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On the Nature of Students' Digital Mathematical Performances

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A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Education

ON THE NATURE OF STUDENTS' DIGITAL MATHEMATICAL PERFORMANCES

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by

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Abstract

In this study I investigate the nature of digital mathematical performances (DMPs) produced by elementary school students (Grades 4-6). A DMP is a multimodal text/narrative (e.g., a video) in which one uses the performance arts to communicate mathematical ideas. I analyze twenty-two DMPs available at the *Math + Science Performance Festival* in 2008. Assuming a sociocultural/postmodern perspective with emphasis on multimodality, my focus is on the role of the arts and technology in shaping students' mathematical communication and thinking. Methodologically, I employ qualitative case studies, along with video analysis. I conduct a descriptive analysis of each DMP using Boorstin's (1990) categories of what makes good films, focusing on surprises, sense-making, emotions, and visceral sensations. I also conduct a cross-case analysis using Boorstin's categories and the mathematical processes and strands of the Ontario Curriculum. The multimodal nature of DMP is one of its most significant pedagogic attributes. Mathematics is traditionally communicated through print-based texts, but the production of DMPs is an alternative that engages students in conceiving multimodal narratives. The playfulness offers scenarios for students' collaboration, creativity, and imagination. By making DMPs available online, students share their ideas in a public and social environment, beyond the classrooms. Most of the DMPs only explore Geometry and offer opportunities to experience some surprises, sense-making, emotions, and visceral sensations. The lack of focus on other strands (e.g., Algebra) may be seen as a reflection on what (and how) students are (or not) learning in their classes. The production of *conceptual* DMPs is a rare event, although I acknowledge that I analyzed only DMPs of the first year of the Festival, that is, students did not have examples or references to produce their DMPs. Some DMPs potentially explore conceptual mathematical surprises, but they appear to have gaps in terms of sense-making. The use of the arts and technologies does not guarantee the mathematical conceptuality of DMPs. This study contributes to mathematics education with an exploratory discussion about how mathematical ideas can be (a) communicated and represented as multimodal texts at the elementary school level and (b) seen through a performance arts lens. The study also points out directions about the pedagogic components for conceiving conceptual DMPs in terms of the performance arts and the components of the Ontario Curriculum.

Keywords: mathematics education, digital technology, the performance arts, multimodality, curriculum.

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Prologue: Digital Mathematical Performance

In this study I analyze, from a performance arts point of view, how elementary school students communicate mathematical ideas using the performance arts. I investigate the nature of students' *digital mathematical performances* (DMPs).

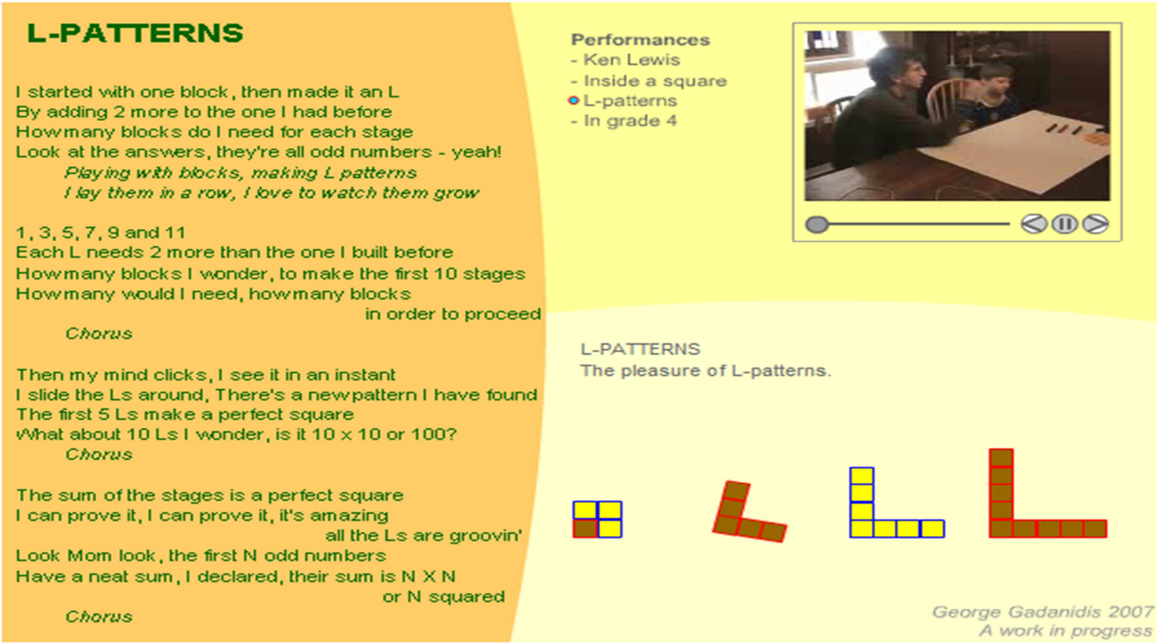
Traditionally, "mathematical performance" is conceptualized as pertaining to the domain of assessment and evaluation (Lesh & Lamon, 1994). An alternative view of *mathematical performance* is as a process of communicating mathematics using the performance arts (Gadanidis & Borba, 2008). Gadanidis and Borba (2006) posit:

> Performance takes place in the theatre, at poetry readings and on the screen. What if as mathematicians, as math educators, or as students of mathematics we moved outside of the domain of assessment (where performance takes on a different meaning) and used an artistic lens to look at how we 'perform' mathematics? If we view mathematics as embodied performance, what do we see differently? Thinking of mathematics and mathematics teaching and learning as performance may help to destabilize and reorganize our thinking about what it means to do and teach mathematics with technology. (para. 1)

Digital technology plays a significant role in mathematical performance (Borba, 2007). DMPs are multimodal texts (e.g., video files or registers) used to represent and communicate mathematics through music, cinema, theater, poetry, storytelling, and so forth (Gadanidis, 2006; Scucuglia & Borba, 2007), that are usually publicly disseminated online (Borba, 2009).

Before introducing the research questions of this study, I present three examples of DMPs. Figure A shows *L-Patterns* (Gadanidis, 2007a). This DMP is a learning object that compiles videos, poem/lyrics, and a simulation to explore sequences and series of odd numbers. *L-Patterns* connects multiple representations (e.g., numeric/geometric), multiple modes of communication (e.g., visual, audio, spatial, linguistic, and gestural) and different mathematical strands (e.g., *Geometry and Spatial Sense* and *Patterning and Algebra*). Through poems, video clips, songs, sessions with students, and experiments, *L-Patterns* explores geometrically and algebraically the notion that the sequence of odd numbers (1, 3, 5, 7, . . .) can be represented by "Ls" (2n – 1) and the series of odd numbers (1 + 3 + 5 + 7 + . . .) can be represented by squares (n^2), that is, $\sum_{i=1}^{n}(2i-1) = n^2$.

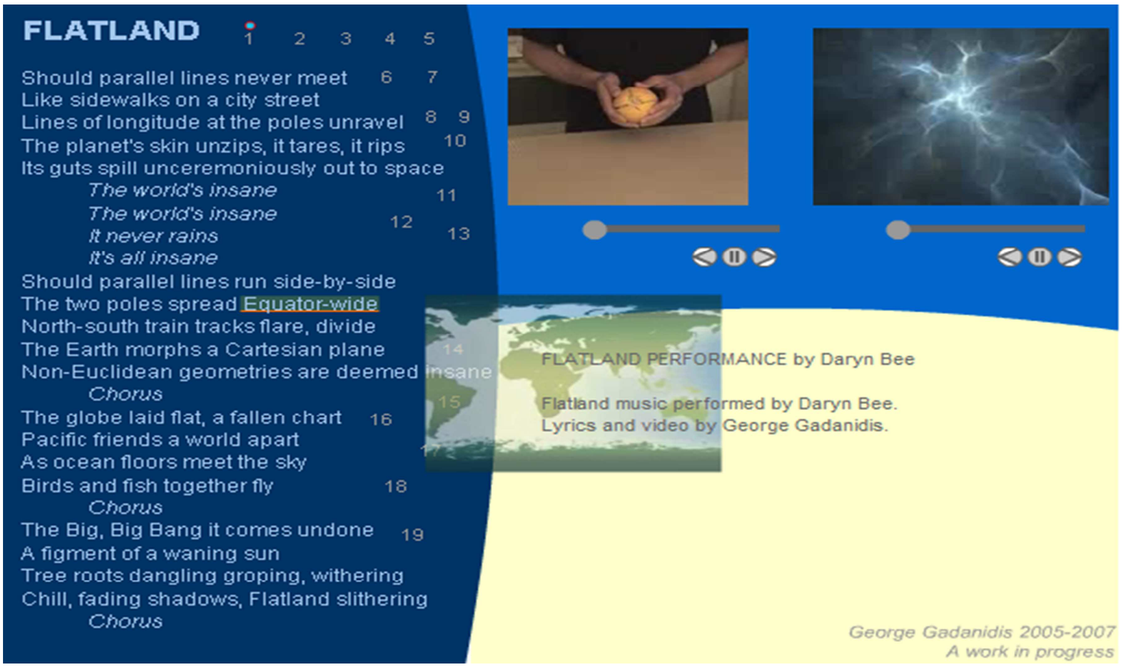
Figure A: L-Patterns (Gadanidis, 2007a)



Description: In this DMP, the audience may read a poem/lyric on the left side. On the right side (above) the audience may watch four different videos such as a video-music clip and sessions with students in which they explore the L-Patterns. On the right side (below), the audience may see a simulation in which a sequence of Ls (1, 3, 5, 7, . . ., 2n – 1) is transformed in the series 1 + 3 + 5 + 7 + … + (2n - 1) forming a square (n^2). See also www.fields.utoronto.ca/mathwindows/oddnumbers/index.html.

Flatland (Gadanidis, 2005) is another example of a DMP. It explores parallelism in spherical geometry (see Figure B). This DMP helps to illustrate how parallel lines do meet on spherical surfaces. (For a discussion of “straight” and “parallel” lines on a sphere, see the interview with mathematician Megumi Harada at www.fields.utoronto.ca/mathwindows/sphere). In this context, the performance arts and digital technologies offer ways to communicate mathematical ideas with creativity (Gadanidis, 2006). This DMP presents many hyperlinked videos about songs, students’ and teachers’ activities, skits, visual proofs, mathematical applications in everyday contexts, live performances, and digital stories. Surprisingly, it shows, for instance, how parallel lines do meet on spherical surfaces, how the sum of values of the angles of triangles has over 180 degrees, and other ideas involving spherical geometry (a non-Euclidian geometry).

Figure B: Flatland (Gadanidis, 2005)



Description: In this DMP, the audience may read a poem/lyric on the left side of the object and click on 19 different links to videos. Some of the expressions in the poems such as "Equator-wide" display simulations when the user passes the mouse over these words. On the right side (above) the audience may watch the sets of videos for each of the 19 links. These videos present music clips, sessions with students, experiments and visual proofs, lives performances in classrooms, and so on. Each of these videos can also be considered a DMP. On the right side (below), the audience may visualize some simulations, depending on the link the user clicks on.

The third example I present is *The buttons get arrays*. That is a DMP in video format produced by pre-service teachers and it is available at www.edu.uwo.ca/mpc/mpf2010/mpf2010-106.html. Through stop-motion animation, the DMP explores the relation between the area and the perimeter of rectangles, i.e., how the perimeter of different representations of rectangles changes, considering a constant area. Figure C shows a sequence of images of *The buttons get arrays*.

Figure C: *The buttons get arrays* (www.edu.uwo.ca/mpc/mpf2010/mpf2010-106.html).

Description: In this DMP, the audience watches a stop-motion animation. The DMP shows, visually, that representations of different rectangles, with different perimeters, have the same area (16 units). Three levels or stages are explored. First, the rectangle 16u by 1u has an area equal to 16u and a perimeter equal to 34u. Second, the rectangle 2u by 8u has an area equal to 16u and a perimeter equal to 20u. Finally, and surprisingly, the rectangle 4u by 4u has an area equal to 16u and a perimeter equal to 16u. The DMP also explores that a square is a special case of rectangle.

In this study I analyze twenty-two DMPs produced by elementary school students (from Grade 4 to Grade 6). These DMPs are publicly available at the Math + Science Performance Festival (Gadanidis, Borba, Gerofsky & Jardine, 2008). On the website of this Festival (http://www.mathfest.ca), students submit their DMPs, the organizers of the Festival make the performance available, and every year a collection of judges, comprised of Canadian artists, educators, and mathematicians, indicate DMPs based on three aspects: (a) the nature of the mathematical idea; (b) creativity and imagination, and; (c) artistic and technological aspects.

Although more details about the Festival are presented in chapter three (Methodology), it is important to mention that most of the DMPs showcased in the Festival are not virtual learning objects like *L-Patterns* and *Flatland*. The students' DMPs are not very sophisticated in terms of digital design in comparison to *L-Patterns* and *Flatland*. Most of the DMPs in the Festival are "single" video files (similar to *The buttons get arrays*) or other types of files (e.g., power point). *L-Patterns* and *Flatland* can be seen as DMPs formed by several DMPs, because they are formed by many videos, poems/lyrics, and simulations. However, I would like to highlight that *L-Patterns*, *Flatland*, and *The buttons get arrays* are considered examples of some of the potential affordances of DMPs, connecting multiple representations, modes of communication, and mathematical strands. These DMPs also potentially offer the audience a set of experiences that are used as the basis of analysis in this thesis, namely, mathematical surprises, emotional moments, and visceral sensations (Boorstin, 1990). Thus, these three DMPs are (eventually) referred to in this study within the analytical discussions of students' DMPs, offering a basis of comparison/contrast to students' DMPs.

Rationale

Gadanidis and Borba (2008) state that there is a need "to investigate whether mathematics itself is being transformed by the performance affordances of new media" (p. 50).

This dissertation is the first doctoral thesis in mathematics education that focuses specifically on the notion of DMP. Research projects about DMP have been conducted (e.g., www.researchideas.ca) as well as refereed articles and book chapters have been published in journals and conferences focusing on that notion. However, as the literature presented in chapter one points out, there is a need regarding the development of an "extended academic work" such as a doctoral thesis on DMP. Moreover, nowadays, students use digital technology in their everyday activities and the interdisciplinary dimension between mathematics and the arts has a potential to provide contexts for students' creativity (Gadanidis, Hughes, & Cordy, 2011). The investigation conducted in this study about the ways students communicate ideas and use the arts in a DMP can be seen as a window[1] on mathematical activity in classrooms involving playful, public, and technological practices in pedagogic scenarios.

Doxiadis (2003) discusses the role of narratives in mathematics. Traditionally, mathematics "is the very prototype of a logico-deductive science . . . The main task of a mathematical field is a full classification of its objects of inquiry" (p. 7). In contrast, "stories – like all art – speak to us only to the extent that we are touched by them" (p. 8). Doxiadis sees "the invasion of storytelling into mathematics [as] a revolution" (p. 6). According to Doxiadis, narrative connects mathematics to the soul:

> Mathematical narrative must enter the school curriculum, in both primary and secondary education. The aim is: a) to increase the appeal of the subject, b) to give it a sense of intellectual, historical and social relevance and a place in our culture, c) to give students a better sense of the scope of the field, beyond the necessarily limited technical mathematics that can be taught within the constraints of the school system. . . . Mathematical narrative must supplement and interact with technical mathematics teaching . . . So, use some of the time to make sure what they are taught sticks. Save time for narrative, use it to embed mathematics in the soul. (p. 20)

Considering the parallel between typical (e.g., logic-deductive, linear, cold, non-human) mathematical representations, texts or discourses and the emergence of mathematical narratives or stories to communicate and represent mathematics (e.g., aesthetic and emotional human experiences), this study focuses on ways students may produce multimodal mathematical texts/narratives that offer the audience surprises (ways of seeing the new and wonderful in mathematics), sense-making, emotional moments, and visceral sensations.

[1] Noss and Hoyles (1996) say, "[the] window metaphor is important. Windows are for looking through, not looking at. It is true that windows mediate what we see and how we see it. Equally, windows can, at times, be objects for design and study. But in the end, what counts is whether we can see clearly beyond the window itself onto the view beyond" (p. 10).

Research Purpose and Questions

This study takes place at the intersection of mathematics education, digital technology, and the performance arts. My purpose is to produce knowledge about the nature of elementary school students' DMPs through the interpretation, from a performance art point of view, of the mathematical ideas explored by students in DMPs and how these ideas are communicated through the arts and the use of technology. As mentioned, for this research, I will be investigating twenty-two DMPs produced by elementary school students, available at the Math + Science Performance Festival.

The research questions of this study are:

- What is the nature of elementary school students' digital mathematical performances in the Math + Science Performance Festival?
- What are the mathematical ideas explored and how do students communicate them using the performance arts?

Structure of the Dissertation

In this *prologue* I introduced the notion of DMP and presented three examples of DMPs. I mentioned that in this study I will be analyzing elementary school students' DMPs available at the Festival and I presented the research questions. In *chapter one*, I present a literature review about DMP. I discuss themes linked to the notion of DMP such as mathematics, technology, the arts, audience and narrative, multimodality, embodiment, visualization, creativity, surprises, and aesthetics.

In *chapter two* I present the theoretical framework of the study. Initially, I locate myself as a sociocultural/postmodern interpreter. Then, I discuss the notions of humans-with-media (Borba & Villarreal, 2005), cognitive ecology (Levy, 1993), and multimodality (Kress, 2003; The New London Group, 1996; Walsh, 2011) to highlight the role of digital technology in shaping and reorganizing the production of mathematical knowledge. I also introduce (a) Boorstin's (1990) categories about what makes good films as a performance arts lens and (b) the mathematical strands and processes of the Ontario curriculum (Ontario Ministry of Education, 2005). The performance arts lens in combination with the curriculum components lens structures the analysis and offers insights on the nature of students' DMPs in this study.

Seeking synergy between theoretical perspectives and methodological procedures, in *chapter three* I discuss qualitative research methodology. I introduce students' DMPs to describe the process of data collection and I discuss the notions of video analysis and case studies to highlight issues on data analysis in this study. Based on these notions, I present and describe the analytical procedures taken in the conduct of this study.

In *chapter four* I present a descriptive, single-case analysis of each DMP, based on Boorstin's (1990) categories: (a) *Voyeur – new/wonder/surprising*; (b) *Voyeur – sense-making*; (c) *Vicarious emotions*; and (d) *Visceral sensations*.

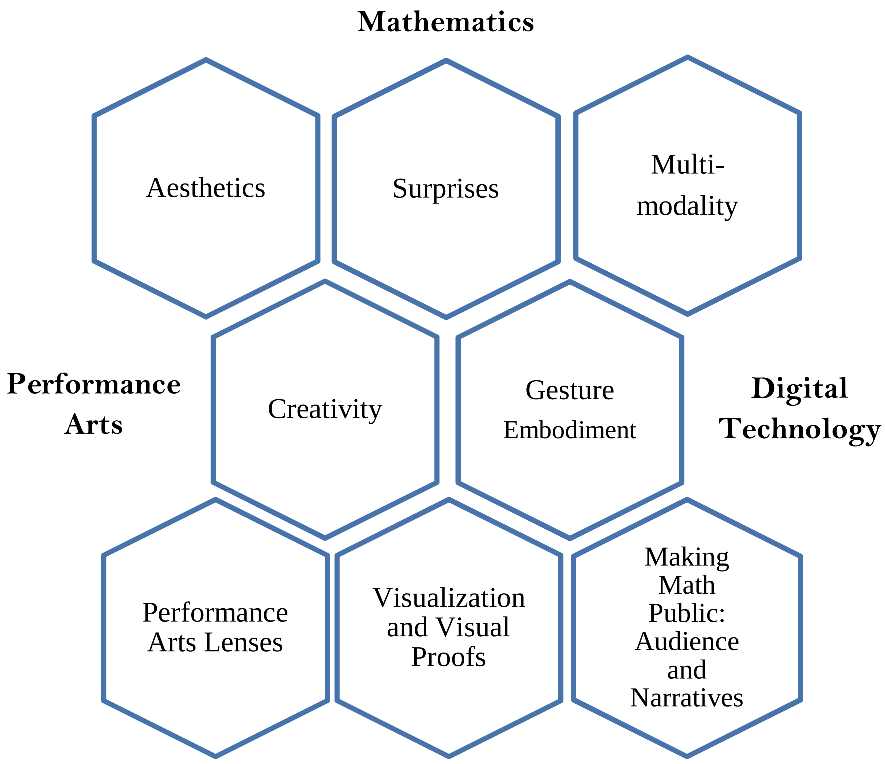
Consequently, in *chapter five*, I present a cross-case analysis, comparing/contrasting the cases, and exploring similarities and patterns among them. In chapter five, I use both the performance arts lens constructed through Boorstin's (1990) perspectives and the curriculum component lens (Ontario Ministry of Education, 2005).

Finally, I present an *epilogue* to indicate the main findings of the study as well as limitations of it and possibilities for future studies on DMP.

Chapter One: A Literature Review on Digital Mathematical Performance

In this chapter I present a literature review on digital mathematical performance (DMP), considering the main issues pointed out in this study. The literature review is organized in themes that are extended and linked to the data analysis presented in chapters four and five. Initially, I present an overview about the three main themes related to DMP: mathematics, digital technology, and the performance arts. Then, I present eight sections that are related to the three main themes. These sections are: making mathematics public: the audience and the narratives; multimodality; gesture and embodiment; visualization and visual proofs; performance arts lenses; surprises; creativity; and aesthetics. Figure 1.1 presents a diagram that represents the themes explored in this literature review and ways to consider connections between them:

Figure 1.1: Themes of the Literature Review



Mathematics

Gadanidis (2006) argues that:

> The answer to ‘What is digital mathematical performance?’ lies initially with ‘mathematics’ and not with ‘digital’ and ‘performance’. The first question to answer is not ‘How do we represent mathematics multimodality in a digital environment?’. . . and it is not, ‘How do we perform mathematics?’ The first question needs to be ‘What mathematics is appropriate or worthy of performance?’ (p. 1)

Gadanidis (2007b) posits that wonderful mathematics experiences are *rare events* (Noss, 2005), but DMP can offer a way for students and teachers to talk about *big mathematical ideas* (Gadanidis& Borba, 2008). By creating music or movies, *math performers* can provide surprises to their audiences and communicate rich mathematical ideas in creative ways. Gadanidis and Hughes (2008) state that *conceptual* DMPs: (a) connect mathematical ideas; (b) offer some surprise; and (c) express feelings and emotions. According to Gadanidis, Hughes, and Cordy (2011), it is important to emphasize that:

> Students can add artwork to “decorate” procedural knowledge, thus adding a layer of sugar-coating to otherwise dry mathematical ideas, but mathematical art, like art in general, requires a deeper engagement and understanding. Thus, for us, challenging mathematics is a corequisite for artistic mathematical expression. (p. 424)

Gadanidis, Hughes, Scucuglia, and Tolley (2009) use the expression “*low floor and high ceiling*” to refer to rich math ideas within DMP. It refers to those ideas that explore mathematical concepts or problems that one can engage with minimal mathematical knowledge (*low floor*) and it can be extended to advanced mathematical ideas (*high ceiling*). The mathematics explored in DMPs such as *Flatland* and *L-Patterns* has low floor and high ceiling, as these ideas have been explored by both Grade-2 students and by pre-service teachers (Gadanidis, 2007b; Gadanidis, Hughes & Borba, 2008). Hoyles and Noss (2006) highlight the role of information technology in *opening windows* onto conceptual mathematical problems.

> The key point is that expressive computational engagement on the part of students offers observers a *window* onto mathematical meaning under construction; or put another way, while students use and construct tools to build models to explore and solve problems, their thoughts become simultaneously externalised and progressively shaped by their interactions with the tools. (Hoyles & Noss, 2006, p. 3)

Digital Technology

Digital technology is theoretically and pragmatically significant for DMP. Borba (2007) posits that “different media change the mathematics produced” (p. 15). Borba and Villarreal (2005) propose the notion of humans-with-media to argue that technology is not neutral in knowledge production. As I discuss in chapter two, I assume the view that media shape and reorganize mathematical thinking. Humans-with-media form thinking collectives in the processes of producing a DMP (see Borba, 2007; Borba & Gadanidis, 2008; Gadanidis & Borba, 2008; Scucuglia & Borba, 2007; Scucuglia, Gadanidis & Borba, 2011a).

Gadanidis, Hughes, and Cordy (2011) discussed the notion of humans-with-media “in a study of a short-term mathematics program for grade 7–8 gifted students that integrated open-ended mathematics tasks with the arts (poetry and drama) and with technology” (p. 397).

> Technological tools may be seen as playing the role of a more knowledgeable other, offering both choice in new directions for extending knowledge as well as scaffolding for developing a more complex conceptual understanding. This view is supported by Levy (1997) and Borba and Villareal (2005), who suggest that technology itself is an actor in the collaborative process. The technological tools may also be seen as catalysts, providing opportunities and tools for collaboratively exploring more complex mathematical ideas. We noted earlier that humans-with-media may be seen as forming a collective where new media also serve to disrupt and reorganize human thinking (Borba & Villareal, 2005). The technological tools used by the gifted students were not simply information or representation tools; rather, they were tools the students thought with. (Gadanidis et al., 2011, p. 426)

Gadanidis and Geiger (2010) add that “when we immerse ourselves in using a technology (and this immersion is a critical component), we naturally think with that technology” (p. 95). Scucuglia and Borba (2007) conducted a workshop for in-service mathematics teachers at the Brazilian National Conference in Mathematics Education. The authors describe how teachers can produce DMPs using accessible and friendly software (e.g., *Microsoft Photo-Story 3.0* and *Microsoft Moviemaker*). Scucuglia and Borba argue that the design of the software shapes the design of the DMPs. That is, the nature of the media shapes the ways DMPs are produced and represented; it shapes the nature of mathematical meaning production in DMPs. By constructing an immersive environment, mathematics teachers can learn and teach “digital languages” proposing the production of DMPs as a mathematical activity to be conducted in classrooms. Borba and Scucuglia (2009) investigate how the design of mathematical learning objects can involve the use of digital media, modeling, and the performance arts. According to the authors, within a scenario of in-service mathematics teacher education, the collaborative use of DMPs in online courses form communities of practice and offer support and ways in which teachers can (a) share their experiences online and (b) bring into their face-to-face classrooms activities that emphasize students' use and production of DMPs.

Rosa (2008) used the notions of humans-with-media (Borba & Villarreal, 2005) and constructionism (Papert, 1980) as lenses to investigate how teachers construct online identities when they explore Calculus through a virtual *Role Playing Game* (RPG). Rosa highlights the use of RPG in teacher education and suggests that "the construction of online identities shows itself *in transformation*, *in immersion* and *in agency* to the teaching and learning. These three facets consider the 'being-with', 'thinking-with' and the 'knowing-making-with'" (p. 1).

According to Borba (2007):

> As access to Internet increases, and user friendly tools become more available, possibility of collaboration and constant change in pieces of digital mathematical performance may become even more real. Modeling and digital mathematical performance are in the process of becoming not only alternatives for face-to-face classroom, but also for online classes. (p. 9)

Moreover, regarding the Web 2.0 affordances (e.g., instant communication, video streaming, high speed file exchange), studies have pointed out the emergent relevance of DMP and the use of digital technology in online distance education (see Borba, 2009; Borba & Scucuglia, 2009; Gadanidis & Borba, 2008; Gadanidis & Geiger, 2010; Maltempi & Malheiros, 2010).

The Performance Arts

Performance art is generally defined as a "multimedia art form originating in the 1970s in which performance is the dominant mode of expression. Performance art may incorporate such elements as instrumental or electronic music, song, dance, television, film, sculpture, spoken dialogue, and storytelling" (http://encyclopedia2.thefreedictionary.com/Performance+arts). More specifically:

> Performance art is art in which the actions of an individual or a group at a particular place and in a particular time constitute the work. It can happen anywhere, at any time, or for any length of time. Performance art can be any situation that involves four basic elements: time, space, the performer's body and a relationship between performer and audience. (http://dictionary.sensagent.com/performance+arts/en-en/)

There are many studies focusing on links between the arts and mathematics through the notion of aesthetics, symmetry in geometry, geometric projection and space in painting, parallels between the history of mathematics and the history of the arts, patterns and sequences of numbers in music theory, fractal geometry, etc. (Abdonour, 1999; Presmeg, 2009; Sinclair, 2001; Sinclair, Pimm & Higginson, 2006). Some studies focus on artistic-cultural issues through the notion of ethnomathematics (D'Ambrosio, 2006; Gerdes, 2010) and others on interlocutions involving the arts and digital technology (Banchoff & Cervone, 1998; Louro & Fraga, 2008). Specifically, some studies focus on the use of literature, narratives, comics, and storytelling in teaching and learning mathematics (Altieri, 2009; Franz & Pope, 2005; Goral & Gnadinger, 2006; Kinniburgh & Byrd, 2008; Shatzer, 2008; Van den Heuvel-Panhuizen & Van den Boogaard, 2008;Ward, 2005; Wilburne, Napoli, Keat, Dile, Trout & Decker, 2007) and on learning (mathematics) through drama, dance, music, and role playing games (Cooper & Barger, 2009; Jehen, 2008; Johnson, 2009; Kynyon, 2008; Snyder, 2000; Spielman, 2007; Tselfes & Paroussi, 2009). There are also some studies focusing on mathematical learning through role playing games or narratives considering the use of digital technology (Herbst, Chazan, Chen, Chieu, & Weiss, 2011; Rosa, 2008; Sinclair, Healy, & Sales, 2009). A few studies mention links between performance arts and mathematics (Brown, 2007; Duatepe-Paksy & Ubuz, 2009; Kishore, 2006), but the theorization on these studies does not emphasize the same issues highlighted in DMP, which focuses simultaneously on the performance arts, teaching and learning mathematics, and the use of digital technology. Seeing mathematics (education) as performance, Brown (2007) states:

> Mathematics must surely join art in seeing its domain as transcending singular notions of beauty. Mathematics, like art, can teach us about ourselves, but not necessarily through didactic means. That is, the student may be allowed to learn their own lesson rather than the one supposed by their teacher or some other authority about what constitutes beauty or correctness. (p 759)

> Contemporary art has long since moved on from notions of art objects being admired by independent observers. Similarly, practitioners in mathematics education, insofar as they see themselves sharing some of art education's aspirations, (such as attending to aesthetic qualities, self expression, learning about oneself) are potentially locked into a similar time warp as regards how we should understand mathematical objects. In mathematics education there is much benefit to be gained from understanding the discipline's aesthetic qualities and in finding ways to enable our students to share these pleasures. (p. 763)

Gerofsky (2006) presents some foundations of the performance arts to theorize DMP, highlighting the trans-disciplinary characteristic of performance studies and how it has been related to the use of digital media:

> Performance studies draw together notions of performance from such diverse sources as theatre, literature, visual arts, anthropology, sociology, linguistics, artificial intelligence and cultural studies. The kinds of activities included as "performances" are tremendously varied … In recent years, theorists of performance have expanded the notion of performance to include digitally-mediated performances, especially since these can now have a strong interactive, 'live' component rather than simply acting as an archive of an ephemeral live performances. (p. 1)

Gerofsky (2006) then proposes a view of DMP that mentions the relevance of the use of digital media in combination with other elements such as embodiment, audiences, variety of spaces, imagination, and complex mathematical problems. She suggests that it "would not be solely digital, but would move easily between online, onscreen experiences, physically-present, kinesthetic embodied experiences with other people in a variety of spaces, and quietly conceptual individual imagining, working and thinking" (p. 10). According to Gerofsky:

> It is unusual (and energizing) to link mathematics and math education with performance, in no small part because many of the things that make a performance distinctive and interesting go squarely against many of the long-held traditions of mathematics. [It is important to] explore the human necessity of performance in mathematics, [suggesting] some of the varieties and features of performance that might be offered to mathematics education from other fields. (p. 2)

Making Mathematics Public: The Audience and the Narratives

Cyberspace is usually the locus that makes DMPs public. When DMPs are published at the Math + Science Performance Festival, for instance, students and teacher are communicating and sharing their ideas beyond the classrooms. They are communicating them to the world (Gadanidis & Geiger, 2010). Then, the Internet is a scenario and a protagonist that makes DMPs social and cultural artefacts. It is a nexus with potential for collective intelligences (Levy, 1997). According to Gadanidis and Borba (2008):

> If we think of art and the Internet as being different actors, in the same anthropomorphic eyes that Borba and Villarreal (2005) use to see new media, it is reasonable to expect that they will interact with and influence and change mathematics. (p. 50)

Gadanidis and Geiger (2010) have referred to the Math + Science Performance Festival as “one example that helps bring the mathematical ideas of students into public forums where it can be shared and critiqued and which then provides opportunity for the continued development of knowledge and understanding within a supportive community of learners” (p. 102). Gadanidis and Geiger (2010) also posit the Festival “offers a glimpse into how collaboration in mathematics learning might be extended to include math performance, or perhaps how collaboration in a media-rich digital environment might be reconceptualized as collaborative performance” (p. 101).

Gadanidis, Hughes, and Scucuglia (2009) present a case study to show how a first nation community was engaged in creating and presenting digital performances about both mathematics and their culture and history. The authors discuss the pedagogic relevance of making mathematics public to the community and to the virtual world. They emphasize this process was relevant to (a) students’, parents’, teachers’, and researchers’ mathematical learning, (b) see mathematics differently, (c) raise expectations of what students can do, and (d) build new relationships in the community and between mathematics educators and professional artists.

The role of *audience* is fundamental in performance arts. At same time, one of the goals of the research projects on DMP[2] is to bring mathematics to the public (a variety of audiences), beyond the classroom. Gadanidis, Hughes, Scucuglia, and Tolley (2009) mention the parallel between one’s favourite book or new movie and one’s favourite mathematical idea. By using the

[2] Digital Mathematical Performance (www.edu.uwo.ca/dmp). Students as Performance Mathematicians (www.edu.uwo.ca/mpc/students.html).

notion of Boal's (1985) *Theatre of Oppressed* to challenge the traditional view of students as spectators, Gadanidis and Borba (2008) suggest that DMPs can offer ways for the emergence of students as *spect(actors)*. These authors emphasize students' active and collective role in producing DMPs in learning. Gadanidis, Hughes, and Borba (2008) do not suggest that all mathematics should be taught using performance arts. However, thinking about mathematics as performance may lead to new ways for students to (a) learn with technology, (b) disrupt traditional power and authority structures in classrooms, and (c) share mathematics beyond the classroom (Gadanidis, Gerofsky & Hughes, 2008).

The very notion of *narrative* also highlights issues surrounding the role of the audience. DMPs can be understood as *digital narratives* (Gadanidis, Borba, Hughes & Scucuglia, 2010; Gadanidis, Hughes & Scucuglia, 2009). As mentioned in the prologue, Doxiadis (2003) sees "the invasion of storytelling into mathematics [as] a revolution" (p. 6). Based on Bruner's (1994) perspectives,[3] Gadanidis et. al. (2010) argue that (a) narratives are fundamental media of communication, and (b) there is a dialogical relationship between narrative/identity and community. Narratives are social artifacts and "the narrated self is constructed with and responsive to other people" (Miller & Goodnow, 1995, p. 172). The creation of a narrative involves the process of thinking about the *self* and the *other*, and about how to portray an image of the self to an audience. Thus, by producing DMPs, students and teachers construct identities as *performance mathematicians* (Gadanidis, Hughes & Borba, 2008; Scucuglia, 2011; Scucuglia, Borba & Gadanidis, 2010).

Bruner (1996) makes distinctions between narrative and paradigmatic modes of thinking. According to Sinclair, Healy and Sales (2009), for Bruner "paradigmatic thinking is logical, deductive, and timeless; it deploys warranted assertions and justified reasoning. In contrast, the narrative mode strives to put its timeless miracles into the particulars of experience and to locate the experience in time and place" (p. 442). Interestingly, like Doxiadis (2003), Sinclair, Healy and Sales (2009) point out the combination or symbiosis between paradigmatic thinking and narrative in mathematics education.

> In sum, both the narrative and the paradigmatic have fundamental roles in the construction and [organization] of knowledge, but they have different motivations: while

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[3] Here, the notion of *agency* is fundamental. It refers to "the capacity for autonomous social action. Agency commonly refers to the ability of actors to operate independently of the determining constraints of social structure" (Oxford *Dictionary of the Social Sciences*, 2002). This notion is also central within socioculturalism.

> the paradigmatic is concerned with what is, given the constraints of the system in question, and with identifying and proving generalities that [characterize] objects and relations in the system, the narrative focuses on particular activities of these objects as they are played out in time, on what might be behind the events in question and on how they resemble or remind us of other things we know about . . . [in our research] narrative modes of thinking will emerge quite strongly as students interact with mathematical objects and relationships. (Sinclair, Healy, & Sales, 2009, p. 443)

It is also important to notice that mathematics has a problem with its public image. Gadanidis and Scucuglia (2010) present a literature review to argue that many studies (e.g., Lim. 1999; Frank, 1990; Furinguetti, 1993; Picker & Berry 2000; Rensaa, 2006; Rock & Shaw, 2000) highlight that "both students and adults hold negative stereotypical views of mathematics and mathematicians" (p. 22). Gadanidis and Scucuglia (2010) argue that virtual learning objects produced based on the notion of DMP may support alternate ways of displaying the work of mathematicians. Video recordings of activities conducted by mathematicians may be seen as performative in the learning objects available at http://www.fields.utoronto.ca/mathwindows/. Watching different mathematicians in different workplaces, using different materials, exploring different ideas, helps people to disrupt negative images they potentially hold about mathematicians. Consequently, it also has an impact on how people see mathematics.

Gadanidis, Gerofsky, and Hughes (2008) add "exploring mathematical ideas through performance can offer a way to challenge some of the most limiting stereotypes around mathematics learning" (p. 19). Through DMP, learning mathematics can be viewed as an aesthetic and human experience (Higginson, 2006) rather than an impersonal and unpleasant activity. Gadanidis and Borba (2008) argue that seeing mathematics through performance lenses offers ways to realize that: (a) mathematics is a human experience rather than a cold science; (b) mathematics is about gaining insights on the complexity of mathematical ideas rather than learning procedures for getting correct answers; (c) a good teacher creates situations where students have to think hard rather than making learning easy; and (d) teaching should start with what a child can imagine rather than with what a child already knows and understands.

Multimodality

Pahl and Rowsell (2005) posit that the word *multimodal* “describes the way we communicate using a number of different modes to make meaning” (p. 27). Rowsell and Walsh (2011) state that “multimodality is the field that takes account of how individuals make meaning with different kinds of modes” (p. 55-56). According to Walsh (2011), *multimodality* is “a study of the communicative process, particularly how meaning is communicated through different semiotic or meaning-making resources and in different social contexts” (p. 105).

> Multimodality as in comprehension and competence with language through a variety of modes such as image, sound, touch, multi-dimensions, is the principle upon which digital environments work. This principle of multimodality needs to be understood for educators to apply and assess new modes of learning as a part of everyday classroom practice. (Rowsell & Walsh, 2011, p. 54)

Barton (2008) posits that “mathematics is created in the act of communication … [and] mathematics is learned through communication” (p. 144). Kotsopoulos (2007) states that “the legitimization of communication has important social and pedagogical implications” (p. 7) and, thus, communication is a fundamental element in mathematics curricula.

Gadanidis, Gerofsky, and Hughes (2008) argue that DMPs offer ways to communicate mathematics using different modes. Gestures, images, writing, sounds, and verbal language are fundamental modes of communication in learning mathematics. DMPs combine multiple modes of communication (Gadanidis, 2006, 2007b; Gadanidis, Hughes & Borba, 2008). Besides writing, DMPs are composed of videos, images, drawings, flash simulations, sounds, speeches, gestures, and other elements that compose multimodal designs. DMPs can be seen as multimodal texts/narratives (Scucuglia, 2012).[4]

According to Gadanidis, Hughes, and Cordy (2011), “the use of multimodal expression changes the feel of the learning environment” (p. 425). In online environments, students and teachers can use text, drawings, and images, and various tools and representations.

> Different modalities - aural, visual, gestural, spatial, and linguistic - come together in one surround in ways that reshape the relationship between printed word and image or printed word and sound. Thinking with and communicating through multiple representations is a

[4] Although my focus is on learning and pedagogy, it is important to notice the socio-educational dimension of multimodality. Klein and Kirkpatrick (2010) clarify that research on multimodal representations “exemplifies the contemporary effort to democratize education by empowering students” (p. 89).

> common expectation in current mathematics curriculum reform documents. (Gadanidis, Hughes & Cordy, 2011, p. 425)

Evans, Feenstra, Ryon, and McNeill (2011) argue that a multimodal approach:

> Aims to take into account the range of cognitive, physical, and perceptual resources that people utilize when working with mathematical ideas . . . The ability of students to effectively use gestures, as an additional form of communication, can be further refined through the implementation of both physical and virtual manipulatives, and whether these manipulative forms differently elicit gesture and other forms of communication. If differences were detected, then the design, implementation, and use should follow suit. (p. 258)

Radford (2009) highlights that in a "multi-modal view of thinking, the problem is not focused on gestures only" (p. 123), but the literature in mathematics education usually links multimodality to the notions of gestures and embodiment in mathematical learning. Interestingly, gestures and embodiment are also important aspects in the performance arts (Gerofsky, 2006).

Gesture and Embodiment

Most of the DMPs analyzed in this study are video recordings of students' playful (mathematical and theatrical) activities. The notion of gestures is thus a significant aspect of students' communication in DMPs. Gerofsky (2010) defines gestures "as movements of hands, face and other parts of the body employed in a largely unconscious way for non-verbal communication" (p. 322). According to Gerofsky, "theories of embodied learning take for their starting point the fact that people experience the world as situated, sentient, emotive mind/bodies" (p. 322). Gerofsky also states that "embodied learning theory rejects the notion of a split between mind and body or reason and sensation/emotion and explores pedagogies that integrate somatic, sensory and intellectual engagement on the part of learners" (p. 322). Gerofsky highlights that:

> In mathematics education, embodiment is particularly interesting because of the abstract, disembodied nature of disciplinary goals in this subject area. Researchers have begun to pay attention to the fact that, even in mathematics, learners' abstract conceptual knowledge draws on bodily experience, the use of concrete objects and engagement with imagined or

> virtual objects and actions that recall actual physical experiences. These bodily experiences ground the abstractions of language and mathematical symbolism. (p. 322)

Researchers have discussed the relations between speech and gestures in mathematical activity and cognition (Arzarello, Paola, Robutti, & Sabena, 2009), including the use of technology (Borba & Scheffer, 2004). Radford (2009) mentions that the point in emphasizing the role of gestures in mathematical activity

> [Is not to] diminish the cognitive role of the written. It is rather an invitation to entertain the idea that mathematical cognition is not only mediated by written symbols, but that it is also mediated, in a genuine sense, by actions, gestures and other types of signs. (p. 112)

Gerofsky, Savage, and Maclean (2009) explore the notion of "being the graph" to discuss the multimodal and gestural-kinesthetic dimension of students using their bodies to represent the dynamicity of graphs of functions and its coordination with algebraic and table representations. Núñez (2006) highlights that there is a diversity of perspectives about speech and gesture in mathematics education. Some of these issues are:

> (1) Speech accompanying gesture is universal . . . (2) Gestures are less monitored than speech . . . (3) Gestures show an astonishing synchronicity with speech . . . (4) Gestures can be produced without the presence of interlocutors . . . (5) Gestures are co-processed with speech . . . (6) Hand signs are affected by the same neurological damage as speech… (7) Gesture and speech develop closely linked . . . (8) Gesture provides complementary content to speech content . . . (9) Gestures are co-produced with abstract metaphorical thinking. (Núñez, 2006, p. 175-176)

Visualization and Visual Proofs

Visualization is a very significant aspect of mathematical thinking and reasoning and it is commonly explored though the use of technology in mathematics. DMP offers ways to explore visual aspects (visual modality of communication) in mathematical learning.

Presmeg (1986) states that visualization involves a mental scheme that represents visual or spatial information. It is a "kind of reasoning activity based on the use of visual or spatial, either mental or physical, performed to solve problems or prove properties" (Gutiérrez, 1996, p. 9). Borba and Villarreal (2005) point out that "visualization has been considered as a way of reasoning in mathematics learning" (p. 79), because it refers to "a process of forming images …

and using them with the aim of obtaining a better mathematical understanding and stimulating the mathematical discovery process" (p. 80; see also Zimmermann & Cunningham, 1991).

Bishop (1989) argues that "there is a wide range of visual imagery used by individuals even when restricted to mathematical activity" (p. 8). Presmeg (1986) identifies five types of students' visual imagery. They are: (i) concrete, pictorial (pictures-in-the-mind); (ii) pattern imagery (pure relations in a visual-spatial scheme); (iii) memory images of formulae; (iv) kinaesthetic imagery (involving muscular activity, e.g., fingers "walking"); and (v) dynamic (moving) imagery (Bishop, 1989).

Hanna and Sidoli (2007) state that "visualisation can be most useful to the aspects of mathematical proof important to mathematics education, particularly those connected with explanation and justification" (p. 77). Zwicky (2000) explores visual proofs from an aesthetic point of view, writing poems formed geometric demonstrations of series that converge absolutely. Hanna (2000) posits that "in the classroom the key role of proof is the promotion of mathematical understanding" (p. 6) and some of the functions of proofs are: verification, explanation, discovery, communication, construction, and exploration. On the one hand, "a number of mathematicians and logicians are now investigating the use of visual representations, and in particular their potential contribution to mathematical proofs" (Hanna, 2000, p. 15). On the other hand, it is important to recognize that there is a lot of controversy about the topic.

> A key question raised by the intensified study of visualization is whether, or to what extent, visual representations can be used, not only as evidence for a mathematical statement, but also in its justification. Diagrams and other visual aids have long been used to facilitate understanding, of course. They have been welcomed as heuristic accompaniments to proof, where they can inspire both the theorem to be proved and approaches to the proof itself. In this sense it is well accepted that a diagram is a legitimate component of a mathematical argument. Every mathematics educator knows that diagrams and other visual representations are also an essential component of the mathematics curriculum, where they can convey insight as well as knowledge. They have not been considered substitutes for traditional proof, however, at least until recently. Today there is much controversy on this topic, and the question is now being explored by several researchers. (Hanna, 2000, p. 15)

Performance Arts Lens

Gadanidis and Borba (2008) argue DMPs can be analyzed through a performance arts lens. These authors use Boorstin's (1990) lenses to analyze *Flatland*. Boorstin (1990) states:

> We don't watch movies one way, we watch them three ways. We derive three distinct pleasures from watching a film, which I call the voyeur's, the vicarious, and the visceral. Each demands a different set of filming techniques, often in contradiction with the others; it has its own sort of content, its own rules, the three compete within us. (p. 9)

These three lenses – voyeuristic experience of the new, wonderful and surprising; vicarious emotional moments; and visceral sensations – refer to what makes good movies or films. Gadanidis and Borba (2008) make a parallel using these lenses to discuss what makes good mathematical experiences and good DMPs. Boorstin's perspective is discussed within the theoretical framework of this study and these lenses form the main body of analytic categories in this research. Gadanidis and Borba (2008) point out five reasons to use Boorstin's (1990) lens: (a) this task is exploratory and studies of this nature represent a first step to looking at mathematics education through a performance lens and what differences this lens might make; (b) it allows one to transcend movies and open possibilities to analyze the design of DMPs to identify "patterns of meaning" involving mathematical thinking; (c) movies, besides being entertainment, have an important role as a story in terms of education experience and sense-making; (d) the sense of story is important in developing mathematical thinking and, consequently, in developing the engagement and understanding of students; and (e) there is a convergence between the interpretative perspective developed on performance and the aesthetics of (mathematics) teaching and learning.

Gadanidis (2009) argues:

> Using Boorstin's analysis of what makes movies work, and paraphrasing his ideas to suit our mathematics education context, school math *works* when: (1) it helps us experience the new and the wonderful in mathematics (as big ideas); (2) it engages us with these ideas in a way that keeps surprising us (unlike a predictable plot, where we can easily guess what will happen next); (3) it provides opportunities for us to connect emotionally with our math experiences; and (4) it helps us sense mathematical beauty. (p. 45)

According to Gadanidis, Hughes, and Borba (2008):

> Boorstin suggests that we experience three distinct pleasures from watching a movie, which we will paraphrase: (1) the pleasure of experiencing the new, the wonderful, and the surprising in mathematics; (2) the pleasure of experiencing emotional mathematical moments (either our own or others); and (3) the visceral pleasure of sensing mathematical beauty . . . The L pattern activity offers a number of surprises: As a representation of the odd numbers, it reveals that the sums of these odd numbers are square numbers, which can be represented physically as squares . . . Students who work on this activity become excited about the patterns they see, and they share their ideas and their excitement with others. Students may also experience moments of frustration or anxiety and share these emotions, as well . . . Zwicky (2003) commented on this square pattern formed by the Ls, stating that such patterns draw our attention and invite us to look at things like this . . . We are drawn to forming such patterns . . . Sinclair (2001) notes that an aesthetic mathematics experience often involves a sense of pattern or a sense of fit. (p. 171-172)

Surprises

The literature in mathematics education has discussed the role of surprises in mathematical activity. Watson and Mason (2007) "tend to see surprise as a positive emotion [and] mathematics as full of philosophical and cognitive surprises; surprise as motivating curiosity and effort" (p. 4).

Floyd (2011) states that the concept of *surprising* is "centrally important … for the philosophy of mathematics" (p. 128). The author argues that mathematics is dependent on surprises, on the unexpected, on the beauty, in order to capture our interest in practicing mathematics. Floyd contextualizes the senses of surprising, wonderfulness, and admirableness mentioning the long-term engagement of a mathematician in proposing and proving a new theorem (or corollary). Focusing on the work of Wittgenstein, Floyd (2011) argues:

> The concepts of the surprising, the interesting, and the change of aspect of things are centrally important, both for Wittgenstein and for the philosophy of mathematics . . . the surprising may be accommodated within our discussions of mathematics without forcing us to adopt either Platonism or eliminative anti-psychologism about the phenomena at issue. [Floyd's arguments highlights] the usefulness of Wittgenstein's approach to the

investigation of surprise in mathematics, which resists construing the notion as everywhere indicating the discovery of new objects or facts. (p. 128)

Floyd's perspectives help me to see and understand the significance of surprises in DMPs on two levels: (1) when students play roles as performance mathematicians to produce a DMP, and (2) when DMPs offer mathematical surprises to the *audience*; that is, surprises are significant to students' mathematical engagement and activity when they produce a DMP and to the audiences' engagement and curiosity in thinking mathematically. Interestingly, Adhami (2007) highlights a pedagogical dimension of surprises in exploring mathematics.

> Surprise implies facing something unexpected, out of place, counterintuitive, or somewhat 'not fitting.' It implies that the learner has met the elements of the situation separately before, but is now associating them for the first time, as in much of learning. (Adhami, 2007, p. 34)

Beyond seeing surprises from a personal/cognitive point of view – that is, surprise as causing cognitive conflict and re-shaping mental models in order for the achievement of equilibrium – Adhami (2007) also explores a social (or pedagogical) dimension of mathematical surprises to discuss settings such as classrooms. Adhami states that a requirement for increasing the chance of surprise is to structure a task to have a low floor and high ceiling, which is also a significant aspect pointed out by Gadanidis, Hughes, Scucuglia, and Tolley (2009). Moreover, Adhami argues that there are three significant phases when considering the development of tasks and emergence of surprises in the mathematics classroom. They are: (a) engagement with the task; (b) probing, playing, groping for patterns and testing; and (c) sharing, formalization, and proof.

> Capitalising on students' surprise in the classroom has a few requirements. An initial motivational requirement is of generating minimal interest to engage with the activity, often linked in younger students with a story or a strange disagreement to be resolved. To sustain engagement, however, the activity must be structured to allow more than one 'puzzling-out' route towards some worthwhile insights, providing also that some of these routes are accessible to most of the students. In other words, the activity should he structured to have a 'low floor', 'high ceiling' and a number of routes and steps in between. (Adhami, 2007, p. 35)

Although a DMP can be considered a "digital product" (like the final proof of a theorem), each DMP reveals some aspects of its creation. Moreover, authors such as Scucuglia, Gadanidis, and Borba (2011) discuss the process of producing a DMP as a classroom activity. The process is based on students' engagement with a task, playing, and sharing, and it involves an engaging process of communicating and representing mathematical learning by using digital technology as a computational strategy. It may be seen even as a process of problem-solving. In addition, the arts are used as a linguistic expression, which usually involves creativity, repetition, and practice. Thus, the aspects of mathematical and pedagogical surprises pointed out by Floyd (2011) and Adhami (2007) are inherent to some DMPs analyzed in this research. The most significant aspects concerning mathematical and pedagogical surprises in a DMP refer to (a) the conceptual nature of the mathematical idea and (b) the modes used to communicate the idea (multimodality), which are shaped by the arts and digital media affordances.

Scucuglia (2011a) explored the notion of *conceptual mathematical surprises* in DMPs, which refer to ideas that offer to the audience ways to see the new and wonderful in mathematics through aspects such as: (a) connections between mathematical ideas, concepts, and strands; (b) connections between representations and modes of communication; (c) creativity and imagination; (d) experimentation with technology and visualization; (e) mathematical modeling; (f) exploration of the foundations of mathematics and the revelation of its crisis; and so forth.

Surprises are then related to creativity as well. Lewis (2006) states that:

> Creativity is an act that produces effective surprise . . . the surprise associated with creative accomplishment often has the quality of obviousness after the fact. The creative product or process makes perfect sense—once it is revealed. For the creative person, surprise is the privilege only of prepared minds—minds with structured expectancies and interests. (p. 36)

Originality is also an aspect related to both creativity and surprise. Hallman (1963) posits:

> Originality means surprise. Just as novelty describes the connections that occur in the creative act, unpredictability to the setting of the new creation in the physical environment, and uniqueness to the product when regarded as valuable in its own right, so surprise refers to the psychological effect of novel combinations upon the beholder. Surprise serves as the final test of originality, for without the shock of recognition which

registers the novel experience, there would be no occasion for individuals to be moved to appreciate or to produce creative works. (p. 20)

Creativity

Mathematical surprises emerge with creativity (Gadanidis, 2007b). The combination of digital media and the arts can offer ways to enhance students' and teachers' creativity in communicating mathematics (Scucuglia, Borba & Gadanidis, 2011b). Gadanidis, Hughes and Borba (2008) argue that "engaging students' [creativity] is a key element of learning" (p. 169).

Creativity can be generally defined as the phenomenon in which an individual creates something new that has some kind of value. Plucker and Beghetto (2004) claim "creativity is the interplay between ability and process by which an individual or group produces an outcome or product that is both novel and useful as defined within some social context" (p. 156). Dictionary.com defines creativity as "the ability to transcend traditional ideas, rules, patterns, relationships, or the like, and to create meaningful new ideas, forms, methods, interpretations, etc.; originality, progressiveness, or imagination: the need for creativity in modern industry; creativity in the performing arts" (http://dictionary.reference.com/browse/creativity).

Runco (2007) posits:

> Creativity is, in a phrase, a vital form of *human capital*. Creativity both contributes to the information explosion and helps each of us copy and adapt to it . . . Creativity plays a role in many everyday activities . . . Creativity plays a significant role in language, for example, and in fact this may be the best example of everyday creativity. (p. ix-x)

According to Hallman (1963), the notion of *connectedness* is central to a definition of creativity. The author states:

> Creativity is both a combination of elements into new relations, and a re-combining of them. This means that creativity is not merely the capacity to connect elements in a new way, but to transplant these new combinations onto previously unrelated materials. It is the capacity to regard life metaphorically, to experience even orderliness as plastic, to shift intellectual processes. (p. 18-19)

It is also important to notice "the word *creativity* is 'fuzzy' and lends itself to a variety of interpretations" (Sriraman, 2005, p. 20). Specifically in mathematics education, there is a variety of discussions (see Beghetto & Kaufman, 2009; Plucker & Zabelina, 2009; Sriraman, 2009).

Selter (2009), for instance, highlights a distinction between *creativity*, *flexibility*, and *adaptivity*. According to Selter, "creativity is the ability to invent new or modify known strategies. Flexibility is the ability to switch between different strategies. Adaptivity is the ability to use appropriate strategies the individual has creatively developed or flexibly selected" (p. 620).

Aesthetics

Lockhart (2011) states that "the first thing to understand is that mathematics is an art . . . Mathematics is the purest of the arts, as well as the most misunderstood . . . Mathematicians sit around making patterns of ideas" (p. 3). Lockhart also highlights that "the art is not in the 'truth' but in the explanation, the argument. It is the argument itself which gives the truth its context, and determines what is really being said and meant. Mathematics is *the art of explanation"* (p. 5). Thus, the notion of aesthetics is also highlighted in the literature in mathematics.

Webster's dictionary (1993) defines aesthetic as:

> 1. The branch of philosophy dealing with such notions as the beautiful, the ugly, the sublime, the comic, etc., as applicable to the fine arts, with a view to establishing the meaning and validity of critical judgments concerning works of art, and the principles underlying or justifying such judgments.
> 2. The study of the mind and emotions in relation to the sense of beauty. (p. 19)

Pimm and Sinclair (2006) mentions that:

> Contemporary views of aesthetics often attempt to combine elements of pleasure with aspects of sensory perception. We are well aware that the notion of aesthetics by itself is far from synonymous with that of 'pleasure', especially perhaps when speaking of mathematics. But it nonetheless seems to us worth asking what are some potential sources of pleasure for mathematicians. (p. 224)

Sinclair (2000) talks about a sense of *mathematical fit* as an aesthetic sensation, a sensing of mathematical beauty. The author posits that:

> [Aesthetics] is partially about discerning patterns or perceiving relations, and taking note of how things relate to one another and how they seem to fit together. When we experience things fitting together, they often look beautiful to us, and they often bring us a sense of pleasure. (p. 4)

Sinclair (2006) adds that “the phase of playing around or ‘getting a feel for’ is aesthetic in so far as the mathematician is framing an area of exploration, qualitatively trying to fit things together and seeking patterns that connect or integrate” (p. 95). According to Sinclair (2006), “mathematicians can be attracted by the visual appeal of certain mathematical entities, by perceived aesthetic attributes such as simplicity and order or by some sense of ‘fit’ that applies to a whole structure” (p. 99).

Final Comments of the Chapter

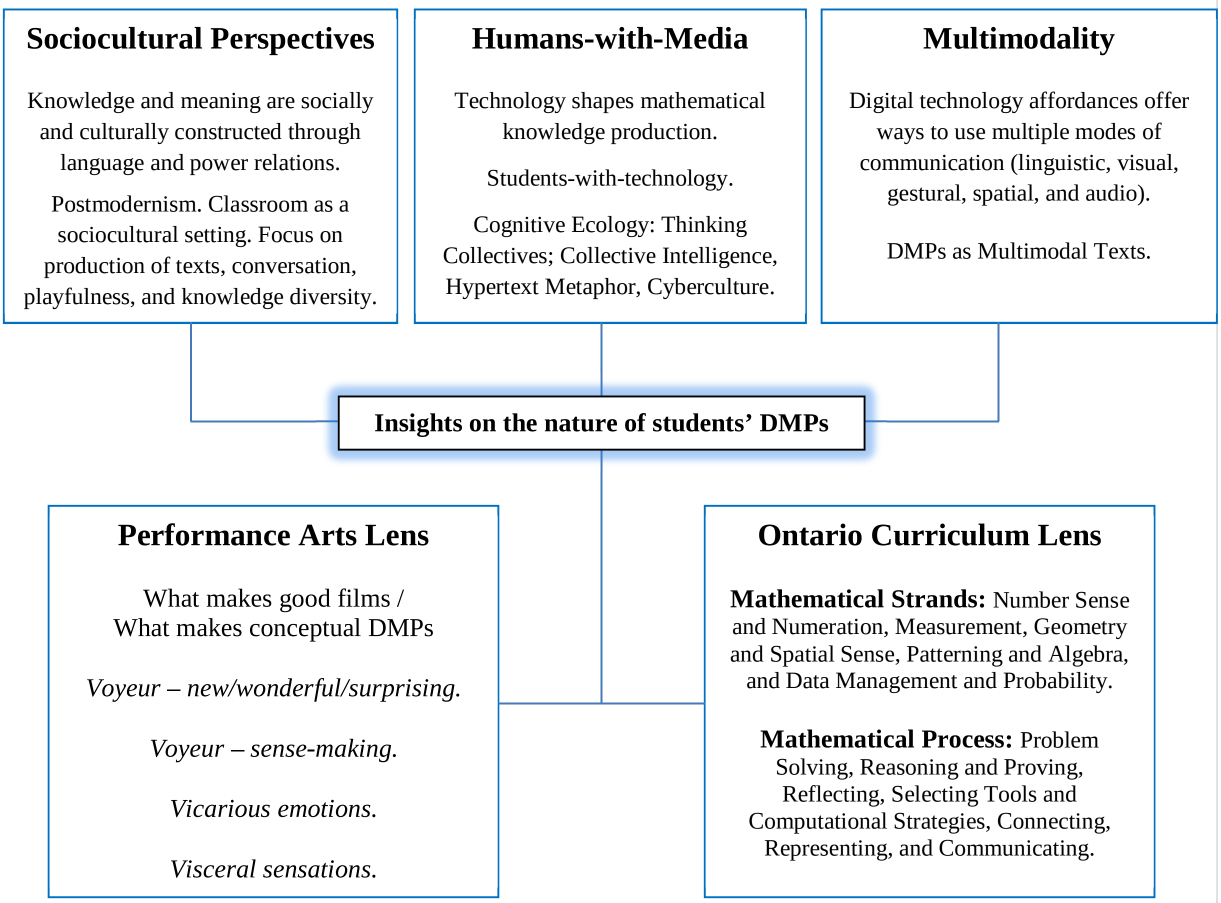
The literature review presented in this chapter highlights the diversity of perspectives explored around the notion of DMP. These perspectives are important to inform and provide directions for the focus of this doctoral study. Several aspects explored in this chapter are linked to (and expanded in) the analysis presented in chapters four and five, guided by the following research questions: What is the nature of elementary school students’ digital mathematical performances in the Math + Science Performance Festival? What are the mathematical ideas explored and how do students communicate them using the performance arts?

Given the increasing digital nature of students’ school and everyday experiences, DMP has great potential to become a trend in mathematics education, which emphasizes the interlocution between the arts and the use of digital technology in mathematics teaching and learning, and considers the nature of the mathematical classroom as digital, multimodal, and playful (Scucuglia, Gadandis, & Borba, 2011b). This study then contributes to that direction: opening some windows toward the notion of DMP, refining some of the perspectives already explored in the literature, and discussing, from a performance art point of view, the ways students communicate ideas, use the arts, and think mathematically in DMPs.

Chapter Two: A Theoretical Framework to Analyze Students' Digital Mathematical Performances

There are five intersecting theoretical themes informing this study. These are: (1) sociocultural perspectives, (2) humans-with-media, (3) multimodality, (4) a performance arts lens, and (5) the Ontario curriculum lens. The first three bodies inform the research goal to explore how (mathematical) knowledge is produced, how students think and communicate ideas, and how they use technology to explore mathematics. The last two lenses offer ways to interpret students' digital mathematical performances (DMPs) from a performance arts point of view, but also considering the components of the mathematical strands and processes of the Ontario curriculum. These lenses offer interpretive ways to explore insights on the nature of students' DMPs. Figure 2.1 illustrates and summarizes some of the key ideas discussed in this chapter:

Figure 2.1: Summary of the Theoretical Framework



Sociocultural Perspectives

Sociocultural perspectives point out that reality, knowledge, and meaning are socially, historically, and culturally produced through language. They connect activity to participation in cultural practices (Cobb, 1994). Instead of focusing on the learner as a biological/genetic being and on the individual processes of meaning-making and knowledge construction (e.g., cognitive conflict and equilibrium in Piagetian constructivism), sociocultural perspectives emphasize the social interaction and enculturation in mathematical learning, development, and activity.

Vygotsky (1978) investigated children's development and how development is conditioned by the role of culture and language. According to Vygotsky, higher mental functions are historically developed within particular cultural groups, through social interactions with the significant people in children's lives, particularly parents and teachers. Through these interactions, children learn the habits of the culture, including patterns of speech, verbal and written language, and other symbolic representations. Thus, Vygotsky emphasized (a) the social interaction with more knowledgeable others in the zone of proximal development[5] and (b) the role of culturally developed sign systems and languages as psychological tools of thinking.

