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Abstract

Price effects of an output tax in Sector 1 and an equivalent production subsidy in the second industry are analyzed in a general equilibrium model. Each commodity satisfies final demand and uses two primary inputs and the other good as an intermediate product. Production coefficients are variable and demand is elastic. In a Leontief model with fixed coefficients, Metzler showed that price of the taxed good will rise and price of the subsidized good will fall. This result generally holds under less restrictive conditions also although it is sometimes reversed. How the topic relates to the tax-incidence literature is also discussed.

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1. Introduction

The aim of this paper is to determine how output prices respond if a tax is levied on the production of one commodity and an equivalent subsidy is allowed to another when the two goods are used as final products as well as intermediate inputs in the production of each other. Behavior of commodity prices in such a case is generally difficult to predict because it depends on a number of factors that often conflict with each other. First, there is the primary effect of the tax and the subsidy which is at least partially offset by the role of these goods as intermediate products. Then there are attempts by consumers to substitute one good for the other in consumption and by producers to vary the proportions in which intermediate products are combined with primary factors in production. In a general equilibrium setting, such attempts will generate additional forces that will affect commodity prices directly and indirectly through variations in factor prices.

The starting point of this paper is a study by Metzler (1951) which proved that, in a fixed coefficients model with inelastic demand, holding factor prices and commodity-composition of output constant, price of the taxed commodity will rise and that of the subsidized good will fall. Allen (1972) considered the role of a budgetary surplus and deficit in this model, and Atsumi (1981) relaxed some of the Metzler restrictions and found that this result held even if input-output coefficients were flexible and output-mix was allowed to vary. However, Atsumi's framework is still rather restrictive in many respects. He assumes that value-added per unit of output is constant, which is only slightly better than Metzler's assumption of fixed production.
coefficients. There is no explicit role for primary factors, so that possibilities of substitution between such factors and intermediate goods have to be ignored.\footnote{1} Moreover, Atsumi does not show how final demand will change or how production coefficients will alter. His results, therefore, cannot be interpreted in terms of elasticities of demand ($\varepsilon$) and substitution ($\sigma$'s), or capital-labor ratios, which might be empirically estimated.

How taxes affect output prices is really one aspect of the incidence of such taxes - "forward shifting" in the jargon of tax-incidence studies - and, like other issues that arise in analyzing the burden of a tax, it is useful to consider this problem in a general equilibrium model with as few restrictions as possible. Besides providing an alternative approach to the problem, the analysis here leads to several cases in which the Metzler conclusion needs to be qualified and even reversed; for instance, if only the assumption about inelastic demand is relaxed, a tax will lower the price of the taxed good if it is relatively capital intensive (Result 2). There are other situations in which the Metzler result holds under much weaker conditions.

Metzler assumed $n$ commodities. Here only two goods are allowed, but in every other respect the framework employed in this paper is more general than the Leontief model used by Metzler: two primary factors, labor and capital, are explicitly introduced into the production process, all input-output ratios are variable, and a demand function is specified. Models of this sort have been developed for several taxes in Bhatia (1982a, 1982b) and they show that when taxes of the type Metzler considered are levied, factor prices generally change, depending on capital-labor ratios in the two industries and various elasticities of substitution and demand in the economy. None of these could
be considered in the Metzler model because of its assumptions of fixed
coefficients and unchanged factor prices. As the analysis later will show,
most of the new results here emerge when these assumptions are relaxed.

In the next section is set out the structure of the model which is
solved in Section 3. The question of taxes, subsidies and commodity prices is
taken up in Sections 4 and 5, and the main conclusions are summarized in
Section 6.

2. The Model

It is assumed that there are two goods \( X_1 \) and \( X_2 \) which are used both as
intermediate and final products. Labor (\( L \)) and capital (\( K \)) are the two
primary factors with fixed endowments. Each good is produced with the help of
capital, labor, and the other commodity. Total output consists of \( x_i \), which

goes toward meeting final demand, and \( X_{ij} \) (i, j = 1, 2; i ≠ j), which is used as an
intermediate good. Constant returns to scale, full employment, perfect
mobility of factors between sectors, perfect competition in all markets, and
complete flexibility of all prices and input-output coefficients are also
assumed. An inter-industry flows model of this type was set up and solved in
Bhatia (1982a) for a value-added tax, so there is no need to specify all the
equations de novo. The structure of production is summarized in equations (1)
and (2) which also ensure full employment of primary factors, and the final
demand function is given by (3).

\[
\lambda^{*}_{L1} x_1^{*} + \lambda^{*}_{L2} x_2^{*} = L^{*} - (\lambda^{*}_{L1} L_1^{*} + \lambda^{*}_{L2} L_2^{*}) \tag{1}
\]

\[
\lambda^{*}_{K1} x_1^{*} + \lambda^{*}_{K2} x_2^{*} = K^{*} - (\lambda^{*}_{K1} K_1^{*} + \lambda^{*}_{K2} K_2^{*}) \tag{2}
\]
\[ x_1^* = \epsilon (p_1^* - p_2^*) \] 

(3)

where asterisks denote proportional changes, \( \lambda_{ij} \) is the proportion of \( i \)th primary factor used directly or indirectly (via the intermediate good) in producing one unit of the \( j \)th final good, \( R \)'s represent gross input-output ratios based on total usage of each factor of production, and \( \epsilon \) is the income compensated elasticity of demand. 2

**The Price Equations**

Since the focus of this paper is on output prices, let us consider the price equations in some detail. There is a tax, \( T \), per unit of output on Commodity 1 and an equivalent subsidy on Commodity 2. Under perfect competition, output prices reflect unit costs. Accordingly,

\[ a_{L1} w + a_{k1} r + a_{21} p_2 + T = p_1. \] 

(4)

\[ a_{L2} w + a_{k2} r + a_{12} p_1 - T = p_2 \] 

