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AND ASYMMETRIC INFORMATION

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THE OPTIMUM QUANTITY OF MONEY
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1. INTRODUCTION

When households have a positive time preference, the holding of money entails a real cost. Often it is argued that this may be socially inefficient and that welfare could be increased by reducing the opportunity cost of holding money so as to avoid a wasteful economizing on holding cash. This idea is the basis for Milton Friedman's theory of an optimum quantity of money. According to Friedman (1969), the quantity of money is optimal when there is a real return on money equal to the rate of time preference.

Friedman's theory is surprisingly wide accepted among laissez-faire economists even though it has the following implication: whereas the market system itself cannot achieve a first best equilibrium, so that money is needed as an additional institution, the government is supposed to have the ability to restore efficiency simply by using the governments power to tax in order to pay interest on money.

The problem with Friedman's theory is that it is not based on a model which makes explicit why money is needed in the first place. A careful examination of his theory would require a precise description of reasons which prevent the market from performing its role (of achieving an efficient allocation) without money.\(^1\)

This calls for an explicit modelling of some friction which makes it impossible to achieve an Arrow-Debreu allocation. Such a "microfoundation" of money has to explain why certain contracts cannot be binding and thus are not enforceable. It should also provide exact conditions for feasible government interventions.
The standard way to introduce frictions is the overlapping generations approach. In an overlapping generation model, however, with mortality being the only friction, the introduction of money can restore first best efficiency and thus completely overcomes the friction. So this approach is not appropriate to analyse the optimum quantity of money problem.

The present paper takes a different route to introducing frictions into the standard Arrow-Debreu model. Money is modelled as a social institution that helps to overcome frictions which are due to asymmetric information among agents. The structure of the model is similar to a framework set up by Bewley (1983) and Hellwig (1982). In contrast to their papers the frictions that impede the enforcement of Arrow-Debreu contracts will be made explicit. Money as compared with insurance saves on information costs.

The explicit formulation of the informational frictions can clarify the contradiction between Bewley (1983) and Benhabib/Bull (1983) about the validity of the optimum quantity theory. Financing the interest payments on money at the rate of time preference would require lump-sum taxes. The government, however, being subject to exactly the same restrictions that make the institution of money necessary in the first place, is not able to impose true lump-sum taxes.

The reason is that taxes which are not based on the individual endowment create a separate demand for holding extra money balances: to avoid bankruptcy, households plan to finance the taxes completely out of the resulting interest payments. Then, of course, the government budget constraint would be violated. Thus, the government would have to impose lump-sum taxes independent of the actual endowment. To do that, however, it would need exactly the information whose absence caused the demand for
money in this model.

This argument makes clear why in Bewley's model the optimum quantity theory is not working. In contrast, in Benhabib's and Bull's model the quantity theory seems to be confirmed. Yet this is so only because these authors do not constrain government activities in a way equivalent to the frictions which ensure a positive price of money in their model. They simply do without modelling the economic reasons which give cause to their transactions technology.

2. **THE BASIC MODEL**

In the model to be presented here, households in each period face individual endowment risks. The actual realization of a random endowment of a non-storable good can be observed without cost only by the individual household himself. Therefore, contingent Arrow-Debreu contracts are not incentive-compatible. Two social institutions will be discussed which may contribute to overcome this friction: insurance and money. For other agents it is possible to obtain the true information about an individual realizations at a fixed cost. In order to economize on these costs and centralize risk-sharing, insurance companies arise as intermediaries. The verification costs constitute the main difference between this model and the frictionless classical model: they keep full insurance from being attractive. There exists an "insurance region": only for high risks (low realizations) is it worthwhile to bear the fixed cost.

Households, however, may reduce their insurance region and, instead, hold precautionary money. Thus the introduction of money into this economy saves on information costs by reducing the insurance region. In periods of
high endowment households exchange the good against money, and they sell the money in periods of low endowment. Yet holding money also implies a real cost in terms of deferred consumption due to time preference - as long as there is no payment of interest on money. Therefore, some combination of insurance and money holding is optimal. In contrast to the institution-free Arrow-Debreu model, these two institutions will be active as a response to informational frictions.

The basic structure of the model is as follows: in the economy there is only one (non-storable) consumption good. All households are identical, risk-averse and live infinitely long. A representative household maximizes the time-additive discounted expected utility of consumption \( W = \sum_{t=1}^{\infty} \delta^{t-1} EU(c_t) \). \( U \) is assumed to be strictly concave and continuously differentiable.

Each household \( h \) receives a random endowment \( z_h \) in each period. \( z_h \) is independently and identically distributed over time. The risks between households are assumed to be independent. Thus there is no social risk, e.g. the average endowment per person is constant during all periods (\( E(z) \) for all \( t \)).

It is straightforward to characterize an Arrow-Debreu allocation: depending on his individual realization the household will either have to deliver part of his endowment or receive a compensation such that in each period he consumes the average endowment \( E(z) \) per person.

