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Robust Contracts in Common Agency*

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Abstract

Business activities often involve a common agent managing a variety of projects on behalf of investors with potentially conflicting interests. The extent of the agent's actions is also often unknown to investors, who have to design contracts that provide incentives to the manager despite this lack of crucial knowledge. We consider a game between several principals and a common agent, where principals know only a subset of the actions available to the agent. Principals demand robustness and evaluate contracts on a worst-case basis. This robust approach allows for a crisp characterization of the equilibrium contracts and payoffs and provides a novel proof of equilibrium existence in common agency by constructing a pseudo-potential for the game. Robust contracts make explicit how the efficiency of the equilibrium outcome relative to collusion among principals depends on the principals' ability to extract payments from the agent.

JEL: C72, D81, D82, D86, H21.

Key Words: Common agency, robustness, worst case, efficiency.

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Business enterprises often involve various projects that must be carried out in tandem. For instance, having offices or plants in multiple locations as is the case for multinational operations and (vertically or horizontally) integrated businesses. These projects are often funded by different financiers or investors but are jointly managed by a single agent. This situation can bring the financiers' interests into conflict. In this context, financiers design contracts to incentivize the manager to maximize their own profits. The optimal contracts depend on the actions the manager can take, for instance favoring one of the projects over others, but not all of the manager's potential actions are usually known by the financiers. This can be because of physical distance from the projects or lack of expertise in their management. How should a contract be designed to be robust to unforeseen agent actions?[1]

We study this type of situation in a moral hazard common agency game where various principals design contracts that are *robust* to misspecification of the agent's action set.[2] Specifically, principals seek to maximize the minimum guaranteed payoff across possible action sets as in Carroll (2015). The game has two stages. First, risk-neutral principals (the financiers) simultaneously and non-cooperatively offer contracts to a risk-neutral agent (the manager) that is protected by limited liability. Second, the agent takes an unobserved and costly action that affects the distribution of output across each principal's project. Contracts specify payments to the agent contingent on the realization of output across the projects, which is observed by all principals.

We provide an explicit characterization of the optimal contracts and payoffs. We show that, for any given contracts offered by the other principals, robust contracts take the form of linear revenue sharing contracts. These contracts align incentives by tying the principal's

[1]Similar situations arise for governments taxing multinationals, where the firm's actions change while the tax system remains unchanged, or for firms with a common supplier (e.g., marketing agencies, Bernheim and Whinston, 1985; Mezzetti, 1997) with unknown complementarities in the production of the goods.

[2]Common agency, as introduced in Bernheim and Whinston (1986a,b), has been applied in a variety of settings. Multiple lobbyists influencing a politician (Grossman and Helpman, 1994; Dixit, Grossman and Helpman, 1997; Le Breton and Salanie, 2003; Martimort and Semenov, 2008). A firm being taxed by the local, state and federal government (Martimort, 1996; Bond and Gresik, 1996). An agent performing complementary tasks for two principals (Mezzetti, 1997). A public good provider eliciting payment from consumers (Laussel and Le Breton, 1998). An auctioneer facing multiple bidders (Milgrom, 2007).

payments to the agent's linearly, making any action that increases the agent's payment also increase the principal's payment. This result parallels the "sharecropping" arrangements in Hurwicz and Shapiro (1978) and the linearity result in robust contracting presented in Carroll and Walton (2022),[3] but differs in that the optimal contracts depend on the competing contracts offered by other principals through their effect on the agent's payments.

When all principals offer linear revenue sharing contracts, contracts are affine in total output, so that all players (the principals and the agent) receive a share of total output. That is, seeking robustness, the financiers would make it so that payoffs depend equally on each project's output, rather than trying to incentivize the manager to favor their own project (potentially at the expense of others). The agent's limited liability shapes the contracts preventing the principals from offloading all risk onto the agent.[4]

We show that a pure strategy Nash equilibrium of this game always exists. To show this, we lever on the characterization of optimal contracts and payoffs that we provide, while imposing minimal assumptions over the action set of the agent. We show that the game has a pseudo-potential, similar to that of the standard Cournot competition model, and use it to establish the existence of an equilibrium as in Monderer and Shapley (1996) and Dubey, Haimanko and Zapechelnyuk (2006). This approach is new in the common agency literature and complements previous results on the existence of equilibrium (e.g., Bernheim and Whinston, 1986a; Fraysse, 1993; Carmona and Fajardo, 2009).

In our equilibrium analysis we assume that principals correctly predict the behavior of the other principals and focus on robustness with respect to the agent's action set. This setup captures many situations in which the behavior of principals is known at the time of contracting but the behavior of the agent is only known afterward. For instance, when

[3]See also Chassang (2013), Antic (2021), Garrett (2014), and Frankel (2014), among others. Dai and Toikka (2022) study an analogous problem of moral hazard in teams (one principal and multiple agents). They find that the optimal contract for the principal is to give each agent a share of total output.

[4]Limited liability makes moral hazard common agency games intractable in general. Our robust contracting approach makes it possible to overcome this issue. See Martimort and Stole (2012) for an application of common agency with limited liability. Their model, without robustness concerns, can be seen as a special case of ours, where output is perfectly correlated between all principals.

jointly financing a new project proposed by a (previously unknown) entrepreneur, or when contracting is subject to financial disclosure laws that make the agent's other dealings known to interested parties. Our focus on robust contracts makes the analysis tractable without restricting the principals' choice set as in Dixit (1996) and Maier and Ottaviani (2009). Moreover, our results do not depend on the details of the agent's action set or the information structure as is usual in other common agency setups (Martimort, 2006).

We further explore the efficiency properties of the equilibrium, as captured by the sum of (expected or guaranteed) payoffs across players, that we refer to as surplus. We show that the share of output that each player receives under linear revenue sharing contracts also corresponds to their share of total surplus. Moreover, total surplus increases monotonically on the agent's share, as it is the agent who is making decisions over output and bearing the cost of actions. The outcome is of course not efficient in general, unless the agent appropriates all output at the expense of the principals.

We use these results to revisit the classical question of the efficiency of competitive outcomes relative to collusion among principals (Bernheim and Whinston, 1986b). When principals collude, the problem reduces to that of a principal incentivizing a multi-project agent, an extension of the problems in Holmstrom and Milgrom (1987) and Carroll (2015). The solution is a contract that gives the agent a share of total output across projects. This result makes it possible to compare the outcome of the common agency game to that of the game where principals collude and offer a joint contract.

Surplus is higher under collusion than under competition among principals because the agent's share of output is lower when the principals compete. This is because principals impose an externality on each other when they compete, reducing the willingness of other principals to provide incentives to the agent.[5] When a principal increases their share of total output, they decrease the agent's share, lowering total surplus and thus the payoffs of other

[5]This result is similar to those in Bernheim and Whinston (1986a), Holmstrom and Milgrom (1988), and Martimort and Stole (2012) under moral hazard, Martimort and Stole (2012) and Bond and Gresik (1996) under adverse selection, where free-riding among principals reduces the agent's effort.

principals. Principals take into account the effects on their own and the agent's payoff, but ignore the effects on other principals. This is the case when different projects of the same business enterprise are funded by different investors.[6]

Finally, we show that limited liability plays a central role in shaping equilibrium contracts and their efficiency properties. We consider two cases. First, we consider a stronger form of limited liability that operates over each individual contract and not over the agent's total payment. That is, each project's payments are considered separately, for instance, as in a multinational operation where limited liability applies to each project residing in a different jurisdiction. In this case, equilibrium contracts are such that each principal gets a share of total output for a fee. This fee is proportional to the share of total output that the principals appropriate for themselves, effectively pricing the share for the principals. This results in the agent's equilibrium share of total output being higher than under collusion, overturning the relative efficiency of collusion over competition.

Second, without limited liability, the equilibrium outcome is efficient because it is optimal for the principals to offload all risk onto the agent. The optimal contracts demand a fixed payment from the agent and make them the residual claimant of all output, as in Bernheim and Whinston (1986a). We see this limiting case as instructive of the role of limited liability in shaping the agent's incentives.

1 Model setup

Consider a game played between two principals, indexed by $i \in \{1, 2\}$, and one agent A, all risk-neutral.[7] This can be the case of an agent jointly managing the projects of two financiers, as in multinational operations. The game has two stages. First, principals simultaneously and non-cooperatively offer contracts to the agent specifying payments contingent on realized output. Second, the agent takes an action in their action set, $\mathcal{A}$. Actions stochastically affect

[6] The presence of multiple investors for the same project can be explained with frictions as those in soft budget constraints (Dewatripont and Maskin, 1995).

[7] We extend the results to N principals in Appendix E.

the projects' output at a cost to the agent. Then, output realizes, and payments take place.

Principals observe output in both projects but do not observe the agent's action. Moreover, they do not know the complete set of actions, $\mathcal{A}$, available to the agent. However, each principal knows only a subset $\mathcal{A}_0^i$ of $\mathcal{A}$, for $i \in \{1, 2\}$, as in Carroll (2015). In the case of multiple projects funded by different financiers, this reflects financiers' lack of information about what the manager can do in the projects. The lack of information can come from the financiers' lack of expertise in the workings of the projects, or physical or temporal distance from the projects, that can be located in different jurisdictions and take place in the future when new actions can be taken.

Agent's actions and output. The agent's actions stochastically affect the projects' output, y_1 and y_2 respectively. The output space for the projects is $Y \subset \mathbb{R}^2$, with Y_i, the projection onto $\mathbb{R}$ (the set of real numbers), assumed compact with $\min\{Y_i\} = 0$ and $\max\{Y_i\} = \overline{y}_i$. The agent chooses an action from a compact action set $\mathcal{A} \subset \Delta(Y) \times \mathbb{R}_+$, where $\Delta(Y)$ denotes the space of Borel distributions on Y, and $\mathbb{R}_+$ the set of positive real numbers. An action is a pair $(F, c) \in \mathcal{A}$, where F is a probability distribution over output $y = (y_1, y_2)$ and $c \geq 0$ is the cost of the action. We endow the space $\Delta(Y)$ with the weak-$\star$ topology and $\Delta(Y) \times \mathbb{R}$ with the natural product topology.

We impose the following three assumptions on the known action sets $\mathcal{A}_0^1$ and $\mathcal{A}_0^2$.

Assumption 1. ***(Inaction)*** *$(\delta_0, 0) \in \mathcal{A}_0^i$, for $i \in \{1, 2\}$, where $\delta_0 \in \Delta(Y)$ is the distribution assigning probability 1 to output $(0, 0) \in Y$.*

Assumption 2. ***(Positive Cost)*** *For all $(F, c) \in \mathcal{A}_0^i$, $i \in \{1, 2\}$, if $c = 0$, then $F = \delta_0$.*

Assumption 3. ***(Non-triviality)*** *There exists $(F, c) \in \mathcal{A}_0^i$ for $i \in \{1, 2\}$ such that $E_F[y_1 + y_2] - c > 0$.*

Assumption 1 says that the agent can always choose not to produce (generating the minimum output with certainty) at no cost. Assumption 2 requires the agent to pay a cost in order to produce. Assumption 3 ensures that the principals and the agent will, potentially, find it beneficial to participate in the game.

Contracts and limited liability. A contract is a continuous function $w_i : Y \to \mathbb{R}$ specifying payments conditional on realized output. A contract scheme is a vector of contracts $w = (w_1, w_2)$. The agent is protected by limited liability which places restrictions over the type of contracts that principal can offer. We follow Martimort and Stole (2012) in imposing the following assumption on the aggregate payments to the agent:

Assumption 4. ***(Limited Liability)*** $w_1(y) + w_2(y) \geq 0$ *for all* $y \in Y$.

Assumption 4 protects the agent rather than individual projects, reflecting cases in which the projects can cross-subsidize payments (for instance if they are part of the same conglomerate). That is, the manager can only pay from the projects' output but not from their own pocket. We discuss other limited liability assumptions in Sections 6 and 7, including a stronger form of limited liability that operates over projects (for instance if they operate in different jurisdictions) and the related *private* common agency case where principals are restricted to contract only on the output of their own project, as opposed to the *public* common agency considered throughout the paper.

Actions and payoffs. Given a contract scheme w and an action set $\mathcal{A}$, the agent will choose an action (F, c) to maximize their expected payoff. The set of optimal actions and the value they give to the agent are, respectively:

$$A^{\star}(w|\mathcal{A}) = \underset{(F,c)\in\mathcal{A}}{\operatorname{argmax}}\ E_F[w_1(y) + w_2(y)] - c \tag{1}$$

$$V_A(w|\mathcal{A}) = \max_{(F,c)\in\mathcal{A}} E_F[w_1(y) + w_2(y)] - c. \tag{2}$$

The value of a principal, given a contract scheme w, is the minimum payoff guarantee offered by the contract scheme (Carroll, 2015):[8]

$$V_i\left(w\right)=\inf_{\mathcal{A}\supseteq\mathcal{A}_0^i} V_i\left(w|\mathcal{A}\right), \tag{3}$$

where $V_i\left(w|\mathcal{A}\right)$ is the principal's value for a given action set $\mathcal{A}$:

$$V_i\left(w|\mathcal{A}\right)=\min_{(F,c)\in A^\star(w|\mathcal{A})} E_F\left[y_i-w_i\left(y\right)\right]. \tag{4}$$

The principals do not know which action in $A^\star$ the agent will choose, so they consider the minimum payoff across those actions.[9] Any other tie-braking rule can lead to cases where the expected payoff the principal actually gets is lower than $V_i\left(w|\mathcal{A}\right)$. This is because, in general, the same action that generates high output for one principal can imply low output for another.

The best response of principal i to a contract w_j is:

$$\mathrm{BR}_i\left(w_j\right)=\underset{w_i}{\operatorname{argmax}}\, V_i\left(w_i,w_j\right). \tag{5}$$

We call the contracts in the best response *robust*, because they maximize the principal's guaranteed payoff across all possible action sets. The principal's actions and payoffs are independent of the agent's action set, $\mathcal{A}$. They instead depend on $\mathcal{A}_0^i$.

We can now define a Nash equilibrium for the game. We seek to capture situations in which the behavior of principals is known at the time of contracting but the behavior of the agent is not, focusing the uncertainty of the principals on the unknown action set of the agent. This can happen when financiers jointly finance a new multi-project endeavour proposed by a entrepreneur, or when contracting is subject to financial disclosure laws that

[8] In Appendix G, we allow for a lower bound on the cost that the agent faces, imposing restrictions on the action sets $\mathcal{A}\supseteq\mathcal{A}_0^i$ considered by the principals. Doing so does not change our main results. In particular, a version of Theorem 1 can be proven and Proposition 1 goes unchanged.

[9] We depart from the usual assumption in the robustness literature, where the principal believes that the agent will take the best action for the principal among those in $A^\star\left(w|\mathcal{A}\right)$ (e.g., Frankel, 2014; Carroll, 2015).

make the agent's other dealings known to interested parties. The difference in the principal's knowledge of the agent affect their choices, captured by differences in $\mathcal{A}_0^i$.

Definition 1. A pure strategy Nash equilibrium is a contract scheme $w^\star = (w_1^\star, w_2^\star)$ such that $w_i^\star \in \mathrm{BR}_i\left(w_j^\star\right)$ for $i, j \in \{1, 2\}$ and $i \neq j$, along with an action choice by the agent $A^\star\left(w^\star|\mathcal{A}\right)$ given an action set $\mathcal{A}$.

2 The robustness of linear revenue sharing contracts

We now characterize the principals' best responses. We propose a set of contracts that imply linear revenue sharing (LRS) between the principal and the agent and show that they are robust to misspecification of the agent's action set. That is, they maximize the principal's guaranteed payoff so that there is always a LRS contract in the principals' best response.

Definition 2. (Linear Revenue Sharing Contracts) Given a contract w_j, a contract w_i is a LRS contract for principal i if it ties the principal's ex-post payoff linearly to the agent's total revenue. That is, for some $\alpha_i \in (0, 1]$ and $k_i \in \mathbb{R}$

$$y_i - w_i\left(y\right) = \frac{1-\alpha_i}{\alpha_i}\left(w_1(y) + w_2\left(y\right)\right) + k_i, \quad j \neq i. \tag{6}$$

The share $\alpha_i \in (0, 1]$ plays an important role by tying the agent's revenue $(w_1(y) + w_2\left(y\right))$ to the principal's $(y_i - w_i\left(y\right))$. This link will underpin the relationship between the principals' and the agent's payoffs in equilibrium, when all principals offer LRS contracts. We return to this in Section 3, where we show that, in equilibrium, each principal appropriates a share $\theta_i\left(\alpha_1, \alpha_2\right) \in (0, 1]$ of total output.

We can characterize LRS contracts further. From (6), any LRS contract w_i satisfies

$$w_i\left(y\right) = \alpha_i y_i - \left(1-\alpha_i\right) w_j\left(y\right) - \alpha_i k_i, \tag{7}$$

for $i \in \{1, 2\}$, $j \neq i$, all $y = (y_1, y_2) \in Y$, and some $\alpha_i \in (0, 1]$ and $k_i \in \mathbb{R}$. The first term of the contract in (7) is reminiscent of the max-min optimality of linear contracts in principal-agent settings (e.g., Hurwicz, 1977; Hurwicz and Shapiro, 1978; Chassang, 2013; Carroll, 2015). The second term resembles the behavior of principals in the common agency setup of Bernheim and Whinston (1986a), where a principal first offsets the payments of other principals to then design their preferred incentive scheme. However, under LRS contracts the payments of other principals are only partially offset. The principal claims a fraction of both their project's output and of the payments made by the other principals to the agent.[10] This results in the sharing of the agent's total revenue. In this way, LRS contracts deal with the dual objective of the principal: providing incentives to the agent to increase their output and competing against the offers made by other principals.

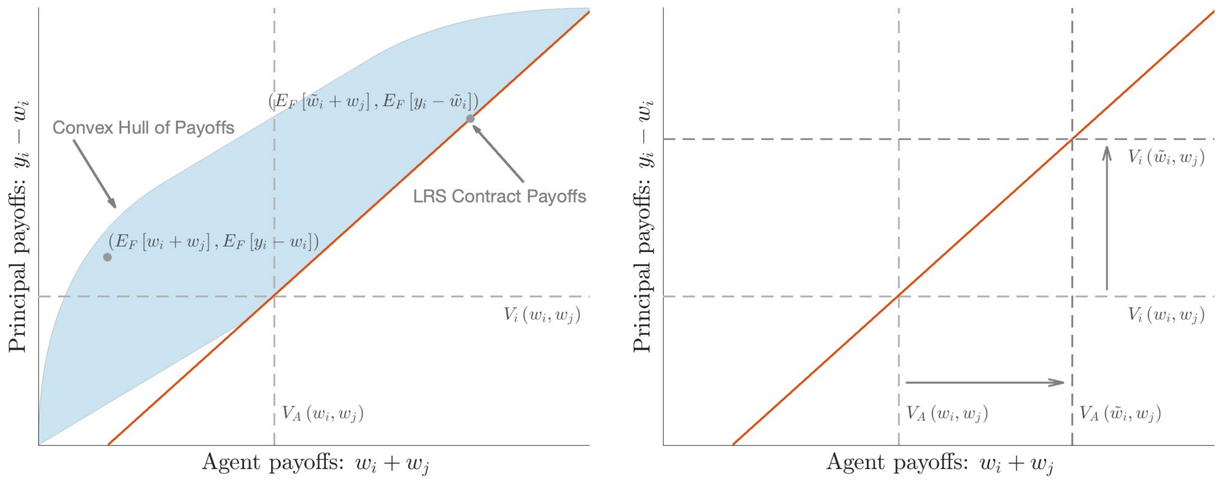
The main result of this section establishes the robustness of LRS contracts by showing that offering a LRS contract is always a best response for the principals. The proof of this result follows the arguments in Carroll (2015) and we provide it in full in Appendix A.1.

Theorem 1. *For any contract w_j, there exists a LRS contract $\overline{w}_i$ such that $\overline{w}_i \in BR_i(w_j)$ and $\min_{y \in Y} \{\bar{w}_i(y) + w_j(y)\} = 0$. That is, there is always a LRS contract that is robust for principal i.*

The key for establishing the robustness of LRS contracts is the affine link they imply between the principal and the agent's payoffs. To see this, consider a scenario where the principals offer some contract scheme (w_1, w_2). Figure 1a, depicts the payoffs implied by the contract scheme. The convex hull of these payoffs captures all possible distributions on the projects' output. The guaranteed payoff of the principal is the lowest (expected) payoff that the agent could induce given their known actions $(\mathcal{A}_0^i)$.[11] It is possible to improve principal i's guaranteed payoff by offering a LRS contract $\tilde{w}_i$ that (weakly) increases the agent's guaranteed payoff. The payoffs implied by the LRS contract lie on the solid line.

[10] It is instructive to think of the principal's problem in two steps: (i) Undoing other principals' payments, and (ii) offering the agent an aggregate contract $\tilde{w}_i$. When w_i is a LRS contract the aggregate contract is: $\tilde{w}_i(y) = \alpha_i(y_i + w_j(y)) - \alpha_i k_i$. Principal i receives $1 - \alpha_i$ of the payoff $(y_i + w_j(y))$ and the agent receives the rest. k_i acts like a transfer between the principal and the agent, determined by limited liability.

[11] The agent only takes actions such that $E_F[w_1 + w_2] \geq V_A(w_1, w_2 | \mathcal{A}_0^i)$. See Lemma 1 in Appendix A.1.



(a) Contract scheme (w_i, w_j)

(b) LRS contract $\tilde{w}_i$

Figure 1: Principal i and Agent's Ex-Post payoffs

Note: *The figure shows the ex-post payoffs for a principal (i) and the agent. The left panel presents the convex hull of payoffs under an arbitrary contract scheme (w_i, w_j) and under an alternative scheme where principal i offers a LRS contract $\tilde{w}_i$. The right panel shows how the guaranteed payoffs of the principal and the agent increase with the LRS contract. $\tilde{w}_i$ offers weakly higher payoffs for the agent than w_i which increases the agent's guaranteed payoff. This, in turn, increases the guaranteed payoff of the principal. The figures are generated for $Y_1 = Y_2 = [0,1]$, $w_j(y) = y_j^2/2$ and $w_i(y) = y_i^2/2 + 1 - y_j^2/5$.*

Linear revenue sharing implies that an increase in the agent's payoff leads to a proportional increase in the principal's payoff, and vice versa, so that the agent cannot exploit the contract in a way that would hurt the principal. The logic behind Figure 1 applies in general and constitutes the core of the proof of Theorem 1.

3 Equilibrium in linear revenue sharing contracts

We now establish the existence of an equilibrium in pure strategies where principals offer LRS contracts. We do so by first providing an explicit characterization of LRS contract schemes and their associated payoffs. Then, we show that the common agency game has a pseudo-potential function as in Dubey, Haimanko and Zapechelnyuk (2006). This approach avoids the typical challenges posed by the failure of convexity of the principals' best responses (see Bernheim and Whinston, 1986a, Fraysse, 1993, and Carmona and Fajardo, 2009).

Proposition 1 characterizes LRS contracts schemes that satisfy the limited liability condition in Assumption 4 with equality. When both principals offer LRS contracts, each principal appropriates a share, θ_i, of total output $(y_1 + y_2)$. The shares $\{\theta_1, \theta_2\}$ depend on the parameters $\{\alpha_1, \alpha_2\}$ that characterize the LRS contracts offered by the principals. Hereafter, we focus only on the shares $\{\theta_1, \theta_2\}$ because they characterize contracts in a LRS contract scheme and determine payoffs as shown below.

Proposition 1. *Let w be a LRS contract scheme satisfying Assumption 4 with equality. There exist $\{\theta_i, k_i\}_{i \in \{1,2\}}$ such that contracts satisfy*

$$w_i(y) = (1-\theta_i)\, y_i - \theta_i y_j - k_i, \text{ for } i \in \{1,2\},\ j \neq i, \tag{8}$$

where $k_1 = -k_2$ and $\theta_i \in [0, 1-\theta_j]$. Moreover, principal i's guaranteed payoff satisfies

$$V_i(w) = \theta_i \max_{(F,c)\in\mathcal{A}_0^i} \left\{ E_F[y_1 + y_2] - \frac{c}{1-\theta_1-\theta_2} \right\} - k_i. \tag{9}$$

The liability constraint imposed in Assumption 4 implies that each principal is constrained by the other principal's actions. In this way, the strategy space of each principal in terms of their choice over their share of output (θ_i) depends on others' strategies (θ_j). This makes our framework into a *quasi-game* in the sense of Debreu (1952). We provide further discussion of this in Appendix B.[12]

Contracts in a LRS contract scheme balance the principals' dual objectives of incentivizing the agent and competing with the other principal by giving the agent a fraction $(1-\theta_i)$ of the principal's project output and taking a share θ_i of the other project. This implies that the payoffs of all players depend only on total output, with principal i receiving a share θ_i of it (i.e., ex-post payoff satisfy $y_i - w_i(y) = \theta_i (y_1 + y_2) + k_i$) and the agent receiving a share $(1-\theta_1-\theta_2)$. By tying all payoffs to total output, the objectives of the principals

[12] The original choice of the principals in terms of α_1 and α_2 is not constrained in this way, with each principal choosing $\alpha_i \in (0,1]$. However, it is convenient to cast the problem as choosing $\theta_i \in [0, 1-\theta_j]$ because it is θ_i directly characterizes the equilibrium wages and payoffs.

are made congruent. This makes explicit the intuition behind the solution to the common agency game in Bernheim and Whinston (1986a, p. 929):

> "[W]e underscore the need to make principals' objectives congruent in equilibrium: since all principals can effect the same changes in the aggregate incentive scheme, none must find any such change worthwhile. One can think of this congruence as being accomplished through implicit side payments among principals."

Under LRS contracts, there are no distributional concerns coming from the agent's actions. Even though the agent can, in principle, favor one project's output, the LRS contract scheme makes this irrelevant because payoffs depend only on total output. Moreover, the agent's actions depend only on the shares θ_1 and θ_2 and not on k_1 and k_2. In this sense, k_1 and k_2 act as transfers between the principals channeled through the agent.[13]

We proceed by further characterizing contracts and payoffs in an equilibrium in LRS contracts. Following Proposition 1, we focus on the role of the shares θ_1 and θ_2 that define LRS contract schemes. The principals' actions depend on the set of the agent's actions that they know, $\mathcal{A}_0^1$ and $\mathcal{A}_0^2$. As mentioned above, equation (9) ties the principals' payoffs to a distorted version of total surplus (note that the cost of the action is inflated). The actions in $\mathcal{A}_0^i$ dictate how large principal i believes this surplus can be, and therefore how much they want to incentivize the agent by setting the value of θ_i. In this way, differences in $\mathcal{A}_0^i$ result in differences in the contracts offered by different principals. Principals that believe the agent to be able to generate a higher surplus will set a higher θ_i. Proposition 2 presents the results.

