1998

Optimal Taxation in Life-Cycle Economies

Andres Erosa

Martin Gervais

RESEARCH REPORT 9812

Optimal Taxation in Life-Cycle Economies

by

Andrés Erosa   Martin Gervais

November 1998

Department of Economics
Social Science Centre
University of Western Ontario
London, Ontario, Canada
N6A 5C2
econref@sscl.uwo.ca

Optimal Taxation in Life-Cycle Economies

Andrés Erosa
The University of Western Ontario
Department of Economics
London, Canada N6A 5C2

aerosa@julian.uwo.ca

Martin Gervais*
The University of Western Ontario
Department of Economics
London, Canada N6A 5C2

gervais@julian.uwo.ca

November 9, 1998

Abstract

This paper studies optimal taxation in an overlapping generations economy. We characterize the optimal path of fiscal policy, both in the long run and in the transition to the steady state. The implications of this study are in sharp contrast with the prescription offered by infinitely-lived agent models. First, the government's desire to tax initial holdings of capital at confiscatory rates is endogenously curtailed by intergenerational redistributional considerations. Second, because of life-cycle elements, capital income taxes are in general different from zero even in the steady state. The tax rate on capital income should only be zero if it is optimal to tax consumption goods uniformly over the lifetime of individuals. The conditions for uniform taxation of consumption depend, in turn, on preferences, the age-profile of labor productivity, and the set of taxes available to the government. (JEL E62, H21)

1

A
giv
of
Ch
ent
equ
inf
by

are
tur
and
lon
tax
cap
In
pos
solu
imp

in w
utili
the
at d
Pare
that

taxes
as in
are a
other
can i
unde
First
gene
zero.
prope
coinc
to lu
state,

1 Introduction

A classic problem in public finance concerns the optimal manner in which to finance a given stream of government purchases in the absence of lump-sum taxation. In the context of a standard neoclassical growth model with infinitely-lived individuals (or dynasties), Chamley (1986) and Judd (1985) have demonstrated that an optimal income-tax policy entails taxing capital at confiscatory rates in the short-run and setting capital income taxes equal to zero in the long-run. The optimality of this initial levy of capital relies heavily on the infinitely-lived agent abstraction, in which altruistic individuals are more than compensated by the elimination of distortionary taxation in the future.

The purpose of this paper is to re-examine the question of optimal taxation when there are overlapping generations of individuals who attach zero weight to the well-being of future generations. In this environment, the government faces a trade-off between equity and efficiency. The individuals on whom the burden of a front-loading policy fall are no longer the same individuals who in the long-run benefit from the elimination of distortionary taxation. The government's desire to resort to confiscatory taxation of initial holdings of capital is thus endogenously curtailed by intergenerational redistributional considerations. In contrast to the infinitely-lived agent model, there is no need in this framework to impose exogenous bounds on feasible capital income taxes to circumvent the arguably trivial solution outlined above (Chamley (1986), Jones et al. (1993, 1997), Chari et al. (1994) and implicitly Lucas (1990)).

In order to study optimal taxation in this environment, we formulate a Ramsey problem in which social welfare is defined as the discounted sum of successive generations' lifetime utility (as in Samuelson (1968) and Atkinson and Sandmo (1980)). This formulation of the government's objective respects the valuations placed by individuals on consumption at different dates. Consequently, an allocation solving the Ramsey problem is constrained Pareto efficient, in the sense that it cannot be Pareto dominated by any other allocation that is a competitive equilibrium for some fiscal policy.

We show that optimal taxation in overlapping generations economies generally features taxes on capital and labor income that are different from zero during the transition as well as in the steady state. We study Ramsey taxation for two tax systems: one where tax rates are allowed to depend on the age of the individual supplying the factors of production, the other with age-independent taxes. We show that the set of allocations that the government can implement with age-independent taxes is a subset of the set of implementable allocations under an age-dependent tax system. Nevertheless, both tax systems share many properties. First, consumption taxes are redundant and, therefore, can be ruled out without loss of generality. Second, taxes on both capital and labor income are in general different from zero. Third, the steady state capital-labor ratio has the modified golden rule property. This property implies, as was originally shown by Samuelson (1968), that the capital labor-ratio coincides with that of a first best allocation, that is, one where the government has access to lump-sum taxes. Fourth, if the Ramsey equilibrium allocation converges to a steady state, then the steady state is independent of the transition path. We emphasize that this

property does not hold in infinitely-lived agent models. In particular, the exogenous bounds constraining taxes during the transition determine the final steady state.

We find the conditions for zero taxation of capital income in overlapping generations economies to be closely related to the uniform commodity taxation results in the public finance literature (see Atkinson and Stiglitz (1980)). For instance, the tax rate on capital income should only be zero if it is optimal to tax the consumption of goods uniformly over the lifetime of individuals. The conditions for uniform taxation of consumption depend, in turn, on preferences, the age-profile of labor productivity, and the set of tax instruments available to the government. We also find conditions under which the relative tax rates on labor income at different ages are inversely related to the relative income elasticities of labor supplied at those ages. When taxes are not allowed to depend on age, the government can reduce the total deadweight loss of distortionary taxation by taxing capital income. The government can thus find optimal to tax capital income even if optimal capital income taxes are zero in an age-dependent tax system. In other words, the conditions for zero taxation of capital income are much more stringent under an age-independent tax system.

Atkinson and Sandmo (1980) provide one of the first studies of optimal taxation in overlapping generations economies. They show that a first-best steady state allocation can be achieved if the government can use lump-sum taxes. They also show, however, that in the absence of government debt, consumption taxes, and lump-sum taxes, the government does not have enough instruments to achieve the modified golden rule capital-labor ratio. Park (1989) shows that in a model with intra-generational heterogeneity the government might use distortionary taxation to redistribute income across individuals with different innate abilities. In our paper we abstract from intra-generational redistributional issues. Our model builds on the framework developed by Auerbach and Kotlikoff (1987) to study fiscal policies. Escolano (1991) computes numerically optimal Ramsey taxes in this framework. Unlike previous studies of optimal taxation in overlapping generations models, we formulate the Ramsey problem using the primal approach (Atkinson and Stiglitz (1980), Lucas and Stokey (1983)). Under this approach, rather than choosing a sequence of tax rates, the government directly chooses a feasible resource allocation subject to a series of constraints guaranteeing that the allocation can be decentralized as a competitive equilibrium for some fiscal policy. This formulation of the Ramsey problem leads to an explicit characterization of the Ramsey policy. Our study also sheds some light on how life-cycle elements affect the optimal fiscal policy.

The rest of the paper is organized as follows. The next section presents the economic environment. In section 3 we formulate the Ramsey problem and characterize optimal fiscal policies. Section 4 discusses age-independent taxes. In section 5 we present some numerical examples. Two striking properties emanate from the Ramsey equilibrium allocations generated in these examples. First, the allocations under age-dependent and age-independent taxes are very similar. Second, they are extremely sensitive to changes in the intergenerational discount factor.

2 The Economy

Consider an economy in which there are an infinite number of time periods, t, indexed by positive integers. The economy is populated by overlapping generations of identical individuals. Individuals live $(J+1)$ periods, from age 0 to age J. At each time period a new generation is born. The population is assumed to grow at constant rate n per period. Consequently, the share of age-j individuals in the population, μ_j, is time invariant and satisfies $\mu_j = \mu_{j-1}/(1+n)$, for $j = 1, \ldots, J$, where $\sum_{j=0}^{J} \mu_j = 1$. We subsequently use the shares μ_j to express aggregate variables in per capita terms. Each generation is indexed by its date of birth. At date $t = 0$ the generations alive are $-J, -J+1, \ldots, 0$. Individuals derive utility from consumption goods and leisure. The objective of an individual born in period $t \geq -J$ is given by

$$\sum_{j=j_0(t)}^{J} \beta^{j-j_0(t)} U\left(c_{t,j}, 1 - l_{t,j}\right), \tag{1}$$

where $c_{t,j}$ and $l_{t,j}$ respectively denote consumption and time devoted to work in period $(t+j)$ by an age-j individual born in period t. The age of individuals alive at date zero is denoted $j_0(t)$. For generations $t \geq 0$, $j_0(t) = 0$, so that in general $j_0(t) \equiv \max\{-t, 0\}$. We assume $\beta > 0$ and that the utility function $U(\cdot, \cdot)$ is increasing in both arguments, strictly concave, and satisfies standard Inada conditions.