But the individual realization of the random endowment can be observed without cost only by the household himself. Therefore, there is no mechanism to attain an Arrow-Debreu allocation: the agent has no incentive to report his actual endowment truthfully. Given the information structure, it
would always be optimal for him to understate his own realization to get a higher compensation. The Arrow-Debreu contracts are not incentive-compatible. Even though ex ante risk-sharing agreements would be in everybody's interest, they cannot be binding, because reporting the truth ex post is not individually rational.

Social institutions arise as a substitute for these contracts which are not enforceable. Anybody wishing to verify the statement of a household about his endowment realization could do so at a fixed cost $k$. To economize on these costs, risk neutral insurance agencies will specialize in offering one-period contracts which—according to Townsend (1979)—have the following properties: the household pays a fixed premium $b_t$. For high realizations there is no verification and the household consumes the random endowment $z_t-b_t$. There is, however, an insurance region for low realizations ($0 \leq z_t \leq x_t$). Here the agency verifies the household's statement by incurring the cost $k$, and pays a compensation depending on $z_t$ such that, within this region, the household can consume the constant amount $a_t-b_t$. There is a zero-profit constraint on the agency due to free entry.

Because buying insurance is costly, self-insurance via holding money (so accepting a loss out of foregone time preference) may be worthwhile for a household to reduce $x_t$.

3. **INDIVIDUAL OPTIMIZATION**

3.1 **General Problem**

A representative household chooses in each period $t=1,2,\ldots$ his consumption $c_t$, his end of period holding of real money $m_t = M_t/p$ and the next period's insurance contract $I_{t+1} = (x_{t+1}, a_{t+1}, b_{t+1})$. Since in this paper I
am not concerned with non-stationary monetary equilibria (with a changing price \( p_t \)), I consider only the individual optimization problem with a given constant money-good price \( p \). The household can choose among insurance contracts \( I_t \) with varying insurance region \([0, x_t]\), gross payment \( a_t \) and premium \( b_t \).

At \( t \), he observes his random endowment \( z_t \) and knows the past realizations of the random process \( \{ z_s \} \) up to \( s = t \). His choice depends on this information and his expectations about the future. He maximizes the present value of expected utility of consumption

\[
\begin{align*}
\text{Max} & \quad \sum_{t=1}^{\infty} \sum_{t=1}^{t-1} \text{EU}(c_t) \\
\text{subject to} & \quad \begin{align*}
1) & \quad (c, m, x, a, b) \\
2a) & \quad c_t + m_t = m_{t-1} + i_t(z_t) \quad \text{for all } t; \quad m_0, i_1(z_1) \text{ given} \\
2b) & \quad i_t(z_t) = \begin{cases} \\
\frac{a_t - b_t}{z_t - b_t} & \text{for } 0 \leq z_t \leq x_1 \\
\frac{a_t - b_t}{z_t - b_t} & \text{for } z_t > x_t \\
\end{cases} \\
2c) & \quad (x, a, b) \text{ such that: } \{ (a + k - z_t)f(z_t) = b \} \quad \text{for all } t; \\
(k \text{ being small}) \\
2d) & \quad c_t \geq 0; \quad m_t \geq 0; \quad a_t \geq x_t \geq 0; \quad x_t - b_t \geq 0 \quad \text{for all } t. \\
2e) & \quad V(m_t) \geq 0 \int_0^{x_t} u(m_t + z)dF(z) \quad \text{for all } t. \\
U(c_t) \text{ is assumed to be strictly increasing, strictly concave and continuously differentiable with } \lim_{c \to 0} U'(c) = \infty. \text{ There is a positive time preference} \\
\end{align*}
\]
The random endowments $z_t$, $t=1,2,...$, are independent and identically distributed with the continuous distribution function $F(z)$ with $0 \leq z \leq \hat{z}$ and the mean $E(z)$. (2a) represents the sequence of budget constraints and (2b) the insurance net payments depending on the choice of the insurance region $x_t$ and the zero-profit constraint of the insurance (2c): the expected value of gross payments $a_t - z_t$ plus verification cost $k$ in the insurance region must be equal to the gross premium $b_t$. $a_t \geq x_t$ is an incentive compatibility constraint. The indirect utility of money holding $v(m_t)$, if the household buys insurance, must never be less than without insurance (2e)).

This infinite horizon problem will be solved in two steps: first, an indirect utility function will be derived for a two-period problem, and then the properties of the infinite horizons solution will be discussed.

