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ENDOGENOUS MONEY AND GOODS PRODUCTION IN A SEARCH MODEL

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Abstract

This paper presents a search theoretic model of money in which the production of both goods and money is endogenous. Money in this model is a private money such as gold. Money must have a higher production cost than consumption goods if it is to circulate as a medium of exchange. Production, storage, and depreciation costs must not be too large in order for a money to be acceptable. Multiple media of exchange are possible provided no money has strictly higher costs of use. Optimal government policy may be to either tax private money production costs or to introduce a fiat money in order to decrease the supply of the commodity money.

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1. Introduction:

Many monetary models define money as an intrinsically useless, unbacked, and exogenously given flat currency. At many times in history, however, people have used commodities as media of exchange. The intrinsic properties of these commodities have long been discussed (see Jevons (1877)) but little effort has been made to incorporate them into the basic structure of monetary models. Kiyotaki and Wright (1989) have done some work on modelling the emergence of a commodity money as a medium of exchange in a search model. It was generally true in that model that the commodity chosen as the medium of exchange was that commodity with the lowest storage cost. There were, however, also examples of a 'speculative equilibrium' in which certain agents sometimes chose a high storage cost good as the medium of exchange in addition to the low cost good.

The purpose of this paper is to further expand this area of the literature that studies commodity money. A potential medium of exchange will be endowed with certain intrinsic properties, other than just the storage costs analyzed by Kiyotaki and Wright (1989), and only under specific conditions will it emerge as an endogenous and generally accepted money. Monetary production decisions are endogenous in this model so that agents can choose not only which commodity to use as money but also how much they wish to have circulate in the economy.

Jevons (1877) has provided probably one of the best and earliest descriptions of the properties that a money must possess if it is to circulate as a medium of exchange. These characteristics can be summarized briefly as follows: utility, portability, indestructibility, homogeneity, divisibility, stability of value, and cognizability. These properties are fairly evident. Jevons argued that the commodities that emerge as media of exchange must also provide some direct utility. Gold, for instance, provides utility when used in ornamentation. He believed that habit or convention could maintain an established money in equilibrium but that initially agents would need to value the commodity for other than its exchange
services. Commodity monies need to be easily carried or transferred between agents. They must be relatively indestructible or durable and be of homogeneous quality. The commodities must be divisible so that they can be used for transactions of all sizes. There must be a stable value so that agents can be certain that it won't be dropped from the economy while held in inventory. Finally, the agents must be able to recognize the commodity and its value if they are to accept it as a medium of exchange.

After Jevons (1877) and Menger (1892), little work seems to have been done on the emergence of money as a medium of exchange and its inherent characteristics until Brunner and Meltzer (1971), Jones (1976), King and Plosser (1986), and Kiyotaki and Wright (1989). Some of the great economists of the early 20th century, notably Keynes (1936), Hicks (1967), Samuelson (1947), and Wicksell (1935), have discussed the general properties of money and the history of its evolution into its modern form. These authors reiterated the same basic characteristics of money proposed by Jevons and of others before him back to Adam Smith in 1776. This work however tended to be from a broad perspective and involved little in the way of formal modelling.

These authors and many¹ since then have completed much work studying the relative merits of commodity money systems (the Gold Standard) and fiat money economies. The general purpose of this work has been to determine whether commodity monies really can yield a more stable price system or to explain why commodity monies are more expensive in terms of resources and hence are less efficient than fiat monies. The Sargent and Wallace (1983) paper is probably one of the more comprehensive works in terms of formal modelling of commodity money and consideration of the main policy questions. Although their model does not entail an endogenous medium of exchange (it uses the OLG framework), it does determine the endogenous supply of gold. They show that the commodity money equilibrium is inefficient because of the resources used to produce gold. There can also be two commodity monies in

¹ See Friedman (1960,1963), Sargent and Wallace (1986), Nickelsburg (1985), and Fischer (1986) to list a few.
their model if they possess equal depreciation rates. The commodity monies with higher depreciation rates will have a zero supply in equilibrium. My paper generates a similar result since any commodity with higher production, storage, and depreciation costs will never circulate as an endogenous medium of exchange. If there are multiple commodity monies, then each will have lower relative costs of one type but higher costs of another.

