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A DYNAMIC MODEL OF ADVERTISING
AND MARKET CONTESTABILITY

by

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Abstract

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Advertising in the pre-entry period confers a lagged benefit to incumbent firms. Thus they hold captive a portion of the market in the post-entry period, leaving the remaining portion contestable by all firms, including new entrants. The captive portion of the post-entry market is neither fixed nor is it unilaterally determined by incumbents' pre-entry advertising intensities. Rather, it is subject to erosion and counter-erosion by post-entry advertising of new entrants and incumbents, respectively. In this dynamic model of market contest, we arrive at a rich set of solutions that encompasses as special cases both the result of Schmalensee's (1983) advertising model and one that is reminiscent of the traditional investment-capacity models of entry deterrence.
A Dynamic Model of Advertising
and Market Contestability

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A Dynamic Model of Advertising and Market Contestability

I. Introduction

In a world of imperfect information, the price signal alone is insufficient to convey adequate information about the existence, location, and product quality of firms. This is well-understood both in the marketing literature as well as in general business practice where advertising is an integral part of the firm's activities. Apart from simply disseminating information, advertising also aims at persuading potential customers of the differential advantages of the product. Thus, even where price and product quality differ negligibly between firms, advertising can account for differences in the market shares of individual firms.

In general, there is a good deal of evidence that points to a strong positive relationship between advertising and market shares and to the fact that the advertising impact lasts for some time. As a result, advertising by incumbent firms can perform an important strategic function in deterring entry of new firms (e.g. Spence (1977, 1980), Cubbin (1981), Baldini and Masson (1981) and Schmalensee (1983)).

In his exploratory paper, Schmalensee (1983) warns against drawing the analogy between entry-deterrence models of advertising and those motivated by other factors, such as those based on investment in productive capacity (e.g. Spence (1977), Dixit (1980)). The main result of theoretical work on the latter stresses that incumbent firms, facing the threat of new entry, will always find it optimal to expand productive capacity to deter entry. Schmalensee's model arrives at the exact opposite, that is, that incumbents will never find it optimal to advertise more when they are faced with the threat of new entry.
Schmalensee cautions, however, that this result (though internally consistent with his model) also appears to be at odds with other theoretical and empirical research findings on brand loyalty (see for example Schmalensee (1982), and references therein). Given this anomaly, he suggests that "more work is clearly required to see exactly what features of [his] model presented drive its results and, more importantly, to construct more general models that can delineate the conditions which overadvertising can optimally and credibly deter entry."

This paper attempts to address precisely this issue. To do so, we must clarify the crucial difference that exists between entry-deterrence models of advertising and those where cost-side variables confer advantages to incumbent firms. Under specific conditions, the latter advantages such as capacity expansion and unit cost reduction cannot be emulated and thus cannot be eroded away by counter-activities of new entrants. In sharp contrast, advertising is a demand-side factor and, as a means of persuasion, the advantage it confers is ephemeral and open to contest at all times by counter-persuasive advertising of rival firms, including new entrants. Consequently, even if incumbent firms can generate strong psychological attachments to their pioneering brands, such advantages of pre-entry advertising can be eroded if new entrants counter with advertising campaigns. It is this difference that divides the two classes of entry-deterrence models.

Thus Schmalensee's (1983) result is not at all counter-intuitive. It is grounded on his assumption that the persuasive impact of advertising is biased in favour of the new entrant. Given the strong "erodability" of the advertising impact, it is not surprising that incumbents in his model will choose to advertise less with the threat of entry. A more general and
flexible treatment of this peculiar nature of the advertising impact should be able to accommodate both his result as well as cases where advertising still serves as an effective entry-deterrent.

We develop in this paper a dynamic two-period model with \( M \) incumbent firms in the first period. The market share of firms is generally determined by their advertising intensity during the period in question. However, since the impact of the incumbents' first-period advertising lasts into the second period (pioneering brand advantage), a portion of the post-entry market is held captive for the incumbents and therefore not contestable by new entrants. This leaves only the remaining portion open to capture by incumbents and new entrants through their second period advertising. In general, the incumbents' captive market in the second period need not be fixed but can be subject to erosion and counter-erosion in accordance with the post-entry advertising intensities of new entrants and incumbents. Thus the degree of contestability in the second period is not unilaterally determined by incumbents, as it is in the productive-capacity models of entry deterrence, but rather is mutually determined by incumbents and new entrants.

In our model, the number of new entrants is determined endogenously in the post-entry equilibrium. The notion of degree of entry is defined as the aggregate market share captured by new entrant. Since most entry-deterrence models assume the number of new entrants to be fixed, this means that higher entry-deterrence is identified by a lower market share for each entrant. In our model, however, changes in the entrants' market share are divided into changes in the number of entrants and in the market share per entrant.