[13]The values of θ_1 and θ_2 can be computed separately from those of k_1 and k_2 because of the effect of k_i on the payoff of principal i is independent of θ_i (see 9). However, the values of k_1 and k_2 are not pinned down (Bernheim and Whinston, 1986a). The values can be pinned down by imposing a participation constraint for the agent and an outside payoff for the principals if the agent does not participate. If each principal can induce the agent not to participate, the value of transfers is set to ensures that each principal receives at least their outside payoff.

Proposition 2. *A pure strategy Nash equilibrium in LRS contracts satisfying Assumption 4 with equality is characterized by a pair of shares (θ_1, θ_2) and transfers (k_1, k_2), such that, for $i \in \{1, 2\}$, the share θ_i satisfies*

$$(1 - \theta_1 - \theta_2)^2 = \frac{(1 - \theta_j)\, c_i}{E_{F_i}[y_1 + y_2]} \tag{10}$$

for an action

$$(F_i, c_i) \in \underset{(F,c)\in\mathcal{A}_0^i}{argmax} \left\{ \left(\sqrt{(1 - \theta_j)\, E_F[y_1 + y_2]} - \sqrt{c} \right)^2 \right\}, \tag{11}$$

and transfers satisfy $k_1 = -k_2$.

The equilibrium contracts satisfy (8) and the principals' guaranteed payoffs are

$$V_i(w) = \left(\sqrt{(1 - \theta_j)\, E_{F_i}[y_1 + y_2]} - \sqrt{c_i} \right)^2, \; i \in \{1, 2\}, \; j \neq i. \tag{12}$$

Finally, Propositions 1 and 2 make it possible to prove that an equilibrium in LRS contracts always exists by characterizing contracts and payoffs in terms of the shares of total output appropriated by the principals. That is, finding a pair of shares (θ_1, θ_2) such that θ_i maximizes principal i's guaranteed payoff, V_i, given θ_j, for $i \in \{1, 2\}$ and $j \neq i$.

Theorem 2. *A pure strategy Nash Equilibrium in LRS contracts with $\theta_1, \theta_2 > 0$ exists.*

To prove Theorem 2, we use the characterization of the principals' guaranteed payoffs in (9) to construct an ordinal potential for our common agency game (Monderer and Shapley, 1996; Dubey, Haimanko and Zapechelnyuk, 2006). This function induces the same order over θ_i as V_i does for each principal and thus, any pair (θ_1, θ_2) that maximizes the potential characterizes a Nash equilibrium of the game for any pair of transfers (k_1, k_2) satisfying $k_1 = -k_2$. We then establish the existence of equilibria by proving that the potential function achieves an maximum for some $\theta_1, \theta_2 > 0$.

4 Collusive contracts

We now describe the outcome of the game when principals collude. Collusion serves as a natural reference point for contrasting the efficiency properties of common agency games where principals compete with one another. When colluding, principals seek to maximize their joint guaranteed payoff, for instance when there is a single investor (or group of investors) financing a multi-project endeavor. In this case, principals offer a single contract that depends on realized output and satisfies limited liability, $w : Y \to \mathbb{R}_+$. Given an action set $\mathcal{A}$ and a contract w, the agent's optimal actions are

$$A^\star (w|\mathcal{A}) = \underset{(F,c)\in\mathcal{A}}{\operatorname{argmax}} \, E_F [w(y)] - c. \tag{13}$$

The joint guaranteed payoff for the principals across all action sets $\mathcal{A} \supseteq \mathcal{A}_0$ is:

$$V_P (w) = \inf_{\mathcal{A}\supseteq\mathcal{A}_0} V_P (w|\mathcal{A}), \tag{14}$$

where the guaranteed payoff given an action set $\mathcal{A}$ is:

$$V_P (w|\mathcal{A}) = \min_{(F,c)\in A^\star(w|\mathcal{A})} E_F [y_1 + y_2 - w(y_1, y_2)]. \tag{15}$$

The optimal contract under collusion is linear in total output $(y_1 + y_2)$ and ties the principals payoff to the agent's as in Theorem 1 and Proposition 1. By making the contract depend on total output the principals leave to the agent the decision of which project (y_1 or y_2) to favor. The decision depends on the agent's true action set, which is unknown to the principals when contracting. Even if the principals want to prioritize a project, say because the one of them is more profitable under the known action set $\mathcal{A}_0$, the same incentives do not generalize across all possible action sets of the agent, and thus do not provide the best guaranteed payoff for the principals. We summarize these results in Theorem 3 and present the proof in Appendix A.2.

Theorem 3. *The optimal contract under collusion is*

$$w_c(y) = (1-\theta_c)(y_1+y_2), \tag{16}$$

where the share θ_c *satisfies* $1-\theta_c = \sqrt{c^\star / E_{F^\star}[y_1+y_2]}$ *for an action* $(F^\star, c^\star) \in \underset{(F,c)\in\mathcal{A}_0}{\textit{argmax}} \left\{ \left(\sqrt{E_F[y_1+y_2]} - \sqrt{c} \right)^2 \right\}$. *Moreover, for any contract of the form* $w(y) = (1-\theta)(y_1+y_2)$ *that guarantees a positive payoff,* V_P *can be expressed as:*

$$V_P(w|\mathcal{A}_0) = \frac{\theta}{1-\theta} \max_{(F,c)\in\mathcal{A}_0} \left\{ (1-\theta) E_F[y_1+y_2] - c \right\}. \tag{17}$$

The principals' problem under collusion is a generalization of the problem in Carroll (2015) to a multi-project principal-agent problem where the agent controls two projects or tasks (y_1, y_2). This type of problem has received extensive attention in the literature. A key question is how the incentives should depend on the different tasks controlled by the agent. Holmstrom and Milgrom (1987) find that the optimal scheme is not generally linear in total output, instead it rewards the agent differently for different tasks.[14] They specifically note (Holmstrom and Milgrom, 1987, p.306):

> "The optimal scheme for the multidimensional Brownian model is a linear function of the end-of-period levels of the different dimensions of the process. ...If... the compensation paid must be a function of profits alone ..., or if the manager has sufficient discretion in how to account for revenues and expenses then the optimal compensation scheme will be a linear function of profits. *This is a central result, because it explains the use of schemes which are linear in profits even when the agent controls a complex multi-dimensional process."* [Emphasis added]

In our model, robustness leads to linearity in total output (profits) regardless of the

[14]Specifically, they consider an agent who controls the drift of a multi-dimensional Brownian motion, the principal chooses how to reward the agent given the terminal value of the Brownian motion.

complexity of the multi-dimensional process that the agent controls. The alignment of incentives between the principal and the agent requires linearity in output.

5 Efficiency

In this section we examine the efficiency properties of the equilibrium and collusive contracts, as captured by total (expected or guaranteed) payoffs. We show that competition between principals leads to a less efficient outcome than collusion.[15] In equilibrium, principals' receive a share of total output. This gives rise to a free-rider problem because principals do not internalize the effect of an increase in their share of total output on the other principals' payoffs. Consequently, the share of output accruing to the agent is lower than under collusion. We show that this implies a less efficient outcome.

We consider two notions of efficiency:

Definition 3. (Total expected surplus, TES) Given a contract scheme w and an action set $\mathcal{A}$, total expected surplus measures the sum of the expected payoffs of all players. This is,

$$\text{TES}\left(w|\mathcal{A}\right) = \left\{ E_F\left[y_1 + y_2\right] - c \,\middle|\, (F,c) \in \underset{(F,c)\in\mathcal{A}}{\operatorname{argmax}} \left\{w_1\left(y\right) + w_2\left(y\right) - c\right\} \right\}, \tag{18}$$

and under collusion, where principals offer a joint contract w_c,

$$\text{TES}\left(w_c|\mathcal{A}\right) = \left\{ E_F\left[y_1 + y_2\right] - c \,\middle|\, (F,c) \in \underset{(F,c)\in\mathcal{A}}{\operatorname{argmax}} \left\{w_c\left(y\right) - c\right\} \right\}. \tag{19}$$

Total expected surplus is a standard notion of efficiency in the literature. Given an action set $\mathcal{A}$, we compute TES for any action the agent would take from that set. However, principal i only knows a minimal action set $\mathcal{A}_0^i$, which leads us to consider total guaranteed surplus

[15] This efficiency result parallels finding in the literature, see for instance Bernheim and Whinston (1986a), Holmstrom and Milgrom (1988), and Martimort and Stole (2012), as well as the adverse selection models of Martimort and Stole (2015, 2012), Martimort and Moreira (2010) and Bond and Gresik (1996). In common agency games of complete information, the issue of efficiency was tackled by considering *truthful* equilibria (Bernheim and Whinston, 1986b), which are always efficient.

as a second notion of efficiency.[16] In what follows, we assume that $\mathcal{A}_0^i = \mathcal{A}_0$ to facilitate comparison across principals.

Definition 4. (Total guaranteed surplus, TGS) Given a contract scheme w and the known action set $\mathcal{A}_0$, total guaranteed surplus measures is the sum of guaranteed payoffs, this is,

$$TGS\left(w\right) = V_1\left(w|\mathcal{A}_0\right) + V_2\left(w|\mathcal{A}_0\right) + V_A\left(w|\mathcal{A}_0\right), \tag{20}$$

where V_i is principal i's guaranteed payoff as in (3) and V_A is the agent's payoff as in (2), and under collusion, where principals offer a joint contract w_c,

$$TGS\left(w_c\right) = V_P\left(w_c|\mathcal{A}_0\right) + V_A\left(w_c|\mathcal{A}_0\right), \tag{21}$$

where V_P is the principals' joint guaranteed payoff as in (14).

We can further characterize TGS when principals offer LRS contracts or the optimal linear contract under collusion, then TGS depends exclusively on the share of output going to the agent, $\theta_A = 1 - \theta_1 - \theta_2$ if principals compete and $\theta_A = 1 - \theta_c$ if they collude. Using (9) and (17) to replace V_i and V_p on (20) and (21), we get

$$\mathrm{TGS}\left(w\right) = \frac{1}{\theta_A} \max_{(F,c)\in\mathcal{A}_0} \left\{\theta_A E_F\left[y_1 + y_2\right] - c\right\}. \tag{22}$$

Moreover, the principal's share of total output (θ_1, θ_2, or θ_c) is equal to their share of total guaranteed surplus (before transfers)

$$V_i\left(w\right) = \theta_i \mathrm{TGS}\left(w\right) - k_i,\ i \in \{1,2\},\ \text{or}\ \ V_P\left(w\right) = \theta_c \mathrm{TGS}\left(w\right). \tag{23}$$

Theorem 4 states the main result of this section, namely that collusion leads to a more efficient outcome than competition between principals.

[16] The payoff function of principal, V_i, is quasi-linear in lump sum transfers. This allows us to consider the sum of payoffs as a measure of efficiency.

Theorem 4. *Let w be a Nash equilibrium in LRS contracts and w_c be an optimal collusion contract. Total expected and guaranteed surplus are weakly higher under the collusive contract. That is, for any known action set $\mathcal{A}_0$ and action set $\mathcal{A} \supseteq \mathcal{A}_0$, and surpluses $s_{TES}^N \in TES(w|\mathcal{A})$ and $s_{TES}^C \in TES(w_c|\mathcal{A})$, it holds that $s_{TES}^N \leq s_{TES}^C$ and $TGS(w) \leq TGS(w_c)$.*

The key for comparing the equilibrium and collusive outcomes is that the agent gets paid a share of total output under both scenarios. That is, the agent's problem under an LRS contract scheme or a linear collusive contract reduces to:

$$\max_{(F,c)\in\mathcal{A}} \left\{\theta_A E_F\left[y_1 + y_2\right] - c\right\}, \tag{24}$$

for some share $\theta_A \in [0,1]$. The agent's actions are, in general, not efficient (in the sense that they do not maximize TES). However, we can establish how total (expected and guaranteed) surplus varies with the contracts. Contracts that offer the agent a larger share of realized output are more efficient. We formalize this argument in Proposition 3.

Proposition 3. *Let w and $w^{'}$ be contract schemes such that the agent receives a share of total output given by θ_A and $\theta_A^{'}$ respectively. Total expected and guaranteed surplus are weakly increasing in the share of total output going to the agent. That is, for any known action set $\mathcal{A}_0$ and action set $\mathcal{A} \supseteq \mathcal{A}_0$, let $s_{TES} \in TES(w|\mathcal{A})$, and $s_{TES}^{'} \in TES\left(w^{'}|\mathcal{A}\right)$. If $\theta_A < \theta_A^{'} \leq 1$ then $s_{TES} \leq s_{TES}^{'}$ and $TGS(w) \leq TGS\left(w^{'}\right)$.*

Proposition 3 allows us to compare the efficiency of the pure strategy Nash equilibria and collusion contracts by comparing the agent's share of output. Under collusion, the agent's share of output ($\theta_A = 1 - \theta_c$) satisfies (16) as in Theorem 3. This condition also arises from the problem of a principal (i) under competition facing $\theta_j = 0$; see equation (9) in Proposition 1. Then, it is sufficient to show that the share that principal i wants to induce for the agent is decreasing in the share of principal j to establish that the collusive outcome is more efficient.

Proposition 4 shows that the agent's share is higher under collusion than in the Nash equilibrium in LRS contracts. Intuitively, when competing, each principal i only internalizes $1-\theta_j$ of the increases in output because of the other principals' actions.[17] Therefore, the principals do not want to give as much incentives to the agent as they would under collusion, knowing that their gains are dampened by the share of total output being appropriated by their competitors. This is the same force at the heart of the "free-rider" problem described in Bernheim and Whinston (1986a), Holmstrom and Milgrom (1988), Maier and Ottaviani (2009) and Martimort and Stole (2012).

Proposition 4. *Let w and w' be LRS contract schemes such that principal j's shares of total output satisfy $\theta_j < \theta_j'$ and principal i's contracts are best responses to principal j's contracts. Then, the agent's shares of total output satisfy $1-\theta_1-\theta_2 \geq 1-\theta_1'-\theta_2'$.*

Proposition 3 and 4 imply that the collusive contract provides a higher surplus than the Nash equilibrium LRS contracts, as stated in Theorem 4. When principals compete they induce a lower share of output for the agent than when they collude (Proposition 4), which reduces both total expected and guaranteed surplus (Proposition 3). We provide all proofs in Appendix C. In Section 6, we revisit this result highlighting the role that limited liability plays in generating a lower surplus when principals compete.

6 Limited liability and efficiency

We now show that the ability of principals to implicitly make side-payments through the agent is crucial for the efficiency result in Theorem 4. These side-payments are possible under the limited liability restriction imposed in Assumption 4. To highlight the role that limited liability plays, we consider two other scenarios. First, a more restrictive form of

[17] The principal's problem in (9) can be framed as that of a single principal facing a multitasking agent. The choice is over a share $\tilde{\theta}_i \in [0,1]$ out of the "reduced" output space $\tilde{Y} \equiv (1-\theta_j)\,Y$. The problem is: $\max_{\theta_i \in [0,1-\theta_j]} \; \theta_i \left(E_F\left[y_1+y_2\right] - \frac{c}{1-\theta_j-\theta_i}\right) - k_i = \max_{\tilde{\theta}_i \in [0,1]} \; \tilde{\theta}_i \left(E_F\left[\tilde{y}_1+\tilde{y}_2\right] - \frac{c}{1-\tilde{\theta}_i}\right) - k_i$, where $\tilde{\theta}_i \equiv \theta_i/1-\theta_j$ and $\tilde{y}_h \equiv (1-\theta_j)\,y_h$ for $h \in \{1,2\}$.

limited liability imposed on individual projects, so that the payments of each principal to the agent have to be non-negative. This prevents principals from making side-payments through the agent and results in a more efficient outcome than collusion as principals incentivize the agent by offering a higher share of total output. Then, a more relaxed scenario without limited liability constraints that results in the efficient outcome being implemented after the principals offload all risk onto the agent.

6.1 Individual limited liability

We start by replacing Assumption 4 with a stronger form of limited liability that applies to each principal's contract. This is the case if the projects are located in different jurisdictions (as is the case for multinationals), or if the projects are registered as separate entities (as is the case for not fully integrated businesses).

Assumption 5. ***(Individual Limited Liability)*** $w_i(y) \geq 0$ *for all* $y \in Y$ *and* $i \in \{1,2\}$.

Assumption 5 constrains the ability of principals to transfer resources through the agent. In this way, individual limited liability limits the ability of a principal to free-ride on the incentives provided by their competitors. LRS contracts are still robust for the principals following the same arguments as in Section 2 (see Theorem 7 in Appendix D.1). However, the form of LRS contract schemes changes under Assumption 5 by pinning down the value of the transfer provided to the agent. In particular, we show in Proposition 5 of Appendix D.2 that LRS contract schemes satisfy

$$w_i(y) = (1-\theta_i)\, y_i + \theta_i \left(\overline{y}_j - y_j\right) \tag{25}$$

for $\theta_1, \theta_2 \in [0,1]$, where $\theta_i \in [0, 1-\theta_j]$ as in Proposition 1 and $\overline{y}_j = \max\{Y_j\}$ as defined in Section 1.[18]

[18]The form of the LRS contract in (25) requires an additional assumption on the output space, namely that $(0, \bar{y}_2), (\bar{y}_1, 0) \in Y$. This pins down the lowest contract for each principal and the value of the transfers to the agent. We formalize this in Appendix D.2.

Principals now incentivize the agent through a combination of high and low powered incentives, in the form of a share of total output $(1-\theta_i)$ and a constant fee $(\theta_i \bar{y}_j)$ respectively. In this way, each principal appropriates a share θ_i of total output for themselves after paying a fee that can be interpreted as the price payed for this share ($\bar{y}_j$ is the price per unit share of total output faced by principal i). This makes it costly for the principal to free-ride when increasing their share θ_i.[19]

The equilibrium contracts under Assumption 5 imply higher (expected and guaranteed) surplus than collusion, as we show in Appendix D.3. The result follows, as before, from comparing the share of total output accruing to the agent in each case. The fee that principals now pay when increasing their share of output is enough to reverse the result of Theorem 4.

The main reason for these results lies in the form of the payments offered by the principals in equilibrium. Under Assumption 4, the principals choose their share of output independently of the constant k. As we discussed above, their choices induce an externality on the payoffs of other principals lowering the agent's share of output. This externality is absent when principals collude, leading to higher surplus. Under Assumption 5 the free riding problem is addressed in a different manner. In order to satisfy individual limited liability, the principals have to increase the fixed payment to the agent (k) as they increase their share of output. This force is absent when principals collude because they do not have to pay any fees to the agent in order to satisfy limited liability. This leads the principals to choose a lower share of output in equilibrium and a higher share for the agent relative to the outcome under collusion. We make this argument precise in Theorem 9.

[19]The dependency of the contract in the maximum output (size) of the competing principal requires stronger conditions on what a principal needs to guarantee herself a positive payoff. In particular, Assumption 3 (non-triviality) is not enough. A necessary condition for principal i to guarantee herself a positive payoff is that there exists an action $(F, c) \in \mathcal{A}_0$ such that $E_F[y_1 + y_2] - c > \overline{y}_j$. We provide conditions for the existence of a pure strategies Nash Equilibrium in Theorem 8 of Appendix D.2.

6.2 No limited liability

Finally, we consider what happens when we dispense with limited liability, instead imposing a participation constraint on the agent, guaranteeing the agent a given expected payoff (normalized here to 0). Without limited liability the outcome of the game is efficient. The solution is the same as in Bernheim and Whinston (1986a), where each principal "sells their firm" to the agent, leaving the agent as sole claimant on total output. To see this, let s_0 denote the total expected surplus under the known action set $\mathcal{A}_0$,

$$s_0 = \max_{(F,c)\in\mathcal{A}_0} \left\{ E_F \left[y_1 + y_2 \right] - c \right\}, \tag{26}$$

and consider a contract by principal j that gives the agent all of project j's output for a price $s_j \leq s_0$, $w_j \left(y \right) = y_j - s_j$. Principal i cannot be guaranteed a payoff higher than $s_0 - s_j$, otherwise the agent's participation constraint would be violated. This payoff is achieved if principal i offers $w_i \left(y \right) = y_i - \left(s_0 - s_j \right)$. Thus, giving the agent all of project i' output is a best response of principal i and the principals divide among themselves all the surplus under the known action set.[20] The outcome without limited liability is therefore efficient, both in terms of total guaranteed and total expected surplus.

7 Discussion and concluding remarks

Taking a robust contracting approach provides a crisp characterization of equilibrium strategies and payoffs in the complicated problem of common agency. The central issue in the literature of how competition among principals affects the efficient provision of incentives can be easily pinned down to one component, namely the share of total output that the agent receives in equilibrium. We show that when principals can make side payments (through the agent) to each other a free-riding problem appears. Free riding

[20]The solution leaves the division of surplus indeterminate as in Bernheim and Whinston (1986b). The same equilibria arise in private common agency, when contracts can only depend on the principal's output.

leads to lower incentives given to the agent, compared to the collusive outcome. When such side payments are not possible, because of individual limited liability, then principals are forced to internalize their externality, which leads to the competitive outcome being more efficient than the collusive outcome.

Our results are themselves robust to several extensions of the setup described in Section 1. First, we consider a game with more than two principals. We show in Appendix E that this does not substantively alter our results. The only difference arises under individual limited liability, where we need to impose a more demanding constraint on the projects' output ($\bar{y}_i$) to guarantee a positive payoff to the agent and the principals (recall that under Assumption 5 contracts depend on the maximum output of other principals). It follows that principals may have negative ex-post payoffs in equilibrium contracts. Alternatively, we can impose a form of "double" limited liability, i.e., impose a cap to principal's payments to the agent, guaranteeing principals a non-negative payoff. We show that a modified version of LRS contracts is still *robust* under these conditions in Appendix F.

Second, we allow principals to have partial knowledge of the agent's possible action sets, rather than considering any set $\mathcal{A} \supseteq \mathcal{A}_0$. In particular, we show in Appendix G that LRS contracts are still *robust* when principals know of a lower bound on cost that is a function of the expected project outputs. This makes it clear that our results do not depend on unreasonable flexibility on the potential actions the agent can take. This, however, does come at a cost in terms of the tractability of the results.

Third, we consider a private common agency game where principals are restricted to contract only on their project's output. This reflects a variety of situations in which principals cannot observe or contract on the agent's other projects, for instance when an agent represents several celebrities, or a realtor represents several home sellers. In these cases, it is common for celebrities to give their agents a share of their earnings, regardless of the earnings of the agent's other clients, or for home sellers to pay the realtors a share of the value of the house, and do not explicitly reward them for not working for other clients.

We show in Appendix H that the equilibrium contract that arises in this setting provides a rationale for this behavior. The principal's best strategy is to give the agent a share of their project's output. The value of the share depends on the project's of other principals and the agent's known actions.

Finally we want to give a brief overview of how the setup we develop can be used to study different problems where a group of parties is interested in the decision of a single agent. We go into these problems in detail in Appendices I and J.

Taxation of multinationals. We reinterpret the common agency game we have studied as the problem of two governments (the principals) designing a tax system on the profits (y_1, y_2) that a multinational (the agent) generates in each country. This is more easily understood after a simple change of notation, letting $t_i(y) \equiv y_i - w_i(y)$ be the taxes payed in country i and the total (ex-post) payoff of the agent be $\sum_i y_i - t_i(y) = \sum_i w_i(y)$. In this formulation the agent "owns" all of the profits and pays a portion to the principals as taxes.

Governments need for robustness can reflect their inability to adjust the tax system in response to changes in corporate practices, or their lack of knowledge of the firm's technology. Tax systems cannot be tailored to specific situations and it is thus desirable for them to perform across the widest variety of possible situations.

The limited liability restrictions we considered above (Assumptions 4 and 5) map to the degree of enforceability that governments have. That is, whether or not they can tax the multinational beyond their borders. Individual limited liability implies that $t_i(y) \leq y_i$, restricting taxes to be at most the firm's domestic profits.

We show that it is optimal to implement a worldwide tax, where the firm's global profits are taxed at a constant rate θ_i, allowing for the full deduction of taxes payed to country j, and a potential tax incentive (in the form of a lump sum subsidy k_i):[21]

$$t_i(y) = \theta_i(y_1 + y_2 - t_j(y)) + k_i. \tag{27}$$

[21] The enforceability regime, i.e, limited liability, determines the value of k_i.

This is the tax system proposed in the Bipartisan Tax Fairness and Simplification Act of 2011 by Senators Wyden and Coats (Senate Bill 727, 2011). Interestingly, it also coincides with the taxes found by Feldstein and Hartman (1979).[22]

Procurement Auction. We also consider a setup where two competing firms bid for a government contract (e.g., for the provision of services, construction, or the privatization of a government asset). It is known that the government faces a cost $c > 0$ awarding the contract, representing the costs of evaluating and screening bids. However, firms have reasons to doubt the announcement. For instance, the government can (secretly) favor one of the firms. It is also possible that the government can lower the cost by randomizing between the firms, this might be the case if bids are hard to assess, or if technicalities can arise that create the chance of a lower bid to be awarded the contract.[23]

In a perfect information setting, this setup is that of a first price auction. The bids in the auction are undefined because the firm with the highest highest valuation would try to marginally outbid the other firm. In contrast, we show that there two equilibria of the game in LRS contracts. In both cases the government awards the contract to the firm with the highest valuation and the bids are pinned down. The difference between the equilibria lies in the bid of the lowest valuation firm (ℓ), which is indifferent between bidding their valuation ($w_\ell(y) = \bar{y}_\ell$) and not bidding at all ($w_\ell(y) = 0$). In each case, the bid of the winning firm is a share θ_h of their valuation $\bar{y}_h$ that we derive in closed form in Appendix J.3.

Provision of public goods. A similar setup applies to the provision of public goods. In this case, output is perfectly correlated across principals, who vary in their (private) valuation of the public good produced by the agent. This setup is typically subject to free rider problems

[22]Feldstein and Hartman (1979) assume complete information and restrict attention to linear taxes. Their "full taxation after deduction" result rests on concerns on the optimal allocation of capital between countries.

[23]Randomness in who is assigned the contract can also arise from last minute changes in the rules (not uncommon in developing countries), or from challenges made in courts to the rules or the decision of the government. It is worth pointing out that randomization is not itself necessary for our results. The firms could simply be worried that the government can allocate the good with certainty to the other contractor. This is in fact the worst case scenario they face.

that prevent the public from being provided even when the valuation of the principals is enough to cover the provision costs. In contrast, we show that when the principals act robustly (not knowing what the real cost of providing the cost would be) the efficient outcome is implemented and the good is always provided (when the known cost warrants it).

Nevertheless, the equilibrium has each principal "partially" free riding on the other by lowering compensation by a fraction of the other principal's payoff, while guaranteeing that the agent optimally chooses to provide the good. An interesting feature of this equilibrium is that all principals get the same share of expected output and the same guaranteed payoff regardless of their valuation.

