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**Relative Prices, Complementarities
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of Aggregate Expenditures**

by

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February 1994

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Relative Prices, Complementarities and Co-movement Among Components of Aggregate Expenditures

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Abstract

Recent work suggests the standard real business cycle framework has difficulty accounting for co-movement among aggregate expenditure components when they are disaggregated even slightly to include business and household investment. I report that introducing empirically plausible relative price variability exacerbates this difficulty. The possibility that complementarities in production may help account for observed co-movement is considered. The results of formal estimation and testing suggest that complementarities improve the framework's empirical performance.

JEL Classification: E13, E23, E32

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1. Introduction

Ability to account for the observed pattern of positive co-movement among consumption, investment and output is used as a measure of success for business cycle models. In fact the canonical Real Business Cycle (RBC) model's ability to account for this aspect of the data is regarded as one of its desirable features.¹ Work by Greenwood and Hercowitz (1991) and Benhabib, Rogerson and Wright (1991) has made it clear that this model has difficulty in accounting for the pattern of co-movement among less aggregated components of expenditures. In particular, a model with the standard technology shock and a conventional specification of preferences and technology implies much less positive co-movement of consumer durables expenditures and residential investment with other components of aggregate expenditures than is evident in post war U.S. data.² If RBC models driven by technology shocks are to provide an empirically plausible basis for studying business cycles, they should be consistent with this basic feature of the data. The objective of this paper is to evaluate, using formal econometric methods, the ability of a particular class of RBC models to account for the pattern of co-movement of expenditures on household durable goods with nondurable consumption expenditures, business investment and aggregate output.

Two extensions to the standard RBC model that are particularly relevant for co-movement issues are introduced. The first draws from the empirical fact documented here that relative prices between the goods in question have displayed substantial variation over the post-war period. I model this by allowing for less than perfectly correlated shocks to the technology of producing these goods. An improvement in the relative efficiency of producing a good, other things equal, encourages reallocation of expenditures toward that good. In a conventional one-shock model this source of reallocation is not present. This suggests that models of this kind that fail to account for the pattern of co-movement that is focused on here, understate their own failure.

The second extension relates to the fact that standard specifications of technology assume

¹See, for example, Kydland and Prescott (1982), Hansen (1985) and King, Plosser and Rebelo (1988).

²The intuition for this result is as follows. We can regard services derived from owning durable consumption goods as being a substitute for nondurable goods and services available to households in the marketplace. If production of goods for sale in the market becomes more efficient due to a technology shock, capital will be built up by firms relative to households to take advantage of the efficiency gains. Investment in household durable goods will thus tend to co-move negatively with other components of aggregate expenditures.

that goods are perfect substitutes in production, *i.e.* it is costless to reallocate resources away from producing one good and toward producing another. In the canonical RBC model this is a manifestation of the assumption that the transformation frontier for consumption and investment goods is linear. Here, the standard production technology is modified to allow for nonlinearities in the transformation frontier.³ In particular, the possibility that there are complementarities in the production of business and household capital goods is considered.⁴ From the perspective of firms, this makes the reallocation of resources between producing business and household capital goods a costly activity. Rather than substituting factors of production away from producing one toward the other in response to shocks to technology, firms find it more efficient to produce the goods in tandem.

To gauge the importance of these extensions for explaining co-movement, I formally test how well different versions of the model can account for the data. Making use of relative price data, I estimate the parameters of the model via Hansen's (1982) Generalized Method of Moments (GMM) procedure. Econometric tests of the model's ability to account for co-movement and other features of the data are then conducted using methods developed by Christiano and Eichenbaum (1992). The predictions of the model are sensitive to particular assumptions regarding the sources of aggregate uncertainty. Versions of the model that embody alternative interpretations of the data in this regard are tested against one another using a GMM specification test developed by Singleton (1985).

The main findings of the paper can be summarized as follows. First, whether or not there is a single aggregate shock or there are good-specific shocks, complementarities are an important source of co-movement. Second, a standard aggregate technology shock also appears to be an important source of co-movement. Third, good-specific shocks are important for accounting for the observed variation in relative prices. Fourth, versions of the model with good-specific shocks can account for the significant negative correlation between business investment and the relative price of business capital reported here. Finally, using relative price data to identify good-specific technology shocks, it is not possible to account for the relative volatility of the two kinds of investment. Neither is it possible to account for the near zero correlation between household investment and the relative price

³Sims (1989) works with a nonlinear transformation frontier in the context of a monetary model.

⁴The complementarities examined here contrast with those studied, for example, by Diamond (1982) and Cooper and John (1989). Here the competitive equilibrium is unique and Pareto optimal; all complementarities in production are internalized by firms.

of household capital reported here. These last two findings could be interpreted as indicating that supply side mechanisms alone are not sufficient to account for the observed behavior of household goods expenditures over the business cycle.

The present work is related to several other papers in the literature. Lucas (1977) emphasized the potential for uncorrelated shocks to technology in individual market sectors to influence outcomes regarding co-movement. He pointed to the probable existence of such shocks as a reason for why we might expect there to be little or no co-movement at all. In the canonical RBC model the assumption of a single aggregate technology shock amounts to assuming that the technologies for producing different kinds of goods are perfectly correlated. The relative price variability documented here is one indication of the falsehood of this assumption. If the technologies for producing different goods are not perfectly correlated, what implications does this have for business cycle analysis? In particular, does inference based on the one-shock model provide an incomplete picture of the ability for an RBC model to account for basic features of the business cycle? Some progress toward answering these questions is made here.

Long and Plosser (1983) account for co-movement across many sectors without an aggregate shock by assuming that production of any good requires inputs of other goods. Hornstein and Praschnik (1993) study the ability of this kind of mechanism for accounting for some of the observations emphasized in this paper. The framework presented here can be seen as a parsimonious implementation of this mechanism since it amounts to assuming that goods are complements in production. An advantage of the approach taken here is that it is more amenable to formal empirical analysis.

McGrattan, Rogerson and Wright (1993) as well as Hornstein and Praschnik make use of complementarities in the utility function between the services derived from household capital goods and nondurable goods and services purchased directly from firms in accounting for the positive co-movement of household and business investment. This strategy is not pursued here because it is viewed as being inconsistent with balanced growth observations. In Fisher (1994) I report results that support the hypothesis that expenditure shares for nondurable and service consumption, household investment and business investment are stationary over the post-war period. This, combined with a pronounced trend in the price of household capital goods relative to nondurables and services over the same period places restrictions on preferences (documented in appendix B)

that these authors violate.

Greenwood and Hercowitz provide an alternative explanation for co-movement than the one examined here. They work with a home production environment in which non-market time and capital combine to yield a home produced nondurable good. They identify a shock to labor-augmenting technological change in home production with the standard technology shock, implicitly assuming the two shocks are perfectly correlated. In addition, they suppose capital and labor are difficult to substitute in home production. An implication of these assumptions is that the demand for household capital rises in response to a positive technology shock because the productivity of capital in the home is also improved and it is difficult to substitute capital into the market. The tendency for capital to flow to the more productive business sector is thus mitigated and co-movement can result.

This resolution of the co-movement issue is problematic principally because it relies on the existence of an exogenous shock process that is probably unidentifiable. Solow residual accounting can, in principle, be used to estimate the conventional technology shock. However, because output from home production is not observed, it cannot be applied to identify the shock to home production they rely on. This makes testing the hypothesis of perfect correlation problematic, and thus testing their resolution to the co-movement puzzle also.⁵ Their analysis contrasts with the approach taken here, since here all exogenous shocks are econometrically identified.

The rest of the paper proceeds as follows. In the next section I develop the model. In section 2 I discuss estimation and diagnostic methods and the data used to implement them. In section 3 the empirical performances of three versions of the model are analyzed. I consider a version of the model with only an aggregate technology shock, a version with good-specific shocks as well as an aggregate shock, and a version that includes good-specific shocks only. I present formal tests of the ability of the various versions of the model to match sample statistics associated with co-movement and other features of the business cycle. In the final section of the paper I conclude.

⁵A similar criticism applies to Benhabib, *et. al.* when they suggest a resolution to the hours-productivity puzzle using a shock to home production.

2. Theoretical framework

In this section I describe the model used to analyze the interaction of aggregate and good-specific shocks and production complementarities as they pertain to co-movement. The model is a modified version of the divisible labor RBC model studied in Hansen (1985). A household capital good is introduced by supposing households yield utility from the flow of services the good provides. This approach to modeling household demand for a durable good follows Bernanke (1985), Dunn and Singleton (1986), Eichenbaum and Hansen (1990) and Mankiw (1982, 1985). It is also consistent with the home production framework described by Benhabib, *et. al.* and Greenwood and Hercowitz.⁶ Preferences are also defined over a distinct nondurable good and leisure. The non-durable and durable good are both produced by firms. The production technology of firms differs from standard treatments in two respects. First, the function describing how goods can be transformed from one into another is allowed to be nonlinear here. Second, stochastic components to the marginal rates of transformation are introduced. These features of the production technology summarize the notions of complementarities and good-specific shocks that were discussed in the introduction.

The model implies certain balanced growth restrictions that can be tested. In Fisher (1994) I report that these balanced growth restrictions cannot be rejected at conventional significance levels using the testing strategy employed by King, Plosser, Stock and Watson (1991). In addition to the balanced growth tests, I also report in Fisher (1994) evidence that the price of household capital relative to nondurable consumption goods displays a pronounced trend over the post-war period. The balanced growth and price observations together impose certain restrictions on preferences that are described in appendix B. The preferences employed here are consistent with these restrictions.

⁶These authors emphasise the interaction of household capital with labor supply by postulating home production functions that include time and capital as inputs. There are two ways the preferences employed here can be viewed as arising from a home production structure. One view involves supposing labor's share in home production is zero. The other involves supposing households' preferences are logarithmically separable in the home produced good and that the home production function is Cobb-Douglas. The approach taken here is founded on the assumption that labor market supply and household capital demand are not interrelated in any substantive way. This of course is contrary to the views taken by these authors.

2.1. The model

The economy is populated by a large number of infinitely lived agents, each of which has momentary utility given by

$$\ln C_t + \eta \ln K_{h,t} + \gamma \ln L_t.$$

In this expression C_t denotes a nondurable good, $K_{h,t}$ denotes a durable good and L_t denotes hours of leisure. The parameters η and γ are positive scalars. The household has a fixed endowment of time, T , from which it may supply H_t to the business sector. Thus we have,

$$T - L_t - H_t \geq 0. \quad (2.1)$$

The stock of household capital evolves according to

$$K_{h,t+1} = (1 - \delta_h)K_{h,t} + I_{h,t}. \quad (2.2)$$

In this expression δ_h and $I_{h,t}$ denote the rate of household capital depreciation and gross investment in household capital, respectively.

