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OPTIMAL EX ANTE COMMITMENT AND
EX POST ENTRY DETERRENCE IN
A STOCHASTIC ENVIRONMENT

by

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ABSTRACT

This paper analyzes a model of entry-deterrence under a stochastic demand environment wherein established firms may indulge in ex post production adjustment after their initial ex ante production commitment. Although potential entrants, who being latecomers, do not have the incumbents' advantage of being able to set up a prior capacity investment, they nevertheless have the ex post flexibility of choosing when to enter the industry. The presence of potential ex post entry and competition alters the stochastic demand distribution facing the incumbents and hence their optimal ex ante capacity investment which in turn affects the outcomes of the ex post competitive equilibrium. The set of possible outcomes is shown to be much richer compared to those initially conjectured by Bain [1956] in his famous classification of entry-deterrence.
I. **INTRODUCTION**

The earliest models on entry-deterrence postulate that an established firm, when faced with potential entry, would attempt to deter entry by maintaining a high constant output (Sylos Postulate: see also Bain [1956], and Modigliani [1958]) or by high excess capacity (Excess Capacity Hypothesis: see Hicks [1954], Pashigian [1968], Wenders [1971] and Spence [1977]).

Recent developments (e.g. Osborne [1973], Needham [1978], Dixit [1979, 1980], and Spulber [1981]), however, have shown that the above hypotheses contain some fundamental problems that pertain to the rationality of both the established firm and entrants:¹ (a) It may not be rational for the established firm to deter entry even when it is feasible to do so. (b) If entry actually occurs, it is not rational for the established firm to maintain output or excess capacity; consequently both these strategies do not serve as sufficiently credible threats that would deter a rational potential entrant. (c) The assumed post-entry Stackleberg game, which assumes Stackleberg leadership on the part of the established firm, is dubious if in fact there is absent any asymmetrical advantage between both parties in the post-entry phase.

In a recent paper, Dixit [1980] appropriately addresses the above interrelated issues by employing a model which upheld the rationality of both the established firm and entrants and where threats to deter entry are made credible because of irreversible² capacity investment. Thus, given any rule of the post-entry game, the established firm, by its choice of its prior capacity investment, may alter the outcome of the game to its advantage. It was then shown that optimality need not necessarily imply
complete entry-deterrence but it may accommodate various outcomes such as (according to Bain's [1956, pp. 21-22] famous classification) "blockaded entry", "effectively impeded entry" or "ineffectively impeded entry".  

Spulber [1981] extended Dixit's [1980] analysis to a dynamic (two-period) model but still maintains the latter's assumption of a non-stochastic environment. However, he went on to emphasize that "the question of strategic capacity investment and entry needs to be re-examined when market uncertainty is present. The presence of risk will serve to emphasize strongly the advantage of flexibility which an entrant possesses as compared to an established firm." (Spulber [1981], p. 513.)

We fully concur with Spulber's observation for the following reason. Although established firms are historically endowed with the "first move" and therefore have the advantage over entrants (late-comers) in being able to set up prior capacity investment, entrants, nevertheless, usually have the ex post flexibility of choosing when to enter the industry; and it is to be expected that entry will occur only in the better states of demand. To explain this important observation, it is essential that a stochastic demand environment be explicitly modeled.

The task of this paper is therefore to extend Dixit's [1980] analysis of entry-deterrence to consider a stochastic demand environment wherein entrants have the ex post flexible choice of entry. Several interesting results are obtained which are unique to models of uncertainty. The main result, however, is that we are able to generate a far richer set of possible outcomes compared to Bain's classification.

In Section II, we present a model of the monopolist under demand uncertainty and in the absence of potential entry. Unlike most of the conventional models in the literature which artificially restrict the monopolist to
only \textit{ex ante} production (before demand is revealed), this model allows for the possibility of \textit{ex post} production adjustment (after demand is revealed). Section III then opens the model to allow for potential entry. Besides the historical endowment of the "first move" given to the incumbent, no other asymmetrical advantage exists between the incumbent and entrants in the \textit{ex post} entry phase. Consequently, given the potentially large number of \textit{ex post} entrants, it is natural to use the perfectly competitive equilibrium as the appropriate rule of the \textit{ex post} game. The problem facing the incumbent then reduces to that of choosing the optimal \textit{ex ante} commitment given the threat of potential \textit{ex post} entry and competition. This \textit{entry-constrained optimum} (with potential entry) is then compared to the \textit{entry-free optimum} (without potential entry) in Section II; and it is through this comparison that we generate the rich set of possible outcomes. Finally, some concluding remarks are gathered in Section IV.