Proofs and Supplemental Results

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A Principals' Best Responses

A.1 Principals' best response under Assumption 4

We now turn to establishing the optimality of LRS contracts for the principal. We ignore the index i on the known action set as there is no confusion in what follows on which principal is making a choice. We start by defining the set of eligible contracts as those that outperform the "lowest" contract the principal can offer. That is, the contract that undoes all payments and offer the agent zero payoffs with certainty, i.e., $w_i(y) = -w_j(y)$. Under assumption 2, the agent's unique optimal action, under any action set, given this contract is to choose inaction. This provides us with a lower bound on the payoff of the principal, which we use to define eligible contracts.

Definition 5. (Eligible Contracts) A contract w_i is eligible for principal i if it satisfies:

$$V_i(w) > w_j(0,0). \tag{A.1}$$

Next, we consider the problem of characterizing the agent's actions given a contract scheme. Despite the principal's lack of knowledge over the agent's actual action set ($\mathcal{A}$), it is possible to impose restrictions on the actions the agent will consider given the incentives provided by a contract scheme w. Intuitively, the agent will only induce actions that generate a payoff higher than the guaranteed payoff under the principal's known action set ($\mathcal{A}_0$). We formalize this idea in the following lemma.

Lemma 1. *Let w be a contract scheme, $\mathcal{A} \supseteq \mathcal{A}_0$ be an action set, and $(F, c) \in A^\star(w|\mathcal{A})$ an optimal action for the agent. Then, it holds that*

$$F \in \mathcal{F} \equiv \{F \in \Delta(Y) \,|E_F[w_1(y) + w_2(y)] \geq V_A(w|\mathcal{A}_0)\}. \tag{A.2}$$

Proof. Consider $(F, c) \in A^\star(w|\mathcal{A})$, then it holds that:

$$E_F[w_1(y) + w_2(y)] \geq E_F[w_1(y) + w_2(y)] - c \geq V_A(w|\mathcal{A}) \geq V_A(w|\mathcal{A}_0) \tag{A.3}$$

Then $F \in \mathcal{F}$.

□

Lemma 1 makes it possible to characterize the principal's payoff for a given contract scheme using the set $\mathcal{F}$. Crucially, $\mathcal{F}$ only depends on the contract scheme and the known set of actions $\mathcal{A}_0$. In this way we replace the complexity of the definition of $V_i(w)$ in (3) with an object that depends only on known elements. Lemma 2 formalizes this idea.

Lemma 2. *Let w be an eligible contract scheme for principal i. Then*

$$V_i(w) = \min_{F \in \mathcal{F}} \; E_F[y_i - w_i(y)]. \tag{A.4}$$

Moreover, if $F \in \underset{F \in \mathcal{F}}{argmin} E_F[y_i - w_i(y)]$ then $E_F[w_1(y) + w_2(y)] = V_A(w|\mathcal{A}_0)$.

Proof. Let w be an eligible contract scheme and $\mathcal{A} \supseteq \mathcal{A}_0$ be an action set.

From the definition of $V_i(w|\mathcal{A})$ in (4) and Lemma 1 we can write:

$$V_i(w|\mathcal{A}) \equiv \min_{(F,c)\in A^\star(w|\mathcal{A})} E_F\left[y_i - w_i(y)\right] \geq \min_{F\in\mathcal{F}} E_F\left[y_i - w_i(y)\right], \tag{A.5}$$

where the right hand side establishes a lower bound for $V_i(w|\mathcal{A})$ that is independent of $\mathcal{A}$. Then, from the definition of $V_i(w)$ in (3) we get:

$$V_i(w) \equiv \inf_{\mathcal{A}\supseteq\mathcal{A}_0} V_i(w|\mathcal{A}) \geq \min_{F\in\mathcal{F}} E_F\left[y_i - w_i(y)\right]. \tag{A.6}$$

To prove equality suppose that $V_i(w) > \min_{F\in\mathcal{F}} E_F\left[y_i - w_i(y)\right]$, and let $F' \in \operatorname{argmin}_{F\in\mathcal{F}} E_F\left[y_i - w_i(y)\right]$. From Lemma 1, $E_{F'}\left[w_1(y) + w_2(y)\right] \geq V_A(w|\mathcal{A}_0)$. There are two options:

1. F' does not place full support on the values of y that maximize $w_1 + w_2$.

   Let $\hat{y} \in \operatorname{argmax}\{w_1(y) + w_2(y)\}$, and $\hat{F} = \delta_{\hat{y}}$ be a distribution that gives probability 1 to $\hat{y}$. Let $\epsilon \in [0,1]$ and $F_\epsilon \equiv (1-\epsilon)F' + \epsilon\hat{F}$. For all ϵ there exists a $\xi_\epsilon > 0$ such that: $E_{F_\epsilon}\left[w_1(y) + w_2(y)\right] - \xi_\epsilon > V_A(w|\mathcal{A}_0)$.

   Now consider the action set $\mathcal{A}_\epsilon \equiv \mathcal{A}_0 \cup \{(F_\epsilon, \xi_\epsilon)\}$. The unique optimal action of the agent in $\mathcal{A}_\xi$ is $(F_\epsilon, \xi_\epsilon)$. Then:

   $$V_i(w) \leq V_i(w|\mathcal{A}_\epsilon) = E_{F_\epsilon}\left[y_i - w_i(y)\right] = (1-\epsilon)E_{F'}\left[y_i - w_i(y)\right] + \epsilon E_{\hat{F}}\left[y_i - w_i(y)\right]. \tag{A.7}$$

   This condition holds for all $\epsilon > 0$. Letting $\epsilon \to 0$ we arrive at a contradiction:

   $$V_i(w) \leq E_{F'}\left[y_i - w_i(y)\right] = \min_{F\in\mathcal{F}} E_F\left[y_i - w_i(y)\right] < V_i(w). \tag{A.8}$$

2. F' places full support on the values of y that maximize $w_1 + w_2$.

   There are still two possible cases:

   (a) $E_{F'}\left[w_1 + w_2\right] > V_A(w|\mathcal{A}_0)$. Then there exists $\xi > 0$ and an action set $\mathcal{A}' \equiv \mathcal{A}_0 \cup \left\{\left(F', \xi\right)\right\}$ such that $\left(F', \xi\right)$ is the unique optimal action for the agent in $\mathcal{A}'$ and:

   $$V_i(w) \leq V_i\left(w|\mathcal{A}'\right) = E_{F'}\left[y_i - w_i(y)\right] = \min_{F\in\mathcal{F}} E_F\left[y_i - w_i(y)\right] < V_i(w) \tag{A.9}$$

   (b) $E_{F'}\left[w_1 + w_2\right] = V_A(w|\mathcal{A}_0)$. This implies $V_A(w|\mathcal{A}_0) = \max_{y\in Y}\{w_1(y) + w_2(y)\}$ which can only be satisfied if F' is available in $\mathcal{A}_0$ at zero cost. By Assumption 2 this implies that $F' = \delta_{(0,0)}$ and that $w_1(0,0) + w_2(0,0) = \max_{y\in Y}\{w_1(y) + w_2(y)\}$. In this case the unique optimal action for the agent under any action set is $(\delta_0, 0)$, so the value of the principal is $V_i(w) = -w_i(0,0) \leq w_j(0,0)$, where the inequality follows from limited liability (Assumption 4). This contradicts eligibility (Definition 5).

This establishes the first claim in the lemma.

We now establish the second claim. Let $F^{'} \in \underset{F \in \mathcal{F}}{\operatorname{argmin}} E_F\left[y_i - w_i\left(y\right)\right]$ and suppose for a contradiction that $E_{F'}\left[w_1\left(y\right) + w_2\left(y\right)\right] > V_A\left(w|\mathcal{A}_0\right)$. Then we can define $F_\epsilon = \left(1-\epsilon\right)F^{'} + \epsilon\delta_0$ for some $\epsilon \in [0,1]$. For a low enough ϵ it holds that: $E_{F_\epsilon}\left[w_1\left(y\right) + w_2\left(y\right)\right] > V_A\left(w|\mathcal{A}_0\right)$. Then there exists $\xi_\epsilon > 0$ such that $A^\star\left(w|\mathcal{A}_\epsilon\right) = \{\left(F_\epsilon, \xi_\epsilon\right)\}$, where $\mathcal{A}_\epsilon = \mathcal{A}_0 \cup \{\left(F_\epsilon, \xi_\epsilon\right)\}$. The payoff to the principal under $\mathcal{A}_\epsilon$ is then:

$$
\begin{aligned}
V_i\left(w|\mathcal{A}_\epsilon\right) &= \left(1-\epsilon\right)E_{F'}\left[y_i - w_i\left(y\right)\right] + \epsilon\left(-w_i\left(0,0\right)\right) \\
&= \left(1-\epsilon\right)V_i\left(w\right) + \epsilon\left(w_j\left(0,0\right) - \left(w_i\left(0,0\right) + w_j\left(0,0\right)\right)\right) \\
&\leq V_i\left(w\right) - \epsilon\left(V_i\left(w\right) - w_j\left(0,0\right)\right) \\
&< V_i\left(w\right).
\end{aligned}
$$

This gives a contradiction so that $E_{F'}\left[w_1\left(y\right) + w_2\left(y\right)\right] = V_A\left(w|\mathcal{A}_0\right)$.

□

The following lemma takes the results of Lemma 2 one step further and is at the core of the linearity argument. It shows that there exists an affine link between the principal's and the agent's guaranteed payoffs, $V_i\left(w\right)$ and $V_A\left(w|\mathcal{A}_0\right)$ respectively. This link allows the principal to increase her own guaranteed payoff by controlling the payoff given to the agent.

Lemma 3. *Let w be an eligible contract scheme. There exist $k \in \mathbb{R}$ and $\alpha \in (0,1]$ such that:*

$$
\begin{aligned}
w_i\left(y\right) &\leq \alpha y_i - \left(1-\alpha\right)w_j\left(y\right) - \alpha k, \quad \textit{for all } y \in Y && \text{(A.10)} \\
V_i\left(w\right) &= k + \frac{1-\alpha}{\alpha}V_A\left(w|\mathcal{A}_0\right) && \text{(A.11)}
\end{aligned}
$$

Proof. The result follows from applying the separating hyper-plane theorem.

We start by defining the following two convex sets:

1. Let $S \subseteq \mathbb{R}^2$ be the convex hull of all pairs $\left(w_1\left(y\right) + w_2\left(y\right), y_i - w_i\left(y\right)\right)$ for $y \in Y$.
2. Let $T \subseteq \mathbb{R}^2$ be the set of all pairs $\left(u,v\right)$ such that $u > V_A\left(w|\mathcal{A}_0\right)$ and $v < V_i\left(w\right)$.

We proceed by establishing that $S \cap T = \emptyset$. Let $\left(u,v\right) \in T$, and let $F \in \underset{F \in \mathcal{F}}{\operatorname{argmin}} E_F\left[y_i - w_i\left(y\right)\right]$, where $\mathcal{F}$ is defined as in Lemma 1. Then, by definition of T and Lemma 2:

$$
u > V_A\left(w|\mathcal{A}_0\right) = E_F\left[w_1\left(y\right) + w_2\left(y\right)\right], \text{ and } v < V_i\left(w\right) = E_F\left[y_i - w_i\left(y\right)\right]. \tag{A.12}
$$

Now, suppose for a contradiction that $\left(u,v\right) \in S$, then there exists $F^{'} \in \Delta\left(Y\right)$ such that

$$
u = E_{F'}\left[w_1\left(y\right) + w_2\left(y\right)\right] \text{ and } v = E_{F'}\left[y_i - w_i\left(y\right)\right]. \tag{A.13}
$$

$F^{'}$ guarantees a payoff to the agent larger than $V_A\left(w|\mathcal{A}_0\right)$ so $F^{'} \in \mathcal{F}$ but

$$
E_F\left[y_i - w_i\left(y\right)\right] > E_{F'}\left[y_i - w_i\left(y\right)\right], \tag{A.14}
$$

which contradicts minimality of F. Then $S \cap T = \emptyset$

We can now apply the separating hyperplane theorem. There exist constants (k, λ, μ) such that $(\lambda, \mu) \neq (0,0)$ and:

$$k + \lambda u - \mu v \leq 0 \qquad (u,v) \in S \tag{A.15}$$
$$k + \lambda u - \mu v \geq 0 \qquad (u,v) \in T \tag{A.16}$$

Now consider $F^{\star} \in \underset{F \in \mathcal{F}}{\text{argmin}} E_F \left[y_i - w_i\left(y\right)\right]$. The pair $\left(E_{F^\star}\left[w_1\left(y\right) + w_2\left(y\right)\right], E_{F^\star}\left[y_i - w_i\left(y\right)\right]\right)$ lies in the closures of both S and T. Then:

$$k + \lambda E_{F^\star}\left[w_1\left(y\right) + w_2\left(y\right)\right] - \mu E_{F^\star}\left[y_i - w_i\left(y\right)\right] = 0. \tag{A.17}$$

Using equation (A.17), we can derive equation (A.10).

It is left to show that $\lambda, \mu > 0$. Note that $(u,v) \in T$ admits u arbitrarily high and v arbitrarily low. So for (A.16) to hold it must be that $\lambda \geq 0$ and $\mu \geq 0$, with at least one strict inequality. There are then two cases to rule out:

1. Suppose $\mu = 0$, then it must be that $\lambda > 0$. From (A.15) and (A.16)

$$u \leq -\frac{k}{\lambda} \quad (u,v) \in S \qquad \text{and} \qquad u \geq -\frac{k}{\lambda} \quad (u,v) \in T \tag{A.18}$$

So, $\max_{y \in Y}\left[w_1\left(y\right) + w_2\left(y\right)\right] = \max_{u \in S} u \leq -\frac{k}{\lambda} \leq \inf_{u \in T} u = V_A\left(w|\mathcal{A}_0\right)$. Which implies:

$$\max_{y \in Y}\left[w_1\left(y\right) + w_2\left(y\right)\right] = V_A\left(w|\mathcal{A}_0\right). \tag{A.19}$$

Equation (A.19) only holds if the agent takes an action with zero cost. Assumption 2 implies that $F = \delta_{(0,0)}$ and that $w_1\left(0,0\right) + w_2\left(0,0\right) = \max_{y \in Y}\left\{w_1\left(y\right) + w_2\left(y\right)\right\}$. In this case the unique optimal action for the agent under any action set is $(\delta_0, 0)$, so the value of the principal is $V_i\left(w\right) = -w_i\left(0,0\right) \leq w_j\left(0,0\right)$, where the inequality follows from limited liability (Assumption 4). $V_i\left(w\right) \leq w_j\left(0,0\right)$ contradicts eligibility, so, it follows that $\mu > 0$.

2. Suppose $\lambda = 0$ and $\mu > 0$. From (A.15) and (A.16)

$$v \geq \frac{k}{\mu} \quad (u,v) \in S \qquad \text{and} \qquad v \leq \frac{k}{\mu} \quad (u,v) \in T \tag{A.20}$$

So $\min_{y \in Y}\left[y_i - w_i\left(y\right)\right] = \min_{v \in S} v \geq \frac{k}{\mu} \geq \sup_{v \in T} v = V_i\left(w\right)$, then:

$$V_i\left(w\right) \leq \min_{y \in Y}\left[y_i - w_i\left(y\right)\right] \leq \min_{y \in Y}\left[y_i + w_j\left(y\right)\right] \leq w_j\left(0,0\right) \tag{A.21}$$

which violates eligibility (the second inequality follows from limited liability, Assumption 4). So $\lambda > 0$.

To finalize the proof we normalize $\mu = 1$, giving from (A.15):

$$k + \lambda\left(w_i\left(y\right) + w_j\left(y\right)\right) - \left(y_i - w_i\left(y\right)\right) \leq 0, \; y \in Y. \tag{A.22}$$

And from (A.17):

$$V_i(w) = k + \lambda V_A(w|\mathcal{A}_0). \tag{A.23}$$

Equations (A.10) and (A.11) follow from rearranging and denoting $\alpha \equiv \frac{1}{1+\lambda} \in (0,1]$.

$\square$

The affine link between the agent's payoff and the principal's payoff is a crucial element in providing incentives for the agent. Given the partial knowledge over the agent's set of actions the principals' optimal strategy is to tie their payoff to that of the agent, thus aligning the agent's objectives with their own. This is the same mechanism at the heart of the optimal contracts in Hurwicz and Shapiro (1978) and Carroll (2015), and will be crucial in establishing the optimality of affine (LRS) contracts in our setting.

Equation (A.10) can be exploited by the principal to construct an alternative contract that dominates the original one, in the sense that it weakly increases principal i's guaranteed payoff. This contract is a LRS contract, as defined in (7). The following two Lemmas formalizes the process.

Lemma 4. *Let w be an eligible contract scheme. There exist $k \in \mathbb{R}$ and $\alpha \in (0,1]$ such that the contract*

$$w_i^{'}(y) = \alpha y_i - (1-\alpha) w_j(y) - \alpha k \tag{A.24}$$

satisfies limited liability with equality and $V_i\left(w_i^{'}, w_j\right) \geq V_i(w)$.

Proof. From Lemma 3, there are $k \in \mathbb{R}$ and $\alpha \in (0,1]$ so that w_i satisfies equations (A.10) and (A.11). Use the same α and k to define an alternative contract $w_i^{''}$ as

$$w_i^{''}(y) = \alpha y_i - (1-\alpha) w_j(y) - \alpha k. \tag{A.25}$$

Rearranging gives:

$$\left(y_i - w_i^{''}(y)\right) = k + \frac{1-\alpha}{\alpha}\left(w_i^{''}(y) + w_j(y)\right). \tag{A.26}$$

Then, take any $\mathcal{A} \supseteq \mathcal{A}_0$ and $(F,c) \in A^\star\left(w_i^{''}, w_j|\mathcal{A}\right)$. Taking expectations gives:

$$E_F\left[y_i - w_i^{''}(y)\right] \geq k + \frac{1-\alpha}{\alpha} V_A\left(\left(w_i^{''}, w_j\right)|\mathcal{A}_0\right) \geq k + \frac{1-\alpha}{\alpha} V_A(w|\mathcal{A}_0) = V_i(w), \tag{A.27}$$

where the first inequality follows from the definition of the agent's guaranteed payoff, $E_F\left[w_i^{''}(y) + w_j(y)\right] \geq V_A\left(\left(w_i^{''}, w_j\right)|\mathcal{A}_0\right)$, the second inequality from (A.10), $w_i^{''}(y) \geq w_i(y)$, and the last equality from (A.11). This process applies to any optimal action for the agent in any $\mathcal{A} \supseteq \mathcal{A}_0$, so the right-hand-side is a lower bound for the payoff principal i under the alternative contract scheme $\left(w_i^{''}, w_j\right)$. This gives the desired result: $V_i\left(w_i^{''}, w_j\right) \geq V_i(w)$.

It is only left to handle limited liability. The contract scheme $\left(w_i^{''}, w_j\right)$ satisfies limited liability, i.e., $\min_{y \in Y}\left\{w_i^{''}(y) + w_j(y)\right\} \geq 0$, because $w_i^{''}(y) \geq w_i(y)$ for all $y \in Y$ from equation (A.10). The alternative contract can be modified by subtracting a constant, making it satisfy limited liability with equality, weakly increasing the principal's payoff. We define $w_i^{'}$ as:

$$w_i^{'}(y) = w_i^{''}(y) - \min_{y \in Y} \left\{ w_i^{''}(y) + w_j(y) \right\} \tag{A.28}$$

This new contract (w_i') can be written as in (A.24) by appropriately redefining k.

□

Taking stock, we have shown that an eligible contract that satisfies limited liability is weakly dominated by a LRS contract satisfying limited liability with equality, i.e., with $k_i = \min_{y \in Y} \{y_i + w_j(y)\}$. We have two more results to prove, establishing the form of the principal's payoffs under LRS contracts and the existence of an optimal contract in that class.

Lemma 5. *Let w be an eligible contract scheme, such that w_i satisfies (7) for some $\alpha_i \in (0,1]$ and $k_i \in \mathbb{R}$, and satisfies limited liability (Assumption 4) with equality for some output level. Then:*

$$V_i(w) = \frac{1-\alpha_i}{\alpha_i} V_A(w|\mathcal{A}_0) + k_i = \max_{(F,c) \in \mathcal{A}_0} \left((1-\alpha_i) E_F[y_i + w_j(y)] - \frac{1-\alpha_i}{\alpha_i} c \right) + \alpha_i k_i. \tag{A.29}$$

Proof. Let $F \in \underset{F \in \mathcal{F}}{\operatorname{argmin}} E_F[y_i - w_i(y)]$. By Lemma 2, there exist $\alpha_i \in (0,1]$ and $k_i \in \mathbb{R}$:

$$V_i(w) = E_F[y_i - w_i(y)] = \frac{1-\alpha_i}{\alpha_i} E_F[w_1(y) + w_2(y)] + k_i = \frac{1-\alpha_i}{\alpha_i} V_A(w|\mathcal{A}_0) + k_i \tag{A.30}$$

The second equality follows by replacing $V_A(w|\mathcal{A}_0)$ and w_i from (7).

□

Remark. When $\alpha_i = 0$ the principal offsets the other principal's payments to the agent, i.e., $w_i(y) = -w_j(y)$. In this case the agent's unique optimal action under any $\mathcal{A} \supseteq \mathcal{A}_0$ is $(F,c) = (\delta_0, 0)$ because of Assumption 2. Then, $V_i(w) = w_j(0,0)$. The result is consistent with Lemma 5 if we interpret the term $\frac{1-\alpha_i}{\alpha_i} c$ as 0 when $c = 0$ and ∞ for $c > 0$. In this way we can treat the result of Lemma 5 more generally as applying to $\alpha_i \in [0,1]$.

Lemma 6. *In the class of LRS contracts that satisfy limited liability (Assumption 4) with equality there exists an optimal one for principal i.*

Proof. From Lemma 5 we can express $V_i(w)$ directly as a function of α_i as in (A.29). Recall that $k_i = \min_{y \in Y} \{y_i + w_j(y)\}$ is independent of α_i. Moreover, the function $(1-\alpha_i) E_F[y_i + w_j(y)] - \frac{1-\alpha_i}{\alpha_i} c$ is continuous in α_i, thus its maximum over $\mathcal{A}_0$ is continuous in α_i as well. Continuity implies that the right-hand-side of equation (A.29) is continuous in α_i it achieves a maximum in $[0,1]$. This α_i gives the optimal guarantee over all LRS contracts that satisfy limited liability with equality.

□

We can now state the complete proof of Theorem 1. We can strengthen some of our results under an additional assumption on the agent's actions:

Assumption 6. ***(Full Support)*** *For all $(F, c) \in \mathcal{A}_0$, if $(F, c) \neq (\delta_0, 0)$ then supp$(F) = Y$.*

Theorem 1. *For any contract w_j, there exists a LRS contract $\overline{w}_i$ such that $\overline{w}_i \in BR_i(w_j)$ and $\min_{y\in Y}\{\bar{w}_i(y) + w_j(y)\} = 0$. That is, there is always a LRS contract that is robust for principal i.*

Moreover, if $\mathcal{A}_0$ satisfies Assumption 6, any robust contract is a LRS contract or $\max_{w_i} V_i(w_i, w_j) = w_j(0, 0)$.

Proof. Consider a contract w_j by the competing principal. By Lemma 4 any eligible contract, $\hat{w}_i$, is weakly dominated by a LRS contract satisfying limited liability with equality. By Lemma 6 there is a contract that is optimal in the class of LRS contracts satisfying limited liability with equality, call it $w_i^\star$. This applies to any eligible contract, so that $V_i(w_i^\star, w_j) \geq V_i(\hat{w}_i, w_j)$.

Alternatively, any ineligible contract, $\tilde{w}_i$, satisfies $V_i(\tilde{w}_i, w_j) \leq w_j(0, 0) = V_i(-w_j, w_j) \leq V_i(w_i^\star, w_j)$, where the first inequality follows from Definition 5, the equality from the remark above, and the inequality from the fact that $w_i = -w_j$ is a LRS contract satisfying limited liability with equality with $\alpha_i = 0$.

It follows that $w_i^\star$ weakly dominates any eligible or ineligible contract, so that $w_i^\star \in \mathrm{BR}_i(w_j)$.

Finally, we turn to the second clause of the theorem. Consider a contract w_j by the competing principal. Suppose that there exists an eligible contract, then any contract in the best response is eligible. Let w_i be an optimal contract for principal i. Define a LRS contract $w_i^{'}$ as in Lemma 4 with respect to the eligible contract scheme $w \equiv (w_i, w_j)$. The contract scheme $w^{'} \equiv \left(w_i^{'}, w_j\right)$ satisfies:

$$E_F\left[y_i - w_i^{'}(y)\right] \geq k + \frac{1-\alpha}{\alpha} V_A\left(w^{'}|\mathcal{A}_0\right) = V_i(w) + \frac{1-\alpha}{\alpha}\left(V_A\left(w^{'}|\mathcal{A}_0\right) - V_A(w|\mathcal{A}_0)\right) \geq V_i(w), \tag{A.31}$$

where the equality follows from replacing for k using equation (A.11) from Lemma 3, and the second to last inequality from the fact that $V_A\left(w^{'}|\mathcal{A}_0\right) \geq V_A(w|\mathcal{A}_0)$, with strict inequality unless $w_i^{'}$ is identical to w_i. To see this, recall that $w_i^{'}(y) \geq w_i(y)$ for all $y \in Y$ from Lemma 3 and that $\mathcal{A}_0$ satisfies the full support property, Assumption 6.

Equation (A.31) holds for all F. It follows that $V_i\left(w_i^{'}, w_j\right) \geq V_i(w)$, with strict inequality when w_i is not identical to $w_i^{'}$. Then $w_i = w_i^{'}$, LRS contracts, or else optimality would be contradicted. □

A.2 Principals' best response under collusion

The proof of Theorem 3 follows from Theorem 1 in Carroll (2015). We proceed in a similar way as in the proof of Theorem 1 in Appendix A.1 by establishing a series of Lemmas that allow us to apply Carroll (2015)'s result. Lemma 1 applies unchanged to the collusion setting because it depends on the aggregate contract the agent faces (which is $w_1(y) + w_2(y)$ when principals compete and $w(y)$ when they collude). Lemmas 2 and 3 also have analogues for collusion that we state below. The proofs of these lemmas remains unchanged after taking into account the change in payoffs.

Lemma 7. *Let w be an eligible contract then*

$$V_P(w) = \min_{F\in\mathcal{F}} E_F[y_1 + y_2 - w(y)].$$

Moreover, if $F \in \underset{F\in\mathcal{F}}{argmin} E_F[y_1 + y_2 - w(y)]$ then $E_F[w(y)] = V_A(w|\mathcal{A}_0)$.