Sargent and Wallace (1983) model the depreciation of gold as the only intrinsic property of the commodity money while Nickelsburg (1985) emphasizes the durability of the commodity backing a government issued representative money. Fischer (1986) and Whitaker (1979) model commodity monies without any consideration of their characteristics. They focus on policy issues that come into importance once the market has already selected the media of exchange in some manner. Only with the more recent literature that focuses on explaining the emergence of money in an endogenous framework has serious consideration been given once again to the properties of money. Brunner and Meltzer (1971) emphasize the low relative marginal cost of acquiring information about money in comparison to other commodities while Jones (1976) focuses on the relative degree of acceptability of a commodity in determining whether it will emerge as a medium of exchange. King and Plosser (1986) hypothesize that if the quality of gold is verifiable in a world of generally uncertain qualities then it may emerge as a medium of exchange. Finally, Kiyotaki and Wright (1989), upon which my work is most closely based, study the storage costs of commodities and the beliefs of the agents as the important factors determining whether a particular commodity could circulate as a money.

One of the purposes of this paper here is to further expand the list of characteristics of money that have been studied in a framework of endogenous media of exchange. The production cost of money, although alluded to as an important factor by Keynes (1936) and Wicksell (1935), does not seem to have previously been formally modelled. This cost is an important factor in this paper for determining the supply of money and the value of accepting money in trade. The depreciation rates and storage costs of
money have been considered by various authors as discussed above but never together within a single model.

Although a number of the models discussed above generate an endogenous supply of money (see Fischer (1986), Sargent and Wallace (1983), King and Plosser (1986), and Kiyotaki and Wright (1989)), they do not mention the possibility of an overproduction of money that is a basic result of my model. In fact, King and Plosser (1986) assert that the competitive equilibrium is equivalent to the social planners optimal allocation in their framework. As long ago as Adam Smith (1776) some economists believed that there could be an excess supply of private money if agents were free to produce it. Smith observed that the banks of his era tended to over-issue their private bank notes which were backed by gold. He argued, however, that this was not a sustainable policy and the banks would either fail or be forced to reduce their issues of bank notes. Therefore, there could be no equilibrium with an oversupply of money even though agents may attempt to achieve this. In contrast, it is possible, using the search model of this paper, to generate a sustainable steady state equilibrium in which agents produce too much money and, as a result, lower their overall welfare.

Until the recent work explaining the endogenous emergence of money as a medium of exchange, much of the work involving commodity monies has focused on the issue of whether such a money is less efficient and just how many resources are used up by such a system. They have generally taken the existence of the commodity money for granted and have only discussed in a broad and general sense the evolution or emergence of the money. This paper here is a part of the recent but expanding literature beginning at a more fundamental level and explaining first why there may be a commodity money before examining the questions regarding its relative merits.

This paper does not attempt to examine all the possible characteristics that a money should exhibit but instead concentrates on only a few properties. The commodities considered for money in this model will be of homogeneous quality, have a stable value since all prices are fixed at one, and will be easily
recognized by all agents. For simplicity, assume money is indivisible. The properties most closely modeled in this paper are portability, through the introduction of storage costs, and indestructibility, by subjecting all commodities to exogenous depreciation. The production of all goods and commodity monies is assumed to require the payment of fixed costs. These can be used to approximate the scarcity of a commodity. Scarcer products have higher production costs and quite often provide higher value.

Some of the basic predictions of the model are given here. Only potential monies that are more costly per unit to produce than consumption goods will circulate as media of exchange. This is interpreted as implying that commodity monies must be sufficiently scarce in order to be valued. Even some commodities that have greater costs in all categories than consumption goods can be used as money provided that agents are willing to do so and believe that others will also accept them in trade. Two media of exchange cannot simultaneously circulate if one strictly dominates the other in terms of lower costs of production, storage, and depreciation. If the costs of one commodity money decline, this can force agents to stop using certain types of other commodity monies. This can be thought of as being opposite to a traditional Gresham’s Law since in this model the ‘good’ money (i.e. the money with lower costs) can drive out the bad monies. A generally accepted commodity money can coexist with a fiat money even though the fiat money may have strictly lower costs. The advantage of being able to produce the commodity money yourself induces agents to use the more costly commodity money rather than wait to find the fiat money.

If the double coincidence of wants problem is sufficiently large then the production of the commodity money will be welfare improving for the agents. The money supply and output may be positively or negatively correlated in response to changes in the parameters of the model. The correlation was positive only if the initial level of goods inventories was sufficiently high.

The next section of the paper outlines the basic model with only one type of commodity money and no fiat money. Section III examines government intervention to attempt to maximize welfare.
Section IV examines the interaction of a commodity money and a fiat money. The final section provides a summary and conclusion to the paper.

