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Abstract

Asset price data imply a large degree of international risk sharing, while aggregate consumption data do not. We show that a model with trade in goods and endogenously segmented asset markets can account for this puzzling discrepancy. *Active households*—who pay a fixed cost to transfer money into or out of assets—share risk within and across countries, and their marginal utility growth prices assets, so asset prices imply high risk sharing. *Inactive households* consume their current income and do not share risk, so aggregate consumption (which averages across all households) reflects lower risk sharing. The model also provides a resolution to the Backus-Smith-Kollmann puzzle.

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Keywords: international risk sharing, real exchange rates, segmented asset markets, limited asset market participation, consumption-real exchange rate anomaly, Backus-Smith-Kollmann puzzle

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1 Introduction

How much do countries share risk through international financial markets, and how large are the gains from doing so? The answers to these questions depend on how risk sharing is measured. Brandt, Cochrane, and Santa-Clara (2006) show that measures of risk sharing based on asset price data imply significant international risk sharing, while measures based on aggregate consumption data display much less risk sharing.[1] As a consequence, as shown by Lewis (2000), welfare gains from risk sharing based on stock returns are higher than those based on aggregate consumption.

The discrepancy in these risk sharing measures is puzzling. In standard international macro models, consumption determines asset prices, rendering consumption-based and asset price-based measures of risk sharing identical. Resolving this puzzle involves either changing the preferences used in standard models or modifying the asset market structure, as discussed by Brandt, Cochrane and Santa-Clara. In this paper, we take the latter route, and evaluate the extent to which frictions that endogenously limit participation in asset markets can account for the discrepancy between the asset price-based and consumption-based measures of international risk sharing.

We analyze a two-country monetary model with international trade in assets along the lines of Alvarez, Atkeson and Kehoe (2002, 2009) (henceforth AAK02 and AAK09), in which households pay heterogenous fixed costs to exchange money for interest-bearing assets. Households face idiosyncratic income shocks, and asset markets are endogenously segmented because only a fraction of households at any point in time find it beneficial to pay the fixed cost to access their assets.[2] We enrich this model with real aggregate shocks and trade in goods, features which are essential for generating movements in aggregate consumption and risk sharing across countries. Indeed, without trade, consumption is constrained by domestic resources and there is no risk sharing.

Limited asset market participation leads to differences in the asset price-based and consumption-based risk sharing measures. Households that actively adjust their asset holdings share risk among each other, both within and across countries. Since these households' marginal utility growth determines asset prices, there is a high degree of international risk sharing based on asset prices. On the other hand, these households account for a time-varying fraction of aggregate consumption in each country, so measures of consumption imply a low

[1] In particular, Brandt, Cochrane, and Santa-Clara (2006) show that stochastic discount factors derived from stock prices are highly correlated across countries, indicating significant international risk sharing, while marginal utility growth derived from aggregate consumption is weakly correlated across countries, indicating much less risk sharing.

[2] In AAK02, households have idiosyncratic income shocks, while in AAK09, households have heterogeneous fixed costs. We incorporate both these features of household heterogeneity in our model.

degree of international risk sharing at the aggregate level.

We quantify this mechanism by calibrating our model to match the cross-sectional variance of household income and consumption in US data and the fraction of households actively adjusting their assets, along with the time series properties of aggregate traded and non-traded output in the US and an aggregate of 21 OECD trading partners. We compute risk sharing measures in the model based on stochastic discount factors derived from asset prices and based on aggregate consumption. The model predicts a high cross-country correlation of the equity-price based discount factors—about 0.85—and a cross-country correlation of aggregate consumption that is much lower—about 0.32. These values are in line with the empirical findings in Brandt, Cochrane, and Santa-Clara (2006). Both asset market segmentation and trade in goods are necessary to generate the discrepancy between consumption-based and asset price-based measures of risk sharing in our framework. To illustrate this, we consider two alternative market structures that eliminate, in turn, these key features. In a frictionless, complete markets model (with no asset market segmentation), the risk sharing measures are both above 0.9, reflecting relatively high risk sharing. In a segmented markets model with no trade in goods, the ordering of the risk sharing statistics is opposite: aggregate consumption reflects better risk sharing than asset prices.

In our model, underlying the difference between asset-price based and consumption-based measures of risk sharing is the prediction that active households' consumption is more highly correlated across countries than inactive households' consumption. We provide suggestive evidence for this implication using micro data from the US (the Consumer Expenditure Survey, or CEX) and the UK (the Family Expenditure Survey, or FES), showing that assetholders in the two countries have more highly correlated consumption than non-assetholders.

Our model also generates a negative correlation between the real exchange rate and the ratio of aggregate consumption across countries, offering a resolution to the Backus-Smith-Kollmann puzzle (Backus and Smith (1993), Kollmann (1995)).[3] In the standard complete markets model, the ratio of two countries' aggregate consumption is perfectly correlated with the real exchange rate, but in the data this correlation is often negative (see calculations for G7 countries in Table 2). In our model, the ratio of *active* households' consumption, not aggregate consumption, is perfectly correlated with the real exchange rate. We find the correlation of relative aggregate consumption with the real exchange rate is -0.8. Traded output shocks and monetary shocks generate this negative correlation, because they move the ratios of relative aggregate consumption and relative active consumption in opposite directions. Non-traded shocks move the ratios of relative consumption in the same direction,

[3]The Backus-Smith-Kollmann puzzle is also referred to in the literature as the consumption-real exchange rate anomaly.

but lead to a low cross-country correlation in aggregate consumption.[4,5]

A version of our model with an exogenous division of active and inactive households yields both a negative consumption-real exchange rate correlation (as also shown in Kollmann (2012)) and a discrepancy between asset-price based and consumption-based risk sharing measures, but our model with endogenously segmented markets has additional important implications. In particular, the fraction of households actively participating varies over time in response to shocks that change the incentive to participate. One implication is that endogenous movements in the fraction of inactive households and their average income actually reinforce risk sharing and slightly raise the consumption-real exchange rate correlation. A related cross-sectional implication is that higher average inflation in our model is associated with more frequent asset market activity, as inflation reduces the value of inactive households' income, leading to better international risk sharing. Using data for 86 countries, we document that average inflation is strongly positively associated with the correlation between the real exchange rate and consumption relative to the US. Our model with endogenous segmentation is consistent with this fact, while the exogenously segmented model is not.

Given that our benchmark model can address the discrepancy between the asset-price based and consumption-based measures of risk sharing, we employ it to evaluate the welfare effects of access to international financial markets. We construct alternative measures of welfare gains relative to financial autarky based on actual ex ante average utility across households, utility of aggregate consumption, or utility of active households' consumption. We find that active consumption-based welfare gains are significantly larger compared to aggregate consumption-based gains, consistent with Lewis (2000)'s findings that asset prices imply higher welfare gains than aggregate consumption. With a coefficient of relative risk aversion of two, the welfare gain measured from active consumption is an order of magnitude larger than the gain measured from aggregate consumption.

Other papers that explain the discrepancy between consumption-based and asset price-based measures of risk sharing include Colacito and Croce (2011) and Lewis and Liu (2015). In these papers, Epstein-Zin preferences and long-run consumption risk result in a separation between asset prices and contemporaneous aggregate consumption levels, which generates

[4] The role of non-traded goods in reducing international consumption correlations has been well-studied, going back to, e.g. Stockman and Dellas (1989), Tesar (1993), and Stockman and Tesar (1995). However, while adding non-traded goods per se reduces consumption correlations, it does not account for a difference in consumption-based and asset price-based risk sharing measures or a low consumption-real exchange rate correlation in the absence of asset market frictions.

[5] An alternative asset market friction that has been used to explain the low consumption-real exchange rate correlation is incomplete markets (e.g., Corsetti, Dedola, and Leduc (2008)). However, Devereux, Smith, and Yetman (2012) (and references therein) show that a key prediction of an incomplete markets model—that conditional forecasts of the real exchange rate and the ratio of consumption are perfectly correlated—is not borne out in the data.

cross-country asset price correlations that are larger than consumption correlations. We complement these papers by evaluating how much we can explain with standard time-separable preferences and a single asset market friction—segmented markets due to fixed costs—with otherwise complete asset markets.

Our use of an endogenously segmented asset markets model with heterogeneous households to study international risk sharing is novel.[6] AAK02 and AAK09 both analyze a two-country model with only non-traded goods and monetary shocks, and their focus is on interest rates and exchange rates, while aggregate consumption is essentially constant due to the absence of real shocks. Both papers conjecture that segmented asset markets can explain the Backus-Smith-Kollmann puzzle, but do not explore the implications of their model for consumption. In contrast to these papers, our model has real shocks and trade in goods to allow for movements in aggregate consumption and international risk sharing. In an appendix, AAK09 consider an extension with traded goods, but no real shocks.

In addition to the exogenously segmented market model in Kollmann (2012), other work that incorporates household heterogeneity to address international risk sharing and the Backus-Smith-Kollmann puzzle includes Sungur (2004), and Kocherlakota and Pistaferri (2007).[7] Sungur finds support for the AAK02 model's relationship between the real exchange rate and active households' consumption in Italian regional data. Kocherlakota and Pistaferri test the implications of a model with private information using US and UK survey data and show that the ratio of higher moments of the consumption distribution across countries is linked to the real exchange rate.

The remainder of the paper is organized as follows. Section 2 presents our benchmark segmented asset markets model with traded and non-traded goods and characterizes equilibrium. Section 3 contains all the quantitative analysis, including results on measures of international risk sharing and welfare gains from financial markets. Section 4 concludes.

2 Model

We extend the two-country environment in AAK02 and AAK09, to include two main features. *Real shocks* generate movements in aggregate consumption, and *trade in goods* is essential for generating risk sharing across countries, while non-traded goods generate real exchange

[6]Endogenously segmented asset markets models have been used recently in studying the effects of monetary shocks on inflation and interest rates. In addition to Alvarez, Atkeson, and Kehoe (2002), recent examples are Alvarez, Atkeson, and Edmond (2009), Khan and Thomas (2015), and Dotsey and Guerron-Quintana (2016).

[7]A related set of papers characterize differences in the consumption behaviour of stockholders and non-stockholders (e.g., Vissing-Jørgensen (2002a)), or active and inactive stock market participants (e.g., Bonaparte and Cooper (2009)) in closed economy settings.

rate movements. We also incorporate households who do not hold assets as well as those who hold assets but are sometimes inactive in asset markets.

We examine an infinite horizon, two-country, cash-in-advance economy with three goods—one internationally traded good and two non-traded goods.[8] We refer to the two countries as "home" and "foreign", and label foreign variables with an asterisk ($*$). Each country has a government and a continuum of households of measure one. Each household receives endowments of traded and non-traded goods, consisting of an idiosyncratic component and an aggregate component. Home households use the home currency (hereafter referred to as dollars) and foreign households use the foreign currency (euros) to purchase goods. Exogenous fluctuations in aggregate endowments and in money growth rates are the sources of aggregate uncertainty.

Assetholding households can buy and sell assets to insure against idiosyncratic and aggregate shocks. However, they must pay a fixed cost to transfer money into or out of their asset balances. This fixed cost is a stand-in for frictions that explain why households do not actively participate in asset markets every period, such as transactions costs for converting interest-bearing assets into cash, as in Baumol (1952) and Tobin (1956).[9] This segmentation of households into active participants and non-participants in the asset market disconnects asset prices from aggregate consumption.

2.1 Timing and Uncertainty

In each period $t = 0, 1, 2, \ldots$, there are three aggregate shocks and one idiosyncratic shock in each country. The aggregate shocks in the home country are the endowments (Y_{Tt}, Y_{Nt}) and the change in the money supply, $\mu_t = M_t/M_{t-1}$, and in the foreign country they are $(Y^*_{Tt}, Y^*_{Nt}, \mu^*_t)$. The idiosyncratic shock y_t determines an individual household's endowments, i.e. $(Y_{Tt}y_t, Y_{Nt}y_t)$ in the home country and $(Y^*_{Tt}y_t, Y^*_{Nt}y_t)$ in the foreign country. y_t is drawn i.i.d. across households and over time from a distribution with density f_y, with a mean of 1.

We let s_t denote the realization of the six aggregate shocks, $s_t = (Y_{Tt}, Y_{Nt}, \mu_t, Y^*_{Tt}, Y^*_{Nt}, \mu^*_t)$, and define the aggregate *state* $s^t = (s_0, s_1, \ldots, s_t)$ as the history up to date t of these shocks, with s_0 given. We let $g(s^t)$ denote the density of the aggregate state, s^t. We define $y^t = (y_0, y_1, \ldots, y_t)$ as the history of idiosyncratic shocks for any household and, abusing

[8] We use a single traded good to focus on the difference between traded and non-traded goods, as in Tesar (1993) and Backus and Smith (1993). Adding multiple traded goods as in, for example, Stockman and Tesar (1995), would introduce an additional risk sharing channel through movements in the terms of trade (as in Cole and Obstfeld (1991)).

[9] Another motivation for such a cost is that there is a fixed cost to ensuring repayment of private debt, as described by Chatterjee and Corbae (1992). A fixed cost like this is also related to the stock market participation cost considered by Luttmer (1999).

notation, we let $f_y(y^t)$ denote the density of the idiosyncratic history, y^t. In what follows, the argument of f_y will make it clear whether it refers to the density over histories or over current realizations of the idiosyncratic shock.

2.2 Households

Households differ *ex ante* along two dimensions: first, a fraction ω are assetholders, and the remaining $1-\omega$ are non-assetholders. Second, among assetholders, the fixed cost of accessing the asset market, γ, is drawn from a distribution $f_\gamma(\gamma)$. The fixed cost for each household is constant over time. We describe the decision problem of a home country assetholder first.

A home country household enters period t with money balances from selling its endowments from the previous period, $t-1$, equal to $(P_T(s^{t-1})Y_{Tt-1} + P_N(s^{t-1})Y_{Nt-1})\, y_{t-1}$, where P_T and P_N denote the dollar prices of traded and non-traded goods. The household then splits into a worker and a shopper. The worker sells the period-t endowment while the shopper decides whether to make any transfers to or from the asset market account and purchases goods for consumption. The shopper's cash-in-advance constraint is:

$$\begin{aligned} P(s^t)C(s^t, y^{t-1}, \gamma) \leq & \left(P_T(s^{t-1})Y_{Tt-1} + P_N(s^{t-1})Y_{Nt-1}\right) y_{t-1} \\ & + z(s^t, y^{t-1}, \gamma)\left[\tau(s^t, y^{t-1}, \gamma) - P(s^t)\gamma\right] \end{aligned} \tag{1}$$

where $P(s^t)$ is the consumption price index, $z(s^t, y^{t-1}, \gamma)$ is an indicator equal to 1 if the household makes a transfer and zero otherwise, and $\tau(s^t, y^{t-1}, \gamma)$ is the amount transferred. If $\tau > 0$, the household withdraws money from the asset market, and if $\tau < 0$, the household saves some of its current income. Transferring to or from the asset market requires the payment of a fixed amount γ of consumption goods. If $z(s^t, y^{t-1}, \gamma) = 0$, the household spends all of its available money in the current period and doesn't change the amount of assets it holds.

