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OPTIMAL SCHOOLING, EXTERNALITIES AND THE MARRIAGE MARKET*

by

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and

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*Jim Davies, Michael Hoy, Glenn MacDonald, Chris Robinson and other participants in the Labour Economics Workshop at U.W.O. provided helpful comments. The usual caveat applies.

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I. INTRODUCTION

In contemporary Western society educational institutions perform a variety of functions. In addition to augmenting productive skills (human capital) and screening individuals according to ability, educational institutions also provide a characteristic by which individuals sort in the marriage market. Although the former functions of education have been extensively analyzed by economists, the latter function has been largely ignored. Nevertheless it may be of considerable importance. Coeducational institutions provide a potential meeting place for prospective spouses in the years which typically precede marriage. Less directly, educational attainment appears to be an important characteristic by which prospective spouses are ranked.

The existing empirical evidence supports this view. The correlation between the education of spouses in the U.S. has been approximately 0.6 throughout the post-war era, considerably higher than the correlation between the family backgrounds of spouses (Hollingshead (1950) and Blau and Duncan (1967)). McCormick and Macrory (1943-4) found that out of approximately 20 traits, education was ranked third by women, behind ambition and intelligence, as a desirable trait in prospective spouses. Finally, a recent study by Taylor and Glenn (1976) found that women with greater education marry men with higher occupational status. The purpose of this paper is to present a formal model in which, in addition to providing earnings-related skills, education also serves as a characteristic by which individuals sort in the marriage market.

It is important to distinguish between alternative roles of education, since it is well known that alternative views regarding education lead to
differing diagnoses concerning whether there is over- or underinvestment in education from a social viewpoint. If education, in addition to augmenting individual productive skills and hence earnings, generates benefits to society through more intelligent voting behavior, less criminal activity, etc., not fully internalized by the investor, then private educational investments will be socially sub-optimal. Furthermore, several authors have argued that because human capital investment decisions are typically undertaken in a family context there may be underinvestment in education. Ishikawa (1975) has shown that if parental concern for children terminates when offspring achieve independence parents will underinvest in the human capital of their progeny. Lazear (1983) argues that parents may underinvest in their own human capital if they seek to maximize the net present value of lifetime earnings, ignoring the effects of their human capital on their children's ability and hence costs of investment. Lastly Nerlove, Razin and Sadka (1982) demonstrate that underinvestment may occur even if parents are concerned with the lifetime well-being of their children. In the model presented by Nerlove, et al., an externality arises because random marriage links family lines and both spouses benefit from a bequest by one set of parents. This situation is analogous to a multiple-donor model of charitable contributions in which donors ignore the psychic benefits accruing to other donors as a result of their transfers to the recipient.

In contrast to these externality arguments, models of educational signalling and screening typically generate an overinvestment result (e.g., Spence (1973) and Stiglitz (1975)). More importantly, for the present discussion, Welch (1974) has asserted that when education is a valuable trait in the marriage market there will be overinvestment in education:
"[T]he idea is that more productive marriage partners are preferred and that, on balance, net productivity in marriage is positively related to [own] education. Increased education serves to increase an individual's chance of a good marriage, of marriage to a more educated spouse. If an individual's social product is independent of his spouse's education, then the marriage-go-round results in overinvestment in education."


Marriage plays an important role in the models of Nerlove, et al. (1982) and Welch (1974). However the former conclude there is underinvestment while the latter concludes that overinvestment will arise. This paper re-examines this issue.

In the following section we present a formal model in which, in addition to providing earnings-related skills, education also serves as a characteristic by which individuals sort in the marriage market. Section III examines, in the context of this model, the important question of whether there is over- or underinvestment in schooling from a social viewpoint. We demonstrate that privately optimal levels of schooling may either exceed or fall short of the social optimum level. The final section gathers our conclusions.

II. A MODEL OF INDIVIDUAL INVESTMENT

Consider a simple economy composed of $2N$ individuals: The set $I \supseteq i=1,\ldots,N$ males and the set $J \supseteq j=1,\ldots,N$ females. In each set, individuals differ only in their productivity and the two sets of individuals are perfectly identical except for their differing sex.

At the beginning of life each individual, $k = i$ or $j$, is endowed with resources $R$. During the initial period, in which all individuals are single, resources may be spent either on consumption $z_k$ or on investment in schooling $e_k$ at a constant unit cost $s$. Returns to schooling, in the form of higher earnings, accrue in the following period. The resource con-
straint is therefore
\( R \geq z_k + se_k \) \quad \forall k \in I \lor j \in J

At the end of the initial period, all individuals enter into a monogamous marriage for the remainder of their lifetime. The identity of eventual spouses is, however, unknown when schooling is undertaken. Hence an individual can form only probabilistic beliefs concerning the characteristics of his or her spouse.