Lemma 8. *Let w be an eligible contract. There exits $k \in \mathbb{R}$ and $\theta_c \in (0,1]$ such that*

$$\begin{aligned} w(y) &\leq (1-\theta_c)(y_1+y_2) - (\theta_c - 1)k && \text{(A.32)} \\ V_P(w) &= k + \frac{\theta_c}{1-\theta_c} V_A(w|\mathcal{A}_0) && \text{(A.33)} \end{aligned}$$

With Lemmas 7 and 8, we can use the framework developed in Carroll (2015) to obtain:

Theorem 3. *The optimal contract under collusion is*

$$w_c(y) = (1-\theta_c)(y_1+y_2), \tag{A.34}$$

where the share θ_c satisfies $1 - \theta_c = \sqrt{c^\star / E_{F^\star}[y_1+y_2]}$ for an action $(F^\star, c^\star) \in \underset{(F,c)\in\mathcal{A}_0}{argmax} \left\{ \left(\sqrt{E_F[y_1+y_2]} - \sqrt{c} \right)^2 \right\}$. Moreover, for any contract of the form $w(y) = (1-\theta)(y_1+y_2)$ that guarantees a positive payoff, V_P can be expressed as:

$$V_P(w|\mathcal{A}_0) = \frac{\theta}{1-\theta} \max_{(F,c)\in\mathcal{A}_0} \{(1-\theta) E_F[y_1+y_2] - c\}. \tag{A.35}$$

Finally, If $\mathcal{A}_0$ satisfies assumption 6, then all optimal contracts satisfy (16).

Proof. The results follows from Lemmas 7 and 8 along with Lemmas 2, 4, 5 and 6 in Carroll (2015). The argument is the same as in Carroll (2015, Thm. 1) replacing his y for $y_1 + y_2$.

□

B Existence of Nash equilibria

We start by providing a proof of Proposition 1 that characterizes LRS contract schemes and their payoffs.

Proposition 1. *Let w be a LRS contract scheme satisfying Assumption 4 with equality. There exist $\{\theta_i, k_i\}_{i\in\{1,2\}}$ such that contracts satisfy*

$$w_i(y) = (1-\theta_i)\, y_i - \theta_i y_j - k_i, \text{ for } i \in \{1,2\}, \; j \neq i, \tag{B.1}$$

where $k_1 = -k_2$ and $\theta_i \in [0, 1-\theta_j]$. Moreover, principal i's guaranteed payoff satisfies

$$V_i(w) = \theta_i \max_{(F,c)\in\mathcal{A}_0} \left\{ E_F[y_1 + y_2] - \frac{c}{1-\theta_1-\theta_2} \right\} - k_i. \tag{B.2}$$

Proof. Consider a contract scheme (w_1, w_2) such that, given w_j, the contract w_i satisfies definition 2 for $i \in \{1,2\}$ and $j \neq i$. Then, there exist shares $\alpha_1, \alpha_2 \in [0,1]$ and constants $k_1, k_2 \in \mathbb{R}$ such that:

$$w_i(y) = \alpha_i y_i - (1-\alpha_i) w_j(y) - \alpha_i k_i, \; i \in \{1,2\}, j \neq i. \tag{B.3}$$

The aggregate contract offered to the agent is thus

$$w_1(y) + w_2(y) = \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2-\alpha_1\alpha_2}(y_1 + y_2 - k_1 - k_2). \tag{B.4}$$

We arrive at (8) by defining $\theta_i \equiv \frac{(1-\alpha_i)\alpha_j}{\alpha_1+\alpha_2-\alpha_1\alpha_2} \in [0, 1-\theta_j]$ and solving the system of equations formed by (B.3) for $i \in \{1,2\}$.

The relationship between the constants k_1 and k_2 follows from satisfying Assumption 4 with equality, which implies that $k_1 = -k_2$. Recall that $\min_{y\in Y} y_i = 0$ for $i \in \{1,2\}$, so limited liability requires $w_1(0,0) + w_2(0,0) = 0$.

We simplify the expression for the aggregate contract offered to the agent to:

$$w_1(y) + w_2(y) = (1-\theta_1-\theta_2)(y_1 + y_2). \tag{B.5}$$

The ex-post payoff of principal i follows from replacing w_i as in (B.3). The principals' guaranteed payoffs are obtained using equation (A.29) and Lemma 5.

□

Proposition 2. *A pure strategy Nash equilibrium in LRS contracts satisfying Assumption 4 with equality is characterized by pairs of shares (θ_1, θ_2) and transfers (k_1, k_2), such that, for $i \in \{1,2\}$, the share θ_i satisfies*

$$(1-\theta_1-\theta_2)^2 = \frac{(1-\theta_j)\, c_i}{E_{F_i}[y_1 + y_2]} \tag{B.6}$$

for an action

$$(F_i, c_i) \in \underset{(F,c)\in\mathcal{A}_0^i}{\mathit{argmax}} \left\{ \left(\sqrt{(1-\theta_j)\, E_F\left[y_1+y_2\right]} - \sqrt{c} \right)^2 \right\}, \tag{B.7}$$

and transfers satisfy $k_1 = -k_2$*. The equilibrium contracts satisfy* (8) *and the principals' guaranteed payoffs are*

$$V_i\left(w\right) = \left(\sqrt{(1-\theta_j)\, E_{F_i}\left[y_1+y_2\right]} - \sqrt{c_i} \right)^2, \; i \in \{1,2\}\,,\; j \neq i. \tag{B.8}$$

Proof. Consider a LRS contract scheme w satisfying Assumption 4 with equality. The contracts in w are characterized by two shares (θ_1, θ_2) and two transfers (k_1, k_2) satisfying $k_1 = -k_2$ (Proposition 1). Moreover, The values of θ_1 and θ_2 can be computed separately from those of k_1 and k_2 because of the effect of k_i on the payoff of principal i is independent of θ_i (see 9).

For w to be a Nash Equilibrium of the common agency game (Definition 1), the contract of each principal must maximize their guaranteed payoff taking the competing contract as given. From (9), this amounts to the share of principal i, θ_i, solving

$$\max_{(F,c)\in\mathcal{A}_0^i} \max_{\theta\in[0,1-\theta_j]} \left\{ \theta E_F\left[y_1+y_2\right] - \frac{\theta}{1-\theta-\theta_j} c \right\}. \tag{B.9}$$

For a fixed $(F_i, c_i) \in \mathcal{A}_0$, the solution to the inner max in (B.9) is characterized by:

$$(1-\theta_i-\theta_j)^2 = \frac{(1-\theta_j)\, c_i}{E_{F_i}\left[y_1+y_2\right]},\;\; 0 \leq \theta_i,\;\; \theta_j \leq 1. \tag{B.10}$$

When both principals satisfy (B.10) we have

$$1-\theta_1-\theta_2 = \sqrt{\frac{(1-\theta_j)\, c_i}{E_{F_i}\left[y_1+y_2\right]}} = \sqrt{\frac{(1-\theta_i)\, c_j}{E_{F_j}\left[y_1+y_2\right]}}. \tag{B.11}$$

We obtain (F_i, c_i) by replacing (B.10) in (B.9) which gives the rest of the result

$$(F_i, c_i) \in \underset{(F,c)\in\mathcal{A}_0^i}{\mathrm{argmax}} \left\{ \left(\sqrt{(1-\theta_j)\, E_F\left[y_1+y_2\right]} - \sqrt{c} \right)^2 \right\}. \tag{B.12}$$

□

Proposition 1 implies that we can represent the principal's choices over LRS contracts as choices over their share of output, θ_i. Similarly, the guaranteed payoffs depend on the principal's choices only through the shares $\{\theta_1, \theta_2\}$, so we write $V_i\left(\theta_i, \theta_j\right)$ for principal i's guaranteed payoff. As mentioned in the main text, doing this makes our framework into a *quasi-game* in the sense of Debreu (1952). Before proving the existence of a Nash Equilibrium in pure strategies for the common agency game, we state a formal definition of the game.

Definition 6. (Quasi-Game) Consider a game with three players, the agent (A), and principals 1 and 2. We denote the players by the subscript, $i = A, 1, 2$. The agent's action set is $\Gamma_A \subset \Delta(Y) \times \mathbb{R}_+$. Each principal's action set is $\Gamma_i \equiv \{w : Y \to \mathbb{R} | \, w \text{ is continuous}\}$, $i \in \{1,2\}$, the set of continuous functions mapping Y into $\mathbb{R}$. The 3-tuple of actions, $a = (a_A, a_1, a_2)$, is an element of $\Gamma = \Gamma_A \times \Gamma_1 \times \Gamma_2$. Player i's payoff is a function $v_i : \Gamma \to \mathbb{R}$. For each player i we define the set of other players' actions: $\overline{\Gamma}_A = \Gamma_1 \times \Gamma_2$ and $\overline{\Gamma}_i = \mathcal{A} \times \Gamma_j$ for $i \in \{1,2\}$ and $j \neq it$, with typical elements $\overline{a}_A = (a_1, a_2) \in \overline{\Gamma}_A$, $\overline{a}_1 = (a_A, w_2) \in \overline{\Gamma}_1$, and $\overline{a}_2 = (a_A, w_1) \in \overline{\Gamma}_2$. Given $\overline{a}_i$ (the actions of all other players except player i), player i's choice is restricted to a non-empty set $S_i(\overline{a}_i) \subseteq \Gamma_i$; for $i \in \{1,2\}$ this is $S_i(\overline{a}_i) = \{w_i \in \Gamma_i | w_1(y) + w_2(y) \geq 0 \, \forall \, y \in Y\}$. Player i chooses a_i in $S_i(\overline{a}_i)$ so as to maximize $v_i(\overline{a}_i, a_i)$. Following Debreu (1952), $a^\star$ is an equilibrium point if for all $i \in \{A,1,2\}$, $a_i^\star \in S_i(\overline{a}_i^\star)$ and $v_i(a^\star) = \max_{a_i \in S_i(\overline{a}_i^\star)} v_i(\overline{a}_i^\star, a_i)$.

We now provide the proof of Theorem 2.

Theorem 2. *A pure strategy Nash Equilibrium in LRS contracts with $\theta_1, \theta_2 > 0$ exists.*

Proof. Consider a LRS contract scheme w satisfying Assumption 4 with equality. The contracts in w are characterized by two shares (θ_1, θ_2) and two transfers (k_1, k_2) satisfying $k_1 = -k_2$ (Proposition 1). The principals' guaranteed payoffs are as in (9).

A pure strategy Nash Equilibrium in LRS contracts is then a pair of values for (θ_1, θ_2), such that $\theta_i \in [0, 1-\theta_j]$ maximizes principal's i guaranteed payoff taking θ_j as given, along with any pair of transfers (k_1, k_2) satisfying $k_1 = -k_2$.

To prove the existence of an equilibrium, we follow Monderer and Shapley (1996) and construct an ordinal potential function for the game:

$$P(\theta_1, \theta_2) = \theta_1 \theta_2 G(\theta_1 + \theta_2), \tag{B.13}$$

where we define $G : \mathbb{R}_+ \to \mathbb{R}$ as follows:

$$G(x) \equiv \begin{cases} \frac{1}{1-x} \max_{(F,c) \in \mathcal{A}_0} \{(1-x) E_F[y_1 + y_2] - c\} & \text{if } x < 1 \\ 0 & \text{if } x \geq 1 \end{cases}. \tag{B.14}$$

G is continuous.

Given some transfers (k_1, k_2) satisfying $k_1 = -k_2$, the function P is an ordinal potential for the game for shares $\theta_1, \theta_2 > 0$ because the function P induces the same order over θ_i as the function V_i, that is for all $\theta_j > 0$ and $\theta, \theta' \in [0,1]$:

$$V_i(w(\theta, \theta_j, k_1, k_2)) - V_i\left(w\left(\theta', \theta_j, k_1, k_2\right)\right) > 0 \quad \Longleftrightarrow \quad P(\theta, \theta_j) - P\left(\theta', \theta_j\right) > 0. \tag{B.15}$$

where $w(\theta, \theta_j, k_i, k_j)$ is a LRS contract scheme. The result follows from manipulating (9). Maximizing the principals' guaranteed payoffs is equivalent to solving

$$\tilde{V}_i(\theta, \theta_j) \equiv \max_{\theta \in [0, \theta_j]} \theta G(\theta + \theta_j). \tag{B.16}$$

Thus, (B.15) implies that any maximum of P such that $\theta_1, \theta_2 > 0$ is a pure strategy equilibrium of the common agency game.

We end the proof by verifying that such a maximum is achieved. P attains a maximum in $[0,1]^2$ by Weierstrass' theorem. Assumption 3 ensures that there exist $\theta_1, \theta_2 > 0$ such that $G(\theta_1 + \theta_2) > 0$

(and thus that $P(\theta_1, \theta_2) > 0$). This follows from there being an action in $\mathcal{A}_0$ that generates enough (expected) output to cover the cost of production. Then, all $(\theta_1^\star, \theta_2^\star) \in \text{argmax}_{(\theta_1,\theta_2)\in[0,1]^2} P(\theta_1, \theta_2)$ satisfy $\theta_1^\star, \theta_2^\star > 0$. All these pairs, along with the transfers k_1 and k_2 are taken as given above, characterize contracts that are Nash equilibria of the common agency game.

If Assumption 3 is violated then it is not possible to induce the agent to produce and the game has a trivial solution. An equilibrium still exists. For instance, it is a best response for both principals to set $\theta_i = 0$, implying $\tilde{V}_i = 0$.

$\square$

C Relative efficiency of Nash and collusion contracts

Proposition 3. *Let w and $w^{'}$ be contract schemes such that the agent receives a share of total output given by θ_A and $\theta^{'}_A$ respectively. Total expected and guaranteed surplus are weakly increasing in the share of total output going to the agent. That is, for any known action set $\mathcal{A}_0$ and action set $\mathcal{A} \supseteq \mathcal{A}_0$, let $s_{TES} \in TES(w|\mathcal{A})$, and $s^{'}_{TES} \in TES\left(w^{'}|\mathcal{A}\right)$. If $\theta_A < \theta^{'}_A \leq 1$ then $s_{TES} \leq s^{'}_{TES}$ and $TGS(w) \leq TGS\left(w^{'}\right)$.*

Proof. Let w and $w^{'}$ be contract schemes such that the agent receives a share of total output given by θ_A and $\theta^{'}_A$ respectively, with $\theta_A < \theta^{'}_A$.

We first prove that total expected surplus is increasing in θ_A. Consider $(F, c) \in A^{\star}(w|\mathcal{A})$ and $\left(F^{'}, c^{'}\right) \in A^{\star}\left(w^{'}|\mathcal{A}\right)$, then

$$\tilde{V}_A(\theta_A|\mathcal{A}) = \theta_A E_F[y_1 + y_2] - c < \theta^{'}_A E_F[y_1 + y_2] - c \leq \theta^{'}_A E_{F'}[y_1 + y_2] - c^{'} = \tilde{V}_A\left(\theta^{'}_A|\mathcal{A}\right). \quad \text{(C.1)}$$

The first inequality follows from $\theta_A < \theta^{'}_A$ and the second one from (F, c) being feasible at $\theta^{'}_A$.

Furthermore, $E_{F'}[y_1 + y_2] \geq E_F[y_1 + y_2]$, otherwise $(F, c) \notin A^{\star}(\theta_A|\mathcal{A})$. To see this, note that if $E_{F'}[y_1 + y_2] < E_F[y_1 + y_2]$, from the second inequality in (C.1) we get

$$c - c^{'} \geq \theta^{'}_A\left(E_F[y_1 + y_2] - E_{F'}[y_1 + y_2]\right). \quad \text{(C.2)}$$

Then, because $\theta_A < \theta^{'}_A$ and we have assumed that $E_F[y_1 + y_2] - E_{F'}[y_1 + y_2] > 0$, we can write

$$c - c^{'} \geq \theta_A\left(E_F[y_1 + y_2] - E_{F'}[y_1 + y_2]\right). \quad \text{(C.3)}$$

Rearranging gives

$$\theta_A E_{F'}[y_1 + y_2] - c^{'} \geq \theta_A E_F[y_1 + y_2] - c. \quad \text{(C.4)}$$

This violates $(F, c) \notin A^{\star}(\theta_A|\mathcal{A})$ because (F, c) would provide a higher payoff.

Finally, using the second inequality in (C.1) we have

$$c^{'} - c \leq \theta^{'}_A\left[E_{F'}[y_1 + y_2] - E_F[y_1 + y_2]\right]; \quad \text{(C.5)}$$

$$c^{'} - c \leq E_{F'}[y_1 + y_2] - E_F[y_1 + y_2]; \quad \text{(C.6)}$$

$$E_F[y_1 + y_2] - c \leq E_{F'}[y_1 + y_2] - c^{'}. \quad \text{(C.7)}$$

The inequality holds for arbitrary actions in the agent's best response, which proves the monotonicity of expected total expected surplus on θ_A.

We now prove that total guaranteed surplus is increasing in θ_A. Consider $(F, c) \in A^{\star}(w|\mathcal{A}_0)$ and $\left(F^{'}, c^{'}\right) \in A^{\star}\left(w^{'}|\mathcal{A}_0\right)$, then:

$$\theta^{'}_A E_F[y_1 + y_2] - c \leq \theta^{'}_A E_{F'}[y_1 + y_2] - c' \quad \text{(C.8)}$$

by optimality of the agent. From (22) we have:

$$\text{TGS}\,(w) = E_F\,[y_1 + y_2] - \frac{1}{\theta_A} c < E_F\,[y_1 + y_2] - \frac{1}{\theta_A^{'}} c \leq E_{F'}\,[y_1 + y_2] - \frac{1}{\theta_A^{'}} c^{'} = \text{TGS}\left(w^{'}\right). \quad \text{(C.9)}$$

The inequalities follow from $\theta_A < \theta_A^{'}$ and (C.8), respectively.

□

Proposition 4. *Let w and $w^{'}$ be LRS contract schemes such that principal j's shares of total output satisfy $\theta_j < \theta_j^{'}$ and principal i's contracts are best responses to principal j's contracts (i.e., w_i and $w_i^{'}$ attain (9) for w_j and $w_j^{'}$ respectively). Then, the agent's shares of total output satisfy $1 - \theta_1 - \theta_2 \geq 1 - \theta_1^{'} - \theta_2^{'}$.*

Proof. Suppose for a contradiction that the best response of principal i implies a higher share for the agent when responding to $w_j^{'}$ than when responding to w_j: $1 - \theta_i - \theta_j < 1 - \theta_i^{'} - \theta_j^{'}$.

Because w_i is a best response to w_j, principal i's payoff under θ_i is at least as high as under any other share, given fixed transfers (k_1, k_2). Consider an alternative share for principal i: $\tilde{\theta}_i \equiv \theta_i^{'} - \left(\theta_j - \theta_j^{'}\right)$. This alternative share implies that the share of the agent is the same as under $w^{'}$, that is $1 - \tilde{\theta}_i - \theta_j = 1 - \theta_i^{'} - \theta_j^{'}$. It must be that:

$$\theta_i \left(E_F\,[y_1 + y_2] - \frac{c}{1 - \theta_i - \theta_j} \right) \geq \tilde{\theta}_i \left(E_{F'}\,[y_1 + y_2] - \frac{c^{'}}{1 - \theta_i^{'} - \theta_j^{'}} \right). \quad \text{(C.10)}$$

Where $(F, c) \in A^\star\,(w|\mathcal{A}_0)$ and $\left(F^{'}, c^{'}\right) \in A^\star\left(w^{'}|\mathcal{A}_0\right)$. The pair (F, c) is determined by the agent's problem and thus depends only on the share of the agent.

Similarly, $w_i^{'}$ is a best response to $w_j^{'}$ and we can consider an alternative share for principal i: $\tilde{\theta}_i^{'} \equiv \theta_i - \left(\theta_j^{'} - \theta_j\right)$. As before, this alternative share implies that the share of the agent is $1 - \theta_i - \theta_j$. It must be that:

$$\theta_i^{'} \left(E_{F'}\,[y_1 + y_2] - \frac{c^{'}}{1 - \theta_i^{'} - \theta_j^{'}} \right) \geq \tilde{\theta}_i^{'} \left(E_F\,[y_1 + y_2] - \frac{c}{1 - \theta_i - \theta_j} \right). \quad \text{(C.11)}$$

By subtracting these inequalities we get:

$$\left(\theta_i - \tilde{\theta}_i^{'}\right) \left(E_F\,[y_1 + y_2] - \frac{c}{1 - \theta_i - \theta_j} \right) \geq \left(\tilde{\theta}_i - \theta_i^{'}\right) \left(E_F\,[y_1 + y_2] - \frac{c}{1 - \theta_i^{'} - \theta_j^{'}} \right). \quad \text{(C.12)}$$

Using the definitions of $\tilde{\theta}_i$ and $\tilde{\theta}_i^{'}$ we obtain:

$$\theta_i - \tilde{\theta}_i^{'} = \tilde{\theta}_i - \theta_i^{'} = \theta_j^{'} - \theta_j > 0, \quad \text{(C.13)}$$

where the final inequality holds by assumption of the proposition. Then we can write

$$\left(\theta_j^{'} - \theta_j\right)\left(E_F\left[y_1 + y_2\right] - \frac{c}{1 - \theta_i - \theta_j}\right) \geq \left(\theta_j^{'} - \theta_j\right)\left(E_F\left[y_1 + y_2\right] - \frac{c}{1 - \theta_i^{'} - \theta_j^{'}}\right); \tag{C.14}$$

$$\max_{(F,c)\in\mathcal{A}_0}\left\{E_F\left[y_1 + y_2\right] - \frac{c}{1 - \theta_i - \theta_j}\right\} \geq \max_{(F,c)\in\mathcal{A}_0}\left\{E_F\left[y_1 + y_2\right] - \frac{c}{1 - \theta_i^{'} - \theta_j^{'}}\right\}; \tag{C.15}$$

$$\mathrm{TGS}\left(1 - \theta_i - \theta_j\right) \geq \mathrm{TGS}\left(1 - \theta_i^{'} - \theta_j^{'}\right). \tag{C.16}$$

This contradicts Proposition 3 because $1 - \theta_i - \theta_j < 1 - \theta_i^{'} - \theta_j^{'}$.

□

Theorem 4. *Let w be a Nash equilibrium in LRS contracts and w_c be an optimal collusion contract. Total expected and guaranteed surplus are weakly higher under the collusive contract. That is, for any known action set $\mathcal{A}_0$ and action set $\mathcal{A} \supseteq \mathcal{A}_0$, and surpluses $s_{TES}^N \in TES(w|\mathcal{A})$ and $s_{TES}^C \in TES(w_c|\mathcal{A})$,, it holds that $s_{TES}^N \leq s_{TES}^C$ and $TGS(w) \leq TGS(w_c)$*

Proof. Under collusion, the agent's share of output ($\theta_A = 1 - \theta_c$) satisfies (16) as in Theorem 3. This share is also a solution the problem of a principal (i) under competition facing $\theta_j = 0$, see equation (9) in Proposition 1. Proposition 4 establishes that the share of the agent implied by the optimization of principal i is decreasing in the share of the competing principal. From 2 we know that in a Nash equilibrium in LRS contracts $\theta_1, \theta_2 > 0$, so it must be that the share of output going to the agent is lower in any Nash equilibrium in LRS contracts than under collusion. Proposition 3 gives the result.

□

D Individual limited liability

D.1 Principals' best response under Assumption 5

The results obtained in Appendix A.1 under Assumption 4 apply with almost no changes under Assumption 5. We now derive the parallel result to Theorem 1 through a series of lemmas following the arguments Appendix A.1.

First we adapt the definition of eligibility to reflect the change in limited liability. Eligible contracts are those that outperform the "lowest" contract the principal can offer. That is, the zero contract, i.e., $w_i(y) = 0$. This contract makes the ex-post payoff of the principal equal to y_i, and their guaranteed payoff equal to 0 (corresponding to the worst case scenario in which the agent can reduce their cost by not producing for principal i). This provides us with a lower bound on the payoff of the principal, which we use to define eligible contracts.

Definition 7. (Eligible Contracts) A contract w_i is eligible for principal i if it satisfies:

$$V_i(w) > 0. \tag{D.1}$$

The results in Lemmas 1, 2, and 3 apply without changes, even though the proofs are slightly altered to account for the change in limited liability. In the proof of Lemmas 2 and 3, the value of the principal satisfies $V_i(w) = -w_i(0,0) \leq 0$ rather than $V_i(w) = -w_i(0,0) \leq w_j(0,0)$ when the agent takes action $(\delta_0, 0)$. In both cases the inequality follows from limited liability and implies a violation of the eligibility of the contract (Definition 7).

Then, for any contract scheme w, we can then construct an alternative contract for principal i of the LRS form (Definition 2) that satisfies individual limited liability. This result parallels Lemma 4 is only adjusted to account for the change in limited liability in Assumption 5.

Lemma 9. *Let w be an eligible contract scheme. There exist $k \in \mathbb{R}$ and $\alpha \in (0,1]$ such that the contract*

$$w_i^{'}(y) = \alpha y_i - (1-\alpha)\, w_j(y) - \alpha k \tag{D.2}$$

satisfies the limited liability condition in Assumption 5 with equality and $V_i\left(w_i^{'}, w_j\right) \geq V_i(w)$.

Proof. From Lemma 3, there are $k \in \mathbb{R}$ and $\alpha \in (0,1]$ so that w_i satisfies equations (A.10) and (A.11). Use the same α and k to define an alternative contract $w_i^{''}$ as

$$w_i^{''}(y) = \alpha y_i - (1-\alpha)\, w_j(y) - \alpha k, \tag{D.3}$$

just as in Lemma 4. This contract gives principal i a (weakly) higher payoff, $V_i\left(w_i^{''}, w_j\right) \geq V_i(w)$. The proof has no changes relative to that of Lemma 4.

It is only left to handle limited liability. Recall that the original contract satisfies Assumption 5, $w_i(y) \geq 0$ for all $y \in Y$. Then the new contract $w_i^{''}$ also satisfies Assumption 5 because $w_i^{''}(y) \geq w_i(y) > 0$ for all $y \in Y$, where the first inequality comes from equation (A.10). The alternative contract can be modified by subtracting a constant, making it satisfy limited liability with equality, weakly increasing the principal's payoff. We define $w_i^{'}$ as:

$$w_i^{'}(y) = w_i^{''}(y) - \min_{y \in Y} \left\{ w_i^{''}(y) \right\} \tag{D.4}$$

This new contract (w_i') can be written as in (D.2) by appropriately redefining k.

□

As in Appendix A.1, Lemma 9 allows us to establish an affine link between the principal and the agent's guaranteed payoffs. The result in Lemma 5 applies without changes. Finally, we can use these results to establish the existence of an optimal LRS contract among those satisfying Assumption 5.

Lemma 10. *In the class of LRS contracts that satisfy the limited liability (Assumption 5) with equality there exists an optimal one for principal i.*

Proof. From Lemma 5 we can express $V_i(w)$ directly as a function of α_i as in (A.29). For w_i to satisfy Assumption 5 with equality it must be that $k(\alpha) = \min_y \left[y_i - \frac{1-\alpha}{\alpha} w_j(y) \right]$, which is continuous in α for a given w_j. Moreover, the function $(1-\alpha_i) E_F [y_i + w_j(y)] - \frac{1-\alpha_i}{\alpha_i} c$ is also continuous in α_i, thus its maximum over $\mathcal{A}_0$ is continuous in α_i as well. Continuity implies that the right-hand-side of equation (A.29) is continuous in α_i it achieves a maximum in $[0,1]$. This α_i gives the optimal guarantee over all LRS contracts that satisfy limited liability (Assumption 5) with equality.