Consumption is a composite of traded and non-traded goods, with constant elasticity of substitution $\sigma > 0$ and weight $a \in (0,1)$ on traded goods, $C = (aC_T^{\frac{\sigma-1}{\sigma}} + (1-a)C_N^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$. Standard derivations yield demands for traded and non-traded household consumption,

$$C_T(s^t, y^{t-1}, \gamma) = \left(\frac{P_T(s^t)}{aP(s^t)}\right)^{-\sigma} C(s^t, y^{t-1}, \gamma) \tag{2}$$

$$C_N(s^t, y^{t-1}, \gamma) = \left(\frac{P_N(s^t)}{(1-a)P(s^t)}\right)^{-\sigma} C(s^t, y^{t-1}, \gamma) \tag{3}$$

and a similar division of the fixed cost γ into traded and non-traded parts, $\gamma_T(s^t)$ and $\gamma_N(s^t)$. The consumption price index is $P(s^t) = (a^\sigma P_T(s^t)^{1-\sigma} + (1-a)^\sigma P_N(s^t)^{1-\sigma})^{\frac{1}{1-\sigma}}$.

In the asset market, assetholding households trade aggregate- and idiosyncratic-state-contingent assets with an international financial intermediary. Households only trade assets of their own currency (absence of arbitrage ensures that this is without loss of generality). The asset market budget constraint for a household with state (s^t, y^{t-1}) in $t \geq 1$ is

$$\int\int q(s^t, s_{t+1}, y^{t-1}, y_t)B(s^t, s_{t+1}, y^{t-1}, y_t, \gamma)ds_{t+1}dy_t \\ +z(s^t, y^{t-1}, \gamma)\tau(s^t, y^{t-1}, \gamma) \leq B(s^t, y^{t-1}, \gamma) \tag{4}$$

where $q(s^t, s_{t+1}, y^{t-1}, y_t)$ is the price of a claim to one unit of home currency in the asset market in state (s^{t+1}, y^t), and $B(s^t, s_{t+1}, y^{t-1}, y_t, \gamma)$ is the quantity of these claims that a household with fixed cost γ purchases in state (s^t, y^{t-1}). The payoff from asset holdings, $B(s^t, y^{t-1}, \gamma)$, is allocated toward new asset purchases and transfers to the goods market.

Assetholders are initially endowed in period 0 with government debt. A home country household with fixed cost γ has $B_{h0}(\gamma)$ dollars of home government debt and $B_{f0}(\gamma)$ euros of foreign government debt. As in AAK09, these initial endowments of government debt are contingent on the idiosyncratic fixed cost so as to make households initially identical within and across countries. In period 0, there is trading in the asset market, but not the goods market, so the asset market budget constraint is

$$\int\int q(s_1, y_0)B(s_1, y_0, \gamma)ds_1dy_0 \leq B_{h0}(\gamma) + e_0B_{f0}(\gamma) \tag{5}$$

where e_0 is the exchange rate in period 0, in dollars per euro.

Households in the home country have preferences given by:

$$\sum_{t=0}^{\infty}\int_{s^t}\int_{y^t}\beta^t U\left(C(s^t, y^{t-1}, \gamma)\right) g(s^t)f(y^{t-1})ds^t dy^{t-1} \tag{6}$$

where $\beta \in (0, 1)$, $U(C) = C^{1-\eta}/(1-\eta)$ with $\eta > 0$.

A home household's problem is to choose consumption, asset holdings, and money transfer decisions to maximize utility (6) subject to the sequence of goods market budget constraints, (1), and asset market budget constraints, (4) and (5). We assume that households do not hold cash from one period to the next, either in the goods market or in the asset market, spending all cash in the goods market on consumption, and allocating all cash in the asset market to state-contingent assets. AAK02 develop sufficient conditions for not holding cash to be optimal.

Assetholders in the foreign country face analogous constraints. The foreign price index P^* and consumption levels C_T^*, C_N^*, γ_T^*, γ_N^* are defined similarly as in the home country, given prices $P_T^*(s^t), P_N^*(s^t)$. A foreign household with fixed cost γ in state (s^t, y^{t-1}) has the cash-in-advance constraint:

$$\begin{aligned} P^*(s^t)C^*(s^t, y^{t-1}, \gamma) \leq & \left(P_T^*(s^{t-1})Y_{Tt-1}^* + P_N^*(s^{t-1})Y_{Nt-1}^*\right) y_{t-1} \\ & + z^*(s^t, y^{t-1}, \gamma)\left[\tau^*(s^t, y^{t-1}, \gamma) - P^*(s^t)\gamma\right] \end{aligned} \tag{7}$$

while the foreign asset market budget constraints are:

$$\begin{aligned} \int\int q^*(s^t, s_{t+1}, y^{t-1}, y_t)B^*(s^t, s_{t+1}, y^{t-1}, y_t, \gamma)ds_{t+1}dy_t & \\ + z^*(s^t, y^{t-1}, \gamma)\tau^*(s^t, y^{t-1}, \gamma) \leq B^*(s^t, y^{t-1}, \gamma) & \end{aligned} \tag{8}$$

$$\int\int q^*(s_1, y_0)B^*(s_1, y_0, \gamma)ds_1dy_0 \leq \frac{B_{h0}^*(\gamma)}{e_0} + B_{f0}^*(\gamma) \tag{9}$$

2.2.1 Non-assetholders

Non-assetholders face a version of the cash-in-advance constraint (1) with no possibility of saving or withdrawing money, so they are analogous to the hand-to-mouth households considered in Kollmann (2012). Their consumption, denoted with a subscript I, is given by:

$$C_I(s^t, y_{t-1}) = y_{t-1}\frac{P_T(s^{t-1})Y_{Tt-1} + P_N(s^{t-1})Y_{Nt-1}}{P(s^t)} \tag{10}$$

and the division into traded and non-traded consumption, C_{IT} and C_{IN}, is similar to (2)-(3).

2.3 Asset market

There is a world financial intermediary that trades assets with households and the governments. The intermediary has no wealth, so the net value of total trades equals zero. The intermediary's profits when the aggregate state is s^t are given by adding up the transactions of (s_{t+1}, y_t)-contingent assets of both currencies, at prices $q(s^t, s_{t+1}, y^{t-1}, y_t)$ and $q^*(s^t, s_{t+1}, y^{t-1}, y_t)$, minus purchases of government debt at prices $q(s^t, s_{t+1})$, $q^*(s^t, s_{t+1})$, which are prices of state-contingent claims to one unit of each currency in the next period. We spell out the full intermediary's maximization problem in appendix A.1, and highlight here the following no-arbitrage conditions that this problem yields:

$$q(s^t, s_{t+1}, y^{t-1}, y_t) = q(s^t, s_{t+1}) f(y_t) \tag{11}$$

$$q^*(s^t, s_{t+1}, y^{t-1}, y_t) = q^*(s^t, s_{t+1}) f(y_t) \tag{12}$$

$$q(s^t, s_{t+1}) = q^*(s^t, s_{t+1}) \frac{e(s^t)}{e(s^{t+1})} \tag{13}$$

Here, $e(s^t)$ is the nominal exchange rate, in dollars per euro. Equations (11) and (12) state that the price of one unit of national currency for a household in either country in state (s^{t+1}, y^t) must equal the price of one unit of currency weighted by the probability of the idiosyncratic shock y_t. Equation (13) confirms that restricting households to only hold assets of their own currency is without loss of generality.

2.4 Market Clearing and Equilibrium

An equilibrium consists of goods prices and asset prices along with consumption quantities and asset holdings that solve households' problems and satisfy the no-arbitrage conditions (11)-(13), market clearing conditions for goods, money, and assets, and the governments' budget constraints. Market clearing for traded goods is:

$$\begin{aligned} Y_{Tt} + Y^*_{Tt} = &\int \left(\omega \int \left[C_T(s^t, y^{t-1}, \gamma) + \gamma_T(s^t, \gamma) z(s^t, y^{t-1}, \gamma) \right] f_\gamma(\gamma) d\gamma \right. \\ &\left. + (1-\omega) C_{IT}(s^t, y_{t-1}) \right) f_y(y^{t-1}) dy^{t-1} \\ &+ \int \left(\omega \int \left[C^*_T(s^t, y^{t-1}, \gamma) + \gamma^*_T(s^t, \gamma) z^*(s^t, y^{t-1}, \gamma) \right] f_\gamma(\gamma) d\gamma \right. \\ &\left. + (1-\omega) C^*_{IT}(s^t, y_{t-1}) \right) f_y(y^{t-1}) dy^{t-1} \end{aligned} \tag{14}$$

We include the remaining equilibrium conditions for the home country, with the foreign versions given by the obvious analogues. For non-traded goods in the home country,

$$\begin{aligned} Y_{Nt} = &\int \left(\omega \int \left[C_N(s^t, y^{t-1}, \gamma) + \gamma_N(s^t, \gamma) z(s^t, y^{t-1}, \gamma) \right] f_\gamma(\gamma) d\gamma \right. \\ &\left. + (1-\omega) C_{IN}(s^t, y_{t-1}) \right) f_y(y^{t-1}) dy^{t-1} \end{aligned} \tag{15}$$

The government's budget constraint is:

$$B(s^t) = M(s^t) - M(s^{t-1}) + \int q(s^t, s_{t+1}) B(s^t, s_{t+1}) ds_{t+1} \tag{16}$$

In the asset market, bond holdings summed across all households equals government debt:

$$\int\int\int B(s^t, s_{t+1}, y^{t-1}, y_t, \gamma) f_y(y_t) f_y(y^{t-1}) f_\gamma(\gamma) dy_{t+1} dy^t d\gamma = B(s^t, s_{t+1})$$

and the home money market clearing condition is:

$$\begin{aligned}\int\int \Big[\big(P_T(s^{t-1}) Y_T(s^{t-1}) + P_N(s^{t-1}) Y_N(s^{t-1}) \big) \, y_{t-1} \\ + \omega z(s^t, y^{t-1}, \gamma) \left(\tau(s^t, y^{t-1}, \gamma) + \gamma \right) \Big] f_y(y^{t-1}) f_\gamma(\gamma) dy^{t-1} d\gamma = M(s^t)\end{aligned} \tag{17}$$

2.5 Characterizing Equilibrium

We follow a procedure similar to AAK02 to characterize an equilibrium with a few simple, static conditions determining active and inactive households' consumption allocations and asset market participation decisions. We leave most of the detailed derivations to Appendix A.1, while Appendix A.2 describes the computation method.

Active households pool their income within a period and have equal consumption (because they are *ex ante* identical), denoted $C_A(s^t)$, which satisfies the following condition:

$$\beta^t U'(C_A(s^t)) g(s^t) = \lambda P(s^t) Q(s^t) \tag{18}$$

Here, λ is the multiplier on the date-0 budget constraint and $Q(s^t)$ is the date-0 price of a dollar. Equation (18) is a standard first-order condition given that active households can smooth consumption across dates and states: it equates the marginal utility of consumption in any state in which the household is active to the date-0 price of consumption weighted by the marginal value of wealth.

Inactive assetholding households and non-assetholders consume the value of their income, given by (10). Using the goods market clearing conditions (14)-(15) and the budget constraint (1), we can write the money market clearing condition (17) as $M(s^t) = P_T(s^t) Y_{Tt} + P_N(s^t) Y_{Nt}$. Then, inactive conusmption (10) can be written:

$$C_I(s^t, y_{t-1}) = y_{t-1} \frac{n(s^t)}{\mu_t} \tag{19}$$

with $n(s^t) = \frac{M(s^t)}{P(s^t)}$. Equation (19) shows that inactive households' consumption is reduced by money growth, as some of the real value of their inherited money balances is inflated away each period.

Now, we consider the choice of whether to transfer into or out of the asset market account. For a household with fixed cost γ in state (s^t, y^{t-1}), the net benefit of becoming active can be written as:

$$h(y_{t-1}; s^t, \gamma) = U\left(C_A(s^t)\right) - U\left(y_{t-1}\frac{n(s^t)}{\mu_t}\right) - U'\left(C_A(s^t)\right)\left[C_A\left(s^t\right) + \gamma - y_{t-1}\frac{n(s^t)}{\mu_t}\right] \quad (20)$$

The first two terms of the function h in (20) give the increase in consumption from switching from inactive to active. The third term gives the net cost of the change in asset balances necessary to get to the active consumption level $C_A(s^t)$: an active household increases or reduces asset balances, and this change is valued at the marginal utility of active consumption.

A household is active if the expression in (20) is positive. For any $\gamma > 0$, it is straightforward to verify the following properties of h:

- $h(\cdot; s^t, \gamma)$ has a minimum at $y_{t-1} = C_A(s^t)\frac{\mu(s^t)}{n(s^t)}$
- $h(\cdot; s^t, \gamma)$ is decreasing for $y_{t-1} < C_A(s^t)\frac{\mu(s^t)}{n(s^t)}$ and increasing for $y_{t-1} > C_A(s^t)\frac{\mu(s^t)}{n(s^t)}$
- $h(\cdot; s^t, \gamma)$ is convex
- For CRRA utility, $U(C) = C^{1-\eta}/(1-\eta)$ with $\eta > 0$, $\lim_{y_{t-1}\to 0} h(y_{t-1}, s^t, \gamma) = \infty$ and $\lim_{y_{t-1}\to\infty} h(y_{t-1}, s^t, \gamma) = \infty$

These properties imply that h is U-shaped, with two zeros. We refer to the two zeros of $h(\cdot; s^t, \gamma)$ as $y_L(s^t, \gamma)$ and $y_H(s^t, \gamma)$, with $y_L < y_H$. These cutoffs define households' asset market participation decision: if $y_{t-1} \in [y_L(s^t, \gamma), y_H(s^t, \gamma)]$, the cost of transferring outweighs the benefit, so a household is inactive in period t, and is active if $y_{t-1} < y_L(s^t, \gamma)$ or $y_{t-1} > y_H(s^t, \gamma)$.

