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The Collapse and Recovery of the Capital Share in East Germany After 1989*

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Abstract

After the 1990 unification, East Germany's capital income share plunged to 15.2 percent in 1991, then increased to 37.4 percent by 2015. To account for these large changes in the capital share, I model an economy that gains access to a higher productivity technology embodied in new plants. As existing low productivity plants decrease production, the capital share varies due to the non-convex production technology: plants require a minimum amount of labor to produce output. Two policies—transfers and government-mandated wage increases—have opposite effects on output growth, but contribute to lowering the capital share early in the transition.

Keywords: technological change, capital share, labor share, transfers, union markups

JEL Classification: E20, E25, O11

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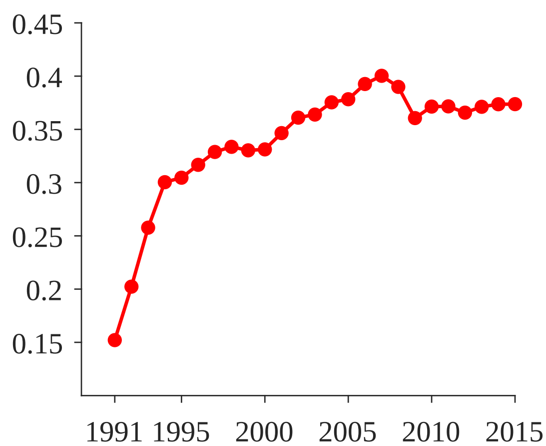
1 Introduction

The stability of the capital and labor income shares over time and across countries is well documented (e.g., Kaldor (1961), Gollin (2002)), and disciplined the choice of aggregate production function in macroeconomic models (e.g., Cooley and Prescott (1995)). This paper documents a dramatic departure from the constancy of factor income shares in East Germany following the 1990 unification with West Germany. The capital share in East Germany was 15.2 percent in 1991, doubled to 30.5 percent in 1995, and settled around 37 percent in the early 2000s (Figure 1).[1] While data prior to unification is incomplete, available information suggests the capital share was high in 1989 in East Germany.[2]

Figure 1: EAST GERMANY'S CAPITAL INCOME SHARE



NOTES: Data for East Germany cover five Bundesländer: Brandenburg, Mecklenburg-Vorpommern, Sachsen, Sachsen-Anhalt and Thüringen. The capital income share is defined as is standard in the literature (see equation 1 in Section 2). The data used to construct the capital share are from VGRdL (see Appendix A.1).

What accounts for East Germany's low capital income share following unification? To answer this question, I build a dynamic general equilibrium model in which an economy transitions from a low to high productivity technology embodied in plants. This experiment is motivated by the opening up of East Germany, which allowed replacing outdated capital with more productive technologies.[3] I incorporate two policies that set East Germany apart from other transition economies: large transfers from West Germany and

[1]Section 2 provides details on the computation of the capital income share in East Germany since 1991.

[2]Two components of GDP suggest the (gross) capital share was high in East Germany in 1989. First, the compensation of employees to GDP ratio was about 40 percent in East Germany in 1989, as documented by Sinn and Sinn (1992, pp. 209-216). Second, East Germany's consumption of fixed capital—one component of gross capital income—was 15 percent of GDP in 1988, as reported by United Nations (1988, Table 1.3,p. 611). This value is close to the average depreciation to GDP ratio for East Germany during the 1980s.

[3]For example, Sinn and Sinn (1992, p. 44) argue that East Germany's capital stock was "fairly obsolete"

dramatic wage increases. While technology change delivers time-variation in the capital share, the policies are mainly responsible for the low capital share early in the transition.

In the model, plants require a minimum amount of labor to produce output, as in Hansen and Prescott (2005). Thus, plants may be operated or left idle at any time. Above the minimum scale, plants operate a decreasing returns technology. Building new plants requires one unit of capital investment. New plants are built only if the expected discounted value of profits equals the cost of one unit of capital. Since the cost of new capital is not plant specific and since high productivity plants have higher profits, their stock accumulates over the transition, while no investments are made in low productivity plants.

The model's capital share varies endogenously due to declining profit shares at low productivity plants. The key feature driving this result is the non-convexity in the plant production technology. When plants employ more than the minimum required labor, plant labor productivity is proportional to the wage, and profits are a constant share of output. However, when the minimum labor constraint binds, a plant's marginal product of labor is lower than the wage, and the profit share is low. During the transition, low productivity plants have declining profit shares, since they mostly operate at the minimum scale where labor productivity is fixed, but wages grow during the transition.[4] High productivity plants have a constant profit share, as they always operate above the minimum scale. As low productivity plants are left idle or exit, the economy's capital share— computed as a weighted average of plant profit shares—displays U-shaped dynamics. In the new steady state, only high productivity plants operate and the capital share is constant.

Two government policies are incorporated in the model, to mirror East Germany's experience. To capture the extensive financial support received from West Germany, I model transfers from abroad that allow the economy's total use of resources to expand beyond domestic production. These transfers are available for either consumption or investment, consistent with the data. To capture the dramatic wage increases in the early years of the East German transition, I model union markups, whose magnitude is mandated by the government.[5] The data on transfers and wage increases are presented in Section 2. Both transfers and union markups raise the cost of labor, reduce the profit share of low produc-

and "meant for methods of production that have little in common with the kind of specialization required for an internationally competitive economy".

[4]In the calibrated model, low productivity plants operate at the minimum scale by period three.

[5]Burda and Hunt (2001) argue that unions were able to choose wages in East Germany early in the transition. Akerlof et al. (1991) argue that the West German government supported the wage increases since it thought this would prevent migration to the West and the ensuing reduction in Western wages. Sinn (2007) documents that the head of the *Treuhandanstalt*—the Trust set up to privatize the East German companies—was told, by the federal government, to stay away from wage bargaining. He argues the German government bears responsibility for allowing the wage increases.

tivity plants, and contribute to lowering the capital share further during the transition.[6]

To evaluate the model's success in delivering a decline in the capital share, I examine an experiment in which the transfers and wage markups are exogenous time-varying inputs calibrated to the size of financial transfers and wage increases in East Germany. In the initial period (which corresponds to 1989), the model's capital share equals the calibrated steady state value of 0.365. In the third period (1991), the capital share declines to 0.256, and then rises throughout the transition. I perform counterfactual experiments to decompose the relative importance of technology change and government policies in delivering the 11 percentage point drop in the capital share. Technology change in conjunction with the non-convexity of the plant production technology generates small variations (of up to 5 percentage points) in the capital share throughout the transition. However, the timing is inconsistent with data, since the capital share in 1991 remains at 0.365 in this experiment. As a result, the large transfers and wage increases account for all of the 11 percentage point decline in the capital share. Transfers alone deliver a capital share of 0.35 in 1991, while wage markups alone yield a capital share of 0.318. Together, the two policies interact nonlinearly and amplify their effects on the capital share.

The model's predictions for output and expenditure shares line up reasonably well with East German data. Output declines in the first two years of the model—by one third of the drop in the data—due to a large decline in labor supply, which is consistent with data. Thereafter, the model closely tracks East Germany's output growth from 1991 to 2015. The two government policies have offsetting effects on output growth. Transfers have a positive impact on growth, as they allow for more investment in the high productivity capital.[7] Wage increases depress output growth by deterring capital accumulation (as suggested, for example, in Akerlof et al. (1991) and Sinn (2007)) and by depressing labor supply. Lastly, the model's predictions for expenditure shares of output closely track East German data. The model's mean consumption to output ratio is 1.01, consistent with data, an outcome made possible through the large transfers.

The 1990 German unification saw large migration flows from East to West Germany. In this paper, I model East Germany as an economy that receives transfers from abroad and sees large wage increases, but I abstract from migration flows (see Hunt (2006), Uhlig (2006) or Fuchs-Schündeln and Izem (2012) for facts on migration, and Raffelhüschen (1992) for very early estimates on migration). To gauge the quantitative importance of the closed economy assumption, I incorporate exogenously shrinking population in the

[6] Sinn (2007, pp. 162-163) argues the dramatic increase in East German wages altered investment prospects and kept investors away. He documents that only 9 percent of the privatized East German companies were sold to foreign investors, and attributes this to expectations of low profits due to high wages.

[7] Fischer, Sahay, and Végh (1996) also show that foreign aid is conducive to higher GDP growth.

model. While population changes give an incomplete view of the impact of migration—since they do not capture changes in the age composition and skill of the non-immigrant population—I find that the large declines in East Germany's population only contribute an additional 0.5 percentage points to the initial decline in the capital share. For this reason, I leave more detailed explorations of migration and its quantitative impact on the capital share to future research.

A recent literature has documented a substantial and prolonged decline in the labor share in the United States of 5 to 6 percentage points (Karabarbounis and Neiman (2014), vom Lehn (2018)). In contrast, the East German experience features sharp changes in the capital share—an initial decline and subsequent spike—that are about 3 times larger. Similar to Grossman et al. (2017), this paper explores the implications that productivity has on the capital share. I show that while technology change alone delivers relatively small variations in the capital share (of about 5 percentage points), policies such as transfers and government-mandated wage increases can significantly amplify the effects.

This paper also contributes to the literature on transition economies, by showing that a significant part of the initial decline in output observed in East Germany was due to government policies that led to a large fall in labor supply. Existing work has explored common factors that account for the sharp declines in output in transition economies in the early 1990s.[8] However, it is well documented that East Germany experienced the largest downturn in output and decline in labor supply among Eastern European Economies.[9,10] The quantitative analysis in this paper shows that the government policies implemented in East Germany's transition reduced labor supply and led to a sizable decline in output.

The remainder of the paper is organized as follows. Section 2 presents facts about the East German transition. Section 3 develops the theoretical framework. Section 4 presents the quantitative results and the extension with population changes. Section 5 examines the nonlinear impact of government policies on the capital share, Section 6 performs sensitivity analysis and Section 7 concludes.

[8]Declines in output for a number of transition economies are documented, for example, in Blanchard and Kremer (1997) and Roland and Verdier (1999). For a survey of the literature on the various explanation for these initial output declines, see Campos and Coricelli (2002). Some quantitatively important factors are costs associated with the adoption of new technologies of production (as in Atkeson and Kehoe (1993)) or collapses in trade (Rodrik (1994) and Gorodnichenko, Mendoza, and Tesar (2012)).

[9]Dornbusch and Wolf (1994, Table 5.3) compare changes in gross domestic product for East Germany, Hungary, Czechoslovakia and Poland from $1989 - 1991$ to emphasize the severity of East Germany's initial downturn. Also see Sinn and Sinn (1992, pp. 29-31). I reach the same conclusion when comparing gross domestic product per working-age person across a number of Eastern European economies (Appendix A.1).

[10]Blanchard, Commander, and Coricelli (1995) document changes in employment for a number of economies, while Dornbusch and Wolf (1994) report it for East Germany. I compute hours worked per working-age person for East Germany and a number of other transition economies (see Appendix A.1 for data sources). I find the decline in hours was largest in East Germany.

2 Facts about the East German Transition

Of the many economies that underwent economic restructuring since the early 1990s, East Germany stands out due to two government policies: the large transfers received from West Germany, and the dramatic increases in wages negotiated by the West German unions with the support of the German government (see, for example, Sinn (2002)).

Net transfers to East Germany in 1991 were 53.7 percent of East German GDP (Uhlig (2008, Figure 2)).[11] Despite their decline over time, net transfers still amounted to 32 percent of East German GDP in 2003. While some transfers were used to subsidize investment, a large portion were channeled to consumption through social benefit programs (see, for example, Jensen (2004)). As a result, East Germany's consumption to output ratio averaged 1.01 over $1991-2015$.

The increases in wages in East Germany were equally dramatic. Real hourly wages increased by about 12.5 percent from $1989-1990$ and by about 32 percent from $1990-1991$ (Table 1). The high wage increases were made possible through the intervention of West German unions with the government's support. One of the reasons for raising wages was the fear that East Germany would become an attractive location for foreign investors, due to low wage costs, and would provide unwanted competition for firms in West Germany (see, for example, Sinn (2007) and Burda and Funke (2001)).[12]

Table 1: EAST GERMANY'S ANNUAL HOURLY WAGE GROWTH (IN PERCENT)

1989 − 1990	1990 − 1991	1991 − 1995	1995 − 2005	2005 − 2015
12.5	32.3	5.7	1.7	1.7

NOTES: Data for $1989-1990$ is from Krueger and Pischke (1995) and, although it measures nominal monthly wage growth, it reflects real growth due to a stable average consumer price index (CPI). Specifically, Akerlof et al. (1991) report that average CPI in East Germany was almost unchanged through the end of 1990. Since data for $1989-1990$ is not hourly, it likely represents an underestimate of the true hourly wage growth, because hours worked declined during that period. Data for $1990-1991$ is real hourly wage growth from Hunt (2001). Hourly wage growth rates for $1991-2015$ are real and deflated using the consumption deflator from national accounts (see Appendix A.1 for more details). Data over specific intervals of years (e.g., $1991-1995$, $1995-2005$, $2005-2015$) represent averages of annual growth rates in hourly wages.

[11] Transfers are transactions in which an entity receives goods, services or cash payments from another entity without providing anything in return. Net transfers are the difference between transfers received from the rest of the world and transfers paid to the rest of the world.

[12] Other reasons for the wage increases are discussed in Akerlof et al. (1991). For details on the wage bargaining process and the rapid increase in wages in East Germany see Krueger and Pischke (1995), Burda and Hunt (2001), Burda and Funke (2001), Dornbusch and Wolf (1994) and Sinn (2007, pp. 161-163). The last two sources also discuss the replacement of the East German mark with the Deutschmark. They argue that the one-to-one currency conversion was not the main mistake made by the German government, but the wage increases that came after this conversion were more problematic. Particularly, without the ensuing wage increases, the currency conversion would have left East German wages relatively low.

Is spite of the large transfers, East Germany did not experience the growth miracle many had hoped for.[13] After a two-year contraction that shrunk the economy to about two-thirds of its 1989 level, output per working-age ($15-64$) person in East Germany grew rapidly from $1991-1995$, at an average yearly growth of 9.9 percent, before abruptly slowing to 2.3 percent over $1995-2015$. While East Germany's rapid growth stood out over $1991-1995$, other transition economies that did not receive extensive financial support fared better in terms of per capita GDP growth in the last two decades.[14] In hindsight, this outcome is not surprising. Canova and Ravn (2000) argue the weak growth was due to transfers. Burda (2006) and Snower and Merkl (2006) argue that East Germany's performance was crippled by the transfers received and the large negotiated wage increases. The wage increases likely depressed output growth by deterring capital accumulation (see, for example, Akerlof et al. (1991) and Sinn (2007)).