II. \textbf{MODEL WITHOUT POTENTIAL ENTRY}

Consider first the situation where there is no potential entry, and existing firms act as a perfect cartel or a "monopoly". Let the stochastic industry demand condition be given by

\begin{equation}
(1) \quad p(q, \theta) \quad \text{with } \frac{\partial p}{\partial q} < 0 \text{ and } \frac{\partial p}{\partial \theta} > 0
\end{equation}

where $p$ and $q$ are, respectively, the demand price and total output; and $\theta$ is a random variable with probability distribution $\Phi(\theta)$.

At this stage, it is worth noting that the problem of the monopolist under uncertain demand has been the subject of much analysis in recent years (e.g., Baron [1971], Leland [1972], Lim [1980, 1981]) but all these models artificially constrain the firm to have no possibility of \textit{ex post} production
adjustment after demand is revealed. In general, firms are normally observed to precommit to some \textit{ex ante} production but they also reserve the option to adjust their output \textit{ex post} whenever the better states of demand occur. To capture this realistic feature, we shall let \( y \geq 0 \) describe the firm's \textit{ex ante} production commitment and \( z \geq 0 \) be the \textit{ex post} production adjustment. Thus, total output then becomes

\[ q = y + z, \quad y \geq 0, \ z \geq 0 \]  

(Note that \( z \) is a random variable depending on the states of demand whereas \( y \) is not.)

Whether the firm's optimal mix between \( y \) and \( z \) consists only of \textit{ex ante} production \((y > 0, z = 0)\), or only \textit{ex post} production \((y = 0, z \geq 0)\), or some combination of both \((y > 0, z \geq 0)\) depends crucially on the cost conditions underlying the \textit{ex ante} and the \textit{ex post} production processes. Turnovsky [1973]\textsuperscript{4} convincingly argued that production cost of \( z \) should in general exceed that of \( y \) for the simple reason that \textit{ex post} adjustment usually involves the use of costlier overtime labor or the greater organizational cost of putting together a faster production process. A simple version of the total cost function can be expressed as

\[ C(y, z) = c_y y + c_z z \quad \text{where } c_z > c_y \]  

from which the marginal cost schedule is graphed in Figure 1.
This simplified cost function is analogous to that employed by Dixit [1980] except for the difference in interpretation. In his model of entry-deterrence, \( y \) is the capacity output chosen by the established firm such that for \( q \leq y \) the constant average variable cost is \( c_y \). The entrant in his model, who has no prior commitment to capacity, must acquire capacity if he were to enter to produce \( z > 0 \). His constant average cost of production is therefore \( c_z \) which is made up of the constant average variable cost \( c_y \) plus the constant average capacity-acquiring cost \( c_z - c_y \). In this section where there is no potential entry, \( c_z - c_y \) can therefore be interpreted as either the marginal ex post production adjustment cost or the marginal capacity-acquiring cost of the firm if he wishes to produce over and above his prior commitment \( y \).

The important point to note is that the ex ante commitment \( y \) is a choice variable, and so is \( z \) in this model of no potential entry. They are solutions to the expected profit-maximizing problem:

\[
\begin{align*}
(4) \quad & \max_{y \geq 0} \max_{z \geq 0} E[\max \{ R(y + z, \theta) - c_y y - c_z z \}] \\
\text{where } & \ R(y + z, \theta) = p(y + z, \theta)(y + z) \text{ is the revenue function which is assumed to be strictly concave in } (y + z). \text{ The problem can be split up into two stages. First, given any level of } y, \text{ optimal } z \text{ solves the problem }
\end{align*}
\]

\[
\begin{align*}
(5) \quad & \max_{z \geq 0} R(y + z, \theta) - c_y y - c_z z \equiv \pi(y, \theta) \\
\text{and optimal } y \text{ then solves the problem }
\end{align*}
\]

\[
\begin{align*}
(6) \quad & \max_{y \geq 0} E[\pi(y, \theta)].
\end{align*}
\]
Ex post Production Adjustment

Defining $MR(y + z, \theta) \equiv \partial R(y + z, \theta)/\partial q$ to be the marginal revenue function, the ex post solution (to problem (5)) can be written as $z(y, \theta)$ which implicitly solves

$$(7) \quad MR(y + z, \theta) = c_z \Rightarrow z(y, \theta) = \begin{cases} > 0 & \text{if } MR(y, \theta) > c_z \\ = 0 & \text{if } MR(y, \theta) \leq c_z \end{cases}$$

Because $R(y + z, \theta)$ is strictly concave in $(y + z)$, condition (7) is both necessary and sufficient for optimum. Evaluated at output level $y$, if the ex post marginal revenue $MR(y, \theta) > c_z$ (the ex post marginal production cost), then $z > 0$. Otherwise, if $MR(y, \theta) \leq c_z$ then $z = 0$ because of the inequality constraint that $z \geq 0$.