(5)

where \( w \) is the wage rate, \( r \) represents rental per unit of capital, and \( a_{ij} \) is the quantity of input \( i \) used directly in producing a unit of the \( j \)th good. By substituting for \( p_2 \) in (4) and for \( p_1 \) in (5), these equations can also be written in terms of total factor requirements \( R_{ij} \)'s:

\[ R_{L1} w + R_{k1} r + (1 - a_{21}) T / (1 - a_{12} a_{21}) = p_1 \] 

(6)

\[ R_{L2} w + R_{k2} r - (1 - a_{12}) T / (1 - a_{12} a_{21}) = p_2 \] 

(7)

In these equations, \( R_{ij} \) is the amount of the \( i \)th primary factor required directly or indirectly for producing one unit of the \( j \)th final good. For example, \( R_{L1} = (a_{L1} + a_{L2} \cdot a_{21}) / (1 - a_{12} \cdot a_{21}) \) where \( a_{L1} \) denotes labor used directly in producing a unit of Commodity 1 and \( a_{L2} \cdot a_{21} \) is labor needed for producing \( x_{21} \) which is an intermediate good in the first industry. Since \( a_{ij} \)'s are positive, \( R_{ij} \)'s will also be positive if we assume that \( a_{12} a_{21} < 1 \).
From these price equations, assuming that firms minimize unit costs, we get:

\[ \rho_{L1}^* w^* + \rho_{K1}^* r^* + \rho_{21}^* p_2^* + T^* = p_1^* \] (8)
\[ \rho_{L2}^* w^* + \rho_{K2}^* r^* + \rho_{12}^* p_1 - T^* = p_2^* \] (9).

Here \( \rho_{ij}^* \) is the direct share of the ith input in industry j, derived from \( a_{ij}^* \)'s. For example, \( \rho_{K1}^* = a_{K1}^* r / p_1^* \). For each industry, there are three \( \rho \)'s which will add to unity.

In equations (8) and (9) the primary effect of the taxes and subsidy is indicated by \( T^* \), and the secondary effect is captured by the terms involving \( \rho_{21}^* \) and \( \rho_{12}^* \). These two effects are what Metzler considered, and that obviously will be the whole story if factor prices somehow did not change \( (w^* = r^* = 0) \). The raison d'être for the present paper is that when commodity taxes and subsidies are introduced, factor prices do change. We shall see in the next section that the wage-rental ratio does not remain constant even when severe restrictions are imposed on various elasticities of substitution and demand. Generally speaking, thus, there will be an additional "factor price" effect which could reinforce or offset the two effects considered by Metzler. In some instances Metzler's result about output prices will hold a fortiori, whereas in others it would have to be qualified.

**Commodity Taxes and Factor Prices**

When the tax-cum-subsidy is introduced, all prices change in response to a series of often complex interactions on both the supply and demand sides of the economy. Producers will try to contract output in the taxed industry and expand it in the other (an output effect), attempts will be made throughout the economy to increase the use of the subsidized intermediate good by
substituting it for both labor and capital wherever possible (a factor-substitution effect), and consumers will tend to substitute the subsidized good for the taxed commodity (a demand response). This general equilibrium process can be circumscribed by placing restrictions on the behavior of various economic agents. For example, if consumer demand is completely inelastic, there will be no demand response. Output in each industry could still vary because intermediate usage of each good would respond to changes in relative input prices such as \( w/p_1 \) and \( r/p_2 \). These could alter even if the wage-rental ratio does not. Of course, if all input-output coefficients are fixed and elasticity of demand is zero, primary-factor prices cannot possibly change. In the absence of such restrictions, movements in factor prices will depend on capital-labor ratios in the two industries and the many elasticities of substitution and demand. These, in turn, will also affect output prices. One could justifiably concentrate on the two effects discussed by Metzler only in the rare case in which factor prices did not change. Otherwise, the factor-price effect must also be taken into account.

Before anything more can be said about output prices, therefore, the model needs to be solved for movements in \( w \) and \( r \). Changes in relative factor rewards are given by \( (w^* - r^*) \). Once solutions for \( R^* \)'s are obtained, there remain six unknowns \( x_1^*, x_2^*, w^*, r^*, p_1^* \) and \( p_2^* \), and five equations \( (1), (2), (3), (8), \) and \( (9) \). Without loss of generality, the wage rate, \( w \), is chosen as the numéraire in terms of which all prices are expressed which implies that \( w^* \) is zero in equations \( (8) \) and \( (9) \). Changes in the wage-rental ratio, therefore, are indicated by \( r^* \) rather than \( (w^* - r^*) \). Also, because of the assumption of fixed factor endowments, \( L^* \) and \( K^* \) can be set equal to zero.
which will simplify equations (1) and (2) and the manipulations needed to solve the model.

3. **The Solution for r**

The crucial step in solving the model for \( r^* \) is to equate proportionate change in the supply of \( x_1 \) to the corresponding change in its demand after duly incorporating the full employment conditions and the price equations. First, equations (8) and (9) can be simplified by substituting for \( p_1^* \) and \( p_2^* \) to obtain:

\[
\begin{align*}
\theta_{L1} w^* + \theta_{K1} r^* + (1 - \rho_{21}) \gamma T^* &= p_1^* \quad (10) \\
\theta_{L2} w^* + \theta_{K2} r^* - (1 - \rho_{12}) \gamma T^* &= p_2^* \quad (11)
\end{align*}
\]

where \( \rho \)'s represent the share of the intermediate good in each industry (e.g., \( \rho_{12} = a_{12} \cdot p_1/p_2 \)), \( \theta_{ij} \) denotes the total, i.e., direct plus indirect distributive share of the \( i \)th primary factor in industry \( j \) (for instance, \( \theta_{K2} = R_{K2} r/p_2 \)), and \( \gamma = 1/(1 - \rho_{12} \rho_{21}) \).