Finally, in this dynamic competitive setting, post-entry demand
conditions facing new entrants depend on the incumbents' pre-entry behaviour while the incumbents must take into account the reaction of new entrants in arriving at their pre-entry decisions. Entry behaviour is therefore rational and the entry-deterrrent threat of incumbents perfectly credible.

II. The Model

Consider a two-period model. Let the set of incumbents that are in the market in period 1 be \( I = \{ i = 1, \ldots, M \} \), where \( M \) is given to the analysis. New entrants that enter the market in period 2 are defined as \( J = \{ j = 1, \ldots, N \} \). The number of new entrants, \( N \), is to be determined as an internal solution of the model.

The period-1 and period-2 demand functions are given by

\[
Q^1 = Q^1(p^1), \quad Q^1_{p1} < 0, \quad Q^2 = Q^2(p^2), \quad Q^2_{p2} < 0. \tag{1}
\]

The demand facing each incumbent in period 1 is

\[
q^1_i = b^{i1}.Q^1(p^1), \quad \forall i \in I, \tag{3}
\]

where \( b^{i1} \in [0,1] \) denotes the incumbent \( i \)'s period-1 market share.

At this juncture, we should emphasize that since all firms have access to the same production technology, it is not the production activities that determine market share. Rather, the market share of each firm is to be determined by the incumbent's activities in sales effort through advertising in relation to such activities of all other firms. Thus, the market share of each incumbent \( i \) in period 1 is its relative share of
aggregate advertising expenditure, $X^1$, in that period,

$$b^i = x_i^1/X^1, \quad \forall i \in I,$$

where $X^1 = \sum x_i^1$.  \hspace{1cm} (4)

In order to focus on the impact of advertising, we shall assume that the cost of production of both the incumbent $i$ and new entrant $j$ are identical functions of quantity produced $q$ in a given period $t$.

$$c^{kt} = c(q^{kt}), \quad \text{for } t = 1, 2; \ k \in I, J. \hspace{1cm} (6)$$

In period 2, the market is open to contest by new entrants. Let $b^{i2}$ and $b^{j2}$ denote the market shares of incumbent firm $i$ and new entrant $j$, respectively. In terms of total market demand $Q^2$, the market demand of the incumbent firm $i$ in period 2 is

$$q^{i2} = b^{i2}Q^2, \quad \forall i \in I, \hspace{1cm} (7)$$

and that of the new entrant $j$ is

$$q^{j2} = b^{j2}Q^2, \quad \forall j \in J. \hspace{1cm} (8)$$

The crucial feature in our model is that $b^{i2}$ and $b^{j2}$ are likely to be non-identical, in the sense that $b^{i2}$ is greater than $b^{j2}$ even if $x^{i2}$ is equal to $x^{j2}$. This owes to the assumption that persuasion through advertising has lagged benefits. Incumbents, having already advertised in the market in period 1, would have established some form of market advantage, reputation or familiarity, with the customers. Thus, some degree of market advantage exists for incumbents in period 2.

One simple way to capture this in a formal manner is as follows. Owing to its advertising in period 1, each incumbent holds captive a
certain market share in period 2 equal to proportion $r^i$ of its market share in period 1. Thus, even without any post-entry advertising, incumbents can lay claim to a portion $r$ of the period-2 market, with $r$ being equal to the weighted average of all individual $r^i$'s, or

$$z = \sum r^i b^{i1}, \quad 0 \leq r \leq 1.$$  \hspace{1cm} (9)

Only a portion $(1-r)$ of total market demand in period 2 is open to contest. To be sure, each incumbent is still free to fight for a share of the non-captive or contestable portion, $(1-r)$, of the period-2 market, but it can only do this on an equal footing with the new entrants.

Define $x^2$ as the total marketing expenditure by all firms (incumbents and new entrants) in period 2,

$$x^2 = \sum x^{i2} + \sum x^{j2}. \hspace{1cm} (10)$$

We can write the incumbent $i$'s market share in period 2 as

$$b^{i2} = r^i b^{i1} + (1-r) x^{i2}/x^2, \quad \forall i \in I.$$  \hspace{1cm} (11)

The first term in (11) refers to firm $i$'s captive market in period 2, which arises from its period-1 advertising, $x^{i1}$. The second term in (11) refers to its share ($x^{i2}/x^2$) of the contestable $(1-r)$ portion in the period-2 market that owes to its period-2 advertising effort relative to those of all firms.

Correspondingly, the new entrant's market share of the contestable portion of the period-2 market is

$$b^{j2} = (1-r) x^{j2}/x^2, \quad \forall j \in J,$$  \hspace{1cm} (12)

in order that the market shares of all incumbents and new entrants satisfy
the adding-up constraint,
\[ \sum_i b_i^2 + \sum_j b_j^2 = 1. \]  \hspace{1cm} (13)

To summarize, the pre-entry market share of the incumbent firm
is a positive function of its share of total advertising expenditures. Its
post-entry market is: (a) a share of the non-contestable portion of this
market, which is a positive function of its pre-entry market share; and (b)
a share of the contestable portion of this market, which is a positive
function of its share of total post-entry advertising expenditures. Finally,
the new entrant's share of the contestable portion of the post-entry
market is a positive function of its share of total post-entry advertising.

