Ex Ante and Ex Post Firms and Equilibrium Price Fluctuations

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I. INTRODUCTION

"There can be no doubt that in this country ever since 1790 our price structure has included a large number of prices that remained unchanged for months or years at a time, side by side with prices that changed monthly, weekly, daily, or in recent years even hourly" (Tucker (1938), p. 47). Furthermore, between agricultural and manufactured goods, it is evident that "for at least a century...recorded prices of manufactured goods [have been] more rigid than those of farm products" (Tucker (1938), p. 53).

Adam Smith (1776) himself was also aware of the greater volatility of agricultural prices and attributed it to the relative uncontrollability, due to weather conditions, of agricultural production. His presumption, of course, is that demand curves are stable. If demands for all commodities are instead unstable and fluctuating, it is immediately apparent that stochastic supply in agricultural production alone is neither necessary nor sufficient\(^1\) for farm product prices to persistently exhibit greater fluctuations than the prices of non-farm products. Consequently, the issue of differential price fluctuations among commodities in general still remains an important open question.

This question became a focus of a lively debate in the thirties. On the one hand, Gardiner Means (1935) accrued the relative price rigidity of some commodities to 'administered' (or perhaps monopoly) pricing of large-scale enterprises and consequently challenged the operation of the flexible price model of classical economics. On the other hand, classical economists (e.g. Tucker (1938)) were also quick to defend the robustness of their model: citing that there are a multitude of factors, affecting both the slopes and fluctuations of the supply and demand conditions, that may cause differential price fluctuations between commodities.
However, it should be recognized that the issue at hand is why some commodities persistently (over a long period) exhibit greater price fluctuations than others, so that the problem to be addressed is intrinsically one of long-run equilibrium. Consequently many of the ad hoc conjectures concerning plausible short-run price fluctuations can be put to rest.

The principal task of this paper is to construct a model of long-run equilibrium price fluctuations. While its main concern is to identify the key determinants of equilibrium price fluctuations, the analysis also reveals several interesting implications which are summarized as follows.

Firstly, we demonstrate that the nature of long-run equilibrium could take several forms which include (i) complete price rigidity; (ii) complete quantity rigidity; (iii) combination of segments containing (i) and (ii); (iv) combination of segments containing (ii) and partly flexible prices (accompanied by output adjustment); and (v) combination of segments containing (i), (ii) and partly flexible prices. Which equilibrium configuration will result depends strictly on the ex post adjustment cost and (less intuitively) the set-up cost of doing business. Consequently, previous equilibrium models which implicitly assume the ex post adjustment cost to be either non-existent (e.g., Dreze and Gabszewicz (1967), Sheshinski and Dreze (1976), and Lippman and McCall (1981)—henceforth know as DGSLM) or prohibitively large (e.g. Tisdell (1963), and Appelbaum and Lim (1981)) are but two polar extreme cases of our model.

Secondly, we indicate that set-up costs, which are usually a neglected item in the previous literature, are a non-trivial element in two important aspects. First, the important excess-capacity result of DGSLM cannot survive without the presence of set-up costs. Second, the presence of set-up costs, no matter how small, drastically alter the nature of the equilibrium price
fluctuations.

Thirdly, we demonstrate that under a very plausible set of equilibrium conditions, and for all values of \textit{ex post} adjustment costs and set-up costs, there is non-existence of an equilibrium that supports only \textit{ex post} firms (who do not precommit themselves to production before prices are revealed). Any equilibrium that exists must entertain either the existence of only \textit{ex ante} firms (who precommit themselves to some production \textit{ex ante}; but who also have the option of producing \textit{ex post} to the revelation of prices) or the co-existence of \textit{ex ante} and \textit{ex post} firms. This result is empirically appealing. It suggests that in any market that is subject to an uncertain environment, it is more than likely that we will observe at least some firms who precommit themselves to some production even though such actions are unalterable \textit{ex post}. Our result puts into question the DGSLM models which assume (rather than derive) the existence of an equilibrium where only \textit{ex post} firms exist.

In fact, the assumption that only \textit{ex post} agents exist trivializes the whole problem of uncertainty since all decisions can be postponed until the state of nature is known with certainty. This in turn makes irrelevant all the interesting problems that are associated with risk-bearing. Our non-existence result rescues the general notion that with uncertainty, there must exist some precommitments before the state of nature is revealed; and it is precisely the \textit{ex post} unalterability of such precommitments that provide the precondition for risk-bearing.

Finally, we draw some important implications which are relevant to the question of efficient price stabilization.

The rest of the paper is organized as follows. In section II we shall discuss the development of the models in the literature regarding the behavioral modes of the firm under price uncertainty. In section III, the most general
of these behavioral modes will be used for the construction of the long-run equilibrium conditions. In section IV, we present the qualitative nature of the equilibrium price fluctuations for various values of \textit{ex post} adjustment cost given that set-up costs are non-existent. Equilibrium price fluctuations in the presence of set-up costs are presented in section V. Lastly, section VI will conclude with some implications and suggestions for further extensions.

II. BEHAVIORAL MODES OF THE FIRM UNDER PRICE UNCERTAINTY

To build a model of equilibrium price fluctuations, the first crucial step is to examine how firms would react to an uncertain environment of fluctuating prices.

In the literature on the imperfectly competitive firm, it is by now familiar that in an uncertain environment of fluctuating demands, the firm could choose from a multitude of behavioral modes (see Baron (1971), Leland (1972), and Lim (1980, 1981)). This issue of choice of behavioral modes, however, is still not often appreciated in the literature on the competitive firm.