The characterization of consumption and asset market activity decisions is analogous in the foreign country, where active households consume $C_A^*(s^t)$. Combining the first order condition (18) with its foreign analogue yields the following risk sharing condition:

$$e(s^t)\frac{P^*\left(s^t\right)}{P\left(s^t\right)} = \frac{\lambda^*}{\lambda}\frac{U'\left(C_A^*(s^t)\right)}{U'\left(C_A(s^t)\right)} \quad (21)$$

This condition relates the ratio of marginal utilities to the real exchange rate, $x(s^t) = e(s^t)\frac{P^*\left(s^t\right)}{P(s^t)}$. The marginal utility of home country active households relative to foreign country active households' rises in proportion to the appreciation of the home real exchange rate. Active households therefore share risk internationally. In the standard complete markets model, equation (21) holds for aggregate consumption, but not in our model. This distinction allows the model to account for the discrepancy between asset price-based and consumption-based measures of risk sharing, and to explain a negative correlation between relative aggregate consumption and the real exchange rate. Active households' consumption is a time-varying fraction of aggregate consumption:

$$C(s^t) = \omega \int \left[m_A(s^t,\gamma) C_A(s^t) + \frac{n(s^t)}{\mu_t} \int_{y_L(s^t,\gamma)}^{y_H(s^t,\gamma)} y f_y(y) dy \right] f_\gamma(\gamma) d\gamma + (1-\omega) \frac{n(s^t)}{\mu_t} \tag{22}$$

where $m_A(s^t,\gamma) = F_y\left(y_L(s^t,\gamma)\right) + 1 - F_y\left(y_H(s^t,\gamma)\right)$ is the fraction of assetholding households with fixed cost γ that are active, with F_y denoting the CDF associated with the density f_y.

In the next section, we calibrate the model and quantify how large is the difference in risk sharing measures, and we illustrate how risk sharing among active households is transmitted to aggregate consumption.

3 Quantitative Analysis

We parameterize and simulate the model and compare its predictions to a variant with no trade in goods and to a frictionless model. We show that only the model with trade and segmented asset markets generates asset price-based risk sharing measures higher than the consumption-based measures. Section 3.1 describes the parameters used in our quantitative experiments. Section 3.2 discusses the main results on the asset price-based and the consumption-based risk sharing measures and the Backus-Smith-Kollmann puzzle. We illustrate the risk sharing mechanisms in our model in Section 3.3 and provide suggestive evidence for the model's mechanisms in Section 3.4. Welfare analysis is in Section 3.5.

3.1 Parameterization

A model period corresponds to one quarter. Since we focus on evaluating the implications of an asset market friction, we set the parameters governing preferences to values commonly used in international macro models. We set the discount factor $\beta = 0.99$ and the coefficient of relative risk aversion $\eta = 2$. We set the elasticity of substitution σ between traded and non-traded goods to 0.5, and we set the share a on traded goods in consumption so that the fraction of expenditures on traded goods is 50%. These are both close to the estimates in Stockman and Tesar (1995).

We choose the distributions of fixed transfer costs and idiosyncratic income shocks to match statistics on income and consumption inequality and asset market activity in the US. Since our model assumes a time-invariant cross-sectional variance of income, we pick average measures of inequality in the US Consumer Expenditure Survey (CEX) over $1980Q1-2003Q4$.[10] Using CEX data from Heathcote, Perri, and Violante (2010), we estimate residual variances of quarterly income and consumption unexplained by household characteristics. We regress income and consumption on the following characteristics of the reference person: sex, race, education, experience (proxied by age), interaction terms between experience and education, and dummies for region of residence, following Krueger and Perri (2006). From $1980Q1$ to $2003Q4$, these characteristics explain, on average, about 23 percent of the cross-sectional variance of income and consumption. The variance of the log residual income is 0.37, so we choose the distribution of idiosyncratic shocks in the model to be lognormal with mean 1 and variance of log income of 0.37.

We set the fraction of assetholders to $\omega = 0.83$, which is equal to the average fraction of households that report positive financial wealth in the CEX data for 1980-2003. Previous empirical studies focusing on specific classes of assets have shown that only a fraction of households own stocks and bonds. For example, Vissing-Jørgensen (2002a) shows that only 20 percent of households hold stocks, while 30 percent hold bonds. Taking into account indirect stock holding through pension funds and IRAs, Haliassos and Bertaut (1995) calculate that 37 percent of households hold stocks.[11] Our larger number comes from inferring assetholding status based on the presence of financial income rather than direct reports of assetholdings. We consider sensitivity of the results to other values of ω in Appendix A.5.

Varying the distribution of fixed costs among assetholders $f_\gamma(\gamma)$ allows us to match a given cross-sectional variance of consumption and fraction of active households. In the CEX, the variance of consumption unexplained by household characteristics is, on average, 0.23. Vissing-Jørgensen (2002b) calculates that between 29% and 53% of US households adjust their holdings of risky assets each year. A very low mean value of γ implies that most households are active in any period, and hence the variance of consumption is close to zero, while a very high mean value of γ means that few households are active, so that the variance of consumption approaches the variance of income. Holding fixed the mean of γ, increasing the dispersion of the distribution lowers the variance of log consumption while increasing the fraction of households that are active. Holding fixed the variance of γ, increasing the mean raises the variance of log consumption while lowering the fraction of households active. We choose the distribution of γ to be lognormal, and we choose its mean and variance so

[10] Appendix A.4 contains details on the microdata.

[11] Vissing-Jørgensen (2002a) uses the US Consumer Expenditure Survey for 1980-1996, and Haliassos and Bertaut (1995) use the 1983 Survey of Consumer Finances.

that in the steady state, the model's variance of log consumption equals 0.23 and 40% of asset-holding households are active annually.[12] The parameters of f_γ imply that the average fixed cost is 0.981 and the standard deviation is 0.651.

The stochastic process of shocks is given by a first order vector autoregressive process:

$$\log \hat{s}_{t+1} = A \log \hat{s}_t + \varepsilon_{t+1} \tag{23}$$

where $\hat{s}_t = [\hat{Y}_{Tt}, \hat{Y}_{Nt}, \hat{\mu}_t, \hat{Y}^*_{Tt}, \hat{Y}^*_{Nt}, \hat{\mu}^*_t]'$ denotes deviations of the exogenous state variables from their long-run averages, $[\bar{Y}_T, \bar{Y}_N, \bar{\mu}_t, \bar{Y}^*_T, \bar{Y}^*_t, \bar{\mu}^*_t]'$. A is a 6×6 matrix of coefficients, and $\varepsilon_t \sim N(0, \Sigma)$. We estimate a symmetric version of equation (23) on data from the US and a trade-weighted aggregate of 21 OECD countries. We use annual data from the OECD STAN database for $1988 - 2015$ on GDP of the manufacturing, mining, agriculture, and utilities sectors for Y_T, and the remainder (services and construction) for Y_N. We use data on $M1$ from the OECD Main Economic Indicators to calculate money growth μ for the same period. Imposing symmetry means:

$$A = \begin{bmatrix} A_1 & A_2 \\ A_2 & A_1 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} \Sigma_1 & \Sigma_2 \\ \Sigma_2 & \Sigma_1 \end{bmatrix} \tag{24}$$

where A_1 and A_2 are each 3×3 matrices and Σ_1 and Σ_2 are each 3×3 symmetric matrices. The estimated values for A are:

$$A_1 = \begin{bmatrix} 0.306 & -0.047 & -0.033 \\ 0.152 & 0.469 & 0.020 \\ -0.135 & -0.176 & 0.353 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0.001 & -0.004 & -0.014 \\ 0.011 & -0.061 & 0.142 \\ 0.365 & -0.391 & -0.055 \end{bmatrix}$$

and for Σ,

$$\Sigma_1 = 10^{-4} \times \begin{bmatrix} 6.892 & 2.287 & -2.413 \\ 2.287 & 1.176 & -1.160 \\ -2.413 & -1.160 & 8.198 \end{bmatrix}, \ \Sigma_2 = 10^{-4} \times \begin{bmatrix} 3.842 & 1.360 & -3.110 \\ 1.360 & 4.884 & -6.969 \\ -3.110 & -6.969 & 2.008 \end{bmatrix}$$

In our symmetric two-country economy, we set $\bar{Y}_T = \bar{Y}^*_T = 1$ and $\bar{Y}_N = \bar{Y}^*_N = 1$ and $\bar{\mu} = \bar{\mu}^* = 1.04$, which is the average money growth factor for the US in our sample period. We adjust the annual estimates to make them quarterly as in Edmond and Veldkamp (2009).

[12] A household is inactive for one year with probability $(1 - \omega \bar{m}_A)^4$, where $\bar{m}_A$ is the mass of active households each quarter on average in the steady state. So the implied value of $\bar{m}_A$ is $\frac{1-0.6^{1/4}}{\omega} = 0.144$.

3.2 International Risk Sharing Through the Lens of the Model

We show that the model generates high risk sharing when measured using asset prices and low risk sharing when measured using aggregate consumption. In addition, the correlation between the real exchange rate and the ratio of consumption across countries is negative.

The asset price-based risk sharing measures in Brandt, Cochrane, and Santa-Clara (2006) are constructed from stochastic discount factors (SDFs) inferred from data on real domestic and international equity returns. To construct a similar measure in our model, we first construct real equity and risk-free returns that satisfy the Euler equation for an active household in each country, then we recover the SDF that prices these assets using the Hansen and Jagannathan (1991) approach. In our complete markets environment, equity and risk-free bonds are redundant assets, so introducing them does not change the allocation.

The real equity index in the home country pays a dividend equal to n_t, the real value of the aggregate endowment. The real equity price v_t and the real risk-free price v_t^{rf}, denominated in units of consumption, satisfy the following Euler equations:

$$v_t = \beta E_t \frac{u'(C_{At+1})}{u'(C_{At})}(v_{t+1} + n_{t+1}) \tag{25}$$

$$v_t^{rf} = \beta E_t \frac{u'(C_{At+1})}{u'(C_{At})} \tag{26}$$

where for ease of notation, we drop the history dependence.[13] The risk-free return is denoted $R_t^{rf} = \frac{1}{v_t^{rf}}$. For the foreign country, v_t^*, v_t^{rf*}, and R_t^{rf*} are defined analogously. For each country, we construct returns on three risky assets: domestic equity, international equity, and an international bond. In the home country, these returns are:

$$R_{dt+1} = \frac{v_{t+1} + n_{t+1}}{v_t}$$

$$R_{it+1} = \frac{x_{t+1}}{x_t} \frac{v^*_{t+1} + n^*_{t+1}}{v^*_t}$$

$$R_{xt+1} = \frac{x_{t+1}}{x_t} R_t^{rf*}$$

[13] Appendix A.2 contains the numerical details of how we solve equations (25) and (26).

In the foreign country, the three returns are:

$$R^*_{dt+1} = \frac{v^*_{t+1} + n^*_{t+1}}{v^*_t}$$
$$R^*_{it+1} = \frac{x_t}{x_{t+1}} \frac{v_{t+1} + n_{t+1}}{v_t}$$
$$R^*_{xt+1} = \frac{x_t}{x_{t+1}} R^{rf}_t$$

Following Brandt, Cochrane, and Santa-Clara (2006), we apply the Hansen and Jagannathan (1991) construction to the vector of simulated excess returns on the three assets—e.g. in the home country, $R^e_{t+1} = [R_{dt+1} - R^{rf}_t, R_{it+1} - R^{rf}_t, R_{xt+1} - R^{rf}_t]'$. The minimum-variance SDF that prices these excess returns is

$$m_{t+1} = \frac{1}{R^{rf}_t} - \frac{1}{R^{rf}_t} E[R^e_{t+1}]\Sigma_e^{-1}(R^e_{t+1} - E[R^e_{t+1}]) \tag{27}$$

where Σ_e is the covariance matrix of the excess returns. In equation (27), $E[\cdot]$ denotes the sample mean of a simulated series. We define the foreign SDF m^*_{t+1} analogously.

Table 1 presents results for our benchmark model, and for two alternative market structures: a frictionless, complete markets model with all households active, and a segmented markets model with no trade in goods. We report risk sharing statistics based on three different series: all households' consumption, active households' consumption, and the SDF computed from (27). In addition to cross-country correlations, we report a risk sharing index developed by Brandt, Cochrane, and Santa-Clara (2006) given in equation (28) and labeled BCS risk sharing index in the table.

$$1 - \frac{var(m_{t+1} - m^*_{t+1})}{var(m_{t+1}) + var(m^*_{t+1})} \tag{28}$$

In equation (28), m_{t+1} is either the SDF from equation (27), the intertemporal marginal rate of substitution (MRS) for active home households (i.e., $\beta^t \left(C_{At+1}/C_{At}\right)^{-\eta}$), or the intertemporal MRS for aggregate home consumption (i.e., $\beta^t \left(C_{t+1}/C_t\right)^{-\eta}$), while m^*_{t+1} is the analogue for the foreign country. The numerator of the fraction in equation (28) measures how much risk is not shared across countries, while the denominator measures how much risk there is to share. The index lies between -1 and 1, and measures how far countries are from perfect risk sharing, corresponding to an index value of 1 when $m_{t+1} = m^*_{t+1}$.

The first column of Table 1 contains our benchmark model results, showing a substantial difference between asset price-based measures of risk sharing and the measures based on aggregate consumption. The first five rows report volatilities, showing that the real exchange

Table 1: RESULTS FROM BENCHMARK MODEL AND ALTERNATIVE ENVIRONMENTS

	Asset Market Segmentation			
	Endogenous			
	Benchmark	No trade in goods	Exogenous	Frictionless complete markets
STANDARD DEVIATION (%)				
Real exchange rate	1.17	4.14	1.49	0.57
Real income	0.75	0.75	0.75	0.75
Aggregate consumption	0.93	0.87	0.84	0.68
Active consumption	0.94	1.37	1.34	0.68
SDF	5.64	4.74	4.79	5.57
INT'L CORRELATIONS				
Real income	0.56	0.56	0.56	0.56
Aggregate consumption	0.32	0.52	0.27	0.91
Active consumption	0.81	−0.14	0.84	0.91
SDF	0.85	0.17	0.65	0.94
BCS RISK SHARING INDEX				
Aggregate consumption	0.47	0.62	0.40	0.95
Active consumption	0.77	−0.28	0.83	0.95
SDF	0.97	0.22	0.84	0.99
CORRELATION $\frac{C}{C^*}$ AND x	−0.79	−0.30	−0.83	1.00

NOTES: Statistics are Hodrick-Prescott filtered, logged, and averaged over 10000 simulations of 800 quarters. SDF refers to stochastic discount factors.

rate in the benchmark model is more volatile than real income, aggregate and active consumption have similar volatilities, and the SDF that prices equity and international bonds is more volatile than either consumption measure. Although the true stochastic discount factor in the model is the intertemporal marginal rate of substitution for active households, the SDF constructed from (27) is more volatile because the set of assets it prices is very restricted compared to the complete asset markets of the model. Nevertheless, the international correlations of the SDF and active consumption are similar—0.85 and 0.81—and are higher than the correlation of aggregate consumption, at 0.32. The BCS index also shows that risk sharing measured from asset prices is higher than when measured from aggregate consumption (0.97 vs. 0.47). Using data from the US, UK, and Japan, Brandt, Cochrane, and Santa-Clara (2006) show that their risk sharing index based on asset prices is around 0.99, while the analogue of the index constructed from aggregate consumption data is in the range $0.3 - 0.4$, and we show in Table 2 that the average bilateral correlation of consumption across G7 countries is 0.27. Thus, our model is successful in generating high risk sharing based on asset prices and low risk sharing based on aggregate consumption.