The production side of the economy can be described as follows. An intermediate good $\hat{Y}_t$ is produced according to the Cobb-Douglas technology

$$\hat{Y}_t = K_{b,t}^\alpha (z_{a,t} H_t)^{1-\alpha}, \quad (2.3)$$

where $\alpha \in (0, 1)$, $K_{b,t}$ is the stock of capital in the business sector and $z_{a,t}$ is the exogenous, possibly stochastic, level of labor augmenting technology. The stock of business capital evolves according to

$$K_{b,t+1} = (1 - \delta_b)K_{b,t} + I_{b,t}. \quad (2.4)$$

Here δ_b denotes the rate of depreciation of business capital and $I_{b,t}$ denotes gross business investment. The intermediate good is transformed into C_t , $I_{h,t}$ and $I_{b,t}$ according to the transformation relation

$$\hat{C}_t + (\hat{I}_{m,t}^\kappa + \hat{I}_{h,t}^\kappa)^{1/\kappa} \leq \hat{Y}_t. \quad (2.5)$$

Here, $\kappa \in [1, \infty)$, $\hat{C}_t \equiv C_t/z_{c,t}$, $\hat{I}_{m,t} \equiv I_{m,t}/z_{m,t}$ and $\hat{I}_{h,t} \equiv I_{h,t}/z_{h,t}$. The exogenous, possibly stochastic, terms $z_{c,t}$, $z_{h,t}$ and $z_{b,t}$ represent potential differences in the relative progression of factor productivity in the production of the different goods. The more conventional $z_{a,t}$ term represents technology that affects the production of all goods simultaneously. The value taken by κ determines the elasticity of substitution in production of household and business capital.

If we think in terms of a social planning problem, particular choices of $\hat{C}_t$, $\hat{I}_{m,t}$ and $\hat{I}_{h,t}$ determine how the planner allocates a given supply of capital and labor toward the production of the goods. For a fixed amount of capital and labor devoted to producing the nondurable consumption good, say, particular values of $z_{c,t}$ determine the amount of the final good produced in efficiency units. In this way $z_{c,t}$, $z_{h,t}$ and $z_{b,t}$ help to determine the efficiency of capital and labor in producing the different kinds of goods.⁷

Since the simultaneous econometric identification of $z_{a,t}$, $z_{c,t}$, $z_{h,t}$ and $z_{b,t}$ is problematic, I study special cases of the environment in which various combinations of the terms are operational. The first case I consider is where only the aggregate process $z_{a,t}$ is operational. I refer to the model with this restriction as the OS Model, where OS stands for one-shock. The second case considered has $z_{a,t}$, $z_{h,t}$ and $z_{b,t}$ operational. In this case both aggregate and good-specific shocks drive the economy. I refer to the model with this restriction as the MS-A Model, where MS stands for multiple-shock. The third case, called the MS-C model, involves only the good-specific terms.

We are now in a position to describe how the exogenous productivity terms evolve over time. Define the vector z_t to include the terms operational for a given version of the model being studied. So, in the OS model $z_t = z_{a,t}$, in the MS-A model $z_t = [z_{a,t}, z_{h,t}, z_{b,t}]'$ and in the MS-C model, $z_t = [z_{c,t}, z_{h,t}, z_{b,t}]'$. The evolution of z_t is described by

$$\ln(z_t) = \ln(z_{t-1}) + \epsilon_t, \quad (2.6)$$

where $\epsilon_t \sim \text{iid } N(\epsilon, \Sigma)$. Here $\epsilon_t = \epsilon_{a,t}$, $\epsilon_t = [\epsilon_{a,t}, \epsilon_{h,t}, \epsilon_{b,t}]'$ and $\epsilon_t = [\epsilon_{c,t}, \epsilon_{h,t}, \epsilon_{b,t}]'$, for the OS model,

⁷There are other interpretations of the stochastic terms $z_{c,t}$, $z_{h,t}$ and $z_{b,t}$. One might view $z_{b,t}$ as determining the level of capital augmenting technology in the business sector. Greenwood, Krusell and Hercowitz (1993) examine a model of vintage capital which in its reduced form includes a term like this. Measured changes over time in $z_{c,t}$ could reflect improvements in the quality of existing consumption goods or the introduction of new consumption goods that yield a given amount of utility with less input of capital and labor in production. Similarly, changes over time in $z_{h,t}$ could represent changes in the efficiency of new capital in home production.

the MS-A model and the MS-C model respectively. Also, ϵ and Σ are chosen to be conformable with ϵ_t . In versions of the model where a particular exogenous term is assumed to be nonoperational I adopt a deterministic version of (2.6) to describe its evolution over time. In doing so I fix the associated innovation term to a constant. When $z_{a,t}$ or $z_{c,t}$ are nonoperational I set the constant to zero *a priori*. When they are operational, the mean of $\epsilon_{a,t}$, ϵ_a , and of $\epsilon_{c,t}$, ϵ_c , can be nonzero to accommodate the trend in total factor productivity. When $z_{h,t}$ and $z_{b,t}$ are nonoperational I allow the mean of $\epsilon_{h,t}$, ϵ_h , and of $\epsilon_{b,t}$, ϵ_b , to be nonzero to accommodate trends in relative prices.

In the presence of complete markets, the competitive equilibrium of each of the three economies corresponds to the solution to the relevant version of the following social planning problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \{ \ln C_t + \eta \ln K_{h,t} + \gamma \ln(T - H_t) \}$$

subject to (2.2)–(2.6), with $K_{h,-1}$, $K_{b,0}$ and z_{-1} given. The maximization is with respect to choice of contingency plans $\{C_t, K_{h,t}, K_{b,t+1}, H_t : t \geq 0\}$. I have used (2.1) to substitute out L_t . Also $\beta \in (0, 1)$ is the subjective discount factor and E_0 is the time zero conditional expectations operator.

The solution to this problem is found by solving a transformed problem in which variables converge to stationary random variables. To write this problem down, I make use of the definitions

$$\begin{aligned} \tilde{\beta} &= \exp(-[\epsilon_{a,t} + \frac{\epsilon_{m,t}}{1-\alpha}])\beta; \quad c_t \equiv \frac{C_t}{a_t z_{c,t}}; \quad k_{h,t} \equiv \frac{K_{h,t}}{a_t z_{h,t}}; \quad k_{b,t+1} \equiv \frac{K_{b,t+1}}{a_t z_{b,t}}; \\ i_{h,t} &\equiv k_{h,t} - (1 - \delta_h) \frac{k_{h,t-1}}{\exp[\epsilon_{a,t} + \frac{\alpha}{1-\alpha} \epsilon_{b,t} + \epsilon_{h,t}]}; \quad i_{b,t} \equiv k_{b,t+1} - (1 - \delta_b) \frac{k_{b,t}}{\exp(\epsilon_{a,t} + \frac{1}{1-\alpha} \epsilon_{b,t})}. \end{aligned}$$

Here $a_t = z_{a,t} z_{b,t}^{\alpha/(1-\alpha)}$.⁸ The aggregate resource constraint can now be written

$$c_t + (i_{m,t}^{\kappa} + i_{h,t}^{\kappa})^{1/\kappa} \leq \exp(-\alpha[\epsilon_{a,t} + \frac{1}{1-\alpha} \epsilon_{b,t}]) k_{b,t}^{\alpha} H_t^{(1-\alpha)}. \quad (2.7)$$

It follows now that finding the solution to the above problem corresponds to solving

$$\max E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \{ \ln c_t + \eta \ln k_{h,t} + \gamma \ln(T - H_t) \}$$

⁸The term $z_{b,t}^{\alpha/(1-\alpha)}$ appears in a_t to account for a possible trend in capital augmenting technical change (see, for example, Greenwood, Hercowitz and Krussel).

subject to (2.5), (2.6), (2.7) and $k_{h,-1}$, $k_{b,0}$, z_{-1} given. This maximization is with respect to contingency plans $\{c_t, k_{h,t}, k_{b,t+1}, H_t : t \geq 0\}$.

I approximate the planner's policy functions using log-linear functions of the state. This involves computing a first order Taylor series expansion of a given model's Euler equations around its non-stochastic steady state. The log-linear Euler equations can be mapped into a linear state space framework in which solution methods are well known (see, for example, Blanchard and Kahn [1980]). The method is analogous to King, Plosser and Rebelo's (1988).

2.2. Discussion

I now briefly describe some of the mechanisms at work in this model. A useful device for doing this is the impulse response function. Using parameter values reported in section 4, impulse response functions (in terms of per cent deviations from the balanced growth path) for household investment and business investment are shown in figure 1. From top to bottom, the rows of this figure correspond to responses to (uncorrelated) one per cent innovations in the aggregate technology shock, the household capital specific shock and the business capital specific shock.⁹ The columns of this figure correspond to alternative settings of the parameter κ . In the left hand column $\kappa = 1$, which corresponds to a linear transformation frontier. In the right hand column $\kappa = 1.08$ (this value is close to values reported in section 4) so that there are complementarities in the production of household capital and business capital. The impulses occur in period 2.

In the top left hand graph in this figure we can see quite clearly the fundamental problem with a model that has only the standard technology shock and no complementarities. In response to a permanent innovation to the overall level of business production possibilities, it is optimal to shift resources into the business sector and away from the household sector. This is accomplished by sharply reducing household investment in the period of the shock and increasing business investment. In later periods when business production possibilities are abundant, household investment rises above its balanced growth trend along with business investment. However, the initial responses are so strong that, as we shall see below, the pattern of co-movement of household investment with

⁹ Responses to the consumption good-specific shock are not displayed in this figure. Household and business investment are unresponsive and consumption responds immediately and permanently in the direction of the innovation. This pattern emerges since the consumption good-specific shock has only an income effect. It does not affect the ability of the household to intertemporally substitute at all (see Barro and King (1984)). Complementarities have no effect on these responses.

other expenditure components is easily rejected by the data.

One explanation for this failure is because it is costless for firms to reallocate production from one good to another. An indication of the effect of the perfect substitution assumption is the size of the impact responses of the two kinds of investment in the top left hand panel of figure 1. If κ is greater than unity then there are costs associated with changing the production mix of capital goods. The implications of this are evident in the top right hand graph. The affects of introducing complementarities are twofold. First, household and business investment now move together. Second, the absolute values of the impact responses to the innovations are smaller. When there are complementarities, there are costs associated with reallocating production from one good to another. The desire to shift production towards the business sector is still present following the shock, only now its effects are mitigated by these costs.

In the bottom two rows of the figure the importance of the aggregate shock for generating co-movement in the presence of good-specific shocks is evident. In the first column we observe how good-specific shocks do nothing to improve the co-movement problem described above. For example, a positive impulse to the efficiency in production of business capital makes household capital relatively costly to produce. There is a shift of resources to producing the business capital good in response and the two kinds of investment move in opposite directions. Because it is costless to reallocate production between goods, the responses in the period of the shock appear to be very large. In the second column we see that this tendency for negative co-movement is so strong that complementarities do much less to reverse it when compared to the aggregate shock example. Finally, the volatility reduction effect of complementarities, noted previously, is present in these graphs as well.