\underline{Ex ante Versus Ex post Production}

For instance, the competitive firm in Oi (1961), and DGSLM is assumed to be able to exercise complete \textit{ex post} flexibility; that is, facing a stochastic price, the firm need not commit itself to any production \textit{ex ante} but can simply afford the luxury of waiting for the revelation of price \textit{ex post} whence it instantaneously produces at price equal to marginal cost. The formal problem in this behavior is

\begin{equation}
\mathbb{E}\left[ \max_{q} pq - c_{f}(q) \right]
\end{equation}

where \( E \) is an expectations operator on the stochastic price \( p \); \( q \) is output quantity; and \( c_{f}(\cdot) \) is the cost function for the \textit{ex post} flexible behavior. The optimal output (which equates \( p = c'_{f}(q) \)) is obviously \textit{ex post} variable.
depending on the level of \( p \) revealed \textit{ex post}.

On the other extreme, the competitive firm in Baron (1970), Sandmo (1971), and Tisdell (1963) is assumed to be \textit{ex post} non-flexible: that is the firm somehow cannot afford the luxury of waiting for price to be revealed but must instead commit itself to production before the revelation of price. Formally, this behavior is equivalent to

\[
\text{Max} \ E[pq - c_n(q)] \\
q
\]

where \( c_n(\cdot) \) is the cost function for the \textit{ex post} non-flexible behavior. The optimal output in this case (which equates \( E[p] = c'_n(q) \)) is \textit{ex post} unalterable regardless of what price level is revealed \textit{ex post}.

To rationalize these two polar models, it is instructive to realize that Blackwell's principle of greater informativeness (see Marschak and Miyasawa (1968)) implies

\[
E[\text{Max} \ pq - c_f(q)] \geq \text{Max} \ E[pq - c_n(q)] \text{ if } c_f(\cdot) = c_n(\cdot). \quad (3)
\]

That is, if the cost, and hence profit functions are identical for the two behavioral modes, the one with greater \textit{ex post} flexibility will always be preferred. If, however, \( c_n(\cdot) \neq c_f(\cdot) \), Blackwell's dominance in (3) may not follow. In particular, if \( c_f(\cdot) > c_n(\cdot) \), then it is possible that the \textit{ex post} flexible behavior, inspite of its advantage of greater informativeness, might be less desirable than the \textit{ex post} non-flexible behavior because of the higher cost of \textit{ex post} production.

There are several reasons why additional costs are associated with the \textit{ex post} flexible behavior. Recall that with this behavior the firm can simply await the revelation of price before it produces. This implies a \textit{greater speed} of production which is often accompanied by perhaps a more costly technology; or higher input costs due to, for example, the use of overtime work; or a greater organizational cost in putting together a production process at a
higher speed.

The presence of a trade-off between the benefit (due to greater informativeness) and costs (due to the additional cost of a speedier production process) of the \textit{ex post} flexible behavior is overwhelmingly convincing. Otherwise, it is difficult to rationalize why many firms, especially those whose technology entails long production lags, continue to opt for the less informative \textit{ex post} non-flexible behavior.

\textbf{Ex ante and Ex post Production}

Using the idea of this trade-off, Turnovsky (1973) presented a model of the firm which entertains varying degrees of \textit{ex post} flexibility so that the models discussed above are included as two polar extremes. He considered a model where the firm

\[
\max_{y,z} \mathbb{E} \left[ \max_{y,z} p(y+z) - c(y,z) \right] \tag{4}
\]

where \(y\) and \(z\) are levels of \textit{ex ante} and \textit{ex post} production respectively, and where the cost function has the property that \textit{ex post} production is more costly than \textit{ex ante} production so that \(c(0,y+z) > c(y,z) > c(y+z,0)\) for \(y \neq 0\) and \(z \neq 0\). This more general formulation is especially desirable since it encompasses the completely \textit{ex post} flexible and non-flexible behavior as two polar extreme cases: Note that (4) reduces to (1) whenever \(y = 0\), and to (2) whenever \(z = 0\).

\textbf{Long-Run Equilibrium}

Turnovsky (1973), however, confined his inquiry solely to the investigation, at the firm level, of how price uncertainty affects the firm's output responses, and never extended his model to the study of \textit{long-run equilibrium}. Existing
works on equilibrium are therefore limited to special models which assume either complete \textit{ex post} non-flexibility (as in Tisdell (1963), and Appelbaum and Lim (1981)) or complete \textit{ex post} flexibility as in the DGSLM model.

In the following section, a variant of Turnovsky's (1973) general behavioral mode will be used to construct the conditions for equilibrium.

III. FIRM'S BEHAVIOR IN LONG-RUN EQUILIBRIUM

Cost Functions

As in section II, it is assumed that a firm may produce \textit{ex ante} (output level $y$) and/or \textit{ex post} (output level $z$). Let the cost function be

$$c(y,z) = c(y) + \alpha c(z)$$  \hspace{1cm} (5)$$

where the cost function $c(\cdot)$, with $c(0) = 0$, has the usual U-shaped average-cost property, and $\alpha \in [1, \infty]$ is the \textit{ex post} adjustment cost parameter describing the additional cost of \textit{ex post} production over the \textit{ex ante} production. (Thus $1/\alpha \in [0,1]$ defines the degree of \textit{ex post} adjustment.)

The separability of the cost function between \textit{ex ante} and \textit{ex post} production is an innocuous assumption for the following reasons. First, since the primary objective of this paper is to generate the qualitative properties of the equilibrium price fluctuations, invoking non-separability only complicates the analysis and does not seriously affect the results. Second, there is no presumption as to why \textit{ex ante} and \textit{ex post} production technologies should be interdependent; growing a crop under natural sunlight, for instance, involves a completely different technology compared to one that uses artificial light. Third, if the primary difference between \textit{ex ante} and \textit{ex post} production is due to higher effective prices of factors (perhaps because of overtime factor usage) in \textit{ex post} production, then the separability condition (5) is a natural
consequence of the fact that cost functions are in general homogeneous of
degree one in factor prices.  

