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RELATIVE PRICES, THE TRADE BALANCE,
AND THE BALANCE OF PAYMENTS

by

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Abstract

A small scale general equilibrium model is constructed to explain
the joint behavior of the trade balance, balance of payments, relative
price of nontraded goods, and the real exchange rate. The model can be
used to obtain a set of predictions about the response of these four
variables to various exogenous disturbances, such as movements in the
terms of trade or changes in government expenditure. Since different
exogenous shocks imply different patterns of comovement between the
trade balance, balance of payments, relative price of nontraded goods,
and the real exchange rate, the general relationship between movements
in these variables is theoretically ambiguous.

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1. INTRODUCTION

Several recent papers have used small scale choice-theoretic intertemporal general equilibrium models to address problems of interest in international finance. For instance, Stockman (1980, 1982) uses these models to examine the relationship in a two country world between the nominal exchange rate, the real exchange rate, the terms of trade, and various other economic variables. He has also investigated the question of Ricardian equivalence in an open economy and the implications of this for the sterilization of reserves flows. Greenwood (1982) and Sachs (1982) use these models to analyze the joint determination of the current account and the exchange rate in a small open economy. They obtain a set of predictions about the pattern of comovement between the exchange rate, the current account, and certain key economic variables such as real income, government spending, and the money supply. Finally, Helpman (1981) uses these models to look at the choice of exchange rate regimes.

A fundamental property of these neoclassical models is that agents' decision making is undertaken in a rational manner, based upon forward-looking expectations about real income, real rates of return, government spending, and money supplies. By adopting a choice-theoretic approach, mis-specifications about agents' decision rules are avoided. In some cases, this has led to considerable doubt being cast upon some orthodox views in international finance. Stockman, for example, arrives at the conclusion that the scope for
sterilizing fluctuations in the balance of payments is severely limited -- perhaps even nonexistent. Helpman challenges the traditional basis for believing that there is a fundamental difference between fixed and flexible exchange rate regimes. In other cases, these models explain so-called empirical anomalies. Stockman provides an equilibrium explanation of the long-observed correlation between the nominal and real exchange rates -- a fact which many claimed conclusively proved price rigidity in the goods market. Greenwood's and Sach's papers provide a theoretical explanation of the lack of an observed correlation between the current account and the exchange rate across countries and time periods.

Finally, while the above papers address different issues, they all use a fairly standard microeconomic-oriented general equilibrium model as a basis for analysis. The common framework of analysis adopted by these papers makes it relatively easy to compare and contrast the conclusions at which they arrive. This has not always been the case in international finance. Many of the traditional models employed disparate and ad hoc specifications of agents' decision rules which made it hard to compare and contrast the results obtained by these models in a systematic manner.

This paper undertakes an examination of the codetermination of the current account, the balance of payments, and the relative price of nontraded goods in a small open economy with a fixed exchange rate. The paper may be regarded as an extension of Greenwood (1982) to include nontraded goods and to allow for a discussion of the
balance of payments rather than the exchange rate. A set of predictions is obtained about the responsiveness of these variables to shocks in such things as the terms of trade and government spending. It turns out that the correlation between the current account and either the balance of payments or the relative price of nontraded goods is ambiguous. For example, an anticipated improvement in the future terms of trade leads to a worsening of the current account, an improvement in the balance of payments, and a rise in the relative price of nontraded goods. A temporary increase in government spending on imports also causes a deterioration in the current account, but now leads to a worsening in the balance of payments and a fall in the relative price of nontraded goods.

2. THE MODEL'S SETTING

Imagine a small open economy, with a lifespan of two periods, that has adopted a system of fixed exchange rates. This economy is inhabited by a representative agent who is blessed with perfect foresight, and who desires to maximize his lifetime utility, \( U(\cdot) \), as given by

\[
U(\cdot) = \sum_{t=1}^{2} \beta^{t-1} U(N_t, Z_t)
\]

where \( \beta \) is his subjective rate of time preference, and \( N_t \) and \( Z_t \) are his consumption of a nontraded and an imported good, respectively, in period \( t \). The momentary utility function, \( U(\cdot) \), is assumed to be strictly quasi-concave and twice differentiable with both non-tradeables and imports being normal goods.
In each period $t$ the representative agent is endowed with a certain quantity of the nontraded good, $N^t$, and an exported good, $X^t$. Nontraded goods sell within the domestic economy during period $t$ at a relative price in terms of imports of $p_N^t$. Also, during this period the export good may be freely sold to the rest of the world at the world terms of trade, $p_X^t$, by which is meant the relative price of exports in terms of imports. Thus, when measured in terms of the imported good, the individual's real income, $y^t$, would be equal to $p_N^{t-1} + p_X^{t-1}$.

Domestic residents can also freely participate on an international bond market. In the first period the representative agent can purchase or sell real bonds which are denominated in terms of the imported good and which pay the fixed internationally determined real rate of return, $r^*$. For instance, if during the first period the agent purchased one unit of real bonds, he would receive the equivalent of $1+r^*$ units of the imported good during period two.

Now, the individual may choose to hold domestic currency so as to economize on his transactions costs of exchange. Specifically, in each period the fraction $v$ of the agent's real income, $y$, is absorbed in transactions costs. This fraction $v$ is assumed to be a decreasing convex function of the ratio of the agent's nominal money balances, $M$, to his nominal income, $Py$, where $P$ is the nominal price of the imported good. Thus, for a given level of nominal income, there are diminishing returns to holding money. By increasing his holdings
of money, the individual can economize on the proportion of his real income which is being absorbed in transactions costs. However, as the ratio of money, $M$, to nominal income, $Py$, rises, the reduction in the fraction, $v$, of real income brought about by holding an extra unit of money is reduced. In other words,

$$v_t = v(M^t/P^ty_t) \quad v_t = 1,2 \quad \text{with} \quad v^t < 0; \quad v^"t > 0; \quad \text{and} \quad 0 < v < 1$$

Lastly, there is a government in this "small" open economy. In each period the government undertakes a certain amount of unproductive expenditure. In particular, during $t$ the government purchases the quantities $g^t_N$ of the nontraded good and $g^t_Z$ of the imported good. Therefore, the total value of real government spending on goods during period $t$ when measured in terms of the imported good, or $g^t$, would be

$$g^t = \frac{p^t}{N} g^t_N + g^t_Z$$

Also, during period $t$, the government gives the individual a real transfer payment in the amount $\mu^t$. This transfer payment is unrelated to the agent's holdings of real balances.

Like any other actor in the economy, the government must satisfy a budget constraint in each period. Its budget constraints for the two periods are

(1)  
$$\frac{M^1_s}{P^1} = g^1 + \mu^1 + b^1 - \tau^1$$

and

(2)  
$$\frac{(M^2_s - M^1_s)}{P^2} = g^2 + \mu^2 - (1+r*)b^1 - \tau^2$$

where $M^1_s$ and $M^2_s$ are the quantities of domestic currency that the
government issues in periods one and two, \(\tau^1\) and \(\tau^2\) are the real amounts of taxes that it levies on domestic residents in these periods, and \(b^1\) is the quantity (which may be negative) of the import denominated real bond that the government purchases from the rest of the world. For simplicity, it will be assumed that the government uses current taxation to meet its current expenditure on goods. That is, let \(g^t = \tau^t\). The variable \(b^1\) can now be regarded as what is commonly thought of as the government's holdings of foreign reserves.

3. THE INDIVIDUAL'S MAXIMIZATION PROBLEM

The representative agent's constrained maximization problem is shown below with the agent's decision variables being \(N^1\), \(N^2\), \(Z^1\), \(Z^2\), \(M^1/P^1\), and \(M^2/P^2\).

\[
\begin{align*}
(3) \quad & \text{Max } U(N^1, Z^1) + \beta U(N^2, Z^2) \\
& \text{s.t. } \frac{1}{P^1} N^1 + \frac{1}{P^1} M^1 \leq \left(\frac{1}{1+\tau^*}\right) \left[\frac{1}{P^2} N^2 + Z^2 + \frac{M^2}{P^2} - \frac{M^1}{P^2}\right] \\
& \quad \quad \quad = (1-\nu(P^1))y^1 + \mu^1 - \tau^1 + (1-\nu(P^2)y^2 + \mu^2 - \tau^2) \\
\end{align*}
\]

The above optimization problem implies that if the individual is to hold real balances efficiently in the first period the following condition must hold:

\[-v'(M^1/P^1y^1) = (\pi+\tau^*)/(1+\tau^*) \quad \text{where } \pi = (P^2-P^1)/P^2\]

The lefthand side of the above expression represents the marginal product of a unit of real balances in the first period. The righthand side of this expression is the opportunity cost of holding a unit...
of real balances in the first period. Obviously, the efficient utilization of money requires the marginal product of the last unit of real balances held to equal its marginal cost. By inverting the above equation, a standard-looking demand for money function for the agent can be derived

\[
\frac{M^1}{P^1} = k^1((\pi^*+r^*)/(1+r^*))y^1 \quad k^1(\cdot) \equiv v^{-1}(-(\pi^*+r^*)/(1+r^*));
\]

\[k^1' < 0\]

This equation will be useful in analyzing the impact of various shocks on the balance of payments.