II. The Model:

The model will follow those used previously in the literature, primarily that of Kiyotaki and Wright (1990). Their model can be adapted quite readily to consider the issues described above. Assume there is a unit mass of infinitely lived agents uniformly distributed on a circle of circumference two. Let there also exist a continuum of goods that are indexed by the points of the same circle. An agent’s type is identified by a particular good on the circle. Agents are assumed to receive utility from consuming any good within an exogenously specified distance $x \in [0,1]$ of the good that defines their type. For simplicity, let agents receive a fixed utility of $u$ from consuming any good within this distance $x$ of their type.

Assume also that there is a transaction cost of $\varepsilon > 0$ in utility terms that must be paid by any agent who receives a consumption good in trade. This is useful in ruling out the possibility of indirect barter trades between agents. Since it is commodity monies that are primarily being considered here, a money trade is essentially an indirect barter trade except that the intermediate step always uses the same commodity. The $\varepsilon$ rules out indirect barter trades other than those using the proposed commodity money. This is one advantage that commodities used as media of exchange have over consumption goods. Without a transaction cost, the production cost structure of the model would ensure that no consumption good emerged as a generally accepted medium of exchange. Without $\varepsilon > 0$, however, degenerate equilibria with no goods production or mixed strategy equilibria with consumption good media of exchange are possible. These equilibria are not interesting given the goal of this paper so the transaction cost is used to rule them out. This issue will be discussed in more detail below.

Assume now that an agent’s production good is known and is chosen from an independent uniform distribution on the same circle as the consumption goods. Therefore, there is a probability $x$ that a
randomly selected agent with a good will have something that a particular agent will consume. Similarly, there is a probability $x$ that a particular agent’s production good is consumable by another randomly selected agent. This implies a meeting between two agents, both of whom have goods, will result in a double coincidence of wants with probability $x^2$.

Agents are assumed not to be able or willing to consume the good that they produce themselves. As a result, they must search for trading partners in order to acquire something they will be able to consume. Agents are assumed to meet pairwise and randomly according to a Poisson process with a fixed arrival rate of $\beta > 0$. Assume that there is no mechanism by which credit may arise in the model.

Agents are assumed to be able to store only one unit of one commodity at any given time (i.e. agents with a unit of a good cannot also store another good or money). A storage cost must be paid to hold a commodity in inventory. The flow disutilities, $s_m$ and $s_g$, are paid to store money and consumption goods, respectively. These can be thought of as storage costs net of any flow utility agents may experience from storing the commodity. If $s_m > 0$, then agents pay a cost to store money. However, if $s_m < 0$, then agents receive a flow utility from simply holding the commodity money. A negative storage cost on money could approximate either a rate of return or possibly, as proposed by Jevons, the direct utility required by a commodity in order for it to serve as a medium of exchange. There will be monetary equilibria regardless of whether $s_m$ is greater or smaller than $s_g$ provided that the difference is not too large.

Assume that each agent is capable of producing both his own production good and another commodity called gold. Every agent has access to the same production technology for gold. Goods are commodities that yield utility when consumed by certain agents in the economy while gold is assumed to yield no consumption utility but may circulate as a medium of exchange if the agents choose. This model shows that a commodity may serve as a medium of exchange even if it does not yield direct utility. This is contrary to what was proposed by Jevons. The level of consumption utility provided by a
commodity may affect whether or not the agents would initially consider it as a commodity money. Utility, however, was not a requirement in this framework once agents began to use it as a medium of exchange. The benefit of improved trading opportunities outweighed any need for utility from using the money.

An agent cannot produce both goods and gold simultaneously due to the inventory restrictions that exist. Hence, if there are to be both money and goods produced in equilibrium, the agents must be indifferent between producing the two commodities. Let the variable $\theta$ represent the probability with which agents choose to produce goods and $1-\theta$ the probability they produce money. The return from each activity will be identical in an equilibrium with money.

Production is assumed to be instantaneous after the payment of a fixed cost which is measured in terms of utility. All consumption goods are subject to the same fixed production cost, $c_g>0$. A cost of $c_m>0$ must be forgone by an agent in order to produce a unit of gold. Since production is instantaneous an agent will always be able to participate in the trading market.