Given the symmetry between I and J, it suffices initially to consider explicitly only the behavior of a typical individual \( i \in I \). Later it will become necessary to also consider \( j \in J \). For each individual \( i \), lifetime expected utility, evaluated at the initial period, is given by
\[
V_i = u(z_i) + \left( \sum_{ij} u(c_{ij}) + \delta \sum_{ij} u(c_{ji}) \right), \quad 1 \geq \delta > 0, \quad \forall i \in I.
\]

The utility function \( u(\cdot) \) (with \( u' > 0 \) and \( u'' \leq 0 \) so that the individual may be risk-neutral or risk-averse) is identical for all individuals. The first term represents \( i \)'s utility from own consumption \( z_i \), while single. The second term (abstracting from discounting factor)\(^2\) describes the expected utility of the remainder of \( i \)'s lifetime whence he is married to prospective spouse \( j \in J \) whom he may marry with probability \( \pi_{ij} \). It consists of \( \sum_{ij} u(c_{ij}) \) which is \( i \)'s expected utility from own consumption, \( c_{ij} \), while married to \( j \); and \( \delta \sum_{ij} u(c_{ji}) \) which captures \( i \)'s altruistic concern for the expected utility his prospective spouse derives from her consumption, \( c_{ji} \), while married to \( i \).\(^3\) The second component depends on the altruism coefficient \( \delta \in [0,1] \), Edgeworth's "coefficient of effective sympathy", which is assumed the same for all individuals.
Individual i's consumption while married to spouse j, \( c_{ij} \), and spouse j's consumption while married to i, \( c_{ji} \), are described respectively by

\[
(3) \quad c_{ij} = \alpha E_i + \beta E_j \quad \text{and} \quad c_{ji} = \alpha E_j + \beta E_i \quad \forall i \in I, \ j \in J
\]

where \( E_i \) and \( E_j \) are the earnings of i and j, respectively. The weights, \( 0 < \alpha \leq 1 \) \( 0 \leq \beta \leq 1 \), specify the dependence of marital consumption on the individual's own earnings and those of his or her spouse, respectively. Becker (1981, Chapter 4) presents a model where these weights are endogenously determined in the marriage market and shows that they depend in general on who is matched with whom. However, in the real world, the values of these parameters may be severely constrained. Social conventions may be operative which govern the sharing of resources between spouses. Moreover, and more importantly, there are many aspects of family life which involve joint consumption between spouses. Typically the husband does not live in the woodshed and the wife in a mansion (or vice versa); rather both consume the same heating, air-conditioning, home furnishings, etc. Public goods aspects of consumption within the family place constraints on the degree of inequality in the consumption of spouses and may cause the actual values of \( \alpha \) and \( \beta \) to deviate substantially from those determined by a marriage market. In the model presented below we examine the limiting case in which \( \alpha \) and \( \beta \) are parametrically fixed due to either social customs governing sharing between spouses, or the public goods aspects of household "technology".\(^4,5\)

The dependence of marital consumption on spouse's earnings (\( \beta > 0 \)) creates an incentive for individuals to seek a marriage partner possessing higher potential earnings with whom they can enjoy a higher level of consumption. Furthermore, altruism (\( \delta > 0 \)) between spouses creates an additional incentive since a more educated spouse j will earn and consume more, enjoying a higher level of utility, which is partially internalized by the altruistic spouse i.
In the model we have specified education is assumed to perform two distinct roles. On the one hand, it enhances personal earnings through the earnings function\(^6\)

\[
E_k = r_k(e_k) \text{ with } r'_k(e_k) > 0 \text{ and } r''_k(e_k) < 0, \forall k \in I \text{ or } j \in J
\]

On the other hand, there is substantial evidence (see McCormick and Macrory (1943-4) and Taylor and Glenn (1976)) indicating the importance of education as a marriage-sorting device. In our model, education performs this role by affecting the probabilities of meeting and marrying prospective spouses. In general, the sorting probabilities can be written as

\[
\pi_{ij} = \pi_{ij}(e_i, e_j) = \pi_{ji}(e_i, e_j) \equiv \pi_{ji}, \quad \forall i \in I, j \in J
\]

where \(e_i = (e_{i1}, \ldots, e_{iN})\) and \(e_j = (e_{j1}, \ldots, e_{jN})\) are respectively the vectors of educational units of both males and females. The second equality in (5) follows from the assumed perfect symmetry between males and females. By the definition of probabilities, it must also follow that

\[
\sum_j \pi_{ij} = 1 \quad \text{and} \quad \sum_i \pi_{ji} = 1.
\]