III. The Battle over the Degree of Captivity

There is no reason to expect a fixed degree of market captivity of
incumbent \( i \), \( r_i \). First, it is natural to expect that \( r_i \) will be enhanced by
the advertising expenditure of incumbent \( i \), \( x_{i1} \) (see also Baldani and
Masson (1981)). Second, however, \( r_i \) is open to erosion by the aggregate
advertising activity of the new entrants, \( X^{N2} \). Finally, this erosion of \( r_i \)
can be reduced by the advertising expenditure of the incumbent in period 2,
\( x_{i2} \). To capture these three aspects, we write
\[ r_i = r_i(x_{i1}, X^{N2}, x_{i2}), \]
where \( r_{i,x_{i1}} > 0, r_{i,X^{N2}} < 0, r_{i,x_{i2}} > 0, \) \hspace{1cm} (14)
where \( X^{N2} = \sum_j x_{j2} \). \hspace{1cm} (15)
From hereafter, it is useful to recognize that symmetry exists in the sense that among all incumbents
\[ b^{i1} = 1/M, \]
\[ r^i = r, \]
and
\[ b^{i2} = b^{k2}, \quad \forall i, k \in I, \quad (16) \]
and among all new entrants,
\[ b^{j2} = b^{k2}, \quad \forall j, k \in J \quad (17) \]
This assumption is justified given that production costs and demand conditions are identical for all firms, the only asymmetry being that between incumbents, \( i \in I \), and new entrants, \( j \in J \).

Furthermore, to facilitate the exposition of the model, we shall introduce a specific function which satisfies equation (14), namely,
\[ r = \Omega(1-e^{-h(x^{i1})})e^{-k(x^{i2})}x^{N2}, \quad (18) \]
where \( h_{x^{i1}} > 0, k_{x^{i2}} < 0 \).

Specification (18) has certain desirable properties. First, \( \Omega \in [0,1] \) is the fundamental characteristic that defines the potential captive market effect for a particular type of product. As we shall see below, the maximum possible degree of market captivity \( r \) is equal to \( \Omega \). One should expect that for established and homogeneous products, whose quality is well known, the potential impact of pre-entry advertisement tends to be negligible, so that \( \Omega \) tends toward zero. However, for new products, especially, consumer products where consumers are wary of new entrants, \( \Omega \) can be quite large.

Second, given \( x^{N2} \) and \( x^{i2} \), a rise in \( x^{i1} \) enhances the degree of
market captivity \( r \), owing to the factor \( h(x^{i1}) \).

Third, at given levels of \( x^{i1} \) and \( x^{i2} \), an increase in aggregate advertising expenditure by new entrants, \( X^{N2} \), will erode \( r \) at a constant rate, \( k(x^{i2}) \).

Finally, an increase in the sales expenditure of incumbents in period 2 has the effect of counteracting the rate of erosion of \( r \) caused by a rise in \( X^{N2} \).

IV. The Behaviour of New Entrants

In period 1, each of the \( M \) incumbents chooses \( x^{i1}, x^{i2}, p^1, p^2 \), to maximize the present value of profits in the two periods, subject to the constraint that part of the period-2 market is open to contest by new entrants. This constraint on the incumbents is derived from the incumbents' rational expectations regarding the new entrants' behaviour that is in reaction to the incumbents' choice of advertising and price levels in periods 1 and 2 in the first place.

We can proceed to examine the behaviour of new entrants as follows. Given the incumbents'

\[ \emptyset = \{M, x^{i1}, x^{i2}, p^1, p^2\}, \]

(19)
each new entrant chooses its advertising expenditure \( x^{i2} \) (and thus its market share) to maximize profits, that is,

\[ \text{Max. } \pi_j^2(x^{i2}, N, \emptyset) = p^2.b^{i2}.Q^2 - c(b^{i2}.Q^2) - x^{i2}, \quad \forall j \in J. \]

(20)
The necessary condition for this is that:

\[ 0 = \pi_{xj}^2(x^{i2}, N; \emptyset) = (p^2-c^x).Q^2.[b_{xj}^{i2}] - 1, \quad \forall j \in J. \]

(21)
The second-order condition requires that

\[ 0 > \pi_{xjxj}^{i2} = (p^2-c^x).Q^2.[b_{xjxj}^{i2}] - c^x.(Q^2.[b_{xj}^{i2}])^2. \]