From the demand equation (3), after substituting (10) and (11) we get:

\[
x_1^* = \varepsilon (\theta_{K1} - \theta_{K2}) r^* + \varepsilon (2 - \rho_{12} - \rho_{21}) \gamma T^* \quad (12)
\]

On the supply side, applying Cramer's rule to equations (1) and (2) yields:

\[
x_1^* = (\lambda_{L2} \lambda_{K1} R^*_{K1} - \lambda_{K2} R^*_{L1}) + \lambda_{L1} \lambda_{K2} R^*_{K2} - \lambda_{K2} R^*_{L2})/(\lambda_{L1} \lambda_{K2} - \lambda_{K1} \lambda_{L2}) \quad (13)
\]

Or,

\[
x_1^* = [R^*_{L2} - R^*_{K2} + (L_1/L_2) R^*_{L1} - (K_1/K_2) R^*_{K1}] / A \quad (14)
\]

where \( A = (K_1/K_2) - (L_1/L_2) \) reflects the relative factor intensity of the two industries in terms of the total usage of the primary factors of production. If \( x_1 \) is relatively capital (labor) intensive, \( A \) will be positive (negative). For further simplification, solutions for \( r^* \)'s are needed. These, derived in the Appendix, are as follows:

\[
\begin{align*}
R^*_{L1} &= (\alpha + \rho_{21} \beta) r^*/\Omega_L + [\rho_{21} (1 - \rho_{12}) (\rho_{K1} \theta_{L2} \sigma_{K2} - \rho_{L1} \theta_{K2} \sigma_{L2})
\end{align*}
\]
$+$ $\rho_{12} \rho_{21} (1-\rho_{21}) (\rho_{L2} \theta_{L1} \sigma_{L1}^2 - \rho_{K2} \theta_{K1} \sigma_{K1}^2) \Gamma^*/\Omega_L$

$R^*_{K1} = -(\alpha+\rho_{21} \beta) r^*/\Omega_K + (\rho_{21} (1-\rho_{12}) (\rho_{L1} \theta_{K2} \sigma_{L2}^2 - \rho_{K1} \theta_{L2} \sigma_{L1}^2)) \Gamma^*/\Omega_K$

$+$ $\rho_{12} \rho_{21} (1-\rho_{21}) (\rho_{K2} \theta_{L1} \sigma_{K1}^2 - \rho_{L2} \theta_{K1} \sigma_{L1}^2) \Gamma^*/\Omega_K$

$R^*_{L2} = (\beta+\rho_{12} \alpha) r^*/\eta_L + (\rho_{12} (1-\rho_{21}) (\rho_{L2} \theta_{K1} \sigma_{L1}^2 - \rho_{K2} \theta_{L1} \sigma_{K1}^2)) \Gamma^*/\eta_L$

$+$ $\rho_{12} \rho_{21} (1-\rho_{12}) (\rho_{K1} \theta_{L2} \sigma_{K2}^2 - \rho_{L1} \theta_{K2} \sigma_{L2}^2) \Gamma^*/\eta_L$

$R^*_{K2} = -(\beta+\rho_{21} \alpha) r^*/\eta_K + (\rho_{21} (1-\rho_{12}) (\rho_{L1} \theta_{K2} \sigma_{L2}^2 - \rho_{K1} \theta_{L2} \sigma_{K2}^2)) \Gamma^*/\eta_K$

$+$ $\rho_{12} (1-\rho_{21}) (\rho_{K2} \theta_{L1} \sigma_{K1}^2 - \rho_{L2} \theta_{K1} \sigma_{L1}^2) \Gamma^*/\eta_K$

where

$\alpha = \rho_{L1} \rho_{21} \theta_{K2} \sigma_{L2}^2 + \rho_{K1} \rho_{21} \theta_{L2} \sigma_{K2}^2 + \rho_{L1} \rho_{K1} \sigma_{L1}^2$

$\beta = \rho_{L2} \rho_{12} \theta_{K1} \sigma_{L1}^2 + \rho_{K2} \rho_{12} \theta_{L1} \sigma_{K1}^2 + \rho_{L2} \rho_{K2} \sigma_{L2}^2$

$\Omega_L = \rho_{L1} + \rho_{L2} + \rho_{21} > 0$, $\Omega_K = \rho_{K1} + \rho_{K2} \rho_{21} > 0$,

$\eta_L = \rho_{L2} + \rho_{L1} \rho_{12} > 0$, and $\eta_K = \rho_{K2} + \rho_{K1} \rho_{12} > 0$.

In all the above terms $\sigma_{ik}^j$ denotes the partial elasticity of substitution between factors $i$ and $k$ in the $j$th industry, as defined by Allen (1969). These $\sigma$'s can be positive or negative in general although Allen shows that for a linear homogenous production function with three factors, either all $\sigma_{ik}$'s ($i \neq k$) are positive (which makes $\alpha$ and $\beta > 0$) or at most only one of
the partial elasticities of substitution can be negative. However, even if some \( \sigma \)'s are negative, for the type of production functions being used here, \( \alpha \) and \( \beta \) will be positive. In the following sections we shall assume that, unless stated otherwise, all \( \sigma \)'s are positive, i.e., capital, labor and the intermediate goods are gross substitutes.

The economic meaning of some expressions derived above deserves elaboration. Notice first that \( \alpha \) is the weighted sum of all the elasticities of substitution in \( X_1 \) and \( \beta \) denotes the same for \( X_2 \). The \( R^* \)'s represent total change in the use of each factor input, as a result of the tax and tax-induced changes in relative factor prices. Every \( R^* \) has two types of terms---one involving \( \alpha \), \( \beta \) and \( r^* \), and the other including \( T^* \). A change in the wage-rental ratio will alter all factor proportions whereas a revision of the tax rate, for given factor prices, will affect the use of intermediate goods only. This is the reason why all \( \sigma \)'s appear in \( r^* \) terms (via \( \alpha \) and \( \beta \)), whereas \( \sigma \)'s involving intermediate goods alone show up in \( T^* \) terms. If all production coefficients are fixed, it can be verified that \( \alpha \), \( \beta \) and all the \( R^* \)'s will be zero.

