A Comment on Ordover and Willig's "On the Optimal Provision of Journals qua Sometimes Shared Goods"

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A COMMENT ON ORDOVER AND WILLIG'S
"ON THE OPTIMAL PROVISION OF JOURNALS
QUA SOMETIMES SHARED GOODS"

by

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In a recent issue of this journal, Ordoover and Willig (O-W) present a model of the provision of journals to libraries and individuals. While the provision of journals is of interest to academics, the concepts presented have ready extension to a broad class of quasi-public goods. O-W treat journals as "sometimes shared goods" which essentially means that some units of the commodity are consumed privately while others will be consumed collectively. In a regime of increasing returns to scale, the behavior of agencies which arrange collective consumption will have an obvious effect on those who do or might choose to consume the good privately. With any increase in the share of total consumption that is provided collectively, the number of units produced will fall and the price to those who continue to consume privately will rise. This externality implies that some institutional arrangements other than the status quo might improve social welfare. Of course the costs of such institutions must be considered in making any welfare judgements.

While we find the concept of sometimes shared goods to be very important and the model of Ordoover and Willig to be quite interesting, there are assumptions implicit in the model which seriously impair its generality. As a result, we find that the model is inappropriate to the policy purposes for which it has been used and those other uses for which one might wish to use a general model of shared goods.

Our comment proceeds as follows. The next section outlines elements of the model which are essential to our presentation. Section II presents some simple analytics of the type of user fees considered by Ordoover and Willig. This introduces the policy implications drawn by Ordoover and Willig and provides the first suggestion that their results might depend on rather stringent assumptions. Section III presents our central point, which is that
the model requires an assumption that the ranking of libraries by willingness to pay is invariant over changes in a number of key parameters and that this "invariance" assumption imposes severe restrictions on consumer demands. It seems unlikely that Ordover and Willig would have used the model in the manner in which they do, had they been aware of these restrictions. Section IV comments on additional limitations of the model which further preclude its application to policy.

I. The Model

Ordover and Willig present a model in which libraries are initially assumed to be perfect purveyors of journals. That is, they can somehow capture the full value that their readers place on library availability of any journal. The willingness to pay of a library is written $W(m)$ and is the sum over all library readers of the value of journal use minus the inconvenience and cost of using the journal in the library. The term $m$ is an arbitrary continuous index of libraries' populations, ordered in such a way that $W(m)$ is increasing in $m$. Libraries "finance their acquisitions through lump sum fees which do not affect individuals' choices of reading modes". The marginal library has $m = m^*$ so that the willingness to pay of the marginal library is $W(m^*)$. Libraries are charged $P_L$ for a journal subscription and individual subscribers are charged $P_S$. The equilibrium condition for the marginal library is therefore

$$W(m^*) = P_L$$ (0-4)

The number of library readers in a library with index $m$ is $LR(m)$. Among library readers are those who would subscribe if the library were not to subscribe (potential subscribers) and those who would only read the
journal if it were available in the library (browsers). This partition of course depends on prices. The number of subscribers is $N^S$, the number of libraries is $N^L$, and the total number of library readers is $N^T$.

Ordover and Willig consider the possible extension of copyright to the use of journals\(^1\) by introducing user fees. With user fees, each user is charged the price $P_u$ to use the journal in the library. Ordover and Willig first consider an arrangement in which each library is obligated to fund some fraction, $\alpha$, of the subscription price from user fees. Under this arrangement, $\alpha$ and $P_L$ are uniform across libraries while $P_u$ clearly must vary. Later they consider $P_u$ to be constant across all libraries.

II. Simple Analytics of User Fees

The final three propositions of the paper by Ordover and Willig form the basis for their copyright policy prescription. Regarding these propositions, they state:

Overall, we find that a copyright based library usage fee is a practically feasible instrument which is desirable when properly employed (underlining ours), p. 333.

Also;

For small usage fees employed to partially finance library acquisitions, the positive effect on publisher profit from increased private subscriptions outweighs the undesirable effects. P. 332.

The most general of the propositions which supports copyright is proposition nine. Proposition nine states:

\(^1\)While Ordover and Willig refer to their institutional proposal as copyright, it in fact would not be an extension of copyright, which only gives the owner control over production of physical manifestations of the work. The right which O-W would enforce is called a public lending right, an institution which exists in some countries.
The introduction of a positive $\alpha$ increases consumer welfare with Ramsey-optimal subscription prices above marginal cost, whenever there are any marginal prospective subscribers in any of the subscribing libraries.