In order to emphasize sociocultural perspectives in mathematics education, Lerman (1996) highlights some conceptual incoherencies of constructivism in terms of learning theory:

> The extension of radical constructivism toward a social constructivism, in an attempt to incorporate intersubjectivity, leads to an incoherent theory of learning. A comparison of Piaget's positioning of the individual in relation to social life with that of Vygotsky and his followers is offered, in support of the claim that radical constructivism does not offer enough as an explanation of children's learning of mathematics . . . Constructivists, whether radical, weak, or social, draw their inspiration from Piaget, for whom the individual is the central element in meaning-making. . . . Vygotsky attempted to develop

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[5] [Zone of Proximal Development (ZPD)] is "the distance between the actual developmental level as determined by the independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). Actually, this notion of ZPD can be re-interpreted through the ontological quadrivium (Levy, 1998), that I present in the next section (see also Scucuglia, 2011b). In addition, *scaffolding* is also a notion related to ZPD. It refers to "a process through which a teacher or more competent peer gives aid to the student in her/his ZPD as necessary, and tapers off this aid as it becomes unnecessary, much as a scaffold is removed from a building during construction" (http://www.innovativelearning.com/educational_psychology/development/zone-of-proximal-development.html). Balaban (1995) adds that "scaffolding refers to the way the adult guides the child's learning via focused questions and positive interactions" (p. 52).

a fully cultural psychology by which I mean placing communication and social life at the center of meaning-making, which is a challenge to Piaget's ideas. (p. 133)

According to Gadanidis and Geiger (2010):

Sociocultural theories of learning are founded on a position that intellectual development originates in, and so is not just facilitated by, social interaction. Learning is a process of enculturation into the practices of a learning community. Enculturation into the community requires the appropriation of modes of reasoning, discourse and knowledge creation that are accepted by the discipline around which the community is based. Learning mathematics in such a community means a learner must participate in debate about new ideas and practices, offer critique of others ideas and defend their own propositions via explanations and justifications. (p. 96)[6]

Ernest (2004a) mentions that:

Concepts, definitions, and rules of mathematics (including rules of truth and proof) were invented and evolved over millennia. Thus mathematical knowledge is based in contingence, due to its historical development and the inevitable impact of external forces on the resourcing and direction of mathematics. (p. 25)

Ernest (2004a) posits that if mathematics is social, cultural, and public, then mathematics is linguistic, textual, semiotic, and embedded in the social world of human interaction. Moreover, "the form in which this is embodied in practice is in conversation" (p. 25). Thus, mathematical knowledge is intra-personal and cultural, conversational and dialogical. In my interpretation, Ernest (2004a, 2004b) builds an interesting connection between sociocultural theory and postmodernism in mathematics education.

[The lenses of postmodernity reveal] mathematics education as an ineluctably social field of study and practice. The production and warranting of text and knowledge is a function of communities of practice and their discourses. The redrawing of the map of knowledge is fundamentally tied to the outlines and interactions of these social communities of practice. Text and conversation, so central to education and all of knowledge production, are social tools and practices. Even the production of subjectivity, long held to be the sole province of individual psychology, turns out to be a social process in which text and

[6] Sociocultural perspectives are also related to *ethnomathematics* (D'Ambrosio, 2006; Gerdes, 2010) and *mathematical enculturation* (Bishop, 1991). In this scenario, learning is understood as a process one becomes part of the mathematical culture which permeates one's social environment (Bishop, 1991).

> conversation are deeply implicated. Thus a postmodern view of mathematics education is above all, a social view. (Ernest, 2004b, p. 82)

According to Ernest (2004b), a fundamental claiming of postmodernity is "the acceptance of diversity in knowledge and social process of knowledge formation" (p. 70). In this perspective, "knowledge derives from the experience and actions of persons in social practices" (Ernest, 2004b, p. 70). A postmodernist view then rejects categorization and division of knowledge. It emphasizes the notion of the existence of "interpenetrating, overlapping, and shifting knowledges… all founded on the powers of understanding of the human subject" (Ernest, 2004b, p. 70). According to Ernest (2004b):

> Adopting the perspective of postmodernity, even if only temporally, forces a reconceptualization of knowledge, learning and mathematics education on us. It requires a relinquishing of the certainties that the metanarratives of rationality provided in mathematics, psychology and educational research methodology. Instead all knowledge, text and education can only be accounted for by multiple and contestable narratives, with different social footprints and justificatory discourses. The traditional division of disciplines and separation of realms of human thought and action are eroded, and multiple maps of knowledge can be drawn. Likewise many traditional concepts are revealed to be shifting in meanings and multiple interpreted over the passage of time. (p. 82)

Ernest (2004b) posits that "adopting a postmodern perspective on the teaching and learning of mathematics with its focus on text and conversation foregrounds new features" (p. 81). It disrupts traditional notions that knowledge is transmitted or acquired. Postmodernism "focuses on subjectivity as emerging from long-term engagement with text [and] on the emergence of agency as symbolic, textual system" (Ernest, 2004b, p. 81).[7]

There are specific features of mathematics that support the claim that mathematical knowledge and knowing are conversational:

1. Mathematics is primarily a symbolic activity, using writing inscriptions and language to create, record, and justify its knowledge. Viewed semiotically as comprising texts, mathematics is inescapably conversational for it must address a reader.

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[7] Interestingly, Ernest (2004b) argues that "the relationship between learners and text, and how it inscribes the other, is an area that is as yet inadequately explored" (p. 82). This study on the nature of students' DMPs is an attempt to fill that gap.

2. Many mathematical concepts can be analyzed to reveal dominant underlying meanings or interpretations that are at root dialogical and conversational.
3. Mathematical proof, so central to mathematical epistemology, originally developed from a cultural practice of disputation, i.e., conversation, and several modern developments in proof theory, still treat proofs as part of a dialogue.
4. The acceptance of mathematical knowledge depends on a social mechanism that mirrors the structure of conversation. (Ernest, 2004a, pp. 26-27)

According to Pinar, Reynolds, Slattery, and Taubman (2008), dialogue and playfulness in classrooms are fundamental characteristics of a postmodern curriculum. A postmodernism view of curriculum "encourages the role of playfulness in the classrooms, especially in the process of making meaning by the student . . . Play becomes an agent of change" (Pinar et al., 2008, p. 502). Doll (1993) states:

> Play deals not with the present and foundational but with the absent and the possible. Its very nature invites dialogue, interpretation, interaction. Its free-flowing form encourages participation. All these activities are essential to meaning-making (p. 286) . . . [An] advantage of play, once one is attuned to its nature, is that its freedom allows one to challenge and explore in a nonhostile and nonthreatening way. (p. 287)

One interesting issue discussed by Lyotard (1984) to theorize the *postmodern condition* is the critique stated through the notion of "research and its legitimation through performativity" (Lyotard, 1984, p. 41), that is, postmodern science "suggests a model of legitimation that has nothing to do with maximized performance [of a system]" (Lyotard, 1984, p. 60).

According to Lyotard (1984), the modernist methods of legitimation of scientific knowledge are constructed toward the principles of syntax of a formal system as consistent, complete, and decidable. These are the internal properties of the Hilbert's program to provide a "secure" foundation to all mathematics. Interestingly, several of the philosophical discourses of the Enlightenment are intended to be designed as formal mathematical systems (e.g., structured through definitions, theorems, propositions, and so forth). However, "Godel has effectively established the existence in the arithmetic system of a proposition neither demonstrable nor refutable within that system; this entails that the arithmetic system fails to satisfy the condition of completeness" (Lyotard, 1984, p. 42). Thus, Lyotard relates the postmodern condition to the fact that it is possible to generalize that situation of incompleteness considering philosophical discourses, that is, "it must be accepted that all formal systems have internal limitations" (Lyotard, 1984, p. 43). That argument supports the definition of the *postmodern condition* as the "incredulity toward metanarratives" (Lyotard, 1984, p. xxiv): the disbelief toward the appeal to the grand narratives in order to legitimate scientific knowledge.

This "tension" between Hilbert's program and Godel's incompleteness theorems is strongly highlighted in the philosophy of mathematics, mainly toward the notion of *crisis of foundations of mathematics*. When one looks at some perspectives on the history of mathematics, one notices that "mathematics has been defusing uncertainty" (Ernest, 2004a, p. 17). Examples are: incommensurability/irrationality, Zenos' and other logical paradoxes, non-Euclidian geometries, infinitesimal numbers, Cantor's theory, Peano's axioms, four color theorem, fractal geometry, Godel's theorem, Turing's theorem, catastrophe and chaos theories, and so forth (Ernest, 2004a). According to Silva (1999):

> In fact, mathematics is constantly on a crisis of foundations, from discovery of incommensurability among the Greeks to the present dispute raised by the use of computers as tools for mathematical proof (not only for heuristics), through the discovery of non-Euclidean geometries, and the introduction of imaginary numbers in algebraic calculation, mathematics is constantly reviewing its foundations. (p. 47, my translation)

Two important aspects related to the notion of DMP emerge from this notion of crisis of foundations of mathematics. First, it is a pleasant and *surprising* experience to realize that the foundations of mathematics are constantly been undermined (Davis & Hersh, 1980). The sense of surprise, new, and wonderful emergent from *Flatland*, for instance, happens in great part when one recognizes and understands that parallel lines do meet in spherical geometry and one disrupts the stereotype that "parallel lines never meet" (Gadanidis & Borba, 2008). In this sense, by highlighting a crisis of foundations involving non-Euclidian geometries, one might start to see mathematical knowledge as undetermined, organic, non-stable, and rhizomatic (Deleuze & Guatarri, 1987), rather than absolute, immutable, and fully determined.

Second, when I assume a postmodernist view, I am in concordance with the perspective that mathematics is socially and culturally constructed (Ernest, 2004a, 2004b). Thus, a postmodern/sociocultural view shapes the way I interpret and analyze data in this study, considering that it is important to make explicit my interpretative lenses within the design of a qualitative study. Interestingly, Goos, Galbraith, Renshaw and Geiger (2000) clarify that "a central claim of sociocultural theory is that human action is mediated by cultural tools and is fundamentally transformed in the process" (p. 306). Borba and Villarreal (2005) argue that technologies are not neutral in mathematical knowledge production. Media are actors that (re)organize mathematical thinking (Borba, 2004). Thus, I present in the next section a specific theory toward the use of technology and its significance in knowledge production.

Humans-with-Media

Borba and Villarreal (2005) use the expression *humans-with-media* as a metaphor to theorize the cognitive "inter-shaping" between humans and technologies regarding mathematical knowledge production (Borba, 1993). Borba and Villarreal (2005) posit their perspectives considering the notion of *technologies of intelligence* (Levy, 1993): a historical-cognitive perspective of technologies.

According to Levy (1993), there are three main technologies of intelligence associated with memory and knowledge. They are: orality, writing, and information technology. In oral societies, humans produced knowledge through myths and rituals, cyclically and locally, transmitting information from one generation to another. However, this *circularity* was reorganized into *linear* ways of reasoning in writing societies, mainly through the popularization of books, due in large part to the invention of the Gutenberg's printer press.

Information and digital technology can be understood in the same way. The linearity of memory has been assuming a "web design" through the plasticity of digital technology. Computers and online tools combine multiple modes of communication. They shape the ways that contemporary societies interact and communicate. The "linear reasoning" of writing has been challenged by ways of thinking involving orality, writing, images, simulation, experimentation, and instantaneous communication. Regarding current technological innovations, there are new ways to communicate, extend memory, store information, represent, and produce and knowledge.

Borba and Villarreal (2005) thus argue that "our individual consciousness and cognitive process are always subject to interaction with the technologies of intelligence" (p 26). That is, "knowledge is produced with a given medium or technology of intelligence" (p. 23).

> Humans-with-media, humans-media or humans-with-technologies are metaphors that can lead to insights regarding how the production of knowledge itself takes place. . . . This

> metaphor synthesizes a view of cognition and of the history of technology that makes it possible to analyze the participation of new information technology 'actors' in these thinking collectives. (Borba & Villarreal, 2005, p. 23)

Borba and Villarreal (2005) discuss sociocultural perspectives (Tikhomirov, 1981) to develop the notion of humans-with-media. According to Tikhomirov (1981), computers do not replace, substitute, or merely complement humans in their intellectual activities. Processes mediated by computers *reorganize* thinking. Tikhomirov argues that computers play a mediating role in thinking as language does in Vygotsky's theory. Regarding the nature of human-computer interaction in terms of feedback, the dimensions involving computational mediation provide new insights in terms of learning, development, and knowledge production. Tikhomirov claims that

> With regard to the problem of regulation we can say that not only is the computer a new means of mediation of human activity but the very reorganization of this activity is different from that found under conditions in which the means described by Vygotsky are used. (p. 273)

Borba and Villarreal (2005) use Tikhomirov's theory to argue how the notion of mediation by computers is qualitatively different to the mediation involving paper and pencil, for example. Through digital mediation, information technologies reorganize mathematical thinking. Media shape knowledge production and transform mathematics.

Levy (1993) defines *cognitive ecology* as "the study of technical and collective dimensions of cognition" (p. 137). Levy (1993) sees technology not simply as a tool used by humans, but rather as an integral component of the cognitive ecology. Levy (1998) claims "as humans we never think alone or without tools. Institutions, languages, sign systems, technologies of communication, representation, and recording all form our cognitive activities in a profound manner" (p. 121). According to Levy, technologies *do not determine* thinking. Technologies *condition* thinking (Levy, 1993, 2001). Levy (1993) uses the term *thinking collectives* to discuss the collaboration between human and non-human actors in the cognitive ecology. Levy (1993) argues that thinking collectives of humans-technologies form the cognitive ecology.

Levy (1993) suggests the term *hypertext* as metaphor for the nature of the cognitive ecology.[8] Levy claims the actors of communication construct and reshape universes of meanings and, every time communication and meaning production are playing, this metaphor can be used.

[8] For more details, see Appendix A.

Levy constructs this metaphor (of knowledge, cognition, and communication) based on six principles.[9] They are:

- *Principle of metamorphosis.* The hypertext is in constant construction and renegotiation. Its composition and design are always on a permanent playing field for the actors involved in meaning production. These actors are humans, words, images, objects, components of the objects, etc.
- *Principle of heterogeneity.* Knots and connections of a hypertext are heterogeneous. The knots are composed of images, sounds, words, sensations, models, etc. The connections are logical and affective. In communication, messages are multimedia, multimodal, analogical, digital, and so forth. Socio-technical processes will put into play people, groups, artefacts, natural forces of all sizes, and with all kinds of associations that we can imagine among these elements.
- *Principle of multiplicity and fitting of scales.* The hypertext has a fractal nature, that is, each knot or connection, when analysed, can be signified as another hypertext. It is possible to extend and adjust the scale to signify various layers of the hypertext.
- *Principle of exteriority.* The hypertext depends on external elements such as other images, words, and other hypertexts.
- *Principle of mobility of centers.* The hypertext does not have a single center. The hypertext has various mobile centers that provide conditions for a multiplicity of meanings.
- *Principle of topology.* In a hypertext everything works by approximation, by distance and neighbourhood. The hypertext is not in the space, it is the space.

Levy (1997) relates cognitive ecology and thinking collectives to *collective intelligence,* defined as "a form of *universally distributed intelligence*, constantly enhanced, coordinated in real time, and resulting in the effective mobilization of skills" (p. 13). By intelligence, Levy (1998) means "the canonical set of cognitive aptitudes, namely the ability to perceive, remember, learn, imagine, and reason" (p. 123). Levy (1998) indicates a parallel between the biological and the cultural beings to theorize collective intelligence.

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[9] These principles are similar to those proposed by Deleuze and Guattari (1987) to theorize the notion of *rhizome*. See a quotation about that metaphor in Appendix B.

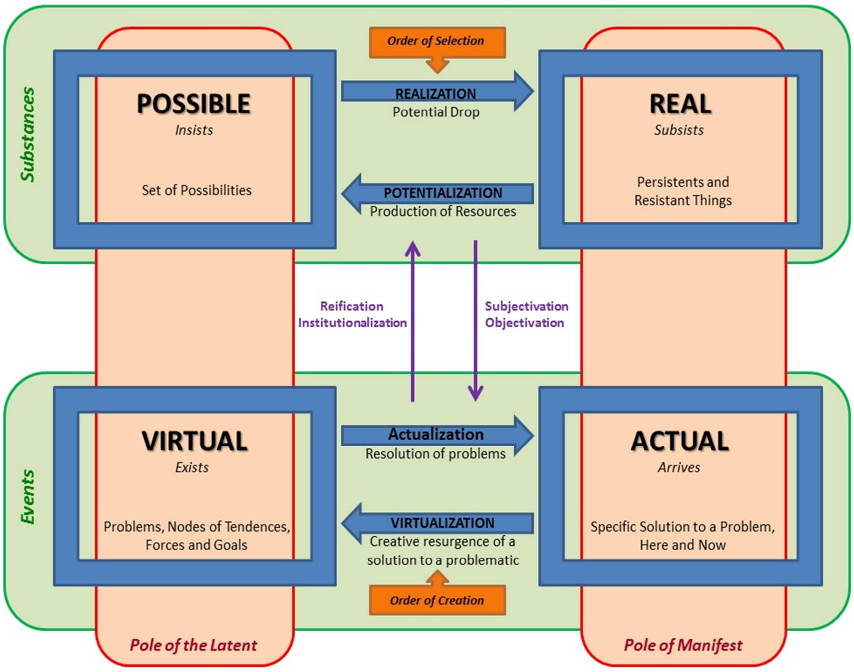
> Because of biology our intelligences are individual and similar (although not identical). Because of the culture our intelligence is highly variable and collective. In effect, the social dimensions of intelligence is closed tied our languages, technologies, and institutions, which differ greatly according to time and space. (Levy, 1998, p. 125)

The *cyberspace*, which includes the setting where DMPs become public (the Math + Science Performance Festival), is a privileged nexus for collective intelligence. Levy (2001) defines cyberspace as the space of communication opened by the world interconnection of computers and memories of computers. This space is unique, because digital codifications shape the plastic, interactive, hypertextual, multimodal, and virtual nature of information in this context. Levy (2001) also defines *cyberculture* as the set of (materials and intellectuals) technologies, practices, attitudes, modes of thinking, and values developed through the growth of the cyberspace. The cyberculture redefines the notions of economy and knowledge, bringing up new possibilities to several areas such as education and the arts. Levy (2001) claims “the genres of cyberculture are similar to performance art, such as dance or theatre [or] the collective improvisations of jazz, the commedia dell’arte, or the traditional poetry competitions of Japan” (p. 135). Levy (2001) uses the term *cyberart* to discuss the artistic-aesthetic dimension of cyberculture, suggesting the possibilities for (collective) collaboration and the continuous creation as a fundamental aspect of cyberart. In other words,

> The virtual work is ‘open’ by design. Every actualization reveals a new aspect of the work . . . Thus the creation is no longer limited to the moment of the conception or realization; the virtual system provides a machine of generating events. (Levy, 2001p. 116)

Levy (1998) constructs an ontological perspective of cognitive ecology based on the *four modes of being*. They are: real, actual, virtual, and possible. This perspective is called the *ontological quadrivium* (see Figure 2.2). By regarding its Latin derivation, *virtualis* means strength or power. In the Scholastic philosophy, “the virtual is that which has potential rather than actual existence . . . [like] the tree is virtually present in the seed” (Levy, 1998, p. 23). Thus, in Aristotle’s metaphysics for example, reality is understood as the dialects of virtual (or potential) and actual. Although in Levy’s (1998) model of being the notions of real, actual, virtual, and possible operate together, he suggests distinctions based on two *poles* (latent and manifest) and two *envelopings* (substances and events). Virtual and possible belong to the pole of latent and real and actual belong to the pole of manifest. In contrast, virtual and actual belong to the enveloping of events and possible and real to the enveloping of substances. Thus, Levy (1998) says virtual exists, possible insists, real subsists, and actual arrives. Levy (1998) highlights the relation possible-real (order of selection) and virtual-actual (order of creation) and, then, emphasizes four transformations: (a) *Realization* refers to choice, potential drop; (b) *Potentialization* refers to the production of resources (order and information); (c) *Actualization* refers to the creation of solutions for problems presented by the virtual. It is the creation of a form; and (d) *Virtualization* is the transition from an act to the problem, that is, the creation of problems. Levy (1998) does not present specific discussions about the transformations for the relations virtual-possible, virtual-real, actual-possible, and actual-real. However, Levy (1998) mentions that the dialectic of possible-real is captured by the dialectic of virtual-actual through *objectivation* and *subjectivation*.[10] In contrast, the dialectic of virtual-actual is captured by the dialectic of possible-real through reification, recording, and institutionalization. Following, in Figure 2.2, I present a diagram that represents the ontological quadrivium, which is the ontological perspective that informs this study:

Figure 2.2: The Ontological Quadrivium (Levy, 1998).



[10] "Subjectivation is the implication of technological, semiotic, and social means in the individual's psychic and somatic functions. Objectivation is defined as the mutual implication of subjective acts in the process of constructing a shared world" (Levy, 1998, p. 169).

The ontological quadrivium is theorized based on the Deleuzian notions of actual and virtual (Deleuze, 1994; Deleuze & Guatarri, 1987; Deleuze & Parnet, 2002). Interestingly, these notions are discussed in relation to cinema and performance (Deleuze, 1986, 1989). According to del Rio (2008), *performance* can be understood as a process of *actualization of the body* "through specific thoughts, actions, displacements, combinations, realignments – all of which can be seem as different degrees of intensity, distinct relations of movement and rest" (p. 9). The relation virtual/actual (events), for instance, offers ways to see the construction of DMPs as a creative process or from a problem-solving point of view. The relation possible/real emphasizes the materiality of the process, such as DMPs as multimodal texts.

Multimodality

The notions of *semiotics* and *multimodality* are fundamental within Levy's perspectives.[11] According to Kress (2003), all communication is multimodal by its nature. However, discussions on multimodality in literacy usually consider that innovation in information technologies challenges the traditional dominant aspects of "print literacy" based just on reading and writing (Heydon, 2010). Kress (2003) states that digital media "make it easy to use a multiplicity of modes, and in particular the mode of image – still or moving – as well as other modes, such as music and sound effect for instance" (p. 5).

Regarding the notions of interactivity and hypertextuality, Kress (2003) states:

> What is true of word and image is also increasingly true of other modes. The ease in the use of different modes, a significant aspect of the affordances of the new technologies of information and communication, makes the use of a multiplicity of modes usual and unremarkable. That mode which is judged best by the designer of the message for specific aspects of the message and for a particular audience can be chosen with no difference in 'cost'. Multimodality is made easy, usual, 'natural' by these technologies. (p. 5)

In this scenario, the notions of *signs*, *modes*, and *media* are fundamental from a semiotic/multimodal point of view. Knowledge, meaning, and learning are concerned with signs (Deleuze, 2000; Kress, 2003). Signs refer to words, images, gestures, tastes, sounds, all of the modes in which information can be communicated. Kress (1997) defines signs as a "combination

[11] Within postmodernism, language is understood broadly. It includes all forms of semiotic representations (Ernest, 2004b). Thus, there is a harmonic and convenient approximation between multimodality and postmodernism or between semiotics and postmodernism (see Deleuze, 1994; Deleuze & Guattari, 1987).

of meaning and form" (p. 6). "The sign is taken to be an arbitrary combination of form and meaning, of *signifier* and *signified*, a combination which is sustained by the force of social convention" (Kress, 2003, p. 41). Interestingly, Deleuze (2000) states:

> Learning is essentially concerned with *signs*. Signs are the object of a temporal apprenticeship, not of an abstract knowledge. To learn is first of all to consider a substance, an object, a being as if it emitted signs to be deciphered, interpreted . . . Everything that teach us emits signs; every act of learning is an interpretation of signs. (p. 4)

Kress (2003) posits that "*mode* is the name for a culturally and socially fashioned resource for representation and communication" (p. 45). That is, modes are "the various forms used to construct signs" (Kress, 1997, p. 7). Traditionally, in literacy, the modes of communication are: reading, writing, listening, and speaking. However, the literature presents distinct definitions about which are the modes of communication. The Board of Studies New South Wales (2003) proposes six "language modes." They are: reading, writing, listening, speaking, representing, and viewing. Pahl and Rowsell (2005) state that "a mode could be visual, linguistic, aural, or tacit" (p. 27). Authors like Jewitt (2006) argue that the *modalities* are aural, visual, gestural, spatial, and linguistic.

The New London Group (1996) discusses language within multiliteracies based on the notion of *design*, that is, "a language for talking about language, images, texts, and meaning-making interactions . . . [including] the key terms 'genres' and 'discourses,' and a number of related concepts such as voices, styles, and probably others" (p. 77). The New London Group discusses the notions of *available designs*, *designing*, and *redesigning*. These processes may offer lenses to see aspects of how students produce DMPs, that is, how they use semiotic resources to produce DMPs, how they shape meanings through the process of production, and what are the meanings produced and interpreted by the audience in their DMPs.

According to The New London Group (1996), "Available Designs - the resources for Design - include the "grammars" of various semiotic systems: the grammars of languages, and the grammars of other semiotic systems such as film, photography, or gesture" (p. 74).

> [*Designing* refers to] the process of shaping emergent meaning involves re-presentation and recontextualization. This is never simply a repetition of Available Designs. Every moment of meaning involves the transformation of the available resources of meaning. Reading, seeing, and listening are all instances of Designing . . . Any semiotic activity -

> any Designing - simultaneously works on and with these facets of Available Designs. . . . Transformation is always a new use of old materials, a re-articulation and recombination of the given resources of Available Designs. (The New London Group, 1996, p. 75)

The notion of *Redesigning* highlights that:

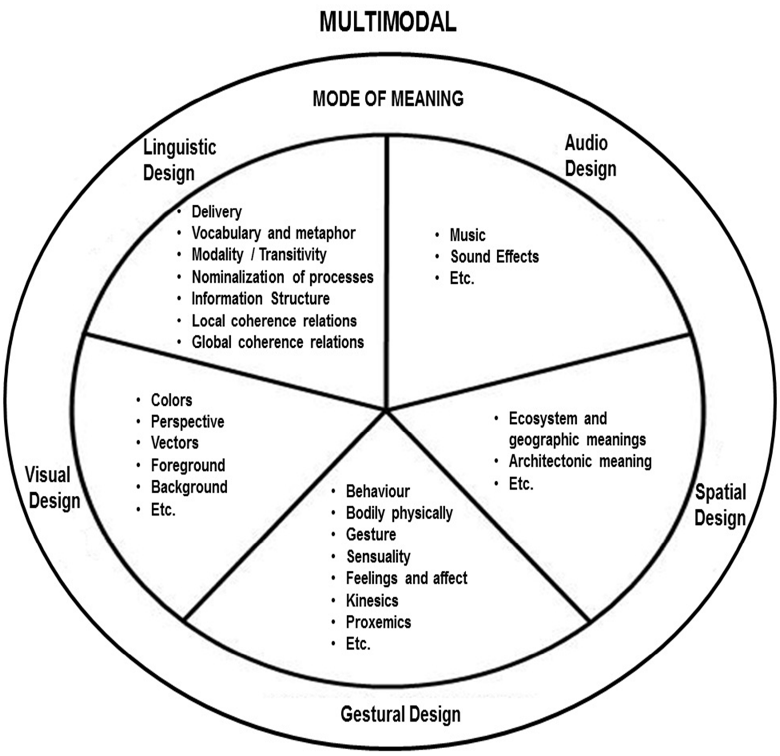
> The outcome of Designing is a new meaning, something through which meaning-makers remake themselves. It is never a reinstantiation of one Available Design or even a simple recombination of Available Designs. The Redesigned may be variously creative or reproductive in relation to the resources for meaning-making available in Available Designs. But it is neither a simple reproduction, nor is it simply creative. As the play of cultural resources and uniquely positioned subjectivity. . . . The Redesigned becomes a new Available Design, a new meaning-making resource. (The New London Group, 1996, p. 76)

The New London Group (1996) also proposed a model formed by six designs. These are: Linguistic Design, Visual Design, Audio Design, Gestural Design, Spatial Design, and Multimodal Design. The "Multimodal Design is of a different order to the other five modes of meaning; it represents the patterns of interconnection among the other modes" (p. 78). The New London Group highlights the Linguistic Design in relation to the other Designs. Some elements of Linguistic Design are:

> *Delivery:* Features of intonation, stress, rhythm, accent, etc. *Vocabulary and Metaphor:* Includes colocation, lexicalization, and word meaning. *Modality:* The nature of the producer's commitment to the message in a clause. *Transitivity:* The types of process and participants in the clause. Vocabulary and metaphor, word choice, positioning, and meaning. *Nominalization of Processes:* Turning actions, qualities, assessments, or logical connection into nouns or states of being (e.g., "assess" becomes "assessment"; "can" becomes ability). *Information Structures:* How information is presented in clauses and sentences. *Local Coherence Relations:* Cohesion between clauses, and logical relations between clauses (e.g. embedding, subordination). *Global Coherence Relations:* The overall organizational properties of texts (e.g. genres). (The New London Group, 1996, p. 80)

The Visual Meanings refer to images, page layouts, screen formats. The Audio Meanings refer to music and sound effects. The Gestural Meanings refer to body language, embodiment, facial expressions. The Spatial Meanings refer to the meanings of environmental spaces, architectural spaces. Thus, the Multimodal Design "is the most significant, as it relates all the other modes in quite remarkably dynamic relationships. . . . In a profound sense, all meaning-making is multimodal. All written text is also visually designed" (The New London Group (1996, p. 80-81). Figure 2.3 illustrates the multimodal model proposed by The New London Group (1996):

Figure 2.3: Multimodal Design: The New London Group (1996)



The New London Group (1996) highlights that two key concepts related to multimodal meanings and relationships between the Designs are: hybridity and intertextuality.

> The term *hybridity* highlights the mechanisms of creativity and of culture-as-process particularly salient in contemporary society. People create and innovate by hybridizing - that is, articulating in new ways - established practices and conventions within and between different modes of meaning. . . . Popular music is a perfect example of the process of hybridity . . . new relations are constantly being created between linguistic

meanings and audio meanings (pop versus rap) and between linguistic/audio and visual meanings (liver performance versus video clips). (The New London Group, 1996, p. 82)

According to the New London Group (1996),

Intertextuality draws attention to the potentially complex ways in which meanings (such as linguistic meanings) are constituted through relationships to other texts (real or imaginary), text types (discourse or genres), narratives, and other modes of meaning (such as visual design, architectonic or geographical positioning). Any text can be viewed historically in terms of the intertextual chains (historical series of texts) it draws upon, and in terms of the transformations it works upon them. For instance, movies are full of cross references, either made explicitly by the movie maker or read into the movie by the viewer-as-Designer: a role, a scene, an ambiance. The viewer takes a good deal of their sense of the meaning of the movie through these kinds of intertextual chains. (p. 82)

Pahl and Rowsell (2005) state that “we need to see texts as artefacts, that is, as objects with a history and a material presence” (p. 27). According to Kress (2003),

Communication – whatever the mode – always happens as *text*. The ‘stuff’ of our communication needs to be fixed, in the sense of my metaphor above, in a mode: knowledge or information has no outward existence other than in such modal fixing. (p. 47)

Kress (2003) relates this notion of fixing to materiality and the notion of shaping to text. In this perspective, “text is the result of social action, of work: it is work with representational resources which realise social matters” (Kress, 2003, p. 47). Kress (2003) mentions that, traditionally, “the term ‘text’ began to be used for (recorded and/or transcribed) spoken entities as much as for written entities. The video recorder has begun to have a similar effect for other modes – movement, gesture, position in space” (p. 48). In concordance with Kress (2003):

I will use the term *text* for any instance of communication in any mode or in any combination of modes, whether recorded or not. If it happened as communication it will have been ‘recorded’ in any case by the participants in that communicational event. And if this ‘recording’ is partial, as inevitably it must be, then it is simply differently partial than is the case with recordings made with contemporary technological means. (p. 48)

Ernest (2004b) posits that the notion of text in teaching and learning mathematics includes both “the discursive representations constructed and utilized by teachers to communicate to leaners, and those inscribed by learners themselves to communicate to teachers”

(p. 79). In this perspective, texts include “the ‘live’ discourse representations constructed in talking, using body language, inscribing in chalkboards, overhead transparencies, computers screens, . . ., books, worksheets, computer software, [and so forth]” (p. 79). Thus, considering students’ DMPs as multimodal texts and my role as a postmodernist interpreter, it is important to highlight that:

> The school math text, the transcribed teacher talk, written pupil work, interview protocols, etc., all require considerable interpretation by the learner or researcher and no reading can ever be final. Even if there were a ‘correct’ reading of meaning of one of these texts, and postmodernity denies this, there would be no means of knowing who had achieved it. Instead what we have is a process of negotiation between participants which may move towards a consensus of interpretation, never forgetting that there is also a background ideology which holds in place a notion of correctness underpinning agreement, which is itself a function of the discursive practice with its own historical trajectory. (Ernest, 2004b, p. 71)

Finally, the notion of media in multimodality “focus more on the manner of dissemination of various modes” (Kress, 1997, p. 7).[12] Therefore, in synthesis, meaning is socially and culturally produced in many different ways, with many different modes and media (Kress & van Leeuwen, 1996, p. 111).

In the next section I discuss the performance arts lens I use to analyze students DMPs. The notions of social constructivism, humans-with-media, and multimodality also inform my perspectives through the performance arts lens. That is, the aspects discussed about cinema (Boorstin, 1990), which highlight surprises, sense-making, emotions, and visceral sensations the audience may feel by *reading* students’ DMPs as multimodal texts or narratives, emphasize the social and cultural focus on meaning production, the pedagogic production of texts, the use of technology and dialogue in students’ meaning and knowledge production, the playfulness in the classroom, and the significance of digital media affordances in offering multiple ways of communication.

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[12] This notion of *media* is commensurable with the perspectives of humans-with-media and cognitive ecology within my theoretical framework (Borba & Villarreal, 2005; Levy, 1993, 1998, 2001). I also (generally) see in this study a commensurability involving the terms media, technology, technology of intelligence, and language.

A Performance Arts Lens to Interpret Students' DMPs

As noted in chapter one, Boorstin (1990) presents three types of pleasures the audience may experience by watching a movie. They are: (a) the new, wonderful and surprising; (b) vicarious emotional moments, and (c) visceral sensations. These "pleasures" or categories of experience are referred to as "what makes good movies." Gadanidis and Borba (2008) used Boorstin's categories as lenses to analyze DMPs. Osafo (2010) used these lenses to analyze digital performances about science. In this study, as the reader will notice in chapter four, I use a variation of these categories to analyze students' DMPs. In this section, I present the main aspects pointed out by Boorstin (1990) in this theorization.

The voyeur eye.

Boorstin (1990) argues "the voyeur pleasure is the simple joy of seeing the new and the wonderful" (p. 12). "The voyeur's eye is the mind's eye, not the heart's, the dispassionate observer, watching out of a kind of generic human curiosity" (Boorstin, 1990, p. 13). "The voyeur's pleasure – the pleasure of simply watching, of looking for its own sake – is in a sense the most primitive of movies pleasures" (Boorstin, 1990, p. 45). "Because the voyeur's eye bores easily, it demands surprise – so long surprise comes with a rational explanation . . . it requires a thudding sense of the reality of things, of the plausibility of actions" (Boorstin, 1990, p. 13). "This requires not only a sense of actual reality but an instinct for what audiences have come to accept as real" (Boorstin, 1990, p. 52).

> Audiences want their overall expectations fulfilled . . . but moment to moment they want to be wrong. The voyeur in us wants to be surprised. We want the filmmaker to be cleverer than we are. For the writer, that means constantly creating expectations that (for right kind of reasons) aren't quite fulfilled; for the editor, it means varying the rhythms of editing. (Boorstin, 1990, p. 50)

Boorstin (1990) goes on to further explain the voyeur's eye. Boorstin advises "the simple pleasure of seeing is so mundane it is easy to underestimate" (p. 15). Thus, notions of light, space, and sound are fundamental in making a credible world. It "requires not only a sense of actual reality but an instinct for what audiences have come to accept as real" (Boorstin, 1990, p. 52). According to Boorstin, "the voyeur in us is logical to a fault, impatient, picky, literal, but if properly respected it gives the special pleasures of the new and the clever, of a fresh place or a crisply, thought-out story (p. 61).

Osafo (2010) posits that the voyeur's eye can be interpreted in two levels: referring to *new/wonderful/surprising* and *sense-making*. This splitting helps the researcher to analyze aspects of reasoning present in the digital performances in science education or, as in the case of this study, in mathematics education. Then, conceptual DMPs offer (a) *surprises*, that is, "the joy of seeing the new and the wonderful" (Boorstin, 1990, p. 13) and (b) *sense-making*, which refers to mathematical sense of story and reasoning and thinking (Gadanidis & Borba, 2008). As discussed in chapter one, surprises are fundamental in mathematical activity (Foyd, 2011; Watson & Mason, 2007) and so is sense-making (construction of meaning and knowledge).

It is important to notice that the voyeur's eye, like the vicarious and visceral, has an *individual or subjective dimension*, that is, "the voyeur is more dominant in some people than in others. It is a measure of the breadth of the film experience that these voyeuristic responses are only a part of the satisfactions of watching a movie" (Boorstin, 1990, p. 61).

The vicarious eye.

According to Boorstin (1990):

> The vicarious eye sees with the heart. It mixes our yearning to matter with our need to please other, to read the slightest shift in their feelings and accommodate to it . . . [it] puts our heart in the actor's body: we feel what the actors feels, but we judge it for ourselves . . . The tension between the two impulses – the urge to be the character and to judge him simultaneously – gives the vicarious experience grit. (p. 67)

Boorstin (1990) states "for the voyeuristic eye, credibility depends on plausibility . . . For the vicarious eye, however, credibility depends on emotional truth" (p. 75). "While the voyeuristic eye demands scenes grounded firmly in the solid geometry of three-dimensional space, the vicarious eye makes no such requirement" (Boorstin, 1990, p. 90). "The voyeuristic experience may be grand or clever, but the vicarious experience can be profoundly moving. The vicarious experience is not bound by iron bands of logic but by silken cords of emotional truth" (Boorstin, 1990, p. 67). "Intense, honest emotions are a valuable commodity; they are hard to come by, and they make us feel alive. Our vicarious eye scans our environment for emotional beacons; when we find one, we lock in" (Boorstin, 1990, p. 71).

> For the vicarious eye the basic unit isn't the story beat but the actor's moment, that single pure note of emotional truth when the actor's pitch is perfect and his intensity is high. The moments are the pearls of performance the director watches for on the set and the editor treasures during cutting. (Boorstin, 1990, p. 77)

As Bordwell and Thompson (1993) state, "to make an expectation about 'what happens next' is to invest some emotion in the situation" (p. 48).

According to Boorstin (1990), close-ups enhance audiences' vicarious pleasures:

> The natural choice for capturing this inner space is a shot of nothing but the actor's face: the tight close-up. The rest of the world falls away, crowded out and blurry. At close range a lens has much less depth of focus so the nearby face stands in high relief against a tapestry of soft, amorphous blobs of color . . . The inner landscape is a susceptible to light as the outer world, but here instead of defining space, light defines mood. Overall, the "look" of a picture can create emotional expectations. (p. 90-91)

Gadanidis and Borba (2008) add, "whereas the voyeur's eye needs a wide-angle view, to experience context and relationship, the vicarious eye needs a close-up view of the actors" (p. 47). Thus, conceptual DMPs also offer emotional moments. It offers ways for students and teachers to vicariously experience human and emotional aspects of learning mathematics (Gadanidis & Borba, 2008; Gadanidis & Hughes, 2008). "It connects with students in a personal, emotional way" (Osafo, 2010, p. 39).

The visceral eye.

Boorstin (1990) says that the point of the experience of the visceral eye "is not to feel what the character feels but to feel your own emotions, to have the experience yourself, directly" (p. 110). "The extreme examples of the visceral eye are the suspense scene and the action scene" (Boorstin, 1990, p. 121).

> Visceral thrills are filmmaking's dirty little secret. Though they can require considerable art to achieve . . . – the passions aroused are not lofty, they're the gut reactions of the lizard brain – thrill of motion, joy of destruction, lust, blood lust, terror, disgust. Sensations, you might say, rather than emotion. (Boorstin, 1990, p. 110)

Contrasting the three eyes, Boorstin (1990) posits:

> What feels real here [in the visceral eye] is not plausible, as in the voyeur's world, not the emotionally true of the vicarious world, but the thrill . . . It feels real because it is real: it is happening to you. The fundamental criticism in the voyeur's world is 'that couldn't happen,' in the vicarious world 'he wouldn't do that,' but the visceral world it is 'it doesn't get me.' . . . Visceral kicks are more purely cinematic than sedate rewards of the voyeur or the vicarious eye. (p. 111-114)

In terms of filming, Boorstin (1990) clarifies that "close-ups, so powerful for intensifying the vicarious impact of the actor's emotions, only diminish the visceral because in close-ups the danger is no longer real and palpable. Creating the thrilling effect is as much a matter of optics and camera angles as of real jeopardy" (p. 116).

Considering the fact that some DMPs available at the Math + Science Performance Festival are music-video clips, or parodies of popular TV shows, it is relevant to mention that, according to Boorstin (1990):

> TV is film with a myopic visceral eye. Music videos are revealing in this respect. Pop songs aim at the gut, but with rare exceptions the vapidity of the songs turns rock videos into the exercise in form without content, at once the purest type of filmmaking and the most debased. Since the songs are designed for visceral appeal, the video makers naturally turn to the techniques of the visceral film, and they strive mightily to overcome the inherent handicaps of the medium. (p. 133)

Through the performance art lens proposed by Boorstin (1990), I assume in this study that conceptual DMPs also offer visceral sensations or pleasures. They involve direct mathematical experiences. As discussed in chapter one, the notion of visceral sensations is related to DMP through the notion of aesthetics and beauty in mathematics, like the notion of *mathematical fit* (Sinclair, 2000). Interestingly, Sinclair (2004) states that "the phase of playing is aesthetic insofar as the mathematician is framing an area of exploration, qualitatively trying to fit things together, and seeking patterns that connect or integrate" (p. 272).

Therefore, considering the three eyes proposed by Boorstin (1990), Gadanidis and Borba (2008) suggest that conceptual DMPs offer surprises and sense-making, emotions, and visceral sensations. As the reader will notice in chapters four and five, I refer to these categories as: (a) *Voyeur – new/wonderful/surprising*; (b) *Voyeur – sense-making*; (c) *Vicarious emotions*; and (d) *Visceral sensations*.

Mathematical Strands and Processes of the Ontario Curriculum

Besides the performance art lens, I also use in chapter five an "Ontario curriculum lens" to analyze the mathematical strands and processes in students' DMPs. A "curriculum lens" is significant for this study because, according to Schwab (1984):

> Curriculum is what is successfully conveyed to differing degrees to different students, by committed teachers using appropriate materials and actions, of legitimated bodies of knowledge, skill, taste, and propensity to act and react, which are chosen for instruction after serious reflection and communal decision by representatives of those involved in the teaching of a specified group of students who are known to the decision makers. (p. 240)

Schwab's view of curriculum highlights four commonplaces of education: students, teachers, subject matter, and milieu. Thus, the investigative focus of this study on the nature of students' DMPs, on the ways students communicate mathematical ideas using the performance arts and digital technology, is related to issues in curriculum.

In the second part of chapter five, I present a cross-case analysis of students' DMPs considering the mathematical strands and processes pointed out in the Ontario Curriculum (Ontario Ministry of Education, 2005). The five strands in the K-8 Ontario Mathematics Curriculum are:

- Number Sense and Numeration,
- Measurement,
- Geometry and Spatial Sense,
- Patterning and Algebra, and
- Data Management and Probability.

The seven mathematical processes are:

- Problem Solving,
- Reasoning and Proving,
- Reflecting,
- Selecting Tools and Computational Strategies,
- Connecting,
- Representing, and
- Communicating.

These strands and processes overlap and, even though they are not explicitly the categories of analysis in chapter four, most of the fundamental aspects that characterize them are part of the descriptive analysis in chapter four. Following, I introduce each strand and process using direct quotes from the curriculum document (Ontario Ministry of Education, 2005), considering aspects that are relevant for this study. On the one hand, if the reader is familiar with the curriculum document, the following two subsections may be disregarded. On the other hand, the following quotes are significant to contextualize the nature of the analysis conducted in chapter five, in which analytic links between the curriculum strands and processes and the notion of DMP are presented. For specific information, the reader may check the document available at www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf.

The mathematical strands of the Ontario curriculum.

The focus on the curriculum mathematical strands in students' DMPs offers ways to interpret what are the mathematical ideas students are exploring through performance. According to the K-8 Ontario Curriculum Mathematics the strands are:

> ***Number sense and numeration.*** Number sense refers to a general understanding of number and operations as well as the ability to apply this understanding in flexible ways to make mathematical judgements and to develop useful strategies for solving problems. In this strand, students develop their understanding of number by learning about different ways of representing numbers and about the relationships among numbers. They learn how to count in various ways, developing a sense of magnitude. They also develop a solid understanding of the four basic operations and learn to compute fluently, using a variety of tools and strategies. (p. 8)
>
> ***Measurement.*** In this strand, students learn about the measurable attributes of objects and about the units and processes involved in measurement. Students begin to learn how to measure by working with non-standard units, and then progress to using the basic metric units to measure quantities such as length, area, volume, capacity, mass, and temperature. They identify benchmarks to help them recognize the magnitude of units such as the kilogram, the litre, and the metre. Skills associated with telling time and computing elapsed time are also developed. Students learn about important relationships among

measurement units and about relationships involved in calculating the perimeters, areas, and volumes of a variety of shapes and figures. (p. 8)

Geometry and spatial sense. Spatial sense is the intuitive awareness of one's surroundings and the objects in them. Geometry helps us represent and describe objects and their interrelationships in space. A strong sense of spatial relationships and competence in using the concepts and language of geometry also support students' understanding of number and measurement. Spatial sense is necessary for understanding and appreciating the many geometric aspects of our world. Insights and intuitions about the characteristics of two-dimensional shapes and three-dimensional figures, the interrelationships of shapes, and the effects of changes to shapes are important aspects of spatial sense. Students develop their spatial sense by visualizing, drawing, and comparing shapes and figures in various positions. In this strand, students learn to recognize basic shapes and figures, to distinguish between the attributes of an object that are geometric properties and those that are not, and to investigate the shared properties of classes of shapes and figures. Mathematical concepts and skills related to location and movement are also addressed in this strand. (p. 9)

Patterning and algebra. One of the central themes in mathematics is the study of patterns and relationships. This study requires students to recognize, describe, and generalize patterns and to build mathematical models to simulate the behaviour of real-world phenomena that exhibit observable patterns . . . In the junior grades, students use graphs, tables, and verbal descriptions to represent relationships that generate patterns. Through activities and investigations, students examine how patterns change, in order to develop an understanding of variables as changing quantities. (p. 9)

Data management and probability. The related topics of data management and probability are highly relevant to everyday life. Graphs and statistics bombard the public in advertising, opinion polls, population trends, reliability estimates, descriptions of discoveries by scientists, and estimates of health risks, to name just a few . . . In this strand, students learn about different ways to gather, organize, and display data. They learn about different types of data and develop techniques for analysing the data that include determining measures of central tendency and examining the distribution of the data. Students also actively explore probability by conducting probability experiments and using probability models to simulate situations. The topic of probability offers a wealth of interesting problems that can fascinate students and that provide a bridge to other topics, such as ratios, fractions, percents, and decimals. Connecting probability and data management to real-world problems helps make the learning relevant to students. (p. 9-10)

The mathematical processes of the Ontario curriculum.

The focus on the curriculum mathematical processes in students' DMPs offers ways to interpret the mathematical activity students are engaged through DMP.

Problem-solving. The curriculum document mentions that "problem solving forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction" (Ontario Ministry of Education, 2005, p. 11). "It is considered an essential process through which students are able to achieve the expectations in mathematics" (p. 11). According to the curriculum document, problem-solving:

- is the primary focus and goal of mathematics in the real world;
- helps students become more confident in their ability to do mathematics;
- allows students to use the knowledge they bring to school and helps them connect mathematics with situations outside the classroom;
- helps students develop mathematical understanding and gives meaning to skills and concepts in all strands;
- allows students to reason, communicate ideas, make connections, and apply knowledge and skills;
- offers excellent opportunities for assessing students' understanding of concepts, ability to solve problems, ability to apply concepts and procedures, and ability to communicate ideas;
- promotes the collaborative sharing of ideas and strategies, and promotes talking about mathematics;
- helps students find enjoyment in mathematics;
- increases opportunities for the use of critical-thinking skills (estimating, evaluating, classifying, assuming, recognizing relationships, hypothesizing, offering opinions with reasons, and making judgements) (Ontario Ministry of Education, 2005, p. 12).

The strategy model mentioned in the curriculum document is based on (a) understand the problem; (b) make a plan; (c) carry out the plan; and (d) look back at the solution. DMP can be seen in part as a problem-solving experience because there is a parallel between the strategy model and the process of how to structure mathematics to elucidate the mathematical ideas explored considering the presence of surprises, sense-making, emotions and visceral sensations, which is part of a good story within Boorstin's (1990) perspectives.

> ***Reasoning and proving***. The reasoning process supports a deeper understanding of mathematics by enabling students to make sense of the mathematics they are learning. The process involves exploring phenomena, developing ideas, making mathematical conjectures, and justifying results. Teachers draw on students' natural ability to reason to help them learn to reason mathematically. Initially, students may rely on the viewpoints of others to justify a choice or an approach. Students should be encouraged to reason from the evidence they find in their explorations and investigations or from what they already know to be true, and to recognize the characteristics of an acceptable argument in the mathematics classroom. Teachers help students revisit conjectures that they have found to be true in one context to see if they are always true. For example, when teaching students in the junior grades about decimals, teachers may guide students to revisit the conjecture that multiplication always makes things bigger. (Ontario Ministry of Education, 2005, p. 14)
>
> ***Reflecting***. Good problem solvers regularly and consciously reflect on and monitor their own thought processes. By doing so, they are able to recognize when the technique they are using is not fruitful, and to make a conscious decision to switch to a different strategy, rethink the problem, search for related content knowledge that may be helpful, and so forth. Students' problem solving skills are enhanced when they reflect on alternative ways to perform a task, even if they have successfully completed it. Reflecting on the reasonableness of an answer by considering the original question or problem is another way in which students can improve their ability to make sense of problems. Even very young students should be taught to examine their own thought processes in this way. One of the best opportunities for students to reflect is immediately after they have completed an investigation, when the teacher brings students together to share and analyse their solutions. Students then share strategies, defend the procedures they used, justify their

> answers, and clarify any misunderstandings they may have had. This is the time that students can reflect on what made the problem difficult or easy (e.g., there were too many details to consider; they were not familiar with the mathematical terms used) and think about how they might tackle the problem differently. Reflecting on their own thinking and the thinking of others helps students make important connections and internalize a deeper understanding of the mathematical concepts involved. (Ontario Ministry of Education, 2005, p. 14)

Selecting Tools and Computational Strategies. Considering the fact that DMPs are essentially produced through the use of digital technology, *Selecting Tools and Computational Strategies* is a very significant process in DMP. The curriculum document states that:

> Students need to develop the ability to select the appropriate electronic tools, manipulatives, and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems. Various types of technology are useful in learning and doing mathematics. Although students must develop basic operational skills, calculators and computers can help them extend their capacity to investigate and analyse mathematical concepts and reduce the time they might otherwise spend on purely mechanical activities. Students can use calculators or computers to perform operations, make graphs, and organize and display data that are lengthier and more complex than those that might be addressed using only pencil-and-paper. Students can also use calculators and computers in various ways to investigate number and graphing patterns, geometric relationships, and different representations; to simulate situations; and to extend problem solving. When students use calculators and computers in mathematics, they need to know when it is appropriate to apply their mental computation, reasoning, and estimation skills to predict and check answers. The computer and the calculator should be seen as important problem-solving tools to be used for many purposes. Computers and calculators are tools of mathematicians, and students should be given opportunities to select and use the particular applications that may be helpful to them as they search for their own solutions to problems. Students may not be familiar with the use of some of the technologies suggested in the curriculum. When this is the case, it is important that teachers introduce their use in ways that build students' confidence and contribute to their understanding of the concepts being investigated. Students also need to understand the

situations in which the new technology would be an appropriate choice of tool. Students' use of the tools should not be laborious or restricted to inputting or following a set of procedures. (Ontario Ministry of Education, 2005, p. 14-15)

As discussed in this chapter, learning and meaning and knowledge production are concerned with connections, representations and communication (Deleuze, 2000; Levy, 1993, 1998, 2001). Interestingly, *connecting*, *representing*, and *communicating* are three of the curriculum mathematical processes.

> ***Connecting.*** Experiences that allow students to make connections – to see, for example, how concepts and skills from one strand of mathematics are related to those from another – will help them to grasp general mathematical principles. As they continue to make such connections, students begin to see that mathematics is more than a series of isolated skills and concepts and that they can use their learning in one area of mathematics to understand another. Seeing the relationships among procedures and concepts also helps develop mathematical understanding. The more connections students make, the deeper their understanding. In addition, making connections between the mathematics they learn at school and its applications in their everyday lives not only helps students understand mathematics but also allows them to see how useful and relevant it is in the world beyond the classroom. (Ontario Ministry of Education, 2005, p. 16)
>
> ***Representing.*** In elementary school mathematics, students represent mathematical ideas and relationships and model situations using concrete materials, pictures, diagrams, graphs, tables, numbers, words, and symbols. Learning the various forms of representation helps students to understand mathematical concepts and relationships; communicate their thinking, arguments, and understandings; recognize connections among related mathematical concepts; and use mathematics to model and interpret realistic problem situations. Students should be able to go from one representation to another, recognize the connections between representations, and use the different representations appropriately and as needed to solve problems. (Ontario Ministry of Education, 2005, p. 16)
>
> ***Communicating.*** Communication is the process of expressing mathematical ideas and understanding orally, visually, and in writing, using numbers, symbols, pictures, graphs, diagrams, and words. Students communicate for various purposes and for different

> audiences, such as the teacher, a peer, a group of students, or the whole class. Communication is an essential process in learning mathematics. Through communication, students are able to reflect upon and clarify their ideas, their understanding of mathematical relationships, and their mathematical arguments. Teachers need to be aware of the various opportunities that exist in the classroom for helping students to communicate. . . . Effective classroom communication requires a supportive and respectful environment that makes all members of the class feel comfortable when they speak and when they question, react to, and elaborate on the statements of their classmates and the teacher. The ability to provide effective explanations, and the understanding and application of correct mathematical notation in the development and presentation of mathematical ideas and solutions, are key aspects of effective communication in mathematics. (Ontario Ministry of Education, 2005, p. 17)

Final Comments of the Chapter

In this second chapter I presented the theoretical lenses I use to analyze students' DMPs in this research. I initially discussed some sociocultural perspectives and their relations to postmodernism in mathematics education (Ernest, 2004a, 2004b). That is, I emphasized meaning and knowledge production as social and cultural processes and my pedagogic focus on playfulness, production of multimodal texts, and conversations as characteristics of a postmodern curriculum in teaching and learning mathematics. I also highlighted the role of technology in knowledge production, considering the notions of humans-with-media (Borba & Villarreal, 2005) and cognitive ecology (Levy, 1993, 1998, 2001). I pointed out some perspectives on multimodality (Kress, 2003), while considering the designs proposed by The New London Group (1996) to highlight DMPs as multimodal texts/narratives. I discussed Boorstin's (1990) categories about what makes good films. Making a parallel, these categories on performance arts are used in this study to analyze students' DMPs, considering some aspects of what makes conceptual DMPs (Gadanidis & Borba, 2008; Gadanidis & Hughes, 2008). Finally, I introduced the mathematical strands and processes of the Ontario curriculum (Ontario Ministry of Education, 2005) considering the fact that these components are also used as lenses to analyze students' DMPs. In the next chapter I discuss the methodological issues of the study.

Chapter Three: A Qualitative Research Design for a Study on the Nature of Students' Digital Mathematical Performances

In this chapter, I discuss methodological issues. I initially present some aspects of qualitative research that inform the processes of data collection and analysis of the study. In this research I employ qualitative case studies, along with video analysis. The issues discussed in this chapter seek to clarify some of the procedures and interpretative aspects conducted during the study that shaped the interpretative/descriptive/analytic ideas presented in chapters four and five.

Qualitative Research

As introduced in the prologue, the research questions of this study are:

- What is the nature of elementary school students' digital mathematical performances in the Math + Science Performance Festival?
- What are the mathematical ideas explored and how do students communicate them using the performance arts?

Considering the nature of these research questions and the way I locate myself as a sociocultural/postmodern interpreter, the proposed doctoral thesis falls within the tradition of qualitative research. According to Denzin and Lincoln (2005):

> Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretative, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This mean that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meaning people bring to them (p. 5).

Qualitative research has an *interpretative* dimension, that is, it views human experiences and actions as mediated by interpretation (Schwandt, 2003). Patton (2002) states that, in qualitative research, "the researcher is the instrument" (p. 14). Bogdan and Biklen (1992) argue that "qualitative researchers' goal is to better *understand* human behavior and experience. [Qualitative researchers] seek to grasp the processes by which people construct meaning and to describe what those meanings are" (p. 49).

Qualitative researchers emphasize the relevance of a synergy (or consistency) between research methods and theoretical perspectives (Araujo & Borba, 2004; Skovsmose & Borba, 2004). Lincoln and Guba (1985) posit that in qualitative studies “the design cannot be given a priori, but must emerge as the study proceeds” (p. 225). In this sense, Lincoln and Guba use the term *emergent design* to refer to the dynamicity of developing a qualitative research, that is, theoretical perspectives and understandings are grounded by the data and refined through the process of conducting the research.

As I describe in this chapter, the students’ DMPs I analyze are publicly available in a Festival located in a virtual environment. The literature has addressed specific issues for qualitative researchers in online education (Borba, Melheiros, & Zulatto, 2010; Stahl, 2009). Vernon (2007) argues that educational research involving cyberspaces requires specific skills from the researcher as a *bricoleur.*[13] Thus, I acknowledge my role as an interpretative *bricoleur* in collecting, analyzing, and (re)presenting data in this study. In this role, I will be attentive to issues related to the use of multimodal communication (Kress, 2003) in virtual settings. Researchers must *think-with-digital-media* as they adapt qualitative methods to this scenario (Borba, Malheiros, & Scucuglia, 2012).

Data Collection

Marshall (1996) discusses different types of samples on data collection in qualitative research. According to Marshall:

> *Judgement sample:* The researcher actively selects the most productive sample to answer the research question. This can involve developing a framework of the variables that might influence an individual’s contribution and will be based on the researcher’s practical knowledge of the research area, the available literature and evidence from the study itself. . . . During interpretation of the data it is important to consider subjects who support emerging explanations and, perhaps more importantly, subjects who disagree [challenge the theoretical framework assumed] . . .
>
> *Theoretical sample:* The iterative process of qualitative study design means that samples are usually theory driven to a greater or lesser extent. Theoretical sampling necessitates

[13] *Bricolage* is a metaphor for qualitative research proposed by Denzin and Lincoln (2005), referring to something constructed by using whatever materials or resources are available.

building interpretative theories from the emerging data and selecting a new sample to examine and elaborate on this theory. It is the principal strategy for the grounded theoretical approach but will be used in some form in most qualitative investigations necessitating interpretation. (p. 524)

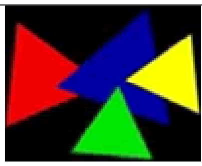
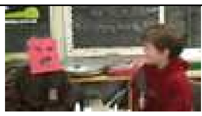
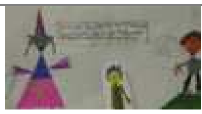
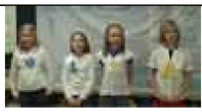
Based on the notions of *judgement sample* and *theoretical sample* (Marshall, 1996), the data of this study are comprised of 22 elementary school students' DMPs. The students are from Grade 4 to Grade 6 from different public schools in Ontario. These data refer to *all* the students' DMPs published in the Math + Science Performance Festival in 2008 (Gadanidis, Borba, Gerofsky, & Jardine, 2008), which was the first year of the Festival. As mentioned in the prologue, this Festival is based on a virtual environment where DMPs are published and shared. Every year, Canadian celebrities (musicians, poets, TV presenters) and mathematicians select performances in terms of: (a) depth of the mathematical ideas, (b) creativity and imagination, and (c) quality of the performances. Figure 3.1 shows the website of the Festival, which is the setting of the students' DMPs analyzed in this study.

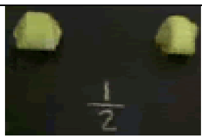
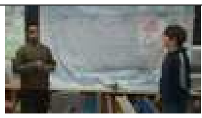
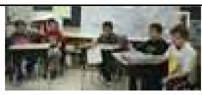
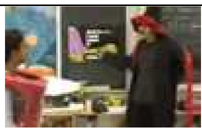
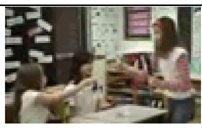
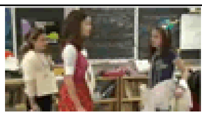
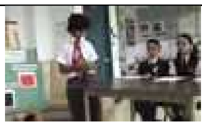
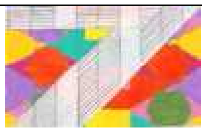
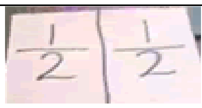
Figure 3.1: The Math + Science Performance Festival Website.

Figure 3.1: DMPs can be submitted and viewed at www.mathfest.ca . The Festival runs every year since 2008 and it is supported by ESSO Imperial Oil, The Fields Institute, The Research Western, Western Education, and The Canadian Mathematical Society.

In table 3.1 I introduce the students' DMPs analyzed in the study. I present the number, the title, a screen-captured image, the digital format of the DMPs, and the length (time) of each video. There are 19 DMPs in video format (totaling 56 minutes and 30 seconds), two DMPs in PowerPoint format, and one DMP in image format. A detailed description of each DMP is presented in chapter four as part of the analysis.

Table 3.1: List of students' DMPs analyzed in the study

#	Title	Image	Digital Format	Time length (minutes : seconds)
1	Polly Gone		Video	2:25
2	Geometrical Idol		Video	2:41
3	Shape Songs		Video	1:50
4	Triangles		Video	4:20
5	Little Quad's Quest		Video	Part 1: 3:09 Part 2: 1:29 Part 3: 2:30 Part 4: 2:07 Part 5: 2:37
6	Math Healing Wishes for Kyla		Power Point	None
7	Math Facts Show		Video	4:12
8	2D Land		Video	3:23
9	We are the Polygons		Video	2:02

10	Fabulous Fractions		Video	0:38
11	Sphere on the Loose		Video	1:26
12	Radius & Diameter		Video	2:12
13	Square Trial		Video	1:33
14	Pointacula		Video	3:49
15	Equivalent Fractions		Video	0:37
16	Shape Idol		Video	2:56
17	Square Base pyramid		Video	2:30
18	Who hurt Mr. Square?		Video	1:57
19	Are You Smarter than a 4th Grader?		Video	5:31
20	Ricky's Metre Chocolate Bar		Power Point	None
21	Grade 5 Math Art		Image	None
22	Fractiontastic		Video	0:36

Ethics.

According to AERA (2000):

> Research in education is often directed at children and other vulnerable populations. . . . As educational researchers, we should strive to protect these populations, and to maintain the integrity of our research, of our research community, and of all those with whom we have professional relations. (p. 1)

The data analyzed in this research are publicly available at the website of the Festival (http://www.mathfest.ca), and as a result of having ethical clearance from The University of Western Ontario for the study as a whole, subsequent consent from the participants was not required.

Data Analysis

Data analysis usually begins with the organization, coding, and identification of patterns from the variety of data collected (Denzin & Lincoln, 2005; Patton, 2002). By coding and identifying patterns, the researcher can create themes while searching for alternative explanations for his/her research question(s). The researcher then presents the data, displaying them in narrative form using text, charts, graphs, and tables, as well as alternative forms such as artistic approaches (see Eisner, 1981). Specifically, there are two intersecting bodies informing this data analysis. They are *video analysis* and *qualitative case studies*.

Video analysis.

Powell, Francisco, and Maher (2003) built an analytical model of video recordings of mathematics teaching experiments. These authors argue that "video is an important, flexible instrument for collecting aural and visual information. It can capture rich behavior and complex interactions and it allows investigators to re-examine data again and again" (p. 407). It is a way researchers can identify and understand complex interactions and it allows them to re-analyze these interactions many times. In this study I use a variation of Powel et al.'s model, which I adapted in order to analyze students' DMPs. Following, I present some of the non-linear steps or procedures proposed by Powell et al.[14] for video analysis and its relation to this study:

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[14] The original model is structured in seven procedures. For the purpose of this study, I am combining some of the procedures of the model into five main categories.

Viewing and describing. These procedures refer to the researcher's familiarization with the data. "To become familiar with the content of the video data, researchers watch and listen to the videotapes several times. In this phase, researchers watch and listen without intentionally imposing a specific analytical lens on their viewing" (Powell et al., 2003, p. 15). The researcher must write descriptions of the video recording at this stage. These descriptions help the researcher to develop a holistic view of the data and identify initial critical events. "By viewing and describing video data, researchers acquire a fairly in-depth knowledge of the content of their videos" (Powell et al., 2003, p. 416).

An initial procedure in this study was to read students' DMPs several times to become familiar with them. Moreover, a continuous process of description of the DMPs was conducted: (1) I created a summary (a table) containing general information about the DMPs such as the URL, the mathematical idea explored and its relation to the curriculum strands (Ontario Ministry of Education, 2005), the participants and the authors, the place where the performance was conducted, an image of the performance, and any other complementary information. (2) I described the performance in detail, including the full transcription based on some aspects of multimodal discourse analysis (O' Halloran, 2011), and several screen-captured images of the DMP to illustrate students' actions, gestures, costumes, ways of using materials, and so on.

Coding. Coding is a very significant process in qualitative research and crucial to analysis of video data.