Individuals are endowed with one unit of time in each period of their life. We assume that an age-j individual can transform one unit of time into z_j efficiency units of labor (for $j = 0, \ldots, J$). The date-t aggregate labor input, expressed in efficiency units, is given by

$$l_t = \sum_{j=0}^{J} \mu_j z_j l_{t-j,j}.$$

At each date there is a unique produced good that can be used for private consumption, government consumption, or as capital. The technology to produce goods is represented by a neoclassical production function with constant returns to scale:

$$y_t = f(k_t, l_t), \tag{2}$$

where y_t, k_t and l_t denote the levels of aggregate output, capital, and effective labor, respectively. Feasibility requires that total consumption plus investment be less than or equal to aggregate output

$$c_t + (1+n)k_{t+1} - (1-\delta)k_t + g_t \leq y_t, \tag{3}$$

where $0 < \delta < 1$ is the depreciation rate of capital, c_t denotes aggregate private consumption at date t, g_t stands for date-t government consumption, and all aggregate variables are expressed in per capita terms.

4

To finance its exogenous stream of expenditures, we assume that the government has access to a set of fiscal policy instruments and a commitment technology to implement its fiscal policy. The set of instruments consists of proportional taxes on consumption, labor and capital income, as well as government debt. At each date, the tax rates on factor services are allowed to depend on the age of the individual supplying the services. The date-t tax rates on capital and labor services supplied by an age-j individual (born in period $(t-j)$) are denoted by $\tau_{t-j,j}^a$ and $\tau_{t-j,j}^w$, respectively. Similarly, consumption taxes are allowed to depend on the age of the consumer, and we use $\tau_{t-j,j}^c$ to represent the date-t tax rate on consumption of an age-j individual. Equivalently, the government chooses the after-tax prices of consumption goods, $q_{t-j,j}$, labor services, $w_{t-j,j}$, and capital services, $r_{t-j,j}$, with these prices depending on the age of the consumer or the supplier of the factor service. In addition, the government can issue debt in order to match any imbalance between expenditures and revenues in any given period. It is assumed that the government taxes the return on capital and debt at the same rate, making debt and capital perfect substitutes. We denote total asset holdings of an age-j individual at date t by $a_{t-j,j}$. The government budget constraint at date $t \geq 0$ is given by

$$(1+\hat{r}_t)b_t + g_t = (1+n)b_{t+1} + \sum_{j=0}^{J}(q_{t-j,j}-1)\mu_j c_{t-j,j} + \sum_{j=0}^{J}(\hat{r}_t - r_{t-j,j})\mu_j a_{t-j,j} +$$

$$\sum_{j=0}^{J}(\hat{w}_t - w_{t-j,j})\mu_j z_j l_{t-j,j}, \quad (4)$$

where $q_{t,j} \equiv (1+\tau_{t,j}^c)$, $w_{t,j} \equiv (1-\tau_{t,j}^w)\hat{w}_{t+j}$, and $r_{t,j} \equiv (1-\tau_{t,j}^a)\hat{r}_{t+j}$. Note that the producer price of consumption goods has been normalized at one. Before-tax prices for capital and labor are given by $\hat{r}_t = f_k(k_t, l_t) - \delta$ and $\hat{w}_t = f_l(k_t, l_t)$.

An individual born at period $t \geq -J$ solves the following problem:

$$\max \sum_{j=j_0(t)}^{J} \beta^{j-j_0(t)} U\left(c_{t,j}, 1 - l_{t,j}\right), \quad (5)$$

$$q_{t,j}c_{t,j} + a_{t,j+1} = w_{t,j}z_j l_{t,j} + (1+r_{t,j})a_{t,j}, \quad j = j_0(t),\dots,J, \quad (6)$$

$$a_{t,j_0(t)} \text{ given and equal to 0 if } t \geq 0.$$

Let U^t denote the indirect utility function of generation t as a function of the government tax policy.

The government takes individuals' optimizing behavior as given and chooses a fiscal policy to maximize its objective function. Following Samuelson (1968) and Atkinson and Sandmo (1980), social welfare is defined as the discounted sum of individual lifetime welfares. In other words, the government's objective is the maximization of

$$\sum_{t=-J}^{\infty} \gamma^t U^t,$$

5

where $0 < \gamma < 1$ is the intergenerational discount factor.

This formulation of the government's objective respects the valuations placed by individuals on consumption at different dates. Consequently, an allocation solving the Ramsey problem is constrained Pareto efficient, in the sense that it cannot be Pareto dominated by any other allocation that is a competitive equilibrium for some fiscal policy. The standard trade-off between equity and efficiency becomes transparent with the explicit (relative) valuation of all generations affected by fiscal policy. This is in sharp contrast with optimal taxation problems in infinitely-lived agent models. As emphasized by Escolano (1991), when interpreted as a guide for fiscal policy, infinitely-lived agent models tend to put as much burden on currently alive individuals as possible. In particular, the government can perfectly imitate a lump-sum tax by taxing heavily initial holdings of financial assets. Judd (1985), Chamley (1986), Chari et al. (1994), Jones et al. (1993,1997), and other authors in this literature introduce upper bounds on feasible capital income taxes to avoid initial confiscatory taxes. The bounds assumed determine the magnitude of the welfare gains achieved by switching to the taxes prescribed by the optimal Ramsey problem. Indeed, with sufficiently high bounds a Pareto optimal equilibrium can be achieved. In overlapping generations economies, however, the explicit valuation of different generations' felicity effectively limits the government's desire to resort to a front-loading policy and, consequently, there is no need to impose exogenous bounds on capital income taxes.

3 Optimal Fiscal Policy

In this section we show that, in general (in a sense that will be specified below), the solution to the Ramsey problem features non-zero taxes on capital and labor income. In particular, and contrary to infinitely-lived agent models, if the Ramsey allocation converges to a steady state solution, optimal capital income taxes will in general be different from zero even in that steady state. These results are obtained by identifying conditions under which optimal capital or labor income taxes are zero. We then find parameter restrictions for two classes of utility functions that satisfy the condition for the optimality of zero capital income taxation. We begin the analysis by formulating the Ramsey problem.

3.1 The Ramsey Problem

The Ramsey problem consists of choosing a set of taxes so that the allocation that arises when prices and quantities are determined in competitive markets maximizes a given welfare function. The set of allocations that the government can implement thus consists of the allocations chosen by individuals for any arbitrary fiscal policy. The allocations belonging to this set are formally defined below.

Definition 1 (Implementable Allocation) *Let $\{g_t\}_{t=0}^{\infty}$ be a given sequence of government expenditures. Given initial aggregate endowments $\{k_0, b_0\}$ and initial individual asset*

holdings $\{a_{-j,j}\}_{j=1}^{J}$ such that

$$k_0 + b_0 = \sum_{j=1}^{J} \mu_j a_{-j,j},$$

an allocation $\left\{ \{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}, k_{t+J+1} \right\}_{t=-J}^{\infty}$ is implementable if there exist a fiscal policy arrangement $\left\{ \{q_{t,j}, r_{t,j}, w_{t,j}\}_{j=j_0(t)}^{J}, b_{t+J+1} \right\}_{t=-J}^{\infty}$ and a sequence of asset holdings $\left\{ \{a_{t,j}\}_{j=j_0(t)}^{J} \right\}_{t=-J}^{\infty}$ such that

D1a. given prices from the fiscal policy arrangement, $\{c_{t,j}, l_{t,j}, a_{t,j+1}\}_{j=j_0(t)}^{J}$ solves the consumer problem given by (5)–(6) for $t = -J, \ldots$;

D1b. factor prices are competitive: $\hat{r}_t = f_k(k_t, l_t) - \delta$ and $\hat{w}_t = f_l(k_t, l_t)$, $t = 0, 1, \ldots$;

D1c. the government budget constraint (4) is satisfied at $t = 0, 1, \ldots$;

D1d. aggregate feasibility (3) is satisfied at $t = 0, 1, \ldots$.

Proposition 1 below establishes that there is a family of fiscal policies that implement the same allocation, or, equivalently, that one of the fiscal policy instruments is redundant. For instance, without loss of generality, government debt can be set to zero provided the following conditions are met. First, the after-tax prices of consumption goods are adjusted to maintain constant the value of each consumers' assets in terms of the after-tax prices of consumption goods. Second, the after-tax prices of labor services are adjusted so that they remain constant in terms of the after-tax prices of consumption goods. Third, the return on capital is adjusted in order to keep the relative price of present versus future consumption unchanged. Alternatively, taxes on consumption goods can be set to zero by picking the appropriate amount of government debt and after-tax prices of capital and labor services.

Proposition 1 Let $\{g_t\}_{t=0}^{\infty}$ be a given sequence of government expenditures and let $\left(k_0, b_0, \{a_{-j,j}\}_{j=1}^{J} \right)$ be initial endowments such that

$$k_0 + b_0 = \sum_{j=1}^{J} \mu_j a_{-j,j}.$$

Suppose that the fiscal policy $\left\{ \{q_{t,j}, r_{t,j}, w_{t,j}\}_{j=j_0(t)}^{J}, b_{t+J+1} \right\}_{t=-J}^{\infty}$ and the sequence of asset holdings $\left\{ \{a_{t,j}\}_{j=j_0(t)+1}^{J} \right\}_{t=-J}^{\infty}$ implements the allocation $\left\{ \{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}, k_{t+J+1} \right\}_{t=-J}^{\infty}$.