Entry Conditions: Set-Up Costs

A firm may enter the industry to produce *ex ante* only; both *ex ante*
and *ex post*; or *ex post* only. In a competitive model with atomistic firms, it
is customary to assume 'free entry' in the sense that in an environment of
equally accessible information and non-cooperation, existing firms cannot
beneficially pursue strategies to prevent entry. This notion, however, is
not incompatible with the fact that there always exist some 'entry fee' or
set-up costs in participating in the industry. Operating a firm often incur
much more than the factor costs of production. Lump-sum transactional costs and
costs of organizing factors, which we broadly define as set-up costs, are often
unavoidable costs associated with firm formation. Hence, we shall require that
all firms, whether they choose to produce *ex ante* and/or *ex post*, incur set-up costs

$$T(y+z) = \begin{cases} 
T > 0 & \text{if } y + z > 0 \\
0 & \text{if } y + z = 0 
\end{cases} \quad (6)$$

for all $y \geq 0$, $z \geq 0$. This definition is formally equivalent to Baumol and
Willig's (1981) "long-run fixed costs...[which] can be eliminated in the long
run by cessation of production. ...[This is to be contrasted with] sunk costs
that cannot be eliminated, even by total cessation of production."

Firm's Decisions

Let $\Pi_y (y) \equiv py - c(y)$ and $\Pi_z (z) \equiv pz - \alpha c(z)$ be the 'variable' profits
from *ex ante* and *ex post* productions respectively; and $p \in [p^-, p^+]$ be the stochastic
price whose probability distribution is denoted by $\Psi(p)$. The problem facing
a typical (risk-neutral) firm is then to

$$\max_{y, z} \mathbb{E} \left[ \max \{ \Pi_y (y) + \Pi_z (z) - T(y + z) \} \right]. \quad (7)$$
By backward induction, this problem can be solved in two stages.

Firstly, given any *ex ante* level of output $y \geq 0$, the firm's *ex post* decision is to $\max_z \Pi_z (z) - T(y + z)$. The solution $z$ will in general depend on $y$. In the second stage, this solution can be substituted back into the profit function so that *ex ante* production can be chosen to solve (7).

However, because of the discontinuity of the set-up cost function in (6), the *ex post* problem should be treated with caution. For instance, if $y = 0$, then $T(y + z) = T(z)$ so that whether or not $T$ is incurred depends strictly on whether $z$ is positive or zero; thus $T(z) = T$ is a 'variable' cost as far as *ex post* decision is concerned. On the other hand, if $y > 0$, then $T(y + z) = T(y)$ irrespective of the levels of *ex post* production; thus $T(y) = T$ becomes a 'sunk' cost where *ex post* decision is concerned. Whether a firm will produce $y > 0$ or $y = 0$ is an important question which shall be addressed shortly. Meantime, it is useful to look at these two cases separately.

**Ex post Firm**

If $y = 0$, we shall henceforth call the firm an *ex post* firm whose only problem is to decide on its level of $z$:

$$\max_z pz - \alpha c(z) - T(z) \quad \text{when } y = 0. \tag{8}$$

This yields the solution

$$z_T = \begin{cases} 
z(p, \alpha) > 0 & \text{if } p \geq p^*(\alpha, T) \\
0 & \text{if } p < p^*(\alpha, T)
\end{cases} \tag{9}$$

where

$$z(p, \alpha) \text{ satisfies } p = \alpha c'(z) \tag{10}$$

and

$$p^*(\alpha, T) = \min_z \frac{\alpha c(z) + T}{z}. \tag{11}$$
For future reference, we shall denote
\[ z_T^0 \equiv z(p^0(\alpha, T), \alpha). \] (12)

The solutions in (9)-(12) are illustrated in Figure 1(a). They yield profit level
\[
\Pi_z(z_T) - T(z_T) = \begin{cases} 
  p[z(p, \alpha) - \alpha c(z(p, \alpha)) - T] & \text{if } p \geq p^0(\alpha, T) \\
  0 & \text{if } p < p^0(\alpha, T)
\end{cases}
\] (13)

so that the \textit{ex post} firm's expected profit becomes
\[
E[\Pi_z(z_T) - T(z_T)] = \int_{p^0(\alpha, T)}^p [p(z(p, \alpha) - \alpha c(z(p, \alpha)) - T)] d\psi(p) \geq 0. \] (14)
**Ex ante Firm**

If \( y > 0 \), the firm shall be henceforth called an *ex ante* firm. It produces *ex ante* but retains the option of producing *ex post* as well. Its *ex post* decision, however, differs from that of the *ex post* firm since when \( y > 0 \), \( T(y + z) = T(y) = T \) is a 'sunk' cost where its *ex post* decision is concerned. Thus the *ex post* decision of an *ex ante* firm is to

\[
\text{Max } \frac{p z - \alpha c(z)}{z} \quad \text{when } y > 0
\]

which yields the solution

\[
z = \begin{cases} 
  z(p, \alpha) > 0 & \text{if } p \geq p^0(\alpha) \\
  0 & \text{if } p < p^0(\alpha)
\end{cases}
\]  

(16)

where \( z(p, \alpha) \) satisfies (10) and

\[
p^0(\alpha) = \min \frac{\alpha c(z)}{z}.
\]

(17)

Again, for future reference, we let

\[
z^0 = z(p^0(\alpha), \alpha).
\]

(18)

The solutions in (16)-(18) are illustrated in Figure 1(b).