(22)
Moreover, free entry in period 2 ensures that the number \( N \) of new entrants is such that each entrant's profit is zero:

\[
0 = \Pi^j_2(x^j_2, N; \emptyset) = p^2.b^j_2.Q^2 - c(b^j_2.Q^2) - x^j_2, \quad \forall j \in J. \quad (23)
\]

To focus on the impact of advertising on the demand conditions, we shall assume the simplest possible production condition, that is, that constant variable cost obtains (\( c^c \) is zero) in period 2 and 1. Since \( p^2 > c^c \), from (21), the second-order condition means that

\[
b^j_2 \times x^j_2 \times x^j_2 < 0 \quad \Rightarrow \quad (1-r)/x^2 - k.r > 0. \quad (24)
\]

This condition in turn implies

\[
0 > \Pi^j_2_N = (p^2-c^c).Q^2.b^j_2_N
\]

\[
= -(p^2-c^c).Q^2.((x^j_2)^2/x^2).[(1-r)/x^2 - k.r], \quad (25)
\]

which is simply the standard stability condition requiring the profit of new entrants to fall (rise) when the number of new entrants \( N \) rises (falls).

Equations (21) and (23) simultaneously determine \( N \) and \( x^j_2 \), and thus \( x^{N_2} \). New entrants are assumed to be price followers so that \( p^2 \) is left to be determined by incumbents. The impact of \( \emptyset \) on \( x^j_2 \) and \( N \) now can be found by totally differentiating equations (21) and (23), i.e.

\[
\begin{bmatrix}
\Pi^j_2 x^j_2 x^j_2 & \Pi^j_2 x^j_2 N \\
\Pi^j_2 x^j_2 & \Pi^j_2 N
\end{bmatrix}
\begin{bmatrix}
dx^j_2 \\
dN
\end{bmatrix}
= 
\begin{bmatrix}
-\Pi^j_2 x^j_2 \emptyset . d\emptyset \\
-\Pi^j_2 \emptyset . d\emptyset
\end{bmatrix} \quad (26)
\]

where the determinant of the LHS matrix:
\[ \Delta^i = \Pi_{xj2}^{j2} \cdot \Pi_{N}^{j2} > 0, \]  
by the Routh-Hurwicz condition for stability of the system (noting that \( \Pi_{xj2}^{j2} = 0 \) from (21)).

From these are derived the new entrants’ reaction functions, which will be crucial for the incumbent’s solution:

\[ \frac{dN}{dx^{i1}} = -b^{i2}_{xj1} \cdot b^{i2}_{xj2} / D < 0, \]  
(28)

\[ \frac{dN}{dx^{i2}} = -b^{i2}_{xj2} \cdot b^{i2}_{xj2} / D < 0, \]  
(29)

\[ \frac{dx^{i2}}{dx^{i1}} = (1/D) \cdot [b^{i2}_{xj1} \cdot b^{i2}_{xj2} - b^{i2}_{N} \cdot b^{i2}_{xj2} / D < 0, \]  
(30)

\[ \frac{dx^{i2}}{dx^{i2}} = (1/D) \cdot [b^{i2}_{xj2} \cdot b^{i2}_{xj2} - b^{i2}_{N} \cdot b^{i2}_{xj2} / D < 0, \]  
(31)

where \( D = (\Delta^i) \cdot (b^{i2}_{xj2})^2 > 0, \)

The stability condition (24) ensures that (28) and (29) are negative, that is, a rise in incumbents’ advertising, \( x^{i1}, x^{i2}, \) decreases the number of new entrants \( N. \) In this limited sense, incumbents’ advertising unambiguously deters entry. However, the “entry deterrence” of incumbents’ advertising is usually defined as limiting not only \( N \) but the new entrants’ market share, or what is proportional to this, \( N \cdot x^{i2} \) (or \( X^{i2} \)).

To address this issue (which requires specification of the conditions governing the signs of (30) and (31)), we substitute the general function of \( r \) (18) into (28) to (31). Thus,

\[ \frac{dN}{dx^{i1}} = [(x^{i2} / X^{2}) / D] \cdot [dr / dx^{i1}] \cdot [b^{i2}_{xj2} / x^{i2}] < 0, \]  
(28')

\[ \frac{dN}{dx^{i2}} = [(x^{i2} / X^{2}) / D] \cdot [dr / dx^{i2} + (1-r) / X^{2}] \cdot [b^{i2}_{xj2} / x^{i2}] < 0, \]  
(29')

\[ \frac{dx^{i2}}{dx^{i1}} = -[(x^{i2} / X^{2}) / D] \cdot [dr / dx^{i1}] \cdot [(1-r) / X^{2} - k], \]  
(30')

\[ \frac{dx^{i2}}{dx^{i2}} = -[(x^{i2} / X^{2}) / D] \cdot [dr / dx^{i2} + k \cdot r] \cdot [(1-r) / X^{2} - k]. \]  
(31')