Table 2: AGGREGATE CONSUMPTION CORRELATIONS AND CONSUMPTION-REAL EXCHANGE RATE CORRELATIONS IN G7 DATA

	AGGREGATE CONSUMPTION CORRELATIONS, CORR(C, C^*)					
	France	Germany	Italy	UK	Canada	Japan
US	0.32	0.28	0.02	0.45	0.60	0.25
France		0.38	0.45	0.31	0.21	0.35
Germany			0.21	0.11	0.09	0.18
Italy				0.17	0.15	0.13
UK					0.42	0.37
Canada						0.12
Average consumption correlation = 0.27						

	CONSUMPTION-REAL EXCHANGE RATE CORRELATIONS, CORR$(\frac{C}{C^*}, RER)$					
	France	Germany	Italy	U.K.	Canada	Japan
US	0	-0.03	-0.12	-0.34	-0.08	0.23
France		0.05	0.15	-0.12	-0.05	0.12
Germany			0.10	-0.11	-0.05	0.05
Italy				-0.13	-0.03	0.04
UK					-0.02	-0.20
Canada						0.05
Average consumption-real exchange rate correlation = -0.02						

NOTES: G7 data from the OECD spans $1960Q1 - 2015Q4$. Consumption data are private final consumption expenditures from the OECD Quarterly National Accounts. Consumption is expressed per population ages $15 - 64$, from United Nations, World Population Prospects. Consumption-real exchange rate correlations are between bilateral real exchange rates and bilateral relative consumption for the seven countries. Real exchange rates (RER) are computed using nominal exchange rates and consumer price indices from OECD Main Economic Indicators. Consumption and real exchange rates are logged and Hodrick-Prescott filtered.

Our model also resolves the Backus-Smith-Kollmann puzzle as the correlation between the real exchange rate and the cross-country ratio of aggregate consumption in the benchmark model is negative, at -0.79, significantly lower than 1 (see the last row of Table 1), and consistent with the prevalence of negative values observed in G7 data (see Table 2). In the next subsection, we explore the mechanisms that transmit international risk sharing to aggregate consumption and that explain why the ratio of aggregate consumption is negatively correlated with the real exchange rate.

In the second column of Table 1, we consider an alternative goods market structure, with no international trade in goods, as in AAK02. With no trade in goods, the order of the risk sharing measures is flipped: for example, aggregate consumption is more correlated across countries than active consumption or the SDFs. The correlation of aggregate consumption is essentially the same as that of aggregate income, since aggregate consumption is constrained by domestic resources; since monetary shocks reallocate consumption between active and inactive households, active consumption moves opposite to aggregate consumption, generating a slightly negative cross-country correlation in active consumption.

In the third column of Table 1, we report results from a version of the model with exogenously segmented asset markets. We assume that a constant fraction of households is always active, equal to the steady state fraction of active households in the benchmark model, $\bar{m}_A = \int \left[F_y\left(\bar{y}_L(\gamma)\right) + 1 - F_y\left(\bar{y}_H(\gamma)\right)\right] f_\gamma(\gamma) d\gamma$, and the remainder are always inactive. This model is similar to Kollmann (2012)'s model with "hand-to-mouth" consumers. The patterns in the table are essentially the same as the benchmark model. In section 3.3, we show how aggregate consumption risk sharing in our benchmark model is affected by the endogenous movements in the set of active households, and in section 3.4 we show that the endogenously segmented model is consistent with cross-sectional data on inflation and consumption risk sharing, while the exogenous model is not. For this reason, the endogenously segmented markets model better fits the data on international risk sharing.

The last column of Table 1 displays the results from the frictionless complete markets model. This model is consistent with a high degree of risk sharing implied by asset prices in the data, but inconsistent with a low degree of risk sharing implied by aggregate consumption data. Since all households are active in this model, the risk sharing equation (21) applies to aggregate consumption, and the correlation between the real exchange rate and the ratio of aggregate consumption across countries is equal to one.

3.3 The Role of Endogenous Asset Market Segmentation

The quantitative results in Table 1 show that active consumption is highly correlated across countries, and equation (21) implies that the ratio of active households' consumption across countries is perfectly correlated with the real exchange rate. In this section, we illustrate how traded, non-traded, and monetary shocks interact with endogenous asset market segmentation to generate positive comovement in aggregate consumption and a negative consumption-real exchange rate correlation.

We can write aggregate consumption in period t, from equation (22) as:

$$C_t = \omega m_{At} C_{At} + \frac{n_t}{\mu_t}(\omega y_{It} + 1 - \omega) \tag{29}$$

where $m_{At} = \int \left[F_y\left(y_{Lt}(\gamma)\right) + 1 - F_y\left(y_{Ht}(\gamma)\right)\right] f_\gamma(\gamma) d\gamma$ is the mass of active assetholders and $y_{It} = \int \int_{y_{Lt}(\gamma)}^{y_{Ht}(\gamma)} y f_y(y) dy f_\gamma(\gamma) d\gamma$ is the fraction of income held by inactive assetholders.

In Appendix A.3, we show that, to a first-order approximation near a symmetric steady state, changes in relative consumption across countries are given by:

$$\begin{aligned} \hat{C}_t - \hat{C}_t^* = {} & (\phi_I + \phi_{NA}) \left[\psi_T(\hat{Y}_{Tt} - \hat{Y}_{Tt}^*) + \psi_N(\hat{Y}_{Nt} - \hat{Y}_{Nt}^*) - \hat{\mu}_t + \hat{\mu}_t^*\right] \\ & + \phi_I \left[\hat{y}_{It} - \hat{y}_{It}^*\right] + \phi_A \left[\frac{1}{\eta}\hat{x}_t + \hat{m}_{At} - \hat{m}_{At}^*\right] \end{aligned} \tag{30}$$

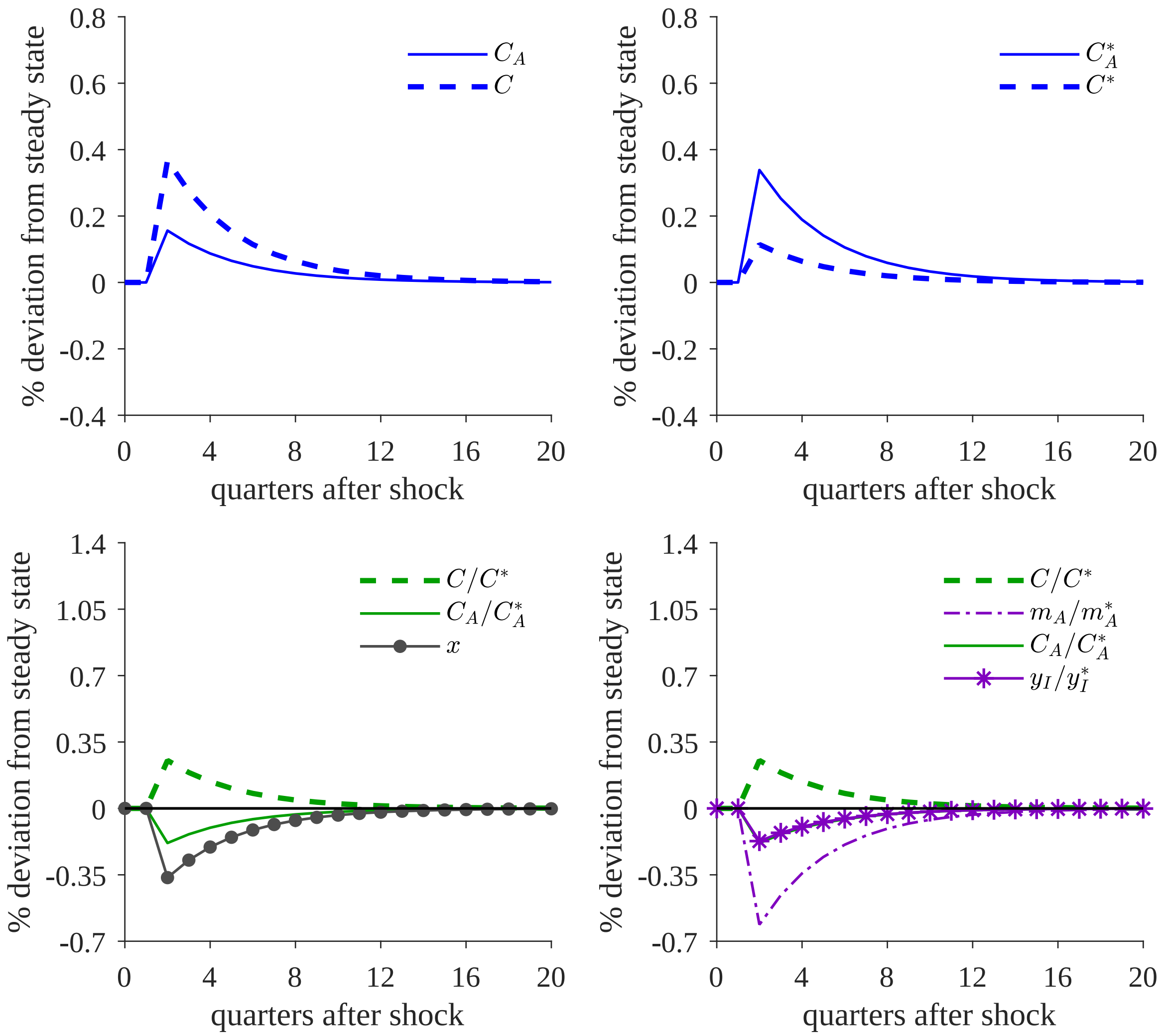
where all variables with hats are log deviations from the steady state, e.g., $\hat{C}_t = \log \frac{C_t}{\bar{C}}$. We refer to the left hand side as relative aggregate consumption growth. Here, $\phi_A = \omega \frac{\bar{m}_A \bar{C}_A}{\bar{C}}$ is active assetholders' steady state share of consumption, $\phi_I = \omega \frac{\bar{n}}{\bar{\mu}\bar{C}} \bar{y}_I$ is inactive assetholders' steady state share of consumption, and $\phi_{NA} = 1 - \phi_A - \phi_I$ is non-assetholders' steady state share of consumption. The constants ψ_T and ψ_N are the steady state expenditure shares on traded and non-traded goods.

We use equation (30) to illustrate the effects of shocks to traded and non-traded output and money growth on relative aggregate consumption growth. The first two terms are the contribution of households not participating in asset markets in period t. If all households were either always inactive, $\phi_I = 1$, or non-assetholders, $\phi_{NA} = 1$, then $\hat{y}_{It} = \hat{y}_{It}^* = 0$, and relative aggregate consumption growth would simply reflect relative traded and non-traded income growth, weighted by the appropriate consumption expenditure shares, and relative money growth, i.e. $\hat{C}_t - \hat{C}_t^* = \psi_T(\hat{Y}_{Tt} - \hat{Y}_{Tt}^*) + \psi_N(\hat{Y}_{Nt} - \hat{Y}_{Nt}^*) - \hat{\mu}_t + \hat{\mu}_t^*$. On the other extreme, if all households were always active assetholders, then $\phi_A = 1$ and $\hat{m}_{At} = \hat{m}_{At}^* = 0$, and relative aggregate consumption growth would be perfectly correlated with the real exchange rate, reflecting perfect cross-country risk sharing, i.e. $\hat{C}_t - \hat{C}_t^* = \frac{1}{\eta}\hat{x}_t$.

Asset market segmentation has two effects on consumption risk sharing relative to either of these two extremes. First, relative aggregate consumption growth is a weighted average of contributions from active and inactive households, so it is not perfectly correlated with the real exchange rate. This is true even with exogenous market segmentation, in which case equation (30) is $\hat{C}_t - \hat{C}_t^* = (\phi_I + \phi_{NA})[\psi_T(\hat{Y}_{Tt} - \hat{Y}_{Tt}^*) + \psi_N(\hat{Y}_{Nt} - \hat{Y}_{Nt}^*) - \hat{\mu}_t + \hat{\mu}_t^*] + \phi_A \frac{1}{\eta}\hat{x}_t$.

Second, with endogenously segmented markets, movements in the fraction of active households and the average income of inactive households have additional effects that reinforce risk sharing. We illustrate these effects through impulse responses to 1% shocks to home country traded and non-traded output and home country money growth. The impulse responses are plotted in Figures 1 - 3. For this analysis, we set the off-diagonal elements of the matrix A to zero, to isolate the effects of each shock in the absence of spillovers to other shocks. Impulse responses incorporating these spillovers are shown in Appendix A.5, and reflect similar patterns as Figures 1 - 3.