Given the solution \( z \geq 0 \) and \( y > 0 \), the *ex ante* firm's profit is

\[
\Pi_y(y) - T(y) + \Pi_z(z) = \begin{cases} 
  [py - c(y) - T] + [pz(p, \alpha) - \alpha c(z(p, \alpha))] & \text{if } p \geq p^0(\alpha) \\
  [py - c(y) - T] & \text{if } p < p^0(\alpha)
\end{cases}
\]

(19)

Taking expectations over \( p \), optimal \( y \) is then chosen to

\[
\text{Max } \int_y [py - c(y) - T] d\gamma(p) + \int_{\frac{p}{p^0(\alpha)}} [pz(p, \alpha) - \alpha c(z(p, \alpha))] d\gamma(p)
\]

(20)

which yields the *ex ante* solution

\[
\tilde{y} = y(E[p]) \text{ satisfying } E[p] = c'(\tilde{y})
\]

(21)
Condition Separating the \textit{Ex ante} and \textit{Ex post} Firm

It is important to emphasize that throughout the above discussion, the decision is not restricted to whether to produce \( z > 0 \) \textit{ex post} or to produce \( y > 0 \) \textit{ex ante}. It is this type of decision that contributed to the two polar models where firms are assumed to produce either \textit{ex post only} (Oi (1963), and DGSLM) or \textit{ex ante only} (Baron (1970), Sandmo (1971), Tisdell (1963), and Appelbaum and Lim (1981)). Rather, the appropriate decision, which this model examines, is whether to enter late as an \textit{ex post} firm (producing \( z > 0 \)) or to enter early as an \textit{ex ante} firm (producing \( \textit{ex ante} \ y > 0 \) as well as the option of producing \textit{ex post} \( z ≥ 0 \)).

In what follows, we shall discuss the condition determining the preference between early or late entry. Substituting \( \bar{y} \) from (21) into (20) and subtracting from it (14), we have

\[
\frac{\int [p_y - c(y) - T \cdot \bar{y}(p, \alpha, T)] d\bar{y}(p)}{p} + \frac{\int [p_z(p, \alpha) - \alpha c(z(p, \alpha))] d\bar{y}(p)}{p^O(\alpha)} \leq 0 \tag{22}
\]

which if positive (equal to zero) (negative) implies early entry would be preferred (indifferent) (dispreferred) to late entry.

For future reference, the above condition can be simplified as follows.

Define

\[
P^O(\alpha, T)
\]

\[
G(T; \bar{y})^4 \equiv \int_{p^O(\alpha)} [p_z(p, \alpha) - \alpha c(z(p, \alpha))] d\bar{y}(p) \geq 0 \tag{23}
\]

and

\[
P^{O}(T; \bar{y})^5 \equiv \min_{y} c(y) + T \cdot \bar{y}(p^O(\alpha, T)) - G(T; \bar{y}), \tag{24}
\]

condition (22) can now be rewritten as

\[
E[p] > (\equiv) \min_{y} P^{O}(T; \bar{y}) \tag{25}
\]
which if \( > (\leq) \) holds implies early entry as an \textit{ex ante} firm (to produce \( \tilde{y} > 0, z \geq 0 \)) is preferred (indifferent) (dispreferred) to late entry as an \textit{ex post} firm (to produce \( z_T > 0 \)). In the event that participation as an \textit{ex ante} firm is desirable, the solution \( \tilde{y} > 0, z \geq 0 \) is illustrated in Figure 2(a) (solution for \( \tilde{y} \)) and 2(b) (solution for \( \tilde{y} + z \)).

![Figure 2](image)

For any arbitrary price distribution, it is not apparent whether or not early entry (as an \textit{ex ante} firm) or late entry (as an \textit{ex post} firm) is preferred. While the \textit{ex ante} firm has the advantage of the lower cost of production in producing \( y \) and also the option of producing \( z \) \textit{ex post}, it has the disadvantage of having to absorb the full set-up cost \( T \) regardless of whether \( z \) is produced or not. On the other hand, the \textit{ex post} firm has to incur \( T \) only when \( z_T \) is produced. Whichever strategy is preferable depends critically on
the parameters $\alpha$ and $T$, and also on the price distribution $w(p)$.

In this paper, we shall be interested only in the equilibrium price distribution.

Equilibrium Conditions

With free entry, equilibrium requires

E(1): Both *ex ante* and *ex post* firms, if they were to produce, yield non-positive expected profits with at least one class of firms producing at zero expected profit.

If a firm's optimal output levels result in negative expected profit, it will not participate in the market. Hence the condition that at least one class of firms produces in equilibrium rules out the possibility that the market never opens to trade in *every state of the world*. This is equivalent to ruling out the possibility where the whole demand distribution is so low (in relation to cost conditions) such that there is no state of demand which will attract the participation of some firms.

While E(1) assumes trading in at least some states of the world, it does not guarantee the market will open to trade in all states of the world. In other words, for a given demand distribution, there still remains the possibility that some low states of demand exist whereby firms find it unprofitable to produce some output. Although we cannot dismiss this possibility on theoretical grounds, it is difficult (but of course not impossible) to find an empirical example of a commodity market in which demand conditions are such that the market opens in some states of the world but not at others. Nearly all important markets we can think of are observed to have an equilibrium supported all the time by a positive level of output. We shall henceforth assume this to be the case.
E(2): In equilibrium, the market opens to trade (at positive levels of output) in every state of the world.

Both equilibrium conditions E(1) and E(2) will now be treated formally. According to E(1), ex post firms must yield non-positive profit, which when applied to (13) and (14), requires either

\[
\tilde{p} = p^0(\alpha, T) = z_T > 0
\]  

(26)

or

\[
\tilde{p} < p^0(\alpha, T) = z_T = 0
\]  

(27)

This says \( \tilde{p} \) \( \in [p, \tilde{p}] \) such that \( p > p^0(\alpha, T) \); otherwise an ex post firm will earn positive profit.