3.2 Two-Period Problem

In period $t-1$, the household has a known wealth $w_{t-1}$. He faces an endowment risk $z_t$ in period $t$. In order to insure against this risk, he can both transfer part of his wealth into period $t$ by holding real money $m_{t-1}$ and buy an insurance contract $(x_t, a_t, b_t)$. His maximization problem is:

1') \[
\text{Max} \quad U(c_{t-1}) + \delta \int_0^{\hat{z}} U(c_t)f(z)dz
\]
subject to:

2'a) $m_{t-1} + c_{t-1} = w_{t-1}$

$c_t = m_{t-1} + i_t(z_t)$
2'b) \( i_t(z_t) = a_t - b_t \quad 0 \leq z_t \leq x_t \)
\[ z_t - b_t \quad z_t > x_t \]

2'c) \[ \int_0^x (a + k - z)f(z)dz = b \]

2'd) \( c_{t-1} \geq 0; \ c_t \geq 0; \ m_t \geq 0; \ a_t \geq x_t \geq 0; \ b_t \geq 0. \)

In a first step, one has to solve for the optimum choice of insurance contract, given money holding \( m_t \). This yields the indirect utility function \( V(m_t) \). Money holding saves on insurance costs, if with increasing \( m_t \) the optimum insurance region (and thus \( x_{t+1} \)) decreases. The Lagrangian for the optimum insurance contract problem is (omitting subscript \( t \)):

3) \[ L = U(m + a - b)F(x) + \int_x^\hat{x} U(m + z - b)f(z)dz \]
\[ - \lambda [(a + k)F(x) - \int_0^x zf(z)dz - b] \]

For an interior solution (with \( v(m) > \int_0^m u(m+z)dF(z) \), the first-order conditions are:

4.1) \[ \int_x^\hat{x} U'(m + z - b)f(z)dz = U'(m + a - b)(1 - F(x)) \]

4.2) \[ U(m + a - b) - U(m + x - b) = U'(m + a - b)(a + k - x) \]

4.3) \[ (a + k)F(x) - \int_0^x zf(z)dz - b = 0 \]

4.4) \( m + x - b \geq 0; \ a \geq x \geq 0; \ b \geq 0 \) and
With \( V(m) \) as indirect utility function of the maximization solution. Equation 4.1 is the familiar optimum condition for partial insurance: the marginal utility of consumption within the insurance region must be equal to the conditional expected marginal utility in the risk region. Equation 4.2 states that there is a discontinuity in consumption at \( x \) due to the set-up nature of the information cost. The change at \( x \) from not being insured to being insured implies a discrete jump: extending the insurance region means an increase in the probability of incurring the verification cost, and that means an increase in the insurance premium: \( \delta b/\delta x = (a + k - x)f(x) \). To be willing to bear this cost, a discrete increase in utility (weighted with the probability density) is required:

\[
\delta b/\delta x \cdot U'(m + a - b) = [U(m + a - b) - U(m + x - b)]f(x)
\]

The fixed cost of verification introduces a non-convexity into the optimization problem. One may expect a non-monotonicity in \( x_{t+1}(m_t) \) as in Gale/Hellwig (1986). A sufficient condition for a monotonic relationship \( \delta x_{t+1}/\delta m_t < 0 \) is

\[
\begin{align*}
\left( \frac{u - u}{a} \right) f(x) &> \frac{u''(m + z - b)f(z)dz}{\int_x^z f(x)dz} \\
\left( \frac{u - u}{a} \right) f(x) &< \frac{u''(m + a - b)f(z)dz}{\int_x^z f(x)dz}
\end{align*}
\]

Provided \( \left| \left( \frac{u - u}{a} \right) f(x) \right| < \left| \int_x^z u''(m + z - b)f(z)dz \right| \)

and \( \left( \frac{u - u}{a} \right) f(x) < \int_x^z u''(m + a - b)f(z)dz \)

(see Appendix 1) (read \( u = u(m + a - b); u = u(m + x - b) \)).
If the utility function has constant absolute risk aversion (e.g. if $U = -e^{-c}$) the optimum insurance contract is independent of $m_t$. (5) holds as equality. Since with constant absolute risk aversion the extent of bearing risk is unchanged by a change in money holding, there is no saving on information costs with increasing money holding.

In the case of uniform distribution, (5) simplifies to

\[
\begin{vmatrix}
 u - u \\
 a \\
 u - u
\end{vmatrix} >
\begin{vmatrix}
 u - u \\
 1 \\
 u - u
\end{vmatrix}
\]

(5')

So $\delta x_{t+1}/\delta m_t < 0$ if the utility function has decreasing absolute risk-aversion.

From now on, I assume that (5) holds and that $V(m_t)$ is continuous, strictly concave and differentiable. The class of non-trivial functions with these properties is non-empty, as the following example shows—it describes the explicit solution of this non-convex problem for the case of a uniform distribution and the utility function $U = c^\frac{1}{c}$.

Example:

$U = c^\frac{1}{c}$; $F(z) = z \quad (0 \leq z \leq 1)$

In this case, by rearranging the first-order conditions (see Appendix 2) one gets a simple characterization of the optimal contract:

The amount of consumption $a-b$ guaranteed by the insurance contract within the insurance region $[0,x]$ depends only on the verification costs: $a - b = \frac{1}{c} - k$. The premium $b$ increases with the insurance region such as $b = 0.5x$. The premium (and thus the insurance region) is determined by the amount of money holding via the condition

$$\frac{1}{c}[U(m+b) + U(m+1-b)] = U(m+\frac{1}{c}-k)$$

(see figure 1).