Figure 1: Impulse responses to 1% increase in home country traded output, Y_T



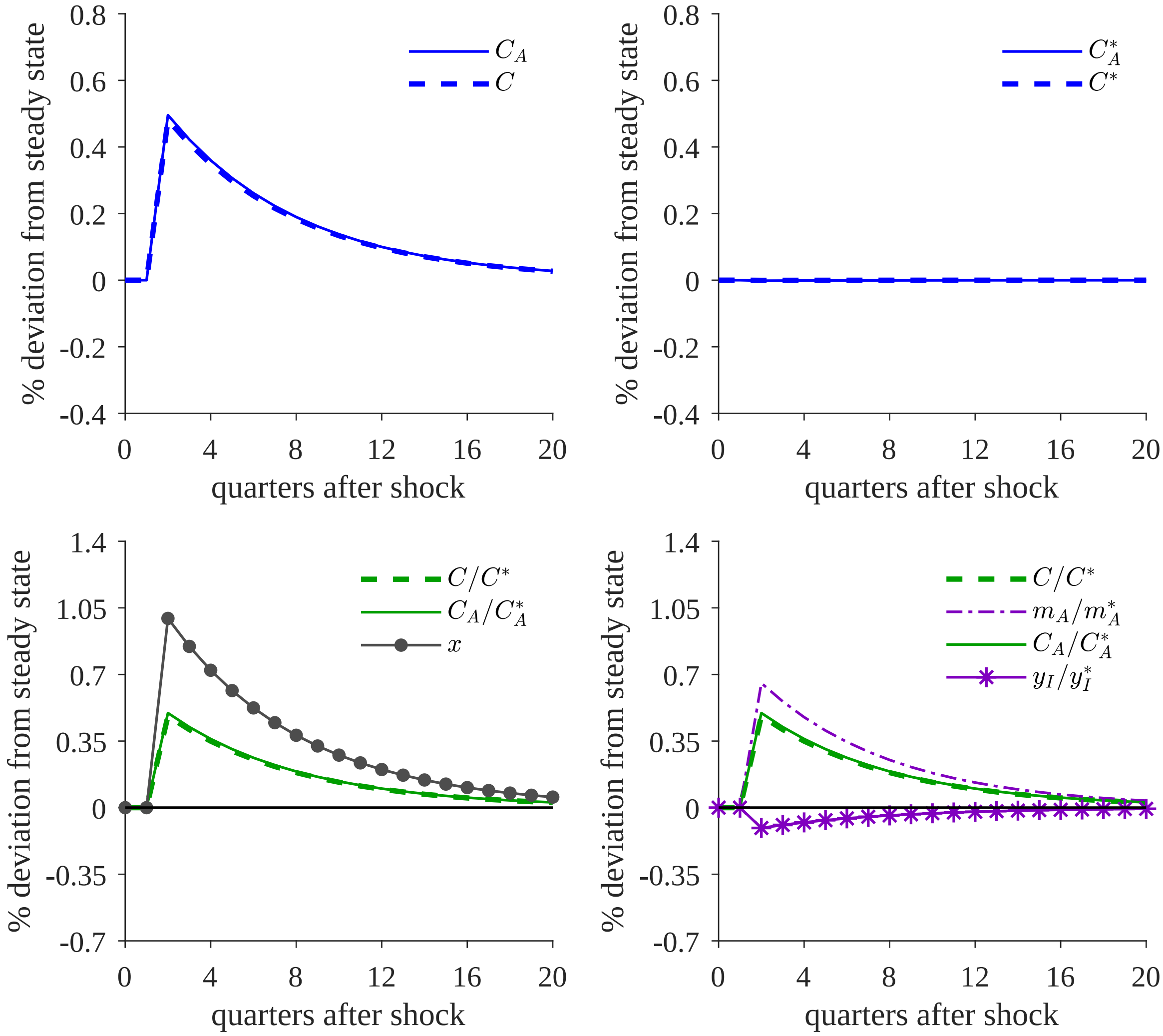
First, consider the responses to a positive shock to home country traded output, Y_T, in Figure 1. The upper-left panel shows home country active and aggregate consumption, C_{At} and C_t. In response to the shock, all households have higher income, but active households smooth by saving, so their consumption rises less than aggregate consumption. In the foreign country, active households' consumption increases by more than aggregate consumption, because they borrow from home households, while inactive households' consumption doesn't move (upper-right panel). Therefore, relative aggregate consumption growth and relative active consumption growth move in opposite directions (lower-left panel). A shock to traded output generates a negative correlation in relative aggregate consumption growth and the real exchange rate, $\operatorname{corr}\left(\frac{C}{C^*}, x\right) < 0$, but cross-country comovement in aggregate consumption that is similar to the comovement of active consumption.

The lower-right panel of Figure 1 shows the effects of changes in the set of active and inactive households. The fraction of households active in the foreign country rises more than in the home country, because foreign households have the most to gain from sharing the shock to home country output. In addition, the income of inactive households goes down in the home country relative to foreign country, because high-income home households who want to save become active, while poor foreign households become active to borrow. These two movements reinforce the risk sharing apparent in aggregate consumption, pulling relative aggregate consumption in the same direction as relative active consumption, as can be seen in equation (30). Overall however, the impact on the consumption of inactive households dominates these effects: a large fraction of home households must consume their additional income while the price of non-traded goods is rising, so $\hat{C}_t - \hat{C}^*_t$ rises while $\hat{x}_t$ falls, inducing a negative correlation between relative aggregate consumption and the real exchange rate.

Next, in Figure 2 we plot the responses to a positive shock to home country non-traded output, Y_N. Active households would like to smooth by lending to foreign households, but the increased non-traded endowment must all be consumed domestically. So active and aggregate consumption move closely together (upper-left panel), foreign consumption does not move (upper-right panel), and relative aggregate and relative active consumption growth are both positively correlated with the real exchange rate (lower-left panel). Since C_A rises much more than C^*_A, m_A/m^*_A increases (lower-right panel). The income of inactive home households moves relatively little, because there are greater numbers of both borrowers and lenders. Here again, movements in the relative fractions of active households in each country, i.e., m_A/m^*_A, reinforce risk sharing (see equation 30). Overall, a shock to non-traded income generates low cross-country comovement in consumption and a positive correlation in relative aggregate consumption and the real exchange rate.

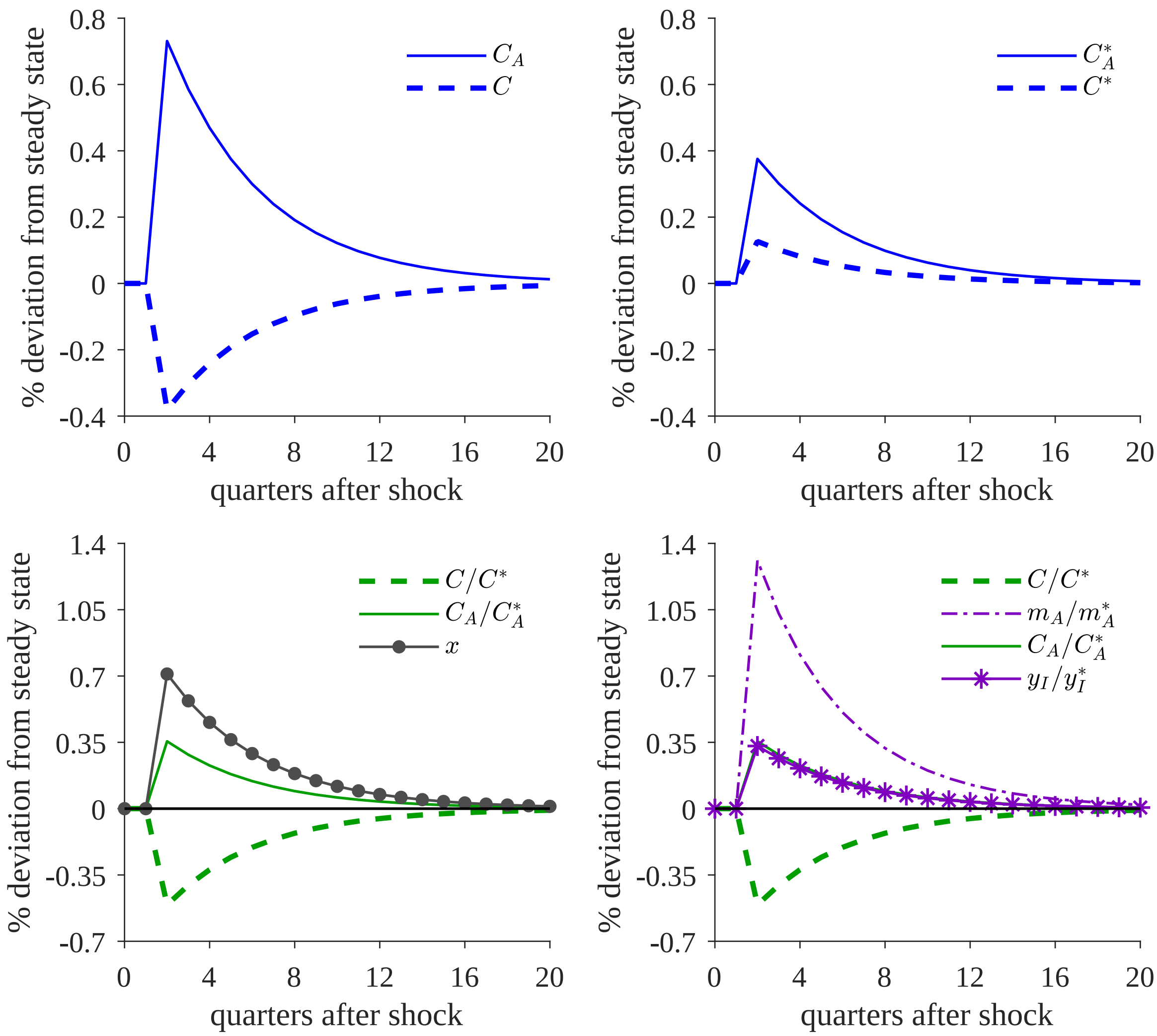
Finally, Figure 3 plots the responses to a positive shock to home country money growth, μ. An increase in money growth reallocates resources from inactive to active households (upper-left panel), as inflation reduces the real value of inactive households' money balances while active households benefit from the money injection. Active home households lend more abroad, so that C^*_A rises while foreign inactive households' consumption doesn't change, so C^* rises less than C^*_A (upper-right panel). As a result, relative aggregate consumption moves opposite to relative active consumption, and the correlation between relative aggregate consumption and the real exchange rate is negative (lower-left panel). The lower-right panel of the figure shows that the effects of endogenous segmentation are roughly opposite to

Figure 2: Impulse responses to 1% increase in home country non-traded output, Y_N



those in response to a traded output shock in figure 1: home households have a greater incentive to become active to avoid the effects of inflation, so m_A/m_A^* increases. Since money growth reduces real money balances, marginal inactive households become active, while marginal active households become inactive. This increases average income of inactive home households, y_I, so y_I/y_I^* rises. As in the case of the shock to Y_T, these changes in the composition of active households in response to a money growth shock mitigate the divergence between relative aggregate consumption and relative active consumption, as seen in equation (30). A monetary shock leads to a negative correlation between relative aggregate consumption growth and the real exchange rate, but generates a negative correlation in aggregate consumption.

Figure 3: Impulse responses to 1% increase in home country money growth, μ



3.4 Suggestive Evidence for Model Mechanisms

The decomposition in equation (30) and the impulse responses in the lower-right panels of Figures 1 - 3 illustrate two effects of incorporating endogenously segmented asset markets. First, active households' consumption reflects better risk sharing than inactive households' consumption, and the same is true of assetholders' consumption and non-assetholders' consumption. Second, *endogenous* segmentation has an offsetting effect on international risk sharing that is not present in models with exogenously segmented asset markets, such as Kollmann (2012). Monetary shocks, in particular, affect the incentive to actively participate in asset markets, and equilibrium movements in the fraction of active households increase the degree of international risk sharing apparent in aggregate consumption, leading to a stronger relationship between relative aggregate consumption and the real exchange rate. A related prediction of the model is that an increase in the average money growth rate raises the fraction of households active on average, and thereby increases aggregate consumption risk sharing.

In this section, we provide suggestive evidence that is consistent with these two implications. We first look at micro data to evaluate international risk sharing across different groups of households, and then we turn to aggregate cross-country data to illustrate the relationship between average inflation and aggregate consumption risk sharing.

3.4.1 International Risk Sharing in Microdata

The model's prediction that international risk sharing for assetholders is better than for non-assetholders is borne out in US and UK survey data over the period 1980-2003. To fully test the cross-country risk sharing implications of endogenously segmented asset markets, we would need to identify households *actively accessing assets*, which would require panel data at a high frequency on assetholders' consumption, income, and asset holdings for two or more countries. Since we do not have such data, we instead compare the model's predictions for average consumption among assetholders and non-assetholders to analogous averages constructed from micro data. The data are described in Appendix A.4.

Table 3: Cross-country Consumption Correlations in US and UK Surveys

	US and UK Data	Benchmark Model
Aggregate consumption	0.08	0.32
assetholders' consumption	0.30	0.38
Non-assetholders' consumption	0.03	0.25

Notes: The table reports correlations of Hodrick-Prescott filtered log non-durable annual consumption over $1980 - 2003$. US data are from the Consumer Expenditure Survey (CEX) and UK data are from Family Expenditure Survey (FES). See Appendix A.4 for further description of the data.

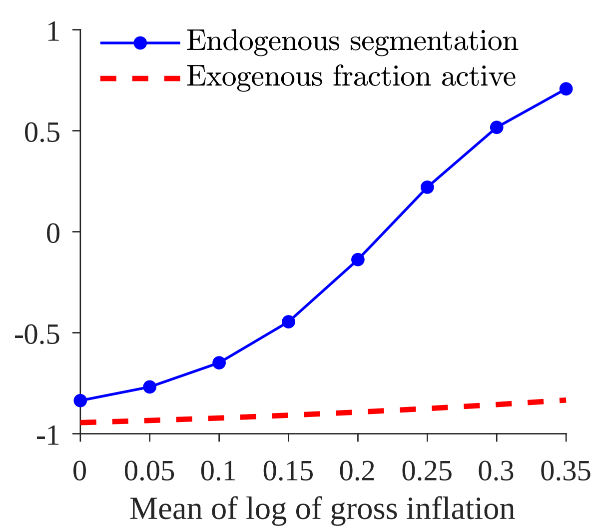
Table 3 reports statistics from the micro data and from the benchmark model. The first column of the table shows that the correlation between aggregate nondurable consumption in the US and UK is 0.08. Assetholders' consumption comoves more strongly across the US and UK than the aggregate, with a correlation of 0.30, while non-assetholders' consumption is essentially uncorrelated. The ranking of these correlations is consistent with the model results: assetholders' consumption is more highly correlated than non-assetholders', as shown in the second column of the table.

The aggregate consumption correlation from micro data provided in Table 3 is significantly lower that the one computed from national accounts data. Indeed, in Table 2, we show that over the period 1960$Q1$ to 2015$Q4$, HP-filtered US and UK aggregate consumption have a correlation of 0.45. If we restrict the time period to $1980-2003$ as in the survey data we use, the correlation changes to 0.44. The discrepancy between the measures of consumption from survey data and from national accounts for the US and the UK has been previously acknowledged in the literature. Krueger et al. (2010) document that in both the US CEX as well as the UK FES per capita consumption growth is significantly slower than the corresponding national accounts measures. Our focus on US and UK data is driven solely by the availability of financial assets variables which allow us to measure consumption for assetholders and non-assetholders. Our findings for the US and UK suggest that documenting differences in international risk sharing among assetholders and non-assetholders across more countries is a promising avenue for future empirical work.

3.4.2 Inflation and International Risk Sharing in Aggregate Data

We highlight the model's prediction that higher average inflation leads to higher measures of risk sharing, and provide novel cross-country evidence for this implication. We also show that the relationship between average inflation and international risk sharing in a version of our model with *exogenously* segmented asset markets is far weaker.

