progressive rate structure unless supplemented by an income averaging scheme.

The consumption tax is similar to the expenditure tax in that a greater tax burden may fall on those whose consumption varies over the life cycle. While the variations in consumption may not be as great as those of expenditure, it is quite difficult to calculate consumption. One must impute
values for service flows from durable goods and from housing.

One might view expenditure and consumption taxes as more just because they tax according to one's "endowment". This is true when inheritances and bequests are included in the endowment. Someone living under an income tax regime who inherits a large fortune and never works a day in his life will pay tax only on the interest and capital gains on his holdings. Under expenditure (and consumption) taxation, inheritance money will be taxed as it is spent (or consumed).

People value time as well as money. Therefore, the word endowment should really include non-market leisure time as well. For example, someone with a high wage rate who works only a little may pay the same tax as someone with a lower wage but the same income who works a lot. Despite the equal incomes the first man is clearly better off, having greater enjoyment of leisure time. None of the taxes discussed so far takes this into account.

The problem of leisure time valuation leads one naturally to consideration of the different labour incentive effects embodied in the various tax systems. People's wages vary over the life cycle causing changes in the opportunity cost of leisure. Work effort will be greatest when wage rates are highest. Both income and simple wage tax systems tend to distort this relationship. Not only is work generally
discouraged in favour of leisure, but it is discouraged more at some points in the life cycle (middle age) than at others.

For similar reasons, there may be a disincentive to invest in human capital through education because of the tax penalty paid by the concentration of earnings over a shorter period that is associated with the earnings profiles of more highly educated workers.

Marginal tax rates will be highest, and the disincentive to work the greatest under an expenditure (or consumption) tax whenever expenditure (or consumption) is highest. This has the effect of discouraging expenditures during family formation, and penalizing those who have large unexpected expenses such as medical costs.

Up to now we have been discussing the relative merits of different forms of personal taxation. A whole range of other issues arises when one considers moving between systems. For instance, in going from a wage tax to an income tax or an expenditure tax, the tax authorities will have to decide whether or not to tax previous accumulations of assets in the year of transition. This can lead to excessive once-over taxation. But if past accumulations are exempted there will be an incentive to "load up" on durable goods and assets prior to the transition. Similar problems may occur with
the treatment of capital gains or losses if they are done on a realization basis. People should postpone the sale of appreciated assets to avoid the taxation of capital gains. People should realize capital losses before the transition to expenditure taxation.

Since the treatment of savings is different under each type of taxation, the question arises about what to do, at the time of transition, with people who have accumulated savings (or gone into debt) under one system and will have to dissave (repay) under another. The problem is most acute for the elderly who have saved under an income or wage tax and have to dissave under an expenditure or consumption tax.

These then, are some of the characteristics of, and problems associated with the basic types of personal taxation. We have seen that there may be inefficient distortions in the labour and savings market, that there are conceptual difficulties in defining equity, and there are practical problems in implementation. Bearing these things in mind, let us now direct our attention to the reform proposals described in the Introduction to this chapter.

3. EXISTING RESEARCH ON THE REFORM PROPOSALS

Much of the existing literature on the reform proposals has focussed on administrative problems. There are still
difficulties associated with the service stream from durables and housing as is pointed out by Mieszkowski (1980, p. 199) and Graetz (1980, p. 184). Mieszkowski (1980, pp. 196-200) also discusses problems which arise in harmonizing the reformed tax system with international tax arrangements. There is also considerable discussion in the literature on the subject of tax avoidance. While these administrative problems are by no means trivial, several other important issues have not yet been addressed. These are main subject material of the thesis.

The main issue is that introduction of the new tax treatment of assets will induce taxpayers to change their life cycle patterns of savings and tax payments. Unless the government introduces other tax adjustments at the same time as it implements tax reform, it may be confronted with large changes in its revenue from the personal sector. These will have distinct short run and long run components.

Even if adjustments are made to avoid large variations in government revenues there will still be winners and losers in the process of reform. These welfare effects depend not only on differences in individual tastes and endowments but also on age.

Several writers including Mieszkowski (1980, p. 193) and Bradford (1980, p. 89) suggest that the reformed system
will have a desirable horizontal equity feature; people with the same present discounted value (PDV) of earnings pay the same PDV of tax. While this is true under certain circumstances, no formal proof has been given.

The horizontal equity feature derives from the tax averaging that taxpayers are able to achieve under the reformed system. The averaging process in itself is quite interesting because for some time economists such as Vickery (1973), Deutsch (1975), Goode (1980) and others have tried to devise cumulative averaging schemes which have proven more complex and costly to administer than this system. The main reason for the cost of previous averaging systems is the necessity of keeping and updating records over long periods of time. Almost no long term record keeping is required under the new system.

4. PLAN OF THESIS AND OUTLINE OF CONCLUSIONS

So far I have described several systems of personal taxation and given a very brief outline of yet another system which involves a choice for the taxpayer about the tax treatment of his assets. In Chapter II, I develop a formal model of this system. This model allows simple and direct comparison of wage taxation, expenditure taxation and the new type of taxation. The theory suggests that the new type of taxation compares quite well on efficiency and equity grounds.
with the other forms of taxation.

As indicated before, the results in Chapter II are based on a simple two period consumption-savings model where the consumer has two savings vehicles corresponding to the two different tax treatments of assets. Formulated as a maximization problem this enables us to see what life cycle patterns of consumption savings and taxes will evolve not only under the wage tax and the reformed tax, but under an expenditure tax as well. I make direct comparisons of the static optima under these three tax systems and show how the results depend on earnings and tastes. In addition, I give a formal proof of the horizontal equity condition and give a set of sufficient conditions which will preserve it in an extended model with inheritances and bequests. Chapter II contains the basic theoretical work of the thesis.

In the third chapter, I look at some of the comparative static properties of the new system. Specifically, I ask what happens to the 'representative man' if there is a change in interest rates, what happens if we tax interest payments and what are the redistributive effects of changing over to the new system from a wage tax. I consider two other things as well, namely problems associated with indexing the tax function for real growth over a long time period and the possibility of government control of portfolio composition
in the new system through a scheme of partial deductibility of one of the assets.

The purpose of Chapter IV is to establish that the simple two period model of Chapter II can readily accommodate a wide range of policy tools without undue complication. In particular, I integrate direct transfers, tax credits, compulsory savings plans, deductions and exemptions into the system. Some compromise is necessary, however. The tax treatment of dividends and capital gains and the tax links between spouses, and between spouses and children, cause some problems and are dealt with in two appendices to Chapter IV. Chapter IV is somewhat technical and may be skipped without loss of continuity.

The remaining chapters are more empirical in content. The objective is to simulate the long run and short run effects of switching from a wage tax system to the new system. To do this we need to know what typical life cycle consumption patterns look like. In Chapter V, I address this question. Using a simple trend-cycle regression and data from the Survey of Family Expenditure (Statistics Canada) I estimate what portion of lifetime consumption falls in each period of life.

In Chapter VI, I develop the simulation model. The simulations are done in a utility maximization framework.
Individuals are given an expected earnings pattern and are endowed with tastes. They must maximize their utility subject to a lifetime income constraint. In Chapter VI, I lay out the utility, earnings and tax parameters. These values of the parameters are justified in the light of previous empirical work and the work in Chapter V.

Chapter VII is the chapter in which most of the empirical results of the tax change are derived. Basically, I am looking at the switch from a wage tax system to the new system. There are five general questions I want to address:

1. What are the changes in steady state taxes and savings profiles?
2. How do variations in the assumed income path and utility parameters influence savings and taxes?
3. How valuable is the income averaging feature of the new system to taxpayers?
4. What happens during transition: who wins and who loses?
5. Are the results sensitive to the indexing assumption?

Chapter VIII summarizes my answers to these questions, reviews some of the weaknesses of the model, draws conclusions about the likely effects of moving towards the new tax system and suggests some further research possibilities. The main conclusions are;
1. The change in the optimal patterns of savings and taxes over the life cycle may be large when we switch from the wage tax to the new type of tax, but the incentive to change may be weak. This makes it difficult to tell how dramatic the actual changes will be.

2. Low retirement period earnings largely determine the general pattern of savings and tax changes after the switch.

3. The steady state changes involve a rise in savings and a fall in taxes during the working years and a rise in taxes and a fall in savings in retirement.

4. Transitional effects are important. There is a double-effect on savings and taxes in the short run which tends to make aggregate savings rise by more and taxes fall by more in the short run than in the long run. This is due to two factors: the fact that the young change their behaviour before the old, in an overlapping generations model, and the fact that some "catch-up" adjustment will be undertaken by those caught in mid-cycle by the transition.

5. The middle-aged will be the winners in terms of inter-generational transfers during transition.

6. A rise in the actual or perceived indexing intentions of the policy makers will induce a rise in savings and a drop in the tax base. The extent of the reaction is difficult to judge.

7. Under the new system, people with the same PDV of earnings pay the same PDV of tax and enjoy the same PDV of consumption.

8. The previous horizontal equity condition can be preserved in a system with bequests.

9. Under the new system, a fall in the rate of interest will definitely increase the demand for one asset and may increase overall savings even if preferences are homothetic.

Let me now turn to the business of building the formal model.
CHAPTER II
THE TWO ASSET TAX SYSTEM

1. INTRODUCTION

In this chapter I will develop a simple model for comparing a wage tax, an expenditure tax and the two asset tax system. In particular, I want to show that the two asset tax can be designed in a way which

1. is progressive in any desired degree.
2. discriminates neither with respect to the time distribution of income nor that of consumption.
3. requires no capital gains evaluation before realization.
4. does not discriminate against purchases of durable goods.
5. taxes according to the present discounted value of income.

In addition, I will discuss methods of handling bequests and inheritances under this type of taxation.

2. A TWO ASSET TAX SYSTEM

Consider the following situation. Taxpayers live for two periods. They earn a certain amount in each period and distribute their consumption between periods in a way which
is determined by their tastes and the intertemporal trade options open to them. When the taxpayer accumulates an asset he must choose whether to declare it as 'R' type or as 'B' type for tax purposes. The B asset is treated as a bond with tax-free interest. The R asset is deductible when accumulated and taxed including the capital gain when liquidated. Any interest paid out on R assets before liquidation counts as taxable income unless it is reinvested in R. For instance, if the taxpayer buys a piece of land, he has a choice of declaring the land to be B or R. If he chooses the B treatment, all rental income from the land is tax free and any capital gains he gets when it is sold are tax free. If he chooses the R treatment, he gets a tax deduction equal to the purchase price of the land but he must declare any rental income from the property (the rental income may be sheltered if it is reinvested in more R). If he sells the land he will pay tax on the proceeds of the sale (less the value of any new R assets he acquires in the year the land is sold). Since interest and capital gains are deducted when reinvested, the R asset really allows for tax free compounding of the return to property.

A person who does all his saving in R assets pays taxes on his income less savings or, in other words, on expenditure. If he alters his life cycle savings pattern his life cycle tax pattern will change. When tax rates are progressive,
the present discounted value (PDV) of his tax payments will change too.

A person who does all his saving in B assets has a fixed PDV of taxes even if tax rates are progressive. His savings pattern and expenditure pattern do not affect the PDV of taxes. The PDV of taxes does depend, however, on the distribution of lifetime earnings.

In general terms, the savings pattern a taxpayer adopts is influenced by two considerations: to have a low lifetime tax bill; and to have a desirable pattern of expenditure. If only R assets are available (an expenditure tax system) the two objectives cannot be pursued independently. If only asset B is available the taxpayer has no power to move towards the first objective. If both assets are available and the taxpayer can freely choose how much of each to hold, both objectives can be pursued independently. Not surprisingly, it turns out that in this 'hybrid' system the taxpayer will in fact minimize the PDV of his tax bill and will choose his expenditure pattern independently of the minimizing process. I now turn to the issue of finding the optimal B and R holdings when only R is available, when both B and R are available and when only B is available.

Let us assume for the moment that all earned income falls in period one. The endowment is then represented by E
Diagram 2.1 Taxes and Savings in the Simple Model
in Diagram 2.1. By trading in the asset $R$, taxable income pair $YT_1, YT_2$, along $L_1$ may be reached. Notice that

\begin{align*}
(2.1) & \quad YT_1 = E - R \\
(2.2) & \quad YT_2 = R(1 + r)
\end{align*}

which imply

\begin{equation}
\frac{dYT_2}{dYT_1} = - (1 + r) = \text{slope } L_1
\end{equation}

The curved line, $L_2$, gives post-tax disposable income ($YD$) levels in each period corresponding to different levels of $R$. Notice that

\begin{equation}
(2.4) \quad YD_1 = E - R - t \left[ E - R \right]
\end{equation}

and

\begin{equation}
(2.5) \quad YD_2 = R(1 + r) - t \left[ R(1 + r) \right]
\end{equation}

which imply

\begin{equation}
\frac{dYD_2}{dYD_1} = - \frac{1-t'[R(1+r)]}{1-t'[E-R]} . (1 + r) = \text{slope } L_2
\end{equation}

where $t'[.]$ is the marginal tax function. If $YT_1 = YT_2$, marginal tax rates are equal in each period and the slope of $L_2$ equals $1 + r$ (which is also the slope of $L_1$).

Points along $L_2$ are attainable consumption points using only asset $R$. $L_2$ represents the opportunity set for an individual with endowment $E$ subject to an expenditure tax. It is concave reflecting the tax penalty paid by those with
TABLE 2.1  Taxes and Savings in Simple Model  
(See Diagram 2.1)

<table>
<thead>
<tr>
<th>System</th>
<th>Asset Holdings</th>
<th>Taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R</td>
<td>B</td>
</tr>
<tr>
<td>R</td>
<td>QE</td>
<td>O</td>
</tr>
<tr>
<td>B-R</td>
<td>AE</td>
<td>HF</td>
</tr>
<tr>
<td>B</td>
<td>O</td>
<td>GN</td>
</tr>
<tr>
<td>B adjusted</td>
<td>O</td>
<td>HD</td>
</tr>
</tbody>
</table>
uneven expenditure patterns. Tangency with the relevant indifference curve at, say, \( C_2 \) determines consumption. If \( P \) is the taxable income position corresponding to \( C_2 \) then the optimal \( R \) holdings in the \( R \) system are equal to the distance \( QE \). The PDV of taxes is \( GE \).

By using the asset \( B \), a better consumption point than \( C_2 \) may be reached by trading in a north-westerly direction along \( L_3 \). In fact, if we are going to use both assets we can do even better than that. If we had chosen \( J \) instead of \( P \) as our taxable income position, the bond trading locus now open to us would have been \( L_4 \). Clearly, \( J \) dominates all other taxable income positions on \( L_1 \) because it implies the highest bond trading locus (\( L_4 \)). The best consumption point using both \( B \) and \( R \) is at \( C_1 \). \( M \) is the disposable income position corresponding to \( J \). Here \( Y_{T_1} = Y_{T_2} \) and \( Y_{D_1} = Y_{D_2} \) and taxes are equal in each period (distance \( FA \)). The holdings of \( R \) are given by the horizontal distance \( AE \), while those of \( B \) are given by \( HF \). Both, in this case, are positive. The PDV of taxes is given by the distance \( DE \).

Does the \( B-R \) system have the characteristics listed in points one to five above? As regards point one, changes in the progressivity of the tax function change the degree of curvature of \( L_2 \). The analysis is essentially unaltered. If the PDV of taxes is kept constant while the progressivity of
the rate structure is changed, B and R holdings will be unaffected.

Regardless of where E falls along L₁ or where C₁ falls along L₄, J is the optimal taxable income position. People with the same PDV of income will have the same PDV of taxes and the same PDV of expenditure regardless of the time distribution of income, or the time distribution of desired expenditure. In all cases, the PDV of income is OE, the PDV of taxes is DE and the PDV of expenditure is OD. Therefore, point 2 is satisfied.

Capital gains on R are taxed only when R is sold. Interest income on R is taxed as it is paid out. Interest and capital gains on B are not taxed at all. Therefore, no capital gains evaluation is necessary before realization. This is the observation made in point three.

Point four claims that the purchases of durable goods are not discouraged under the B-R system the way they are under the expenditure system (R). A large durable good purchase, when financed by the sale of some B assets, will not affect taxes.

The last point was that taxes are based on the PDV of income under the B-R system. This has already been established since the distance DE in Diagram 2.1 is independent
of the position of $C_1$ along $L_4$ and $E$ along $L_1$.