3. Econometric Method

I use a variant of the GMM procedure discussed in Christiano and Eichenbaum to estimate the different versions of the model and evaluate their empirical performance. This procedure involves using an exactly identified GMM estimator to estimate the parameters of the model. Given these parameters, statistics implied by the model can be computed. These same statistics can be computed from the data in a way that does not involve the model. If the model is correctly specified and we abstract from sampling uncertainty then the two sets of statistics should be the same. To

test this hypothesis I use the Wald-type statistic employed by Christiano and Eichenbaum.¹⁰ This statistic, which I denote Q , is discussed in appendix C.

An important part of the empirical analysis is a comparison of the performances of the different versions of the model. To formalize this comparison I use a specification test introduced by Singleton (1985). The test statistic he proposed, which I denote λ , incorporates information about a specific non-nested alternative in the test of a null specification. This makes it possible to evaluate formally how well one version of the model can account for a particular list of statistics relative to another version. In contrast to the Q statistic, implementing this procedure requires using an overidentified GMM estimator to estimate the parameters of the model. I discuss the λ statistic in appendix C as well. In the remainder of this section I outline the estimation strategy and describe the dataset.

3.1. Estimation and diagnostics

I focus the discussion of the estimation strategy adopted in this paper on how it is applied in the case of the MS-A model. I then point out the changes in the procedure required to implement the OS and MS-C models. It is convenient to partition the vector of model parameters that must be estimated in the MS-A model, Ψ_1 , as follows

$$\Psi_{1,1} = [\eta, \gamma, \alpha, \delta_m, \delta_h]';$$

$$\Psi_{1,2} = [\kappa, \epsilon_a, \epsilon_h, \epsilon_b, \sigma_a, \sigma_h, \sigma_b, \rho_{ah}, \rho_{ab}, \rho_{hb}]',$$

where $\Psi_1 \equiv [\Psi'_{1,1} \ \Psi'_{1,2}]'$. Here, the terms σ_i , $i = a, h, b$ and ρ_{ij} , $i = a, h$ and $j = h, b$, $i \neq j$, refer to standard deviations and correlation coefficients of the innovations to technology, respectively. They are used to construct the estimate of the covariance matrix Σ . Note that T and β were not estimated. Following Christiano and Eichenbaum, T was fixed at 1369 hours per quarter and β was set to $1.03^{-.25}$ to imply a 3 per cent annual subjective discount rate.

My estimator of Ψ_1 is described formally in appendix C. Here I give an overview. Consider first the elements of $\Psi_{1,1}$. The procedure involves equating certain first moments in the model and the data. A simple interpretation of the procedure is as follows. The depreciation rates correspond

¹⁰Two advantages of this procedure over standard calibration exercises are: (i) the sampling uncertainty in the data generated moments is taken into account; and (ii) the sampling uncertainty in the model implied statistics that arises because of sampling uncertainty in the model parameters is taken into account.

to the sample average rate of depreciation implicit in the associated capital stock and investment series. The estimators of α and η are designed to make the model reproduce the sample mean business capital to output and household capital to output ratios. Finally, the estimator of γ is designed to make the model reproduce the sample average of hours worked per capita.

To describe how the elements of $\Psi_{1,2}$ are estimated I first need to specify how I identify series for $\epsilon_{a,t}$, $\epsilon_{h,t}$ and $\epsilon_{b,t}$. I use standard Solow residual accounting to identify $\epsilon_{a,t}$. Recall that output in this study is measured in terms of consumption goods. This fact implies

$$Y_t = z_{c,t} K_{b,t}^\alpha (z_{a,t} H_t)^{1-\alpha}. \quad (3.1)$$

Taking logs of both sides of (3.1), first differencing and rearranging, we have

$$(1 - \alpha)\epsilon_{a,t} + \epsilon_{c,t} = \Delta \ln(Y_t) - \alpha \ln(K_{b,t}) - (1 - \alpha)\Delta \ln(H_t). \quad (3.2)$$

Here Δ denotes the first difference operator. By replacing Y_t , $K_{b,t}$ and H_t in (3.2) with empirical measures one can identify either $\epsilon_{a,t}$ or $\epsilon_{c,t}$. In the cases of the OS and MS-A models, where $\epsilon_{c,t} = 0$, (3.2) can be used to identify a series for $\epsilon_{a,t}$. In the case of the MS-C model, where $\epsilon_{a,t} = 0$, it can be used to identify a series for $\epsilon_{c,t}$.

The procedure used to identify series for $\epsilon_{h,t}$ and $\epsilon_{b,t}$ involves a similar principle. The first step involves identifying relative prices of capital goods in the model. These prices are identified with marginal rates of transformation. Using the consumption good as the numeraire, it is straightforward to show that the relative prices of household capital, $P_{h,t}$, and business capital, $P_{b,t}$, are given by

$$P_{h,t} = \frac{z_{c,t}}{z_{h,t}} \left[\frac{\hat{I}_{h,t}}{\hat{I}_t} \right]^{\kappa-1}, \quad (3.3)$$

and

$$P_{b,t} = \frac{z_{c,t}}{z_{b,t}} \left[\frac{\hat{I}_{b,t}}{\hat{I}_t} \right]^{\kappa-1}, \quad (3.4)$$

where $\hat{I}_t = (\hat{I}_{m,t}^\kappa + \hat{I}_{h,t}^\kappa)^{1/\kappa}$. These expressions merely state that $P_{h,t}$ and $P_{b,t}$ are given by the time t marginal rate of transformation between $K_{h,t}$ and C_t , and $K_{b,t+1}$ and C_t , respectively.¹¹ Taking

¹¹Notice that linear homogeneity of the transformation frontier implies $z_{h,t}P_{h,t}/z_{c,t}$ and $z_{b,t}P_{b,t}/z_{c,t}$ are stationary random variables. It follows that $\ln(P_{h,t})$ and $\ln(P_{b,t})$ are difference stationary with unconditional mean growth rates

logs and first differencing (3.3) and (3.4) , we arrive at the following:

$$\epsilon_{h,t} - \epsilon_{c,t} = \frac{1}{\kappa} \Delta \ln \left[\frac{I_t}{P_{h,t} I_{h,t}} \right] + \Delta \ln \left[\frac{I_{h,t}}{I_t} \right]; \quad (3.5)$$

$$\epsilon_{b,t} - \epsilon_{c,t} = \frac{1}{\kappa} \Delta \ln \left[\frac{I_t}{P_{b,t} I_{b,t}} \right] + \Delta \ln \left[\frac{I_{b,t}}{I_t} \right]. \quad (3.6)$$

Here, $I_t = P_{h,t} I_{h,t} + P_{b,t} I_{b,t}$ is total gross investment measured in terms of consumption goods.¹²

Conditional on knowledge of κ , we could identify $\epsilon_{h,t}$ and $\epsilon_{b,t}$ directly from (3.5) and (3.6) by using data on $P_{h,t}$, $P_{b,t}$, $I_{h,t}$, $I_{b,t}$ and I_t . In the MS-C model, (3.2), (3.5) and (3.6) could be used to identify $\epsilon_{c,t}$, $\epsilon_{h,t}$ and $\epsilon_{b,t}$, if we had knowledge of κ .¹³ Thus, if κ were known it would be a straightforward exercise to estimate the elements of $\Psi_{1,2}$. Unfortunately this is not the case, which complicates the estimation procedure somewhat. I use κ to gauge the importance of production complementarities for co-movement. With this in mind, I estimate $\Psi_{1,2}$ in two separate ways. The first way incorporates the restriction $\kappa = 1$. In the second I allow κ to be unrestricted. In this case I use the model's prediction for the contemporaneous correlation between business and household investment, denoted $\rho(I_h, I_b)$, to identify κ . This approach is motivated by the discussion at the end of the last section.

For the restricted cases, the interpretation of how I estimate the elements of $\Psi_{1,2}$ is straightforward. The sample means and the sample variance-covariance matrix of the series for $\epsilon_{a,t}$, $\epsilon_{h,t}$ and $\epsilon_{b,t}$ are used. In the unrestricted cases I proceed in the same way, with the following modification. Letting $M(\Psi_1)$ denote the probability limit of the model's prediction for $\rho(I_h, I_b)$, I identify κ when it is unrestricted by imposing the condition¹⁴

$$E\{\sigma_{I_h} \sigma_Y M(\Psi_1) - Y_t^f I_{h,t}^f\} = 0. \quad (3.7)$$

In (3.7) Y_t^f and $I_{h,t}^f$ are understood to be data on output and household investment, respectively,

given by $\epsilon_h - \epsilon_c$ and $\epsilon_b - \epsilon_c$, respectively. This restriction on the relative prices of household and business capital, which depends crucially on the exogenous process assumed in (2.6), is consistent with post-war U.S. data. The tests underlying this conclusion are reported in Fisher (1994).

¹²Output in the models is measured as $Y_t = C_t + I_t$.

¹³Notice that we have only three conditions to identify four variables. This is why implementing a version of the model with all technology terms operational is problematic.

¹⁴I compute $M(\Psi_1)$ based on the log-linear approximation of the planner's policy functions by using a spectral method discussed by Christiano and Eichenbaum (1992).

that have been filtered to remove trends and means, and σ_{I_h} and σ_Y denote the standard deviation of household investment and output (in consumption units), respectively.

The above described procedure for estimating Ψ_1 can be applied for the OS and MS-C models as well. The parameters to be estimated for the OS model are $\Psi_{1,1}$ as it is specified above, and

$$\Psi'_{1,2} = [\kappa, \epsilon_a, \epsilon_h, \epsilon_b, \sigma_a]'$$

For the MS-C model, the parameters to be estimated are $\Psi_{1,1}$ as it is specified above, and

$$\Psi''_{1,2} = [\kappa, \epsilon_c, \epsilon_m, \epsilon_h, \sigma_c, \sigma_m, \sigma_h, \sigma_{cm}, \sigma_{ch}, \sigma_{mh}]'$$

To estimate the elements of $\Psi'_{1,2}$ and $\Psi''_{1,2}$ I proceed in a manner directly analogous to the procedure outlined for $\Psi_{1,2}$. The procedure for estimating the elements of $\Psi_{1,1}$ for these models is unchanged from the procedure outlined for the MS-A model.

To implement the diagnostic procedures, various statistics from the data generating process must be estimated. I emphasize the following three sets of statistics:

$$\Psi_{2,1} = [\rho(Y, C), \rho(Y, I_h), \rho(Y, I_b), \rho(Y, I), \rho(C, I_h), \rho(C, I_b), \rho(I_h, I_b)]';$$

$$\Psi_{2,2} = [\sigma_{P_h}/\sigma_Y, \sigma_{P_b}/\sigma_Y, \rho(P_h, I_h), \rho(P_b, I_b)]';$$

$$\Psi_{2,3} = [\sigma_Y, \sigma_C/\sigma_Y, \sigma_{I_h}/\sigma_Y, \sigma_{I_b}/\sigma_Y, \sigma_I/\sigma_Y, \sigma_H]'$$

Here σ_X denotes the unconditional standard deviation of the variable X and $\rho(X_1, X_2)$ denotes the unconditional contemporaneous cross correlation of X_1 with X_2 . Time subscripts have been dropped from variable names to simplify the notation.