Figure 4: Consumption-Real Exchange Rate Correlation and Inflation in Model with Endogenous and Exogenous Asset Market Segmentation



In Figure 4, the solid line plots the correlation between relative aggregate consumption and the real exchange rate across levels of average inflation (i.e. the time series average of

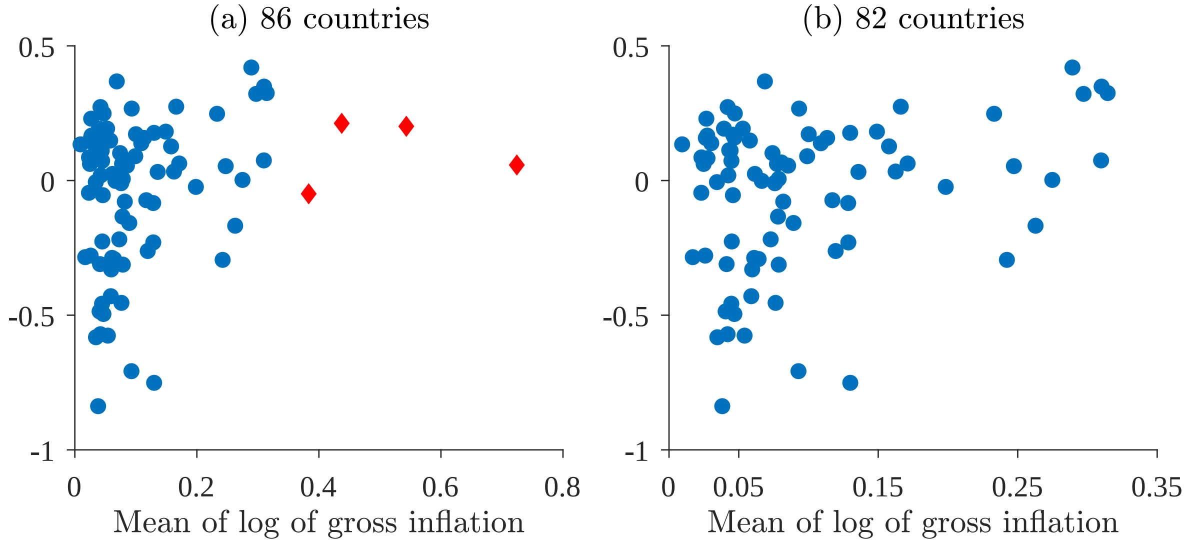
$\log \frac{P_{t+1}}{P_t}$) in the benchmark model with endogenous market segmentation. To construct this figure, we solve the model for different levels of the steady state money growth rate, $\bar{\mu}$, holding fixed the rest of the calibrated parameters. Average money growth translates one-for-one into average inflation. As average money growth increases, the fraction of households active in the steady state increases, since the cost of inactivity rises as higher inflation reduces the value of inherited money balances. The dotted line in Figure 4 is from a version of the model in which we fix the fraction of households that are active equal to the calibrated model's steady state value, $\bar{m}_A = \int \left[F_y\left(\bar{y}_L(\gamma)\right) + 1 - F_y\left(\bar{y}_H(\gamma)\right)\right] f_\gamma(\gamma) d\gamma$. In the model with an exogenous fraction of households active, average money growth has little effect on the extent of risk-sharing.

We turn to cross-country data to document a novel fact: higher inflation is associated with better international risk sharing. In Figure 5, we show that high-inflation countries have higher correlations between their real exchange rates and the ratio of aggregate consumption relative to the US. The left panel of the figure plots the consumption-real exchange rate correlations against mean inflation using HP filtered data for 86 countries.[14] The right panel plots the same points, minus four outliers with very high inflation, to verify that the pattern is not driven by these very high inflation countries. Both plots suggest a strong positive relationship between average inflation and the consumption-real exchange rate correlation.

We confirm the statistical significance of this relationship in Table 4, which shows the results from a cross-section regression of the consumption-real exchange rate correlation on average inflation. The first column is for all 86 countries, while the second column removes the four outliers. Removing the outliers in fact strengthens the relationship, raising the slope from 0.576 to 0.949. While our model is a symmetric two-country world rather than a cross-section of asymmetric countries as in the data, our results indicate that there is a significant relationship between average inflation and international risk sharing in the data,

[14] We use data on nominal exchange rates with the US, consumer price indices (CPI) and household final consumption expenditures from the World Bank's World Development Indicators (WDI) dataset for 86 countries over $1960 - 2017$. We list the countries included in our analysis by regions, as defined by the World Bank: *Europe and Central Asia* (Albania, Armenia, Austria, Belarus, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Kazakhstan, Kyrgyz Republic, Latvia, Lithuania, Luxembourg, Macedonia, Moldova, Netherlands, Norway, Poland, Portugal, Romania, Russian Federation, Serbia, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, Ukraine, United Kingdom), *South Asia* (Bangladesh, India, Pakistan, Sri Lanka), *East Asia and Pacific* (Australia, Brunei Darussalam, Cambodia, China, Hong Kong, Indonesia, Japan, Korea, Macao, Malaysia, New Zealand, Phillippines, Singapore, Thailand, Vietnam), *Latin America and Carribean* (Bahamas, Belize, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Haiti, Honduras, Mexico, Nicaragua, Panama, Paraguay, Peru, Puerto Rico, Uruguay, Venezuela), *North America* (Canada, United States), *Middle-East* (Israel), and *Sub-Saharan Africa* (South Africa). We also construct a data point for the Euro Area starting 1999, using the WDI data for the nominal exchange rate and the CPI, and an aggregate of household consumption over the constituent countries.

Figure 5: CONSUMPTION-REAL EXCHANGE RATE CORRELATION AND INFLATION



NOTES: Data are from the World Development Indicators dataset published by the World Bank. For each country, we compute the correlation between the (HP filtered) log real exchange rate with the US and the (HP filtered) log consumption per capita differences to the US, using annual data for 1960 – 2017. We restrict the sample to years when all data—the nominal exchange rate with the US, the consumer price index (CPI) and the household final consumption expenditures—are available in a given country. We plot the consumption-real exchange rate correlation against mean log gross CPI inflation. The 4 countries marked by diamonds in panel (a) are (reading from left to right along the x-axis): Peru, Ukraine, Belarus, and Brazil and are excluded from panel (b). The full list of countries is provided in footnote 14.

and that a model with endogenously segmented markets model is better able to account for this finding than a model with exogenous asset market segmentation.

Our empirical result is similar in spirit to Bansal and Dahlquist (2000)'s findings, namely that deviations from uncovered interest parity (UIP) are smaller between the US and countries with high inflation compared to countries with low inflation. While the UIP relationship deals only with relative prices (the nominal exchange rate and the interest rate differential), our focus is on the relationship between a relative price (the real exchange rate) and quantities (relative aggregate consumption). Bansal and Dahlquist note that their finding could be interpreted as evidence of segmented markets. Our model results confirm this interpretation in the context of the consumption-real exchange rate relationship.

3.5 Welfare Gains from International Risk Sharing

In this section, we show that our model is consistent with the finding in Lewis (2000) that welfare gains from international financial markets measured using asset prices are higher

Table 4: Consumption-Real Exchange Rate Correlation Regressions

	(1)	(2)
Average inflation	0.576	0.949
	(0.237)	(0.377)
Constant	-0.104	-0.136
	(0.040)	(0.047)
Number of countries	86	82
R^2	0.066	0.073

NOTES: All coefficient estimates are statistically significant at 0.05 level. Standard errors are reported in parentheses. Regression (1) uses data from 1960 – 2017 for the 86 countries listed in footnote 14, while regression (2) drops four countries that are outliers in terms of inflation: Belarus, Brazil, Peru and Ukraine.

than estimates based on aggregate consumption data. We use our model to construct three alternative measure of welfare gains: (*i*) *actual gains* based on the ex-ante utility of a representative household, (*ii*) *active consumption-based gains* measured using the utility of a hypothetical household who is active every period, and (*iii*) *aggregate consumption-based gains* measured using the utility of a hypothetical household who consumes aggregate consumption each period. We interpret the active consumption-based gains as an asset price-based measure of welfare since active households price assets in our environment.[15] We find that these active consumption-based welfare gains are an order of magnitude larger than both the aggregate consumption-based gains and the actual welfare gains.

We consider lifetime consumption-equivalent measures of the gains from international asset trade relative to financial autarky. To express the weighted average of ex-ante expected utility across households with different fixed costs, γ, and different assetholding statuses, we define the per-period average utility of non-assetholders, $u_{NA}(s^t)$, inactive assetholders,

[15] The actual asset-price based measure in Lewis (2000) measures the change in the present discounted value of wealth from moving from a diversified portfolio (with constant shares) of domestic and international equity to one of only domestic equity. Since this calculation departs significantly from our framework, we instead use active households' utility directly as a measure of asset market-based gains.

$u_I(s^t)$, and active assetholders, $u_A(s^t)$ as:

$$u_{NA}(s^t) = \int \frac{\left(y_{t-1}\frac{n(s^t)}{\mu_t}\right)^{1-\eta}}{1-\eta} f_y(y_{t-1})dy_{t-1} \tag{31}$$

$$u_I(s^t) = \int \int_{y_L(s^t,\gamma)}^{y_H(s^t,\gamma)} \frac{\left(y_{t-1}\frac{n(s^t)}{\mu_t}\right)^{1-\eta}}{1-\eta} f_y(y_{t-1})dy_{t-1}f_\gamma(\gamma)d\gamma \tag{32}$$

$$u_A(s^t) = \int m_A(s^t,\gamma)\frac{\left(C_A(s^t)\right)^{1-\eta}}{1-\eta} f_\gamma(\gamma)d\gamma \tag{33}$$

Then the weighted average ex-ante expected utility is:

$$W = \sum_{t=0}^{\infty} \int \beta^t \left[(1-\omega)u_{NA}(s^t) + \omega(u_A(s^t) + u_I(s^t))\right] g(s^t)ds^t \tag{34}$$

To define the actual welfare gain, let $\bar{W}$ denote the lifetime utility level under international financial autarky. Then, the actual welfare gain from international financial markets is given by the factor $\Gamma = \left(W/\bar{W}\right)^{1/(1-\eta)}$, so that $100 \times (\Gamma - 1)$ is the percentage increase in consumption each period that all households in financial autarky would need in order to attain the same weighted average utility level as in the benchmark equilibrium with international asset trade.

The active consumption-based gains are calculated from the utility of a hypothetical household who is active every period:

$$W_A = \sum_{t=0}^{\infty} \beta^t \int_{s^t} \frac{C_A(s^t)^{1-\eta}}{1-\eta} g(s^t)ds^t$$

Similarly, the aggregate consumption-based gains are calculated from the utility of a hypothetical household who consumes aggregate consumption every period:

$$W_C = \sum_{t=0}^{\infty} \beta^t \int_{s^t} \frac{C(s^t)^{1-\eta}}{1-\eta} g(s^t)ds^t$$

Denoting $\bar{W}_A$ and $\bar{W}_C$ the analogues of these two utility levels in financial autarky, the active consumption-based gains are given by $\Gamma_A = \left(W_A/\bar{W}_A\right)^{1/(1-\eta)}$ and the aggregate consumption-based gains are given by $\Gamma_C = \left(W_C/\bar{W}_C\right)^{1/(1-\eta)}$.

Table 5 reports the welfare gains in both the benchmark model and in the frictionless, complete markets model for different measures of risk aversion. In the frictionless, complete markets model, all the measures are the same, and indicate that, if $\eta = 2$, international

Table 5: Welfare Gains in Model: Actual and Consumption-Based

	Benchmark Model		Frictionless complete markets	
	$\eta = 2$	$\eta = 5$	$\eta = 2$	$\eta = 5$
Welfare gains (in percent)				
Actual gains	0.0060	0.0355	0.0031	0.0070
Active consumption-based	0.0428	0.1183	0.0031	0.0070
Aggregate consumption-based	0.0038	0.0043	0.0031	0.0070

Notes: Welfare gains are averaged over 10000 simulations of 800 quarters.

asset trade raises welfare by the equivalent of a 0.0031 percent increase in consumption in every quarter. In the benchmark model, the actual welfare gains are a bit larger, at 0.0061 percent. Within the benchmark model, we find that welfare gains implied by asset prices are indeed larger than welfare gains implied by aggregate consumption, by an order of magnitude (0.0428% vs. 0.0038%). In addition, these results indicate that the aggregate consumption-based measure of welfare gains is much closer to the actual welfare gains from international asset trade than the measure based on active consumption. Table 5 also reports results for a higher value of risk aversion, $\eta = 5$, which magnifies welfare gains and the difference between the aggregate consumption measure and the active consumption measure.

4 Conclusions

We extend the segmented asset markets model of Alvarez, Atkeson, and Kehoe (2002) to incorporate real shocks and trade in goods, and show that it is successful in resolving the puzzling observation that asset prices imply a high degree of international risk sharing, while aggregate consumption suggests a low degree of risk sharing (as documented in Brandt, Cochrane, and Santa-Clara (2006)). While asset market segmentation in principle breaks the link between aggregate consumption and asset prices, our contribution is in quantitatively evaluating this mechanism in a calibrated model. We illustrate how traded shocks, non-traded shocks, and monetary shocks move the ratios of relative aggregate consumption and relative active consumption in opposite directions, generating high risk sharing implied by asset prices but low risk sharing implied by aggregate consumption. Evidence from microdata in the US and UK supports our model's implication that assetholders' consumption is more highly correlated across countries than non-assetholders' consumption.

Our model provides a resolution to the puzzle highlighted by Backus and Smith (1993) and Kollmann (1995), by generating a negative correlation between the cross-country ratio of aggregate consumption and the real exchange rate. Endogenously segmented asset markets generate a consumption-real exchange rate correlation that rises with average inflation, and we show that this is consistent with cross-country evidence.

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A Appendix

A.1 Additional detail on model setup and characterization

A.1.1 Intermediary’s problem

The intermediary’s profits in state s^t are

$$\begin{aligned}\int\int\int\int \Big[& q(s^t, s_{t+1}, y^{t-1}, y_t) B(s^t, s_{t+1}, y^{t-1}, y_t, \gamma) + \\ & e(s^t) q^*(s^t, s_{t+1}, y^{t-1}, y_t) B^*(s^t, s_{t+1}, y^{t-1}, y_t, \gamma) \Big] f_y(y^{t-1}) f_\gamma(\gamma) ds_{t+1} dy_t dy^{t-1} d\gamma \\ & - q(s^t, s_{t+1}) B(s^t, s_{t+1}) - e(s^t) q^*(s^t, s_{t+1}) B^*(s^t, s_{t+1})\end{aligned}$$

where $q(s^t, s_{t+1})$ and $q^*(s^t, s_{t+1})$ are prices of government debt, i.e. the prices of state-contingent claims to one unit of each currency in the next period. The intermediary maximizes these profits subject to the constraint that at every state s^{t+1}, the total value of payments to households equals the interest payments received on government debt,

$$\int\int\int \left[B(s^{t+1}, y^t, \gamma) + e(s^{t+1})B^*(s^{t+1}, y^t, \gamma)\right] f_y(y^t) f_\gamma(\gamma) dy^t d\gamma$$
$$= B(s^t, s_{t+1}) + e(s^{t+1})B^*(s^t, s_{t+1})$$

The first order conditions of the intermediary's problem yield the no-arbitrage conditions in the text, equations (11)-(13).