Similar to the growth experience, the behaviour of East Germany's capital income share seems puzzling at first. As documented in Figure 1, the capital share doubled from 15.2 to 30.5 percent over $1991-1995$ and stabilized around 37 percent since the early 2000s. Following Kravis (1959) and Gollin (2002), the capital income share is defined as:

$$1-\frac{\text{Compensation of Employees + Labor Income of Self-Employed}}{\text{Gross Domestic Product - Indirect Taxes}} \tag{1}$$

In equation 1, the labor income of the self-employed is imputed as in Gollin (2002, p. 468), assuming that a self-employed person earns the average employees' compensation (Appendix A.1). The dramatic increase in East Germany's capital share (Figure 1) is driven by a 36 percent decrease in the compensation of employees to GDP ratio from $1991-2015$. The labor income of self-employed and indirect taxes have a relatively small impact on the overall increase in the capital share since 1991.

Section 3 develops a model of an economy which transitions from a low to high productivity technology embodied in plants. I show that the two government policies—the large transfers and dramatic wage increases—were main drivers of the decline in East Germany's capital share early in the transition.

[13] Hoffmann (1992) documents the differing outlooks that prevailed early in the transition. Some believed that East Germany would grow fast, while others feared it would develop into a depressed region.

[14] Poland and Slovakia grew by more than East Germany since 1995 (see Appendix A.1 for data sources).

3 Model Environment

The economy is populated by a large number of infinitely lived consumers who derive utility from a final good and leisure. Consumers are heterogenous in the type of labor they supply to the market, and are organized in a continuum of unions, based on their labor type. Unions set the wage and face a downward sloping demand for their labor. The final good is produced by high and low productivity plants, which may operate concurrently. The government sets tax rates and limits the maximum markups that unions may charge.

3.1 Production Technology

Aggregate output is produced by plants using capital and a composite of differentiated types of labor. There are two types of plants distinguished by their productivity level, which can be low (L) or high (H). Plants with productivity $z_i \in \{z_L, z_H\}$ are referred to as z_i-plants, where $z_L < z_H$. The output of a z_i-plant at time t is modelled as in Hansen and Prescott (2005) through a decreasing returns technology above a minimum scale:

$$y_{i,t} = \begin{cases} z_i n_{i,t}^{1-\theta} & \text{if } n_{i,t} \geq \bar{n} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Here, $\theta \in (0, 1)$, and $n_{i,t}$ is the quantity of composite labor input employed by a z_i-plant at t.[15] There is a minimum requirement of (composite) labor necessary to operate a plant, which is identical across plant types and is given by $\bar{n}$. Each plant uses one unit of capital which has "one-hoss-shay" depreciation. The capital operates at its full efficiency until the plant exits the economy, with probability δ_i.

The decreasing returns technology guarantees that it is optimal to operate many small plants rather than one large plant. Moreover, all operating plants of the same type will employ the same amount of labor. The requirement that $n_{i,t} \geq \bar{n}$ together with a limited number of potential workers implies an upper bound on the number of plants that can be operated. Labor can be moved across locations at no cost, hence it will only be allocated to plants with $n_{i,t} \geq \bar{n}$. As a result, at any time some plants may be left idle.

New plants are built only if the expected discounted value of profits equals the cost of one unit of capital investment.[16] Since the cost of capital investments is not plant spe-

[15] The plant labor input, $n_{i,t}$, for $i \in \{L, H\}$ is a composite of differentiated types of labor, $l_{i,t}(j)$, $j \in [0, 1]$. Here, $l_{i,t}(j)$ is the labor of type j hired by z_i-plants at time t. The choice of $l_{i,t}(j)$ for a given choice of composite labor input, $n_{i,t}$ is described in footnote 18.

[16] Formally, at any time period s, investing 1 unit of capital gives expected discounted profits of $\sum_{t=s+1}^{\infty} \beta^{t-s} \frac{U'(C_t)}{U'(C_s)} \cdot (1 - \tau_M) \pi_{i,t} \cdot [1 - (1 - \tau_M) \delta_i]^{t-s-1}$. Here, profits for z_i-plants at time t are denoted

cific, and since high productivity plants have higher profits, new capital investments are made only in high productivity plants. As a result, during a transition the stock of high productivity plants accumulates, while there is no investment in low productivity plants.

Let $M_{i,t}$ denote the stock of z_i-plants that can *potentially* operate at time t and N_t the aggregate labor to be employed at t.[17] The aggregate production function is defined in (3), where $m_{i,t}$ is the mass of z_i-plants operated at t and allocated $n_{i,t}$ units of labor input.

$$F\left(N_t, M_{H,t}, M_{L,t}\right) \equiv \max_{\{m_{i,t},n_{i,t}\}_{i\in\{H,L\}}} m_{H,t} z_H n_{H,t}^{1-\theta} + m_{L,t} z_L n_{L,t}^{1-\theta} \tag{3}$$
$$\text{s.t. } m_{H,t} n_{H,t} + m_{L,t} n_{L,t} \leq N_t$$
$$0 \leq m_{i,t} \leq M_{i,t}, \text{ for all } i$$
$$n_{i,t} \geq \bar{n}, \text{ for all } i$$

Finding the maximum output that can be produced at t involves solving a static optimization problem to allocate the aggregate labor input, N_t, across the available plants. If there are enough high productivity plants to employ all the labor supplied, it is optimal to leave all low productivity plants idle. If there is a scarcity of z_H-plants, some or all of the z_L-plants are operated. It is clear that the market clearing condition for aggregate labor in problem (3) holds with equality. The other inequality constraints bind or not, indicating the mass of plants that operate and the amount of labor input assigned to each plant.

Proposition 1 *Let $\alpha \equiv \left[(1-\theta)\, z_H/z_L\right]^{1/\theta}$ and $\rho \equiv \left(z_H/z_L\right)^{1/\theta}$. The aggregate production function is the solution to the maximization problem in (3) and is given by:*

$$F\left(N_t, M_{H,t}, M_{L,t}\right) = \begin{cases} z_L\left(\rho M_{H,t} + M_{L,t}\right)^{\theta} N_t^{1-\theta} & \text{if } N_t \geq \eta_{1,t} \\ z_H M_{H,t}^{\theta}\left(N_t - M_{L,t}\bar{n}\right)^{1-\theta} + z_L \bar{n}^{1-\theta} M_{L,t} & \text{if } \eta_{2,t} \leq N_t \leq \eta_{1,t} \\ A M_{H,t} + z_L \bar{n}^{-\theta} N_t & \text{if } \eta_{3,t} \leq N_t \leq \eta_{2,t} \\ z_H M_{H,t}^{\theta} N_t^{1-\theta} & \text{if } \eta_{4,t} \leq N_t \leq \eta_{3,t} \\ z_H \bar{n}^{-\theta} N_t & \text{if } N_t \leq \eta_{4,t} \end{cases} \tag{4}$$

where $\eta_{1,t} \equiv \left(\rho M_{H,t} + M_{L,t}\right)\bar{n}$, $\eta_{2,t} \equiv \left(G_\alpha M_{H,t} + M_{L,t}\right)\bar{n}$, $\eta_{3,t} \equiv G_\alpha M_{H,t}\bar{n}$, $\eta_{4,t} \equiv M_{H,t}\bar{n}$ and where $G_\alpha \equiv \max\{1, \alpha\}$ and $A = \left[z_H\left(G_\alpha\right)^{1-\theta} - z_L G_\alpha\right]\bar{n}^{1-\theta}$.

Proof. See Appendix *A.2.1*. ■

by $\pi_{i,t}$ and are taxed at rate τ_M (allowing for depreciation) and δ_i is the probability a z_i-plant exits (due to capital depreciation). Since households own the plants, profits at any period t are discounted using the households' marginal utility of consumption relative to period s when the investment occurred.

[17] Equipment is a small fraction (about 15 percent) of the total capital stock in both East and West Germany. Equipment is not modelled explicitly, but rather it is considered tied to the structures (i.e., plants).

Since the plant production function has decreasing returns to scale above $\bar{n}$, plants have an optimal size (equation 2). If the stock of high productivity plants is not high enough to employ all the labor, then some or all of the low productivity plants operate. In the first branch of the aggregate production function in equation (4), when $N_t \geq \eta_{1,t} \equiv (\rho M_{H,t} + M_{L,t})\,\bar{n}$, all plants of both types operate and are allocated more than $\bar{n}$ units of labor (see plant labor choices in Appendix *A.2.2*). For lower N_t, a higher fraction of total labor can be employed at high productivity plants and low productivity plants decrease their scale. First, low productivity plants hire $\bar{n}$ workers (second branch of the production function). Then, some low productivity plants stop operating. In this case, low productivity plants are not a scarce input and they earn no rent (third branch of the production function). Finally, no low productivity plants operate, all labor is hired at high productivity plants and the aggregate production function is Cobb-Douglas. If $N_t < M_{H,t}\bar{n}$, only some high productivity plants operate and the rest are idle. This situation does not occur in equilibrium (Section 3.4) and is reflected in the model's parameterization (Section 4.1).

Remark 2 *The aggregate production function, $F : \mathbb{R}^3 \to \mathbb{R}$ is: (i) continuous, (ii) homogeneous of degree one, (iii) weakly increasing (iv) differentiable everywhere except at $N = \eta_1 = M_H\bar{n}$, and (v) weakly concave.*

The aggregate labor input is a composite of differentiated types of labor, $l_t(j)$, $j \in [0,1]$. Then, $N_t = \left[\int l_t(j)^\nu\, dj\right]^{1/\nu}$, where $1/(1-\nu)$ is the elasticity of substitution between the differentiated types of labor.

There is a representative firm in the economy that operates the plants. The problem of the representative firm can be stated in two parts. First, given factor prices: $r_{H,t}$, $r_{L,t}$ and w_t, the firm chooses N_t, $M_{H,t}$, and $M_{L,t}$ to maximize profits:

$$\max_{\{Y_t, N_t, M_{H,t}, M_{L,t}\}} \{Y_t - w_t N_t - r_{H,t}M_{H,t} - r_{L,t}M_{L,t}\}$$
$$\text{s.t. } Y_t = F(N_t, M_{H,t}, M_{L,t})$$

Second, for any given amount of aggregate labor, N_t, the aggregate demand for each differentiated type of labor, $l_t(j)$, is the solution to:

$$\begin{aligned} w_t N_t &= \min_{\{l_t(j)\}, j\in[0,1]} \int w_t(j)\, l_t(j)\, dj \\ \text{s.t. } N_t &\geq \left[\int l_t(j)^\nu\, dj\right]^{1/\nu} \end{aligned} \tag{5}$$

where $w_t(j)$ is the wage for labor of type j. I set $\nu \leq 1$ so the problem in (5) has a solution.

The demand for type j labor is:

$$l_t(j) = \left(\frac{w_t}{w_t(j)}\right)^{1/(1-\nu)} N_t \tag{6}$$

where $w_t = \left[\int w_t(j)^{\nu(\nu-1)} dj\right]^{(\nu-1)/\nu}$ is the aggregate wage.[18]

3.2 Consumers

Consumers are endowed with a specific type of labor and are organized in a continuum of unions indexed by $j \in [0,1]$. Each union represents consumers with a specific type of labor. Unions are modeled in the spirit of Blanchard and Kiyotaki (1987). Unions set the wage for labor of type j and face a downward sloping demand for labor given by (6).

The preferences of a representative consumer in the jth union are given by:

$$\sum_{t=0}^{\infty} \beta^t U\left[C_t(j), l_t(j)\right] \tag{7}$$

The jth union chooses consumption $C(j)$, non-negative investments in new plants $X_H(j)$ and $X_L(j)$, rents capital stocks $M_H(j)$ and $M_L(j)$, and chooses the wage rate $w(j)$ to maximize (7) subject to the demand for labor given in (6), the budget constraint in (8) and the laws of motion for capital in (9) and (10).

$$\begin{aligned}
(1+\tau_C) C_t(j) + X_{H,t}(j) + X_{L,t}(j) &\leq (1-\tau_N) w_t(j) l_t(j) \\
&\quad + (1-\tau_M)\left[r_{H,t} M_{H,t}(j) + r_{L,t} M_{L,t}(j)\right] \\
&\quad + \tau_M\left[\delta_H M_{H,t}(j) + \delta_L M_{L,t}(j)\right] + T_t \qquad (8) \\
M_{H,t+1}(j) &= (1-\delta_H) M_{H,t}(j) + X_{H,t}(j) \qquad (9) \\
M_{L,t+1}(j) &= (1-\delta_L) M_{L,t}(j) + X_{L,t}(j) \qquad (10)
\end{aligned}$$

Here, rental rates of capital are $r_{H,t}$ and $r_{L,t}$, and depreciation rates are δ_H and δ_L. Taxes on consumption, labor and capital income are τ_C, τ_N, τ_M, respectively. Lump-sum transfers are T_t. The union takes rental rates, tax rates and the aggregate wage, w_t as given.

The cost of creating new plants (i.e., the cost of one unit of capital investment) is the same for high and low productivity plants, as seen in equation (8). New plants are built

[18] Similarly, given the composite labor input, $n_{i,t}$, hired at date t by a z_i-plant, one can derive the demand for each differentiated type of plant labor input $l_{i,t}(j)$, where $i \in \{L, H\}$ and $j \in [0,1]$. The following holds: $l_{i,t}(j) = (w_t/w_t(j))^{1/(1-\nu)} n_{i,t}$. Using equation (6), we find $l_{i,t}(j) = (n_{i,t}/N_t) \cdot l_t(j)$ for all i and j.

only if the expected discounted value of plant profits equals the cost of one unit of capital. Thus, in equilibrium, only high productivity plants are built, as discussed in Section 3.1.