The first two lines of Table 2.1 accompanying Diagram 2.1 give the holdings of $R$ and $B$ and of the PDV of taxes for the $R$ and $B-R$ systems. The third line of the table gives the same information for the $B$ system. Switching from one system to another may involve a change in the PDV of taxes. It is convenient to make comparisons between systems when tax rates are adjusted to make the PDV taxes the same. If no adjustment is made, someone with endowment $E$, restricted to asset $B$, can reach $C_3$ but not $C_1$. He pays tax only in the first period and his disposable income is $G$. His tax is $GE$ instead of $DE$. If tax rates are adjusted so that he pays the same PDV of tax under $B$ as $B-R$ he will be able to reach $C_1$. His total saving will be $HD$ under the tax adjusted $B$ system and $AE$ plus $HF$ under $B-R$. The difference is equal to the distance $KI = DE/(2 + r)$. Therefore, savings are greater under $B-R$. The difference is independent of the location of the consumption point along $L_4$.

The savings differential is not independent of the endowment point however. The closer $E$ is to the $45^\circ$ line the less is the increase in savings involved in moving from $B$ to $B-R$ while holding taxes constant. When $E$ is on the $45^\circ$ line, no saving increase results and no tax adjustment is necessary. If $E$ were to lie above the $45^\circ$ line, savings would fall but
this implies that the greater part of earned income falls in retirement.

Finally, applying the same tax adjustment to all taxpayers regardless of endowment differences when moving from B to B-R, will tend to reward those with skewed income more than those with even income streams. This reflects the fact that an earnings tax (B) penalizes those with skewed income while B-R taxes the PDV of income.

3. THE TWO ASSET SYSTEM WITH BEQUESTS

Consider the problem of taxing bequests in a way which does not affect the PDV of the donor’s taxes. The solution to this problem is to tax bequested R assets as if they had been liquidated by the donor in the year of the bequest, and to let bequested B assets change hands tax free. In the last year of his life, the donor has the choice of liquidating his assets and consuming the proceeds himself or giving the assets to his heirs. The amount of tax paid will be the same either way. If the assets are B assets, the donor pays no tax on the bequest. He would pay no tax if he liquidated the assets for his own consumption. If the assets are R assets, they will be taxed if consumed and they will be taxed if bequested. Bequests are really treated as if they were consumption in the year of the bequest.
The fact that R bequests are taxed and B bequests are not, does not mean that no R bequests will ever be made. Although there is no tax on B bequests, there is no deduction for B saving either. The donor may want to make some of his bequest in the form of R as part of his tax minimizing strategy. If the donor receives all his income in period one and his desired expenditure in period two is smaller than in period one, he may wish to make his entire bequest in the R form. This follows from the fact that taxable income must be the same in both periods if the taxpayer is behaving optimally. The taxpayer must defer taxes from period one to period two. If he intends to make a bequest he will therefore use the R asset. This will lower the tax base in period one and raise it in period two. In general, the composition of the bequest will depend on the time paths of income and desired expenditure and on the size of the bequest. Most likely, the bequest will be made in installments over a number of years. Donors will pass on a little R asset each year, keeping retirement period tax rates in line with those in the working years.

There is no distortion of the bequest motive with this tax treatment. If the donor foregoes one unit of consumption, the recipient can enjoy exactly one more unit of consumption. The tax does not distort the relative prices of donor and recipient consumption. If the donor passes a dollar's worth
of B to his heir, his own potential consumption drops by a dollar and that of his heir rises by a dollar. If the donor passes on a dollar's worth of R, his own potential consumption will decline by one dollar less his marginal tax rate and his heir's potential consumption will rise by the same amount. However, taxing the bequest again as income in the recipient's hands will distort the bequest motive. In this case, the 'cost' of recipient consumption will be higher than that of the donor and bequests will be discouraged.

4. CONCLUSIONS

The two asset tax base has some distinct advantages over the other two tax bases presented here. The fact that it taxes individuals according to the PDV of their lifetime incomes provides an attractive horizontal equity feature. In addition, it does not distort the savings decision by taxing property income, it does not discourage the purchase of durable goods, and it does not distinguish between taxpayers on the basis of their desired life cycle expenditure patterns.

The system can be extended to include bequests in a way which does not destroy the horizontal equity feature and does not distort the bequest motive. While the system without bequests requires no capital gains evaluation before realization, bequested R assets in the extended system must be evaluated because taxes are levied against the donor when
the bequest is made.

The system works by letting each individual do his own income averaging. It is likely that the individual is the best judge of his own income and expenditure paths thus averaging will be more effective under the B-R system than under a more centralized system.

There are some further considerations however. Uncertainty effects will be important. A very high marginal tax rate will be levied on R assets in the estate of someone who dies before he is able to make all his bequest payments. This uncertainty effect may discourage the use of asset R as a bequest vehicle.

Taxpayers cannot be certain about the future course of tax rates or specific exemptions or deductions. Different indexing assumptions in the tax function will have strong implications for the size and especially the composition of the optimal savings portfolio.

The switch from the B base to the B-R base, it was argued, will increase savings when the PDV of taxes is kept constant. It is worth noting that there is no change in the terms of trade in switching from B to B-R. In both cases, the final optimum involves the same marginal rate of substitution between periods (1 + r). This implies that there is
no change in excess burden in shifting from B to B-R. Shifting from R to B-R under a constant PDV of tax will involve a reduction in excess burden as evidenced by the difference in utility between points C_3 and C_2 in Diagram 2.1. The PDV of taxes is GE.

The model employed here has a fixed labour supply and a fixed wage pattern. Changes in the system of taxation may well have work effort effects which are not accounted for in this formulation. Under a progressive income or wage tax, the return to labour is taxed more heavily in the middle years than in the early or late years. The move to the B-R type of taxation which applies a uniform labour incentive effect over the life cycle will therefore tend to increase work effort in the middle years and perhaps reduce the average retirement age. The combined effects of less taxation and greater work effort in the middle years as well as a shorter working life will all tend to increase the savings rate of those in the peak earning years. This is analogous to the labour incentive effects of public pension plans identified by Burbidge and Robb (1980) and Daly (1980). Applying this type of analysis to the B-R system reveals one of the advantages of this type of tax. Most tax systems distort the labour-leisure choice. But the B-R tax applies a uniform incentive over the life cycle while the wage tax and the income tax distort the labour-leisure choice at a
variable rate over the life cycle, with the disincentive to work being greatest in the peak earnings years.

The choice of transition analysed here, from a wage tax to a B-R tax, is justified primarily on the grounds of simplicity. It might be argued that an income tax system would be a better starting state. The existing Canadian system is not a pure income tax system however, with exemptions for interest and dividend income, half rates on capital gains income and no taxation of housing capital gains or implicit rental income. An exact duplication of the current system for purposes of simulation is a prohibitively costly and complex task. For these reasons I have analysed the more simple wage tax case.
APPENDIX 2.1
THE MODEL IN CONTINUOUS TIME

This appendix gives a formal derivation of the optimal savings paths in a two asset model with a progressive tax function. The assets are distinguished by their tax characteristics, the first being like a bond which is neither deductible nor subject to an interest tax and the second being like an RRSP which is taxed upon liquidation and is tax deductible when accumulated. The problem is formulated in continuous time. Zero boundary conditions are imposed on the two assets at birth and death. The utility function is assumed to be separable. The problem is then to maximize the following integral.

\[ \int_0^T e^{-\rho t} U[w_t-I(w_t-u_R+tR)-u_R-u_B+(R+B)r]dt \]

where \( R_t = u_R \) = accumulation of RRSP's
\( B_t = u_B \) = accumulation of bonds
\( I(\cdot) \) = a progressive income tax function
\( I' \) = the marginal tax rate
\( U[\cdot] \) = a utility function
\( \rho \) = the rate of time preference
\( r \) = the interest rate
\( w_t \) = the wage rate.

The expression inside the square brackets in (1) is
consumption at time t. It equals wage income less taxes less asset purchases plus interest income. The expression in round brackets after I is taxable income and is equal to wage income less RRSP accumulation plus interest income paid out of the RRSP fund.

The Hamiltonian for this system is

\[ H = e^{-\text{RHO}.t} U \left[ w_t - I(w_t - u_R + R_R) - u_R - u_B + (R+B)r \right] \]

\[ + \lambda R \cdot u_R + \lambda B \cdot u_B \]

The necessary conditions are

\[ \dot{R} = u_R \]
\[ \dot{B} = u_B \]

\[ \frac{d(\lambda_B)}{dt} = -\frac{\partial H}{\partial u} = -\text{RHO}.t \\]

where \( \dot{\cdot} \) denotes the first derivative

\[ \frac{d(\lambda_R)}{dt} = \frac{\partial H}{\partial R} = \text{RHO}.t \]

\( I' - 1) \]

\[ \frac{\partial H}{\partial u} = e^{-\text{RHO}.t} \left( I' - 1 \right) + \lambda_R = 0 \]

\[ \frac{\partial H}{\partial u} = e^{-\text{RHO}.t} + \lambda_B = 0 \]

Although this is a control problem with two controls which would usually require solving two simultaneous differential equations in B, B, R and R, we can, by a few simple
manipulations, get an equivalent pair of equations which can be solved recursively. To show this we need two facts. First, marginal tax rates will be constant along the optimum path and second, optimum consumption moves according to

\[ \circ C = \frac{U'}{U''} (r - \text{RHO}) \]

The first proposition can be seen by noting that equations (7) and (8) together imply that

\[ \lambda_B (1 - I') = \lambda_R \]

And therefore,

\[ \frac{d(\lambda_B)}{dt} (1 - I') = \frac{d(\lambda_R)}{dt} \cdot \frac{dI'}{dt} = \frac{d(\lambda_R)}{dt} \]

\[ \frac{d\lambda_B}{dt} (1 - I') = \frac{d\lambda_R}{dt} \]

Now (11) and (12) imply

\[ \frac{dI'}{dt} = 0 \]

Or marginal tax rates are constant.

The familiar looking proposition (9) follows from setting the time derivative of (8) equal to (6);

\[ \text{RHO} \cdot e^{-\text{RHO} \cdot t} \cdot U' + \circ e^{-\text{RHO} \cdot t} = \lambda_B = -re^{rt} U' \]

Or

\[ \circ C = \frac{U'}{U''} (r - \text{RHO}) \]
Since marginal tax rates are constant, taxable income (YT) must be constant so.

\[ YT_t = w_t - R_t + rR_t \]

and \[ Y_{t} = 0 \]

So that

\[ R_t - rR_t = w_t \]

This is a second order, non-homogeneous equation. Notice it contains no terms in B or its derivatives. Once (16) is solved, we can substitute back into the consumption relation, (9), and get a first order equation in B and \( \dot{B} \) only.

It may be instructive to look at the optimal paths for B and R in the simple case where \( w_t = \bar{w} \), \( \text{RHO} = r \), \( B_0 = R_0 = B_T = R_T = 0 \). Since R begins and ends at zero and since taxable income must be constant, a constant wage implies that there can never be any change in holdings of R, i.e. R=0 for all t. Equation (9) tells us that consumption is constant when \( \text{RHO} = r \). Since \( w_t = \bar{w} \) and \( R_t = 0 \), \( B_t \) must also be zero throughout. The intuition is that since the interest rate equals the rate of time preference people do not wish to enter the bond market. Since resources may be transferred between periods i and j using B and R at rates

\[ (1+r)^{j-i} \text{ and } (1+r)^{j-i} \frac{1-I_j}{1-I_i} \text{ respectively,} \]
any path where $I_i' \neq I_j'$ can be improved on by a portfolio adjustment. When $w_t = \bar{w}$ tax rates will be equal across periods iff $R_t = 0$ for all $t$.

Let us now solve the more general problem where $\rho \neq r$ and wages are growing at some rate $g_w$. To begin, we must solve the homogeneous part of (16).

\[(18) \quad \ddot{R} - R = 0\]

We need two basic solutions. One is

\[(19) \quad R_t = e^{rt}\]

Any solution to $\dot{R} - R = k_1$ will also solve (18). Hence $R$ equal a constant is another independent solution. The general solution to (18) is

\[(20) \quad R_t = C_1 + C_2 e^{rt}\]

To complete the solution of (16) we need a particular solution to

\[(21) \quad \ddot{R} - R = w_0 e^{gwt}\]

\[(22) \quad \text{If } \quad R_t = k_3 e^{gwt} \]

\[(23) \quad \text{then } \quad \dot{R}_t = g_w k_3 e^{gwt}\]

\[(24) \quad \text{and } \quad \ddot{R}_t = g_w^2 k_3 e^{gwt}\]
Using (21) we have

\[-k_3 g_w (r-g_w) e^{g_w t} = w_0 e^{g_w t}\]

\[k_3 = \frac{w_0}{g_w (r-g_w)}\]

The general solution to (16) is

\[(25) \quad R_t = C_1 + C_2 e^{r t} + \frac{w_0}{g_w (r-g_w)} e^{g_w t}\]

Imposing \(R_0 = R_T = 0\) we have

\[(26) \quad C_1 + C_2 + \frac{w_0}{g_w (r-g_w)}\]

\[(27) \quad C_1 + C_2 e^{r t} + \frac{w_0}{g_w (r-g_w)} e^{g_w t}\]

\[(28) \quad C_1 = \frac{w_0}{g_w (r-g_w)} \cdot \frac{e^{r t} e^{g_w t}}{1-e^{r t}}\]

\[(29) \quad C_2 = \frac{w_0}{g_w (r-g_w)} \cdot \frac{e^{g_w t} - 1}{1-e^{r t}}\]

And finally writing \(R_t\) and \(R_T\) out in full

\[(30) \quad R_t = \frac{w_0}{g_w (r-g_w)} (e^{g_w t} - 1) - (e^{r t} - 1) \frac{1-e^{g_w t}}{1-e^{r t}}\]

\[(31) \quad R_T = \frac{w_0}{g_w (r-g_w)} \frac{r e^{g_w t} - 1}{1-e^{r t}} e^{r t} + g_w e^{g_w t}\]

The term in square brackets in (30) is positive for \(g, r > 0\) if
\[
\frac{1-e^{r_{\text{gt}}}}{1-e^{r_{\text{r}}} - t} > \frac{1-e^{r_{\text{t}}}}{1-e^{r_{\text{t}}}} \quad t = 1 \ldots T-1
\]

which holds iff \( r > g_w \). Since the other term in (30) is positive when \( r < g_w \) and negative when \( r > g_w \), \( R_t \) is negative for all \( t \) between 0 and \( T \). If \( g_w < 0 \) all results are reversed and \( R_t > 0 \) for all \( t \).

We can now evaluate taxable income \( YT_t \), by substituting the expressions (30) and (31) for \( R_t \) and \( R_t \) into equation (15).

\[
YT_t = w_t - R_t + rR_t
\]

(33)

\[
YT_t = \frac{rW_0}{g_w(r-g_w)} \cdot \frac{e^{r_{\text{t}}-g_{\text{gt}}}}{1-e^{r_{\text{t}}}}
\]

which is positive and constant. The income after taxes and net of changes in \( R \) is given by \( TD \).

(34)

\[
YD_t = w_t - R_t - rR_t - I(w_t - R + rR) = YT_t - I(YT_t)
\]

Therefore \( YD \) is constant and independent of \( \text{RHO} \).

The results obtained so far have been fairly general in that all we have required of the tax and utility functions is

\[
U' > 0 \quad , \quad U'' < 0 \quad \text{and temporal additivity}
\]
\[ T' > 0, \quad T'' > 0 \]

To facilitate the discussion of bond holdings, I now assume a form for \( U \).

\[ U(C_t) = \frac{C_t^{\text{ALPHA}}}{\text{ALPHA} - 1} \quad 0 < \text{ALPHA} < 1 \]

For this case (9) becomes

\[ C_t = C_t \frac{\text{r-RHO}}{\text{ALPHA} - 1} \]

or

\[ C_t = C_0 e^{\frac{\text{t-RHO}}{\text{ALPHA} - 1} \cdot t} \]

The required bond holdings are determined by the condition that the flat YD stream be transformed into the consumption stream of (36). It follows immediately that if \( r = \text{RHO} \), \( B_t \) will be zero. In general,

\[ B_t e^g - B_t = C_0 e^{egct} - YD_t, \quad g_c = \frac{\text{r-RHO}}{\text{ALPHA} - 1} \]

This equation is similar to (15). Its solution for \( B_0 = B_T = 0 \) is

\[ B_t = \frac{YD}{r} \frac{e^{egct} - 1}{e^{egct} - e^{rt}} e^{egct} + \frac{1 - e^{rt}}{e^{egct} - e^{rt}} - 1 \]

which is zero for all \( t \) when \( \text{RHO} = r \) and is \( > 0 \) according to \( r > \text{RHO} \).