The explicit solution yields the following relation between insurance region
and money holding:

\[ x = 1 - 4 \sqrt{k \left( m + \frac{1}{2} - k \right) \frac{1}{2}}. \]

If \( k=0 \), complete insurance is optimal. If \( k \geq \frac{1}{2} \) verification costs are too high (25% of the maximum endowment) and it is not worth while to buy insurance even if \( m=0 \). For \( 0 < k < \frac{1}{2} \) the insurance region decreases continuously with increasing \( m \) until \( x=0 \) at

\[ m = \frac{(0.5 - 2k)^2}{4k} \]

For \( m > m^* \), the household buys no more insurance.

As is shown in appendix 2, the indirect utility function \( V(m) \) at \( m^* \) (that is at the level of money holding at which the household switches to buying no more insurance, being too costly relative to money holding) is continuous and differentiable. Furthermore, \( V''(m) < 0 \). So \( V(m) \) is continuous, differentiable, and concave for all \( m \), and the optimum money holding can be calculated maximizing the expected utility, given the initial wealth \( w_t \):

\[ T(w) = \max \{ u(w - m_t) + \delta V(m_t) \} \]

with first-order conditions:

\[ \{ U'(w_t - m_t) - \delta V'(m_t) \} m_t = 0; \; m_t \geq 0; \; U'(w_t - m_t) \geq \delta V'(m_t) \]
3.3 Infinite Horizon Problem

Extending the time horizon extends the stochastic programming problem. Consumption today has to be weighted against the future period's maximum, expected utility from money holding. With $V(m_t)$ being strictly concave, strictly increasing and differentiable, the problem can be solved applying the method of Schechtman/Escudero (1977).

With time horizon $T$, the first-order conditions give a sequence of Langrange-multiplier $\lambda_{tT}(z_1, \ldots, z_t)$ of the budget constraints for periods $t=1, \ldots, T$. $\lambda_{tT}$ is the marginal utility of money in period $t$, given a time horizon $T$ and the information about previous endowment realizations $z_1, \ldots, z_t$ (confer Bewley (1983)).

The Lagrange-multiplier satisfy:

$$
\lambda_{tT}(z_1, \ldots, z_t) \geq \delta \mathbb{E} \lambda_{t+1, T}(z_1, \ldots, z_t)
$$

= if $m_{tT} > 0$

$$
\lambda_{tT}(z_1, \ldots, z_t) \geq u'(c_{tT}(z_1, \ldots, z_t))
$$

= if $c_{tT} > 0$

$m_{tT}(c_{tT})$ is the utility maximizing money holding (resp. consumption) in period $t$, given $z_1, \ldots, z_t$, if the time horizon is $T$. Extending the time horizon, one gets:

$$
\lambda_{t,T+1} > \delta \mathbb{E} \lambda_{t,T} \quad \text{with} \quad \lambda_{t,T+1} \geq \lambda_{tT}
$$

Schechtman/Escudero (1977) show that with $\delta=1$, the household will accumulate
arbitrarily large real money holdings if the time horizon goes to infinity:

$$\lim_{t \to \infty} \frac{m}{t} = \infty$$

Since there is no cost in money holding in the sense of foregone time preference, households in the long run insure themselves simply by holding enough money. That means in this model: insurance, being costly, cannot be competitive in the long run.

But if $\delta < 1$ the loss of time preference makes money holding costly. Households now accumulate money to the point where the marginal self-insurance benefit of additional money holding components for the loss in foregone time preference. The maximum amount of money holding is bounded away from infinity. If $k$ is low enough and/or $\delta$ low enough, there will be both insurance and money holding even in the long run.

The wealth $w_t$ in each period is a random variable depending on the last period's wealth $w_{t-1}$ and the realization of $z_t$. As Schechtman/Escudero (1977) show, for $\delta < 1$ this stochastic wealth process converges to a unique distribution function $F_w(w)$. In the same way, the wealth process generates a distribution function $F_c$ of the consumption path, $F_m$ of money holding and $F_x$ of insurance contracts.

4. **Market Equilibrium**

I analyse an economy without social risk. There is a continuum of identical households, all with the same utility function $U$ and discount parameters $\delta$, each facing an endowment risk $z_t$ ($t=1,2,...$). $z_t$ is
identically and independently distributed, with the same distribution function \( F(z) \) for each household. The set of households is modelled as a measure space \((H, \mu, \nu)\) with \(\nu(H) = 1\). In each period \(t\), the distribution of endowment across households is equal to the distribution function \( F(z) \), with average endowment almost surely \( E(z) \). Being primarily interested in the long run behavior of this stationary economy, abstracting from initial conditions, I furthermore assume that the distribution of initial wealth \( w^h \) across households is equal to the limiting distribution function \( F_w(w) \) of the individual wealth process. This, effectively, is a special assumption about the initial distribution of money—being such as if generated by a long run stationary process. This assumption ensures that the distribution of wealth across households in each period is equal to the distribution function \( F_w(w) \). There is no fluctuation of the wealth-distribution on the aggregate level.