Given \( \tilde{p} \leq p^0(\alpha, T) \), non-positive expected profit for ex ante firms requires \(^7\) the inequality (\( \leq \)) to hold in (25). Consequently, we must have

\[
E[p] = p^0_y(T; \bar{y} | \tilde{p} \leq p^0(\alpha, T)) = (\bar{y} > 0, z \geq 0)
\]  

(28)

or

\[
E[p] < p^0_y(T; \bar{y} | \tilde{p} \leq p^0(\alpha, T)) = (y = 0, z = 0)
\]  

(29)

Equilibrium condition E(1) therefore suggests that the equilibrium configuration may take one of the three following possibilities. First, combining (26) and (28), we have

\[
\begin{cases}
\tilde{p} = p^0(\alpha, T) \\
E[p] = p^0_y(T; \bar{y} | \tilde{p} = p^0(\alpha, T))
\end{cases} \Rightarrow \begin{cases}
z_T > 0 \\
(\bar{y} > 0, z \geq 0)
\end{cases}
\]  

(30)*

so that both ex post and ex ante firms earn zero expected profit and therefore co-exist. Second, combining (27) and (28), we have

\[
\begin{cases}
\tilde{p} < p^0(\alpha, T) \\
E[p] = p^0_y(T; \bar{y} | \tilde{p} < p^0(\alpha, T))
\end{cases} \Rightarrow \begin{cases}
z_T = 0 \\
(\bar{y} > 0, z \geq 0)
\end{cases}
\]  

(31)*

so that only ex ante firms exist to produce \( (\bar{y} > 0; z \geq 0) \). Third, combining (26) and (29), we have
\[
\begin{align*}
\{ \tilde{p} &= p^O(\alpha,T) \\
\text{E}[p] &< p^O_y(T;\tilde{y}|\tilde{p} = p^O(\alpha,T)) \Rightarrow \begin{cases} 
z^O_T > 0 \\
(y = 0, z = 0) \end{cases}
\end{align*}
\]

(32)

in which only \textit{ex post} firms exist to produce \(z^O_T > 0\).

It shall now be shown that among the three possibilities in (30)*, (31)*, and (32), the last can be ruled out as a feasible equilibrium candidate because of assumption E(2). To prove this, let us assume (32) holds, i.e. only \textit{ex post} firms exist in equilibrium. Then it follows that the equilibrium price distribution must degenerate into a unique price (with probability equal to one) \(p^* = p^O(\alpha,T)\). Otherwise if \(\exists p > p^O(\alpha,T)\), \textit{ex post} firms earn positive profits and if \(\exists p < p^O(\alpha,T)\), this range of prices (at which no \textit{ex post} firms produce) must, according to E(2), be supported by the outputs of \textit{ex ante} firms which are assumed not to exist in the first place. But at the same time, if equilibrium price is \(p^* = p^O(\alpha,T)\), then because (from (11) and (24)) \(p^O(\alpha,T) \geq p^O_y(T;\tilde{y})\) for all \(T \geq 0\) and \(\alpha \geq 1\), we also must have \(p^* \geq p^O_y(T;\tilde{y}|p^* = p^O(\alpha,T))\); which violates condition (32). In other words, if only \textit{ex post} firms exist in equilibrium, then E(2) implies that equilibrium price structure must be the unique price \(p^* = p^O(\alpha,T)\) at which \textit{ex ante} firms can earn positive profit--this of course cannot qualify as an equilibrium candidate. Thus, under E(2), there is no equilibrium whereby only \textit{ex post} firms exist. This non-existence result puts into question the equilibrium models of DGSIM which assume the existence of only \textit{ex post} firms.

In summary, every equilibrium that exists must therefore accommodate the existence of \textit{ex ante} firms. In particular, the feasible equilibrium candidates are either (30)* (where both \textit{ex ante} and \textit{ex post} firms co-exist) or (31)* (where only \textit{ex ante} firms exist). Which of these two will result
will depend crucially on the parameters of T and \(\alpha\) in relation to the specifications of the exogenous demand distribution. In the following two sections, we shall consider an arbitrary demand distribution such that equilibriums \(E(1)\) and \(E(2)\) can be realized and derive the nature of the equilibrium price distribution for various values of \(T\) and \(\alpha\).

IV. EQUILIBRIUM PRICE FLUCTUATIONS WITHOUT SET-UP COSTS

In the equilibrium models of DGSIM, it is usually the case that set-up costs are a neglected item so that the properties of equilibrium that were derived were made to seem insensitive to it. We shall show, however, that this is the contrary.

We proceed by considering three special cases, (a) \(\alpha = 1\); (b) \(\alpha \to \infty\); and (c) \(\infty > \alpha > 1\).

Case (a): \((T=0, \alpha=1)\)

With \(\alpha=1\), which implies no \textit{ex post} adjustment cost, Blackwell's dominance in (3) immediately suggests that between the decisions to produce \textit{ex ante} or \textit{ex post}, firms will always choose the latter. This is precisely the implicit assumption which led to the DGSIM model in which firms only produce \textit{ex post}. Using this behavioral mode, the DGSIM model was then used to derive two main results: (i) In equilibrium, firms on the average operate at excess capacity,\(^{9}\) and (ii) increasing demand uncertainty increases the equilibrium number of firms.

These results are, however, questionable on two grounds. First, even if \(\alpha=1\), we have shown before that no equilibrium exists which sustain only \textit{ex post} firms. Second, if \(\alpha=1\) and \(T=0\), the above results (i) and (ii) cannot survive because the equilibrium price distribution degenerates into a unique
certainty price \( p^* \) (equal to minimum average cost). Consequently, there is no excess capacity in equilibrium and furthermore, increasing exogenous demand uncertainty cannot affect the equilibrium number of firms since equilibrium price \( p^* \) is insensitive to demand fluctuations.

The proof proceeds as follows. When \( T = 0 \) and \( \alpha = 1 \), note from (11), (17), and (24), we have

\[
p^0(\alpha=1,T=0) = p^0(\alpha=1) = p^0_y(T=0;\gamma)^0 = \min_y \frac{c(y)}{y}.
\]

We claim that the equilibrium price distribution is such that \( \bar{p} = p^0(\alpha=1,T=0) \). The reason is if \( \bar{p} < p^0(\alpha=1,T=0) \), then (from (33)) also must

\[E[p] < p^0_y(T=0;\gamma)\]

which together imply negative expected profits for ex post and ex ante firms. Thus, it must be \( \bar{p} = p^0(\alpha=1,T=0) \) so that the only equilibrium possible is (30)* with the exclusion of (31)*.