The focus of the study is on co-movement as represented by the statistics in $\Psi_{2,1}$. Notice that when κ is unrestricted in the estimation, $\rho(I_h, I_b)$ must be dropped from $\Psi_{2,1}$ since it is used to identify κ . The statistics in $\Psi_{2,2}$ and $\Psi_{2,3}$ are studied to evaluate the models' overall performances. The statistics in $\Psi_{2,2}$ have been selected to evaluate how well the models can account for relative price variability and the interaction of relative prices with quantities. The statistics in $\Psi_{2,3}$ have been chosen since these are statistics typically stressed in the RBC literature. Since the data

display trends, some stationary inducing transformation must be adopted to ensure the moments in $\Psi_2 = [\Psi'_{2,1} \ \Psi'_{2,2} \ \Psi'_{2,3}]'$ exist. To this end I detrend both the time series that emerge from the model and actual data using the Hodrick and Prescott (1980) filter. Thus the population moments in Ψ_2 correspond to Hodrick-Prescott filtered versions of the data.

To conclude the description of the estimation procedure, notice that any estimator for Ψ_1 and Ψ_2 will be based on the same data set. Thus estimators of Ψ_1 and Ψ_2 will have a non-zero covariance. To obtain the correct sampling distribution for my estimator, I estimate these parameters simultaneously (see appendix C).

3.2. The data

A time period in the model is assumed to be a quarter, so all the data in the study are quarterly. The sample period is 1955:3 to 1988:2. Real expenditure series are based on 1982 prices. All quantity series were measured in per capita terms using an efficiency weighted population measure based on Hansen's (1992) efficiency weights.

$P_{b,t}$ was measured as the ratio between a constructed implicit price deflator for business investment and an implicit price deflator for nondurable and service consumption. $P_{h,t}$ was measured in the same way using household investment. These and other deflators used in this study were constructed in the usual way by first forming nominal and real expenditure series for the two kinds of investment and consumption. The data used to form the expenditure series for business investment included expenditures on private and government investment in producer structures and equipment. The data used to form the expenditure series for household investment included private and government residential investment and durable consumption expenditures. All data except for the government investment data is from the Citibase data tape. The government investment data was provided the Bureau of Economic Analysis. The consumption expenditure series used to derive the consumption good deflator were nondurable consumption, government consumption and service consumption less the imputed value of housing services. The government consumption series was defined as government purchases of goods and services minus government investment.

The series used for C_t , $I_{h,t}$ and $I_{b,t}$ correspond to the real expenditure versions of the series outlined above. Gross output was measured as the sum of real consumption, the product of the household capital relative price and real gross household investment, the product of the market cap-

ital relative price and real gross market investment, and the nominal value of inventory investment divided by the consumption deflator. The measures of $K_{h,t}$ and $K_{b,t}$ were chosen to match the real investment series. $K_{b,t}$ was measured as the sum of the stock of producer and government structures and equipment. $K_{h,t}$ was measured as the sum of the stock of consumer durables and government and private residential capital. The government capital stock data is from various issues of the Survey of Current Business as is the private residential capital stock data. The data on the private net stock of producer structures and equipment is from the NIPA accounts. I used two measures of hours worked. The first based on establishment surveys and the second on household surveys. These data were used to quantify measurement error in hours worked data that Prescott (1986) and others have argued may distort Solow residual based measures of technology. A description of the treatment of measurement error in hours worked data is provided in appendix C and a more complete description of the dataset is provided in appendix A.

4. Empirical results

The parameter estimates for the κ restricted and κ unrestricted versions of the OS, MS-A and MS-C models are displayed in tables 1 and 2 under the headings 'R' and 'U' respectively. Table 1 contains parameter estimates (standard errors) for all parameters except for those pertaining to the forcing processes and table 2 contains these latter parameters estimates.

Three features of the estimation results are of particular interest. First, the point estimate for α is significantly less than usually estimated. Christiano and Eichenbaum (1992), for example, estimate α to be no less than 0.339 (standard error 0.006). The estimate here is uniformly about 0.237 (0.011). The difference is explained by the fact that the measure of business capital used here is smaller than usual. The effect of this lower estimate of α on the models' properties is to increase the elasticity of demand for labor (by flattening the marginal product of labor schedule). However, the quantitative impact of this is small.

The second feature is the seemingly small estimates for κ . These estimates, with the exception of the OS Model, are reasonably precise. They imply elasticities of substitution between the two kinds of capital of 66.7, 14.7 and 9.0 for the OS, MS-A and MS-C models, respectively. As we shall see, the behavior of the models changes quite dramatically when we allow for these seemingly small complementarities in production.

The third feature regards the estimates of the correlation coefficients among the good-specific and aggregate shocks as seen in table 2. Consider first the MS-A model. The point estimates and standard errors for the correlation coefficients between the aggregate shock and the household capital and business capital shocks in both the restricted and unrestricted cases indicate either no correlation at all (business capital) or slightly negative correlation (household capital). The point estimates of the technology shock correlations in the MS-C model are for the most part closer to unity than the in the MS-A model. Given the way I identify the shocks in this case (see (3.5) and (3.6)), this result is not surprising. What is important to notice, however, is that the correlations of the good-specific shocks with the aggregate shock and with each other are substantially less than unity. This is in contrast to the situation in the OS model in which all the shocks are implicitly perfectly correlated. It suggests, as will be confirmed below, that the OS and the MS models will behave quite differently.

4.1. Univariate diagnostics

In tables 3, 4 and 5 data and model statistics associated with co-movement, relative prices and quantity volatilities are reported. In these tables standard errors are in round brackets. To test the null hypothesis that sample and model statistics are drawn from the same data generating process, I make use of the Q statistic discussed previously. Numbers in square brackets denote the corresponding probability values.

Consider first the co-movement statistics in table 3. The U.S. data column indicates significantly positive correlations for all the statistics listed. That is, the various components of aggregate expenditures display a pattern of positive co-movement. The restricted OS model shows rejection of the hypothesis that the model and sample statistics are drawn from the same data generating process at the five-percent level for all co-movement statistics except for $\rho(C, I_b)$. Consistent with the discussion of impulse response functions, the most pronounced failures are with respect to statistics involving I_h . For example the $\rho(I_h, I_b)$ statistic implied by the model is -0.876 (0.013) compared to 0.364 (0.121) in the sample. However, even the correlations of consumption and total investment with output are rejected. The model predicts *too much* positive co-movement among these variables.

The restricted MS models do not fare much better. With respect to consumption and total

investment co-movement with output they display an obvious improvement. This is not surprising given the less than perfect correlation between the shocks in these models. However, the predicted $\rho(I_h, I_b)$ statistic is made worse in the MS models. It falls to -0.953 (0.010) and -0.958 (0.022) in the MS-A and MS-C models, respectively. Also, with one exception, there is no improvement with the other statistics. We may conclude that the benchmark restricted OS model understates the failure of this class of models with respect to co-movement among disaggregated components of expenditures.

As seen from the table, the impact of allowing for complementarities generally improves the co-movement properties of all three versions of the model. The unrestricted OS model cannot be rejected at the 95 per cent level with respect to $\rho(C, I_h)$ and at the 70 per cent level with respect to $\rho(I_h, Y)$. For the other statistics it is still rejected at the 1 per cent level. The MS-A and MS-C models perform similarly. If we use the liberal 1 per cent rejection criterion then these models cannot be rejected with respect to any of the statistics displayed in this table. These models do best with respect to $\rho(C, I_b)$, $\rho(I_h, Y)$, $\rho(I_b, Y)$, $\rho(C, Y)$ and $\rho(I, Y)$ and most poorly with respect to $\rho(C, I_h)$. An interesting question is how these models do in comparison to each other. I delay this comparison until later when I discuss multivariate diagnostics.

While co-movement is the focus of the study, a comprehensive assessment of the different versions of the model requires we evaluate them along other dimensions of the data. The relative price statistics in table 4 are helpful for this purpose. The statistics in the U.S. data column indicate substantial and roughly equal volatility in relative prices for the two kinds of capital relative to output. Household investment is close to being uncorrelated with the price of household capital with a correlation coefficient of 0.196 (0.099). On the other hand, the correlation coefficient for business investment and the price of business capital is significantly negative at -0.307 (0.133).

Since the restricted OS model implies relative prices are constant, it is no surprise that it cannot account for these statistics. Even allowing for complementarities, the OS model does not fare much better. The only success is with $\rho(I_h, P_h)$, but this is because the standard error of the model implied statistic is so large that there is no value of the sample correlation that it could be rejected against. The MS models perform considerably better. It is hard to reject the hypothesis that both the unrestricted and the restricted MS models' predictions for relative price variability are consistent with the data. In addition, both the MS models, whether or not κ is restricted, appear

to be consistent with the price-quantity correlation for business investment. This is perhaps not very surprising since the negative correlation in the sample is indicative of a significant role for supply shocks in determining business investment. The only failure of the MS models in this table is with the price-quantity correlation for household investment. This may reflect the absence of 'demand' shocks in the models.

The quantity volatility statistics in table 5 can be used to gauge the performance of the class of models studied here with other RBC models in the literature. Looking down the U.S. data column we see that a rough characterization of the data is $\sigma_{I_h} > \sigma_{I_b} > \sigma_Y > \sigma_C$. Consumption is less than half as volatile as output, business investment is more than one-and-a-half times as volatile as output, household investment is close to three times as volatile as output. In addition hours are somewhat less volatile than output (I use a household survey measure of hours to estimate σ_N) and total investment is almost twice as volatile as output, lying between σ_{I_h} and σ_{I_b} in relative volatility.¹⁵

Generally speaking, the models with an aggregate shock do well at matching output volatility, regardless of complementarities. In no case with an aggregate shock can the hypothesis that output volatility in the model and in sample are from the same data generating process be rejected at even the 20 per cent level of significance. The MS-C model performs poorly in comparison. Consumption volatility is uniformly too high in the models. The MS-C model performs the best along this dimension and it can still be rejected at the 2 per cent level. The models do well at accounting for the volatility of total investment. Neither of the restricted versions can be rejected with respect to this statistic at the 10 per cent level and the unrestricted models cannot be rejected at even the 20 per cent level. The models all do poorly with respect to hours volatility. This last finding is not surprising since the standard RBC model is well known to perform poorly along this dimension. Taken together these results are not at variance with the conventional model (see Hansen (1985)).

The volatility behavior of disaggregated investment in the three models indicated in table 5 illustrates two points. First, complementarities are important for smoothing investment over the cycle. Second, despite the presence of good-specific shocks that could in principle account for the

¹⁵Differences with statistics usually reported emerge from two facts. First, output is measured here in terms of a base year consumption bundle instead of 1982 dollars. Second, relatively volatile residential investment and durable goods consumption is not included in the measure of business investment as is usually the case.

relative volatility of disaggregated components of investment, no version of the model can account for the observation that household investment is more volatile than business investment. These points are made as follows.