Together, equations (30) and (38) comprise the solution to our problem. Since only wage and interest parameters
enter (30), we know that \( R \) is unaffected by changes in taste or tax parameters. For a given set of parameters we know \( YD \) in equation (38) to be a positive constant. Since tax parameters affect nothing else in (38), changes in \( B_t \) due to a tax change may be identified with the changes in \( YD \). In other words, \( B_t \) responds as it would to changes in lifetime income. If tastes are homothetic, the time distribution (though not the level) of bond holdings will be independent of the tax function and the time path of wages.

In the present formulation tastes are homothetic and the pattern of bond holdings (and of consumption) changes only by a scale factor when income or tax parameters change. Notice that separability of the utility function is neither necessary nor sufficient to guarantee this result. Assume for instance that we regard simple discounting as too constraining a functional form to capture all life-cycle consumption effects and use instead a function where \( \text{ALPHA}_t \) is time dependent.

\[
U = \sum_{t} \text{ALPHA}_t C_t
\]

This form is separable but not homothetic. The pattern of consumption and of bond holdings will be dependent on tax and income parameters in a way which is not independent of scale. Separability is satisfied but scale neutrality is not.
On the other hand, if our utility function is

\[ U = \Pi C_t^\alpha ] \]

(40)

all the results of Chapter II are preserved. Consumption and B holdings are constant to a scale factor in the face of wage or tax changes but we do not have separability.
CHAPTER III
SOME COMPARATIVE STATIC PROPERTIES
OF THE TWO ASSET SYSTEM

1. INTRODUCTION

In this chapter I look at the steady state changes in the level and composition of savings when there are changes in policy variables and parameters of the system. In particular, I show that a fall in the rate of interest will raise optimal R holdings while having an ambiguous effect on B; that taxation of interest income from B will discourage B holdings and will involve a rising pattern of marginal tax rate over the life cycle; and that the uncompensated welfare effects of shifting from B to B-R will be positive and will vary more than in proportion to income.

In addition, I look at the way a non-negativity constraint on R holdings is likely to affect life cycle savings. Then I argue that a "partial deductibility" scheme of taxation gives the government some control over the composition of the optimal portfolio. And finally, I examine some special problems connected with the indexation of the tax system for real growth.

2. INTEREST RATE CHANGES (Diagram 3.1)

If we assume interest rates paid on B and R to be the
Diagram 3.1 Effects of a Change in Interest Rates
same and this common rate then declines, there may be changes in the size and composition of savings. Diagram 3.1 which is a modified form of Diagram 2.1, illustrates these movements. \( L_1 \) gives the taxable income possibilities and \( L_2 \) is the post-tax or consumption locus. When the interest rate declines, both these lines rotate. If period two is the retirement period with zero income, \( L_1 \) and \( L_2 \) will rotate about points on the horizontal axis. \( L_1' \) and \( L_2' \) are the new positions. The horizontal distance between \( E \) and \( A \) gives the level of \( R \). Point \( A \) moves leftward to \( A' \) indicating that \( R \) holdings will rise. \( C \) and \( C' \) are consumption points and \( B \) and \( B' \) are post-tax positions corresponding to \( A \) and \( A' \).

The effect on \( B \) holdings depends on the relative horizontal distances between \( B \) & \( C \) and \( B' \) & \( C' \). Three forces are at work here. First, the shift of \( B \) to \( B' \) is unequivocally a leftward shift and therefore tends to reduce \( B \) holdings (increase debt). The shift from \( C \) to \( C' \) involves two components. The income effect will be leftward, assuming consumption is normal, and will tend to increase \( B \) holdings (reduce debt). The substitution effect operates the other way: \( B \) holdings tend to fall (or debt rise). It is not possible a priori to state whether \( B \) holdings rise or fall on balance. But if the income effect equals the substitution effect (as at \( C' \)), \( B \) holdings will necessarily fall (by the horizontal distance from \( B \) to \( B' \)). Since the
horizontal distance from A to A' is greater than that between B and B', R rises by more than B falls and total savings would be greater after a fall in interest rates.

The somewhat counterintuitive notion that savings of the retirement security, R, rise as r falls derives from the income averaging role of R. As the interest rate declines, future income from R and B holdings falls, lowering future taxable income and marginal tax rates. Current holdings of R rise to equalize tax rates over time.

3. TAXATION OF BOND INTEREST

This provision alters the equation which gives the rate at which consumption can be transferred between periods. The return to a unit of B equals that on a unit of R and they are

\[ \frac{dc_2}{dc_1}_B = \frac{dc_2}{dc_1}_R \]

\[ 1 + r - T_2' r = \frac{1}{1 - T_1} (r + r) \]

This implies that

\[ r(1 - \frac{1}{1-T_1}) = \frac{1}{1-T_1} - \frac{1}{1-T_2} \]

The lefthand side must be negative since marginal tax rates lie between zero and one. This implies that T_1 is smaller
than $T_2'$. The taxation of bond interest (or deduction of debt service) will tend to cause reductions in bond holdings or increases in debt. To maintain some balance in consumption, people will want to save more R's. The increased R accumulation lowers tax rates in the early years and raises them in retirement.

4. DISTRIBUTIVE EFFECTS (Diagram 3.2)

R's benefit the rich, that is, the net discounted benefit of these tax arrangements, varies positively with the scale of income. In Diagram 3.2 below, we have the situation for two individuals identical in all respects but income. Let us assume for the moment that all income falls in the first period and that tastes are homothetic.

As we raise income, progressivity of the tax function implies that the outward movement of the post-tax (curved) line is less than in proportion to the shift of the pre-tax (through $E_1$ and $E_2$). Let $C$ be, the relevant consumption ray. With the use of R the poor and the rich man are $C_1$ and $C_2$ respectively. Without access to R tax provisions, the consumption points would be $A_1$ and $A_2$. Measured in terms of period one consumption, the gain attributable to R legislation for the poor man is $P_1D_1$ and $P_2D_2$ for the rich man.

The easiest way to see that $P_2D_2 > P_1D_1$ is to calculate
Diagram 3.2 Effects of the Scale of Income
the general points \( P \) and \( D \) for any \( E \). I have done this for a specific form of the tax function but the nature of the distributional effect will be the same for many progressive tax functions.

Let \( YT_1 = \) taxable income in period \( i \)

\( T_i = \) tax paid in period \( i \)

\( E = \) endowment (period 1)

\( A(YT_i)^B = \) tax function \( B > 1 \).

We have

\[
(3.4) \quad YT_2 = (E - YT_1)(1+r)
\]

and since \( YT_1 = YT_2 \),

\[
(3.5) \quad YT_1 = E \frac{(1+r)}{(2+r)}
\]

\[
(3.6) \quad T_1 = T_2 = A(E(1+r)^B)
\]

Therefore point \( D \) is

\[
(3.7) \quad D = YT_1 - A \cdot YT_1^B + \frac{YT_1^B}{1+r}
\]

which after substituting for \( YT_1 \) and \( YT_2 \) is

\[
D = E - AE^B \cdot \frac{(1+r)^B - 1}{(2+r)^B}
\]

\[
(3.8) \quad P = E - AE^B
\]

\[
(3.9) \quad D - P = AE^B \left( 1 - \frac{(1+r)^B - 1}{(2+r)^B} \right)
\]

Since \( B > 1 \), the gain \( (D - P) \) of having \( R \) varies more than..
in proportion to income. We have a similar result if E falls entirely in period two, although R borrowing will be necessary. When E falls on the 45 degree line, R's are of no use to either man.

If the endowment point moves outwards along the horizontal axis (or along any straight line through the origin) it should be clear by looking at Diagram 2.1 and Table 2.1 that R saving will change in direct proportion to earned income (E) and B saving will change in direct proportion to consumption (C₁). Thus, when taxes are progressive, the proportionate change in R is greater than that of B. If income does not increase proportionately in each period or if preferences are not homothetic, these relations may not hold.

The increase in total savings as we raise income depends on what happens to R and B individually. The following propositions can be seen to be true when tastes are homothetic, earned income is increased proportionately, total savings are positive, and taxes are progressive.

1. If R > 0 and B < 0 savings will rise more than in proportion to income.

2. If R > 0 and B > 0 savings will rise less than in proportion to income.
5. NON-NEGATIVE R

If R, like an RRSP, is restricted to non-negative values, taxes may be deferred forwards but not backwards. Someone whose wage rises throughout his life has no use for the R asset and will behave as if there were only B. If his income rises until retirement at age N, and then falls to zero, it will be to his advantage to defer payment of some of his taxes. The typical time path of taxable income will be rising up to a certain age and will be constant after that. Early in life, when income is low, people will not be able to borrow R and save B. This means that their taxable incomes will be lower in this period than the average over the lifetime.

6. PARTIAL DEDUCTIBILITY

Consider what happens if some percentage of R is deducted (or taxed). If income falls entirely in period one, the curved post-tax line will be tangent to the B trading locus at a point north-west of the 45° line. Provided the tax required is otherwise unchanged, taxes, consumption and total savings will be unchanged. R will rise and B will fall by equal amounts regardless of the position of the consumption point. Notice that this mechanism gives the government a way of influencing the composition of savings without affecting the level. As it stands now, foreign securities
and those not listed on the Toronto Stock Exchange are the only securities excluded from R portfolios. However, the potential for influencing the credit flows to the public and private sectors clearly exists.

7. INDEXING FOR REAL GROWTH

We have seen that under the B-R system an individual will try to keep his marginal tax rates constant over his life cycle. If the tax function is indexed this may involve having a rising tax and taxable income over the life cycle.

Where there is growth in real per capita income and the government fails to adjust the tax function average tax rates will rise and the government's share of total income will rise. If real incomes are growing at 2% we should deflate taxable incomes by 2% (or raise the tax brackets by 2%) to ensure a constant average tax rate. The exemption levels must also be raised by 2%.

This form of indexing is neutral in the sense that proportionate growth in the distribution of the tax base leads to proportionate growth in the distribution of taxes. If someone currently has twice the average income and pays three times the tax paid on the average income, proportional (or deflation) indexing preserves this relationship as incomes grow generally.
There is a fairly serious objection to be raised against this sort of indexing in spite of the advantages of a constant government share and distributional neutrality. When the rate structure is indexed there is a strong incentive to defer taxes. Setting inflation to zero and allowing for only 2% real growth, the tax minimizing strategy will involve almost $2\frac{1}{2}$ times the taxable income and tax at age 70 as at age 25. Taxable income should rise at 2% a year so that marginal tax rates are constant. While earnings typically grow at an average lifetime rate of about 2%, most of this growth is concentrated in the earlier years. Earnings drop off after 65. This is what causes a "shortage" of taxable income in old age.

Once the tax minimizing taxable income pattern is established using the tax deferral mechanism, conventional borrowing and lending can be used to get the desired consumption profile.

Taxes themselves will rise at 2% per year which means the greatest tax will be paid in old age. If we revise the indexing assumption from 2% to 3%, the ratio of taxes at 25 to taxes at 70 will go from $2\frac{1}{2}$ to $3\frac{3}{4}$. The life cycle distribution of taxes is strongly pushed towards old age by indexing.

There are other ways of ensuring that progressivity of
the tax function and income growth do not conspire against the size of private sector disposable income. Exemption levels can be raised so as to keep aggregate taxes constant. But this process will not preserve the distributional neutrality of the taxes. In the extreme as time progresses one man (the richest) will pay all the tax. This is because exemption levels will have to grow faster than incomes. Each year the portion of people who pay no tax will increase.

To keep his marginal tax rate constant under the "exemption indexing" system, the taxpayer contrives to have a constant taxable income and therefore a constant amount of tax each year. By contrast, under the "deflation indexing" scheme, taxable incomes and taxes must rise over the life cycle to keep marginal tax rates constant. In one case, individual taxes are constant over the life cycle and successive generations pay higher taxes. In the other case, successive generations pay higher taxes and those taxes are paid at an increasing rate over the life cycle. The switch from exemption indexing to inflation indexing will involve a large transitional drop in the tax base. This is because the young will adapt quickly, lowering their early period taxes. The larger future tax payments will not show up for at least a generation.

Let us look more generally at methods of indexing.
(3.10) \[ T_t = f(Y_t, t) \]

Taxes are a function of income at time \( t \) and of \( t \) itself. The marginal tax function is

(3.11) \[ f_1(Y_t, t) \]

Marginal tax rates will be constant when taxes are minimized so that (3.11) is equal to a constant and its derivative with respect to time is zero.

(3.12) \[ f_{11} \frac{dy_t}{dt} + f_{12} = 0 \]

If average income is growing at \( dy_t/dt \) then any function \( f \) satisfying (3.12) will keep government's share constant. Deflation indexing and exemption indexing are just two ways of building functions satisfying (3.12). Notice that if the tax function were proportional, \( f_{11} \) would be zero. \( f_{12} \) would therefore be zero as well. \( f_{12} \) tells us how strongly indexed the function is. It will typically be negative indicating that a fixed income attracts a lower marginal tax rate as time progresses. \( f_{12} \) will get larger (negative) as the degree of progressivity increases and as the rate of income growth increases. Indexing by deflation can be represented as

(3.13) \[ f(Y_t, t) = e^{gt} h(Y_t/e^{gt}) \]
where \( h', h'' > 0 \)

and \( g = \text{rate of growth of average income}. \)

Under these conditions, taxes for the individual will grow at \( g \) per year over the life cycle.

There are several interrelated factors here. The degree of progressivity of the tax function \((f_{ii} \text{ in equation 3.12})\), the size of the government's share over time, the pattern of life cycle tax payments and the drift in the distributive characteristics of the tax system over time; none of these things is independent of the others. We are now in a position to state the problem.

Given that we want a progressive tax system and we are unwilling to have government's share of income growing constantly, can we design a tax function that will keep the redistributive properties of the existing system without shifting taxes strongly towards old age?

Suppose incomes of people age 'a' in year \( t \) are given by

\[
Y_{at} = Y_{ao} e^{gt}
\]

where \( Y_{ao} \) is the income at age 'a' in some base year.

Successive generations will have income profiles which
are expanding at rate \( g \). Let the tax function be

\[
T_{at} = f(Y_{at}, t) = h(Y_{at}/e^{g(t-a)})e^{g(t-a)}
\]

\[h', h'' > 0\]

We want to demonstrate two things. First, this tax function will induce people to pay tax at a constant rate over their lifetimes and secondly, each generation will pay the same portion of its lifetime income in taxes. If lifetime incomes rise by 2% from year to year taxes will rise by 2% annually.

Consider the first point. For an individual, both \( t \) and a rise by one year. The \( e^{g(t-a)} \) terms in (3.15) are constant. The tax function does not change over time. The marginal tax rate will be constant only if \( Y_{at} \) is constant. Taxes are minimized when \( f_1 \) is constant. Taxes will therefore be paid out at a constant rate over the life cycle.

The second point can be demonstrated two ways. We should be able to prove that people of the same age in different years pay different tax and that people of different ages in the same year pay different taxes and that these differences are exactly proportional to the differences in earnings profiles for the different cohorts. In the first case, the ratio of taxes of different cohorts at the same age is
\[
\frac{T_{at}}{T_{as}} = \frac{h(Y_{ao} e^{g(t-a)}/e^{g(t-a)} - e^{g(s-a)}}{h(Y_{ao} e^{g(s)/e^{g(s-a)}} - e^{g(s-a)}} = e^{g(t-s)}
\]

This implies that taxes of successive generations rise at the same rate \(g\) as income. In the other case, taxes of people with different ages at the same point in time are related by

\[
\frac{T_{at}}{T_{bt}} = \frac{h(Y_{ao} e^{g(t-a)}/e^{g(t-b)} - e^{g(t-b)}}{h(Y_{ao} e^{g(t-a)}/e^{g(t-b)} - e^{g(t-b)}} = e^{g(b-a)}
\]

But we know that taxable incomes are constant over the life cycle and rise between generations at rate \(g\). Therefore, \(Y_{ao} = Y_{bo} e^{g(b-a)}\) and (3.17) becomes

\[
\frac{T_{at}}{T_{bt}} = e^{(b-a)}
\]

Differences in tax are proportional to differences in age.