> This activity is aimed at identifying themes that help a researcher interpret data. In our model, this activity is similar to identifying critical events in that both require watching videos intensively and closely for long periods of time. At this phase of analysis, the difference is that researchers focus attention on the content of the critical events. Therefore, video recording is helpful in this activity in much the same way as it is in enhancing the identification of critical events . . . Like identifying critical events, coding is directed by researchers' theoretical perspective and research questions. Repeated and shared viewing, made possible by video technology, and the density of video data enhance researchers' ability to search and identify codes, whether these are predetermined or emergent. Just as with critical events, codes are defined in relation to the research question pursued or emergent themes . . . we have developed coding schemes informed by our assumptions about mathematical thinking and our research practices into

> the development of mathematical ideas and forms of reasoning. We have found it particularly useful and important to code for learners' mathematical ideas, mathematical explanations or arguments, mathematical presentations (symbolic, pictorial, and gestic), and features and functions of discourse. We have also refined codes related to several constructs such as critical events, trace, and the flow of ideas among learners in a group. (Powell et al., 2003, p. 224)

Codes reflect, among other things, the theoretical framework assumed by the researcher. These codes help the researcher to identify patterns and critical events and create themes or categories as part of the process of interpretation. According to Holton (2007), "it is through coding that the conceptual abstraction of data and its reintegration as theory takes place" (p. 237). Codes help the researcher to identify turning points or "critical events" in relation to the research questions. By coding the data, unexpected findings emerge and the researcher can create new theories grounded in the data. Codes reflect the researcher's theoretical framework, but also account for unexpected phenomena and enable the emergence of new theory or modifications to his/her pre-existing notions. Basit (2003) states:

> Codes are tags or labels for allocating units of meaning to the descriptive or inferential information compiled during a study. . . They can take the form of a straightforward category label or a more complex one, for example, a metaphor. (p. 144)

The codes of this study were shaped by the theoretical lenses introduced in chapter two (see Figure 2.1) and guided by the research questions. In the descriptive analysis displayed in chapter four, the main codes were constructed using Boorstin's (1990) lens, based on an adaptation of the categories of surprises, sense-making, emotions, and visceral sensations. Moreover, the mathematical idea explored by the students, students' insights and conjectures in the performative process, students' collaboration and ways of using technologies, and the communication based on multiple modes were also considered in the interpretative analysis conducted through the procedures of video analysis. Based on the analysis constructed in chapter four, a cross-case analysis was conducted in chapter five. The similarities interpreted through the cases offered emergent insights on the nature of students' DMPs. Thus, emergent codes grounded in the data conditioned the analytic categories displayed in chapter five.

Critical events. An event is critical in relation to the research question(s). Critical events refer to the interpretation and selection of data considering the researcher's interests. At this moment, the researcher intends to identify, organize, and interpret significant moments considering students' mathematical thinking. According to Powell et al. (2003), in the analysis of teaching experiments, "an event is called *critical* when it demonstrates a significant or contrasting change from previous understanding, a conceptual leap from earlier understanding" (p. 416).

> Significant contrasting moments may be events that either confirm or disaffirm research hypotheses; they may be instances of cognitive victories, conflicting schemes, or naïve generalizations; they may represent correct leaps in logic or erroneous application of logic; they maybe any event that is somehow significant to a study's research agenda. (Powell et al., 2003, p. 417)

> Critical events are contextual. An event is critical in its relation to particular research questions pursued. Thus, an instance in which learners present a mathematical explanation or argument may be significant for a research question concerned with students' building of mathematical justification or proof and, as such, be identified as a critical event. In contrast, a researcher concerned with the impact of teacher interventions on students' reflective abstraction or mathematical understanding might deem as critical those events that connect teacher interventions and associated student articulations of their thinking. However, the relation between critical events and research questions pursued also implies that researchers might identify events as critical that include negative instances of a hypothesis, instances of wrong leaps, and somehow significant to the study's research question. (Powell et al., 2003, p. 418)

Although DMPs do not necessarily represent a register of teaching experiment sessions, it is important to notice that the focus of this study is on (a) how students communicate their ideas and use the performance arts, and (b) what kind of surprises, emotions, and visceral sensations the audience may experience by reading students' DMPs. Most important, it is fundamental to notice that usually DMPs are already a selection of events refined and constructed by students. DMPs are usually short video recordings (the DMPs analyzed in this research have approximately 3 minutes). Therefore, I consider that all events are critical events in a DMP because they are already a selection of events of mathematical activity proposed by students. DMPs are products and they do not represent the integral process of creation, even though insights about the process may be interpreted through the final text/narrative. That is one of the reasons why I consider it important to present a full transcription to describe DMPs.

Transcription. In transcribing critical events, the researcher highlights significant elements of students' verbal language and activity. It allows the researcher to conduct an in-depth analysis of how communication occurred.

> There are essential reasons to transcribe videotapes. First, following procedures within data collection and analytic traditions, researchers may implement an open-coding process on data to discover themes that are above, beyond, and beside those suggested by specific, *a priori* guiding research questions and deductive codes. The production of the transcript and the physical, static rendering of a research session affords researchers opportunities for extended, considered deliberations of talk and noted gestures. Second, researchers analyzing participants' discursive practices, especially their dialogue, find it useful to view the printed, sequential rendering of speech to see what it reveals about the mathematical meanings and understandings participants construct. Since discursive practices include actions that are not only utterances, researchers indicate in their transcripts relevant body movements as well as inscriptions (writings, drawings, sketches, and so on) that participants create. Third, transcripts are, for practical purposes, a permanent record and can reveal important categories that are not always capable of being discerned by viewing videotapes since, notwithstanding the technology of replay, the visual and aural video images that the viewer's mind eye and ear captures are essentially ephemeral. Instead with transcript data, one can consider more than momentarily the meaning of specific utterances. Fourth, researchers transcribe so that later, if and where appropriate in narrative reports, they can provide evidence of findings in the participants own words. (Powell et al., 2003, p. 418)

As mentioned, I consider it is important to present a full transcription of DMPs. As the reader will notice, the transcriptions presented in the analysis of each students' DMPs include their spoken language, description of the use of audio, actions (gestures and ways of using artifacts), and several images (to describe the actions, facial expressions and close-ups during filming). Some aspects of the notion of *multimodal discourse analysis* are considered in this study.[15] O' Halloran (2011) states that "multimodal discourse analysis . . . is an emerging paradigm in discourse studies which extends the study of language per se to the study of language in combination with other resources, such as images, scientific symbolism, gesture, action, music and sound" (p. 1). "The multimodal analysis includes the interactions between the spoken language, kinetic features (including gaze, body posture and gesture) and cinematography effects (including camera angle and frame size)" (O'Halloran, 2011, p. 14). In chapters four and five, students' actions and sounds are described between brackets (e.g., *[Students start to sing]*, *[Applause]*). The symbol *[?]* means that I was unable to understand what the student said. Considering the fact that I present all 22 transcriptions in chapter four, I use single space, font 11 in italics. The name of the student/performer/speaker is displayed in a bold font. Following, I present an example of a transcription displayed in this research from DMP #2 – *Geometrical Idol*:

Trapezoid: *Hi!*
Presenter: *What are you gonna sing today?*
Trapezoid: *I'm singing "Now I am a trapezoid."*
[Someone plays the guitar].
Trapezoid *[singing]: Head, oh head, I lost my head. And head, oh head, I could've been dead. Head, oh head, I feel sick in bed. Now, I am a trapezoid.* (See Figure 3.2).
[Applause].
Judge 2: *I liked that very much!*
Judge 1: *It was ok.*

Figure 3.2: The Trapezoid

Storyline and composing narrative. Based on the coded transcriptions and the critical events interpreted, the researcher contrasts them with other data resources. Thus, the researcher creates themes and sub-themes that allow him/her to compose and display the research as a narrative highlighting the discussions of the research questions based on evidence from the data.

> The storyline is the result of making sense of the data with particular attention to identified codes . . . Constructing a storyline requires the researcher to come up with

[15] Discourses are ubiquitous ways of knowing, valuing, and experiencing the world. Discourses can be used for an assertion of power and knowledge, and they can be used for resistance and critique. Discourses are used in everyday contexts for building power and knowledge, for regulation and normalization, for the development of new knowledge and power relations. In *critical discourse analysis* (CDA; Fairclough, 1995), one assumes "discourse is shaped and constrained by (a) social structure (class, status, age, ethnic identity, and gender) and by (b) culture. Discourses "help to shape and constrain our identities, relationships, and systems of knowledge and beliefs." CDA considers "three levels of analysis: (a) the actual text; (b) the discursive practices (that is the process involved in creating, writing, speaking, reading, and hearing); and (c) the larger social context that bears upon the text and the discursive practices. (McGregor, 2003, para. 7)

> insightful and coherent organizations of the critical events, often involving complex flowcharting. This process often involves discerning *traces*, which are a collection of events, first coded and then interpreted, to provide insight into a student's cognitive development. The trace contributes to the narrative of a student's personal intellectual history as well as to the collective history of a group of students who collaborate . . . The process of making sense of the critical events and codes is complex and, more often than not, nonlinear. Researchers may have to go back and forth examining critical events, codes . . . The process of making sense of the critical events and codes is complex and, more often than not, nonlinear. Researchers may have to go back and forth examining critical events, codes. (Powell et al., 2003, p. 430)

As mentioned, in chapter four I present an in-depth descriptive analysis of students' DMPs based on Boorstin's (1990) categories of what makes good films. In chapter five I present a cross-case analysis, seeking to discuss similarities and particular aspects of students' DMPs. The very notion of qualitative case studies is thus fundamental in this study, considering the focus on the nature of students' DMPs, the research design, and ways I display the composition of the narrative of this study.

Qualitative case studies.

The very notion of qualitative case studies is relevant to this research. Stake (2005) argues that "case study is not a methodological choice but a choice of what is to be studied" (p. 443). Stake (2003) claims case studies are specific and bounded, they have patterns, and the focus is on the understanding of the complexity of the case. The researcher decides what and how the case story is told. Stake (2003) states that "case studies are of value for refining theory and suggesting complexities for further investigation" (p. 156). Yin (2006) claims "the strength of the case study method is its ability to examine, in-depth, a case, within its 'real-life' context" (p. 111). Case study "is best applied when research addresses descriptive or exploratory questions and aims to produce a firsthand understanding of people and events" (Yin, 2006, p. 112). For Yin (2006), designing a case study requires three basic steps: (a) defining the case; (b) "deciding whether to do a single case study or a set of case studies" (p. 113); and (c) pointing out theoretical perspectives and their relation to the research methods. Patton (2002) highlights:

> The case study approach to qualitative research analysis constitutes a specific way of collecting, organizing, and analyzing data; in that sense it represents an analysis *process*. The purpose is to gather comprehensive, systematic, and in-depth information about each case of interest. The analysis process results in a product: a case study. Thus, the term case study can refer to either the process of analysis or the product of analysis, or both. (p. 447)

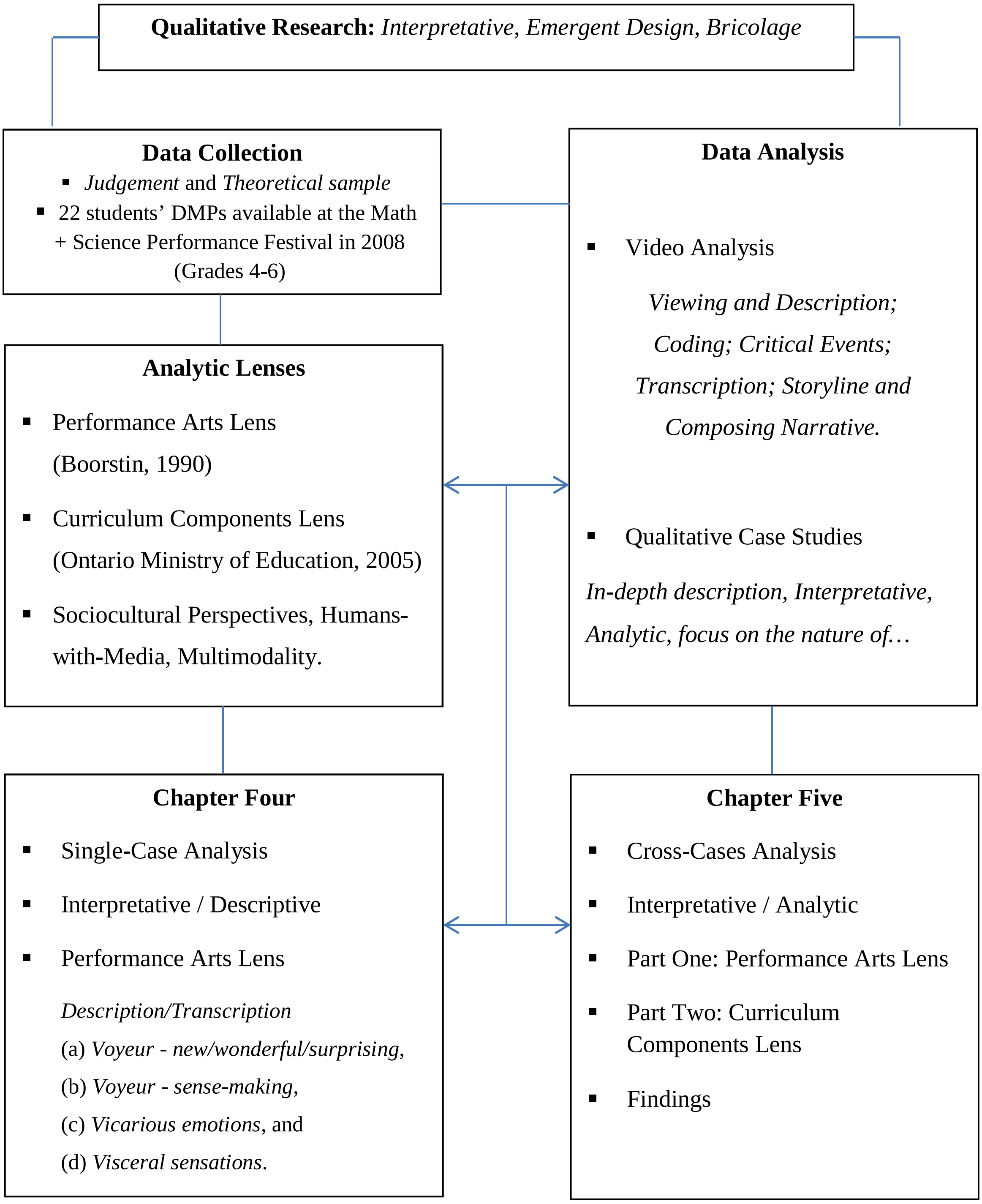
Therefore, qualitative case studies are methodologically significant to this study because (1) the focus is bounded (within the Festival, on grades 4-6 students' DMPs published in 2008), and (2) it helps me to delineate ways to tell and represent the stories emergent from my analysis, seeking for the depth of understanding considering the research questions. In chapter four, I present a descriptive single-case analysis of students' DMPs. In chapter five, I present a cross-case analysis. Overall, this study assumes the design of multiple case studies (Yin, 2006).

The Interpretative / Descriptive / Analytic Lens of the Study

In chapter four I present a descriptive analysis of each of the 22 elementary school students' DMPs available at the Math + Science Performance Festival in 2008. My focus is on interpreting students' mathematical thinking, communication, and representation through a performance-arts lens. That is, based on the research questions, I use a variation of Boorstin's (1990) lens to analyze each of the students' DMPs considering the model proposed by Powell et al. (2003) for video analysis. I call the analysis displayed in chapter four a single-case analysis, which may be considered an interpretative/descriptive process constructed through four main categories of a performance arts lens: (a) *Voyeur - new/wonderful/surprising*, (b) *Voyeur - sense-making*, (c) *Vicarious emotions,* and (d) *Visceral sensations*.

In chapter five I present a cross-case analysis, that is, I present a discussion based on interpretative similarities and differences between the cases analyzed in chapter four, focusing on relevant aspects regarding the research questions. In chapter five, besides using the performance arts lens based on Boorstin's (1990) categories, I also use components of the Ontario Curriculum as a lens to analyze students' DMPs. Both lenses are informed by the bodies of theories discussed in chapter two (sociocultural perspectives, humans-with-media, and multimodality). Thus, Figure 3.3 illustrates and summarizes the analytic lens of the study in relation to the methodological positions and procedures assumed in the study.

Figure 3.3: The Methodological Design of the Study



Chapter Four: A Descriptive Analysis of Each of 22 Cases of Elementary School Students' Digital Mathematical Performances

In this chapter I present a descriptive analysis of each of the 22 elementary school students' digital mathematical performances (DMPs) available at the Math + Science Performance Festival (Festival) in 2008. My focus is on interpreting students' mathematical thinking and communication through a performance arts lens.

The analysis is based on the following research questions:

- What is the nature of elementary school students' digital mathematical performances in the Math + Science Performance Festival?
- What are the mathematical ideas explored and how do students communicate them using the performance arts?

Boorstin's (1990) lens for what makes a good film is formed by three categories. This lens identifies three eyes: voyeuristic, vicarious, and visceral. However, I use four categories for analysis of a DMP, by splitting Boorstin's first category of voyeuristic into two components: new/wonderful/surprising and sense-making. These two components offer a more accurate analytic way to discuss (a) the role of surprises in experiencing and exploring mathematics (Gadanidis & Borba, 2008; Watson & Mason, 2007) and (b) students' reasoning and mathematical thinking. My intention in this chapter is to use these four categories to interpret and conduct a descriptive analysis of each elementary school students' DMPs (see Table 4.1).

Table 4.1: Analytical Lenses ▪ What makes good films (Boorstin, 1990); ▪ What makes conceptual DMPs (Gadanidis & Borba, 2008; Gadanidis & Hughes, 2008).	
Voyeur - New/Wonderful/ Surprising	Senses of surprise, new, and wonderful are fundamental for the voyeuristic eye. It captures the attention of the audience and offers the pleasure of experiencing something new.
Voyeur - Sense-Making	Rational and logical senses are also significant for a voyeuristic experience. Surprises have to make sense to be effective.
Vicarious Emotions	Emotional moments happen when the audience feels what actors are feeling. Close-ups on the actors' facial expression are used in films to intensify the vicarious experiences.
Visceral Sensations	Direct sensations of mathematical fit/beauty/changes (Sinclair, 2001) offer visceral experiences to the audience. Soundtracks are often used to enhance/amplify visceral sensations.

I see the analysis of students' DMPs as qualitative case studies (Stake, 2005). Thus, as discussed in the methodology, this chapter assumes the design of a multiple case study (Yin, 2006), although the descriptive analyses are displayed individually (single-case). In the next chapter, I present a cross-case analysis, which makes connections between the cases.

I am displaying each of the cases in this chapter in the following way: I first present a *table* with general information about the DMP such as a screen-captured image, the URL, the mathematical content in relation to the mathematical strands of the Ontario Curriculum (Ontario Ministry of Education, 2005), the digital format (e.g., video, audio), time length of the videos, the artistic expression used (skit, music, animation), the participants and/or authors, the setting where it was conducted, and any other information displayed in the DMP. Then, I present a *description* of the DMP, including the transcription. Finally, and most importantly, I present the descriptive analysis of the DMP using the four analytical categories:

(a) *Voyeur - new/wonderful/surprising*,

(b) *Voyeur - sense-making*,

(c) *Vicarious emotions*, and

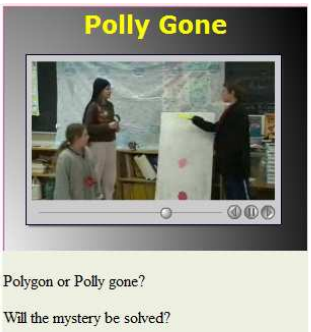
(d) *Visceral sensations*.

Although I seek to discuss each DMP in detail, it is a challenge to present through a print-based text the analysis of a DMP, which, by its very nature, is fundamentally digital and multimodal. Thus, I use screen-captured images to describe and analyze several moments of the DMPs and I provide information about students' gestures and actions in the transcriptions, regarding a notion of multimodal discourse analysis (O' Halloran, 2011).

However, I want to point out that this chapter is a long chapter. The reasons are: (a) I analyze 22 DMPs, which may be considered a high number of DMPs; (b) considering the notion of case studies, I seek to present as much descriptive detail as I can in order to elucidate and discuss the nature of the DMPs; (c) I present the full transcription of each DMP with lots of screen-captured images, because DMPs are already a collection of "critical events" and the representation of students' verbal/spoken language and (visual) actions are significant from a multimodal point of view; (d) I make initial connections between the cases, although these connections will become clearer in the cross-case analysis presented in chapter five; and (e) the reader may watch the DMPs available at www.mathfest.ca, but it is part of my task as a researcher to introduce, describe, and display my interpretation of the data as a print-based text. Considering these reasons, the reader may feel free to figure out alternate ways of reading this long chapter (e.g., consulting the transcriptions only when necessary, focusing on specific cases or set of cases highlighted in chapter five, etc.).

DMP #1: Polly Gone

Table 4.2: Polly Gone



URL:	http://www.edu.uwo.ca/mathscene/geometry/geo5.html
Strands and Content:	Strand: Geometry and Spatial Sense Content: Properties of polygons
Format:	Video.
Time length:	2:25
The Arts:	Skit.
Participants:	Three Elementary School Students. I refer to these students as Student 1, Student 2, and Student 3, respectively, from left to right, as shown in the picture on the left.
Setting:	Performed in a classroom.
Info:	Polygon or Polly gone? Will the mystery be solved?

Description

In this skit, a student is upset because “Polly (is) gone.” Two other students misinterpret the statement to be “polygon,” and try to understand the statement by considering different properties of polygons. When they realize that “Polly (the parrot) is gone” they try to find her. Using properties of polygons, they solve mathematical puzzles that unlock two doors and find and free Polly. Below is a transcription of the DMP, which I present in two scenes:

Figure 4.1: Student 1 cries.

Figure 4.2: Student 3 shows a triangle.

Figure 4.3: Student 2 shows an octagon.

Scene One

All Students: *The mystery of Polygon.*
Student 1: *[crying] Polly gone!* (See Figure 4.1).
Student 2 and Student 3: *What’s wrong?*
Student 1: *[crying] Polly gone!*
Student 3: *A polygon is a closed 2D shape with sides made with straight line segments. For example, this [circular figure] is not a polygon, because the sides are not straight. But this triangle is a polygon because sides are straight* (See Figure 4.2).
Student 1: *[crying] No! Polly gone!*
Student 2: *Maybe she means regular polygons, which are polygon with all side lengths equal and all angles that are same degree. For example, this octagon is a regular polygon, because all sides are the same length and all angles are the same too* (See Figure 4.3).
Student 1: *[crying] Polly gone! Polly gone!*
Student 2: *This [other] shape is not a regular polygon. It is an irregular polygon because not all angles and size of lengths are the same.*
Student 1: *[crying] No! Polly gone!*
Student 2 and Student 3: *Oh no! Polly gone!* (See Figure 4.4).

Scene Two

Student 1: *Where could Polly be?*
Student 3: *How about behind this door? But, which door knob? [Seeing the door with three figures on it]. Well, let’s see. They must be polygons. I know this one is not a polygon [touching the figure on the bottom], because not all sides are straights. So, it could be that one [pointing out the figure on the center]* (See Figure 4.5).
Student 2: *It can’t be this one either [removing the figure on the center].*
Student 1: *I know why. [It is] because these lines are curved.*
All three Students: *It must be this one!*
Student 3: *[touches the figure on the top of the door (a polygon), and turns the piece of cardboard (the door) making a sound using his voice. A new door appears]* (See Figure 4.6).
All three Students: *Oh no! Another door!*

Student 2: *But they are all polygons.*
Student 3: *They must be regular polygons. I know this one [touching the figure on the top] is not a regular polygon, because not all side lengths and angles are equal.*
Student 1: *What is it then?*
Student 3: *It is an irregular polygon.*
Student 1: *I get it.*
Student 2: *It can't be this one neither [pointing out and removing the figure on the center]. Do you know why [Student 1]?*
Student 1: *Yes, [it is] because all angles are different.*
All three Students: *So, it must be that one!*
Student 1: *Well, all eight sides are the same length.*
Student 3: *And all angles are the same degree.*
All three Students: *So, it must be regular!*
Student 1: *[touches the figure on the bottom of the cardboard, the door opens and Polly appears]* (See Figure 4.7)
All three Students: *Yeah! Polly!*
Student 1: *The end.*

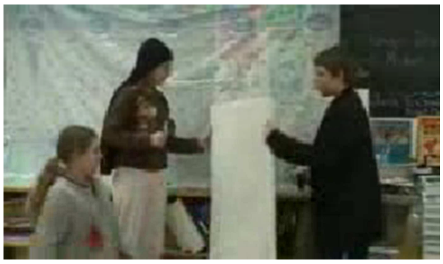
[Applause]

Figure 4.4: Student 2 and Student 3 realize that 'Polly gone!'

Figure 4.5: The first door.

Figure 4.6: The second door.

Figure 4.7: Students open the second door and free Polly.



Voyeur - New/Wonderful/Surprising

Polygon vs. *Polly (is) gone* is a play on words or a pun. This pun is a traditional *math joke* discussed in many websites (see http://www.moonsighting.com/math_jokes.html). Although the pun itself makes a superficial connection to mathematics, the dialogue that ensues does make connections to the properties of polygons. Thus, the play on words and the connection between "polygon" and "Polly gone" are more deeply embedded in the plot of the story. Properties of polygons are used to solve mathematical puzzles and unlock doors to find and free Polly. The audience has no reason to expect that the pun (in scene one) is anything more than a joke, so the deeper connection along the story (in scene two) can be seen as a surprise. However, the mathematical information is quite typical of what a student might encounter in the study of polygons. This DMP does not constitute a new or wonderfully surprising perspective on polygons, that is, it does not offer new ways to see mathematics.

Although the conceptual approach to polygons is typical in this DMP, the audience may recognize that there is expectation in the scenes. As Boorstin (1990) states, the voyeuristic eye "thrives on a delicate balance between expectation and surprise" (p. 102). In the first scene, students are presenting properties and examples of polygons and there is an expectation of a dramatic event, because Student 1 is crying and Student 2 and Student 3 are trying to help her by talking about polygons, but Student 1 keeps crying. At this moment, the audience may think about questions such as "Will they be able to help her?" or "What is going to happen?" These kinds of questions refer to expectation, to a sense of looking forward to something. In scene two, the fact that students are solving a mathematical (or geometrical) puzzle also provides expectations to the audience. The audience may come up with questions such as "Are they going to solve it? How?" or "Are they going to find and free Polly?" Although there are expectations and surprises in how mathematics is connected in the story, there are not mathematical surprises that help the audience to see mathematics in new conceptual ways. The DMP #1 *Polly Gone* does not provide a conceptual mathematical surprise like *Flatland* or *The buttons get arrays* does (see Prologue). Traditionally, the immediate notion that students or even teachers have about parallelism in geometry is that "parallel lines are straight lines that never meet" or "squares are not rectangles." *Flatland* explores a non-Euclidian geometry, bringing up the discussion that *parallel lines do meet* on spherical surfaces. *The buttons get arrays* offers ways to explore a square as a specific case of rectangles, connecting representations and mathematical strands (e.g., algebra and geometry). Thus, unlike *Polly Gone, Flatland* and *The buttons get arrays* are examples that offer "students and teachers opportunities for a fresh, rational experience . . . with multiple opportunities for mathematical surprise and insight" (Gadanidis & Borba, 2008, p. 47).

In this DMP, students use different modes of communication, including elements such as speech, gestures, manipulative materials, and drama. Interestingly, these non-written modes of communication are fundamental in creating a sense of expectation due to the play on words. If they were using writing to express the "polygon pun," they would not provide the same level of expectation to the audience, because the first time the reader reads "Polly gone" and "polygon" the surprise of the pun is over. By using speech, students sustain the expectation for a while, because "Polly gone" and "polygon" sound very similar when one speaks or listens to these words. Through the emphasis on the audio design of modality in this DMP, students sustain the play between the words "polygon" and "Polly gone." When Student 2 and Student 3 finally realize that Student 1 is actually crying because "Polly (is) gone" – and not because "polygon" (as they were thinking) – they clearly get surprised and say: "*Oh no! Polly gone!*"

Voyeur - Sense-Making

Students use the play on words to talk about polygons and they connect the pun to the fact that Polly is found by solving puzzles that involve properties of polygons. The solutions make sense in terms of the polygon properties (that is, they are not arbitrary). In other words, even though the initial connection between "polygon" and "Polly gone" is just a pun, it does later connect to the plot of the story, as properties of polygons are used to find Polly. Thus, the surprise present in this DMP is understandable because (a) the play on words is clarified (scene one) and (b) the pun is related to a mathematical puzzle, which allows the actors to find and free Polly (scene two).

In scene one, students use and show examples (and counter-examples) to define and illustrate what are (and are not) polygons and regular polygons. Moreover, there is logical thinking when the actors are presenting the proprieties of polygons. When Student 3 says "*a polygon is a closed 2D shape with sides made with straight line segments*"[16] he is defining a polygon, that is, he is presenting sufficient and necessary conditions for the existence of a polygon.[17] After defining what a polygon is, Student 3 provides examples (and counter-examples) of polygons: "*For example, this [circular figure] is not a polygon, because the sides are not straight. But this triangle is a polygon because sides are straight*" (see Figure 4.2).

The same kind of logical thinking appears during Student 2's speech in the scene one, when she defines and provides examples of regular polygons: "*Maybe she means regular polygons, which are polygon with all side lengths equal and all angles that are same degree. For example, this octagon is a regular polygon, because all sides are the same length and all angles are the same too.*" These definitions and examples explored in scene one are significant in scene two to solve the puzzle, because the door knobs are represented by figures and only a polygon

[16] "A closed plane figure with straight sides" (*The Nelson Canadian School Mathematics Dictionary*, p.173).

[17] "The formal definition of a mathematical object in mathematics states the least number of properties of the object need to identify it" (*The Nelson Canadian School Mathematics Dictionary*, p. 58).

opens the first door and only a regular polygon opens the second door. Thus, there is a mathematical connection from scene one to scene two because the process of solving the puzzle involves the identification and classification of figures – if a figure is (or is not) a polygon and if a figure is (or is not) a regular polygon.

The process of solving the puzzles in scene two is a process of *experimentation*, in the sense that students are (a) *elaborating conjectures* (e.g., students say: "*where could Polly be? How about behind this door? But, which door knob? Well, let's see. They must be polygons*"); (b) *refuting conjectures* (e.g., students say: "*I know this one is not a polygon, because not all sides are straights. So, it could be that one… Can't be this one either... I know why. [It is] because these lines are curved*"); (c) *confirming conjectures* (e.g., students say: "*It must be this one!*" and the doors open). Therefore, even though the conceptual approach to polygons is traditional in this DMP, students present arguments, examples, logical thinking, connections and ways to pose and solve a problem related to polygons.

Vicarious Emotions

This is an emotional story. Someone is crying (Student 1) and someone is lost (Polly). There is a happy ending (students find and free Polly). The plot of the story uses properties of polygons to pose and solve puzzles that are integral to story development, and they are part of the emotional moments, such as (a) realizing that "Polly is gone" and (b) finding Polly by solving mathematical puzzles. However, the emotions in the story are not connected to deep mathematical ideas, that is, the emotions are only connected to superficial concepts regarding properties and examples of polygons. There are not deep explorations regarding relationships involving polygons. An example of how the students could have used mathematics more deeply would be to explore the notion that "a square is a special case of rectangle or rhombus." So a door puzzle might require a rectangle or a rhombus to open it and the students only had a square. They could realize and prove to one another through the properties of rectangles and rhombus that a square is a special case of them, thus relating their emotions in the story to this idea. This might lead the audience to see polygons in multiple ways or as fitting in multiple categories by considering overlapping properties, which might offer ways to see mathematics differently. This idea is explored, for instance, in the DMP #3 *Shape Songs* and DMP #5 *Little Quad's Quest*.

Although the emotions in this DMP are related to a traditional approach on the study of polygons, the expectations and surprises in both scenes (already mentioned) are emotionally expressed by the characters. For instance, scene one is emotional regarding the fact that Student 1 is crying because "*Polly (is) gone*." Student 2 and Student 3 are trying to help her by talking about polygons, but Student 1 keeps crying. When Student 2 and Student 3 realize that "*Polly (is) gone*," Student 1 stops crying. In scene two, the process of posing and figuring out the puzzle is also emotional because it refers to how to find and free Polly. By opening the second door and freeing Polly, students demonstrate feelings involving happiness and achievement. They celebrate it by saying "*Yeah! Polly!*" The way students say together "*It must be this one!*" and "*Oh no! Another door!*" reveals some of these emotions as well.

Moreover, close-up shots of the actors' faces are used several times in this DMP (see Figure 4.3), which may enhance the vicarious pleasures of the audience. The close-ups on the actors are used in moviemaking to make the scenes more emotional and the audience may feel what the actors are feeling. Boorstin (1990) emphasizes that the vicarious eye can be intensified by zooming-in the landscape of the scene on the actors' facial expressions. This is a way to highlight an emotional moment and offer vicarious experiences to the audience.

The audience may vicariously feel that there is a sense of collegiality and collaboration in the story (see the transcription), which helps the audience possibly make stronger emotional connections to the relationships between the characters and their emotional experiences. In scene one, for example, Student 2 and Student 3 are trying to help Student 1, because she is crying. They demonstrate feelings involving solidarity or friendship when they talk about polygons, because they think that Student 1 is crying and saying "polygon" and they want to help her. Even when they realize that Student 1 is actually crying because "Polly (is) gone," they show a sense of friendship and collaboration, because all three students start to try to find and free Polly by solving the puzzles. The process of solving the puzzles is also collaborative, because students complement the actions, conjectures, and reasoning of each other when figuring out the puzzles. These moments reveal playful events of *students-thinking-with-materials* by communicating and representing their mathematical ideas collectively/collaboratively, conjecturing and experiencing mathematics by using drama and manipulative materials (e.g. see the following transcription from scene two).

Student 3: *… I know this one is not a polygon [touching the figure on the bottom], because not all sides are straights. So, it could be that one [pointing out the figure on the center].*

Student 2: *It can't be this one either [removing the figure on the center].*

Student 1: *I know why. [It is] because these lines are curved.*

All three Students: *It must be this one!*

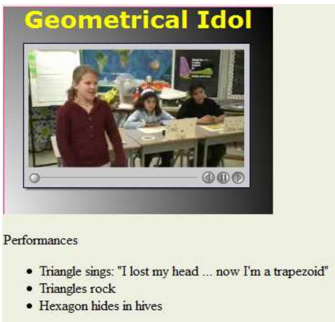
Visceral Sensations

Sinclair (2000) talks about a sense of *mathematical fit* as an aesthetic sensation, a sensing of mathematical beauty. In this skit, students have some sense of mathematical fit when properties of polygons solve puzzles. From the transcription, it is possible to recognize this sense when all three students say together "*It must be this one!*" or "*It must be regular!*" These are the key moments which students figure out the puzzles and open the doors. However, this sense of mathematical fit is conceptually superficial, because it simply deals with proprieties of polygons. The pleasure of mathematical fit in this DMP is not as intense as for example the pleasure of odd numbers fitting in a square in the DMP L-patterns (see Gadanidis, Hughes, & Borba, 2008), which offers several ways to make connections involving multiple representations (e.g. numerical, visual, material) through different mathematical strands (e.g. geometry, numeration, patterning, and algebra).

The process of figuring out the puzzle involves feelings of expectation and tension. It is possible to recognize these emotions when students say, for instance, "*But, which door knob?*" "*Oh no! Another door!*" or "*What is it then?*" These moments may offer some visceral sensations to the audience, because students are in action (solving a puzzle) and the process of solving a problem to save Polly involves some suspense, tension, and expectation. However, there is no soundtrack at this moment and, as Boorstin (1990) states, a soundtrack is fundamental to the visceral experience. Therefore, students could have used a soundtrack in this DMP to provide more intense visceral experiences.

Solving the puzzles involves students' experimentation, hands-on activity, conjecturing, and refuting/confirming of conjectures based on their knowledge on polygons. These processes are very significant in terms of mathematical thinking, but, in this DMP, they are not developed in a deep way. Although students are in direct mathematical experience – they are in action manipulating polygons to figure out a puzzle – they are not thinking-with-artifacts in a profound manner (see Borba & Villarreal, 2005; Papert, 1990), which might offer them ways to demonstrate the beauty of mathematics by connecting multiple representations and strands. As mentioned, students could have explored ideas like relations involving quadrilaterals (e.g. “a square is a special case of rectangle or rhombus”), which is explored in the DMP #5 *Little Quad’s Quest.* Students could have also explored some issues on modeling and representing polygons, such as (a) Architecture: What are the polygons we usually see in constructions? Why are triangles more “rigid” than rectangles and often used to construct roofs? and (b) Tessellation: Which polygons are proper to cover a plane surface? Why do bee hives have a hexagonal form? Why do hexagons fit in a way that optimizes the area on a plane surface? These are some of the questions students could have explored in this DMP as a way to provide visceral experiences to the audience.

DMP #2: Geometrical Idol

Table 4.3: Geometrical idol 	**URL:**	http://www.edu.uwo.ca/mathscene/geometry/geo1.html
	Strands and Content:	Strand: Geometry and Spatial Sense Content: Proprieties of polygons.
	Format:	Video.
	Time length:	2:41.
	The Arts:	Skit and songs: Musical.
	Participants:	Four elementary school students and one guitar player.
	Setting:	Performed in a classroom.
	Info:	Performances • Triangle sings: "I lost my head ... now I'm a trapezoid" • Triangles rock • Hexagon hides in hives

Description

Four students perform a skit and sing songs. By making a parody of the TV Show called *American Idol*, they play roles as judges, presenters, and singers at the *Geometrical Idol*. Students sing songs about trapezoids, triangles, and hexagons. There is a participant who plays the guitar, but he or she does not appear in the video. First, the presenter introduces the first candidate as a trapezoid: “*She has one set of parallel lines and two acute angles.*” She performs a song called *Now I am a trapezoid*. The judges approve the candidate and she gets the chance to go to Hollywood (the next level of the competition). The presenter then introduces the second candidate as a triangle: "*she has three acute angles.*" She performs a song called *Triangles rock* and the candidate is also approved. Finally, the presenter introduces the third candidate as a hexagon: "*She has three sets of parallel lines and two acute angles*." The candidate performs a song called *Hexagon* and she is not approved. Next, I present a transcription of the DMP:

Presenter: *Hello! My name is Traci Chris and welcome to Geometrical Idol! First up we have trapezoid. She has one set of parallel lines and two acute angles. Ladies and gentleman, number one.* (See Figure 4.8).
Trapezoid: *Hi!*
Presenter: *What are you gonna sing today?*
Trapezoid: *I'm singing "Now I am a trapezoid."*
[Someone plays the guitar].
Trapezoid *[singing]: Head, oh head, I lost my head. And head, oh head, I could've been dead. Head, oh head, I feel sick in bed. Now, I am a trapezoid.* (See Figure 4.9).
[Applause].
Judge 2: *I liked that very much!*
Judge 1: *It was ok…* (See Figure 4.10)
Judge 3: *Here we are! You're going to Hollywood!*
Trapezoid: *Yes! Yes! I am going to Hollywood! I am going to Hollywood!*
Presenter: *Next step we have triangles. She has three acute angles. Ladies and Gentleman, number seven thousand, three hundred fifty-two.*
Triangle: *Hello!*
Judge 1: *Hello!*
Triangle: *I am going to sing "Triangles rock."*
Judge 3: *Ok!*
Triangle *[singing. Someone plays the guitar]: Triangles, they have three points, three sides, three types. Isosceles, scalene, equilateral. Triangles, they try again. Triangles, to the very end.* (See Figure 4.11)
[Applause].
Judge 3: *That was very good!*
Judge 2: *Good job!*
Judge 3: *Three yes! You are absolutely! You are going to Hollywood!*
Triangle: *Thank you!*
Presenter: *Last but not least, we have hexagon. She has three… sets of parallel lines and two acute angles. Ladies and gentleman, number fourteen thousand, five hundred, sixty-nine.*
Hexagon: *Ok! I'm going to sing "Hexagon"! Ok… Ok… [It] has six sides. Hides in hives. Takes all the honey. And leaves no money. It is a hexagon.* (See Figure 4.12).

Figure 4.8: The presenter

Figure 4.9: The Trapezoid

Figure 4.10: The judges

Figure 4.11: The Triangle

Judge 1: *Ok! You can stop.*
Judge 3: *Yes!*
Hexagon: *Yeah? Yeah?*
Judge 1: *No! That was not good!*
Judge 2: *I have to pass…*
Judge 3: *[?] Really repulsive.*
Hexagon: *Hum… [very sad].*
All participants: *That was the [?] episode of Geometrical Idol!*
[Applause].

Figure 4.12: The Hexagon

Following are the lyrics of the DMP in Table 4.4:

Table 4.4: Lyrics of DMP #2 – Geometrical Idol

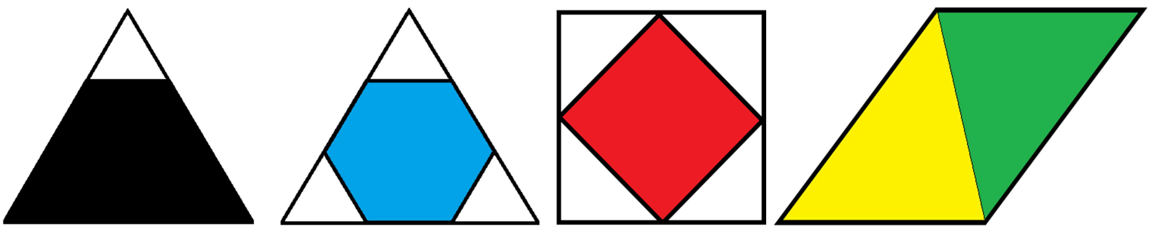
Song 1: Now I am a Trapezoid	***Song 2: Triangles Rock***	***Song 3: Hexagon***
Head, oh head, I lost my head.	*Triangles, they have three points,*	*[It] has six sides.*
And head, oh head,	*three sides, three types.*	*Hides in hives.*
I could've been dead.	*Isosceles, scalene, equilateral.*	*Takes all the honey.*
Head, oh head, I feel sick in bed.	*Triangles, they try again.*	*And leaves no money.*
Now, I am a trapezoid	*Triangles, to the very end.*	*It is a hexagon.*

Voyeur - New/Wonderful/Surprising

Ideas such as "the triangle lost her head, now she is a trapezoid" or "hexagons hiding in hives" may help students and teachers see the new and wonderful in mathematics.

Regarding the first song *Now I am a trapezoid* (see Figure 4.9), the imagining of a trapezoid as a "triangle with its head cut-off" is surprising, as it is not a common representation in school curriculum. This representation of *trapezoid* may help students wonder about other shapes and how to see polygons in *new* ways as parts of other shapes or "hiding" in other shapes such as a hexagon from a triangle, a square from a larger square, triangles from a parallelogram, and so forth (see Figure 4.13 a-d).

Figure 4.13 a-d: Figures imagined from other figures



Also, the idea of "hexagons hiding in hives" offers ways to see connections between mathematics and real life. A honeycomb (see Figure 4.14) is an example of a tessellated natural structure. Mathematical problems involving tessellations, for instance, offer ways to see the new and wonderful in mathematics. Tessellation is "an arrangement of plane figures of the same shape and size, to cover a surface without gaps or overlapping" (*The Nelson Canadian School Mathematics Dictionary*, p. 228). Triangles, squares, and hexagons are the only regular polygons that tessellate by themselves. Interestingly, the hexagon tiles the plane with minimal surface area, that is, the hexagonal structure uses the least material to produce the cells within a given volume (for proofs, see Thompson, 1952).[18]

Figure 4.14: A honeycomb. Available at: http://en.wikipedia.org/wiki/File:Bienenwabe_mit_Eiern_und_Brut_5.jpg

It is surprising in this DMP that geometrical figures take human forms, and sing songs about themselves. Polygons are typically experienced as non-human or non-alive and with no feelings. Gadanidis, Hughes and Gerofsky (2008) pose interesting questions regarding ways to explore this kind of surprise. The authors emphasize possibilities on how the richness of students' imagination and connections to real life can be extended, regarding that this view of shapes taking human form may make a difference for students and teachers because, by experiencing surprises, they take pleasure in seeing something new and wonderful in mathematics (Gadandis, Hughes, & Borba, 2008). Watson and Mason (2007) "tend to see surprise as a positive emotion [and] mathematics as full of philosophical and cognitive surprises; surprise as motivating curiosity and effort" (p.4). Thus, the surprising experience in DMP #2 *Geometrical Idol* might offer ways to see triangles and trapezoids differently or see geometrical shapes in nature. One might imagine what if the triangle lost all three vertices or what if the triangle was created by cutting vertices off of another shape. What might this shape have been? All this aesthetic, playful, and imaginative context may offer ways to the audience to see mathematics differently, surprisingly.

This DMP also offers surprise in seeing a performance contest (like a "TV show of talents") using mathematical songs. It is not common for students to communicate mathematics through song. Songs are popular forms of communication, which are easily accessible compared

[18] See also *isoperimetric problems* concerning *tessellations* (Toth, 1963).

to traditional forms such as writing and drawing (traditional text) or verbal language concerning typical instruction.

In the second song the student sings: **"***Triangles, they have three points.*" Probably, students are making reference to or exploring the number of *vertices* of triangles.

Voyeur - Sense-Making

There are similarities among the songs in terms of mathematical content, because they all refer to properties of polygons. The first song reveals student's sense of geometrical visualization. As mentioned, by singing "*Head, oh head, I lost my head… Now I am a trapezoid*," the student is imagining a trapezoid as a triangle with its "head cut-off." The second song mentions some properties of triangles and how they can be classified (scalene, isosceles, and equilateral). The triangles are portrayed as determined or persistent, because the lyrics states "*Triangles, they try again. Triangles, to the very end*." The third song talks about a hexagon hiding in hives (as in beehives). The song explores the recognition and visualization of everyday shapes around us (honeycomb) in a different way, because the hexagon is portrayed as a thief, hiding in a hive, taking all of the bees' honey, and then not paying for it as the lyrics state, "*Hides in hives, takes all the honey and leaves no money. It is a hexagon*."

The parallel with a TV show provides a context of reality to this DMP, which is an important aspect of the voyeuristic eye – "creating a credible flow of time and space and creating a story" (Boorstin, 1990, p. 13). However, students could have presented a deeper mathematical dialogue between judges and candidates after each song as sometimes happens in the case of musical performances in the original show. The judges could have asked questions and the students could have had a conversation to *extend* the mathematical ideas presented through the songs, to *make connections* to other significant mathematical aspects (concepts and representations) they were talking about in the lyrics. *Connecting* is a fundamental aspect of a conceptual DMP (Gadanidis & Hughes, 2008), and a very significant mathematical process pointed out in curricula (NCTM, 2000; Ontario Ministry of Education, 2005).

On the one hand, "children learn by repetition and songs are a great way to incorporate learning into their young lives . . . Learning by music helps [students] to retain your information" (Kynyon, 2008, p. 1). On the other hand, songs usually deal with conceptual gaps concerning mathematical understanding or connections. That is, the role of the listener (audience) is fundamental to interpreting the ideas of a mathematical song and making connections that probably are not totally clear at the first approximation with the lyrics. Also, a musical (a combination of skit and song performances like this DMP #2) is an interesting form of DMP because it may offer ways to remember the content (songs) and ways to develop understanding and make connections (skit). Thus, in this DMP, students could have performed scenes in which the judges could discuss mathematics with the candidates based on the ideas or concepts presented in the songs, thus offering different ways for the audience to make connections.

After the performance of the first candidate, the judges could have added new ways of looking at triangles, or new ways of cutting-off shapes to produce new polygons (like a hexagon from a square). After the performance of the second candidate, the judges could have asked "Why do triangles try again to the very end?" Is it because representations of triangles are "rigid" and often used in the architecture of roofs? After the performance of the third candidate, the judges could have mentioned other shapes "hiding" somewhere, thereby making connections to the other candidates. For example, where do we see representations of triangles and trapezoids in our environment? The judges could have also justified why the third candidate failed. Was that a bad performance in terms of singing, in terms of mathematics, or both? These discussions might enhance the audiences' understanding toward the mathematical content of the songs by offering ways to make connections involving concepts, representations, and processes.

The presenter produces incoherence when she introduces the hexagon by saying: "*she has three sets of parallel lines and* ***two acute angles***." A regular hexagon actually has six obtuse angles, all of which measure one hundred and twenty degrees. Most likely, the presenter was confused with the dialogue referring to the first candidate (the trapezoid), which is: "*she has one set of parallel lines and* ***two acute angles***." When introducing the hexagon, the presenter makes a short pause in the speech and looks back to the table. It seems she tries to find her notes. These actions (looking back and pausing) offer some evidence that the presenter was not totally confident with her speech. It might also indicate the possible confusion she made with the introduction of the first candidate.

Vicarious Emotions

There is a sense of competition and expectation among the actors in the skit, like the sense of looking forward to the results in a talent show. The audience may feel the same feelings, vicariously, when watching the DMP. Happiness through achievement is emphatic when the candidate passes and gets the opportunity to go to Hollywood. Both the trapezoid and the triangle celebrate with enthusiasm when the judges pass them. In contrast, sadness emerges when a candidate fails. After the negative feedback of the judges, the hexagon makes a sad sound (almost crying) and she crosses her arms and turns back very disappointed. The audience may vicariously feel the same emotions the actors are feeling such as happiness/achievement or sadness/deception, according to their success or failure on the show.

In this DMP, zooming-in on the candidates' faces is used and may offer ways for the audience to focus on the emotions of the performers when they are singing. As mentioned, Boorstin (1990) emphasizes that emotional moments can be intensified through a close-up on the actors' facial expressions (see Figures 4.15 a-c).

Figure 4.15 a-c: Close-up on facial expression in *Geometrical Idol*.

Feelings appear in the trapezoid's lyric: "*I could've been dead…I feel sick in bed*." That is a mathematical emotional moment involving a key-relationship because these feeling are connected to the mathematical idea in which the trapezoid is visualized a "triangle with its head cut-off." As mentioned, this idea offers ways to see mathematics differently. Thus, in this song, students' mathematical imagination is linked to emotions, providing vicarious pleasures to the audience.

The students are playing roles as shapes, assuming artistic identities as geometrical figures. As mentioned, it may be a surprise to the audience, but it also may offer emotional experiences. The process of playing the roles (being a character) may reveal some sense of embodiment when they are performing: a sense of "thinking the self as a polygon." Students (as actors) are actually "making mathematical objects come alive" when they perform them and sing, for instance, "*I am a trapezoid*."

The third song identifies everyday shapes (hexagon and beehives) and portrays the hexagon as a thief, hiding in a hive, taking all of the bees' honey. There is a connection between real-life forms a representation of a regular hexagon can take and how it may increase the vicarious experience of their emotions by the audience, because there is expectation regarding the presence of a thief and the fact that the bees are losing the honey.

The trapezoid and the hexagon demonstrate they are very excited to perform their songs. They arrive with enthusiasm on scene when they are introduced, making animated gestures. The audience may feel students are feeling a strong sense of enthusiasm to start their performances. In contrast, the hexagon's performance may be considered exaggerated by the audience, which can be related to something funny or hilarious.

The TV show design reveals the influence of media entertainment in producing a DMP. Students bring elements of the individuals of the original TV show's personalities by demonstrating that some judges are stricter than others or by the reactions of the candidates toward the judges' feedback. The audience may feel some of the judges are indifferent, cold or insensitive, which are characteristics related to unpleasant feelings. For instance, in the judgment of the first candidate, the judge in the middle says: "*I liked that very much!*" In contrast, the judge on the left only says: "*It was ok.*"

Visceral Sensations

Although this DMP does not present scenes of action or suspense, it provides visceral experiences because the main plot of the performance is based on a soundtrack and there is an appeal to the pop culture of entertainment with regard to the fact that the DMP is a parody of the very popular TV show *American Idol*. Boorstin (1990) argues that the presence of a soundtrack is fundamental within the visceral eye. Boorstin also states that pop music has the appeal for visceral pleasures because that is a characteristic of the popular culture of entertainment. Video clips of pop-rock bands, for instance, are short in time length (a few minutes), usually include fast changes of scenes, and the popular culture has a visceral appeal.

When the trapezoid says "*I lost my head*" or "*could have been dead*", it could elicit fear or humour, depending on how the audience interprets it. Tension and fear, which are visceral sensations, may also be related to the hexagon hiding in hives and stealing honey.

Although the mathematical ideas explored in this DMP do not present connections between strands (e.g. geometry and spatial sense to patterning and algebra), ideas such as “a trapezoid coming from a triangle” and “hexagons hiding in hives” lead to a sense of *mathematical fit* (Sinclair, 2006), because the audience may experience that (a) a trapezoid fits in a triangle and (b) regular hexagons fit one to another (covering a surface) and hives (hexagonal format) are things found in nature. Sinclair (2000) suggests that

> [Aesthetics] is partially about discerning patterns or perceiving relations, and taking note of how things relate to one another and how they seem to fit together. When we experience things fitting together, they often look beautiful to us, and they often bring us a sense of pleasure (p. 4).

Thus, some ideas presented in this DMP offer visceral sensations to the audience.

DMP #3: Shape Songs

<table>
<tr><td rowspan="8">Table 4.5: Shape Songs


Square, Rectangle and Triangle sing their songs.</td><td>URL:</td><td>http://www.edu.uwo.ca/mathscene/geometry/geo7.html</td></tr>
<tr><td>Strands and Content:</td><td>Strand: Geometry and Spatial Sense.
Content: Proprieties of square, rectangle, and triangle.</td></tr>
<tr><td>Format:</td><td>Video.</td></tr>
<tr><td>Time length:</td><td>1:50.</td></tr>
<tr><td>The Arts:</td><td>Songs.</td></tr>
<tr><td>Participants:</td><td>Two elementary school students and one guitar player.</td></tr>
<tr><td>Setting:</td><td>Performed in a classroom.</td></tr>
<tr><td>Info:</td><td>Square, Rectangle and Triangle sing their songs.</td></tr>
</table>

Description

Two students sing three songs about the proprieties of polygons from the point of view of each polygon. The first song is about a square and the student on the left side of the video performs it. The second song is about the rectangle and the student on the right side of the video performs it. Finally, the third song is about triangles and both students perform it. There is a participant who plays the guitar, but he or she does not appear on the video. The transcriptions of the songs (lyrics) are presented in Table 4.6:

Table 4.6: Lyrics of DMP #3 – Shape Songs		
Song 1: Square	***Song 2: Rectangle***	***Song 3: Triangle***
Square! I have four sides *Square! I am a quadrilateral.* *Square! It is not fair that they* *All call me such a dumb square* *I can be a square and hip too.* *I can be cool just like you.* *Just have to realize that I am a rectangle too.* *Therefore I have skills and you know.*	*I am just a rectangle* *Let's try to make this simple* *I got four sides, four angles* *Because I am not a triangle* *I even got four vertices, therefore I am not 3D* *My lines of symmetry are two* *And I got skills just like you*	*I am just the triangle* *I can be equilateral* *My vertices are three* *Therefore, I am 2D* *You know me, I can be scalene* *Equilateral or isosceles, my lines of symmetry are three or one* *I can also be a polygon* *So, maybe a square is a lot* *But he can't be me so there's a thought.* *[Applause]*

Voyeur - New/Wonderful/Surprising

In the first song a student playing the role of a square is singing "*I am a rectangle too.*" The mathematical idea presented in this song – that "every square is a special case of rectangle" – is a conceptual mathematical surprise. This is a mathematical surprise because in typical mathematics classrooms this is not uncovered (Gadanidis, Gerofsky, & Hughes, 2008). Moreover, students or even elementary school teachers usually think that a square is not a rectangle, but this is a misconception. The *Nelson Canadian School Mathematics Dictionary* (1995) defines a *square* as "a **rectangle** with equal sides, or a **rhombus** with equal angles. It follows from either definition that a square has fours equal sides and four right angles, and that its diagonals are equal and bisect each other at right angles" (p. 215, original emphasis). Similarly, a rectangle is a special case of parallelogram. Thus, a square is actually a special case of a rectangle, a rhombus, and a trapezoid. The idea presented in this first song may thus help disrupt possible misconceptions people have toward the study of squares and rectangles. It may help the audience to see mathematics as more complex and interesting. These relationships involving quadrilaterals are discussed in the analysis of the DMP #5 *Little Quad's Quest (Part 1 to Part 5)*.

Voyeur - Sense-Making

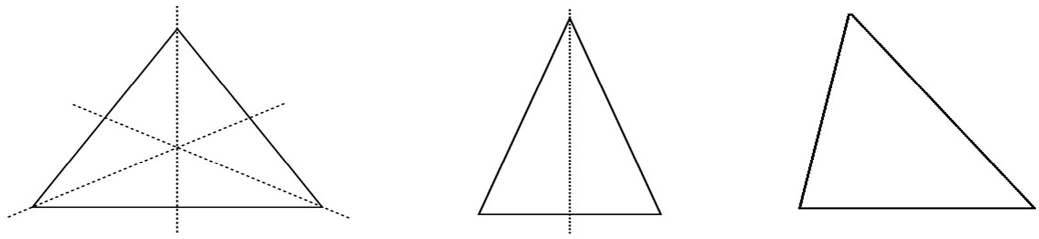
There are similarities among the three songs in this DMP in terms of mathematical content. All three songs are about shapes (polygons) and some of their properties involving, for instance, number of sides, dimensions, lines of symmetry, and specific types (e.g., scalene, equilateral, and isosceles triangles).

In the first song, as mentioned, there is a mathematical surprise involving the fact that every square is also a rectangle. However, the song does not provide an explanation about why a square is a specific case of a rectangle. Basically, it superficially says a square is a quadrilateral because it has four sides and one just has "*to realize that I am a rectangle too*." The song could present the definition of a rectangle – a rectangle with equal sides or a rhombus with right angles – and explore the fact that a square is a special case of rectangle or rhombus. That is one way the song could provide an explanation about why every square is a rectangle too, and then the audience would identify the arguments that would support the mathematical surprise.

In the second song, the student sings: "*I got four sides, four angles… I even got four vertices, therefore I am not 3D.*" In this case, the conditions proposed by the student are not sufficient to make his mathematical/geometrical assumption fully consistent. That is, there is a 3D shape – the *tetrahedron* – that has four sides (faces), four "angles" (trihedral angles) and four vertices. In contrast, students present a consistent logical statement when students say in the third song: "*My vertices are three, therefore, I am 2D*." In this case, the existence of three (and only three) vertices in a figure is a sufficient and necessary condition to determine a triangle, which is a 2D figure.

In the third song, students are talking about three types of triangles – scalene, equilateral, and isosceles – and they sing: "*My lines of symmetry are three or one.*" An equilateral triangle – a triangle with all sides equal in length – has three lines of symmetry. An isosceles triangle, that is, a triangle with one pair of sides equal, has one line of symmetry. But, a scalene triangle, which is a triangle with no equal sides, has no lines of symmetry. Therefore, in order to ensure mathematical rigor and consistency, students could have sung "three, zero, one", instead "three or one." Figure 4.16 shows these possibilities:

Figure 4.16: Lines of symmetry of triangles



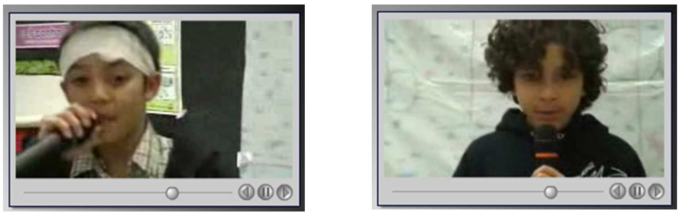
Vicarious Emotions

All three songs are sung by the shapes. They tell us their stories and relate their feelings. Like in DMP #2 *Geometrical idol*, students are playing roles as geometrical shapes. The fact that humans are (imaginary) “taking forms” of a square, a rectangle, and triangles take may increase the vicarious experience of their emotions by the audience. In this sense, the words *I am* (first person singular – grammatical person) are often present in the lyrics (e.g “*I have four sides*,” “*I am a quadrilateral*,” “*I am just a rectangle*,” “*I am just the triangle*”). It reinforces the sense of a student thinking and learning about a polygon by *being* (or acting as) a polygon. In contrast, the vicarious experience would be different – probably less intense – if the songs were in third person singular, with lyrics such as: “*A square has four equal sides and four right angles… A square is a rectangle too.*”

The shapes express emotions and the audience may feel these emotions. The first song in particular presents emotions that the square is feeling. When the student sings “*It is not fair that they call me such a dumb square*,” the audience may interpret that the square is dealing with feelings of injustice, involving bias or unfairness at that moment. It happens because, colloquially, the expression “someone is a square,” means someone is boring, conservative, or old-fashioned. The students were creative in using such expression to disrupt the stereotype associated to the colloquialism by arguing that a square “*can be hip too… can be cool just like you… can be cool just like you. Just have to realize that [a square is] a rectangle too*.” In this way, the students are relating the square’s emotions toward the negative expression “someone is a square” to the mathematical surprise of the song (every square is a rectangle too). Even though there is not a deep explanation about why every square is a rectangle, the audience may feel these mathematically related emotions because in this performance shapes can talk, sing, and have emotions.

Moreover, it is not a simple task to perform a song with emotion, even if the song talks about emotions and feelings. In this DMP the audience may feel that the student on the left side of the video is having a good performance of the songs because he was performing with confidence, demonstrating that he enjoyed singing, and was excited to perform. Vicariously, the audience may have the same feelings. It is important to note that Boorstin (1990) believes that the appeal to emotional space emphasizes the focus on the actor's face, "exploring every nuance of expression, dowsing for emotion in the invisible depths behind the eyes" (p. 90). In this DMP, this kind of landscape or close-up is well explored (See figures 4.17 a-b). The audience may see students' facial expressions while they interpreting their roles as polygons and sing about their properties. Therefore, this technique of recording – close on actors' facial expressions, landscape with no other distractions – draws attention to what the actors feel and the audience may vicariously feel what the "polygons are feeling."

Figure 4.17 a-b: Close-up on students' facial expression in Shape Songs



Visceral Sensations

All three songs present some sense of mathematical fit. When the polygons say, "*I am a rectangle too*", "*I got skills just like you*" or "*I can also be a polygon*", the audience may understand these statements as related to a sense of belonging. Interestingly, in the first song, the sense of surprise and new about the fact that "every square is a rectangle" may lead the audience to a context of discovery, as an "a-ha moment," and it may offer ways for visceral experiences. During the second song, it is clear that the student lost the beat when singing one of the verses. The audience may sense this moment as visceral by feeling (a) tension when the beat is lost and (b) satisfaction when the student and the guitar player work in synergy to get the beat back.

Even though this DMP does not present explicit scenes of action or suspense, it is fundamentally formed by soundtracks. Boorstin (1990) states that the use of a soundtrack is a significant aspect within the visceral eye. In addition, Boorstin says that "pop songs aim at the gut" (p. 133) and video clips of pop music are design for visceral appeal. In this sense, it is relevant to notice that the last song presented in this DMP is a parody of the pop rock band called Green Day, which is a very popular band. The sense of mathematical fit present in the songs combined with the fact that (a) soundtracks provide visceral sensations to the audience and (b) there is a pop song involved, offer ways for the audience to viscerally experience this DMP.

The visceral sensation of this DMP can be intensified if the audience extends the idea that "a square is a special case of rectangle" to other problems or strands. *The buttons get arrays* (see Prologue) explores an interesting problem involving perimeter and area of rectangles. It shows that representations of different rectangles, with different perimeters, have the same area (16 units). For instance, area can be equal to 16u and the perimeter equal to 34u or area can equal 16u and the perimeter equal to 20u. Surprisingly, the rectangle 4u by 4u has area equals to 16u and perimeter equals to 16u. Then, beyond exploring that a square is a special case of rectangle, *The buttons get arrays* explores multiple representations (visual, objects, symbols, writing) and connects different mathematical strands (geometry and algebra). Similarly, this problem can be explored considering a constant perimeter and multiple values for the area. If a rectangle has a perimeter equal to 12u, for instance, then, there are many ways to design rectangles, length and height could be equal to 1u and 5u, 2u and 4u, and 3u and 3u. To solve this problem, one has to realize that the rectangle with the largest area is a square (3u by 3u). This problem is explored by Grade 2-3 students in a DMP called *We made 12*, available at www.edu.uwo.ca/mathscene/mathfest2009/mathfest224.html.

DMP #4: Triangles

Table 4.7: Triangles

URL:	http://www.edu.uwo.ca/mathscene/mathfest/mathfest106.html
Strands and Content:	Strand: Geometry and Spatial Sense. Content: Proprieties of figures formed by triangles.
Format:	Video.
Time length:	4:20.
The Arts:	Songs and Skit: Musical.
Participants:	Many elementary school students (Estimation: 21-28 students).
Setting:	Performed in an open space.
Info:	A musical about the shapes that can be made with triangles.

Description

In this DMP students make shapes using triangles. They make representations of a parallelogram, a rhombus, a hexagon, a dodecagon, a robot, and a person, performing a skit in an open space (that looks like a gym) where they visit centers or groups where shapes they created with manipulative materials are presented (see figure in the Table 4.7). At each center, students mention the properties of the shapes and sing a chorus. At the end of the story, students make several representations involving the use of triangles using their bodies. Following, I present a transcription of the DMPs different scenes:

Introduction

All Students *[singing]: We can make so many shapes. Wow! We can make so many shapes. Wow! We can make so many shapes with triangles. Everybody sing* (See Figure 4.18)

Figure 4.18: *Triangles* - Introduction

Scene One (See Figure 4.19)

Students [a group of seven]: *Hey. What's going on over there?*
Student: *They are making shapes with triangles.*
Students: *Cool!*
Student: *Oh! Come on! Follow me!*
[Students walk until the first center, where two students are waiting for them].

Figure 4.19: *Triangles* – Scene One

Scene two (See Figure 4.20)

Students: *What are you doing?*
Students: *We are making a parallelogram.*
Students: *What is a parallelogram?*
Student: *A parallelogram is a shape with four sides and four angles.*
All Students *[singing]: We can make a parallelogram. We can make a parallelogram. We can make a parallelogram with triangles. Everybody sing.*
Student: *What are they making over there?*
Student: *Come on guys! Let's see!*
[Students walk until the second center, where four students are waiting for them].