From (30') and (31'), whether the effect of incumbents’
advertising, \( x^{i1} \) and \( x^{i2} \), on \( x^{i2} \) is negative or positive depends on whether
the erosion factor \( k \) on the incumbents' captive market is smaller or larger
than \( (1-r) / \chi^2 \). This is persuasive. If the incumbents' captive market is
less (more) easily eroded by new entrants' advertising, this implies that
new entrants would choose a smaller (larger) \( x^{i2} \) than otherwise.

\[ \text{V. The Behaviour of Incumbent Firms} \]

The behaviour of new entrants as described in the previous section
is taken as the constraint by incumbents in formulating their best course
of action. Given our symmetry assumption, each incumbent \( i \in I \) faces the
same constraint when he chooses the prices, \( P^1 (= p^{i1} \forall i) \), \( P^2 (= p^{i2} \forall i) \),
and advertising, \( x^{i1}, x^{i2} \), to maximize the present value of his profits:

\[
\begin{align*}
\text{Max. } & \quad \Pi_i = \Pi^{i1} + \delta \Pi^{i2} \\
& \quad (P^1, P^2, x^{i1}, x^{i2}) = [b^{i1}.P^1.Q^1 - c(b^{i1}.Q^1) - x^{i1}] \\
& \quad + \delta [b^{i2}.P^2.Q^2 - c(b^{i2}.Q^2) - x^{i2}], \quad \forall i \in I, \quad (32)
\end{align*}
\]

where \( \delta \) is the discount factor. The solutions for \( P^1 \) and \( P^2 \) require the
following necessary conditions:

\[
\begin{align*}
0 &= \Pi^1_{p1} = b^{i1}.Q^1.(p^1.(1-1/\mu_1) - c'_1), \quad \forall i \in I, \quad (33) \\
0 &= \Pi^1_{p2} = b^{i2}.Q^2.(p^2.(1-1/\mu_2) - c'_2), \quad \forall i \in I. \quad (34)
\end{align*}
\]

Consequently, \( P^1 \) and \( P^2 \) depend solely on the price elasticities of industry
demand, \( \mu_1, \mu_2 \). Since we assume that aggregate advertising expenditure
does not affect the total size of market demand, both price levels are
independent of advertising. It should be interesting to relax this
assumption in future works. Our present effort chooses to avoid this to
focus on the issue of non-price competition over market shares. Price in
this model does not play any strategic role in determining market shares.

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Turning now to the solutions for $x^{i1}$ and $x^{i2}$, the necessary conditions are:

$$0 = \Pi_{xi1} = (p^1 - c_1).Q^1.b^{i1}_{xi1} + \delta.(p^2 - c_2).Q^2.b^{i2}_{xi1} - 1$$
$$+ \delta.(p^2 - c_2).Q^2.\left[\sum_j b^{i2}_{xj2}.dx^{i2}/dx^{i1} + b_N^{i2}.dN/dx^{i1}\right]$$

$$0 = \Pi_{xi2} = \delta[(p^2 - c_2).Q^2.b^{i2}_{xi2} - 1]$$
$$+ \delta.(p^2 - c_2).Q^2.\left[\sum_j b^{i2}_{xj2}.dx^{i2}/dx^{i2} + b_N^{i2}.dN/dx^{i2}\right]$$

(35)

(36)

Given our symmetry assumption and the fact that

$$b^{i2}_N = x^{i2}.b^{i2}_{xj2}.$$  

(37)

the above can be rewritten and rearranged as follows:

$$(p^1 - c_1).Q^1.b^{i1}_{xi1} + \delta.(p^2 - c_2).Q^2.b^{i2}_{xi1} - 1$$
$$= - \delta.(p^2 - c_2).Q^2.b^{i2}_{xj2}.(N.dx^{i2}/dx^{i1} + x^{i2}.dN/dx^{i1})$$

(35)'

$$\delta[(p^2 - c_2).Q^2.b^{i2}_{xi2} - 1]$$
$$= - \delta.(p^2 - c_2).Q^2.b^{i2}_{xj2}.(N.dx^{i2}/dx^{i2} + x^{i2}.dN/dx^{i2}).$$

(36)'

The LHS term in equations (35)' and (36)' is the direct marginal effect of $x^{i1}$ and $x^{i2}$ (excluding the effect due to new entrants’ reaction) on the incumbent firm’s present value of profit. The RHS, however, is the marginal opportunity cost due to the new entrants’ reaction. This owes to the fact that the sum of new entrants’ advertising expenditures, $N.x^{i2}$, adjusts to the incumbent’s advertising expenditures $x^{i1}$ and $x^{i2}$, and in turn affects the incumbent’s market share and thus profit in period 2.

Let us now examine how the expectation of contest by new firms in period 2 influences the behaviour of incumbent firms.