We shall consider equilibria in which there is an ongoing production of both goods and money. Since gold is not consumed by any agent its supply would thus tend to increase over time given agents are continuously producing it. Thus, to ensure that the money supply simply does not increase to its maximum value, assume that gold is subject to an exogenous depreciation rate to approximate the rate at which it wears out through its use as a medium of exchange. Let the Poisson arrival rate $\gamma_{in}>0$ denote the rate at which this occurs. Gold may be used fully as a medium of exchange up to that point that it perishes. After that, the gold becomes completely unusable and the agent holding it at that time is returned to production.²

² There may also be equilibria in which all of the gold desired by the agents is produced in the first instant of time. After that agents would produce only goods. For this type of equilibrium to be a steady state there could be no depreciation of gold because this would slowly erode its supply over time until none remained. This type of equilibrium will not be considered here, since we wish to examine steady state equilibria and the characteristics of money (including depreciation).
Let the arrival rate $\gamma > 0$ represent the depreciation rate for consumption goods. A good yields full utility until the time that it depreciates. After that, it is valueless to everyone and will be thrown away. The agent must then decide once again whether to produce money or goods. It is quite interesting to note that there are equilibria for this model in which the agents may endogenously choose to use a commodity as a medium of exchange that is less durable than the goods the money is used to purchase. The gold’s usefulness as a medium of exchange can outweigh the cost of depreciation.

One of the innovations of this paper is that agents are now able to produce the money used as a medium of exchange. Since agents are free to choose the amount of their production, the supply of gold is endogenously determined within the model. Agents chose not only whether or not there will be a medium of exchange but also the supply of that money which will circulate in the system. Agents can only hold zero or one unit of money, so the aggregate money supply will simply be the fraction of agents in the system who have gold in inventory. Let $m$ represent this fraction. The only other state in which an agent may be found is a goods trading state (given by the fraction $g = 1 - m$) when the agent has one unit of his production good in inventory.

The next step in defining the model is to set up the value functions for the agents in each state. Let $r$ denote the private rate of time preference. The value of having a single unit of a consumption good in inventory is then given by $V_g$ in equation (1). This agent type can perform double coincidence of wants barter trades for another good or it can trade for money. With an arrival rate of $\beta$, the agent meets trading partners. With probability $g$, the partner also has a good in inventory and, with probability $x^2$,

$$
V_g = \beta x^2 \left[ (1 - \epsilon) + \max_\theta \left\{ \theta (V_g - c_g) + (1 - \theta) (V_m - c_m) \right\} - V_g \right] + \beta \max_\pi \max_\theta \pi \left( V_m - V_g \right)
$$

$$
+ \gamma \left[ \max_\theta \left\{ \theta (V_g - c_g) + (1 - \theta) (V_m - c_m) \right\} - V_g \right] - s_g + V_g
$$

(1)

there is a double coincidence of wants so that the agents will trade. The agent consumes the new good
for a net utility of $u - \epsilon$ and then must decide once again whether to a unit of gold or another unit of the production good. A fraction $\theta$ of the agents (or similarly a fraction $\theta$ of the times an agent must produce over his lifetime) will produce a new good for a return of $(V_g - c_g)$ while the remaining agents will produce a unit of money for a return of $(V_m - c_m)$. The expected capital loss from producing after having been a goods trader is thus $[\theta(V_g - c_g) + (1 - \theta)(V_m - c_m)] - V_g]$. This shows plainly why an equilibrium with both money and goods requires identical returns to production or $(V_g - c_g) = (V_m - c_m)$. If $(V_g - c_g) > (V_m - c_m)$ then only goods are produced and if the opposite is true then only money is produced.

Agents will not participate in indirect barter trades for consumption goods because all goods provide the same trading opportunities and the transaction cost makes for a negative return to accepting a non-consumable good.

There is a probability $m_x$ that the trading partner has money, or gold, instead of a good and also wants the agent's good. Then the agent must decide with probability $\pi$ whether or not to accept the money in trade for his good. If the gold is accepted, then the agent moves into the money holding state with a net capital gain of $V_m - V_g$. In a monetary equilibrium with a universally accepted medium of exchange $\pi$ will equal 1 signifying that money is always accepted in trade. This model cannot generate mixed strategy monetary equilibria with $0 < \pi < 1$ unless production costs for money and goods are identical, i.e. $c_m = c_g$.

The rate at which consumer goods perish or depreciate is approximated by the arrival rate $\gamma_g$. After a good depreciates the agent must either produce a new good or a unit of money in order to continue trading. Depreciation represents a capital loss to the agents since the fixed production costs must to be paid again. The variable $s_g$ represents the flow storage cost paid by the agents in order to hold their production good in inventory.