It is also worth noting that in this model factor-intensities in the two industries will have the same rank both in net and gross terms. In other words, if \( a_{K1}/a_{L1} > a_{K2}/a_{L2} \), \( R_{K1}/R_{L1} \) will also exceed \( R_{K2}/R_{L2} \). Moreover, the determinant of the matrix of production coefficients (\( |R| \)) will have the same sign as the determinant of the matrix of factor shares (\( |\theta| \)), where

\[
|\theta| = \begin{vmatrix} \theta_{L1} & \theta_{K1} \\ \theta_{L2} & \theta_{K2} \end{vmatrix} = \theta_{L1}\theta_{K2} - \theta_{L2}\theta_{K1},
\]

which implies, \textit{inter alia}, that if the first industry is relatively capital intensive (\( A > 0 \)), \( \theta_{K1} > \theta_{K2} \), and \( |\theta| \) will be negative. In general
$|\theta| < 0$ as $R_{K1}/R_{L1} > R_{K2}/R_{L2}$. These results will be helpful in the next section when effects of taxes on commodity prices are discussed.

Using these values of $\tilde{R}^*$'s, equation (14) can be set equal to (13) to solve for $r^*$:

$$\tilde{r}^* = N\gamma T^*/D$$

(15)

where

$$W = \delta_1(\rho_{12}^{-1})\rho_{21}(\rho_{K1}^{1}\theta_{L2}^{1}\sigma_{K2}^{2}-\rho_{L1}^{1}\theta_{K2}^{2}\sigma_{L2}^{1})+\delta_2(\rho_{21}^{-1})\rho_{12}(\rho_{L2}^{1}\theta_{K1}^{1}\sigma_{L1}^{2}-\rho_{K2}^{2}\theta_{L1}^{2}\sigma_{K1}^{1})$$

$$-A\epsilon(\rho_{12}^{1}+\rho_{21}^{1})$$, and

$$\delta_1 = (L/L_2)/\Omega_L + (K/K_2)/\Omega_K + \rho_{12}(\eta_{L} + \eta_{K})/\eta_{L}\eta_{K}$$

$$\delta_2 = \rho_{21}[(L/L_2)/\Omega_L + (K/K_2)/\Omega_K] + (\eta_{L} + \eta_{K})/\eta_{L}\eta_{K}$$

$$D = A(\theta_{K2}^{1} - \theta_{K1}^{1}) + (\beta + \rho_{12}\alpha)(\eta_{K} + \eta_{L})/\eta_{L}\eta_{K}$$

$$+ (\alpha + \rho_{21}\beta)(L/L_2)/\Omega_L + (K/K_2)/\Omega_K$$, and $\alpha$, $\beta$, $\gamma$, and $A$ are as defined above. It can be shown that $D$ is positive (because $\epsilon < 0$, $\alpha$ and $\beta > 0$, and $(\theta_{K2}^{1} - \theta_{K1}^{1})$ and $A$ have opposite signs). We know that since $\rho_{12}$ and $\rho_{21}$ are less than unity, $\gamma$ is positive, the sign of $r^*$ thus will be the same as the sign of $N$, and $N$ can be zero, positive, or negative, depending on the values of $\sigma$'s, $A$ and $\epsilon$ in equation (15).

Before the question of commodity prices is considered, notice that elasticities of substitution involving intermediate goods alone appear in $N$. The other two elasticities, $\sigma_{LK}^1$ and $\sigma_{LK}^2$, appear in the denominator of $r^*$ as components of $\alpha$ and $\beta$ which are positive, so they do not affect the sign of $r^*$. This is due to the fact that the tax and subsidy being considered here directly affect the intermediate usage of the two goods which is not affected by any substitution possibilities between labor and capital per se. If instead of the tax-cum-subsidy we consider a partial factor tax, say, a tax on labor used in $x_{1}^{1}$, $\sigma_{LK}^1$ will influence the sign of $r^*$ along with $\sigma$'s involving intermediate goods.
It is also worth noting that in the present case when $\sigma_{K2}^1$, $\sigma_{L2}^1$, $\sigma_{L1}^2$, $\sigma_{K1}^2$ and $\epsilon$ are zero, i.e., fixed proportions and inelastic demand as Metzler assumed, $N$ and $r^*$ will be zero. Of course, $r^*$ can be zero in other situations as well. For example, very large values of $\frac{1}{KL}$ and $\frac{2}{KL}$, which appear only in $D$ (via $\alpha$ and $\beta$ respectively) will also suffice: $r^* \to 0$ as $\frac{1}{KL}$ or $\frac{2}{KL} \to \infty$. Several results along these lines will emerge in the next section.

4. Changes in Commodity Prices

The expression for $r^*$ derived in (15) can now be substituted into equations (10) and (11), setting $w^* = 0$, to determine how $p_1$ and $p_2$ will be affected.

$$p_1^* = \frac{\theta_{KL} \gamma T^*}{N} + (1 - \rho_{21}) \gamma T^*$$

$$p_2^* = \frac{\theta_{KL} \gamma T^*}{N} - (1 - \rho_{12}) \gamma T^*$$

Since $N$ can have any sign, the tax and subsidy being discussed here will not have the same effect on output prices in all cases. However, several general results can still be derived.

1. If input-output coefficients involving intermediate goods are fixed, and demand for final goods is perfectly inelastic, price of the taxed commodity must rise and that of the subsidized good must fall.

This is a restatement of the result proved by Metzler. If all $\sigma$'s relating to intermediate goods and $\epsilon$ are set equal to zero, $N$ becomes zero, so $p_1^* = (1 - \rho_{21}) \gamma T^*$ and $p_2^* = -(1 - \rho_{12}) \gamma T^*$. Clearly the elasticity of the price of the taxed commodity with respect to the tax rate ($\frac{p_1^*}{T^*}$) is positive and corresponding elasticity for the subsidized good will be negative.
Secondary effects due to the presence of intermediate goods do not dominate the primary effects of the tax-cum-subsidy.