Then any other fiscal policy $\left\{\{\tilde{q}_{t,j}, \tilde{r}_{t,j}, \tilde{w}_{t,j},\}_{j=j_0(t)}^{J}, \tilde{b}_{t+J+1}\right\}_{t=-J}^{\infty}$ *and sequence of asset holdings* $\left\{\{\tilde{a}_{t,j}\}_{j=j_0(t)+1}^{J}\right\}_{t=-J}^{\infty}$ *satisfying*

$$\frac{1+r_{t,j_0(t)}}{q_{t,j_0(t)}} = \frac{1+\tilde{r}_{t,j_0(t)}}{\tilde{q}_{t,j_0(t)}}, \qquad t = -J, \dots, \tag{P1a}$$

$$\frac{w_{t,j}}{q_{t,j}} = \frac{\tilde{w}_{t,j}}{\tilde{q}_{t,j}}, \qquad t = -J, \dots, \quad j = j_0(t), \dots, J, \tag{P1b}$$

$$\frac{(1+r_{t,j+1})\,q_{t,j}}{q_{t,j+1}} = \frac{(1+\tilde{r}_{t,j+1})\,\tilde{q}_{t,j}}{\tilde{q}_{t,j+1}}, \quad t = -J, \dots, \quad j = j_0(t)+1, \dots, J \tag{P1c}$$

$$\frac{a_{t,j+1}}{q_{t,j}} = \frac{\tilde{a}_{t,j+1}}{\tilde{q}_{t,j}} \qquad t = -J, \dots, \quad j = j_0(t)+1, \dots, J, \tag{P1d}$$

also implements the allocation.

Proof. We need to show that any alternative fiscal policy and sequence of asset holdings which satisfy conditions (P1a) – (P1d) also satisfy conditions (D1a – D1d) in Definition 1. Notice that conditions (D1b) and (D1d) (factor prices and feasibility) are trivially satisfied under the alternative fiscal policy.

Using the consumer's budget constraint under the initial fiscal policy and conditions P1b and P1c we obtain

$$q_{t,j}c_{t,j} + a_{t,j+1} = \tilde{w}_{t,j}\frac{q_{t,j}}{\tilde{q}_{t,j}}z_j l_{t,j} + (1+\tilde{r}_{t,j})\frac{\tilde{q}_{t,j-1}}{\tilde{q}_{t,j}}\frac{q_{t,j}}{q_{t,j-1}}a_{t,j}.$$

Multiplying this expression by $\tilde{q}_{t,j}/q_{t,j}$ and using condition P1d we get

$$\tilde{q}_{t,j}c_{t,j} + \tilde{a}_{t,j+1} = \tilde{w}_{t,j}z_j l_{t,j} + (1+\tilde{r}_{t,j})\tilde{a}_{t,j}.$$

Thus, any allocation $\{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}$ satisfying the consumer's budget constraints under the initial fiscal policy also satisfies the budget constraints under the alternative fiscal policy (with asset holdings for generation $t \geq -J$ given by $\{\tilde{a}_{t,j}\}_{j=j_0(t)+1}^{J}$). Since the converse is also true the two budget sets are identical. And since consumers face the same decision problem under both fiscal policies, we conclude that $\left\{\{c_{t,j}, l_{t,j}, \tilde{a}_{t,j+1}\}_{j=j_0(t)}^{J}\right\}_{t=-J}^{\infty}$ solves the consumer problem when prices are given by the alternative fiscal policy. Condition D1a is thus satisfied.

It remains to show that the government budget constraint (4) is satisfied under the alternative fiscal policy. Using feasibility (3) and the fact that the production function (2) is homogeneous of degree one, this constraint can be written as

$$\sum_{j=0}^{J}(1+\tilde{r}_{t-j,j})\mu_j \tilde{a}_{t-j,j} = (1+n)\tilde{a}_{t+1} + \sum_{j=0}^{J}\tilde{q}_{t-j,j}c_{t-j,j} - \sum_{j=0}^{J}\tilde{w}_{t-j,j}\mu_j z_j l_{t-j,j}.$$

Using conditions P1a – P1d this expression is equivalent to

$$\sum_{j=0}^{J}(1 + r_{t-j,j})\frac{q_{t-j,j-1}}{q_{t-j,j}}\frac{\tilde{q}_{t-j,j}}{\tilde{q}_{t-j,j-1}}\mu_j a_{t-j,j}\frac{\tilde{q}_{t-j,j-1}}{q_{t-j,j-1}}$$

$$= (1 + n)a_{t+1}\frac{\tilde{q}_{t-j,j}}{q_{t-j,j}} + \sum_{j=0}^{J}\tilde{q}_{t-j,j}c_{t,j} - \sum_{j=0}^{J}w_{t-j,j}\frac{\tilde{q}_{t-j,j}}{q_{t-j,j}}\mu_j z_j l_{t-j,j}.$$

Multiplying both sides of the previous expression by $q_{t-j,j}/\tilde{q}_{t-j,j}$ we obtain the government budget constraints under the initial fiscal policy. Since this constraint holds at all dates by assumption under the initial fiscal policy, we conclude that the government budget constraints also hold under the alternative fiscal policy, and condition D1c is also satisfied. ∎

It is interesting to consider this redundancy Proposition in light of the findings in Summers (1981). This author shows that since the timing of tax collections over the lifetime of individuals is very different under labor income or consumption taxation, the two sources of taxation give rise to very different age-savings patterns. This leads to Summers' finding that wage taxation significantly reduces the economy's capital intensity relative to consumption taxation. Proposition 1 shows that this result does not hold in our economy. The reason is that in our economy the government is allowed to borrow and lend. As a result, any difference in aggregate savings between tax regimes is absorbed by government debt and the capital stock is unaffected by the choice between consumption and wage taxation.

Without any loss of generality, we set $\tau_{t,j}^c$ equal to zero for all t and j throughout the rest of the paper.

Let $p_{t,j}$ denote the Lagrange multiplier associated with the budget constraint (6) faced by an age-j individual born in period t. The necessary and sufficient conditions for a solution to the consumers' problem are given by (6) and

$$\beta^{j-j_0(t)}U_{c_{t,j}} - p_{t,j} = 0, \tag{7}$$

$$\beta^{j-j_0(t)}U_{l_{t,j}} + p_{t,j}w_{t,j}z_j \leq 0, \quad \text{with equality if } l_{t,j} > 0, \tag{8}$$

$$-p_{t,j} + p_{t,j+1}(1 + r_{t,j+1}) = 0, \tag{9}$$

$$a_{t,J+1} = 0, \tag{10}$$

$j = j_0(t), \ldots, J$, where $U_{c_{t,j}}$ and $U_{l_{t,j}}$ denote the derivative of U with respect to $c_{t,j}$ and $l_{t,j}$ respectively.[1]

Using the consumers' optimality conditions, we can construct a sequence of implementability constraints which will allow us to define a Ramsey problem in which the government chooses allocations rather than after-tax prices. The time-t implementability constraint is obtained by multiplying the budget constraints (6) by $p_{t,j}$, summing over

---

[1]The Inada conditions guarantee that consumption and leisure will be strictly positive in each period.

$j \in \{j_0(t), \dots, J\}$, and using (7) – (9) to substitute out prices.[2] The implementability constraint associated with the cohort born in period t is

$$\sum_{j=j_0(t)}^{J} \beta^{j-j_0(t)}(U_{c_{t,j}} c_{t,j} + U_{l_{t,j}} l_{t,j}) = U_{c_{t,j_0(t)}}(1 + r_{t,j_0(t)})a_{t,j_0(t)}. \qquad (11)$$

Proposition 2 *An allocation $\left\{\{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}, k_{t+J+1}\right\}_{t=-J}^{\infty}$ is implementable if and only if it satisfies feasibility (3) and the implementability constraint (11).*

Proof. By construction, implementable allocations satisfy feasibility and implementability. We now show that the converse is also true.

Suppose that $\left\{\{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}, k_{t+J+1}\right\}_{t=-J}^{\infty}$ satisfies the feasibility and implementability constraints (3) and (11). Define before-tax prices as $\hat{r}_t \equiv f_k(k_t, l_t) - \delta$ and $\hat{w} \equiv f_l(k_t, l_t)$. Define the sequence of after-tax prices $\left\{\{w_{t,j}, r_{t,j+1}\}_{j=j_0(t)}^{J}\right\}_{t=-J}^{\infty}$ as follows:

$$w_{t,j} \equiv -\frac{U_{l_{t,j}}}{z_j U_{c_{t,j}}},$$

$$r_{t,j+1} \equiv \frac{U_{c_{t,j}}}{\beta U_{c_{t,j+1}}},$$

and let $p_{t,j} = \beta^j U_{c_{t,j}} > 0$ for $j = j_0(t), \dots, J$ and $t \geq -J$. Then, by construction, $\{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}$ satisfies the consumer's first order conditions (7) – (9) for all $t \geq -J$.