Furthermore, it is assumed that education level, \(e_i\), of any individual \(i\) affects his marriage probabilities \(\pi_{ij}\) in the following manner. Aligning \(e_i = (e_{i1}, e_{i2}, \ldots, e_{iN})\) and \(e_j = (e_{j1}, e_{j2}, \ldots, e_{jN})\) in increasing order: \(e_1 < e_2 < \ldots < e_N\), it is then assumed that

\[
\left\{
\begin{align*}
\frac{\partial \pi_{ij}}{\partial e_i} & \leq 0 & \text{for } j \leq i \\
\frac{\partial \pi_{ij}}{\partial e_i} & \geq 0 & \text{for } j > i
\end{align*}
\right.
\]

Furthermore, by the definition of probability measure, we must have

\[
\sum_j \frac{\partial \pi_{ij}}{\partial e_i} = 0.
\]
Assumption (7) states that for any individual \( i \in I \), a marginal increment in education \( e_i \) has the effect of not increasing his probabilities of marrying spouses \( j \in J \) who have equal or less education than he; while his chances of marrying a spouse with a higher education than his, may increase. This assumption is not unreasonable given the evidence of a high correlation between the education of spouses (Hollingshead (1950) and Blau and Duncan (1967)).

**Privately Optimal Behavior**

The problem facing each individual \( i \) is to choose \( z_i \) and \( e_i \) to maximize his expected lifetime utility (2) subject to resource constraint (1). The necessary and sufficient condition for an interior optimum is:

\[
(9) \left[ \sum_j \pi_{ij} u'(c_{ij}) + \delta \sum_j u'(c_{ji}) r_i'(e_i) + \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(c_{ij}) + \delta u(c_{ji}) \right] = u'(z_i)s, \forall i \in I
\]

This simply states that the privately optimal level of education \( e_i \) must equate the marginal utility loss from foregone consumption due to one unit of educational expenditure (R.H.S. of (9)) with the expected marginal utility gains from schooling (L.H.S. of (9)). The expected gains from schooling consist of two components. The first term (in L.H.S. of (9)) identifies the direct returns to education to the investor himself \( \left( \sum_j \pi_{ij} u'(c_{ij}) r_i'(e_i) \right) \) and to his prospective spouse \( \left( \delta \sum_j u'(c_{ji}) r_i'(e_i) \right) \) toward whom he is altruistic. This term is always positive since \( u'(\cdot) \) and \( r_i'(\cdot) \) are positive.

The second term (in L.H.S. of (9)) captures the expected gains to schooling via marriage sorting. The first part of this term \( \left( \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(c_{ij}) \right) \) represents the effects of sorting on individual \( i \)'s own marital consumption \( c_{ij} \); whereas the second part \( \left( \delta \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(c_{ji}) \right) \) measures the effects of \( i \)'s
sorting on his spouse's consumption $c_{ji}$ and hence her utility which is inter-
nalized by $i$. This term is non-negative on account of assumption (7). In
assumption (7), the effect of increasing $e_i$ is to shift the probability
weights from (marrying) lower-educated to higher-educated spouses. The
effect of this is to cause the new probability distribution $\pi_{ij}(e_1,\ldots,e_i + \delta e_i,\ldots,e_N; e_J)$ to have stochastic dominance over the old probability
distribution $\pi_{ij}(e_1,\ldots,e_i,\ldots,e_N; e_J)$ so that, as long as the utility
functions $u(c_{ij})$ and $u(c_{ij}')$ are monotonic, the second L.H.S. term of (9)
will always be non-negative (see Hadar and Russell (1969), and also
Rothschild and Stiglitz (1970) for the proof to this result).

The gains to education depend in general also on the parameters $\alpha$, $\beta$
and $\delta$. These parameters influence not only the direct returns to education
from the labour market, but also the indirect returns via the marriage
market. An exogenous decrease in the degree of altruism ($\delta$), for instance,
diminishes the gains to education and in the limiting case where $\delta = 0$;
that is, all individuals are egoistic, equation (9) becomes

$$\alpha_i'(e_i) \sum_j \pi_{ij} u'(c_{ij}) + \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(c_{ij}) = u'(z_i) \text{ if } \delta = 0, \beta > 0, \forall i \in I$$

Notice that even in the absence of altruism, the incentive to sort in the
marriage market remains as long as the contribution ($\beta$) of spouse's earnings
to the individual's consumption is positive.