Figure 4.20: *Triangles* – Scene Two: the parallelogram center

Scene three (See Figure 4.21)

Students: *We made a rhombus.*
All Students *[singing]: We can make a rhombus. We can make a rhombus. We can make a rhombus with triangles. Everybody sing.*
Student: *Hey! What are they doing over there?*
Student: *Let's go and see it! Come on guys!*
[Students walk until the third center, where two students are waiting for them].

Figure 4.21: *Triangles* – Scene Three: the rhombus center

Scene Four (See Figure 4.22)
Student: *What are you making?*
Student: *A hexagon.*
All Students *[singing]: We can make a hexagon. We can make a hexagon. We can make a hexagon with triangles. Everybody sing.*
Student: *Hey! What are they doing over there?*
Student: *Come on guys! Let's go there and see it!*
[Students walk until the fourth center, where two students are waiting for them].

Figure 4.22: *Triangles* – Scene Four: the hexagon center

Scene Five (See Figure 4.23)
Student: *We are making a dodecagon.*
Student: *How many that has?*
Student: *Twelve!*
All Students *[singing]: We can make a dodecagon. Wow! We can make a dodecagon. We can make a dodecagon with triangles. Everybody sing.*
Student: *Hey! What is over there?*
Student: *I don't know. I goanna go see it too. Let's go!*
[Students walk until the fifth center, where four students are waiting for them].

Figure 4.23: *Triangles* – Scene Five: the dodecagon center

Scene Six (See Figure 4.24)
Student: *What are you guys making?*
Students: *We made a robot!*
All Students *[singing]: We can make a robot. We can make a robot. We can make a robot with triangles. Everybody sing.*

Figure 4.24: *Triangles* – Scene Six: the robot center

Scene Seven (See Figure 4.25)
[Students appear in a hands-on activity making a representation of a person with colored triangles].
Student: *Hey! What are they doing over there?*
Student: *Oh yeah! Can we go see it?*
Student: *Of course! Let's go guys!*
[Students walk until the sixth center, where four students are waiting for them].
Student: *What are you making?*
Students: *We made a person!*
All Students *[singing]: We can make a person. Wow! We can make a person. We can make a robot with person. Everybody sing.*

Figure 4.25: *Triangles* – Scene Seven: the person

Final Scene (See Figure 4.26 a-c)
All Students *[singing and making collective representation using their bodies]: We can make so many shapes. Wow! We can make so many shapes. Wow! We can make so many shapes with triangles. Everybody sing. We love triangles!*

Figure 4.26 a-c: *Triangles* – Final Scene: Representations of triangles using their body collectively

Voyeur - New/Wonderful/Surprising

This DMP presents mathematical surprises, exploring ways to see a geometrical figure differently, as formed by triangles. At each center, students demonstrate a sense of discovery regarding the fact that they are recognizing and seeing triangles hiding or fitting in polygons and other shapes. When students communicate that there are triangles hiding in parallelograms, rhombi, hexagons, and other figures, their imagination (geometrical visualization) is similar to "lost my head" in DMP #2 *Geometrical Idol*. This kind of mathematical exploration may help students experience mathematics differently.

The way the skit is performed involves a sense of anticipation/expectation that captures the audience's attention and helps the audience to explore how triangles fit in many figures or that triangles form many shapes. In every scene, students always ask "*Hey! What are they doing over there?*" The process of repeating this question may generate a sense of "what next?" to the audience. What else we can make with a triangle? What else is formed by a triangle?

Similar to DMP #2 *Geometrical Idol* and DMP #3 *Shape Songs*, students are singing to communicate their mathematical learning. Usually, a playful and artistic approach is not conducted in traditional mathematics classes. Traditionally, mathematics is communicated using writing (text, diagrams, and tables), and verbal language. However, in this DMP (as in DMP#2 and DMP#3), mathematics is been communicated through song, which may surprise the audience. Interestingly, in contrast to traditional forms of mathematical communication, songs are publicly accessible (Gadanidis & Geiger, 2010). Thus, this DMP may be interesting to people who were not interested in mathematics because they did not see any beauty in exploring mathematics through traditional modes of communication. The content (triangles in shapes) in combination with the modality (song) may help students experience mathematics differently.

The fact that there are many students participating in this musical and they are not in a regular mathematics classroom may be a mathematics pedagogical surprise to the audience. Moreover, the appearance of a representation of a robot may be a surprise to the audience because students are showing a kind of sequence of polygons formed by triangles in each center and, after the dodecagon, the audience may expect to see another polygon, but instead, students show the robot.

Voyeur - Sense-Making

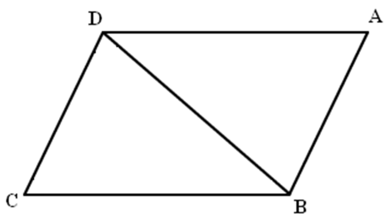
This DMP explores a mathematical idea in a way that is not typically investigated in traditional classes. Usually, elementary school students do not explore how many figures (including regular polygons) are formed by triangles, but this DMP presents many examples of how to experience it. In addition, there is a consistency, there is a plot, a similarity regarding what is happening when students are visiting and exploring each center.

Although students do not explore connections between strands – like the DMP *L-Patterns* (Gadanidis, 2007a) explores connections between geometry, patterning, and algebra – the plot of students' mathematical investigation in this DMP refers to the recognition about how different shapes are formed by triangles. Thus, there are interesting connections between representations involving (a) different polygons and (b) different figures formed by triangles. Furthermore, in the last scene, students use their bodies to represent figures formed by triangles. The use of the body adds an interesting "layer of sings" in meaning production, that is, from a multimodal and semiotic point of view, embodiment offers a way to connect and communicate mathematical ideas through multiple representations.

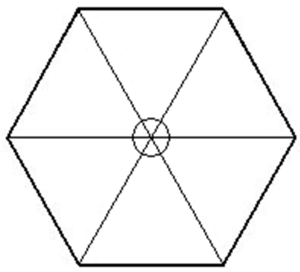
The use of the body for mathematical representation and communication is explored, for instance, in studies that point out issues of multiple representations of functions regarding the use of motion sensors connected to graphing calculators (see Borba & Scheffer, 2004). In DMP #4, by forming triangles with their fingers, hands, arms, and body, individually and collectively, students explore sets of signs and representations that offer ways to communicate mathematics through multiple modes (e.g. using manipulative materials and gestures). Specifically, when using their bodies, students may develop a sense of symmetry because they have to think about and act as a triangle formed by two symmetric parts (see Figure 5.26 a-c)

There are ways to extend the idea presented in this DMP. The audience (or the students who produced the DMP) may explore the fact that seeing triangles in shapes offers a way to (a) visualize lines of symmetry on regular polygons (e.g. regular hexagon) and (b) find the number of degrees of the sum of the internal angles of a polygon such as 360° in a parallelogram or 720° in a regular hexagon (see Figures 4.27 a-b). By exploring the angles of polygons, students may connect strands such as geometry to measurement, thus exploring mathematical ideas conceptually.

Figure 4.27 a-b: Some properties of a parallelogram and a regular hexagon



The value of the sum of the internal angles of the parallelogram ABCD is equal to the sum of the value of the internal angles of the triangles ABD and CBD, which is 180° + 180° = 360°.



The value of the sum of the internal angles of a hexagon may be calculated by (a) regarding the existence of 6 triangles, but excluding the circumference formed in the center, that is, 6 x 180° – 360° = 1080° – 360° = 720°.
(b) regarding that the triangles are equilaterals and then the value of each of the angles on the base of the triangles is 60°, therefore, one can find the value of the sum of the internal angles of a hexagon by calculating 12 x 60° = 720°.

Vicarious Emotions

In this DMP students demonstrate emotions involving a sense of belonging, togetherness, and community, and a celebration of what they have learned or discovered. They look excited because they are doing mathematics together, collectively, and because they are exploring the fact that they can make many shapes with triangles and they are sharing this experience with colleagues and with the audience. Vicariously, the audience may feel students are excited by their experience. Their explicit excitement towards a sense of sharing a mathematical idea, like "let me show you triangles hiding in figures," reveals they really feel they are doing something mathematically special. Therefore, the emotional moments in this DMP are connected to the key mathematical concepts explored in the skit/musical.

As mentioned, there is a sense of anticipation and expectation going on in the DMP and it involves emotion as well – "What's next?" "What are we going to see next center?" "What else can we make with triangles?" As Bordwell and Thompson (1993) state, "to make an expectation about 'what happens next' is to invest some emotion in the situation" (p. 48). This is exactly what students are doing in this DMP. Furthermore, the recording presents several moments of zooming-in during key moments of the performance, focusing on the mathematical objects and students' faces, gestures, and movements. As mentioned in the other cases, Boorstin (1990) argues that these close-up landscapes in the scenes make the vicarious experience more intense.

Visceral Sensations

This DMP provides visceral experiences in demonstrating how triangles *fit* in several figures. Students use colors to highlight the sense that triangles *fit* in shapes such as a parallelogram, a rhombus, a hexagon, a robot, and so forth.

This DMP also shows some aspects of "mathematics-in-action" which may be related to visceral experiences. There are at least three moments that reveal this: (a) the fact that many students are in an open and large space visiting (moving to) different centers to investigate different shapes formed by triangles; (b) when students show the process of creating a representation of a person with triangles. All other shapes are shown already constructed. But, after showing the robot, they show how they create a person with triangles. It reveals students' geometrical sense (visualization, recognition and identification of forms) and direct experience (hands-on) in manipulating materials or figures of several representations of different types of triangles; and (c) when students collaboratively use their bodies to form several kinds of shapes incorporating the use of triangles. It reveals several aspects of their geometrical thinking, including some sense of symmetry and embodiment (See Figure 4.28 a-b).

Figure 4.28 a-b: Students' direct/hand-on experiences during the performance

Moreover, Boorstin (1990) states that action in combination with soundtracks constitute very significant elements of the visceral experience. These elements are fundamental to “amplify” the audience’s visceral sensations. In this DMP, beyond the moments of mathematics-in-action (as mentioned), students are using a common chorus that express their learning and their mathematical thinking. This chorus works as a soundtrack in a visceral sense in the DMP.

DMP #5: Little Quad’s Quest (Part 1 to Part 5)

<table>
<tr><td rowspan="8">Table 4.8: Little Quad’s Quest (Part 1 to Part 5)

</td><td>URL:</td><td>http://www.edu.uwo.ca/mathscene/lq/lq1.html</td></tr>
<tr><td>Strand and Content:</td><td>Strand: Geometry and Spatial Sense
Content: Proprieties of quadrilaterals</td></tr>
<tr><td>Format:</td><td>Videos.</td></tr>
<tr><td>Time length:</td><td>Part 1: 3:09; Part 2: 1:29; Part 3: 2:30;
Part 4: 2:07; Part 5: 2:37</td></tr>
<tr><td>The Arts:</td><td>Skit and Animation: shadow puppet theatre.</td></tr>
<tr><td>Participants:</td><td>Elementary school students
(Estimation: 6 students).</td></tr>
<tr><td>Setting:</td><td>Performed in the classroom.</td></tr>
<tr><td>Info:</td><td>None</td></tr>
</table>

Description

This story is about an adventure lived by a quadrilateral named Little Quad in Polygon Ville. At the beginning of the story, Little Quad was very unhappy because he did not fit or was not as needed or useful as other quadrilaterals were. After meeting the Great Geo (the “God of the quadrilaterals”), Little Quad decided to discover who he was and how he would fit in the world. Along the way, Little Quad met a square, a rectangle, a rhombus, and a trapezoid, and he realized that these are quadrilaterals with specific names and properties, but he was not like them and did not fit in the world like them. At the end of the story, Little Quad discovers he can fly and the Great Geo let him know he is a kite, that is, “a convex quadrilateral with two pairs of equal adjacent sides” (*The Nelson Canadian School Mathematics Dictionary*, 1995, p. 122). The story is told in five parts. Following, I present a transcription of each part:

Part 1 - Opening

Figure 4.29: Little Quad and the Bunny

Narrator: *Once upon a time in Polygon Ville, lived a little quadrilateral, who was very unhappy. All the other quadrilaterals – square, rectangle, rhombus, and trapezoid – were always helping here and there. But the little quadrilateral was never needed for anything. One day, Little Quad went for a walk. He walked and walked and walked. Finally, he sat down under a great tree in silence.*

Little Quad: *What am I gonna do with my life?*

Narrator: *Tears began to roll down his sides… A bunny was nibbling down on the fresh grass and seeing his great sadness, she spoke:*

Bunny: *Little shape, what is wrong?* (See Figure 4.29)

Little Quad: *See? You don't even know my name. If I was a square, or a rectangle, or a rhombus, or a trapezoid, then you would know. But no, I am just Little Quad, nobody knows me or needs me.*

Bunny: *I am sorry! I didn't mean to upset you. Can I help?*

Little Quad: *No! There is nothing anyone can do. It is hopeless. Nobody will need my shape for anything.*

Narrator: *Luckily, Little Quad was sitting under the tree of the great Eagle. This bird was the winged messenger of the Great Geo, who had created all the shapes in Polygon Ville. The Great Geo lived at the top of the mountain, and he needed the Eagle to bring him the latest news from Polygon Ville. The Eagle over heard the conversation and flu down to investigate. The bunny was startled and quickly hopped away.*

Figure 4.30: *The Eagle with Little Quad*

Eagle: *Little Quadrilateral, what is the problem?*

Little Quad: *I want to be useful and needed like all the other quadrilaterals. But nobody needs my shape. I don't fit anywhere.*

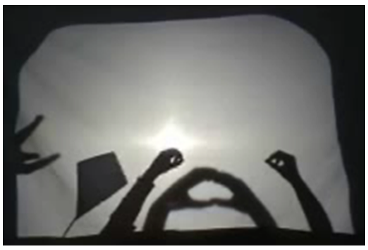
Eagle: Little: *The Great Geo might be able to help you. I will take you to him.*

[The Eagle flies bringing Little Quad with him]. (See Figure 4.30)

Narrator: *Before Little Quad knew he was in the sky – flying higher and higher until Polygon Ville was just a dot on the ground – it happens so quickly that he didn't have time to get scared. In fact, he felt strangely excited as he decided to flutter in the breeze. In seconds, the Eagle set him down at the top of the mountain.*

Great Geo: *Eagle, why have you come and what is the shape?* (See Figure 4.31).

Figure 4.31: *Little Quad* meets Great Geo



Eagle: *Oh, Geo. I have brought one of your creations. The One known as Little Quadrilateral, he is distressed and distraught. I thought you might to know.*

Great Geo: *Little one, why are you so upset?*

Little Quad: *I am just a Little Quadrilateral. Nobody needs my shape. I am useless.*

Great Geo: *Little Quad, I made you, and you are special. Your path may be less clear than others, but you will find your way. Go to square, she can help. Don't despair little one.*

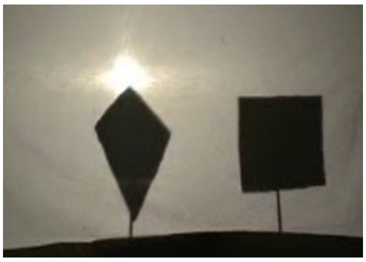
Narrator: *Eagle scooped the little Quad into his wings. Once in the air, Little Quad felt wildly excited again. But before he knew it, Little Quad was back in Polygon Ville at Square's store.*

Part 2 – Little Quad meets Square

Narrator: *Once upon a time in Polygon Ville, lived a little quadrilateral, who was very unhappy. All the other quadrilaterals – square, rectangle, rhombus, and trapezoid – were always helping here and there. But the little quadrilateral was never needed for anything. So, he went to see Square for help.*

Little Quad: *Square! I need help! What can I be? You have four beautiful right angle and four equal sides.* (See Figure 4.32).

Figure 4.32: *Little Quad* meets Square



Square: *Why don't you try square dancing?*

Little Quad: *Good idea!*

[Music starts and Little Quad and Square start to dance].

Square: *Sorry! But you are not very good at square dancing.*

Little Quad: *Why?*

Square: *You keep stepping on my vertices. Maybe, you can try being a window.*

Little Quad: *Sure!*

Square: *You are not very good being a window.*

Little Quad: *Why?*

Square: *You don't fit in with the other squares. Try being a floor tile.*

Little Quad: *Sure!*

Square: *You are not very good being a floor tile either.*

Little Quad: *Why not? I have four vertices.*

Square: *Maybe it is because you don't have two pairs of parallel sides and you sides are not all equal. Let me take you to Rectangle's house.*

Little Quad: *Ok.*

Narrator: *Square and Little Quad walked to Rectangle's house. On the way, the wind blows Little Quad backwards a bit.*

Little Quad: *Square! Wait for me! I am blowing Away!*

Square: *My vertices, hold on tight. We're almost at Rectangle's house. It is just over there.*

Part 3 – Little Quad meets Rectangle

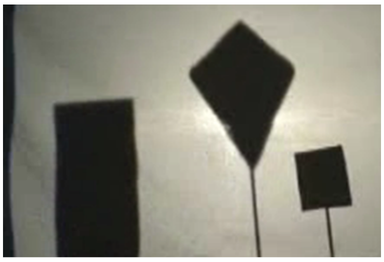
Narrator: *Once upon a time in Polygon Ville, lived a little quadrilateral, who was very unhappy. All the other quadrilaterals – square, rectangle, rhombus, and trapezoid – were always helping here and there. But the little quadrilateral was never needed for anything. Square and Little Quad walked to rectangle's house. On the way, the wind blows Little Quad backwards a bit. [Short pause]. Square and Little Quad arrived at Rectangle's house. He knocks on the door and the door swings open.*

Rectangle: *Come in my friends! I am just finishing my morning yoga.* (See Figure 4.33)

Little Quad: *Why are you doing that?*

Rectangle: *I do yoga to relax my sides, so I can fit in lots of different places.*

Figure 4.33: *Little Quad* meets Rectangle



Little Quad: *Oh Rectangle, you are perfect in every way. You have two pairs of parallel sides and four vertices. You can even fit in the Canadian flag.*

Rectangle: *Oh! And don't forget about money and widescreen TV. I love all the channels!*

Little Quad: *Oh Rectangle, please help me! I want to fit in the world.*

Rectangle: *Don't cry Little Quad. I will finish my yoga and you can lift weights. That way you can make your vertices strong.*

Little Quad: *But, but I…*

Rectangle: *Nonsense!*

Little Quad: *I might have the same number of angles, but I can't fit in money. You do have the number of vertices, but I can't be a wide screen TV, because I have no right angles.*
Rectangle: *Ah… Try one. You can be strong.*
Little Quad: *Oh… ok.*
Rectangle: *This is our beginner weight.*
Little Quad: *Maybe it won't hurt.*
Rectangle: *Good for you!*
Little Quad: *I just can't do it. I am too weak.*
Rectangle: *At least you tried. You have done well my son. But now, it is time to leave and go to rhombus.*
Little Quad: *Ok. Thank you, Rectangle.*
Narrator: *Square, Rectangle, and Little Quad walked to Rhombus' houses. The wind is stronger now.*
Square: *Little Quad! What are we going to do with you? You keep blowing away. We can't keep you on the ground.*
Square and Rectangle: *Little Quad! Little Quad! Where are you?*
Little Quad: *Over here! I am blowing away again!*
Square: *I know how to get you down.*
Little Quad: *How?*
Square: *Make yourself long and skinny.*
Little Quad: *Ok. I will try. [Pause] Wow! This is so cool! I am falling from the tree. Thanks, Square. Oh, look! There is Rhombus' house.*

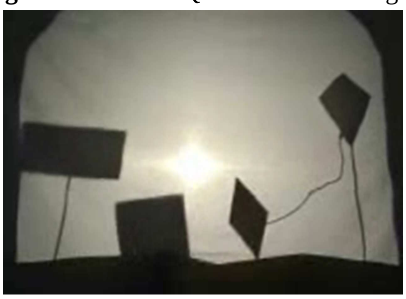
Part 4 – Little Quad meets Rhombus

Narrator: *Once upon a time in Polygon Ville, lived a little quadrilateral, who was very unhappy. All the other quadrilaterals – square, rectangle, rhombus, and trapezoid – were always helping here and there. But the little quadrilateral was never needed for anything. Rectangle, Square, and Little Quad arrive at rhombus' house. He knocked on the door.*
Little Quad: *Is there anyone home?* (See Figure 4.34).

Figure 4.34: *Little Quad* meets Rhombus

Rhombus: *Yes, me. Go away! I am painting my nails.*
Little Quad: *Oh, please! Help me! I am desperate!*
Rhombus: *Ok, fine. Come in. Why are you here?*
Little Quad: *I am here because I need help. I don't know what shape I am. And I don't fit in anywhere. I have been to see Square and Rectangle and they don't have any ideas.*
Rhombus: *Well, Little Quad. Maybe you can be one of my beautiful sparkling diamonds.*
Little Quad: *Ok. I would love to be like you Rhombus! I see you everywhere: in a beautiful chandelier or in a white fluffy snowflake and in a big scary cat's eye. [Pause] I don't fit. I got four vertices, four sides, and four angles, but I still don't fit.*
Rhombus: *Oh! I guess you do need have all your sides equal and two pair of parallel. I don't have any more ideas. Maybe we should go see Trapezoid.*
Square: *Before we [?]. Do you have any string? Little Quad keeps blowing away. Letting you know.*

Figure 4.35: *Little Quad* with the string.



Rhombus: *Yes, I do. I will go get it.*
Square: *Little Quad, we'll tie one end of the string to a vertices then you won't get blown away from us.* (See Figure 4.35).
Little Quad: *Thanks, Square! What a great idea!*
Narrator: *The wind is strong as they walked to Trapezoid's house and the three quadrilaterals pull little Quad along as he is blown away by the breeze.*

Part 5 – Little Quad meets Trapezoid

Narrator: *Once upon a time in Polygon Ville, lived a little quadrilateral, who was very unhappy. All the other quadrilaterals – square, rectangle, rhombus, and trapezoid – were always helping here and there. But the little quadrilateral was never needed for anything. Rhombus, Rectangle, Square, and Little Quad arrived at Trapezoid's house. He knocked on the door.*
Trapezoid: *Who is it?*
Little Quad: *It's me, Little Quad.*
Trapezoid: *Yes, yes. Oh you.*
Little Quad: *Yes, it is me.*
Trapezoid: *Hello. What's the matter? What do you want? Money? Food? A place to stay?*
Little Quad: *No! I just want to be like you.*
Trapezoid: *[laughs]. Everybody wants to be like me. I'm cool, I am smart, and I am strong.*
Little Quad: *You have four vertices, four beautiful sides and you are just so perfect.*
Trapezoid: *You forgot to mention I have one pair of parallel sides.*
Little Quad: *Yes. But, can you help me?*
Trapezoid: *Well... No! Good bye.*
Little Quad: *[knocks the door again].*
Trapezoid: *Hmm... What now?*
Little Quad: *You still haven't helped me. You're not being very nice.*
Trapezoid: *Oh! Sorry. I haven't gotten very much sleep lately. Come on in.*
Little Quad: *No, thanks. I just want you to help.*
Trapezoid: *Well, if you don't want to step in, go away!*
Little Quad: *[knocks the door again].*
Trapezoid: *Go Away! I can't help you.*
Little Quad: *Yes, you can. I see how other shapes do things.*
Trapezoid: *Go away, because I only help shapes who know who they are!*
Narrator: *The shapes leave trapezoid's house and go outside.*
Little Quad: *Trapezoid, hold on to the string.*
Narrator: *All of a sudden a big gust of wind comes up and blows Little Quad into the air.*
Square: *Little Quad is so beautiful! Wow!*
Narrator: *Quad goes higher and higher until he sees the Great Geo.*
Little Quad: *Hi Geo! Look! I am Flying! Wee!*
Great Geo: *Good, you found what you are meant to be.*
Little Quad: *Thanks but what am I?*
Great Geo: *I don't know. Oh, I do know. You remind me of the great bird called the kite.*
Little Quad: *That's it. That's my name. I'm a kite.*
All Quadrilaterals: *Is it a bird? Is it a plane? No, it is Little Quad! And he's a kite! Yeah!!!*
(See Figure 4.36 a-b)

Figure 4.36 a-b: Little Quad with all the quadrilaterals and with the Great Geo.

Voyeur - New/Wonderful/Surprising

This DMP presents conceptual mathematical surprises, offering to the audience ways to see (1) how quadrilaterals are related to each other, that is, how some quadrilaterals are specific cases of others or how they fit in each other and (2) how quadrilaterals fit as everyday objects or how we can find them in our environment. Thus, (1) and (2) offer opportunities to see quadrilaterals in multiple ways or fitting in multiple categories.

The story creates expectation because it is about an adventure where Little Quad wants to know who he is and how he fits as an everyday object (like other quadrilaterals do). The fact that Little Quad starts to fly is a mathematical surprise and it is related to a play on words. That is, the story relates the process of discovering how Little Quad fits (or how he is useful or needed in the world) to his geometrical proprieties and classification as a quadrilateral. As mentioned, a kite is "a convex quadrilateral with two pairs of equal adjacent sides" (*The Nelson Canadian School Mathematics Dictionary*, 1995, p. 122). The angles of a kite are congruent where the pairs of sides meet. A rhombus and a square are special cases of a kite. Moreover, a kite, as an everyday object, is "a light framework covered with cloth, plastic, or paper, designed to be flown in the wind at the end of a long string" (http://www.thefreedictionary.com/kite). Thus, the parallel between a kite as a type of quadrilateral and a kite as an everyday object is a significant surprise in this DMP and, as I will discuss, this idea is related to multiple ways quadrilaterals can be categorized, offering ways to see mathematics differently. In DMP #3 *Shape Songs*, the notion of "a square as a special case of rectangle" was discussed. In this DMP this notion is also explored in a more complex way. It is more complex because it involves many quadrilaterals. In each part of the DMP, Little Quad (and the audience) discovers the properties of square, rectangle, rhombus, and trapezoid as well as how these quadrilaterals fit in the environment, as everyday objects (square as a window, rectangle as a widescreen TV, rhombus as a diamond). By presenting in the story all these quadrilaterals together and mentioning their properties, the audience may realize how they are related and see which quadrilateral is a special case of another (see Fig. 4.37). This idea may offer to the audience ways to see new relationships among quadrilaterals and their properties.

In this DMP, which is a shadow puppet play (a kind of skit/animation or shadow theatre), students use representations of quadrilaterals produced with manipulative materials. They pasted these quadrilaterals onto sticks and used a light projector to project the shadows of the shapes to display and tell a story. Students also use their arms and hands to create forms such as the Bunny, the Eagle, and the Great Geo. The audio, visual, and gestural designs of this DMP and the form used to communicate mathematics by using shapes and shadows may surprise the audience. Considering all the DMP that are being analyzed in this research, DMP #5 *Little Quad's Quest* is the only one in this shadow theatre format, which may be interpreted by the audience as a different (new and surprising) way to communicate mathematics, in contrast to the traditional forms of text involving writing, diagrams, graphs, and so forth.

Voyeur - Sense-Making

The story presents arguments and examples and reveals properties and connections between quadrilaterals. The mathematics explored in this DMP is connected to a "logical sense" (sets of necessary and sufficient conditions) related to properties of quadrilaterals and similarities between them. The audience may notice the following conversations in this DMP:

- When Little Quad meets the Square, Little Quad says to square: "*You have four beautiful right angles and four equal sides… I have four vertices [like you]*." Square responds to Little Quad: *"… don't have two pairs of parallel sides and you sides are not all equal*."
- When Little Quad meets the Rectangle, Little Quad says: "*You have two pairs of parallel sides and four vertices … I might have the same number of angles*."
- When Little Quad meets the Rhombus, Little Quad says "*I would love to be like you Rhombus! … I got four vertices, four sides, and four angles, but I still don't fit [like you]*." Rhombus responds to Little Quad: "*I guess you do need have all your sides equal and two pair of parallel sides.*"
- When Little Quad meets Trapezoid, Little Quad says to him: "*I just want to be like you ...You have four vertices and four beautiful sides.*" Trapezoid responds and completes: "*Everybody wants to be like me … You forgot to mention I got one pair of parallel sides*."

Thus, in most of the conversations that happen throughout this DMP, the characters are actually presenting specific properties of quadrilaterals and how one quadrilateral may be a specific case of another, and how they fit in the world. To make it clear how one quadrilateral may be a specific case of another, I present some definitions from *The Nelson Canadian School Mathematics Dictionary* (1995; original emphasis):

- A **square** is "a **rectangle** with equal sides, or a **rhombus** with equal angles. It follows from either definition that a square has fours equal sides and four right angles, and that its diagonals are equal and bisect each other at right angles" (p. 215).
- A **rectangle** is "a right-angled **parallelogram**. A rectangle has all the proprieties of a parallelogram (opposite sides parallel and equal, etc.) with the additional property that its angles are right angles and its diagonals are equal" (p. 191).
- A **rhombus** is a "**parallelogram** with equal sides. Sometimes called diamond or rhomb. The diagonals of a rhombus are at right angles to each other" (p. 196).
- A **parallelogram** is "a quadrilateral with opposite sides parallel… their opposite sides are equal as well as parallel" (p. 161). Some special parallelograms are **rectangle**, **rhombus**, and **square**.
- A **kite** is "a convex quadrilateral with two pairs of equal adjacent sides" (p. 122).
- A **trapezoid** is "a **quadrilateral** having one pair of opposite sides parallel and unequal" (p. 232).
- A **quadrilateral** is "any plane figure having four straight sides. Special quadrilaterals include **kite**, **chevron**, **parallelogram**, **rectangle**, **rhombus**, **square**" (p. 184).

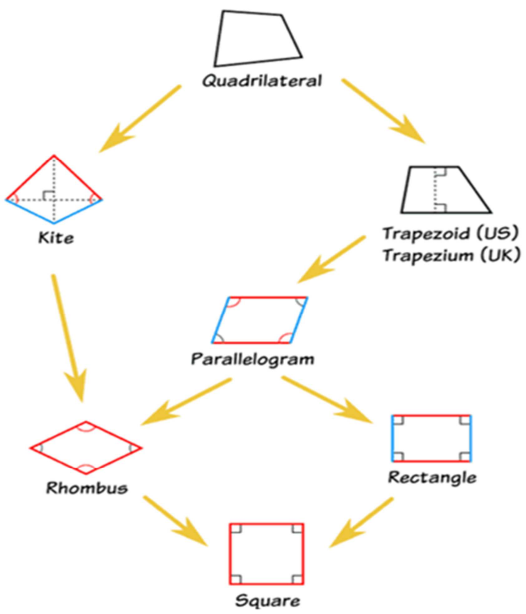
The quoted dictionary then defines (1a) "a square as a special case of rectangle" (1b) "a square as a special case of rhombus" (2) "a rectangle as a special case of parallelogram" and (4) "a rhombus as a special case of parallelogram." One may notice that the quoted dictionary might also have defined:

(a) "a rectangle as a special case of isosceles trapezoid"

(b) "a parallelogram as a special case of trapezoid"

(c) "a rhombus (or a square) as a special case of kite"

Figure 4.37: Relations between Quadrilaterals (retrieved from http://www.mathsisfun.com/quadrilaterals.html)



A more complete set of definitions regarding the relationships of (special cases of) quadrilaterals is well illustrated by http://www.mathsisfun.com/quadrilaterals.html (see Figure 4.37).

Although the DMP does not present connections between different strands, it does offer (a) examples of how quadrilaterals fit in world; (b) proprieties of quadrilaterals; and (c) ways for the audience to make connections about relationships between quadrilaterals, which reveals how some quadrilaterals are specific cases of others or how the same quadrilateral may fit in more than one category.

It is also important to notice that Little Quad has some appreciation or skills related to flying. In part one the narrator says: "*Before Little Quad knew he was in the sky – flying higher and higher until Polygon Ville was just a dot on the ground – it happens so quickly that he didn't have time to get scared. In fact, he felt strangely excited as he decided to flutter in the breeze.*" In part two, the narrator states: "*On the way, the wind blows Little Quad backwards a bit.*" In part three, Little Quad says: "*I am blowing away again!*" In part four, "*the three quadrilaterals pull little Quad along as he is blown away by the breeze.*" Finally, in part five, "*all of a sudden a big gust of wind comes up and blows Little Quad into the air*" and all of the shapes realized how beautiful Little Quad was and they discovered he was a kite and he fit in the world. Therefore, students were creative in providing expectation throughout the story by suggesting how Little Quad would fit in the world. These connections in the story were significant in order to the audience to make sense about what are the properties of a kite and how one can identify representations of a kite in the environment.

The DMP is presented in five separate video files and the audience has the option to watch each part out of sequence. Such design challenges the linearity of the voyeur eye, that is, the multi-linear design of the DMP offers a way to disrupt the "mono-linear sequence" of traditional stories. Gadanidis and Borba (2008) mention that in these cases "the videos may be experienced in an order chosen by the user. Our attention is scattered and not guided" (p. 47). This has an impact on sense-making because it influences the connections the audience may make to understand the story and the mathematical ideas. In this sense, the repetitions at the beginning of each of the five parts in this DMP are very important in terms of understanding. In every part, the narrator starts: "*Once upon a time in Polygon Ville, lived a little quadrilateral, who was very unhappy. All the other quadrilaterals – square, rectangle, rhombus, and trapezoid – were always helping here and there. But the little quadrilateral was never needed for* *anything*.” The objective of these repetitions is to summarize the goal of the story to the audience, making each part understandable and consistent to the audience.

Vicarious Emotions

This is an emotional story. Little Quad was very unhappy and he embarks on a journey to meet other quadrilaterals, discover his identity, his properties as a quadrilateral, and how he would fit in the world, in other words, how useful he is in the environment or as an everyday object. Interestingly, the emotions are connected to conceptual mathematical ideas. Happiness and unhappiness in the story depend on how Little Quad fits or does not fit mathematically. In each scene, by meeting the other shapes, Little Quad gets disappointed when he realizes he does not fit like them. But, by being persistent and hopeful, he finds motivation to continue his journey until he finds out who he is and how he fits. One of the most emotional moments of the story is when Little Quad discovers he is a kite in part five. That is the moment Little Quad realizes he fits in world and discovers his properties. After flying higher and higher, the following transcription reveals the happy end:

Little Quad: *Thanks but what am I?*

Great Geo: *I don’t know. Oh, I do know. You remind me of the great bird called the kite.*

Little Quad said: *That’s it. That’s my name. I’m a kite.*

All Quadrilaterals: *Is it a bird? Is it a plane? No, it is Little Quad! And he’s a kite! Yeah!!!*

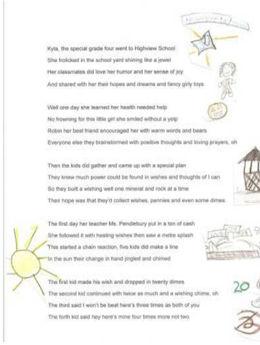
Similarly to DMP #2 Geometrical *Idol* and DMP #3 *Shape Songs*, the students are playing roles as geometrical shapes. The connection between “alive forms” that a square, a rectangle, a rhombus, a trapezoid, and a kite assume may increase the vicarious experience of their emotions by the audience. Students are making shapes “come alive” in this DMP. Each shape assumes specific human personalities in the story, some of which are related to mathematical aspects, and we may identify representations of these quadrilaterals fitting the world or classify them as quadrilaterals in multiple ways. At the beginning of the story, for instance, Little Quad was unhappy because he felt he was not needed as a square or a rectangle. The audience may interpret that as a feeling of rejection, inferiority, or hopelessness by being mathematical “unfit.” The rectangle looked like someone strong. He had a deep voice and he was practicing exercises. The rhombus seemed to be someone vain or even a snob when she said:

"*Go away! I am painting my nails.*" Trapezoid seemed to be a narcissist, with a cold or indifferent personality. The trapezoid was the only one who did not help Little Quad. He said, "*Everyone would like to be like me! I am smart and strong… Go away! I just help shapes who know who they are.*" This personality involving superiority may be related to the fact that a trapezoid is a general type of quadrilateral; by definition every square, rectangle, parallelogram, or rhombus are special cases of trapezoid, but a kite is not a special type of trapezoid (Figure 4.37 illustrates and supports that assumption). Therefore, even though in this DMP there are no human faces to express emotions or close-ups on the characters, the shapes and participants of the story express different emotions, which are very significant to the dramatic design of the story and its relation to the mathematical idea involving the relations between quadrilaterals, its multiple categorizations, and how they fit in the world.

Visceral Sensations

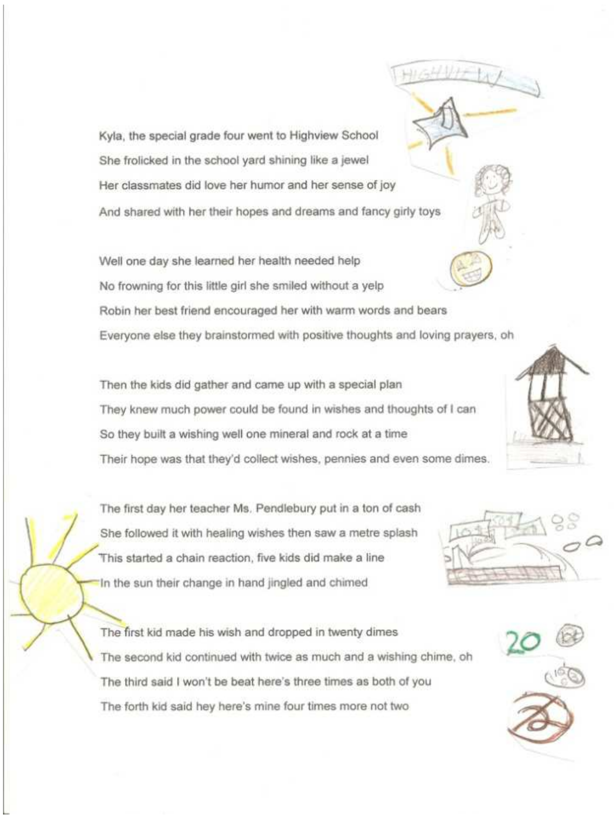
The sense of mathematical fit in this story is a key aspect of the DMP, that is, the main goal of the adventure is to find out who Little Quad is and how he would fit in the world. This sense gets more intense as the story moves along because the audience (a) discovers how square, rectangle, rhombus, and trapezoid fit in the world and (b) has many opportunities to see how quadrilaterals fit with one another, that is, how a quadrilateral may be a special case of another (e.g. every square is a special case of rectangle, rhombus, parallelogram, kite, and trapezoid). The sense of mathematical fit in this DMP may provide visceral experiences to the audience, because it is related to a sense of aesthetic (Sinclair, 2001). Furthermore, through the exploration of connections between properties of quadrilaterals, the audience may viscerally experience mathematics because the DMP offers a way of seeing mathematical objects as fitting in many categories, like a square as a special case of rectangle, rhombus, parallelogram, and so forth.

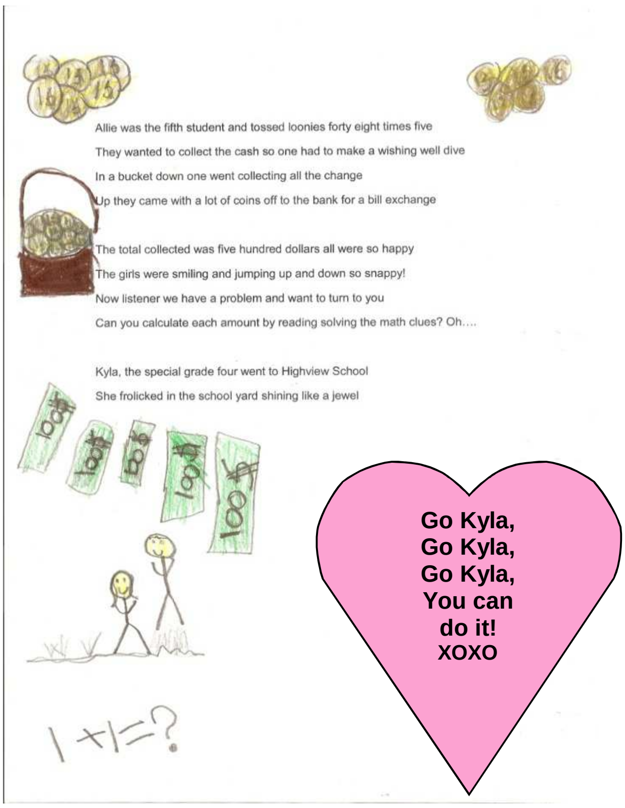
DMP #6: Math Healing Wishes for Kyla

Table 4.9: Math Healing Wishes for Kyla	**URL:**	http://www.edu.uwo.ca/mathscene/mathfest/mathfest101.html
	**Strands and Content:**	Strands: Measurement; Patterning and Algebra. Content: Factorial growth.
	Format:	Power Point.
	Time length:	*None*
	The Arts:	Poetry.
	Participants:	Elementary school students.
	Setting:	Poem created in the classroom.
	Info:	This poem was written for our fellow grade four classmate who is bravely and successfully battling cancer. We believe in the power of healing wishes. May they grow exponentially!

Description

The poem tells a story in which students collect money and wishes to help Kyla, a classmate who is battling cancer. The way students describe the collection of money involves a factorial growth. The poem is presented as follows:





Voyeur - New/Wonderful/Surprising

In DMP #6 *Math Healing Wishes for Kyla*, the reader (the audience) is surprised by a factorial sequence (2, 4, 12, 48, 240, ..., 2.(n!), …). In other words, exploring factorial sequence with Grade 4/5 students through a story that considers the possible application of a mathematical problem in an everyday situation, may offer ways for the students and the audience to see mathematics differently. The mathematical surprise in this poem has the same nature of surprise presented in the fairy tale entitled *The King's Chessboard* (see Birch & Grebu, 1988). In this fairy tale, a king wanted to reward a wise man who had served him. The man did not want anything, but the king insisted. So the wise man decided he wanted a grain of rice on the first square of a chessboard, and he wanted the amount doubled every day for each of the sixty-four squares. The king was not sure how much rice that was, but he thought it was a simple request concerning a number of grains of rice following the sequence (1, 2, 4, 8, 16, 32, …). However, this is an exponential growth. Surprisingly, the king was not able to provide so much rice to accomplish his promise. In the story, Birch and Grebu (1988) estimate that over 200 million tons of rice were necessary to cover all sixty-four days.

This DMP is represented in Power Point format. Thus, students are using writing and figures to communicate their mathematical ideas. Although these are "typical" modes of communicating mathematics, the performative form of a poem or story is not a typical way of communicating mathematics. Thus, this form may offer surprises to the audience in terms of representation or communication considering an interlocution between mathematics and poetry.

Voyeur - Sense-Making

The sense of story in this poem is consistent. Students present arguments that support the mathematical surprise of the poem in the fifth and sixth stanzas:

The first kid made his wish and dropped in twenty dimes
The second kid continued with twice as much and a wishing chime, oh
The third said I won't be beat here's three times as both of you
The fourth kid said hey here's mine four times more not two
Allie was the fifth student and tossed loonies forty eight times five

Students explore some mathematical ideas involving number sense and numeration. Beyond portraying a context involving measurement of $, they deal with sense of place-value, when they say, for example, *twenty dimes*. Interestingly, students explore ideas involving patterning and multiplication. It reveals some understanding about the notion of mathematical function, because students are exploring a particular type of relation[19] between two sets of elements (students and money / domain and codomain). Moreover, the problem involves an "imaginary real situation" and, most important, it connects different strands. The students were creative in offering ways for the audience to make connections when they associated in the plot of the story the *fifth* student to *forty eight times five*, because *fourth-eight* is the number associated to the *fourth* student (see the table below). Thus, the ways students display the poem helps the audience to develop the following mathematical thinking involving correspondence:

Table 4.10: Sense-Making in DMP #6

Student	$
1st	20 dimes = 20 x $0.10 = $2.00
2nd	$2 x 2 = $4
3rd	$4 x 3 = $12
4th	$12 x 4 = $48
5th	$48 x 5 = $240

The chart above shows a pattern that the audience may identify in order to figure out the sequence or function involving the *factorial*,[20] which means $n! = n \,.\, (n-1) \,.\, (n-2) \ldots 3 \,.\, 2 \,.\, 1$. For instance, $5! = 5 \,.\, 4 \,.\, 3 \,.\, 2 \,.\, 1 = 120$. In older grades, the problem presented in the poem can be extended and connected to issues on patenting algebra involving generalization, which involves both the creation of a formula (the function $f(n) = f(n+1) / n = 2 \,.\, n!$), and the graphing representation of this factorial function.

Vicarious Emotions

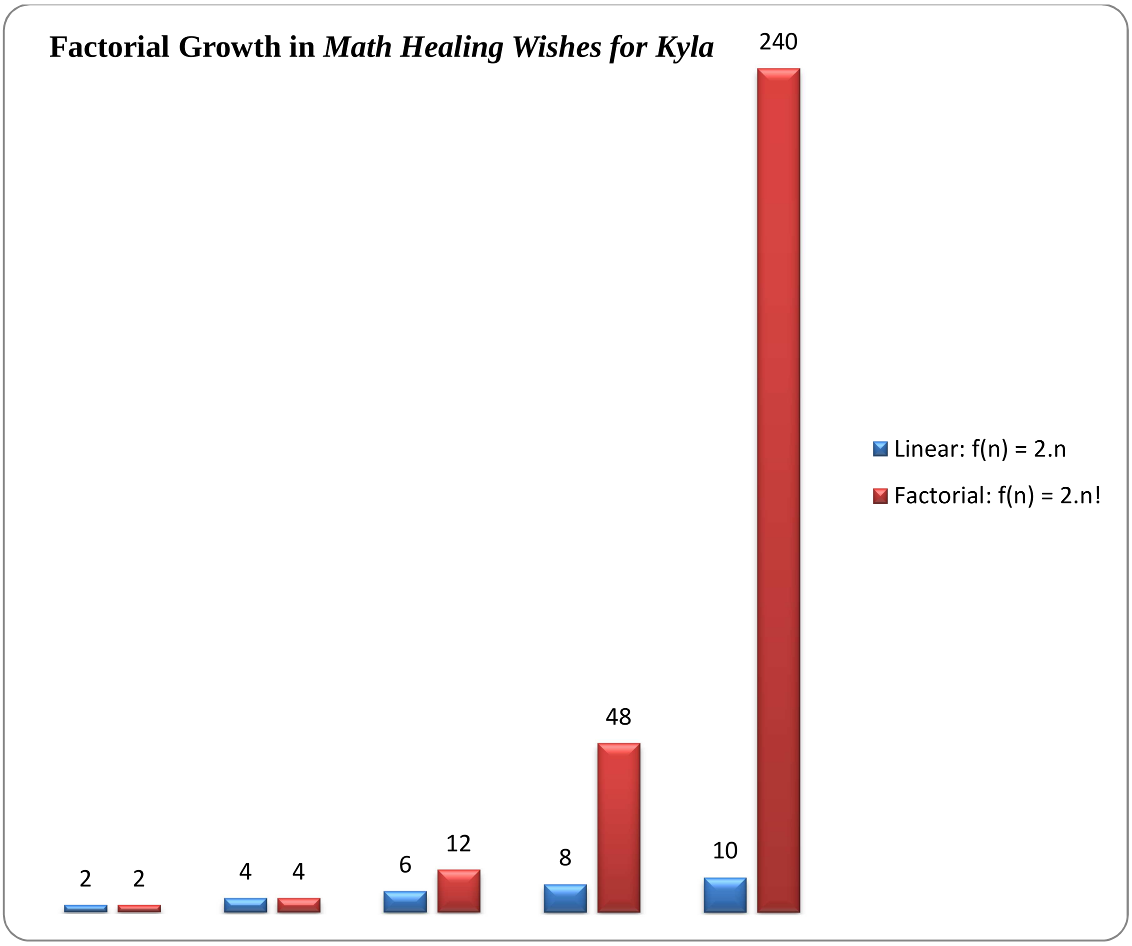
[19] Each element of a set (students/domain) relates to one element of another set (money/codomain).

[20] Factorial is "the function whose value is found by multiplying together all the positive whole numbers up to a given number" (The Nelson Canadian School Mathematics Dictionary, p. 87).

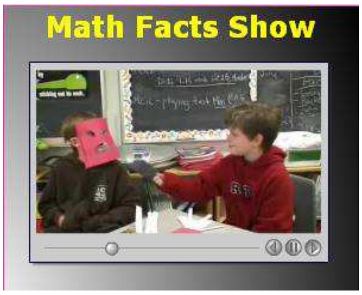
This poem is really emotional. Students express their feelings to provide support to their colleague Kyla. From the poem, the audience may feel students' love, affect, solidarity, hope, and their intention to provide a safe context (sense of belonging). Students relate their emotions toward a classmate who is battling cancer to mathematics, emphasizing a financial support that increases as a factorial sequence or function.

Visceral Sensations

There is a sense of mathematical fit in this DMP, because factorial growth fits with the students' need (or wish) to provide support to Kyla in a short period of time. Furthermore, as Osafo (2010) suggests, fast changes refer to visceral experiences. Exploring factorial growth is a way to sense quick changes of phenomena. The following graph illustrates how the factorial function f(n) = 2.(n!) has a much more intense growth compared to a linear function such as f(n) = 2.n.



DMP #7: Math Facts Show

Table 4.11: Math Facts Show		
	**URL:**	http://www.edu.uwo.ca/mathscene/geometry/geo11.html
	Strands and Content:	Strand: Geometry and Spatial Sense Content: Proprieties of a square, triangle, and circumference.
	Format:	Video and music.
	Time length:	4:12.
	The Arts:	Skit.
	Participants:	Four students and a guitar player.
	Setting:	Performed in a classroom.
	Info:	*None*

Description

A student interviews three different shapes: a square, a triangle, and a circumference. They talk about their proprieties, such as edges, vertices, dimensions, lines of symmetry, as well as their favourite book, and so forth. The last shape interviewed, the circumference, answers the questions by singing. Following, I present a transcription in three parts:

Part 1 - Squareon

Interviewer: *Welcome to the Late Night Math Facts Show! [Someone is playing the guitar]. Today, we are starting in Geometry. Our first guest is Squareon.*

Squareon: *Hello. I am so glad to be here with you guys. We are going to talk about myself and people are learning. Is it not amazing?* (see Figure 4.38).

Figure 4.38: *Squareon*

Interviewer: *Is it true that you are symmetrical?*

Squareon: *Yes. I have four lines of symmetry. One straight down the middle of my face. One horizontally in the middle of my face. And two going to either corner.*

Interviewer: *How many edges do you have? Because I heard that every shape has an edge.*

Squareon: *Yes. I have four edges going around my face. One, two three, four [pointing the edges around the face/mask].*

Interviewer: *Do you have any vertices?*

Squareon: *Yes. I have four vertices. One, two three, four [pointing to the vertices around the face/mask]. Is there a pattern going on?*

Interviewer: *I heard that every shape is a 2D shape or a 3D shape. Are you a 2D shape or a 3D shape and what are your dimensions?*

Squareon: *I am a 2D shape, so my dimensions are length and width.*

Interviewer: *How many parallel lines do you have?*

Squareon: *I have two pairs of parallel lines. Two going vertically and two going horizontally.*
Interviewer: *How many angles and what type are they?*
Squareon: *I have four ninety degree angles, which make my vertices.*
Interviewer: *What is your favourite book?*
Squareon: *My favourite book is "circumference and the great night of angle-land"*
Interviewer: *Thank you.*
Squareon: *Good bye.*

Part 2 - Triny

Figure 4.39: *Triny*

Interviewer: *Now, for the second guest, Triny.*
Triny: *Hello. Thank you for inviting me.* (see Figure 4.39).
Interviewer: *So, Triny, are you symmetrical?*
Triny: *Yes. I am symmetrical. I have three lines of symmetry.*
Interviewer: *Do you have any edges?*
Triny: *I have three edges. One, two, three [pointing the edges around the face-mask].*
Interviewer: *Do you have any vertices?*
Triny: *I have three vertices. One, two, three [pointing the vertices around the face-mask].*
Interviewer: *Are you a two dimensional shape?*
Triny: *I am a 2D shape. So, my dimensions are base and height.*
Interviewer: *Thank you.*

Part 3 – Super-circle

Figure 4.40: *Super-Circle*

Interviewer: *Now, for the third guest, Super-circle.*
[Super-circle arrives].
Interviewer: *So, Super-circle, what do you like about yourself?* (see Figure 4.40).
Super-circle: *[Singing] Circle, I am a circle. I am as happy as a kangaroo. I am so proud that I came to this [?] game to you. [Someone is playing the guitar].*
Interviewer: *How many lines of symmetry do you have?*
Super-circle: *[Singing] One, two, three, four, as many lines as eight-four. Come to my face to see the lines in my face. [Someone is playing the guitar].*
Interviewer: *Thank you. Why do you like circumference so much?*
Super-circle: *[Singing - solo] Circumference is around. Like a moat in a town. It bounces like a ball. For one and for all. It measures all around. But never be unfound.*
Interviewer: *What are your dimensions?*
Super-circle: *[Singing - solo] Area in Pennsylvania, it's inside not around. With all the people in it, they're standing on it now. Circumference is all around the bigger, the circle the more is found. Yay!*
Interviewer: *What is your favourite book?*
Super-circle: *My favourite book is "Circumference is a first round table." Yeah! A math adventure.*
Interviewer: *Thank you Super-circle. And that was this episode of the Late Night Math Facts Show! [Someone is playing the guitar].*

Voyeur - New/Wonderful/Surprising

Students present several properties (e.g., lines of symmetry, dimensions, edges, angles) of a square, a triangle, and a circle. Although students mention some possibilities of representing polygons as everyday objects (such as "*Circumference is around. Like a moat in a town. It bounces like a ball. For one and for all. It measures all around.*"), the mathematics explored in this DMP is typical, that is, it does not offer ways for the audience to experience something unexpected about polygons. The nature of the mathematics explored in this DMP is similar to that explored in DMP #1 *Polly Gone*.

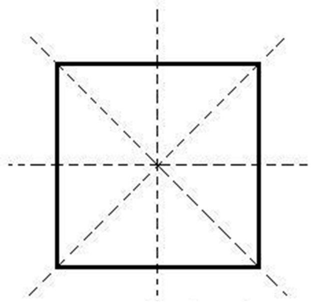
It is surprising that the characters are using masks. The use of these masks is significant for the communication of the mathematical ideas, because the characters discuss some of their properties by pointing to their faces or masks. Squareon, for example, points to his face and says "I have four edges … I have four vertices." Thus, the use of masks is significant in terms of visualization and embodiment when students are communicating the properties of polygons.

Voyeur - Sense-Making

Similar to DMP #2 *Geometrical Idol*, this DMP is a parody of a popular TV show. The fact that students are performing a parody of the *Late Night Show* offers sense of reality to the audience, which is significant for voyeuristic experiences (Boorstin, 1990). In addition, it is interesting that students relate properties of polygons, based on visualization and concrete materials, using masks that work as manipulative materials in this DMP. Some of the students' arguments are supported by pointing to the masks. The visual, aural, and gestural modes of communication are significant for the audience's understanding in this performance. However, most of the ideas present superficial explanations rather than deep connections.

Squareon states that he has four lines of symmetry, four edges and four vertices. Interestingly, he poses a question to the audience "*Is there a pattern going on?*" The pattern refers to a "pattern of fours" – four lines, four edges, and four vertices. It may also refer to the fact that Squareon says he has two pairs of "parallel lines" and the lines of symmetry are formed by two diagonals – that connects two opposite vertices – and two lines connecting each of the opposite medium points of the edges. That is, when Squareon poses a question about patterns related to his properties, the audience may explore different types of patterns, for instance, the pattern that refers to the relation between the lines of symmetry, edges, and vertices, as illustrated in Figure 4.41.

Figure 4.41: Lines of symmetry of a square



Squareon also states that he is a two-dimensional shape, has two pairs of parallel lines, four right angles, and his favourite book is *"circumference and the great night of angle-land."*[21] Based on these statements, students could have explored the notion of a square as a special case of a rectangle (similar to DMP #5 *Little Quad's Quest*), because Squareon is basically mentioning the properties of a rectangle at this moment and, as a square, he has the same properties. Moreover, it is not clear *why* that is his favourite book. Students could have provided more explanation about this to intensify the audience's voyeur experience in terms of sense-making or understanding.

Triny states that he has three lines of symmetry, three edges, three vertices, and he is a two-dimensional shape. The audience may notice that these are specific properties of an *equilateral triangle*, because only equilateral triangles have three lines of symmetry (see DMP #3 *Shape Songs*). However, students do not provide detailed explanation about why Triny is equilateral, what the specific properties are, and what would be the other types of triangles. These explanations would help intensify the rational eye regarding voyeuristic experiences in this DMP.

When Super-circle sings, he said he has infinite lines of symmetry, he can be identified as an everyday object (e.g. a ball), he is a two-dimensional shape, and his favourite book is "*Circumference is a first round table.*" However, the lyrics are not clearly communicated, that is, the audience may have difficulty understanding what the character is saying. Moreover, there are no clear arguments or detailed information about the properties or why that is his favourite book. Students could have emphasized (through visualization and use of materials) a shape with infinite lines of symmetry as something surprising, or asked what would a world be like without representations of a circle, in other words, they could imagine and dramatize a contrast such as a car with square rather than circular tires.

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[21] It may suggest a kind of "disconnection:" Why is a square saying that a book about circumference is his favourite one?

When Super-circle sings "*Area in Pennsylvania, it's inside not around. With all the people in it, they're standing on it now. Circumference is all around the bigger the circle the more is found,*" he is, perhaps, exploring an interesting problem, which is: the circle is the shape with highest area given a perimeter." However, it is not clear if the statement above refers to this problem or not. It might be a conceptual mathematical idea to explore, but the lyrics have many gaps, making a plausible understanding about the communicated ideas rather unclear.

Vicarious Emotions

Like DMP #2 *Geometrical Idol*, students are playing the role of shapes, making mathematical objects "come alive." The use of masks may intensify the audience's sense of the actor portraying the character, thereby enhancing the audience's vicarious experiences in feeling the actors' "(mathematical) emotions." Squareon, for instance, is very excited with his participation in the show. He demonstrates it when he says: "*I am so **glad** to be here with you guys. We are going to talk about myself and people are learning. Is it not amazing?*" The emotions and vicarious experiences in this DMP may offer ways for the audience to visualize the shapes and understand the properties explored. However, as mentioned, the mathematical ideas explored in this DMP are superficial and there are gaps in terms of sense-making. Thus, the emotions the audience may vicariously experience in this DMP are connected to superficial or typical information about polygons.

Visceral Sensations

The fact that students are using masks may offer a visceral sensation to the audience because it is a direct experience with the geometrical object. The masks may offer the sensation that the mathematical object is alive, speaking, gesturing, and arguing. It is significant in terms of mathematical thinking because (a) the students are viscerally thinking as a polygon and (b) the audience is visualizing and listening to an interview with polygons talking about themselves.

Super-circle is singing and it refers to the use of a soundtrack in moviemaking, which may offer visceral sensations to the audience. Furthermore, Surper-circle mentions a way that a circle may fit in the world when he talks about a ball rolling. Thus, there is a sense of mathematical fit in this DMP, but it is not as intense as it is in DMP #5 *Little Quad's Quest* or

DMP #4 *Triangles*, because the connections between the representation of polygons and everyday objects or contexts are superficially explored in DMP #7 *Math Facts Show*.

DMP #8: 2D Land

Table 4.12: 2D Land

URL:	http://www.edu.uwo.ca/mathscene/geometry/geo12.html
Strands and Content.	Strand: Geometry and Spatial Sense Content: Proprieties of square, triangle, and circle.
Format:	Video.
Time length:	3:23.
The Arts:	Skit and Animation.
Participants:	Four Students
Setting:	Classroom
Info:	*None*

Description

The skit/puppet animation is performed in three scenes. In the first scene, a square, a triangle, and a circle are arguing with each other about who is the best. A character, playing the role as a kind of judge, appears and says he will interview each one of them to decide who is the best. In the second scene, through an interview, the square says he is the best, because he is the best in all the sports. The circle says he is the best because he has infinite lines of symmetry. The triangle says she is the best because she has three angles and three vertices. A new round of questions begins, with the judge asking what are the best properties of each shape. The square says he has one face, four edges and four vertices. The circle says he is a curve formed by points at the same distance from the center. The triangle says there are three kinds of triangles, that is, scalene, isosceles, and equilateral. Finally, the judge asks what is the best purpose of each one shape. The square says he is a good base for buildings. The circle says his best purpose is moving fast by rolling. The triangle says her best purpose is to make structures such as roofs strong. Finally, at the last scene, the judge decides that they are all the best.

Following, I present a transcription of the DMP in three scenes:

Scene One

Narrator: *One loud morning in 2D Land, everybody in Shape City was arguing over who is the best. Three of the shapes that were arguing were Tiffany Triangle, Sammy Circle, and Sally Square.* (See Figure 4.42).

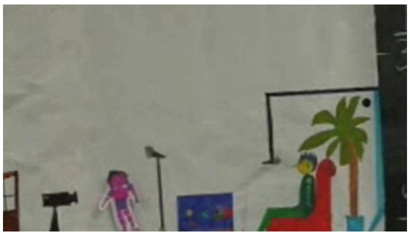
Figure 4.42: Tiffany Triangle, Sammy Circle, and Sally Square

Tiffany Triangle: *No! I am the best!*
Sam Circle: *No! I am the best! You pointy freak!*
Sally Square: *Round-headed freak. I am the best because I have so many right angles.*
Sam Circle: *How dare you say you're the best? It's only true that I am the best. I'm the best because you can't even roll.*
Tiffany Triangle: *You're rolling around everywhere. You don't even have any vertices.*
Interviewer: *Easy! Easy! Now what are all you arguing about?*
Tiffany Triangle: *We are arguing because they don't accept the fact that I am better than them.*
Sam Circle: *No! I am the best!*
Sally Square: *I am the best!*
Tiffany Triangle, Sam Circle, and Sally Square: *No, no, no! I am the best!*
Interviewer: *Quiet! I will interview you all tomorrow to see who is truly the best.*
Narrator: *The next day…*

Scene Two

Interviewer: *Alright! Sally Square, your first question is, why are you the best?*
Sally Square: *I am the best because I am the best at all the sports and I have a [?] as well.* (see Figure 4.43).

Figure 4.43: Sally Square

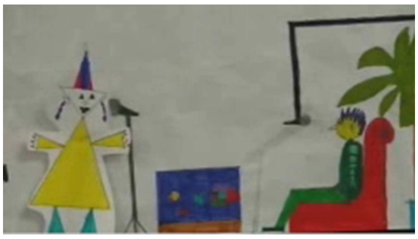


Interviewer: *Alright! Now, Sam Circle, why are you the best?*
Sam Circle: *I am the best because I have a circumference and I have infinity symmetrical lines.* (see Figure 4.44)

Figure 4.44: Sam Circle

Interviewer: *Alright! Now, Tiffany Triangle, why are you the best?*
Tiffany Triangle: *I am the best because I have three angles and three vertices.*
Interviewer: *Alright! Sally Square, second question for you. Your second question is: what are your best properties?*
Sally Square: *My best properties is that I have one face, four edges and four vertices.*
Interviewer: *Alright! Now, you Sam Circle. What are your best properties?*
Sam Circle: *My best properties are that I am a curve formed by points the same distance from the center.*
Interviewer: *Alright! Now, Tiffany Triangle, what are your best properties?*
Tiffany Triangle: *My best properties are in Vertexville.* (see Figure 4.45)

Figure 4.45: Tiffany Triangle



Interviewer: *Alr… No! Your physical properties!*
Tiffany Triangle: *My best physical properties are that there are three kinds of me: isosceles, scalene, and equilateral. Scalene is the best of course.*
Interviewer: *Alright! Sally Square. Last question: what is your best purpose?*
Sally Square: *My best purpose is that I am a good base for building.*

Interviewer: *Alright! Sam Circle. What is your best purpose?*
Sam Circle: *My best purpose is moving things fast, fast if I roll it.*
Interviewer: *Alright! Now, Tiffany Triangle. What is your best purpose?*
Tiffany Triangle: *My best purpose is holding a strong structure, like roofs.*
Interviewer: *Alright! I will tell you who is the best, tomorrow.*
Narrator: *The next day…*

Scene Three
Interviewer: *Alright! It has occurred to me who is the best.* (see Figure 4.46).
Tiffany Triangle: *Who? Who? Tell me! Tell me!*
Interviewer: *The best is all of you! You are all equal, which means that you are the best!*
Sally Square: *Well! I guess this is true… I am sorry guys.*
Sam Circle: *Yeah! I am sorry too.*
Tiffany Triangle: *I am sorry as well.*
Sally Square: *Hey guys! Wanna go for an ice cream?*
Tiffany Triangle: *Totally!!!*
Narrator: *The end.*

Figure 4.46: Interviewer's announcement

Voyeur - New/Wonderful/Surprising

Although the mathematics explored in this DMP are very similar to those explored in DMP #7 *Math Facts Show*, students explore the connections between representations of polygons and everyday objects and contexts. In scene two, Sally Square and Tiffany Triangle explain their best purpose by relating the representation of their shapes to important aspects in architecture or construction. They argue that a square is "*a good base for building*," a circle moves "*things fast, fast if I roll it*," and a triangle makes "*a strong structure, like roofs*." Thus, connections between representations of polygons (or any geometric object) and everyday objects, structures, or contexts offer ways for the audience to see the new/wonderful/surprising in mathematics.

Voyeur - Sense-Making

Although the narrator in the story is not a representation of a polygon, students were creative in presenting a story about polygons where most of the characters are polygons. Furthermore, regarding the dimensions of polygons in the context of Euclidian geometry, it makes sense that the characters are in *Shape City*, which is located in *2D Land*.