The same is true for the distribution of money holding \( F_m \), of consumption \( F_c \) and of insurance contracts \( F_x \) — all being generated by the distribution \( F_w \). In each period, market aggregates are stationary.

In the absence of money, insurance is the only way for stabilizing the consumption path. The wealth process in this case is generated only by the household's choice of insurance policy. All households being identical, they choose the same policy \((x,a,b)\), generating the random wealth

\[ \tilde{w}_t = I(z_t) \]

Without the possibility of intertemporal wealth transfer, all households consume their period's wealth \( c_t = \tilde{w}_t \). The aggregate consumption is

\[ c = \int c \, f_c \, dc \quad \text{with} \quad F_c = F_w. \]

Due to the zero-profit constraint on insurance companies, aggregate consumption equals aggregate endowment minus verification costs:
\[ \int_c^f c \, dc = \int_z^f z \, dz - k F(x) \]

With money holding and \( dx/dm < 0 \), households will demand money in order to save on information costs.

Individual excess demand for the consumption good is \( e^h_c(z_t) = c^h(z_t) - i^h(z_t) \).

Individual excess demand for money is: \( e^h_m(z_t) = m^h_t(z_t) - m^h_{t-1} \).

In equilibrium, the aggregate excess demand for money must be equal zero in each period:

\[ \int m_t \, f_m \, dm = \int m_{t-1} \, f_m \, dm \]

In periods of high endowment households are willing to trade the consumption good against money in order to sell money in periods of low endowment. Since on the aggregate level there is no risk and the initial wealth distribution is equal to \( F_w \), the distribution of aggregate money holding and of aggregate consumption across households is stationary in each period, whereas the distribution of an individual's money holding, consumption, and insurance follows the random distribution \( F_m' \), \( F_c' \) and \( F_x' \).

So, for each nominal money supply \( M \), there exists a stationary self-fulfilling monetary rational expectations equilibrium with a constant price \( p_t = p \), if the initial distribution of nominal money holding \( F_M \) across households is equal to \( F_{mp} \). (Of course, there also exists a non-monetary stationary rational expectations equilibrium with \( p_t = 0 \) and \( m^h_t = 0 \) for all \( t, h \)).

In the monetary equilibrium, the average verification costs per household are \( K = k \int_0^x F(x) f_0 \, dx < kF(x) \).

If \( \delta = 1 \), each household's long run money holding (serving as costless means of self-insurance) would be infinite, and so no monetary equilibrium exists in this case.
5. **THE OPTIMUM QUANTITY OF MONEY PROBLEM**

Having specified frictions within the economy which lead to a positive demand for money, it is natural to assume that the government is subject to the same informational frictions as all private agents. What actions are then available for the government in order to restore the optimal allocation by providing some optimal quantity of money?

First-best efficiency (the Arrow-Debreu allocation) would be easily realized if the government could tax households with high endowments and transfer these taxes to households with low endowments. Yet such a perfect insurance, of course, would require complete knowledge about the random realizations of all households and thus is not feasible if the government also has to pay the verification cost $k$.

Thus the only way to restore first-best efficiency seems to be to impose true lump-sum taxes in order to finance a real return on money. But is it really feasible to realize a first-best allocation this way?

Payment of interest on money holding at the rate $r$, together with a lump-sum tax $T$, changes the households budget constraint as follows:

\[ 2a') \quad c_{t+1} + m_{t+1} - m_t (1+r) - T + i(z_{t+1}) \quad \text{for all } t \]

with the first-order condition:

\[ 8') \quad [U'(w_t - m_t) - \delta (1+r) V'(m_t (1+r))]m_t = 0; \]

\[ m_t \geq 0; \quad U'(w_t - m_t) \geq \delta (1+r) V'(m_t (1+r)). \]
The effect of a change in the rate of interest \( r \) on \( m_t \) is indeterminate due to the income effect. Yet, since consumption in all periods is a normal good (utility being time-additive), \( m_t (1+r) \) definitely rises (if \( m_t > 0 \)). So according to Part 3 - an increase in \( r \) will reduce the insurance region, given \( \frac{dx_{t+1}}{dm_t} < 0 \). This seems to confirm Friedman's idea: if \( r \) equals the rate of time preference \( (r = 1/\delta - 1) \), there is no cost involved in holding money. Insurance, being costly, will be given up in the long run.

But if a lump-sum tax is imposed independent of the actual endowment realization, the household must adjust both money holding and insurance in order to avoid bankruptcy (e.g. his wealth being smaller than the tax payment in some period). There will arise an autonomous demand for money and/or insurance which enables the household to pay the taxes even during a long period of low realizations.

The household must ensure that

\[
m_{t-1} (1+r) + x_t - b_t \geq T \text{ for all } t.
\]

With \( r \) being small, this will result both in an increase of the insurance region (and of the verification costs) and in a demand for money in addition to the precautionary money holding. With \( r + 1/\delta - 1 \), however, the cost in terms of foregone time-preference for money holding as self-insurance goes to zero, and so the lump-sum tax will be completely financed out of extra money holding (insurance being not attractive). In that case, as in Hellwig (1982) and Bewley (1983), the tax creates an autonomous demand for money such that it can be financed out of its own return: \( m^a_t \cdot r = T \). So the lump-sum tax will finance exactly the return \( m^a_t \) on this minimum level of money holding, but does not suffice to pay the return on precautionary money which it was designed for.