In Figure 2(a), we illustrate the solution for the case where $\theta$ is binomially distributed as

$$\theta = (\theta_1, \theta_2) \text{ with probability } (q, (1 - q)),$$

where $\theta_2 > \theta_1$ indicates that state 2 is the better state. Therein, let $y'$ and $y''$ satisfy $MR(y', \theta_1) = c_z$ and $MR(y'', \theta_2) = c_z$. Then it is easily inferred that the solution $z(y, \theta)$ takes on the form:

$$(8) \quad z(y, \theta) = \begin{cases} [(y' - y), (y'' - y)] & \text{with probability } (q, (1 - q)) \text{ if } y \leq y' \\ [0, (y'' - y)] & \text{with probability } (q, (1 - q)) \text{ if } y' \leq y \leq y'' \\ 0 & \text{with probability } 1 \quad \text{if } y'' \leq y \end{cases}$$

Optimal Ex ante Production Commitment Without Potential Entry

Substituting $z(y, \theta)$ for $z$ in (5) and then taking expectations with respect to $\theta$, the expected profit now becomes
(9) \[ E[\pi(y, \theta)] = \begin{cases} q[R(y', \theta_1) - c_z y' + (c_z - c_y) y] + (1 - q)[R(y'', \theta_2) - c_z y'' + (c_z - c_y) y] & \text{if } y' \leq y'' \\ q[R(y, \theta_1) - c_y y] + (1 - q)[R(y, \theta_2) - c_y y] & \text{if } y'' \leq y \end{cases} \]

where \[ A \equiv q[R(y', \theta_1) - c_z y' + (1 - q)[R(y'', \theta_2) - c_z y''] \text{ and } B \equiv (1 - q)[R(y'', \theta_2) - c_z y''] \]

are constants that are independent of \( y \). Now maximizing (9) with respect to \( y \) yields the necessary and sufficient condition:

(10) \[ 0 = E[\text{MR}_o] - c_y \equiv \begin{cases} [q_c + (1 - q)c_z] - c_y & \text{if } y' \leq y'' \\ [qMR(y, \theta_1) + (1 - q)c_z] - c_y & \text{if } y' \leq y \leq y'' \\ [qMR(y, \theta_1) + (1 - q)MR(y, \theta_2)] - c_y & \text{if } y'' \leq y \end{cases} \]

This simply states that optimal \( y \) must equate the effective expected marginal revenue \( E[\text{MR}_o] \) (the bracketed \([·]\) terms in R.H.S. of (10)) with the marginal cost \( c_y \).

In Figure 2(b), the \( E[\text{MR}_o] \) schedule is shown to consist of three line segments with kinks occurring at \( y' \) and \( y'' \). The explanation for it is as follows. For \( y \leq y' \), note that from (8) the optimal ex post production adjustment is \( z(y, \theta_1) = y' - y \geq 0 \) in state 1 and \( z(y, \theta_2) = y'' - y \geq 0 \) in state 2. Thus, regardless of the value of \( y \) in the region \( y \leq y' \), the presence of ex post adjustment in both states makes the total output \( y + z \)
constant at \( y' \) (in state 1) and at \( y'' \) (in state 2); consequently the expected revenue is constant being unaffected by \( y \). However, since an increase in one unit of \( y \) implies a corresponding unit reduction in \( z \) in both states, there is cost-saving of \( c_z \) in both states for every unit increase in \( y \). Thus, the effective expected marginal revenue \( E[MR_o] \) of \( y \) production in the region \( y \leq y' \) is \( qc_z + (1 - q)c_z = c_z \) (the \{\cdot\} term in line 1 of (10) or the segment \( c_z k' \) in Figure 2(b)).