The agent’s optimization problem implies that his compensated demand functions for nontraded and imported goods will take the following forms

\[
N^1 = N^1(p^1_N, p^2_N/(1+r^*), 1/(1+r^*), \omega)
\]

\[
N^2 = N^2(p^1_N, p^2_N/(1+r^*), 1/(1+r^*), \omega)
\]

\[
Z^1 = Z^1(p^1_N, p^2_N/(1+r^*), 1/(1+r^*), \omega)
\]

\[
Z^2 = Z^2(p^1_N, p^2_N/(1+r^*), 1/(1+r^*), \omega)
\]

where \(\omega\) is an index of his real welfare level. The sign under an argument in one of these demand functions shows the sign \(^1\), implied by the consumer’s problem (3), of the partial derivative of that demand function with respect to the argument in question. It is needless to say that the actor’s level of real welfare, \(\omega\), is dependent on such things as his endowments of nontraded and exported
goods, the relative prices of nontraded and exported goods, the levels of real taxes and transfer payments, and the rates of return on holding money, \(-\pi\), and bonds, \(r^*\). The nature of this dependence is discussed in fuller detail later.

The structure of the agent's optimization problem implies that his demand for money decision influences his consumption of the nontraded and traded goods in each period only through an income effect brought about by a reduction in the individual's transactions costs of exchange (net of the cost of holding money) due to the fact that he holds money. Consequently, \(\pi\) does not appear directly in the above compensated demand functions. The influence of \(\pi\) on the various good's consumption is felt indirectly through \(\omega\). As the rate of return on holding money, \(-\pi\), changes, the individual adjusts his holdings of real balances. This leads to movements in (net) transactions costs which have income effects that cause \(\omega\) to shift. Appendix A contains further details.

4. THE MODEL'S GENERAL EQUILIBRIUM

In the model's general equilibrium the money market must always clear. Consequently,

\[
M^t = M^t_S \quad t = 1,2
\]

Also, the nontraded goods market must clear domestically each period, implying that

\[
N^t = (1-v(M^t/P^t y^t))N^t - g_N^t \quad t = 1,2
\]
The lefthand side of the above equation shows the demand for non-traded goods by the representative agent in period t. The righthand side shows the net supply of nontraded goods available to private citizens in that period. This is equal to the total supply of nontraded goods minus that portion of this supply which is absorbed in the transactions costs of exchange and less the amount of nontraded goods which is purchased by government.

An economy-wide budget constraint may be obtained by substituting the government's budget constraints (1) and (2) into the individual's one. This yields

\[
(8) \quad p_N^1 l^1 N^1 + l^1 + g^1 + \left( \frac{1}{1+r^*} \right) \left[ p_N^2 N^2 + Z^2 + g^2 \right] = (1-v\left( \frac{M^1}{p^1 y^1} \right)) y^1 + \left( \frac{1}{1+r^*} \right) (1-v\left( \frac{M^2}{p^2 y^2} \right)) y^2
\]

By imposing equilibrium in the nontraded goods market, the above economy-wide budget constraint can be rewritten to obtain the following relationship which states that trade must balance intertemporally

\[
(9) \quad t^1 = (1-v\left( \frac{M^1}{p^1 y^1} \right)) p_x^1 z^1 - l^1 - g^1 = -\left( \frac{1}{1+r^*} \right) [(1-v\left( \frac{M^2}{p^2 y^2} \right)) p_x^2 z^2 - l^2 - g^2]
\]

The term in brackets on the lefthand side of the above equality sign represents the current period's trade balance, \( t^1 \), while the one in brackets on the righthand side represents the second period's trade balance, \( t^2 \). As can be then seen, \( t^1 = -(1/(1+r^*))t^2 \).

Now, the law of one price states that

\[
p^t = e^{t} p_f^t \quad t = 1, 2
\]
where \( e \) is the fixed domestic currency price for a unit of foreign currency and \( P^t_F \) is the period \( t \) foreign nominal price of the imported good. The above law and the definition for \( \pi \) imply

\[
\pi = \frac{(P^2_F - P^1_F)}{P^2_F}
\]

As can be seen, because this economy is a small open one with a fixed exchange rate, \( e \), its domestic nominal price of imported goods in both periods, \( P^1 \) and \( P^2 \), and, consequently, the rate of return on holding money, \( -\pi \), are exogenous datum determined from abroad. Thus, the opportunity cost of holding \((\pi+r*)/(1+r*)\) is determined exogenously from outside the economy.

Finally, the government's budget constraint (1) implies that today's balance of payments, \( b^1 \), may be written as

\[
b^1 = \frac{M^1_s}{P^1} - \mu^1 \quad \text{(Recall that } g^t = t^t \text{ yt)}
\]

However, for this small open economy with a fixed exchange rate, the current supply of money, \( M^1_s \), is an endogenous variable determined by the demand for it, \( M^1 \). Thus, by imposing equilibrium in the money market, or equation (6), one may rewrite the above expression as

\[
(10) \quad b^1 = \frac{M^1}{P^1} - \mu^1
\]

\[
= k^1((\pi+r*)/(1+r*))y^1 - \mu^1 \quad \text{(by using (4))}
\]
5. THE IMPACT EFFECT OF SHIFTS IN THE TERMS OF TRADE

It is interesting to analyze how movements in the terms of trade impact on the relative prices of nontraded goods, the trade balance, and the balance of payments. Before proceeding it will be mentioned that sometimes it is useful to carry out the analysis using the following initial conditions as a benchmark:

\[ \beta = 1/(1+r^*) \quad p^1_x = p^2_x \]
\[ \bar{N}^1 = \bar{N}^2 \quad \bar{x}^1 = \bar{x}^2 \]
\[ g^1_N = g^2_N \quad (1-v(\cdot 1)) = (1-v(\cdot 2)) \]
\[ g^1_Z = g^2_Z \]

(11)

These initial conditions make, in the model's general equilibrium, the first and second periods identical from the representative agent's perspective.

To start with, suppose that there is an anticipated improvement in the future terms of trade. In other words, suppose that \( \hat{p}_x^2 > 0 \) while \( \hat{p}_x^1 = 0 \), where the "\(^\wedge\)" over a variable denotes that its proportionate rate of change is being discussed, so, for example, \( \hat{p}_x^2 = dp_x^2/p_x^2 \). As a consequence of this beneficial change in the future terms of trade, the representative agent immediately realizes an improvement in his real welfare, \( \omega \), of the amount

\[ dw = (1-v(\cdot 2)) \frac{p_x^2}{(1+r^*)} \hat{p}_x^2 \]

(12)

(see Appendix B for details).

Thus, the change in the individual's real welfare ensuing from a
relative change in the future terms of trade is strictly proportional to the discounted value of his net endowment of the exported good in the future period.

This improvement in the actor's real welfare will of course lead, at the original set of relative prices, to increases in his demands for nontraded and imported goods in both periods, a fact which is easily discerned by "eyeballing" the set of demand functions (5). However, the nontraded goods market must clear domestically and the supply of nontraded goods in each period is fixed. Consequently, the relative prices of nontraded goods in each period, \( p_N^1 \) and \( p_N^2 \), must adjust so as to maintain equilibrium in the nontraded goods market in face of the gain in the agent's real welfare. The proportionate changes in the relative prices of nontraded goods, \( \hat{p}_N^1 \) and \( \hat{p}_N^2 \), can be uncovered by subjecting the system of equations (7), describing equilibrium in the nontraded goods market, to the usual comparative statics exercise. The results of this exercise are

\[
(13) \quad \hat{p}_N^1 = \frac{\left[ n_{2}^N \eta_{1} N \left( p_{x}^2 / p_{N}^1 \right) (1+r^*) + n_{2}^N \eta_{2} N \left( p_{x}^2 / p_{N}^1 \right) \right]}{\Delta} \quad (1-v(\cdot)) \hat{p}_x^2 > 0
\]

\[
\equiv A(1-v(\cdot)) \hat{p}_x^2 > 0
\]

and

\[
(14) \quad \hat{p}_N^2 = \frac{\left[ n_{2}^N \eta_{1} N \left( p_{x}^2 / p_{N}^2 \right) + n_{1}^N \eta_{1} N \left( p_{x}^2 / p_{N}^1 \right) \right]}{\Delta} \quad (1-v(\cdot)) \hat{p}_x^2 > 0
\]

\[
\equiv B(1-v(\cdot)) \hat{p}_x^2 > 0
\]

with \( \Delta \equiv n_{1}^N \eta_{1}^2 - n_{2}^N \eta_{1}^2 > 0 \).
and where $r_s^t$ is the elasticity\(^4\) -- defined to be positive -- of the demand function for \(t^{th}\) period nontraded goods with respect to its \(s^{th}\) argument and \(m_N^t\) is the marginal propensity to consume \(t^{th}\) period nontraded goods.