Other things equal, a decrease in $\beta$, due perhaps to the diminished
importance of joint consumption within the family, will decrease the gains
to education since less consumption will be generated for $i$'s spouse and not
only so, $i$ has less to gain from being matched with a more educated spouse.
In the extreme case where $\beta = 0$, (9) becomes:

"
\[ c' \left( e_i \right) u' \left( c \left( e_i \right) \right) + \sum_j \frac{\partial n_j}{\partial e_i} u \left( c_j \left( e_i \right) \right) = u' \left( z_i \right) s \quad \text{if } \beta = 0, \delta > 0, \forall i \in I \]

Even in this limiting case there is still an incentive to sort by education as long as there is interspouse altruism (i.e., \( \delta > 0 \)). This incentive arises because, although own consumption is independent of spouse's earnings, marriage to a more educated spouse represents marriage to an individual with higher utility which is internalized by \( i \). We note also that the parameters \( \beta \) and \( \delta \) interact, so that the larger is \( \delta \) the greater the impact of a change in \( \beta \) on the total returns to education and vice versa.

Only in the limiting case where \( \beta = \delta = 0 \) will there be absolutely no incentive to sort by education in the marriage market. To see this, observe that (9) reduces to

\[ c' \left( e_i \right) u' \left( c \left( e_i \right) \right) = u' \left( z_i \right) s \quad \text{if } \beta = \delta = 0, \forall i \in I \]

In this individualistic world, the individual spends just enough \( e_i \) to equate his own private returns from one unit of schooling to its cost. No extra \( e_i \) is spent in order to sort out his mate since there is now absolutely no incentive to do so.

**Equilibrium**

We shall now characterize the equilibrium in this simple economy. The equilibrium solution consists of the set of educational investments:

\[ e^*_i = (e^*_i, \ldots, e^*_N) \text{ for males and } e^*_j = (e^*_i, \ldots, e^*_N) \text{ for females such that} \]

(a) (9) is satisfied for every \( i \in I \) and the female counterpart of (9)

is satisfied for every \( j \in J \)

(b) the probability function \( \pi_{ij} \) and \( \pi_{ji} \) satisfy (6), (7), and (8)

(c) the economy-wide constraint given by:
\( (10) \frac{1}{N} \sum_{i,j} \dot{r}_{ij}(e^*_i, e^*_j) \cdot r_j(e^*_j) = \frac{1}{N} \dot{r}_j(e^*_j) = \frac{1}{N} \sum_{i,j} \dot{r}_{ij}(e^*_i, e^*_j) r_i(e^*_i) \quad \forall i \in I, j \in J \)

holds.

Condition (a) states that \( e^*_i \) and \( e^*_j \) must be chosen optimally for every \( i \in I \) and \( j \in J \). Condition (b) has been discussed. The final condition (c) is the economy-wide equation analogous to the requirement that the aggregate expected demand must equal the supply available. In particular, for all \( i \in I \), the average expected income of their spouses: \( \frac{1}{N} \sum_{i,j} (e^*_i, e^*_j) \cdot r_j(e^*_j) \), must equal the average available in the economy, \( \frac{1}{N} \dot{r}_j(e^*_j) \). The equality within parentheses is the analogous statement for the other sex group, and the second equality simply follows from the assumed symmetry between I and J.

The equilibrium characterized by (a), (b) and (c) can be looked upon as a rational expectations equilibrium. If individual beliefs \( \pi_{ij} \) and \( \pi_{ji} \) are such that, for all individuals taken together, condition (10) is violated, then our equilibrium concept requires such beliefs to be revised until (10) is restored.

The existence of an equilibrium matching of spouses in a situation such as this has been demonstrated by Gale and Shapley (1962). These authors prove that for any set of preference rankings, there exists a 'stable' set of assignments in which no two individuals not currently married to each other would prefer being matched with one another rather than to their current spouses.
III. PARETO-OPTIMALITY

We shall now consider whether the equilibrium level of private educational investment is above or below the social optimal.

Consider the problem of a planner who maximizes a Benthamite social welfare function:

\[
\text{Max } \sum_{i \in I} V_i + \sum_{j \in J} V_j \text{ s.t. } 2NR \geq \sum_{i \in I} (z_i + se_i) + \sum_{j \in J} (z_j + se_j).
\]

The Pareto-optimal level of \(e_i\) for individual \(i \in I\) can be written as:

\[
\left[ \sum_{j \in J} (\alpha u'(c_{ij}^*) + \delta \beta u'(c_{ij}^*)) \right] r_i^* + \left[ \sum_{j \in J} (\beta u'(c_{ij}^*) + \delta \alpha u'(c_{ij}^*)) \right] r_j^* + \sum_{ij} \frac{\partial \Pi_{ij}}{\partial e_{i*}} (u(c_{ij}) + \delta u(c_{ij})) + \sum_{ij} \frac{\partial \Pi_{ij}}{\partial e_{j*}} (u(c_{ij}) + \delta u(c_{ij})) = \mu'(z_i^*) \cdot s \quad \forall i \in I
\]

(The symmetrical equation for all \(j \in J\) is suppressed to avoid redundancy.)