3.3 Government

The government runs a balanced budget every period. It taxes consumption, labor income and capital income. It permits depreciation allowances as given by $\tau_M \delta_H M_{H,t}(j) + \tau_M \delta_L M_{L,t}(j)$ for every j. Transfers from the rest of the world, $\overline{Tr}_t$, are an additional source of government revenues. The revenues collected by the government are lump-sum rebated to the households and denoted by T_t.

$$\begin{aligned} T_t &= \int \left[\tau_C C_t(j) + \tau_N w_t(j) l_t(j)\right] dj \\ &\quad + \int \left[\tau_M (r_{H,t} - \delta_H) M_{H,t}(j) + \tau_M (r_{L,t} - \delta_L) M_{L,t}(j)\right] dj + \overline{Tr}_t \end{aligned} \tag{11}$$

In addition, the government restricts the monopoly power of unions and the wage markups charged. This policy is modeled as a provision to make unions set wages competitively if markups exceed $1/\bar{\nu}_t$, where $\bar{\nu}_t \geq \nu$. As a result, the markup charged is $1/\bar{\nu}_t$. This policy is meant to capture the government's support for the large wage increases negotiated by the West German unions in East Germany (see Section 2 for details).

3.4 Equilibrium

In this section, I define an equilibrium and discuss key properties of the transition from a low to high productivity technology and the new steady state.

Definition 3 *An equilibrium are allocations* $\left\{Y_t, N_t, M_{H,t}, M_{L,t}, \left\{l_t^d(j)\right\}_{j\in[0,1]}\right\}_{t=0}^{\infty}$ *and*

$\left\{\left\{C_t(j), l_t^s(j), X_{H,t}(j), X_{L,t}(j), M_{H,t}(j), M_{L,t}(j)\right\}_{j\in[0,1]}\right\}_{t=0}^{\infty}$ and prices $\left\{r_{H,t}, r_{L,t}, w_t, \left\{w_t(j)\right\}_{j\in[0,1]}\right\}_{t=0}^{\infty}$ such that:

1. Given parameters $\left\{\tau_{C,t}, \tau_{N,t}, \tau_{M,t}, \overline{Tr}_t, \bar{\nu}_t\right\}_{t=0}^{\infty}$, initial conditions $M_{L,0}$, $M_{H,0}$ and prices $\{r_{H,t}, r_{L,t}, w_t\}_{t=0}^{\infty}$, the allocation $\{C_t(j), l_t^s(j), X_{H,t}(j), X_{L,t}(j), M_{H,t}(j)\, M_{L,t}(j)\}_{t=0}^{\infty}$, and $\{w_t(j)\}_{t=0}^{\infty}$ solve the jth union's problem for every $j \in [0,1]$.
2. Given prices, $\left\{Y_t, N_t, M_{H,t}, M_{L,t}, \left\{l_t^d(j)\right\}_{j\in[0,1]}\right\}_{t=0}^{\infty}$ solves the firm's problem.
3. The government balances its budget given in equation (11) for all t.

4. The markets clear and the resource constraint holds for all t.

$$l_t^d(j) = l_t^s(j) = l_t(j) \text{ for all } j$$
$$\int M_{i,t}(j)\,dj = M_{i,t} \text{ for all } i \in \{L, H\}$$
$$\int \left[C_t(j) + X_{H,t}(j) + X_{L,t}(j)\right] dj \leq Y_t + \overline{Tr}_t$$

A transition occurs because the economy gains access to a higher productivity technology embodied in new plants. This experiment is motivated by the 1990 unification which provided East Germany access to the world technology frontier. Initially, at $t = 0$, the economy has only z_L-plants and the stock of new plants is zero, i.e., $M_{H,0} = 0$. However, the high productivity technology becomes available and there are positive investments into it, i.e., $X_{H,0} > 0$. Due to the availability of more productive z_H-plants, investments in low productivity plants cease. This follows from the fact that building plants of either type is equally costly (see equation (8)), but the z_L-plants are less productive and so the expected discounted value of their profits is lower (see discussion in footnote 16 of Section 3.1).[19] As a result, the stock of z_L-plants depreciates over time at rate $\delta_L \geq 0$, while the more productive z_H-plants accumulate during the transition.

The first order conditions of the jth union problem are summarized by $X_{L,t}(j) = 0$, the budget constraint, the laws of motion for capital, as well as:

$$\begin{aligned}
\frac{U_C(C_t(j), l_t(j))}{U_C(C_{t+1}(j), l_{t+1}(j))} &= \beta \frac{1+\tau_{C,t}}{1+\tau_{C,t+1}} \left(1 + (1-\tau_{M,t+1})(r_{H,t+1} - \delta_H)\right) \\
w_t(j) &= \min\left\{\frac{1}{\nu}, \frac{1}{\bar{\nu}_t}\right\} \cdot \frac{(1+\tau_{C,t})}{(1-\tau_{N,t})} \frac{-U_l[C_t(j), l_t(j)]}{U_C[C_t(j), l_t(j)]} \qquad (12) \\
\lim_{t\to\infty} \beta^t U_C[C_t(j), l_t(j)]\, M_{H,t+1}(j) &= 0
\end{aligned}$$

where $1/\nu$ is the wage markup that the union chooses, and $1/\bar{\nu}_t$ is the markup allowed by the government. The intratemporal condition (12) is derived in detail in Appendix *A.2.4*.

I assume $\nu \leq \bar{\nu}_t \leq 1$ for all t. The wage markup in equilibrium is thus $1/\bar{\nu}_t$, meaning that the wage increases are mandated by the government. The markup is an exogenous policy in numerical experiments, and the values for $\bar{\nu}_t$ are discussed in Section 4.1.

The first order conditions of the representative firm's problem are:

$$r_{i,t} = \frac{\partial F(N_t, M_{H,t}, M_{L,t})}{\partial M_{i,t}},\ i \in \{L, H\}, \quad w_t = \frac{\partial F(N_t, M_{H,t}, M_{L,t})}{\partial N_t}$$

[19]It is also easy to show using (4) that the marginal product of high productivity plants, $M_{H,t}$, is strictly higher than the marginal product of low productivity plants, $M_{L,t}$, whenever all z_H-plants operate.

Given the symmetry of the unions, they all make the same choices. In particular, $w_t(j) = w_t$ and $l_t(j) = N_t$. For the remainder of the paper, I drop the j subscripts.

Proposition 4 *Along the transition path, as the stock of low productivity plants approaches zero asymptotically, the aggregate production function in (4) becomes:*

$$\lim_{M_{L,t}\to 0} F(N_t, M_{H,t}, M_{L,t}) = \begin{cases} z_H M_{H,t}^{\theta} N_t^{1-\theta} & \text{if } N_t \geq M_{H,t}\bar{n} \\ z_H \bar{n}^{-\theta} N_t & \text{if } N_t \leq M_{H,t}\bar{n} \end{cases}$$

Proof. See Appendix $A.2.3$. ■

Proposition 5 *In a steady state, there are no idle z_H-plants. The aggregate production function is Cobb-Douglas, $F(N_t, M_{H,t}, M_{L,t}) = z_H M_{H,t}^{\theta} N_t^{1-\theta}$, and $\bar{n} \leq \left[\frac{z_H\theta(1-\tau_M)}{\beta^{-1}-1+(1-\tau_M)\delta_H}\right]^{1/(\theta-1)}$.*

Proof. Suppose to the contrary that a steady state is reached at time T, and some z_H-plants are idle. It follows that it is impossible to allocate at least $\bar{n}$ units of labor to all z_H-plants, i.e., $N_T < M_{H,T}\bar{n}$. Using Proposition 4, the aggregate output is $z_H\bar{n}^{-\theta}N_T$. Since z_H-plants are not a scarce input in production, they earn no rents. There will be no investments undertaken: $X_{H,T} = 0$. Thus, the capital stock depreciates: $M_{H,T+1} < M_{H,T}$. This contradicts the fact that a steady state was reached at time T. Hence, in a steady state all z_H-plants operate and $N^* \geq M_H^*\bar{n}$, where the asterisk (*) denotes steady state. Using the steady state Euler equation and the first order condition for the return to capital, it is easy to show that having no idle z_H-plants entails $\bar{n} \leq \left[\frac{z_H\theta(1-\tau_M)}{\beta^{-1}-1+(1-\tau_M)\delta_H}\right]^{1/(\theta-1)}$. ■

In numerical experiments, $\bar{n}$ satisfies the parameter restriction in Proposition 5. Along the transition path and in the new steady state, all z_H-plants operate. There is no uncertainty in the model and due to the trade-off between consumption and savings, it is not optimal to accumulate too many z_H-plants and then leave some idle.

4 Quantitative Analysis

I calibrate the model to the East German economy using national accounts and revenue statistics. The benchmark model lines up well with East German data on output, expenditure shares, factor income shares, and hours worked. I perform counterfactual experiments to illustrate that the model's success in delivering a large initial drop in the capital share is due to the government policies: the transfers and wage markups. The two policies have nonlinear effects, as their joint impact on reducing the capital share is almost double the contributions of each policy. Lastly, I show that incorporating declining population has a quantitatively small impact on the capital share.

4.1 Parameter Choices

Table 2 presents the benchmark model's parameters and the data used in the calibration. The time-varying inputs—the net transfers to East Germany and its wage markups—are presented in Figure 2. Appendix A.1 provides detailed data sources.

Table 2: BENCHMARK MODEL PARAMETERS AND TIME-VARYING INPUTS

PARAMETERS	VALUES	TARGETS
Capital income share	$\theta = 0.365$	Average for Germany, $1991 - 2015$
Discount factor	$\beta = 0.96$	Steady state capital stock of 2.7
Capital depreciation rates	$\delta_H = \delta_L = 0.08$	Average for Germany, $1991 - 2015$
Tax rate on consumption	$\tau_C = 0.16$	Average for Germany, $1991 - 2015$
Tax rate on labor income	$\tau_N = 0.41$	Average for Germany, $1991 - 2015$
Tax rate on capital income	$\tau_M = 0.24$	Average for Germany, $1991 - 2015$
z_H-type capital stock in 1989	$M_{H,0} = 0$	EG capital stock was obsolete, see Sinn and Sinn (1992, p. 43-44)
z_L-type capital stock in 1989	$\frac{M_{L,0}}{Y_0} = 3.5$	EG capital to output ratio estimates range from 1.6 to 4.9, see Sinn and Sinn (1992, p. 43, 209)
Minimum labor requirement	$\bar{n} = 0.94 \cdot \frac{N^*}{M_H^*}$	Sensitivity analysis
Leisure parameter	$\psi = 0.84$	EG aggregate hours in 1989
Low productivity level	$z_L = 1$	Normalization
Ratio of plant productivities	$z_H/z_L = 1.65$	EG TFP growth, $1991 - 2015$
TIME-VARYING INPUTS	VALUES	TARGETS
Transfers, $\overline{Tr}_t$	Figure 2	Net Transfers to EG
Wage markups, $1/\overline{\nu_t}$	Figure 2	EG hourly wage growth, $1989 - 91$

NOTES: EG denotes East Germany. Detailed data sources are provided in Appendix A.1.

The model's steady state capital income share θ is set to 0.365, the average for unified Germany over $1991 - 2015$. The capital share is computed using equation 1. The large variation in the capital share in East Germany has a small impact on the share for unified Germany, because East Germany is small relative to the West (e.g., about 13 percent in total GDP terms). Indeed, the capital shares of unified Germany and West Germany fluctuate little and track each other closely, with Germany's share about 0.7 percentage points lower, on average. The discount factor targets a steady state capital to output ratio of 2.7. The annual depreciation rate of the capital stocks is set to be the same for both stocks and targets the average depreciation to GDP ratio for Germany.

Tax rates on consumption, labor and capital income are computed using the methodology of Mendoza, Razin, and Tesar (1994). In the model, tax rates do not vary over time and are set to the averages for Germany over $1989 - 2015$.

Given the values of $\theta, \beta, \delta_H, \delta_L, \tau_C, \tau_N, \tau_M$, the remaining parameters from Table 2 are determined jointly. We use data provided by Sinn and Sinn (1992) to set the capital stock to output ratios.[20] Consistent with the idea that East Germany's capital at unification was outdated (see footnote 3), the stock of high-productivity plants in 1989 is set to zero. The stock of low-productivity plants is set to 3.5 times GDP, which is the midpoint of the Sinn and Sinn (1992) estimates of the net stock of fixed assets (excluding land) of 1.6 and 4.9 GDPs in 1989. The high-end estimate is based on an exchange rate of 1 to 1 between the Mark and Deutschmark, while the low-end estimate is obtained by writing-off 67 percent of the capital stock to accommodate West German accounting rules. In the model, the stock of low productivity plants becomes outdated once the new technology is available, and it depreciates gradually, as there are no further investments into it. In Section 6, I perform sensitivity analysis to the initial stock of low-productivity plants.

The utility function is $U(C, l) = \log(C) + \psi \log(1 - l)$, where ψ is chosen to match aggregate hours worked in East Germany in 1989 (see Appendix A.1). The minimum labor requirement $\bar{n}$ generates a plant level non-convexity and influences which plants operate in equilibrium. In the steady state, all z_H-plants operate above the minimum scale and $\bar{n} \leq N^*/M_H^* = \left[\frac{z_H\theta(1-\tau_M)}{\beta^{-1}-1+(1-\tau_M)\delta_H}\right]^{1/(\theta-1)}$ (see Proposition 5). Along the transition path, since there is no reason to accumulate too many z_H-plants and leave some idle, it also must be that $\bar{n} \leq N_t/M_{H,t}$ for all t. I set $\bar{n} = 0.94 \cdot N^*/M_H^*$ to ensure these constraints hold.[21] This choice of $\bar{n}$ results in low productivity plants operating at the minimum labor constraint early in the transition. In Section 6, I perform sensitivity analysis to show that for very low values of $\bar{n}$, all z_L-plants operate above the minimum scale for a longer period during the transition $(n_L > \bar{n})$, which is inconsistent with evidence on plant closures in East Germany after unification (e.g., see Brücker (1995) or Kaser (2007)).