So the Friedman-rule of equalizing the return on money to the rate of time preference is not feasible, given the informational frictions.
The government would require more information than is available to the other agents in the economy. Thus the theory of the optimum quantity of money turns out to be valid only if the government has abilities which would make the institution of money unnecessary.

This result contradicts the paper of Benhabib/Bull (1983). They model an ad-hoc transactions technology which gives money a role "in reducing the costs of transactions more efficiently than any alternative means, for example the establishment of a centralized, economy-wide clearing house." (p. 102).

In their model, lump-sum taxes financing a real return on money equal to the time preference, can restore first-best efficiency. Yet the authors give no reason why imposing such taxes is feasible whereas other central agencies are assumed to be inefficient. By contrast, the results of this paper suggest that, if transaction costs were modelled in a more satisfactory way — i.e. by working out the complete economic structure — the government would be subject to restrictions similar to the constraints the transaction technology imposes on the market system.

The verification costs modelled here may be interpreted as a formulation of some aspects of transactions technologies. In that view, Bewley and Benhabib/Bull do get contradictory results, not because they treat distinct motives for money holding (precautionary versus transactions demand), but because the latter authors do not constrain government activities in an appropriate way.

As shown, the Friedman rule (payment of interest equal to the rate of time preference) cannot be realized in the economy modelled in this paper. Only to the extent that true lump-sum taxes are feasible, the government could improve upon the "laissez-faire" monetary equilibrium, paying a (second best)
welfare maximizing rate of interest strictly lower than the rate of time preference. This can be seen most obviously if $k$ is so large that one-period insurance contracts are never attractive ($m = 0$). In this case, if the minimum endowment is equal to zero, taxation will always lead to an autonomous demand for money $m^*$ so as to be able to finance the tax out of the interest rate income. Thus, there is no way to improve upon a zero-rate of interest equilibrium. If, however, the minimum endowment is strictly positive ($g > 0$), no autonomous demand for money will arise out of taxation, as long as the tax does not exceed this minimum endowment ($T \leq g$). (So $g$ gives an upper bound for the maximum feasible rate of interest $r < 1/\delta - 1$.) In this case the government has the (limited) ability of imposing a true lump-sum tax, not available to insurance companies with one-period contracts. This seeming advantage of the power of taxation, however, does not take into account the possibility of long-period insurance contracts, which will be analyzed in the next section.

6. **LONG-TERM CONTRACTS AS A SUBSTITUTE FOR MONEY**

In the preceding paragraphs, money has been modelled as an institution that saves on information costs—costs which make it impossible to establish a complete-market system. By accepting money in exchange for the good, there is no need to spend verification costs. In contrast, a contract promising to pay some amount of goods in the future, runs the risk of bankruptcy of the trading partner. In that case, again verification of the individual state would require to incur the cost $k$. But the theory of incentives shows that there is another way to overcome the friction of asymmetric information, e.g. long-term contracts. There exists now a vast literature analysing long-term
relationships (as super games) - cf. Rubinstein/Yaari (1983) and Townsend (1982). The idea is that not telling the truth today will be punished in later periods. An insurance agency conditioning future payoffs on the (observable) past reports gives an incentive to honest behavior - in contrast to one-period contracts.

If households do not discount the future, infinitely-long-term contracts can, in the long run, ensure a first-best allocation with no need for verification. When the household's average report about his past realizations is below the expected value of the endowment he will be punished by getting lower compensations as long as his average reports are below some limit.3

This strategy gives an incentive to tell the truth and so a first-best allocation can be reached in the long run. Of course, that no longer works as time preference increases (as \( \delta \) goes to 0 there is no longer an incentive to refrain from one-shot gains). Exactly as with money, long-term contracts will establish a first-best solution only if there is no positive time preference. Townsend (1982) shows in a simple model that long-term contracts with finite horizon will always do better than one-period contracts. Yet in his paper he overlooks an interesting restriction which reveals a further symmetry between money and long-term contracts: Incentive-compatible contracts cannot be contingent on the realization of the last period (there is no way to punish a false statement about the last period's realization). To punish false reports about former periods the insurance agency has to be able to charge the household a positive amount in the last period in some events. This charge, however, must not exceed the minimum realization of endowment (being zero in this model).
So with a finite contract period there is no way to sanction wrong statements about former periods in the last one. That also applies to the period before the last one and so on. Thus, in this model, there is no gain out of finite-horizon long-term contracts when each period there is a positive probability of a zero endowment.