The signs of the above two expressions for $\sigma_N^1$ and $\sigma_N^2$ are both unambiguously positive, as proved in Appendix C. (Since all the terms in the numerators of both expressions are positive, this amounts to saying that the denominator of both expressions, or $\Delta$, is positive\(^6\), which is, in fact, the case.) Consequently, an anticipated gain in the future terms of trade leads to an increase in the relative price of nontraded goods in both periods. As has been mentioned, when the future terms of trade improve, the individual feels wealthier, a fact (12) shows. Thus, at the original set of relative prices, the individual will try to increase his consumption of both goods in both periods. However, the supply of nontraded goods is fixed and this upsurge in the demand for them can only be choked off by a rise in both periods' relative prices for nontraded goods.

Tangentially, it will be noted for later use that if the initial conditions mentioned at the beginning of this section were imposed the relative price of nontraded goods in each period rises by the same proportionate amount. That is $\sigma_N^1 = \sigma_N^2$. This occurs because now $\tilde{A} = \tilde{B}$, where the tilde over a variable denotes that its magnitude at the initial conditions\(^7\) is being discussed. The result is not surprising since, when the two periods are initially identical from the agent's perspective, anticipated gains in the future terms of
trade affect the demands for nontraded goods in each period in exactly
the same manner.

A more detailed examination of (13) reveals that the proportionate rise in today's relative price of nontraded goods, or $p^1_N$, is
an increasing function of $m^1_N$, $m^2_N$, $\eta^1_2$, $\eta^2_1$, $p^2_N X^2 / p^1_N N^1$, and $p^1_{X^1} / p^2_N N^2$, 
but a decreasing function of $\eta^1_1$ and $\eta^2_2$. This can be readily explained intuitively. At the original set of relative prices, when the
individual's wealth increases, so does his demand for first period nontraded goods. This upward shift in demand will be greater the
larger is the agent's marginal propensity to consume first period nontraded goods or the larger $m^1_N$ is. This will cause the relative
price of first period nontraded goods to rise. The extent to which
the relative price of first period nontraded goods rises in response
to the upward shift in demand will be governed by the own elasticity of demand for nontraded goods in this period, $\eta^1_1$. In particular,
the larger is this elasticity, or the more willing individuals are
to substitute away from the consumption of first period nontraded
goods in response to an increase in their relative price, the smaller
will be the proportionate rise in their price due to a gain in the
future terms of trade. The quantity $p^2_{X^2} / p^1_N N^1$ measures the size of
the second period export market vis-à-vis the first period nontraded
goods market. The bigger this datum is, the bigger will be the up-
surge in demand for nontraded goods due to the improvement in real
welfare relative to the fixed net supply of first period nontraded
goods available to private citizens, and consequently, the larger
$p^1_N$ will have to be.
However, the story is not yet over. By applying the reasoning in the above paragraph to the situation prevailing in the second period one would expect the relative price of second period nontraded goods, \( p_N^2 \), to rise. Specifically, \( \hat{p}_x^2 \) on this account should be positively related to \( m_N^2 \) and \( \frac{p_x}{p_N^2} \) but inversely related to \( n_2^2 \). Now, recall that the first period demand for nontraded goods is positively related to the future relative price of nontraded goods. Therefore, as the relative price for future nontraded goods rises so does the demand for current nontraded goods. The extent of this increase in demand is regulated by the cross elasticity of demand for current nontraded goods, \( \eta_1^1 \). This increase in demand for current period nontraded goods leads to an upward movement in their price, \( p_N^1 \). This story would lead one to expect that \( \hat{p}_N^1 \) should be positively dependent on \( \eta_1^1 \), \( m_N^2 \), and \( \frac{p_x}{p_N^2} \) but negatively related to \( n_2^2 \) as is indeed the case.

Lastly, there is a slight twist to the above scenario. Note that when discussing the relative price of future nontraded goods, \( p_N^2 \), the role of the relative price of current nontraded goods, \( p_N^1 \), in determining the demand for future nontraded goods was omitted. The effect of an increase in \( p_N^1 \) on the demand for future nontraded goods and their relative price, \( p_N^2 \), depends positively on the cross elasticity of demand for future nontraded goods, \( \eta_1^2 \). But again, a rise in \( p_N^2 \) will in turn cause a further rise in \( p_N^1 \). Thus, \( p_N^1 \) should be positively dependent on \( \eta_1^2 \).

Next, an investigation will be undertaken of the impact that an anticipated gain in the future terms of trade has on today's real
trade balance, \( t^1 \). Recall that

\[
\begin{align*}
t^1 &= (1-v(\cdot,1)) \frac{1-\lambda}{\lambda} p_x^{1-\lambda} - Z^1 - \frac{1}{\beta Z}
\end{align*}
\]

so that in the situation under analysis\(^9\)

\[
\begin{align*}
t^1 &= -\left( \xi^1_1 \hat{p}^1_N + \xi^1_2 \hat{p}^2_N + m^1_Z \right) (1-v(\cdot,2)) \left[ \frac{p_x^{2X^2}/(1+r^*)}{p_x^2} \right] < 0
\end{align*}
\]

with \( \hat{t}^1 = dt^1/dZ^1 \),

and where \( \xi^1_1 \) is the elasticity -- whose sign is unknown -- of demand for current imports with respect to the relative price of current nontraded goods, \( \xi^1_2 \) is the elasticity -- which is positive -- of the demand for current imports with respect to the relative price of future nontraded goods, and \( m^1_Z \) is the marginal propensity to consume -- again, positive -- current imports. The sign of the above expression is unambiguously negative\(^10\), a fact which is demonstrated in Appendix C. Therefore, an anticipated improvement in the future terms of trade causes today's trade balance to worsen.

As can be seen easily, there is a greater propensity toward a trade balance deficit the larger \( \xi^1_2 \) and \( m^1_Z \) are, ceteris paribus. The bigger \( m^1_Z \) is, the bigger will be the upshift in demand for current imports due to the improvement in real welfare, \( \left[ \frac{p_x^{2X^2}/(1+r^*)}{p_x^2} \right] \), brought about by the gain in the future terms of trade. Also, the greater \( \xi^1_2 \) is, the greater will be the increase in the demand for current imports occurring because of the rise in the relative price of future nontraded goods, \( \hat{p}^2_N \). Finally, since the sign of \( \xi^1_1 \) is unknown, nothing can be said about the effect of the rise in the relative price of current nontraded goods, \( \hat{p}^1_N \), on the demand for current imports and, consequently, today's trade balance.
It proves fruitful to evaluate equation (15), describing the relative change in today's trade balance, at the initial conditions mentioned at the beginning of this section. By expressing equation (9), which states that trade must balance intertemporally, in terms of proportionate rates of change while employing the initial conditions, one obtains
\[
\{\xi_1 \pi_1^2 + \xi_2 \pi_2^2 + m_2 (1-v(\cdot 2)) [p_x^2 / (1+r^*) z^2] p_x^2 \}
\]
\[
= [(1+r^*)/(2+r^*)] (1-v(\cdot 2)) [p_x^2 / (1+r^*) z^2] p_x^2
\]
Therefore,
\[
\tilde{t}^1 = - (1+r^*)/(2+r^*) (1-v(\cdot 2)) [p_x^2 / (1+r^*) z^2] p_x^2 < 0
\]
The above solution for the relative change in the trade balance has intuitive appeal. Once again, the improvement in the agent's welfare due to the anticipated gain in the future terms of trade is \((1-v(\cdot 2)) [p_x^2 / (1+r^*) z^2] p_x^2\). Now, the optimizing agent would like to use this increase in his wealth to increase his consumption of both goods in both periods. However, since nontraded goods are fixed in supply their relative prices must rise in the model's general equilibrium so as to choke off any increased demand for them. Consequently, the increase in the agent's wealth must, in the end, be vented solely on the consumption of imported goods. Since initially both periods were absolutely identical from the agent's perspective, one would expect that he would increase his consumption of the imported good
in each period by the same amount. This is in fact what he does.