What remains to be shown is that in (30)*, \( \exists \ p < \bar{p} = p^0(\alpha=1,T=0) \). Assume the contrary is true, i.e. \( \exists \ p < \bar{p} = p^0(\alpha=1,T=0) \), then it follows from (33) that \( E[p] < p^0_y(T=0;\gamma) \) which violates (30)*.

Hence, with \( \bar{p} = p^0(\alpha=1,T=0) \) and \( \exists \ p < p^0(\alpha=1,T=0) \), the equilibrium price distribution \( \gamma^* \) must be such that it contains a single price \( p^* = p^0(\alpha=1,T=0) \). Equilibrium (30)* can now be rewritten as

\[
\begin{align*}
\left\{ p^* = p^0(\alpha=1,T=0) = \min \frac{c(z_T)}{z_T} \right\} \Rightarrow \left\{ z_T = z^0 > 0 \right\}
\end{align*}
\]

\[
\begin{align*}
\left\{ p^* = p^0_y(T=0;\gamma^*) = \min_y \frac{c(y)}{y} \right\} \Rightarrow \left\{ (\gamma^* = z^0 > 0, z = z^0 > 0) \right\}
\end{align*}
\]

and the equilibrium price distribution is illustrated in Figure 3(a). Notice that there is no differentiation in this case between ex ante and ex post production. This is because given a rigid equilibrium price \( p^* \), all uncertainty about price fluctuations disappears.
Figure 3: Equilibrium Price Fluctuations When T = 0

Case (b): (T = 0, α → ∞)

In the opposite extreme where the \textit{ex post} adjustment cost is prohibitively large, i.e., α → ∞, we move into the models considered by Tisdell (1963), and Appelbaum and Lim (1981).

With α → ∞, note that \( p^0(\alpha \rightarrow \infty, T = 0) = \infty \). Thus, given any distribution of finite demand curves that are bounded from above, \( \exists \ p \geq p^0(\alpha \rightarrow \infty, T = 0) \).

This is equivalent to saying \( \bar{p} < p^0(\alpha \rightarrow \infty, T = 0) \) which immediately excludes (30)* as an equilibrium possibility. The only equilibrium is therefore (31)* in which \( \bar{p} < p^0(\alpha \rightarrow \infty, T = 0) \) implying non-existence of \textit{ex post} firms, i.e., \( z_T = 0 \).

Furthermore because \( p^0(\alpha, T = 0) = p^0(\alpha) \), it also follows \( \bar{p} < p^0(\alpha \rightarrow \infty) \) implying \textit{ex ante} firms that exist do not produce \textit{ex post}: \( z = 0 \). The only
output produced in equilibrium is therefore $\bar{y} > 0$ which is supported by

equilibrium price distribution $\{Y^*|\bar{p} < p^0(\alpha \rightarrow \infty)\}$ such that

$$E[p]^* = p^0_y(T=0; Y^*)|\bar{p} < p^0(\alpha \rightarrow \infty) = \min_{Y} \frac{c(y)}{y}$$

(see Figure 3(b)).

Case (c): $(T=0, \alpha > 1)$

Suppose there is a finite ex post adjustment cost $\alpha > 1$ which is, however, not prohibitively large as to make ex post production unprofitable.

In allowing ex post production and hence (because of $T=0$) ex post firms to exist, then it must be that $\bar{p} = p^0(\alpha, T=0)$ so that equilibrium is restricted to (30)* where both ex post and ex ante firms co-exist. The equilibrium price distribution is therefore $\{Y|\bar{p} = p^0(\alpha, T=0) = p^0(\alpha)\}$ such that (from (30)*)

$$\begin{align*}
\{\bar{p} = p^0(\alpha, T=0) = \min_{z_T} \frac{\alpha c(z_T)}{z_T} \} & \Rightarrow \begin{cases}
\quad z_T = z^0 > 0 \\
(\bar{y} > 0, z = z^0 > 0)
\end{cases}
\end{align*}$$

The equilibrium price distribution (illustrated in Figure 3(c)) consists of two distinct segments AB and BC. While the rigid-price segment BC results from the entry of ex post firms under favorable demand conditions, the rigid-quantity segment AB is a consequence of the absence of ex post output adjustment under less favorable demand conditions.

V. EQUILIBRIUM PRICE FLUCTUATIONS WITH SET-UP COSTS

In general, the conduct of business involves much more than the factor cost of production. Lump-sum transactional costs and costs of organizing factors of production, which we broadly define as set-up costs, often pose as critical factors that differentiate between individuals (with low set-up costs) who participate as firm-owners and those (with high set-up costs) who
participate merely as employees.

In this model, the presence of set-up costs \( T > 0 \), as in the \textit{ex post} adjustment cost \( \alpha \), will obviously affect the entry decision of the firm regarding whether to enter early or late. However, compared to the effects of \textit{ex post} adjustment cost, set-up costs will produce a different kind of equilibrium price fluctuations.

The analysis that follows will consider in general the case where \( T > 0, \alpha \geq 1 \). In particular, the discussion of the special case \( T > 0, \alpha = 1 \) (when compared to case \( T = 0, \alpha = 1 \) in the preceding section) will be revealing in two important aspects: Firstly, it demonstrates the acute sensitiveness of equilibrium price fluctuations to the presence of set-up costs—however small these costs may be. Secondly, it indicates that the presence of set-up costs is crucial in resuscitating the excess capacity result.