In all three models without complementarities, investment volatility is grossly counterfactual. While the conventional one shock RBC model without household capital produces business investment volatility that is in rough accordance with the data, here the OS model performs poorly along this dimension. Household capital in the model encourages large swings in the two kinds of investment as resources are reallocated to their most efficient use. Goods-specific shocks in the MS-A and MS-C models merely serve to amplify the swings in investment. Thus, these models, without complementarities, perform worse than the OS model.

As we saw when we examined impulse response functions, complementarities tend to dampen the large swings in investment by making production reallocation costly. In the OS model they do not bring business investment down enough and bring household investment down too far. Both relative volatility statistics can be rejected outright as matching the data. Complementarities have a similar effect in the MS models. The main difference is that the household capital good-specific shocks ensure that household investment volatility is significantly higher in these models than in the OS model. We can still easily reject the match of these models to the data along these two dimensions for the MS-A model. The unrestricted MS-C model cannot be rejected at the 75 per cent level with respect to the volatility of household investment relative to output and at the 2 per cent level for business investment. Nonetheless, despite the presence of capital good specific shocks, both MS models fail to account for the ordering of these investment volatilities relative to each other.

4.2. Multivariate diagnostics

Two distinguishing features of this study are its emphasis on co-movement and its use of relative price observations to evaluate models. A useful complement to the univariate diagnostics discussed above is an evaluation of the overall performance of the models with respect to co-movement and relative prices. Q -statistics [p -values] to test several joint co-movement and relative price hypotheses are shown in table 6. Each test involves the hypothesis that a particular set of statistics in a given model and in sample are from the same data generating process. The hypothesis H_1 involves,

$\rho(C, Y)$, $\rho(I_h, Y)$, $\rho(I_b, Y)$, $\rho(C, I_h)$ and $\rho(C, I_b)$, H_2 involves $\rho(I_h, Y)$, $\rho(C, I_h)$ and $\rho(C, I_b)$, H_3 involves $\rho(C, Y)$ and $\rho(I, Y)$ and H_4 involves σ_{P_h}/σ_Y and σ_{P_b}/σ_Y .

The tests of H_1 all show significant rejection of all models. However, Monte Carlo evidence reported by Burnside and Eichenbaum (1993) indicates high dimensional Q tests over-reject in small samples by a considerable amount. Thus the failures indicated here may be exaggerated. When the list of co-movement statistics is shortened somewhat, as with H_2 , the only model not rejected at the 5 per cent level is the unrestricted MS-A model. Both an aggregate shock and complementarities seem to be important for co-movement as defined by this hypothesis. When we examine co-movement just in terms of total consumption and total investment (H_3), the OS model is easily rejected. The MS models do considerably better, especially when there are complementarities. In this case neither MS model can be rejected at the 45 per cent level. With respect to relative price variability, the unrestricted OS model is easily rejected. Neither of the MS models, whether or not complementarities are present, are rejected with respect to H_4 at the 10 per cent level.

Thus far, comparison between models has been at a relatively informal level. A convenient tool for formalizing comparisons is Singleton's λ -statistic. Recall that λ can be used to evaluate a given null model with respect to a specific non-nested alternative by combining information from both.¹⁶ The main step in computing λ is to estimate the null and alternative model using an overidentified version of the GMM procedure described in section 3. The overidentifying restrictions imposed are that the unconditional expected value of particular sample statistics and the probability limit of the same statistics in the models are the same. So a byproduct of computing λ is that Hansen J -statistics of these overidentifying restrictions are computed. J -statistics (p -values) for hypotheses H_2 , H_3 and H_4 are reported in table 7 for unrestricted versions of the models. In addition λ -statistics for the same hypotheses are shown for tests of each of the unrestricted models against the remaining two.

Consider first the J -statistics. We see that the overidentifying restrictions are rejected easily for the OS model. For the MS-A model, overidentifying restrictions associated with H_3 and H_4 cannot be rejected at the 10 per cent level and the restrictions associated with H_2 cannot be rejected at the 3 per cent level. The MS-C model does not perform as well. All overidentifying restrictions

¹⁶The three versions of the model considered here are not, strictly speaking, nested. Thus more conventional specification tests for nested alternatives, such as likelihood ratio type tests, are not applicable.

can be rejected at the 3 per cent level.

Perhaps more interesting are the specification tests. The λ -statistics for the OS model show that this model is always rejected in favor of the two alternatives. When the MS-A model is tested against the OS model it is never rejected at even the 15 per cent level. This model also does well in terms of co-movement and relative prices when tested against the MS-C model. The MS-C model does not fare as well as its MS counterpart. While it cannot be rejected in terms of the statistics included in H_2 against the OS model at the 5 per cent level, it is rejected in terms of H_3 . The MS-C model of course dominates the OS model in terms of relative prices (H_4). The evidence against the MS-C model is even stronger when it is compared to the MS-A model. With respect to the disaggregated co-movement hypothesis, H_2 , the models are perhaps tied since the MS-C model cannot be rejected relative to the MS-A model at the 10 per cent level. However, the MS-C model is easily rejected in favor of the other MS model when H_3 is considered and is rejected at the 3 per cent level when H_4 is considered. To summarize, when the models are tested against each other in terms of co-movement and relative price variability, the MS-A model seems to perform the best.

5. Concluding remarks

A quantitative business cycle model with capital disaggregated between households and firms, which combines good-specific technology shocks, the standard aggregate technology shock and production complementarities, has been studied. Various versions of the model have been estimated and their ability to account for certain aspects of the data has been tested. The study was motivated by the fact that the canonical RBC model, once it is augmented to include a household capital good, cannot account for the pattern of co-movement among components of aggregate expenditures. In addition, evidence of significant relative price variability for household and business capital goods suggests this failure may be more pronounced than conventional one-shock models would suggest.

Once relative price variability is taken into account, it is found that complementarities in the production of capital goods is helpful in accounting for the pattern of co-movement. However, the aggregate technology shock is still vital in this respect. That is, a version of the model that is based on an alternative identification of technology shocks that assumes there is no aggregate shock is outperformed by a model based on a more standard identification scheme that assumes there is an aggregate shock. Complementarities in production alone, as estimated here, are insufficient to

account for the estimated volatility of relative prices. Good-specific shocks are crucial for accounting for these observations.

The model has been successful at accounting for certain basic features of the data. Its failures along two key dimensions are perhaps more revealing however. The model is unable to account for the relative volatility of the two forms of investment and for the near zero correlation between household investment and the relative price of household capital. A maintained hypothesis of this study is that all aggregate fluctuations are due to supply disturbances. When it comes to accounting for the behavior of variables like durable goods expenditures and residential investment, conventional wisdom seems to be that demand factors, such as those associated with monetary disturbances, are important. While this hypothesis has not been tested here, the failure of the supply shock model to account for these simple statistics leaves open the possibility that monetary factors are crucial for understanding the behavior of household investment.

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Figure 1. Responses of I_h and I_b to innovations in technology

Notes: The left hand column corresponds to the case where $\kappa = 1$ and the right hand column corresponds to the case where $\kappa = 1.08$. The impulse response functions correspond to one percentage point innovations.

Table 1. Preference and technology parameter estimates with standard errors

Parameter	OS Model		MS-A Model		MS-C Model	
	R	U	R	U	R	U
T	1369	1369	1369	1369	1369	1369
β	$1.03^{-.25}$	$1.03^{-.25}$	$1.03^{-.25}$	$1.03^{-.25}$	$1.03^{-.25}$	$1.03^{-.25}$
δ_b	0.0214 (0.0002)	0.0214 (0.0002)	0.0214 (0.0002)	0.0214 (0.0002)	0.0214 (0.0002)	0.0214 (0.0002)
δ_h	0.0219 (0.0004)	0.0219 (0.0004)	0.0219 (0.0004)	0.0219 (0.0004)	0.0219 (0.0004)	0.0219 (0.0004)
α	0.237 (0.011)	0.237 (0.011)	0.237 (0.011)	0.237 (0.011)	0.237 (0.011)	0.237 (0.011)
κ	1	1.015 (0.014)	1	1.068 (0.031)	1	1.111 (0.068)
η	0.390 (0.008)	0.390 (0.008)	0.390 (0.008)	0.390 (0.008)	0.390 (0.008)	0.390 (0.008)
λ	5.206 (0.086)	5.206 (0.088)	5.206 (0.088)	5.206 (0.088)	5.206 (0.088)	5.206 (0.088)

Notes: Standard errors are reported in parentheses only for estimated parameters. Other parameters were set a priori. The one shock model estimates and standard errors are in the column with the header 'OS Model'. Similarly estimates and standard errors for the multiple shock model with the aggregate shock identification scheme are in the columns with the header 'MS-A Model'. The entries in the columns with the header 'MS-C Model' are for the multiple shock model and the consumption sector shock identification scheme. The header 'R' indicates entries in the column are for restricted models, that is with κ restricted to equal unity. The header 'U' is for unrestricted models with κ estimated via the method-of-moments procedure outlined in the text.

Table 2. Forcing process parameter estimates with standard errors

Parameter	OS Model		MS-A Model		MS-C Model	
	R	U	R	U	R	U
ϵ_i	0.308 (0.180)	0.308 (0.180)	0.308 (0.180)	0.308 (0.180)	0.235 (0.180)	0.235 (0.180)
ϵ_h	0.278 (0.087)	0.278 (0.087)	0.278 (0.089)	0.278 (0.090)	0.513 (0.139)	0.512 (0.154)
ϵ_b	0.077 (0.127)	0.077 (0.126)	0.077 (0.131)	0.077 (0.127)	0.311 (0.195)	0.313 (0.182)
σ_i	1.421 (0.153)	1.421 (0.153)	1.421 (0.157)	1.421 (0.157)	0.659 (0.222)	0.659 (0.222)
σ_h	NA	NA	0.896 (0.073)	0.888 (0.071)	0.748 (0.227)	0.884 (0.154)
σ_b	NA	NA	0.947 (0.119)	0.913 (0.112)	1.198 (0.237)	1.069 (0.275)
ρ_{ih}	NA	NA	-0.349 (0.118)	-0.276 (0.119)	0.193 (0.466)	0.511 (0.278)
ρ_{ib}	NA	NA	0.050 (0.157)	-0.014 (0.173)	0.616 (0.161)	0.545 (0.239)
ρ_{hb}	NA	NA	0.304 (0.116)	0.381 (0.103)	0.452 (0.206)	0.511 (0.185)

Notes: See the notes to table 1 for an explanation of the column headings. Means and standard deviations of the forcing processes are measured in per cent. NA denotes not applicable. The subscript 'i' refers to either 'A' or 'C', depending on the model in question. It stands for 'A' in the OS Model and the MS-A Model columns. It stands for 'C' in the MS-C Model columns.