In the region \( y' \leq y \leq y'' \), ex post adjustment (from (8)) in state 1 is \( z(y, \theta_1) = 0 \) and in state 2 is \( z(y, \theta_2) = y'' - y \geq 0 \). Consequently, while the value of \( y \) in this region affects the revenue in state 1, it does not affect the state-2 revenue since the presence of ex post adjustment makes the total output \( y + z(y, \theta_2) \) constant at \( y'' \). Nevertheless, an increase in one unit of \( y \) still yields a marginal benefit (due to cost-saving from a corresponding unit reduction in \( z(y, \theta_2) \)) of \( c_z \) in state 2, while the marginal revenue in state 1 is \( MR(y, \theta_1) \). Thus, the effective expected marginal revenue \( E[MR_o] \) from \( y \) production is \( qMR(y, \theta_1) + (1 - q)c_z \) (the bracketed \{\cdot\} term in line 2 of (10) or the segment \( k' k'' \) in Figure 2(b)).

Lastly, for \( y \geq y'' \), we have (from (8)) \( z(y, \theta_1) = z(y, \theta_2) = 0 \). There is no positive ex post adjustment in both states. Hence the expected marginal revenue of \( y \) is simply \( qMR(y, \theta_1) + (1 - q)MR(y, \theta_2) \) (the bracketed \{\cdot\} term in line 3 of (10) or the segment \( k'' N \) in Figure 2(b)).

It is important to emphasize at this stage that the area under the effective expected marginal revenue \( E[MR_o] \) schedule in Figure 2(b) does not at all represent the expected total revenue. This is because the expected revenue (the bracketed \{\cdot\} term in the R.H.S. of (9)) consists of constants
such as A and B which are not captured when we perform the integral over the $E[MR_o]$ function contained in (10).

The properties of the optimal solution for $y$ and $z$ can now be easily inferred from Figure 2(b). First, since the *ex post* adjustment cost $(c_z - c_y) > 0$, the first line of equation (10) can never hold, and optimal $y^*$ is restricted to the range $y^* > y'$; and from (8), optimal $z(y^*, \theta_1) = 0$ indicating $y^*$ will always be large enough such that *ex post* production adjustment, if it ever occurs, will occur only in the better state.

Second, if the *ex post* adjustment cost $(c_z - c_y) > 0$ is small such as when $c_y$ (e.g., $c_y^a$) lies between $K'$ and $K''$ in Figure 2(b), then $y^*$ (e.g., $y_{\alpha}^a$) lies in the region $y' < y^* < y''$ so that from (8), *ex post* production adjustment $z$ occurs only in the better state. Moreover, in the range $y' < y^* < y''$, optimum $y^*$ is not affected by the demand in the better state since $\theta_2$ does not enter as an argument describing the segment $K'K''$ of the $E[MR_o]$ schedule. This is indeed surprising since if $\theta_2$ were to change one would expect a change also in optimal $y^*$. Our result, however, indicates the contrary. The reason is because the *effective* expected marginal revenue $E[MR_o]$ is defined not only for the *ex ante* production $y$ but also for *ex post* production adjustment $z$ (which, of course, depends on $y$). In the range $y' < y^* < y''$, the optimality of $y^*$ fully preempts *ex post* production adjustment $z$ in the good state; hence changes in $\theta_2$ will only affect the level *ex post* adjustment $z$ and not the *ex ante* production level $y^*$.

Finally, if the *ex post* adjustment cost $(c_z - c_y) > 0$ is large such as when $c_y$ (e.g., $c_y^b$) lies below $K''$ in Figure 2(b), then $y^*$ (e.g., $y_{\beta}^b$) $> y''$ which, from (8), implies *ex post* production adjustment will not occur. In
this case, the firm takes the advantage of the low cost of \textit{ex ante} production to produce a high level of \textit{y} and, because of the large \textit{ex post} adjustment cost \((c_z - c_y)\), refrains from any \textit{ex post} production adjustments.

III. \textbf{MODEL WITH POTENTIAL EX POST ENTRY}

The model in the preceding section is now extended to allow for potential entry. As in nearly all the literature on entry-deterrence, it is assumed that for historical reasons the "monopolist" (or established firms acting as a perfect cartel) is in the market first so that it has the advantage of being able to install the \textit{ex ante} capacity. Its constant average variable cost of production is \(c_y\). Alternatively \(c_y\) can be interpreted as the constant average cost of the slower \textit{ex ante} production technology.

Entrants, who by definition are late comers, may enter \textit{ex post} or after demand conditions are revealed. However, their constant average production cost \(c_z\) is greater than \(c_y\) since it consists not only of the constant variable production cost \(c_y\) but also the average capacity-acquiring cost \((c_z - c_y)\). Alternately \(c_z (> c_y)\) can be interpreted as the constant average cost of the costlier \textit{ex post} production technology since it entails a speedier production process.