A.1.2 Details on characterization of equilibrium

With complete asset markets, a household's sequence of asset market budget constraints collapses to a single date-0 budget. Let $Q(s^t) = q(s^0, s_1)q(s^1, s_2)\cdots q(s^{t-1}, s_t)$ denote the date-0 price of a dollar at state s^t. Using the no-arbitrage condition (11), the sequence of budget constraints for home households (4) with fixed cost γ can be written:

$$\sum_{t=0}^{\infty}\int\int Q(s^t) f_y(y^{t-1}) z(s^t, y^{t-1}, \gamma)\tau(s^t, y^{t-1}, \gamma) ds^t dy^{t-1} \leq B_{h0}(\gamma) + e_0 B_{f0}(\gamma) \tag{35}$$

A household's problem is then to choose consumption, $C(s^t, y^{t-1}, \gamma)$, transfer decisions $z(s^t, y^{t-1}, \gamma)$, and transfers $\tau(s^t, y^{t-1}, \gamma)$ to maximize expected utility (6) subject to the date-0 budget constraint (35) and the cash-in-advance constraint (1). As in Alvarez, Atkeson, and Kehoe (2009), we assume that $B_{h0}(\gamma)$ and $B_{f0}(\gamma)$ are chosen so that households all have the same Lagrange multiplier on the date-0 budget constraint (35) in this maximization problem.

The first order conditions for $C(s^t, y^{t-1}, \gamma)$ and $\tau(s^t, y^{t-1}, \gamma)$, in states when $z(s^t, y^{t-1}, \gamma) = 1$, yield:

$$\beta^t U'\left(C(s^t, y^{t-1}, \gamma)\right) g(s^t) = P(s^t)\lambda Q(s^t) \tag{36}$$

where λ is the multiplier on the date-0 budget constraint. Equation (36) shows that $C(s^t, y^{t-1}, \gamma)$ is independent of y^{t-1} if $z(s^t, y^{t-1}, \gamma) = 1$. That is, idiosyncratic risk is pooled among all active households, and they all consume the same level, $C_A(s^t)$.

If a household chooses $z(s^t, y^{t-1}, \gamma) = 0$, then its consumption is the same as that of a non-assetholder, $C_I(s^t, y_{t-1})$, defined in the text in equation (19).

Now, we consider the choice of whether to transfer into or out of the asset market account. If $z(s^t, y^{t-1}, \gamma) = 1$, then using the goods market budget constraint (1), the amount transferred is: $\tau(s^t, y^{t-1}, \gamma) = P(s^t)\left(C_A(s^t) + \gamma - C_I(s^t, y_{t-1})\right)$. We can then write the household's problem as:

$$\max \sum_{t=0}^{\infty} \int \int \beta^t \Big[z(s^t, y^{t-1}, \gamma) U(C_A(s^t)) \\ + \left(1 - z(s^t, y^{t-1}, \gamma)\right) U\left(y_{t-1}\frac{n(s^t)}{\mu_t}\right)\Big] g(s^t) f_y(y^{t-1}) ds^t dy^{t-1}$$

subject to:

$$\sum_{t=0}^{\infty} \int \int Q(s^t) f_y(y^{t-1}) z(s^t, y^{t-1}, \gamma) P(s^t)\left[C_A(s^t) + \gamma - y_{t-1}\frac{n(s^t)}{\mu_t}\right] ds^t dy^{t-1} \\ \leq B_{h0}(\gamma) + e_0 B_{f0}(\gamma)$$

In the Lagrangian of this problem, the value in state (s^t, y^{t-1}) of setting $z(s^t, y^{t-1}, \gamma) = 1$ is given by:

$$\beta^t U\left(C_A(s^t)\right) g(s^t) f_y(y^{t-1}) - \lambda Q(s^t) f_y(y^{t-1}) P(s^t)\left[C_A(s^t) + \gamma - y_{t-1}\frac{n(s^t)}{\mu_t}\right] \tag{37}$$

And the value of setting $z(s^t, y^{t-1}, \gamma) = 0$ is:

$$\beta^t U\left(y_{t-1}\frac{n(s^t)}{\mu_t}\right) g\left(s^t\right) f\left(y^t\right) \tag{38}$$

Lastly, the value of λ is from the first order condition when $z(s^t, y^{t-1}, \gamma) = 1$, equation (36).

The net gain of setting $z(s^t, y^{t-1}, \gamma) = 1$ versus setting $z(s^t, y^{t-1}, \gamma) = 0$—the difference between expressions (37) and (38)—is positive whenever:

$$U\left(C_A(s^t)\right) - U\left(y_{t-1}\frac{n(s^t)}{\mu_t}\right) - U'\left(C_A(s^t)\right)\left[C_A\left(s^t\right) + \gamma - y_{t-1}\frac{n(s^t)}{\mu_t}\right] > 0 \tag{39}$$

which leads to the definition of the function h in (20) in the text.

An equilibrium allocation is characterized by active household consumption levels and cutoffs determining the set of active households in each country. All equilibrium variables depend only on the current realization of $s_t = (Y_{Tt}, Y_{Nt}, \mu_t, Y^*_{Tt}, Y^*_{Nt}, \mu^*_t)$ and not on its history. Given a vector of shocks s_t, we use the risk sharing condition (21); the demand functions (2)-(3) and their foreign analogues; the goods market clearing conditions (14)-(15) and the foreign analogue; and the conditions $h(y_L(s^t, \gamma)) = h(y_H(s^t, \gamma)) = h(y^*_L(s^t, \gamma)) = h(y^*_H(s^t, \gamma)) = 0$ to solve for the active consumption levels, the thresholds for households to make transfers, and equilibrium prices. In practice, we solve for the equilibrium decision rules on a grid of s_t values and interpolate them. The next subsection provides more detail on the computation. We solve for an equilibrium in which all home and foreign households are identical in period 0, so that $\lambda = \lambda^*$ in (21).

A.2 Model Solution

In this section, we provide details on the equilibrium conditions and our solution method. Equilibrium variables in each period solve a static system of equations given the exogenous shocks. We define prices of traded and non-traded goods relative to the average price index, $p_{Tt} = \frac{P_{Tt}}{P_t}, p_{Nt} = \frac{P_{Nt}}{P_t}$, and likewise for the foreign country, $p^*_{Tt} = \frac{P^*_{Tt}}{P^*_t}, p^*_{Nt} = \frac{P^*_{Nt}}{P^*_t}$. The risk sharing condition is

$$x_t = \frac{U'(C^*_{At})}{U'(C_{At})} \tag{40}$$

where $x_t = e_t \frac{P^*_t}{P_t}$ is the real exchange rate. The law of one price for traded goods can be written as:

$$x_t p^*_{Tt} = p_{Tt} \tag{41}$$

The definitions of the home and foreign consumption price indices in terms of the relative prices $p_{Tt}, p_{Nt}, p^*_{Tt}, p^*_{Nt}$ imply:

$$1 = (a^\sigma p_{Tt}^{1-\sigma} + (1-a)^\sigma p_{Nt}^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{42}$$

$$1 = (a^\sigma (p^*_{Tt})^{1-\sigma} + (1-a)^\sigma (p^*_{Nt})^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{43}$$

We can the write the market clearing conditions as:

$$\begin{aligned} Y_{Tt} + Y^*_{Tt} = &\left(\frac{p_{Tt}}{a}\right)^{-\sigma} \int \left(m_{At}(\gamma)(C_{At}+\gamma) + \frac{n_t}{\mu_t} y_{It}(\gamma)\right) f_\gamma(\gamma) d\gamma \\ &+ \left(\frac{p^*_{Tt}}{a}\right)^{-\sigma} \int \left(m^*_{At}(\gamma)(C^*_{At}+\gamma) + \frac{n^*_t}{\mu^*_t} y^*_{It}(\gamma)\right) f_\gamma(\gamma) d\gamma \end{aligned} \tag{44}$$

$$Y_{Nt} = \left(\frac{p_{Nt}}{1-a}\right)^{-\sigma} \int \left(m_{At}(\gamma)(C_{At}+\gamma) + \frac{n_t}{\mu_t} y_{It}(\gamma)\right) f_\gamma(\gamma) d\gamma \tag{45}$$

$$Y^*_{Nt} = \left(\frac{p^*_{Nt}}{1-a}\right)^{-\sigma} \int \left(m^*_{At}(\gamma)(C^*_{At}+\gamma) + \frac{n^*_t}{\mu_t} y^*_{It}(\gamma)\right) f_\gamma(\gamma) d\gamma \tag{46}$$

where real aggregate money balances, fractions of households active in each γ group, and average incomes of inactive households in each γ group are given by:

$$n_t = p_{Tt}Y_{Tt} + p_{Nt}Y_{Nt} \tag{47}$$

$$n_t^* = p_{Tt}^*Y_{Tt}^* + p_{Nt}^*Y_{Nt}^* \tag{48}$$

$$m_{At}(\gamma) = F(y_{Lt}(\gamma)) + 1 - F(y_{Ht}(\gamma)) \tag{49}$$

$$m_{At}^*(\gamma) = F(y_{Lt}^*(\gamma)) + 1 - F(y_{Ht}^*(\gamma)) \tag{50}$$

$$y_{It}(\gamma) = \int_{y_{Lt}(\gamma)}^{y_{Ht}(\gamma)} y f_y(y) dy \tag{51}$$

$$y_{It}^*(\gamma) = \int_{y_{Lt}^*(\gamma)}^{y_{Ht}^*(\gamma)} y f_y(y) dy \tag{52}$$

The cutoffs $y_{Lt}(\gamma), y_{Ht}(\gamma)$ are the two solutions to the equation:

$$0 = U(C_{At}) - U\left(\frac{n_t}{\mu_t} y_{Lt}(\gamma)\right) - U'(C_{At})\left(C_{At} + \gamma - \frac{n_t}{\mu_t} y_{Lt}(\gamma)\right) \tag{53}$$

and $y_{Lt}^*(\gamma), y_{Ht}^*(\gamma)$ solve the foreign analogue,

$$0 = U(C_{At}^*) - U\left(\frac{n_t^*}{\mu_t^*} y_{Lt}^*(\gamma)\right) - U'(C_{At}^*)\left(C_{At}^* + \gamma - \frac{n_t^*}{\mu_t^*} y_{Lt}^*(\gamma)\right) \tag{54}$$

Given a realization of the state variables, $(Y_{Tt}, Y_{Nt}, \mu_t, Y_{Tt}^*, Y_{Nt}^*, \mu_t^*)$, we solve the system of 7 equations (40) through (46) for the 7 variables $(C_{At}, C_{At}^*, p_{Tt}, p_{Tt}^*, p_{Nt}, p_{Nt}^*, x_t)$, while defining the variables given in equations (47)-(52) and solving for the cutoffs using equations (53) and (54).

Given an initial stock of money, M_{-1}, we define the money stock, price level, and nominal exchange rate in any period t from: $M_t = \mu_t M_{t-1}$, $P_t = \frac{M_t}{\mu_t}$, and $e_t = x_t \frac{P_t}{P_t^*}$.

There are two steps that deserve additional explanation: computing the integrals and solving for the cutoffs. We compute the integrals for γ and for y using Gauss-Hermite quadrature, so we compute each variable that is a function of γ for a discrete grid $\{\gamma_i\}$. For each value of γ_i, we solve for the home cutoffs (and analogously for the foreign cutoffs) using Newton's method in one dimension on equation (53) twice, with different initial values each time. As described in the text, the right hand side of equation (53), which is the function $h(y; s^t, \gamma)$ defined in the text, is convex, has a minimum value below 0 when $y = C_{At}\frac{\mu_t}{n_t}$, and crosses zero twice. So as long as the initial guess for $y_{Lt}(\gamma)$, say $y_{Lt}^0(\gamma)$, satisfies $y_{Lt}^0(\gamma) < C_{At}\frac{\mu_t}{n_t}$ and $h(y_{Lt}^0(\gamma); s^t, \gamma) > 0$ and the initial guess for $y_{Ht}(\gamma)$, say $y_{Ht}^0(\gamma)$, satisfies $y_{Ht}^0(\gamma) > C_{At}\frac{\mu_t}{n_t}$, then Newton's method from these two starting values quickly and reliably converges to two distinct solutions to equation (53) with $y_{Lt}(\gamma) < y_{Ht}(\gamma)$.

In solving the model, we use Chebyshev polynomial interpolation to solve for the equilibrium decision rules on a grid for the state variables and interpolate them. We simulate sequences of shocks, and then compute the interpolated values of equilibrium variables. We use 4 grid points for each of the 6 state variables, corresponding to the Chebyshev interpolation nodes for intervals up to 3 standard deviations above and below the steady state value of each state variable. We interpolate using 3$^{\text{rd}}$ degree Chebyshev polynomials. Increasing the degree of the polynomial approximation or the width of the grid does not change any of our results.

After having solved for equilibrium variables, we use a collocation method to solve the asset pricing equations (25) and (26). For each value of the state vector s, letting $g(s,\varepsilon) = (I-A)\log\bar{s} + A\log s_i + \varepsilon$ denote the vector of possible values next period, and letting $\pi(\varepsilon)$ denote the distribution of the vector of innovations, equation (25) states:

$$v(s)u'(C_A(s)) - \beta E_\varepsilon[u'(C_A(g(s,\varepsilon)))(v(g(s,\varepsilon)) + n(g(s,\varepsilon)))] = 0 \tag{55}$$

We solve for the coefficients of the Chebyshev polynomial approximation to the function v so that equation (55) holds exactly on the grid for s, using Gauss-Hermite quadrature to compute the expectation over ε. The computation of v_{rf} is analogous.

A.3 Aggregate Consumption Risk Sharing Equation

In this section, we log-linearize the model's equations around a symmetric steady state, in order to derive a first order approximation of the cross country differences in the growth rates of aggregate consumption, i.e., $\hat{C}_t - \hat{C}_t^*$, where $\hat{C}_t \equiv \log C_t - \log\bar{C}$ and $\bar{C}$ denotes the steady state level of aggregate consumption in the home country.