This expression, which equates the marginal cost of education (R.H.S. of (12)), with the social marginal gains (L.H.S. of (12)) in general differs from the privately-optimal condition given by (9).

Recall that in (9) the private marginal benefit to education consists of two terms wherein the first captures the private returns (that contribute to higher consumption for oneself and one's spouse), and the second represents the indirect returns via marital sorting. In (12), however, the social marginal benefit to education consists of four terms: The first term, depicting the private direct returns, is identical to the corresponding term in (9). The second term in (12), which is missing in (9), is the positive externalities associated with human capital investment which an individual fails to fully internalize in his private decision in (9). These externalities arise because individual \(i^*\), irrespective of whether or not he is altruistic, does not fully consider the impact of his earnings on the marital consumption and
hence utilities of his potential spouses.\textsuperscript{12} The failure of the private
decision (9) to consider these externalities contributes to the under-
investment result considered by Ishikawa (1975) and Lazear (1983).\textsuperscript{13}

The last two terms in (12) are respectively the social marital-
sorting effect on all $i \in I$ and $j \in J$ reflecting the fact that educational
investment of any one individual $i* \in I$ affects the marital probabilities
of all individuals of both sexes. In contrast to the private gains to
sorting in (9) which has been shown to be non-negative for any monotonic
utility function the social gains to marital sorting will in general
depend on the form of the utility function.

\textbf{Risk-neutral Individuals}

In the case where the individuals are all risk-neutral (i.e., $u' > 0$
and $u'' = 0$), the social marital effect will vanish:\textsuperscript{14}

\begin{equation}
(13) \quad 0 = \sum \sum \frac{\partial \pi_{ij}}{\partial e_{i*}}(u(c_{ij}) + \delta u(c_{ij})) + \sum \sum \frac{\partial \pi_{ij}}{\partial e_{i*}}(u(c_{ij}) + \delta u(c_{ij})) \quad \text{if} \quad u'' = 0 \quad \forall \ i* \in I
\end{equation}

The reason for the divergence between the private (second term in (9)) and the
social marital-sorting effect (13) is as follows. Although each individual of
either sex may be able to use increased investment to improve, through sorting,
his or her marital consumption and hence utility, this cannot be true for society
as a whole. The fact that the incomes of one sex are independent of the
investment of the other sex must imply that the aggregate income of spouses
is invariant with respect to the investment strategies of the other sex.
Consequently, one individual's expected gain, through sorting, for a higher
income spouse must imply an offsetting expected loss for the rest of
individuals of the same sex. Condition (13) is simply a statement of the
zero-sum game involved in marriage sorting.
Thus, whether or not the market investment represents an under- or over-investment depends on the strengths of the externality effect (which gives rise to the under-investment) and the marriage-sorting effect (which gives rise to the over-investment). To see this more clearly, subtract (12) (after using (13)) from (9), to get

\[(14) \quad -[\sum_{j} \pi_{i*}^{*} u'(c_{j*}^{*}) + \delta \omega'(c_{i*}^{*})] r_{i*}^{*} + \sum_{j} \frac{\partial \pi_{i*}^{*}}{\partial e_{i*}^{*}} u(c_{i*}^{*}) + \delta u(c_{i*}^{*})] \leq 0 \quad \forall i* \in I\]

where a positive sign represents over-investment and a negative sign under-investment. While the first term is in general non-positive due to the externality effects, the second term is in general non-negative due to marriage sorting. In light of these two counteracting effects, even under risk neutrality, it is in general unclear whether there is under- or overinvestment in education.

Some special cases, however, are worth noting. In one limiting case where \( \delta = 0 \) and \( \beta > 0 \), expression (14) reduces to

\[ -\beta \sum_{j} \pi_{i*}^{*} u'(c_{j*}^{*}) r_{i*}^{*} + \sum_{j} \frac{\partial \pi_{i*}^{*}}{\partial e_{i*}^{*}} u(c_{i*}^{*}) \leq 0 \quad \text{if} \quad \delta = 0, \beta > 0 \quad \forall i* \in I\]

Thus, even in the absence of altruism, the question of optimality of investment remains unresolved. As long as \( \beta > 0 \), the under-investment (negative first term) that arises from not fully internalizing the positive external benefit derived by spouses must still be weighed against the overinvestment (positive second term) due to marital sorting.