It is important to emphasize that except for providing the incumbent the advantage of the "first move", there are no other contrived informational or cost differences between the incumbent and the entrants in the \textit{ex post} entry phase. For instance, although entrants have the \textit{ex post} flexibility of choosing when to enter and producing at marginal cost \(c_z\), the incumbent has the same \textit{ex post} flexibility to produce, at the same marginal cost \(c_z\), its \textit{ex post} production adjustment \(z\).
The question to be addressed shortly is how the presence of potential entry affects the incumbent's ex ante commitment \(y\). More precisely, the rule of the ex post game to be considered is one of perfectly competitive price (Bertrand-Nash) equilibrium. However, the incumbent may exercise leadership by altering the initial conditions, through its choice of prior commitment \(y\), to affect the outcome of the ex post equilibrium. It will be shown that, depending on the underlying cost and demand parameters, optimal \(y\) (in this model of uncertainty and ex post flexibility) may generate a richer set of outcomes compared to Bain's classification on entry-deterrence.

**Ex post Competitive Price Equilibrium**

We begin the analysis by considering first the nature of the ex post equilibrium with entry. Given any level of ex ante production level \(y\) of the established firm, ex post entry may occur as long as the ex post demand price \(p(y, \theta) > c_z\) (the marginal production cost of entrants); and entry will proceed until the total ex post output \(z > 0\) is such that price is driven down to \(p(y + z, \theta) = c_z\). The ex post rule of the game is therefore the ex post competitive equilibrium at which zero profit prevails for all ex post production of \(z\) regardless of whether they are produced by entrants or by the incumbent. Thus in Figure 3(a) let us define \(y_1\) and \(y_2\) to be such that \(p(y_1, \theta_1) = c_z\) and \(p(y_2, \theta_2) = c_z\), respectively. Then the ex post equilibrium level of \(z\) produced can be written as

\[
\begin{align*}
  z(y, \theta) &= \begin{cases} 
  [y_1 - y, y_2 - y] & \text{with probability } [q, (1-q)] \text{ if } y \leq y_1 \\
  [0, y_2 - y] & \text{with probability } [q, (1-q)] \text{ if } y_1 \leq y \leq y_2 \\
  0 & \text{with probability } 1 \text{ if } y_2 \leq y 
  \end{cases}
\end{align*}
\]

which differs from that expressed in (8).
Figure 3(a)

$E[MR_n]$ in thick (discontinuous) line segments
$E[MR_o]$ in thin line segments

Figure 3(b)
Optimal *Ex ante* Production Commitment with Potential *Ex post* Entry

Given *ex post* competitive equilibrium in $z$ production, the resulting stochastic price distribution confronting the incumbent can be derived as follows.

In Figure 3(a), $p(y, \theta_i)$ ($i=1,2$) illustrates the *primitive* demand price in state $i$. However, given the *ex post* competitive equilibrium where all *ex post* production $z$ must yield only zero profit, the *effective* demand price distribution $p_n(y, \theta)$ facing the incumbent is

$$p_n(y, \theta) = \begin{cases} [c_z, c_z] & \text{with probability } [q, (1-q)] \text{ if } y \leq y_1 \\ [p(y, \theta_1), c_z] & \text{with probability } [q, (1-q)] \text{ if } y_1 \leq y \leq y_2 \\ [p(y, \theta_1), p(y, \theta_2)] & \text{with probability } [q, (1-q)] \text{ if } y_2 \leq y \end{cases}$$

(12)

To verify this, observe in Figure 3(a) that if $y \leq y_1$ we have $p(y, \theta_i) \geq c_z$ which means *ex post* entry will occur in all states ($i=1,2$) until *ex post* price is depressed to equal $c_z$; hence we have line 1 in (12). In the region $y_1 \leq y \leq y_2$, we have $p(y, \theta_1) \leq c_z \leq p(y, \theta_2)$ so that *ex post* entry does not occur in state 1 but entry occurs in state 2 until state-2 price is driven down to $c_z$; hence we have line 2 in (12). Lastly, for $y \geq y_2$, $p(y, \theta_i) \leq c_z$ for all $i$ so that entry will not occur with the result that the incumbent faces $p(y, \theta_i)$ for all state $i$; thus verifying line 3 of (12).