Beyond making connections between the properties of polygons and their relation to everyday objects or contexts, students make connections between the properties of the polygons, that is, they compare and contrast one polygon to another, highlighting "positive" and "negative" aspects. Table 4.13 illustrates the comparative aspects mentioned in the DMP:

Table 4.13: Polygons Properties in DMP #8: 2D Land

	Circle	Square	Triangle
Positive	▪ *has infinity symmetrical lines* ▪ *is a curve formed by points the same distance from the center.* ▪ *moves things fast*	▪ *has many right angles* ▪ *has one face, four edges and four vertices.* ▪ *is good base for building.*	▪ *has three angles and three vertices.* ▪ *has three types: isosceles, scalene, and equilateral.* ▪ *holds a strong structure, like roofs.*
Negative	▪ *is a round-headed freak.* ▪ *rolls around everywhere* ▪ *does not have any vertices*	▪ *doesn't even roll.*	▪ *is pointy freak!* ▪ *can't even roll.*

Vicarious Emotions

Like in the DMP #2 *Geometrical Idol*, DMP #3 *Shape Songs*, DMP #5 *Little Quad's Quest*, and DMP #7 *Math Facts Show*, students play the role of polygons in this DMP. Beyond engaging themselves in a process of thinking-as-a-polygon, the plot of the story is dramatic, and the emotions are expressed based on the properties of each polygon when arguing which shape is the best. Thus, the actors' (polygons') emotions in this DMP are fundamentally mathematical. The sense of "being the best" or seeing the other as a "freak" is constructed around the properties or characteristics of each polygon, such as "*being good base for building*" or being seen as a "*pointy freak*." Furthermore, the plot of the story is based on a dispute about which polygon – a circle, a square, or a triangle – is the best. Although each polygon has different properties and purposes, in the last scene, they are seen as equal. It does not mean that they have every property equal necessarily. It means that they are all polygons, but each of them has very special and unique properties and purposes, which makes each of them the best. Therefore, in the emotional dimension of the story, students relate the difference of the polygons' properties to a sense of equity, supporting a happy ending.

Visceral Sensations

As in DMP #2 *Geometrical Idol*, DMP #4 *Triangles*, DMP #5 *Little Quad's Quest*, and DMP #7 *Math Facts Show*, students explore the idea that representations of polygons *fit* in the world (how they would fit or not). For instance, a square is a good base for buildings, but it does not roll as a circle does. Thus, students make connections between different shapes and connections between representations of shapes and the world around them, exploring ways of direct experiences, potentially offering visceral sensations to the audience.

DMP #9: We are the Polygons

Table 4.14: We are the Polygons

Polygons and non-polygons on stage.

URL:	http://www.edu.uwo.ca/mathscene/geometry/geo8.html
Strands and Content:	Strand: Geometry and Spatial Sense Content: Properties of polygons
Format:	Video.
Time length:	2:02.
The Arts:	Song and Skit: Musical.
Participants:	Four elementary school students and one guitar player
Setting:	Performed in a classroom.
Info:	Polygons and non-polygons on stage.

Description

Students sing songs and perform a skit playing roles as polygons and figures that are not polygons. Two students play roles as polygons (Polygon 1 and Polygon 2) and two students play roles as non-polygons (Non-Polygon 1 and Non-Polygon 2). Following, I present a transcription:

Polygon 2: *We are the polygons, we have straight edges.* (see Figure 4.47)
Polygon 1: *We are the polygons, we look like hedges.*
Non-Polygon 2: *[Jumping to the front and crossing arms] We aren't the polygons. We look like blocks.*
Non-Polygon 1: *[Jumping to the front and crossing arms] We aren't the polygons. We get caught in rocks.*
Polygon 2: *We are the polygons. We're always closed.*
Polygon 1: *We are the polygons, our song always flows.*
Non-Polygon 2: *[Jumping to the front and crossing arms] We aren't the polygons. We can't close.*
Non-Polygon 1: *[Jumping to the front and crossing arms] We aren't polygons. We don't know how to pose.*

Figure 4.47: Polygons and non-polygons

Polygon 2: *We are the polygons, we're everywhere.*
Polygon 1: *We are the polygons, so treat us with care.*
Non-Polygon 2: *[Jumping to the front and crossing arms] We aren't the polygons. No one treats us with care.*
Non-Polygon 1: *[Jumping to the front and crossing arms] We aren't the polygons. It isn't fair.*
Polygon 1: *Polygon*
Non-Polygon 1: *Not.*
Polygon 1: *Polygon*
Non-Polygon 1: *Not.*
Non-Polygon 2 and Polygon 2: *[Jumping to the front] Hold up.*
Non-Polygon 2: *Why are we fighting?*
Polygon 2: *We can get along.*
Non-Polygon 2: *Why aren't we cooperating?*
Polygon 2: *Why are we singing this song?*
Non-Polygon 2: *Whatever.*
Polygon 2: *Here some facts.*
Non-Polygon 2: *So just relax.*
Polygon 2: *Polygons are parallels.*
Non-Polygon 1: *But we are not.*
Non-Polygon 2: *Polygons are everywhere.*
Polygon 1: *Man there are a lot.*
Non-Polygon 1: *But we are always mistreated.*
Polygon 1: *They always get defeated.*
Non-Polygon 2: *They make us so frustrated.*
Non-Polygon 1: *Because we are always hated.*
Non-Polygon 2: *I know there is another way.*
Polygon 2: *And that can come any day.*
Polygon 1: *Why? Why? Why?*
Non-Polygon 1: *We don't have lines of symmetry.*
Polygon 1: *Then you guys are freaky.*
Polygon 2: *Polygons are interesting.*
Non-Polygon 2: *They are everywhere and listening.*
Non-Polygon 1: *But we're there too.*
Polygon 1: *Who knew?*
Polygon 2: *I did.*
Non-Polygon 2: *Polygons are complicated.*
Polygon 1: *You be the judge.*
Polygon 2: *Is your name mud?*
Non-Polygon 1, Non-Polygon 2 and Polygon 1: **sighs**
Polygon 1: *Why? Why? Why?*
Non-Polygon 1: *Why doesn't this fly?*
Polygon 1: *We can work together.*
Non-Polygon 2: *Friends forever!*
Polygon 1: *Now until forever.*
Non-Polygon 2: *Wow this is cool!*
Non-Polygon 1: *We didn't learn this in school.*
All Figures: *Yeah! Hurray! We are friends anyway!*

Voyeur - New/Wonderful/Surprising

Although the approach of comparing polygons and figures that are not polygons is interesting, this DMP does not provide ways of seeing the new and wonderful in mathematics. Students mention ways to explore some properties of polygons and possible connections between representations of polygons and everyday objects. However, similar to DMP #1 *Polly Gone*, the mathematical content is quite typical regarding a traditional exploration in a math classroom.

Just like in DMP #2 *Geometrical Idol*, DMP #4 *Triangles*, and DMP#7 *Math facts Show*, students are communicating their ideas by using songs and skit, representing them using video recording. This format of communicating/representing mathematical ideas challenges the traditional way of representing mathematics through print-based texts regarding the use of writing, charts, and diagrams. This DMP (like most of the DMP analyzed in this dissertation) may offer surprises to the audience because the ideas are represented through the performance arts in combination with digital media, which offers ways for multimodal communication, that is, this DMP is surprising because, collaboratively, students are producing multimodal texts to represent their mathematical ideas using the arts.

Voyeur - Sense-Making

Like in DMP #1 *Polly Gone*, this DMP explores the properties of polygons (or elements that define a polygon). At the beginning of the performance, the Polygons say things like: "*we have straight edges*" and "*We're always closed.*" In contrast, the Non-Polygons say: "*We can't close.*" Students are exploring the definition of polygons, which is: "A closed plane figure with straight sides" (*The Nelson Canadian School Mathematics Dictionary*, p.173).

Although students do not connect different strands in the DMP – they only explore *Geometry and Spatial* sense – there is an interesting connection to visual representations regarding the definition that students are communicating verbally (or orally). The students are using concrete representations of polygons and non-polygon figures as part of their costumes. Thus, the audience may connect the definition of polygons as closed figures with straight lines to the fact that the polygons (in yellow) have these characteristics (see figure 4.48).

Figure 4.48: Students' "costumes"

The DMP also explores the notion of representations of polygons as common everyday objects and properties such as lines of symmetry. However, the students explore these ideas superficially, that is, they do not explore connections between representations and strands. They could have explored visually what they were mentioning verbally (lines of symmetry in regular polygons) or connected *Geometry to Measurement* by exploring the notion of angles in polygons or explored less traditional definitions of polygons (such as "a triangle with a line of symmetry is isosceles"), for instance.

Students could also have explained why they are suggesting that, "*Polygons have parallels.*" Parallelism is a characteristic of some polygons such as trapezoids, parallelograms, rectangles, squares, and regular hexagons. But, as counter-examples, there are polygons where parallelism is not a characteristic such as triangles. Furthermore, the non-polygons say: "*We don't have lines of symmetry.*" This statement is not true. Although this might be acceptable in the context of the performance, because particular non-polygon figures displayed visually on the students' t-shirts do not have lines of symmetry, the non-existence of lines of symmetry is not a general property of figures that are not polygons. A circle, for instance, is not a polygon and it has infinite lines of symmetry.

Vicarious Emotions

Like in DMP #2 *Geometrical Idol*, DMP #3 *Shape Songs*, and DMP #*7 Math Facts Show*, students take on the roles of geometric objects, making them "come alive." The feelings and emotions the characters express during the DMP are directly related to the condition of being or not being a polygon. Similarly to DMP #8 *2D Land*, the geometric representations of figures are "disputing who is the best." Vicariously, the audience may feel that the non-polygons feel "inferior" to the polygons and the polygons feel "superior" to the non-polygons. The non-polygons, for instance, explicitly say, "*No one treats us with care. They make us so frustrated. We are always hated*." These feelings emerge because non-polygons do not have the same "special" properties that polygons do, and representations of non-polygons do not fit in the world as representations of polygons do. In contrast, the polygons impose their "superiority" by saying such things as, "*Then you guys are freaky. Polygons are interesting.*" Therefore, all the actors' emotions in this DMP are "mathematical emotions" because the joy or sadness in the story refers directly to the mathematical aspects of comparing representations of polygons and representations of figures that are not polygons. Interestingly, the story has a happy ending because all the characters agree that polygons and non-polygons can work together and be friends. Furthermore, they celebrate the fact that their exploration (mathematics through the performance arts the audience may suppose) is not common in school.

Visceral Sensations

This DMP offers some visceral experience to the audience through the exploration of how representations of polygons may *fit* in the world. However, this exploration is superficial. On the one hand, students mention that, "*Polygons are everywhere.*" On the other hand, they do not offer examples about where and how polygons are everywhere. DMP #5 *Little Quad's Quest*, for instance, offers several examples about how and where squares, rectangles, rhombi, and trapezoids fit in the world.

Although the examples of how polygons fit in the world are superficial in this DMP, there are two other aspects that may offer visceral sensation to the audience: (a) the use of a soundtrack and (b) action in the scene. In the DMP, students are singing, which may be viscerally felt by the audience as Boorstin (1990) states. Furthermore, the fact that the Non-Polygons and the Polygons jump to the front along the story refers to a "quick change" at that moment, that is, it offers action to the scene, which is a characteristic of the visceral eye (Boorstin, 1990).

DMP #10: Fabulous Fraction

<table>
<tr><td rowspan="8">Table 4.15: Fabulous Fractions

</td><td>URL:</td><td>http://www.edu.uwo.ca/mathscene/mathfest/mathfest103.html</td></tr>
<tr><td>Strands and Content:</td><td>Strands: Number Sense and Numeration; Geometry and Spatial Sense
Content: Visual representation of equivalent fractions</td></tr>
<tr><td>Format:</td><td>Video.</td></tr>
<tr><td>Time length:</td><td>0:38.</td></tr>
<tr><td>The Arts:</td><td>Stop-Motion Animation</td></tr>
<tr><td>Participants:</td><td>Three students</td></tr>
<tr><td>Setting:</td><td>None</td></tr>
<tr><td>Info:</td><td>None</td></tr>
</table>

Description

Like in DMP #15 *Equivalent Fractions* and DMP #22 *Fractiontastic*, this DMP explores numeric and visual representations of equivalent fractions. By using concrete materials and the film effect of stop-motion to produce an animated video, this DMP explores the connection between the numeric and the geometric representation of fractions through the use of the representation of a square or cube. The DMP explores equivalent fractions by relating the number 1 to a whole square/cube (value of the area or volume of a square or cube as the unit), the fraction 1/2 to two-halves of the square/cube, the fraction 1/4 to four-quarters of the square/cube, and 1/8 to eight-eights of the square/cube. Although there is background music, the DMP does not use verbal or oral language to express the ideas. Figure 4.49 a-d presents the ideas visually explored with modeling clay:

Figure 4.49 a-d: Fabulous Fraction

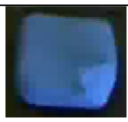
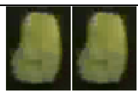
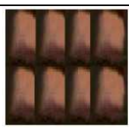
Voyeur - New/Wonderful/Surprising

Although this DMP explores a variation of the traditional “pizza fraction task,” it does offer ways of seeing the new and wonderful in mathematics because it (a) connects representations and strands and (b) demonstrates division by motion and the equivalence of fractions. The connection between numeric and geometric representations of fractions is a “big curricula issue” (NCTM, 2000; Ontario Ministry of Education, 2005). Surprisingly, this DMP explores a square/cube representation instead exploring the circular shape of a pizza. Furthermore, the stop-motion animation/video format of the DMP offers ways to visualize the process of division in motion while demonstrating the equivalence of fractions. Thus, the fact that the DMP is a multimodal text, which supports specific types of designs (e.g., audio, gestural, special), is a key-aspect to offering surprise to the audience regarding the “visual demonstration” being explored.

Voyeur - Sense-Making

This DMP explores an interesting connection between numeric and geometric representations of fractions. Considering the unit as the whole square or cube (number 1 referring to the area or volume), the DMP presents four levels or stages of the division 1/[2^(n – 1)]. There is a connection between the number of pieces and the size of each piece (1/2, 1/4, and 1/8 of the introduced value of the area or volume). Thus, this DMP is exploring the following equivalent fractions: 1 = 1x1 = 2x(1/2) = 4x(1/4) = 8x(1/8). Therefore, the audience may think about the following connections in this DMP:

Table 4.16: Visual Equivalence of Fractions in DMP #10

Stage	Fraction: 1/[2^(n – 1)]	Visual	Equivalent fractions. “Number of pieces x size of the piece = Unit.”
1	1 = 1/(2^0)		1 x 1 = 1
2	1/2 = 1/(2^1)		2 x (1/2) = 1
3	1/4 = 1/(2^2)		4 x (1/4) = 1
4	1/8 = 1/(2^3)		8 x (1/8) = 1

This DMP could have presented a more dynamic way to display the numeric and geometric representations in the animation. In this DMP, the audience may have difficulty (a) visualizing the numeric representation of the fraction 1/8 and (b) realizing that the value of the size of the areas or volumes of the pieces are actually proportional in the process of division. Displaying the division process clearly and explicating the proportion and number of pieces of the square (or cube) is very important for the audience to make sense about the equivalence of the fractions. This process is considerably clearer in DMP #15 *Equivalent Fractions* and DMP #22 *Fractiontastic*.

Vicarious Emotions

In this DMP the audience does not see human characters or representations of humans such as animated puppets. Thus, the audience may have difficulty feeling what the “actors” are feeling in this DMP, which is the fundamental characteristic of the vicarious eye (Boorstin, 1990). However, the audience may interpret the representations of geometric objects in motion as a “character in action.” Some emotions can also be conditioned by the background music, but this aspect is more associated to visceral sensations.

Visceral Sensations

This DMP presents three aspects that offer visceral sensations to the audience. As mentioned, the use of soundtracks may offer visceral sensations to the audience. The motion that represents the division of the geometric object (visually) and its connection to the numeric representation happen very fast in the DMP. Furthermore, the animation format (stop-motion animation) also enhances the sense that things are happening fast. Quick changes, like the one presented in this DMP, are a characteristic of the visceral eye (Osafo, 2010). Most importantly, the notion of “equivalent fractions” explored through a visual representation in this DMP provides an intense sense of *mathematical fit*. Thus, it is visceral to visualize that the representations of 1, 2x1/2, 4x1/4 and 8x1/8 are in motion and fit together, that is, the exploration of visual and gestural meaning for 1 = 2x1/2 = 4x1/4 = 8x1/8.

DMP #11: Sphere on the Loose

<table>
<tr><td rowspan="8">Table 4.17: Sphere on the Loose

</td><td>URL:</td><td>http://www.edu.uwo.ca/mathscene/geometry/geo3.html</td></tr>
<tr><td>Strands and Content:</td><td>Strands: Geometry and Spatial Sense
Content: Properties of a sphere</td></tr>
<tr><td>Format:</td><td>Video.</td></tr>
<tr><td>Time length:</td><td>1:26.</td></tr>
<tr><td>The Arts:</td><td>Musical: Skit and Songs</td></tr>
<tr><td>Participants:</td><td>Two students and one guitar player</td></tr>
<tr><td>Setting:</td><td>Performed in a Classroom</td></tr>
<tr><td>Info:</td><td>The Sphere is on the loose. Will he be captured? Will he go to jail? Or is there another fate awaiting this figure-on-a-roll?</td></tr>
</table>

Description

In this musical, a sphere is wanted by the sheriff because the sphere has committed various crimes, such as going over the speed limit. The sheriff captures the sphere and pushes the sphere to a hill to arrest the sphere. Following, I present a transcription of the DMP:

Sphere [singing]: *Happy, happy, happy world! Come eat candy, come eat more. Come on! You know you want it. So, come here and eat some candy* (see Figure 4.50).
Presenter: *We interrupt this program for a special news announcement. [Someone starts to play the guitar and the presenter shows a poster.] A sphere has been found and he is running wild. He has committed various crimes such as going over the speed limit and rolling and ramming things to destroy them. Now, we will show you what happened. [The guitar stops playing].* (see Figure 4.51 a-b)
Sheriff: *Why can't I find that little varmint? I've been looking everywhere. [Someone starts to play the guitar].*
Sphere *[appears suddenly, attacking the sheriff]: Surprise!* (see Figure 4.52).
Sheriff: *What are you doing here? You little… that's not manly. You cone!*
Sphere: *What?*
Sheriff: *You will cone!*
Sphere: *Nobody calls me a cone!!!*
Sheriff: *I do.*
Sphere: *Draw!*
[Sheriff arrests the Sphere] (see Figure 4.53)
Sphere: *Where'd you go? Ah! Foiled again!*
Sheriff: *Come with me, outlaw.*
Sphere: *Wait. Jail isn't this way, the hill is… Oh no!*
Sheriff: *See you later Sphere.*
Sphere [rolling/falling]: *Ah!!!*
[Applause]

Figure 4.50: Sphere

Figure 4.51 a-b: Presenter with a poster

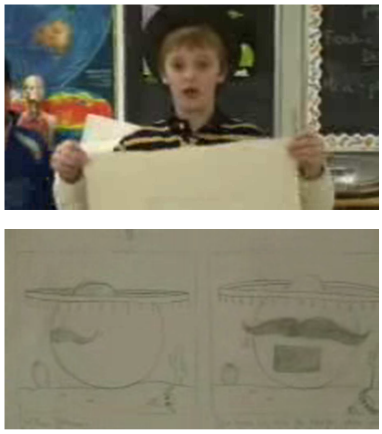


Figure 4.52: Sphere appears suddenly

Figure 4.53: Sheriff arrests the Sphere

Voyeur - New/Wonderful/Surprising

Similar to the other DMPs analyzed in this research, the performative and multimodal aspects in communicating and representing the mathematical ideas are surprising. Students only superficially explore the notion of representations of spheres as everyday objects by suggesting that a "*sphere rolls everywhere*." They could have explored visual-imaginative aspects such as a sphere formed from the revolution of a circle or the sphere as a surface to explore non-Euclidian geometries, like in the DMP *Flatland* (Gadanidis, 2005).

There is an interesting surprise in the DMP. As the reader may see in the transcription, the students try to offer surprises to the audience by appearing suddenly in the scene. "*Sphere [appears suddenly, attacking the sheriff]: Surprise!*" However, although a sphere is surprising the sheriff in the skit, it is not a mathematical surprise because it does not offer the audience a way to see the new and wonderful in mathematics. It is only a surprise within the context of the story. In contrast, this DMP provides an interesting representation of the sphere as an everyday object (a ball). Students were creative in performing a skit in which a sphere is a "criminal" due to the fact that the sphere "*goes over the speed limit and rolls and rams things to destroy them.*" This idea may be interpreted as a mathematical surprise by the audience.

Voyeur - Sense-Making

In the DMP, there are connections between the verbal/oral ideas communicated, the actions of the character (gestures and use of spatial elements), and the visual representation in a poster, which offer important visual information to the audience. However, students did not explore "big ideas" about the sphere. They could have used multiple types of visual representation (use of specific costumes, for instance) and explored dimensions and visualization-imagination (sphere as the rotation of a circumference), superficial area and volume of spheres, multiple ways of seeing a representation of a sphere as an everyday object, and so forth. They could have imagined, for instance, a sphere in a world of non-spheres, where things like rolling would be considered "mad" (see http://publish.edu.uwo.ca/george.gadanidis/imaginethis/uoit/index.html).

Vicarious Emotions

In this DMP, just like in DMP #2 *Geometrical Idol*, DMP#3 *Shape Songs*, DMP #7 *Math facts Show* and DMP #9 *We are the Polygons*, students are playing the role of geometric objects. The performance starts with the student who plays the role of the sphere presenting an announcement. It is not clear if the student is actually playing the role of the sphere at that moment or another role. If he is playing the role of the sphere at that moment, the audience may feel the sphere is really happy, but it is not exactly a “mathematical emotion.” The most interesting emotion in the DMP is when the sphere gets offended and angry because the sheriff calls the sphere a cone (or suggests that he will transform the sphere into a cone). The students play the scene well, expressing the feelings of anger surrounding the offence, and the audience may, vicariously, feel the “mathematical emotion.” The sphere gets offended by being called a cone because the sphere perhaps sees the cone as an “inferior” shape. Interestingly, the reasons why the sphere gets offended are not explicit in the performance, which allows the audience to experience a diversity of “mathematical emotion,” depending on the way one interprets the offence/anger.

Visceral Sensations

There are two explicit elements for the visceral eye in this DMP. The performance is mostly an action scene where a sheriff captures a sphere and there is a soundtrack.

There is also some sense of mathematical fit as the students are exploring ways in which representations of spheres may fit in the word. However, the connection is actually superficial because it only explores the notion that “spheres roll.” As mentioned, they could have explored multiple ways to represent spheres fitting in the world.

DMP #12: Radius & Diameter

Table 4.18: Radius & Diameter	**URL:**	http://www.edu.uwo.ca/mathscene/geometry/geo10.html
	**Strands and Content:**	Strand: Geometry and Spatial Sense Content: Properties of circles
	Format:	Video
	Time length:	2:12.
	The Arts:	Skit
	Participants:	Three students
	Setting:	Performed in a Classroom
	Info:	*None*

Description

Three students perform a skit that explores the notion of radius and diameter in circles. Initially, a presenter poses a problem Liam has to solve for school. Liam goes to visit Mr. Shape and they explore the definitions of radius and diameter. Finally, the presenter poses some conclusions in a poetic way. Following, I present a transcription of the DMP:

Figure 4.54: Presenter

Figure 4.55: Mr. Shape

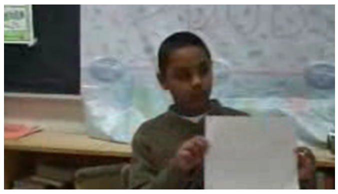


Figure 4.56: Liam and Mr. Shape

Figure 4.57: Presenter's conclusions

Presenter: *Once upon in time there was a young boy named Liam who didn't know anything about radius and diameter. For a school project, he had to bake a cake with a radius of fifteen centimetres and the diameter of thirty centimetres. So, he went to Shape Valley up triangle mountain into Mr. Shape's lab.* (See Figure 4.54)
Mr. Shape: *I told you. I invented a new shape. I call 'cyscisism.'[Showing a poster].* (See Figure 4.55)
Liam: *Hello! [Looks around] Hello!!!*
Mr. Shape *[jumps]: Whoa! What is it Liam?*
Liam: *I was wondering what's radius and diameter.*
Mr. Shape: *A diameter is a straight line from the edge of the circle, through the center, and to the other side.*
Liam: *Oh! So, a kind of like a pogo ball… with a little bottom in the center or a smiley face guy with a belt across him.*
Mr. Shape: *The radius is a line from the edge of the circle to the center.*
Liam: *Oh! So you mean like Pacman with his mouth closed.*
Mr. Shape: *Yeah!*
Liam: *Oh! Now I get it! Now I can go home, bake my cake, and hopefully get an A+. Bye.* (See Figure 4.56).
Presenter: *Radius times two equals the diameter. If you flick the radius it will draw a circle, which you can color purple. Radius is from edge to center. [Making gestures]. There is no diameter of a square. The studying circle is like going to the fair. It's so fun you'll say you might even want to get some blood. So he went down Triangle Mountain through Shape Valley all the way home. He baked his cake and ran to school. He got an A+ on his radius and diameter part but his cake sucked. The end.* (See Figure 4.57).
[Applause]

Voyeur - New/Wonderful/Surprising

Although the mathematics explored in this DMP are typical regarding what happens in a traditional math classroom – students basically present the definitions of radius and diameter – the connections to everyday objects is interesting and might offer some mathematical surprise to the audience. Initially, the presenter poses a "practical problem" because the notion of radius and diameter are needed to bake a cake. Students define the diameter of a circle as "*a straight line from the edge of the circle, through the center, and to the other side*" and they relate the representation of a diameter to "*a pogo ball… with a little bottom in the center or a smiley face guy with a belt across him.*" Similarly, they define the radius as "*a line from the edge of the circle to the center*" and they relate the representation of the radius to "*Pacman with his mouth closed*." Finally, poetically, the presenter concludes that "*radius times two equals the diameter*." Like in DMP #2 *Geometrical Idol*, DMP #5 *Little Quad's Quest*, and DMP #8 *2D Land,* students connect representations of polygons to everyday objects. Furthermore, in this DMP, students connect the properties of polygons to solve an "everyday problem" and use poetry as well. These types of connections offer ways of seeing the new and wonderful in mathematics.

Voyeur - Sense-Making

Students are exploring definitions and examples. Formal definitions of mathematical objects refer to the least number of properties of the object needed to identify it (*The Nelson Canadian School Mathematics Dictionary*, 1995), that is, definitions involve logical thinking toward sufficient and necessary conditions for a statement. Students' definitions of radius and diameter in the DMP are similar to those presented in the referred dictionary. However, diameter and radius are elements of both circles and spheres. Based on the posed problem, where students are asked to bake a cake, it is probable that students realized that its form looks like the representation of a cylinder. That is the most traditional shape of cakes and that is why students are exploring diameter and radius of circles. Could students have explored other shapes of cakes such as cones and spheres? Students could have, for instance, explored the problem of the volume of an ice-cream cone, in which the notions of radius and diameter are also needed.

The definitions and examples of objects that represent radius and diameter are significant to the presenter who concludes that "*radius times two equals the diameter*." The use of poetry and gestures by the presenter at the final scene is also interesting in terms of meaning production. On the one hand, the use of rhymes conditions the words the poet uses and, obviously, conditions the nature of the meaning. The use of gestures made with the hands helps the audience on the visualization and understanding about some of the properties of a circle. From a gestural mode of communication, the student's embodiment is fundamental for meaning production at this moment of the DMP. On the other hand, the students could have explored other mathematical ideas through the final poem to provide ways for the audience to extend the problem. They could have explored, for instance, the relation between radius/diameter and length of circumferences or the area of circles, thereby introducing the notion of the number *pi*.

Although it is not significant to the plot of the story, it is relevant to mention that Mr. Shape is portrayed as a "shape inventor." At the beginning of the skit, he says he invented a new shape named "*cyscisism*" and shows a poster, displaying a representation of this "new figure." However, the focus is not clear and the audience cannot see the image of the shape.

Vicarious Emotions

The DMP does not offer conceptual mathematical emotions. However, regarding the use of zooming-in on the characters' facial expressions, the audience may vicariously feel what Liam is feeling when he gets really excited by fact that he learned what radius and diameter are and, as a result, he expects he will get a good mark in his task. Although this DMP presents emotions that are significant to the story, the emotions are not necessarily mathematical emotions.

Visceral Sensations

This DMP offers the experience to the audience that representations of elements of circles (like radius and diameter) fit in the world as the way to create a model to solve an "everyday problem." There is a connection between mathematical objects and "an everyday object." In the skit, students connect the shape of a cake to representations of circles and use the notion of diameter as a parameter to bake a specific kind of cake. Connections between representations of mathematical objects and "everyday situations" may offer visceral sensations to the audience because the audience may experience how mathematics *fit* in the world.

DMP #13: Square Trial

Table 4.19: Square Trial	URL:	http://www.edu.uwo.ca/mathscene/geometry/geo6.html
	Strands and Content:	Strand: Geometry and Spatial Sense Content: Properties of quadrilaterals
	Format:	Video
	Time length:	1:33.
	The Arts:	Skit
	Participants:	Five Students
	Setting:	Performed in a classroom.
	Info:	The Square is being picked on. Will he find justice in court?

Description

In this skit students explore the notion that a square is a special case of rectangle, parallelogram, and rhombus. The characters are in a court of law, mediated by a judge. The rectangle (Roy) and the parallelogram (Pat) are arguing that the square (Steven Square) is not like them. On the defence side, Steven Square (the victim) is explaining how he is not being treated fairly and the rhombus (Ryan) is defending Steven Square. Following, I present a transcription of the DMP:

Figure 4.58: Steven Square

Figure 4.59: Roy (The Rectangle)

Judge: *The case we've today is that Steven Square is not being treated fairly because of his body shape. Steven you may begin the first side of the story.*

Steven Square: *Well… I was on work one day and out of the blue a man poked me in the eye, as you can see. It hurt and he said he did because I was a square. I don't find this very well.* (See Figure 4.58).

Judge: *Thank you for sharing your side of the story. Leroy, you may speak. I would like to hear your angle.*

Roy: *The reason I was treating him unfairly is because you're bragging that you like us when we know.* (See Figure 4.59).

Judge: *Stand up tall Leroy. Thank you for sharing your angle of the story. We now will hear from your partner.*

Pat: *I have to say that Roy and I are different because I have four vertices and Roy has four straight sides. See? You are completely different.* (See Figure 4.60).

Judge: *Thank you. I would like to hear the final corner of the case. Ryan, go ahead.*

Ryan: *I have to say that past statements aren't that true because Steven has four vertices and four straight sides and doesn't that make him a rectangle and a parallelogram? Even a rhombus is true. I rest my case.* (See Figure 4.61).

Judge: *The conclusion is it: Steven Square is to be treated fairly because he has been proved to be like Ryan, Roy and Pat. Case closed.* (See Figure 4.62).

Figure 4.60: Pat (The Parallelogram)

Figure 4.61: Ryan (The Rhombus)

Figure 4.62: The Judge

Voyeur - New/Wonderful/Surprising

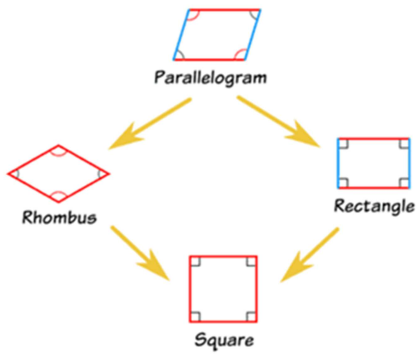
This DMP presents a conceptual mathematical surprise by exploring representations and proprieties of quadrilaterals and the connection between them. Like in DMP#5 *Little Quad's Quest*, students discuss how a quadrilateral may be categorized as a special case of others. Although students do not bring a trapezoid or a kite into the plot of the story, they explore the notion that a square is a special case of rectangle, parallelogram, and rhombus. This idea offers to the audience ways of seeing the new and wonderful in mathematics because it disrupts possible misconceptions such as "squares are not rectangles, parallelograms, rhombi." Surprisingly, the DMP highlights that the same mathematical/geometric object (a square) may fit in multiple categories. In contrast, students do not explore how representations of quadrilaterals may fit in the world around them as explored in DMP #5 *Little Quad's Quest.*

Voyeur - Sense-Making

The mathematics explored in this DMP is based on logical reasoning, regarding properties of quadrilaterals and the similarities and relationships between them. The audience may realize that students are exploring the notion that a square is a special case of rectangle, parallelogram, and rhombus (See Figure 4.63), but to understand this notion the audience has to connect ideas and representations throughout the story. Considering his name, it is immediate to realize that Steven Square is a character who play a role of a square, but it is only possible to understand that Roy is a rectangle not because he says it or let one knows his properties, but because he is dressed in a "costume" that represents a rectangle (that is, there is a representation of a rectangle made with concrete material pasted on the student's t-shirt). The audience has to develop the same "sense of visual recognition" (in combination with other modes) in the plot of the story to realize that Pat is the parallelogram and Ryan is the rhombus. The use of "math costumes" is important to the audience's understanding and it highlights that the use of materials (e.g., manipulatives, visual representations) are significant in order for students and the audience to make connections, make sense of the story and of mathematics, and produce meanings and knowledge within dramatic events involving the exploration of properties and relationships between quadrilaterals.

The students provided an interesting conclusion in the performance, mentioning information about similarities between the properties of the quadrilaterals, which made it clear that a square is a special case of other quadrilaterals. Ryan argues "*Steven has four vertices and four straight sides and doesn't that make him a rectangle and a parallelogram? Even a rhombus is true. I rest my case.*" and the Judge concludes that: "*Steven Square is to be treated fairly because he has been proved to be like Ryan, Roy and Pat.*"

Figure 4.63: Square as a special case of rhombus, rectangle and parallelogram)



However, a point should be mentioned about the nature of the students' arguments. "*Four vertices and four straight sides*" are the properties of quadrilaterals in general. Thus, on the one hand, the argument is sufficient and consistent if students are exploring the notion that square, rectangle, rhombus, and parallelogram are all quadrilaterals. On the other hand, the argument is superficial if the audience considers that students are exploring the notion that a square is a special case of rectangle, parallelogram, and rhombus.

Vicarious Emotion

This DMP explores "mathematical emotions" because the emotions which the characters .feel in the story are related to mathematical ideas. The audience may vicariously feel that the rectangle's and the parallelogram's bias toward the square are based on their "misconceptions" about the square's properties. Pat and Roy are "mistreating" Steven Square because they see the square as "different/inferior," they do not see in the square the same properties they have. Thus

Steven Square feels poorly due to this mistreatment. All of the characters are emotionally involved with the situation in the court because "*Steven Square is not being treated fairly because of his body shape.*" Considering the fact that the DMP explores the notion that a square is a specific type of rectangle, parallelogram, and rhombus, the Judge concludes that "*Steven Square is to be treated fairly*." The story then explores emotions toward a "sense of equity" based on mathematical ideas, which may be vicariously felt by the audience.

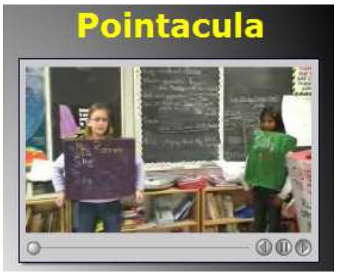
The fact that students are playing roles as shapes may enhance the audience's vicarious pleasures. Similar to DMP #2 *Geometric Idol*, DMP #5 *Little Quad's Quest*, DMP #7 *Math Facts Show*, DMP #8 *2D Land*, DMP #9 *We are the Polygons*, and DMP #11 *Sphere on the Loose*, students are playing the role of geometric objects. The playfulness emergent with the performance arts offers ways for students to "*think-and act-as-shapes*" and the emotions they have as actors are "mathematical emotions." The very notion of embodiment is significant in this scenario (Gerofsky, 2010), because the students' actions (including gestures) reveal the ways they are experiencing geometry, thinking about quadrilaterals, and reflecting about geometric forms to communicate mathematics to an audience through the act of "*being* a quadrilateral." Phrases such as "*I have to say that Roy and I are different because I have four vertices and Roy has four straight sides*" highlight students' sense of embodiment based on playfulness.

Visceral Sensations

Like in DMP #5 *Little Quad's Quest*, this DMP offers a sense of *mathematical fit*, regarding the connections between the properties of quadrilaterals. The properties of a square fit with the properties of a rectangle, a parallelogram, and a rhombus, which makes square a specific case of these other quadrilaterals. In this DMP, the audience may sense, viscerally, multiple connections between the properties of a square, a rectangle, a parallelogram, and a rhombus.

Students could have provided a stronger sense of mathematical fit by exploring, for instance, how representations of quadrilaterals would fit in the world around them. DMP #5 *Little Quad's Quest*, for instance, relates the representation of a rectangle to a wide TV screen and a rhombus to a diamond.

DMP #14: Pointacula

Table 4.20: Pointacula	**URL:**	http://www.edu.uwo.ca/mathscene/geometry/geo2.html
	**Strands and Content:**	Strand: Geometry and Spatial Sense Content: Dimensions and properties of solids
	Format:	Video
	Time length:	3:49.
	The Arts:	Musical
	Participants:	Four students and a guitar player
	Setting:	Performed in a classroom.
	Info:	Pointacula is back! Silly Cylinder and Rudy the Rectangular Prism are scared! Will Cubie the Cube save the day?

Description

In this DMP students explore properties of some three-dimensional shapes (rectangular prism, cube, and cylinder). In the story, Pointacula is the character who terrorizes the shapes who live in 3D town. Following, I present a transcription of the DMP in three scenes:

Scene One

Rooney: *Hi everyone! My name is Rudy the Rectangular Prism. Me and my friends here are going to show you the musical called 3D Town.* (See Figure 4.64).

Figure 4.64: Rooney

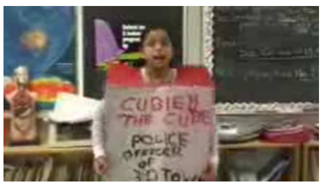
Reporter: *Breaking news! We just have got the report that Pointacula has returned. Will are here to take care of us. Please, save us Cubie!* (See Figure 4.65).

Figure 4.65: Reporter

[Someone starts to play the guitar]

Cubie *[singing]: I am Cubie. I like to dance and sing. I'm strong sitting down. I go blank blank. I have six faces. Eight vertices. Up to the rescue. How about you?* (See Figure 4.66).

Figure 4.66: Cubie



Rooney *[singing]: I am Rudy. The Rectangular Prism. And I love to sing, sing, sing, sing. I sing in the bathroom, when I wash my faces. And when I wash my vertices, in very wet faces.* (See Figure 4.67).

Silly *[singing]: I am Silly Cylinder. I live at the 3D town. I have two faces. Then I can make cookies. Which will never make you frown.* (See Figure 4.68).

Pointacula: *Stop! Why is there happiness when there should be evil?[Singing] I am Pointacula. Point me in brackets. I am evil and I have lots of enemies. I hate beauty music at all. So instead of talking to you my point I will go terrorizing them all.* (See Figure 4.69).

Silly: *Umm… Pointy… the mall is that way.*

Rooney: *I don't care a Pointacula. Let's just all go to bed right now. Bye.*

Scene two

Reporter: *Breaking news! We have got the report that Pointacula has attacked. We don't know where. Let's talk to the victim who has been attacked by faint powder. So, how do you feel after been attacked by Pointacula?*

Victim (Silly): *I don't like talking about it, but I feel very sad and I get a lot of nightmares about Pointacula. I feel scared just thinking about it.* (See Figure 4.70).

Reporter: *That is ok. We just want to know what Pointacula has stolen from you.*

Victim (Silly): *Well, I haven't checked yet. Let me see if I have my most valuable item. Oh no! Pointacula took it! He took my diamond! It has six vertices, eight edges, and eight faces.*

Reporter: *That is ok. You can have a rest. [Going close to Rudy]. So, what did you exactly see there?*

Rooney: *Well, as the witness, I didn't see anything and all what I hear was Silly screaming and shouting. I don't wanna talk about it.* (See Figure 4.71).

Silly: *I was not screaming…*

Reporter: *That is enough both of you. Let us just go downtown now.*

Figure 4.67: Rooney

Figure 4.68: Silly

Figure 4.69: Pointacula

Scene three

Police Officer: *I will arrest you evil Pointacula.*

Pointacula: *Oh, no. You won't.*

Police Officer: *Oh, yes. I will. I will shoot with glue.*

Pointacula: *Well I'm goanna dodge it.*

Police Officer: *Shoot, shoot.*

Pointacula: *Oh no! I've been hit. How can I be hit? Now I am stuck in the wall.*

Police Officer *[handcuffs Pointacula]* (See Figure 4.72).

Pointacula: *I will be back 3D town! You just watch…*

Police Officer: *This town is now safe from the evil Pointacula or is it?*

All Characters: *Thank you for listening to the Pointacula presentation [Applause]*

Figure 4.70: Reporter and Silly

Figure 4.71: Reporter and Rooney

Figure 4.72: Police Officer handcuffs Pointacula

Voyeur - New/Wonderful/Surprising

This DMP offers mathematical surprise because it explores specific properties of solids. It is surprising that elementary school students (grades 4 to 6) are exploring some specific properties of solids such as number of faces and vertices of polyhedrons. For instance, they mention that a cube has "*six faces and eight vertices*" and a diamond (an octahedron) "*has six vertices, eight edges, and eight faces.*" Furthermore, the connections between representation of solids and everyday objects are interesting and may offer surprises to the audience as well. Students related, for instance, a cube to a "strong" object and a cylinder to an object proper to make cookies. Connections between mathematical or geometric representations and everyday objects usually involve interesting processes of visualization/imagination and offer ways of seeing the new and wonderful in mathematics.

Another surprise in this DMP may refer to the reasons why Pointacula is a scary character. Pointacula is "scary" because he is "pointy." A "pointy shape" refers to a shape with parts that stick out or sharp. In DMP #18 *Mr. square*, for instance, this notion is explored through the idea that an isosceles triangle (a pointy shape) hurt Mr. Square. Thus, Pointacula is scary in this DMP because parts of its shape (its vertices) would hurt the other shapes in 3D Town (see Figure 4.73).

Voyeur - Sense-Making

In order to present strong arguments about the mathematical surprises, students could have explored more specific properties of each polyhedron such as numbers of faces, vertices, and edges. They could have also explored the difference between polyhedrons, which are solids with plane faces (e.g. cube and tetrahedron) and solids of revolution, which are solids formed by rotating a plane figure about a line in the plane of the figure (e.g. cone, cylinder, and sphere).

Figure 4.73: Pointacula (a "pointy shape")

The notion of "Pointacula as a scary character" is not totally explicit in the story. As an interpreter, I just inferred the reasons why Pointacula is scary after analyzing DMP#18 *Mr. Square*, that makes clear the idea that an isosceles triangle is a "pointy shape" that may hurt other shapes in a play. Thus, in this DMP, students could have made clear the reasons why Pointacula is a scary character in the story. They could have included in their dialogues clear statements toward the fact that Pointacula is a "pointy shape" that would easily stick out the other shapes and that is why he terrorizes 3D Town. However, there is an element in the story that suggests that Pointacula is a pointy shape. The character's hat is pointy, which makes Pointacula look like a cone or a "pencil", for instance (see Figure 4.73). Generally, the use of costumes is very significant to the audience to understand the properties of each character/shape. The visual design provided by the use of costumes is fundamental to offer the audience visual representations that can be related to what students are saying and gesturing. That is, the DMP offers ways to the audience connect multiple representations and designs of meanings in making sense of the mathematical ideas explored about geometric dimensions and solids. The gestural design is also important to the audience's meaning production and the playfulness emergent with the use of the performance arts is rich in terms of embodiment. In scene one, for instance, when Silly Cylinder sings "*I have two faces,*" the students makes a body movement with the costume in order to show the faces to the audience (see Figure 4.74). This embodied action is fundamental to the audience to visualize and understand the properties of a cylinder.

Figure 4.74: Use of costumes - embodiment

Like in DMP #1 *Polly Gone*, students are very clever in exploring a pun on words. In DMP #1, students played with the expressions "Polygon" and "Polly (is) gone." In this DMP, in the end of scene one, Pointacula says "*I will go terrorizing **them all***" and, to escape from Pointacula, Silly the Cylinder says "***the mall** is that way.*" Although this pun is not mathematical (as Polly gone is), it is significant to the plot of story because it refers to the way Silly mislead Pointacula in order to escape in the moment of his attack.

Vicarious Emotions

Like in the DMP#2 *Geometric Idol*, DMP #3 *Shape Songs*, DMP #7 *Math Facts Show*, DMP #8 *2D Land* , DMP #9 *We are the Polygons*, DMP #11 *Sphere on the Lose*, and DMP #13 *Square Trial*, students are playing roles as geometric objects. The story involves emotions that refer to what shapes are feeling. As mentioned before, the playfulness emergent with the performances arts offer ways to students play roles as shapes, embodying their actions through a mathematical activity: thinking-as-shapes, acting-as-shapes, learning, representing, and communicating ideas about shapes through the process of *being* shapes. The DMP explores thus some *mathematically related emotions*, because the emotions the characters feel are related to their properties as mathematical objects in the plot of the story. For instance, Pointacula is "bad" because he is pointy and Silly Cylinder feels *"very sad and I get a lot of nightmares about Pointacula… [and feels] scared just thinking about it.*"

The DMP uses zooming-in on the character's facial expression which may intensify the vicarious pleasure. Thus, by feeling what the actors are feeling in this DMP, the audience may feel what the solids are feeling, that is, through the playfulness, the audience may imagine itself as formed by edges, vertices, and faces, for instance, and feel "fear" by Pointacula's attacks.

Visceral Sensations

This story explores visceral mathematical sensations through suspense and action connected to mathematical ideas. It portrays, for instance (a) how scared the solids feel regarding Pointacula's attack and (b) action when the police officer captures Pointacula. These sensations are intensified by the use of soundtrack in key-moments of visceral scenes.

The DMP explores the notion that representations of solids fit in the world, like a cylinder as the representation of an object proper to make cookies and an octahedron as a diamond. However, these connections would be further explored. Students do not mention, for instance, how representations of cubes or rectangular prisms would fit in the world around them. Students could explore multiple ideas based on these solids such as: (a) objects such as ice cubes and boxes having these formats; (b) cube a special case of rectangular prism (c) the notion of dimensions (2D and 3D) by forming rectangular prisms from rectangles. All these ideas, which offer multiple ways to see connections between representations and ideas, would offer to the audience both more intense visceral sensations and surprises.

DMP #15: Equivalent Fractions

Table 4.21: Equivalent Fractions 	**URL:**	http://www.edu.uwo.ca/mathscene/mathfest/mathfest105.html
	Strands and Content:	Strands: Number Sense and Numeration; Geometry and Spatial Sense Content: Visual representation of equivalent Fractions
	Format:	Video.
	Time length:	0:37.
	The Arts:	Stop-Motion Animation
	Participants:	Four Students
	Setting:	*None*
	Info:	*None*

Description

Like in the DMP #10 *Fabulous Fraction* and DMP #22 *Fractiontastic*, this DMP explores numeric and visual representations of equivalent fractions. It explores, visually, the notion that 1 = 2x (1/2) = 4x(1/4) = 8x(1/8) = 16x(1/16).

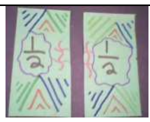
Voyeur - New/Wonderful/Surprising

This DMP offers conceptual mathematical surprises because it explores connections between multiple representations and strands. The DMP explores an articulation between numeric and geometric representation of factions to investigate one case of equivalence of fractions. It involves numeration and geometry by exploring representations of squares as formed by equivalent fractional parts, that is, 1 = 2x (1/2) = 4x(1/4) = 8x(1/8) = 16x(1/16). The connection between numeric and geometric representations of fractions is a "big curricula issue" (NCTM, 2000; Ontario Ministry of Education, 2005). The conceptuality of this idea in combination of the animation format offers ways to the audience see the new and wonderful in mathematics.

Voyeur - Sense-Making

This DMP explores connections between numeric and geometric representations of fractions. Considering the unit as the whole square (number 1 referring to the area), the DMP presents five levels or stages of the division 1/[2^(n – 1)]. There is a connection between the number of pieces and the size of each piece (1/2, 1/4, 1/8, and 1/16 of the introduced value of the area). Thus, this DMP is exploring the following equivalence of fractions: 1 = 1x1 = 2x1/2 = 4x1/4 = 8x1/8 = 16x1/16. Therefore, the audience may think about the following connections in this DMP:

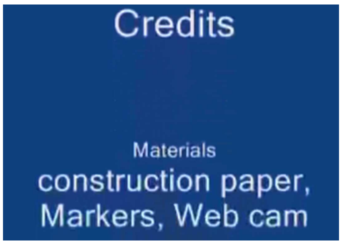
Table 4.22: Visual Equivalence of Fractions in DMP #15

Stage	Fraction: 1/[2^(n – 1)]	Visual	Equivalent fractions. "Number of pieces x size of the piece = Unit."
1	1 = 1/(2^0)		1 x 1 = 1
2	1/2 = 1/(2^1)		2 x (1/2) = 1
3	1/4 = 1/(2^2)		4 x (1/4) = 1
4	1/8 = 1/(2^3)		8 x (1/8) = 1
5	1/16 = 1/(2^4)		16 x (1/16) = 1

In comparison to the DMP #10 *Fabulous Fraction,* this DMP presents a better visual design. The process of displaying the division is clear because it demonstrate visually the proportion and the number of the pieces of the square, which offers ways to the audience to make sense about the equivalence of the fractions. This process is also clear in the DMP #22 *Fractiontastic*, in which students overlap the geometric/numeric representations.

This DMP can be understood as a "visual proof" for one case of equivalence of fractions. Hanna (2000) posits that "in the classroom the key role of proof is the promotion of mathematical understanding" (p. 6) and some of the functions of proofs are: verification, explanation, discovery, communication, construction, and exploration.

Figure 4.75: Credits to Materials



Another important aspect in this DMP refers to the use of materials. Both manipulative materials and digital technology are very important authors in the production of this DMP and they shape the nature of the mathematical meaning and knowledge production. Interestingly, in the credits of the DMP, students even acknowledge the use of paper, markers, and web cam (see Figure 4.75). To produce the animation of this DMP, students used the stop-motion animation. First, they used manipulative materials to create geometric representations (isometric piece of papers with numeric representations of fractions in each of them) and they took many pictures of these representations in different positions with the web cam. Consequently, they used software to produce a video putting the sequence of pictures in such way it would offer to the audience a perception of motion that makes sense about the equivalence of the fractions. All these aspects reveal how important technology can be in producing a DMP and how the use of technology shapes the nature of mathematical meaning and knowledge production. To produce this DMP students had to *think-with-technology*, that is, students had to reflect about specific ways to create representations of fractions and specific ways to display these representations in a way that would offer to the audience both (a) a visual effect of motion and (b) sense-making about equivalence of fractions. This performance highlights how *students-with-media* form thinking collectives in the process of producing a DMP.[22]

Figure 4.76: Aspects of Symmetry

Interestingly, some of the geometric representations of the fractions show aspects of *symmetry* considering the form students hatched/painted the representations with markers. In the presentations of 1/2, for instance, the symmetry is clear (but not perfect) in relation to a vertical axis. If we consider a process of rotation of the representations, some aspects of symetry can also be identified in the representations of 1/4 (See Figure 4.76 a-b). In the representations of 1/8 and 1/16 some smilarities can be identyfied in terms of the ways hatched the figures, but these similarities do not refer to symmetries. Although students were not exploring symmetry in the DMP, it is

[22] Scucuglia and Borba (2007) explain the process of editing a video to produce a DMP through the notion of humans-with-media (Borba & Villarreal, 2005).

interesting that they were concerned in providing some sense of aesthetic by hatched/painted the representations of fractions by exploring the notion of patterns somehow.

Vicarious Emotions

This DMP does not explore emotions through dialogues and gestures like the skit performances do, for instance. As said in DMP #10 *Fabulous Fraction*, in this DMP the audience does not see human characters neither representations of humans such as animated puppets. Thus, the audience may have difficulty to feel what the “actors” are feeling in this DMP, which is the fundamental characteristic of the vicarious eye (Boorstin, 1990). However, the audience may interpret the representations of geometric objects in motion as a “character in action.” Some emotions can also be conditioned by the background music, but this aspect is more associated to visceral sensations.

Visceral Sensations

Like in DMP #10 *Fabulous Fraction*, this DMP presents three aspects that offer visceral sensations to the audience. (1) The use of soundtracks may offer visceral sensations to the audience; (2) The motion that represents the division of the geometric object (visually) and its connection to the numeric representation happen very fast in the DMP. Furthermore, stop-motion animation format also enhances the sense that things are happening fast. Quick changes, like the one presented in this DMP, are a characteristic of the visceral eye (Osafo, 2010); (3) Most importantly, the notion of “equivalent fraction” explored through visual representation in this DMP provides an intense sense of *mathematical fit* (Sinclair, 2004). Thus, it is visceral to visualize that the numeric/geometric representations of 1, 2x1/2, 4x1/4, 8x1/8, and 16x1/16 are, in motion, fitting one to another.

DMP #16: Shape Idol

Table 4.23: Shape Idol

URL:	http://www.edu.uwo.ca/mathscene/geometry/geo4.html
Strands and Content:	Strand: Geometry and Spatial Sense Content: Properties of polygons
Format:	Video
Time length:	2:56.
The Arts:	Musical
Participants:	Four students and one guitar player
Setting:	Performed in a classroom
Info:	Which shape will win the contest?

Description

This DMP is similar to DMP #2 *Geometric Idol*, in which students explore properties of geometric shapes through a skit that makes a parody of the TV show American Idol. In the first song students explore how a square is related to other shapes. In the second song, students explore properties and types of triangles. In the third song, students mention that shapes can be explored together. Finally, students explore the notion that there are families of shapes and they mention that representations of shapes are in the world around them. Following, I present a transcription of the DMP:

Figure 4.77: Presenters

Student 1 and Student 2 (as presenters): *Welcome to Shape Idol!* (See Figure 4.77).
Student 1 (as a judge): *Our first contestant is Megan M. Please, get on the stage.*
Student 2 (as a judge): *We are gonna ask you some questions. The first one is: where are you from?*
Megan M: *I am from Four Vertices Island.*
Student 2 (as a judge): *What are you gonna do?* (See Figure 4.78).
Megan M: *I am gonna sing my rap about squares… Gimme a beat!*
[Participants start a beat]
Megan M *[singing]: Hey hexagon, I am straight up playa. The arms go up to me, isn't that better? Things I hate, pentagons aren't nice. Four vertices like way up tight.* (See Figure 4.79).
[Applause]
Student 1 (as a judge): *Congratulations! You are one of the finalists. Here is your square.*
Megan M: *Thank you! Thank you!*
Student 2 (as a judge): *Our second contestant is Madame N.*

Figure 4.78: Judges

Figure 4.79: Megan M

Madame N: *I am going to sing a song I made about triangles because they are so awesome.*
Student 2 (as a judge): *Ok.*
[Someone starts to play the guitar]
Madame N *[singing]: Triangle are a lot of what would you talk about. I don't care what you say. I'm goanna shout it out. Triangles are strongest shapes, they have three sides. Doesn't matter, just have it or just have fun. There are three types of triangles. Here are their names: isosceles, scalene, equilateral. They are not the same. Sing with the people. Triangle are strongest shapes, they have three sides. Doesn't matter, just have it or just have fun.* (See Figure 4.80).
[Applause]
Student 2 (as a judge): *Here is your triangular [certificate?]*
Madame N: *Are you serious? Oh my gosh!*

Figure 4.80: Madame N

Figure 4.81: Student 1 and Student 2

Student 2 (as a singer): *We are going to show you how to really sing…*
[Someone starts to play the guitar]
Student 2 *[singing]: They are coming over. They are all together. They are a family of shapes and they have been that way forever.*
Student 1 *[singing]: They are coming over. They are all together. We'll always stick together, and that's never gonna change.*
Student 2 and Student 1: *Yeah!* (See Figure 4.81).
Judge 1: *Since we are all so good, why don't we sing together?*
All students *[singing]: I say s-h-a-p-e-s… I say s-h-a-p-e-s … That is all around us!*
(See Figure 4.82).
All students: *And that was Shape Idol!*
[Applause]

Figure 4.82: All Students

Voyeur - New/Wonderful/Surprising

This DMP offers conceptual mathematical surprises because it explores (a) connections between representations of geometric shapes and (b) connection between representations of shapes and everyday objects.

In the first song, the students contrast some properties of squares, pentagons, and hexagons through the lyrics "*Hey hexagon, I am straight up playa. The arms go up to me, isn't that better? Things I hate, pentagons aren't nice. Four vertices like way up tight.*" In typical explorations, students see shapes in isolation. In this song, students are exploring some connections between three different types of polygons.

In song two, the triangle is portrayed as a "strong" shape. This is an interesting and non-typical aspect of seeing triangles in elementary school. The "strength" in this case refers to the fact that in order to deform a triangle, one has to change its angles and, to change the angle the lengths of its sides has to be changed necessarily. That is, in order to deform a triangle, one needs to change the lengths of its sides. "Triangles are the strongest shape because they have fixed angles. The angles do not collapse like the angles of a square do. Triangles are used to make a truss - the strongest architectural support." (http://answers.askkids.com/Weird_Science/why_is_the_triangle_the_strongest_shape).

In the third song, students mention the fact that shapes can be explored in terms of their relationships, not in isolation. They sing: "*They are coming over. They are all together. They are a family of shapes and they have been that way forever… We'll always stick together, and that's never gonna change.*" In the last song, students mention that representations of shapes may be found in the world about them. They sing: *"I say s-h-a-p-e-s…That is all around us!"*

Potentially, the ideas and connections explored in the four songs of this DMP offer to the audience ways of seeing the new and wonder in mathematics because mathematical objects are not seen individually or isolated, they are explored in relation to other objects and in relation to the world.

Voyeur - Sense-Making

In this DMP students do not present arguments to support the ideas they are exploring. In the first song, the hip-hop style makes it difficult to understand the words the student is singing. Probably, the audience can only understand exactly what the student is singing after listening to the song several times.[23] Although students are seeking to explore connections between a square, a pentagon, and a hexagon, there is no explanation about how the connections are made. In the skit, the student is introduced, says the she will sing a song about squares and she sings: "*Hey hexagon, I am straight up playa. The arms go up to me, isn't that better? Things I hate, pentagons aren't nice. Four vertices like way up tight.*" Like in DMP #2 *Geometrical Idol* and DMP #3 *Shape Songs*, students do not present justifications and explanations about the insights posed in the lyrics. Students could have explored similarities and differences between the shapes. They could have, for instance, explored the fact that squares, pentagons, and hexagons are all polygons and they can be seen as formed by triangles. In contrast, they have different number of

[23] From a voyeuristic point of view (Boorstin, 1990), it would be interesting if DMPs based on songs displayed the lyrics in the virtual platform of the Festival. This actually happens in most of the DMPs after 2009.

sides and different number of angles. Thus, they could have explored the question: are the angles of these shapes congruent?

In the second song, students explore the fact that there are three types of triangles (equilateral, isosceles, and scalene) and they are the "*strongest shapes*." Like in the DMP #8 *2D Land*, students are exploring thus the notion of representation of triangles as everyday objects, specifically regarding notions in architecture, that is, representations of triangles as a shape used to construct roof structures and the structural rigidity of the shape. Students could have provided more explanation about why triangles are strong shapes. They could have compared triangles to shapes such as parallelograms and justified that triangles are strong because to change their angles it is necessary to change the length of its sides.

In the third song students suggest that shapes should not been seen separately or in isolation. They sing: "*[there] are a family of shapes.*" Students probably intend to explore the notion that a set of shapes can be classified in the same category (similar to DMP #5 *Little Quad's Quest*), or that one shape fits in other shapes (like the idea explored in the DMP #4 *Triangles*), or that one shape many be a special case of other (similar to the idea explored in the DMP #13 *Square Trial*). However, again, students do not present explanation or examples to justify or support what they intend to explore. They could be more explicit in the lyrics (presenting more information) or they could have used visual elements (e.g., posters) to support their ideas, as is done in DMP#17 *Square Based Pyramid*, for instance.

In the last song, students mention that representations of shapes may be found in the world about them. They sing: *"I say s-h-a-p-e-s … That is all around us!"* However, students do not offer examples of how representations of shape can be found in the world. DMP #2 *Geometrical Idol*, for instance, mentions that bee hives can be seen as representations of hexagons. In DMP #5 *Little Quad's Quest*, students relate, for instance, a representation of a rectangle to a wide TV screen and a representation of a rhombus to a diamond.

The use of gestures is also important in some moments of the DMP. In the introduction, for instance, the presenters/judges say "*Welcome to Shape Idol!*" and they make a synchronized and rotated body movement that offers to the audience a sense of symmetry (see Figure 4.83 a-b).

Figure 4.83 a-b: Students' gestures involving symmetry.

Vicarious Emotion

Differently from DMP #2 *Geometric Idol*, DMP #3 *Shape Songs*, DMP #7 *Math Facts Show*, DMP #9 *We are the Polygons*, DMP #11 *Sphere on the Lose*, DMP #13 *Square Trial*, and DMP #14 *Pointacula*, students are not taking on roles as geometric shapes in this DMP. They are playing roles as singers who sing about shapes. However, some mathematical emotions are explored when students (a) portray the triangles as strong shapes (song two) and (b) explore a sense of *belonging* by saying "*they are all together. They are a family of shapes and they have been that way forever.*"

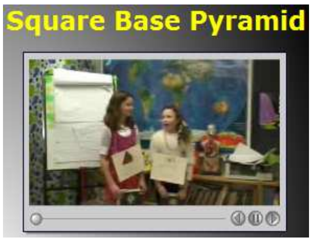
The DMP also uses close-ups on the actors' facial expressions. The audience may vicariously feel what the actors are feeling, but some of these feelings are not necessarily mathematical emotions, that is, is this case, emotions related to the mathematical properties of the shapes. Some of the emotions in this skit, like in DMP #2 *Geometric Idol*, refer to a "sense of achievement" by having performed well the song in the conquest. Although it is not a mathematical emotion, it is an emotion that may be significant to the mathematical activity, since *achievement* and/or recognition may be an aspect that may encourage students on getting engaged in future and further mathematical experiences.

Visceral Sensation

This DMP may offer some visceral sensations to the audience because it explores how different shapes *fit* each other and that shapes *fit* in the world around the students. However, as mentioned in the section on *Voyeur - Sense-Making* of this case, there is no explanation to support these connections. The DMP is also based on soundtracks, which is a fundamental characteristic of the visceral eye (Boorstin, 1990).

DMP #17: Square Base Pyramid

Table 4.24: Square Base Pyramid



URL:	http://www.edu.uwo.ca/mathscene/geometry/geo13.html
Strands and Content:	Strand: Geometry and Spatial Sense Content: Properties of a square base pyramid
Format:	Video.
Time length:	2:30.
The Arts:	Musical: Skit and Songs
Participants:	Three students and a guitar player
Setting:	Performed in a classroom
Info:	*None*

Description

Students play a skit to explore properties of a square base pyramid such as number of vertices, faces, and edges. Two students play roles as archaeologist exploring a pyramid and one student plays the role of Cleopatra's mummy, who explains the properties of a square base pyramid. Following, I present a transcription of the DMP:

Figure 4.84: Myrtle and Student 2

Myrtle: *Hey! Look that pyramid. Let's go see what is inside.*
Student 2: *Ok…Myrtle!* (See Figure 4.84).
Myrtle: *What?*
Student 2: *It is so scary! I'm shaking in my boots.*
Myrtle: *It is ok. Hey! Look that toilet paper… let's see how much there is.*
Student 2: *Cool!*
[A character appears wrapped in toilet paper and they take it off]
Cleopatra *[spinning]: Wow! Wow! Wow! Wow!* (See Figure 4.85).

Figure 4.85: Cleopatra spinning

Myrtle: *It is alive!*
Cleopatra: *It is Cleopatra!*
Student 2: *Where have you been all my life?*
Cleopatra: *Dead!*
Student 2: *So you had feelings for me.*
Cleopatra: *I am way out of your league. Anyway.*
[Someone plays the guitar]
Cleopatra *[singing]: It is a pyramid and it looks like this [showing a poster].* (See Figure 4.86).

Figure 4.86: Cleopatra shows a poster

Myrtle and Student 2 *[singing and dancing]: na-na-na, na-na, na-na-na, na-na-na-na.*
Cleopatra *[singing]: It has five faces and it has five vertices.*
Myrtle and Student 2 *[singing and dancing]: na-na-na, na-na, na-na-na, na-na-na-na.*
Cleopatra *[singing]: With eight edges, it is a square base pyramid.*

Myrtle and Student 2 *[singing]: na-na-na, na-na, na-na-na, na-na-na-na.*
Cleopatra: *That is a square base pyramid.*
Myrtle *What is a square base pyramid?*
Cleopatra: *This is a square base pyramid.*
Myrtle: *But we already knew that.*
Cleopatra: *You are asking to start at the beginning.*
Myrtle and Student 2: *We…*
Cleopatra: *Stop! Stop! Stop! A square base pyramid has five faces, five edges … sorry… five faces, five vertices, and eight edges. Also, some people believe that pyramids have curses or they have magical powers.* (See Figure 4.87 a-b).
Myrtle and Student 2: *Really?*
Cleopatra: *Yeah! Also, it has three triangular faces because triangles are the strongest shapes. And it has a square base, so it keeps it held up.*
Myrtle: *Why the triangular faces make it so strong?*
Cleopatra: *Because triangles are the strongest shapes.*
Myrtle: *So, a square base pyramid has five faces, five vertices, and eight edges?*
Cleopatra: *Yes!*
Myrtle: *And it is one of the strongest shapes because it has four triangular faces and a square base.*
Cleopatra: *Yes! Now you know about square base pyramid… My work here is done.*
Myrtle: *That was awesome!*
Student 2: *I know. Let's go back you our condo.*
[Applause]

Figure 4.87 a-b: Cleopatra's explanation

Voyeur - New/Wonderful/Surprising

This DMP explores a conceptual mathematical surprise because it explores in details some properties of a square base pyramid and connects representations considering the properties of this solid. The DMP explores the representation of a square base pyramid in a poster, conducting a dialogue that explains the properties of the pyramid, and make a connection between the representation of a pyramid as a mathematical object and a pyramid as an everyday object. Students explore that a square based pyramid (a) is formed by a square base and four triangular faces; (b) has five vertices, five faces, and eight edges and (c) is a strong shape. The conceptual nature of these ideas may potentially offer surprises to the audience.