It appears the tax authorities face a problem wherein certain desirable objectives are set against each other: Using traditional methods of adjusting the rate structure and exemption levels to achieve a reasonable flow of tax revenue there must necessarily be either a disruption of the existing distributive nature of the system or a strong inducement to defer taxes until retirement, or both.
One might solve this problem by using equation (3.15) as the tax function. However, this involves the use of politically unpalatable age discrimination. Essentially what (3.15) does is apply a tougher tax function to older taxpayers than to young ones. While this ensures that the older taxpayers, who are on lower lifetime earnings paths, pay the same portion of their lifetime earnings in taxes, it will be seen as unfair. Sixty-five year old taxpayers would like to be filling out the tax forms which apply to 30 year olds. This effect is stronger the more progressive are the tax rates.

The way equation (3.15) was set designed it generates a lifetime tax burden which is proportional to lifetime income for individuals of different generations occupying the same relative income positions in their respective cohorts. In addition, the individual is induced to use tax averaging savings devices so as to pay a constant tax over his life. In principle, the system can be refined to allow for any optimal pattern of lifetime taxes. One which might be better would involve optimal taxes rising gradually to 65 and dropping off after that. As the systems now exist in Canada and the U.S., very large amounts of tax exempt saving would have to be done to raise retirement period taxable income sufficiently to capture all the benefits of tax averaging. Age deductions and pension income deductions tend to exacerbate
the situation.

8. CONCLUSIONS

In this chapter, I set out to demonstrate several comparative steady state propositions. I have shown that a fall in the interest rate will stimulate R-saving and may increase or reduce B saving. The possibility of a perverse effect of the interest rate on savings exists (interest rate up, savings down).

Secondly, taxation of the interest on B naturally discourages B saving and also distorts the optimal pattern of taxes and marginal tax rates. Both will rise over the life cycle rather than remaining constant.

Thirdly, I have shown that shifting from B to B-R will benefit the rich more than the poor if no compensation is made. The benefits are more than proportional to income. R holdings change in proportion to earned income under the B-R system, while B holdings change in proportion to consumption.

Fourthly, I argued that a non-negativity constraint on R holdings may induce a rising and then constant pattern of taxes and tax rates over the life cycle.

Fifthly, I showed how the government can control the
composition of the optimal portfolio through a partial deductibility mechanism.

Lastly, I gave a method for indexing the tax function for real growth which does not imply a sharply rising pattern of optimal taxes the way conventional indexing does.
CHAPTER IV

TAX AND TRANSFER POLICIES

IN THE TWO ASSET SYSTEM

1. INTRODUCTION

In this chapter I seek to establish that a fairly wide range of policy tools can be conveniently integrated into the B-R system. I show that many of the Canadian tax-transfer programs are equivalent to adjusting the taxpayer's taxable income and/or disposable income in one or more periods. These programs generally involve a shift in the optimal B and R savings patterns which will partially or completely offset the government's initiative.

This framework gives a simple method of reducing policy options into two dimensions. It also makes it easy to judge the savings, tax and welfare effects of different policy measures. The policies which I wish to consider are grouped under four headings:

1. Direct Transfers
   - Old Age Security (OAS)
   - Guaranteed Income Supplement (GIS)

2. Compulsory Savings Plans
   - Canada Pension Plan (CPP)
   - Employer Pension Plans

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3. **Tax Exemptions and Deductions**

   Pension Income Deduction
   Interest Dividend and Capital Gain Deduction
   Age Exemption
   Basic Personal Exemption
   Standard Exemption

4. **Tax Links**

   Marriage Exemption
   Dependents Exemption
   Deductions Transferred From Spouse
   Spousal RRSP Contributions

2. **WHAT ARE B AND R?**

   The asset R is intended to represent RRSP's and other Registered Pension Plans (RPP). If we regard employer contributions to employee plans as coming directly out of wages then the employee does not pay tax on this withheld income and his pension plan is credited with the money. The process has the same effect on taxes and savings as a direct purchase of RRSP's so RRSP's and RPP's may be taken as identical.

   There are ceilings on RPP and RRSP accumulation. $5500 or 20% of earned income for RRSP's and a combined ceiling for RRSP's plus RPP of $3500 or 20% of income for those in employee plans. The lower rate for those with employee plans reflects the fact that employers are also contributing.
As noted before, it is not possible to have negative RRSP holdings. This means a floor of zero must be imposed on R since taxpayers might otherwise move to a negative position in R early in life when earned (and taxable) income is low. Whether or not these constraints are binding depends of course on individual circumstances.

The B asset represents bonds. Interest on bonds becomes taxable in amounts in excess of $1000. Therefore, a tax must be levied against those with bond holdings which exceed, say, $10,000 at 10%. Dividend and capital gain bearing assets do not fit conveniently into the B-R system. Some of the problems in this connection are addressed in Appendix 4.1.

Bearing in mind these provisions we can investigate the implications of changing parameters in the programs listed above. The task is simplified by the fact that most programs can be easily classified in terms of what they do to alter either taxable income, disposable income or both in each period of life. In each case, there is a simple change in the optimal B and R portfolio.

3. DIRECT TRANSFERS AND TAX CREDITS

Direct transfer programs are of two types. Ones like the OAS and GIS increase taxable income in one or more
periods of life. This stimulates offsetting adjustments in R to re-establish the equality of marginal tax rates. At this point the value of the transfer has been apportioned equally between all periods. If the consumer does not wish to allocate the extra consumption equally to all periods he will then use B to allocate the extra consumption perhaps in proportion to the existing consumption pattern. R keeps marginal tax rates in line, B maintains the desired consumption pattern.

The second type of direct transfer increases disposable, but not taxable, income in one or more periods. The optimal response to such a program involves an offsetting change in B. R does not change because taxable income and marginal tax rates are unaffected. B changes simply to allocate the extra consumption in the best way.

Tax credits like the Refundable Child Tax Credit do not affect marginal tax rates. They have the same influence on optimal B and R holdings as an increase in disposable income of an equal amount (assuming taxes are not zero).

4. COMPULSORY SAVINGS PLANS

Plans like the CPP have the effect of changing both taxable and disposable income in several periods. The CPP reduces taxable income and disposable income in the working
years and increases both in the retirement years. This relative shifting of young versus old taxable incomes will cause R to fall.

The introduction of the CPP involves an income effect which extends the lifetime consumption set. That is, the CPP deductions and payments promise an implicit rate of return higher than that for other securities.* If the CPP has an implicit return equal to that for other securities, R holdings would exactly offset the premiums and benefits. B holdings would be unaffected. If the implicit return is higher for the CPP, R will fall by more than CPP premiums and by less than discounted benefits. To the extent that the consumption set is improved, B holdings change to allocate the extra consumption, perhaps proportionately, to each period.

Employer and Registered Pension plans are much the same. The rate of return on them (they must by law be actuarially sound though in practice they are suffering large and growing experience deficiencies) cannot be much in excess of the rate

*We should, at the aggregate level, consider the Barro (1974) observations about wealth and the government debt. If the CPP involves a government debt then either the debt must be incorporated into individuals' tax expectations or we must assume individuals' tax expectations to be subject to error and therefore to be reformulated periodically according to some scheme.
on privately held RRSP's*. Contributions are deductible, benefits are taxed and if we regard the employer's contribution as coming out of employee wages, then Employer Registered Pension Plans act very much like an RRSP. Both the employer and employee contributions reduce the employee's taxable income, and all benefits are taxed.

5. DEDUCTIONS AND EXEMPTIONS

An increase in the Basic Personal or Standard Deductions will lower taxes paid in each period by an equal amount. No change in R results. (This assumes it was possible to equalize tax rates before.) B holdings will change to apportion the consumption gain according to the desired consumption profile (no change if consumption is uniform).

An increase in exemptions in a single period, like the age or dependents' exemption, provides an inducement to shift money into R to be liquidated in the exempt period and thereby reap a tax gain. The effect on B holdings is twofold. First, the shift in R must be offset or the consumption pattern will be twisted. Second, any extra consumption resulting from the overall tax reduction must be distributed across periods. The change in B for a given period will be

* Inflation erodes the rate of return on some pension fund portfolios. Accommodation of this shows up as a reduction in benefits often through restrictions on vesting and portability (Pesando and Rea, 1977).
minus the change in R for that period plus an amount due to the allocation of extra consumption.

The Pension Income Deduction (PID) applies to payments out of an RRSP or RPP. The CPP, GIS, OAS and some other small items are excluded. There is a deduction ceiling of $1000. A spouse's taxes may be affected by the amount of this deduction claimed.

Notice that if you pay taxes at any point in your life, it is not sensible to leave all or part of a potential $1000 PID unexploited. You can reduce taxes at no loss to future consumption by buying R and selling B. The Interest Income Deduction is very similar, having a $1,000 limit and linking the taxes of married couples. It is based on interest, dividends and realized capital gains.

Let me summarize the situation for a single individual. In the years before retirement, the taxpayer is eligible for several deductions and exemptions. His income consists basically of wages, bond and dividend income and transfer payments. The Deductions and Exemptions consist of the Basic Personal Exemption, the Standard Deduction, CPP contributions, RRSP and RPP contributions, Interest Dividend and Capital Gains Deductions and some other smaller items. In retirement, the taxpayer has these same deductions plus the Age Exemption and the Pension Income Deduction. No real
complications arise in the single individual case. He will try to spread taxable income evenly over all periods of his life. It is unlikely that his compulsory retirement income in the form of OAS, GIS and CPP exceed the retirement period total exemption level by enough to give rise to a marginal tax rate that is higher than in his working years. This means he will be able to add to retirement income using RPP's and RRSP's until tax rates are equal. Notice, however, that since he has higher exemptions in the retirement period and to the extent that he requires a lower than average level of consumption when old, he may have to borrow B to save the tax equalizing amount of R.

For married couples, the tax arrangements are quite a bit more complicated. Because of tax linkages, the effective post-65 marginal tax rate curve has a discontinuity in it. The tax rates of the husband and wife may not be equal unless the level of retirement savings is quite high. Furthermore, it is not generally possible to arrange to have equal tax rates before 65. These issues are examined in Appendix 4.2.
APPENDIX 4.1

DIVIDENDS AND CAPITAL GAINS

In reality, dividend and capital gain bearing assets form a substantial part of wealth held outside of institutional pensions and RRSP's. These two income streams have distinct tax treatments. Bond income over the limit is treated as ordinary taxable income. It attracts greater tax than the other two forms. Tax is paid on capital gains income at half the rate on bond income and then only when realized. When taxes are paid on an accrual basis, the asset holder is not able to compound the stream as quickly. The deferral of the tax is therefore advantageous to the capital stock holder.

Dividends generally attract the least tax. The recipient of dividend income pays tax at 1 1/2 times the value of the income at the full MTR but is credited for taxes on the same amount paid at a 25% MTR. Thus, if the taxpayer's MTR was 25%, he would avoid tax on the income altogether.

The value of the Dividend Tax Credit depends on the investor's MTR. If it is in excess of 25%, he pays tax on the income at a higher rate than he is credited for and a small net tax is implied. Someone with a taxable income of $20,000 in 1979 had a MTR of 28%. He would pay (0.28-0.25)x1.5 = 4.5% on the dividend income. Note that while
such income is less than $1000 it also qualifies for the
Dividend Income Deduction. In this range, a rise in dividends
may lower taxable income.

Clearly, the existing Canadian tax system allows for a
greater variety of assets and asset income types than is
captured by B and R alone. But greater detail can be had
only at the cost of much greater complexity when it comes to
calculating optimal tax strategy.
APPENDIX 4.2

TAX LINKS

There are three main ways the tax affairs of a husband and wife are linked. First, the spouse with the higher income, the husband, say, may claim the Married Exemption (ME), subject to a $2320 maximum. This deduction is reduced when the spouse's Net Income is greater than $430 and disappears altogether at Net Income of $2750.

Second, there are Deductions Eligible for Transfer (DET) which depend on the wife's Age exemption, Pension Income Deduction, Interest Income Deduction and her Net Income.

These factors along with the Spousal RRSP arrangements make for some rather complicated joint decision making in the household. One method for analyzing the problem is to establish for given time paths of R and B:

1. Who should claim interest income from B
2. Who should claim pension income from R
3. Who should contribute to R
4. Who should contribute to B.

If the total amounts of R and B are given in each period, the decision rules derived below indicate how the totals are best divided between the spouses. The tax calculation can then be done for each spouse and the amount of
disposable income for the family unit can be determined. A simulation could be done for the family the same way as is done in Chapter VII for a single individual, that is, by maximizing a family utility function with respect to total B and R.

Consider first the case of a couple where the wife has no income in the retirement period and when the husband wants to save using RRSP's. Net Income is given by

4. Al $\text{Net Income} = \text{Earned Income} + \text{CPP benefits}
+ \text{OAS} + \text{GIS} + \text{Interest Income}
+ \text{RRSP withdrawals}
- \text{CPP Contributions}
- \text{RRSP Contributions}
- other (UIC, tuition).

The first place for the husband to shelter his retirement income is in his own RRSP. He will be able to shelter $8290 by using the deductions shown in column one of Table 4. Al.

If he wants he can shelter $430 more in his wife's plan and also get $430 more tax room for himself through the DET as indicated in the second column of Table 4. Al. This gives him a tax free total income of $9150. Further savings in the husband's plan will lead to taxation. Savings in the wife's plan will result in a reduction of the ME and an offsetting rise in the DET. Savings made in this way do not
<table>
<thead>
<tr>
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<th>Husband</th>
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<td>Transfer (DET)</td>
<td>1660</td>
<td>1660+430</td>
<td>1660+1000</td>
</tr>
<tr>
<td>Total</td>
<td>$8290</td>
<td>$9150</td>
<td>$9720</td>
</tr>
</tbody>
</table>

The table above lists the tax sheltering of retirement income. Once the $1000 limit on the wife's Pension Income Deduction is reached however, the couple is going to have to pay tax. The maximum tax free level for them is $9720 (column 3). Contributions to the wife's plan beyond this point will reduce the ME with no offsetting changes in DET. The husband will be exposing his own pension to taxation by contributing to his wife's. But he can do no better by contributing directly to his own plan.

When the couple's retirement income is $9720 the wife has $1000 Net Income (her RRSP). When her Net Income reaches $2650 the DET are reduced by the amount of any further increases. Between $2650 and $2750 the husband exposes two dollars of his income to tax for every dollar he shelters in his wife's plan. He loses the ME and the DET. The ME is exhausted after the wife's Net Income passes $2750. Further pension income results in the same taxation regardless of which spouse holds it. Eventually, the wife's DET are reduced.
to zero. At a Net Income of $5300 the wife has no tax advantage to give to the husband. The husband has by this time $4310 in taxable income. Since further income in the wife's hands is taxable at her (lower) marginal tax rates with no effect on the husband's taxes, the couple will start accumulating RRSP's in her name. If they save enough, the wife's marginal tax rate will rise to the level of the husband's. The couple will then contribute to both plans equally and marginal tax rates will rise at only half the rate.

Notice that at a Net Income of $5310 the wife is using up all of her Personal Exemption ($2650), her Age Exemption ($1660) and her Pension Income Deduction ($1000). She may have other deductions left like the $100 Standard Deduction so there will be a certain amount of retirement income which collects tax free above $5310.

These relations are summarized in Diagram 4.A1. The bottom horizontal axis measures joint retirement income from RRSP's and RPP's, the left side vertical axis is husband's taxable income. Wife's taxable income is measured downwards from the upper right corner. Her Net Income is measured along the top. Notice the tax free maximum income level of $9720. $L_1$ and $L_2$ are the taxable income functions for the husband and wife respectively. The jog in $L_1$ at $2650$ and a gain at $2750$ represent the beginning of DET reductions and the end of ME reductions. The husband can avoid this jog if
he contributes only to his own plan so that the wife's Net Income never exceeds $2650. But since she is going to have other income for OAS, and GIS at least, the husband will probably be forced up to the higher branch. (In the following analysis I assume that the ME and the Personal Exemption are both equal to $2650. This eliminates the jog and reduces the tax free maximum by $100 to $9620.)