The productivity of the old production technology, z_L, is normalized to 1. The productivity of the new production technology, z_H, is set to match the increase in total factor productivity (TFP) over the period $1991 - 2015$. Total factor productivity is calculated—as is standard in the literature—as the Solow residual from a Cobb-Douglas production function: $TFP_t \equiv Y_t / \left[(M_{H,t} + M_{L,t})^\theta N_t^{1-\theta}\right]$, where $M_{H,t} + M_{L,t}$ represents the total capital stock at time t. In calculating TFP growth in the data, we first detrend TFP by $1.012^{(1-\theta)}$, where 1.2 percent represents the average annual growth rate in GDP per working-age person in West Germany over $1991 - 2015$. Since the model doesn't feature exogenous

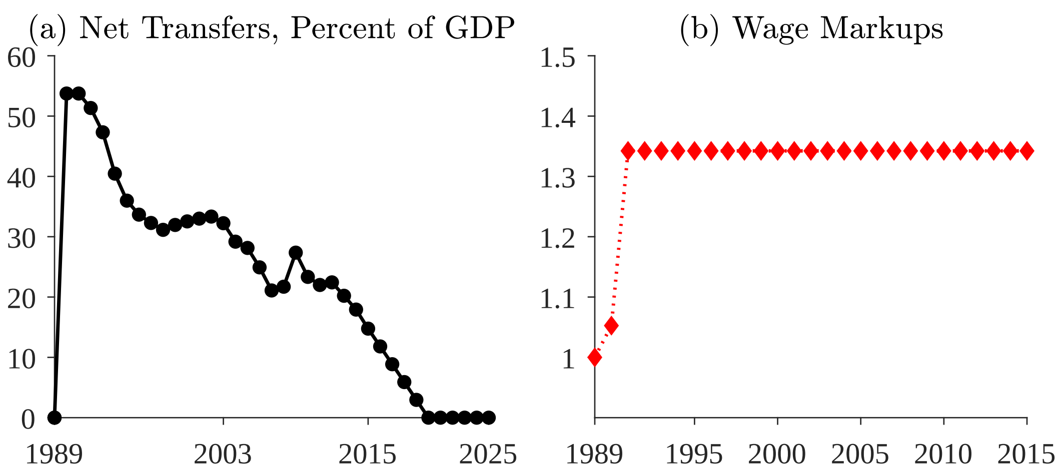
[20]See Appendix A in Sinn and Sinn (1992) on pages $214 - 216$ for data and discussion on pages $43 - 44$.

[21]During a transition, the stock of z_H-plants overshoots its steady state level slightly, although these plants operate above the minimum scale at all times. As a result, the ratio $N_t/M_{H,t}$ dips below its steady state value during the transition. For this reason, $\bar{n}$ is set strictly below the ratio N^*/M_H^*.

growth in technology, this transformation allows the model and data to be comparable.

Net transfers are an exogenous input into the model (Figure 2.a). Transfers are zero in the first period, which corresponds to 1989. Net transfers from West to East Germany for $1990-2003$ are from Uhlig (2008, Figure 2) and are comparable to data reported by the European Commission (2002). Since data for $2004-2015$ is not readily available, I use the East German trade balance deficit to GDP ratio—a proxy for the costs of unification suggested by IWH (2014)—to infer the rate of change in net transfers. This calculation implies net transfers in 2015 are about 15 percent of East German GDP. After 2015, I let transfers decline linearly to zero by 2020. This choice is motivated by the scheduled end date of 2019 of the Solidarity Pact II, signed by the federal states and aimed at promoting East Germany's development. While this package is just part of the transfers received, I assume all other transfers stop as well, and perform sensitivity analysis in Section 6.

Figure 2: MODEL INPUTS: TRANSFERS AND WAGE MARKUPS



NOTES: Net transfers to East Germany for $1990-2003$ are from Uhlig (2008). For $2004-2015$, transfers shrink at the annual growth rates of East Germany's trade balance to GDP ratio. Thereafter, transfers decline linearly to zero in 2020. Wage markups match East Germany's wage increases between 1989 and 1991.

The government-mandated wage markups, $1/\bar{\nu}_t$, are chosen so that the model matches the large increases in wages between 1989 and 1991 (see Table 1). In setting the values of $\bar{\nu}_t$, wages from East German data are detrended by the annual growth factor 1.012. I set $\bar{\nu}_{1989}=1$, $\bar{\nu}_{1990}=0.92$ and $\bar{\nu}_{1991}=0.72$. Thereafter, I keep markups unchanged (Figure 2.b). In Section 6, I perform sensitivity analysis by allowing the wage markups, $1/\bar{\nu}_t$, to decline gradually over time after 1991.

4.2 Numerical Experiments

I use the calibrated model to examine the East German transition. I perform counterfactual experiments to decompose the relative importance of technology change and government policies to the decline in East Germany's capital income share.

Figure 3 plots results from the benchmark experiment in which the transfers and wage markups are exogenous time-varying inputs calibrated to the size of financial support received and the large wage increases in East Germany. Total factor productivity and hourly wages grow endogenously in the model during the transition from low to high productivity technology. As discussed in Section 4.1, the model was calibrated to deliver an increase in TFP of about 45 percent over $1991-2015$, consistent with East German data (Figure 3.a). The more productive plants accumulate gradually over time, so the model does not deliver the sharp TFP increase observed between 1991 and 1995. The model's wage markups were calibrated to match the large increases in hourly wages between 1989 and 1991. The behavior of the wage rate following 1991 is determined endogenously, and it matches the data well (Figure 3.b).

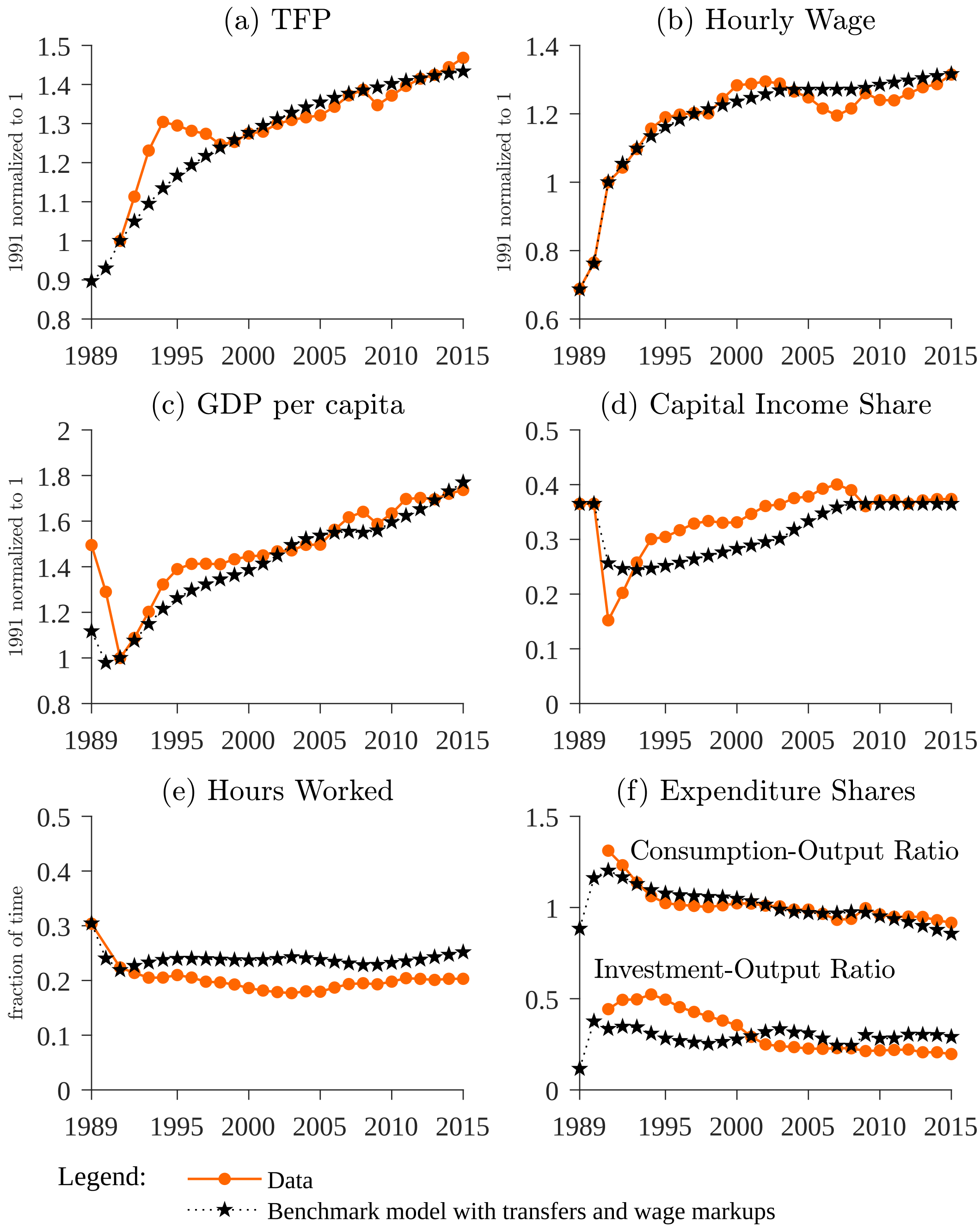
The model's GDP closely tracks East Germany's GDP per working-age person (Figure 3.c). The data is detrended to allow comparisons to the model which does not feature exogenous growth in technology.[22] I find the model's output declines by 10 percent over $1989-1991$, about one third of the decline observed in the data. This drop in output is driven entirely by the decline in hours worked.[23] The large increases in hourly wages between 1989 and 1991 depress hours worked in the model. Moreover, the transfers received generate a wealth effect which further reduces hours. After 1991, the model's output grows, albeit initially at a lower pace compared to East German data, because the productivity improvements in the model occur gradually. The overall output growth in the model over the period $1991-2015$ is similar to the data.

The model delivers time-variation in the capital income share (Figure 3.d). In the first two years of the model, the capital share equals the calibrated value of $\theta = 0.365$. Afterwards, the capital share declines since the accumulation of the more productive capital pushes low productivity plants to reduce their scale. Due to the binding minimum scale constraint, low productivity plants have a lower capital share since their labor productivity is fixed, while wages grow during the transition. Transfers and wage increases raise the cost of labor and reduce the profits of the z_L-plants. The model delivers an 11 percent-

[22] As detailed in Section 4.1, East German GDP is detrended with an annual trend growth factor of 1.012.

[23] Changes in capital stocks do not depress model output. While $M_{L,t} + M_{H,t}$ decreases slightly initially, output in the initial two years of transition equals $z_L\left(\rho M_{H,t} + M_{L,t}\right)^{\theta} N_t^{1-\theta}$ (see equation 4), and the productivity-adjusted measure of capital, $\rho M_H + M_L$, does not decline.

Figure 3: EAST GERMANY: DATA AND BENCHMARK MODEL PREDICTIONS



NOTES: East Germany's TFP, hourly wages and GDP per capita are detrended as described in Section 4.1.

age points drop in the capital share, from a value of 0.365 in 1990 to 0.256 in 1991. Thereafter, the capital share increases as less output in the economy is produced by z_L-plants. The model predicts that by 2008 no low productivity plants are used for production, and the aggregate capital share is back to 0.365. The stock M_L is not completely depreciated by that time, but the remaining z_L-plants are left idle and their marginal product is zero.[24]

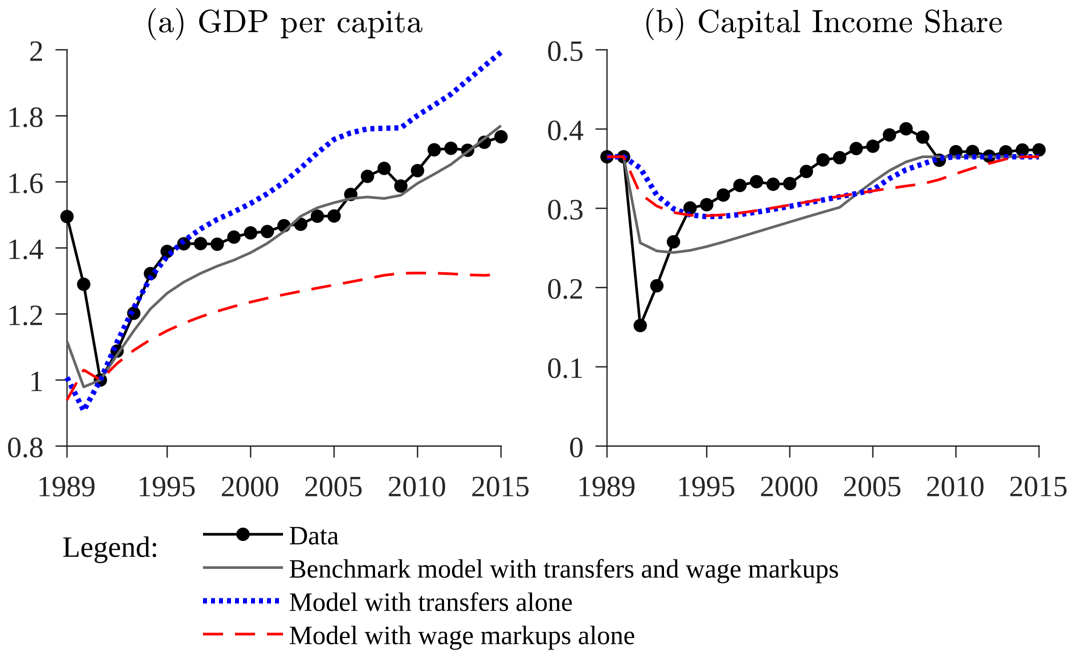
Large transfers and wage increases contribute to a 29 percent decline in hours worked over $1989-1991$, in line with East German data (Figure 3.e). After 1991, the model predicts a slight increase in hours due to the decline in transfers over time. In addition, the model's predictions for expenditure shares match the data well (Figure 3.f). Since unification, East Germany's consumption and investment exceeded production, as the consumption to output ratio averaged 1.01 over $1991-2015$, while the investment to output ratio averaged 0.32. This was made possible through the large transfers received. In the model, while transfers are available for consumption and investment, most get allocated to consumption, and only some go to investment, consistent with the data. The model delivers an average consumption share of output of 1.01 and an average investment share of 0.29 for the period $1991-2015$. The counterparts in a model with no transfers $\left(\text{i.e., } \overline{Tr}_t = 0 \text{ for all } t\right)$ are an average consumption-output ratio of 0.74 and investment-output ratio of 0.26.

I perform counterfactual experiments to decompose the contribution of government policies to output growth and the capital share. First, I remove the government-mandated wage increases from the model by setting $\bar{\nu}_t = 1$ for all t. This experiment labeled "Model with transfers alone" in Figure 4, delivers a smaller decline in hours worked and output early in the transition. Throughout the transition, hours are higher, the z_H-plants accumulate faster, and output grows by more compared to the benchmark experiment with both transfers and wage markups (Figure 4.a). The drop in the capital share is smaller, and its value is 0.35 in 1991 compared to 0.256 in the benchmark (Figure 4.b). In the experiment with wage markups alone, $\overline{Tr}_t = 0$ for all t, output is depressed compared to the benchmark, while the capital share is 0.318 in 1991.