As can be seen, of the increase in the agent's wealth, he spends the fraction \((1+r\lambda)/(2+r\lambda)\) on current imported goods. That is, he increases his consumption of current imported goods by the amount 
\[(1-v(\lambda))(p^n_x^2/(2+r\lambda)) \hat{p}^2_x.\] This would imply that, of the increase in the agent's wealth, he would still have the amount 
\[(1+1+r\lambda)(1-v(\lambda))(p^n_x^2/(2+r\lambda)) \hat{p}^2_x\] left over. But this would be worth 
\[(1-v(\lambda))(p^n_x^2/(2+r\lambda)) \hat{p}^2_x\] in the future period, which is what he would spend on imported goods then.

How does the increase in the future terms of trade affect today's (real) balance of payments? Once again, the balance of payments in the first period, or \(b^1\), is expressed as

\[b^1 = M^1/P^1 - \mu^1 = k^1((\pi^\lambda+r\lambda)/(1+r\lambda)) y^1 - \mu^1\]

Thus, in the current situation

\[b^1 = \hat{y}^1\] where \(\hat{b}^1 \equiv db^1/(M^1/P^1)\)

\[= (p_n^1/y^1)\hat{p}_n^1 > 0 \quad \text{since}\ \hat{p}_n^1 > 0\]

Therefore, an anticipated gain in the future terms of trade causes a balance of payments surplus today. This result occurs because an increase in the future terms of trade leads to the relative price of current nontraded goods rising. This, in turn, causes current real income, \(y^1\), to increase which leads to an upward shift in current
transactions costs of exchange. In order to economize on these increased transactions costs, the agent acquires more real balances, $M^1/P^1$, which tends to put the balance of payments in surplus.

Now, an examination will be made of the impact a temporary movement in the current terms of trade — in other words $\hat{p}_x^1 > 0$ and $\hat{p}_x^2 = 0$ — has on the relative prices of current and future nontraded goods, the trade balance, and the balance of payments. The discussion will be brief since the important features of this comparative statics exercise are similar to those already covered in the analysis of the impact that a gain in the future terms of trade has. When the current terms of trade temporarily improve, the agent realizes an improvement in his real welfare, $\omega$, of the amount

$$d\omega = (1-N(1))p_x^{1-1}\frac{\hat{p}_x^1}{\hat{p}_x^2}$$

Note that an improvement in the current terms of trade affects the relative prices of current and future nontraded goods only via a wealth effect on the demand for these commodities. Following from the previous analysis, one would expect the relative prices of current and future nontraded goods to rise in response to the increased demand for these commodities generated by the improvement in the actor's real welfare. In fact, one would expect the expressions for the resulting response in $p_N^1$ and $p_N^2$ to be very similar to those obtained when discussing the effect of a change in the future terms of trade on these variables.
Indeed, they turn out to be almost identical. The most important difference between the expressions obtained in the two cases is that they differ by a multiplicative factor of \((1+r^*)\). The expressions obtained in the current case are

\[
\hat{p}_N^1 = (1+r^*)(1-v(\cdot 1)) \frac{\hat{p}_x^{1}}{\hat{p}_x^{2-2}} \hat{p}_x^1 > 0
\]

and

\[
\hat{p}_N^2 = (1+r^*)(1-v(\cdot 1)) \frac{\hat{p}_x^{1}}{\hat{p}_x^{2-2}} \hat{p}_x^2 > 0
\]

Ceteris paribus, the individual would prefer a temporary improvement in the current terms of trade to an improvement in the future terms of trade. This is because a unit of real income today is worth \(1+r^*\) units of real income tomorrow. Thus, the wealth effect ensuing from a temporary gain in the terms of trade is \(1+r^*\) times stronger than the one following from a gain in the future terms of trade.

Consequently, the induced changes in \(\hat{p}_N^1\) and \(\hat{p}_N^2\) are \(1+r^*\) times larger in response to a movement in the current terms of trade, \(\hat{p}_x^1\), vis-à-vis the future terms of trade, \(\hat{p}_x^2\).

A temporary improvement in the current terms of trade results in a propensity toward a trade balance surplus today. By expressing equation (9), describing the trade balance, in terms of proportionate rates of change, one finds that

\[
\hat{c}^1 = (1-m_{\frac{1}{2}})(1-v(\cdot 1))[\hat{p}_x^{1 \cdot 1}/Z^{1 \cdot 1}]\hat{p}_x^1 - \xi^1 \hat{p}_N^1 - \xi^2 \hat{p}_N^2 > 0
\]
The method employed in Appendix C can be used to show that this expression is unambiguously positive. Again, it proves fruitful to evaluate the above expression at the initial conditions presented at the beginning of this section. One now obtains the following result

$$\hat{\tau}_1 = (1-\nu(\cdot 1))(1/2+r^*) (p_{X}^{\frac{1}{X}}/Z)^{1} \hat{p}_X^1 > 0$$

When the current terms of trade temporarily improve the net value of the agent's current exports, and consequently his wealth, increase by the amount $(1-\nu(\cdot 1)) p_{X}^{\frac{1}{X}} \hat{p}_X^1$. Recall that since non-traded goods are fixed in supply, it must be the case that in the model's general equilibrium the agent will use his increased wealth to increase his consumption of imports in either or both periods. Since both periods are now identical from the agent's perspective it seems reasonable to presume that he will increase his consumption of imports by the same amount in each period. In fact, this is what the agent does.

The agent spends only the fraction $[(1+r^*)/(2+r^*)]$ of the increase in his wealth on extra import consumption today. That is, he increases his consumption of current imports by the amount $[(1+r^*)/(2+r^*)](1-\nu(\cdot 1)) p_{X}^{\frac{1}{X}} \hat{p}_X^1$. Therefore, he must be saving the amount $[1/(2+r^*)](1-\nu(\cdot 1)) p_{X}^{\frac{1}{X}} \hat{p}_X^1$ toward the consumption of future imports. This is the amount by which today's trade balance improves. Thus, the agent will be able to increase his consumption of future
imports by \( (1+r^*)/(2+r^*) \) \( (1-\nu(\cdot 1))p_{X^1}^{1-1\cdot 1} p_{X}^{1-1\cdot 1} \). Finally, notice since the value of current exports has risen by more than the value of current imports, a tendency toward a trade balance surplus will emerge.

Today's balance of payments, \( b^1 \), improves when the current terms of trade gain temporarily. In the situation under analysis

\[
\hat{b}^1 = \frac{\hat{b}}{y} = \left[ \left( \frac{p_{N^1}}{y^1} \right)^{\hat{p}_N^1} + \left( \frac{p_{X^1}}{y^1} \right)^{\hat{p}_X^1} \right] > 0
\]

(since both \( \hat{p}_N^1 \) and \( \hat{p}_X^1 > 0 \))

When the current terms of trade temporarily improve, current real income, \( y^1 \), rises due to both the increase in the relative price of current exports, or \( \hat{p}_X^1 \), and the subsequent rise in the relative price of current nontraded goods, \( \hat{p}_N^1 \). As current real income, \( y^1 \), rises so does the demand for current real balances, \( \hat{M}^1/\hat{P}^1 \), and this leads to a tendency toward a balance of payments surplus.

Finally, suppose that the increase in the current terms of trade had been permanent -- that is, let \( \hat{p}_X^1 = \hat{p}_X^2 = \hat{p}_X^3 \). Now, how would the relative prices of current and future nontraded goods, the trade balance, and the balance of payments be affected? Not surprisingly, a permanent improvement in the terms of trade causes the relative prices of both current and future nontraded goods to rise. In particular

\[
\hat{p}_N^1 = \left[ (1+r^*) (1-\nu(\cdot 1)) \left( p_{X^1}/p_{X^2} \right) + (1-\nu(\cdot 2)) \right] \hat{p}_X > 0
\]

(19)
\[(20) \quad \hat{p}_N^2 = [(1+r^*)(1-v(\cdot))(p_x^{1-1}/p_N^{1-2})+(1-v(\cdot2))]B \hat{p}_x > 0\]

Note that when the initial conditions are imposed, the increases in \(p_N^1\) and \(p_N^2\) are \(2+r^*\) times as large as those occurring when just the future terms of trade improve. This is because the wealth effect occurring from a permanent gain in the terms of trade is then \(2+r^*\) times greater than that occurring when just the future terms of trade improve. As can be seen, in general a permanent increase in the terms of trade leads to bigger upward movements in the relative prices of current and future nontraded goods than does either a temporary gain in the current terms of trade or an anticipated improvement in the future terms of trade. This is because the wealth effect occurring from a permanent increase in the terms of trade is the sum of the wealth effects occurring in the other two cases.