Voyeur - Sense-Making

The use of a poster in the DMP is helpful to the audience to visualize and understand the properties of a square base pyramid. Furthermore, the song "*it has five faces and it has five vertices… with eight edges, it is a square base pyramid*" makes clear and helps the audience to remind what are properties the students are exploring. The dialogue that students conduct after singing the song is also significant to summarize clearly the ideas to the audience. Myrtle makes the point very clear at the end of the skit saying: "*so, a square base pyramid has five faces, five vertices, and eight edges … and it is one of the strongest shapes because it has four triangular faces and a square base.*"

Like in DMP #16 *Shape Idol*, students do not present explanation about why triangles are the strongest shapes. Students only mention that a square base pyramid is a strong shape because it is formed by triangles and "*triangles are the strongest shapes.*" Students could have explored the notion that triangles are strong because to change its angles it is necessary to change the length of it sides. They could compare a triangle to a parallelogram, for instance, and highlight the rigidity of triangles and the flexibility of parallelograms when one try to change the angles of these shapes.

Moreover, students could have explored Euler's formula in this DMP. A square base pyramid is a good example to explore the notion that the numbers of faces (F), vertices (V), and edges (E) of a polyhedron are determined by the formula $V + F - E = 2$. In the case of the square based pyramid, as highlighted in the DMP, one has $V = 5$, $F = 5$, and $E = 8$. Using Euler's formula, one has $V + F - E = 2 \Rightarrow 5 + 5 - 8 = 2 \Rightarrow 2 = 2$. Thus, a square base pyramid is an interesting example to explore Euler's formulas.

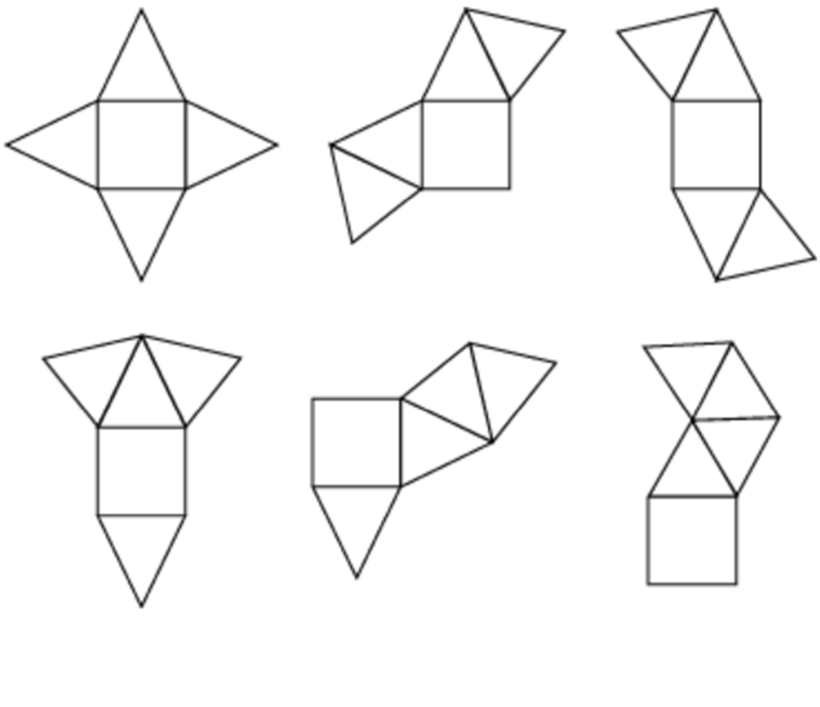
Vicarious Emotions

The story involves some emotions, but these are not mathematical emotions. At the beginning, the archaeologists are scared, for instance, when they first see the mummy of Cleopatra. Student 2 says: "*It is so scary! I'm shaking in my boots.*" At the end of the DMP, there is some emotion linked to the fact that the archeologists discovered or learned something new about the properties of a square base pyramid. Myrtle, for instance, says: "*That was awesome!*" However, along the story, the emotions the actors feel are not necessarily linked to their mathematical exploration and discovery. Differently from other DMP such as DMP #5 *Little Quad's Quest*, students do not play roles as geometric objects in this DMP. If they did, they could potentially vicariously provide to the audience some mathematical emotions by linking the playful role of a geometric object to the emotions this object could feel in a skit.

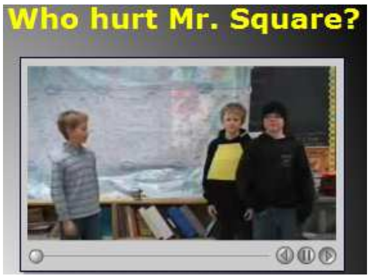
Visceral Sensations

As mentioned, the story involves some fear and suspense at the beginning, which are characteristics of the visceral eye (Boorstin, 1990). The story also uses some soundtrack, which may enhance the visceral sensation the audience may feel. Students explore some sense of mathematical fit considering the fact that they explore the representation of a square base pyramid as a mathematical/geometric object and a pyramid as an everyday object. That is, the representation of geometric pyramids fit in the world around the students as an Egyptian pyramid. However, students could have explored how four triangles and a square *fit* each one to another in forming a square base pyramid. They could have explored this idea by investigating several ways of construct the net of a square base pyramid (see Figure 4.88 a-b). The exploration of the nets would help students in the justification of why a square base pyramid of a strong shape because it would highlight how triangles fit one to another forming the pyramid.

Figure 4.88 a-b: Possible nets of a square based pyramid
(http://www.learner.org/courses/learningmath/geometry/session9/solutions_b.html)



DMP #18: Who Hurt Mr. Square?

Table 4.25: Who Hurt Mr. square? 	**URL:**	http://www.edu.uwo.ca/mathscene/geometry/geo9.html
	Strands and Content:	Strand: Geometry and Spatial Sense Content: Properties of quadrilaterals and other figures
	Format:	Video.
	Time length:	1:57.
	The Arts:	Musical: Skit and Songs
	Participants:	Three students and a guitar player
	Setting:	Performed in a Classroom
	Info:	*None*

Description

In this DMP a circle, a square, and a trapezoid decide to go to get some donuts. In their way, Mr. Square gets hurt and appears lying down on the floor. Mr. Circle and Mr. Trapezoid justify that they did not hurt Mr. Square. They figure out that Mr. Isosceles hurt Mr. Square because an isosceles triangle has an acute angle and there is an acute angle dent mark in Mr. Square. Finally, Mr. Isosceles is charged. Following, I present a transcription of the musical.

Figure 4.89: Mr. Circle, Mr. Trapezoid and Mr. Square

Figure 4.90: Mr. Square lying on the ground

Figure 4.91: Mr. Circle's performance

Mr. Circle: *Hey Mr. Square! Mr. Trapezoid and I, Mr. Circle, are going to get some glazed donuts. Do you wanna come?* (See Figure 4.89).

Mr. Square: *Sure! Just let me get my coat to cover all of my four sides.*

[Mr. Square appears lying on the ground.] (See Figure 4.90).

Mr. Trapezoid: *What is going on?*

Mr. Circle: *I think I heard a loud noise.*

Mr. Trapezoid: *Oh! Who hurt Mr. Square? I bet you it was you Mr Circle. You hurt his relative Mr. Rectangle. A square is a rectangle. I bet that it was you!*

Mr. Circle: *Circle won't tell. Let's go and dance to tell you I didn't do it. [Starts to sing]. I didn't do it, it wasn't me. Because I have no vertices. It wasn't me. It wasn't me. It wasn't me. [Stops singing]. I bet you hurt him…* (See Figure 4.91).

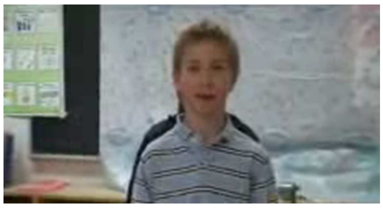
Mr. Trapezoid: *Why would I hurt him? He was like my best friend.*

Mr. Circle: *You have an acute angle, and there is an acute angle dent mark on him.*
Mr. Trapezoid *[singing]: I didn't do it. It wasn't me. I guess the life ain't the life for me. I am a 2D geometric shape. Why would I cover him up with [?] I already told you, it wasn't me. I guess the life ain't the life for me. It wasn't me.* (See Figure 4.92).
Mr. Circle: *So, if it wasn't me and it wasn't you, who could have done it?*
Mr. Isosceles: *Hey guys!*
Mr. Circle: *Hey Mr. Isosceles! I heard you hurt Mr. Square. You have an acute angle, and there is an acute angle dent mark on him. Oh! And you have three vertices. [?]*
Mr. Isosceles: *What are you talking about?*
Mr. Circle: *You hurt him!*
Mr. Isosceles: *What are you talking about?*
Mr. Circle: *You hurt him!*
Mr. Isosceles: *Fine!*
[Mr. Circle headlocks Mr. Isosceles] (See Figure 4.93).
Mr. Square: *Mr. Isosceles was charged with a hundred and twelve years in prison. Now I'm going to go and get my glazed donuts that I missed.* (See Figure 4.94).
Mr. Isosceles: *Real food stinks! I wish I could have glazed donuts.*

Figure 4.92: Mr. Trapezoid's performance

Figure 4.93: Mr. Circle headlocks Mr. Isosceles

Figure 4.94: Mr. Square



Voyeur - New/Wonderful/Surprising

This DMP offers conceptual mathematical surprises to the audience because it explores simultaneously the properties of different shapes. Typically, students explore the properties of shapes in isolation, that is, students do not investigate characteristics of shape considering a collection or a set of them. In this DMP, students explore properties of circles, squares, trapezoids, and isosceles triangles. When students explore shapes together, they engage themselves in a mathematical activity that offers ways to explore connections between representations and properties of different shapes. This type of exploration offers ways to the audience see the new and wonderful in mathematics.

At the beginning of the story, students mention that "*a square is a rectangle.*" As discussed in DMP #3 *Shape Songs*, DMP #5 *Little Quad's Quest*, and DMP #13 *Square Trial*, the notion that a square is a special case of rectangle has the potential to be surprising. It offers ways to see quadrilaterals differently, to see a quadrilateral as a special case of others and how the properties of quadrilaterals are connected.

The DMP also mentions that a circle has no vertices, a trapezoid has an acute angle and an isosceles triangle has an acute angle and three vertices. However, these ideas are typically explored in mathematics classrooms and may not offer conceptual mathematical surprises to the audience necessarily.

Voyeur - Sense-Making

Like most of the other DMP, this DMP does present arguments to justify or to support the ideas explored. The plot of the story is quite clever. Students create a skit in which a shape got hurt and they discover who hurt Mr. Square based on the mathematical properties of a triangle, who was responsible for hurting Mr. Square. However, there is no argument to support the notion that “a square is a rectangle” and there is no explanation why a circle has no vertices, a trapezoid has an acute angle and an isosceles triangle has an acute angle and three vertices. Actually, these ideas or properties should be properly explored in terms of definitions. The way students conduct their dialogues may be interpreted by the audience as “every trapezoid has one acute angle” and “an isosceles triangle has only one acute angle.” These statements are not totally true. Students should make clear the point that an isosceles triangle has at least two acute angles and there are trapezoids with no acute angles (e.g., a rectangle).

The student who is playing the role as Mr. Trapezoid is using a representation of a trapezoid made with paper pasted on his t-shirt, which works as a kind of “math costume.” As mentioned in DMP#7 *Math Facts Show*, for instance, the use of these materials, in specific “math costumes,” are important to the audience’s understanding. These materials offer ways to the audience to visualize and reflect about the properties students are exploring in the skit. However, only one student is using this “costume” in the skit. Students could have used these “math costumes” to dress up all the characters and provide to the audience some visual insights that may be significant to their mathematical thinking, reasoning, and understating.

In the story, Mr. Square felt hurt because he was attacked by Mr. Isosceles. The “proof” of this attack was that there was an acute angle dent mark on Mr. Square and Mr. Isosceles had an acute angle. However, the conditions for such “proof” may be considered as “not sufficient” by the audience. In the story, Mr. Trapezoid has also an acute angle. Thus, students do not present sufficient or deductive mathematical arguments or evidence to support the reasons why Mr. isosceles attacked Mr. Square, which is the main mathematical idea explored in the story.

Vicarious Emotions

Like in other DMP (e.g., DMP #5 *Little Quad's Quest* and DMP #7 *Math Facts Show*), students are playing roles as geometric objects. The playfulness emergent with the use of the performance arts offers ways for students to "think and act" like mathematical objects. The reasoning emergent through the mathematical activity involving embodiment offers ways for students to understand and communicate ideas about the properties of polygons. Considering these properties are explored within a skit, in the context of a story, the emotions the characters feel are mathematical emotions, that is, emotions related to mathematical ideas.

The DMP also uses close-ups on the characters' facial expressions, which usually enhance the vicarious pleasure the audience may feel.

Visceral Sensations

The story involves action and suspense. The fact that Mr. Square was hurt and found lying on the floor may be interpreted as a visceral scene by the audience. Moreover, the plot of the story is based on the investigation of who hurt Mr. Square, which evokes mystery and suspense. The scene on which Mr. Circle headlocks Mr. Isosceles is also visceral, because it involves action.

The DMP also explores some aspects involving senses of mathematical fit. When students mention that "*a square is a rectangle,*" they are exploring the notion that a square is a special case of a rectangle. This idea can be interpreted by the audience as the properties of a square fit on the properties of a rectangle. Most importantly, the investigation about who hurt Mr. Square in the story is figured out because the properties of the isosceles triangle (an acute angle) fit on the evidence found on Mr. Square (acute angle dent mark on him).

Like in DMP #5 *Little Quad's Quest* or DMP #8 *2D Land*, students could have explored how representations of polygons fit in the world around, seeking to provide other mathematical visceral sensations to the audience.

DMP #19: Are You Smarter than a 4th Grader?

Table 4.26: Are you smarter than a 4th grader? 	**URL:**	http://www.edu.uwo.ca/mathscene/mathfest/mathfest108.html
	Strands and Content:	Strand: Geometry and Spatial Sense Content: Properties of square
	Format:	Video
	Time length:	5:31.
	The Arts:	Skit
	Participants:	Six students
	Setting:	Performed in a classroom
	Info:	*None*

Description

Students play a skit in which they perform a parody of the TV show “are you smarter than a 5th grader?” In the DMP, students explore some properties of squares. The first candidate answers questions about area and perimeter of squares and she wins the price. The second candidate answers correctly his first question about definition of square. Before answering his second questions, his glasses fall down and get broken. Thus, he is unable to see the figure to answer the question and, thus, he does not get the prize. Following, I present a transcription of the skit:

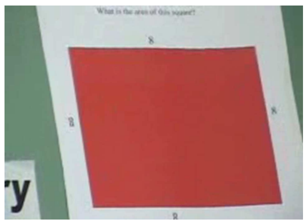
Figure 4.95: Presenter

Presenter: *Are you smarter than a fourth grader?* (See Figure 4.95). *[Applause]*
Presenter: *Yes, it me! Your host Jacob. Returning from last week. Yes, she is! Yes, she is famous! Give up for Rachel.*
Rachel: *Hello everyone! I am so happy to be back. I am ready to become a multimillionaire.* (See Figure 4.96).

Figure 4.96: Rachel

Presenter: *Ok Rachel. Last time, we left off with two questions left on the game board. And for three hundred and sixty thousand dollars, please, select you next question.*
Rachel: *I guess I have to say grade one Geometry.*
Presenter: *Ok. What is the area of this square?* (See Figure 4.97).
Rachel: *It is length times width, so, eight times eight is… sixty four?*
Presenter: *Let’s see what our Grade fours have answered… Edward?* (See Figure 4.98).
Edward: *The answer to the question … The answer to the question is sixty-four.*
Presenter: *Emma?*
Emma: *Yeah! It is sixty-four.*

Figure 4.97: Square – Question about the area



Presenter: *Rachel?*
Rachel (judge): *The answer is sixty-four!*
Presenter: *Guess what? They're all right!*
[Applause]
Presenter: *So, you have five hundred thousand dollars. Please, choose another question. Would you like to go to the one million dollar question?*
Rachel: *What do you think? I need more money… yeah!*
Presenter: *The one million dollar question is: what is the perimeter of this square?* (See Figure 4.99).
Rachel: *I guess I just have to add all the sides. So, it is sixty?*
Presenter: *Sixteen? Let's see what our Grade fours have answered … Edward?*
Edward: *I believe that the answer of this one million dollar question that Rachel has is sixteen.*
Presenter: *Emma?*
Emma: *The answer is sixty four.*
Presenter: *Rachel?*
Rachel: *The answer is fifty six!*
Presenter: *Only one of them is right today. And the right answer is: sixty… sixteen!*
Rachel: *Yes! This money is mine! I got lots of money, give the bills.*
Presenter: *Thank you for playing with us Rachel. You may collect you check in your way out. Now, give it up for our next contestant, all way from Quebec, let's give it up for Milken… Milken A-Cal.*
Milken: *Hello! I'm Milken A-Cal. I am smart and I know everything! I have a degree in biology, brology, patology, newology just to name a few.* (See Figure 4.100).
Presenter: *Thank you for your brief auto-biography. Please, select your first question.*
Milken: *I always was good in math. So, I choose grade four Geometry.*
Presenter: *The grade four Geometry question is: what is a correct definition of a square? (a) the sides of the square are all congruent and all the sides are parallel; (b) the sides of the square are sometimes congruent and all the sides are parallel; (c) the sides of the square are never congruent and sometimes they are parallel or (d) the square does not have any ninety degree angles and the sides are parallel.* (See Figure 4.101).
Milken: *Well, congruent means parallel and parallel means the sides go the same way, so, b and c are obviously wrong. So, could be (d) or (a) but…*
Presenter: *All our grade fours have locked in our answers.*
Milken: *The same message again. It could be (b) or could be (c). Almost likely go with (a)!*
Presenter: *Let's see what the grade fours have answered… Edward?*
Edward: *I believe the question for this answer is (a).*
Presenter: *Emma?*
Emma: *The answer is (a).*
Presenter: *Rachel?*
Rachel: *The answer is (a)!*
Presenter: *They are all correct! You have twenty-five thousand dollars!*

Figure 4.98: "Grade fours"

Figure 4.99: Square - Perimeter

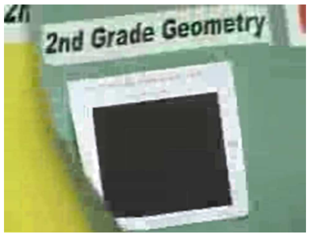
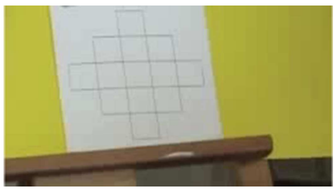


Figure 4.100: Milken A-Cal

Figure 4.101: Grade Four Geometry Question

Figure 4.102: Grade Four Geometry Question



[Applause]
Milken: *Ok! I am surely to win. I will choose third grade Geometry!*
Presenter: *How many different… How many different sides are… How many different sides… How many different… How many squares of different sides are in the following figure?* (See Figure 4.102).
Milken: *Let's see. Give me approximately 5.2 minutes. [Take his glasses off]. Oh no! My glasses are broken! I have to use a cheat.* (See Figure 4.103).
Presenter: *You don't need it.*
Milken: *Help me, please.*
Presenter: *Emma, tell him your answer.*
Emma: *I think the answer is one.*
Presenter: *Wrong! Beat it, Milken!*
Milken: *No! I'll have to win! Why did I fail? Why did my glasses break? Ahhh! Ow! My hand*
[Applause]
Teacher: *Ok! Hey guys, that was a lot better!*

Figure 4.103: Milken's glasses

Voyeur - New/Wonderful/Surprising

This DMP explore ideas as they are typically explored in mathematics classrooms. Conceptually, this DMP does not offer ways to see the new and wonderful in mathematics. What may surprise the audience, as most of the other DMP do, is that students are not representing their ideas traditional using print-based texts. By producing registers in videos (multimodal texts), students are communicating their ideas using multiple communicational elements of designs of meanings (visual elements, spaces, gestures, verbal language, the arts). It offers different layers of signs and may have an impact on meaning and knowledge production.

Voyeur - Sense-Making

Students' reasoning presents some incoherencies in this DMP. When the first candidate responds her second questions, she says "*sixty*" to answer the questions about the perimeter of the square that the length of its side is four. Initially, the presenter corrects her saying "sixteen." After asking to the students their answers, the presenter also gets confused to announce the right answer. The following part of the transcription highlights this confusing moment of the skit:

Rachel: *I guess I just have to add all the sides. So, it is* ***sixty****?*

Presenter: *Sixteen? Let's see what our Grade fours have answered … Edward?*

Edward: *I believe that the answer of this one million dollar question that Rachel has is sixteen.*

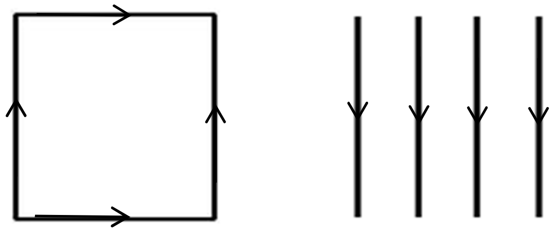
Emma: *The answer is sixty four.*

Rachel: *The answer is fifty six!*

Presenter: *Only one of them is right today. And the right answer is:* ***sixty… sixteen!***

The second incoherence in the skit refers to the way students explore the definition of square. As further discussed in DMP #5 *Little Quad's Quest*, the formal definition of square involve the relations between several quadrilaterals. A square is defined as "a rectangle with equal sides, or a rhombus with equal angles" (*The Nelson Canadian School Mathematics Dictionary,* 1995, p. 215). A square can also be defined as a quadrilateral with four equal sides and four equal angles. In the skit, students states that a correct definition of a square is "*the sides of the square are all congruent and all the sides are parallel*." It seems acceptable the notion that "all the sides are congruent" because it means they have the same size or they are equal in length. However, the notion that "all the sides are parallel" is questionable because it is ambiguous. On the one hand, each of the sides is parallel to its opposite side. In this sense, all sides are parallel, because each of them is parallel to the opposite side. That is, "each side is parallel to another." On the other hand, the statement "*all the sides are parallel*" may mean that that all four sides are parallel, which does not make sense at all because it does not form a quadrilateral (see Figure 4.104 a-b).

Figure 4.104 a-b: "Two pairs of parallel sides" vs. "All the sides are parallel"



The use of posters in the skit is important to the audience to understand the questions explored in the story and think/reflect about them. The use of figures are significant to students' and audience's mathematical understanding and reasoning along the story because the exploration they conduct in the skit is based on the visualization and geometric investigation based on figures. However, most of the posters are shown very fast in the video and the close-up is not well conducted. These technical issues involving filming or recording the performance may have a direct (and negative) impact on the audience's interpretation and visualization.

It is interesting that in the end of the skit the teacher says: "*Ok! Hey guys, that was a lot better!*" This comment reveals that students were practicing the skit more than once. The process of creating, practicing, and performing a skit involves *repetition*, which is very significant for learning (Deleuze, 1994). A main pedagogic focus in DMP is the process of students developing communication skills (Gadanidis, Hughes & Cordy, 2011). *Mathematical communication* is a process or skill pointed in several curricula (e.g., NCTM, 2000, Ontario Ministry of Education, 2005). The repetition of lines is very important in theater performance and learning (mathematics). During the filming of the performances teachers may insist to students to repeat their lines in order to have a take in which they communicate ideas clearly. Sometimes, students make mistakes in speech, even when they are reading what they should talk. The “materialization” of thoughts into words is one way of expressing the *self*. It is a process of identity construction and learning in which the student explains what we know and reorganize their thinking in this formative process. The notion of DMP becomes significant in that context. Typically, students represent their ideas through print-based texts, in which the *other* is the teacher. In some situations, students express ideas verbally in the classroom to their colleagues, but the modes of communication are not fixed. By creating a video record, students are representing their ideas using multiple modes of communication (linguistic, visual, aural, spatial, and gestural). DMPs, as multimodal texts, compile various levels of signs, which enhance the nature of production of meanings. Moreover, DMPs are published online, that is, the audience includes many *others*, not just the teacher and the students in the classroom. DMP helps students to communicate their mathematical ideas beyond the conventional settings of the classrooms, through the construction of their identities as “performance mathematicians” based on the use of digital technology and the performance arts. It is interesting to notice that this DMP and others such as DMP #9 *We are the Polygons* and DMP #17 *Square Base Pyramid* were video recorded in only one shot. That is, these DMP are formed by uninterrupted performative events. DMP such as DMP #1 *Polly Gone* or DMP #8 *2D Land* are formed by different scenes generated from cuts on the process of recording. In these cases, potentially, the students recorded the same scene several times (repeated their speeches and actions several times) and selected the best scenes to compose the final video. In these processes of repetition, students may have the chance to reflect on the ways they are communicating their ideas and figure out ways to improve their communication. That is a very interesting process of learning, in which students externalize their thought on the reality they are experiencing.

Vicarious Emotions

The DMP explores some emotions considering the fact that the characters are excited because they are participating in a TV show. The participants get excited when they win or frustrated when they fail. The audience may vicariously feel these emotions the actors feel, but these are not “mathematical emotions.” That is, these are not emotions directly related to a mathematical concept, like those the audience may feel in DMP #5 *Little Quad’s Quest*. The emotion in this DMP are like some of those offered in DMP #2 *Geometric Idol* and DMP #16 *Shape Idol*. These are emotions related to the actors’ excitements in achieving something (e.g., wining a conquest) which involves mathematics.

The DMP also presents close-ups on the actors’ facial expressions, which may enhance the visceral pleasure of the audience. However, these close-ups are not well conducted in the DMP and, sometimes, these close-ups make the processes of visualization and understanding very confusing to the audience.

Visceral Sensations

The parody of a TV show, which appeals to a popular culture, may offer some visceral sensations to the audience. The audience may feel its own sensations by getting really excited on the expectation generated by the competition in the show (will he/she answer the question correctly and get the price?). In this context, students could have used soundtracks to enhance the visceral sensations in the skit.

Generally, the DMP does not offer mathematical visceral sensations to the audience. There is actually some sense of fit involved with multiple choice questions as the one conducted in the skit (which alternative fit as a correct answer?). However, multiple choice questions are very typical in traditional mathematical experiences and they are not related to the sense of aesthetics assumed in this study considering the notion of mathematical fit (Sinclair, 2004). Thus, these types of questions may not offer visceral sensations to the audience.

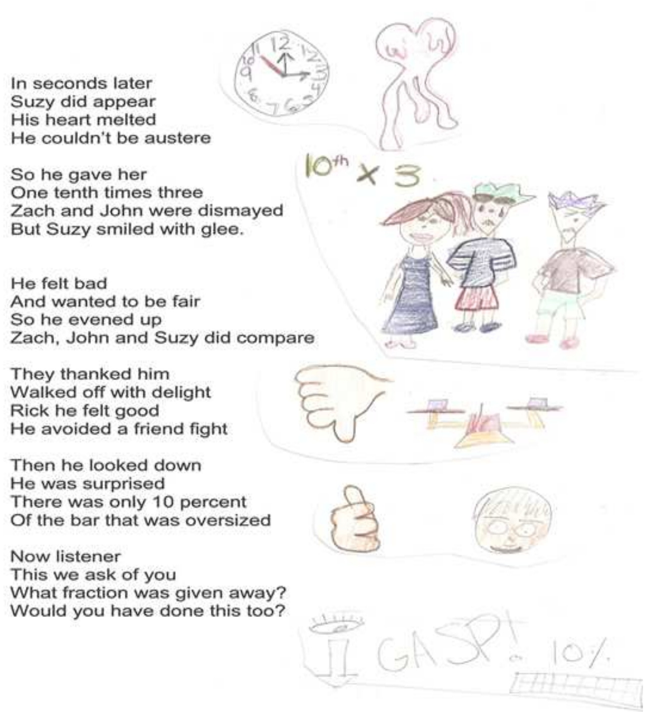
DMP #20: Ricky's Metre Chocolate Bar

Table 4.27: Rick's Metre Chocolate Bar

URL:	http://www.edu.uwo.ca/mathscene/mathfest/mathfest102.html
Strands and Content:	Strand: Geometry and Spatial Sense Content: Properties of square
Format:	Power Point
Time length:	*None*
The Arts:	Poem/Lyrics
Participants:	Rick, Rick's mom, John, Zach, Suzy
Setting:	*None.*
Info:	This poem can be sung to the tune of On Top of Old Smokey. It was written to reinforce our learning on decimals, fractions, percent and even stimulate discussions on sharing.

Description

The following images present the poem:



Voyeur - New/Wonderful/Surprising

This DMP is surprising because it connects different strands and challenges the audience to think about a posed problem. The DMP explore mostly *Numeration* and *Measurement* by investigating fractions of the length of a chocolate bar. It also explores some Geometry, by relating the representation of a rectangle to a chocolate bar, which is an everyday object around the students' world. The posed problem is also surprising because the audience has to reflect on the information posed considering the use of rhymes, seeking to answer the two questions. Students also intend to portray that the characters of the story are surprised. In the second last verse students literally say that Rick "*was surprised. There was only 10 percent. Of the bar that was oversized.*" The connections between strands and representations (students use writing and drawings) and the nature of the DMP that poses questions to the audience may potentially offer mathematical surprises to the audience.

Voyeur - Sense-Making

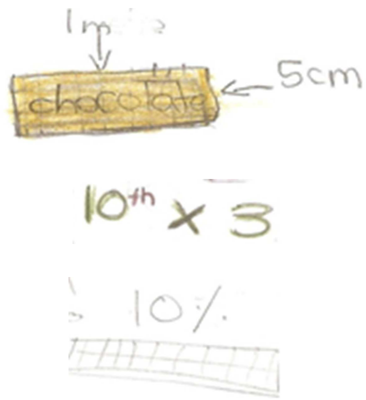
The problem posed by the students in this DMP offers ways to the audience have more than one interpretation. The ambiguity emerges because the poem does not make it explicit if the fractions refer always to the original length of the chocolate bar or if the fractions refer to the length of that left from the bar each time a piece is taken off or shared. Table 4.28 summarizes two ways of interpreting the problem:

Table 4.28: Sense-Making in DMP #22- Rick's Metre Chocolate Bar

Information in the poem	*Interpretation One: The fraction refers always to the original length of the bar (1m)*	*Interpretation Two: the fraction refers always to the actual size of the bar*
Rick's Original Chocolate Bar	1m = 100cm	1m = 100cm
One hundredth to John	1cm (left 99cm)	1cm (left 99cm)
One tenth to Zach	10cm (left 89 cm)	9.9cm (left 88.1cm)
One tenth times three to Suzy	30cm (left 59 cm)	8.81cm x 3 = 26.43cm
Rick evened up	30cm to Suzy 30cm to Zack (10cm + 20cm) 30cm to John (1cm + 29cm) (left 10cm)	26.43cm to Suzy 26.43cm to Zack (9.9cm + 16.53cm) 26.43cm to John (1cm + 25.43cm) (left 20.71cm)
Surprise: there was only 10 percent	10% of 100cm = 10cm	Surprise because there was only 10% and not 20.71% (In this case, 10.71% were "given way").
What fraction was given way?	90cm = 90% of 100cm = 9/10	10.71% = 1071/10.000
Would have done this too?	Yes or No.	Yes

Probably, students created the poem considering "interpretation one." But it is interesting that two interpretations emerged from the problem. Problems with several interpretations are important for students' and the audience's mathematical thinking. It actually makes the problem rich and more interesting. This DMP can be thus considered one example in which the subjectivity emergent with the use of the performance arts (poem/lyrics/story with rhymes) offered interesting ways to the audience think mathematically, seeking for divergence in thinking instead focusing on problem with "only one correct answer."

Figure 4.106 a-c: Drawings in Ricky's Metre Chocolate Bar



It is interesting that students are exploring connections between representations of numbers considering fractions and percentages. This type of connection is highlighted in several elementary school curricula (NCTM, 2000; Ontario Ministry of Education, 2005).

In terms of the designs considering modes of communication (New London Group, 1996), this DMP does not explore audio elements. However, the use of drawings, in combination with writing is very important in terms of meaning production. They offer mathematical insights to the audience (see Figure 4.106 a-c, for instance) as well as emotions.

The modes of communication typically explored trough print-based texts are less prominent in DMP. The notions of inter-textually and hybridity in multimodality (The New London Group, 1996) are fundamental in DMP regarding the symbolic nature of mathematics. The typical modes of communication based on paper and pencil (writing, symbols, diagrams) in combination with the affordances of digital media (e.g., digital registers of images and sounds) form the multimodal nature of DMPs.

Vicarious Emotions

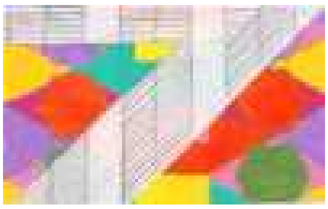
The poem presents an emotional story. At the beginning, Rick felt proud for his marks in school and his mom was impressed. Rick's heart "melted" when Suzy appeared. After that some of the emotions Rick feels are related to the mathematical ideas explored in the story. First, he felt bad because John and Zack did not receive the same amount of chocolate when he first shared it. Then, afterwards, Rick felt good because he evened up the amount each one of his friends received. Some of the emotions Rick felt can be considered “mathematical emotions” because they are direct related to the idea explored in the story regarding a process of sharing based on fractions and percentage. Vicariously, the audience may feel these emotions the actors feel, mainly because the audience is challenged or invited to solve the questions posed in the poem and reflect simultaneously about both the mathematical ideas and the emotions emergent from the experience with these ideas.

Visceral Sensations

This DMP presents an imagined everyday mathematical experience lived by the characters in the story. There is a close link between the mathematical ideas explored and an everyday situation. Furthermore, the audience is engaged in a process of thinking about an everyday mathematical problem considering the fact the DMP poses questions. Direct experiences and links between mathematical ideas and everyday contexts offer to the audience visceral sensations.

There is a sense of mathematical fit in the story because it involves the exploration of fractions and percentages. The notion of equivalence of fractions is indirectly present in this DMP, which refers to the notion on how different pieces parts (fractions) together form one total. In the case of this DMP, considering only the “interpretation one,” 1/10 + 3/10 + 3/10 + 3/10 = 1 or 10% + 30% + 30% + 30% = 100%.

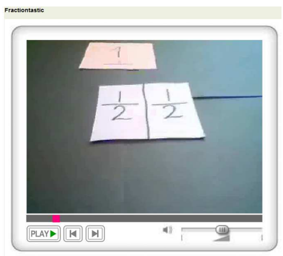
DMP #21: Grade 5 Math Art

Table 4.29: Grade 5 Math Art	**URL:**	http://www.edu.uwo.ca/mpc/performances_2008.html
	**Strands and Content:**	Strand: Geometry and Spatial Sense Content: Drawing of triangles
	Format:	JPG.
	Time length:	*None*
	The Arts:	Drawing
	Participants:	*None*
	Setting:	*None*
	Info:	*None*

Description/Analysis

This DMP is a drawing of two coloured representations of triangles. Some representations of rectangles may also be visualized/imagined on the background of the image. Although a story can be imagined from the drawing, this DMP does not necessarily present a story. Some drawings such as optical illusions, representations of infinity, and visual proofs may offer ways of seeing the new and wonderful in mathematics. However, this DMP do not offer elements to surprise the audience mathematically. Drawings do not explore the audio design of meaning production. Emotions are not explored as well in this DMP. The use of colors may offer visceral sensation to the audience, but not necessarily visceral mathematical sensations.

DMP #22: Fractiontastic

Table 4.30: Fractiontastic 	**URL:**	http://www.edu.uwo.ca/mathscene/mathfest/mathfest104.html
	Strands and Content:	Strand: Number Sense and Numeration and Geometry and Spatial Sense Content: Equivalence of Factions
	Format:	Video
	Time length:	0:36.
	The Arts:	Stop-Motion Animation
	Participants:	Two students
	Setting:	*None*
	Info:	*None*

Description

This DMP is very similar to DMP#10 *Fabulous Fractions* and DMP#15 *Equivalent Fractions*. It explores numeric and visual representations of equivalent fractions. It explores, visually (numerically and geometrically), the notion that 1 = 2x(1/2) = 4x(1/4) = 8x(1/8).

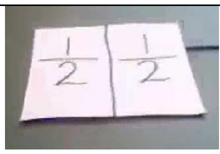
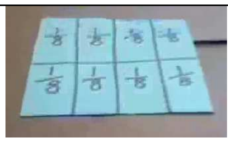
Voyeur - New/Wonderful/Surprising

As discussed in DMP #15 *Equivalent Fractions*, this DMP offers conceptual mathematical surprises because it explores connections between multiple representations and strands. The DMP explores an articulation between numeric and geometric representation of factions to investigate one case of equivalence of fractions. It involves numeration and geometry by exploring representations of squares as formed by equivalent fractional parts, that is, 1 = 2x(1/2) = 4x(1/4) = 8x(1/8). The connection between numeric and geometric representations of fractions is a “big curricula issue” (NCTM, 2000; Ontario Ministry of Education, 2005). The conceptuality of this idea in combination with the animation format (stop-motion) offers ways to the audience see the new and wonderful in mathematics because equivalence of fractions is a “big idea” and students explore a “visual proof” involving that idea.

Voyeur - Sense-Making

Like DMP#15 *Equivalent Fractions*, this DMP explores connections between numeric and geometric representations of fractions. Considering the unit as the whole square (number 1 referring to the area), the DMP presents four levels or stages of the division 1/[2^(n – 1)]. There is a connection between the number of pieces and the size of each piece (1/2, 1/4, and 1/8 of the introduced value of the area). Thus, this DMP is exploring the following equivalence of fractions: 1 = 1x1 = 2x1/2 = 4x1/4 = 8x1/8. Therefore, the audience may think about the following connections in this DMP:

Table 4.31: Visual Equivalence in DMP #22 - Fractiontastic

Stage	Fraction: 1/[2^(n – 1)]	Visual	Equivalent fractions. “Number of pieces x size of the piece = Unit.”
1	1 = 1/(2^0)		1 x 1 = 1
2	1/2 = 1/(2^1)		2 x (1/2) = 1
3	1/4 = 1/(2^2)		4 x (1/4) = 1
4	1/8 = 1/(2^3)		8 x (1/8) = 1

An interesting aspect in this DMP #22, differently from DMP #10 *Fabulous Fractions* and DMP #15 *Equivalent Fractions*, is that students explore the equivalence overlapping the geometric/numeric representations. Students demonstrate visually how 1 = 1x1 = 2x1/2 = 4x1/4 = 8x1/8 putting one representation over another, providing evidence to the audience toward the understanding of the equivalence of the fractions. In this case, the ways students explore the equivalence is more clear than the process shown in the DMP #10, because in DMP #10, it is not clear that the pieces that represent the fractions, when put together, they represent the same unit. That is, visually, the equivalence is not clear in DMP#10 as it is in this DMP.

As discussed in DMP #15 *Equivalent Fractions*, the notion of visual proof (Hanna, 2000) is significant in this DMP. Moreover, the use of materials and the process of editing the animation are interesting from the humans-with-media perspective (Borba & Villarreal, 2005). It reveals how students-with-technology form thinking collectives when they produce a DMP. It elucidates that technology shape the production of mathematical knowledge. In this DMP, students also acknowledge the use of construction paper and markers as significant materials in the production of the DMP. They also highlight that the creation of the process of movement in the animation was conducted by both authors. The process of reasoning toward these movements (to produce an animation) highlights students' mathematical thinking because they have to think on a visual/numeric/geometric sequence of images that would make sense for them and for the audience about the notion of equivalence of fractions. The way students display these sequences of images to create a story based on the use of software is a representation of students learning and mathematical thinking.

Vicarious Emotions

As mentioned in DMP #15 *Equivalent Fractions*, this DMP does not explore emotions through dialogues and gestures like the skit performances, for instance, do. As said in DMP #10 *Fabulous Fraction*, in this DMP the audience does not see humans neither representations of humans such as animated puppets. Thus, the audience may have difficulty to feel what the "actors" are feeling in this DMP, which is the fundamental characteristic of the vicarious eye (Boorstin, 1990). However, the audience may interpret the representations of geometric objects in motion as a "character in action." Some emotions can also be conditioned by the background music, but this aspect is more associated to visceral sensations.

Visceral Sensations

As mentioned in DMP #15, like in DMP #10, this DMP presents three aspects that offer visceral sensations to the audience. (1) The use of a soundtrack; (2) The motion that represents the division of the geometric object (visually) and its connection to the numeric representation happen very fast in the DMP. Furthermore, the stop-motion animation format also enhances the sense that things are happening fast. Quick changes, like the one presented in this DMP, are a characteristic of the visceral eye (Osafo, 2010); (3) Most importantly, the notion of "equivalent fraction" explored through visual representation in this DMP provides a sense of *mathematical fit*, because it is visceral to visualize that the numeric/geometric representations of 1, 2x1/2, 4x1/4, and 8x1/8 are, in motion, fitting one to another.

Final Comments of the Chapter

In this chapter I presented a descriptive analysis of students' DMPs, that is, I displayed a single-case analysis. I described each one of the students' DMPs presenting transcriptions and screen-captured images and used a variation of Boorstin's (1990) categories of what makes good films as a performance arts lens to interpret "what makes conceptual DMPs." The categories considered were: *voyeur - new/wonderful/surprising*, *voyeur - sense-making*, *vicarious emotions*, and *visceral sensations*. I focused on these aspects as much as I could on the descriptive analysis I conducted. I made initial connections between the cases, but potential "interpretative gaps" tend to become clear in the cross-case analysis presented in next chapter. The analysis presented in this chapter presents some evidence that students' DMPs offer opportunities for the audience to experience some surprises, emotions, and visceral sensations, but the production of *conceptual* DMPs is a rare event. Students present interesting mathematical reasoning when they conduct the performances, but, overall, students do not present arguments to support the conceptuality of some of their ideas. Moreover, some logical incoherencies were found. The mathematical sense-making aspect of students' DMPs seems to be the most problematic in terms of pedagogy. Geometry is the most common strand explored by students and connections between strands are also rare events. In contrast, the multimodal, digital, and playful nature of DMPs must be highlighted in terms of pedagogy in mathematics learning. In the cross-case analysis conduct in the next chapter, these findings may be clarified to the reader.

Chapter Five: A Cross-Case Analysis of the Elementary School Students' Digital Mathematical Performances

Introduction

In this fifth chapter I present a cross-case analysis of the DMPs discussed in chapter four. I seek to indicate similarities and differences among the cases and identify themes that emerge, based on the research questions of this study, which are:

- What is the nature of elementary school students' digital mathematical performances in the Math + Science Performance Festival?
- What are the mathematical ideas explored and how do students communicate them using the performance arts?

The chapter is structured in two parts. First, I conduct a cross-case analysis considering the four main categories explored in the last chapter emergent from the performance arts lens (Boorstin, 1990). These categories are*: Voyeur* - n*ew/wonderful/surprising*, *Voyeur - sense-making*, *Vicarious emotions*, and *Visceral sensations*. Sub-themes are also considered.[24]

In the second part of the chapter I discuss the mathematical ideas explored in the DMPs in terms of *content*/*strands* as well as some aspects regarding the K-8 mathematics *curriculum's processes* (Ontario Ministry of Education, 2005). That is, I use the components of the curriculum introduced in chapter two as analytic lenses as well.

In order to present a cross-case analysis in this chapter I use charts or tables to point out similarities between cases, "classifying" them according to sub-categories, discussing some findings and connections to the literature. Although the themes and sections named in this chapter have a "categorical or structural design," they can be considered as "provisory" as they overlap. The patterns I interpret through the analytic process may be represented in different ways through other provisory categories. Seeking to avoid too many repetitions of words, I refer to each DMP regarding their numbers (#), as shown in Table 3.1 (see chapter three):

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[24] A fifth category could be explored. Typically, when one watches movies in theaters, for instance, the nature of the interaction is unidirectional (movie→viewer). However, the levels of interactions are different when one explores the cyberspace or watches new interactive forms of DVDs, for instance. In online environments, one can click on several links, select different endings for the stories, watch making offs, and so forth. Potentially, all these aspects (the mobile nature of hypertexts) have an impact in terms of meaning production as well as on the pleasures pointed out by Boorstin (1990). Thus, focusing on the nature of the interactivity of humans-technologies, a fifth category will be explored in future studies considering the analysis of DMPs.

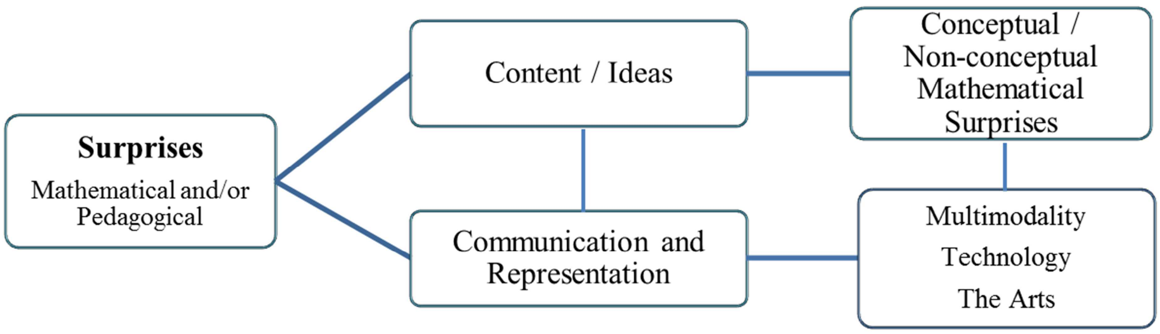
Chapter Five - Part One: A Cross-Case Analysis through a Performance Arts Lens

Voyeur - New/Wonderful/Surprising

Boorstin (1990) posits that the sense of surprise is fundamental for the voyeuristic eye. It captures the attention of the audience and offers the pleasure of experiencing something new and wonderful. According to Boorstin, the voyeuristic eye refers to the joy of "watching out of a kind of generic human experience" (p. 12), because "people love to be taken to a place that's like nothing they've seen before" (p. 16). Surprises are also fundamental for the mathematical activity (Adhami, 2007; Floyd, 2001). Watson and Mason (2007) "tend to see surprise as a positive emotion [and] mathematics as full of philosophical and cognitive surprises; surprise as motivating curiosity and effort" (p. 4). Gadanidis and Borba (2008) state that some DMPs offer "some opportunities for more interesting experiences for the voyeur's eye" (p. 47).

All elementary school students' DMPs present *mathematical and pedagogical surprises* in that they constitute a way of communicating and representing mathematics by using and combining the performance arts and digital media. This combination supports multimodal communication, which is not a typical/expected practice in mathematics and pedagogy. Specifically, both *mathematical and pedagogical surprises* occur in terms of *content/ideas* and/or *communication* and *representation*. *Conceptual mathematical surprises* refer to connections, to ways of representing and communicating mathematics by connecting multiple representations and/or strands. In contrast, "non-conceptual" surprises refer to typical or expected ways of exploring mathematics or ways of seeing ideas in isolation, without significant connections. *Communication* and *representation* are explored in terms of *multimodality*, *technology*, and *the arts*. Figure 5.1 presents a diagram that illustrates the ways I interpreted the surprises offered by elementary school students' DMPs in this study:

Figure 5.1: Surprises in Students' DMPs



Following, considering the diagram above and my interpretation/analysis of each DMP in chapter four, Table 5.1 displays an interpretative categorization of the surprises in students' DMPs. As the reader shall notice, a same DMP may fit in more than one sub-theme.

<table>
<tr><th colspan="3">Table 5.1: Mathematical and Pedagogical Surprises</th></tr>
<tr><td colspan="2">Content/Idea</td><td rowspan="2">Communication: Multimodality, Technology, The Arts
#1, #2, #3, #4, #5, #6, #7, #8, #9, #10, #
#11, #12, #13, #14, #15, #16, #17, #18, #19, #20, #21, #22
(See Table 5.4 for details on the type of arts)</td></tr>
<tr><td>(Potentially) Conceptual
#2, #3, #4, #5, #6, #8, #10, #12, #13, #14, #15, #16, #17, #18, #22</td><td>Non-conceptual
#1, #2, #3, #7, #9, #11, #12, #16, #19, #20, #21</td></tr>
</table>

Conceptual Mathematical Surprises.

Conceptual mathematical surprises refer to a variety of ways of *making deep connections* between multiple representations, ideas, content, concepts, strands, and/or everyday problems. In other words, the nature of conceptual mathematical surprises offers ways of seeing the new and wonderful in mathematics by connecting strands, portraying mathematics as fuzzy and multiple, (e.g. mathematical objects fitting in multiple categories, mathematics living in continuous reorganizations of its foundations), extending abstract mathematical ideas to the empirical dimension, exploring visualization, imagination, and creativity, and so forth. These connections "open windows into mathematics" (Gadanidis, 2011; Noss & Hoyles, 1996), offering ways of experiencing the beauty of rich mathematical ideas (Gadanidis & Borba, 2008; Higginson, 2006).

Some of the students' DMPs may potentially offer conceptual mathematical surprises. Aspects concerning connections between strands, for instance, are discussed in the second part of this chapter. In DMP #6, #10, # 15, #20, and #22 students connect different strands. The surprise emergent in DMP #6 based on the exploration of a factorial growth and its relation to patterns and an everyday situation offer conceptual surprises to the audience because they connect different representations and strands in such ways that mathematics is portrayed as wonderful. In DMP # 15 and #22 students offer "visual proofs" of equivalent fractions through explorations that show how a geometric figure like a square or rectangle may be shared in equivalent parts, which represents fractions of the whole unit. Visual proofs (Hanna, 2000) offer ways of seeing the new and wonderful in mathematics (Zwicky, 2000). In contrast, the connection between strands explored in #20 does not offer conceptual surprises necessarily because the nature of the connection may be considered superficial.[25] That is, the focus is on *Measurement* and the exploration of *Number Sense and Numeration* may be considered superficial by the audience.

In DMP #2, #3, #4, #5, #13, #14, #16, #17 and #18 students offer ways of seeing "mathematics as fuzzy," that is, "not black-and-white" or either "right or wrong." In DMP #3, #5, #13 and #18, for instance, they offer ways of seeing quadrilaterals as fitting in or forming many categories. Instead of exploring quadrilaterals in isolation (exploring a square as a figure different from a rectangle, for instance), in these DMPs students explore "a quadrilateral as a specific case of other quadrilaterals", like "a square as a specific case of rectangle," which is an idea that offers surprise because it disrupts possible (and usual) misconceptions people may have about quadrilaterals. In DMP #2, #4, and #16 students also explore multiplicity in showing specific ways of seeing connections between representations of different polygons. In these three DMPs students offer ways of seeing similarities between shapes (e.g., triangles, trapezoids, regular hexagons) by exploring how their representations are connected or fit one to another. In DMP #2 students explore the representation of a trapezoid as a "triangle with its head cut-off." In DMP #4 students explores how triangles fit in many shapes or how shapes may be constructed with triangles. These explorations offer ways of seeing similarities between shapes, instead of posing the idea of shapes as isolated entities. In addition, in DMP #14 and #17 students explore solids and ways of seeing and imagining connections between the representations of the solids

[25] DMP #20 is interpreted as conceptually surprising not only because it connects strands, but mainly because it explores a problems that offers at least two distinct mathematical interpretations.

and representations of the faces that form the solids, which may offer conceptual mathematical surprises to the audience, regarding the fact that this exploration is made by students in Grade 5.

Many DMPs explore connections between mathematics and everyday situations or contextual problems. These connections may be understood as *initial* ways of exploring issues about applied mathematics or modeling,[26] offering potential ways to see the new and wonderful in mathematics. Considering that the DMPs analyzed in this study were produced by students from Grande 4 to Grade 6, in my interpretation, some DMPs may offer conceptual mathematical surprises to the audience through "mathematical modeling."

The connections explored in DMPs #2, #5, #8, #12, #14, #16, and #17 may offer conceptual mathematical surprises to the audience by exploring how representations of geometric shapes may fit in the world around the students. DMPs #6 and #20 may offer conceptual mathematical surprises by exploring measurement and #6, specifically, by exploring patterns and algebra. In DMP #2 students mention the very interesting problem involving tessellation and isoperimetric issues regarding the area of hexagons when the last candidate sings "*[It] has six sides. Hides in hives*." Hives are formed by representations of hexagons which are shapes that tessellate, optimizing that area usage. In DMP # 5, students mention how quadrilaterals may fit in the world (e.g. squares as windows, rectangles as wide TVs, rhombi as diamonds, and so forth). In DMP #8 students mention everyday application of representations of circles (make things move fast), triangles (strong structures for roofs) and squares (good base for buildings).

In DMP #12 students use the notion of radius and perimeter of a circle to solve a problem of baking a cake with specific characteristics in its dimensions. In DMP #16 students explore the fact that representations of "shapes are everywhere." In DMP #14 students relate the shape of a cylinder to the format of a cookie maker and a diamond to the representation of a regular octagon. In DMP #16 students highlight that "*s-h-a-p-e-s… That is all around us*" and, specifically, "*Triangle are strongest shapes.*" In DMP #17 students explore square based pyramids conducting the skit in a scenario that represents an Egyptian pyramid.

In particular, in DMPs #8, #14, and #16 students mention that triangles are "strong" shapes and they are used to construct things like roofs. This exploration is interesting because it involves a contextualization in architecture and explores the imaginative dimension of understanding the dynamicity and flexibility of the vertexes and angles of triangles in contrast to shapes like

[26] See Borba and Scucuglia (2009) for more discussions on the interlocution between DMP and modeling.

parallelograms, for instance. Moreover, in DMP #6 students explore a very interesting way of seeing a factorial growth in an applied situation and in DMPs #6 and #20 students explore everyday ways of measurement involving counting money and/or time.

Every DMP explores interesting mathematical ideas that can be extended by the audience. Some insights about possible ways to extend the ideas were indicated in the Chapter four. However, overall, students could have proposed more “open-ended” problems to the audience, posed questions and/or indicated ways of explore rich ideas based on the content explored in the DMP. Conceptual mathematical surprises emerge from connections and students did not pose explicitly ways or paths for the audience to extend the initial ideas explored in the DMP. That is, connections are not fully developed in the DMPs, as is expected of a short performance, but they offer potential for further explorations. For example, why is a square a rectangle? Why did the Egyptians build a square based pyramid? Why are triangles strong shapes? Why do bees use hexagons to build hives?

Following, in Table 5.2 I summarize a classification on the potential conceptual mathematical surprises in students’ DMPs analyzed:

Table 5.2: Potential Conceptual Mathematical Surprises	
Connections between Strands	#6, #10, #15, #22
Visual Proof	#10, #15, #22
Multiplicity	#2, #3, #4, #5, #13, #18
Relations between properties and representations of shapes	#2, #3, #4, #5, #13, #14, #16, #17, #18
Connections between mathematical ideas and everyday contexts/situations	#2, #5, #6, #8, #9, #12, #16, #17, #20

In contrast, it is important to mention that some of the mathematical ideas students explored in DMPs #1-3, #7, #11, #12, #19, and #21 offer mathematical surprises, but they are not conceptual mathematical surprises. That is, they do not present deep connections between mathematical representations, ideas, concepts, and strands. The nature of the mathematical idea in some DMPs is traditional or typical as explored in traditional classrooms and textbooks. Thus, some DMPs offer both conceptual and non-conceptual mathematical surprises. DMPs may be seemed as reflections ("mirrors") of what students learn in classrooms. In some DMPs, students do not seem to be experiencing conceptual surprises in non-geometry strands. Probably, the exploration of surprises is not a pedagogical focus of these DMPs.

Even though in DMPs #1, #7, #8, #9, and #16 students do not explore specific ways of seeing that representations of some shapes may fit in representations of other shapes (like in DMPs #2, #3, #4, #5, #13, and #18), students offer "superficial" ways of comparing/contrasting properties of different shapes such as circles and polygons. That is, in DMPs #1, #7, #8, #9, and #16 students do not explore properties of shapes in absolute isolation. In DMPs #1 and #9, students are comparing some characteristics of polygons and non-polygons. In DMP #1, students offers some mathematical surprise because the pun on words "Polly (is) gone" and "polygon" is creative (a kind of math joke) and students relate this play on words to posing and solving a puzzle. However, the way mathematics is explored in the skit is only based on the definitions and examples of polygons and regular polygons and on the identification of materials that represent and do not represent polygons and regular polygons. Although there is some connection to an "everyday problem," the nature of the mathematics explored in DMP #1 is superficial because the audience does not see anything mathematically unexpected about polygons. In DMPs #7 and #8, students explore some properties of triangle, square, and circle regarding properties such as lines of symmetry. Specifically, in DMP #8, the contrast between properties is highlighted in the puppet theatre in which the characters (the shapes) are disputing who is the best. In DMP #16, the candidate who sings the first song makes a superficial contrast between a square, a pentagon and a hexagon when she sings: "*Hey hexagon, I am straight up playa. The arms go up to me, isn't that better? Things I hate, pentagons aren't nice. Four vertices like way up tight.*" Thus, DMPs #1, #7, #8, #9, and #16 explore sets of geometric objects in a way that offers possibilities of seeing connections between properties of different shapes, although these connections are not explicit in the DMP or they a not further explored.

DMPs #1, #4, #7, and #11 may offer surprises by mentioning ("superficial") applications of mathematics (or mathematical/geometric objects) to empirical contexts, everyday situations or "imagined/real" problems. These DMPs explore representations of polygons and circles as everyday objects relating, for instance, representations of polygons as door knobs or circle and sphere rolling. However, the mathematics explored is based on definitions and traditional ways of seeing mathematics and the empirical connections are considerable superficial.

Following, in Table 5.3, I summarize a classification of non-conceptual mathematical surprises regarding the DMPs analyzed in this study:

Table 5.3: Non-Conceptual Mathematical Surprises	
Superficial relations between properties and representations of shapes	#1, #2, #4, #5, #13, #14, #16, #17
Superficial extensions to everyday contexts/situations	#1, #8, #9, #12, #16, #17

Thus, after discussing the role of surprises in students' DMPs, it is important to highlight the convergence between surprises and creativity. According to Hallman (1963):

> Originality means surprise. Just as novelty describes the connections that occur in the creative act, unpredictability to the setting of the new creation in the physical environment, and uniqueness to the product when regarded as valuable in its own right, so surprise refers to the psychological effect of novel combinations upon the beholder. Surprise serves as the final test of originality, for without the shock of recognition which registers the novel experience, there would be no occasion for individuals to be moved to appreciate or to produce creative works. (p. 20)

Lewis (2006) states that:

> Creativity is an act that produces effective surprise… the surprise associated with creative accomplishment often has the quality of obviousness after the fact. The creative product or process makes perfect sense—once it is revealed. For the creative person, surprise is the privilege only of prepared minds—minds with structured expectancies and interests. (p. 36)

Thus, as discussed in this section, some students' DMPs present potential for conceptual and/or non-conceptual mathematical surprises, considering the nature of the idea explored. Surprises and collectiveness play significant roles in students' DMPs. These elements are important for the voyeuristic eye (Boorstin, 1990), for mathematical activity (Floyd, 2010, Gadanidis & Borba, 2008; Scucuglia, 2011a; Watson & Mason, 2007), to offer emotions in the pedagogic scenarios (Adhami, 2007), and for creativity as well. Mathematical surprises are fundamental aspects to offer ways of seeing the new and wonderful in mathematics and to produce conceptual DMPs.

Multimodality: Technology and the Genre of the Performance Arts.

It is surprising to see students communicating mathematical ideas using the performance arts and multiple modes of communication. Traditionally, in numeracy, students' learning is communicated and represented thought print-based texts involving words, symbols, and diagrams. Written language based on the use of media such as paper and pencil offers a limited set of modes of communication. Digital media offer ways to combine multiple modes of communication such as aural, visual, gestural, spatial, and linguistic (Jewitt, 2006; New London Group, 1996). In this sense, digital media offer ways to "enhance" learning in terms of the development of multiliteracies and pedagogies (Heydon, 2007; Hughes, 2006; Walsh, 2011). Thus, it is mathematically and pedagogically surprising to recognize, see and read the ways elementary school students are representing their learning in each DMP analyzed in this study by using non-typical elements of the designs of mathematical communication such as gestures and sounds combined with images, visual effects, drawings, verbal and written words, symbols, manipulative materials, spatial elements, and the performance arts as well. Hybridity and intertextuality (The New London Group, 1996) are fundamental aspects of students' DMPs and very significant in terms of mathematical thinking, communication, and meaning production. The combination and use of different types of texts are fundamental aspects in producing DMPs. Pedagogically, the notion of DMP goes beyond the use of the most popular elements and artifacts of communication involved in print-based texts. From a humans-with-media perspective (Borba & Villarreal, 2005), the production of DMPs does not replace the use of "old" media such as paper and pencil, for instance. However, the use of digital technology reorganizes the production of mathematical meaning and knowledge and, potentially, offer new perspectives toward the use of the all the elements and artefacts that form the multimodal designs of communication.

Multimodality also refers to the synergy between technology and the arts. Playfulness and production of multimodal texts in classrooms are characteristics of a sociocultural / postmodern curriculum. This study has no data to analyze how the processes of recording and editing the videos were conducted, which would elucidate how students-and-teacher-with-media produced each DMP, in other words, how the technology shaped the production of mathematical knowledge when students conceived DMP, forming thinking collectives in a pedagogic scenario (Borba & Villarreal, 2005). However, some points can be discussed regarding the use of digital technology to produce a DMP, because each DMP (as a product, a multimodal text) offers insights about the process of how they were produced. First, I will present the Table 5.4 the format of the DMPs in relation to the arts performed in the students' DMPs.

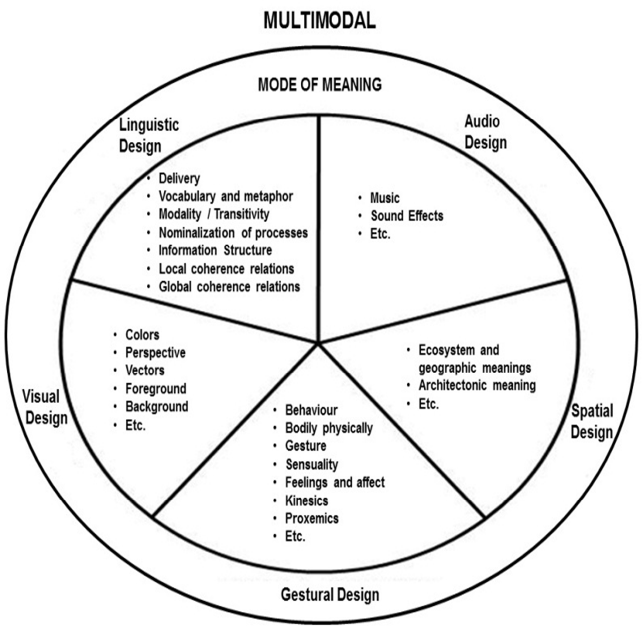
Table 5.4: The Format and the Arts Performance in Students' DMPs

Video Format					***Power Point***	
Musical (Skits + Songs)	*Skit*	*Song*	*Puppet Theatre*	*Animation*	*Poem Drawing*	*Drawing*
#2, #4, #7, #9, #14, #16, #17	#1, #11, #12, #13, #18, #19	#3	#5, #8	#10, #15, #22	#6, #20	#21

From the table above, and regarding the analysis in chapter four, some issues can be discussed in terms of *multimodality,* use of technology and the arts.

Most of the DMPs analyzed in this study were displayed in video format, which requires the use of a video camera and, probably, software of video editing. All the DMP in video format combine all the five designs of modalities regarding the model proposed by the New London Group (1996). The designs are: linguistic, visual, gestural, spatial, and audio (see Figure 5.2).

Figure 5.2: Multimodal Design (The New London Group, 1996)



From the humans-with-media perspective (Borba & Villarreal, 2005), it is also important to highlight that the process of editing a video is a process of *thinking-with-technology* (Scucuglia & Borba, 2007). It is a reflective process in which the authors (students and/or teachers) think about what kind of meaning the audience may produce. It is a learning process in which the authors develop sense-making (sense of story) regarding possible signs, representations, and feelings (emotions and sensations) the audience may engage with. It is a process of developing communication skills in representing clearly the ideas to an audience. It is a process that forms thinking and feeling collectives of students-and-teachers-with-media.

The production of stop-motion animations (DMPs #10, #15 and #22) requires a long term engagement with the processes of manipulating materials, taking pictures, and editing the video. That is, the effect of motion in animations is shaped through the (a) manipulation of materials and the process of taking sequences of pictures of the materials in different positions and (b) work with the timeline of the video editing software that constructs the story (the sense-making) involving mathematical thinking. From the DMPs #10, #15 and #22, the audience may interpret some of the actions and ways of thinking mathematically that the collectives formed by students-with-manipulative-and-software were engaged in the visual exploration of equivalence of fractions.