To show that the budget constraints (6) and the transversality condition (10) are satisfied, given $a_{t,j_0(t)}$ define recursively for $j = j_0(t), \dots, J$

$$a_{t,j+1} = w_{t,j} z_j l_{t,j} + (1 + r_{t,j})a_{t,j} - c_{t,j},$$

and note that given the definition of after-tax prices, the implementability constraint implies that $a_{t,J+1} = 0$ for all $t \geq -J$.

Finally, we need to show that the government budget constraint is satisfied. To do so, multiply the budget constraint of the age-j individual born in period $(t - j)$ by μ_j, and add the resulting equations for $j \in \{0, \dots, J\}$ to get

$$\sum_{j=0}^{J} \mu_j(c_{t-j,j} + a_{t-j,j+1}) = \sum_{j=0}^{J} \mu_j(w_{t-j,j} z_j l_{t-j,j} + (1 + r_{t-j,j})a_{t-j,j}),$$

or

$$c_t + (1+n)a_{t+1} = a_t + \sum_{j=0}^{J} \mu_j(w_{t-j,j} z_j l_{t-j,j} + r_{t-j,j} a_{t-j,j}). \qquad (12)$$

---

[2]The transversality condition (10) allows us to set $a_{t,J+1}$ equal to zero for all t.

10

Using the fact that the production function is homogeneous of degree one, we can write the feasibility constraint as

$$c_t + (1+n)k_{t+1} - (1-\delta)k_t + g_t = (\hat{r}_t + \delta)k_t + \hat{w}_t \sum_{j=0}^{J} \mu_j z_j l_{t-j,j}. \tag{13}$$

Combining equations (12) and (13), we have

$$g_t - (1+\hat{r}_t)k_t + a_t = (1+n)(a_{t+1} - k_{t+1}) + \sum_{j=0}^{J} \mu_j z_j (\hat{w}_t - w_{t-j,j})l_{t-j,j} - \sum_{j=0}^{J} \mu_j r_{t-j,j} a_{t-j,j}.$$

Adding $\hat{r}_t a_t$ on both sides of the previous expression and defining $b_t \equiv a_t - k_t$, the previous expression can be written as

$$(1+\hat{r}_t)b_t + g_t = (1+n)(b_{t+1}) + \sum_{j=0}^{J} \mu_j z_j (\hat{w}_t - w_{t-j,j})l_{t-j,j} + \sum_{j=0}^{J} \mu_j (\hat{r}_t - r_{t-j,j}) a_{t-j,j}.$$

Proposition 2 shows that a feasible allocation can be decentralized as a competitive equilibrium if and only if it satisfies the implementability constraints (11). It should be emphasized that an age-dependent tax system is essential for this proposition to hold. In particular, for any given date, an allocation that satisfies the implementability constraints does not necessarily have the marginal rate of substitution between present and future consumption constant across individuals of different ages. Consequently, an allocation can only be consistent with the consumers' first order conditions if the after-tax interest rates are age-dependent. Similarly, the after-tax wage rate should also be age-dependent. It will be shown in Section 4 that further restrictions need to be imposed on the Ramsey problem for an allocation to be implementable with age-independent taxes.

Let $\gamma^t \lambda_t$ be the Lagrange multiplier associated with generation t's implementability constraint (11) and define W_t as

$$W_t = \sum_{j=j_0(t)}^{J} \beta^{j-j_0(t)} \left[U_{t,j} + \lambda_t \left(U_{c_{t,j}} c_{t,j} + U_{l_{t,j}} l_{t,j} \right) \right] - \lambda_t U_{c_{t,j_0(t)}} (1 + r_{t,j_0(t)}) a_{t,j_0(t)}, \tag{14}$$

where $U_{t,j} = U(c_{t,j}, 1 - l_{t,j})$. Then, the Ramsey problem is

$$\max_{\left\{ \{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}, k_{t+J+1} \right\}_{t=-J}^{\infty}} \sum_{t=-J}^{\infty} \gamma^t W_t$$

subject to feasibility (3) for $t = 0, \ldots$.

11

The objective function W_t includes the implementability constraint associated with the cohort born in period t. The shadow value of relaxing this constraint, in terms of period-t marginal utility of a newborn individual, is given by the costate variable λ_t. Since government debt is unconstrained, the government budget constraint does not effectively constrain the maximization problem and has therefore been omitted from the Ramsey problem. Once a solution is found, the government budget constraint can be used to back out the level of government debt.

3.2 Characterization of Optimal Fiscal Policies

Let $\gamma^t \phi_t$ denote the Lagrange multiplier associated with the time-t feasibility constraint (3). The necessary conditions for a solution to the Ramsey problem are

$$-\gamma^t \phi_t (1+n) + \gamma^{t+1}\phi_{t+1}(1-\delta+f_{k_{t+1}}) = 0 \tag{15}$$

$$\gamma^t W_{c_{t,j}} - \gamma^{t+j}\phi_{t+j}\mu_j = 0 \tag{16}$$

$$\gamma^t W_{l_{t,j}} + \gamma^{t+j}\phi_{t+j}\mu_j z_j f_{l_{t+j}} \le 0 \quad \text{with equality if } l_{t,j} > 0 \tag{17}$$

for $t = -J, \ldots$, and $j = j_0(t), \ldots, J$, where $W_{c_{t,j}}$ and $W_{l_{t,j}}$ denote the derivative of W with respect to $c_{t,j}$ and $l_{t,j}$ respectively.

We now derive necessary conditions under which the Ramsey allocation features zero taxation of labor and capital income. This is done by comparing the optimality conditions from the consumers' problem to those of the Ramsey problem. Combining equations (16) and (17) for the non-trivial case of positive labor supply implies that

$$-\frac{W_{l_{t,j}}}{W_{c_{t,j}}} = \frac{(1+\lambda_t)U_{l_{t,j}} + \lambda_t U_{l_{t,j}}H_{t,j}^l}{(1+\lambda_t)U_{c_{t,j}} + \lambda_t U_{c_{t,j}}H_{t,j}^c} = z_j \hat{w}_{t+j}, \tag{18}$$

where

$$H_{t,j}^c = \frac{U_{c_{t,j},c_{t,j}}c_{t,j} + U_{l_{t,j},c_{t,j}}l_{t,j}}{U_{c_{t,j}}}, \tag{19}$$

$$H_{t,j}^l = \frac{U_{c_{t,j},l_{t,j}}c_{t,j} + U_{l_{t,j},l_{t,j}}l_{t,j}}{Ul_{t,j}}. \tag{20}$$

Since any optimal fiscal policy has to satisfy the consumers' optimality conditions, we can compare (18) to its analogue from the consumers optimization problem. Combining equations (7) and (8) assuming a positive labor supply implies that

$$-\frac{U_{l_{t,j}}}{U_{c_{t,j}}} = z_j w_{t,j} = z_j \hat{w}_{t+1}(1-\tau_{t,j}^w). \tag{21}$$

From equations (18) and (21), the tax rate on labor income for an age-j individual born in period t is given by

$$\tau_{t,j}^w = \frac{\lambda_t(H_{t,j}^l - H_{t,j}^c)}{1 + \lambda_t + \lambda_t H_{t,j}^c}. \tag{22}$$

12

n write the

(13)

$r_{t-j,j}a_{t-j,j}.$

he previous

$j,j)a_{t-j,j}.$

competitive
It should be
to hold. In
y constraints
and future
llocation can
nterest rates
ident. It will
isey problem

lementability

$t,j_0(t),$ (14)

Since λ_t is in general different from zero when lump-sum taxes are not available, this tax rate on labor income will be equal to zero only if $H_{t,j}^l = H_{t,j}^c$.

Similarly, consider the ratio of equation (16) at age j and $j+1$. Using (15), this ratio is given by

$$\frac{W_{c_{t,j}}}{W_{c_{t,j+1}}} = \frac{(1+\lambda_t)U_{c_{t,j}} + \lambda_t U_{c_{t,j}} H_{t,j}^c}{\beta\left[(1+\lambda_t)U_{c_{t,j+1}} + \lambda_t U_{c_{t,j+1}} H_{t,j+1}^c\right]} = 1 + \hat{r}_{t+j+1}. \tag{23}$$

Using equation (7) from the consumers' problem at age j and $j+1$ together with (9), the individual counterpart of (23) is

$$\frac{U_{c_{t,j}}}{\beta U_{c_{t,j+1}}} = 1 + r_{t,j+1}. \tag{24}$$

Dividing (23) by (24) we have

$$\frac{1+\hat{r}_{t+j+1}}{1+r_{t,j+1}} = \frac{1+\lambda_t + \lambda_t H_{t,j}^c}{1+\lambda_t + \lambda_t H_{t,j+1}^c}, \tag{25}$$

which implies that the tax rate on capital income is different from zero unless $H_{t,j}^c = H_{t,j+1}^c$.