The difference in the *primitive* price distribution $p(y, \theta)$ and the *effective* price distribution $p_n(y, \theta)$ in (12) emphasizes the discipline that entrants impose on the incumbent. Faced with *effective* price $p_n(y, \theta)$ in (12), the incumbent's expected profit can be expressed as
\[ E[\eta_n(y, \theta)] = \begin{cases} q[c_z - c_y]y + (1-q)[c_z - c_y]y & \text{if } y \leq y_1 \\ q[p(y, \theta_1) - c_y]y + (1-q)[c_z - c_y]y & \text{if } y_1 \leq y \leq y_2 \\ q[p(y, \theta_1) - c_y]y + (1-q)[p(y, \theta_2) - c_y]y & \text{if } y_2 \leq y \end{cases} \]

The necessary and sufficient condition for optimal \( y \) is therefore

\[ 0 = E[MR_n] - c_y = \begin{cases} qc_z + (1-q)c_z - c_y & \text{if } y \leq y_1 \\ qMR(y, \theta_1) + (1-q)c_z - c_y & \text{if } y_1 \leq y \leq y_2 \\ qMR(y, \theta_1) + (1-q)MR(y, \theta_2) - c_y & \text{if } y_2 \leq y \end{cases} \]

This simply means that with potential entry, optimal \( y \) must equate the effective expected marginal revenue \( E[MR_n] \) (the bracketed \([\cdot]\) term in the R.H.S. of (14)) and the marginal cost \( c_y \). (Note that \( MR_n \) is the marginal revenue given demand price \( p_n(\cdot) \) whereas \( MR \) is derived with respect to demand price \( p(\cdot) \).)

Comparing the effective expected marginal revenue \( E[MR_n] \) (for an incumbent who faces potential entry) in (14) with \( E[MR_o] \) (for a monopolist facing no threat of entry) in (10), note that they are identical except for the regions of \( y \) at which they apply. This becomes more transparent in Figure 3(b) where we superimpose the \( E[MR_n] \) schedule (consisting of the thick discontinuous line segments \( c_zK_1, L_1K_2, \) and \( L_2N \)) on the \( E[MR_o] \) schedule (the thin line segments \( c_zK', K'K'' \) and \( K''N \) taken from Figure 2(b)). Notice that while the \( E[MR_o] \) schedule has kinks at \( y' \) and \( y'' \), the \( E[MR_n] \) schedule is discontinuous at \( y_1 \) and \( y_2 \).

This remarkably simple and illuminating diagram in Figure 3(b) shall now serve as the focus of analysis in our comparison between the entry-free optimum (without potential entry) and the entry-constrained optimum (with potential entry).
For the sake of not further cluttering the diagram, the insertion of the horizontal \( c_y \) (\(<c_z\)) line is left to the reader. Let \( y^{**} \) define the entry-constrained optimum (which satisfies (14) or the intersection between \( c_y \) and the \( E[MR_n] \) schedule) and \( y^* \) the entry-free optimum (which satisfies (10) or the intersection between \( c_y \) and the \( E[MR_o] \) schedule). With these definitions, a few general results are in order.

First, since \( E[MR_n] \geq E[MR_o] \), it is always true that \( y^{**} \geq y^* \). In other words, the presence of potential entry causes the incumbent's prior commitment to be at least as large as when there is no threat of entry.

In other words, the presence of potential entry will, in some cases, induce the monopolist to behave 'more competitively' by expanding its prior commitment (i.e., \( y^{**} > y^* \)); but there also exist cases where the monopolist will continue to maintain its status quo behavior (i.e., \( q^{**} = q^* \)).

Second, since \( c_y < c_z \), the first line of condition (14) can never hold so that \( y^{**} \) must lie in the range \( y^{**} \geq y_1 \), which from (11) implies ex post entry in state-1, \( z(y^{**}, q_1) = 0 \). In other words, with potential entry, the incumbent's prior commitment will always be high enough such that ex post entry will never occur in the bad state of demand. Entry, if it ever occurs, does so only in the better state. This emphasizes the point that while entrants hold the disadvantageous position as late-comers who do not have the opportunity to set up a prior capacity, they nevertheless have the ex post flexibility of choosing to enter the market only in the better state of demand.

Turning now from the above general results to more specific cases, it can be seen from Figure 3(b) that as we vary \( c_y \) downward from \( c_y = c_z \), optimal \( y^{**} \) accommodates an extremely rich set of possible outcomes:

**Case A: Easy Entry** (Bain's terminology): This happens when \( c_y = c_z \) so that
existing firms have no cost advantage over potential entrants. In long-run 
equilibrium \( p_n(\ast, \theta_i) = c_z = c_y \) for all \( i \) so that regardless of the exogenous 
fluctuations in the primitive demand functions \( p(\ast, \theta) \), the equilibrium price 
reduces to \( c_z = c_y \) with probability 1 or with certainty. In equilibrium, 
all uncertainty disappears despite fluctuations in the primitive demand 
function \( p(\ast, \theta) \) and also there is no differentiation between \textit{ex ante} and 
\textit{ex post} production; all firms behave as in the certainty model of perfect 
competition with each earning zero profit.