A sufficient condition for the Metzler result to hold in this model is that relative factor prices remain unchanged, and that can happen even if some of the restrictions imposed in deriving Result 1 are relaxed. For example, if demand is not inelastic, but σ's specified above are zero, r∗ will still be zero as long as the two industries have the same gross capital-labor ratio (A=0). Again, as noted above, if the elasticity of substitution between the two primary factors is infinity in either industry, r∗ will be zero.

2. If all production coefficients are fixed, a tax will lower the price of the taxed good if it is relatively capital intensive.

If all σ's are zero, r∗ = γT∗(ρ12 + ρ21 - 2)/(θK1 - θK2). The wage-rental ratio then will depend on relative factor shares θK1 and θK2 because γ is positive and ρ12 and ρ21, being shares of intermediate goods, lie between 0 and 1: r∗ > as θK1 < θK2. In particular, if the taxed industry is relatively capital intensive, an increase in the rate of tax will lead to a higher wage-rental ratio or a lower relative return to capital.

Substituting for r∗ into (16) we get:

\[ p_1^*/T^* = γ[\theta_{K1}(ρ_{12} - 1) + ρ_{21}(1 - 1)]/(θK1 - θK2) \]  

(18)

If the taxed industry is relatively capital intensive, (θK1 > θK2), p1*/T* will be negative because 0 < ρ_{12}, ρ_{21} < 1. Curiously, in this case, p2*/T* is also given by (18) so that there is no change in relative output prices. In this case p1* - p2* = 0; both p1 and p2 will rise (fall) if the taxed industry happens to be relatively labor (capital) intensive.

We know from earlier discussion that at unchanged factor prices, the price of the taxed good cannot fall. This result, therefore, provides an example in which the "factor-price" effect dominates the two effects which
lead to the Metzler result. It is perhaps better to treat it as an
illustration of the straightjacket into which the economy is put because of
the assumption of fixed production coefficients everywhere. Why should the
two prices move in lock step, though? The reason is that so long as the two
industries use inputs in fixed but different proportions, full employment can
be attained at only one set of outputs. Since demand depends on relative
commodity prices alone in this model, it is imperative that they should not
change to preclude any excess demand or supply for goods.

3. If intermediate goods are used in fixed proportions but capital and
labor can be substituted for each other, price of the taxed commodity
will rise.

If all \( \sigma \)'s except \( \sigma_{KL}^1 \) and \( \sigma_{KL}^2 \) are zero, \( r^* = -\gamma T^* A \varepsilon (\rho_{12} + \rho_{21} - 2)/D \). It has been established above that \( D \) is positive, \( \gamma > 0 \), and \( \varepsilon < 0 \). Since \( (\rho_{12} + \rho_{21} - 2) < 0 \), the sign of \( r^* \) will be the opposite of the sign of \( A \). If the taxed industry is relatively capital (labor) intensive, \( r^* \) will
be negative (positive), as in Result 2. With this expression for \( r^* \),

\[
P_1^* / T^* = \gamma [(1 - \rho_{21}) - \theta_{KL} A \varepsilon (\rho_{12} + \rho_{21} - 2)/D]
\]

(19)
which is definitely positive as long as \( A \leq 0 \). This can be explained by
noting that when \( A \leq 0 \), \( r^* \geq 0 \). We know that even when \( r^* = 0 \), the price of
the taxed good will rise (Result 1). When \( r^* > 0 \), i.e., capital becomes
relatively more expensive, the direct effect of the commodity tax is reinforced.

Turning to the subsidized good,

\[
P_2^* / T^* = -\gamma \theta_{KL} A \varepsilon (\rho_{12} + \rho_{21} - 2)/D + (1 - \rho_{12})
\]

(20)
which is negative so long as \( A \geq 0 \). The explanation is analogous to the tax
case: \( r^* \leq 0 \) which will reinforce the direct effect of the subsidy or simply reproduce Result 1 \((r^* = 0)\).

It is interesting to compare this result with Result 2. The only change in the underlying parameters is that now the two primary factors can be substituted for each other, but that makes all the difference. From (19) and (20) we can see that \( p_1^* \) and \( p_2^* \) won't have the same sign even when the two industries have the same gross capital-labor ratio \((A = 0)\) or demand is inelastic \((\varepsilon = 0)\). Moreover, if \( A > 0 \), \( p_1^*/T^* \) has an ambiguous sign, and the same is true of \( p_2^*/T^* \) when \( A < 0 \). That is so in spite of the fact that \( r^* \) has the same sign in both Results 2 and 3. The explanation for this ambiguity, therefore, must lie in the magnitude of \( r^* \).

If the expression for \( r^* \) in Result 2 is multiplied and divided by \(-A\), it becomes clear that it has the same numerator as in the present case, but the denominator, \( D \), differs considerably. There \( D \) is simply \( A\varepsilon (\theta_{K2} - \theta_{K1}) \) while here it will have two additional positive terms involving \( \sigma_{LK}^1 \) and \( \sigma_{LK}^2 \) which can be written down by referring to the definition of \( D \) earlier in the paper. It follows that, in the present case, movements in the relative price of capital will be less than in Result 2. In terms of the three effects discussed above, in Result 2, the primary effect of the tax was dominated by the combined force of the secondary effect of the subsidy and the factor substitution effect, whereas here, because of the smaller decline in \( r^* \) when \( A > 0 \), the sign of \( p_1^*/T^* \) cannot be determined \textit{a priori}. For analogous reasons, when \( A > 0 \), the sign of \( p_2^*/T^* \) will be ambiguous while it was positive in Result 2.
There are two important corollaries to this result, one about relative output prices, and the other about the role of demand.

Corollary 1. If intermediate goods are used in fixed proportions, but elasticity of substitution between the primary factors is not zero, the relative price of the taxed good must rise.