Let $\dot{V}_g$ represent the rate of change of $V_g$ over time. This will be zero in the steady state equilibria to be examined in this paper. It will be argued later that the model's equilibrium will be stable.
and tend toward this steady state.

The value of holding money is represented by $V_m$ in equation (2) while $\dot{V}_m$ represents the rate of change of this value. Agents with money will also meet trading partners at the rate $\beta$. With probability $g_\Pi$ (the upper case $\Pi$ represents the strategy of the other agents) the arriving agent will be a goods trader storing a good that the money holder wants and who is willing to trade for gold. After the trade, the money holder immediately consumes the new good for a utility of $u - \varepsilon$ net of transaction costs and then produces either a good or another unit of money. With probability $\theta$ the agent will produce a unit of his production good while, with probability $1 - \theta$, a unit of gold is produced. The gold will depreciate at rate $\gamma_m$ after which the money holder suffers a capital loss and must once again produce something.

To close the model, it is necessary to specify the laws of motion governing the population fractions of agents. Using the identity $1 = g + m$ only one law of motion need be specified. There is a net outflow of agents from the goods trading state by those goods traders who produce money after barter trades or goods depreciation. The outflow of goods traders who accept money is also partially offset by those money traders who produce goods after giving up their money. Therefore, after barter trades (which occur at rate $\beta g^2 z^2$), the depreciation of goods (occurring at rate $g \gamma_g$), and money trades (occurring at rate $\beta g m x \pi$) there is a $(1 - \theta)$ probability of an outflow of an agent from the goods trading state. The inflow of agents to the goods trading state is by money traders who produce a good after their money has depreciated. This occurs at rate $m \gamma_m \theta$.

\[
\dot{g} = -[\beta g^2 z^2 + \beta g m x \pi + g \gamma_g](1 - \theta) + m \gamma_m \theta
\]  

(3)

Only the steady state equilibria of this model will be considered. This implies that, in an
equilibrium, the rate of change of the population fractions is zero so that \( g \) is calculated by setting equation (3) to zero.

A pure strategy steady state monetary equilibrium will specify a production strategy of \( 0<\theta<1 \), a money acceptance probability of \( \pi=1 \), a distribution of agents by inventories (\( g \) and \( m \)), and a pair of value functions, \( V_g \) and \( V_m \), such that: i) the return to accepting money \( (V_m - V_g) \) is positive and the agents are indifferent between money and goods production, i.e. \( (V_m - c_m) = (V_g - c_g) \), given \( g, m, V_g \) and \( V_m \), and ii) given \( \pi \) and \( \theta \), the population fractions (\( g \) and \( m \)) are in steady state (i.e. equation (3) is zero) and \( V_g \) and \( V_m \) represent the optimized values of storing goods and money, respectively. A barter equilibrium will be characterized by a production strategy of \( \theta=1 \) (i.e. a dominance of goods production over money production \( (V_g - c_g) > (V_m - c_m) \)) and a refusal to accept money (i.e. \( \pi=0 \)) because \( (V_m - V_g) \) is negative.

Assume that the initial inventories of money and goods are zero. It is possible that the economy can jump immediately to the steady state equilibrium since production decisions are made before any trade occurs and because we are considering rational expectations equilibria.

Equation (4) represents the return, \( (V_m - V_g) \), for a goods trader who sells his inventory for money. It must be assumed that utility, net of transaction costs and expected production costs, is positive, i.e. \( u = e - \theta c_g - (1-\theta)c_m > 0 \). If this restriction were violated then there would never be any production or trading and a degenerate equilibrium would result. There will at least always be barter trades when the restriction is true. The return to accepting money in equation (4) will then be positive when agents believe in the value of money (\( \pi=1 \)) and money depreciation and storage costs are smaller or at least not too much larger than the comparable goods costs.

\[
[r + \theta (\beta g x (\pi - \gamma_m) + (1-\theta)(\beta g x^2 + \gamma_x) + \beta m x x) (V_m - V_g)]
\]

\[
= \beta g x (\pi - \gamma) [u - e - \theta c_g - (1-\theta)c_m] - (\gamma_m - \gamma_x) [(\theta c_g + (1-\theta)c_m) + s_g] - s_m + \dot{V}_m - \dot{V}_g
\]  

(4)