The non-convexity in the plant production technology is crucial for generating variations in the capital share. Without transfers or wage markups (i.e., $\overline{Tr}_t = 0$ and $\bar{\nu}_t = 1$ for all t), the capital income share varies little initially, and by at most 5 percentage points throughout the transition, due to the change in technology. Introducing either of the two government policies yields further declines in the capital share. Transfers alone deliver a capital share of 0.35 in 1991, while wage markups alone yield to a capital share of 0.318 in 1991 (Figure 4.b). When the two policies are considered together, they have a nonlinear effect on the capital share, which declines to 0.256 in 1991, as shown in the benchmark experiment. Section 5 discusses these nonlinearities in more detail.

[24]In the benchmark, the gross marginal product of $M_{H,t}$ capital (denoted by $F_{M_{H,t}}$) drops sharply from 41 to 21 percent in the first 3 periods, and then gradually reaches its steady state value of 13.5 percent. Gross returns to $M_{L,t}$ (denoted by $F_{M_{L,t}}$) decline from 10 percent to zero. The mean return over the transition for the two types of capital—computed as $(F_{M_{H,t}} \cdot M_{H,t} + F_{M_{L,t}} \cdot M_{L,t})/(M_{H,t} + M_{L,t})$—is 12.6 percent.

Figure 4: East Germany: Data and Predictions from Different Experiments



Notes: East Germany's GDP per capita is detrended as described in Section 4.1.

4.2.1 An Extension with Population Changes

To evaluate the effects of a declining population in East Germany following unification, I extend the model to incorporate population growth. While the decline in population doesn't capture some aspects of migration, such as the age composition or skill of the migrants, it impacts the aggregate labor input, and contributes to further declines (albeit small) in the capital income share, relative to the benchmark.

Incorporating population growth entails relatively simple changes to the consumer problem. First, consumers' utility becomes $\sum_{t=0}^{\infty} \beta^t U\left(C_t, l_t\right) \cdot \Lambda_t$, where Λ_t denotes the population at time t, and where I simplified the notation to reflect the symmetry of unions (i.e., I dropped j subscripts). Second, the laws of motion for capital stocks become: $\frac{\Lambda_{t+1}}{\Lambda_t} \cdot M_{i,t+1} = \left(1 - \delta_L\right) M_{i,t} + X_{i,t}$, for $i \in \{L, H\}$.

The working-age $(15 - 64)$ population in East Germany shrunk by 3.3 percent from $1989 - 1990$, by 1.4 percent from $1990 - 1991$, and by an average of 0.8 percent from $1991 - 2016$. I treat the yearly population growth rates from $1989 - 2016$ as an exogenous model input. After 2016, I let population growth equal the average since 1991. I recalibrate the model to match the same calibration targets as in Section 4.1 (only ψ is changed).

The quantitative impact of the declining population on the capital share is small. As before, the capital share equals θ in the first two periods of the model. By the third period (which corresponds to 1991), the capital share declines to 0.25 in the model with population change relative to the benchmark value of 0.256. In both experiments, the capital share returns to $\theta = 0.365$ by 2009. For the periods when it is lower than θ, the aggregate capital share is on average 0.6 percentage points lower with declining population compared to the benchmark with no change in population. While population change is an imperfect proxy for migration, given the results of this section, I leave more detailed explorations of migration and its quantitative impact on the capital share to future research.

5 A Closer Look at the Model's Factor Income Shares

In this section, I discuss the model's success in generating time-varying factor shares. I illustrate that the non-convexity in the plant-level production technology is crucial in allowing for variation in factor shares. In the presence of the non-convexity, introducing transfers and wage markups affects factor shares non-linearly, leading to declines in the capital share that are of larger magnitude and that occur sooner in the transition.[25]

5.1 Technology Change and the Dynamics of Factor Income Shares

I illustrate the impact that technology change alone can have on the dynamics of factor income shares. To this end, I examine an economy in which transfers are zero and wages are perfectly competitive, i.e., $\overline{Tr}_t = 0$ and $\bar{\nu}_t = 1$ for all t. I relate the profit shares of plants that operate during a transition from a low to a high productivity technology to the aggregate capital share. I show that variation in the profit share for plants that operate at the minimum scale translates in a time-varying aggregate capital share.

Let $\pi_{i,t}$ denote total profits of a z_i-plant at time t, where $i \in \{H, L\}$. The profit share of output is denoted by $\phi_{i,t} = \pi_{i,t}/y_{i,t}$.

Proposition 6 *The plant profit shares satisfy $\phi_{i,t} \leq \theta$ for all t, with equality if $n_{i,t} > \bar{n}$.*

Proof. For each t, z_i-plants maximize profits subject to hiring at least $\bar{n}$ units of labor.

$$\pi_{i,t} \equiv \max_{n_{i,t} \geq \bar{n}} z_i n_{i,t}^{1-\theta} - w_t n_{i,t}$$

The first order condition is $w_t \geq (1-\theta) z_i n_{i,t}^{-\theta}$, with equality if $n_{i,t} > \bar{n}$. Thus, profits are given by $\pi_{i,t} \leq z_i n_{i,t}^{1-\theta} - \left[(1-\theta) z_i n_{i,t}^{-\theta}\right] n_{i,t} = \theta z_i n_{i,t}^{1-\theta} = \theta y_{i,t}$, with equality if $n_{i,t} > \bar{n}$. Equivalently, $\phi_{i,t} \equiv \frac{\pi_{i,t}}{y_{i,t}} \leq \theta$, with equality if $n_{i,t} > \bar{n}$. ■

[25] Without the non-convex plant technology, the government policies have no effect on the capital share.

If the minimum labor constraint does not bind, plant labor productivity is proportional to the wage rate and the plant profit share is constant. However, if plants are constrained to hire $\bar{n}$ units of labor, the profit share is lower than θ.

During a transition, all high productivity plants operate and they hire more than the minimum required labor. Hence, $\phi_{H,t} = \theta$ for all t. Low productivity plants initially hire more than $\bar{n}$ units of labor, but later switch to operating at the minimum scale, as more labor is hired at the z_H-plants. Thus, $\phi_{L,t} \leq \theta$ with strict inequality for some t.

Let ϕ_t denote the aggregate capital income share at t. Then, $\phi_t = (\Pi_{H,t} + \Pi_{L,t})/Y_t$, where $\Pi_{i,t}$ are the total profits for plants of type z_i at t and Y_t is aggregate output at t. Letting $Y_{i,t}$ be the total output produced by plants of type z_i at t, the aggregate capital income share can also be written as the weighted average of plant profit shares.

$$\phi_t = \frac{\Pi_{H,t}}{Y_{H,t}}\frac{Y_{H,t}}{Y_t} + \frac{\Pi_{L,t}}{Y_{L,t}}\frac{Y_{L,t}}{Y_t} = \phi_{H,t}\frac{Y_{H,t}}{Y_t} + \phi_{L,t}\frac{Y_{L,t}}{Y_t}$$

The second equality is obtained using $\Pi_{i,t}/Y_{i,t} = \pi_{i,t}M_{i,t}/y_{i,t}M_{i,t} = \phi_{i,t}$ for all i and t. Moreover, using the result $\phi_{H,t} = \theta$ for all t, the aggregate capital income share becomes:

$$\phi_t = \theta - (\theta - \phi_{L,t})\frac{Y_{L,t}}{Y_t} \tag{13}$$

Equation (13) shows that if the profit share of z_L-plants equals θ or the total output produced by these plants is zero, the aggregate capital share is constant and equals θ. However, if $\phi_{L,t} < \theta$ and $Y_{L,t} > 0$, then $\phi_t < \theta$. Low productivity plants have a profit share lower than θ whenever they hire only $\bar{n}$ units of labor (Proposition 6). Hence, the non-convexity in the plant-level technology is crucial for generating variation in factor shares. The aggregate capital share is lower than θ if (i) all low productivity plants operate and they hire $\bar{n}$ labor, or if (ii) only some low productivity plants operate and hire $\bar{n}$ labor.

In an economy that starts out with only z_L-plants, and where the more productive technology becomes available, a growing fraction of total labor is hired at these plants. Low productivity plants reduce their scale, until they all hire only $\bar{n}$ labor.[26] One of two scenarios can occur. If the productivity difference between the plants is large enough, i.e., $\alpha \equiv [(1-\theta)z_H/z_L]^{1/\theta} > 1$, only a fraction of the low productivity plants continue to operate and hire $\bar{n}$ units of labor, while the rest are left idle.[27] If the difference in plant level productivities is smaller, i.e., $\alpha < 1$, all z_L-plants operate throughout the transition

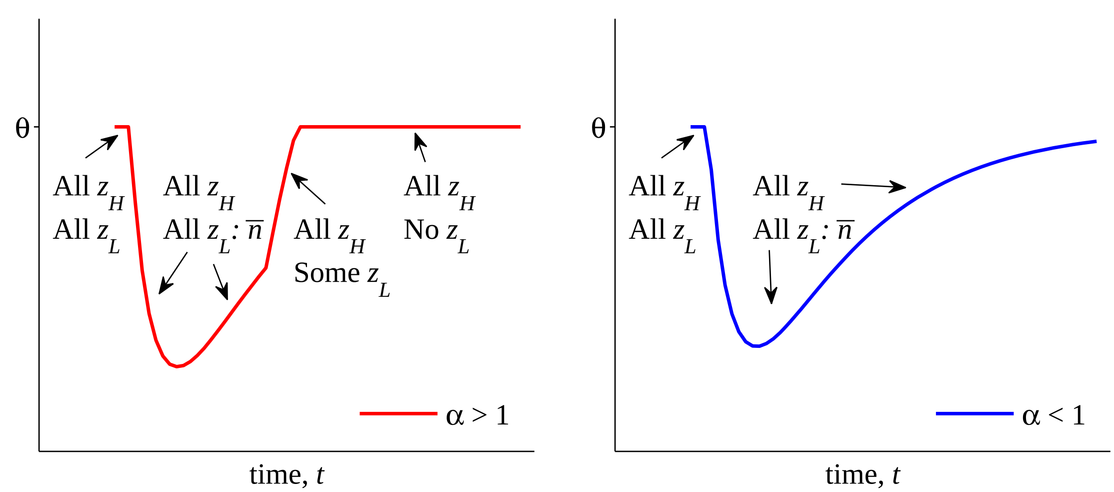
[26]For low $\bar{n}$, it is possible that all z_L-plants operate above the minimum scale during the transition. This outcome is in sharp contrast to the experience of transition economies and is not considered here.

[27]The condition $\alpha > 1$ is equivalent to $z_H/z_L > 1/(1-\theta)$. For $\theta = 0.365$, $\alpha > 1$ if the high productivity technology is about 57 percent more productive than the low productivity technology.

at the minimum scale and only exit the economy as they depreciate. The intuition is that when $\alpha > 1$, the high productivity capital stock accumulates faster during the transition and these plants hire more labor compared to $\alpha < 1$. Low productivity plants cannot shrink their size below $\bar{n}$ and, as a result, some of them become idle. Ultimately, in both scenarios, only high productivity plants operate in the new steady state.

The dynamics of the aggregate capital income share are shown in Figure 5. The left side panel plots the aggregate capital share in an economy with $\alpha > 1$. Initially, all plants operate and hire more than $\bar{n}$ units of labor and the aggregate capital share equals θ. As the stock of z_H-plants accumulates over time, the low productivity plants reduce their scale. First all z_L-plants operate at $\bar{n}$, then some of them stop operating. During this process, the capital income share falls below θ. In periods when some z_L-plants are idle, they earn no rents because they are not a scarce input in production. The capital share increases rapidly during these periods, as plants with low profit share exit. In the steady state of the economy, the capital share is constant again and equal to θ. The U-shaped dynamics for the aggregate capital share are slightly different when $\alpha < 1$ (see the right side panel of Figure 5). As discussed previously, when the productivity difference between the two types of plants is smaller, so that $\alpha < 1$, all z_L-plants operate throughout the transition and they exit the economy as they depreciate. The aggregate capital share drops below θ, as low productivity plants operate at the corner. As the economy replaces these plants, the capital share returns gradually to the value θ.

Figure 5: THE CAPITAL SHARE IN A MODEL WITH TECHNOLOGY CHANGE ONLY



NOTES: The figure plots the dynamics of the capital income share in an economy where the aggregate production function is given in equation 4, for different values of $\alpha \equiv [(1-\theta)\, z_H / z_L]^{1/\theta}$.

5.2 Nonlinear Impact of Transfers and Wage Markups on Factor Shares

Section 5.1 showed that technology change combined with the non-convexity in the plant-level production technology generates U-shaped dynamics of the aggregate capital income share. Here, I show that introducing transfers or wage markups may lead to larger declines in the capital share, earlier in the transition. Moreover, the two policies interact non-linearly to amplify their impact on the capital share.

To illustrate these results, I perform a range of experiments in which there is a one-time permanent change in either transfers or wage markups. First, I consider the calibrated economy from Section 4, but I set wages perfectly competitively (i.e., $\bar{\nu}_t = 1$ for all t), and I let transfers increase from zero at $t = 0$ to $\overline{Tr} \in [0, 70]$ percent of output starting period $t = 1$. I contrast these experiments to the economy with zero transfers and perfectly competitive wages discussed in Section 5.1, to highlight the impact on the capital share.

The permanent increase in the flow of transfers at $t = 1$ has a positive income effect which leads to a reduction in aggregate labor input and an increase in the wage rate relative to a model with no transfers. If transfers are large enough, the drop in labor at time $t = 1$ when the policy is introduced is associated with a decline in output. Thereafter, a model with transfers generates higher output growth compared to a model with zero transfers because transfers allow for a faster accumulation of high productivity capital.

The decline in the capital share may occur *sooner* in the transition and is of *larger magnitude* compared to a model with zero transfers. Transfers lower aggregate labor and plant labor, $n_{i,t}$. If transfers are large enough, low productivity plants operate at the minimum scale $\bar{n}$ sooner than in a model with zero transfers. The profit share at these plants declines and the economy's capital share is less than θ, earlier in the transition.