It now can be seen immediately that today's balance of payments, \(b_1\), improves upon a permanent gain in the terms of trade. Specifically,

\[
\hat{b}_1 = \hat{y}_1 \\
= [(p_N^{1-1}/y^1)\hat{p}_N^1 + (p_x^{1-1}/y^1)\hat{p}_x^1] > 0
\]

This improvement in the balance of payments is larger than in either of the previous two cases. This is because current income, \(y_1\), now jumps upward by a greater amount due to the larger rise in the relative price of current nontraded goods, \(\hat{p}_N^1\).

Finally, what impact does a permanent increase in the terms of
trade have on today's trade balance, $t^1$? It once again proves benefi-
cial to use the initial conditions as a benchmark from which to
pose this question. The answer one gets to this question is

$$
\hat{t}^1 = 0
$$

Perhaps somewhat surprisingly, a permanent change in the terms of
trade does not affect the trade balance when the initial conditions
hold. When the terms of trade permanently improve, the value of
exports rises in both periods by the same amount. The individual
now increases his consumption of imports in each period one-to-one
with the increase in the value of exports in each period. As a result,
there is no impact on the trade balance from a permanent gain in the
terms of trade.

6. THE IMPACT EFFECTS OF GOVERNMENT SPENDING SHOCKS

The impact effects that changes in government expenditure on
imported goods have on the relative prices of current and future
nontraded goods, the trade balance, and the balance of payments are
quantitatively very similar to those that shifts in the terms of trade
have. For instance, suppose that future government expenditure on
imported goods rises — that is, assume $\hat{g}_Z^2 > 0$ while $\hat{g}_Z^1 = 0$. The
individual realizes that this new government expenditure will be
financed by increases in future taxes. Thus, the agent suffers a
welfare loss of the amount

$$
dw = - \frac{g_Z^2}{(1+r^*)} \hat{g}_Z^2
$$
An increase in future government spending on imports leads to a loss in the agent's real welfare in just about the same manner as would a worsening in the future terms of trade. Consequently, one would expect the relative prices of current and future nontraded goods to drop in response to the reduction in demand for them generated by the fall in real welfare. Algebraically, in this case, one finds that

\[
\hat{p}_N^1 = -A\left(\frac{g_Z}{p_x^2}\right)g_Z^2 < 0 \\
\hat{p}_N^2 = -B\left(\frac{2g_Z}{p_x^2}\right)g_Z^2 < 0
\]

Now, it can be seen that a proclivity toward a balance of payments deficit will result from the increase in future government spending. Here,

\[
\hat{b}^1 = \hat{y}^1 = (p_N^1/y^1)\hat{p}_N^1 < 0 \quad \text{since } \hat{p}_N^1 < 0.
\]

There is a tendency toward a deficit in the balance of payments because individuals now desire less real balances, $M^1/P^1$, due to the fact that their current real income, $y^1$, has fallen since the relative price of current nontraded goods, $p_N^1$, has dropped in response to the reduction in the agent's real welfare. A trade balance surplus will ensue from this increase in future government spending on imports. In particular,
\[ \hat{\tau}^1 = -\left( \xi^1 p_N^1 \xi^2 p_N^2 - m_z (g_Z^2/z^1 (1+r^*) g_Z^{2}) \right) > 0 \]

(Note that both \( \hat{p}_N^1 \) and \( \hat{p}_N^2 \) are negative.)

The above expression can be shown to be unambiguously positive.

However, as it did before, it proves insightful to evaluate this expression for the relative change in today's trade balance, \( \hat{\tau}^1 \), at the initial conditions presented at the beginning of the previous section. By doing this, one obtains the following equation

\[ \hat{\tau}^1 = \left[ (1+r^*)/(2+r^*) \right] (g_Z^2/((1+r^*)z^1) g_Z^2 > 0 \]

This result obtains because the agent would like to spread out over both periods the impact of the loss in his real welfare,

\[-[g_Z^2/(1+r^*)] g_Z^2, \]
due to the increase in future government spending.

That is, the agent reduces his consumption of current imports by

\[(1/2+r^*) g_Z^2 g_Z^2. \]

Since the value of current imports, \( p_{x}^{1-1} \), and current government expenditure on imports, \( g_Z^1 \), are unaffected by this shift in future government spending on imports, a propensity toward a balance of payments surplus arises today. This positive gain in the balance of payments reflects the increased savings individuals are currently undertaking so as to maintain a more constant level of future import consumption in face of the increased future tax hikes.

By following the line of reasoning employed above, it can be shown that a temporary increase in current government spending on imports causes the relative price of nontraded goods in both periods to fall, and a worsening of both the trade balance and the balance...
of payments to occur. A permanent increase in government spending on imports also causes the relative price of nontraded goods in each period to fall and, again, causes a deterioration in the balance of payments but now has no impact on the trade balance (when the initial conditions hold).

How do shifts in government expenditure on nontraded goods affect the relative prices of current and future nontraded goods and today's trade balance and balance of payments? Unfortunately, the answers to these questions are not always unambiguous. To begin with, suppose that current government expenditure on nontraded goods rises. In other words, assume that $\hat{g}_N^1 > 0$ and $\hat{g}_N^2 = 0$. The welfare loss that individuals suffer in face of this event is

$$d\omega = -\frac{1}{p_N} \hat{g}_N^1 \hat{g}_N$$

Not surprisingly, the welfare loss resulting from a temporary increase in government spending on nontraded goods is similar to the welfare loss suffered when there is a temporary increase in government spending on imports. One might think, therefore, that changes in this type of government spending should impact on the relative prices of current and future nontraded goods in the same manner as did a temporary change in current government expenditure on imports. This, however, is not the case.

Recall that the nontraded goods market must clear domestically each period. Thus, in the current period both the demand and supply
of nontraded goods for private citizens has been perturbed. By undertaking the appropriate comparative statics exercise on the system of equations (7) describing equilibrium in the nontraded goods market, one obtains the following results

\[
\hat{p}_N^1 = - \frac{\left[ n_{2}^{1,2} \left( \frac{p_N^{1,1}}{p_{N1}^{1,1}} + n_{2}^{1,2} (1+r^*) \left( \frac{p_{N1}^{1,1}}{p_{N1}^{2,2}} \right) \right] \hat{g}_N^1}{\Delta} \\
+ \frac{n_{2}^{2} \left( \frac{p_N^{1,1}}{p_{N1}^{1,1}} \right) \hat{g}_N^1}{\Delta}
\]

\[
= -(1+r^*) A \left( \frac{p_{N1}^{1,1}}{p_{N1}^{2,2}} \right) \hat{g}_N^1 + \left[ \frac{n_{2}^{2} \left( \frac{p_N^{1,1}}{p_{N1}^{1,1}} \right)}{\Delta} \right] \hat{g}_N^1 > 0
\]

and

\[
\hat{p}_N^2 = - \frac{\left[ n_{1}^{1,2} (1+r^*) \left( \frac{p_N^{1,1}}{p_{N1}^{1,1}} \right) + n_{1}^{1,2} (1+r^*) \left( \frac{p_{N1}^{1,1}}{p_{N1}^{2,2}} \right) \right] \hat{g}_N^1}{\Delta} \\
+ \frac{n_{1}^{2} \left( \frac{p_N^{1,1}}{p_{N1}^{1,1}} \right) \hat{g}_N^1}{\Delta}
\]

\[
= -(1+r^*) B \left( \frac{p_{N1}^{1,1}}{p_{N1}^{2,2}} \right) \hat{g}_N^1 + \left[ \frac{n_{1}^{2} \left( \frac{p_N^{1,1}}{p_{N1}^{1,1}} \right)}{\Delta} \right] \hat{g}_N^1 < 0
\]

As can be seen, when current government expenditure on nontraded goods temporarily rises, so does today's relative price of nontraded goods, \( p_N^1 \). (The sign of the above expression for \( \hat{p}_N^1 \) is proven in Appendix C.) The first part of the expression for \( \hat{p}_N^1 \), which is by now familiar, shows the negative impact on the relative price for current nontraded goods brought about by a reduction in the demand for them caused by the deterioration in the individual's welfare. This deterioration in the individual's real welfare results from the increased government expenditure on current nontraded goods. The second part of the expression illustrates the positive effect on the
relative price of current nontraded goods that a contraction in the net supply of them available to private citizens has. It turns out that this second effect dominates so that the relative price of current nontraded goods rises, as was probably expected a priori. 