In the other case where \( \beta = 0 \) and \( \delta > 0 \), expression (14) becomes

\[ -\delta \omega'(c_{i*}^{*}) r_{i*}^{*} + \sum_{j} \frac{\partial \pi_{i*}^{*}}{\partial e_{i*}^{*}} u(c_{i*}^{*}) \leq 0 \quad \text{if} \quad \beta = 0, \delta > 0 \quad \forall i* \in I\]

Although with \( \beta = 0 \) there is no incentive to sort to improve one's own marital consumption, the sorting effect (positive second term that implies over-investment) is still present because altruism provides the
incentive to match with higher-income spouses. However, the first
(negative) term which represents the failure to internalize the reci-
procal altruism of spouses, by itself, implies under-investment. Thus
the overall effect remains ambiguous even in this special case.

In the extreme case when $\beta = \delta = 0$ expression (14) equals
zero in which case the private and social optima coincide.

Risk-averse Individuals

When individuals are risk averse, the social marital-sorting effect
given by the expression in (13) does not in general reduce to zero. Nevertheless,
the question of optimality of educational investment remains ambiguous. The
general condition indicating whether the private education represents an
over- or underinvestment is then given by (substracting (12) from (9)):

$$
(15) \quad [-\sum_{j} \pi_{ji}^*(u'(c_{ji}^*) + \delta u'(c_{i^*j}))] r_{i^*}^* 
+ \sum_{i \neq i^*} \sum_{j} \frac{\partial \pi_{ji}}{\partial e_{i^*}} (u(c_{ji}) + \delta u(c_{j^*i}) + \sum_{j} \frac{\partial \pi_{ji}}{\partial e_{i^*}} (u(c_{ji}) + \delta u(c_{j^*i})) \leq 0 \text{ for } V_{i^*} I
$$

where a positive (negative) sign represents an over- (under-) investment.

The first term is, as before, non-positive because of the failure to
internalize all the positive externalities associated with the accumulation
of private human capital. The second term is the externality effects of
marital sorting on all other $i$ ($\neq i^*$) individuals of the same sex (first
component) and on all other $j$ individuals of the opposite sex (second component).

Unless some strong restrictions are invoked in addition to (7) regarding the
impact of individual investment on the marital-probabilities of the rest of
the population, the second term is in general of ambiguous sign. The reason
is because marginal utility weights differ across income groups. Consequently,
even though the social impact of an individual's marital-sorting investment
on the total expected income is zero, the effect on total expected utilities of all individuals could be positive or negative depending on the distribution of the resulting marital-probability weights affecting the different individuals.

Thus, the use of education as a signalling device in the marriage market does not necessarily lead to an overinvestment as asserted by Welch (1974). Even in the case of risk-neutral individuals, it has been shown whenever $\beta > 0$ or $\delta > 0$ so that there is an incentive to sort, the overinvestment due to sorting must necessarily be accompanied by under-investment due to the non-internalizing of the other externality effects. Welch has obviously over-emphasized the marital-sorting effects and failed to recognize the other externality effects.

For precisely the opposite reason to Welch, the model of Nerlove, Razin and Sadka (1982) fails to recognize the marital-sorting effects and demonstrates an under-investment result. In their model, it is assumed that: (i) parents are concerned about the total bequests received by their child and his (or her) spouse; and (ii) marriage is completely random, being independent of parental bequests. They conclude that under-investment would necessarily follow because parents, in bequeathing to their own child, fail to internalize the external benefit of this bequest on the utility of the parent of the child's spouse. In the real world, most bequests are embodied in human capital (education) so that in the context of our model, their assumption (i) is analogous to having $\alpha > 0$ and $\beta > 0$. But with $\beta > 0$, we have shown that there must necessarily exist the incentive to sort for a spouse with a larger bequest (or education). This implication is however inappropriately ruled out by their assumption
(ii) of complete random marriage. Thus, unless sorting costs are prohibitively large, or marital search is very unproductive, marital sorting cannot be ignored. This implication is further confirmed by the empirical evidence that shows a high correlation between spouses' education. In light of this, there is serious doubt concerning the under-investment result derived by these authors. In general, the presence of marital-sorting effects together with the other externalities discussed above would cause the divergence between the private and social optima. But whether or not there will be under- or overinvestment cannot be determined a priori but remains an open empirical question.

IV. SUMMARY AND CONCLUSIONS

Since the appearance of the signalling and screening models of Spence (1973) and Stiglitz (1975), it is now well understood that education, in addition to its traditional role of producing human capital, plays an important role in conveying information to firms regarding individual abilities.

In contrast this paper has emphasized the fact that outside the labor market, education may play a similar role in matching spouses. Empirical evidence (McCormick and Macrory (1943-4), Taylor and Glenn, 1976) supports this view.