Although we provide an example in section 3.4 of a utility function under which $H_{t,j}^c = H_{t,j+1}^c$, it is clear that neither this condition nor the equivalent condition for labor income taxes to be zero hold for general utility functions. This amounts to the following proposition.

Proposition 3 *At each date the optimal tax rate on labor income is different from zero unless $H_{t,j}^l = H_{t,j}^c$ and the optimal tax rate on capital income is different from zero unless $H_{t,j}^c = H_{t,j+1}^c$.*

Proposition 3 shows that the government will in general use non-zero taxes on both capital and labor income. In contrast to infinitely-lived agent models, capital income taxes are non-zero even if the solution to the Ramsey problem converges to a steady state. Since consumption and leisure are, in general, not constant over the lifetime of an individual, there are no reasons to expect that $H_{t,j}^c = H_{t,j+1}^c$ for any t, not even in the steady state. On the contrary, the function $H_{t,j}^c$ in infinitely-lived agent models becomes a constant in the steady state since these models abstract from life-cycle elements.

The steady state solution, if it exists, is characterized by the following equations:

$$1 - \delta + f_k = \frac{1+n}{\gamma}, \tag{26}$$

$$(1+\lambda)\beta^j U_{c_j} + \lambda\beta^j U_{c_j} H_j^c = \gamma^j \phi \mu_j, \tag{27}$$

$$(1+\lambda)\beta^j U_{l_j} + \lambda\beta^j U_{l_j} H_j^l \leq \gamma^j \phi \mu_j z_j f_l \quad \text{with equality if } l_j > 0, \tag{28}$$

13

for $j = 0, \ldots, J$. Of course, the feasibility and implementability constraints (3) and (11) should hold as well. Thus, a steady state is characterized by $(2(J+1)+3)$ equations in $(2(J+1)+3)$ unknowns $\left(\{c^j, l^j\}_{j=0}^{J}, k, \phi, \lambda\right)$. Notice that in steady state the marginal product of capital (net of depreciation) equals the effective discount rate applied to different generations $(1+n)/\gamma - 1$. This condition implies, as was originally shown by Samuelson (1968), that the capital-labor ratio coincides with that of the first best allocation, that is, it coincides with the capital-labor ratio that would be achieved if the government had access to lump-sum taxation. In other words, the steady state capital-labor ratio has the modified golden rule property.

The steady state allocation is also independent of the transition path leading to it. In particular, for each value of γ we can solve for the final steady state path independently of the transition that leads to this steady state. Associated with each value of γ, or final steady state, there is an optimal amount of public debt which can be backed out of the government budget constraint. This amount of government debt is accumulated during the transition path from the initial to the final steady state allocation. The higher γ is, the lower are the accumulated government debt and the welfare of generations alive during the transition. This property is not shared by infinitely-lived agent models, where the transition path determines the steady state allocation of the Ramsey problem. In particular, the steady state allocation depends on the exogenous bounds imposed on capital income taxes during the transition.

We can characterize further the optimal path of Ramsey taxes for two forms of utility functions. First we consider CES utility functions in which leisure enters multiplicatively. This class of preferences includes those that are necessary for an economy to have a balanced growth path. Second we study optimal taxation for CES utility functions for consumption with additively separable leisure. Both forms of utility functions have been widely used in the public finance literature.

3.3 CES Preferences

Consider utility functions of the form

$$U(c, 1-l) = u(c)v(l), \tag{29}$$

where $u(\cdot)$ is homogeneous of degree $(1 - \sigma_c)$ and $U(\cdot, \cdot)$ satisfies the Inada conditions. For this class of utility functions, the following Corollary to Proposition 3 identifies restrictions on parameters under which capital income taxes are zero in the long run.

Corollary 1 *Let the utility function be of the form given by (29) and suppose that the Ramsey allocations converge to a steady state. In that steady state, the tax rate on capital income is zero if and only if $\gamma = \beta(1+n)$ and $z_j = z$, $j = 0, \ldots, J$.*

14

Proof. From Proposition 3, a necessary condition for zero taxes on capital income is $H_j^c = H_{j+1}^c$. Under utility functions of the form given by (29),

$$H_j^c = \frac{u''(c_j)v(l_j)c_j + u'(c_j)v'(l_j)l_j}{u'(c_j)v(l_j)}$$
$$= \frac{u''(c_j)}{u'(c_j)}c_j + \frac{v'(l_j)}{vl_j}l_j.$$

Since u is homogeneous of degree $(1 - \sigma_c)$, u' is homogeneous of degree $-\sigma_c$ and $u''(c)c = -\sigma_c u'(c)$. It follows that the first term in the above expression is a constant—the negative of the coefficient of relative risk aversion $(-\sigma_c)$. Consequently, $H_j^c = H_{j+1}^c$ if and only if

$$\frac{v'(l_{j+1})}{vl_{j+1}}l_{j+1} = \frac{v'(l_j)}{vl_j}l_j,$$

which for utility functions that satisfy the Inada conditions only holds if $l_{j+1} = l_j$. But if $l_j = l$ and capital is not being taxed, then (7) at j and $j + 1$ implies that

$$\frac{u'(c_j)v(l_j)}{\beta u'(cj+1)v(l_{j+1})} = \frac{u'(c_j)}{\beta u'(cj+1)} = (1+r) = (1+\hat{r}) \tag{30}$$

which in turn implies that $c_j = c$, $j = 0, \dots, J$. Now for (30) to hold, we need $\beta(1+\hat{r}) = 1$. From (15), this will only be the case if $\gamma = \beta(1 + n)$. Finally, (22) implies that labor income taxes will also be age-independent. But for (21) to hold under age-independent taxes, $z_j = z$, $j = 0, \dots, J$, needs to be imposed.

The converse follows immediately. ∎

This proposition highlights that with CES preferences, capital income taxes are zero in the long run only under very restrictive assumptions on parameters. For utility functions analyzed in the next subsection, zero taxation on capital income holds much more generally.

3.4 Additively Separable Preferences

Consider utility functions of the form

$$U(c, 1-l) = u(c) + v(l), \tag{31}$$

where $u(\cdot)$ is homogeneous of degree $(1 - \sigma_c)$ and $U(\cdot, \cdot)$ satisfies the Inada conditions. Our second Corollary to Proposition 3 shows that for this class of utility functions, capital income taxes will not be used during the transition nor in the long run.

Corollary 2 *For utility function of the form given above (31), the Ramsey problem prescribes zero taxes on capital income from time period 1 and thereafter.*

15

Proof. Again, this follows directly from Proposition 3. Under utility functions of the form (31),

$$H_{t,j}^c = \frac{u''(c_{t,j})}{u'(c_{t,j})} c_{t,j},$$

which corresponds to the negative of the coefficient of relative risk aversion $(-\sigma_c)$. It follows that $H_{t,j}^c = H^c$ for all t and j, and (25) implies that $\tau_{t,j}^a = 0$.[3] ∎

Under this class of utility functions, we show that the long run labor income tax profile over an individual's life depends directly on the income elasticity of labor supplied at different ages.

Proposition 4 *Let the utility function be of the form given by (31) and suppose that the Ramsey allocations converge to a steady state. In that steady state, the relative tax rates on labor income at different ages are inversely related to the relative income elasticities of labor supplied at those ages.*

Proof. Combining equations (16) at age j and (17) at age $j+1$ for the non-trivial case of positive labor supply together with their counterparts from the consumers' problem (7) at age j, (8) at age $j+1$ and (9), the tax rate on labor income at age $j+1$ is

$$\tau_{j+1}^w = \frac{\lambda(H_{j+1}^l - H_j^c)}{1 + \lambda + \lambda H_j^c}. \tag{32}$$

Using (22), it follows that

$$\frac{\tau_{j+1}^w/(1 - \tau_{j+1}^w)}{\tau_j^w/(1 - \tau_j^w)} = \frac{H_{j+1}^l - H_j^c}{H_j^l - H_j^c}. \tag{33}$$

Now let m denote non-factor income, and let $l_j(w, r, m)$ denote the supply of labor at age j, where $w \equiv \{w_j\}_{j=0}^J$ and $r \equiv \{r_j\}_{j=0}^J$. The first order condition for labor (8) assuming a positive supply at age j can then be written

$$-\beta^j v'(l_j(w, r, m)) = p_j(w, r, m) w_j z_j. \tag{34}$$

Differentiating (34) with respect to m and rearranging implies that

$$\frac{v''(l_j)}{v'l_j} \frac{\partial l_j}{\partial m} = \frac{\partial p_j}{\partial m} \frac{1}{p_j} \tag{35}$$

---

[3]Note that this result only holds from date 1 on. At date 0, the first order condition (16) is different from all other dates since $W_{c_{t,j_0(t)}}$ for $j(0) \neq 0$ includes extra terms to reflect the right hand side of the implementability constraint (11).