Ordover and Willig tell an "intuitive" story to explain their result. They claim that as $\alpha$ (the percentage of the library subscription price which a library has to generate through a uniform user fee) increases from zero, some library users who were on the margin between subscribing and using the library, will switch with no welfare loss to private subscriptions. Since they assume that journal prices are above marginal cost, this increase in private subscriptions increases publisher profits, ceteris paribus. This increase in profits allows publishers to lower all subscription prices and therefore increase welfare. Thus they claim the "surprising" result that "perfect purveyance of shared units of a sometimes shared good is not generally optimal". We share their surprise.

Using economic reasoning, a contrasting result seems inescapable. Imposing any positive infinite $\alpha$ will impair perfect appropriability of the shared good and therefore reduce the willingness to pay of the marginal library below $P_L$, the libraries' subscription price. All marginal libraries fail, which results in a finite loss in revenue which would have to be traded off against the revenue gains from increased personal subscriptions. Consideration of the welfare effects of user fees requires the usual consideration of costs and benefits, and without additional structure on demands there can be no assurance that marginal benefits exceed marginal costs, even at $\alpha=0$. Of course, in the world of discrete libraries it may be unrealistic to assume that there are libraries which are exactly on the margin. But similarly it would be unrealistic to have marginal subscribers.
So, the pedestrian concern with costs and benefits re-emerges.

If the economic logic of user fees seems inescapable, so does the mathematics.\(^2\) With user fees determined by \(\alpha\), the equilibrium condition for the marginal library is

\[
W(m^*) = (1-\alpha)P_L \tag{O-W: A6}
\]

0-W differentiate with respect to \(\alpha\) to obtain

\[
\frac{dm^*}{d\alpha} = - \frac{\frac{\partial W}{\partial P_u}(m^*)}{\frac{\partial P_u}{\partial \alpha}} - P_L \frac{\frac{\partial W}{\partial m}(m^*)}{\partial \alpha} \frac{\partial P_u(m^*)}{\partial m} \tag{O-W: A7}
\]

Substitution of the following two equations shows the numerator to be zero.

\[
\frac{\partial W(m)}{\partial P_u} = - LR(m) \tag{O-W: A8}
\]

\[
\frac{\partial P_u(m)}{\partial \alpha} = P_L / LR(m), \text{ for } \alpha = 0 \tag{O-W: un-numbered}
\]

\(^2\) We should perhaps qualify this statement somewhat. While the math appears correct there are some puzzling features in O-W equation (A7), reproduced in the text below. It is intuitively obvious that \(dm^*/d\alpha\) can never be negative. However, when \(\alpha > 0\) the numerator of (A7) becomes

\[
\alpha P_L \frac{\partial (\frac{1}{LR(m^*)})}{\partial \alpha}, \text{ which is not likely to be zero. The denominator of A7 can be of either sign so that a negative value of } dm^*/d\alpha \text{ appears possible unless constraints considerably stronger even than those we discuss below are imposed. While we are disturbed by this puzzle, it plays no role in our criticism of the model.}
Having established that the marginal library is (instantaneously) unchanging with $\alpha$, it is a fairly simple matter to establish that net benefits are positive at the margin when $\alpha = 0$, so that an optimal $\alpha$ will be greater than zero.

How may these contradictory results be reconciled? The answer is in a very strong continuity assumption implicit in the mathematical formulation. The numerator of equation (3) is the instantaneous loss of willingness to pay net of user fees, the triangle ABC in Figure 1.

![Figure 1](image_url)

As $P_u$ is increased from zero, triangle ABC does grow at a rate instantaneously equal to zero. If there is a continuous relationship between willingness to pay and $m^*$, the marginal library changes at a rate instantaneously equal to zero. It is the continuity of this relationship which rules out a "jump" in $m^*$ as $\alpha$ changes. Formally, this enters through the assumptions necessary for application of the implicit function theorem, which is invoked in moving from equation (2) to equation (3). Use of the implicit function theorem requires that $W(m) - (1 - \alpha)P_L$ is (locally) continuous in all relevant arguments, namely, $P_s$, $P_u$, $\alpha$, $m$. It is the continuity of $m$ which is suspended in the economic discussion above.