Table 3. Model predictions for co-movement

Statistic	U.S. Data	OS Model		MS-A Model		MS-C Model	
		R	U	R	U	R	U
$\rho(I_h, I_b)$	0.364 (0.121)	-0.876 (0.013) [0]	0.364	-0.953 (0.010) [0]	0.364	-0.958 (0.022) [0]	0.364
$\rho(C, I_h)$	0.706 (0.099)	0.0004 (0.040) [0]	0.720 (0.563) [0.98]	0.116 (0.111) [0]	0.232 (0.158) [0.01]	0.274 (0.371) [0.22]	-0.141 (0.349) [0.02]
$\rho(C, I_b)$	0.504 (0.134)	0.461 (0.015) [0.74]	0.890 (0.048) [0.008]	0.048 (0.131) [0.008]	0.479 (0.104) [0.87]	-0.209 (0.415) [0.08]	0.086 (0.332) [0.26]
$\rho(I_h, Y)$	0.850 (0.046)	0.014 (0.015) [0]	0.643 (0.577) [0.72]	0.020 (0.020) [0]	0.581 (0.139) [0.043]	0.041 (0.041) [0]	0.521 (0.184) [0.042]
$\rho(I_b, Y)$	0.660 (0.085)	0.468 (0.013) [0.025]	0.946 (0.048) [0.005]	0.239 (0.034) [0]	0.837 (0.032) [0.048]	0.204 (0.096) [0.001]	0.745 (0.128) [0.63]
$\rho(C, Y)$	0.828 (0.045)	0.977 (0.002) [0.001]	0.986 (0.002) [0.001]	0.821 (0.053) [0.91]	0.817 (0.059) [0.89]	0.661 (0.204) [0.45]	0.600 (0.271) [0.42]
$\rho(I, Y)$	0.946 (0.015)	0.996 (0.0002) [0.001]	0.997 (0.0003) [0.001]	0.969 (0.008) [0.21]	0.958 (0.013) [0.57]	0.942 (0.036) [0.93]	0.913 (0.054) [0.57]

Notes: See table 1 for an explanation of the column headings. Results are from using the HP filter. The numbers in the 'U.S. Data' column are point estimates of based on U.S. data for the statistic. The numbers in parenthesis are associated standard errors. The numbers in the remaining columns correspond to probability limits of the statistics for the indicated model. Numbers in parenthesis are standard errors computed in the manner described in the text. The numbers in square brackets are p -values associated with the Q -test described in the text. In each case the test is whether the model implied statistic is equal to the data generated statistic. Standard errors and p -values are not included in the entries for $\rho(I_b, I_h)$ in the columns with the header 'U'. This follows from the fact that this moment was used to estimate the transformation relation parameter κ in these cases. See the text for details.

Table 4. Model predictions for relative prices

Statistic	U.S. Data	OS Model		MS-A Model		MS-C Model	
		R	U	R	U	R	U
σ_{P_h}/σ_Y	0.430 (0.104)	0	0.023 (0.010) [0]	0.494 (0.056) [0.38]	0.547 (0.068) [0.16]	0.788 (0.235) [0.12]	0.935 (0.343) [0.13]
σ_{P_b}/σ_Y	0.543 (0.104)	0	0.024 (0.012) [0]	0.522 (0.078) [0.86]	0.530 (0.081) [0.91]	0.833 (0.257) [0.29]	0.874 (0.307) [0.30]
$\rho(I_h, P_h)$	0.196 (0.099)	NA	-0.043 (0.971) [0.81]	-0.143 (0.030) [0.005]	-0.649 (0.064) [0]	-0.172 (0.055) [0.003]	-0.724 (0.127) [0]
$\rho(I_b, P_b)$	-0.307 (0.133)	NA	0.946 (0.021) [0]	-0.342 (0.045) [0.82]	-0.534 (0.145) [0.28]	0.833 (0.070) [0.53]	-0.720 (0.151) [0.076]

Notes: See the notes to table 1 for an explanation of the column headings. See the notes to table 3 for an explanation of the table entries. In the case of the Restricted OS Model relative prices are constant. Thus the volatility statistics are identically zero in this case and the correlation coefficient is not defined.

Table 5. Model predictions for quantity volatility

Statistic	U.S. Data	OS Model		MS-A Model		MS-C Model	
		R	U	R	U	R	U
σ_Y	2.44 (0.28)	2.28 (0.25) [0.66]	2.11 (0.24) [0.36]	2.34 (0.26) [0.79]	2.03 (0.23) [0.24]	1.47 (0.45) [0.049]	1.20 (0.44) [0.009]
σ_N	1.64 (0.15)	1.03 (0.12) [0.001]	0.89 (0.10) [0]	1.19 (0.14) [0.02]	0.96 (0.11) [0]	0.84 (0.21) [0.001]	0.67 (0.16) [0]
σ_{I_h}/σ_Y	2.78 (0.19)	6.85 (0.50) [0]	1.06 (0.30) [0]	13.3 (1.9) [0]	1.83 (0.15) [0]	15.7 (5.7) [0.02]	2.55 (0.74) [0.76]
σ_{I_b}/σ_Y	1.61 (0.18)	8.16 (0.61) [0]	3.27 (0.47) [0]	14.8 (2.2) [0]	3.31 (0.59) [0.008]	17.5 (6.7) [0.02]	3.73 (0.85) [0.02]
σ_C/σ_Y	0.42 (0.03)	0.46 (0.01) [0]	0.49 (0.01) [0]	0.52 (0.03) [0]	0.58 (0.04) [0.008]	0.58 (0.10) [0.02]	0.67 (0.14) [0.02]
σ_I/σ_Y	1.78 (0.06)	1.92 (0.05) [0.14]	1.86 (0.06) [0.41]	2.01 (0.10) [0.10]	1.95 (0.10) [0.22]	2.12 (0.25) [0.24]	2.15 (0.38) [0.37]

Notes: See the notes to table 1 for an explanation of the column headings. See the notes to table 3 for an explanation of the table entries.

Table 6. Joint tests of model predictions for co-movement and relative prices

Hypothesis	OS Model		MS-A Model		MS-C Model	
	R	U	R	U	R	U
H_1	597.1 [0]	19.7 [0.001]	390.0 [0]	50.0 [0]	323.2 [0]	38.5 [0]
H_2	405.3 [0]	17.7 [0.001]	240.9 [0]	6.6 [0.086]	213.9 [0]	11.7 [0.008]
H_3	12.9 [0.002]	13.7 [0.001]	4.9 [0.087]	1.5 [0.47]	1.6 [0.45]	0.7 [0.72]
H_4	NA	42.9 [0]	2.0 [0.37]	4.5 [0.11]	3.3 [0.19]	3.8 [0.15]

Notes: See the notes to table 1 for an explanation of the column headings. The entries without square brackets are Q -statistics computed as described in the text. The entries in square brackets are the associated p -values. The test statistics are χ^2 random variables with degrees of freedom equal to the number of statistics included in the null hypothesis. The hypotheses tested involve the following lists of statistics:

H_1 : $\rho(C, Y)$, $\rho(I_h, Y)$, $\rho(I_b, Y)$, $\rho(C, I_h)$ and $\rho(C, I_b)$

H_2 : $\rho(I_h, Y)$, $\rho(C, I_h)$ and $\rho(C, I_b)$

H_3 : $\rho(C, Y)$ and $\rho(I, Y)$

H_4 : σ_{F_h}/σ_Y and σ_{F_b}/σ_Y

Table 7. Specification tests

H_0	H_1	J_2	J_3	J_4	λ_2	λ_3	λ_4
OS Model	MS-A Model	15.2 [0.002]	19.1 [0]	13.4 [0.001]	11.8 [0.001]	17.9 [0.0006]	11.7 [0]
	MS-C Model				14.0 [0.0001]	20.5 [0]	11.5 [0.0007]
MS-A Model	OS Model	9.1 [0.03]	3.5 [0.17]	3.7 [0.16]	0.46 [0.50]	2.0 [0.16]	1.8 [0.19]
	MS-C Model				0.01 [0.90]	2.4 [0.12]	1.6 [0.20]
MS-C Model	OS Model	10.1 [0.02]	2.2 [0.02]	8.8 [0.01]	2.8 [0.09]	12.1 [0.0005]	0.54 [0.46]
	MS-Y Model				2.3 [0.12]	16.9 [0]	5.4 [0.02]

Notes: See the notes to table 1 for an explanation of the model names. H_0 is the null hypothesis for the λ -test described in the text and H_1 is the specified alternative. The columns with the headers J_i , $i = 2, 3, 4$ contain Hansen J -statistics for the overidentifying restrictions associated with a particular specification test. These entries in a particular row correspond to the null hypothesis in that row. The columns with the headers λ_i , $i = 2, 3, 4$ contain Singleton λ -statistics for specification tests associated with different sets of statistics. The three sets used correspond to the hypothesis with the same subscripts described in table 6. Entries without square brackets are test statistics and entries in square brackets are p -values. Both test statistics are χ^2 random variables. The Q -tests have degrees of freedom equal to the number of statistics included in the test and the λ -tests have one degree of freedom.

A. A description of the dataset

In this appendix I elaborate on how I constructed the dataset used in this study. As much as possible, the model has been used as a guide for how to construct empirical measures for variables in the model. In contrast to most existing quantitative business cycle studies, relative prices of contemporaneously dated goods are not assumed to be constant. This plays a major part in how the series are put together.

I begin by defining series that form the basis for the actual series used in the study. Unless otherwise noted, Citibase series names are given in parenthesis. Time subscripts are dropped to simplify the notation. Most of the series extend beyond the 1955:3-1988:2 sample of the study. This sample period was chosen because it was the maximal one that contained all the series. The household hours series used covers this period and was the main constraint when deciding on the sample period to use.

- C^N : the sum of nominal series for nondurable consumption (GCN), service consumption (GCS) less housing services (GCSH).
- C^R : the sum of 1982 constant dollar series for nondurable consumption (GCN82), service consumption (GCS82), less housing services (GCSH82).
- P_C : the ratio of C^N to C^R .
- I_h^N : the sum of nominal series for durable consumption (GCD), private residential investment (GIR) and government residential investment (from various issues of the Survey of Current Business, and series provided by the Department of Commerce in private correspondence).
- I_h^R : the sum of 1982 constant dollar series for durable consumption (GCD82), private residential investment (GIR82) and government residential investment (from various issues of the Survey of Current Business, and series provided by the Department of Commerce in private correspondence).
- I_b^N : the sum of nominal series for investment in producer durables goods (GIPD), producer structures (GIS) and government nonresidential investment (from various issues of the Survey of Current Business, and series provided by the Department of Commerce in private correspondence).
- I_b^R : the sum of 1982 constant dollar series for investment in producer durables goods (GIPD82), producer structures (GIS82) and government nonresidential investment (from various issues of the Survey of Current Business, and series provided by the Department of Commerce in private correspondence).
- V^N : nominal business inventories (GVF+GVU)
- POP : efficiency weighted population measure provided by Gary Hansen and described in Hansen (1992).

Using these definitions, the empirical measures of the variables in the model are as follows:

- $C = C^R / POP$
- $I_h = I_h^R / POP$
- $I_b = I_b^R / POP$
- $P_h = (I_h^N / I_h^R) / P_C$
- $P_b = (I_b^N / I_b^R) / P_C$
- $I = P_h I_h + P_b I_b$
- $Y = C + I + (V^N / POP) / P_C$.