16

Since under separable preferences

$$H_j^l = \frac{v''(l_j)l_{t,j}}{v(l_j)},$$

(35) implies that

$$\frac{H_{j+1}^l}{H_j^l} = \frac{\eta_{l_j}}{\eta_{l_{j+1}}},$$

where η_j is the income elasticity of l_j. It follows from (33) that the income from relatively inelastically supplied labor is to be taxed proportionally more than the income from relatively more elastically supplied labor. ∎

Proposition 4 can be viewed as an application to a life-cycle framework of the public finance principle that necessities should be taxed more than luxuries (Átkinson and Stiglitz (1980) and Kehoe (1991)). In other words, labor income should be taxed relatively more heavily when it is relatively more income-inelastic. It can also be shown that the income elasticity of the labor supply depends on the labor productivity profile and the discount rate of individuals relative to the intergenerational rate of time preferences (which determines the steady state interest rate). To see this, notice that when $\gamma = \beta(1 + n)$ and $z_j = z$ for all j, the steady state consumption and labor supply are constant throughout individuals' lives. It then follows from (32) that labor taxes are age-independent. In other words, these restrictions imply that the income elasticity of the labor supply is constant across ages. Any deviation from these restrictions will change the relative income elasticity of labor supplied at different ages and age-dependent taxes will be used.

4   Age-Independent Taxes

In this section we study Ramsey taxation when taxes on labor and capital income are not allowed to depend on the age of the individual supplying the factor of production. We begin by characterizing the set of implementable allocations under an age-independent tax system. Next, we show that if the solution to the Ramsey problem converges to a steady state allocation, then the steady state allocation satisfies two properties that were shown to hold under an age-dependent tax system. Finally, we show that the necessary conditions for zero taxation of capital income in the steady state of an age-independent tax system are much more stringent than under an age-dependent tax system.

We show that the set of allocations that the government can implement with age-independent taxes is a proper subset of the set of implementable allocations under an age-dependent tax system. Intuitively, with age-independent taxes the government has very few instruments to make a given allocation satisfy the consumer's first order conditions.

17

The consumer's first order conditions $(7) - (9)$ imply that $\tau^w_{t-j,j} = \tau^w_t$ if and only if

$$\frac{U_{l_{t-j,j}}}{U_{c_{t-j,j}} z_j} \leq \frac{U_{l_{t,0}}}{U_{c_{t,0}} z_0}, \quad \text{with equality if } l_{t-j,j} > 0, \quad j = 1, \dots, J,$$

and that $\tau^a_{t-j,j} = \tau^a_t$ if and only if

$$\frac{U_{c_{t-j,j+1}}}{U_{c_{t-j,j}}} = \frac{U_{c_{t,1}}}{U_{c_{t,0}}}, \quad j = 1, \dots, J-1.$$

Thus, at each point in time an allocation has to satisfy two extra conditions in order to be implementable with an age-independent tax system. First, the marginal rate of substitution between consumption and leisure needs to be constant across individuals of different ages. Second, the marginal rate of substitution between present and future consumption also needs to be constant across individuals of different ages.

The above necessary conditions for age-independent taxes can be expressed in the following compact way

$$R^1_{t-j,j} \leq 0, \quad \text{with equality if } l_{t,j} > 0 \quad j = 1, \dots, J, \tag{36}$$
$$R^2_{t-j,j} = 0, \quad\quad\quad\quad\quad\quad j = 1, \dots, J-1, \tag{37}$$

where for all $t \geq 0$

$$R^1_{t-j,j} \equiv \log U_{c_{t,0}} z_0 - \log U_{l_{t,0}} + \log U_{l_{t-j,j}} - \log U_{c_{t-j,j}} z_j, \quad j = 1, \dots, J,$$
$$R^2_{t-j,j} \equiv \log U_{c_{t,1}} - \log U_{c_{t,0}} + \log U_{c_{t-j,j}} - \log U_{c_{t-j,j+1}}, \quad j = 1, \dots, J-1.$$

Proposition 5 *An allocation* $\left\{ \{c_{t,j}, l_{t,j}\}^J_{j=j_0(t)}, k_{t+J+1} \right\}^\infty_{t=-J}$ *is implementable with age-independent taxes if and only if it satisfies feasibility (3) and the implementability constraints (11) and (36) – (37).*

The Ramsey problem then becomes

$$\max \sum_{t=-J}^{\infty} \gamma^t W_t,$$

subject to

$$\sum_{j=0}^{J} \mu_j c_{t-j,j} + (1+n) k_{t+1} - (1-\delta) k_t + g_t - f\left(k_t, \sum_{j=0}^{J} \mu_j z_j l_{t-j,j}\right) = 0, \quad t = 0, \dots.$$

$$R^1_{t-j,j}(c_{t,0}, l_{t,0}, c_{t-j,j}, l_{t-j,j}) = 0, \quad j = 1, \dots, J, \quad t = 0, \dots$$
$$R^2_{t-j,j}(c_{t,0}, c_{t,1}, c_{t-j,j}, c_{t-j,j+1}) = 0, \quad j = 1, \dots, J-1, \quad t = 0, \dots$$

18

where W_t is defined as in (14)

In Appendix A we show that if the solution to the Ramsey problem converges to a steady state, then the steady state allocation satisfies the following two properties: first, the steady state capital-labor ratio has the modified golden rule property, that is, the steady state marginal product of capital (net of depreciation) is equal to the effective discount rate applied to different generations $((1 + n)/\gamma - 1)$. Second, the steady state allocation is independent of the transition path leading to it.

The first order conditions of the Ramsey problem also imply that the conditions for zero capital income taxation are much more stringent with age-independent taxes. In fact, when the utility function is of the form (31) (additively separable in consumption and leisure) capital income taxes are zero in the steady state only if $\gamma = \beta(1 + n)$ and $z_j = z$ for all j. Furthermore, in contrast to an optimal age-dependent tax system, capital income taxes will be different from zero throughout the transition path to the final steady state allocation.

This result illustrates how the optimal tax rates crucially depend on the set of fiscal instruments available to the government. When either $\gamma \neq \beta(1 + n)$ or $z_j \neq z$ for some j, the income elasticity of labor supply is not constant across ages. In this case the government would like to tax labor income at different rates over the lifetime of an individual. When labor income taxes are not allowed to depend on age, the government can affect the way individuals substitute labor intertemporally by resorting to non-zero capital income taxes and use this margin to tax labor income more heavily at ages where it is relatively income inelastic.

5 Numerical Examples

In this section we present some numerical examples in order to gain further insights about the properties of Ramsey taxation. The model economy is parameterized so that its initial steady state mimics important features of the U.S. economy for some initial tax rates on capital and labor income. At date zero taxes are set optimally and we compute the associated steady state Ramsey equilibria under both age-dependent and age-independent tax systems.[4]

The utility function used in these examples is additive separable in consumption and leisure

$$u(c, l) = \frac{c^{1 - \sigma_c}}{1 - \sigma_c} + \frac{(1 - l)^{1 - \sigma_l}}{1 - \sigma_l}.$$

The production function is assumed to be of the Cobb-Douglas form, with the capital share represented by α. The parameter values assumed for this exercise are shown in Table 1, and we use Hansen's profile as the labor productivity profile.

---

[4]In our experiments we use the fact that the final steady state is independent of the transition path. Notice that computing the steady state path involves solving a highly non-linear system of equations.

Table 1: Parameter Values

β	σ_c	σ_l	α	δ	n	g
0.985	2.0	5.0	0.36	0.08	0.015	0.22*

*As a percentage of GDP in the initial steady state

Table 2 presents the main results for three values of the intergenerational discount factor. We selected $\gamma = .966$ as our benchmark discount factor. For this value of γ, the steady state allocations of the initial equilibrium and the age-dependent Ramsey equilibrium share the same consumption growth rate along individuals' life. The other two values for γ were chosen slightly below ($\gamma = .96$) and slightly above ($\gamma = .970$) the benchmark level.