In this paper we have constructed a model in which education not only enhances earnings in the labor market, but also represents a trait by which individuals select marriage partners. Within the context of our model we conclude that privately optimal levels of schooling will in general differ from the social optimum. However, it is not possible in general to conclude a priori whether there is under- or overinvestment.
On the one hand, the failure to fully internalize the positive externalities in consumption accruing to other members of the family is a source of sub-optimal investment. This possibility is emphasized by Ishikawa (1975), Lazear (1983) and Nerlove, et al. (1982). On the other hand, however, the possibility of overinvestment arises when education is used as a marital sorting device--as conjectured by Welch (1974). The former group of authors neglect the role of education in marital sorting, while Welch appears to overlook the external benefits of education. Our model incorporates both these effects and shows that they are intimately related, since greater external benefits increase the incentives to sort for a spouse having higher education. In order to evaluate whether there is over- or underinvestment in education knowledge of the magnitude of the underlying parameters would be required.
1 We assume that all educational investments are self-financed and that schooling is the only means by which resources can be transferred over time. This characterization captures well-known imperfections in financial markets related to investment in education. Allowing for perfect financial markets, however, would not change our results.

2 For the purpose of this paper where we are interested merely in the comparison between the private and social optimal levels of investment, the discount factor is inessential and its inclusion would only complicate the notation.

3 Notice that the above formulation of altruism, i's ex ante expected utility depends on the utility that spouse j would derive from her consumption while married to i. A slightly different variant would make the spouse's utility during marriage the object of altruism, i.e.

\[ V_1 = u(z_1) + \left\{ \sum_j \pi_{ij} u(c_{ij}) + \delta \sum_j \pi_{ij} V_j' \right\} \]

where \( V_j' = u(c_{j1}) + \delta[V_1 - u(z_1)] \) is the utility of the jth woman conditional on being married to male i. Substituting into (2)' gives:

\[ V_1 = u(z_1) + [1 - \delta^2 \sum_j \pi_{ij}]^{-1} \left\{ \sum_j \pi_{ij} u(c_{ij}) + \delta \sum_j \pi_{ij} u(c_{ji}) \right\} \]

This "reduced form" utility function differs from (2) only in the factor of proportionality \([1 - \delta^2 \sum_j \pi_{ij}]^{-1}\) attached to marital utility. Note that \( \sum_j \pi_{ij} = 1 \), and for the expected utility to be bounded, \( \delta < 1 \). In other words, altruism must be less than "perfect" in that spouse's utility is discounted relative to own utility (see also Becker (1974)).
Becker (1981, pp. 84-7) argues that when the division of marital income is inflexible, lump sum transfer payments will take place in the form of 'bride prices' or 'dowries' to achieve the equilibrium distribution determined by the marriage market. However, neither bride prices nor dowries are prevalent features of contemporary western societies even though joint consumption within the family seems to us to be pervasive.

The coefficients \( \alpha \) and \( \beta \) parameterize the importance of intra-family public goods. From (3) the total consumption of spouses \( i \) and \( j \) is:

\[
c_{ij} + c_{ji} = (\alpha + \beta)(E_i + E_j)
\]

If \( \beta = 1 - \alpha \) the total consumption of the couple equals total earnings—all consumption is private. If, however, \( \beta > 1 - \alpha \) then \( (c_{ij} + c_{ji}) > (E_i + E_j) \), and some aspects of consumption are joint. The magnitude of \( (\alpha + \beta - 1) \geq 0 \) indexes the importance of public goods.

In this model, the only source of heterogeneity among individuals is their differences in productivity (or ability) which is represented by the function \( r_k(e_k) \) for \( k = i \) or \( j \). We assume that the differences are such that if \( r_m(\cdot) > r_e(\cdot) \) then also \( r_m'(\cdot) > r_e'(\cdot) \), i.e., ability and schooling are complements. Hence sorting by education implies sorting by ability.

Furthermore, note that we have assumed that individual \( i \)'s earnings, \( r_i(e_i) \), are independent of his spouse's education \( e_j \). Therefore, we exclude any inter-spouse productivity effect considered by Benham (1974) and Behrman and Wolfe (1981). Introducing this interdependence does not at all affect the results of this paper.
Sufficiency is assured by the concavity of \( u(\cdot) \) and the strict concavity of \( r_k(\cdot) \) functions.

If the utility function is given by equation (2)' in footnote 3, the first order condition differs from (9) in that the right-hand-side is multiplied by \((1 - \delta^2)\). Consumption while single would therefore be smaller, and educational investment greater, than implied by (9), other things equal.