Perhaps somewhat surprisingly, a temporary increase in current government spending has an ambiguous impact on the relative price of future nontraded goods. The first term in the expression for $\frac{\eta_1^2}{\eta_N}$ shows the depressing effect on the relative price of future nontraded goods that a reduction in demand for them, caused by the welfare loss generated by the increased government expenditure, has. Again, this effect is familiar. The second term shows the positive impact that a reduction in the net supply of current nontraded goods has on the relative price of future nontraded goods. This effect is perhaps somewhat subtle. It is operational because, as mentioned, a drop in the net supply of current nontraded goods available to private citizens increases the price for them. However, this increase in the relative price for current nontraded goods leads to individuals substituting away from the consumption of current nontraded goods to consuming future nontraded goods -- and future imports as well. This has a positive impact on the relative price of future nontraded goods. As can be seen, this effect is larger the bigger $\eta_1^2$ is, which represents the elasticity of demand of future nontraded goods with respect to the relative price of current nontraded goods. The net impact of these two effects is theoretically ambiguous.
However, something more can be said about the sign of the expression for $\tilde{p}_N^2$. In particular, it can be shown (see Appendix C) that the sign of this expression depends positively on the sign of $U_{12}(N^1, Z^1)$. Whether $U_{12}(N^1, Z^1)$ is positive or negative determines whether nontraded or imported goods are complements or substitutes, in the (nonstandard) Edgeworth-Pareto sense, in the momentary utility function. Now, for example, when $U_{12}(N^1, Z^1)$ is negative the initial effect of a reduction in the current net supply of nontraded goods available to the representative agent is to increase the marginal utility of current imports. This leads the agent to desire to consume more current imports. However, to do this, the agent must withdraw expenditure from the second period. Provided that future nontraded goods are normal goods -- which was assumed -- this leads to a reduction in the demand for them. Given the fixed supply of future nontraded goods, their relative price must thus fall.

Next, the impact that a temporary increase in current government expenditure on nontraded goods has on today's trade balance will be analyzed. Expressing the trade balance equation (9) in terms of proportional rates of change yields in this situation that

$$
\tilde{t}^1 = -\xi_1 \tilde{p}_N^{-1} \tilde{p}_N^{-2} + m_N \left( p_N \tilde{e}_N / Z^1 \right) \tilde{e}_1 \geq 0
$$

Unfortunately, the sign of the above expression is theoretically ambiguous. It can be shown, though, that the sign of this expression depends positively on the sign of $U_{12}(N^1, Z^1)$ -- see Appendix C. By evaluating the above equation at the initial conditions (11), one
obtains the following condition 18

\[ \hat{\tau}^1 = -(1/2+r\tau)[\hat{\xi}_1 - (1+r\tau)\hat{\xi}_2] (\hat{p}_N - \hat{p}_N) \hat{\tau}^2 < 0 \]

Note from (21) and (22) that \( \hat{p}_N - \hat{p}_N \) is a positive number when the initial conditions hold. Therefore, the sign of the change in today's trade balance, \( \hat{\tau}^1 \), is minus the sign of \( \hat{\xi}_1 - (1+r\tau)\hat{\xi}_2 \). Now, by delving into the consumer's problem it is easy to show that, at the initial conditions, the sign of \( \hat{\xi}_1 - (1+r\tau)\hat{\xi}_2 \) depends negatively on the sign of \( U_{12}(N^1, Z^1) \). Thus, the sign of \( \hat{\tau}^1 \) depends positively on the sign of \( U_{12}(N^1, Z^1) \). Whether \( U_{12}(N^1, Z^1) \) is positive or negative in turn depends on whether current nontraded and imported goods are complements or substitutes with each other, in the Edgeworth-Pareto sense, in the momentary utility function.

That the change in the trade balance should depend positively on the sign of \( U_{12}(N^1, Z^1) \) makes intuitive sense. Again, suppose that \( U_{12}(N^1, Z^1) \) is negative. Hence, a decrease in the supply of current nontraded goods available to private citizens will initially increase the marginal utility of current imported goods. This leads to an upsurge in the demand for current imported goods and a consequent deterioration in the trade balance.

An improvement in the balance of payments occurs in response to the temporary increase in current government spending on imports. This is because such a change in government spending leads to a rise in the relative price of current nontraded goods, \( \hat{p}_N \), which in turn
causes current real income, $y^1$, to increase, and consequently the
demand for real balances, $\frac{M^1}{P^1}$, to rise. Algebraically, the ex-
pression one gets for the relative change in the balance of payments
is

$$\frac{B^1}{y^1} = \frac{P_N^{1.1}}{\frac{y^1}{P_N^{0.1}}} > 0$$

since $P_N^{1.1} > 0$ as (21) shows

Finally, it can be shown that, while an increase in future govern-
ment spending on imports unambiguously increases the relative price
of future nontraded goods, it has an ambiguous impact on the rela-
tive price of current nontraded goods. The effect of such a govern-
ment spending change on both the trade balance and the balance of
payments is ambiguous. A permanent increase in government spending
-- at the initial conditions -- causes the relative prices of both
current and future nontraded goods to rise and, therefore, is asso-
ciated with a balance of payments surplus. The trade balance is un-
affected by such an increase in government spending.

7. CONCLUSIONS AND EXTENSIONS

In this paper a small-scale microeconomic-oriented general equi-
librium model was constructed to explain the joint behavior of the
trade balance, the balance of payments, and the relative price of
nontraded goods. A set of predictions was obtained about the relation-
ship between movements in the trade balance, the balance of payments,
and the relative price of nontraded goods and important exogenous variables such as the terms of trade and government spending. Here it was vital to distinguish between movements in these exogenous variables that were transitory or permanent in character, and also between events that were currently happening and those which were expected to happen in the future.

The present model can be easily extended to analyze the determination of the real exchange rate. Suppose that the domestic aggregate price index, \( \Phi^*_1 \), is some homogeneous function of degree one in the domestic nominal prices of the imported good and the nontraded good. If this was the case, one could write the current domestic aggregate price level as \( \Phi^*_1 = P^1 f(p^*_N) \). Using the law of one price, it then follows that \( \Phi^*_1 = e \frac{P^*_F}{P^*_F} f(p^*_N) \). Now, as is commonly done to get a measure of the real exchange rate, divide the domestic aggregate price index by \( e \frac{\Phi^*_F}{\Phi^*_F} \) where \( \Phi^*_F \) is the foreign aggregate price index in the current period. Thus, the measure being used to reflect the real exchange rate is \( \Phi^1/e \frac{\Phi^*_1}{\Phi^*_F} = P^1 f(p^*_N) / \Phi^*_F \). Note that all foreign prices, and consequently the foreign aggregate price level, \( \Phi^*_F \), are unaffected by any shocks emanating within the domestic economy since by assumption the domestic economy is a small open one. Therefore, any domestic shocks which lead to a change in the relative price of nontraded goods, \( p^*_N \), will cause a movement in today's real exchange rate. For example, a temporary increase in current government spending on imports will cause today's real exchange rate to drop
since the relative price of nontraded goods falls, while a temporary increase in current government spending on nontraded goods will cause an appreciation in today's real exchange rate since the current relative price of nontraded goods rises.

Lastly, this paper has ignored the role that investment spending plays in determining the trade balance. This is worrisome since Sachs (1981) has noted that, empirically, investment may explain a substantial portion of the variation in the trade balance. Modeling investment is no easy matter, but it seems that a thorough understanding of the trade balance depends on a detailed study of the determinants of investment spending in an open economy, as Sachs has emphasized.
APPENDIX A

The agent's optimization problem (3) implies that if he is to hold money efficiently in the second period, the following first order condition must be satisfied

$$-v'\left(\frac{M^2}{P^2 y^2}\right) = 1$$

By inverting the function $v'(\cdot)$, one can see that the agent's demand for real balances in the second period, $M^2/P^2$, may be written as

$$M^2/P^2 = k^2 y^2 \quad \quad k^2 \equiv v^{-1}(-1)$$

Now, by using the above relationship and equation (4) in the text, it follows that the agent's optimization problem (3) may be rewritten in the manner shown below where $N^1$, $Z^1$, $N^2$, and $Z^2$ are now the choice variables.