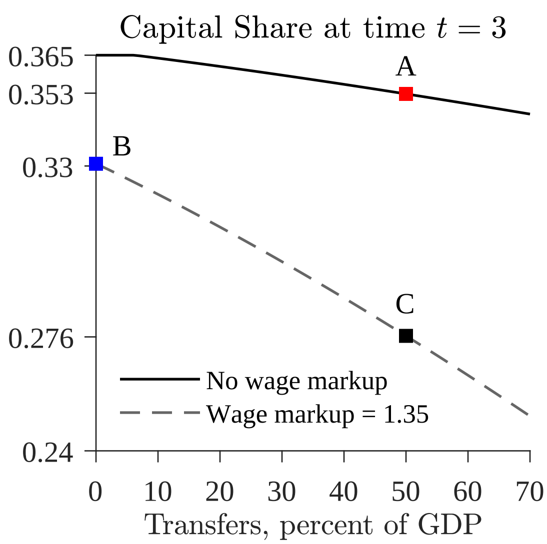
Moreover, the decline in the capital share is of larger magnitude in the economy with transfers. Recall that the capital share is $\phi = \theta - (\theta - \phi_{L,t}) \cdot Y_{L,t}/Y_t$ (see derivation in Section 5.1). First, the profit share of low productivity plants, $\phi_{L,t}$, is lower compared to a model with zero transfers. When z_L-plants operate at $\bar{n}$, their output is fixed $z_L \bar{n}^{1-\theta}$. The higher wage rate that prevails in an economy with transfers thus leads to lower profits, $\pi_{L,t} = z_L \bar{n}^{1-\theta} - w_t \bar{n}$, and a lower profit share. Second, the share of output produced by low productivity plants, $Y_{L,t}/Y_t$, is higher compared to the zero transfer model. The z_H-plants decrease their optimal labor input and hence their output in response to the higher wages. Low productivity plants are constrained to operate at $\bar{n}$. All else equal, this leads to a larger share of output being produced by low productivity plants.

Wage markups produce qualitatively similar effects on the capital share. I consider the calibrated economy from Section 4, but eliminate the transfers. I let wages be perfectly competitive at time $t = 0$, $\bar{\nu}_0 = 1$, and consider a permanent increase in wage markups,

$\bar{\nu}_t < 1$ for all $t \geq 1$. Wage markups lower output growth during the transition compared to a perfectly competitive economy, as aggregate labor and the accumulation of capital are depressed. The capital share falls below θ earlier in the transition, and declines by more in the presence of wage markups, similar to the impact of transfers.

When the two government policies—transfers and wage markups—are considered together, they have nonlinear effects on the capital income share. To illustrate this result, I show—using the numerical experiments discussed in this section—that the capital income share is a concave function of the policies. Thus, the joint impact of the two policies on reducing the capital share is larger than the sum of contributions of each policy.

Figure 6: THE CAPITAL SHARE AS A FUNCTION OF POLICIES



NOTES: Notes

Figure 6 plots the capital income share at time period $t = 3$ of a transition, in a range of experiments with one-time permanent changes in transfers and markups. The solid line plots results from an economy with no wage markups (i.e., $\bar{\nu}_t = 1$ for all t), as transfers vary from 0 to 70 percent of output. The dashed line plots the capital income share as transfers vary when the wage markup is $1/\overline{\nu_t} = 1.35$ for all $t \geq 1$. When both markups and transfers are zero, the aggregate capital income share at $t = 3$ is 0.365, the calibrated value of θ. Introducing transfers of 50 percent of GDP, reduces the capital share at $t = 3$ to 0.353 (point A on Figure 6). Alternatively, keeping transfers at zero, but introducing a wage markup $1/\overline{\nu}_t = 1.35$ reduces the capital share to 0.33 (point B in the figure). Point C shows the joint impact of 35 percent wage markups and 50 percent transfers on the capital share at time $t = 3$. In particular, with both policies the capital share is 0.276, a decline of

8.9 percentage points from the value of θ, nearly double the sum of contributions of the two policies (which is 4.7 percentage points).

The nonlinear effects observed are driven by asymmetric responses of high and low productivity plants to increased wages and depressed aggregate labor induced by transfers and markups. High productivity plants lower their labor input and maintain a profit share of θ. However, low productivity plants operate at the minimum scale, $\bar{n}$, and thus have a profit share lower than θ. Since the aggregate capital share is given by $\phi = \theta - (\theta - \phi_{L,t}) \cdot Y_{L,t}/Y_t$ (equation 13), the impact of transfers and markups on ϕ depends on the fraction of output produced by low productivity plants and their profit share. Thus, policies affect the aggregate capital share in a nonlinear fashion.

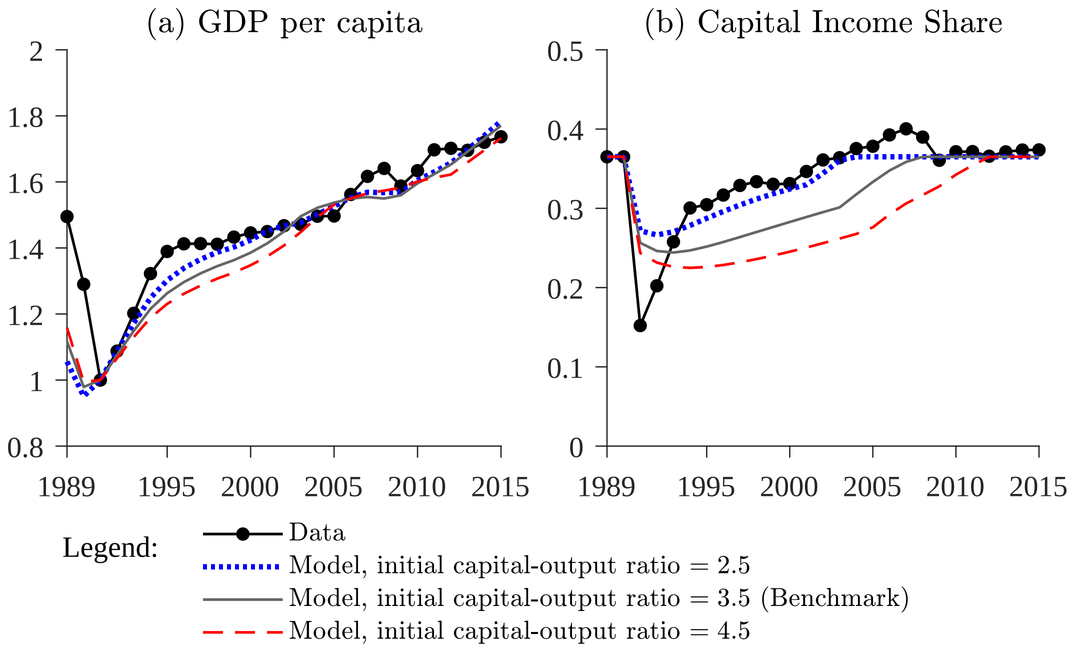
6 Sensitivity Analysis

In this section, I perform sensitivity analysis with respect to the initial capital output ratio, the minimum scale parameter and the time-varying inputs used in the benchmark model.

The range of estimates for the initial capital stock to output ratio in East Germany is wide, from 1.6 to 4.9 (see Section 4.1). The benchmark model is calibrated to match an initial capital output ratio of 3.5. Figure 7 presents the model's results for output and the capital income share when the initial capital to output ratio is 2.5, 3.5 or 4.5. In each case, other model parameters are recalibrated to match the targets described in Table 2. The predictions for output are similar across these experiments. With an initial capital stock to output ratio of 2.5, the decline in the capital share is lower than in the benchmark (9.3 percentage points compared to 11), but the recovery is faster and tracks the data closely for the remaining years.

Figure 8 plots the model's results for output and the capital income share, when the ratio of $\bar{n}$ to N^*/M_H^* takes three different values: 0.94, 0.84 and 0.44. In the benchmark, z_L-plants hit the minimum scale in period 3 of the model (i.e., 1991), and they stop operating in period 20 (i.e., 2008), when the capital share equals θ. Lowering $\bar{n}$ to $0.84 \cdot N^*/M_H^*$ also delivers z_L-plants that operate at the minimum scale in period 3, but these plants never shut down, and they exit the economy very slowly as the stock of M_L depreciates. As a result, the recovery in the capital share is more gradual (dotted line in Figure 8.b). Finally, when $\bar{n}$ equals $0.44 \cdot N^*/M_H^*$, z_L-plants initially operate above $\bar{n}$ for longer (5 periods). Afterwards, more of the economy's labor is hired at high-productivity plants, so having z_L-plants operate at the minimum scale has a smaller impact on the aggregate capital share compared to the experiments with higher $\bar{n}$. The prediction that all z_L-plants operate during the transition when the ratio of $\bar{n}$ to N^*/M_H^* is lower (i.e., 0.84 or 0.44)

Figure 7: East Germany: Data and Benchmark Model Sensitivity to Initial Capital



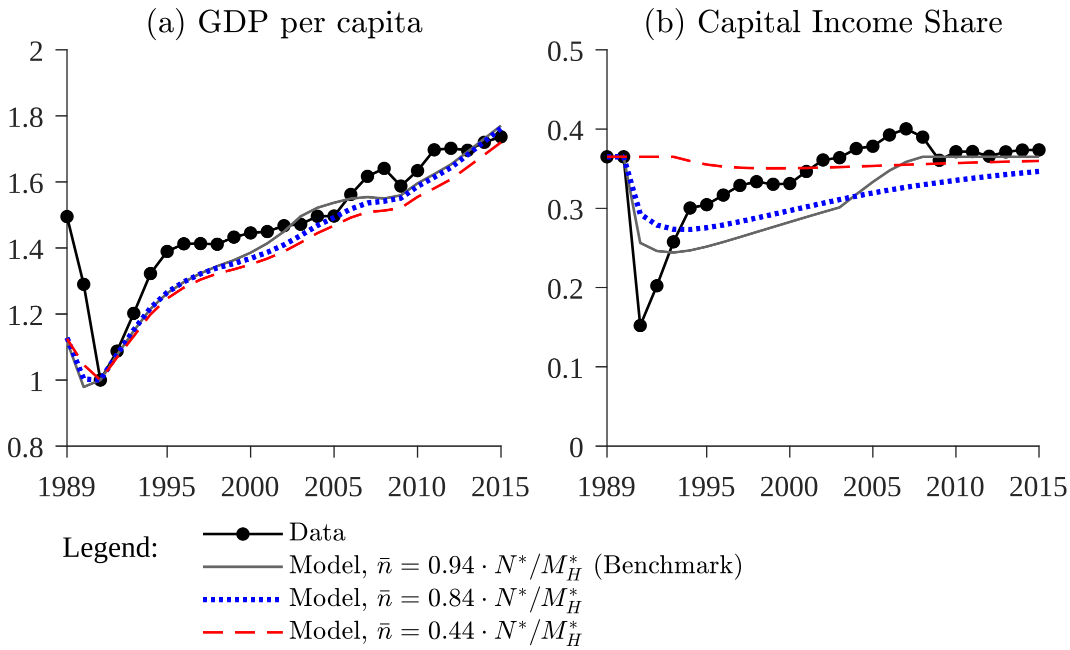
Notes: East Germany's GDP per capita is detrended as described in Section 4.1.

is inconsistent with evidence on plant closures in East Germany after unification (see Brücker (1995) or Kaser (2007)). This motivates the choice of $\bar{n}$ in the benchmark model.

In the benchmark experiment of Section 4, the wage markups were chosen to match the wage increases between $1989 - 1991$ and then held fixed thereafter (Figure 2). I consider alternate wage markups that are the same for $1989 - 1991$, but decrease linearly to no markup in 2040, ($\bar{\nu}_{2040} = 1$). The predictions from this experiment are nearly identical to the benchmark for the period $1989 - 2003$. In the benchmark, 2004 is the first year when some z_L-plants remain idle. The main difference in the experiment with declining markups is a smaller increase in wages during the transition. This leads to a larger increase in hours worked and a higher GDP per capita after year 2004. A higher labor input means that z_L-plants are used in production for a longer time, and they become idle and exit the economy later. As a result, the capital share in this model is a bit lower following 2004 compared to the benchmark, but in both experiments it equals 0.365 by 2014.

I also perform sensitivity analysis by allowing transfers to linearly decline to zero by year 2040, as opposed to 2020 in the benchmark. The predictions from this experiment are almost identical to the benchmark. The main differences occur after 2020, when due to positive transfers, the labor input is lower.

Figure 8: East Germany: Data and Benchmark Model
Sensitivity to $\bar{n}$



Notes: East Germany's GDP per capita is detrended as described in Section 4.1.

7 Conclusions

It is well-documented that the shares of income accruing to capital and labor are fairly constant in many countries, but show small variations over time. This paper documents a dramatic departure from the constancy of factor income shares in East Germany following the 1990 unification with West Germany. The capital share of income in East Germany plunged to 15.2 percent in 1991, then increased to 37.4 percent by 2015.

To account for this fact, and motivated by the opening up of the East German economy, I build a dynamic general equilibrium model in which an economy transitions from a low to high productivity technology embodied in plants. The model's implied aggregate production function delivers endogenous variations in the capital income share, due to a non-convexity in the plant level production technology. Two government policies that set East Germany apart from other transition economies—large transfers from West Germany and dramatic wage increases—amplify the decline in the capital share early in the transition. While this paper analyzes East Germany, the finding that technology change induces variations in the capital share has applicability to other economies.

References

Akerlof, George A., Andrew K. Rose, Janet L. Yellen, and Helga Hessenius. 1991. "East Germany in from the Cold: The Economic Aftermath of Currency Union." *Brookings Papers on Economic Activity* 1991 (1): 1–87.

Atkeson, Andrew, and Patrick J. Kehoe. 1993. "Industry Evolution and Transition: The Role of Information Capital." *Federal Reserve Bank of Minneapolis, Staff Report*, vol. 162.

Bird, Edward J., Johannes Schwarze, and Gert G. Wagner. 1994. "Wage Effects of the Move Toward Free Markets in East Germany." *Industrial and Labor Relations Review* 47 (3): 390–400.

Blanchard, Olivier J., Simon Commander, and Fabrizio Coricelli. 1995. "Unemployment and Restructuring in Eastern Europe and Russia." In *Unemployment, Restructuring and the Labor Market in Eastern Europe and Russia*, edited by Simon Commander and Fabrizio Coricelli, 289–329. The World Bank, Washington D.C.