Surprisingly, Table 2 shows that, given a value of γ, the utility achieved by a newborn individual and the economy's aggregate variables do not vary significantly across the steady states of the age-dependent and the age-independent Ramsey equilibria. Indeed, the maximum difference in steady state output between the two types of equilibria, across the three values of γ reported in Table 2, does not exceed .6 percent. Similarly, for all values of γ reported, the differences in utility for newborn agents do not represent more than .01 percent of consumption of the initial steady state (Ccomp). Notice, however, that tax rates do vary across the two tax systems. When taxes are age-dependent, capital income taxes are zero (see Corollary 2) and labor income taxes vary significantly along the life of individuals (see Figure 1). These tax rates tend to follow, with a lag, the shape of the age-profile of labor productivity. Proposition 4 is useful in understanding this finding. This Proposition shows that it is optimal to tax labor income at relatively high rates at ages where it is relatively income inelastic. It is easy to show that, *ceteris paribus*, the income elasticity of labor supplied at age j is inversely related to the age-j labor efficiency level. Since the after-tax interest rates in all the equilibria computed are higher than individuals' rate of time preference, it is also the case that, *ceteris paribus*, labor is more income inelastic the lower the age of an individual. Contrary to the age-dependent Ramsey equilibrium, the tax rates on capital income are positive—albeit small, 2 to 3 percent—when taxes are not allowed to depend on age (see section 4). In addition, it is interesting to note that the difference between the average tax rate on labor income with age-dependent taxes and the tax rate with age-independent taxes does not exceed .5 percent for any of the three values of γ reported.

The sensitivity of the Ramsey equilibrium to the value of the intergenerational discount factor γ is striking. Table 2 shows that with $\gamma = .96$ a newborn individual living in the initial steady state can forego about 2 percent of his lifetime consumption and yet achieve the steady state utility level of the Ramsey equilibrium. When $\gamma = .97$, newborn individuals will only be indifferent between the two allocations if their consumption of goods in the initial steady state is supplemented by about 7 percent. The enormous impact of γ on steady state utility is easily understood when we observe the negative association between the steady state government debt and the intergenerational discount factor. When

20

$\gamma = .97$ the government owns more than 30 percent of the capital stock. This implies that generations alive when the economy reaches the steady state face lower tax rates on their income. This comes at the expense, of course, of the individuals alive during the transition to the Ramsey steady state. These generations are the ones financing the purchases of capital by the government.

6   Conclusion

This paper studies optimal taxation in an overlapping generations economy. It characterizes the optimal path of fiscal policy, both in the long run and in the transition to the steady state. The implications of this study in terms of capital income taxation are in sharp contrast with the Chamley-Judd results. In particular, it is shown that the optimal rate of capital income taxation will in general be different from zero, even in the long run.

An important drawback in our study, which permeates most of the literature on optimal taxation, is that the fiscal policies we consider are not time consistent (Kydland and Prescott (1977)). Although this problem is not as acute in overlapping generations economies as is it in infinitely-lived agent models, a more satisfactory characterization of optimal taxation would focus on time consistent policies.

An interesting avenue for future research is to introduce human capital accumulation in our environment. We conjecture that the main properties of optimal taxation derived in this paper would remain unaltered by the presence of human capital accumulation.

mplies that
:es on their
e transition
urchases of

laracterizes
the steady
:e in sharp
mal rate of
:un.

:ure on op-
: (Kydland
generations
erization of

cumulation
1 derived in
ion.

A Age-Independent Taxes

Under age-independent taxes, the Ramsey problem is given by:

$$\max \sum_{t=-J}^{\infty} \gamma^t W_t,$$

where W_t is defined in (14), subject to feasibility (3), and the implementability constraints (36) and (37), to which we associate the Lagrange multipliers $\gamma^t \phi_t$, $\gamma^t \epsilon^1_{t-j,j}$, and $\gamma^t \epsilon^2_{t-j,j}$, respectively.

Since we only study this problem in steady state, we only provide the first order conditions of this problem for $t \geq J$. These first order conditions are as follows:

$$\gamma^t \phi_t (1+n) - \gamma^{t+1} \phi_{t+1} (1 - \delta + f_{k_{t+1}}) = 0. \qquad\qquad\qquad \text{`}k_{t+1}\text{'}$$

$$\gamma^t W_{c_{t,0}} - \gamma^t \phi_t \mu_0 - \sum_{j=1}^{J} \gamma^t \epsilon^1_{t-j,j} \partial_{c_{t,0}} R^1_{t-j,j} - \sum_{j=1}^{J-1} \gamma^t \epsilon^2_{t-j,j} \partial_{c_{t,0}} R^2_{t-j,j} = 0 \qquad \text{`}c_{t,0}\text{'}$$

$$\gamma^{t-1} W_{c_{t-1,1}} - \gamma^t \phi_t \mu_1 - \gamma^t \epsilon^1_{t-1,1} \partial_{c_{t-1,1}} R^1_{t-1,1} - \gamma^t \epsilon^2_{t-1,1} \partial_{c_{t-1,1}} R^2_{t-1,1}$$
$$- \sum_{j=1}^{J-1} \gamma^{t-1} \epsilon^2_{t-1-j,j} \partial_{c_{t-1,1}} R^2_{t-1-j,j} = 0 \qquad\qquad \text{`}c_{t-1,1}\text{'}$$

$$\gamma^{t-j} W_{c_{t-j,j}} - \gamma^t \phi_t \mu_j - \gamma^t \epsilon^1_{t-j,j} \partial_{c_{t-j,j}} R^1_{t-j,j} - \gamma^t \epsilon^2_{t-j,j} \partial_{c_{t-j,j}} R^2_{t-j,j} \qquad\qquad \text{`}c_{t-j,j}\text{'}$$
$$- \gamma^{t-1} \epsilon^2_{t-j,j-1} \partial_{c_{t-j,j}} R^2_{t-j,j-1} = 0 \qquad j = 2, \ldots, J-1$$

$$\gamma^{t-J} W_{c_{t-J,J}} - \gamma^t \phi_t \mu_J - \gamma^t \epsilon^1_{t-J,J} \partial_{c_{t-J,J}} R^1_{t-J,J}$$
$$- \gamma^{t-1} \epsilon^2_{t-J,J-1} \partial_{c_{t-J,J}} R^2_{t-J,J-1} = 0 \qquad\qquad \text{`}c_{t-J,J}\text{'}$$

$$\gamma^t W_{l_{t,0}} - \gamma^t \phi_t \mu_0 z_0 \hat{w}_t - \sum_{j=1}^{J} \gamma^t \epsilon^1_{t-j,j} \partial_{l_{t,0}} R^1_{t-j,j} = 0 \qquad\qquad \text{`}l_{t,0}\text{'}$$

$$\gamma^{t-j} W_{l_{t-j,j}} - \gamma^t \phi_t \mu_j z_j \hat{w}_t - \gamma^t \epsilon^1_{t-j,j} \partial_{l_{t-j,j}} R^1_{t-j,j} = 0. \qquad \text{`}l_{t-j,j}\text{'}, j = 1, \ldots, J$$

The steady state Ramsey path is thus characterized by a system of $(4J+4)$ equations—the above $(2J+3)$ first order conditions, R^1_j for $j = 1, \ldots, J$, R^2_j for $j = 1, \ldots, J-1$, feasibility and implementability—in $(4J+4)$ unknowns $\left(\{c^j, l^j\}_{j=0}^{J}, \{\epsilon^1_j\}_{j=1}^{J}, \{\epsilon^2_j\}_{j=1}^{J-1}, k, \phi, \lambda \right)$. Some properties of the steady state are immediate. First, the steady state is independent of the transition path. In particular, government debt is not part of the above system of equations and it can be obtained from the government budget constraint once the steady state solution is found. Second, the steady state capital-labor ratio has the modified golden rule property.