Log-linearizing aggregate consumption in the home country, equation (29), we have:

$$\bar{C}\cdot\hat{C}_t = \omega\bar{m}_A\bar{C}_A(\hat{m}_{At} + \hat{C}_{At}) + \omega\frac{\bar{n}}{\bar{\mu}}\bar{y}_I(\hat{y}_{It} + \hat{n}_t - \hat{\mu}_t) + (1-\omega)\frac{\bar{n}}{\bar{\mu}}(\hat{n}_t - \hat{\mu}_t) \tag{56}$$

From the definition of n_t,

$$\bar{n}\hat{n}_t = \bar{p}_T\bar{Y}_T(\hat{p}_{Tt} + \hat{Y}_{Tt}) + \bar{p}_N\bar{Y}_N(\hat{p}_{Nt} + \hat{Y}_{Nt}) \tag{57}$$

We define the steady state traded fraction of income as $\psi_T \equiv \frac{\bar{p}_T\bar{Y}_T}{\bar{p}_T\bar{Y}_T + \bar{p}_N\bar{Y}_N}$, the steady state non-traded fraction of income as $\psi_N \equiv \frac{\bar{p}_N\bar{Y}_N}{\bar{p}_T\bar{Y}_T + \bar{p}_N\bar{Y}_N}$, the steady state fraction of aggregate consumption for inactive assetholding households as $\phi_I \equiv \omega\frac{\bar{n}}{\bar{\mu}}\frac{\bar{y}_I}{\bar{C}}$; the fraction for active assetholding households as $\phi_A \equiv \omega\frac{\bar{m}_A\bar{C}_A}{\bar{C}}$; and the fraction for nonassetholding households as $\phi_{NA} \equiv (1-\omega)\frac{\bar{n}}{\bar{\mu}\bar{C}}$. From the definition of the price index and the steady state market clearing conditions (traded and non-traded income shares are equal to expenditure shares in the steady state),

$$0 = \psi_T \hat{p}_{Tt} + \psi_N \hat{p}_{Nt} \tag{58}$$

The log-linear equation becomes:

$$\hat{C}_t = (\phi_I + \phi_{NA})\left[\psi_T \hat{Y}_{Tt} + \psi_N \hat{Y}_{Nt} - \hat{\mu}_t\right] + \phi_I \hat{y}_{It} + \phi_A\left[\hat{C}_{At} + \hat{m}_{At}\right] \tag{59}$$

The log-linearized risk sharing condition (21) is:

$$\hat{x}_t = -\eta(\hat{C}^*_{At} - \hat{C}_{At}) \tag{60}$$

Subtracting the analogue for $\hat{C}^*_t$ from equation (59) and using equation (60) yields equation (30) in the text for $\hat{C}_t - \hat{C}^*_t$.

A.4 Micro Data

We use data from the US Consumer Expenditure Survey provided by Heathcote, Perri, and Violante (2010)—henceforth HPV—and from the UK Family Expenditure Survey provided by Blundell and Etheridge (2010). Both datasets are available for download at: https://www.economicdynamics.org/si-cross-facts/. For the calibration, we estimate residual variances of income and consumption unexplained by household characteristics, as described in Section 3.1, using the "CEX sample b" provided by HPV.

For the statistics in Table 3, for the US we instead use the CEX data from Krueger and Perri (2006), since it contains information on financial asset holdings. We classify a household as an asset holder if their financial wealth is positive. We construct aggregate average nondurable consumption per capita (i.e. per adult equivalent as defined by OECD equivalence scale) and average consumption per capita for non-assetholders (i.e. households with zero financial wealth) and assetholders (i.e. households with positive financial wealth). The UK survey data is annual, so for the US, we aggregate quarterly consumption into an annual average. We then compute the US$-$UK correlations of the Hodrick-Prescott filtered log consumption for three groups: aggregate, assetholders and non-assetholders.

A.5 Sensitivity Analysis

We illustrate that differences between the asset-price based and the consumption-based measures of risk sharing reported in Table 1 are robust to changes in the coefficient of relative risk aversion, η, the elasticity of substitution σ, and the fraction of assetholders ω. The first column of Table 6 reports the benchmark model results where $\sigma = 0.5$, $\eta = 2$, $\omega = 0.83$ and where there are spillovers across shocks. As we increase the elasticity of substitution from $\sigma = 0.3$ to 0.5 and to 1.3, the discrepancy between the international SDF correlations and the international correlations of aggregate consumption widens. The same is true as we increase the coefficient of relative risk aversion, or as we reduce the fraction of assetholders.

We also illustrate the effects of shocks on active and aggregate relative consumption and on active and inactive households, when spillovers are allowed. We report impulse responses to 1 percent shocks to home country traded and non-traded output, and home country money growth, while letting the off-diagonal elements of matrix A be non-zero. The impulse responses are plotted in Figures $6 - 8$. A noticeable difference from the impulse responses plotted in the text (where spillovers are shut down) is in the impact of a non-traded shock on foreign consumption. This is because with spillovers, the non-traded shocks in the home country is transmitted to other shocks which have a non-zero impact on foreign consumption.

Table 6: Sensitivity of Results to σ, η and ω; Results with No Spillovers of Shocks

	Benchmark model	Vary σ		Vary η		Vary ω		No Spillovers of Shocks
		$\eta = 2$, $\sigma = 0.3$	$\eta = 2$, $\sigma = 1.3$	$\eta = 1.2$, $\sigma = 0.5$	$\eta = 5$, $\sigma = 0.5$	$\omega = 0.5$	$\omega = 1$	
Standard Deviation (%)								
Real exchange rate	1.17	1.61	0.56	0.90	1.52	1.40	1.15	1.15
Real income	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
Aggregate consumption	0.93	0.91	0.95	0.87	0.93	0.95	0.93	0.91
Active consumption	0.94	0.98	0.90	0.86	1.17	1.22	0.91	0.85
SDF	5.64	5.66	5.62	5.52	7.05	5.79	5.63	5.71
Int'l Correlations								
Real income	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56
Aggregate consumption	0.32	0.36	0.25	0.41	0.23	0.25	0.32	0.43
Active consumption	0.81	0.66	0.95	0.62	0.97	0.83	0.80	0.77
SDF	0.85	0.79	0.92	0.88	0.84	0.82	0.85	0.85
BCS Risk Sharing Index								
Aggregate consumption	0.47	0.52	0.40	0.56	0.38	0.41	0.47	0.44
Active consumption	0.77	0.60	0.94	0.56	0.96	0.82	0.76	0.77
SDF	0.97	0.94	0.99	0.98	0.97	0.96	0.97	0.97
Correlation $\frac{C}{C^*}$ and $\frac{P*}{P}$	−0.79	−0.76	−0.82	−0.72	−0.83	−0.84	−0.78	−0.82

Notes: Statistics are Hodrick-Prescott filtered, logged, and averaged over 10000 simulations of 800 quarters. SDF refers to stochastic discount factor. In the benchmark model, $\eta = 2$, $\sigma = 0.5$, and $\omega = 0.83$, and there are spillovers between shocks (i.e., off-diagonal elements in matrix A are non-zero). When changing the values of σ or η, or when imposing no spillovers of shocks, we recalibrate the mean and variance of the fixed cost distribution to match the same targets as in the benchmark model: a variance of log consumption of 0.23 and 40 percent of asset-holding households active (or a third of *all* households active since $\omega = 0.83$). When changing the values of ω, we recalibrate to target the same variance of log consumption and a third of all households active.

Figure 6: Impulse responses to 1% increase in Y_T, with spillovers to other shocks

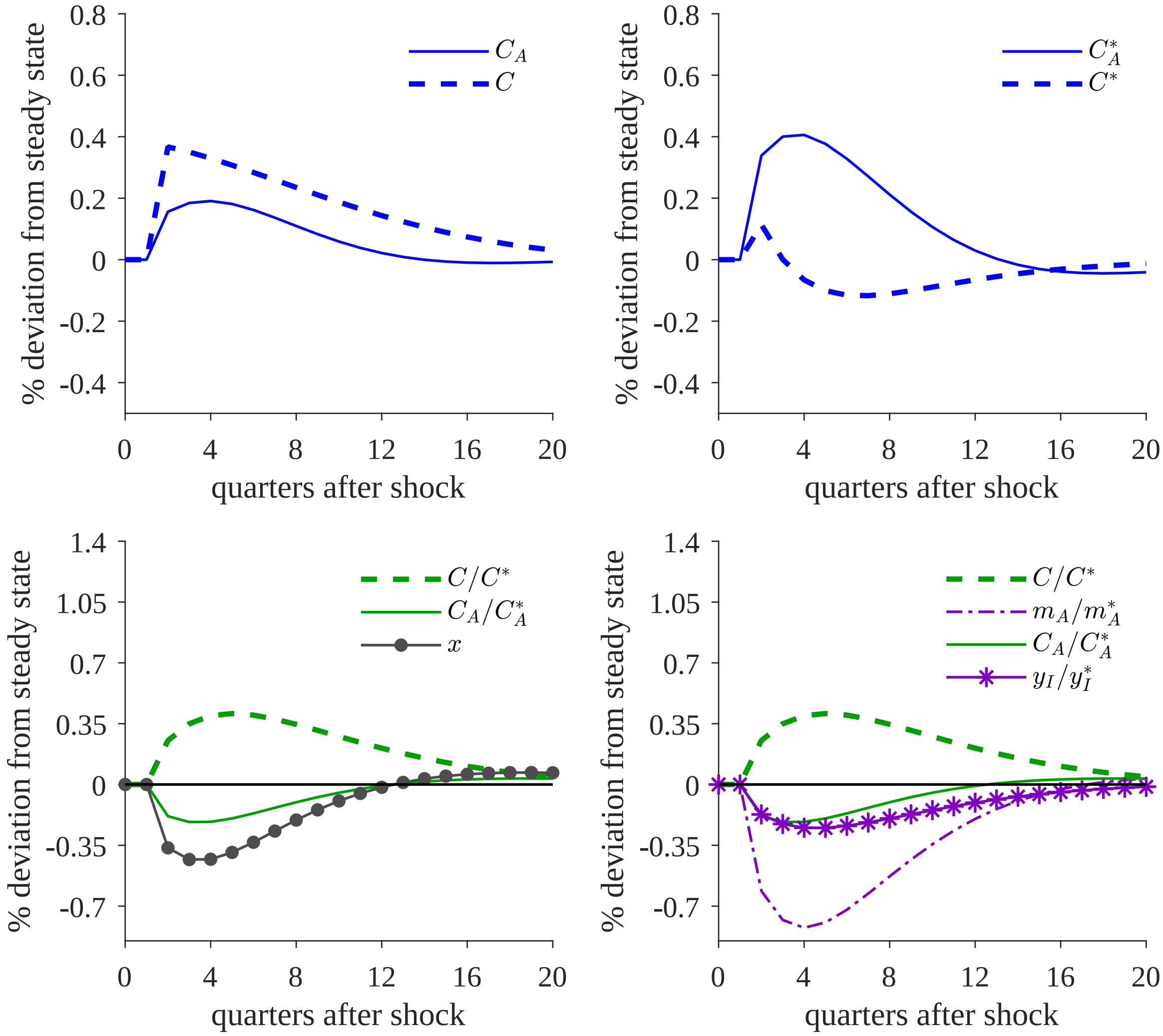

Figure 7: Impulse responses to 1% increase in Y_N, with spillovers to other shocks

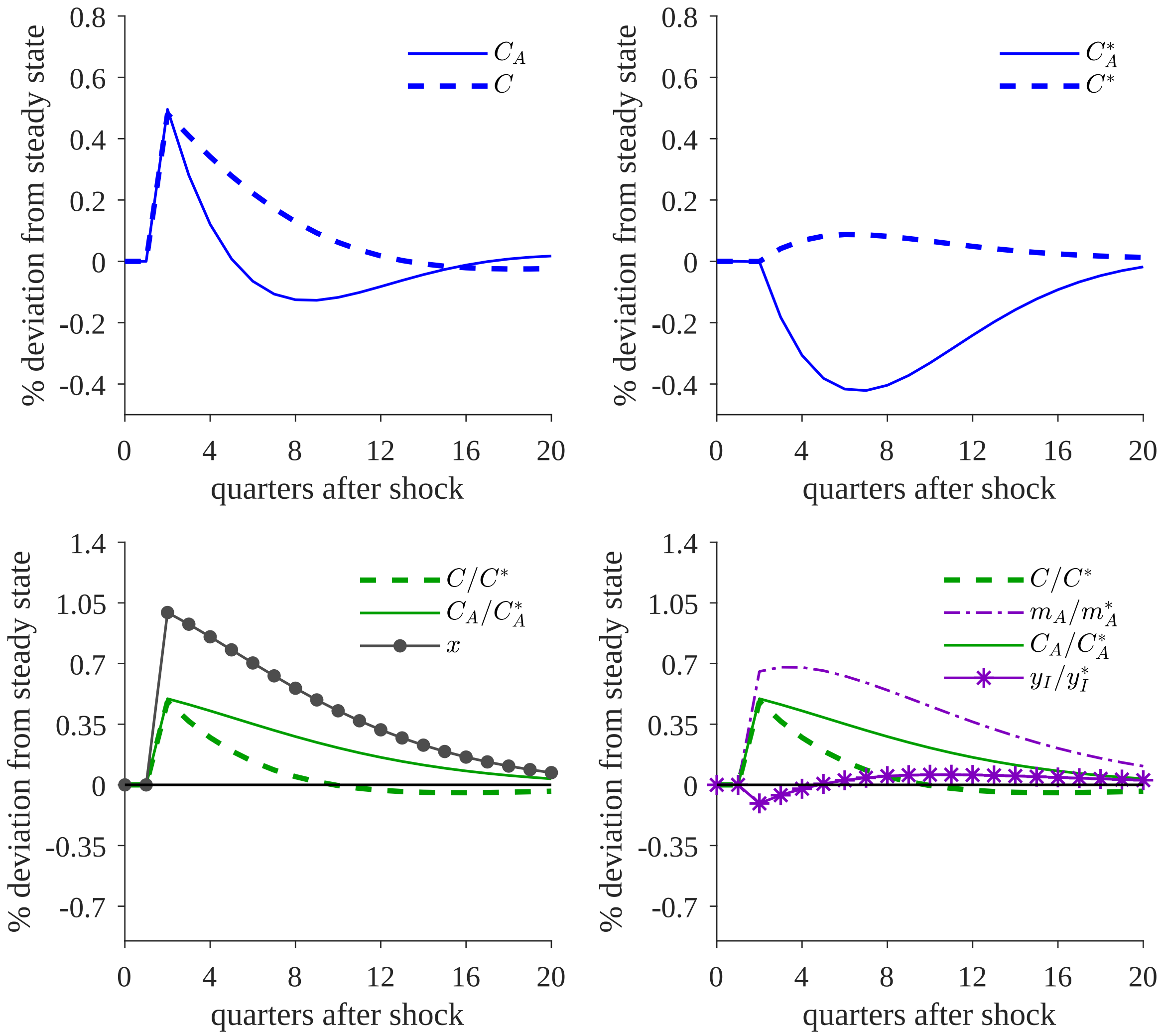

Figure 8: Impulse responses to 1% increase in μ, with spillovers to other shocks