A. No New Entry

'In the absence of new entrants, the RHS terms of equations (35)'
and (36)' vanish and we have only internal competition among incumbents in both periods. For the sake of simplicity, assume that the two demand curves in both periods are identical so that the optimal \( p^1 = p^2 \). It then can be verified from the two equations that the solution of each incumbent is characterized by

\[ x^{i1}_0 / x^{i2}_0 = (1-r)/(1+\delta r), \tag{38} \]

which implies that \( x^{i1}_0 \geq x^{i2}_0 \). In the absence of new entry, the lagged term effect of advertising makes \( x^{i1} \) a more powerful strategic variable than \( x^{i2} \). The optimal solution \((x^{i1}_0, x^{i2}_0)\) is denoted by \( Z \) in Figure 1.

Moreover, this power is accentuated by a larger degree of potential market captivity \( \Omega \), and a larger rate of discount \( \delta \). Only in the extreme case where \( \Omega \) is zero, that is, when there is absolutely no lagged advantage from earlier advertising, will we obtain the symmetrical result that \( x^{i1}_0 \) equals \( x^{i2}_0 \). Otherwise \( x^{i1}_0 \) is strictly greater than \( x^{i2}_0 \).

**B. Presence of New Entry**

Where new entry is allowed to occur, the incumbents' choice of optimal \( x^{i1} \) and \( x^{i2} \) in (35)' and (36)' must now take into account the new entrants' reaction functions, given by (28)' to (31)'. These reaction functions are embodied in the RHS terms of equations (35)' and (36)' . They display the manner in which the new entrants' aggregate advertising expenditure, \( N \cdot x^{i2} \), reacts to \( x^{i1} \) and \( x^{i2} \).

Equations (28)' to (31)' yield the following:

\[ d(N \cdot x^{i2})/dx^{i1} = N \cdot (dx^{i2}/dx^{i1}) + x^{i2} \cdot (dN/dx^{i1}) \]

\[ = -[(x^{i2}/x^{i1})^2 / D] \cdot [dr/dx^{i1}] \cdot [A + B], \tag{39} \]

\[ d(N \cdot x^{i2})/dx^{i2} = N \cdot (dx^{i2}/dx^{i2}) + x^{i2} \cdot (dN/dx^{i2}) \]
\[
= -[(x^2)^2/D][\text{dr/dx}^2 + k.r][A + B]
+ [(1-r)/(x^2-k.r)B]
= -[(x^2)^2/D][\text{dr/dx}^2][A + C.B],
\]

where \( A = N.(x^2/x^2)\cdot((1-r)/x^2-k) \),
\( B = -b_{xj2xj2}/x^2 > 0 \),
and \( C = 1 + [(1-r)/(x^2-k.r)/[\text{dr/dx}^2 + k.r] > 1 \).

Thus,
\[ d(N.x^2)/dx^1 \geq 0 \text{ for } (A + B) \leq 0, \]
and \[ d(N.x^2)/dx^2 \geq 0 \text{ for } (A + C.B) \leq 0. \]

Now \((A + B) \geq 0 \) implies \((A + C.B) > 0 \); this constitutes case (i).

However, when \((A + B) < 0 \), we can have either \((A + C.B) \leq 0 \), which is case (ii), or \((A + C.B) > 0 \), which is case (iii). These three cases exhaust all possible configurations. They are summarized as follows:

**Case (i):** \((A + B) \geq 0 \Rightarrow (A + C.B) > 0\),
\[ d(N.x^2)/dx^1 \leq 0, \quad d(N.x^2)/dx^2 < 0. \]

This case occurs when the erosion factor \( k \) is relatively small, that is, equal to or smaller than \((B.x^2/(N.x^2) + (1-r)/x^2)\). A small erosive impact of new entrants' advertising means that new entrants would choose to reduce their advertising \( x_{i2} \) in reaction to higher levels of incumbent advertising \( x_{i1} \) and \( x_{i2} \). This effect reinforces the negative relationship between the number of new entrants and incumbent advertising.

From (35)' and (36)', it can be shown that the optimal expenditures of the incumbents must be
\[ x_{i1}^* \geq x_{i1}, \quad x_{i2}^* > x_{i2} \text{.} \]

It is favourable for the incumbent firm to take an aggressive position by increasing advertising expenditures in both periods since both of these have the effect of reducing the total advertising of new entrants, \( N.x^2 \), in...
period 2. This is shown as the north-east quadrant in Figure 1.