Case B: \textbf{Ineffectively impeded entry} (Bain's terminology): This happens, 
according to Bain, when the incumbents' cost advantage is small so that 
it is preferable to allow entry. In our model, this is captured by the 
case when \( c_y \) lies in the gap \( K_1L_1 \) in Figure 3(b) so that \( y' < y* < y_1 \) and 
\( y^* < y** = y_1 \). The presence of potential entry causes the incumbent to 
expand their prior commitment from \( y^* \) to \( y** = y_1 \) but which is insufficient to 
block entry. In fact, from (11), it allows \textit{ex post} entry by the amount 
\( z(y_1, \theta_2) = y_2 - y_1 > 0 \) in the good state.

Case C: \textbf{Unimpeded entry} (our terminology): This case, which Bain did not 
consider, happens when \( c_y \) cuts the segment \( L_1K' \) where both \( E[MR_0] \) and \( E[MR_n] \) 
coincide. Thus we have \( y^* = y** \) and \( y_1 < y^* = y** < y' \). Herein, two 
interesting observations are worth noting. First because \( y^* = y** \), the 
presence of potential entry does not at all alter the prior commitment of 
the monopolist despite the fact that entry will occur in state 2 (since 
\( y_1 < y** < y_2 \)). It is important to emphasize the difference between this 
status quo behavior and that of Bain's "blockaded entry" (to be discussed 
later under Case F). Therein, it will be shown that the reason for the 
monopolist's unaltered optimum even in the face of potential entry is 
that the threat of entry is never realizable since the monopolist's optimum 
output is large enough to force prices below the limit price \( c_z \). In our 
case, status quo behavior is maintained despite realizable threat of entry.
in state 2. The reason for this is that in the segment $L_1K'$, $E[MR_0] = E[MR_n] = \{qMR(y, \theta_1) + ((1 - q)c_z)\}$ (see line 2 of (14) and (10)) which implies that with or without potential entry, the effective marginal revenue in state 2 remains unchanged at level $c_z$. Second, because $\theta_2$ does not affect the schedule $E[MR_n](= E[MR_o])$ in the segment $L_1K''$, changes in the good state of demand does not alter the optimal prior commitment regardless of whether or not there is threat of ex post entry.

Case D: Ineffectively impeded entry (again): This occurs when $c_y$ falls between $K''$ and $K_2$. Without potential entry, the monopolist produces $y^*$ in the region $y^* > y''$ which implies ex post adjustment does not occur at all in both states. However, with potential entry, it expands its prior commitment from $y^*$ to $y^{**}$; but because $y_1 < y^{**} < y_2$ such expansion is insufficient to deter ex post entry in state 2. This case differs from Case B only to the extent that the entry-free optimum in Case B ($y' < y^* < y''$) entertains ex post adjustment in state 2 whereas the entry-free optimum here ($y'' < y^*$) does not accommodate any ex post adjustment.

Case E: Effectively impeded entry (Bain's terminology): This occurs when $c_y$ falls between $K_2$ and $L_2$. Without potential entry, the monopolist's optimum $y^*$ occurs in the region $y'' < y^* < y_2$. Because $y'' < y^*$, its optimum does not entertain any ex post adjustment in both states. With potential entry, however, it expands $y^*(< y_2)$ to $y^{**} = y_2$ which effectively deters entry in all states of the world. This is the uncertainty analogue of the "limit-output" or "limit-price" model.

Case F: Blockaded entry (Bain's terminology): This happens when $c_y$ is small such that it cuts the segment $L_2N$. Since the $E[MR_o]$ and $E[MR_n]$ schedules coincide in this segment, we have $y^* = y^{**}$. The presence of potential entry, as
in Case C, does not alter the monopoly optimum $y^*$, but unlike Case C, the optimum here is already sufficiently large ($y^* > y_2$) as to completely block entry in all states of the world. In other words, potential entry is never realized.

Cases A to F that issue from Figure 3 represent the largest possible set of outcomes. The reader is encouraged to check that by varying $c_z$ and/or the parameters of the demand distributions, many other interesting sets of outcomes will emerge but these outcomes are merely the subset of what have been described above.