Max $U(N^1, Z^1) + \beta U(N^2, Z^2)$

s.t.

$$p_N^{-1}N^1 + Z^1 + \left(\frac{1}{1+r^*}\right)\left[p_N^{-2}N^2 + Z^2\right]$$

$$= (1-v(k^{-1} (\frac{\pi+r^*}{1+r^*}))y^1 + \mu^1 - \tau^1 + \left(\frac{1}{1+r^*}\right)[(1-v(k^2))y^2 + \mu^2 - \tau^2]$$

$$- \left(\frac{\pi+r^*}{1+r^*}\right)\frac{M^1}{p^1} - \left(\frac{1}{1+r^*}\right)\frac{M^2}{p^2}$$

$$= (1-v(k^{-1} (\frac{\pi+r^*}{1+r^*}))y^1 + \mu^1 - \tau^1 + \left(\frac{1}{1+r^*}\right)[(1-v(k^2))y^2 + \mu^2 - \tau^2]$$

$$- \left(\frac{\pi+r^*}{1+r^*}\right)k^{-1}(\frac{\pi+r^*}{1+r^*})y^1 - \left(\frac{1}{1+r^*}\right)k^2 y^2$$
The right-hand side of the agent's new budget constraint represents his real disposable wealth (net of transactions costs) in period 1. This is an exogenous variable from the perspective of the current maximization problem. It can now be seen clearly that changes in the rate of return on holding money, $-\pi$, only influence the agent's choice of $N^1$, $Z^1$, $N^2$, and $Z^2$ via income effects generated by shifts in his real disposable wealth.
APPENDIX B

In this appendix, a demonstration will be undertaken of the real welfare gain the agent realizes today when the future terms of trade, $p_x^2$, are anticipated to improve. Recall that the agent's lifetime utility is

$$U = U(N^1, Z^1) + \beta(U^2, Z^2)$$

Therefore,

$$dU = U_1(\cdot 1)dN^1 + U_2(\cdot 1)dZ^1 + \beta U_1(\cdot 2)dN^2 + \beta U_2(\cdot 2)dZ^2$$

where the notation $\cdot t$ within a function implies that the arguments of that function are being evaluated at their date $t$ values. Now, the change in the agent's real welfare, $dw$, when measured in terms of current imports is

$$dw \equiv \frac{dU}{U_2(\cdot 1)} = \frac{U_1(\cdot 1)}{U_2(\cdot 1)} dN^1 + dZ^1 + \beta \frac{U_1(\cdot 2)}{U_2(\cdot 2)} dN^2 + \beta \frac{U_2(\cdot 2)}{U_2(\cdot 1)} dZ^2$$

Now, the agent's first order conditions from his optimization problem (3) tell one that $U_1(\cdot 1)/U_2(\cdot 1) = \frac{1}{p_N}$, $\beta U_1(\cdot 2)/U_2(\cdot 1) = \frac{p_N^2}{(1+r^*)}$, and $\beta U_2(\cdot 2)/U_2(\cdot 2)$. Thus,

$$dw = \frac{1}{p_N} dN^1 + dZ^1 + \frac{p_N^2}{(1+r^*)} dN^2 + \frac{dZ^2}{(1+r^*)}$$

Recall that equation (8) in the text states that in the model's general equilibrium the following relationship holds.
\[
\frac{1}{p_N} [N^1 \cdot g^1 + \frac{1}{1+r^*} (p_N^2 + z^2 + g^2)] = (1-v(\cdot 2)) y^1 + \frac{1}{1+r^*} (1-v(\cdot 2)) y^2
\]

This relationship plus condition (7), which states that the nontraded goods market must always clear domestically each period, imply that in the current situation

\[
\frac{1}{p_N} dN^1 + \frac{1}{(1+r^*)} dN^2 + \frac{1}{(1+r^*)} d^2 = \frac{(1-v(\cdot 2)) x^2}{(1+r^*)} \frac{d^2}{dP_x}
\]

Consequently,

\[
d\omega = \frac{(1-v(\cdot 2)) x^2}{(1+r^*)} \frac{d^2}{dP_x}
\]

which is equation (12) given in the text.
APPENDIX C

It is easy to see from the consumer's problem and the various equilibrium conditions in the model that the following five equations completely characterize the determination of $p_N^1$, $p_N^2$, $Z^1$, $Z^2$, and $\lambda$ in the model's general equilibrium -- $\lambda$ is the Lagrange multiplier associated with the agent's constrained maximization problem.

(C1) $U_1[(1-v(\cdot 1))N^{-1}-g_N^1, Z^1] = \lambda p_N^1$

(C2) $U_2[(1-v(\cdot 1))N^{-1}-g_N^1, Z^1] = \lambda$

(C3) $\beta U_1[(1-v(\cdot 2))N^{-2}-g_N^2, Z^2] = \lambda p_N^2/(1+r^*)$

(C4) $\beta U_2[(1-v(\cdot 2))N^{-2}-g_N^2, Z^2] = \lambda/(1+r^*)$

(C5) $Z^1+g_N^1+(1/(1+r^*))Z^2+g_N^2 = (1-v(\cdot 1))p_x^1X^1+(1/(1+r^*))[(1-v(\cdot 2))p_x^2X^2$

Now, how will an anticipated gain in the future terms of trade, $p_x^2$, impact on today's consumption of imports, $Z^1$, and consequently today's trade balance, $t^1$? The answer to this equation is easily obtained by subjecting equations (C2), (C4), (C5) and (9) to the usual sort of comparative statics exercise. The results of this exercise are

(C6) $\frac{\partial Z^1}{\partial p_x^2} = \frac{(1+r^*)\beta U_{22}^2(x)}{[U_{22}^1(x)+\beta(1+r^*)^2U_{22}^2(x)]} (1-v(\cdot 2))X^2 > 0$

and
\[ \frac{3t^{-1}}{3p^2_x} = - \frac{3z^1}{3p^2} < 0 \]

Consequently, the solution for \( t^1 \), as given by equation (15) in the text is unambiguously negative as was stated.

Next, equations (C1), (C2), and the above solution for \( \frac{3z^1}{3p^1_x} \) can be used to see how \( p^1_N \) is affected by a shift in \( p^2_x \). One finds that

\[ \frac{3p^1_N}{3p^2_x} = - \left[ \frac{p^1_N U_{22}(-1) - U_{12}(-1)}{U_2(-1)} \right] \frac{3z^1}{3p^2_x} > 0 \]

(recall that \( \frac{3z^1}{3p^1_x} > 0 \))

where the above expression is unambiguously positive due to the assumption that nontraded goods are normal, which implies that \( [p^1_N U_{22}(-1) - U_{12}(-1)] < 0 \). An immediate implication of the above result is that the solution for \( p^1_N \) in this case, as given by (13) in the text, must also be unambiguously positive. Since all of the terms in the numerator of (13) are positive, it must therefore follow that (13)'s denominator, or \( \Delta \), is positive, too.

Finally, how does a change in \( p^2_x \) impact on \( p^2_N \)? Equations (C3) and (C4) can be used to provide an answer to this question. The answer is that

\[ \frac{3p^2_N}{3p^2_x} = - \left[ \frac{p^2 N U_{22}(-1) - U_{12}(-2)}{U_2(-1)} \right] \frac{3z^2}{3p^2_x} > 0 \]
where the expression in brackets is negative due to the fact that nontraded goods are normal goods, and where (C5) and (C6) can be used to show that

$$\frac{\partial z^2}{\partial p_x^2} = \frac{U_{22}(\cdot 1)}{U_{22}(\cdot 1)+\beta(1+r^*)^2U_{22}(\cdot 2)2} \frac{(1-\nu(\cdot 2))x^2}{(1-\nu(\cdot 2))x^2} > 0$$

Since $\frac{\partial p_N^2}{\partial p_x^2}$ is unambiguously positive, it must be the case that the solution (14) given in the text for $p_N^2$ in this circumstance must also be positive.

Now, the effect on today's consumption of imports, $z^{\cdot 1}$, of a temporary shift in current government spending on nontraded goods, $g_N^{\cdot 1}$, will be investigated. This effect can be uncovered through the use of (C2), (C4), and (C5). One finds that

(C7) \[
\frac{\partial z^{\cdot 1}}{\partial g_N^{\cdot 1}} = \frac{U_{21}(\cdot 1)}{U_{22}(\cdot 1)+\beta(1+r^*)^2U_{22}(\cdot 2)} > 0
\]

and, thereby, through (9) that

$$\frac{\partial t^{\cdot 1}}{\partial g_N^{\cdot 1}} = -\frac{\partial z^{\cdot 1}}{\partial g_N^{\cdot 1}} > 0$$

Therefore, $\frac{\partial z^{\cdot 1}}{\partial g_N^{\cdot 1}} > 0$ iff $U_{21}(\cdot 1) > 0$, while $\frac{\partial t^{\cdot 1}}{\partial g_N^{\cdot 1}} > 0$ iff $U_{21}(\cdot 1) > 0$. Thus, the sign of $t^{\cdot 1}$, which is shown by equation (23) in the text, depends positively on the sign of $U_{21}(\cdot 1)$ as was mentioned.
How does the temporary increase in $g_N$ affect $p^2_N$? By undertaking the required comparative statics exercise on equations (C3), (C4), and (C5), it can be seen that

$$\frac{\partial p^2_N}{\partial g_N} = \left[ \frac{p^2_N U_{22}^{\cdot 2} - U_{21}^{\cdot 2}}{U_2^{\cdot 2}} \right] (1 + r^*) \frac{\partial z}{\partial g_N} > 0$$

Thus, $\frac{\partial p^2_N}{\partial g_N} > 0$ as $U_{21}^{\cdot 1} < 0$. This implies that expression (22) in the text describing $p^2_N$ in this situation must be positively related to the sign of $U_{12}^{\cdot 1}$.