If \( \beta = 0 \), and \( \delta > 0 \), (9) reduces to

\[
\begin{align*}
    u'(z_i) & = \alpha x_i' \sum_j \pi_{ij} u'(\alpha x_i) + \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(\alpha x_i) + \delta \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(\alpha x_j) \\
    & = \alpha x_i' u'(\alpha x_i) + \delta \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(\alpha x_j)
\end{align*}
\]

since \( \sum_j \pi_{ij} = 1 \) and \( \sum_j \frac{\partial \pi_{ij}}{\partial e_i} = 0 \).

This social welfare function takes into account the altruistic attitudes of each individual embodied in (2) (or alternatively (2)' in footnote 3).

We could define a social welfare function which ignores altruism:

\[
(11)' \quad S = \sum_i \{ u(z_i) + \sum_j \pi_{ij} u(c_{ij}) \} + \sum_j \{ u(z_j) + \sum_i \pi_{ij} u(c_{ji}) \}
\]

However, this social welfare function implies myopia on the part of the planner.

If individual preferences are characterized by (2)' (in footnote 3), rather than (2), then maximizing (11) yields an equation which differs from (12) by having the R.H.S. multiplied by \((1 - \delta^2)\).

If the social welfare function is given by (11)' in the previous footnote, rather than (11) the Pareto optimum \( e_i^* \) is given by:

\[
(12)' \quad \sum_j \pi_{ij} \{ (u'(c_{ij})) r_i' + \sum_j \pi_{ij} \{ (\beta u'(c_{ji}')) r_j' + \sum_i \frac{\partial \pi_{ij}}{\partial e_i} u(c_{ij}) + \frac{\sum_j \partial \pi_{ij}}{\partial e_i} u(c_{ji}) \} \} = u'(z_i^*) \forall i \in I.
\]

This is identical to (12) when \( \delta = 0 \).
The qualitative results remain intact if individual utility functions are given by equation (2)' (in footnote 3) and these preferences are reflected in the social welfare function (11).

These authors consider models where parents, who are investing in their own, or their children's human capital, fail to fully internalize the returns to these investments which accrue to children. Their underinvestment result is therefore due to an intergenerational externality whereas in this model, it arises from an intragenerational (inter-spouse) externality.

Let \( \hat{c}_{ij} = \alpha \bar{r}_i + \beta \bar{r}_j \) where \( \bar{r}_j \) and \( \bar{r}_i \) are respectively the average earnings of \( i \in I \) and \( j \in J \) respectively. Then by Taylor's expansion, the first component of expression (12) can be written as

\[
\sum_{ij} \frac{\partial \pi_{ij}}{\partial e_{i*}} u(c_{ij}) = \sum_{ij} \frac{\partial \pi_{ij}}{\partial e_{i*}} [u(\hat{c}_{ij}) + u'(\hat{c}_{ij}) (\alpha (\bar{r}_i - \bar{r}_i) + \beta (\bar{r}_j - \bar{r}_j))]
\]

\[
= u(\hat{c}_{ij}) \sum_{ij} \frac{\partial \pi_{ij}}{\partial e_{i*}} + u'(\hat{c}_{ij}) \sum_{i} \alpha (\bar{r}_i - \bar{r}_i) \sum_{j} \frac{\partial \pi_{ij}}{\partial e_{i*}}
\]

\[
+ \beta \sum_{j} (\bar{r}_j - \bar{r}_j) \sum_{i} \frac{\partial \pi_{ij}}{\partial e_{i*}}
\]

\[
= 0
\]

since \( \sum_{j} \frac{\partial \pi_{ij}}{\partial e_{i*}} = \sum_{i} \frac{\partial \pi_{ij}}{\partial e_{i*}} = 0 \). It can easily be shown that all the other marital sorting components in expression (13) also reduce to zero by the above reasoning.
If \( \beta = 0 \), \( c_{ij} = \alpha x_i + \beta x_j = \alpha x_i \) and \( c_{ji} = \alpha x_j + \beta x_i = \alpha x_j \) so that (14) becomes (after using \( i \) for \( i^* \) to simplify the notation)

\[
-\delta \sum_j \pi_{ji} u'(\alpha x_i) \delta \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(\alpha x_i) + \delta \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(\alpha x_j)
= -\delta \alpha u'(\alpha x_i) \sum_j \pi_{ji} r_i^+ \delta \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(\alpha x_i) + \delta \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(\alpha x_j)
= -\delta \alpha u'(\alpha x_i) r_i^+ \delta \sum_j \frac{\partial \pi_{ij}}{\partial e_i} u(\alpha x_j)
\]

since \( \sum_j \pi_{ji} = 1 \) and \( \sum_j \frac{\partial \pi_{ij}}{\partial e_i} = 0 \) because \( \sum_j \pi_{ij} = 1 \).
REFERENCES