□

We can now state the main result of this Appendix establishing the optimality of LRS contracts under Assumption 5.

Theorem 7. *For any contract w_j, there exists a LRS contract $\overline{w}_i$ such that $\overline{w}_i \in BR_i(w_j)$ and $\min_{y \in Y} \overline{w}_i(y) = 0$ for $i \in \{1,2\}$. That is, there is always a LRS contract that is robust for principal i. Moreover, if $\mathcal{A}_0$ satisfies Assumption 6, any robust contract is a LRS contract or $\max_{w_i} V_i(w_i, w_j) = 0$.*

Proof. Consider a contract w_j by the competing principal. By Lemma 9 any eligible contract, $\hat{w}_i$, is weakly dominated by a LRS contract satisfying limited liability with equality. By Lemma 10 there is a contract that is optimal in the class of LRS contracts satisfying limited liability with equality, call it $w_i^\star$. This applies to any eligible contract, so that $V_i(w_i^\star, w_j) \geq V_i(\hat{w}_i, w_j)$.

Alternatively, any ineligible contract, $\tilde{w}_i$, satisfies $V_i(\tilde{w}_i, w_j) \leq 0 = V_i(0, w_j) \leq V_i(w_i^\star, w_j)$, where the first inequality follows from Definition 7, the equality from the remark above, and the inequality from the fact that $w_i = 0$ is a LRS contract satisfying Assumption 5 with equality with $\alpha_i = k_i = 0$.

It follows that $w_i^\star$ weakly dominates any eligible or ineligible contract, so that $w_i^\star \in \mathrm{BR}_i(w_j)$.

The proof of the second clause of the theorem is the same as in Theorem 1.

□

D.2 Nash equilibria under Assumption 5

We focus on characterizing equilibria in LRS contracts as in Section 3. The main difference with our previous results is that individual limited liability pins down the transfers of the principals to the agent (k_1, k_2). Proposition 5 states the result (which parallels that of Proposition 1)

Proposition 5. *Let w be a LRS contract scheme satisfying Assumption 5 with equality. There exist $\theta_1, \theta_2 \in [0,1]$ such that contracts satisfy*

$$w_i(y) = (1-\theta_i)\, y_i + \theta_i \left(\overline{y}_j - y_j\right), \tag{D.5}$$

where $\theta_i \in [0, 1-\theta_j]$. Moreover, principal i's guaranteed payoff satisfies

$$V_i(w) = \theta_i \max_{(F,c)\in\mathcal{A}_0} \left\{ E_F[y_1 + y_2] - \frac{c}{1-\theta_1-\theta_2} \right\} - \theta_i \overline{y}_j. \tag{D.6}$$

Proof. Consider a contract scheme (w_1, w_2) such that, given w_j, the contract w_i satisfies definition 2 for $i \in \{1,2\}$ and $j \neq i$. Then, there exist shares $\alpha_1, \alpha_2 \in [0,1]$ and constants $k_1, k_2 \in \mathbb{R}$ such that:

$$w_i(y) = \alpha_i y_i - (1-\alpha_i) w_j(y) - \alpha_i k_i, \;\; i \in \{1,2\}, j \neq i. \tag{D.7}$$

Solving the system of equations formed by (D.7) for $i \in \{1,2\}$ and defining $\theta_i \equiv \frac{(1-\alpha_i)\alpha_j}{\alpha_1+\alpha_2-\alpha_1\alpha_2} \in [0, 1-\theta_j]$ we arrive at

$$w_i(y) = (1-\theta_i)\, y_i - \theta_i (y_j - k_i - k_j) - k_i. \tag{D.8}$$

The resulting contract is increasing in y_i and decreasing in y_j, so $\min w_i(y)$ is attained at $y_i = 0$ and $y_j = \overline{y}_j$.[24] In order to satisfy Assumption 5 it must be that $\min w(y) = w\left(0, \overline{y}_j\right) = 0$. This implies $k_i = -\frac{\theta_i}{1-\theta_1-\theta_2}\left((1-\theta_j)\,\overline{y}_j + \theta_j \overline{y}_i\right)$. Replacing for k_1 and k_2 we get (25).

The aggregate contract faced by the agent is: $w_1(y) + w_2(y) = (1-\theta_1-\theta_2)(y_1+y_2) + \theta_1\overline{y}_2 + \theta_2\overline{y}_1$. The ex-post payoff of principal i follows from replacing w_i as in (D.7). The principals' guaranteed payoffs are obtained using equation (A.29) and Lemma 5.

□

Before proceeding, it is useful to understand the characteristics of the game that induce a principal to offer high powered versus low powered incentives. A lower θ gives the principal a lower share of total output, and, all else equal increases the share of the agent. It also reduces the fee that the principal pays. Hence incentives are 'high powered'. Conversely a higher θ gives the agent a smaller share of output, and it increases the fee the principal pays to the agent. Hence incentives are 'low powered'.[25] This allows for understanding the effect

[24] The points $(0, \overline{y}_2)$ and $(\overline{y}_1, 0)$ are in Y by the assumption that Y is a cross product. This assumption is not a necessary one, and is just convenient for determining the values of (y_1, y_2) for which $\min_y w_i(y)$ is attained. If the assumption is lifted only the constants k_1 and k_2 are directly affected. For instance if output is perfectly and positively correlated $\min w_i(y)$ is attained when $y_1 = y_2 = 0$ and $k_1 = k_2 = 0$.

[25] The payment of fees to the agent implies that the ex post payoffs of the principals can be negative. Yet, our results do not rely on the ability of principals to make unbounded payments to the agent. In the online appendix we augment the model by adding limited liability on the principal's side.

of competition and productivity on the use of 'high powered' incentives by simply analyzing how they affect the share of output θ.

We now provide sufficient conditions for the existence of a pure strategy Nash equilibrium in LRS contracts under Assumption 5.

The first condition in Theorem 8 allows us to use the potential approach of Theorem 2 by making the principals' contracts in (25), and payoffs in (D.6), symmetric. The condition is not overly restrictive. Only the maximum output that can be produced is required to be the same across principals, leaving the rest of the output space unconstrained and imposing no constraints on the agent's known actions. For instance, the agent can be known to favor production for one of the principals, or one of the principals can have just extreme realizations of output (only high and low values of y_i in Y_i). The second condition ensures that the principals' best responses are single valued by making the agent's (implied) cost function convex enough. Single-valuedness of the principals' best responses is sufficient to ensure existence of a pure strategy Nash equilibrium.

We can also show existence of equilibrium outside of the conditions in Theorem 8 for several special cases. For instance, when the cost function is linear in expected total output, or when the agent is indifferent between actions.[26] The latter condition is well suited to describe situations such as auctions or lobbying, where the agent is expected not to have preferences over actions.

Theorem 8. *A pure strategy Nash equilibrium in LRS contracts that satisfy Assumption 5 with equality for some $y \in Y$ exists if either of the following conditions hold:*

i. ***(Symmetry)*** *The output space is such that* $\max\{Y_1\} = \max\{Y_2\} = \overline{y}$.

ii. ***(Convexity of $\mathcal{A}_0$)*** *The known action set $\mathcal{A}_0$ satisfies the following properties:*

a) The projection of $\mathcal{A}_0$ onto $\Delta(Y)$, $\mathcal{F}_{\mathcal{A}_0} = \{F \in \Delta(Y) \mid (F,c) \in \mathcal{A}_0\}$, is convex and so is the set of expected total output that can be achieved under $\mathcal{A}_0$, $X_{\mathcal{A}_0} = \left\{x \in \mathbb{R} \mid \exists_{F \in \mathcal{F}_{\mathcal{A}_0}} x = E_F[y_1 + y_2]\right\}$.

b) The function $f : X_{\mathcal{A}_0} \to \mathbb{R}$ defined for each $x \in X_{\mathcal{A}_0}$ as

$$f(x) = \min\left\{c \mid \exists_{F \in \mathcal{F}_{\mathcal{A}_0}} (F,c) \in \mathcal{A}_0 \text{ and } E_F[y_1 + y_2] = x\right\} \tag{D.9}$$

is a continuous function and its square root is a convex function.

Proof. Consider the first condition in Theorem 8. As in the proof of Theorem 2, we show that the function $G : \mathbb{R}_+ \to \mathbb{R}$ defined in (B.14) can be used to construct an ordinal potential for the game.

We can use G to express the the guaranteed payoff of each principal i (V_i as in D.6) as

$$V_i(\theta_i, \theta_j) = \theta_i \left(G(\theta_1 + \theta_2) - \overline{y}\right). \tag{D.10}$$

[26] Bernheim and Whinston (1986a) also establish the existence of an equilibrium of the common agency game and show that it implements the efficient outcome for the case in which the agent is indifferent between actions. We reproduce their results under this condition in the online appendix.

Then, as in Monderer and Shapley (1996), we define an ordinal potential function P that satisfies (B.15) for $\theta_1, \theta_2 > 0$. That is,

$$P(\theta_1, \theta_2) = \theta_1 \theta_2 \left(G(\theta_1 + \theta_2) - \overline{y}\right). \tag{D.11}$$

For $\theta_j > 0$, the function P induces the same order over θ_i as the function V_i. Moreover, P attains a maximum in $[0,1]^2$. Any such maximum characterizes a pure strategy Nash equilibrium in LRS contract scheme that satisfies Assumption 5 with equality.

There are two cases to consider for the maximum of P.

1. There exists an action $(F, c) \in \mathcal{A}_0$ such that $E_F[y_1 + y_2] - c > \overline{y}$. Then, there is an action that generates enough (expected) surplus to cover the cost that the principals pay in fees. So, there exists $\theta_1, \theta_2 > 0$ such that $G(\theta_1 + \theta_2) - \overline{y} > 0$, which implies $P(\theta_1, \theta_2) > 0$. Then, the maximum of P on $[0,1]^2$ is not attained in the boundary, that is, for all $(\theta_1^\star, \theta_2^\star) \in \underset{(\theta_1,\theta_2)\in[0,1]^2}{\operatorname{argmax}} P(\theta_1, \theta_2)$, it holds that $\theta_1^\star, \theta_2^\star > 0$.

2. There is no action $(F, c) \in \mathcal{A}_0$ such that $E_F[y_1 + y_2] - c > \overline{y}$. Then the principals cannot guarantee themselves a positive payoff. A trivial equilibrium exists where $\theta_1 = \theta_2 = 0$.

Consider now the second condition in Theorem 8. From Proposition 5, only expected total output is relevant in determining payoffs for LRS contracts schemes. Hence, it is without loss to have the agent choose expected total output, x, and an associated cost, c. Naturally, if two actions have the same expected total output the agent will choose the one with lower cost. These actions form the lower envelope of the action set in the (x, c) space and imply the cost function of the agent given by (D.9).

When principals offer a LRS contract scheme we can cast the problem of principal i as that of choosing the share of output going to the agent $\theta_A \equiv 1 - \theta_1 - \theta_2$. The value of principal i in D.6 can be written as

$$V_i(w) = \max_{(F,c)\in\mathcal{A}_0} \max_{\theta_A\in[0,1-\theta_j]} \left\{ (1 - \theta_j - \theta_A)\left(E_F[y_j + y_i] - \overline{y}_j\right) - \frac{1 - \theta_j - \theta_A}{\theta_A} c \right\}. \tag{D.12}$$

Given an action (F, c), there is a unique solution for θ_A

$$\theta_A = \begin{cases} \sqrt{\frac{(1-\theta_j)c}{E_F[y_j+y_i]-\overline{y}_j}} & \text{if } (1-\theta_j)\left(E_F[y_j + y_i] - \overline{y}_j\right) \geq c; \\ 1 - \theta_j & \text{otherwise.} \end{cases} \tag{D.13}$$

Replacing back into the principal's guaranteed payoff and imposing (D.9) gives

$$V_i(w) = \max_{\left\{x | x = E_F[y_1+y_2]; F\in\mathcal{F}_{\mathcal{A}_0}\right\}} \left\{ \left(\max\left\{ \sqrt{(1-\theta_j)\left(x - \overline{y}_j\right)} - \sqrt{f(x)} \quad , \quad 0 \right\} \right)^2 \right\}, \tag{D.14}$$

where we consider choices over expected total output available in $\mathcal{A}_0$, with the relevant cost given by f. The continuity and convexity of $\sqrt{f}$ imply that $\sqrt{(1-\theta_j)\left(x - \overline{y}_j\right)} - \sqrt{f(x)}$ is continuous and strictly concave and hence admits a unique global maximum on the set $\tilde{x}(\theta_j) \in \{x | x = E_F[y_1 + y_2]; F \in \mathcal{F}_{\mathcal{A}_0}\}$. We define the argmax of V_i as: $x^\star(\theta_j) \equiv \max\{\tilde{x}(\theta_j), \underline{x}\}$, where $\underline{x}$ is the lowest value of x for which $\sqrt{(1-\theta_j)\left(x - \overline{y}_j\right)} - \sqrt{f(x)} = 0$, so that $V_i \geq 0$. The

Theorem of the Maximum implies that $x^{\star}(\theta_j)$ is a continuous function. The best response of principal i is then:

$$\theta_i = \mathrm{BR}_i(\theta_j) = (1-\theta_j) - \sqrt{\frac{(1-\theta_j) f(x^{\star}(\theta_j))}{x^{\star}(\theta_j) - \overline{y}_j}} \tag{D.15}$$

which is also a continuous function.

Finally, consider the function $g : [0,1]^2 \to [0,1]^2$ defined by

$$g(\theta_1, \theta_2) = (\mathrm{BR}_1(\theta_2), \mathrm{BR}_2(\theta_1)) \tag{D.16}$$

This function is continuous and maps a compact convex subset of an Euclidean space into itself. By Brouwer's fixed point theorem it has a fixed point. That is $(\theta_1^{\star}, \theta_2^{\star}) \in [0,1]^2$ such that $\theta_i^{\star} = \mathrm{BR}_i\left(\theta_j^{\star}\right)$ for $i \in \{1,2\}$, $j \neq i$. These shares define a LRS contract scheme that is an equilibrium of the game.

□

D.3 Efficiency under Assumption 5

Theorem 9. *Let w be a Nash equilibrium in LRS contracts satisfying Assumption 5 and w_c be an optimal collusion contract. Total expected and guaranteed surplus are weakly higher under the Nash Equilibrium contract scheme. That is, for any known action set $\mathcal{A}_0$ and action set $\mathcal{A} \supseteq \mathcal{A}_0$, and surpluses $s_{TES}^N \in TES(w|\mathcal{A})$ and $s_{TES}^C \in TES(w_c|\mathcal{A})$, it holds that $s_{TES}^C \leq s_{TES}^N$ and $TGS(w_c) \leq TGS(w)$.*

Proof. As in the proof of Theorem 4, Proposition 3 implies that it is sufficient to compare the share of output accruing to the agent in a Nash equilibrium in LRS contracts under Assumption 5 with their share under collusion to prove the theorem. The results in Proposition 3 still apply because fees do not play a role in the agent's decisions and are net out when computing total payoffs.

Let w be a contract scheme in LRS contracts as the one in equation (25) characterized by shares (θ_1, θ_2). The share of output going to the agent is: $\theta_A^N = 1 - \theta_1 - \theta_2$. Assume further that w is a Nash equilibrium of the game. Under collusion, the principals offer a contract w_c that gives them a share θ_c of output and the agent a share $\theta_A^C = 1 - \theta_c$, see (16) in Theorem 3. The problem of the agent is then equivalent to that in (24).

Before proceeding with the proof, we define the following shorthand for the guaranteed payoffs of the agent and the principals as functions of the shares (θ) that define contracts. The agent's guaranteed payoff depends only on their share of total output (see 24), so we write

$$\tilde{V}_A(\theta_A) \equiv \max_{(F,c)\in\mathcal{A}} \{\theta_A E_F[y_1 + y_2] - c\} \tag{D.17}$$

The principals' payoffs depend only on their share and that of the agent (see D.6), so we write

$$\tilde{V}_i(\theta_i, \theta_A) \equiv \frac{\theta_i}{\theta_A} \tilde{V}_A(\theta_A) - \theta_i \overline{y}_j. \tag{D.18}$$

Now, suppose that w and w_c are such that $\theta_A^N < \theta_A^C$. We will show that this leads to a contradiction. There are five cases to consider.

Case 1. Both principals can reduce their share of output so as to give the agent the same share of output as under collusion. This is,

$$\theta_i \geq \theta_A^C - \theta_A^N, \; i \in \{1,2\}. \tag{D.19}$$

Thus, any principal can unilaterally induce the collusive outcome by reducing their own share, θ_i, to $\theta_i^C \equiv \theta_i - \left(\theta_A^C - \theta_A^N\right)$. Doing so would decrease the principal's payoff by

$$0 \geq \tilde{V}_i\left(\theta_i^C, \theta_A^C\right) - \tilde{V}_i\left(\theta_i, \theta_A^N\right) = \left(\theta_A^C - \theta_A^N\right)\bar{y}_j + \frac{\theta_i^C}{\theta_A^C}\tilde{V}_A\left(\theta_A^C\right) - \frac{\theta_i}{\theta_A^N}\tilde{V}_A\left(\theta_A^N\right) \tag{D.20}$$

because w is a Nash equilibrium. This applies to both principals and implies that

$$0 \geq \left(\tilde{V}_1\left(\theta_1^C, \theta_A^C\right) - \tilde{V}_1\left(\theta_1, \theta_A^N\right)\right) + \left(\tilde{V}_2\left(\theta_2^C, \theta_A^C\right) - \tilde{V}_2\left(\theta_2, \theta_A^N\right)\right) \tag{D.21}$$

$$= \left(\theta_A^C - \theta_A^N\right)\left(\bar{y}_1 + \bar{y}_2\right) + \frac{\theta_1^C + \theta_2^C}{\theta_A^C}\tilde{V}_A\left(\theta_A^C\right) - \frac{\theta_1 + \theta_2}{\theta_A^N}\tilde{V}_A\left(\theta_A^N\right) \tag{D.22}$$

$$= \left(\theta_A^C - \theta_A^N\right)\left(\bar{y}_1 + \bar{y}_2\right) + \left(\frac{1-\theta_A^C}{\theta_A^C} - \frac{\theta_A^C - \theta_A^N}{\theta_A^C}\right)\tilde{V}_A\left(\theta_A^C\right) - \frac{1-\theta_A^N}{\theta_A^N}\tilde{V}_A\left(\theta_A^N\right) \tag{D.23}$$

$$= \left(\theta_A^C - \theta_A^N\right)\left(\bar{y}_1 + \bar{y}_2 - \frac{\tilde{V}_A\left(\theta_A^C\right)}{\theta_A^C}\right) + \left(\frac{1-\theta_A^C}{\theta_A^C}\tilde{V}_A\left(\theta_A^C\right) - \frac{1-\theta_A^N}{\theta_A^N}\tilde{V}_A\left(\theta_A^N\right)\right) \tag{D.24}$$

Where the second equality follows from the definition of θ_i^C,

$$\theta_1^C + \theta_2^C = \theta_1 + \theta_2 - 2\left(\theta_A^C - \theta_A^N\right) = 1 - \theta_A^N - 2\left(\theta_A^C - \theta_A^N\right) = \left(1 - \theta_A^C\right) - \left(\theta_A^C - \theta_A^N\right) \tag{D.25}$$

The second term in the right hand side of the inequality is positive,

$$\frac{1-\theta_A^C}{\theta_A^C}\tilde{V}_A^C - \frac{1-\theta_A^N}{\theta_A^N}\tilde{V}_A^N \geq 0, \tag{D.26}$$

because the principals maximize $\frac{1-\theta}{\theta}\tilde{V}_A\left(\theta|\mathcal{A}_0\right)$ when they collude, see (17) in Theorem 3. The first term is also non-negative. $\theta_A^C - \theta_A^N \geq 0$ by assumption and

$$\bar{y}_1 + \bar{y}_2 > \frac{\tilde{V}_A^C}{\theta_A^C} \tag{D.27}$$

because Assumption 2 (positive cost) prevents the agent from guaranteeing a payoff equal to the maximum output under the known action set $\mathcal{A}_0$ (all $(F, c) \in \mathcal{A}_0$ satisfy $E_F\left[y_1 + y_2\right] \leq \bar{y}_2 + \bar{y}_2$ and $c \geq 0$ with at least one strict inequality). This contradicts the original inequality, violating the assumption that w is a Nash equilibrium. At least one principal has a profitable deviation.

Case 2. Only one principal, say principal i, can reduce their share of output so as to give the agent the same share of output as under collusion and the other principal's contract satisfies $\theta_j > 0$. This is

$$\theta_i \geq \theta_A^C - \theta_A^N > \theta_j > 0. \tag{D.28}$$

Consider then the following expression

$$0 \geq \left(\tilde{V}_i\left(\theta_i^C, \theta_A^C\right) - \tilde{V}_i\left(\theta_i, \theta_A^N\right)\right) + \left(\theta_j - \left(\theta_A^C - \theta_A^N\right)\right)\left(\frac{1}{\theta_A^C}\tilde{V}_A\left(\theta_A^C\right) - \bar{y}_i\right) - \tilde{V}_j\left(\theta_j, \theta_A^N\right), \tag{D.29}$$

where the first term is less than or equal to zero because w_i is a best response to w_j as in Case 1, the second term is also less than zero because of (D.28), and the third term is also less than or equal to zero because w_j is eligible for principal j and must therefore provide a positive guaranteed payoff (Definition 7).

We can expand the first and last terms using (D.18) and (D.20) as in Case 1 to get

$$0 \leq \left(\theta_A^C - \theta_A^N\right)\left(\bar{y}_1 + \bar{y}_2 - \frac{\tilde{V}_A\left(\theta_A^C\right)}{\theta_A^C}\right) + \left(\frac{1-\theta_A^C}{\theta_A^C}\tilde{V}_A\left(\theta_A^C\right) - \frac{1-\theta_A^N}{\theta_A^N}\tilde{V}_A\left(\theta_A^N\right)\right). \tag{D.30}$$

However, this leads to the same contradiction as in Case 1. Hence, it must be that $\tilde{V}_i\left(\theta_i^C, \theta_A^C\right) - \tilde{V}_1\left(\theta_i, \theta_A^N\right) \geq 0$, proving that principal i has a profitable deviation. Then w^N is not an equilibrium.

Case 3. None of the principals can reduce their share of output so as to give the agent the same share of output as under collusion $\left(\theta_1, \theta_2 < \theta_A^C - \theta_A^N\right)$ and $\theta_1, \theta_2 > 0$. We can consider

$$0 \geq \left(\theta_i - \left(\theta_A^C - \theta_A^N\right)\right)\left(\frac{1}{\theta_A^C}\tilde{V}_A\left(\theta_A^C\right) - \bar{y}_j\right) - \tilde{V}_i\left(\theta_i, \theta_A^N\right),\ i \in \{1,2\},\ j \neq i, \tag{D.31}$$

where the inequality follows as in Case 2 from $\theta_i < \theta_A^C - \theta_A^N$ along with the fact that $\tilde{V}_A\left(\theta_A^C\right) \geq 0$ (Assumption 1) and that w is eligible, so that $\tilde{V}_i\left(\theta_i, \theta_A^N\right) > 0$. By summing (D.31) for $i = 1, 2$ we get (D.30) as in Cases 1 and 2, which again implies a contradiction.

Case 4. One principal, say j, has no share of the surplus, so that $\theta_j^N = 0$. Then, the problem of principal i is equivalent to choosing the agent's share. Because w is a Nash equilibrium, it must be that

$$\theta_A^N = \underset{\theta_A \in [0,1]}{\operatorname{argmax}}\left\{\frac{1-\theta_A}{\theta_A}\tilde{V}_A\left(\theta_A\right) - \left(1-\theta_A\right)\bar{y}_j\right\}. \tag{D.32}$$

Consider the case where $\theta_A < \theta_A^C$, then

$$\frac{1-\theta_A}{\theta_A}\tilde{V}_A\left(\theta_A\right) - \left(1-\theta_A\right)\bar{y}_j \leq \frac{1-\theta_A^C}{\theta_A^C}\tilde{V}_A\left(\theta_A^C\right) - \left(1-\theta_A^C\right)\bar{y}_j \tag{D.33}$$

because $\frac{1-\theta_A}{\theta_A}\tilde{V}_A\left(\theta_A\right) \leq \frac{1-\theta_A^C}{\theta_A^C}\tilde{V}_A\left(\theta_A^C\right)$ for $\theta_A \neq \theta_A^C$ from the collusion problem (17) as in Case 1, and $-\left(1-\theta_A\right)\bar{y}_j < -\left(1-\theta_A^C\right)\bar{y}_j$ by assumption. It then follows that $\theta_A^N \geq \theta_A^C$.

Case 5. Finally, we consider the case where both principals have no share of the surplus in equilibrium, so that $\theta_1 = \theta_2 = 0$, implying that $\theta_A^N = 1$, which contradicts $\theta_A^C > \theta_A^N$.

□

E Multiple principals

The model considered in Section 1 can be extended to multiple principals preserving all of our main results. In what follows, we denote the number of principals by N, and given a principal i, we define the vector of competing contracts as: $w_{-i}(y) = (w_1(y), \ldots, w_{i-1}(y), w_{i+1}(y), \ldots, w_N(y))$. We first extend the definition of LRS contracts (Definition 2) to an environment with N principals.

Definition 8. (Linear Revenue Sharing Contracts with N Principals) Given a vector of competing contracts w_{-i}, a contract w_i is a LRS contract for principal i if it ties the principal's ex-post payoff linearly to the agent's payment. That is, for some $\alpha_i \in (0,1]$ and $k_i \in \mathbb{R}$

$$y_i - w_i(y) = \frac{(1-\alpha_i)}{\alpha_i}\left(\sum_{n=1}^{N} w_n(y)\right) - k_i. \tag{E.1}$$

A version of Theorem 1 applies so that there is always a LRS contract in each principal's best response.

Theorem 10. *For any set of contracts w_{-i}, there exists an LRS contract $\overline{w}_i$ such that $\overline{w}_i \in BR_i(w_{-i}) \equiv \underset{w_i}{argmax}\, V_i(w_i, w_{-i})$ and $\overline{w}_i$ satisfies either Assumption 4, $\min_{y\in Y}\left\{\bar{w}_i(y) + \sum_{j\neq i} w_j(y)\right\} = 0$, or Assumption 5, $\min_{y\in Y}\{\bar{w}_i(y)\} = 0$. If $\mathcal{A}_0$ satisfies the Assumption 6, then any robust contract for principal i is a LRS contract or they cannot guarantee a payoff higher than $\sum_{j\neq i} w_j(0,0)$ under Assumption 4 or 0 under Assumption 5.*

Proof. The proof is virtually identical to that of Theorems 1 and 7. Lemmas 1 to 6 follow unchanged by defining the aggregate competing contract $w^c(y) = \sum_{j\neq i} w_j(y)$.