At the outset, it should be quite apparent that when \( T > 0, \alpha \geq 1 \), the equilibrium configuration is not unique and may take the form of either (30)* or (31)* depending on the values of \( T \) and \( \alpha \). If, on the one hand \( T \) and \( \alpha \) are both small such that \textit{ex post} firms find it profitable to participate, then equilibrium will take the form of (30)* where both \textit{ex ante} and \textit{ex post} firms co-exist. Otherwise, if \( T \) and/or \( \alpha \) is large, it is more likely that \textit{ex post} firms will not participate so that the only equilibrium is (31)* where only \textit{ex ante} firms exist. These two possible equilibria will be discussed in turn.

**Equilibrium With Co-existence of Ex ante and Ex post Firms**

Let \( T > 0 \) and \( \alpha \geq 1 \) be both small such that \textit{ex post} firms find it profitable to participate in the market. Then the zero profit condition for these firms implies the equilibrium price distribution is \( \{ \mathbf{y} | \mathbf{p} = p^0(\alpha, T) \} \) and hence equilibrium condition (30)* applies. In such an equilibrium, \textit{ex post}
firms produce $z_T = z_T^0 > 0$; and *ex ante* firms produce ($\tilde{y} > 0$, $z > 0$) where its *ex post* production $z > 0$ satisfies

$$z = \begin{cases} z(p,\alpha) & \text{for } p^0(\alpha) \leq p \leq p^0(\alpha, T) \\ 0 & \text{for } p \leq p < p^0(\alpha) \end{cases}$$

and its *ex ante* production $\tilde{y} > 0$ satisfies the condition that the equilibrium expected price

$$E[p]^* = p_T^0(T; \tilde{y}^* | \bar{p} = p^0(\alpha, T)) = \min_{\tilde{y}} \frac{c(\tilde{y}) + T \tilde{y} + \bar{p} - G(T; \tilde{y}^* | \bar{p} = p^0(\alpha, T))}{\tilde{y}} < \min_{\tilde{y}} \frac{c(\tilde{y})}{\tilde{y}}$$

since $\tilde{y}^*(\bar{p} = p^0(\alpha, T)) = 1$ and (from (23)) $G(T; \tilde{y}^* | \bar{p} = p^0(\alpha, T)) > 0$ for $T > 0$.

The equilibrium solutions for the *ex ante* firm are shown in Figure 4(a) and the equilibrium price distribution for the industry supporting both *ex ante* and *ex post* firms is shown in Figure 4(b).

---

**Figure 4:** Equilibrium with Co-existence of *Ex Ante* and *Ex Post* Firms when $(T > 0, \alpha \geq 1)$
In Figure 4(a), it is seen that \textit{ex ante} firms produce $\bar{y} > 0$ at excess capacity (i.e. output is less than that which Min $\frac{c(y)}{y} + T$). But this excess capacity result is with respect to a deterministic level of \textit{ex ante} production whereas in the DGSLM model (which assumes no \textit{ex ante} production) excess capacity is with respect to the 'average' of \textit{ex post} production. In either case, although excess capacity arises only if $T > 0$, the reasons for excess capacity differ between the two models. In the DGSLM model, it is a consequence of the convexity of the profit function in prices \footnote{13} whereas in our more general model, it arises because \textit{ex ante} firms in equilibrium are willing to absorb loss in their \textit{ex ante} production since they are adequately compensated by the gains in \textit{ex post} production.

With regard to the equilibrium price distribution, note how the presence of set-up costs--no matter how small--drastically alters the equilibrium price fluctuations from one of rigid price (when $(T=0, \alpha=1)$ in Figure 3(a)) to one (in Figure 4(b)) which contains four segments, AB, BC, CD and DE. While ABCD captures the equilibrium supply of $\bar{y}$ (segment AB) and $z$ (segments BC and CD) by \textit{ex ante} firms, the upper rigid-price segment DE reflects the equilibrium supply of $z_T$ by \textit{ex post} firms.

\textbf{Equilibrium With Existence of Only \textit{Ex ante} Firms}

When $T$ and/or $\alpha$ is large, the less likely will \textit{ex post} firms participate in the market. In this case, the equilibrium price distribution can be written as \{\textit{ex ante} $| p < p^0(\alpha,T)$ \} which from (27) implies $z_T = 0$. Consequently, equilibrium (31)* applies in which only \textit{ex ante} firms exist to produce ($\bar{y} > 0$, $z \geq 0$).

There are, however, two possibilities even with the sole existence of \textit{ex ante} firms. One arises whenever \{\textit{ex ante} $| p^0(\alpha) < \bar{p} < p^0(\alpha,T)$ \} in which case
ex ante firms produce ($\tilde{y} > 0$, $z > 0$) where $z > 0$ satisfies

$$z = \begin{cases} 
\xi(p, \alpha) & \text{for } p^0(\alpha) < p < \bar{p} < p^0(\alpha, T) \\
0 & \text{for } p < p^0(\alpha)
\end{cases}$$

and $\bar{y} > 0$ satisfies

$$E[p]^* = p_y^0(T; \forall^*(p^0(\alpha), \bar{p} < p^0(\alpha, T))$$

$$= \min \frac{c(y) + T \cdot \forall^*(p^0(\alpha, T)) - G(T; \forall^* | p^0(\alpha) < \bar{p} < p^0(\alpha, T))}{y}$$

$$= \min \frac{c(y) + T}{y}$$

because $\forall^*(p^0(\alpha, T)) = \forall^*(\bar{p}) = 1$ for $\bar{p} < p^0(\alpha, T)$, and (from (23))

$G(T; \forall^* | p^0(\alpha) < \bar{p} < p^0(\alpha, T)) > 0$. The equilibrium price distribution for this case where ex ante firms produce both ex ante and ex post is illustrated in Figure 5(a). Note that because of the non-existence of ex post firms, the upper rigid-price segment as appeared in Figure 4(b) is now absent. Furthermore, the excess capacity result continues to hold for ex ante production since $E[p]^* < \min \frac{c(y) + T}{y}$.