In an attempt to account for measurement error in hours worked data (which may lead to overstating the volatility of technology as measured by the Solow residual), two measures of hours were used. These were an establishment hours series from Citibase (LPMHU) and a series based on household surveys conducted by the Bureau of Labor Statistics. This latter series has been efficiency weighted by Gary Hansen and it is the efficiency weighted series I use here. See Hansen (1992) for a discussion of this series.

The way I identify the price of business capital here deserves some discussion. Gordon (1990) has argued that BEA measures of durable goods prices do not take enough into account quality changes in capital goods. He constructed a measure of durable goods prices, which he argues persuasively does a much better job at taking into account quality changes, that displays a much greater trend than the measure used here. The series Gordon constructed is annual and so was not used to construct the benchmark dataset. It is hoped that while the BEA series may miss the long run trend, its high frequency components are acceptable. As a gauge of the robustness of the findings here to this measurement issue I conducted some limited experiments with a data set based on Gordon's series. Gordon's series was interpolated using related series using the BEA deflator, a constant and a trend (the method is due to Fernandez). Since it ends in 1983, this series is spliced to the series described above (the BEA producer durable deflator). This new producer durable deflator was then used to construct a constant 1982 dollar producer durables measure which was used to form an alternative series to I_b^R described above. The rest of the dataset was left unchanged. When the empirical analysis described in the body of the paper was conducted using this modified set of data, the qualitative findings were unchanged.

B. Restrictions on preferences implied by balanced growth and trends in relative prices

In this appendix I show how results from Fisher (1994) place restrictions on preferences. Two key results reported there are as follows. First, expenditure shares for nondurable consumption and services, household investment and business investment are consistent with the balanced growth hypothesis. That is, they are stationary random variables despite the apparent nonstationarity in the ratio of the quantity of household investment to output. Second, the relative price of household capital, measured in the manner described in appendix A, displays a pronounced downward trend.

For preferences to be consistent with balanced growth, King, Plosser and Rebelo (1988) show

the momentary utility function must have the following form:¹⁷

$$\begin{aligned} & \frac{\tilde{C}_t^{1-\sigma}-1}{1-\sigma} V(T-H_t), \quad 0 < \sigma, \sigma \neq 1 \\ & \ln \tilde{C}_t + V(T-H_t), \quad \sigma = 1, \end{aligned}$$

where $\tilde{C}_t$ is a bundle of consumption services. To establish the further restrictions on preferences implied by the trend in the relative price of household capital, it is convenient to adopt the following functional form for consumption services:

$$\tilde{C}_t = [C_t^\nu + \eta K_{h,t}^\nu]^{1/\nu}. \quad (\text{B.1})$$

Here η is a positive scalar and $\nu \geq 1$. The parameter ν governs the elasticity of substitution between the services from household capital and consumption goods purchased from firms. The result established in this appendix is that for preferences to be consistent with balanced growth and the trend in the price of household capital, it must be the case that $\nu = 0$.

To establish this result, first assume, without loss of generality, that $V(T-H_t) = 1$. Now, consider the (nonstochastic) Euler equation for household capital accumulation:

$$\tilde{C}_t^{-\sigma} \left(\frac{C_t}{\tilde{C}_t} \right)^{\nu-1} P_{h,t} = \beta \tilde{C}_{t+1}^{-\sigma} \left[\left(\frac{C_{t+1}}{\tilde{C}_{t+1}} \right)^{\nu-1} P_{h,t+1} (1-\delta_h) + \eta \left(\frac{K_{h,t+1}}{\tilde{C}_{t+1}} \right)^{\nu-1} \right]. \quad (\text{B.2})$$

In a competitive equilibrium,

$$P_{h,t} = \frac{z_{c,t}}{z_{h,t}} \left[\frac{\hat{I}_{h,t}}{\hat{I}_t} \right]^{\kappa-1}.$$

A trend in the relative price of household capital requires $z_{c,t}/z_{h,t}$ to have a trend. Without loss of generality, assume $z_{c,t} = 0, \forall t$. Then, after substituting the resulting expression for $P_{h,t}$ into (B.2) and dividing the equation through by

$$C_{t+1}^{-\sigma} \left(\frac{C_{t+1}}{\tilde{C}_{t+1}} \right)^{\nu-1} \frac{1}{z_{h,t+1}}$$

we arrive at

$$\begin{aligned} & \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{\sigma+\nu-1} \left(\frac{C_t}{C_{t+1}} \right)^{\nu-1} \frac{z_{h,t+1}}{z_{h,t}} \left[\frac{\hat{I}_{h,t}}{\hat{I}_t} \right]^{\kappa-1} \\ & = \beta \left[(1-\delta_h) \left[\frac{\hat{I}_{h,t+1}}{\hat{I}_{t+1}} \right]^{\kappa-1} + \eta z_{h,t+1} \left(\frac{K_{h,t+1}}{C_{t+1}} \right)^{\nu-1} \right]. \end{aligned} \quad (\text{B.3})$$

Along a balanced growth path endogenous variables in (B.3) will all grow at constant rates. In the present model this implies that:

$$C_t = z_{a,t} z_{b,t}^{\alpha/(1-\alpha)} c;$$

$$K_{h,t+1} = z_{a,t} z_{b,t}^{\alpha/(1-\alpha)} z_{h,t} k_h;$$

$$\hat{I}_t = z_{a,t} z_{b,t}^{\alpha/(1-\alpha)} \hat{i};$$

¹⁷The notation in this appendix follows that in the text.

$$\hat{f}_{h,t} = z_{a,t} z_{b,t}^{\alpha/(1-\alpha)} i_h;$$

$$\tilde{C}_t = z_{a,t} z_{b,t}^{\alpha/(1-\alpha)} z_{h,t} \tilde{c}.$$

Here I denote by small case letters the non-stochastic steady state values of variables from the stationary representation of the model. The last expression in this list follows from the fact that at t gets large, $K_{h,t}$ will dominate in the expression defining $\tilde{C}_t$.

If we substitute these expressions into (B.3) and cancel terms we arrive at

$$\begin{aligned} & \left(\frac{z_{a,t} z_{b,t}^{\alpha/(1-\alpha)} z_{h,t}}{z_{a,t+1} z_{b,t+1}^{\alpha/(1-\alpha)} z_{h,t+1}} \right)^{\sigma+\nu-1} \left(\frac{z_{a,t} z_{b,t}^{\alpha/(1-\alpha)}}{z_{a,t+1} z_{b,t+1}^{\alpha/(1-\alpha)}} \right)^{\nu-1} \frac{z_{h,t+1}}{z_{h,t}} \left[\frac{z_{a,t} z_{b,t}^{\alpha/(1-\alpha)} i_h}{z_{a,t+1} z_{b,t+1}^{\alpha/(1-\alpha)} i} \right]^{\kappa-1} \\ &= \beta \left[(1 - \delta_h) \left[\frac{z_{a,t} z_{b,t}^{\alpha/(1-\alpha)} i_h}{z_{a,t+1} z_{b,t+1}^{\alpha/(1-\alpha)} i} \right]^{\kappa-1} + \eta \frac{z_{h,t+1}}{z_{h,t}} z_{h,t}^{\nu} \left(\frac{k_h}{c} \right)^{\nu-1} \right]. \end{aligned}$$

Since the technology terms grow at constant rates, the entire left hand side of this equation must be a constant along a balanced growth path (in the stochastic economy it would be a stationary random variable). However, if there is a trend in the relative price of household capital the right hand side will be growing (will be a nonstationary random variable), unless $\nu = 0$. Thus, for a balanced growth path to exist as a competitive equilibrium in this economy, it must be the case that $\nu = 0$.

This restriction underlies the choice of preferences in the main text. The principle implication this restriction has for co-movement is that it restricts to unity the elasticity of substitution between business produced consumption goods and services from household capital. Without this restriction, complementarities in preferences could in principle account for the pattern of co-movement found in the data. It should be pointed out that the restriction $\nu = 0$ could be circumvented if, for example, a particular trend is assumed in the productivity of household capital relative to market-produced nondurables and services in the service flow expression (B.1). This trend would have to *exactly* offset the trend in the relative price of household capital, which would require that household capital become progressively *less* efficient in home production over time. This does not seem to be a reasonable hypothesis.

C. Details of the estimation and diagnostic procedures

In this appendix I accomplish three tasks. First, I describe the GMM procedures used to estimate Ψ_1 and Ψ_2 . Second, I describe the Q , λ and Hansen J statistics. Third, I describe how measurement error in hours worked data was incorporated in to the analysis.

For all versions of the model, the following list of unconditional moment conditions was used to estimate $\Psi_{1,1}$

$$E \left\{ \delta_h - 1 + \frac{K_{h,t+1}}{K_{h,t}} - \frac{I_{h,t}}{K_{h,t}} \right\} = 0; \quad (C.1)$$

$$E \left\{ \delta_b - 1 + \frac{K_{b,t+1}}{K_{b,t}} - \frac{I_{b,t}}{K_{b,t}} \right\} = 0; \quad (C.2)$$

$$E \left\{ \beta^{-1} - (1 - \delta_h) \frac{P_{h,t+1} C_t}{P_{h,t} C_{t+1}} - \eta \frac{C_t}{P_{h,t} K_{h,t+1}} \right\} = 0; \quad (C.3)$$

$$E \left\{ \beta^{-1} - \frac{C_t}{C_{t+1}} \left[\alpha \frac{Y_{t+1}}{P_{b,t} K_{b,t+1}} + (1 - \delta_b) \frac{P_{b,t+1}}{P_{b,t}} \right] \right\} = 0; \quad (C.4)$$

$$E \left\{ (1 - \alpha) \frac{T - N_t}{N_t} \frac{Y_t}{C_t} - \gamma \right\} = 0. \quad (C.5)$$

These equations follow directly from assumptions made in the text. Equations (C.3) and (C.4) follow from applying the law of iterated expectations to the Euler equations for household and business capital respectively.

The unconditional moment conditions used to estimate $\Psi_{1,2}$ depend on the version of the model being considered. Since obvious modifications apply, I only list the conditions used for the MS-A model:

$$E\{\epsilon_{i,t} - \epsilon_i\} = 0, i = a, h, b; \quad (C.6)$$

$$E\{(\epsilon_{i,t} - \epsilon_i)^2 - \sigma_i^2\} = 0, i = a, h, b; \quad (C.7)$$

$$E\{(\epsilon_{i,t} - \epsilon_i)(\epsilon_{j,t} - \epsilon_j) - \sigma_i \sigma_j \rho_{ij}\} = 0, i = a, m, j = m, h, i \neq j. \quad (C.8)$$

The unconditional moment conditions used to estimate the elements of Ψ_2 were

$$E\{X_t^2 - \sigma_X^2\} = 0, X = Y, H; \quad (C.9)$$

$$E\{Y_t^2[\sigma_X/\sigma_Y] - X_t^2\} = 0, X = C, I_h, I_b, I, P_h, P_b; \quad (C.10)$$

$$E\{\sigma_X \sigma_Y \rho(C, X) - C_t X_t\} = 0, X = I_h, I_b, Y; \quad (C.11)$$

$$E\{\sigma_X \sigma_Y \rho(X, Y_t) - X_t Y_t\} = 0, X = I, I_h, I_b. \quad (C.12)$$

$$E\{\sigma_X \sigma_Y \rho(I_h, I_b) - I_{h,t} I_{b,t}\} = 0; \quad (C.13)$$

Here I have made use of the fact that data transformed using the HP filter have zero mean by construction. For the case where κ is restricted, $\rho(I_h, I_b)$ is estimated via (C.13). For the case where κ is unrestricted, $\rho(I_h, I_b)$ is not estimated separately because it is used to identify κ . In this second case (3.7) replaces the $\rho(I_h, I_b)$ condition represented by (C.13).