□

We can also extend the characterization of LRS contract schemes provided in Propositions 1 and 5, with LRS contracts schemes being characterized by a share, θ_i, of total output and total guaranteed surplus going to principal i. Guaranteed surplus is computed relative to the payoffs under inaction. However, we show in Proposition 6 that in order to have $\theta_i > 0$ in equilibrium under Assumption 5, the agent needs to have access to an action (F, c) such that $E_F[y_i] > \sum_{j\neq i} E_F\left[\overline{y}_j - y_j\right]$. This condition is stronger than non-triviality (Assumption 3) and increasingly difficult to satisfy as the number of principals increases. Intuitively the LRS contract compensates the agent for their forgone earnings from other principals, this requires principal i's payoff to be large in order to guarantee a positive payoff.

Proposition 6. *Let w be a LRS contract scheme satisfying limited liability with equality. There exist $(\theta_1, ..., \theta_N)$ and $(k_1, \ldots, k_N)$ such that the for all $i \in \{1, 2, \ldots, N\}$ contracts are:*

$$w_i(y) = (1-\theta_i)\, y_i - \theta_i \sum_{j\neq i} y_j - k_i \qquad \textit{Under Assumption 4} \tag{E.2}$$

$$w_i(y) = (1-\theta_i)\, y_i + \theta_i \sum_{j\neq i} \left(\overline{y}_j - y_j\right) \qquad \textit{Under Assumption 5} \tag{E.3}$$

where $\sum_{i=1}^{N} k_i = 0$ *under Assumption 4.*

Proof. Let w be a LRS contract scheme satisfying limited liability with equality. Each contract w_i has the following form

$$w_i\left(y\right) = y_i - \frac{1-\alpha_i}{\alpha_i}\sum_{n=1}^{N} w_n\left(y\right) - k_i. \tag{E.4}$$

Then, the sum of contracts satisfies

$$\sum_{n=1}^{N} w_n\left(y\right) = \frac{\sum\left(y_n - k_n\right)}{1+\sum_{n=1}^{N}\frac{1-\alpha_n}{\alpha_n}}. \tag{E.5}$$

Letting $\theta_i \equiv \frac{1-\alpha_i}{\alpha_i}/1+\sum_{n=1}^{N}\frac{1-\alpha_n}{\alpha_n}$, we arrive at (E.2). Assumption 4 implies that $\sum_{n=1}^{N} k_n = 0$. Under Assumption 5, it must be that $\min w_i\left(y\right) = 0$, the minimum is achieved when $y_i = 0$ and $y_j = \overline{y}_j$ for $j \neq i$. We solve for k_i in that case

$$k_i = -\theta_i\sum_{j\neq i}\overline{y}_j + \theta_i\left(\sum_{n=1}^{N} k_n\right). \tag{E.6}$$

Replacing back into principal i's LRS contract we get (E.3). From Lemma 5 we can establish that the share of total guaranteed surplus going to principal i in equilibrium is equal to θ_i. Principal i's payoff given inaction is $-\theta_i\sum_{j\neq i}\overline{y}_j$ and total surplus given inaction is zero by construction. Then we have:

$$\theta_i = \frac{V_i\left(w\right) + \theta_i\sum_{j\neq i}\overline{y}_j}{\sum_n V_n\left(w\right) + V_A\left(w|\mathcal{A}_0\right)} \tag{E.7}$$

$$= \frac{\frac{1-\alpha_i}{\alpha_i}V_A\left(w|\mathcal{A}_0\right) + k_i + \theta_i\sum_{j\neq i}\overline{y}_j}{\left(1+\sum_n\frac{1-\alpha_n}{\alpha_n}\right)V_A\left(w|\mathcal{A}_0\right) + \sum_n k_n} = \frac{\frac{1-\alpha_i}{\alpha_i}V_A\left(w|\mathcal{A}_0\right) - \theta_i\sum_{j\neq i}\overline{y}_j + \theta_i\left(\sum_n k_n\right) + \theta_i\sum_{j\neq i}\overline{y}_j}{\left(1+\sum_n\frac{1-\alpha_n}{\alpha_n}\right)V_A\left(w|\mathcal{A}_0\right) + \sum_n k_n} = \theta_i \tag{E.8}$$

$\square$

F Double limited Liability

Under individual limited liability (Assumption 5), equilibrium contracts require principals to pay a fee to the agent. This fee depends on the maximum potential payment that other principals can make, thus, in equilibrium, principals offer potentially large payments to the agent. Thus, principals can have negative ex post payoffs. In this Section we explore the implications of allowing for this payoffs by introducing limited liability on the principals. We show that the core of our results does not rely on the principals offering unbounded rewards to the agent.

Assumption 7. ***(Principals' Limited Liability)*** *$y_i - w_i(y) \geq 0$ for all $y \in Y$, $i \in \{1, 2\}$.*

Imposing limited liability on the principals amounts to restricting contracts so that $y_i - w_i(y) \geq 0$ for all $y \in Y$. Under assumption 7 only the definition of LRS contracts changes, adding a cap to the amount that the principal can pay to the agent.

Definition 9. (Linear Revenue Sharing Contracts under Assumption 7) Given a contract w_j, a contract w_i is a LRS contract for principal i if it ties the principal's ex-post payoff linearly to the total revenue of the agent. That is, for some $\alpha_i \in (0, 1]$ and $k_i \in \mathbb{R}$

$$y_i - w_i(y) = \min\left\{\frac{(1-\alpha_i)}{\alpha_i}(w_1(y) + w_2(y)) + k_i\,,\, 0\right\},\;\; j \neq i. \tag{F.1}$$

We show that LRS contracts remain optimal. Lemmas 1 and 2 remain unchanged. Crucially, the argument in 3 also goes through unchanged as it does not impose any restrictions on how high the payments stipulated by contracts can be. The following Lemma establishes that (A.11) applies for LRS contracts satisfying Assumption 7.

Lemma 11. *Let w be an eligible contract with $w_i(y) = \min\{\alpha y_i - (1-\alpha) w_j(y) - \alpha k\,,\, y_i\}$ for some $\alpha \in (0, 1]$ and $k \in \mathbb{R}$. Then*

$$V_i(w) = k + \frac{1-\alpha}{\alpha} V_A(w|\mathcal{A}_0) \tag{F.2}$$

Proof. Let $F^* \in \underset{F \in \mathcal{F}}{\operatorname{argmin}} E_F[y_i - w_i]$. By Lemma 2 we have that

$$\begin{aligned} k + \frac{1-\alpha}{\alpha} V_A(w|\mathcal{A}_0) - V_i(w) &= k + \frac{1-\alpha}{\alpha} E_{F^\star}[w_1 + w_2] - E_{F^\star}[y_i - w_i] && \text{(F.3)} \\ &= k + \frac{1}{\alpha} E_{F^*}[w_i] + \frac{1-\alpha}{\alpha} E_{F^*}[w_j] - E_{F^*}(y_i) && \text{(F.4)} \\ &= k + \frac{1}{\alpha} E_{F^*}[\min\{\alpha y_i - (1-\alpha) w_j(y) - \alpha k\,,\, y_i\}] \\ &\quad + \frac{1-\alpha}{\alpha} E_{F^*}(w_j) - E_{F^*}(y_i) && \text{(F.5)} \end{aligned}$$

Suppose for a contradiction that F^* places some positive probability, $\delta > 0$, on a set $\bar{Y} \subset Y$ such that such that $\alpha y_i - (1-\alpha) w_j(y) - \alpha k > y_i$ for $y \in \overline{Y}$. Rearranging, $0 > (1-\alpha)(y_i + w_j(y)) + \alpha k$.

Rearranging again, $-\frac{\alpha}{1-\alpha}k > y_i + w_j(y)$, where the right hand side is the agents payment if output is y.

Now, consider $\hat{y} \in Y$ for which $\alpha\hat{y}_i - (1-\alpha)w_j(\hat{y}) - \alpha k = \hat{y}_i$. Rearranging,

$$\hat{y}_i + w_j(\hat{y}) = \alpha\hat{y}_i - (1-\alpha)w_j(\hat{y}) - \alpha k + w_j(\hat{y}) \tag{F.6}$$

$$(1-\alpha)(\hat{y}_i + w_j(\hat{y})) = -\alpha k \tag{F.7}$$

$$\hat{y}_i + w_j(\hat{y}) = -\frac{\alpha}{1-\alpha}k \tag{F.8}$$

$$\hat{y}_i + w_j(\hat{y}) > y_i + w_j(y) \tag{F.9}$$

It must be that F^* puts positive probability on a $\tilde{y} \in Y$ for which the payoff to principal i is positive by eligibility.

Now, consider an alternative distribution F', which is equal to F^* except that it shifts all the weight δ in $\overline{Y}$ to $\hat{y}$. Then $E_{F'}[w_i + w_j] > V_A(w|\mathcal{A}_0)$ because of (F.9). Further, consider F'' that is the same as F' but shifts a small but positive weight from $\tilde{y}$ to $\hat{y}$ such that we still have $E_{F''}[w_i + w_j] \geq V_A(w|\mathcal{A}_0)$. It holds that $F'' \in \mathcal{F}$. But then, the payoff to principal i under F'' is worse than that under F' and F^* which violates the minimality of F^*.

Hence, it must be that F^* places full support on $y \in Y$ for which $\alpha y_i - (1-\alpha)w_j(y) - \alpha k \leq y_i$. Then we have from (F.5)

$$\begin{aligned}
k + \frac{1-\alpha}{\alpha}V_A(w|\mathcal{A}_0) - V_i(w) &= k + \frac{1}{\alpha}E_{F^*}[\alpha y_i - (1-\alpha)w_j(y) - \alpha k] + \frac{1-\alpha}{\alpha}E_{F^*}(w_j) - E_{F^*}(y_i) \\
&= k + E_{F^*}\left[y_i - \frac{1-\alpha}{\alpha}w_j(y) - k\right] + \frac{1-\alpha}{\alpha}E_{F^*}(w_j) - E_{F^*}(y_i) \\
&= 0.
\end{aligned} \tag{F.10}$$

The last equality gives the result.

□

We can now use Lemmas 3 and 11 to construct a LRS contract that improves over any contract considered by principal i.

Lemma 12. *Let $w = (w_i, w_j)$ with w_i satisfying (A.10) and (A.11) from Lemma 3. Then, the contract*

$$w_i^{'}(y) = \min\{\alpha y_i - (1-\alpha)w_j(y) - \alpha k\, ,\, y_i\} \tag{F.11}$$

where k is such that $\min_y w_i^{'}(y) = 0$ satisfies $V_i\left(w_i^{'}, w_j\right) \geq V_i(w)$.

Proof. Following the arguments in Lemma 9, we use (A.10) to define an auxiliary contract

$$w_i^{''}(y) = \alpha y_i - (1-\alpha)w_j(y) - \alpha k^{''}, \tag{F.12}$$

where $k^{''}$ is such that $\min_y \left\{w_i^{''}(y)\right\} = \min_y \left\{\alpha y_i - (1-\alpha)w_j(y) - \alpha k^{''}\right\} = 0$ This contract improves principal i's guaranteed payoff, $V_i\left(w_i^{''}, w_j\right) \geq V_i(w)$.

Contract $w_i^{'}$ is characterized by the same *alpha* and by $k \geq k^{''}$, and satisfies

$$w_i^{'}(y) \leq \alpha y_i - (1-\alpha) w_j(y) - \alpha k \tag{F.13}$$

$$k + \frac{1-\alpha}{\alpha}\left(w_i^{'}(y) + w_j(y)\right) \leq y_i - w_i^{'}(y). \tag{F.14}$$

Now, let $\mathcal{A} \supseteq \mathcal{A}_0$ and $(F,c) \in A^{\star}(w|\mathcal{A})$. Taking expectations we get

$$E_F\left[y_i - w_i^{'}(y)\right] \geq k + \frac{1-\alpha}{\alpha} E_F\left[w_i^{'}(y) + w_j(y)\right] \tag{F.15}$$

$$= k + \frac{1-\alpha}{\alpha} V_A\left(\left(w_i^{'}, w_j\right)|\mathcal{A}_0\right) \tag{F.16}$$

$$= k^{''} + \frac{1-\alpha}{\alpha} V_A\left(\left(w_i^{'} + \frac{\alpha}{1-\alpha}\left(k-k^{''}\right), w_j\right)|\mathcal{A}_0\right) \tag{F.17}$$

This applies to any optimal (F,c) under any action set, so this guarantees a payoff for principal i.

Moreover,

$$w_i^{'} + \frac{\alpha}{1-\alpha}\left(k-k^{''}\right) = \min\left\{\alpha y_i - (1-\alpha) w_j(y) - \alpha k\,,\, y_i\right\} + \frac{\alpha}{1-\alpha}\left(k-k^{''}\right) \tag{F.18}$$

$$> \min\left\{\alpha y_i - (1-\alpha) w_j(y) - \alpha k\,,\, y_i\right\} + \alpha\left(k-k^{''}\right) \tag{F.19}$$

$$= min\left\{w_i^{''}\,,\, y_i\alpha\left(k-k^{''}\right)\right\} \tag{F.20}$$

$$\geq w_i. \tag{F.21}$$

The inequality holds for all $y \in Y$ because $w_i^{''} \geq w_i$ and w_i satisfies principals limited liability, $w_i \leq y_i \leq y_i + \alpha\left(k-k^{''}\right)$. So, the agent is always at least as well off under $w_i^{'} + \frac{\alpha}{1-\alpha}\left(k-k^{''}\right)$ as under w_i, $V_A\left(\left(w_i^{'} + \frac{\alpha}{1-\alpha}\left(k-k^{''}\right), w_j\right)|\mathcal{A}_0\right) \geq V_A(w|\mathcal{A}_0)$. Joining with (F.17) gives

$$E_F\left[y_i - w_i^{'}(y)\right] \geq k^{''} + \frac{1-\alpha}{\alpha} V_A(w|\mathcal{A}_0) = V_i(w) \tag{F.22}$$

which holds for all $(F,c) \in A^{\star}(w|\mathcal{A})$ so that $V_i\left(\left(w_i^{'}, w_j\right)|\mathcal{A}\right) = \min_{F \in A^{\star}(w|\mathcal{A})} E_F\left[y_i - w_i^{'}(y)\right] \geq V_i(w)$ which gives the desired result $V_i\left(w_i^{'}, w_j\right) \geq V_i(w)$.

□

We can now establish the optimality of LRS contracts.

Theorem 11. *For any contract w_j, there exists a LRS contract*

$$\overline{w}_i(y) = \min\left(\alpha y_i - (1-\alpha) w_j(y) - \alpha k, y_i\right) \tag{F.23}$$

such that $\overline{w}_i \in BR_i(w_j)$ and $\overline{w}_i$ satisfies Assumptions 5 and 7. That is, there is always a LRS contract that is robust for principal i. Moreover, if $\mathcal{A}_0$ satisfies Assumption 6, any robust contract is a LRS contract or $\max_{w_i} V_i(w_i, w_j) = 0$.

Proof. By Lemma 11 the value of a principal under an LRS contract satisfies (F.2). This function is continuous on α by interpreting the term $\frac{1-\alpha}{\alpha}c$ as 0 when $c = 0$ and ∞ for $c > 0$. Recall that the value of k that ensures Assumption 5 is satisfied with equality is also a continuous function of α. Then, there is a $\alpha^{\star}$ that maximizes (F.2) and characterizes the optimal LRS contract $w_i^{\star}$.

Consider an arbitrary contract w_i. By Lemma 12, there is always a LRS contract that weakly improves over w_i. This contract is itself improved upon by $w_i^{\star}$. Then, the $w_i^{\star}$ in the best response.

Now, impose Assumption 6 and suppose w_i is an optimal contract for principal i. Define $w_i^{'}$ as in Lemma 12 and let $\mathcal{A} \supseteq \mathcal{A}_0$ and $(F, c) \in A^{\star}(w|\mathcal{A})$. From equation F.17,

$$E_F\left[y_i - w_i^{'}(y)\right] = k^{''} + \frac{1-\alpha}{\alpha} V_A\left(\left(w_i^{'} + \frac{\alpha}{1-\alpha}\left(k - k^{''}\right), w_j\right)|\mathcal{A}_0\right), \tag{F.24}$$

where $k - k^{''} \geq 0$ as in the proof of Lemma 12. Contract w_i satisfies Equation (A.11) from Lemma 3 which we use to replace for k and obtain

$$E_F\left[y_i - w_i^{'}(y)\right] \geq V_i(w) + \frac{1-\alpha}{\alpha}\left(V_A\left(\left(w_i^{'} + \frac{\alpha}{1-\alpha}\left(k - k^{''}\right), w_j\right)|\mathcal{A}_0\right) - V_A(w|\mathcal{A}_0)\right). \tag{F.25}$$

As in the proof of Lemma 12, $w_i^{'} + \frac{\alpha}{1-\alpha}\left(k - k^{''}\right) \geq w_i(y)$ point wise and therefore $V_A\left(\left(w_i^{'} + \frac{\alpha}{1-\alpha}\left(k - k^{''}\right), w_j\right)|\mathcal{A}_0\right) \geq V_A(w|\mathcal{A}_0)$, with strict inequality unless $w_i^{'} + \frac{\alpha}{1-\alpha}\left(k - k^{''}\right)$ is identical to w_i. Then, using Lemma 2 and the lower bound for $E_F\left[y_i - w_i^{'}(y)\right]$ in the last equation we obtain

$$V_i\left(w_i^{'}, w_j\right) \geq V_i(w) + \frac{1-\alpha}{\alpha}\left(V_A\left(\left(w_i^{'} + \frac{\alpha}{1-\alpha}\left(k - k^{''}\right), w_j\right)|\mathcal{A}_0\right) - V_A(w|\mathcal{A}_0)\right) > V_i(w) \tag{F.26}$$

where the strict inequality follows when w_i is not identical to $w_i^{'} + \frac{\alpha}{1-\alpha}\left(k - k^{''}\right)$. Then, $w_i = w_i^{PLL} + \frac{(k^{''} - k)}{\lambda}$, or else optimality would be contradicted. This proves the result.

□

G Lower bound on costs

In our common agency model, Section 1, principals do not know the agent's true action set $\mathcal{A}$. In Section 2 we show that LRS contracts offer the best guaranteed payoff possible across all possible action sets $\mathcal{A} \supseteq \mathcal{A}_0$. This includes action sets where large amounts of output produced for free, making the distributions that induce the worst case guarantee have zero cost. This assumption is convenient for the exposition of the problem, but it stands to reason that production is costly. To address this, we now assume that the principal knows a lower bound on the cost of producing any given level of expected output. We also assume full support (Assumption 6) and the stronger form of limited liability (Assumption 5), although the proofs can be modified to apply under Assumption 4. We prove that LRS contracts are still a best response to LRS contracts under these conditions.

Let $b : \mathbb{R} \to \mathbb{R}_+$ be a convex function satisfying $b(0) = 0$. An action set is a compact set $\mathcal{A} \subset \Delta(Y) \times \mathbb{R}_+$ such that for any $(F, c) \in \mathcal{A}$ we have that $c \geq b(\mathbb{E}_F[y_1 + y_2])$. This holds also for any $(F, c) \in \mathcal{A}_0$ with a strict inequality (i.e., $c > b(\mathbb{E}_F(y))$ if $(F, c) \in \mathcal{A}_0$). This is similar to the positive cost assumption when there was no lower bound on costs.

The following Lemma parallels Lemma 1 and relates the expected payments to the agent under any action set with its value under $\mathcal{A}_0$.

Lemma 13. *Let $(F, c) \in A^\star(w|\mathcal{A})$. For $\mathcal{A} \supseteq \mathcal{A}_0$, it holds that:*

$$E_F[w_1(y) + w_2(y)] \geq V_A(w|\mathcal{A}_0) + b(E_F[y_1 + y_2])$$

Moreover, if $(F, c) \in A^\star(w|\mathcal{A})$ then $F \in \mathcal{F}$ where:

$$\mathcal{F} = \{F \in \Delta(Y) \,|E_F[w_1(y) + w_2(y)] \geq V_A(w|\mathcal{A}_0) + b(E_F[y_1 + y_2])\}$$

Proof. To see the first inequality let $(F, c) \in A^\star(w|\mathcal{A})$ for $\mathcal{A} \supseteq \mathcal{A}_0$:

$$E_F[w_1(y) + w_2(y)] - b(E_F[y_1 + y_2]) \geq E_F[w_1(y) + w_2(y)] - c \geq V_A(w|\mathcal{A}) \geq V_A(w|\mathcal{A}_0)$$

where the first inequality holds since $c \geq b(E_F[y_1 + y_2])$. Then $F \in \mathcal{F}$.

□

Lemma 14. *Let w be an eligible contract for principal i (Definition 5), then $V_i(w) = \min_{F \in \mathcal{F}} E_F[y_i - w_i(y)]$. Moreover if $F \in \underset{F \in \mathcal{F}}{argmin} E_F[y_i - w_i(y)]$ then $E_F[w_1(y) + w_2(y)] = V_A(w|\mathcal{A}_0) + b(E_F[y_1 + y_2])$.*

Proof. Let w be an eligible contract. It must be that: $V_i(w) \geq \min_{F \in \mathcal{F}} E_F[y_i - w_i(y)]$, to see this, use the definition of V_i in (3),

$$V_i(w) = \inf_{\mathcal{A} \supseteq \mathcal{A}_0} \min_{(F,c) \in A^\star(w|\mathcal{A})} E_F[y_i - w_i(y)] \geq \min_{F \in \mathcal{F}} E_F[y_i - w_i(y)], \tag{G.1}$$

where the inequality follows from Lemma 13 because if $(F, c) \in A^\star(w|\mathcal{A})$ then $F \in \mathcal{F}$.

Now we can establish the equality in the first statement of the Lemma. Suppose not, then it must be that $V_i(w) > \min_{F\in\mathcal{F}} E_F[y_i - w_i(y)]$. Then, for $F \in \operatorname{argmin}_{F\in\mathcal{F}} E_F[y_i - w_i(y)]$, we have that $E_F[w_1(y) + w_2(y)] \geq V_A(w|\mathcal{A}_0) + b(E_F[y_1+y_2])$. Finally, consider the action set $\mathcal{A}' \equiv \mathcal{A}_0 \cup \{(F, b(E_F[y_1+y_2]))\}$. It follows that $(F, b(E_F[y_1+y_2])) \in A^\star\left(w|\mathcal{A}'\right)$, which implies

$$V_i(w) \leq V_i\left(w|\mathcal{A}'\right) = \min_{(F,c)\in A^\star\left(w|\mathcal{A}'\right)} E_F[y_i - w_i(y)] \leq \min_{F\in\mathcal{F}} E_F[y_i - w_i] < V_i, \tag{G.2}$$

a contradiction.

Now, for the second result in the Lemma, let $F \in \operatorname{argmin}_{F\in\mathcal{F}} E_F[y_i - w_i]$ and suppose for a contradiction that $E_F[w_1(y) + w_2(y)] > V_A(w|\mathcal{A}_0) + b(E_F[y_1+y_2])$.

Let $\epsilon \in [0,1]$ and consider $F_\epsilon = (1-\epsilon)F + \epsilon\delta_{(0,0)}$ and $\mathcal{A}_\epsilon = \mathcal{A}_0 \cup \left\{\left(F_\epsilon, b\left(E_{F'}(y_1+y_2)\right)\right)\right\}$. It follows that $\left\{\left(F_\epsilon, b\left(E_{F'}(y_1+y_2)\right)\right)\right\} = A^\star(w|\mathcal{A}_\epsilon)$ for low enough ϵ because the agent's payoff is strictly greater choosing F_ϵ at a cost of $b\left(E_{F'}(y_1+y_2)\right)$, than choosing any $(F,c) \in \mathcal{A}_0$. By convexity, $b(E_{F_\epsilon}(y_1+y_2)) \leq (1-\epsilon)b(E_F[y_1+y_2]) + \epsilon b(0)$. Principal i' payoff is

$$V_i\left(w|\mathcal{A}'_\epsilon\right) = (1-\epsilon)E_F[y_i - w_i(y)] - \epsilon w_i(0,0) < E_F[y_i - w_i(y)] = V_i(w) \leq V_i\left(w|\mathcal{A}'_\epsilon\right), \tag{G.3}$$

which contradiction the definition of V_i. The strict inequality follows from $E_F[y_i - w_i(y)] > 0$ by eligibility and $w_i(0,0) \geq 0$ by the agent's limited liability.

□

We are interested in LRS contract schemes. So, suppose that principal j offers a contract of the form, $w_j(y) = (1-\theta_j)y_j + \theta_j(\bar{y}_i - y_i)$, as in Proposition 5 and that principal i's contract, $w_i : Y \to \mathbb{R}_+$, is eligible but is not an LRS contract. In particular, there does not exist $\theta_i \in [0, 1-\theta_j]$ and k such that $w_i(y) = (1-\theta_i)y_i + \theta_i(\bar{y}_j - y_j) + k$. Our objective is to show that there exist an alternative LRS contract w'_i that dominates w_i, where $w'_i(y) = \left(1-\theta'_i\right)y_i + \theta'_i(\bar{y}_j - y_j)$ for some $\theta'_i \in [0, 1-\theta_j]$.

The same separation argument as in Lemma 3 applies. However, the separation is done in outcome space and not in payoff space.

Define the function

$$t(x) = \max\{b(x) + V_A(w|\mathcal{A}_0), (1-\theta_j)x + \theta_j\bar{y}_i - V_i(w)\}$$

Clearly t is a convex function.

Now let $S \in \mathbb{R}^2$ be the convex hull of pairs $(y_1+y_2, w_i(y_1,y_2) + w_j(y_1,y_2))$ for all $(y_1,y_2) \in Y$, and let $T \in \mathbb{R}^2$ be the set of all pairs (x,z) such that x lies in the convex hull of points y_1+y_2, and $z > t(x)$.[27] These sets are convex and disjoint.[28] If there weren't disjoint there would exist $F \in \Delta Y$ such that $E_F[w_i(y_1,y_2) + w_j(y_1,y_2)] > t(E_F[y_1+y_2])$. Then, the following two inequalities hold

$$E_F[w_i(y_1,y_2) + w_j(y_1,y_2)] > b(E_F[y_1+y_2]) + V_A(w|\mathcal{A}_0), \tag{G.4}$$
$$E_F[w_i(y_1,y_2) + w_j(y_1,y_2)] > (1-\theta_j)E_F[y_1+y_2] + \theta_j\bar{y}_i - V_i(w). \tag{G.5}$$

[27] Formally $T = \left\{(x,z) \in \mathbb{R}^2 | x \in [\min_Y\{y_1+y_2\}, \max_Y\{y_1+y_2\}] \quad \wedge \quad z > t(x)\right\}$.

[28] The first one is a convex hull, so it is convex, the second one is the intersection of the upper contour set of a convex function (a convex set) with two half spaces (convex sets), so it is convex as well.

Replacing $w_j(y) = (1-\theta_j) y_j + \theta_j (\bar{y}_i - y_i)$, the second inequality becomes

$$V_i(w) > E_F[y_i - w_i(y_1, y_2)] \tag{G.6}$$

From Lemma 14 we know that $V_i(w) = \min_{F \in \mathcal{F}} E_F[y_i - w_i(y_1, y_2)]$, but from the first inequality we know that $F \in \mathcal{F}$, this is a contradiction.