Again, one has an exact parallel to the well-known terminal value problem of money in models with finite time horizon. This at first surprising result underlines the formal equivalence of institutions on this abstract level. Interpreting long-term contracts as a cooperative supergame, there exist multiple equilibria of this supergame in the same way as there exist multiple monetary equilibria (the non-monetary, autarchic equilibrium corresponding to the non-cooperative Cournot-Nash solution). The set of feasible punishment strategies depends on the minimum endowment. If it is zero, the severest possible punishment would be to give no insurance payments for some period (the threat of reverting to the Cournot-Nash solution). With positive minimum endowment $z$, more severe punishments (taking away $z$) are feasible. By adopting an optimal severe punishment strategy (a "stick and carrot" strategy as discussed in Abreu (1985)), the insurance company can support a credible second-best efficient allocation. Thus, a government using its power of taxation to pay a welfare maximizing rate of interest could not achieve an allocation superior to long-term contracts.

The formal equivalence between money and long-term contracts should, in my view, not be seen as an argument that the precautionary motive for money holding is not a sufficient basis for monetary theory. On the contrary, in my interpretation it convincingly illustrates the informational advantage of
money as compared to complicated written contracts: if record keeping of all
the messages sent to the insurance company is costly, money can serve as a
means of communication about an individual's endowment history (proof of
liquidity) in a similar spirit as in Gale (1980).

7. **CONCLUSION**

To analyse the quantity theory problem, one must model explicit
frictions which prevent the economy from establishing complete markets and so
give rise to a demand for money. Consequently, in the model presented here
money serves as a substitute for Arrow-Debreu contracts which are not
enforceable because of asymmetric information. For contingent insurance
contracts to be incentive-compatible, verification costs have to be incurred
due to informational frictions. Holding precautionary money saves on these
verification costs.

The frictions introduced also allow a precise description of feasible
government interventions. As it is shown, a government wishing to implement
Friedman's rule (of paying a real return on money equal to the rate of time
preference) would need more information than is available to private agents.
The results indicate that the quantity theory can be confirmed only within
models which do not constrain government activities in a way equivalent to the
constraints needed to ensure a positive price of money. Furthermore, a formal
equivalence between money and long-term contracts as two institutions
overcoming the friction is illustrated.

The idea outlined in this paper also seems to be a promising foundation
to analyse the choice between assets with different returns. Whereas most
approaches impose an ad-hoc transaction technology to ensure that money is
held in the presence of interest-bearing assets, asymmetric information about the quality of (named) assets may be modelled in a similar way as in this paper to give money, being a non-interest bearing, but anonymous asset, an advantage over other assets (confer the approach used in King/Plosser (1986)).
FOOTNOTES

1 Clower (1968) already made the point that statements about an optimal growth rate of money should be based on a model that gives an explicit role for money. His scepticism will be confirmed here by showing that indeed in a rigorous model with positive demand for money the quantity theory does not work.

2 The same argument applies to cash-in-advance models as long as the economic frictions underlying the finance constraint are not explicitly modelled. For a discussion of the optimum quantity theory in such models see Kohn (1984).

Appendix 1 Monotone Decreasing Insurance Region

Total differentiation of equations 4.1 to 4.3 gives:

\[(A.1) \quad [1-F(x)]da + \int_{x}^{z} u''(m+z-b)f(z)dz/u - [(1-F(x)]db - (u-u_x)/u f(x)dx\]

\[= \int_{x}^{z} u''(m+z-b)f(z)dz/u - [(1-F(x)]dm\]

\[(A.2) \quad (a+k-x)f(x)da + \int_{a}^{x} (u-u_x)f(x)/u - (a+k-x)f(x)db - (u-u_x)/u f(x)dx\]

\[= \int_{a}^{x} (u-u_x)f(x)/u - (a+k-x)f(x)dm\]

\[(A.3) \quad F(x)da - db - (a+k-x)f(x)dx = 0\]

By substituting (4.1 to 4.3) define:

\[\alpha = 1-F(x) = \int_{x}^{z} u''(m+z-b)f(z)dz/u > 0\]

\[\beta = (a+k-x)f(x) = (u_a - u_x)/u_a > 0\]

and define:

\[\gamma = \int_{x}^{z} u''(m+z-b)f(z)dz/u > 0\]

\[\delta = (u_a - u_x)f(x)/u_a > 0\]

A.1, A.2, A.3 can be solved as:

\[
\begin{pmatrix}
\alpha & \gamma - \alpha & -\delta \\
\beta & \delta - \beta & -\delta \\
F(x) & -1 & \beta
\end{pmatrix}
\begin{pmatrix}
da \\
db \\
dx
\end{pmatrix}
= \begin{pmatrix}
\gamma - \alpha \\
\delta - \beta \\
0
\end{pmatrix}
dm
\[
\frac{dx}{dm} = \Delta_x / \Delta \\
\text{with } \Delta_x = \alpha \delta - \beta \gamma > 0 \text{ if} \\
\left| \begin{array}{c}
\int_{x}^{u} (u - u_x f(x) dx \\
\int_{x}^{u} u^{\prime}(m + z - b) f(z) dz
\end{array} \right| > \left| \begin{array}{c}
\int_{x}^{u} (m + z - b) f(z) dz \\
\int_{x}^{u} u^{\prime}(m + z - b) f(z) dz
\end{array} \right|
\]

\[\Delta = 2 \alpha \beta \delta - \alpha^2 \delta - \beta^2 \gamma - (\gamma - \delta) s F(x) = - (\alpha - \beta)^2 \delta - \beta^2 (\gamma - \delta) s F(x) < 0\]

\[\text{if } \alpha - \beta > 0 \text{ or } \int_{x}^{u} u^{\prime}(m + z - b) f(z) dz > u - u_x
\]

\[\text{and } \gamma - \delta > 0 \text{ or } \int_{x}^{u} u^{\prime}(m + z - b) f(z) dz / u > (u - u_x f(x) / u
\]