Continuous models are normally argued to be satisfactory approximations of a discrete world, and vice versa. Where continuous and discrete
models conflict, there are usually common sense grounds for choosing between them. The existence of only a finite number of libraries would seem to suggest that the discrete model is appropriate, especially for policy purposes. Fortunately however, it is not necessary to choose on these grounds alone. In the following section it will be shown that the continuity assumption, along with other statements in the Ordover and Willig paper, require additional assumptions which severely restrict the nature of library populations.

III. **Invariance of Library Rankings**

Ordover and Willig have placed a restriction on \( m \) which on first reading seems harmless enough. They state "for convenience, we take \( m \) to be a scalar index defined so that the \( W \) function is increasing in \( m \)." The difficulty with this is that the ranking of libraries by willingness to pay will not, in general, be invariant with regard to parameters of the model which are specifically being altered within the model. An assumption that library rankings are invariant over the parameters \( P_s, P_u \) and \( \alpha \) is a very restrictive one, as will be shown below.\(^3\) First however, we consider the role of this assumption in the O-W paper.

The invariance assumption enters in the definitions of terms which enter the profits function for publishers. For example, the definitions of the number of subscribers, the number of libraries and the aggregate net benefit of libraries require integration from \( m^* \) to infinity, that is, integration over all existing libraries. If rankings of libraries can change, integration from \( m^* \) to infinity is not equivalent to integration over libraries. Simple relabelling of the regions of integration will not restore the model, since continuity of profits, welfare, number of

\(^3\)It is interesting that in this case the mathematical formulation obfuscates restrictive assumptions while fundamental economics makes them quite apparent. This is contrary to the claim usually made that mathematical formulations force the analyst to make his assumptions explicit.
libraries etc., does not hold. Consider Figure 2.

![Figure 2]

Let \( m \) be chosen for \( \bar{P}_s \) so as to satisfy the requirement that \( W \) increases in \( m \). Without invariance, a different \( P_s \) (for example) can result in a path like the one labelled \( W(m, \bar{P}_s) \). In either case, all libraries above the horizontal line labelled \( P_L \) exist. Notice however that for \( \bar{P}_s \), the number of libraries will not be continuous in \( P_L \). As a result, failure of invariance would negate all of the propositions which rely on their specified profits or welfare functions, i.e., propositions one through eight. The failure of the continuity required for proposition nine can be seen from Figure 2 as well. The equilibrium condition that \( W(m^*) = (1-\alpha)P_L \) does not provide a continuous relationship between \( P_L \) and \( m \) for any \( \alpha \).

Propositions ten and eleven involve user fees which are uniform across individuals and are stated as follows:

Proposition 10: Suppose that at the profit maximizing \( P_s \) and \( P_L \), with \( P_u = 0 \), (i) there are some marginal potential subscribers in some subscribing libraries and (ii) the number of journal readers in each marginal library is less than the average number of journal readers in all the subscribing libraries, i.e.,

\[
LR_u^T/N_L > LR(m^*) \tag{O-W: 22}
\]

Then there exist \( P_u', P_s, P_L \) with \( P_u > 0 \) at which both the publisher profit and the consumers' welfare are greater than they are with \( P_u = 0 \) and with \( P_s, P_L \) set profit maximally.
Proposition 11: Suppose that at the Ramsey-optimal \( P_S \) and \( P_L \) with \( P_u = 0 \), (22) holds. Then there exist \( P_S', P_L', P_u \) with \( P_u > 0 \) at which publisher profit is unchanged while consumer welfare is greater than that in the situation above.

Proofs of these propositions involve the assertion that if \( P_L \) is adjusted so that the marginal library is just compensated for its losses from user fees, the number of libraries will not be changed with user fees. Figure 3 shows how this assertion relies on invariance.

![Graph showing \$, W(m) vs. LR(m*)](image)

**Figure 3**

Without invariance, the locus of libraries can take virtually any form, including the one shown. The horizontal line at \( P_L \) is the lower boundary of the feasible region for libraries. Implementation of a user fee, with a compensating reduction in \( P_L \), will rotate this boundary about the point \( (P_L, LR(m*)) \) as shown. Notice that there are several different kinds of marginal libraries, and price adjustments which just compensate any one of them will result in a change in the number of libraries. Invariance would assure that the locus will cross horizontal lines only once, so that the propositions hold.  

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4 Actually a further assumption, that the \( W(m) \) locus is strictly upward sloping at \((LR(m*), P_L)\) is required for the propositions to hold.
How Restrictive is Invariance?