Then, by the separating hyperplane theorem, there exist λ and μ and k with $(\lambda, \mu) \neq (0,0)$ such that

$$\lambda(y_1 + y_2) + \mu z \leq k \qquad \forall ((y_1 + y_2), z) \in S \tag{G.7}$$
$$\lambda(y_1 + y_2) + \mu z \geq k \qquad \forall ((y_1 + y_2), z) \in T \tag{G.8}$$

The second inequality implies that $\mu \geq 0$. Now suppose $\mu = 0$ then it must be that $\lambda = 0$, which is a contradiction. This implies that $\mu > 0$.

The inequality in (G.7) implies that

$$w_i(y_1, y_2) + w_j(y_1, y_2) \leq \frac{k - \lambda(y_1 + y_2)}{\mu}, \tag{G.9}$$

from which we can construct the following contract

$$w_i^{'}(y_1, y_2) = \frac{k - \lambda(y_1 + y_2)}{\mu} - w_j(y_1, y_2) = \theta_i^{'} y_i + \left(1 - \theta_i^{'}\right)(\overline{y}_j - y_j) + k^{'} \tag{G.10}$$

where $\theta_i^{'} = \theta_j - \frac{\lambda}{\mu}$ and $k^{'} = \frac{k}{\mu} - \theta_j \bar{y}_i - \left(1 - \theta_i^{'}\right)\overline{y}_j$. Note that $w_i^{'} \geq w_i$ pointwise, and recall that $w_i \neq w_i^{'}$ by assumption. Now we need to check that $V_i\left(w_i^{'}\right) \geq V_i(w_i)$.

Consider any action set $\mathcal{A} \supset \mathcal{A}_0$. Then we have that $V_A\left(w^{'}|\mathcal{A}\right) \geq V_A\left(w^{'}|\mathcal{A}_0\right) > V_A(w|\mathcal{A}_0)$. The last inequality follows because $\mathcal{A}_0$ has full support (Assumption 6) and $w_i^{'}(y) > w_i(y)$ for some $y \in Y$.

Now, let $(F, c) \in \mathcal{A}$ such that $(F, c) = \underset{(F,c) \in A^\star\left(w^{'}|\mathcal{A}\right)}{\arg\min} E_F\left[y_i - w_i^{'}(y)\right]$. Then, $V_i\left(w^{'}|A\right) = E_F\left[y_i - w_i^{'}(y)\right]$. From equation G.8

$$\begin{aligned}
t(E_F[y_1 + y_2]) &\geq E_F\left(\frac{k - \lambda(y_1 + y_2)}{\mu}\right) && \text{(G.11)}\\
&= E_F\left[w_1^{'}(y) + w_2(y)\right] && \text{(G.12)}\\
&= V_A\left(w^{'}|\mathcal{A}\right) + c && \text{(G.13)}\\
&> V_A(w|\mathcal{A}_0) + c && \text{(G.14)}\\
&\geq V_A(w|\mathcal{A}_0) + b(E_F[y_1 + y_2]). && \text{(G.15)}
\end{aligned}$$

Because the inequality is strict then we have that $t(E_F[y_1 + y_2]) = (1-\theta_j) E_F[y_1 + y_2] + \theta_j \bar{y}_i - V_i(w)$.

Then we have that

$$V_i\left(w'|A\right) = E_F\left[y_i - w_i'(y)\right] \tag{G.16}$$

$$= E_F\left[y_i + w_j(y)\right] - E_F\left[w_i'(y) + w_j(y)\right] \tag{G.17}$$

$$= (1-\theta_j)\, E_F\left[y_1 + y_2\right] + \theta_j \bar{y}_i - E_F\left[w_i'(y) + w_j(y)\right] \tag{G.18}$$

$$= t\left(E_F\left[y_1 + y_2\right]\right) + V_i(w) - E_F\left[w_i'(y) + w_j(y)\right] \tag{G.19}$$

$$\geq V_i(w) \tag{G.20}$$

This holds for all $\mathcal{A} \supset \mathcal{A}_0$, which implies that $V_i\left(w'\right) \geq V_i(w)$. So any contract w_i (as described above) can be dominated by a contract of the form:

$$w_i'(y_1, y_2) = \theta_i' y_i + \left(1 - \theta_i'\right)\left(\overline{y}_j - y_j\right) + k'. \tag{G.21}$$

This contract can be improved upon by dropping the constant k'. Doing so makes it satisfy limited liability with equality (when $y_i = 0$ and $y_j = \overline{y}_j$), it also does not affect the problem of the agent, and it weakly increase the value of the principal (strictly if $k' > 0$).

H Private common agency

We now consider the case where principals are restricted to contract only on their own output. This can be due to their inability to observe the other principal's output, or because of regulation that prohibits contracting on output other than your own.

In the private common agency game, a contract is a continuous function $w_i^r : Y_i \to \mathbb{R}_+$. We show that the principal's best response is to give the agent a share of their output under Assumption 5 on limited liability. The share of output given to the agent depends on the competition between the principals. These linear contracts are different from the LRS contracts discussed in Section 2 (Definition 2). The essential feature of the LRS contracts is that they tie the principal's and the agent's payoff in an affine way (6). This was achieved by partially offsetting competing contracts given to the agent. Forcing the contract to depend only on the principal's output makes this impossible.

Theorem 12. *For any contract w_j^r, there exists $\theta_i \in [0,1]$ such that the contract $w_i^r(y_i) \equiv (1-\theta_i)\, y_i$ is in principal i's best response, $w_i^r \in BR_i\left(w_j^r\right)$. Moreover, if $\mathcal{A}_0$ satisfies Assumption 6, if $w_i^r \in BR_i\left(w_j^r\right)$ then $w_i^r(y_i) = (1-\theta_i)\, y_i$ for some $\theta_i \in [0,1]$.*

When both principals play linear contracts as in 12, the best response of principal i is

$$\mathrm{BR}_i(\theta_j) = \underset{\theta_i \in [0,1]}{\operatorname{argmax}} \left[\max_{(F,c)\in\mathcal{A}_0} \left\{ \frac{\theta_i}{1-\theta_i} E_F\left[(1-\theta_i)\, y_i - (1-\theta_j)\left(\overline{y}_j - y_j\right) - c\right] \right\} \right]. \tag{H.1}$$

An interior solution satisfies

$$1-\theta_i = \frac{c + (1-\theta_j)\left(\overline{y}_j - E_F[y_j]\right)}{(1-\theta_i)\, E_F[y_i]} \tag{H.2}$$

where $(F,c) \in A^\star((\theta_i,\theta_j)\,|\mathcal{A}_0)$.[29] The numerator in (H.2) is the opportunity cost of the agent of taking action (F,c), as perceived by principal i, that is, the accounting cost of the action (c) plus the expected forgone earnings from the other principal. The share of output that principal i gives to the agent is equal to the ratio between this cost and the expected payment that the agent receives from the principal.

The principal increases the share of output given to the agent as the forgone earnings from the other principal increase. This resembles the second term in the equilibrium contract (25) found in Proposition 5. When contracts were not restricted, each principal was able to compensate the agent for the forgone earnings from the other principal. Under the restricted contracting domain this explicit form of competition is not possible. Instead, principals compete with each other by offering higher shares of their own output to the agent.

We now present the proof of Theorem 12. Lemmas 1 and 2 remain unchanged. Lemma 15 parallels Lemma 3 and links the principal's payoff guarantee to the agent's payoff given the known action set $\mathcal{A}_0$ in an affine way. This link allows the principal to increase its own guaranteed payoff by controlling the payoff given to the agent. The lemma also offers a relation between any contract w_i, the outcome y_i and the contract w_j offered by the other principal.

[29] We slightly abuse the notation by writing (θ_i, θ_j) instead of (w_i, w_j).

Lemma 15. *Let w be an eligible contract. There exist k, λ with $\lambda > 0$ such that for all $y \in Y$:*

$$w_i(y_i) \leq \frac{1}{1+\lambda} y_i - \frac{\lambda}{1+\lambda} \overline{w}_j - \frac{1}{1+\lambda} k, \tag{H.3}$$

$$V_i(w) = k + \lambda V_A(w|\mathcal{A}_0), \tag{H.4}$$

where $\overline{w}_j = \max_{y_j \in Y_j} w_j(y_j)$.

Proof. Let $S \subseteq \mathbb{R}^2$ be the convex hull of all points $(w_i(y_i) + \overline{w}_j, y_i - w_i(y_i))$ for $y_i \in Y_i$ and $\overline{w}_j = \max_{y_j \in Y_j} w_j(y_j)$, and $T \subseteq \mathbb{R}^2$ be the set of all pairs (u, v) such that $u > V_A(w|\mathcal{A}_0)$ and $v < V_i(w)$. T is convex. As in Lemma 3, $S \cap T = \emptyset$. To see this, let $(u, v) \in T$ then let $F \in \operatorname{argmin}_{F \in \mathcal{F}} E_F[y_i - w_i]$, by definition of T and Lemma 2:

$$u > V_A(w|\mathcal{A}_0) = E_F[w_i(y_i) + w_j(y_j)] \tag{H.5}$$

$$v < V_i(w) = E_F[y_i - w_i(y_i)] \tag{H.6}$$

now, suppose for a contradiction that $(u, v) \in S$, then there exists $F^{'} \in \Delta(Y)$ such that:

$$u = E_{F'}[w_i(y_i)] + \overline{w}_j \tag{H.7}$$

$$v = E_{F'}[y_i - w_i(y_i)] \tag{H.8}$$

Because $Y = Y_1 \times Y_2$ we can choose $F^{'}$ such that $E_{F'}[w_j(y_j)] = \overline{w}_j$. Then:

$$u = E_{F'}[w_i(y_i) + w_j(y_j)] > V_A(w|\mathcal{A}_0) \tag{H.9}$$

That is, $F^{'}$ guarantees a payoff to the agent larger than $V_A(w|\mathcal{A}_0)$ so $F^{'} \in \mathcal{F}$ but:

$$E_F[y_i - w_i] > E_{F'}[y_i - w_i] \tag{H.10}$$

which contradicts minimality of F. Then $S \cap T = \emptyset$ and, by the separating hyperplane theorem, there exist $(k, \lambda, \mu) \in \mathbb{R}^3$ such that $(\lambda, \mu) \neq (0, 0)$ and

$$k + \lambda u - \mu v \leq 0 \qquad (u, v) \in S \tag{H.11}$$

$$k + \lambda u - \mu v \geq 0 \qquad (u, v) \in T \tag{H.12}$$

Now, let $F^\star \in \operatorname{argmin}_{F \in \mathcal{F}} E_F[y_i - w_i(y_i)]$ such that $E_{F^\star}[w_j(y_j)] = \overline{w}_j$. This $F^\star$ always exists because the objective function $E_F[y_i - w_i(y_i)]$ only depends on y_i. Then, if $F \in \mathcal{F}$, the distribution $F^\star$ with the same marginal over y_i as F and full probability over $\overline{w}_j$ also belongs to $\mathcal{F}$. Therefore, the pair $(E_{F^\star}[w_i(y_i) + w_j(y_j)], E_{F^\star}[y_i - w_i(y_i)])$ lies in the closures of both S and T, implying

$$k + \lambda E_{F^\star}[w_1 + w_2] - \mu E_{F^\star}[y_i - w_i] = 0 \tag{H.13}$$

It is left to show that $\lambda, \mu > 0$. $(u, v) \in T$ admits u arbitrarily high and v arbitrarily low. So for (H.12) to hold it must be that $\lambda \geq 0$ and $\mu \geq 0$. There are then two cases to rule out:

1. Suppose $\mu = 0$, then it must be that $\lambda > 0$. From (H.11) and ((H.12)

$$u \leq -\frac{k}{\lambda} \qquad (u, v) \in S \tag{H.14}$$

$$u \geq -\frac{k}{\lambda} \qquad (u, v) \in T \tag{H.15}$$

So, $\max_{y_i \in Y_i} [w_i(y_i) + \overline{w}_j] = \max_{u \in S} u \leq -\frac{k}{\lambda} \leq \inf_{u \in T} u = V_A(w|\mathcal{A}_0)$, which implies

$$\max_{y_i \in Y_i} [w_i(y_i) + \overline{w}_j] = V_A(w|\mathcal{A}_0) \tag{H.16}$$

This can only happen if the agent has an action $(F, 0) \in \mathcal{A}_0$ such that $E_F[w_1(y_1) + w_2(y_2)] = \overline{w}_1 + \overline{w}_2$, but, by Assumption 2, the only action in $\mathcal{A}_0$ with zero cost is $(\delta_0, 0)$, so $\overline{w}_1 + \overline{w}_2 = w_1(0) + w_2(0)$. This is also the unique action in $A^\star(w|\mathcal{A}_0)$ so $V_i(w) \leq V_i(w|\mathcal{A}_0) = -w_i(0) \leq 0$. This violates eligibility ($V_i(w) > 0$).

2. Suppose $\lambda = 0$, then it must be that $\mu > 0$. From (H.11) and (H.12)

$$v \geq \frac{k}{\mu} \qquad (u, v) \in S \tag{H.17}$$

$$v \leq \frac{k}{\mu} \qquad (u, v) \in T \tag{H.18}$$

So, $\min_{y_i \in Y_i} [y_i - w_i(y_i)] = \min_{v \in S} v \geq \frac{k}{\mu} \geq \sup_{v \in T} v = V_i(w)$. But we know that $\min_{y_i \in Y_i} [y_i - w_i(y_i)] \leq 0 - w(0) \leq 0$ this implies $V_i(w) \leq 0$ which contradicts eligibility. So $\lambda > 0$.

Because λ and μ are greater than zero, we normalize $\mu = 1$ to arrive at (H.3):

$$k + \lambda(w_i(y_i) + \overline{w}_j) - (y_i - w_i(y_i)) \leq 0. \tag{H.19}$$

Finally, from (H.13) and Lemma 2 we get (H.4).

□

We can now extend Lemma 4 to the private common agency case using (H.3) to define an alternative contract and then adjusting to satisfy limited liability (Assumption 5) with equality. This contract is linear in the principal's output,

$$w_i(y_i) = (1 - \theta_i) y_i, \tag{H.20}$$

where we set $1 - \theta_i \equiv {}^1/_{1+\lambda}$ for λ as in Lemma 15. The proof is virtually identical and we omit it for space. Hereafter, we focus on linear contracts because they dominate other contracts available to the principal.

In the last lemma, we characterize the principal's payoffs under linear contracts and the existence of an optimal contract in that class.

Lemma 16. *Let w an eligible contract scheme with $w_i(y_i) = (1 - \theta_i) y_i$ for some $\theta_i \in [0, 1)$.*

$$V_i(w) = \frac{\theta_i}{1 - \theta_i}(V_A(w|\mathcal{A}_0) - \overline{w}_j) = \max_{(F,c) \in \mathcal{A}_0} \left(\frac{\theta_i}{1 - \theta_i}(E_F[(1 - \theta_i) y_i - (\overline{w}_j - w_j(y))] - c) \right) \tag{H.21}$$

This also holds for $\theta_i = 1$ if we interpret the term $\frac{\theta_i}{1-\theta_i}c$ as 0 when $c = 0$ and ∞ for $c > 0$.

Moreover, there exists an optimal linear contract for principal i given contract w_j.

Proof. From (H.4) in Lemma 15, changing variables using $1-\theta_i \equiv 1/1+\lambda$, and setting k to guarantee that $w_i(0) = 0$ in (H.3), we get

$$V_i(w) = \frac{\theta_i}{1-\theta_i}\left(V_A(w|\mathcal{A}_0) - \overline{w}_j\right). \tag{H.22}$$

Replacing for V_A we arrive at the desired result.

The function $\frac{\theta}{1-\theta}\left(E_F\left[(1-\theta)y_i - (\overline{w}_j - w_j(y))\right] - c\right)$ is continuous in θ in $[0,1]$, moreover it is also continuous in (F, c) (because w_j is a continuous function) and $\mathcal{A}_0$ is a compact set (constant with respect to θ). Then V_i is continuous in θ as well (by the Theorem of maximum). Because the RHS is continuous in θ it achieves a maximum in $[0,1]$. $\theta^\star$ characterizes the optimal linear contract. □

Finally, we state the proof of Theorem 12.

Proof. **(Theorem 12)** By Lemma 16 there is an optimal contract among the class of linear contracts for principal i, call it $w_i^\star$. Suppose there is an arbitrary contract w_i that does strictly better than $w_i^\star$: $V_i(w_i, w_j) > V_i(w_i^\star, w_j)$. By Lemmas 15 and 4 there exists a linear contract $w_i^{'}$ such that $V_i\left(w_i^{'}, w_j\right) \geq V_i(w_i, w_j)$. This contradicts $w_i^\star$ being optimal among the linear contracts.

Now, impose Assumption 6 and let w_i be an optimal contract for principal i. Define $w_i^{'}$ using (H.3) as in Lemma 15. $w_i^{'}$ satisfies

$$E_F\left[y_i - w_i^{'}(y_i)\right] \geq k + \lambda V_A\left(\left(w_i^{'}, w_j\right)|\mathcal{A}_0\right), \tag{H.23}$$

and V_i satisfies (H.4) with equality. Replacing for k on (H.4) we get

$$E_F\left[y_i - w_i^{'}(y_i)\right] \geq V_i(w) + \lambda\left(V_A\left(\left(w_i^{'}, w_j\right)|\mathcal{A}_0\right) - V_A\left((w_i, w_j)|\mathcal{A}_0\right)\right). \tag{H.24}$$

It must be that $V_A\left(\left(w_i^{'}, w_j\right)|\mathcal{A}_0\right) \geq V_A\left((w_i, w_j)|\mathcal{A}_0\right)$, with strict inequality unless $w_i^{'}$ is identical to w_i, because $w_i^{'}(y_i) \geq w_i(y_i)$ pointwise and $\mathcal{A}_0$ satisfies Assumption 6. Moreover, the equation above holds for all F, so

$$V_i\left(w_i^{'}, w_j\right) \geq V_i(w) + \lambda\left(V_A\left(\left(w_i^{'}, w_j\right)|\mathcal{A}_0\right) - V_A\left((w_i, w_j)|\mathcal{A}_0\right)\right) > V_i(w) \tag{H.25}$$

where the strict inequality holds when w_i is not identical to $w_i^{'}$.

Then, $w_i = w_i^{'}$, or else optimality would be contradicted. Then w_i is linear in y_i. □

I Robust taxation of multinationals

There is a big debate among policy experts and lawmakers on how to reform the corporate income tax with a particular focus on foreign profits. The debate in the United States has centered on whether to adopt a territorial approach–taxing only the profits generated in the U.S.–or a worldwide approach–taxing all profits, foreign and domestic, the same. The need for tax systems to be robust to profit shifting strategies is evident as tax reforms are slow, and complex processes, hard to adapt to changes in firms' actions.

We apply the setup developed in Section 1 to the problem of taxing multinational companies and show that a worldwide tax with a deduction paid for taxes in the foreign countries is robust changes in the firms' production technologies across countries. This is the tax system proposed in the Bipartisan Tax Fairness and Simplification Act of 2011 by Senators Wyden and Coats (Senate Bill 727, 2011).

Consider two countries $i \in \{1,2\}$ and a multinational firm denoted by A. Let π_i be the firm's profit in country i. The set of possible profits that can be declared in country i is $\Pi_i \subset \mathbb{R}$ with $\min \Pi_i = 0$ and $\max \Pi_i = \bar{\pi}_i$. Also $\Pi = \Pi_1 \times \Pi_2$. The firm's actions are distributions (F) over the profits in Π, and a cost (c) associated with each distribution. The firm's action set $(\mathcal{A})$ is then composed by pairs $(F,c) \in \Delta(\Pi) \times \mathbb{R}_+$. The cost can be interpreted as an economic cost (after accounting costs are deducted) of engaging in transfer pricing between the firm's subsidiaries in each country. Alternatively, the cost can be interpreted as unobservable effort from the firm's manager as in Laffont and Tirole (1986).

Each country's government chooses a tax function to maximize their guaranteed corporate tax revenue when they only know a subset $\mathcal{A}_0 \subset \mathcal{A}$, all assumptions on $\mathcal{A}$ and $\mathcal{A}_0$ are as in Section 1. The tax function for country i is a continuous function $t_i : \Pi \to \mathbb{R}$. These tax functions map to the contracts in Section 1 as $t_i(\pi) = \pi_i - w_i(\pi)$ and the multinational's (agent's) payoff is therefore $\sum_i \pi_i - t_i(\pi) = \sum_i w_i(\pi)$.

We consider two different restrictions over the range of the taxes which are equivalent to the two versions of limited liability imposed in the common agency game in Assumptions 4 and 5. We refer to them as weak and strong enforceability:

Weak Enforceability: Countries have weak enforceability if they can only tax up to the amount of profits declared in their respective territories. This implies: $t_i(\pi_1,\pi_2) \leq \pi_i$.

Strong Enforceability: Countries have strong enforceability if they can collect taxes on all profits generated by the firm. This implies: $t_1(\pi_1,\pi_2) + t_2(\pi_1,\pi_2) \leq \pi_1 + \pi_2$.

Weak enforceability is a reasonable restriction for small countries that have a subsidiary of a big multinational. This restriction is equivalent to individual limited liability, Assumption 5.[30] Strong enforceability is a more reasonable restriction for large countries like the United States where the multinational corporation has most of its activity this restriction. This restriction is equivalent to Assumption 4.

The firm's problem is to maximize after tax profits, given a tax scheme $t = (t_1, t_2)$,

$$A^\star(t|\mathcal{A}) = \underset{(F,c)\in\mathcal{A}}{\operatorname{argmax}} E_F\left[(\pi_1 - t_1(\pi)) + (\pi_2 - t_2(\pi))\right] - c. \tag{I.1}$$

[30]Weak enforceability does not amount to a territorial approach to taxation. A territorial approach would amount to restricting the domain of the taxes, so that $t_i(\pi_1,\pi_2) = t_i(\pi_i)$, as in the private common agency setup of Appendix H.

The payoff of government i depends on their tax revenue and the firm's after tax profits,

$$R_i(t) = \inf_{\mathcal{A} \supseteq \mathcal{A}_0} \left\{ \min_{(F,c) \in A^\star(t|\mathcal{A})} E_F \left[t_i(\pi) + \rho_i \sum_{j=1}^{2} (\pi_j - t_j(\pi)) \right] \right\}, \tag{I.2}$$

where $\rho_i \in [0,1]$ is the weight each country puts on the profits of the multinational company. If $0 < \rho_i < 1$ country i cares about raising some distortionary taxes, so that the shadow value of a tax dollar exceeds that of a unit of factor income. See Bond and Gresik (1996) for a justification of the governments' objective function. When $\rho_1 = \rho_2 = 0$ the problem is isomorphic to the common agency problem considered in Section 1.

Similarly to Theorem 1, we can show that given the tax system of country j, country i's best response contains a *worldwide tax*, the equivalent to LRS taxes.

Definition 10. (Worldwide Tax) A tax function t_i is a worldwide (flat) tax rate if the firm's global profits are taxed at a constant rate $\frac{\alpha_i}{1-\alpha_i \rho_i}$, allowing for the full deduction of taxes payed to country j, and a potential tax incentive (in the form of a lump sum subsidy). That is, for some $\alpha_i \in (0,1]$ and $k_i \in \mathbb{R}$:

$$t_i(\pi) = \left(1 - \frac{\alpha_i}{1-\alpha_i \rho_i}\right)(\pi_1 + \pi_2 - t_j(\pi)) + k_i. \tag{I.3}$$

The enforceability regime, i.e, limited liability, determines the constant k_i.

The tax proposed by Senators Wyden and Coats has this form. It proposes a flat tax rate for all profits independently of country of origin. We show that this tax system possesses a robustness property, that a territorial tax system does not have. This property has been informally articulated among tax policy experts (Hungerford, 2014). We provide a rigorous treatment of the policy debate. Crucially, a worldwide tax is not just an equilibrium outcome of the game. A worldwide tax is a best response for country i to any arbitrary tax system of country j.

Interestingly, the worldwide tax has the same form as the taxes found by Feldstein and Hartman (1979). Unlike us, they have a complete information setup and restrict attention to linear tax functions, and their "full taxation after deduction" result rests on concerns on the optimal allocation of capital between countries.

Another important issue is that of the effects of tax competition and the welfare implications of a tax treaty between the countries. As shown in Section 5, competition between countries—the common agency setup–would lead to a lower (higher) overall tax rate on the multinational, relative to cooperation between countries through a tax treaty—the collusion setup—when countries have weak (strong) enforceability and it would lead to a higher (lower) overall tax rate on the multinationals.

I.1 Optimality of the worldwide tax when countries value domestic profits

We establish the optimality of the worldwide tax following the same steps as in Appendices A.1 and D.1. The notation changes so that $\pi = y$ and the ex-post payoffs of governments (principals) and the multinational (the agent) are, respectively,

$$t_i(\pi) = \pi_i - w_i(\pi); \tag{I.4}$$

$$\sum_i \pi_i - t_i(\pi) = \sum_i w_i(pi). \tag{I.5}$$

In the following proofs we consider a particular form of the countries' payoffs where they only care about the domestic profits:

$$R_i(t) = \inf_{\mathcal{A}\supseteq\mathcal{A}_0} \left\{ \min_{(F,c)\in A^\star(t|\mathcal{A})} E_F\left[t_i(\pi) + \rho_i\pi_i\right] \right\}, \tag{I.6}$$

Lemmas 1 and 2 apply without changes. We present them here without proof, adjusting the notation to the multinational case.

Lemma 17. *Let t be a tax scheme, $\mathcal{A}\supseteq\mathcal{A}_0$ be an action set, and $(F,c)\in A^\star(t|\mathcal{A})$ an optimal action for the multinational. Then, it holds that*

$$F \in \mathcal{F} \equiv \left\{ F\in\Delta(Y) | E_F\left[\sum_{i=1}^{2} \pi_i - t_i(\pi)\right] \geq V_A(w|\mathcal{A}_0) \right\}. \tag{I.7}$$

Lemma 18. *Let t be an eligible tax scheme for country i. Then*

$$R_i(t) = \min_{F\in\mathcal{F}} E_F\left[t_i(\pi) + \rho_i\pi_i\right]. \tag{I.8}$$

Moreover, if $F \in \underset{F\in\mathcal{F}}{argmin} E_F\left[t_i(\pi) + \rho_i\pi_i\right]$ then $E_F\left[\sum_{j=1}^{2} \pi_j - t_j(\pi)\right] = V_A(t|\mathcal{A}_0)$.

As before, we can use a separating argument to design what will end up being the optimal tax function. This is the equivalent of Lemma 3.