Subtracting (20) from (19) and rearranging we get:

\[
\frac{p_1^* - p_2^*}{T^*} = \gamma(2 - \rho_{12} - \rho_{21})[1 - A\varepsilon(\theta_{K2} - \theta_{K1})/D]
\]

(21)

Now, as discussed above, \(\gamma(2 - \rho_{12} - \rho_{21})\) is positive. The second term inside the brackets is negative because \(A\) and \((\theta_{K2} - \theta_{K1})\) have opposite signs, \(\varepsilon\) is < 0 and \(D\) is positive. To complete the proof of this corollary, all we need to show is that this term is less than unity in absolute value, and that can be easily done by recalling from Section 3 that \(D\) consists of three positive terms, the first being \(A\varepsilon(\theta_{K2} - \theta_{K1})\). The two additional positive terms then guarantee that \(D\) will be larger than \(A\varepsilon(\theta_{K2} - \theta_{K1})\). Notice that this outcome is reached irrespective of the capital-labor ratios in the two industries and demand elasticity. In Result 2, relative output prices could not change at all whereas here they must, unless some other restrictions are imposed. It is easy to verify that if \(\sigma_{LK}^1 = \sigma_{LK}^2 = 0\), \((p_1^* - p_2^*) = 0\).

Corollary 2. If production coefficients involving intermediate goods are fixed, substitution possibilities between the primary factors reinforce the role of demand.

Result 2 shows that if production coefficients are fixed, elasticity of demand will not affect the wage-rental ratio or relative output prices. For example, there is no \(\varepsilon\) in equation (18) or in the relevant expression for \(r^*\). Now \(\varepsilon\) appears in all the equations of price change as well as in the solution for \(r^*\). Therefore, the elasticity of demand directly affects the
wage-rental ratio as well as the two output prices. In the previous case, as noted there, because all production coefficients were fixed, full employment could be attained at only one set of outputs which were not affected by demand conditions. The wage-rental ratio did alter, but producers couldn't respond to it. In the present case, there will be substitution between labor and capital and a range of outputs will be consistent with full employment of factors. Output levels and commodity prices, therefore, will depend on both supply and demand considerations.

4. If \( \sigma_{L2}^1 = \sigma_{K2}^1 \) and \( \sigma_{L1}^2 = \sigma_{K1}^2 \), i.e., the elasticity of substitution between the intermediate good and the two primary factors is the same in each industry, a tax will raise the price of the taxed good if it is relatively labor intensive.

This implies that the primary factors are separable from the intermediate good in each industry. From equation (16) we know that \( p_1^*/T^* \) will be positive whenever \( NYT^*/D \) is positive. Under the assumption of separability \( N \) can be rewritten as:

\[
N = \delta_1 (\rho_{12} - 1) \rho_{21} \sigma_{K2}^1 (\rho_{K1} \theta_{L2} - \rho_{L1} \theta_{K2}) + \delta_2 (\rho_{21} - 1) \rho_{12}^2 \sigma_{L1}^2
\]

\[
(\rho_{L2} \theta_{L1} - \rho_{K2} \theta_{L1}) - A\epsilon(\rho_{12} + \rho_{21} - 2).
\]  

(22)

It follows that since \( D \) is positive, \( N \) and \( r^* \) will be positive if \( A \), \( (\rho_{K1} \theta_{L2} - \rho_{L1} \theta_{K2}) \) and \( (\rho_{L2} \theta_{K1} - \rho_{K2} \theta_{L1}) \) are negative. It can be verified, by using the definitions of \( \rho \)'s and \( \theta \)'s given earlier in the paper, that if \( (a_{K1}/a_{L1}) < (a_{K2}/a_{L2}) \), which implies that the taxed good is relatively labor intensive, all three of these terms will indeed be negative.
In this case, equation (17) tells us, the sign of \((p_2^*/T^*)\) will be ambiguous. The subsidized commodity now is relatively capital intensive \((A < 0)\). The subsidy tends to lower \(p_2\) but the factor-price effect and the secondary influence from the higher price of \(p_1\) work in the opposite direction, so the net change in the price of Commodity 2 becomes an empirical matter.

In the context of this result, it is easy to verify that when the subsidized industry is relatively labor intensive, tables are turned: \(p_2^*/T^*\) will be unambiguously negative whereas the corresponding elasticity for the first commodity will have an uncertain sign because \(r^* < 0\). It should also be apparent that these results will be valid regardless of the elasticity of demand for final products.

5. If the two industries have the same capital-labor ratio, effect of a tax-cum-subsidy on output prices depends primarily on the elasticities of substitution between the primary factors and the intermediate goods.

If \(A = 0\), which also implies that \((a_{K1}/a_{L1}) = (a_{K2}/a_{L2})\), it follows from the discussion of Result 4 that \(\rho_{K1}^L \theta_{L2}^K = \rho_{L1}^K \theta_{K2}^L\), and \(\rho_{L2}^L \theta_{K1}^L = \rho_{K2}^K \theta_{L1}^L\).

Accordingly,

\[
\begin{align*}
   r^* &= \gamma^T_{\rho^*} \left[ \delta_1 (1 - \rho_{12}^*) \rho_{21}^L \rho_{K1}^L (\theta_{L2}^K - \eta_{L2}^K) + \delta_2 (1 - \rho_{21}^*) \rho_{12}^L \rho_{L2}^L (\theta_{K1}^L - \eta_{K1}^L) \right] / D \tag{23}
\end{align*}
\]

The sign of \(r^*\) obviously depends on the various \(\sigma\)'s. Several results ensue when (23) is substituted into (16) and (17).