Diagram 4A1 was constructed on the assumption that the wife has no income besides RRSP income and that the husband's compulsory income from OAS, GIS; and CPP does not fully exhaust his deductions. This will not always be true. The wife may have GIS, OAS or CPP payments or Interest Income from her investments. These kinds of income will influence the ME and the DET and therefore the maximum tax free R-income that the couple can shelter.

Let us deal first with the case where the wife has Other Income in the form of OAS, GIS, CPP, or earned income but has no Interest Income. If the Other Income does not exceed $4310 it will not attract taxes even if she has up to $1000 in R-income as well. Her husband will lose an amount equal to her Other Income through the ME and/or DET so that the maximum tax free R-income level is $9620 less the amount of Other Income ($9720 if the jog is included). Furthermore, the amount of R-income beyond the first thousand that the husband can put in his wife's name; before her Net
Income reaches $5310, is reduced by the amount of her Other Income. Once this threshold is reached, additional R-income will be taxed at the wife's marginal tax rate and will cease to influence the husband's tax return. At this point, the husband's taxable income is $4310 less the amount of the wife's Other Income. Notice that the greater is the wife's Other Income the smaller is the range (A to B on Diagrams 4.A1 and 4.A2) over which the spouses will have different marginal tax rates.

To see, in general, whether the couple can equalize their tax rates we must know how much Other Income each member has and how much R-income and B-income they jointly have to declare. We have seen that if the wife has no Other Income and the desired R-income falls between A and B, rates will differ. If the wife has Other Income of $4310 and the husband has Other Income of not more than the sum of his Basic, Personal and Age Exemptions, the couple will be able to equalize tax rates regardless of the given R-income level. If the husband has more Other Income than deductions, he may have to pay tax even with zero R-income. Notice that since the leveling off point for the husband's taxable income rises with excess of income over deductions (so that effectively point B is further to the right), the range of different tax rates becomes larger.

If the wife's income is greater than $4310 and the given
R-income is low, the wife may have a higher tax rate than the husband.

Interest Income is much the same as R-income. The first $1000 counts as Net Income but also qualifies for the interest exemption and the DET. The maximum tax free R-income will be unaffected by Interest Income under $1000. Interest Income above $1000 will reduce the tax free maximum point until the wife's Net Income reaches $5310.

Diagram 4.A2 is drawn to represent the options open to a couple where the wife has $1700 Other Income and Interest Income of $1500. Notice the husband's tax rate levels out at $2110 instead of $4310. This $2200 drop results from the higher Net Income ($3200) and the higher offsetting DET ($1000 in Transferred Interest Deduction). Further Interest Income for the wife will close the distance AB lowering the region of differing tax rates. Further Interest Income for the husband above the $1000 exemption level will reduce the maximum tax free level.

What about the optimal distribution of B-income between the spouses? Notice that for tax purposes interest income from B and annuity income from R are indistinguishable; both are taxable above the first $1000 and both qualify for DET. If it weren't for the necessity of using both forms to claim the full first $2000 of tax free income from these sources,
Diagram 4.A2  Tax Sheltering of Retirement Income Between Spouses (2)
all that would matter would be the total income from R and B. The following sequential exhaustion pattern for given any R and B incomes is general enough to cover all possible combinations of Other Income between the spouses. The wife is the spouse with the lower Other Income.

\[ B \]
1. wife takes $1000 (or remainder) of B-income
2. husband takes $1000 (or remainder) of B-income
3. spouse with higher tax rate takes B-income until it is exhausted or until tax rates are equal after which it is split equally between the spouses

\[ R \]
4. husband takes $1000 (or remainder) of R-income
5. wife takes next $1000 (or remainder)
6. spouse with lower tax rates takes R-income until it is exhausted or tax rates are equal after which it is split equally.

There are of course other optimal sequences. Using the example of Diagram 4.A2, it is clear that since the wife has $1500 in Interest Income, the husband must be taking at least $1000 in Interest Income too. If the husband had a taxable income at this point, the wife would then receive the next $500 in Interest Income because her tax rate would either be lower than her husband's or they would both be zero. With the maximum tax free point and the extent of the leftward shift of the wife's Net Income axis established for the given B-income level, the optimal allocation of R-income can be read from Diagram 4.A2 for any given total level of R-income.

We must now consider the division of R- and B-income
between spouses in the pre-65 period. The only real differences are that the Age Exemption and the Pension Income Deduction disappear so the DET can now reach a maximum of only $1000 (the Interest Income Deduction).

To make sure that savings strategies for fixed R and B in the pre-65 years are consistent with the income declaration strategy in the post-65 years, notice that for R the only relevant question in the pre-65 period is which spouse contributes and in the post-65 period the only relevant question is to which spouse's plan was the contribution made. (Contributions can be made by one spouse to the plan of the other.)

If B is fully transferrable between spouses at no tax penalty, we can separate the decision-making process: given levels of R and B in a pre-65 period will be allocated between spouses in a way which is independent of post-65 Other Income levels or post-65 R and B levels.

The couple will strive to have equal tax rates in the pre-65 period. One sequence which gives an optimal distribution of R and B is very similar to that for the post-65 period.
B - 1. wife claims $1000 (or remainder) in B-income
2. husband takes next $1000 (or remainder) B-income
3. spouse with lower tax rate takes B-income until it is exhausted or tax rates are equal. Further income is split equally.

R - 4. spouse with the higher tax rate makes R contributions until tax rates are equal after which contributions are made equally by both spouses.

This yields a systematic method for determining who saves and who declares income from R and B in each period of life.
CHAPTER V
CURRENT LIFE CYCLE PATTERNS.

1. INTRODUCTION

Chapter IV outlined the budget constraint faced by a single taxpayer forming his lifetime savings plan. To simulate how changes in this constraint affect the consumer, we must know how consumers currently allocate their consumption over time. One might reason, a priori, that the life cycle pattern of consumption, abstracting from family size effects, is U-shaped. That is, if consumption of goods requires time and if the opportunity cost of time is highest in the middle years when wages are at their peak, then consumption will be low in middle age.

Becker (1976) argues that there is another potential factor influencing the shape of the consumption path. If consumption goods are an input to the household production function then consumption goods input will be highest when wages and work effort are highest. If the elasticity of substitution between periods is small for goods as final consumption goods relative to the elasticity of substitution for goods as inputs to household consumption, then the life cycle pattern of "consumption" will be hump shaped.

Which of the above two patterns is correct is an empirical matter. For evidence on this I have used data from
the Survey of Consumer Expenditure (Statistics Canada, various years). I ran consumption on a simple trend cycle regression equation to see whether the cycle component displayed a U-shape or a hump-shape. Certain adjustments had to be made to the data to account for consumption of owner occupied housing. I also ran trend cycle regressions for income and taxes. These variables are available from the same database. These regressions provided some rather interesting results which are given in Table 5.6.

2. A TREND-CYCLE REGRESSION

What we want to do is take pooled time-series cross-section data and estimate what the typical family consumption profile looks like. We can do this by assuming that successive generations behave identically to a scale factor and by allowing for a trend in lifetime income from generation to generation. We have this simple trend-cycle relation.

\[
C_m^i = \alpha \lambda^m (1 + \lambda \delta)^{m-i}
\]

where \(C_m^i\) is the consumption at age \(i\) of a family whose head was born in year \(m\), and \(\lambda\) is the growth rate of lifetime family incomes from one generation to the next. The \(\alpha\) can be interpreted as the relative rates of consumption for a representative family at each age, \(i\), of its life cycle.
Forming the ratio of an i-m family to a j-n family and taking logs we have:

\[(5.2) \quad \log(C_i^m/C_j^n) = \log \text{ALPHA}_1 \log \text{ALPHA}_3 + \log(1 + \text{LAMDA}) (m^n+j-i)\]

Equation (5.2) can be estimated using dummy variables. The equation becomes

\[(5.3) \quad \log(C_i^m/C_j^n) = B_0 (m^n+j-i) + B_1 X_{1} + B_2 X_{2} + B_3 X_{3} + B_4 X_{4}\]

Hence, the B's are estimates of the logs of the ALPHAs and \(B_0\) is the estimate of \(\log(1 + \text{LAMDA})\). The X's are dummy variables where

\[(5.4) \quad X_k = 1 \text{ if } i = k\]
\[X_k = -1 \text{ if } j = k \quad k = 1,4\]
\[X_k = 0 \text{ if } i \neq k, j \neq k \text{ or } i=j\]

In fact, the regression cannot be run in this form because the X's are linearly dependent. But we may set \(B_4 = 0\) (\(\text{ALPHA}_4 = 1\)) since the ALPHAs represent the relative rates of consumption. We can drop \(B_4 X_4\) from (5.4) so that \(\text{ALPHA}_4\) implicitly becomes unity. Now let me turn to the problem of getting data for the dependent variable.

3. GETTING CONSUMPTION DATA FROM EXPENDITURE DATA

A copy of the 1976 Urban Family Expenditure data
appears in Table 5.1. It gives Average Family Expenditure by item in 1976 dollars. Owned Living Quarter Expenditure includes Mortgage Interest payments but not repayment of principle. It also includes maintenance and repair expenditure. The value of improvements is recorded as part of the Net Change in Assets and Liabilities. This latter Asset and Liability category is not an ideal measure of wealth because it does not include pension wealth. Contributions to pension plans of all types are recorded in the Security category. The reason these savings are not reflected in the Net Change in Assets and Liabilities is that the value of such a heterogeneous portfolio is difficult to determine.

The Family Expenditure publication also gives Expenditure by class of tenure (Owned or Rented). This information is useful because it helps to compare the expenditure levels of renter and owner families. Unfortunately, there is no crosstabulation of these data by age. There are data, however, on the number of house owners in each age group.

The object here is to assemble from these data a time series on consumption by age group. Since these are expenditure data an adjustment must be made to account for durables. In the time frame of my simulations, the major durable is housing. First consider the non-housing component of consumption (NHC) in each age group. At age i, average per
<table>
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<th>Income</th>
<th>Single's Consumption</th>
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</table>
capita NHC for Owners (O) and Renters (R) together is

\[ (5.5) \quad NHC_{iT} = \text{Total Current Expenditure} - \text{Rented Shelter Costs} - \text{Owned Shelter Costs} + \text{Gifts and Contributions} \]

It is convenient to split this into renters and owners, i.e.

\[ (5.6) \quad NHC_{iT} = a_i NHC_{iR} + (1 - a_i) NHC_{iO} \]

where \( a_i \) is the proportion of renting families in age group \( i \). Let \( b_i \) be the ratio of average owner NHC to average renter NHC in each age group.

\[ (5.7) \quad b_i = \frac{NHC_{iO}}{NHC_{iR}} \]

Therefore,

\[ (5.8) \quad NHC_{iR} = \frac{NHC_{iT}}{a_i} - NHC_{iO} \left( \frac{1 - a_i}{a_i} \right) \]

\[ = \frac{NHC_{iT}}{b_i \left(1 - a_i + \frac{a_i}{b_i}\right)} \]

And similarly,

\[ (5.9) \quad NHC_{iO} = \frac{NHC_{iT}}{1 - a_i + \frac{a_i}{b_i}} \]

We have data on everything but \( b_i \). The \( b_i \) are likely to be greater than one indicating greater consumption on the part
of homeowners. This is due to the higher lifetime income of house owners. The owner/renter average NHC ratio for the population as a whole will be affected (positively) by concentration of home ownership in the 35-55 year age group where consumption is also high. As noted before, the survey contains no data on expenditure by tenure by age group. But there are data on tenure by age group, expenditure by age group and expenditure by tenure. These data are sufficient to establish $b_i$ if we make the assumption that $b_i$ is the same for all age groups. This implies that a renter of age $i$ and an owner of age $i$ on average have the same NHC in proportionate terms as a renter and owner of age $j$.

NHC and home ownership are both age-related so that the aggregate owner NHC to renter NHC, $N$, will depend on the age distribution of the population.

$$N = \frac{NHC_O}{NHC_R} = \frac{\sum_{i=1}^{n} (NHC_{iO} \cdot w_i \cdot (1-a_i))/(\sum_{i=1}^{n} w_i \cdot (1-a_i))}{\sum_{i=1}^{n} (NHC_{iR} \cdot w_i \cdot a_i)/\sum_{i=1}^{n} w_i \cdot a_i}$$

(5.10)

where $w_i$ is the relative size of age group $i$. Substituting (5.8) and (5.9) for $NHC_{iO}$ and $NHC_{iR}$ and simplifying we get

$$N = \frac{\sum_{i=1}^{n} \frac{b(1-a_i)Z_i}{(1-a_i + \frac{a_i}{b})}/\sum_{i=1}^{n} w_i \cdot (1-a_i)}{\sum_{i=1}^{n} \frac{a_i \cdot Z_i}{(1-a_i + \frac{a_i}{b})}/\sum_{i=1}^{n} w_i \cdot a_i}$$

(5.11)

where $Z_i = NHC_{iT} \cdot w_i$. 
This is a non linear expression which may be solved numerically for \( b \). There may be multiple solutions but we have a good idea of the size of \( b \). The ratio of per capita NHC of owners to renters in 1976 was 1.5. Since this ratio does not account for age distribution we would expect \( b \) to be less than 1.5. It is not likely that \( b \) be less than one given the correlation of lifetime income and home ownership. Notice that if we assume a constant consumption profile, a flat age distribution and a constant renter proportions across age groups \( (z_i = Z, a_i = a \text{ and } w_i = w) \), the right hand side of (5.11) reduces to \( b \) which implies that \( N = b \), i.e.: the owner/renter average NHC ratio is the same in each age group as it is in aggregate.

Using the value of \( b \), derived from (5.11), we can compute the non housing consumption of owners and renters using (5.8) and (5.9). From here, it is easy to calculate the total consumption for renters.

\[(5.13) \quad TC_{iR} = NHC_{iR} + RENT\]

If we assumed home owners draw a proportion \( h \) of total consumption from housing services, total consumption for owners is given by

\[(5.14) \quad TC_{iO} = \frac{NHC_{iO}}{1-h}\]
Let us assume renters and owner families have the same utility functions. In fact, many families are renters at one point in the life cycle and are owners at another. Although the portion of consumption derived from housing services may vary between renter and owner families, tenure cannot in itself account for differences in total consumption. Such differences are attributed solely to lifetime income differentials. The life cycle consumption pattern we want is therefore given by the weighted sum of owner and renter total consumption rates at each age.

\[(5.15) \quad TC_{IT} = a_i \cdot TC_{IR} + (1-a_i) \cdot TC_{IO}\]

\[= a_i \cdot RENT + (\frac{a_i}{d} + \frac{1-a_i}{1-h})/(\frac{a_i}{d} + 1-a_i) \cdot NHC_{iT}\]

In expression (5.15) all the right hand side variables are known except \( h \) which is the portion of total consumption which homeowners derive from housing services. In 1976, the comparable figure for renters was about 18%. To establish a reasonable value for \( h \) for owners we must review the relationship between house value \( (V) \), depreciation \( (\text{DELT}A) \), interest, taxes and other expenses \( (T) \), and the service flow from the stock \( (f) \). If the house owner is to be indifferent between holding a house and a bond then

\[(5.16) \quad f = r + \text{DELT}A + T/V\]
This says that the service stream per dollar of housing must equal the opportunity cost of the dollar (r) plus depreciation, plus the expenses incurred per dollar of housing.

The average Multiple Listing Service house price in 1976 was approximately $60,000. Property taxes and Maintenance and Repair expenses in the Urban Family Expenditure data are set at $1293 or about 2.2% of the average home value. If the depreciation rate is 2% and the real after tax interest rate is 4%, equation (5.16) implies a value of about 8% for f. The elasticity of f with respect to variations in DELTA is small (0.245) and the elasticity with respect to r is fairly small (0.47).

If f is 8%, the rental value of the average home is $4705 per year (or $392 monthly) after expenses. Since we know non-housing consumption and housing consumption of homeowners we can determine h

\[
(5.17) \quad h = \frac{4705}{NHCO + 4705} = 27\%
\]

It is now possible to construct the cross-section of consumption by age group given in equation (5.15). Urban Family Expenditure data are available on a comparable basis for the years 1964, 1967, 1969, 1972, 1974 and 1976. Using these six years and four age categories, equation (5.3) will be a pooled time series cross-section regression with 276 observations; that is, all combinations of i, j, m and n
where \( i, j = 1, 4 \) and \( m, n = 1, 6 \) eliminating cases where \( i=j \) and \( m=n \). Also, if we include an observation where the dependent variable is \( \log \left( \frac{C_b^a}{C_d^c} \right) \) then we must eliminate the observation where the dependent variable is \( \log \left( \frac{C_d^c}{C_b^a} \right) \).