So condition (5) guarantees a decreasing insurance region (\(dx/dm < 0\)), if there is an interior solution, that is, as long as the household buys insurance.

If \(u = -e^{-c}\), \(\Delta_x = 0\) (constant absolute risk aversion).

With a uniform distribution (\(F(z) = z; 0 \leq z \leq 1\)), (5) simplifies to:

\[\alpha - \beta = u_x - u_x / u_x
\]

\[\gamma - \delta = u_x - u_x / u_x
\]

\[\Delta_x > 0 \quad \text{and} \Delta_x > 0 \text{ if} \frac{u - u_x}{u_x} > \frac{u - u_x}{u_x}
\]

which is true if \(u\) has decreasing absolute risk aversion.
Appendix 2  \( U = c^{\frac{1}{2}} \); \( F(z) = z \) \((0 \leq z \leq 1)\)

The first-order conditions 4.1 to 4.3 give:

\[
\frac{1}{2} (m+1-b) - \frac{1}{2} (m+a-b) = \frac{1}{2} (m+a-b)^{1-a-k}
\]

\[
\frac{1}{2} (m+a-b) - \frac{1}{2} (m+x-b) = \frac{1}{2} (m+a-b)^{1-a-k}
\]

\[
(a+k-\frac{1}{2}x)x = b
\]

with \( x \leq a \leq 1 \).

Substitute \( m+1-b = B; m+a-b = A; m+x-b = C \) and get

\[
\frac{1}{2} A - B = \frac{1}{2} (B-A-k)
\]

\[
\frac{1}{2} A - C = \frac{1}{2} (C-A-k)
\]

or \( B - A = k \)

\[
\frac{1}{2} A - C = k \quad \text{and thus} \quad B + C = 2A
\]

Therefore: \( 1-a-k = a+k-x \) or \( 1+x = 2(a+k) \).

Substitution in the budget constraint gives:

\[
a-b = \frac{1}{2} - k
\]

\[
x = 2b
\]

Thus

\[
(m+1-b)^{1/2} + (m+b)^{1/2} = 2(m + \frac{1}{2} - k)
\]

which can be solved as:
\[ x = 2b = 1 - 4\sqrt{k} \left( m + \frac{1}{2} - k \right)^{\frac{1}{2}} \]

This gives the indirect utility function:

\[
V(m) = \begin{cases} 
\left( m + 0.5 - k \right)^{\frac{1}{2}} x + \frac{2}{3} \left( m + 1 - 0.5x \right)^{1.5} - \left( m + 0.5x \right)^{1.5} & \text{if } m \leq m - \\
\int_0^1 (m + z)^{\frac{1}{2}} dz = \frac{2}{3} [(m + 1)^{1.5} - m^{1.5}] & \text{if } m \geq m - 
\end{cases}
\]

Since \( \lim_{m \to m -} V(m) = \lim_{m \to m -} V(m) \), \( V(m) \) is continuous for all \( m \).

The first derivative of \( V(m) \) is:

\[
V'(m) = \frac{1}{2} (m + 0.5 - k)^{-\frac{1}{2}} x +\left( m + 1 - 0.5x \right)^{\frac{1}{2}} - \left( m + 0.5x \right)^{\frac{1}{2}} \text{ for } m \leq m -
\]

(because \( U(m + 0.5 - k) \frac{dx}{dm} - \frac{1}{2} [U(m + 1 - 0.5x) + U(m + 0.5x)] \frac{dx}{dm} = 0 \))

\[
V'(m) = \frac{1}{2} (m + 1)^{\frac{1}{2}} - m \text{ for } m \geq m -.
\]

Since \( \lim_{m \to m -} V'(m) = \lim_{m \to m -} V'(m) \), \( V'(m) \) is continuous, thus \( V(m) \)

is differentiable for all \( m \).

If \( m < m - \), \( V''(m) = u''(m+0.5-k)+u'(m+0.5-k)dx/dm < 0 \)

since \( dx/dm = [u'(m+1-0.5x)-u'(m+0.5x)]/[u'(m+1-0.5x)+u'(m+0.5x)]. \)

Thus, \( V(m) \) is well-behaved even though the optimisation problem is non-convex.
LITERATURE


Hellwig, M. (1982), Precautionary Money Holding and the Payment of Interest on Money, Discussion Paper, University of Bonn.