5.1. Elasticity of demand has no effect on output prices.

There is only one term involving \(\epsilon\), \(\epsilon \sigma_{K1} - \sigma_{K2}\), that appears in both \(N\) and \(D\). This term disappears when \(A = 0\). Also, as mentioned earlier, if the two commodities have the same capital-labor ratio, substitutions
between them on the demand side will not have any effect, which leads to the following additional results:

5.2 If \( \sigma_2^{L2} = \sigma_2^{K1} = 0 \), \( (p_2^*/T^*) < 0 \) but \( p_1^*/T^* \) is ambiguous.

5.3 If \( \sigma_2^{K2} = \sigma_2^{L1} = 0 \), \( (p_1^*/T^*) > 0 \) whereas \( p_2^*/T^* \) is uncertain.

5.4 If \( \sigma_2^{K2} = \sigma_2^{L2} = 0 \) and \( \sigma_2^{L2} = \sigma_2^{K1} \), \( (p_1^*/T^*) > 0 \) and \( (p_2^*/T^*) < 0 \).

All these can be explained by what happens to the wage-rental ratio. In (5.2), \( r^* < 0 \), which reinforces both the primary effect of the subsidy in the subsidized industry and its secondary effect on the taxed industry. In (5.3), \( r^* > 0 \). It adds to the primary effect of the tax and weakens the influence of the subsidized good as an intermediate product. In (5.4), \( r^* = 0 \), there is no factor-substitution effect and the Metzler result holds. Other results can also be derived and explained in a similar manner.

6. If elasticity of substitution between the two primary factors is infinity in either industry, price of the taxed good will rise and that of the subsidized commodity will fall in response to a tax-cum subsidy.

The terms \( \sigma_2^{KL} \) and \( \sigma_2^{KL} \) appear only in \( D \), as components of \( \alpha \) and \( \beta \). As either of these elasticities approaches infinity, \( D \) goes to infinity and \( r^* \) moves toward zero. Therefore \( (p_1^*/T^*) \to \gamma(1-\rho_{21}) \) and \( (p_2^*/T^*) \to -\gamma(1-\rho_{12}) \), which are positive and negative respectively.

7. If capital and the subsidized good are complements in a taxed industry, and labor and the taxed commodity are complements in the subsidized sector, the tax cum-subsidy will raise the price of the taxed good so long as the taxed industry is not relatively capital intensive.

In this case \( \sigma_2^{K2} \) and \( \sigma_2^{L1} \) are negative, which implies that \( r^* > 0 \) as long as \( A < 0 \) because, as discussed earlier, even when some factors are
complements (recall that only one elasticity of substitution in each industry can be negative), $\alpha$, $\beta$ and $D$ continue to be positive (cf. footnote 4). If demand is inelastic, the numerator and denominator of $r^*$ have two terms each which are unambiguously positive, therefore $r^* > 0$, and $p_1^*/T^*$ must also be positive, so the price of the taxed good will increase. Effect on $p_2$ under these conditions will be uncertain.

A parallel result can be derived if $\sigma_{L2}^1$ and $\sigma_{K1}^2$ are negative. Then, as long as $A \geq 0$ or $\epsilon = 0$, $r^* < 0$ and $p_2^*/T^*$ will be unequivocally negative but response of $p_1$ will be uncertain.

A Digression on Tax-incidence

It is well known from the work of Harberger and others that in models of the type used here, $r^*$ determines the incidence of a given tax. If $r^* = 0$, the wage-rental ratio does not change. Therefore, assuming full employment of labor and capital, relative factor shares do not alter and the two primary factors bear the burden of the tax in proportion to their initial contribution to national income. The tax falls more heavily on owners of capital if the relative price of capital falls ($r^* < 0$) whereas labor tends to lose if $r^* > 0$. It follows that many of the results derived above can be restated in terms of tax-incidence.

8. **If the incidence of the tax and subsidy falls on labor and capital in proportion to their initial factor shares, price of the taxed good will increase and that of the subsidized good will fall.**

Direct examples of this point are provided in Results 1, 5.4 and 6. These will hold a fortiori if capital's portion of the tax burden is lower than its initial income share ($r^* > 0$). Similarly, the conclusion about the price of
the subsidized good will be strengthened if the incidence of the tax lands less heavily on labor. From equations (10) and (11) it is apparent that \( p_1^*/T^* > 0 \) if \( r^* \geq 0 \) and \( p_2^*/T^* < 0 \) whenever \( r^* \leq 0 \). Of course, as Result 3 illustrates, additional information about elasticities of substitution, elasticity of demand, and factor intensities is sometimes needed to decide how \( p_1^*/T^* \) is affected when \( r^* < 0 \), and how \( p_2^*/T^* \) alters if \( r^* > 0 \).

5. **Effect on Relative Output Price** \( (p_1^*/p_2^*)_{12} \)

All the results so far have been derived by assuming \( w \) to be the numéraire, i.e., all changes in commodity prices are expressed in terms of the wage rate. While this approach gives clear answers to questions of tax incidence, in some instances, changes in output prices cannot be clearly determined. For instance, in Results 4 and 5.2, \( p_1^*/T^* \) is ambiguous whereas \( p_2^*/T^* \) is uncertain in 5.3. In such situations, often changes in \( p_1/p_2 \) can be ascertained unequivocally, as the following discussion will show. In this context, the Metzler result can be restated: "The tax-cum-subsidy will raise the relative price of the taxed good." In the terminology of the model, \( (p_1^* - p_2^*)/T^* \) will be positive.