Blanchard, Olivier J., and Nobuhiro Kiyotaki. 1987. "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review* 77:647–666.

Blanchard, Olivier J., and Michael Kremer. 1997. "Disorganization." *Quarterly Journal of Economics* 112:1091–1126.

Brücker, Herbert. 1995. "Selling Eastern Germany. On the Economic Rationale of the Treuhandanstalt's Privatisation and Restructuring Strategy." *MOCT-MOST* 5:55–77.

Burda, Michael C. 2006. "Factor Reallocation in Eastern Germany after Reunification." *American Economic Review Papers and Proceedings* 96:368–374.

Burda, Michael C., and Michael Funke. 2001. "Wages and Structural Adjustment in the New German States." In *Employment Policy in Transition: The Lessons of German Integration for the Labor Market*, edited by Regina T. Riphahn, Dennis J. Snower, and Klaus F. Zimmermann, 31–51. Berlin: Springer-Verlag.

Burda, Michael C., and Jennifer Hunt. 2001. "From Reunification to Economic Integration: Productivity and the Labor Market in Eastern Germany." *Brookings Papers on Economic Activity* 2001 (2): 1–71.

Campos, Nauro F., and Fabrizio Coricelli. 2002. "Growth in Transition: What We Know, What We Don't, and What We Should." *Journal of Economic Literature* 40 (3): 793–836.

Canova, Fabio, and Morten O. Ravn. 2000. "The Macroeconomic Effects of German Unification: Real Adjustments and the Welfare State." *Review of Economic Dynamics* 3:423–460.

Cooley, Thomas F., and Edward C. Prescott. 1995. "Economic Growth and Business Cycles." In *Frontiers of Business Cycle Research*, edited by Thomas F. Cooley, 1–38. Princeton University Press.

Dornbusch, Rudiger, and Holger W. Wolf. 1994. "East German Economic Reconstruction." In *The Transition in Eastern Europe*, edited by Olivier Jean Blanchard, Kenneth A. Froot, and Jeffrey D. Sachs, Volume 1, 155–190. The University of Chicago Press.

European Commission. 2002. Germany's Growth Performance in the 1990s. Directorate General for Economic and Financial Affairs, Economic Papers No. 170.

Fischer, Stanley, Ratna Sahay, and Carlos A. Végh. 1996. "Economies in Transition: The Beginnings of Growth." *American Economic Review* 86:229–233.

Fuchs-Schündeln, Nicola, and Rima Izem. 2012. "Explaining the Low Labor Productivity in East Germany A Spatial Analysis." *Journal of Comparative Economics* 40:1–21.

Gollin, Douglas. 2002. "Getting Income Shares Right." *Journal of Political Economy* 110 (2): 458–474.

Gorodnichenko, Yuriy, Enrique G. Mendoza, and Linda L. Tesar. 2012. "The Finnish Great Depression: From Russia with Love." *American Economic Review* 102:1619–1643.

Grossman, Gene M., Elhanan Helpman, Ezra Oberfield, and Thomas Sampson. 2017. "The Productivity Slowdown and the Declining Labor Share: A Neoclassical Exploration." mimeo.

Hansen, Gary D., and Edward C. Prescott. 2005. "Capacity Constraints, Asymmetries and the Business Cycle." *Review of Economic Dynamics* 8:850–865.

Hoffmann, Lutz. 1992. "Integrating East German States into the German Economy: Opportunities, Burdens and Options." In *Economic Aspects of German Unification*, edited by Paul J. J. Welfens, 49–77. Heidelberg: Springer-Verlag.

Hunt, Jennifer. 2001. "Post-Unification Wage Growth in East Germany." *Review of Economics and Statistics* 83 (1): 190–195.

———. 2006. "Staunching Emigration from East Germany: Age and the Determinants of Migration." *Journal of the European Economic Association* 4 (5): 1014–1037.

IWH. 2014. "25 Years after the Fall of the Berlin Wall: The Economic Integration of East Germany." Halle Institute for Economic Research - IWH.

Jensen, Heinz. 2004. "Transfers to Germany's Eastern Lander: A Necessary Price for Convergence or a Permanent Drag?" *European Commission Directorate General for Economic and Financial Affairs, Country Focus* 1, no. 16.

Kaldor, Nicholas. 1961. "Capital Accumulation and Economic Growth." In *The Theory of Capital*, edited by Friedrich A. Lutz and Douglas C. Hague, 177–222. New York: St. Martin's Press (for Internat. Econ. Assoc.).

Karabarbounis, Loukas, and Brent Neiman. 2014. "The Global Decline of the Labor Share." *Quarterly Journal of Economics* 2014:61–103.

Kaser, Michael. 2007. "East Germany's Economic Transition in Comparative Perspective." In *Germany's Economic Performance From Unification to Euroization*, edited by Jens Hölscher, 229–239. Palgrave Macmillan UK.

Kravis, Irving B. 1959. "Relative Income Shares in Fact and Theory." *American Economic Review* 49 (5): 917–949.

Krueger, Alan B., and Jorn-Steffen Pischke. 1995. "A Comparative Analysis of East and West German Labor Markets: Before and After Unification." In *Differences and Changes in Wage Structures*, edited by Richard B. Freeman and Lawrence F. Katz, 405–455. The University of Chicago Press.

Mendoza, Enrique G., Assaf Razin, and Linda L. Tesar. 1994. "Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption." *Journal of Monetary Economics* 34 (3): 297–323.

Raffelhüschen, Bernd. 1992. "Labor Migration in Europe, Experiences from Germany after Unification." *European Economic Review* 36:1453–1471.

Rodrik, Dani. 1994. "Foreign Trade in Eastern Europe's Transition: Early Results." In *The Transition in Eastern Europe*, edited by Olivier Jean Blanchard, Kenneth A. Froot, and Jeffrey D. Sachs, Volume 2, 319–356. The University of Chicago Press.

Roland, Gerard, and Thierry Verdier. 1999. "Transition and the Output Fall." *Economics of Transition* 7 (1): 1–28.

Sinn, Gerlinde, and Hans-Werner Sinn. 1992. *Jumpstart: The Economic Unification of Germany*. Cambridge, Massachusetts: The MIT Press.

Sinn, Hans-Werner. 2002. "Germany's Economic Unification: An Assessment after Ten Years." *Review of International Economics* 10:113–128.

———. 2007. *Can Germany be Saved? The Malaise of the World's First Welfare State*. Cambridge, Massachusetts: The MIT Press.

Snower, Dennis J., and Christian Merkl. 2006. "The Caring Hand that Cripples: The East German Labor Market after Reunification." *American Economic Review Papers and Proceedings* 96:375–382.

Uhlig, Harald. 2006. "Regional Labor Markets, Network Externalities and Migration:The Case of German Reunification." *American Economic Review Papers and Proceedings* 96:383–387.

———. 2008. "The Slow Decline of East Germany." *Journal of Comparative Economics* 36:517–541.

United Nations. 1988. National Accounts Statistics: Main Aggregates and Detailed Tables. Part I, Publishing Division United Nations, New York, N.Y.

vom Lehn, Christian. 2018. "Understanding the Decline in the U.S. Labor Share: Evidence from Occupational Tasks." *European Economic Review* 108:191–220.

A Appendix

A.1 Data Appendix

Data for East Germany cover five states: Brandenburg, Mecklenburg-Vorpommern, Sachsen, Sachsen-Anhalt and Thüringen. East Berlin is excluded due to lack of data. The data used in this paper are from several sources, as outlined below.

(i) Statistisches Ämter Der Bundes und Der Länder, Volkswirtschaftliche Gesamtrechnungen der Länder, henceforth VGRdL (http://www.vgrdl.de),

(ii) The Conference Board, Total Economy Database, May 2016 release, henceforth TED 2016 and the March 2018 release, henceforth TED 2018 (https://www.conference-board.org/data/economydatabase/),

(iii) Statistisches Bundesamt, Genesis-Online Datenbank, henceforth Genesis-Online (https://www-genesis.destatis.de/genesis/online),

(iv) the Organization for Economic Co-operation and Development, henceforth OECD (available at http://stats.oecd.org and http://www.oecd-ilibrary.org/statistics), and

(v) United Nations, World Population Prospects 2017, henceforth UN (https://esa.un.org/unpd/wpp/Download/Standard/Population/.

Real gross domestic product (GDP) for East Germany (covering the five aforementioned states and excluding Berlin) is available from VGRdL since 1991. To obtain real GDP for years 1989 and 1990, I use the growth rates of East Germany's GDP reported in TED 2016 and the level of GDP in 1991 from VGRdL.

The working-age $(15 - 64)$ population for East Germany since 1991 is from Genesis-Online, Series: $12411 - 0011$, Population: Länder, reference date, age. Data for 1989 and 1990 is from Table 3.9 of the 1991 and 1992 Statistical Yearbooks of Germany (see Statistisches Jahrbuch für die Bundesrepublik Deutschland available at https://www.destatis.de).

For comparison with East Germany, I construct GDP per working-age $(15 - 64)$ population starting 1989 for twelve other transition economies (i.e. Bulgaria, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Russia, Slovakia, Slovenia and Ukraine) using GDP data from TED 2018 and working-age population from the UN.

East Germany's capital income share is computed using equation 1 in Section 2, and nominal data from VGRdL. Since the labor income of the self-employed in not reported, I follow an imputation method suggested by Gollin (2002, p. 468) which assumes a self-employed person earns the average employees' compensation. I multiply by the stock of self-employed (from VGRdL or Genesis-Online) to obtain their total labor income.

Net transfers from West to East Germany up to 2003 are from Uhlig (2008, Figure 2).

For $2004-2015$, I compute the growth rates of East Germany's trade balance deficit to GDP ratio using VGRdL data, and assume net transfers shrink at the same annual rates. This proxy for the costs of unification was suggested by IWH (2014).

Total hours worked for East Germany are available from VGRdL since 2000. To compute East German hours for $1991-1999$, I combine OECD and VGRdL data. Letting H denote total hours worked, $H^{\text{East Germany}}_{1991-1999} = H^{\text{Germany}}_{1991-1999} - H^{\text{West Germany}}_{1991-1999} - H^{\text{Berlin}}_{1991-1999}$. I construct hours for Germany and West Germany using VGRdL data on total employed (E) and OECD data on hours worked by employed persons (H/E), as in equation 14. The OECD data have longer availability and are interpolated with VGRdL data.[28]

$$H \equiv \frac{H}{E} \cdot E \tag{14}$$

To construct hours for Berlin, I use data on E from VGRdL, and an estimate of H/E. Since the OECD doesn't provide data on hours worked by employed persons for Berlin, I assume that $(H/E)^{\text{Berlin}}_{1991-1999} = 1.05 \cdot (H/E)^{\text{Germany}}_{1991-1999}$, where 1.05 represents the average ratio of these hours in Berlin relative to Germany in VGRdL data over $2000-2005$.

Total hours worked in East Germany in 1989 are computed as $H^{\text{East Germany}}_{1989} = \frac{H^{\text{East Germany}}_{1991}}{0.955 \cdot 0.735}$, where the denominator captures estimates from the literature on the changes in H/E and E over the two year period. Namely, Bird, Schwarze, and Wagner (1994) report a 4.5 percent decline in hours worked by employed persons in East Germany from 1989 to 1991. Dornbusch and Wolf (1994) report a 26.5 percent drop in East German employment over the two year period. Krueger and Pischke (1995) report a similar number.

East German average hours worked per working-age person are computed as total hours worked divided by the population of ages 15 to 64, and expressed relative to 5200 hours per year (52 weeks times 100 hours each). For comparison with East Germany, I construct average hours worked for the transition economies previously mentioned using total hours worked from the Conference Board's Total Economy Database and population from UN. Hours in TED 2016 are available since 1990, while an older release from June 2009 has hours data starting in 1989. Ukraine hours are not available.

East Germany's hourly wage rates since 1991 are computed as compensation of employees divided by total hours worked by employees. Total employee hours are the number of employees times the average hours worked by employed persons (H/E). Nominal wage rates are deflated using the consumption deflator from national accounts.

Tax rates for Germany are computed using the methodology of Mendoza, Razin, and Tesar (1994) and data from the OECD Revenue Statistics and National Accounts.

[28] Hours worked by employed persons (H/E) for Germany from the OECD and VGRdL are identical over $2000-2015$, while for West Germany OECD data over $2000-2011$ are 1 percent higher, on average.

A.2 Proofs and Derivations of Equilibrium Conditions

A.2.1 Proof of Proposition 1

The aggregate production function has five branches depending on which plants operate and how much labor they employ. To simplify the derivations, I drop time subscripts.

Case 1. Suppose all plants operate and are allocated more than the minimum labor requirement. That is, $m_i = M_i$ for $i \in \{L, H\}$, and $n_i > \bar{n}$ for $i \in \{L, H\}$.

The aggregate production function is:

$$F(N, M_H, M_L) = \max_{\{n_i\}_{i\in\{H,L\}}} M_H z_H n_H^{1-\theta} + M_L z_L n_L^{1-\theta}$$
$$\text{s.t. } M_H n_H + M_L n_L = N$$

The first order conditions yield: $n_H = \rho n_L$, where $\rho \equiv \left(\frac{z_H}{z_L}\right)^{1/\theta}$. The feasibility constraint then gives $n_L = \frac{N}{\rho M_H + M_L}$. Next, I use the expressions for n_H and n_L to obtain:

$$F(N, M_H, M_L) = z_L N^{1-\theta} (\rho M_H + M_L)^{\theta}$$

Recall that I assumed $n_i > \bar{n}$. Thus, $n_L = \frac{N}{\rho M_H + M_L} > \bar{n} \Leftrightarrow N > (\rho M_H + M_L)\,\bar{n}$

To summarize: If $N > (\rho M_H + M_L)\,\bar{n}$, then $F(N, M_H, M_L) = z_L (\rho M_H + M_L)^{\theta} N^{1-\theta}$.

Case 2. Suppose all plants operate. Suppose z_H-plants are allocated more than the minimum labor requirement, while z_L-plants operate at the minimum scale. That is, $m_i = M_i$ for $i \in \{L, H\}$, $n_H > \bar{n}$, and $n_L = \bar{n}$.