To define the joint estimator for Ψ_1 and Ψ_2 (again, I emphasize the MS-A model with the obvious modifications for the other models), consider the following generic representation of the moment conditions

$$E[R_t(\Psi^0)] = 0, \forall t \geq 0.$$

Here Ψ^0 is the true value of Ψ and R_t denotes the vector of moment conditions underlying the estimator of Ψ^0 before expectations are applied. Let g_T represent the vector valued function

$$g_T(\Psi) = (1/T) \sum_{t=0}^{\infty} R_t(\Psi).$$

Here T denotes sample size. Under the conditions set forth in Hansen (1982), Ψ^0 can be consistently estimated by choosing the value of Ψ , say Ψ_T that minimizes the quadratic form

$$J_T = T g_T(\Psi)' S_T^{-1} g_T(\Psi).$$

Here S_T is a consistent estimate of the spectral density matrix of $R_t(\Psi^0)$ evaluated at frequency

zero.¹⁸

A consistent estimator of the variance covariance matrix of Ψ_T is given by

$$\text{var}(\Psi_T) = D_T' S_T^{-1} D_T,$$

where $D_T = \nabla g_T(\Psi_T)$. For exactly identified systems, the minimized value of J_T is zero. For the Singleton specification tests an overidentified system is estimated. We can test the overidentified system by using the fact established in Hansen (1982) that the minimized value of J_T is asymptotically distributed as a χ^2 random variable with degrees of freedom equal to the number of overidentifying restrictions. This is what I refer to as the Hansen J statistic.

The Q -statistic can be described as follows. Let ω denote the $m \times 1$ vector that includes as its elements the population moments to be studied. Given a set of values for Ψ_1 the model implies particular values for ω . Let $G : \mathbb{R}^n \mapsto \mathbb{R}^m$, where n is the dimension of Ψ_1 . Define the function F as

$$F(\Psi) = G(\Psi_1) - A\Psi.$$

Here A is a conformable matrix of zeros and ones with the property $A\Psi = \omega$. The null hypothesis that the model is correctly specified can be written as,

$$H_0 : F(\Psi^0) = 0.$$

Using the delta method we have, under the null,

$$\text{var}[F(\Psi_T)] = \nabla F(\Psi_T) \text{var}(\Psi_T) \nabla F(\Psi_T)'$$

Combined with results in Eichenbaum, Hansen and Singleton (1984) and Newey and West (1987), this implies the test statistic

$$Q = F(\Psi_T)' \text{var}[F(\Psi_T)]^{-1} F(\Psi_T), \quad (\text{C.14})$$

is asymptotically distributed as a χ^2 random variable with m degrees of freedom. Using Q I test hypotheses of the form H_0 .

The Singleton specification test involves incorporating information about a specific alternative when testing the null that a model is correctly specified. Suppose we have a null model under consideration and wish to test it against a specific alternative. To construct the λ statistic one would proceed as follows. First, estimate in the manner described above, both models separately. Denote by $g_{i,T}$, $S_{i,T}$ and $D_{i,T}$ the g_T , S_T and D_T evaluated at the parameter estimates for the null model ($i = N$) and the alternative model ($i = A$). Now define

$$\lambda = T \frac{\{[g_{N,T} - g_{A,T}]' S_{N,T}^{-1} g_{N,T}\}^2}{\sigma_\lambda^2},$$

where,

$$\sigma_\lambda^2 = [g_{N,T} - g_{A,T}]' S_{N,T}^{-1} \Sigma_{N,T} S_{N,T}^{-1} [g_{N,T} - g_{A,T}]'$$

and

$$\Sigma_{N,T} = S_{N,T} - D_{N,T} (D_{N,T}' S_{N,T}^{-1} D_{N,T})^{-1} D_{N,T}'.$$

¹⁸To construct the estimator S_T I used the QS-kernal estimator described in Andrews (1992). The reported results were calculated with a lag window equal to 3.

Singleton shows that λ is distributed χ^2 with one degree of freedom. Large values of λ for any pair of null and alternative models is evidence against the null and in favor of the alternative, since the probability limit of $[g_{N,T} - g_{A,T}]' S_{N,T}^{-1} g_{N,T}$ is a positive number under the alternative.

Now I show how the estimation procedure outlined above must be modified to take into account measurement error in hours worked data. The two measures of hours worked are assumed to be related to true hours in the following way

$$\ln(H_t^e) = \ln(H_t^*) + v_t^e;$$

$$\ln(H_t^h) = \ln(H_t^*) + v_t^h,$$

where H_t^* denotes true hours worked, H_t^e represents establishment hours, H_t^h represents household hours, and v_t^e and v_t^h are iid and orthogonal to each other and to H_t^* . I use household hours in my empirical work so I need an estimate of $\sigma_{v^h}^2$, the variance of v_t^h . This is obtained by adding the following moment condition to the GMM estimator described above

$$E\{\sigma_{v^h}^2 - .5[\Delta \ln(H_t^h)]^2 + .5\Delta \ln(H_t^e)\Delta \ln(H_t^h)\} = 0.$$

I now show how the moment conditions described above must be modified to account for measurement error. The notational convention below is that asterisks denote true measures of variables.

Consider first the OS and MS-A models. Equation (3.2) implies that my measure of the aggregate technology shock process is

$$\epsilon_{a,t} = \frac{1}{1-\alpha}[\Delta \ln(Y_t) - \alpha \Delta \ln(K_{b,t})] - \Delta \ln(H_t)$$

In the OS and MS-A models only equation (C.7) needs to be modified to account for measurement error. The modification follows from

$$\begin{aligned} E\{[\epsilon_{a,t} - \epsilon_a]^2 - \sigma_a^2\} &= E\{[\epsilon_{a,t}^* - \Delta v_t^h - \epsilon_a]^2 - \sigma_a^2\} \\ &= E\{[\epsilon_{a,t}^* - \epsilon_a]^2 - 2\Delta v_t^h[\epsilon_{a,t}^* - \epsilon_a] + \Delta v_t^{h^2} - \sigma_a^2\} \\ &= E\{\Delta v_t^{h^2} - 2\Delta v_t^h[\epsilon_{a,t}^* - \epsilon_a]\} \\ &= E\{\Delta v_t^{h^2}\} \\ &= 2\sigma_{v^h}^2. \end{aligned}$$

So, for the OS and MS-A models, I use the condition

$$E\{[\epsilon_{a,t} - \epsilon_a]^2 - \sigma_a^2 - 2\sigma_{v^h}^2\} = 0$$

to estimate σ_a^2 . The conditions involving $\epsilon_{h,t}$ and $\epsilon_{b,t}$ are unaffected by measurement error. This is because hours worked data are not used to derive the $\epsilon_{h,t}$ and $\epsilon_{b,t}$ series and because of the orthogonality assumptions regarding v_t^h . The only other condition involving hours worked is (C.5). It is straightforward to show that this condition remains valid up to a first order approximation.

Now consider the MS-C model. Equation (3.2) implies that my measure of the consumption good-specific technology shock process is

$$\begin{aligned} \epsilon_{c,t} &= \Delta \ln(Y_t) - \alpha \Delta \ln(K_{b,t}) - (1-\alpha)\Delta \ln(H_t) \\ &= \Delta \ln(Y_t) - \alpha \Delta \ln(K_{b,t}) - (1-\alpha)\Delta \ln(H_t^*) - (1-\alpha)\Delta v_t^h \\ &= \epsilon_{c,t}^* - (1-\alpha)\Delta v_t^h. \end{aligned}$$

Following the steps taken above, we have that the condition used to estimate σ_c is

$$E\{[\epsilon_{c,t} - \epsilon_c]^2 - \sigma_c^2 - 2(1 - \alpha)^2 \sigma_v^2\} = 0.$$

Unlike the other two models, the moment conditions involving $\epsilon_{h,t}$ and $\epsilon_{b,t}$ must be modified here. The condition used to estimate ρ_{cb} is

$$\begin{aligned} & E\{\sigma_c^2 + (\gamma_{b,t} - \gamma_b)(\epsilon_{c,t} - \epsilon_c) - \sigma_c \sigma_b \rho_{cb}\} \\ &= E\{(\epsilon_{c,t}^* - \epsilon_c)^2 + (\gamma_{b,t} - \gamma_b)(\epsilon_{c,t}^* - \epsilon_c) - \sigma_c \sigma_b \rho_{cb}\} \\ &= E\{[(\epsilon_{c,t}^* - \epsilon_c) + (\gamma_{b,t} - \gamma_b)](\epsilon_{c,t}^* - \epsilon_c) - \sigma_c \sigma_b \rho_{cb}\} \\ &= E\{(\epsilon_{b,t}^* - \epsilon_m)(\epsilon_{c,t}^* - \epsilon_c) - \sigma_c \sigma_b \rho_{cb}\} \\ &= 0, \end{aligned}$$

where $\gamma_{b,t}$ equals the right hand side of (3.6) and $\gamma_b \equiv \epsilon_b - \epsilon_c$. A similar condition is used to estimate ρ_{ch} . Finally, the condition used to estimate ρ_{hb} is given by

$$\begin{aligned} & E\{\sigma_c^2 + (\gamma_{b,t} - \gamma_m)(\epsilon_{ct} - \epsilon_c) + (\gamma_{h,t} - \gamma_h)(\epsilon_{c,t} - \epsilon_c) \\ & \quad + (\gamma_{b,t} - \gamma_b)(\gamma_{h,t} - \gamma_h) - \sigma_h \sigma_b \rho_{hb}\} \\ &= E\{[(\epsilon_{c,t}^* - \epsilon_c) + (\gamma_{b,t} - \gamma_b)][(\epsilon_{c,t}^* - \epsilon_c) + (\gamma_{h,t} - \gamma_h)] - \sigma_h \sigma_b \rho_{hb}\} \\ &= E\{(\epsilon_{b,t}^* - \epsilon_m)(\epsilon_{h,t}^* - \epsilon_h) - \sigma_h \sigma_b \rho_{hb}\} \\ &= 0, \end{aligned}$$

where the definitions of $\gamma_{h,t}$ and γ_h are analogous to the definitions of $\gamma_{b,t}$ and γ_b .