4. PATTERNS OF CONSUMPTION TAXES AND INCOME

The results of the regressions are presented in Table 5.2. The age intervals used were under 25, 25 to 45, 45 to 65 and 65 plus. Taking logs of the coefficients on \( X_1 \), \( X_2 \) and \( X_3 \) and recalling that the log of the coefficient on \( X_4 \) is constrained to equal zero, we see that the relative rates of consumption in the four periods are 0.47, 0.89, 1.34 and 1.00 respectively. The annual rate of increase of lifetime family consumption is 2%. The relatively high level of consumption in the third period (45-65) is the result of several factors. Most importantly, the incidence of home ownership rises sharply up to age 55 (64.7%) and settles back to 59.6% by age 65. There is a 15 year difference between the average age of homeowners with versus without mortgages; 43.1 and 58.1 respectively. While income probably rises in real terms throughout the working years, mortgage payments end for most families in the third period freeing up income for consumption and non-house forms of saving. Table 5.3 shows the values of total consumption derived from equation (5.15) in constant 1971 dollars for each of the six years and four age groups.
TABLE 5.2 Trend-Cycle Regressions
(See Table 5.4 for summary of regression results)

Consumption Trend Cycle Regression

\[
\text{Consumption} = 0.020 \text{ LAMDA} - 0.75 X_1 - 0.12 X_2 + 0.29 X_3
\]
\[
(24.95) \quad (-18.31) \quad (-3.64) \quad (16.66)
\]

\[
R^2 = .9642
\]

Tax Trend Cycle Regression

\[
\text{Taxes} = 0.082 \text{ LAMDA} - 3.67 X_1 - 1.99 X_2 - 0.41 X_3
\]
\[
(43.44) \quad (-37.41) \quad (-26.41) \quad (-9.82)
\]

\[
R^2 = .9396
\]

Income Trend Cycle Regression

\[
\text{Income} = 0.034 \text{ LAMDA} - 1.49 X_1 - 0.63 X_2 + 0.079 X_3
\]
\[
(53.43) \quad (-44.70) \quad (-24.57) \quad (5.56)
\]

\[
R^2 = .9805
\]

t-statistics in brackets.
LAMDA = growth rate variable (trend)
X's = dummies for cycle
One might expect life cycle family consumption to reach a peak in the 25-45 period during family formation. This appears not to be the case. In the pooled data consumption peaks in period three. Since the period two families are on higher lifetime consumption paths then, a fortiori, consumption for an individual family should be higher in period three. The drop in consumption in period four in the pooled data is dramatic enough to offset the trend in lifetime incomes so that retirement consumption is only 75% of peak consumption.

The values of b, the ratio of owner NHC to renter NHC, are computed for each of the six time periods and appear in Table 5.3. As expected, they are in the range 1 to 1.5.

It is dangerous to infer from these data that the life cycle consumption patterns of future generations will be the same. The observation period is one which saw significant institutional and demographic changes. Fitting a simple trend-cycle equation to these data tends to bury the once-and-for-all factors. The role of taxes is particularly important in this respect. We know from the theory of earlier chapters that when consumers have both R and B to use as savings vehicles, they will try to equalize tax rates over their lives. Failure to do so must be due either to legal restrictions on B and R transactions, liquidity constraints
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<td>1.49</td>
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</table>
or on thwarted expectations about income or other life.
cycle variables.*

Fitting the same trend-cycle equation to personal taxes
as was done for consumption provides some interesting com-
parisons. Taking the logs of the coefficients on the $X_i$ in
the second equation in Table 5.2, we can see that the rela-
tive tax burdens are 0.026, 0.14, 0.66 and 1.00 for the four
periods respectively. These results certainly do not accord
with the constant marginal tax rate prediction of the model
in Chapters II and III. If there were a binding constraint
on the amount of deductible R investment we might expect the
fourth period tax rate to deviate from the others but these
data indicate the peak tax load in period four. The explana-
tion for this lies in the coefficient on LAMBDA. The 8.2%
average annual increase in taxes swamps the life cycle effect
altogether. Taxes will double in less than ten years so that
a person who earns $10,000 a year between 55 and 65 and
defers $5000 for consumption between 65 and 75 will be paying
over twice as much tax in retirement even if he earns no
interest on his savings.

Two points are relevant here in relation to future pre-
dictions. First, this rate of growth cannot be expected to
continue. Even with a real income growth of 4% per year,

* RRSP's which correspond to R in our system do have maximum
restrictions and were not available prior to 1970.
taxes would eat up 100% of output within 20 years. Second, the growth in taxes in this period was probably not fully anticipated so we are not observing consumers in their steady state positions. When the government raises tax rates or lowers brackets, the optimal strategy for the taxpayer is still to have equal marginal tax rates across periods but he must correctly anticipate the adjustments. This means having a declining taxable income. Underestimating the rate of tax increase results in the ascending pattern of taxes indicated by this second regression. As the tax rate increases decline, and/or expectations rise, the pattern will flatten, although it is not likely that first period rates rise very much, due to the non-negativity constraint on R holdings.

Another once-and-for-all feature of the observation period is the relative income gains in the different age groups. Between 1964 and 1976 the increases in constant dollar average family income from all sources were 71%, 61%, 52% and 35% for the four age groups respectively. A more detailed analysis of the tax data would need to include adjustments for the relative income growth rate of different cohorts. It is curious that the relatively abundant age category, the young baby boom children, experience the greatest gains. Supply and demand reasoning would indicate the reverse, that the unit value of their skills should decline in relative terms. The increase in multiple earner families
and improvements in education are probably significant factors here.

The trend-cycle model was fitted to the income data in the Urban Family Expenditure tables in the same way as was done for consumption and taxes. The results of this regression are given in the last equation of Table 5.2. In Table 5.4, I have summarized the results for all three trend-cycle regressions. The first row gives the average growth rates or trend components and the last four lines give the cycle components expressed in percentage terms. These latter terms correspond to the ALPHA's of equation (5.1).

Since my simulations are done for single individuals rather than family units, it is convenient to compute a singles' equivalent consumption pattern. We know the average family size for each age of head. Statistics Canada publishes a poverty line standard which varies by family size (Income Distribution by Size in Canada, 13-207, p. 27). If we divide consumption of families in each age group by the poverty line index for family size equal to the average for the age group, we achieve a rough conversion from family to individual age-consumption profiles. The result of these computations is shown in column four of Table 5.4. Singles appear to have a greater percentage of consumption in retirement and a smaller percentage in the 25 to 45 age range than do family units.
TABLE 5.4 Summary of Trend Cycle Regressions

<table>
<thead>
<tr>
<th></th>
<th>Taxes</th>
<th>Consumption</th>
<th>Income</th>
<th>Single's Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth rate</td>
<td>8.5%</td>
<td>1.9%</td>
<td>3.5%</td>
<td></td>
</tr>
<tr>
<td>Period 1 0-25</td>
<td>.01</td>
<td>.13</td>
<td>.08</td>
<td>.13</td>
</tr>
<tr>
<td>Period 2 25-45</td>
<td>.08</td>
<td>.25</td>
<td>.19</td>
<td>.21</td>
</tr>
<tr>
<td>Period 3 45-65</td>
<td>.34</td>
<td>.36</td>
<td>.38</td>
<td>.35</td>
</tr>
<tr>
<td>Period 4 65+</td>
<td>.57</td>
<td>.26</td>
<td>.35</td>
<td>.29</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
SUMMARY

It appears that the second of the two hypotheses concerning the pattern of life cycle consumption is borne out by the data. Namely, that a hump-shaped time path results from a more intensive use of consumption goods as inputs to the household production function when wages and work effort are high. This holds even when an adjustment has been made to remove the effects of family size.

The time periods chosen for the relative consumption levels in Table 5.4 are in many ways less than ideal. This was unavoidable because data for two of the years was given on this basis. Data for 1976, however, were recorded by ten year intervals (Table 5.1). These data confirm the lower consumption levels after 65. It is likely that consumption begins to taper off some time before 65. Total expenditure and total consumption in the cross-sectional data are lower for the 55-65 age group than for any other group except the under 25's. When we consider that the trend growth in lifetime incomes, it seems likely that consumption actually peaks at around 50 and declines thereafter.

Combining the 25-35 and 35-45 groups tends to mask the marked behavioral differences of these two groups. The change in incidence of home ownership in income and in consumption between the groups and the strong similarities between the 35-45 and 45-55 groups tends to make one think
that a different breakdown would have been more natural. Even so, it is clear that the peak consumption period and the peak family size age period are separate by about a decade, about 40 and 50 respectively.

It is difficult to estimate exact life cycle consumption profiles. It is impossible, without longitudinal data, to construct typical income paths for low income and high income groups, although this information would be valuable for assessing the distributive effects of many government tax and transfer programs. For instance, costs of pensions, UIC, welfare payments and many other expenditure items depend on the distribution of lifetime income. Add to this the fact that future generations may exhibit very different social behavior in terms of family structure and fertility rates and it becomes clear that estimates of future consumption patterns are educated guesses at best.

In the next chapter, I set up a utility function and set of constraints that yield hump-shaped patterns of consumption suggested by the results in this chapter.
CHAPTER VI

SETTING UP THE SIMULATIONS

1. INTRODUCTION

There are three objectives in this chapter. First, I lay out the parameters underlying the utility function for an individual taxpayer. These parameters are chosen in such a way that the hump-shaped pattern of consumption described in the previous chapter emerges from the maximization process.

Second, through an examination of empirical work by Mincer (1974), I derive a family of earnings profile. I choose a profile which fits within the space spanned by this family for use in the simulations in the next chapter. While only one profile (and multiples of it) are used, it is possible to draw a number of a priori conclusions about the effects of varying the shape of the profile.

The last objective of this chapter is to outline the method used to construct the tax function. This is quite important because it directly influences the nature of the taxpayer-consumer's constraint set and is also central to the question of how important the income averaging feature of the B-R system is.
2. THE UTILITY FUNCTION

One thing we should ask of the utility function is that it generate, in conjunction with the budget constraint, a pattern of life cycle consumption that is roughly hump-shaped, peaking at around 50. Also, the elasticity of substitution embodied in the function must be set to reflect the actual sensitivity of savings to the rate of interest.

One of the forms commonly used in the literature; White (1978), Davies (1980), Blinder (1974), Farber (1978), is given in (6.1)

\[ U = \sum_i A_i C_i^B, \quad B < 0, \quad A_i > 0 \]

(6.1)

The \( A_i \) are discounting weights which in some cases incorporate a family size factor as well. In the formulation used by Davies (1980), the consumption variable is consumption per adult equivalent and \( A_i \) is the number of family members. The elasticity of substitution \( (\sigma_{ij}) \) for the form in (6.1) is independent of the \( A_i \).

\[ \sigma_{ij} = \frac{\frac{dC_i}{C_i} - \frac{dC_j}{C_j}}{\frac{dU_i}{U_i} - \frac{dU_j}{U_j}} = \frac{1}{1-B} \]

(6.2)

There is a variety of evidence on \( \sigma \). Most researchers assume that it is constant over the life cycle. Simulation studies by Auerbach and Kotlikoff (1981) use the special logarithmic utility function where the elasticity of substitution is
unity. Some research indicates that $\sigma$ is less than one. By examining the cross-sectional data on the Financial Characteristics of Consumers and Changes in Family Finances published by the Federal Reserve Board for the years 1962 and 1963, Friend and Blum (1975, p. 900) conclude that "investors require a substantially higher premium to hold equities or other risky assets than they would if their attitudes toward risk were described by logarithmic utility functions."

Blinder (1976) also does simulations based on life cycle models to investigate the determinants of the distribution of wealth. He uses a utility function which has a $\sigma$ value of 0.67. He cites a number of previous studies which estimate values of $\sigma$ of .67 or less.

Ghez and Becker (1975, p. 137) by using cross-sectional estimates of the labor supply and consumption responses to changes in wages and age are able to put an upper bound on $\sigma$ of 0.28. The corresponding value of $B$ in equation (6.1) is -2.57.

As it turns out, the value of $\sigma$ has little bearing on my simulation results because there are no substitution effects in the comparative static experiments that I perform. For this reason, I used only one value for $B$. With $B$ equal to -2.5, the elasticity of substitution lies between 1/3 and 1/4.
Empirical work by Ghez and Becker (1975, p. 60) supports the hump-shaped consumption hypothesis of Chapter V. It indicates that consumption varies with age, rising to about 50 and falling off after that. They ran regressions on 1959 American Survey of Expenditure data. These regressions highlight the cross-sectional dependence of consumption on age and level of education. The education variable is strongly correlated with lifetime income. The results indicate that the higher the level of education (and income) the greater the relative consumption in the mid to late forties. The peak consumption period occurs at approximately the same age across education levels but is two and a half times first period consumption for those with at least high school and slightly less than two times first period consumption for those with less than eight years of schooling. This discrepancy is not that great when you consider that the rich group's average income is almost twice that of the poor group. Thus, an income effect appears to be at work although its strength is not very great. I have used a homothetic form, as have many others, since income effects are probably weak. Since the shift from the B to the B-R system involves no substitution effects and since the consumption function is homothetic, consumption changes will be proportional. The proportions are determined by the choice of the $A_1$ in equation (6.1). With a four period life cycle and $A_1 = 2.3$, 
\( A_2 = 3.4, A_3 = 5.0 \) and \( A_4 = 2.0 \) we get a consumption pattern which is proportional to \( C_1 = 10.4, C_2 = 13.8, C_3 = 18.4 \) and \( C_4 = 16.9 \). This displays the typical hump-shaped pattern described in Chapter V.

3. EARNINGS

Mincer (1974) provides evidence on the age distribution of earnings based on American data on earnings, schooling and age of white non-farm males under 65. The theory behind his regressions is that earnings reflect the level of schooling and self investment in human capital after schooling. Returns to investment will decline as the remaining working period declines and the opportunity cost of investment (particularly of time) will rise as earnings levels bear the fruit of previous investment. For these reasons, the rate of investment should decline through the working years and should generate an earnings profile which rises, but at a declining rate.

Mincer fits several functional forms to the data. These correspond to different specifications of the time path of self investment: linear, parabolic and exponential. In Table 6.1, there is a set of earnings patterns derived from regressions based on the different investment specifications and schooling levels. These are normalized so that period one earnings are one in each case. Notice that for three
out of the four specifications, the rate of growth of earnings is greater for those with higher levels of schooling. This implies that those with high levels of schooling will be better served by final earnings pension plans than those with less schooling.

There is quite a range of patterns within this table. Rather than simulate many different patterns, I have taken just one (and multiples of it to check scale effects). Certain \textit{a priori} conclusions can be drawn about the effects of making variations about this path and these are drawn out in the next chapter. The earnings patterns used are proportional to 1, 1.5, 1.6, 0 for the four age groups 20-35, 35-50, 50-65 and over 65.

4. TAXES AND DEDUCTIONS

To get a tax function, I fit a parabolic form to the set of tax rates by taxable income level from the 1976 income tax forms. The regression is

\begin{equation}
\text{TAX} = 0.0954 \times Y\text{TAX}^{1.271} \\
\text{(32.303)} \quad \text{(180.2)} \\
R^2 = .9999
\end{equation}

where TAX is tax owed and

\begin{equation}
Y\text{TAX} \quad \text{is taxable income}
\end{equation}

The t-statistics appear in brackets. The nonlinear estimation package of TSP was used for estimation.
Table 6.1  Age Earnings Profiles

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>E1</th>
<th>E2</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.000</td>
<td>1.000</td>
<td>1.000</td>
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</tr>
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</tr>
<tr>
<td>14.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>16.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

P1  First Parabolic Specification
P2  Second Parabolic Specification
E1  First Exponential Specification
E2  Second Exponential Specification

The three age intervals are 20–35, 35–50, 50–65
The taxable income variable used in the simulations consists of earned income less certain deductions. I took the deductions and exemptions which are applicable to a single individual in 1976. These include the Basic Personal Exemption, the Age Exemption for those over 65 and the Pension Income Deduction for those over 65 cashing in up to $1000 of R assets. Canadian taxpayers are eligible for a number of other deductions as well as those mentioned above. To capture the incidence of these other deductions by earnings level I ran a regression of deduction items 41-50 plus 52 from Taxation Statistics (Revenue Canada, 1978) against a constant and average earnings in each income class. This equation is reproduced in (6.4).

\[
(6.4) \quad \text{DEDUCTIONS} = 0.077 + 0.040 \times \text{EARNINGS}
\]

\[
(2.753) \quad (20.28)
\]

\[ R^2 = .9952 \]

Since the dependent variable was expressed in thousands of dollars this equation tells us that the other deductions are worth about $77 plus 4% of earnings. In the simulations, taxable income consists of earned income less all of the above deductions and exemptions less all contributions to the Canada Pension Plan plus all receipts from the Canada Pension Plan. CPP contributions are calculated as 3.6% of earnings for contributions while payments are equal to the year's maximum pensionable earnings times the ratio of actual
pensionable earnings to the maximum in the taxpayer's last
year of employment.