The aggregate production function is:

$$F(N, M_H, M_L) = \max_{n_H} M_H z_H n_H^{1-\theta} + M_L z_L \bar{n}^{1-\theta}$$
$$\text{s.t. } M_H n_H + M_L \bar{n} = N$$

The solution is $n_H = \frac{N - M_L \bar{n}}{M_H}$. The aggregate production function is then given by:

$$F(N, M_H, M_L) = z_H M_H^{\theta} (N - M_L \bar{n})^{1-\theta} + z_L \bar{n}^{1-\theta} M_L$$

The restriction $n_H > \bar{n}$ implies that $N > (M_H + M_L)\,\bar{n}$.

To sum up: If $N > (M_H + M_L)\,\bar{n}$, $F(N, M_H, M_L) = z_H M_H^{\theta} (N - M_L \bar{n})^{1-\theta} + z_L \bar{n}^{1-\theta} M_L$.

Case 3. Suppose all z_H-plants operate and are allocated at least the minimum labor requirement. Suppose only some z_L-plants operate and they operate at the minimum scale. That is, $m_H = M_H$, $n_H \geq \bar{n}$, $0 < m_L < M_L$, and $n_L = \bar{n}$.

The aggregate production function is:

$$F(N, M_H, M_L) = \max_{n_H, m_L} M_H z_H n_H^{1-\theta} + m_L z_L \bar{n}^{1-\theta}$$
$$\text{s.t. } M_H n_H + m_L \bar{n} = N$$

The measure of low productivity plants that operate is given by $m_L = \frac{N - M_H n_H}{\bar{n}}$. Then:

$$F(N, M_H, M_L) = \max_{n_H \geq \bar{n}} M_H z_H n_H^{1-\theta} + \left(\frac{N}{\bar{n}} - \frac{M_H n_H}{\bar{n}} \right) z_L \bar{n}^{1-\theta}$$

The first order condition yields $n_H = \alpha \bar{n}$, where $\alpha \equiv \left(\frac{(1-\theta) z_H}{z_L} \right)^{1/\theta}$. Combining the first order condition with $n_H \geq \bar{n}$ yields $n_H = \max\{1, \alpha\} \bar{n}$. Using the expressions for m_L and n_H, the assumption $0 < m_L < M_L$ becomes $\max\{1, \alpha\} M_H \bar{n} < N < [\max\{1, \alpha\} M_H + M_L] \bar{n}$.

To summarize: If $\max\{1, \alpha\} M_H \bar{n} < N < [\max\{1, \alpha\} M_H + M_L] \bar{n}$, then
$F(N, M_H, M_L) = \left[z_L \bar{n}^{-\theta} \right] N + \left[z_H \max\{1, \alpha\}^{1-\theta} - z_L \max\{1, \alpha\} \right] \bar{n}^{1-\theta} M_H$

Case 4. Suppose all z_H-plants operate and are allocated more than the minimum labor requirement. Suppose no z_L-plants operate. That is, $m_H = M_H$, $n_H > \bar{n}$, $m_L = n_L = 0$.

The aggregate production function is:

$$F(N, M_H, M_L) = \max_{n_H} M_H z_H n_H^{1-\theta}$$
$$\text{s.t. } M_H n_H = N$$

The solution is $n_H = N/M_H$ and $F(N, M_H, M_L) = z_H M_H^{\theta} N^{1-\theta}$.

Recall that I assumed $n_H > \bar{n}$. This implies that $N/M_H > \bar{n}$. Moreover, from Case 3, I know that if $\max\{1, \alpha\} M_H \bar{n} < N$ then $m_L > 0$; hence, I can restrict $N \leq \max\{1, \alpha\} M_H \bar{n}$.

To summarize: If $M_H \bar{n} < N \leq \max\{1, \alpha\} M_H \bar{n}$, then $F(N, M_H, M_L) = z_H M_H^{\theta} N^{1-\theta}$.

Case 5. Suppose only some z_H-plants operate, and they hire the minimum amount of labor. Suppose no z_L-plants operate. That is, $m_H < M_H$, $n_H = \bar{n}$, $m_L = n_L = 0$.

The aggregate production function is:

$$F(N, M_H, M_L) = \max_{m_H} m_H z_H \bar{n}^{1-\theta}$$
$$\text{s.t. } m_H \bar{n} = N$$

The solution is $m_H = N/\bar{n}$ and $F(N, M_H, M_L) = z_H \bar{n}^{-\theta} N$.

Recall I assumed $m_H < M_H$. This implies $N/\bar{n} < M_H$.

To summarize: If $N < M_H\bar{n}$, then $F(N, M_H, M_L) = z_H\bar{n}^{-\theta}N$.

So far, I have shown that:

$$F(N, M_H, M_L) = \tag{15}$$
$$\begin{cases} f_1 \equiv z_L(\rho M_H + M_L)^\theta N^{1-\theta} & \text{if } N > (\rho M_H + M_L)\bar{n} \\ f_2 \equiv z_H M_H^\theta (N - M_L\bar{n})^{1-\theta} + z_L\bar{n}^{1-\theta}M_L & \text{if } N > (M_H + M_L)\bar{n} \\ f_3 \equiv AM_H + z_L\bar{n}^{-\theta}N & \text{if } G_\alpha M_H\bar{n} \leq N < (G_\alpha M_H + M_L)\bar{n} \\ f_4 \equiv z_H M_H^\theta N^{1-\theta} & \text{if } M_H\bar{n} \leq N \leq G_\alpha M_H\bar{n} \\ f_5 \equiv z_H\bar{n}^{-\theta}N & \text{if } N \leq M_H\bar{n} \end{cases}$$

where $G_\alpha \equiv \max\{1, \alpha\}$ and $A = \left[z_H(G_\alpha)^{1-\theta} - z_L G_\alpha\right]\bar{n}^{1-\theta}$.
Next, it is easy to show that the following statements hold:

$$\begin{aligned} f_1 &= f_2 \text{ if } N = (\rho M_H + M_L)\bar{n} \\ f_1 &> f_2 \text{ if } N > (\rho M_H + M_L)\bar{n} \\ f_3 &= f_2 \text{ if } N = (G_\alpha M_H + M_L)\bar{n} \\ f_3 &> f_2 \text{ if } N \in ((M_H + M_L)\bar{n}, (\alpha M_H + M_L)\bar{n}) \text{ where } \alpha > 1 \end{aligned} \tag{16}$$

Combining (15) and (16), I find the production function in Proposition (1).

A.2.2 Equilibrium Decision Rules for Plant Labor

Corollary 7 *The derivation of the aggregate production function in Section A.2.1 shows that the decision rules for plant labor are given by:*

$$n_{H,t} = \begin{cases} \frac{\rho N_t}{\rho M_{H,t} + M_{L,t}} & \textit{if } N_t \geq \eta_{1,t} \\ \frac{N_t - M_{L,t}\bar{n}}{M_{H,t}} & \textit{if } N_t \in [\eta_{2,t}, \eta_{1,t}] \\ \max\{1, \alpha\}\bar{n} & \textit{if } N_t \in [\eta_{3,t}, \eta_{2,t}] \\ N_t/M_{H,t} & \textit{if } N_t \in [\eta_{4,t}, \eta_{3,t}] \\ \bar{n} & \textit{if } N_t \leq \eta_{4,t} \end{cases} \quad n_L = \begin{cases} \frac{N_t}{\rho M_{H,t} + M_{L,t}} & \textit{if } N_t \geq \eta_{1,t} \\ \bar{n} & \textit{if } N_t \in [\eta_{2,t}, \eta_{1,t}] \\ \bar{n} & \textit{if } N_t \in [\eta_{3,t}, \eta_{2,t}] \\ 0 & \textit{if } N_t \in [\eta_{4,t}, \eta_{3,t}] \\ 0 & \textit{if } N_t \leq \eta_{4,t} \end{cases}$$

where $\eta_1 \equiv (\rho M_{H,t} + M_{L,t})\bar{n}$, $\eta_2 \equiv (G_\alpha M_{H,t} + M_{L,t})\bar{n}$, $\eta_3 \equiv G_\alpha M_{H,t}\bar{n}$, $\eta_4 \equiv M_{H,t}\bar{n}$ and where $G_\alpha \equiv \max\{1, \alpha\}$.

A.2.3 Proof of Proposition 2

During the transition the stock of z_L-plants depreciates over time while the more productive z_H-plants accumulate. I derive the limit of the aggregate production function as $M_{L,t} \to 0$.

Using equation (4), $\lim_{M_{L,t}\to 0} \eta_{1,t} = \rho M_{H,t}\bar{n}$ and $\lim_{M_{L,t}\to 0} \eta_{2,t} = G_\alpha M_{H,t}\bar{n} = \eta_{3,t}$, the aggregate production function as $M_{L,t} \to 0$ can be written as:

$$\lim_{M_{L,t}\to 0} F\left(N_t, M_{H,t}, M_{L,t}\right) = \begin{cases} z_L \rho^\theta M_{H,t}^\theta N_t^{1-\theta} & \text{if } N_t \geq \rho M_{H,t}\bar{n} \\ z_H M_{H,t}^\theta N_t^{1-\theta} & \text{if } G_\alpha M_{H,t}\bar{n} \leq N_t \leq \rho M_{H,t}\bar{n} \\ A M_{H,t} + z_L \bar{n}^{-\theta} N_t & \text{if } G_\alpha M_{H,t}\bar{n} = N_t = G_\alpha M_{H,t}\bar{n} \\ z_H M_{H,t}^\theta N_t^{1-\theta} & \text{if } M_{H,t}\bar{n} \leq N_t \leq G_\alpha M_{H,t}\bar{n} \\ z_H \bar{n}^{-\theta} N_t & \text{if } N_t \leq M_{H,t}\bar{n} \end{cases} \tag{17}$$

where $G_\alpha \equiv \max\{1, \alpha\}$ and $A = \left[z_H \left(G_\alpha\right)^{1-\theta} - z_L G_\alpha\right] \bar{n}^{1-\theta}$.

Notice that as $M_L \to 0$ the third branch of the production function essentially collapses to one point. At $N_t = G_\alpha M_{H,t}\bar{n}$ the output is:

$$\begin{aligned} A M_{H,t} + z_L \bar{n}^{-\theta} N_t &= \left[z_H \left(G_\alpha\right)^{1-\theta} - z_L G_\alpha\right] \bar{n}^{1-\theta} M_{H,t} + z_L \bar{n}^{-\theta}\left(G_\alpha M_{H,t}\bar{n}\right) \\ &= z_H \left(G_\alpha\right)^{1-\theta} \bar{n}^{1-\theta} M_{H,t} = z_H M_{H,t} \left(\frac{N_t}{M_{H,t}}\right)^{1-\theta} = z_H M_{H,t}^\theta N_t^{1-\theta} \end{aligned}$$

Moreover, using $\rho \equiv \left(\frac{z_H}{z_L}\right)^{1/\theta}$, i.e. $z_H = z_L \rho^\theta$, the output on the first branch of the production function can also be written as: $z_H M_{H,t}^\theta N_t^{1-\theta}$.

To summarize, equation (17), becomes:

$$\lim_{M_{L,t}\to 0} F\left(N_t, M_{H,t}, M_{L,t}\right) = \begin{cases} z_H M_{H,t}^\theta N_t^{1-\theta} & \text{if } N_t \geq M_{H,t}\bar{n} \\ z_H \bar{n}^{-\theta} N_t & \text{if } N_t \leq M_{H,t}\bar{n} \end{cases}$$

A.2.4 Intertemporal and Intratemporal Equilibrium Conditions

I derive the first order conditions of the jth union's problem presented in Section 3.2. For the time being, I ignore the requirement that the markup be at most $1/\bar{\nu}_t$. Let $\lambda_t(j)$ denote the Lagrange multiplier on the budget constraint at time t. The first order conditions are summarized by the budget constraints, the laws of motion for capital, $X_{L,t}(j) = 0$, and:

$$\begin{aligned} \beta^t U_C[C_t(j), l_t(j)] &= \lambda_t(j) \cdot (1+\tau_{C,t}) \\ \beta^t U_l[C_t(j), l_t(j)] \frac{\partial l_t(j)}{\partial w_t(j)} &= -\lambda_t(j) \cdot (1-\tau_{N,t}) \left[l_t(j) + w_t(j) \frac{\partial l_t(j)}{\partial w_t(j)} \right] \\ \frac{\lambda_t(j)}{\lambda_{t+1}(j)} &= (1+(1-\tau_{M,t+1})(r_{H,t+1}-\delta_H)) \end{aligned} \tag{18}$$

and the transversality condition: $\lim_{t\to\infty} \beta^t U_C[C_t(j), l_t(j)] M_{H,t+1}(j) = 0$.

The Euler Equation is:

$$\frac{U_C(C_t(j), l_t(j))}{U_C(C_{t+1}(j), l_{t+1}(j))} = \beta \frac{1+\tau_{C,t}}{1+\tau_{C,t+1}} (1+(1-\tau_{M,t+1})(r_{H,t+1}-\delta_H))$$

To simplify equation (18), I derive $\partial l_t(j)/\partial w_t(j)$. Recall that the demand for labor of type j is given by equation (6), i.e. $l_t(j) = \left(\frac{w_t}{w_t(j)}\right)^{1/(1-\nu)} N_t$. Thus,

$$\frac{\partial l_t(j)}{\partial w_t(j)} = w_t^{1/(1-\nu)} N_t \frac{1}{\nu-1} w_t(j)^{1/(\nu-1)-1} = \frac{1}{\nu-1} \frac{l_t(j)}{w_t(j)}$$

Then, equation (18) becomes:

$$\beta^t U_l[C_t(j), l_t(j)] \frac{1}{\nu-1} \frac{l_t(j)}{w_t(j)} = -\lambda_t (1-\tau_{N,t}) \frac{\nu}{\nu-1} l_t(j)$$
$$\Leftrightarrow \beta^t U_l[C_t(j), l_t(j)] = -\lambda_t (1-\tau_{N,t}) \nu w_t(j)$$

The intratemporal condition can be written as:

$$w_t(j) = -\frac{1}{\nu} \frac{(1+\tau_{C,t})}{(1-\tau_{N,t})} \frac{U_l[C_t(j), l_t(j)]}{U_C[C_t(j), l_t(j)]} \tag{19}$$

where $1/\nu$ is the markup that the union chooses. If the government requires that the markup be at most $1/\bar{\nu}_t$, I obtain equation (12) from the paper.