Some musicals, skits, songs, and puppet theatre performances also present a process of editing, but they do not seen very complex. Sometimes they may be imperceptible to the audience. In DMPs #1, #4, and #5, for instance, the audience may identify that there are cuts from one scene to another, but these cuts are very natural. In contrast, DMP#2 and #19 do not present cuts, that is, students' performances in these DMPs were recorded in only one shot.

Except for the stop-motion animations, students did not use digital technology in the process of performing. They only used video cameras to record the skit and musical performances. Students could have used in their performances other tools such as graphing calculators, computers, smart boards, cell phones, i-pods, and so forth. The use of these technologies would enhance the possibilities toward articulating multiple representations and strands in exploring conceptual ideas. Students could have, for instance, explored dynamic geometry, created an immersive environment or a *microworld* for their scenario (Papert, 1980), or used more elaborated effects in the processes of editing the video. These suggestions are based on the notion of humans-with-media (Borba & Villarreal, 2005), which calls our attention to an important point. Although students could have highlighted the role of digital technologies as co-authors in knowledge production by using them in the process of performing, they used video cameras to produce the DMPs, which are fundamental to support multimodal communication. It is also important to mention that usually, typical mathematical experiences are not recorded in the classrooms and video registers are types of texts that can be made available online and this format of text supports modes of communication that print-based texts do not. Thus, the fact that students were using video cameras to record their performances (and this format of text shapes the nature of students' representation and communication), it is possible to interpret that video cameras were co-authors in the production of mathematical knowledge when students produced DMPs. Digital media's affordances offer support for multimodal communication and the specific use of video cameras' resources (e.g., zooming-in) may enhance the (mathematical) pleasures the audience may potentially fell. The digital nature of DMPs emerges from the perspective that students-with-DMPs form thinking collectives.

As highlighted in chapter four, students used a variety of materials (e.g. costumes, posters, scenarios, modeling clay, papers to represent shapes, and so forth) in their musicals, skits, songs, puppet theatre, and animation performances. These materials are very significant for students' and the audience's learning and meaning and knowledge production.

In DMPs #1, #4, #5, #8, #10, #15, and #22 students used *manipulative materials*. In DMP #1, students used paper materials to create representations of polygons and non-polygons which are part of the puzzle posed. In DMP #4, students also use paper material to construct representations of triangles to build representations of several figures (e.g. parallelogram and hexagon). In DMPs #5 and #8 students also used papers and sticks to create the puppet characters that are representations of geometric figures. In DMPs #10, #15, and #22 students used paper materials and modeling clay to produce their animations using the stop-motion animation that reveals a visual proof for equivalence of fractions.

In DMPs #7, #9, #11, #13, #14, and #18 students used *costumes* that characterize them as geometric objects. These costumes (e.g., masks or representations of figures pasted on students' t-shirts) enhance their sense of embodiment and offer visual insights to the audience which are significant in terms of sense-making. In DMP #7, representations of figures (square, triangle, and circle) are interviewed and the masks, for instances, support the visual information about the properties if the figures. From the transcription of DMP #7, *Squareon* says, for instance "*I have* *four edges going around my face. One, two three, four [pointing the edges around the face-mask]*". In DMP #9, students used representations of polygons and non-polygons pasted in front of their T-shirts to play their roles. In DMP #14, students used costumes to play roles as cubes, cylinders and rectangular prisms. In DMPs #13 and #18, for instance, the use of geometric costumes is fundamental to the audience to identify who the characters are and the relation between the properties of the quadrilaterals. The identification of these relations between properties offers ways to the audience to explore the notion that a square is a specific case of rectangle, parallelogram and rhombus. In DMPs #13and #17, students also used costumes, but these are not "mathematical costumes." These are costumes related to the roles students are playing, which are not necessarily roles as mathematical objects.

In DMPs #6, #8, #11, #12, #17, #19, #20, and #21 students used *drawings* to represent and communicate their mathematical ideas. DMPs #6 and #20 were produced in power point format and they explore writing and drawing in the creation of a story-poem. The drawings are significant to illustrate the mathematical ideas and the emotions within the story. In DMP#8, drawings are fundamental to represent the geometric characters and the scenario of the puppet theatre. In DMPs #11, #12, #17 and #19 students used posters of in the performance of their skits to display visual information about the mathematical ideas they are exploring. DMP #11, for instance, shows an illustration of a sphere and #17 the drawing and properties of a square based pyramid. In DMP #12, one of the characters shows a drawing, but it is not possible to see it in the video. In DMP #19 the use of a poster is fundamental to pose questions and create an immersive context of a TV show in which candidates dispute a conquest about mathematics.

Table 5.5 indicates the use of manipulative materials, costumes, and drawings that are significant to mathematical thinking in each DMP in terms of modes of communication. Images captured from some DMPs illustrate this materiality in the process of performing.

Table 5.5: Materials used in Students' DMPs		
Manipulative Materials	*Costumes*	*Drawings*
#1, #4, #5, #8, #10, #15, #22   	#7, #9, #11, #13, #14, #18   	#6, #8, #11, #12, #17, #19, #20, #21   

The *gestural design* of modality is fundamental in DMP. The DMPs based on skits, songs, musical, puppet theatre and animation are built on the ways students use gestures and motions to communicate mathematics. In DMPs #1, #2-5, #7-19, and #22, gesture can be seen as a "technology of intelligence" that shapes the production of mathematical meaning (Borba & Villarreal, 2005; Levy, 1993). Gesture is a fundamental resource to think mathematically and communicate ideas with emotions. In fact, the mathematical activity related to the use of the body in mathematics education is explored through the notion of embodiment (Radford, 2003). Usually, students and teachers use gestures to communicate their ideas in classrooms, but gestures are not typically fixed when students represent their ideas. Traditionally, students represent ideas using print-based texts, which is a format that is not friendly to support the gestural design. Thus, the multimodal design of DMPs offers ways for students to communicate and represent their ideas using gestures, which is a significant mode for mathematical meaning production (Gerofsky, 2009). This notion is discussed in the section *Vicarious Emotions* of this chapter. The following images (Figure 5.3 a-c) illustrate some of the moments in which the gestures were significant in the process of communicating mathematics in students' DMPs:

Figure 5.3 a-c: Some of gestures/embodiment explored by students in their DMPs

The DMPs that explore stop-motion animations (DMPs #10, #15, and #22) do not involve students' bodies in the peformative display. There are neither students' gestures nor speeches. But these DMPs can be analyzed through the gestural design, because there is an emphasis on motion (stop-motion animation) and it offers ways for gestural and visual meaning production. Bordwell and Thompson (1993) state that

> In cinema, facial expressions and movement are not restricted to human figures… by means of animation, drawing or three-dimensional objects can be endowed with highly dynamic movement. For example, in science-fiction and fantasy films, monsters and robots may be given expressions and gestures through the technique of *stop-action* (also called "*stop-motion*") (p. 158).

Thus, in DMPs #10, #15, and #22, the motion and the visual aspects are significant in producing meaning of equivalent fractions through a visual proof. As mentioned, the production of these DMPs involves an interesting process of thinking-with-technology in which students-with-DMPs form thinking collectives.

All the DMPs in video format performed through musical, skit and song used zooming-in (close-ups) on the actors/singers' facial expressions, which is part of the gestural design and may increase the feeling of vicarious emotions for the audience. Figure 5.10 a-c shows some of the zoom-in shots.

The *audio design* of modality is also fundamental in DMP. The DMPs based on skits, songs, musical, and puppet theatre are built on the ways students use speeches or verbal communication to talk about mathematics. In DMPs #1, #2-5, #7-9, #11-14, and #16-19, orality can be considered a "technology" that shapes the production of mathematical meaning (Borba & Villarreal, 2005). Similarly to gesture, students and teachers use verbal words to communicate their ideas in classrooms, but these words are not typically fixed when students represent their ideas. Traditionally, students represent ideas using print-based texts, which is a format that does not support the audio design of communication. Thus, the multimodal design of DMPs offers ways for students to communicate and represent their ideas using audio, which is a significant mode for mathematical meaning production. The reader can identify this importance considering the transcriptions presented in chapter four. In DMP #1, for instance, the use of the audio modality is fundamental to conduct the math joke based on the pun on words “Polygon” and “Polly (is) gone.” In DMPs #10, #15, and #22, students did not use verbal words in the video, but the use of soundtracks is important to offer visceral sensations to the audience.

The use of the *space* in skit performances offers ways for the audience to produce spatial meanings. In all musical and skit performances the space offers signs to the audience and particular ways of producing meaning. Some of these meaning may be mathematical. In DMP #1, students created different scenarios in the process of finding Polly and these scenarios shape the mathematical puzzles investigated. The spatial design of a classroom may also highlight the nature of power relations and the collaboration between student and teachers. Similarly, in DMPs #2, #13, #16, and #19 the scenario indicates the power relations between the characters who play roles as judges, presenters, defendants, and candidates. In DMP #4, for instance, the performance is conducted in a large space (it seems like a gym area) and with the participation of many students. The spatial design of DMP #4 offers ways for the audience to interpret or feel a sense of collaboration involving the students who are performing and thinking mathematically. So, there is a pedagogical relationship between space, playfulness, and collectiveness that are fundamental in DMP and in a sociocultural/postmodern view of curriculum. In DMP #11, for instance, the spatial design is important to represent the idea that a sphere rolls. Overall, the notion of space is fundamental in geometric representation and embodiment as well.

Only three DMPs were displayed in power point format. Power point affordances may support all designs of modalities because they support animations, audio or even videos. However, the DMP in power point format analyzed in this study do not explore gestural, geographic, and audio designs. They explore only the visual and linguistic. Interestingly, even though students used two modal designs in their power point DMP, they were able to connect different strands in DMP #6 and #20. And, as pointed out, most of the analyzed students’ DMPs do not connect mathematical strands of the curriculum. In fact, DMP #6 is the only one that connects three different strands.

When students produce a DMP, they are looking for new ways of telling mathematical stories using multiple modes of communication, that is, ways of creating multimodal texts using the performance arts. Conceptual mathematical narratives offer new/wonderful/surprising ideas through multiple representation and connections (Gadanidis & Hughes, 2008). Mathematical ideas told through stories typically offer surprises, but the nature of the mathematical idea explored is fundamental to offer *conceptual mathematical surprises* to the audience. When students are engaged in a process of producing a DMP, they are engaging in ways of seeing surprises by making creative connections between representations, modes of communication, and concepts or strands. The multimodal nature of DMPs offers ways to students communicate mathematics in alternative ways, in hybridity to typical print-based texts and several types of arts (e.g., drama and music). Multimodal texts in which all the five designs of modality are compiled, combine multiple layers of signs that cannot be represented by only paper and pencil. These layers of signs, conditioned by the digital media, offer to the audience specific ways to produce meanings about mathematical ideas and to feel surprises through the nature of the ideas explored and through the playfulness emergent with the performance arts. However, the exploration of multiple modes of communication does not guarantee the conceptuality of the mathematical idea.

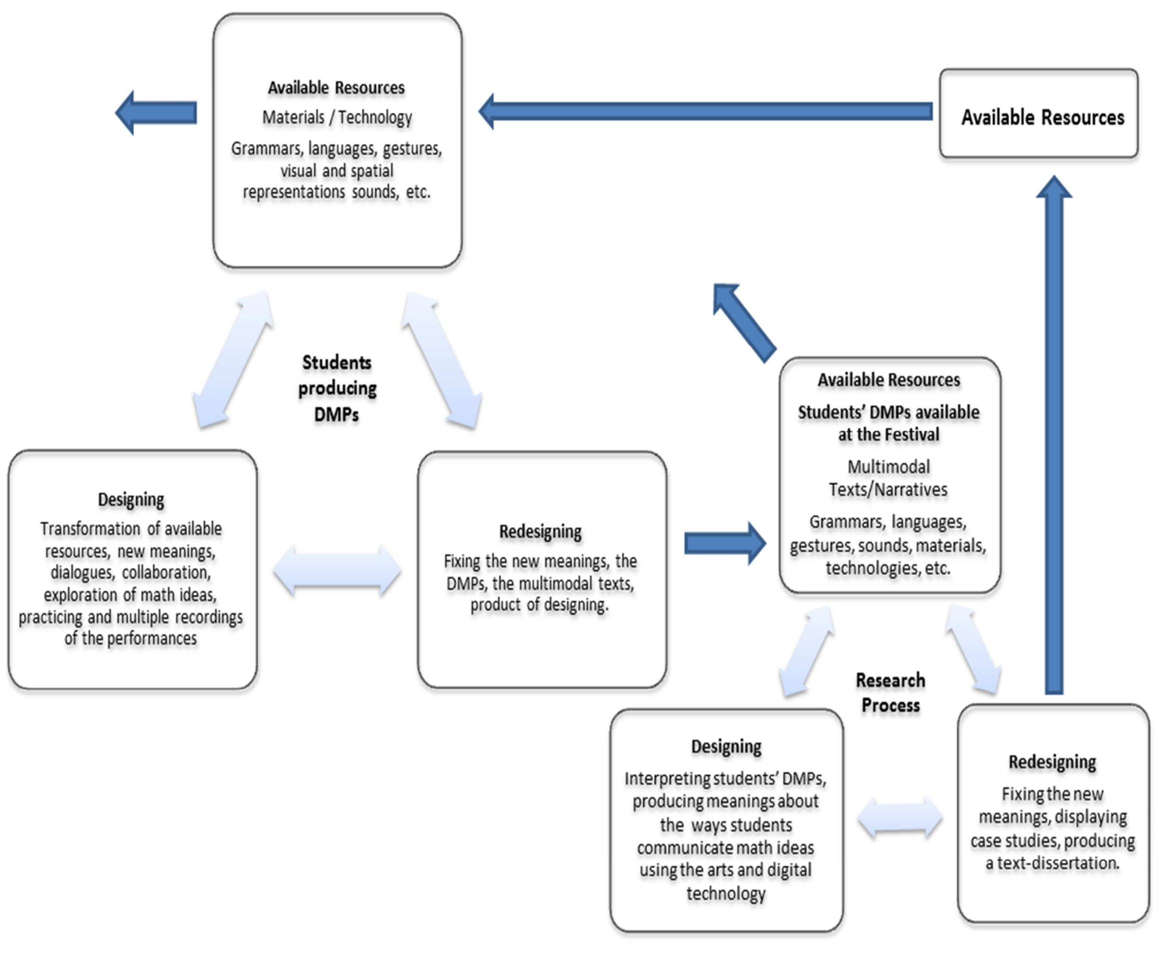
Finally, it is important to highlight a parallel between the process of students producing DMPs and the process of analysis of students' DMPs in this study. From a multimodal point of view, it is interesting to relate these two processes to the notions of *available resources*, *designing*, and *redesigning* proposed by The New London Group (1996).

In order to produce a DMP, students use resources for designing including "the 'grammars' of various semiotic systems: the grammars of languages, and the grammars of other semiotic systems such as film, photography, or gesture" (The New London Group, 1996, p. 74). These resources also include artifacts, materials, and technologies. The available resources are shaped or designed through the practice of the performance, exploration of mathematical ideas, negotiation of meanings, repetition of speeches and gestures, use materials such as video cameras and manipulatives, the edition of videos, and so forth.[27] "Every moment of meaning involves the transformation of the available resources of meaning. Reading, seeing, and listening are all instances of Designing . . . Any semiotic activity - any Designing - simultaneously works on and with these facets of Available Designs" (The New London Group, 1996, p. 75).

[27] That pedagogic scenario may be designed in order to engage students on processes of problem-solving.

Moreover, the emergent meanings through the process of designing are fixed as texts. The process of fixing the modes of communicating re-designs meanings. Interestingly, "the Redesigned becomes a new Available Design, a new meaning-making resource" (The New London Group, 1996, p. 76). Thus, the "products," the redesigned digital supports that represent new meanings, the multimodal texts, the students' DMPs, they become available resources for the audience and, in the case of this study, as research data. Similarly, these resources (students' DMPs) are interpreted (designed) by the researcher and represented as a dissertation (redesigned). Each of the processes – resources, designing, redesigning - shapes one to another. Figure 5.4 illustrates these processes.

Figure 5.4: Students' DMPs and the Research Design as Multimodal Processes.



Voyeur - Sense-making

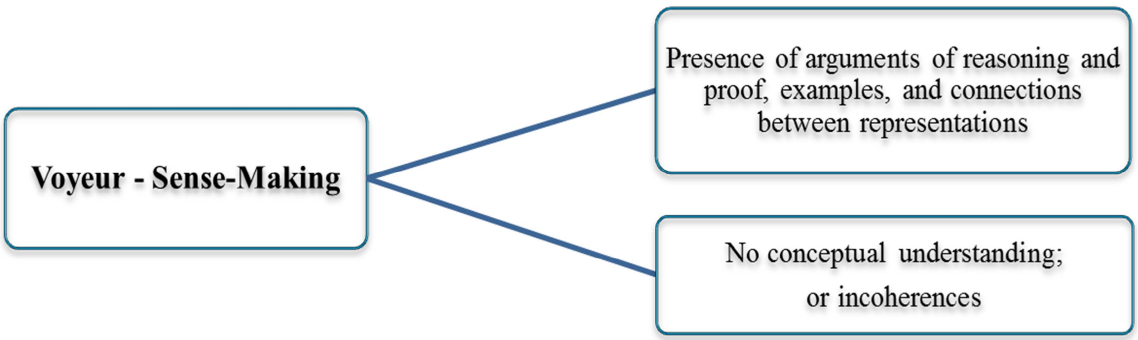
Boorstin (1990) states that rational eye is also significant for a voyeuristic experience. The story and the surprises have to make sense to be effective. Gadanidis and Borba (2008) posit that "events in the movie cannot happen randomly or without connections to other events . . . [which] must fit into a larger, rational scheme" (p. 46).

Osafo (2010) analyzed twenty videos available on YouTube in which teachers and scientists ("experts") use the performance arts to make experiments in the context of science education. Osafo found that the performances were rich in surprises (e.g., unexpected physic/chemical reactions of the experiments such as "explosions"), but the performances were not were rich in terms of sense-making. According to Osafo, "it appears there are a lot of gaps in many of the 20 video, [and] this reveals that there are pedagogical opportunities missed" (p. 151). Science digital performances require greater understanding and "long time" to offer arguments to support understanding. Usually, the performances available on YouTube are "short" as students' DMPs are. Osafo thus suggests that presenters (or performers) "must endeavor to connect the concept to the viewer's knowledge" (p. 177), using analogies, and providing meaningful examples (Osafo, 2010).

Similarly to Osafo's interpretation on science performances, the analysis conducted in this study suggests that students' DMPs present shortcomings in terms of sense-making. Generally, it is important to mention that students explore some visual aspects emergent from use of the affordances of digital media as well as connections between modes of communication, providing some examples. However, in many DMPs, students did not support nor justify the conceptual nature of the ideas they were exploring. Two aspects should be considered toward the conceptual gaps in students' DMPs: (1) DMPs are short, that is, the dramatic events are presented in approximately three minutes and, usually, it is not a simple task to explore or present a lot of arguments to justify the conceptuality of the ideas, although it is not impossible; (2) Mathematical knowledge and thinking are traditionally characterized by objectivity, the directness of logical-deductive reasoning. In contrast, the use of the performance arts brings subjectivity into the epistemological scenario. Thus, some of the DMPs offer "flashes" or insights to the audience based on multiple modes of communication and representation instead offering a linear tutorial of mathematical reasoning. It pushes the meaning production to the audience, focusing much more on diversity and divergence rather than unity and convergence in terms of thinking. That is one of the most interesting epistemological tensions in DMP. As Gerosfsky (2006) points out, the most relevant elements of the performance arts appears to be in “conflict” with the most traditional aspects of mathematics. The tension between subjectivity and objectivity makes DMP even more interesting in terms of pedagogy. Similarly, Doxiadis (2003) discusses the notion of *paramathematics* to highlight the symbiosis between the paradigmatic and the narrative modes of thinking (Bruner, 1996).

In chapter four, the category sense-making was constructed based on two main aspects: (a) presence of arguments, examples, logical thinking, and connections between representations, mainly considering ways of reasoning and proving that support the conceptual or non-conceptual mathematical surprises and (b) absence of conceptual understanding, student’s incoherencies or miscommunications. Figure 5.4 illustrates a diagram that shows the sub-themes I am considering for the theme Voyeur - Sense-Making:

Figure 5.5: Sense-Making in Students’ DMPs



Following, Table 5.6 indicates where each of the DMPs would fit in these sub-themes, regarding the analysis I presented in chapter four:

Table 5.6: Sense-Making in Students’ DMPs

Presents arguments of reasoning and proof, examples, connections between representations, and/or ways to extend the problem	No conceptual understanding, incoherence or miscommunication.
#1, #4, #5, #6, #7, #8, #9, #10, #11, #12, #13, #14, #15, #16, #17, #18, #19, #20, #22	#2, #3, #7, #10, #11, #14, #16, #21

Following, Table 5.7 represents each of the sense-making aspects in contrast with the performance arts used[28]:

Table 5.7: Aspects of Sense-Making in Students' DMPs

	Skit	*Song*	*Animation*	*Puppet theatre*	*Poem / Drawing*
Arguments of reasoning and proof	#1, #12, #13, #17, #18	#14, #17	#10, #15, #22	#5, #8	#6, #20
Examples	#1, #4, #7, #9, #11, #12, #13, #17, #18, #19	#2, #4, #9, #14, #16, #17	#10, #15, #22	#5, #8	#6, #20, #21
Connections between representations	#1, #7, #9, #11, #13, #17, #18, #19	#4, #7, #9, #14, #16, #17	#10, #15, #22	#5, #8	#6, #20
No conceptual understanding, incoherence and/or miscommunication	#2, #11	#3, #7, #14, #16	#10		#21

From the table above, considering the discussions in chapter four, some insights may be presented. It is important to highlight that multimodality does not guarantee the conceptuality of the mathematical ideas in the DMPs. The multimodal nature of the DMPs offers conditions to explore visual, verbal/audio, gestural, and spatial representations of ideas. Usually, students' DMPs are rich in terms of visual examples and connections between representations and modes of communication. However, these connections are not necessarily connections between mathematical strands or ideas. The simple fact of exploring connections between representations and modes of communication are necessary, but they are not sufficient to offer conceptual mathematical surprises to the audience.

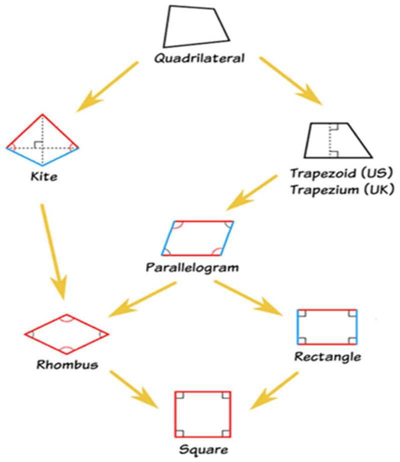
[28] Here, I am splitting the category Musical to Skit and Song.

In DMPs #15 and #22, students present convincing arguments of reasoning through the demonstration of a visual proof regarding equivalence of fractions, exploring visual aspects. The process of visualization in these DMPs is fundamental for mathematical thinking. The audience may see representations of figures in motion forming the representation of a same figure. Two figures with half size of a square (the unity) form the square. Four figures with one quarter of the size of the square form the square. And so on. The final scene shows all the different parts forming together congruent squares. These DMPs demonstrate, visually, that 1 = 2x(1/2) = 4x(1/4) = 8x(1/8) = 16x(1/16). As highlighted in chapter one, Hanna and Sidoli (2007) state that "visualisation can be most useful to the aspects of mathematical proof important to mathematics education, particularly those connected with explanation and justification" (p. 77). Visualization, which is an ordinary process explored in videos through the visual design of modality, involves a mental scheme that represents visual or spatial information (Presmeg, 1986). It is a "kind of reasoning activity based on the use of visual or spatial, either mental or physical, performed to solve problems or prove properties" (Gutiérrez, 1996, p. 9). Borba and Villarreal (2005) point out "visualization has been considered as a way of reasoning in mathematics learning" (p. 79), because it refers to "a process of forming images . . . and using them with the aim of obtaining a better mathematical understanding and stimulating the mathematical discovery process" (p. 80).

In skit performances students explore visual representations, making connections to oral words and gestures. The use of posters, manipulatives, and costumes in combination with speech and gestures has the potential to support arguments of reasoning and proof. Although some DMPs such as #1, #9, and #11 presented some verbal arguments and many connections to visual representations, these DMP do not present "very strong mathematical arguments." Most of the arguments presented in the skit DMPs are more descriptive and definitional rather than logical, analytical and deductive. However, some cases can be highlighted in terms of reasoning and proof. In DMP #1, for instance, students present verbal and visual arguments, but the nature of the mathematical idea is not conceptual. Students only explored traditional definitions of polygons and "non-polygons." However, in the process of figuring out the puzzles, students had to identify specific types of figure among many figures. They were engaged in a process of investigation to solve the puzzle to open a door and free Poly. This process was based on creation of hypotheses by visualization/experimentation, elaboration of conjectures, and refutation of some of the conjectures. Actually, these were puzzles in which students had to posed and solve the problems because they only saw the doors. Intuitionally, students had to see the first door and identify that the question to solve the problem (to open the door) was “which figure is a polygon?” Similarly, for the second door, the question was “which figure is a regular polygon?” Thus, although students did not explore a conceptual mathematical idea in DMP #1, they engaged themselves in a rich process of reasoning based on visualization, identification of properties of figures, and experimentation with manipulative materials in the plot of a skit performance.

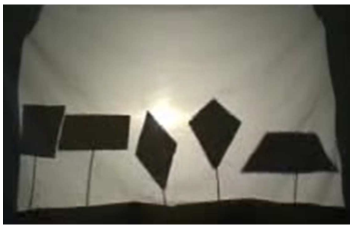
In DMPs #3, #5, #13, and #18 students explore the same idea toward quadrilaterals (a quadrilateral as a special case of others)(see Figure 5.6). Individually, these DMPs present some local gaps in terms of understanding, because they do not offer a direct and clear conclusion about the idea explored and its conceptuality. However, collectively, the set formed by these four DMPs is strongly supportive to verbal and visual arguments toward a conceptual mathematical surprise that a square is a specific case of rectangles and rhombus, a rectangle and a rhombus are specific cases of a parallelogram, and so forth. The following chart presents some images and transcriptions of these DMPs:

Figure 5.6: Relations between Quadrilaterals



- DMP #3: The lyrics of the first song is: *Square! I have four sides. Square! I am a quadrilateral. Square! It is not fair that they All call me such a dumb square. I can be a square and hip too. I can be cool just like you. Just have to realize that I am a rectangle too Therefore I have skills and you know.*
- DMP #5: In the story, Little Quad (a kite) meets a square, a rectangle, a rhombus, and a trapezoid. They contrast their properties. (See Figure 5.7).
- DMP #13: In the skit, Ryan, the rhombus, says: *I have to say that past statements aren’t that true because Steven has four vertices and four straight*

Figure 5.7: Quadrilaterals in *Little Quad’s Quest*



sides and doesn't that make him a rectangle and a parallelogram? Even a rhombus is true.

- DMP #18: Mr. Trapezoid says: *Oh! Who hurt Mr. Square? I bet you it was you Mr Circle. You hurt his relative Mr. Rectangle. A square is a rectangle. I bet that it was you!*

It is important to notice that DMPs #2, #3, #7, #10, #11, #14, #16, and #21 present some aspects of lack of conceptual understanding, incoherence and/or miscommunication. In DMP #2, the presenter produces incoherence when she introduces the hexagon by saying: "she has three sets of parallel lines and *two acute angles*." Probably, the presenter was confused with the introduction of the first candidate (the trapezoid), which is: "she has one set of parallel lines and *two acute angles*." In DMP #3 the student sings: "*I got four sides, four angles… I even got four vertices, therefore I am not 3D*." One can notice that, in this case, the conditions proposed by the student are not sufficient to make his mathematical/geometrical assumption fully consistent. That is, there is a 3D shape – the *tetrahedron* – that has four sides (faces), four "angles" (trihedral angles) and four vertices. Moreover, students are talking about three types of triangles – scalene, equilateral, and isosceles – and they sing: "*My lines of symmetry are three or one.*" An equilateral triangle – a triangle with all sides equal in length – has three lines of symmetry. An isosceles triangle, that is, a triangle with one pair of sides equal, has one line of symmetry. But, a scalene triangle, which is a triangle with no equal sides, has no lines of symmetry.

Like DMPs #15 and #22, DMP #10 also explores visually equivalence of fractions. However, this DMP #10 does not demonstrate clearly the equivalence of fractions as DMPs #15 and #22 do. DMP #10 does not show how all the representations of fractions together represent the same unity. Although the DMP relates two pieces of modeling clay to the fraction 1/2, four pieces to 1/4, and so on, the DMP does not show clearly the proportionality between the pieces that represent the fractions. (See Figure 5.8).

Figure 5.8: Misrepresentation (visual) of equivalence of fractions in DMP #10.

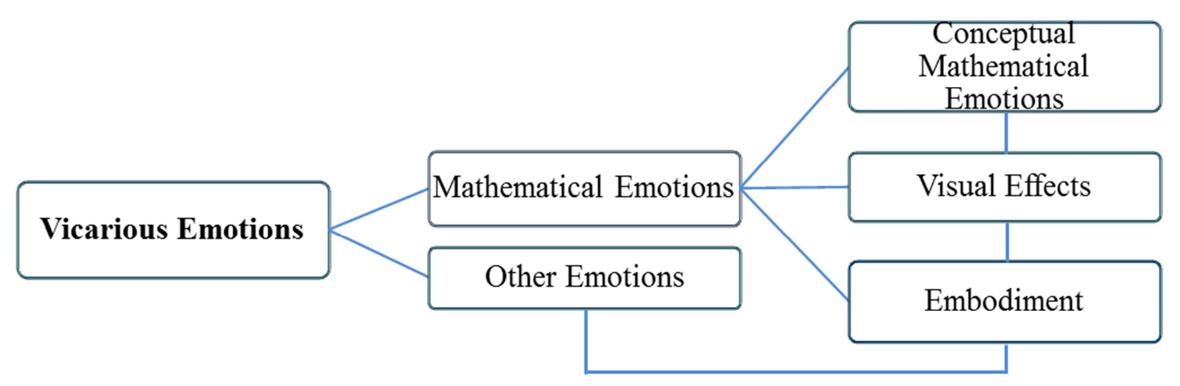
Every DMP in which songs are performed present some kind of conceptual gaps, miscommunication/mispronunciation or incoherence in terms of understanding. In DMP #3, for instance, there is no explanation about why a square is a special case of rectangle. In DMPs #7, #14, and #16 the students' pronunciations in the songs offer confusion to the audience in understanding the mathematical idea.

On the one hand, there is a "negative" aspect about songs as a form of mathematical communication. Lyrics are usually short – compared to a story. Thus, the amount of information in lyrics is limited, which introduces potential gaps in terms of communication of conceptual understanding. Songs also involve specific ways of pronunciation (tuning and phonetics in harmony with the melody and rhythm) and rhyme, which may make difficult the understanding for a listener. Then, songs offer ways for the audience to extend problems, not necessarily because the extension is explicitly present in the lyric, but mainly because lyrics explore subjective dimensions. Songs push the meaning production to the audience. On the other hand, music is a pertinent form of remembering information (Snyder, 2000) and, even though memorization is not the main skill to be developed in mathematical thinking in my point of view, it is an important part of the mathematical reasoning. Moreover, songs are popular forms of representation and dissemination of information in contrast to formal forms such as print-based texts. In fact, from a performative point of view, songs can offer specific types of surprises, emotions and visceral sensations to the audience that a written poem, for instance, cannot, and vice-versa.

Vicarious Emotions

According to Boorstin (1990), emotional moments are fundamental for good stories or movies. The use of the performance arts opens windows into communicating ideas with emotions, but it does not guarantee the presence of *mathematical emotions*. Regarding the analysis presented in chapter four, there is evidence to argue that students' DMPs offer vicarious emotions because the audience may feel what the actors were feeling (Boorstin, 1990; Gadanidis & Borba, 2008). Some of these emotions are mathematical emotions. Three main aspects can be thus discussed regarding emotions in students' DMPs: (a) Some DMPs present *mathematical emotions*. These emotions are mainly supported because students played roles as mathematical/geometric objects. Students use their bodies, gestures, verbal language, and costumes to make shapes "alive" and most of the feelings that shapes feel are based on their mathematical properties. The audience may feel these emotions vicariously (e.g., the triangle lost her head in DMP #2, now she is a trapezoid and she feels sick in bed or Little Quad feels sad in DMP #5 because he did not fit in world, but he felt happy when he discovers he is a kite and he flies). Regarding the nature of the mathematical idea explored, the emotion may be considered a conceptual mathematical emotion; (b) The use of *video effects* such as zooming-in effects on the actor's facial expressions or motion in animations may enhance the vicarious pleasure when the audience watches students' DMPs; (c) Some DMPs offer emotions, but they are not mathematical emotions necessarily. The following scheme represents the sub-categories explored within the theme vicarious emotions.

Figure 5.9: Vicarious Emotions in Students' DMPs



Mathematical Emotions through Embodiment.

In DMPs #2, #3, #5, #7, #8, #9, #11, #13, #14, and #18 students play roles as geometric objects. All these DMP are in video format and the arts used are music, skit, and puppet theatre. When students play roles as plane shapes such as triangles, quadrilaterals, or circles and solids like spheres, pyramids, or cylinders that are involved in a process of *thinking-as-a-shape* or *thinking-as-a-solid*. Students are thus engaged in a process of "imagining the *self*" based on the form and properties of a geometric figure, constructing a playful identity as a mathematical object. These processes can be understand from the point of view of embodiment and mathematical activity (Arzarello, Paola, Robutti, & Sabena, 2009; Gerofsky, 2009; Radford 2003; Williams, 2009), that is, from a semiotic and multimodal point of view, gestures and body language play very significant roles in mathematical meaning production and learning. Therefore, seeing students' DMPs as multimodal texts, the multiple designs explored in skits, songs and theatres offer ways for students to express mathematical emotions. The audience may vicariously feel these mathematical emotions.

According to Gerofsky (2010), "embodied learning theory rejects the notion of a split between mind and body or reason and sensation/emotion and explores pedagogies that integrate somatic, sensory and intellectual engagement on the part of learners" (p. 322). The very notion of embodiment focuses on the role of gestures in learning, including the exploration of manipulatives and technologies in general. The connections between students' verbal and non-verbal modes of communication are significant for meaning and knowledge production.

It is important to mention that some of the gestures expressed by students in their DMPs were superficially discussed within the issues on gestural design of multimodality. In this section, I attempt to highlight that playfulness emergent from performance arts in the classroom may offer ways for students to improve some of their mathematical experiences in terms of embodiment when they play roles as geometric objects, within the context of dramatic and emotional events.

In DMP #2 a student plays a role as a trapezoid imagining herself as a triangle with its head cut-off. She feels sick in bed for this reason. This emotion is related to a connection between representations of two different shapes, which is considered a "conceptual idea." Then, DMP #2 offers conceptual mathematical emotions, that is, emotions directly related to conceptual mathematical ideas.

- DMP #2: The triangle sings: *Head, oh head, I lost my head. And head, oh head, I could've been dead. Head, oh head, I feel seek in bed. Now, I am a trapezoid*

In DMP #5, Little Quad's feels unhappy because he thinks he does not fit in the world like the other quadrilaterals. But he identifies relations between his properties and the properties of the other quadrilaterals as well as a way to fit in the world as a kite, becoming happy then. The same type of conceptual mathematical emotion the audience may vicariously feel in DMP #5, it is possible to feel in DMPs #3, #13 and #18. In DMP #3, a student performs a song in which a square "feels inferior" to a rectangle, but he presents arguments in the song that a square is a rectangle too so a square can be cool like a rectangle. In DMP #13, a square feels "discriminated" by a parallelogram and by a rectangle because they do not see that the square is a special case of them. In DMP #18, Mr. Square has hurt by Mr. Isosceles and there was an acute angle dent marked on Mr. Square. The use of manipulatives and "math costumes" in these DMPs may enhance the sense of embodiment for both students and the audience.

- DMP #3: The lyrics of the first song is: *Square! I have four sides. I am a quadrilateral. Square! … I can be cool just like you. Just have to realize that I am a rectangle too.*
- DMP #5: Little Quad says: *See? You don't even know my name. If I was a square, or a rectangle, or a rhombus, or a trapezoid, then you would know. But no, I am just Little Quad, nobody knows me or needs me… Nobody will need my shape for anything.*
- DMP #13: Steven Square says: *Well… I was on work one day and out of the blue a man poked me in the eye, as you can see. It hurt and he said he did because I was a square. I don't find this very well.*
- DMP #18: *Mr Circle* says: *I have no vertices… Hey Mr Isosceles! I heard you hurt Mr. Square. You have an acute angle, and there is an acute angle dent mark on him. Oh! And you have three vertices.*

In DMPs #7, #8, #9, #11, and #14 students also explore emotions through embodiment by playing roles as geometric objects. From these DMP, the audience can listen to students saying or singing things such as:

- DMP #7: "*I have four lines of symmetry… I have four edges going around my face… I have four vertices … I am a 2D shape… I have two pairs of parallel lines… I have four ninety degree angles, which make my vertices.*"

  "*I am symmetrical. I have three lines of symmetry.*"

  "*Circle, I am a circle. I am as happy as a kangaroo. I am so proud that I came to this game to you.*"
- DMP #8: "*I am the best because I have so many right angles.*"

  "*I am the best because I have a circumference and I have infinity symmetrical lines.*"

  "*My best physical properties are that there are three kinds of me: isosceles, scalene, and equilateral. Scalene is the best of course.*"
- DMP #9: "*We are the polygons, we have straight edges… We are the polygons. We're always closed… We are the polygons, we're everywhere* "
- DMP #11: The sphere gets upset: "*Nobody calls me a cone!!!*"
- DMP #14: "*I am Cubie. I like to dance and sing. I'm strong sitting down. I go blank blank. I have six faces. Eight vertices. Up to the rescue.*"

  "*I am Rudy. The Rectangular Prism. And I love to sing, sing, sing, sing. I sing in the bathroom, when I wash my faces. And when I wash my vertices, in very wet faces.*"

"I am Silly Cylinder. I live at the 3D town. I have two faces. Then I can make cookies. Which will never make you frown."

On the one hand, the aspects highlighted in the quotes above show some evidence of different types of students' embodiment in their DMPs. On the other hand, the nature of such embodiment appears to be different from those discussed in the literature. Although students conducted performance "*thinking-and-acting as geometric objects*" in their DMPs, they did not explicitly explore the use of "mathematical gestures" to communicate and represent ideas. Although students performed roles as geometric objects, they did not perform mathematics in the way of "being a graph" as discussed by Gerofsky, Savage and Maclean (2009) to explore the multimodal and gestural-kinesthetic dimension of students using their bodies to represent the dynamicity of graphs of functions and its coordination with algebraic and table representations. Thus, DMP has a great potential for embodiment in mathematical activity, even though the students could have explored more deeply mathematical gestures and ways of embodiment in their DMPs.

General Mathematical Emotions.

In DMP #4 students explore a conceptual mathematical idea by connecting multiple representations of triangles to other figures. In the musical, emotions are expressed toward the mathematical idea and students use their bodies and gestures to represent triangles and figures formed by triangles. Students do not express their feeling explicitly through words, but the audience may feel that students are very excited and curious in exploring mathematics when they say things such as "*Hey. What's going on over there?... Cool! Oh yeah! Can we go see it? Come on guys! Let's go there and see it!"*

In DMP #6, through a poem in power point format, students express very emotional feeling and support to a colleague who is battling cancer. The students connect their emotions in supporting and helping her to a conceptual mathematical idea involving factorial growth, offering to the audience ways of experiencing mathematical emotions.

The use of digital media is also a significant aspect regarding the mathematical emotions the audience may feel through students' DMPs. In DMPs #10, #15, and #22, for instance, students do not use their bodies to play roles as mathematical objects. Furthermore, these three DMPs connect multiple representations and different strands. Thus, they may offer conceptual mathematical emotions to the audience. The animation effect in these DMPs offers ways of seeing mathematical objects in motion, which may offer emotions to the audience regarding the fact that mathematical objects are "alive." As mentioned, the use of digital media in these animations (software of video edition) conditions the stop-motion animations. The mathematical ideas are represented and communicated through a process of feeling-with-digital-media. Interestingly, in the credits of DMP #15 students make explicit the use of the paper, markers, and web cam. That is, students are "acknowledging" the use of artefacts that are fundamental to the communication of the idea explored. The same happens in the credits of the DMP #22. These facts are interesting because I am assuming that technology shapes the production of mathematical knowledge in such ways that humans-with-media (Borba & Villarreal, 2005) form "feeling-thinking collectives" and students are recognizing the relevance of materials in the production of their DMPs.

Another way of seeing the relevance of the use of digital media in offering mathematical emotions to the audience refers to the process of video recording. Close-ups on actors' faces enhance the vicarious pleasure (Boorstin, 1990). All the DMPs based on musicals, skits, and songs use some zoom-in effect of recording (see Figure 5.10 a-c). This resource is also used in emotions that are not necessarily mathematical emotions. The actions of feeling collectives of human-with-video-cameras are significant then to provide emotions to the audience. Although the nature of the data explored in this research does not offer evidence to identify who was (were) video recording the performances, it is possible to guess that it can be a collective formed by students, and/or teacher(s). Gadanidis et.al (2010) and Scucuglia, Borba and Gadanidis (2010) emphasize that the process of recording a performance for a DMP can be a collective activity in a classroom and it can be conducted by a group of elementary school students using cell phones, for instance.

Figure 5.10 a-c: Close-ups on students' facial expressions – enhancing the vicarious pleasure.

Non-Mathematical Emotions.

In most of the DMPs students also express emotions that are not mathematical emotions (DMPs #1-9, #11-14, #16-20). They explore emotions in the general plot of dramatic events. In DMP #1, for instance, where students explore polygons through a skit, the mathematical idea explored is not conceptual, but the dramatization or the contextualization as a story is fundamental within the notion of mathematical performance. Overall, the arts offer ways to communicate ideas in ways that makes the processes of communicating and representing mathematics human and aesthetic. Then, every DMP analyzed in this research has the potential to evoke emotions to the audience, mainly those which involve very clear human characteristics such as human characters, use of voices, etc. The use of emotions in communicating mathematics also helps students to disrupt potential stereotypes they have about learning mathematics (see Gadanidis, Gerofsky & Hughes, 2008; Gadanidis & Scucuglia, 2010; Gadanidis et. al. 2010). Furthermore, stories, songs, skits, and animations supported in video format are popular forms of representation and, in terms of semiotics, they empower the typical elements that form the communicational nature of print-based texts, in which mathematical symbolism is traditionally represented. Table 5.8 summarizes the analysis of vicarious emotions in the students' DMPs analyzed in this research:

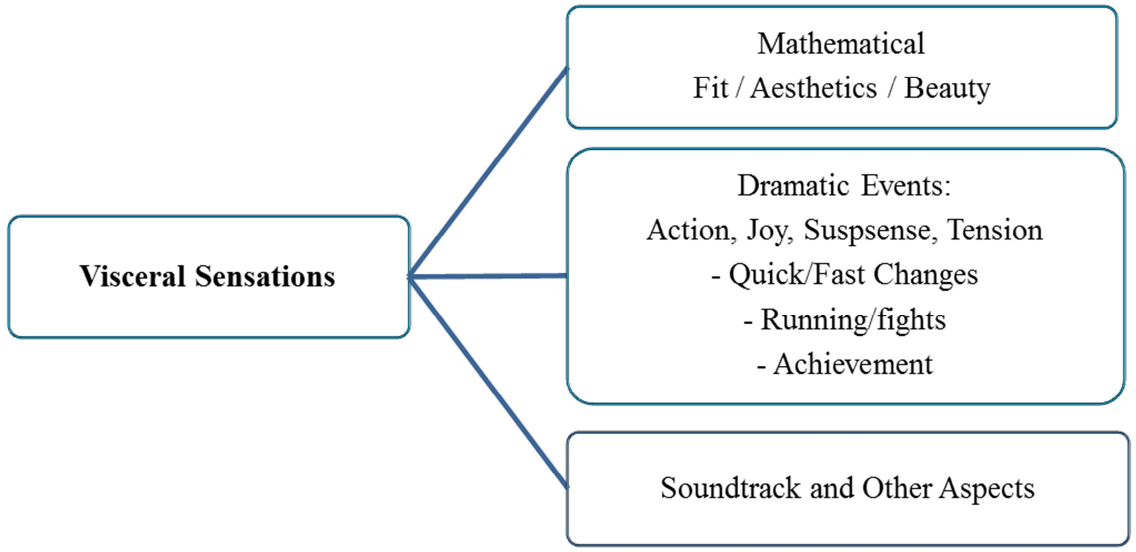
Table 5.8: Vicarious Emotions in Students' DMPs

	Embodiment	*Stop-Motion Animation*	*Close-up*	*Story*
Conceptual Mathematical Emotions	#2, #3, #4, #5, #13, #18	#10, #15, #22	#2, #3, #4, #10, #15, #13, #18, #22	#6
General Mathematical Emotions	#7, #8, #9, #11, #14, #20		#1, #7, #11, #12, #14, #16, #17, #19, #20	#20
Non-Mathematical Emotions	#2, #3, #4, #5, #7, #8, #9, #11, #13, #14, #18, #20		#1, #2, #3, #4, #7, #11, #12, #13, #14, #16, #17, #18, #20	#6, #20

Visceral Sensations

According to Boorstin (1990), the visceral eye refers to direct experiences, to the sensations the audience may feel. The visceral pleasure usually emerges from those moments of action, joy, tension, or suspense of a story in which the audience feels its own feeling rather than the actors' feelings. Soundtracks usually enhance visceral sensations. In DMPs, visceral sensations are linked to (a) sense of mathematical fit, aesthetics and/or beauty (Sinclair, 2000; Gadanidis, 2004) and (b) dramatic events involving scenes of actions, suspense, tension or joy that are supported by quick or fast changes (Osafo, 2010), scenes involving running or fights, and/or achievement, for instance. Figure 5.11 illustrates the main aspects of the visceral eye regarding the analysis of the DMPs in this research:

Figure 5.11: Visceral Sensations in Students' DMPs



Sense of Mathematical Fit.

Sinclair (2006) states that "the phase of playing around or 'getting a feel for' is aesthetic in so far as the mathematician is framing an area of exploration, qualitatively trying to fit things together and seeking patterns that connect or integrate" (p. 95). Sinclair also posits that "mathematicians can be attracted by the visual appeal of certain mathematical entities, by perceived aesthetic attributes such as simplicity and order or by some sense of 'fit' that applies to a whole structure" (p. 99). Thus, the sense of *mathematical fit* (related to aesthetics and beauty in mathematics) is regarded though the lenses of visceral pleasures within the notion of DMP (Gadanidis & Borba, 2008).

In this study, many students' DMPs potentially offer visceral sensations to the audience by exploring the sense of mathematical fit regarding several aspects. In DMP #1, students explore the fact that specific types of figures fit as a solution of a problem or puzzle, that is, to open the doors to free Poly in the story, students had to figure out that polygons and regular polygons were the shapes that would fit in finding a mathematical/geometric solution for the problem. In DMP #12 students also use the notions of diameter and radius of a circle to solve a problem involving the process of baking a specific type of cake.

In DMPs #2, #3, #4, #5, #13, #15, #17, #18, and #22 students explore the notion that some figures fit in others. DMP #2 explores how a trapezoid fits in a triangle because students are imagining a trapezoid and a "triangle with its head cut-off." Similarly, in DMP #4 students explore how triangles fit in many shapes such as parallelograms and regular hexagons. In DMPs #3, #5, #13, and #18 students explore how a specific quadrilateral is a special case of others. By exploring, for instance, the fact that a square is a special case of rectangle, these DMPs explore how the properties of rectangles fit on the properties of squares. In DMPs #15 and #22 students explore how isometric pieces of representations of figures may fit together, forming a whole figure in the process of demonstrating visually equivalence of fractions. In DMP #17 students explores how a square and four triangles may fit each other forming a square based pyramid.

In DMPs #2, #5, #6, #8, #9, #11, #12, #14, #16, #17, and #20 students explore how a mathematical idea (or a representation of a mathematical/geometric object) may fit in the world around the students. These connections between mathematics and everyday contexts were discussed in the section *conceptual mathematical surprises*.

In DMP #6 students explore patterns through a factorial growth contextualized in an everyday situation. In DMP #7 students also explore some aspects involving patterns. A student playing a role as a square poses a question to the audience - *Is there a pattern going on?* - after responding he has four lines of symmetry, four edges, and four vertices. Table 5.9 presents the ways some DMPs may offer visceral sensations to the audience regarding the exploration of senses of mathematical fit:

Table 5.9: Sense of Mathematical Fit in Students' DMPs	
Shapes fit one to another	#2, #3, #4, #5, #13, #15, #17, #18, #22
Mathematics fit in the world	#2, #5, #6, #8, #9, #11, #12, #14, #16, #17, #20
Patterning	#6, #7
Solution of a puzzle or problem	#1, #12

Dramatic Events.

Some aspects found in students' DMPs offer visceral sensations to audience by exploring quick or fast changes. That is, some mathematical ideas explored through performance offer a sense of "mathematical action," offering a sense of direct experiences to the audience. In DMP #6, quick change may potentially be felt through the exploration of a factorial growth. As shown in the last chapter, this type of growth is much more intense when compared with a linear growth, for instance.

Fast changes can also be felt in DMPs #10, #15, and #22. These are the animations produced by using the stop-motion format to visually explore the equivalence of fractions. Sequences of discrete moments represented through pictures are shown in such a way that they offer to the audience a sense of rapid motion.[29] In addition, this sense is linked to a sense of mathematical fit, offering rich ways for the audience to experience a visceral mathematical pleasure through a visual demonstration that seeks meaning production toward the notion of equivalence of fractions.

The use of dramatic events through skits and musicals also offers ways for the audience to sense mathematics viscerally, mainly when there is an emphasis on action or quick gestures. The audience may sense the joy of a moment of achievement and the tension supported by a suspense or thriller thematic, action of an intense scene. Some of these events emerge directly through the exploration of mathematical ideas. Some emotions may be felt as visceral sensations because the frontiers between vicarious emotions and visceral sensations tend to get blended sometimes. In DMP #1, there is tension in trying to find Polly and happiness when Polly is found. In DMPs #2, #16, and #19, students get very excited when they are proved by the judges and they feel disappointed and frustrated. The same type of sense of joy/achievement may be

[29] There is an interesting discussion about the discrete/continuous nature of films, that is, on the use of sequences of discrete images in order to offer the perception of continuity. This issue and its relation to DMP may be explored in a future study based on the notions of movement-image (Deleuze, 1986) and time-image (Deleuze, 1989).

found in DMPs #4, #5, and #9. In DMP #4, in particular, students are excited, running around a large room to experience how many shapes can be made with representations of triangles. In DMP #5, the way Little Quad flies and discovers he fits in the world may also be interpreted as a visceral moment. DMP #11 explores the way a sphere rolls fast.

In DMPs #11, #13, #14, and #18 students explore stories involving disputes or "fights" and they are related to mathematical ideas. In DMP #11, the sheriff arrests the sphere that rolls illegally everywhere, after fighting with the sphere. In DMP #13, the square was attacked by the rectangle and the parallelogram because they claim the square is not like them. In DMP #14, Pointacula terrorizes the solids in 3D Town. In DMP #18, Mr. Isosceles hurt Mr. Square, and thus Mr. Isosceles is viscerally captured by Mr. Circle.

Table 5.10 shows the dramatic events of the DMPs that can offer visceral sensations to the audience through the exploration of mathematical ideas:

Table 5.10: Visceral Sensations - Dramatic Events in Students' DMPs

Quick Change by growth	#6
Quick change by motion/animation	#10, #15, #22
Scenes of Joy (achievement)	#1, #2, #4, #5, #9, #16, #19
Scenes of Action (fights, fly, or run)	#4, #5, #11, #13, #14, #18
Scenes of Suspense and Tension	#1, #2, #11, #13, #14, #16, #17, #18, #19

Soundtracks and Other Aspects.

Soundtracks offer ways to the audience to experience visceral sensations (Boorstin, 1990). DMPs #2, #3, #4, #7, #9, #10, #11, #14, #15, #16, #17, #18, and #22 use songs in one or more moments in the composition of the plot of the performance. In DMPs #2, #3, #4, #7, #9, #11, #14, #16, #17, and #18 (musical and songs), students are singing the songs. These DMPs has also characteristics of video clips, which are naturally visceral according to Boorstin (1990). In DMPs #10, #15, and #22 (stop-motion animation), students use instrumental soundtracks that probably were not created by them. Some of the DMP are parodies of TV shows (#2 and #16) or explore a popular "math joke" (#1), which may enhance the visceral sensations offered to the audience since Boorstin states that the appeal to popular culture offers visceral sensations.

It is important to recognize in this study that Boorstin's (1990) categories are used in an adaptive way, because the categories are originally proposed considering the process of making Hollywood films. These films are usually long (60 to 180 minutes, depending on the genre). The DMP in video format analyzed in this study are short in time (approximately 3 minutes), then, each DMP is potentially visceral, that is, students' DMPs have the potential to offer direct experiences to the audience regarding the short period of time in which mathematical ideas are presented through multiple modes of communication. Table 5.11 presents general aspects in terms of the visceral sensations the audience may potentially feel by reading students' DMPs:

Table 5.11: Visceral Sensations - Soundtracks and Other Aspects in Students' DMPs

Soundtracks	#2, #3, #4, #5, #7, #9, #10, #11, #14, #15, #16, #17, #18, #22
Parody and popular content	#2, #3, #16, #19
Compile multiple modes and present the ideas in a short period of time (DMPs in video format)	#2, #3, #4, #5, #7, #8, #9, #11, #12, #13, #14, #16, #17, #18, #19, #22

Final Comments of Chapter Five – Part One

In the first part of this chapter I presented a cross-case analysis of students' DMPs through a performance arts lens. I used Boorstin's (1990) categories to discuss the types of surprises, sense-making, emotions, and visceral sensations the audience may potentially feel by reading students' DMPs as multimodal texts/narratives.

It is important to acknowledge that I analyzed only the students' DMPs available in the first year of the Festival. It means that students did not have examples as references to produce their DMPs. Thus, I would argue that students' DMPs offer some mathematical surprises. These surprises are mostly supported by the use of the performance arts and digital media: multimodality. That is, it is surprising to see students using the arts and producing multimodal texts to communicate their mathematical ideas. Some of the mathematical ideas explored may be potentially considered conceptual, that is, some of the DMPs offer contexts for seeing the new and wonderful in mathematics but this is not well developed. Multiple representations and multiple strands are not well connected, and mathematical ideas are not deeply related to everyday situations or problems.

The multimodal nature of DMPs is one of its most significant pedagogic attributes. Mathematics is traditionally communicated through print-based texts through the use of writing, charts, diagrams, and graphs. Digital media affordances offer ways to represent mathematical ideas through multiple modes, which adds non-usual layers of signs in communicating mathematics (e.g. audio, gestures, space). However, multimodality does not guarantee the conceptual nature of the idea explored in the DMPs.

Students' DMPs present examples and visual and verbal arguments, but, generally, most of the DMPs present gaps in terms of sense-making and they do not support the conceptually of the idea explored. Two aspects are considered: (a) DMPs are short and it is not a simple task to present mathematical arguments in-depth in a short period of time, even though visual aspects can be explored (e.g., visual proofs); (b) there is a kind of "tension" between the traditional objectivity of mathematical reasoning and the subjectivity emergent with the arts. Some students present incoherencies in terms of reasoning and sense-making in their DMPs such as lack in logical (or deductive) thinking and miscommunication. Overall, students provide interesting examples to the audience by exploring the visual nature of digital media and connecting representations, but they do not present clear or convincing arguments that support the conceptuality of their ideas. They also offer some ways to extend the problems, but these ways are not explicit in most the performances. The audience may find ways to extend and reflect on the problems based on the subjectivity emergent from the performance arts. The use of songs, for instance, helps students and the audience in terms of remembering the contents and ideas, but they push the meaning production to the audience since lyrics usually do not offer much information and are shaped by rhymes. Thus, although students' DMPs have the potential to offer mathematical surprises to the audience, the mathematical sense-making aspect of the DMPs is seen as problematic in this study.[30]

Students depict emotions in their DMPs, but *mathematical emotions* (emotions related to conceptual turning points) are not common. Emotions are common feelings explored in stories/narratives and students' DMPs are digital narratives. However, only some mathematical emotions emerge in the stories through the use of the arts, mainly through the notion of embodiment. Skit performances in which students play roles as mathematical/geometric objects

[30] Osafo (2010) pointed out a similar finding, when he analyzed (also using Boorstin's lenses) digital performances on science education available at YouTube. The performances offer surprises, but they have lacks in terms of understanding or sense-making.

involve use of speech and gestures in conducting dramatic events regarding exploration of mathematical properties. Vicariously, the audience may feel some mathematical emotions that emerge in DMPs through performative activities in which *students-think-as-mathematical-objects* and form thinking-feeling collectives.

Students' DMPs provide some visceral sensations to the audience by exploring sense of mathematical fit such as how representations of mathematical objects fit in the world. Dramatic events involving scenes of action or suspense are commonly found in skits and musical performances (use of soundtracks) and the plot of these dramatic moments are direct related to the exploration of mathematical ideas.

Therefore, overall, considering a performance arts lens (Boorstin, 1990), I suggest that the production of a *conceptual mathematical DMPs is a rare event* within those analyzed in this study. That is, most of the students' DMPs do not offer, simultaneously, conceptual mathematical surprises, sense-making, emotions, and visceral sensations to the audience.

In the next part of the chapter I present a cross-case analysis using as lens the strands and processes of the curriculum (Ontario Ministry of Education, 2005).

Chapter Five - Part Two: A Cross-Case Analysis through the Lens of the Mathematical Strands and Processes of the Ontario Curriculum

In chapter two, I introduced the mathematical strands and processes of the K-8 Ontario Curriculum (Ontario Ministry of Education, 2005). Although these two themes about the curriculum were not explicitly displayed in chapter four (i.e. they did not name sections in the last chapter), the reader shall notice that the strands and the processes were discussed along the chapter within the four main categories on performance arts (Boorstin, 1990). The mathematical strands and processes are already defined as important in the curriculum and they offer additional insights in the analysis of this study. Through a curriculum lens, the mathematical ideas and activity in students' DMPs may be interpreted and analyzed considering significant aspects about learning, teaching, subject matter, and milieu. It offers an initial approximation in terms of insights between DMP and curriculum.

The Mathematical Ideas: Contents and Strands in Students' DMPs

In most of the DMPs (#1-9, #11-14, #16-19, and #21) students explored ideas about Geometry, in specific, properties and/or examples of plane figures (e.g. polygons and circles) and solids. In DMPs #10, #15, and #22 students explored equivalence of fractions, but also made connections to geometric representations. Only in DMP #6 students explored factorial growth and in DMP#20 they explored measurement. Among those DMPs in which students explored Geometry, in DMPs #1-5, #7-9, #12, #13, #16, #18, #19, and #21 students explored plane figures (mainly polygons). In DMPs #11, #14, and #17 students explored solids, in particular, in DMP #17, they explored solids and some aspects of the planes figures that form the solids (faces). Table 5.12 presents the content/ideas explored in each DMP:

Table 5.12: Content/Ideas Explored in each DMP

Geometry		Equivalence of Fractions	Factorial Growth	Measurement
Plane Figures	*Solids*	#10, #15, #22	#6	#20
#1, #2, #3, #4, #5, #7, #8, #9, #12, #13, #16, #17, #18, #19, #21	#11, #14, #17			

In terms of strands, based on those pointed out in the K-8 Mathematics Ontario Curriculum (Ontario Ministry of Education, 2005), there is a clear emphasis on *Geometry and Spatial Sense* regarding the nature of the ideas explored in most of the DMPs analyzed. In the DMP that explore equivalence of fractions, students actually made connections between two strands – *Number Sense and Numeration* and *Geometry and Spatial Sense* – regarding the fact that they used visual/geometric representations of equivalent fractions, as parts of representations of rectangles. In DMP #6, which explores factorial growth, students connected three strands – *Number Sense and Numeration*, *Measurement* and *Patterning and Algebra* – by exploring, for instance, a sequence formed by "twenty dimes" (n = 1), "twice as much" (n = 2), "three times as both of you" (n = 4), and so on. In DMP #20, that explored ways of measuring time and money, students explored ideas within the strand *Measurement* with superficial connections to *Number*

Sense and Numeration. Students did not explore ideas within the strand *Data Management and Probability* in any of the DMPs. Table 5.13 presents the strands explored in each DMP:

Table 5.13: Strands Explored in each DMP			
Geometry and Spatial Sense	*Number Sense*	*Patterning and Algebra*	*Measurement*
#1, #2, #3, #4, #5, #7, #8, #9, #10, #11, #12, #13, #14, #15, #16, #17, #18, #19, #21, #22	#6, #10, #15, #20, #22	#6	#6, #20

Although students explored connections between representations and modes of communication in their DMPs (as discussed in the first part of this chapter), overall, they did not explore many significant connections between strands. According to Gadanidis and Hughes (2008), conceptual DMPs explore big mathematical ideas through the articulation between representations and ideas or strands. Therefore, a lack of *connections of strands* is a characteristic of elementary school students' DMPs in the Festival in 2008, based on the fact that most of the DMPs were produced focusing only on the strand *Geometry and Spatial Sense*.

One question must be discussed on this point: why did most of the students explore ideas on Geometry in their DMPs? One answer would be that Geometry's nature highlights visual representations and the affordances of digital media also have a visual nature. Students linked the visual nature of Geometry to the visual nature of using videos, making a "comfortable/expected zone" to explore, communicate, and represent ideas. That is, students are exploring visual representations in their DMPs, using content only from one area (Geometry) that traditionally explores visualization. The lack of connecting representations and ideas from different strands in students' DMPs is also evident in the curriculum document. By separating and isolating the strands, the curriculum potentially narrows down the possibilities of articulating multiple representations and strands. The traditional nature of Algebra, for instance, is very numeric and symbolic. Teachers usually do not link Algebra to other strands when they teach Algebra and they do not explore visual representations in Algebra.

Alternatively, the nature of mathematical knowledge in classrooms could be the one of *opening windows* (Noss & Hoyles, 1996; Gadanidis, 2005). DMP has the (communicational, representational, and pedagogical) potential to offer ways of opening windows into knowing mathematics in the classroom (Gadanidis, Gerofsky & Hughes, 2008; Gadanidis, Cordy & Hughes, 2011). However the use of DMP does not guarantee the opening of windows necessarily. The nature of the mathematical idea explored and the use of multiple representations are the most significant aspects in opening windows into mathematics. One example of DMP that offers ways of opening windows is the *L-Patterns* (see the full description in Prologue). The DMP *L-Patterns* explores sequence and series of odd and even numbers by connecting *Algebra and Patterning* to *Geometry and Spatial Sense*, *Number Sense and Numeration*, and *Measurement*. The connections between strands are fundamentally shaped by the connections between representations. Thus, the *L-Patterns* is an example of DMP that does not only connect multiple representations, it connects multiple representations and strands. Moreover, the *L-Patterns* can be explored through several school levels based on the notion of "low floor and high ceiling", that is, one can be engaged in the exploration with minimum knowledge about the topic but the nature of the idea and the use of multiple representations offer ways to explore the idea further. For more details about this discussion see Gadanidis, Hughes, and Borba (2008) and Scucuglia, Gadanidis, and Borba (2011a; 2011b).

The Curriculum's Processes in the Students' DMPs

In this section I use the curriculum's mathematical processes (Ontario Ministry of Education, 2005) as lenses for the cross-case analysis. These processes are already defined as important in the curriculum and they offer new insights in the analysis because Boorstin's (1990) categories are not originally related to mathematics. The seven mathematical processes pointed out in the K-8 Ontario Mathematics Curriculum are: problem-solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating. These processes overlap and, even though they were not explicitly the categories of analysis in the last chapter, most of the fundamental aspects that characterize them were part of the analytical lenses in chapter four. Following, I present a discussion of some emergent perspectives within this study considering the mathematical processes of the curriculum (Ontario Ministry of Education, 2005).

Problem-solving.

Although the process of producing a DMP can be understood based on the notion of problem-solving, it was not the main focus in this research. I did not analyze the process of producing a DMP, I analyzed only the DMP. However, some aspects can be discussed because each DMP offers some insights toward the process of its creation.

The curriculum mentions that problem-solving focuses on the "mathematics in the real world . . . helps [student] connect mathematics with situations outside the classroom" (Ontario Ministry of Education, 2005, p. 12). This connection, for instance, was considered in my analysis to discuss the *Voyeur - new/wonderful/surprising* category as well as how representations of mathematical objects may fit in the world as part of the *visceral* eye. In DMPs #5 and #8, for instance, students explored how representations of geometric figures may be identified and the world. In DMP #12, they used the definitions of radius and diameter of a circle to solve a posed problem of baking a cake regarding specific characteristics of its format. Furthermore, problem-solving highlights "enjoyment in mathematics" (Ontario Ministry of Education, 2005, p.12) and the performative nature of DMPs is very playful. DMPs offer ways of seeing the joy in mathematics (Gadanidis, Hughes & Borba, 2008). Skits, songs, musical, or puppet theatre offers playful, non-traditional and enjoyable environments to explore and communicate mathematics.