It is fairly easy to see that invariance does rule out certain very reasonable cases. Figure 4 provides a simple example in which the invariance assumption does not hold.

The diagram shows the demands for journal use in two libraries, 1 and 2. For $P_u = 0$, library 2 provides the greater consumer surplus, but when $P_u \geq P'_u$, the rankings are reversed. Thus the rankings are not invariant over all values of $P_u$. Similar examples can be constructed such that changes in $P_s$ or $\alpha$ will alter the rankings by willingness to pay.\(^5\)

While it is difficult to provide a statement of necessary conditions for invariance which has any useful economic interpretation, various sufficient conditions can be specified. A virtual restatement of the definition of invariance is that at every possible $P_s$, $P_u$, $\alpha$

$$-\frac{\partial^2 W(m)}{\partial m \partial x} \leq \frac{\partial W}{\partial m} \quad x = P_s, P_u, \alpha$$

That is, in every instance, the adverse effect on $W(m)$ increases in $P_u$ and $\alpha$ must not increase with $m$ faster than $W(m)$ increases with $m$ (or the

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\(^5\)Note that in all of this, the demand functions facing libraries are assumed to be independent, as is consistent with the O-W formulation. This rules out the possibility that people would respond to a library's closing by patronizing another library. While we believe this to be an important restriction, we have not focused on it here, since it might be taken as a conventional simplifying assumption.
increase in \( W \) from increases in \( P_s \) must not diminish in \( m \) faster than \( W(m) \) increases.

Some economic characterization of this condition is possible, although we are unable to provide a single simple statement of a necessary condition. Consider first the case in which ranking of libraries for some \( P_u, P_s, \alpha \) will assure only that \( W(m) \) is increasing in \( m \). Then we have that

\[
\frac{\partial^2 W}{\partial m \partial P_u} \leq 0.
\]

However, it is established by Ordover and Willig that \( \frac{\partial W}{\partial P_u} = -LR \). Thus when no strong restriction is initially placed on \( W(m) \), the only general sufficient condition for invariance is that higher ranked libraries have fewer readers. Stronger initial restrictions on \( W(m) \) will allow weaker restrictions on the cross partials. One fairly natural example is the case in which library populations are homogeneous across libraries. That is, all libraries have the same mix of patrons, bigger libraries just have more of them, so bigger libraries have greater willingness to pay. This is admissible because a homogeneous populations assumption imposes strong conditions on \( W(m) \).

It is clear that invariance does impose fairly severe restrictions. Results which rely on this invariance cannot be called "general" as they have been by Ordover and Willig. And of course, policy conclusions drawn from such results have accordingly limited application.

IV. Further Considerations

The O-W paper employs a curious asymmetry in the treatment of the costs associated with institutions. On the one hand, the inconvenience of using books in libraries may be said to run the model. This is the source of the social gains achieved when marginal subscribers leave the libraries. On the other hand, the costs of a user fee system are ignored entirely. The
inconvenience of locking up all the books so that usage can be monitored, of collecting fees, of auditing libraries, etc., are likely to be quite large.

This issue has already surfaced as a practical concern in connection with the use of reprography. The central problem in the collection of copyright fees for reprography has been the high level of transactions costs relative to the royalty fee which might be negotiated. It was clear to the framers of copyright law that these costs would be too high for an effective market to function in many circumstances. This was the basis of the defense to copyright infringement known as fair-use.  

The importance of transactions costs is nowhere more clearly demonstrated than in the Copyright Clearance Center. This organization was set up for the sole purpose of eliminating costly transacting between publishers and those making photocopies of journals. The experience of the Center has been that the revenues generated can barely cover the operating expense of the Center (which has only three employees). This does not include the expense of all those using the Center and bearing the transactions costs of correspondence and self monitoring which is necessitated. Any model which is mute on these costs is incapable of generating reasonable public policies.

Ordover and Willig are hardly alone in what we regard as inconsistent treatment of transactions costs. In fact, it is because this error is so common that we make special note of it. Harold Demsetz' 1969 call for consistent treatment of transactions costs was, as far as we can tell, received with a resounding "we already know that". If we ever did, we seem to have forgotten.  

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5 For a discussion of these points and an analysis of the Copyright Clearance Center, discussed in the next paragraph, see Liebowitz (1980).

6 Dahlman (1979) provides a careful review and extension of the original Demsetz article.
References