Lemma 19. *Let t be an eligible tax scheme for country i. There exist k,λ with $\lambda>0$ such that for all $\pi\in\Pi$:*

$$t_i(\pi) \geq \frac{\lambda-\rho_i}{1+\lambda}\pi_i + \frac{\lambda}{1+\lambda}\pi_j - \frac{\lambda}{1+\lambda}t_j(\pi) + \frac{1}{1+\lambda}k; \tag{I.9}$$

$$R_i(t) = k + \lambda V_A(t|\mathcal{A}_0). \tag{I.10}$$

Proof. Let $S\subseteq\mathbb{R}^2$ be the convex hull of all points $(\pi_1 - t_1(\pi) + \pi_2 - t_2(\pi), \rho_i\pi_i + t_i(\pi))$ for $\pi\in\Pi$, and $T\subseteq\mathbb{R}^2$ be the set of all pairs (u,v) such that $u > V_A(t|\mathcal{A}_0)$ and $v < R_i(t)$.

The rest of the proof follows the same steps of Lemma 3, showing that $S\cup T =$ and applying the separating hyperplane theorem to construct (I.9) and (I.10). $\square$

We can them proceed as in Lemma 4 by using (I.9) to construct an alternative tax function that improves the country's guaranteed payoff and satisfies enforceability (weak or strong) with equality. This alternative contract is a version of the worldwide tax (Definition

10) modified to account for the country only placing value on domestic profits. Letting $\alpha \equiv {}^{1}/_{1+\lambda}$ we can write

$$t_i(\pi) = (1-\alpha)(\pi_1 + \pi_2 - t_j(\pi)) - \alpha(\rho_i y_i + k). \tag{I.11}$$

Moreover, among the class of taxes satisfying (I.11) there is an optimal one for country i, as in Lemma 6. This optimal tax is in the countries best response. The argument is the same as in Theorems 1 and 7, and follows from constructing an alternative tax function to any initial tax schedule using Lemma 19 which weakly dominates the initial tax, then this tax is improved upon by the optimal worldwide tax.

Theorem 13. *For any tax t_j, there exists a worldwide tax $\bar{t}_i$ such that $\bar{t}_i \in BR_i(t_j)$ and $\bar{t}_i$ satisfies (I.11) for some α and k, and (weak or strong) enforceability with equality.*

I.2 Optimality of the worldwide tax when countries value worldwide profits

When countries the multinational's worldwide after tax profits, say for efficiency motives, as in I.2, we can also establish the optimality of worldwide taxes (I.3 in Definition 10). All arguments are the same as before, except for that of Lemma 19 that we modify as follows:

Lemma 20. *Let t be an eligible tax scheme for country i. There exist k, λ with $\lambda > 0$ such that for all $\pi \in \Pi$:*

$$t_i(\pi) \geq \frac{\lambda - \rho_i}{1 + \lambda - \rho_i}(\pi_1 + \pi_2 - t_j(\pi)) + \frac{1}{1 + \lambda - \rho_i} k; \tag{I.12}$$

$$R_i(t) = k + \lambda V_A(t|\mathcal{A}_0). \tag{I.13}$$

Proof. Let $S \subseteq \mathbb{R}^2$ be the convex hull of all points $(\pi_1 - t_1(\pi) + \pi_2 - t_2(\pi), \rho_i(\pi_1 + \pi_2 - t_1(\pi) - t_2(\pi)) + t_i(\pi))$ for $\pi \in \Pi$, and $T \subseteq \mathbb{R}^2$ be the set of all pairs (u, v) such that $u > R_A(t|\mathcal{A}_0)$ and $v < R_i(t)$.

The rest of the proof follows the same steps of Lemma 3, showing that $S \cup T =$ and applying the separating hyperplane theorem to construct (I.12) and (I.13).

□

Just as before, we use (I.12) to construct the worldwide tax. Letting $\alpha \equiv {}^{1}/_{1+\lambda}$ we have

$$\frac{\lambda - \rho_i}{1 + \lambda - \rho_i} = \frac{{}^{1}/_{\alpha} - 1 - \rho_i}{{}^{1}/_{\alpha} - \rho_i} = 1 - \frac{1}{{}^{1}/_{\alpha} - \rho_i} = 1 - \frac{\alpha}{1 - \alpha\rho_i}, \tag{I.14}$$

which gives I.3 after redefining the constant k. We omit all other statements of the modified lemmas for space. The equivalent statement of Theorem 13 applies.

Theorem 14. *For any tax t_j, there exists a worldwide tax $\bar{t}_i$ such that $\bar{t}_i \in BR_i(t_j)$ and $\bar{t}_i$ satisfies I.3 for some α and k, and (weak or strong) enforceability with equality.*

J Applications of common agency under Assumption 5

J.1 Constant marginal cost

To better understand the determinants of the share θ we consider the case where the agent's production technology exhibits constant marginal cost of production in total output, putting structure on the agent's action set $\mathcal{A}$. We characterize the agent's production technology via a cost function, f, as in Theorem 8 of Appendix D.2. In particular, we assume that the lowest cost of inducing a given expected total output is proportional to that expected output.

Assumption 8. *For any $x \in [0, \bar{y}_1 + \bar{y}_2]$ there exists $(F, c) \in \mathcal{A}_0$ such that $E_F[y_1 + y_2] = x$ and*

$$\gamma x = \min \{c | (F, c) \in \mathcal{A}_0 \text{ and } E_F[y_1 + y_2] = x\}, \tag{J.1}$$

where $\gamma < 1$ is the marginal cost.

Assumption 8 allows us to replace the maximization of the agent over $(F, c) \in \mathcal{A}_0$ with one over the expected value of total output $x \in [0, \bar{y}_1 + \bar{y}_2]$. We do this to characterize the equilibrium strategies of the principals and the agent.

Proposition 7. *Impose Assumption 8 and let w be a LRS contract scheme such that principal j plays the contract $w_j(y) = (1 - \theta_j) y_j + \theta_j (\overline{y}_i - y_i)$ for some $\theta_j \in [0, 1]$. Then, principal i best responds with a contract of the form $w_i(y) = (1 - \theta_i) y_i + \theta_i (\overline{y}_j - y_j)$ with:*

$$\theta_i = \begin{cases} (1 - \theta_j) - \sqrt{(1 - \theta_j) \gamma \frac{\overline{y}_1 + \overline{y}_2}{\overline{y}_i}} & \text{if } \theta_j < 1 - \gamma \frac{\overline{y}_1 + \overline{y}_2}{\overline{y}_i} \\ 0 & \text{otherwise} \end{cases} \tag{J.2}$$

Moreover, an equilibrium exists. In that equilibrium, if the true action set is $\mathcal{A} = \mathcal{A}_0$, the agent chooses (F, c) such that $E_F[y_1 + y_2] = \overline{y}_1 + \overline{y}_2$ and $c = \gamma (\overline{y}_1 + \overline{y}_2)$.

Proof. Under Assumption 8, the cost function has the form: $f(x) = \gamma x$ for some constant $\gamma > 0$. The agent's value and optimal action are:

$$V_A(w|\mathcal{A}_0) = \max_{x \in X} \{(1 - \theta_1 - \theta_2 - \gamma) x\} + \theta_1 \overline{y}_2 + \theta_2 \overline{y}_1, \qquad x^\star = \begin{cases} \overline{x} & \text{if } 1 - \theta_1 - \theta_2 > \gamma \\ 0 & \text{if } 1 - \theta_1 - \theta_2 < \gamma \\ X & \text{if } 1 - \theta_1 - \theta_2 = \gamma \end{cases}. \tag{J.3}$$

Then, the best response of principal i is

$$\text{BR}_i(w_j) = \underset{\theta_i \in [0, 1 - \theta_j]}{\text{argmax}} \begin{cases} \theta_i (\overline{x} - \overline{y}_j) - \frac{\theta_i}{1 - \theta_1 - \theta_2} \gamma \overline{x} & \text{if } 1 - \theta_1 - \theta_2 > \gamma \\ -\theta_i \overline{y}_j & \text{if } 1 - \theta_1 - \theta_2 \leq \gamma \end{cases}. \tag{J.4}$$

The function in the first case is strictly concave, its critical value if $\overline{x} > \overline{y}$ is

$$\theta_i^\star = (1-\theta_j) - \sqrt{\frac{(1-\theta_j)\,\gamma\overline{x}}{\overline{x}-\overline{y}_j}}. \tag{J.5}$$

This is an interior solution if $1-\theta_j-\theta_i^\star > \gamma$ and $0 \leq \theta_i^\star \leq 1-\theta_j$. These conditions are satisfied if and only if $\frac{\overline{x}-\overline{y}_j}{\overline{x}} > \frac{\gamma}{1-\theta_j}$, i.e., if expected output is enough to pay for the agent's cost and the fees.

Then, principal i's best response is

$$\mathrm{BR}_i(\theta_j) = \begin{cases} \theta_i^\star & \text{if} \quad (1-\theta_j)\left(\overline{x}-\overline{y}_j\right) > \gamma\overline{x} \\ 0 & \text{otherwise} \end{cases}. \tag{J.6}$$

The best response of each principal is then single valued. As in the proof of Theorem 8 of Appendix D.2, this implies the existence of an equilibrium in pure strategies.

□

When $\theta_i = 0$ principal i's guaranteed payoff, V_i, is zero as well. If this is the case in equilibrium, we say that the principal has been driven out of the game. Effectively, the principal renounces their output by setting $w_i(y) = y_i$. In particular, if $\overline{y}_i < \gamma(\overline{y}_1 + \overline{y}_2)$, the principal cannot guarantee themselves a positive payoff, regardless of θ_j (equation J.2). For a principal to be able to profit in the game, they must be able to cover the (total) production cost of the agent. Clearly, when $w_i(y) = y_i$, the principal can always opt for the zero contract ($w_i(y) = 0$). This is another way to opt out of the game because the principal cannot guarantee herself a positive payoff without incentivizing the agent.

J.2 Constant cost

We now make the agent indifferent between actions. The characterization is very similar to that under Assumption 8. We outline it below.

Assumption 9. *Let $(F, c) \in \mathcal{A}_0$, if $E_F[y_1 + y_2] > 0$, then $c = \gamma > 0$.*

Under Assumption 9, the agent will choose to induce the maximum expected total output, as long as it covers the cost γ. Recall that, under LRS contracts, the agent's payoff is increasing in expected total output.

$$x^\star(\theta_1, \theta_2) = \begin{cases} \overline{x} & \text{if} \quad (1-\theta_1-\theta_2)\,\overline{x} > \gamma \\ 0 & \text{otw} \end{cases}. \tag{J.7}$$

Then, the best response of principal i is

$$\mathrm{BR}_i(w_j) = \underset{\theta_i \in [0, 1-\theta_j]}{\operatorname{argmax}} \begin{cases} \theta_i\left(\overline{x}-\overline{y}_j\right) - \frac{\theta_i}{1-\theta_1-\theta_2}\gamma & \text{if } (1-\theta_1-\theta_2)\,\overline{x} > \gamma \\ -\theta_i\overline{y}_j & \text{if } (1-\theta_1-\theta_2)\,\overline{x} \leq \gamma \end{cases}, \tag{J.8}$$

where the function in the first case is strictly concave, its critical value if $\overline{x} > \overline{y}$ is

$$\theta_i^{\star} = 1 - \theta_j - \sqrt{\frac{(1-\theta_j)\,\gamma}{\overline{x} - \overline{y}_j}}. \tag{J.9}$$

This is an interior solution if $(1 - \theta_j - \theta_i^{\star})\,\overline{x} > \gamma$ and $0 \leq \theta_i^{\star} \leq (1-\theta_j)$, which are satisfied if and only if $\overline{x} - \overline{y}_j > \frac{\gamma}{1-\theta_j}$, i.e., if expected output is enough to pay for the agent's cost and the fees. The best response of principal i is:

$$\mathrm{BR}_i(\theta_j) = \begin{cases} \theta_i^{\star} & \text{if} \quad (1-\theta_j)\left(\overline{x} - \overline{y}_j\right) > \gamma \\ 0 & \text{otherwise} \end{cases}. \tag{J.10}$$

The best response of each principal is then single valued. As in the proof of8 of Appendix D.2, this implies the existence of an equilibrium in pure strategies.

J.3 First price auction

Consider now a setup where two competing firms bid for a government contract (such as a contract for the provision of services to the government, the construction of a public good, or the privatization of a government asset). The government announces that the contracting process has a fixed cost $c > 0$, and that the contract will be awarded with the objective of maximizing the government's profits. The cost of the contract can be interpreted as the social benefit of carrying out the project that the contract stipulates, or the valuation of a government asset that is being privatized. Both firms have their own valuation of the contract, we denote them by $\overline{y}_1$ and $\overline{y}_2$. We assume without loss that $\overline{y}_1 > \overline{y}_2 > c$.

The possible outcomes of the contracting process are that firm 1 is awarded the contract, firm 2 is awarded the contract, or the process is declared null and neither firm gets it. In a perfect information setting, this setup is that of a first price auction.[31] However, if the government is known to be corrupt the firms would have reasons to doubt the announcement. For instance, the government can potentially (and secretly) favor one of the firms. It is also possible that the government is willing to randomize between the firms and lower the cost, this might be the case if bids are hard to assess and the government can lower costs at the expense of adding error to the contracting process, or if technicalities can arise that create the chance of a lower bid to be awarded the contract.[32]

We show that there are two equilibria in robust contracts for this game. The output space is $Y = \{(0,0), (\overline{y}_1, 0), (0, \overline{y}_2)\}$ and the known set of actions for the government (the

[31] The bids in the auction are undefined because firm 1 would try to marginally outbid firm 2.

[32] Randomness in who is assigned the contract can also arise from last minute changes in the rules (not uncommon in developing countries), or from challenges made in courts to the rules or the decision of the government. It is worth pointing out that randomization is not itself necessary for our results. The firms could simply be worried that the government can allocate the good with certainty to the other contractor. This is in fact the worst case scenario they face.

agent) are $\mathcal{A}_0 = \left\{ (\delta_0, 0), \left(\delta_{\overline{y}_1}, c\right), \left(\delta_{\overline{y}_2}, c\right) \right\}$. LRS contracts have the following form

$$w_i = \begin{cases} \theta_i \overline{y}_j & \text{if } y = (0,0) \\ \overline{y}_i - \theta_i \left(\overline{y}_i - \overline{y}_j\right) & \text{if } y = (\overline{y}_i, 0) \\ 0 & \text{if } y = \left(0, \overline{y}_j\right) \end{cases}, \tag{J.11}$$

where $w_i\left(0, \overline{y}_j\right) \leq w_i(0,0) \leq w_i(\overline{y}_i, 0)$ because $\theta_i \geq 0$ and $\overline{y}_j > 0$. That is, the principals always pay more if they win the auction, followed by no one winning and lastly if the auction is won by the other principal.

The government's problem is:

$$V_A(w|\mathcal{A}_0) = \max\left\{ \theta_i \overline{y}_j + \theta_j \overline{y}_i,\ (1-\theta_i)\overline{y}_i + \theta_i \overline{y}_j - c,\ (1-\theta_j)\overline{y}_j + \theta_j \overline{y}_i - c \right\}. \tag{J.12}$$

For any strategy of the firms (θ_1, θ_2) the government will either award the contract to the firm with the highest valuation (firm 1) or not award it at all.

The best response of firm 2 given the government's strategy is to set $\theta_2 = 0$ or to offer the zero contract. This gives rise to two equilibria of the game where the government ends up awarding the contract to the firm with the highest valuation.

i. Firm 2 sets $\theta_2 = 0$, bidding $w_2(y) = y_2$, and firm 1 optimally sets

$$\theta_1 = \begin{cases} 1 - \sqrt{\frac{c}{\overline{y}_1 - \overline{y}_2}} & \text{if } c\left(\frac{\overline{y}_1 - \overline{y}_2}{\overline{y}_1}\right) < \overline{y}_1 \wedge \overline{y}_2 + c < \overline{y}_1 \\ 0 & \text{otherwise} \end{cases}, \tag{J.13}$$

guaranteeing that the government will prefer awarding the contract to firm 1 over declaring the process null, and that the firm 1's valuation is enough to pay the government's cost and compensate it for not awarding the contract to firm 2 (this ensures that $\theta_1 \geq 0$).

ii. Firm 2 walks away from the bid, setting $w_2(y) = 0$, and firm 1 sets $w_1(y) = (1-\theta_1)y_1$ with $\theta_1 = 1 - \sqrt{c/\overline{y}_1}$. For this to be an equilibrium, the zero contract must be a best response for firm 2. That is the case when $\overline{y}_2 < \sqrt{c\overline{y}_1}$.

If $\overline{y}_1 = \overline{y}_2$, there are no eligible contracts for the firms, because the government will be indifferent between them and neither firm can guarantee to be awarded the contract. Because of this the only equilibrium in that case is for both of them to set $w_i(y) = y_i$.

J.4 Provision of public goods

Consider now an agent that produces one unit of a public good with variable quality $q \in [0,1]$ at a cost $f(q) = \gamma q$. There are two principals that value the public good with $y_i = \nu_i q$, $i \in \{1,2\}$. The output space is then

$$Y = \left\{ (y_1, y_2) \in \mathbb{R}_+^2 | \exists_{q \in [0,1]} \quad y_1 = v_1 q \quad \wedge \quad y_2 = v_2 q \right\}. \tag{J.14}$$

Output is perfectly correlated across principals (as opposed to a cross product space as in our baseline model), making so that there is no competition factor as both principals can take advantage of the public good simultaneously. We show that this will only change the intercept of the LRS contract. The efficient outcome is of course to provide the good at highest quality if $\nu_1 + \nu_2 \geq \gamma$.

The equilibrium has each principal "partially" free riding on the other by lowering compensation by a fraction of the other principal's payoff, while guaranteeing that the agent optimally chooses to set $q = 1$. An interesting feature of this equilibrium is that no matter how different the valuations are, all principals get the same share of expected output and the same guaranteed payoff.

Proposition 8. *Let θ be such that ${}^{1-\theta}/_{(1-2\theta)^2} = {}^{\nu_i+\nu_j}/_{\gamma}$. The contracts $w_i(y) = (1-\theta)\, y_i - \theta y_j$ are an equilibrium of the game if $\frac{(\nu_j-\nu_i)^2}{\max\{\nu_i,\nu_j\}} \leq \gamma \leq \nu_1 + \nu_2$.*

Proof. The LRS contracts in equilibrium change because of our assumption on the output space Y. If principal j offers a contract $w_j = (1-\theta_j)\, y_j - \theta_j y_i$, then the LRS contract of principal i, as in (7), is increasing in both y_i and y_j as long as:

$$(\alpha + (1-\alpha)\,\theta_j)\,\nu_i - (1-\alpha)\,(1-\theta_j)\,\nu_j \geq 0 \tag{J.15}$$

In this case, the minimum is achieved when $y_i = y_j = 0$. This implies $k = 0$ and $w_i = (1-\theta_i)\, y_i - \theta_i y_j$, with $\theta_i = (1-\alpha)\,(1-\theta_j)$ and no fees payed to the agent. Condition (J.15) is verified later.

The value of the agent is

$$V_A\left(w|\mathcal{A}_0\right) = \max\left\{0\,,\, (1-\theta_1-\theta_2)\,(\nu_1+\nu_2) - \gamma\right\}. \tag{J.16}$$

The agent will choose either to induce the highest quality of not to produce at all.

The principal's best response are

$$\mathrm{BR}_i\left(w_j\right) = \underset{\theta_i \in [0,1-\theta_j]}{\operatorname{argmax}} \begin{cases} \theta_i\,(\nu_1+\nu_2) - \frac{\theta_i}{1-\theta_1-\theta_2}\gamma & \text{if } (1-\theta_1-\theta_2)\,(\nu_1+\nu_2) > \gamma \\ -\theta_i \overline{y}_j & \text{if } (1-\theta_1-\theta_2)\,(\nu_1+\nu_2) \leq \gamma \end{cases}. \tag{J.17}$$

The interior solution assuming that the agent produces is

$$\theta_i^\star = (1-\theta_j) - \sqrt{\frac{(1-\theta_j)\,\gamma}{\nu_1+\nu_2}} \tag{J.18}$$

Moreover, in equilibrium it must be that

$$\frac{1-\theta_j}{\left(1-\theta_i-\theta_j\right)^2} = \frac{\nu_i+\nu_j}{\gamma} \qquad \wedge \qquad \frac{1-\theta_i}{\left(1-\theta_i-\theta_j\right)^2} = \frac{\nu_i+\nu_j}{\gamma} \tag{J.19}$$

which implies that $\theta_i = \theta_j = \theta$, where θ is such that: $\frac{1-\theta}{(1-2\theta)^2} = \frac{\nu_i+\nu_j}{\gamma}$.

It is left to verify the assumptions, namely condition (J.15) which is satisfied if $\frac{(\nu_j-\nu_i)^2}{\max\{\nu_i,\nu_j\}} \leq \gamma$, and profitability of the agent ($(1-\theta_1-\theta_2)\,(\nu_1+\nu_2) > \gamma$), feasibility of the share θ $\left(0 \leq \theta \leq \frac{1}{2}\right)$ and profitability of the principals, which are always satisfied.

□

References

Antic, Nemanja, "Contracting with Unknown Technologies," 2021. Northwestern Working Paper, https://www.kellogg.northwestern.edu/faculty/antic/ContractingUnknownTech.pdf.

Bernheim, B. Douglas and Michael D. Whinston, "Common Agency," *Econometrica*, 1986, *54* (4), 923–942.

_ and _ , "Menu Auctions, Resource Allocation, and Economic Influence," *The Quarterly Journal of Economics*, 1986, *101* (1), 1–32.

_ and Michael Whinston, "Common Marketing Agency as a Device for Facilitating Collusion," *RAND Journal of Economics*, 1985, *16* (2), 269–281.

Bond, Eric W. and Thomas A. Gresik, "Regulation of multinational firms with two active governments: A common agency approach," *Journal of Public Economics*, January 1996, *59* (1), 33–53.

Breton, Michel Le and Francois Salanie, "Lobbying under political uncertainty," *Journal of Public Economics*, 2003, *87* (12), 2589–2610.

Carmona, Guilherme and José Fajardo, "Existence of equilibrium in common agency games with adverse selection," *Games and Economic Behavior*, 2009, *66* (2), 749 – 760. Special Section In Honor of David Gale.

Carroll, Gabriel, "Robustness and Linear Contracts," *American Economic Review*, 2015, *105* (2), 536–63.

_ and Daniel Walton, "A General Framework for Robust Contracting Models," *Econometrica*, 2022, *Forthcoming.*

Chassang, Sylvain, "Calibrated Incentive Contracts," *Econometrica*, 09 2013, *81* (5), 1935–1971.

Dai, Tianjiao and Juuso Toikka, "Robust Incentives for Teams," *Econometrica*, 2022, *Forthcoming.*

Debreu, Gerard, "A Social Equilibrium Existence Theorem," *Proceedings of the National Academy of Sciences*, 1952, *38* (10), 886–893.

Dewatripont, M. and E. Maskin, "Credit and Efficiency in Centralized and Decentralized Economies," *The Review of Economic Studies*, 10 1995, *62* (4), 541–555.

Dixit, Avinash, Gene M Grossman, and Elhanan Helpman, "Common Agency and Coordination: General Theory and Application to Government Policy Making," *Journal of Political Economy*, 1997, *105* (4), 752–69.

Dixit, Avinash K., *The making of economic policy : a transaction-cost politics perspective*, MIT Press Cambridge, Mass., 1996.

Dubey, Pradeep, Ori Haimanko, and Andriy Zapechelnyuk, "Strategic complements and substitutes, and potential games," *Games and Economic Behavior*, 2006, *54* (1), 77–94.

Feldstein, Martin and David Hartman, "The Optimal Taxation of Foreign Source Investment Income," *The Quarterly Journal of Economics*, 1979, *93* (4), 613–629.

Frankel, Alexander, "Aligned Delegation," *American Economic Review*, January 2014, *104* (1), 66–83.

Fraysse, Jean, "Common Agency: Existence of an Equilibrium in the Case of Two Outcomes," *Econometrica*, 1993, *61* (5), 1225–1229.

Garrett, Daniel, "Robustness of simple menus of contracts in cost-based procurement," *Games and Economic Behavior*, 2014, *87*, 631–641.

Grossman, Gene M and Elhanan Helpman, "Protection for Sale," *American Economic Review*, September 1994, *84* (4), 833–50.

Holmstrom, Bengt and Paul Milgrom, "Aggregation and linearity in the provision of intertemporal incentives," *Econometrica*, 1987, *55* (2), 303–328.

_ and _, "Common Agency and Exclusive Dealing," April 1988. MIT Working Paper, https://economics.mit.edu/files/3779.

Hungerford, Thomas L., "The simple fix to the problem of how to tax multinational corporations - Ending deferral," Issue Brief 378, Economic Policy Institute March 2014.

Hurwicz, Leonid, "On the Interaction between Information and Incentives in Organizations," in Klaus Krippendorf, ed., *Communication and Control in Society*, Gordon and Breach, Science Publishers, 1977, pp. 123–147.

_ and Leonard Shapiro, "Incentive Structures Maximizing Residual Gain under Incomplete Information," *Bell Journal of Economics*, Spring 1978, *9* (1), 180–191.

Laffont, Jean-Jacques and Jean Tirole, "Using Cost Observation to Regulate Firms," *Journal of Political Economy*, 1986, *94* (3, Part 1), 614–641.

Laussel, Didier and Michel Le Breton, "Efficient Private Production of Public Goods under Common Agency," *Games and Economic Behavior*, November 1998, *25* (2), 194–218.

Maier, Norbert and Marco Ottaviani, "Information Sharing in Common Agency: When is Transparency Good?," *Journal of the European Economic Association*, 2009, *7* (1), 162–187.

Martimort, David, "The multiprincipal nature of government," *European Economic Review*, 1996, *40* (3-5), 673–685.

_ , "Multi-Contracting Mechanism Design," in Richard Blundell, Whitney K. Newey, and Torsten Persson, eds., *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress*, Vol. 1, Cambridge University Press, 2006, pp. 57–101.

_ **and Aggey Semenov**, "Ideological uncertainty and lobbying competition," *Journal of Public Economics*, April 2008, *92* (3-4), 456–481.

_ **and Humberto Moreira**, "Common agency and public good provision under asymmetric information," *Theoretical Economics*, May 2010, *5* (2), 159–213.

_ **and Lars Stole**, "Representing equilibrium aggregates in aggregate games with applications to common agency," *Games and Economic Behavior*, 2012, *76* (2), 753–772.

_ **and** _ , "Menu Auctions and Influence Games with Private Information," MPRA Paper 62388, University Library of Munich, Germany February 2015.

Mezzetti, Claudio, "Common agency with horizontally differentiated principals," *The RAND Journal of Economics*, 1997, pp. 323–345.

Milgrom, Paul, "Package Auctions and Exchanges," *Econometrica*, 07 2007, *75* (4), 935–965.

Monderer, Dov and Lloyd S. Shapley, "Potential Games," *Games and Economic Behavior*, 1996, *14* (1), 124 – 143.

Senate Bill 727, *Bipartisan Tax Fairness and Simplification Act of 2011*, 112th Congress (2011-2012), 2011.