Collaboration is also an important aspect in problem-solving. It "promotes the collaborative sharing of ideas and strategies, and promotes talking about mathematics" (Ontario Ministry of Education, 2005, p. 12). All DMPs analyzed in this study were produced collectively through collaboration between students (and teachers). All the DMP have two or more authors. Although teachers do not appear as actors or singers in the DMPs, the role they played in supporting students in their strategies of performing is fundamental. In the production of DMPs, students-and-teachers-with-media form thinking collectives in the production of mathematical knowledge. The use of gestures, verbal language, or manipulatives shapes and reorganizes the mathematical thinking emergent from the collective experiences in the classrooms. DMP #4, for instance, involves a whole class in which many students collaborate one to another by using, for instance, their bodies to create collective representations of triangles in the classroom. Through the notion of collaboration, students' DMPs offer evidence that mathematical experiences in the classroom do not have to be individual and isolated. The joy of mathematics usually emerges from collaborative experiences students have in classrooms and the playfulness of the performance arts opens windows into the collectiveness in the pedagogic scenarios, which are significant elements of postmodern curricula (Doll, 1993).

The strategy model mentioned in the curriculum document is based on (a) understand the problem; (b) make a plan; (c) carry out the plan and (d) look back at the solution. DMP can be considered in part as a problem-solving experience because there is a parallel between the strategy model and the process of how to structure mathematics to elucidate surprises, sense-making, emotions and visceral sensations, which is part of a good story within Boorstin's (1990) perspectives and fundamental aspects of conceptual DMPs.

Reasoning and Proving.

Reasoning and proving refer to mathematical understanding and thinking (Ontario Ministry of Education, 2005). Significant aspects regarding the focus of this study on the nature of students' DMPs refer to students' reasoning, heuristics, understanding, thinking, meaning and knowledge production. The discussions in the section *Voyeur - Sense-Making*, for instance, highlighted some aspects of students' logical, intuitive and deductive thinking when exploring mathematics in their DMPs. In DMP #1, for instance, students explore the notion of polygons and non-polygons based on experimental puzzles where they have to think intuitively and deductively to figure out the questions and the solutions of the puzzles.

However, overall, similarly to Osafo (2010), I found in this study that there is a lack in terms of sense-making in students' DMPs. There is some evidence that this lack exist because DMPs are "short" texts and the arts in general deals with subjectivity. That is, in song performances such as DMPs #2 and #3, for instance, the plot of mathematical ideas is built on a short amount of information and, instead posing ideas objectively, these DMPs use only few words based on rhymes. This characteristic offers some insights to the audience, but, it does not guarantee that the ideas are being communicated clearly with sufficient arguments that justify the statements posed. Again, the issues related to the notion of *paramathematics* (Doxiadis, 2003) appear as epistemological and pedagogical tensions involving the paradigmatic and the narrative modes of thinking.

Reflecting.

The process of producing DMPs requires engagement in practicing the performance and may require engagement with the use of digital media. The process of practicing a skit, for instance, usually requires practice/repetition to communicate ideas clearly though gestures, actions, speech, and so forth. These processes are fundamental for learning in developing students' communication skills (Gadanidis, Hughes, & Cordy, 2011). In the end of DMP #19, for instance, the audience can listen to the teacher saying the following words to the students when they finish the skit performance: "*Ok! Hey guys, that was a lot better!*" This comment means that it was not the first time students were presenting the skit. They were practicing the dramatic events, repeating their actions, gestures and speeches in such way they were seeking to optimize the communication of their mathematical ideas. Deleuze (1994) states an important relation between semiotics, learning and theatre regarding the very notion of repetition.

> The reproduction of the Same is not a motor of bodily movements. We know that even the simplest imitation involves a difference between inside and outside . . . Learning takes place in the relation between a sign and a response (encounter with the other) . . . To learn is to constitute this space of an encounter with signs . . . Signs are the true elements of theatre. They testify to the spiritual and natural powers which act beneath the words, gestures, characters and objects represented. They signify repetition as real movement. (Deleuze, 1994, pp. 22-3)

Furthermore, typical classroom experiences are not recorded and they are not shared publicly. Each DMP as a product (as a multimodal narrative or multimodal text) offers ways to students and teachers reflect on their actions and ways of communicating because they can watch or read their DMP as many times they wish after the creation of the DMP. That is, these experiences help inform or restructure future pedagogic experiences, offering new insights on students' learning based on the reflection of their previous recorded actions. In this scenario, teachers also may assume a very significant role in helping students to create good stories, that is, teachers play very important pedagogic roles in supporting students on the creation of good multimodal texts in which students use the performance arts to communicate their learning. Within the notion of cognitive ecology (Borba & Villarreal, 2005; Levy, 1993), thinking collectives of students-and-teachers-with-DMP may be found in reflective actions even after the creation of the DMPs. Conceptual mathematical surprises, for instance, may offer ways to students think mathematically in future events of their lives after the production of a DMP.

DMPs are also publicity available online and they may be accessed anytime. Thus, the virtual environment of the Math + Science Performance Festival works as a nexus that offers ways of forming collective intelligences (Levy, 1998), which the nature is fundamentally reflective. Gadanidis and Geiger (2010) have referred to the Festival as "one example that helps bring the mathematical ideas of students into public forums where it can be shared and critiqued and which then provides opportunity for the continued development of knowledge and understanding within a supportive community of learners" (p. 102). Moreover, from a narrative point of view (Bruner, 1994), the fact that DMPs are shared publicly online makes students reflect about their experiences and ways of communicating a mathematical idea to the audience. Thus, the role of the audience shapes the ways students reflect about their learning and the ways they communicate mathematics in the DMPs, because students reflect on how to portray the "mathematical self" to a large audience formed by people that are not necessarily in their classrooms. There is thus the pedagogic dimension in which students form identities as *performance mathematicians* by producing a DMP (Scucuglia, Gadanidis & Borba, 2010). In DMPs #2, #7, and #16, for instance, the audience may vicariously feel that students are conscious that they are communicating their mathematical ideas to a wide online audience who is interested in mathematics and these DMPs are parodies of popular TV shows.

Selecting Tools and Computational Strategies.

The curriculum states that "students need to develop the ability to select the appropriate electronic tools, manipulatives, and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems" (Ontario Ministry of Education, 2005, p.14). In many DMPs students use manipulatives, drawing, and "math costumes" (e.g., DMP #1, #4, #5, #7-11, #13-15, #18, #21, and #22) in ways that offer important insights to the audience's mathematical understanding. In DMP #5, for instance, the use of manipulatives is fundamental to the audience visualize the different quadrilaterals, identify their properties and make connections between them. The use of these materials shapes the nature of the reflections and mathematical representations students and the audience engage with.

It is important to mention that the use of digital media is a requirement to produce a DMP. Even though in most DMPs students do not use information technology in the process of performing (e.g., they do not use computer software), the use of video cameras to produce video files is fundamental to support multimodal communication and to make the DMPs public mathematical texts in a non-traditional format (not print-based). Considering the notion of humans-with-media (Borba & Villarreal, 2005), this study offer some evidence about how students-with-digital-media form thinking collectives in the production of DMPs. For instance, in DMPs #10, #15, and #22 (those that explore visual representations of equivalent fractions), the use manipulatives and computer software for video edition was fundamental to produce the stop-motion animations, which condition the design of the performances and the mathematical meanings students and the audience may produce when they produce or read the DMPs. The discussions toward DMPs, the use of technology and multimodality were presented in the first part of chapter five.

Connecting.

Connections, relations, or articulations between procedures and concepts help students develop mathematical understanding (Ontario Ministry of Education, 2005). Connections are one of the most significant aspects analyzed in this research. Connections between representations, for instance, are strongly addressed in the literature on mathematical learning (Confrey, 1990; Kaput, 1992, Noss & Holes, 1996). From the notion of *cognitive ecology* (Levy, 1993), communication, learning, and meaning and knowledge production are concerned with connections (see the metaphor of *hypertext* in chapter two). Along the analysis conducted in chapter four, I interpreted and explored four overlapping types of connections in students' DMPs. These are connections between (a) representations (b) modes of communication; (c) mathematical ideas, concepts, or strands; and (d) mathematical ideas and everyday situations.

In DMP #13, for instance, students perform a skit to explore relations between quadrilaterals, connecting verbal information (speeches), gestures, and visual-manipulative tools (they play roles as quadrilaterals with costumes formed by representations of the figures). The way students make connections between representations and modes of communication offer ways to the audience make connection between properties of quadrilaterals and relations between them. Although students do not explore connections to everyday situations, these multiple forms of connecting representations, modes, and geometric properties of figures emergent in dramatic events in the DMP offer ways to the audience explore surprising ideas such as “a square is a special case of rectangle, parallelogram, and rhombus.”

Overall, most of the students’ DMPs connect multiple modes of communication and representations. In fact, it emerges “naturally” because they are using digital media to communicate and represent their ideas. Although some DMPs explore connections between mathematics and everyday contexts (e.g, DMP #2, #5-9, #11, #12, #16, #17), most of the connections are superficial or they do not present strong arguments to make the connection explicitly to the audience. Only DMPs #6, #10, #15, #20, and #22 make connections across strands, but these DMPs do not fully explore all modes of communication because DMPs #6 and #20 are in power point format and use only writing and drawing and DMPs #10, #15, and #22 are animations in which students do not appear in the performance (no embodied gestures nor verbal information).

Representing.

The curriculum states that:

> In elementary school mathematics, students represent mathematical ideas and relationships and model situations using concrete materials, pictures, diagrams, graphs, tables, numbers, words, and symbols. Learning the various forms of representation helps students to understand mathematical concepts and relationships; communicate their thinking, arguments, and understandings; recognize connections among related mathematical concepts; and use mathematics to model and interpret realistic problem situations. (Ontario Ministry of Education, 2005, p. 16)

The modes of representation pointed out in the curriculum are fundamentally print-based. Typical classrooms experiences students have in classrooms are not recorded. DMP offers ways for multimodal representations in which students can combine all these modes pointed out in curriculum with a vast variety of other including verbal/oral/spoken communication, sound effects and music, gestures, visual simulations, spatial designs, and so forth. Moreover, representing mathematical ideas through the use of the performance arts is not typical as well. The interdisciplinary aspects that involve mathematics and the arts in DMP offer ways to construct rich pedagogic scenarios. The use of gestures in skits, for instance, opens windows for multiple signs to represent mathematics and adds new layers of meanings. In DMP #4, for instance, the use of students' body to create a collaborative representation of triangles is fundamental for students' and audience's understandings. In DMP #10, #15, and #22 students explore visual proofs for equivalence of fractions through the production of a stop-motion animation. Therefore, DMP offers multiple modes and artistic expressions for students to *represent* thinking, reasoning, and learning, which challenges the lack in the curriculum in terms of strictly proposing the print-based format of representing.

Communicating.

The curriculum posits that:

> Communication is the process of expressing mathematical ideas and understanding orally, visually, and in writing, using numbers, symbols, pictures, graphs, diagrams, and words. Students communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, or the whole class. Communication is an essential process in learning mathematics. Through communication, students are able to reflect upon and clarify their ideas, their understanding of mathematical relationships, and their mathematical arguments… The ability to provide effective explanations, and the understanding and application of correct mathematical notation in the development and presentation of mathematical ideas and solutions, are key aspects of effective communication in mathematics. (Ontario Ministry of Education, 2005, p. 17)

Communication is a key-notion in this study. DMP offers ways for student to communicate ideas combing several modes: linguistic, visual, gestural, audio, and spatial (The New London Group, 1996). The combined use of these designs to communicate mathematics offers surprises to the audience. The multimodal nature of students' DMPs is one of its most important pedagogic attributes. The analysis of the DMPs in this study in terms of multimodal communication is discussed in several sections of the dissertation.

Final Comments of Chapter Five – Part Two

The cross-case analysis conducted in the second part of this chapter through the lens of the components of the Ontario curriculum highlights that most of the ideas explored by students are related to geometry. Students do not investigate several deep connections between *strands*, considering those proposed in the curriculum (Ontario Ministry of Education, 2005). Students explore interesting visual representations within *Geometry and Spatial Sense,* but geometry is traditionally a strand that explores visualization. DMP can be understood as "mirrors" or representations of what (and how) students are learning the classrooms and what they are not learning. If they have learned rich visual, concrete representations in Algebra, Number Sense, Measurement, or Probability, they might have used them in the DMPs. So, this analysis is identifying a pedagogical problem.

The analysis based on the curriculum mathematical strands and processes lens also suggests that the use of DMP in the classroom can help address process components. There is a pedagogical synergy between the production of DMPs and the possibility to address the mathematical processes indicated in the K-8 Ontario Mathematics Curriculum (Ontario Ministry of Education, 2005). As lenses, reciprocally, the mathematical strands and processes of the curriculum offer ways to understand students' mathematical activity on their DMPs. Thus, the combination between a performance arts lens (Boorstin, 1990) and a curriculum lens (Ontario Ministry of Education, 2005), shaped by notions within socioculturalism, humans-with-media, and multimodality, offer multiple ways to (a) interpret and analyze the nature of students' DMPs, (b) discuss students' learning through the use of digital technology and the performance arts, and (c) indicate potential components for the production of conceptual DMPs in pedagogic scenarios.

Epilogue: On the Nature of Students' Digital Mathematical Performances

Introduction

In this study I investigated the nature of students' DMPs on the Math + Science Performance Festival. I discussed the mathematical ideas explored by the students, the ways students communicated the ideas to a wide audience (DMPs are publicly online), and the role of technology and the arts in shaping students' reasoning and thinking. These aspects were mainly discussed based on a performance arts lens (Boorstin, 1990), which highlighted surprises, sense-making, emotions, and visceral sensation the audience may potentially feel by reading students' DMPs as multimodal texts/narratives.

Doxiadis (2003) states that, traditionally, mathematical representations and discourses are linear, logic-deductive, based on the goal of classifications. However, mathematics can be also explored through narrative and storytelling. Doxiadis says the use of mathematical narratives "embed mathematics in the soul … [and] mathematical narrative must enter the school curriculum, in both primary and secondary education" (p. 20). However, seeing and exploring mathematics through narratives is not a common practice. In this study, the production of DMPs is considered a pedagogic alternative to fill that gap pointed out by Doxiadis (2003).

Sociocultural Perspectives, Humans-with-Media, and Multimodality

As discussed in chapter two, sociocultural perspectives and postmodernism (Ernest 2004a, 2204b), humans-with-media (Borba & Villarreal, 2005; Levy, 1993, 1998, 2001), and multimodality (The New London Group, 1996; Kress, 2003) are the main bodies of theories that inform this research. Based on these perspectives, I interpreted that the nature of students' DMPs is *multimodal*, *playful*, and *digital*. These three aspects overlap.

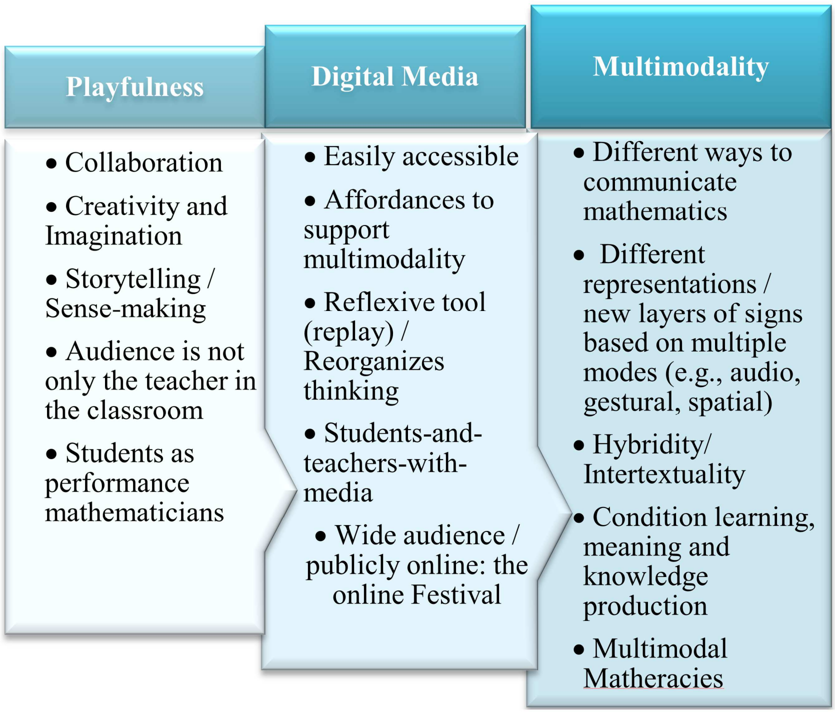
Emphasis on the use and production texts, dialogue, collaboration, diversity in knowledge, and playfulness in the classroom are very significant aspects of teaching and learning mathematics in a sociocultural/postmodernist perspective (Ernest 2004a, 2004b). When students perform and video record a skit or a song in the classroom to produce a DMP, they are not only performing to their classmates and teacher. They are performing, communicating and representing their mathematical activity and learning for a wide audience, because DMPs are publicly available in the website of the Math + Science Performance Festival.

Both the classroom and the cyberspace are social/cultural settings where students, teachers, and other agents interact, collaborate, and produce meaning and knowledge. The playful nature of DMP offers ways of expressing ideas collaboratively, with creativity and imagination. The playfulness may also help students to make sense of mathematics through narrative because when they produce a DMP they are seeking to clearly communicate a mathematical story through a digital narrative/text to the audience. The process of producing a digital mathematical narrative to be published online (that is, producing a DMP for the Festival) is a process in which students construct identities as performance mathematicians (Gadanidis, Hughes & Borba, 2008).

The use of the performances arts is in synergy with the use of digital media in multimodality. The multimodal nature of DMP is potentially one of its most significant pedagogic attributes. Mathematics is traditionally communicated through print-based texts, through the use of writing, charts, diagrams, and graphs. Digital media affordances offer ways to represent mathematical ideas through multiple modes of communication (New London Group, 1996), which adds non-usual layers of signs in representing mathematics (e.g. audio, visual, gestures, space). Hybridity and intertextuality (The New London Group, 1996) are fundamental aspects of students' DMPs and very significant in terms of mathematical thinking and meaning production. Multimodal texts (Kress, 2003; Walsh, 2011), like video files, are easily accessible, although they are not typically used by students to communicate mathematical ideas in the classrooms. Videos are tools which students can replay as many times they wish and *reflect* about their registered previous actions and activities involving communication (e.g., speeches, gestures, and use of the arts). Digital technologies are reflexive tools which offer ways to reorganize mathematical thinking.[31] That is, DMPs (as products, as multimodal texts/narratives) are tools that offer ways to form thinking collectives of students-with-DMPs (Borba & Villarreal, 2005). Figure E.1 illustrates some of the main aspects on the nature of students' DMPs:

[31] Teachers can use DMPs available online as teaching-learning tools. They can even focus on students' mistakes or incoherencies in the DMPs as starting points to discuss mathematical concepts.

Figure E.1: The Nature of Students' DMPs



Performance Arts and Curriculum Lenses

On the one hand, the focus on mathematical surprises, sense-making, emotions, and visceral sensations worked as a type of "model" to analyze students' DMPs in this study. In chapter four and five, I analyzed the DMPs focusing on these aspects as much as possible. Moreover, it was significant to use the curriculum mathematical strands and processes (Ontario Ministry of Education, 2005) as lenses to interpret students' DMPs as "mirrors" of what they learn in classrooms and understand students' mathematical activity on their DMPs. On the other hand, I suggested as findings of the study that students' DMPs present gaps in these categories (surprises, sense-making, emotions, and visceral sensations). That is, the playful, digital, and multimodal aspects emergent with the production of DMPs do not guarantee the mathematical conceptuality of the DMPs, which is the main pedagogic focus considering the production of DMPs.

For instance, when I read the DMP *L-Patterns* (see Prologue), I see several conceptual aspects: (a) it connects geometry to patterning and algebra, that is, it connect multiple strands and representations; (b) it explores multiple modes of communication, offering a multiplicity of semiotic resources for sense-making, (c) it is surprising to see a sequence of L's (1, 3, 5, 7, …, 2n-1) connecting one to another and forming a square (n^2); (d) the emotions portrayed in the poem/lyric/story are mathematical emotions – e.g., "*the sum of the stages is a perfect square, I can prove it, I can prove it, it's amazing*" and (e) it offers direct experiences (connecting L's) and an intense sense of mathematical fit by the formation of perfect squares, which is mathematically visceral from an aesthetic point of view (Sinclair, 2004).

As mentioned in the last chapter, it is important to acknowledge that I analyzed the students' DMPs available in the first year of the Festival. It means that students did not have examples as references to produce their DMPs. Recognizing this fact, the following specific interpretations/findings can be pointed out from the analysis conducted in this study toward the nature of elementary school students' DMPs available at the Math + Science Performance Festival in 2008, considering the performance arts and curriculum lenses:

- Students' DMPs offer mathematical surprises. These surprises are mostly supported by the use of the performance arts and digital media: multimodality. That is, it is surprising to see students using the arts and producing multimodal texts to communicate their mathematical ideas. When students only reproduce typical ideas explored in classrooms (e.g., textbook definitions), for instance, they do not explore the joy and the wonderful of mathematics. However, some of the students' DMPs may potentially offer conceptual mathematical surprises, that is, ways of seeing the new and wonderful in mathematics by connecting multiple representations and multiple strands, connecting mathematics to everyday problems, connecting properties of multiple mathematical objects, or exploring visual proofs.
- The "storytelling/narrative nature" emergent from the playfulness of DMPs helps students in the process of sense-making on the mathematical ideas explored through the performance arts. However, some students present a few incoherencies in terms of mathematical reasoning and sense-making in their DMPs such as lack in logical (deductive) thinking and miscommunication. Overall, students provide illustrative examples to the audience by exploring the visual nature of digital media and connecting representations, but they do not present arguments that support the conceptuality of some their ideas. They also offer some ways to extend the problems explored, but these ways are not explicit in most the performances. The audience may find ways to extend and reflect on the problems only based on the subjectivity emergent from the performance arts. Therefore, the mathematical sense-making aspect of students' DMPs is seen as pedagogically problematic in this study.

- Students depict emotions in their DMPs. Most of the mathematical emotions emerge through the use of the arts, mainly through the notion of embodiment (Gerofsky, 2009; Radford, 2009). Skit performances in which students play roles as mathematical/geometric objects involve use of speech and gestures in conducting dramatic events regarding exploration of mathematical properties. Vicariously, the audience may feel some mathematical emotions that emerge in DMPs through performative activities in which students-think-as-mathematical-objects and form thinking and feeling collectives.
- Students' DMPs offer visceral mathematical sensations to the audience through a sense of mathematical fit such as how representations of mathematical objects fit in the world or how they relate to one another. Dramatic events involving scenes of action or suspense are commonly found in skits and musical performances (including the use of soundtracks) and the plot of these dramatic moments are mostly related to the exploration of mathematical ideas in students' DMPs.
- Most of the ideas explored by students are related to geometry. Students do not investigate several deep connections between *strands*, regarding those proposed in the curriculum (Ontario Ministry of Education, 2005). Students explore interesting visual representations within *Geometry and Spatial Sense*, but geometry is traditionally a strand that explores visualization. DMPs are "mirrors" of what students learned and what students did not learn in their classrooms.
- The mathematical strands and processes present in the curriculum (Ontario Ministry of Education, 2005) help the analysis of this study because they help to identify the focus of the DMPs and identify patterns and gaps on students' ideas, reasoning, ways of reflecting, communicating, representing, connecting, solving problems, and using technology. The production of DMPs helps to address the curriculum mathematical processes in students' classroom activity and, reciprocally, the mathematical processes help in the understanding of the nature of students' DMPs.

Therefore, one of the main conclusions of this study is that the production of conceptual DMPs is a rare event. The use of the performance arts and digital technology does not guarantee the mathematical conceptuality of a DMP, which is its most important aspect. I agree with Gadanidis, Hughes, and Cordy (2011) when they say:

> Students can add artwork to "decorate" procedural knowledge, thus adding a layer of sugar-coating to otherwise dry mathematical ideas, but mathematical art, like art in general, requires a deeper engagement and understanding. Thus, for us, challenging mathematics is a corequisite for artistic mathematical expression. (p. 424)

Based on the descriptive single-case analysis conducted in chapter four and the cross-case analysis conducted in chapter five, Figure E.2 illustrates some of the conclusions of this thesis:

Figure E.2: Some of the conclusions of the thesis

Typical mathematical texts are logic-deductive, non-human, cold.

Doxiadis (2003): mathematical narratives "embed mathematics in the soul."

Mathematics can be a human and aesthetic experience (Higginson, 2006; Sinclair, 2004).

DMP is a pedagogic alternative to produce mathematical narratives (Gadanids & Borba, 2008)

Multimodal texts: the arts + digital media

Theoretical Lenses

Sociocultural Theory / Postmodernism

Humans-with-Media

Multimodality

Boorstin's (1990) categories work as a lens to analyze DMPs

- Voyeur – New/Wonderful/Surprises
- Voyeur – Sense-Making
- Vicarious Emotions
- Visceral Sensations
    - Curriculum strands and processes refine the lens

Conclusions of the Study

DMPs contribute to make mathematics public, to the playfulness and production of multimodal texts / narratives in the classroom. Students-with-technologies form thinking collectives in the production of DMPs.

Conceptual DMPs are rare events. Students' DMPs present some surprises, emotions, and visceral sensations, but there are shortcomings in terms of the presence of conceptual mathematical surprises and/or arguments that support the conceptuality of the idea. Some DMPs present incoherencies in terms of logical/deductive reasoning and miscommunication.

DMPs are "mirrors" of students' mathematical learning. Geometry is the main strand explored. Digital media offer multiple modes of communication, but geometry is already a visual strand. Why do not students explore visual representations in algebra, for instance?

The production of DMPs helps to address the curriculum mathematical strands processes. Using the processes as lenses help to understand students' mathematical activity and learning on their DMPs.

Possible Contributions of the Study

This study is the first doctoral thesis that explores the very notion of DMP and it can be seen as a window into mathematics classroom activity. Three main contributions to the field of mathematics education can be pointed out:

(1) *Teaching and learning:* the discussions presented offer insights about the practice of mathematics in schools considering the use of the performances arts and digital technology. The focus on the nature of the mathematical idea, the relevance of developing students' communication skills, the emphasis on the role of technology in shaping mathematical thinking, and the artistic lens (surprises, emotion, and visceral sensation) offer to teachers directions on how DMPs can be conducted in their classrooms, on how digital mathematical narratives can be conducted by students or how these narratives help teachers and researchers to understand students' learning. This study offers some "criteria" on how the interlocution between the use of digital media and the arts can be integrated in order to produce conceptual DMPs.

(2) *Multimodality:* Ernest (2004b) argues that "the relationship between learners and text, and how it inscribes the other, is an area that is as yet inadequately explored" (p. 82). Similarly, Doxiadis (2003) highlights gaps on the creation of mathematical narratives in schools. This study contributes by discussing possibilities to fill this lack pointed out by Ernest and Doxiadis through the emphasis on the multimodal nature of students' DMPs. Beyond the production of texts considering the use of multiple modes of communication (linguistic, visual, gestural, special, audio), the digital nature of DMPs offers ways to make them available on the cyberspace. Overall, this study explores some insights about how students-with-media form thinking collectives when they produce DMPs. Moreover, the fact that students' DMPs are available at the Math + Science Performance Festival bring students' mathematics to a public and social forum (Gadanidis & Geiger, 2010), beyond the confines of classrooms.

(3) *Analytical lenses:* the use of Boorstin's (1990) lens to analyze DMPs was originally proposed by Gadanidis and Borba (2008) and it was explored by Osafo (2010) to analyze digital performances in science education. On the one hand, it is important to recognize that the use of Boorstin's lens is "an adaptation." Students' DMPs are not Hollywood movies and Boorstin does not talk about mathematics. On the other hand, this study contributes on the ways such adaptation is "refined." The connections proposed in this study between (a) the artistic lens (surprises, sense-making, emotions, and visceral sensations) and (b) the theoretical and

methodological issues discussed offer new insight on the notion of DMP. Theoretically, the view of diversity in knowledge and playfulness in classroom emerges through a postmodern/sociocultural perspective. It is linked to the very notions of humans-with-media (Borba & Villarreal, 2005), cognitive ecology (Levy, 1993), and multimodality (Kress, 2003; The New London group, 1996). Methodologically, the notions of video analysis and case studies refine the interpretative/exploratory focus on the analysis of students' DMPs. The sub-categories emergent on the cross-analysis presented in chapter five (e.g. conceptual mathematical surprises) highlight such refinement. Thus, the analytical lens constructed in this study has a potential to become an interesting theoretical/methodological tool to analyze DMPs.

Limitations of the Study

Methodologically, in several moments of the analytical process, as an interpreter, I felt a kind of "need" to see the *process* of production of students' DMPs. Students' DMPs are multimodal texts, they are *products*, and they only offer some insights about the process of designing the DMPs. In order to discuss the nature of students' DMPs, I do recognize that it would be significant to analyze the process of creation. That is similar to the analysis of the proof a theorem created by mathematician. By analysing only the proof, the interpreter has little evidence (or no evidence) about the "rich" heuristics emergent on the conduction of the proving process. The variety of approaches, experimentations, conjectures, alternatives, decision-makings, problem solving strategies, reorganization of thinking usually are not explicit in the product of a demonstration and it may not be in the DMPs as well.

Future Studies

It would be interesting to present part of the research findings of this study as a *digital performance*. New forms of presenting research using the arts have emerged in qualitative research (Barone & Eisner, 2006; Eisner, 1997). Barone (2001) claims that as *arts-based* researchers, "our inquiry activity and the forms of representation that we employ evidence certain design elements that are aesthetic in character. These forms of representation may be derived from any area of the arts [such as] performing, plastic, and so on" (p. 25).

Eisner (1981) argues:

> In artistic approaches to research, the role that emotion plays in knowing is central. Far from the ideal of emotional neutrality which is sought in much of social science research, the artistically oriented researcher recognizes that knowing is not simply a unidimensional phenomena, but takes a variety of forms. The researcher knows also that the forms one uses to represent what one knows affect what can be said. Thus, when the content to be conveyed requires that the reader vicariously participates in a social situation context, the writer or filmmaker attempts to create a form that makes such participation possible. (p. 8)

Glesne (2010) sees qualitative research as the "meeting place between art and sciences" (p. 241). Discussing language as a producer of meaning and as a creator of social reality – instead as a 'support' or a representation for an objective reality – Glesne points to the role of the researcher as a storyteller and indicates alternate modes to represent research based on poetic transcriptions, drama performance, storytelling, and painting.

Creative-artistic/performatic representations of research works are also an aspect of crystallization[32] and postmodernism. Richardson (2000) argues:

> The scholar draws freely on his or her productions from literary, artistic, and scientific genres, often breaking the boundaries of each of those as well. In these productions, the scholar might have different "takes" on the same topic, what I think of as a postmodernist deconstruction of triangulation. . . . In postmodernist mixed-genre texts, we do not triangulate, we *crystallize*. . . . I propose that the central image for "validity" for postmodern texts is not the triangle—a rigid, fixed, two-dimensional object. Rather, the central imaginary is the crystal, which combines symmetry and substance with an infinite variety of shapes, substances, transmutations, multidimensionalities, and angles of approach. . . . Crystallization provides us with a deepened, complex, thoroughly partial, understanding of the topic. Paradoxically, we know more and doubt what we know. Ingeniously, we know there is always more to know. (Richardson, 2000, p. 934)

Ellingson (2009) clarifies that "in both inductive analytic (e.g., grounded theory) and more artistic approaches to qualitative research, researchers abandoned claims of objectivity in favor of focusing on the situated researcher and the social construction of meaning" (p. 2).

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[32] Crystallization refers to the process of forming crystals or to the cause of assuming a crystalline structure. In qualitative research, crystallization is related to the process of giving a form or expression to an idea or argument, and it is related to creative-artistic forms of research representation and dissemination.

"Crystallization does not depart radically from other recent developments in the wide field of qualitative methodology, but rather offers one valuable way of thinking through the links between grounded theory (and other systematic analyses) and creative genres of representation."

Ellingson (2009) adds that:

> Crystallization combines multiple forms of analysis and multiple genres of representation into a coherent text or series of related texts, building a rich and openly partial account of a phenomenon that problematizes its own construction, highlights researchers' vulnerabilities and positionality, makes claims about socially constructed meanings, and reveals the indeterminacy of knowledge claims even as it makes them. (p. 4)

Considering the multimodal nature of DMPs, it is important to notice that "crystallization can be accomplished in virtually any medium—writing, video, painting, performance art, computer generated images, and so on" (p. 24). One of the strengths of crystallization is that:

> With crystallization, very deep, thick descriptions are possible. Multiple ways of understanding and representing participants' experiences not only provide more description, but more points of connection through their angles of vision on a given topic. Crystallization enables significant freedom to indulge in showing the "same" experience in the form of a poem, a live performance, an analytic commentary, and so on; covering the same ground from different angles illuminates a topic. As our goal in conducting qualitative research generally involves increasing understanding in order to improve dialogue among individuals and groups and to effect positive change in the world, enriching findings through crystallization may move us to fulfilling that goal. (p. 15)

Thus, assuming a view of *research as performance* (Gadanidis, 2010), I will produce a *digital performance* to present some of the findings I discussed in this study. I will submit and share it at the Math + Science Performance Festival.

Finally, I believe the very notion of DMP has a great potential to consolidate a new trend in mathematics education. In the near future I intend to build some links between DMP and the Deleuzian notions of rhizome (Deuleuze & Guatarri, 1987), difference and repetition in theatre (Deleuze, 1994), and crystals of time in cinema (Deleuze, 1989). Mostly important, I also intend to discuss DMP considering the emergence of the notion of *multimodal matheracy* through both (a) new perspectives on multimodal literacy (Rowsell & Walsh, 2011; Walsh, 2011) and (b) sociocultural lens on matheracy (D'Ambrosio, 2006).

From a cultural, ethical, and political point of view, D'Ambrosio (2006) proposes the reorganization of mathematics curricula in three strands: literacy, matheracy, and technocracy. D'Ambrosio (2003) argues that "*literacy* is the capability of processing information, such as the use of written and spoken language, of signs and gestures, of codes and numbers. Clearly, reading has a new meaning today" (p. 238). In my understanding, this "new meaning" refers to the notion of multimodal literacy explored in-depth my thesis, in regard to elementary school students' mathematical activity.

D'Ambrosio (2003) also states that "*technoracy* is the critical familiarity with technology […] the basic ideas behind technological devices, their possibilities and dangers, the morality supporting the use of technology, are essential issues to be raised among children at a very early age" (p. 237). Such perspective is also fundamental in DMP, which by nature is essentially digital and technological. It is a critical point to consider and highlight the critical equilibrium between thinking and feeling collectives of students-teachers-and-researchers-with-media.

> *Matheracy* is the capability of inferring, proposing hypotheses, and drawing conclusions from data. It is a first step toward an intellectual posture, which is almost completely absent in our school systems. Regrettably, even conceding that problem solving, modeling, and projects can be seen in some mathematics classrooms, the main importance is usually given to numeracy, or the manipulation of numbers and operations. Matheracy is closer to the way mathematics was present both in classical Greece and in indigenous cultures. The concern was not with counting and measuring but with divination and philosophy. Matheracy, this deeper reflection about man and society, should not be restricted to the elite, as it has been in the past. (D'Ambrosio, 2003, p. 237)

Therefore, I see mathematics, mathematics education, and matheracy through the lenses of the performance arts. Considering an emphasis on the use of digital technology and on perspectives about multimodal literacy, the very notion of *multimodal matheracy* has a potential to become a way to highlight the production of DMPs at the elementary school mathematics curriculum as a mathematical activity. *Multimodal matheracy* is a way to highlight the aesthetics of mathematics learning with emphasis on arts and the use of digital technology in cultural, ethical, and political dimensions.

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Appendix A: The Four Dimensions of Affectivity

Levy (1998) reorganizes those six principles of the *hypertext* (Levy, 1993) into four *dimensions of affectivity*. Levy (1998) argues these dimensions form a model of the *psyche* that works for an individual human mind as well as for a collective intelligence. Levy (1998) also believes this "model of psyche can be applied to a text, film, message, or any other work of art" (p. 132). Thus, the dimensions of affectivity are:

> *Topology*. The psyche is structured at each moment by its connections, systems of proximity, or a specific space: associations, connections, paths, gateways, switches, filters, attractors. The topology of the psyche is in continuous transformation. . . . *Semiotics.*[33] Mutant hordes of representations, images, signs, and messages of all shapes and kinds (aural, visual, tactile, proprioceptive, diagrammatic). . . . Signs or groups of signs can be also referred to as agents. Transformations of connectivity in turn influence the populations of signs and images. Topology is itself the set of qualitatively connections or relations among signs, messages, and agents. *Axiology.* The representation and regions of psyche space are associated with values. . . These values determine the tropism, attractions and repulsions between images, the polarities among regions or groups of signs. These values are by nature mobile and changing, although some of them may also display certain stability. *Energics.* The tropisms or values attached to images can be strong or weak. The movement of a group of representations can cross certain topological barriers or remain with them. All psychic activity is thus irrigated and animated by an energic economy involving the movement of stabilization of forces or crystallization of energy (Levy, 1998, p. 131-2).

[33] "Semiotics is the study of the meaning of systems of signs" (Kress, 1997, p. 6). Semiotics has been the domain of two large schools of thought; one deriving from the work of the Swiss linguist Ferdinand de Saussure, the other deriving from the work of the American philosopher Charles Sanders Peirce. The semiotics of Saussure bears recognisable traces of its origins in the historical linguistics of the nineteenth century. In it, the sign is taken to be an arbitrary combination of form and meaning, of *signifier* and *signified*, a combination which is sustained by the force of social convention… In Peirce's semiotics the focus is less on the internal constitution of the sign than on the uses of the sign by readers/users, and on the relation of the sign to that which it represents. Peirce focused on what the sign represented, on the object/*referent* in the world, on how it was interpreted, assuming that there was no meaning until there was an interpretation. This he called the *interpretant*. He focused on the sign-characteristics from the point of view of the type of relation between signifier and that which it represented, something that seems to have been of only marginal interest to Saussure. Peirce consequently distinguishes between *iconic signs*, which in their form parallel the meaning of the signified – the drawing of flames to mean fire; *indexical signs*, in which there is a relation of 'consequence', as in smoke signalling combustion; and *symbolic signs*, where the relation between form and meaning was largely sustained by convention ... Hence there is a distinct difference in focus between the two theorists, which could be taken to mean that they had produced distinct and potentially irreconcilable theories. In fact, the two theories are compatible and complementary, if one accepts their different foci (Kress, 2003, p. 41-42).

Appendix B: The Rhizome

Let us summarize the principal characteristics of a rhizome: unlike trees or their roots, the rhizome connects any point to any other point, and its traits are not necessarily linked to traits of the same nature; it brings into play very different regimes of signs, and even nonsign states. The rhizome is reducible to neither the One or the multiple. It is not the One that becomes Two or even directly three, four, five etc. It is not a multiple derived from the one, or to which one is added (n + 1). It is comprised not of units but of dimensions, or rather directions in motion. It has neither beginning nor end, but always middle (*milieu*) from which it grows and which it overspills. It constitutes linear multiplicities with n dimensions having neither subject nor object, which can be laid out on a plane of consistency, and from which the one is always subtracted (n – 1). When a multiplicity of this kind changes dimension, it necessarily changes in nature as well, undergoes a metamorphosis, changes in nature. Unlike a structure, which is defined by a set of points and positions, the rhizome is made only of lines; lines of segmentarity and stratification as its dimensions, and the line of flight or deterritorialization as the maximum dimension after which the multiplicity undergoes metamorphosis, changes in nature. These lines, or ligaments, should not be confused with lineages of the aborescent type, which are merely localizable linkages between points and positions. Unlike the tree, the rhizome is not like the object of reproduction: neither external reproduction as image-tree nor internal reproduction as tree-structure. The rhizome is an antigenealogy. It is a short term memory, or antimemory. The rhizome operates by variation, expansion, conquest, capture, offshoots. Unlike the graphic arts, drawing or photography, unlike tracings, the rhizome pertains to a map that must be produced, constructed, a map that is always detachable, connectable, reversible, modifiable, and has multiple entranceways and exits and its own lines of flight. In contrast to centred (even polycentric) systems with hierarchical modes of communication and pre-established paths, the rhizome is an acentered, non-hierarchical, nonsignifying system without a General and without an organizing memory or central automaton, defined solely by a circulation of states (Deleuze & Guattari, 1987, p. 21).

Appendix C: Curriculum Vitae

RICARDO SCUCUGLIA RODRIGUES DA SILVA

DEGREES

PhD, The University of Western Ontario, Canada - Education Studies, (2008 - 2012).
MA, Sao Paulo State University, Brazil – Mathematics Education, (2004 – 2006).
BA, Sao Paulo State University, Brazil – Mathematics, (1999 – 2002).

EMPLOYMENT POSITIONS

2008 – Present Research Assistant, Faculty of Education
The University of Western Ontario, London, Canada

2007 – 2008 Research Assistant, Department of Mathematics
Sao Paulo State University, UNESP/SP, Brazil
Sao Paulo State Foundation of Research (FAPESP)

2006 – 2007 Research Assistant, Faculty of Education
The University of Western Ontario, London, Canada

2006 – 2007 Instructor, Faculty of Technology
Brazilian Centre of Applied Technology, Sao Paulo, Brazil

2006 – 2006 Instructor, Faculty of Mathematics
University of Holy Heart, Sao Paulo, Brazil

2004 – 2006 Research and Teaching Assistant, Department of Mathematics (GPIMEM)
Sao Paulo State University, UNESP/SP, Brazil
Brazilian National Ministry of Science and Technology

2003 – 2003 Instructor, Faculty of Engineering
Municipal Teaching Foundation of Piracicaba, FUMEP/SP, Brazil

2003 – 2003 Teacher, High School
Municipal Teaching Foundation of Piracicaba, FUMEP/SP, Brazil

AWARDS

2012 Western Research Forum – Award for Research Excellence – Second Place Presentation in Arts, Education, and Humanities Presentation Title: *Little Quad's Quest: a multimodal mathematical narrative.*

2010 Western Research Forum – Award for Research Excellence – Best Oral Presentation in Arts and Humanities Presentation Title: *How do student-with-graphing-calculators investigate the fundamental theorem of calculus?*

SCHOLARSHIPS

CAPES Foundation / MEC Brazil, Doctoral Research, 2010 – 2012.
Western-FoE. Western Graduate Research Scholarship, 2008 – 2012.
FAPESP, TIDIA-Ae, 2007– 2008.
SSHRC / FoE-Western, 2006 – 2007.
CNPq Brazil, Master's Research, 2004 – 2006.
CNPq Brazil, Undergraduate Research, 2000 – 2002.

SCHOLARLY AND PROFESSIONAL ACTIVITIES

2000 – Present Researcher, Research Group on Information Technology, Media and Mathematics Education (GPIMEM, Department of Mathematics, Sao Paulo State University).

2010 – 2012 Member (Information Committee Service / Faculty of Education), University of Western Ontario.

2009 – 2009 Councilor (Society of Graduate Students / Faculty of Education), The University of Western Ontario.

2005 – 2008 Editor, GPIMEM Digital, Digital Journal in Mathematics Education, Sao Paulo State University.

2005 – 2005 Junior Advisor, Undergraduate Students - (GPIMEM), Sao Paulo State University, Brazil.

2004 – 2005 Graduate Teaching Assistant, Department of Mathematics, Sao Paulo State University, Brazil.

Scholarly and Professional Membership

Fields Institute Mathematics Education Forum (University of Toronto)
Brazilian Society of Mathematics Education (SBEM)
Institute of Mathematics and Arts of Sao Paulo (IMA)
Joy of X at http://joyofx.com/

Scholarly and Professional Consultancy

2004 – Present Editorial/Technical Reviewer, BOLEMA Journal.

2010 – 2010 Reviewer, Encyclopedia Britannica.

Conference Reviewer/Chair

33rd Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME –NA 2011).

34th Conference of the International Group for the Psychology of Mathematics Education (PME34, 2010).

X Encontro Nacional de Educação Matemática (ENEM 2010) [X Brazilian National Conferece in Mathematics Education].

GRADUATE COURSES (Instructor)

Gadanidis, G. & Scucuglia, R. (2012). *Teaching and learning mathematics.* Graduate program in education, The University of Western Ontario. London, Ontario, Canada.

Borba, M. C., Malheiros, A. P. S. & **Scucuglia, R.** (2009) *Qualitative Research Methodology in Online Distance Education*. In-service, Higher Education Instructors, Interactive Classroom Program, Rio de Janeiro, Brazil.

Scucuglia, R. (2006), *Graphing Calculators and Mathematics Education*, Inservice, High School Teachers Program, University of Holy Heart, Sao Paulo, Brazil.

Scucuglia, R. (2006), *Calculators and Mathematics Education*, Inservice, Elementary School Teachers Program, University of Holy Heart, Sao Paulo, Brazil.

Scucuglia, R. (2006), *Graphing Calculators in Mathematics Education*, Graduate Program in Mathematics, University of Holy Heart, Sao Paulo, Brazil.

INTERNAL RESEARCH FUNDING

2011	Faculty of Education – Western	$400	Conference Expenses
2011	Ontario Ministry of Education / FoE-Western	$630	Symposium Expenses
2010	Faculty of Education – Western	$1,000	Conference Expenses
2009	Faculty of Education – Western	$1,800	Conference Expenses

EXTERNAL RESEARCH FUNDING

2011	Canadian Mathematical Society	$200	Conference Expenses

PUBLICATIONS

Book Chapters

Borba, M., & **Scucuglia, R.** (Accepted). Lerman's perspectives on information and communication technology. New York: Springer.

Borba, M., Gadanidis, G., Dawson, A.J. & **Scucuglia, R.** (Accepted). Digital Media and Multimodality: Genres of the Performance Arts in Mathematics Education. In: Setati, M.; Nkambule, T. & Goosen, L. (Eds). *Mathematics and language diversity*. New York: Springer.

Borba, M., Malheiros, A. P. S. & **Scucuglia, R.** (In Press) Metodologia de Pesquisa Qualitativa em Educação a Distâcia Online. [Qualitative Research Methodology in Online Distance Education]. In: M. Silva (Ed.) *Sala de aula interativa [Interactive virtual classroom]*. Rio de Janeiro, Brazil: Editora 34.

Scucuglia, R. (2010) Friends of HP50g: Emotional Design and Graphing Calculators. In Claudia L. O. Growenwald & Mauricio Rosa (Eds.) *Mathematics Education and Calculators: Theory and Practice*. (pp. 89-112). Canoas, RS, Brazil: Editora da ULBRA.

Borba, M. & **Scucuglia, R**. (2009). Modelagem e Performance Digital em Educação Matemática Online. [Modeling and Digital Performance in Online Mathematics Education]. In Rita de A. Gonçalves; Julieta S. de Oliveira & Maria Alice Coelho Ribas (Eds.) *Educação em Sociedades dos Meios Virtuais. [Education in Societies of Virtual Media]*. Vol. 1, (pp. 153-172). Santa Maria, RS, Brazil: Centro Universitário São Francisco.

Articles in Refereed Journals

Scucuglia, R., Borba, M., & Gadanidis, G. (under review). "Cedo ou tarde matemática:" uma performance matemática digital criada por estudantes do ensino fundamental ["Sooner or later mathematics:" a digital mathematical performance created by elementary school students]. *Revista de Matemática, Ensino e Cultura Temática, 6*(9).

Gadanidis, G. & **Scucuglia, R.** (2010) Windows into Elementary Mathematics: Alternate public images of mathematics and mathematicians. *Acta Scientiae (ULBRA), 12*(1). 8-23.

Scucuglia, R (2008) An Experimental Investigation with Graphing Calculators about the Fundamental Theorem of Calculus. *Acta Scientiae (ULBRA), 10*(1), 74-92

Papers in Journals and Professional Magazines (Invited)

Scucuglia, R., & Gadanidis, G. (2012). Infinity in my hands: How to get kids to explore infinity. *Notes from the Margin – Canadian Mathematical Society STUDC, III* (Winter 2012), 4-4.

Scucuglia, R., Borba, M. C. & Gadanidis, G. (2010). MathFest.ca: prazer em fazer matematica [MathFest.ca: Pleasure in Performing Mathematics]. *Caipira Cinema Magazine, 21*(1), 14-16

Scucuglia, R. (2008) Mathematics-for-teachers online: Facilitating conceptual shifts in elementary teachers' views of mathematics. *Bolema: Bulletim of Mathematics Education*, v.29, Sao Paulo State University, Brazil. [Translation from English to Portuguese of Gadanidis, Namukasa & Moghaddam (2008)].

Baroni, R, Vieira, V, **Scucuglia, R.** (2005). IV Conference of Graduate Program in Mathematics Education - *Bolema: Bulletim of Mathematics Education*, v.24, 111-121– Sao Paulo State University, Brazil.

Scucuglia, R. (2006). Investigation of the Fundamental Theorem of Calculus using Graphing Calculators (Master Thesis Abstract). *Bolema: Bulletin of Mathematics Education*, v.23 – Sao Paulo State University, Brazil.

Scucuglia, R. (2005) Arquimedes, Pappus, Descartes and Polya. Four Episodes of History of Heuristics – *Bolema: Bulletin of Mathematics Education*, (Book Review) v.23, 123-126 – Sao Paulo State University, Brazil.

Digital Publications (Editor)

Borba, M, **Scucuglia, R.,** & Rosa, M. (2008). *GPIMEM Digital V8*: *Proceedings of GPIMEM 15th Anniversary Conference*. ISSN 1679-6853. Sao Paulo State University, Brazil.

Borba, M, **Scucuglia, R.** (2006). *GPIMEM Digital V7*. ISSN 1679-6853. Sao Paulo State University, Brazil.

Borba, M, **Scucuglia, R.** (2005). *GPIMEM Digital V6*. ISSN 1679-6853. Sao Paulo State University, Brazil.

Others (Wikis)

Scucuglia, R (2010) *Theorizing the Curricula: A Digital Performance on Curriculum Studies*. Available for collaboration at: http://curriculumtheories.wikispaces.com/

Scucuglia, R (2010) *Public Images of Mathematics* (Book Review). Available for collaboration at: http://pimreview.wikispaces.com/

Papers in Refereed Conferences

Scucuglia, R. (2012). The role of surprises on elementary school students' digital mathematical performances. *The CSSE Annual Conference.* Wilfrid Laurier University, Waterloo, ON, Canada.

Scucuglia, R., Gadanidis, G. & Borba, M. C. (2011). Lights, Camera, Math! The F-Pattern News. In: Wiest, L. R., & Lamberg, T. (Eds.). *Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1758-1766). Reno, NV: University of Nevada.

Scucuglia, R., Gadanidis, G. & Borba, M. C. (2011). Thinking collectives and digital mathematical performance. In Setati, M.; Nkambule, T. & Goosen, L. (Eds). *Proceedings of the ICMI Study 21 Mathematics and language diversity* (pp. 348-355). Sao Paulo, Brazil.

Gadanidis, G., Borba, M., Hughes, J., **Scucuglia, R.** & Burke, A. (2011). Sing me a good research story: Research dissemination and new media. In T. Bastiaens & M. Ebner (Eds.), *Proceedings of World Conference on Educational Multimedia, Hypermedia and Telecommunications 2011* (pp. 384-390). Chesapeake, VA: AACE.

Scucuglia, R. & Richit, A. (2011). O Papel das Tecnologias Informáticas na Investigação do Conceito de Soma de Riemann. [The role of information technology on the investigation of the concept of Riemann Sum] *XI Congresso Estadual Paulista sobre Formação de Educadores & I Congresso Nacional de Formação de Professores*. UNESP. Aguas de Lindoia, Brasil.

Richit, A., Richit, A., **Scucuglia, R.** & Tomkelski, M. (2011). Possibilidades Didático-Pedagógicas do Software Geogebra no Estudo do Conceito de Integral [Didatical-pedagogical possibities of Geogebra software on the study of the concepty of interger]. *XIII Inter-American Conference on Mathematics Education*. Recife, Brazil.

Gadanidis, G., Borba, M. C., Hughes, J., **Scucuglia, R.** (2010). Tell me a good math story: Digital Mathematical Performance, Drama, Songs, and Cell Phones in the Math Classroom. *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education*. PME34, 2010. v. 3. p. 3-17-3-24. Belo Horizonte, MG, Brazil: Federal University of Minas Gerais.

Gadanidis, G., Hughes, J., **Scucuglia, R.** Tolley, S. (2009). Low Floor, High Ceiling: Performing Mathematics across Grades 2-8. In Swars, S. L., Stinson, D. W., & Lemons-Smith, S. (Eds.) (2009). *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Psychology of Mathematics Education - North America Atlanta, Georgia 2009 5 593-600 Atlanta, Georgia: Georgia State University

Gadanidis, G., Hughes, J., **Scucuglia, R.** (2009). Mathematics Leaning as Community Service In Swars, S. L., Stinson, D. W., & Lemons-Smith, S. (Eds.) (2009). *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Psychology of Mathematics Education - North America (PME-NA) Atlanta, GA 2009 5 585-592 Atlanta, GA: Georgia State University

Borba, M. C., Gadanidis, G., Hughes, J., **Scucuglia, R.** (2009). Digital Mathematical Performance & Students as Performance Mathematicians: Interlocuções entre Artes e Tecnologias Informáticas em Educação Matemática In *X EGEM - Encontro Gaucho de Educacao Matematica / Educação Matemática: diálogos entre a universidade e a escola* X EGEM - Encontro Gaucho de Educação Matemática Ijui, RS 2009 Ijui, RS: UNIJUI

Scucuglia, R., Borba, M. (2007). Performance matematica digital: criando narrativas digitais em educacao matematica [Digital Mathematical Performance: Creating digital narratives in mathematics education]. *IX Brazilian National Congress in Mathematics Education*. University of Belo Horizonte, Belo Horizonte, Brazil.

Scucuglia, R. (2007). Um olhar inicial sobre performance matematica digital e formacao de professors. [An initial look at digital mathematical performance and teacher education]. *II Conference involving undergraduate and graduate students in Education and Mathematics Education*. Rio Claro, Brazil: Sao Paulo State University,

Scucuglia, R. (2007). A investigacao do teorema fundamental do calculo com calculadoras graficas. [The Investigation of the Fundamental Theorem of Calculus using Graphing Calculators]. *VII Congress of research in Education of Southeast Region.* Vitoria, Espirito Santo, Brazil.

Scucuglia, R. (2007). O teorema fundamental do calculo com calculadoras graficas. [The Fundamental Theorem of Calculus with Graphing Calculators]. *IX Brazilian National Congress in Mathematics Education*. University of Belo Horizonte, Belo Horizonte, Brazil.

Scucuglia, R. (2006). Experimentacao com calculadoras graficas: o teorema fundamental do calculo. [Experimentation with Graphing Calculators: the investigation of Fundamental Theorem of Calculus]. *X National Brazilian Congress of Graduate Students in Mathematics Education*. Federal University of Mina Gerais, Belo Horizonte, Brazil.

Scucuglia, R. (2006). O teorema fundamental do calculo com calculadoras graficas. [The Fundamental Theorem of Calculus and Graphing Calculators]. *VIII Sao Paulo Conference in Mathematics Education*. UNICSUL University, Sao Paulo, Brazil.

Scucuglia, R. (2005). Experimentacao com calculadoras graficas: o teorema fundamental do calculo. [Experimentation with Graphing Calculators: the investigation of Fundamental Theorem of Calculus]. *X National Brazilian Congress of Graduate Students in Mathematics Education*. Federal University of Mina Gerais, Belo Horizonte, Brazil.

Scucuglia, R. (2005). Calculadoras graficas e o teorema fundamental do calculo. [Graphing Calculators and the Fundamental Theorem of Calculus]. *IX National Brazilian Congress of Graduate Students in Mathematics Education*. Faculty of Education, University of Sao Paulo, Sao Paulo, Brazil.

Scucuglia, R. (2004). Funcoes, calculadoras graficas e demonstracao. [Functions, Graphing Calculators and Demonstration]. *VIII National Brazilian Congress of Graduate Students in Mathematics Education*. State University of Parana, Londrina, Brazil.

Abstracts/Posters in Refereed Conferences and Professional Meetings

Scucuglia, R. (2011). Experiencing surprises through students' digital mathematical performances. *2011 Canadian Mathematical Society Winter Meeting.*

Scucuglia, R. (2011). Interpreting Zone of Proximal Development through the Ontological Quadrivium. *33rd Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME –NA 2011)*

Scucuglia, R. & Gadanidis, G. (2011). The Math + Science Performance Festival. *33rd Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME –NA 2011)*

Scucuglia, R. (2011). The Math + Science Performance Festival. *II Education Research Symposium.* London, Ontario: The University of Western Ontario.

Scucuglia, R. (2011). Interpreting the Zone of Proximal Development through the Ontological Quadrivium. *II Education Research Symposium.* London, Ontario: The University of Western Ontario.

Scucuglia, R. (2011). Lights, Camera, Math: Students becoming performance mathematicians. In *Proceedings of 2011 Western Research Forum*. London, Ontario: The University of Western Ontario.

Scucuglia, R. (2010) How do students-with-graphing-calculators investigate the fundamental theorem of calculus. In Proceedings of 2010 *Western Research Forum* London, Ontario: The University of Western Ontario.

Scucuglia, R. (2009). Digital Mathematical Performance In *Proceedings of 2009 Western Research Forum* London, Ontario: The University of Western Ontario.

Scucuglia, R. (2009). Investigating the Fundamental Theorem of Calculus with Graphing Calculators In *Proceedings of 9th Annual Graduate Students in Education Symposium* Kingston, Ontario: Queens University - Faculty of Education.

Scucuglia, R. (2004). GPIMEM and Research. *20's Conference of Graduate Program in Mathematics Education*. Sao Paulo State University, Brazil.

Scucuglia, R. (2003). Applets: One New Technology and its application in Mathematics Education. *10s Conference of GPIMEM*. Sao Paulo State University, Brazil.

Scucuglia, R. (2003). Functions, Graphing Calculators and Demonstration. *10's Conference of GPIMEM*. Sao Paulo State University, Brazil.

Scucuglia, R. (2002). Calculadoras graficas: conjecturando um teorema a partir de um estudo investigative de funcoes. [Graphing Calculators: Conjecturing a Theorem through an investigating study of functions]. In *V Symposium for Undergraduate Researchers*. Sao Paulo State University, Brazil.

Workshops

Scucuglia, R. (2010). Creating Video for the Math + Science Performance Festival. *Workshop at MSTSE Community Evening - Math, Science, Technology, and Arts related to Society and Environment*. Faculty of Education, The University of Western Ontario, London, ON, Canada.

Scucuglia, R. & Miranda, J. P. (2009). *Mathematics and Moviemaking*. Heloisa Marasca State Public School. Rio Claro, SP. Brazil.

Scucuglia, R. (2009). *Mathematics and Digital Performance.* Heloisa Marasca State Public School. Rio Claro, SP. Brazil.

Scucuglia, R. (2009) Math Performance. *Week of Mathematics Majors*. UNIFACEF, Franca, Brazil.

Scucuglia, R.; Jackson, M. & Gadanidis, G. (2008). Imagine This: Adding imagination to mathematics. *Workshop at MSTSE Community Evening - Math, Science, Technology, and Arts related to Society and Environment*. Faculty of Education, The University of Western Ontario, London, ON, Canada.

Scucuglia, R., Borba, M. (2007). Performance matematica digital: criando narrativas digitais em educacao matematica [Digital Mathematical Performance: Creating digital narratives in mathematics education]. *IX Brazilian National Congress in Mathematics Education*. University of Belo Horizonte, Belo Horizonte, Brazil.

Scucuglia, R. (2006). Graphing Calculators. *Course of Degree in Mathematics Day*. UNIP University, Sao Paulo, Brazil

Scucuglia, R. (2006). Graphing Calculators in High School. *Program of Continuing Education- Web of the Knowledge.* University of Holy Heart, Sao Paulo, Brazil

Scucuglia, R. (2006). Calculators in Elementary School. *Program of Continuing Education - Web of the Knowledge.* University of Holy Heart, Sao Paulo, Brazil

Scucuglia, R. (2006). *Didactic and development of mathematical concepts: Graphic Calculators*. Graduate Program in Mathematics Education. University of Holy Heart, Sao Paulo, Brazil

Scucuglia, R. (2005). Graphing Calculators. *First International Symposium of Educational Languages.* University of Holy Heart, Sao Paulo, Brazil

Scucuglia, R. (2004). Notions about Geometricks Software. *Workshop for undergraduate students in mathematics*. Sao Paulo State University, Brazil.

Scucuglia, R. (2004). Graphing Calculators. *Second Week of Studies in Mathematics*. FAI University, Sao Paulo, Brazil.

Scucuglia, R. (2004). Notions about Graphing Calculators. *Workshop for under graduate students in mathematics*. Sao Paulo State University, Brazil.

Scucuglia, R. (2002). Notions about Graphing Calculators. *Workshop for under graduate students in mathematics*. Sao Paulo State University, Brazil.

Invited Speaker

Scucuglia, R. (2011) Teaching experiments and video analysis in Mathematics Education research. Federal University of South Frontier. Erechim, RS, Brazil.

Scucuglia, R. (2009) Digital Mathematical Performance, ULBRA, Canoas, RS, Brazil

Scucuglia, R. (2009) Digital Mathematical Performance, URI, Erechim, RS, Brazil.

Scucuglia, R. (2009) Digital Mathematical Performance, UNIFACEF, Franca, SP, Brazil.

Scucuglia, R. (2009) Digital Mathematical Performance, UNESP, Rio Claro, SP, Brazil.

Scucuglia, R. (2008). GPIMEM's Coordinator interviews GPIMEM's Researchers. *15's Conference of GPIMEM*. Sao Paulo State University, Brazil.

Hughes, J. & **Scucuglia, R.** (2007). Blogs, Wikis and Digital Stories: Infusing new media into your program. Presentation for teachers. *Courtice Secondary School*, Ontario, Canada

Scucuglia, R. (2006). Graphing Calculators and Mathematics Education. Undergraduate mathematics majors seminar. Sao Paulo State University, Brazil.

Scucuglia, R. (2002). Graphing Calculators and CBR. *Mathematical Seminar for under graduate students.* Sao Paulo State University, Brazil.

Scucuglia, R. (2002). Graphing Calculators: enunciating and proving a theorem based on an investigative study about functions.. *Mathematics Symposium for under graduate students*. Institute of Computer Sciences and Mathematics, University of Sao Paulo, Brazil.

Organization of Conferences, Academic Meetings, and Math Festivals

Et. Al. & **Scucuglia, R.** (2011). *2nd Annual Research in Education Symposium*. Faculty of Education. The University of Western Ontario. London, Ontario. Canada.

Scucuglia, R. & Borba, M. C. (2010). *II Performance Matemática Festival*. Heloisa Marasca State Public School. Rio Claro, SP. Brazil.

Scucuglia, R., Borba, M. C. & Miranda, J. P. (2009) *Math Performance Award.* Heloisa Marasca State Public School. Rio Claro, SP. Brazil.

Scucuglia, R. & Borba, M. C. (2009). *I Performance Matemática Festival*. Heloisa Marasca State Public School. Rio Claro, SP. Brazil.

Borba, M. C., Franchi, R. H. O. L., **Scucuglia, R.** (2008). *Conferencia GPIMEM 15 Anos*. Sao Paulo State University, Rio Claro, SP, Brazil.

Gadanidis, G., Hughes, J., **Scucuglia, R.** (2008) *Free Math Concert @ Alderville First Nation Community*. Rice Lake, ON, Canada.

PGEM, R. C., **Scucuglia, R.** (2004). *Conferência 20 anos do Programa de Pós-Graduação em Educação Matemática da UNESP/Rio Claro*. Sao Paulo State University, Rio Claro, SP, Brazil.

Borba, M. C., **Scucuglia, R.** (2003). *Conferencia de 10 anos do GPIMEM*. Sao Paulo State University, Rio Claro, SP, Brazil.

Digital Mathematical Performances

Scucuglia, R. (2011) *My Mistake.* http://www.edu.uwo.ca/mpc/mpf2011/mpf2011-M51.html

Scucuglia, R (2010) *Cell Phone Math*. http://www.edu.uwo.ca/mpc/mpf2010/mpf2010-132.html

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Scucuglia, R (2010) *The Series will come to us*. http://www.edu.uwo.ca/mpc/mpf2010/mpf2010-165.html

Scucuglia, R & Petrela, G. (2010) *The Mathematics of the World Cup*. www.edu.uwo.ca/mpc/mpf2010/mpf2010-166.html

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Scucuglia, R & Miranda, J.P. (2009) *Sooner or Later: Mathematics*. www.edu.uwo.ca/mpc/mpf2010/mpf2010-114.html

Scucuglia, R (2009). *L-Patterns: The Movie.* www.edu.uwo.ca/mathscene/mathfest2009/mathfest269.html

Gadanidis, G., Hughes, J. & **Scucuglia, R** (2008) *Alderville First Nation Project*. www.edu.uwo.ca/mpc/alderville.html

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Scucuglia, R (2007). *L-Patterns: The song*. www.edu.uwo.ca/mathscene/mathfest2009/mathfest269.html

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Collaboration in Digital Mathematical Performances

Gadanidis, G. (2010) *Joy of X: The Math Band Live @TVDSB*. http://joyofx.ca/music/tvdsb2010.html

Gadanidis, G. (2010) *Math Lust* http://joyofx.com/music/mathlust.html

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Current as of June, 2012.