All exemptions and deductions and CPP variables are in-
dexed at 2% per year as is the set of marginal tax brackets.

A real interest rate of 4% is assumed for the return on
all savings of B and R. No tax is charged on this interest.

5. SUMMARY

In this chapter I set out to describe three things
needed for the simulations; a utility function, an earnings
profile and a tax function. The utility function I chose
has a simple isoelastic form:

\[
U = \sum A_i C_i^{2.5}
\]

where \( C_i \) is consumption in period \( i \), and
\[
A_1 = 2.3 \\
A_2 = 3.4 \\
A_3 = 5.0 \\
A_4 = 2.0
\]

The earnings profile is proportional to 1.0, 1.5, 1.6,
0.0. Three earnings paths are actually tested, one starting
at $8,000 per year and the others starting at $13,000 and
$18,000. The effects of varying this pattern are discussed
in the next chapter.
The tax function is based on actual Canadian tax data. It consists of the 1976 rate schedule and the exemptions and deductions which apply to a single individual. These two components of the tax function were given in equations (6.3) and (6.4).

This completes the list of requirements for doing the simulations. We are now in a position to start testing some of the effects of tax policy change on taxpayer behavior.
CHAPTER VII
SIMULATION RESULTS

1. INTRODUCTION

In this chapter I present the results of the simulations. There are five basic areas of interest.

1. What are the changes in steady state tax and savings profiles when the R asset is introduced? (Keeping the PDV of tax constant.)

2. How do variations in the assumed income path and utility parameters influence the steady state changes when R is introduced?

3. How valuable is the income averaging feature of the R asset?

4. What happens during transition? Who gains and who loses?

5. Are the results sensitive to the indexing assumption?

To answer the above questions I ran three sets of simulations. First, I did 'steady state' simulations. These consisted of comparing the savings and tax patterns for individuals who live out their entire lives under a B system with those of individuals who live under a B-R system. Since the PDV of taxes will be different under the two systems, I did a third tax-compensated B-R simulation where the tax
function was multiplied by the constant required to achieve the same PDV of tax under B-R as was the case under the B system. In each case, I ran simulations for three income classes. Taxpayers live for four periods which can be viewed as fifteen year intervals starting at age 20 and ending at 80.

The second set of simulations is intended to capture the transitional effects of bringing in the B-R system. Here I looked only at one income group. The taxpayer experiences an unexpected shift from the B to the B-R system at either the end of period one, two or three. The savings and tax patterns which emerge are different than those of the steady state cases. The tax compensation used here is the same as in the steady state case.

The third set of simulations looks at the steady state, tax-compensated effects of changing the indexing assumption from 2% to 1.5% both under the B system and the B-R system. Again, only one income class is examined.

The simulation model is somewhat more complicated than the simple model of Chapter II. There are four periods to the life cycle instead of just two. The tax function has fixed exemptions in it. There are transfer payments to individuals. A proportional increase in earned income in each period under the B-R system does not mean a proportional
increase in total income and will not result in simple proportional changes in B and R as was the case in Chapter II.

The basic system, however, is very much the same as depicted in Diagram 2.1. It remains true that in switching from B (or wage tax) system to the B-R (or two asset) system:

1. There is no change in the terms of trade between periods.
2. There is no change in the excess burden of taxation.
3. The PDV of the change in taxes equals the PDV of the change in consumption.
4. So long as preferences are homothetic, the size of the elasticity of substitution is irrelevant to the size of savings and tax changes.

2. STEADY STATE COMPARISONS

There are significant changes in the pattern of taxation and saving over the life cycle when R is introduced. To compute these, I ran two simulations with identical earned income paths and utility parameters. In the first simulation, to maximize utility, the taxpayer must basically decide how much B to save. In the second simulation, the taxpayer is allowed to use the R asset. Taxes are adjusted by an equal factor across all periods in a way which ensures that the PDV of taxes is the same as would have occurred
under the B system. This is the steady state comparison. The changes in taxes and savings by income class are given in Table 7.1.

The earned income profiles are the same to a scale factor. This does not mean that total income varies to a scale factor since there are fixed government transfers such as the CPP and the OAS involved. Rises in earned income fall almost entirely in periods 1, 2 and 3 while virtually none falls in period 4. This tends to make the changes in savings and tax more than proportional to earned income. It also tends to make the benefits of averaging more than proportional to income.

However, the table shows that the pattern is basically similar across income classes. Savings increase in the first three periods and fall in the last (greater dissaving). People are deferring taxes and at the same time they are putting away extra cash to cover this future liability. Taxes fall in the first two periods and rise in the last two, especially period 4.

Table 7.1 also reflects some of the constraints on the system. The sum of changes in savings across periods is zero in conformity with the lifetime budget constraint. The discounted sum of tax liabilities is zero reflecting the tax adjustment.
<table>
<thead>
<tr>
<th>Income Class</th>
<th>Tax Changes</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Age</td>
<td>20-34</td>
<td>35-49</td>
<td>50-64</td>
<td>65-79</td>
<td>Discounted</td>
<td>Weighted</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lifetime</td>
<td>Average (1979 wts)</td>
</tr>
<tr>
<td>I</td>
<td>-133</td>
<td>-494</td>
<td>+152</td>
<td>+2019</td>
<td>0</td>
<td></td>
<td>-31</td>
</tr>
<tr>
<td>II</td>
<td>-325</td>
<td>-1067</td>
<td>+183</td>
<td>+5029</td>
<td>0</td>
<td></td>
<td>-74</td>
</tr>
<tr>
<td>III</td>
<td>-558</td>
<td>-1721</td>
<td>+349</td>
<td>+8209</td>
<td>0</td>
<td></td>
<td>-122</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Saving Changes</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Age</td>
<td>20-34</td>
<td>35-49</td>
<td>50-64</td>
<td>65-79</td>
<td>Weighted Average (1979 wts)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>+134</td>
<td>+602</td>
<td>+437</td>
<td>-1172</td>
<td>-1172</td>
<td>+166</td>
</tr>
<tr>
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<td>+325</td>
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<td>+1143</td>
<td>-2791</td>
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<td>+389</td>
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<tr>
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<td>+558</td>
<td>+2169</td>
<td>+1835</td>
<td>-4560</td>
<td>-4560</td>
<td>+644</td>
</tr>
</tbody>
</table>
It is interesting to make some rough calculations using the data in Table 7.1 to see what the implications for government revenues might be at a point in time rather than over the life cycle. We can do this by weighting the columns of Table 7.1 by the current age distribution for males in Canada and by further assuming that incomes and tax rates are indexed by 2% real per year. Using the 1976 population figures from Statistics Canada's Population Projections 1976-2001, we find that the steady state drop in aggregate revenues will be approximately -$31 per taxpayer in income class I and about -$74 and -$122 in classes II and III respectively. Even though all individuals pay the same PDV of tax under the new arrangement they will be paying less in aggregate. This is because people in periods 3 and 4 who are paying more tax currently, are relatively few in number and furthermore are on lower life cycle paths than those in periods 1 and 2. The aggregate figures are therefore dominated by the behavior of the young.

We can perform the same calculation for the savings portion of Table 7.1. Here again, the aggregate numbers are dominated by the younger taxpayers. We see increases in aggregate savings figures at all income levels. These increases in savings are significantly larger than the declines in tax revenue which suggests that the policy change has a very strong savings stimulation effect. The standard macro
theory assumes that only a fraction of each dollar of for-
gone tax revenue is saved. This table suggests that savings
will rise by three or four times the tax loss.

It is important to remember that these are steady state
changes. What they tell us is what differences there would
be in taxes and savings today if the switch to the B-R sys-
tem had been made a long time ago. They tell us very little
what happens to revenues in the short run after tax reform.
They are also dependent on the age distribution of the popu-
lation. Currently, population statistics reflect the large
post-war family formation rate. The aggregate statistics
for savings and tax changes tend to take on the characteris-
tics of the bulge generation. If we apply the population
weights estimated for the year 2001 there is little change
in either the steady state tax or savings figures but by
2026 we find that the aggregate tax change figure is positive
$91 for the middle income level and the savings change
figure is $478. This reflects the high weight that the baby
boom babies will have in old age by the year 2026. Since
the people pay more tax when old and do more dissaving while
old, the 2026 figures show greater aggregate tax revenues.
Aggregate savings rise but not by as much as when the steady
state is reached in 1976.
3. INCOME AND CONSUMPTION VARIATIONS

The changes in savings and taxes are dependent on the income profile assumed. Ignoring transfer payments, a taxpayer who worked from cradle to grave earning 2% (the indexing rate) more each year, would not derive any benefit from using $R$ because his marginal tax rate would be constant already. Table 7.1 for such a person would be filled completely with zeros. It is deviation of actual earnings from the constant-marginal-tax-rate pattern of earnings which determines the pattern of tax and savings changes in 7.1. The major deviation in this case is the low level of retirement period earned income. It induces more tax deferral and more savings prior to retirement when $R$ is introduced.

It is simple to judge how a variation in the time distribution of income will generally affect things. An increase in first period earnings and a reduction of second period earnings which leaves the PDV of earnings unchanged will simply cause exactly offsetting changes in the optimal holdings of $R$ under the B-R system. $R$ savings will rise in period one and fall in period two by exactly the same amounts as earnings change. If the earnings adjustment changes the earnings profile away from the constant-marginal-tax rate profile, changes in savings and taxes arising from the switch to the B-R system will be correspondingly longer than before.

Variations in the utility parameters are also easy to
deal with. If the time distribution of consumption changes, with the PDV of consumption constant, the levels of R saving are unchanged. The B saving changes are equal and opposite to the changes in consumption.

The changes in R saving when we switch from the B to the B-R system are independent of the consumption path. This can be seen by noting that the distance AE on Diagram 2.1 is independent of the position of $C_1$. In fact, the change in B saving, as we switch from B to B-R, does not depend on C either. The difference in B holdings is the difference between HD in the B system and HF in the B-R. This amount is FD and is independent of point $C_1$. Since neither the B nor the R savings changes depend on the consumption pattern, the change in total savings as we go from B to B-R is independent of the consumption profile.

To summarize, when we move from the B system to the B-R system, there will be changes in savings and taxes. These changes are independent of the utility parameters provided the PDV of consumption is kept constant. Changes in B are in fact independent of the pattern of earnings as well. Changes in R will, however, depend on the earnings stream, tending to be large if earnings are significantly different from the constant-marginal-tax rate pattern. In the case presented in Table 7.1, the general pattern of savings and
tax movements derives from the fact that period 4 earnings are low.

4. VALUE OF AVERAGING

In Table 7.2, I give the value of the tax variation involved in keeping utility constant during the introduction of R. This number, which rises rapidly with income, tells the cost in PDV terms of failing to income average through R. The numbers, while quite regressive, are rather small. To the middle income man the PDV of R availability translates into something the order of only $200 a year (PDV of $3801).

This fact is important. Large amounts of R saving are being made in this simulation. The middle income man saves nearly $5000 a year over the 30 year period from age 35 to 65. In spite of the large amounts of R saving, not a great deal of tax benefit is involved. This suggests that other factors such as transactions costs and rate of return differentials between assets classed as B and R types may play an important role in determining how strong the movement into the R asset will be. The more averaging that is done, the less is the incentive to do further averaging. People may go just half way to full averaging or they may do a lot of tax shifting for very little tax gain. The incentive is weak but whether or not the taxpayer chooses to go the full way or not is quite important to policy makers. The size of
<table>
<thead>
<tr>
<th>Income Class</th>
<th>Value of Averaging</th>
<th>Income Elasticity of Tax Reduction</th>
</tr>
</thead>
<tbody>
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<td>I</td>
<td>2977</td>
<td>0.46</td>
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<tr>
<td>II</td>
<td>3801</td>
<td>1.77</td>
</tr>
<tr>
<td>III</td>
<td>6393</td>
<td></td>
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</tbody>
</table>
the tax base in a given year depends on the taxpayers' decisions. Rates must be set according to the size of the base. On the other hand, the tax base the taxpayer chooses will depend on how he anticipates that tax rates will be set now and in the future.

It is interesting to compare these results with those of Robinson (1974). His study gives estimates of average taxes paid at three levels of income and several assumed patterns of lifetime taxable income. He uses this evidence to support the claim that income variations may impose a greater tax burden on the poor than the rich. Unfortunately, he does not specify the exact income patterns nor does he specify how the various income "levels" are related so that exact interpretation is difficult. He suggests that someone whose income is $30,000 and peaks at 40, may pay an average of 6% more tax each year than someone with the same average income equally distributed over the working life. The effect is regressive. At $20,000 average annual income the comparable figure is 8%. At $10,000 it is 16%.

These calculations are done in terms of average rather than discounted income and tax flows and allow for no changes in the tax function over time. They seem to imply that income averaging is more valuable to the poor than the rich. They involve a gain of as much as $750 a year for 40 years. This has well over twice the discounted value of the benefits
of income averaging for the highest income class in my calculations.

To summarize the results on the value of income averaging derived from this section of the simulation we have established the following points.

1. The tax benefits of R are small even when considerable R is used.
2. The extent of tax shifting may be difficult to anticipate given the incentive is weak.
3. The pattern of change is dominated by the low retirement earnings effect.
4. Savings will rise in the first three periods and fall in the last.
5. Taxes will fall in the first two periods and rise in the last two.
6. The tax benefits of introducing R are regressive although small.

5. TRANSITIONAL EFFECTS

The pattern of savings and tax changes in the steady state suggests that transitional effects may be quite large. The short run fall in aggregate tax revenue will be greater than the long run and the short run rise in savings will also "overshoot" initially. This is due in both cases to the delayed (and moderating) tax and savings changes in the
older age categories.

To test the transition effects, I did four simulations all for the middle income group (Table 7.3). The first line of Table 7.3 gives the changes in taxes in the steady state. It is the same as the second line of Table 7.1. The next line is for someone with the same income and tastes who gets to use $R$ only in the second, third and fourth periods of life. There is no change in period 1 savings or taxes because the introduction of $R$ is not anticipated and does not occur until period 2. The next two rows are the same except $R$ becomes available in the third and fourth periods respectively. No changes occur at all when $R$ becomes available in the fourth period because people's savings decisions are already set by this time.

These tables show just how strong the transitional or impact effect may be. Those in the first period of life when $R$ becomes available will raise savings by $325 and pay less in tax by the same amount. Those who are in the second period of life will pay $1413 less tax and save $1328 more. Notice here, that the corresponding Steady State changes in taxes and savings are only $1067 and $1325 respectively. The short run response, especially of taxes, is greater than the long run response. This pattern is repeated in periods 3 and 4. In all cases, the short run change in taxes is more
<table>
<thead>
<tr>
<th>Constraint</th>
<th>Age</th>
<th>20-34</th>
<th>35-49</th>
<th>50-64</th>
<th>65-79</th>
<th>Discounted Lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State</td>
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<td>+183</td>
<td>+5029</td>
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<td>0</td>
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<td>-1551</td>
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<tr>
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<td>0</td>
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<td>0</td>
</tr>
</tbody>
</table>

Change in Savings

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Age</th>
<th>20-34</th>
<th>35-49</th>
<th>50-64</th>
<th>65-79</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State</td>
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<td>+325</td>
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<td>-2508</td>
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<tr>
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<td>0</td>
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<td>-1464</td>
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<td>Three Constraints</td>
<td></td>
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<td>0</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>
negative and the change in savings more positive than in the long run. This is because those caught in mid-life will do some "catch-up" saving, trying to take full advantage of the low past 65 taxable income. On top of this "overshooting" effect, the only people paying higher taxes and saving less are those in period 4 and they will be the last to change.