Case (ii): \((A + C.B) < 0 \Rightarrow (A + B) < 0\),

\[
\frac{d(N.x^{i2})}{dx^{i1}} > 0, \quad \frac{d(N.x^{i2})}{dx^{i2}} < 0.
\]  

(45)

This is the other extreme with \(k\) being equal to or larger than \([C.B.x^2/(N.x^{i2}) + (1-r)/x^2]\). Since the erosive impact of their advertising is large, new entrants would choose to increase their advertising \(x^{i2}\) when incumbents raise their advertising levels \(x^{i1}\) and \(x^{i2}\). At such levels of \(k\), this positive reaction of \(x^{i2}\) is so large as to overwhelm the decline in the number of new entrants \(N\), so that aggregate advertising of new entrants rises with \(x^{i1}\) and \(x^{i2}\).

To reduce new entrants' advertising, the optimal solution for the incumbent firm is to take a relatively passive posture by advertising less in both periods than would be the case with no new entry, that is,

\[
x^{i1*} < x^{i1}_0, \quad x^{i2*} \leq x^{i2}_0.
\]

(45)

This is the south-west quadrant in Figure 1. Schmalensee's (1983) result is but a special case of this. Since he does not explicitly distinguish between pre- and post-entry advertising, his result can be interpreted as simply \(x^{i1*} < x^{i1}_0\).

Case (iii): \((A + B) < 0\), and \((A + C.B) > 0\),

\[
\frac{d(N.x^{i2})}{dx^{i1}} > 0, \quad \frac{d(N.x^{i2})}{dx^{i2}} < 0.
\]

(46)

Here the value of \(k\) lies in the intermediate range between \([B.x^2/(N.x^{i2}) + (1-r)/x^2]\) and \([C.B.x^2/(N.x^{i2}) + (1-r)/x^2]\). As in case (i), a rise in \(x^{i2}\) still has a negative impact on \(x^{i2}\), so it pays for the incumbents to raise the level of \(x^{i2}\) to reduce \(N.x^{i2}\). However, as in case (ii), a rise in \(x^{i1}\) arouses a higher level of advertising by new entrants \(x^{i2}\), resulting in a higher total new entrant advertising, \(N.x^{i2}\). Case (iii) reveals an interesting mixed optimal strategy for incumbents in relation to their
Figure 1. Incumbents' Optimal Solution Set
no-entry solution \((x^{i1}_0, x^{i2}_0)\), that is,

\[
x^{i1}_* < x^{i1}_0, \quad x^{i2}_* > x^{i2}_0.
\]  \(\text{(46)}\)

This solution is contained in Figure 1 as the north-west quadrant.

The result of case (iii) suggests that as a strategic variable, incumbents' period-2 advertising \(x^{i2}\) is more effective than period-1 advertising \(x^{i1}\) in limiting the impact of advertising by new entrants. The reason for this is that \(x^{i1}\) only affects \(N.x^{i2}\) via its effect on \(r\), that is, it increases the portion of the market not contestable to new entrants. However, \(x^{i2}\) affects \(N.x^{i2}\) not only through \(r\) but also by increasing the incumbents' share of the contestable portion of the post-entry market (that is, even when \(r\) is held constant).

\[
d(N.x^{i2})/dx^{i1} = (d(N.x^{i2})/dr).(dr/dx^{i1}), \quad (\ast) \tag{47}
\]

\[
d(N.x^{i2})/dx^{i2} = (d(N.x^{i2})/dr).(dr/dx^{i2}) + d(N.x^{i2})/dx^{i2}
\bigg|_{r \text{ constant}}. \quad (\ast) \tag{48}
\]

\[
d(N.x^{i2})/dr \text{ occurs in both (47) and (48). } d(N.x^{i2})/dx^{i1} > 0 \text{ if and only if } d(N.x^{i2})/dr > 0. \text{ However, when } d(N.x^{i2})/dr > 0, d(N.x^{i2})/dx^{i2} \text{ can still be negative. This explains equation (46) and case (iii).}
\]

By the same token, \(d(N.x^{i2})/dx^{i1} < 0 \text{ if and only if } d(N.x^{i2})/dr < 0.\) When \(d(N.x^{i2})/dr < 0, \text{ we must have } d(N.x^{i2})/dx^{i2} < 0. \text{ Thus, when } d(N.x^{i2})/dx^{i1} < 0, \text{ we cannot have } d(N.x^{i2})/dx^{i2} > 0. \text{ This means that the south-east quadrant of Figure 1 (where } x^{i1}_* > x^{i1}_0 \text{ and } x^{i2}_* < x^{i2}_0 \text{) is inadmissible as a feasible region for the incumbent's optimal solution.}

C. Some Special Models

To highlight the generality of our model, it is useful to consider some special cases along the lines of previous work on entry-deterrence.
Model I: \( r = \Omega \) constant

From (18), the lagged advantage of pre-entry advertising, \( r \), is equal to \( \Omega \) whenever \( h(x^{i1}) = 0 \) for all \( x^{i1} \) and \( k(x^{i2}) = 0 \) for all \( x^{i2} \). This means that \( r \) is independent of the incumbents' pre- and post-entry advertising and of the advertising of new entrants. In this special case, \( d(N,x^{i2})/dx^{i1} = d(N,x^{i2})/dx^{i2} = 0 \), and hence \( (x^{i1*}, x^{i2*}) = (x^{i1}_0, x^{i2}_0) \). Thus, our model demonstrates that incumbents will not change their optimal levels of advertising even with the threat of entry. The rationale for this is that the captive market is essentially a "free gift" that cannot be affected by any action of incumbents and new entrants.