We conclude the Appendix by showing that the conditions for zero taxation of capital income under age-independent taxes are very stringent. We establish that with separable preferences the steady state capital income tax is zero only if $\gamma = \beta(1+n)$ and $z_j = z$ for all j. Assuming separable preferences, the steady state first order conditions become

$$(1+\hat{r}) = \frac{(1+n)}{\gamma} \tag{`k'}$$

$$W_{c_0} = \phi\mu_0 + \frac{u''(c_0)}{u'(c_0)}\left(\sum_{j=1}^{J}\epsilon_j^1 - \sum_{j=1}^{J-1}\epsilon_j^2\right) \tag{`c_0'}$$

$$W_{c_1} = \gamma\phi\mu_1 + \frac{u''(c_1)}{u'(c_1)}\left(-\gamma\epsilon_1^1 + 2(1+\gamma)\epsilon_1^2 + \sum_{j=2}^{J-1}\epsilon_j^2\right) \tag{`c_1'}$$

$$W_{c_j} = \gamma^j\phi\mu_j + \gamma^{j-1}\frac{u''(c_j)}{u'(c_j)}\left(-\gamma\epsilon_j^1 + \gamma\epsilon_j^2 - \epsilon_{j-1}^2\right) \tag{`c_j', $j=2,\ldots,J-1$}$$

$$W_{c_J} = \gamma^J\phi\mu_J - \gamma^{J-1}\frac{u''(c_J)}{u'(c_J)}\left(\gamma\epsilon_J^1 + \epsilon_{J-1}^2\right) \tag{`c_J'}$$

$$W_{l_0} = -\phi\mu_0 z_0\hat{w} - \frac{v''(l_0)}{v'(l_0)}\sum_{j=1}^{J}\epsilon_j^1 \tag{`l_0'}$$

$$W_{l_j} = -\gamma^j\phi\mu_j z_j\hat{w} + \gamma^j\frac{v''(l_j)}{v'(l_j)}\epsilon_j^1. \tag{`l_j', $j=1,\ldots,J$}$$

With $\gamma = \beta(1+n)$ and $z_j = z$ for all j, the steady state path satisfies $\epsilon_j^1 = \epsilon_j^2 = 0$, $\frac{W_{c_j}}{\beta W_{c_{j+1}}} = \frac{U_{c_j}}{\beta U_{c_{j+1}}} = 1+\hat{r} = 1/\beta$, and $\frac{W_{l_j}}{W_{c_j}} = \hat{w}$ for all j. It follows that steady state consumption and leisure are constant over individuals' lifetime. Essentially, under the specified conditions, consumption and leisure are not affected by life-cycle elements. Consequently, it is optimal to tax consumption goods uniformly over the lifetime of individuals and capital income taxes should be zero. For the same reason, labor income tax rates would be age-independent even if they were allowed to vary with age. The values of the multipliers associated with the implementability constraints (36) and (37) are thus zero in the steady state.

References

[1] Auerbach, Alan J. and Kotlikoff, Laurence J. *Dynamic Fiscal Policy*. Cambridge: Cambridge University Press, 1987.

[2] Atkinson, Anthony B. and Sandmo, Agnar. "Welfare Implications of the Taxation of Savings." *The Economic Journal*, Vol. 90, September 1980.

[3] Atkinson, Anthony B. and Stiglitz, Joseph E. *Lectures on Puplic Economics*. New York: McGraw-Hill, 1980.

[4] Chamley, Christophe. "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives." *Econometrica*, Vol. 54 (3), May 1986.

[5] Chari, V.V., Christiano, Lawrence J. and Kehoe, Patrick J. "Optimal Fiscal Policy in a Business Cycle Model." *Journal of Political Economy*, Vol. 102 (4), August 1994.

[6] Escolano, Julio. "Optimal Fiscal Policy in Overlapping Generations Models." Manuscript, The University of Minnesota, 1991.

[7] Jones, Larry E., Manuelli, Rodolfo E. and Rossi, Peter E. "Optimal Taxation in Models of Endogenous Growth." *Journal of Political Economy*, Vol. 101 (3), June 1997.

[8] Jones, Larry E., Manuelli, Rodolfo E. and Rossi, Peter E. "On the Optimal Taxation of Capital Income." *Journal of Economic Theory*, Vol. 73, 1997.

[9] Judd, Kenneth L. "Redistributive Taxation in a Simple Perfect Foresight Model." *Journal of Public Economics*, Vol. 28, 1985.

[10] Kehoe, Patrick J. "Notes on the Primal Approach to Optimal Taxation." Federal Reserve Bank of Minneapolis, Working Paper 479, August 1991.

[11] Kydland, Finn E. and Prescott, Edward C. "Rules Rather than Discression: The Inconsistency of Optimal Plans." *Journal of Political Economy*, Vol. 85, 1977.

[12] Lucas, Robert E. Jr. "Supply-Side Economics: An Analytical Review." *Oxford Economic Papers*, Vol. 42 (2), April 1990.

[13] Park, No-Ho. "Steady-State Solutions of Optimal Tax Mixes in an Overlapping-Generations model." *Jornal of Public Economics*, Vol. 46, 1991.

[14] Samuelson, Paul A. "The Two-Part Golden Rule Deduced as the Asymptotic Turnpike of Catenary Motions." *Western Economic Journal*, Vol. 6, 1968.

[15] Stiglitz, Joseph E. "Pareto Efficient and Optimal Taxation and the New New Welfare Economics," in A.J. Auerbach and M. Feldstein, eds., *Handbook of Public Economics, Vol. 2*. Amsterdam: North-Holland, 1987.

[16] Summers, Lawrence, H. "Capital Taxation and Accumulation in a Life Cycle Growth Model." *American Economic Review*, Vol. 71 (4), September 1981.

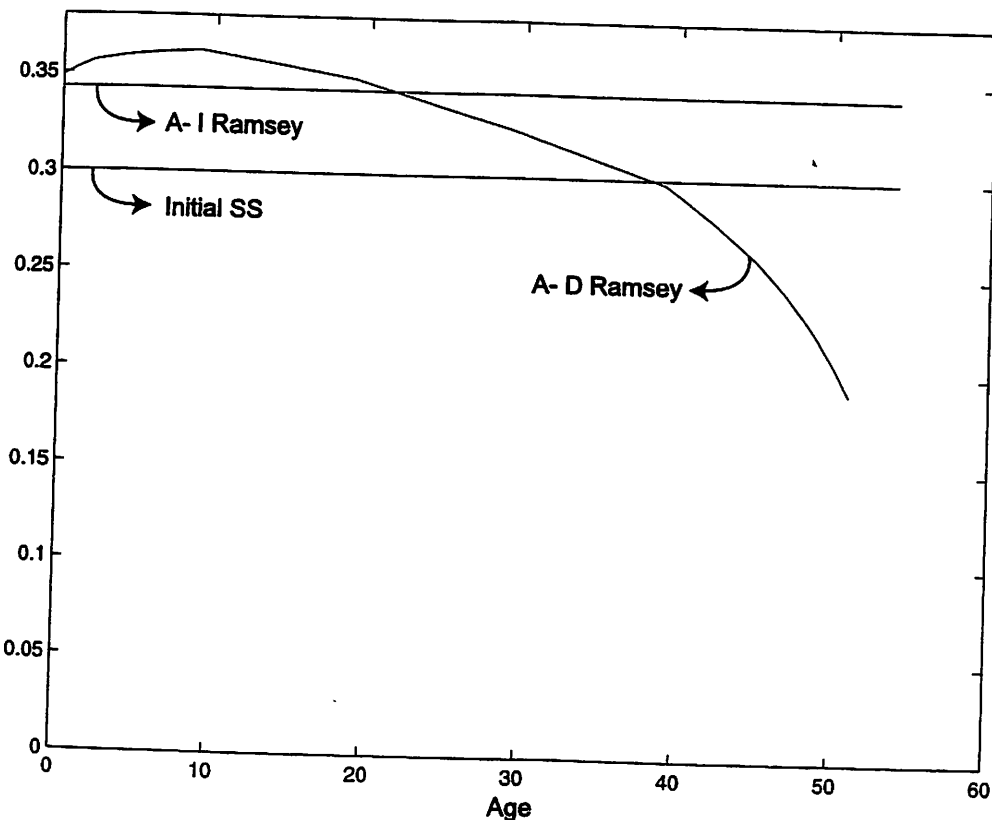## Labor Income Tax Profiles with $\gamma = (1+n)/(1+r) = 0.966$

Figure 1: Labor income tax over the life-time of individuals

Table 2: Results for some values of γ

		Initial SS	Ramsey (A-D)	Ramsey (A-I)
$\gamma = 0.960$	y	1.000	1.022	1.017
	c	0.542	0.548	0.544
	l	0.353	0.352	0.350
	k	2.508	2.679	2.665
	b	0.123	0.032	0.189
	$\tau^k(\%)$	20.000	0.000	2.719
	$\tau^w(\%)$	30.000	33.770*	34.279
	Utility	−179.824	−182.513	−182.383
	CComp (%)	0.000	−2.090	−1.991
$\gamma = 0.966$	y	1.000	1.052	1.049
	c	0.542	0.558	0.555
	l	0.353	0.353	0.351
	k	2.508	2.896	2.886
	b	0.123	−0.361	−0.238
	$\tau^k(\%)$	20.000	0.000	2.149
	$\tau^w(\%)$	30.000	30.674*	31.010
	Utility	−179.824	−175.669	−175.624
	CComp (%)	0.000	3.410	3.448
$\gamma = 0.970$	y	1.000	1.075	1.072
	c	0.542	0.565	0.562
	l	0.353	0.353	0.352
	k	2.508	3.062	3.053
	b	0.123	−0.716	−0.593
	$\tau^k(\%)$	20.000	0.000	2.290
	$\tau^w(\%)$	30.000	28.633*	28.897
	Utility	−179.824	−171.498	−171.437
	CComp (%)	0.000	7.076	7.132

*Total labor taxes collected divided by total before-tax labor income