It is possible here to do some rough calculations for aggregate taxes and savings as was done for the steady state simulations. Again, I assume that incomes rise at 2% per year and the tax function is indexed at 2%. If the tax reform is brought in in 1976 then by applying the 1976 male population weights to the entries in Table 7.3 we find that aggregate taxes drop by $606 per taxpayer and savings rise by $564 per taxpayer for the middle income class. These numbers are substantially larger than in the steady state case (-$74 and $389 respectively). They clearly illustrate the fact that the transitional effects are more important than the long run ones.

Why should the PDV of taxes be lower for those who are constrained and yet remain unchanged for those in the steady state? The tax adjustment is assumed to come in at the same time as the new R comes in. Tax rates must be adjusted upwards to compensate for the advantages of averaging so the later the constraint the better. For instance, the two
constraint fellow pays tax at the lower rates for the first half of his life and still gets to do considerable averaging between periods 3 and 4. His overall tax is lower in PDV terms by $1551. The three constraint fellow cannot do any averaging because he has already entered the last period. He will be subject to the new higher tax rates but since he has no R savings and his fourth period earnings are zero, his Basic Deduction, Old Age Exemption and Other Deductions will be enough that he has no taxable income anyway. Therefore, the last row of the tax and savings tables in 7.3 are all zeros.

The implication here is that during transition, the winners in the intergenerational contest will be those in the middle two periods of life at the time of the change. This assumes that the change is unanticipated and that the tax adjustment is made at the same time as R is introduced and preserves the PDV of Steady State taxes.

Another implication is that the very old may not have any taxable income and will be unaffected by the new asset. If the government temporarily raises tax rates for everyone, to compensate for revenue losses, the old and especially the very young will suffer. To avoid intergenerational transfer effects of the switch, transitional taxes which hit the middle income groups must be adopted. These revenues will help to offset the transitional revenue loss.
There are really two main messages to get from the transition analysis. First, the short run adjustments will "overshoot" because of a conspiracy of "catch up" and vintage effects. And secondly, unless further adjustments are made, the middle age group at the time of change will benefit most from the use of R.

6. INDEXING EFFECTS

Changing the indexing assumption will alter the savings and tax figures differently depending on whether we are under the B or B-R systems. I have been escalating all tax and transfer parameters at 2% real per year. When the indexing assumption is changed it is important to specify exactly which growth rates are changed and which are not. In Table 7.4, I have tabulated the savings and tax changes under both tax systems on the assumption that all deduction and exemption rates are escalated by 1.5% instead of 2%. The schedule of tax rates moves upwards by 1.5% also I have kept the CPP and OAS levels unchanged, however, in an effort to isolate the tax indexing effects from the transfer effects of indexing.

As you would expect in the B-R case the reduction in the indexing rate makes it less attractive to defer taxes. Tax payments for the middle income group lie in the first two periods and fall in the last two, especially the fourth.
TABLE 7.4 Compensated Difference in Taxes and Savings Under Alternative Indexing Assumptions (lower 2% to 1.5%)

<table>
<thead>
<tr>
<th>Age</th>
<th>20-34</th>
<th>35-49</th>
<th>50-64</th>
<th>65-79</th>
<th>Discounted Lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax Change</td>
<td>+165</td>
<td>+8</td>
<td>-183</td>
<td>-663</td>
<td>0</td>
</tr>
<tr>
<td>Savings Change</td>
<td>-167</td>
<td>-136</td>
<td>-63</td>
<td>+365</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax Change</td>
</tr>
<tr>
<td>Savings Change</td>
</tr>
</tbody>
</table>
Savings move in roughly the opposite way.

The situation is reversed under the B system. A reduction in the indexing assumption causes taxes to fall early and rise late, while savings rise early and fall late. In both cases, I adjusted taxes by a constant factor across all periods to keep the PDV of taxes the same under alternative indexing assumptions.

Taxes rise late in life under the B system because the lower indexing assumption effectively means that tax rates rise more quickly over time. Under the B system, taxpayers have no control over the size of the tax base so index reduction causes taxes to rise more rapidly over the life cycle. Taxes fall in the early periods because of the PDV adjustment needed when indexing is reduced.

It is important to note that in this particular simulation, under the B system and 2% indexing, the taxpayer pays no tax in period four. Dropping from 2% to 1.5% the fourth period exemption levels are lower and the taxpayer must, in fact, pay $325 tax each year in retirement. The higher is the taxpayer's income profile the more exemptions he will need and the greater will be the change in fourth period taxes when the indexation is reduced.

How important is this indexing to the taxpayer? This is a difficult question. There are several ways of looking
at it. First of all the taxpayer is really forming an expectation about government action far into the future. While he may be fairly certain that the government is going to take, say, 25% of his income each year, he may be uncertain under what rate structure this will be done. In other words, he may think more in terms of average than marginal tax rates.

Indexation is important in the sense that uncompensated effects on utility may be large. For the B-R case, the reduction in indexation by only a half a percentage point reduces lifetime consumption by almost $5500. In the B case it falls by $3854.

In some sense, the taxpayer cannot be much more worried about indexation change than about income averaging. If averaging is not that important to the PDV of lifetime consumption, a compensated change in indexing will generate a need for savings and tax rescheduling but the incentive for these adjustments will be small.

I underlined the word compensated because, in fact, it may be the income effects of indexing which are most important. Inflation gives a handy method of getting effective 'sunset' legislation for many tax exemptions. In my simulations I have raised the pension income and interest income exemptions by 2% a year. In Canada, however, these have
both been fixed at $1000 for several years despite high inflation. These things may be more important to taxpayers than improving timing of tax payments for averaging purposes.

Several issues have become clearer in this section. First, while an uncompensated change in indexing may have income effects which affect the PDV of lifetime taxes, the incentive to adjust savings and tax patterns is no stronger than the incentive to tax average. Secondly, compensated index changes will have opposite effects under the B and B-R systems: taxes fall and then rise under B but rise and then fall under B-R. Lastly, expectations about indexing are probably formed with less conviction than expectations about average tax rates and this makes it difficult to do sophisticated tax planning which accounts for index effects.

7. SUMMARY

At the outset of this chapter I listed a number of questions of a quantitative nature that I wished to address through the simulations. From the steady state simulations it appears that the value the taxpayer attaches to leaving R available to him may be quite small, even though his holdings of it may turn out to be large. This presents a problem for someone trying to guess at the outcome of introducing the R asset because the incentive mechanism is weak and may not be fully exploited. Nonetheless, it is clear that the
pattern of the changes brought about by the introduction of R is dominated by the low level of earnings in the post 65 period. Savings will rise in the first three periods and fall in the last period. Taxes will fall in the first two periods and rise in the last two. Finally, while the income averaging benefits of having R are regressive, this is not that important because of the small magnitudes involved.

The transitional analysis has demonstrated two main things. First, savings will accelerate and tax revenues will fall in the short run after R is introduced. This is because those in midlife will do some catch-up saving to get in on the income averaging while they still can. Reinforcing this is the vintage effect in which the old (dissavers) react less quickly than the young (savers).

Secondly, it is the middle age group which constitute the "winners" because they live part of their lives under low tax rates (before R is introduced) and are still able to income average. The young live under higher tax rates and the old cannot average.

Changes in the indexing rate will have opposite effects under the B and B-R regimes. An increase in the indexing rate under B-R will induce more saving and less taxes while young, as taxpayers seek to defer more income. Under the B system increasing the index rate will reduce the amount of
tax paid in later life because the tax function becomes less exacting over time.
CHAPTER VIII
SUMMARY, CONCLUSIONS AND
FURTHER RESEARCH

1. RESULTS FROM THE SIMULATION MODEL

In Chapter I, I posed five questions relating to the changes in taxes and savings between the B and B-R systems. Most of these issues were dealt with in Chapter VII. The first conclusion I drew concerned the steady state changes in taxes and saving induced by switching from B to B-R. I observed that even when large amounts of R are saved, the tax saving involved is quite small (less than $4000 over a whole lifetime for the middle income class). This is true even when the taxpayer is saving $5000 of R per year over thirty years. Given that the incentive to use R as a tax averaging device is weak, it is questionable whether people will make full use of it. Transactions cost, interest differentials, risk differences and other things may be more important in determining the composition and size of portfolios.

Although the incentive is weak, the amounts involved are large. I found that the steady state changes in the lifetime pattern of savings and taxes could be quite substantial when the R asset was introduced. In particular, taxes rise in the last two periods and fall in the first two,
while savings fall in the last period and rise in the first three (see Table 7.1). The changes are directly related to the level of income.

The pattern of savings and tax changes is determined largely by the fact that retirement period earnings are low. This means that taxable income must be shifted, using the R asset, towards old age. Less taxes are paid while young because tax rates are high then. Savings rise in the working years because taxes will be higher in retirement. The extra saving is done to cover this higher future tax burden. The extra saving can be done without displacing young consumption, since young taxes are lower.

The last point made about the steady state comparisons was that an uncompensated shift from B to B-R would be regressive, benefitting the rich more than in proportion to income. However, since the gains are small in absolute terms at all levels of income, this may not be too important.

Transitional effects are, I think, the most important aspect of tax reform. Since it is low retirement earnings that drive the changes, tax and savings responses to the new system are strongly age related. This fact tends to accentuate the transitional variations in savings and taxes. There are two factors here. First, roughly speaking, the steady state changes for the young are the opposite of those
for the old. Since the young change before the old, it is the behaviour of the young which will dominate in the short run. Savings will rise by more in the short run than in the long run. Taxes will fall by more in the short run than in the long run. Secondly, taxpayers who are in their middle years and who did not anticipate the tax reform will want to do "catch-up" tax shifting and saving. Those caught in mid-cycle by the tax change save more and pay less tax in periods before retirement than those in the steady state. This effect reinforces the tendency of the system to "overshoot" initially. If the rate of population growth is less than the rate of interest, shifting taxes towards old age, keeping the PDV of taxes constant at the individual level, will result in a rise in aggregate taxes in the steady state. Even so, it is almost certain that short run aggregate taxes will fall because of these transitional effects. All diagonal entries in the tax change part of Table 7.3 are less than or equal to zero. This means that taxes must at least not rise for anyone for the duration of one whole period.

The sum of savings changes is zero across periods in both the steady state and transition cases. This reflects the lifetime budget constraint. In both cases the savings changes are negative in period four and non-negative in periods one to three. Therefore, steady state savings must drop if the population growth rate is greater than zero. In
transition, aggregate savings must drop even more because of
the catch-up and vintage effects noted above.

The last point I wish to make about transition period
after tax reform is that the middle age group is the winner
less the old and especially the young pay for the
intergenerational transfers. Essentially what happens is
that the introduction of R allows people to tax average.
They will pay less tax altogether unless an adjustment is
made at the same time R is made available. When an adjust-
ment is made, those who are very young face higher tax rates
but can tax average. Those who are very old cannot tax ave-
rage and face higher tax rates, but they probably have little
or no taxable income. Those in the middle years can still
do considerable averaging and have faced lower tax rates for
the first half of their lives. Again, the magnitude of
these welfare effects is small because the benefits of ave-
raging are small.

Indexing effects are a bit difficult to assess but
there we can draw four basic conclusions. The first is that
while the uncompensated welfare effects of changing the in-
dexing assumption may be large, the taxpayer probably does
not have a firm idea of the way the tax function is likely
to be indexed over time. It is marginal tax rates across
periods which determine the timing of tax and savings
decisions, but it is probably average tax rates that the taxpayer is best able to predict. Furthermore, a compensated change in the indexing assumption may call for savings and tax adjustments but the incentive to make these changes will be no greater than the incentive to tax average. I have indicated above that this may be small. Taxpayers are probably more worried whether inflation is going to erode nominally fixed exemptions and raise average tax rates than they are about doing the fine tuning to their portfolios necessary to get the best they can out of the averaging device.

Changing the indexing assumption will, nonetheless, alter the optimal savings and tax patterns. The effects are opposite under the B and B-R systems. Under the B-R system, a decrease in indexing puts more tax weight on the final years. Less saving is done early and less tax is paid later than under a high index rate. But under the B system, the reverse is true. Since the B system does not allow for any tax deferral, the decrease indexing puts a greater tax burden on people when they are old. Young savings and old taxes rise, the opposite of the response under B-R.

2. RESULTS FROM THE SIMPLE MODEL

The simple two period model of Chapter II gives some powerful insights into the process of shifting from the B to the B-R system. The model is simple in the sense that there
are no special exemptions or deductions in the tax function, there is no uncertainty and no bequests. Furthermore, preferences are homothetic and there are no limits on borrowing or lending. Under these conditions, I have established that in shifting from B to B-R:

1. There is no change in the terms of trade between periods.
2. There is no change in excess burden.
3. The PDV of the change in taxes equals the PDV of the change in consumption.
4. Tax and savings changes are independent of the elasticity of substitution.

Under the B system:

1. Changes in savings (equal to change in B) are proportional to changes in consumption when there is an equal percentage increase in earnings across periods.

Under the B-R system:

1. Changes in R holdings are proportional to changes in earnings.
2. Changes in B holdings are proportional to changes in consumption.
3. If \( R > 0 \) and \( B < 0 \) savings will rise more than in proportion to earnings.

4. People with the same PDV of earnings pay the same PDV of tax and enjoy the same PDV of consumption. Furthermore, this equity condition can be preserved in a model with bequests if \( R \) assets are taxed at the time of bequest.

5. The degree of progressivity of the tax function does not affect the holdings of \( B \) and \( R \) but will influence the strength of the inactive to do tax averaging.

6. A fall in the interest rate will cause \( R \) holdings to rise. \( B \) holdings may fall only slightly leaving the possibility of a perverse effect of interest rates on savings.

7. Taxation of \( B \) interest discourages \( B \) saving and causes both taxes and tax rates to rise more rapidly over the life cycle.

8. A method for indexing the tax function to keep the government's share of total income from rising as real incomes grow under a proportional rate structure can be designed. But if it is not to induce a sharply rising pattern of taxes over the life cycle it must involve some age discrimination in the tax function.
3. FURTHER RESEARCH

There are a number of extensions to this analysis which might be explored. First, there are several alternative specifications of taxpayer behavior that might yield some insights into aggregate savings and tax changes after tax reform. For instance, the analysis here has assumed myopic expectations on the part of taxpayers. If taxpayers are able to anticipate tax reform, there may be different paths of adjustment than those calculated here. The forces at work however will be broadly similar to those of the myopic expectations model. It is likely that taxpayers will try to "load-up" on certain assets and liquidate others if given a chance to act before reform is made as was mentioned in Chapter I.

Another possible change would involve the introduction of uncertainty into the taxpayer's maximum problem. There may be uncertainty about wage patterns over time, about interest rates, life expectancy, and eligibility for exemptions related to marriage and dependents. The tax function itself may change considerably over a lifetime. I suspect, however, that these changes could add a considerable degree of complexity and even where tractable, the uncertainty problems may not yield unambiguous results. The approach adopted to the maximum problem in this thesis has the advantage of simplicity and analytical tractability in a wide variety of
circumstances.

A third possibility is to allow a variable labour supply in the maximum problem. This would be a somewhat simpler revision than the introduction of uncertainty. The primary focus of this should be the age of retirement. As described at the end of Chapter II, changes to the system of taxation which have a differential impact on work effort over the life cycle can have implications for distribution of earnings over the life cycle. This in turn will influence life cycle patterns of savings and taxes. I feel that introduction of a labour-leisure tradeoff into the utility function is probably the most promising avenue for further research.

Lastly, it would be possible to consider the changes involved in moving to the two asset form of taxation from other initial systems besides the wage tax. Ideally, one might like to use the system of taxation currently in use. This is an immensely complicated body of law to try to put into a simple two period maximum problem however, and some strong simplifications would have to be made.

It is possible to use a pure income tax as the starting point (tax on earnings plus interest and capital gains). This would be a bit closer to the Canadian system although large amounts of income from assets escape taxation in Canada. Some work along these lines has already been done by Daly

I feel the reform proposals deserve further consideration. Amongst the arguments in favour of this type of taxation I think the horizontal equity feature and the relatively administratively simple income averaging feature are the most attractive. In a sense, we have moved part way towards the reformed system already in Canada with the RRSP. We may already have entered the transition period. This makes it all the more important that we fully understand the implications of the tax reform.
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