IV CONCLUSIONS

In this paper, the problem of the monopolist under stochastic demand was analyzed both for the case of no potential entry and for the case where potential entry is imminent.

In the absence of potential entry, this model improves on the existing models in the literature by allowing the firm to exercise the ex post flexibility of production adjustment which firms are often observed to do especially in the better states of demand. The most interesting result here is that there exists a range of optimal ex ante commitment which is invariant to changes in the better states of demand.

When open to potential entry, the model provides a natural setting for extending Dixit's [1980] analysis on entry-deterrence to incorporate the desirable feature of ex post flexibility of entrants. Although entrants, who being late-comers, do not have the incumbent's historical advantage of setting up a prior production capacity, they nevertheless may choose when to enter the industry. Given the potentially large number of ex post entrants
and the absence of any asymmetrical advantage in the *ex post* entry phase between the entrants and the incumbent, it is natural to consider the perfectly competitive equilibrium as the appropriate rule of the *ex post* game. The presence of potential *ex post* entry and competition therefore disciplines the monopolist by altering its demand distributions and hence its optimal prior commitment; which in turn affects the outcome of the *ex post* equilibrium.

In general, potential *ex post* entry induces the monopolist to react in two ways. In some cases, it is forced to behave 'more competitively' by expanding its prior commitment. In other cases, it maintains its status quo position in spite of realizable threat of entry. What is the perhaps most interesting is that in this model of stochastic demand and *ex post* flexible entry, the set of possible outcomes regarding entry deterrence is much richer than the classification originally conjectured by Bain.

The two-state model used here is for the sake of expositional clarity and introducing more states should cause no difficulty other than complicating the notation. Our only misgiving is that this static model is unable to capture much of the interesting dynamics but the compensating advantage is that the role of stochastic demand and *ex post* flexible entry has now become much more transparent.
Footnotes

1 In the dynamic limit-pricing models of Kamien and Schwartz [1971, 1975] and Gaskins [1971], the difficulties associated with entrants' rationality are avoided but only because the entrant's optimal behavior is never investigated. Instead, entry is simply assumed to be some arbitrary function of current price; an assumption which is inconsistent (see Flaherty [1981]) with an entrant's rational behavior.

2 The idea that threats can be made credible by using irreversible decisions originates with Schelling [1960] and has since been applied to entry-deterrence models also by Spence [1977], Salop [1979], Flaherty [1980], and Spulber [1981].

3 In Bain's [1956, pp. 21-22] classification, established firms may cause blockaded entry (monopoly solution is automatically sufficient to block entry) or effectively impeded entry (standard limit-output or limit-price model) or ineffectively impeded entry (allowing entry).

4 Although Turnovsky's [1973] model of the firm under demand uncertainty has the desirable feature of allowing for ex post production adjustment, he has focussed his analysis only for the perfectly competitive firm.

5 This simplifying specification of the cost function enhances expositional clarity and also allows us to maintain the similarity of the cost structure between Dixit's [1980] analysis and our model on entry-deterrence in the next section.

6 $E$ is an expectations operator.

7 The sufficiency condition is ensured because of the strict concavity of $R(q, e)$ in $q$. 
It is important to note that segment K"N which describes \( E[MR_0] = q \cdot MR(y, \theta_1) + (1-q)MR(y, \theta_2) \) entertains the possibility that, especially for larger values of \( y \), \( MR(y, \theta_1) < 0 \). When this happens, the firm obviously could capture a higher revenue in state 1 by disposing of some \( y \), say \( y_d \), such that \( MR(y-y_d, \theta_1) = 0 \). Introducing this feature would mean that the line segment K"N as drawn is not appropriate and should in fact consist of further sub-segments with different slopes. We have, however, chosen not to introduce this further complication since it is not essential to the results we wish to highlight in this paper. The reader, however, should be made aware of this very minor simplification we made in our drawing of segment K"N.

Since \( c_z \) is constant, there is no differentiation between the ex post production \( z \) of entrants or the incumbent in ex post zero-profit equilibrium.

The sufficiency condition again follows from the strict concavity of the revenue \( R(q, \theta) \) function in \( q \).

Since the area under the \( E[MR_0] (\leq E[MR_n]) \) schedule has been pointed out to be an underestimation of the expected revenue in the case of no potential entry, one should not be tempted to draw the implication that expected revenue in the case of potential entry exceeds that when there is no potential entry. This implication is obviously false.
References


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