The other possibility, especially when $T$ and/or $\alpha$ is prohibitively large, equilibrium price distribution $[\forall^* | \bar{p} < p^0(\alpha) < p^0(\alpha, T)]$ in which ex ante firms (because of (10)) do not produce ex post, i.e. $z = 0$. Its ex ante production $\tilde{y} > 0$, however, satisfies

$$E[p]^* = p_y^0(T; \forall^* \bar{p} < p^0(\alpha))^{14}$$

$$= \min \frac{c(y) + T}{y}$$

which implies capacity output. The equilibrium price distribution in this case is shown in Figure 5(b) where only ex ante firms exist to produce only $\tilde{y} > 0$. 

VI. CONCLUDING REMARKS

We shall conclude by drawing some implications and suggesting areas for further extensions.

First, the observation that quantity is rigid in some states of demands does not imply \textit{ex post} adjustment cost must be prohibitively large. This is evident from Figures 3 to 5 where even for small values of $\alpha > 1$ and $T \geq 0$, equilibrium always entertains a rigid-quantity segment (in states of relatively
low demands) being combined with segments where quantity adjusts (in states of relatively high demands).

Second, the present model has obvious implications regarding the question of domestic (or international) price stabilization. To our knowledge, current discussions on efficient price stabilization schemes have concentrated only on short-run models and have overlooked the long-run consequences of these schemes. We find this inappropriate for the reason that follows. Consider, for instance, the case where \( T \) and \( \alpha \) are such that the long-run price fluctuations are given by Figure 3(c). Without this realization, it is conceivable that price stabilization (based on a short-run model) may fix price at a level lower than BC in Figure 3(c) with the consequence that in the long run, all \textit{ex post} productions are choked off. Whether price stabilization schemes should be based on the short-run or long-run model is debatable depending on the time horizon span of policy makers. But even if a short-run view is taken, the design of efficient price stabilization policy cannot avoid the consideration of its long-run consequences.

Thirdly, if the long-run view of price stabilization is taken, then the understanding of long-run equilibrium price fluctuations is indispensable. Furthermore, the model presented here provides an attractive alternative to direct market intervention through price-fixing or imposing price boundaries. By changing the 'entry fee' or 'set-up costs' \( T \)--through lump-sum taxes or subsidies--it has been shown that the degree of price fluctuations could be significantly altered. We suggest that this alternative option of influencing the degree of price instability merits future consideration in the study of efficient price stabilization.
Finally, further extensions of our model might wish to consider the relaxation of our equilibrium assumption $E(2)$ and the set-up cost assumption (6). Although we feel these assumptions are sufficiently general, there might be some special type of markets for which alternate specifications of $E(2)$ or (6) might be more appropriate. Examples of markets that violate $E(2)$ are those where trading does not occur for some states of demand; and examples where (6) is violated are those where set-up costs are incurred regardless of whether output is produced or not.
FOOTNOTES

1 Consider two commodities, A (with stochastic supply and stochastic demand) and B (with only stochastic demand). Then it is clearly possible for B to exhibit greater equilibrium price fluctuations if, for instance, the supply of A is positively correlated with its demand.

2 The two main results of the DGSIM model are as follows: (i) In equilibrium, firms produce (on the average) at excess capacity; (ii) increasing demand uncertainty increases the equilibrium number of firms. We shall also have the opportunity later to critically comment on their result (ii).

3 Let c(w; z) be a general cost function defined over output level z and the vector of input prices w. Then it is well known that c(αw; z) = αc(w; z) because cost functions are homogeneous of degree one in input prices.

4 Note that G(T = 0; Y) = G(T; Y | p ≤ p^0(α)) = 0.

5 Note that p_y^0(T = 0; Y) = p_y^0(T; Y | p ≤ p^0(α)) = \min_y c(y).

6 Henceforth, the symbol "\(\exists\)" denotes "there does not exist"; and "\(\exists\)" means "there exists".

7 Given \(p \leq p^0(\alpha,T)\), expected profit of an ex ante firm in (20) reduces to that in the L.H.S. of (22). Non-negative expected profit therefore requires (22) to hold with inequality \((\leq 0)\) which is the same as having (25) to hold with inequality \((\leq)\).

8 It should be emphasized that this result is also conditional on the assumption that set-up costs either do not exist in the long run or if they ever exist, they do so in the form of the (6) in which they are incurred only if there is positive levels of production. See Baumol and Willig (1981) for an excellent discussion of the relevance and importance of this type of set-up cost in various industries.
9 Capacity output is defined to be the output level where average cost is minimum. In the DGSIM model, output varies with ex post price levels. Hence their excess capacity result may only state that the 'average' of these output levels is below capacity output.

10 See Footnote 5.

11 In all ensuing figures regarding the aggregate equilibrium supply it is assumed that there are a large number of infinitely small firms so that the problem of discontinuity and indeterminacy of equilibrium can be ignored.

12 Let $Q = m(y + z) + nz_T$ be the aggregate output where $m$ and $n$ are respectively the equilibrium number of ex ante and ex post firms. In this paper, we are solely interested in deriving the qualitative nature of the equilibrium price fluctuations and have left open the question of the determination of the equilibrium number of firms.

13 See, for example, Sheshinski and Dreze (1976) for a clear explanation of this result.

14 See Footnote 5.
REFERENCES


Econometrica, 29, 58-64.


Sheshinski, E. and Dreze, J. (1976), "Demand Fluctuations, Capacity Utilization, 

Smith, A. (1776), An Inquiry into the Nature and Wealth of Nations (London), 
reprinted in two volumes, R. H. Campbell, A. S. Skinner, and W. B. Todd 

Tisdell, C. (1963), "Uncertainty, Instability, Expected Profit," Econometrica, 
31, 243-247.

28, 41-54.

Turnovsky, S. J. (1973), "Production Flexibility, Price Uncertainty and the 
Behavior of the Competitive Firm," International Economic Review, 
14, 395-412.