Model II: \( r = \Omega (1 - e^{-h(x^{i1})}) \)

This occurs when \( k(x^{i2}) = 0 \) for all \( x^{i2} \), that is, \( r \) cannot be eroded by new entrants and there is no counter-erosion. However, pre-entry advertising can enhance \( r \). In this case, \( d(N,x^{i2})/dx^{i1} < 0 \) (from (39)) and \( d(N,x^{i2})/dx^{i2} < 0 \) (from the second equality of (40)). Consequently, the incumbents' optimal solution is \( x^{i1*} > x^{i1}_0 \) and \( x^{i2*} > x^{i2}_0 \). Compared to the no-entry solution, the threat of new entry causes incumbents to step up their pre-entry advertising. This is analogous to the traditional result of the investment-capacity model of entry-deterrence. However, our model possesses the additional dimension in that the incumbents also choose to expand their post-entry advertising. The set of optimal solutions to this model is contained only in the north-east quadrant of Figure 1.

Model III: \( r = \Omega (1 - e^{-h(x^{i1})})e^{-k}x^{N2} \)

This is similar to our general model except that \( k(x^{i2}) = k \), a positive constant. New entrants can erode \( r \) but incumbents' post-entry advertising cannot counter the erosion. This model is not qualitatively
different from the general model in the sense that the optimal solution \((x_1^*, x_2^*)\), may lie in any of the three quadrants in the general model. Thus, what drives the result in the general model is not the incumbents' counter-erosion capability; rather, it is the positive value of \(k\), the erosive power of new entrants' advertising.

VI. Concluding Remarks

In general, incumbents possess lagged advantages that may arise from their pre-entry investment in productive capacity or in their pre-entry advertising. By its very nature, the productive capacity advantage of incumbents is not subject to erosion by new entrants, so that incumbents facing threat of new entry will find it optimal to exploit this advantage by expanding pre-entry capacity. This standard result in the literature, however, does not apply to advertising models for the simple reason that advertising can only persuade; and by its very nature, the advantage of persuasion is readily contestable by rivals, new and old. Because of this, while advertising is necessary, it is not a sufficiently powerful tool in tightly securing market shares.

First, to recognize the combative nature of advertising among rival firms, our model assumes that in general the market share of a firm depends on its advertising intensity during that period relative to those of rivals. Second, however, the persuasive impact of advertising, though never permanent, lasts for more than one period. Incumbents' pre-entry advertising confers a lagged benefit in the form of making captive a portion of the post-entry market for themselves. The post-entry market, therefore, is divided into two: the incumbents' captive portion and the remaining part contestable by all firms. Incumbents' pre-entry advertising
only enhances the captive portion of the market but their post-entry advertising has two effects: (i) to counter the erosive impact of new entrants' advertising, and (ii) to do battle for the non-captive part of the market.

This paper makes clear that the response of incumbents to the threat of new entry depends critically on the degree by which new entrants can erode the lagged advantage of incumbents. When the erosion factor $k$ is relatively small, the incumbents will exploit this by expanding their pre- and post-entry advertising levels. This result is the dynamic counterpart of the standard conclusion of productive-capacity models. As $k$ increases to intermediate values, the advantage of pre-entry advertising falls relative to that of their post-entry advertising. The advantage of post-entry advertising tends not to fall as much; though its effect (i) is weakened, its effect (ii) is not. This explains the curious mixed strategy wherein incumbents choose to reduce their pre-entry advertising but raise their post-entry advertising in the face of new entry. Finally, when the erosion factor $k$ is very large, incumbents' lagged advantage from pre-entry advertising is drastically reduced. Their post-entry advertising effect (i) is also significantly weakened to the extent that it overwhelms effect (ii). Consequently, incumbents would choose to reduce both pre- and post-advertising levels when they face the threat of new entry. This can be interpreted as the dynamic counterpart of Schmalensee's (1983) result.

The model obviously can be extended in various directions. One is to allow for advertising to affect the size of aggregate market demand. As it is, we have only focused on the distribution effects among rival firms. Relaxing this assumption would mean that incumbents could use prices in addition to advertising to counter new entry. Second, the
symmetry assumption regarding production cost among firms could be relaxed to allow for cost differentials between incumbents and new entrants, as in the productive capacity models. To highlight the issue of advertising as a weapon of contest among firms, we have chosen to leave these elaborations for future work.
References

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