Finally, how would $p^1_N$ respond to this change in $g^1_N$? Equations (C1), (C4), (C5), and (C7) provide the answer to this question. One finds that

$$\frac{\partial p^1_N}{\partial g_N} = \frac{\{U_{11}^{\cdot 1} U_{22}^{\cdot 1} - U_{12}^{\cdot 1} U_{22}^{\cdot 1}\} + \beta(1 + r^*)^2 U_{22}^{\cdot 2} [U_{11}^{\cdot 1} - p^1_N U_{21}^{\cdot 1}]}{\beta(1 + r^*) [U_{22}^{\cdot 1} + \beta(1 + r^*) U_{22}^{\cdot 2} U_{22}^{\cdot 2}]} > 0$$

The above solution for $\frac{\partial p^1_N}{\partial g_N}$ is unambiguously positive in sign.

(Recall that $[U_{11}^{\cdot 1} - p^1_N U_{21}^{\cdot 1}] < 0$ since imported goods are normal goods.) Consequently, the expression for $p^1_N$ in this circumstance, as given by (21) in the text, must also be unambiguously positive.
FOOTNOTES

1. If the momentary utility function was separable in nontraded and imported goods, then the question marks under the various arguments in these demand functions could be replaced by plus signs.

2. The notation \( t \) within a function is being used to indicate that the arguments of a function are being evaluated at their date \( t \) values. Thus, \( v(\cdot 2) = v(M^2/P_y^2) \).

3. Due to the terminal nature of the second period, it will probably never be the case that \( v(\cdot 1) = v(\cdot 2) \). However, since this two period model is meant to proxy for a multiperiod one, this artifact, due to the terminal nature of the second period, will be ignored.

4. For example, \( \eta_1^1 = \frac{p_N^1}{N^1} \frac{\partial N^1}{\partial p_N^1} \) and \( \eta_2^2 = \frac{p_N^2/(1+r^*)}{N^2} \frac{\partial N^2}{\partial (p_N^2/(1+r^*))} \).

5. In other words, \( m_N^1 = p_N^1 \frac{\partial N^1}{\partial \omega} \) and \( m_N^2 = \frac{p_N^2}{(1+r^*)} \frac{\partial N^2}{\partial \omega} \). Note that the assumption that nontradables are normal goods implies that both \( m_N^1 \) and \( m_N^2 \) are positive.

6. At the initial conditions listed at the beginning of this section it is easy to show that \( \Delta \) is positive. This is because here the consumer's problem implies that \( \tilde{\eta}_1^1 > \tilde{\eta}_2^1 \) and \( \tilde{\eta}_2^2 > \tilde{\eta}_1^2 \), where the tilde over a variable denotes that its value at the initial
conditions is being discussed. Outside of the initial conditions, it is still true that $\Delta$ is positive as Appendix C illustrates.

7. When the initial conditions are imposed, the consumer's problem implies that the following restrictions hold: 

$$\tilde{\eta}_1^1 = \tilde{\eta}_2^2 - (1-\beta)\tilde{\eta}_1^1,$$

$$\tilde{\eta}_1^2 = \tilde{\eta}_2^1 (1+r^*),$$

and that $$\tilde{m}_N^1 = \tilde{m}_N^2 (1+r^*).$$ These restrictions can be used to show that $\tilde{A} = \tilde{B}$.

8. These results are obtained by the straightforward differentiation of the solution for $\tilde{p}_N^1$ as given by (13).

9. Since the current trade balance, $t_1^1$, may be zero -- such as when the initial conditions hold -- current imports, $Z_1^1$, instead of $t_1^1$, have been used to deflate $dt_1^1$.

10. As can be deduced from footnote 1, when the momentary utility function is separable in nontraded and imported goods, $\xi_{1}^1$ is positive. Thus, it is then easy to see from (15) that the current trade balance will worsen, or that $\tilde{c}_1^1 < 0$, as a result of the improvement in the future terms of trade.

11. The following restrictions, obtained from the consumer's problem, hold when the initial conditions are imposed: 

$$\tilde{\xi}_1^1 = \tilde{\xi}_2^2 + (1-\beta)\tilde{\xi}_1^2,$$

$$\tilde{\xi}_1^2 = \tilde{\xi}_2^1 (1+r^*),$$

and that $$\tilde{m}_N^1 = \tilde{m}_N^2 (1+r^*).$$ These restrictions, as well as the fact that now $\tilde{p}_N^1 = \tilde{p}_N^2$, have been used in obtaining the equation given in the text.
12. Since the current balance of payments, $b_1$, may be zero, current real balances, $M^1/P^1$, have been used to deflate $db^1$.

13. The larger the value of current net exports, $(1-v(\cdot 1))p_{X^1}^{1-1}$, vis-à-vis the value of future net exports, $(1-v(\cdot 2))p_{X^1}^{2-1}$, the larger will be the wealth effect ensuing from a temporary gain in the current terms of trade versus the one following from an improvement in the future terms of trade. Thus, for other things to be equal, it should be the case that $(1-v(\cdot 1))p_{X^1}^{1-1} = (1-v(\cdot 2))p_{X^1}^{2-2}$.

14. By using condition (9), it must also be the case that

$$\hat{t}^1 = \left[1/(1+r*)\right](Z^2/Z^1)\{\xi_1^2 + \xi_2^2 p_N^1 \hat{p}_N^2 + m_Z^2 [p_{X^1}^{1-1} / Z^2(1+r*)]p_{X^1}^{2-1}\} > 0$$

Thus, it immediately follows that if the momentary utility function were separable in nontraded goods and imports, so that $\xi_2^2$ was positive, then $\hat{t}^1 > 0$. (Note that $\xi_1^2$ and $m_Z^2$ are always positive.)

15. Note that the system of equations (7), (9), and (10) is linear in its displacements. This implies that to see how the relative price of current nontraded goods is affected by a permanent change in the terms of trade all one has to do is to add the effect of a shift in the future terms of trade on the relative price of current nontraded goods to the effect of a temporary change in the current terms of trade on this variable. The impact of a permanent shift in the terms of trade on the relative
price of future nontraded goods, the trade balance, and the 
balance of payments can be uncovered in a similar fashion.

16. In general, a permanent improvement in the terms of trade has 
an ambiguous effect on today's trade balance. This shouldn't 
be surprising since an anticipated future gain in the terms of 
trade causes a deficit to develop while a current, but temporary, 
one leads to a surplus.

17. Simple differentiation of (21) shows that, in the case under 
discussion, \( \hat{p}_N^1 \) is negatively related to \( \eta_1^1 \), \( m_N^2 \), and 
(\( p_{NN}^1 / p_{NN}^2 \)), but positively associated with \( \eta_1^2 \) and (\( p_{NN}^1 / p_{NN}^2 \)). 
The effect of \( \eta_2^2 \) and \( \eta_2^1 \) on \( \hat{p}_N^1 \) is ambiguous. More specifically, 
whether or not \( \hat{p}_N^1 \) depends positively or negatively on \( \eta_2^2 \) and 
\( \eta_2^1 \) depends directly on the sign of expression (22) for \( \hat{p}_N^2 \).

18. In obtaining this expression, the equations (21) and (22) have 
been used, as well as the restrictions mentioned in footnotes 
(7) and (11).

19. When the momentary utility function is separable in nontraded 
and imported goods, \( \hat{\xi}_1^1 = (1+r^*)\hat{\xi}_2^1 \) so that here \( \hat{\xi}^1 = 0 \).

20. The movement in the price of current nontraded goods depends 
positively on the sign of \( U_{12}(\cdot 2) \).

21. The sign of the change in today's balance of payments is also 
directly related to the sign of \( U_{12}(\cdot 2) \). Finally, the movement 
in the current trade balance is inversely associated with the 
sign of \( U_{12}(\cdot 2) \).
References