From (16) and (17), we get:

\[
(p_1^* - p_2^*)/T^* = (\theta_{K1} - \theta_{K2})N\gamma/D + \gamma(2 - \rho_{21} - \rho_{12}) \tag{24}
\]

The second term in (24), \( \gamma(2 - \rho_{21} - \rho_{12}) \) is always positive. Therefore, the Metzler result will hold whenever the first term is zero or positive: for example when \( \theta_{K1} = \theta_{K2} \), which implies that the two industries have the same capital-labor ratio, or when \( N = 0 \) for whatever reason. This has a direct bearing on Results 5.2 and 5.3 in which the tax and subsidy have an uncertain effect on one of the two output prices. Here \( (p_1^* - p_2^*)/T^* \) will be invariably
positive because $A = (\theta_{K1} - \theta_{K2}) = 0$ by assumption. In general, however, the first term in (24) can be negative and equal to or even larger in absolute value than the second. As pointed out in Result 2, when all $\sigma$'s equal zero there will be no change in relative output price. And, depending on empirical magnitudes, there could be cases in which $(p_1^* - p_2^*)/T^*$ is negative. For example, in the results presented above, there have been several instances in which the taxed industry is relatively capital intensive and $r^* < 0$. Chances of negative values for $(p_1^* - p_2^*)/T^*$ increase if $\sigma_{L2}^1$ and $\sigma_{K1}^2$ are zero or negative because these elasticities appear as components of negative terms in $N$ and with positive terms in $D$. Consequently, when $\sigma_{L2}^1, \sigma_{K1}^2 \leq 0$, $r^*$ will tend to be more negative since $N$ would be larger and $D$ smaller in absolute value.

6. Conclusion and Summary

This paper has analyzed the price effects of a tax on the output of one commodity with an equivalent production subsidy to the second good when each industry uses two primary factors and the other commodity as an intermediate product. Metzler examined this issue in a Leontief model with fixed production coefficients and inelastic demand. He concluded that price of the taxed good will rise and that of the subsidized commodity will fall. Here, a general equilibrium model with variable input-output coefficients and elastic demand is set up.

It is shown that, in general, output prices are affected by changes in factor prices which in turn depend on a demand response, an output effect, and a factor-substitution effect. All these effects do not always work in the same direction, but it is clear that factor rewards generally will not remain unchanged when output taxes and subsidies are levied even when there are rather limited possibilities of varying factor proportion in the economy.
Metzler ruled out the "factor price" effect by assumption. The analysis here proves that the Metzler result will hold under less restrictive conditions as well, although in some cases it will be reversed. The topic is treated as one aspect of the broader issue of tax incidence, and some other conclusions based on incidence theory are also stated.
Footnotes

*Department of Economics, University of Western Ontario, London, Canada, N6A 5C2. Thanks are due to Russ Mellett for his competent research assistance. An earlier version of the paper was presented to the World Congress of the Econometric Society in Aix-en-Provence, August 1980.

1In a number of econometric studies [Hudson and Jorgenson (1974), Berndt and Wood (1975)], elasticities of substitution between primary factors and intermediate goods have been found to be significantly different from zero.

2An alternative way of writing this demand function is

\[ x_1^* - x_2^* = -\sigma_D (p_1^* - p_2^*) \]

where \( \sigma_D \) is the elasticity of substitution in demand (Atkinson and Stiglitz (1980), p. 1968). Equation (3) is the one Harberger (1962) uses, and as he shows \( \varepsilon = \sigma_D [x_1/(x_1 + x_2)] \) when all initial prices are normalized at unity.

3Many readers of an earlier draft of this paper have suggested that \( p_1 \) or \( p_2 \) should be used as the numéraire instead of \( w \). The choice of a numéraire is essentially arbitrary. Here \( w \) is chosen to maintain comparability with the tax-incidence literature where many results have been derived using wage rate as the numéraire.

4For a proof of this statement, see Batra (1973), pp. 177-79. In this connection, note that \( \Omega \)'s and \( \eta \)'s are not the same as the corresponding \( \theta \)'s for each industry. For instance, \( \theta_{K1} = R_{K1} r/p_1 \). After substituting for \( R_{K1} \) we get:

\[ \theta_{K1} = (\rho_{K1} + \rho_2 \rho_{K2} \rho_{21})/(1 - \alpha_{12} \alpha_{21}) \]

which is not the same as \( \Omega_K \).
Proofs of both propositions in this paragraph are straightforward and depend on the definitions of $a_{ij}$, $R_{ij}$, and the $\theta$'s. For details see Batra (1974), pp. 157-160.

See Bhatia (1981), and Harberger (1962) for examples of this result for different types of taxes.
References


Some of the steps required for arriving at $R^*_K$ are given below to illustrate the general approach. Recall that by definition

$$R_K = (a_{K1} + a_{K2}a_{21})/(1 - a_{12}a_{21}).$$

Therefore,

$$\frac{dR}{K_k} = a_{a_{K1} + a_{K2}a_{21} + a_{K1}a_{K2}a_{12}a_{12}} = \frac{a_{a_{K1} + a_{K2}a_{21} + a_{K1}a_{K2}a_{12}a_{12}}}{a_{K1}a_{K2}a_{21}}$$

(A.1)

Now $a_{ij} = a_{ij}(w, r, P_k)(i=L, K, 1, 2; k, j=1, 2; k \neq j)$

By totally differentiating it $a^*_i$'s can be determined. For example, after $w = 0$

$$a^*_{K1} = -\rho_{L1}a_{LK} + \rho_{21}a_{K2}a_{22}$$

(A.2)

$$a^*_{21} = -\rho_{K1}a_{K2} + \rho_{21}a_{22}$$

(A.3)

Allen (p. 505) gives the following stability condition for a linear, homogenous production function with three factors:

$$\rho_{21}a_{22} + \rho_{L1}a_{L2} + \rho_{K1}a_{K2} = 0$$

(A.4)

Using (A.4) and equations (10) and (11) in the paper, (A.2) and (A.3) can be simplified to

$$a^*_{K1} = -(\rho_{L1}a_{LK} + \rho_{21}a_{K2}a_{L2}) + \gamma T\rho_{21}a_{K2}(\rho_{12} - 1)$$

$$a^*_{21} = (\rho_{K1}a_{K2}a_{L2} - \rho_{L1}a_{L2}a_{K2}) + \gamma T(a_{12} - 1)(\rho_{L1}a_{L2} - \rho_{K1}a_{K2})$$

Substituting these into (A.1) and noting that $\theta_{L1} + \theta_{K1} = \theta_{L2} + \theta_{K2} = 1$ we arrive at the value for $R^*_K$ stated in the paper. Other $R^*_i$'s can be determined in the same manner.