However, equation (4) is not the only expression that defines the equilibrium return to accepting money. Recall that in order for there to be an equilibrium with both money and goods production agents
must be indifferent between producing the two types of commodities. This implies \((V_m-c_m)-(V_g-c_g)\) or \((V_m-V_g)=(c_m-c_g)\). The equilibrium return to accepting money is defined by the difference in the fixed production costs of money and goods. There will be a positive return when \(c_m>c_g\) or when there is a saving of production costs by an agent producing a good and then trading for money instead of directly producing money. When this occurs the goods traders will set \(\pi=1\) and always trade for the commodity money when the opportunity arises. Only commodities that are more costly to produce, or possibly scarcer, than consumption goods can serve as generally accepted media of exchange.\(^3\)

The agent's production decision will be determined by the difference between the gain from

\[
[r + \theta(\beta gx\pi + \gamma_m) + (1-\theta)(\beta gx^2 + \gamma_g) + \beta mx\pi ][(V_m-c_m)-(V_g-c_g)]
\]

producing the commodity money and the gain from producing goods, \((V_m-c_m)-(V_g-c_g)\), represented in equation (5). If it is positive then agents produce only money (\(\theta=0\)) while if it is negative they produce only goods (\(\theta=1\)). Only if agents are indifferent will they produce both money and goods (\(0<\theta<1\)).

Substituting \((V_m-V_g)=(c_m-c_g)\) into equation (4) or (5), then setting \(\dot{V}_m-\dot{V}_g=0\) and solving for the steady state equilibrium fraction of agents with goods and money, yields the expressions in equation (6).

The sum of the two equilibrium fractions is of course one. The money supply or fraction of agents with

\(^3\) Consider once again the importance of \(\varepsilon>0\). This parameter is used to rule out consumption goods as media of exchange. Suppose that all agents suddenly have access to the technology to produce a particular consumption good because it will not be considered as a potential medium of exchange (otherwise there is only a measure zero of agents who can produce the money). Because \(V_m-V_g=c_m-c_g\) in equilibrium, a consumption good medium of exchange would yield \(V_m-V_g=0\) since \(c_m=c_g\) for this good. There can thus be no consumption good (or any other commodity) with \(c_m=c_g\) circulating as a generally accepted medium of exchange. However, if \(\varepsilon=0\), \(c_m=c_g\), \(s_m=s_g\) and \(\gamma_m=\gamma_g\) then equation (4) yields a zero return to accepting money if either i) \(g=0\) for any \(\pi\) or ii) \(\pi=x\) for any \(g\). If \(g=0\) there is a degenerate equilibrium with no trade and every agent storing money. If \(\pi=x\), then for any \(g\) and \(\theta\), the consumption good will circulate as a partially acceptable medium of exchange in equilibrium. Any \(\varepsilon=0\) will rule out these indeterminant equilibria and allow us to consider equilibria with generally accepted media of exchange.
money in the economy responds negatively to increases in money storage or production costs or money's depreciation rate and positively to increases in the storage costs or depreciation rate on goods. Changes in the other parameters yield ambiguous results. For low values of x, \( m^* \) is increasing in x but for high values of x it declines as x rises. The velocity of barter trades is \( \beta gx^2 \) while the velocity of money trades is \( \beta gx\pi \). For low x the number of money trades increases relatively faster as x rises. Consequently, agents favour having more money to facilitate trade so \( m^* \) is increasing in x. When x is high however, the number of barter trades increases faster with x so agents favour barter and equilibrium \( m^* \) decreases.

Equation (3) can now be used to define the production strategy \( \theta \) that will achieve the equilibrium steady state mix of goods and money inventories in equation (6). This is given as \( \theta^* \) in equation (7).

\[
\theta^* = \frac{\beta g^2 x^2 + \beta g \gamma x \pi + g \gamma_g}{\beta g^2 x^2 + \beta g \gamma x \pi + g \gamma_g + m \gamma_m} \in [0, 1]
\]

A steady state monetary equilibrium will exist if the agents believe that others will accept money in trade, if the cost of money production is larger than the cost of goods production, and if the population fractions in equation (6) are within the unit interval. If any of these conditions does not hold then a barter equilibrium will result.

Consider now what occurs when the economy first begins to operate. Suppose there is an initial level of goods inventories, say \( g_0 \). Since \( g^* \) sets equation (5) to zero, if \( g_0 > g^* \), we see that \( V_m - c_m > V_g - c_g \) initially. This prompts agents to set \( \theta = 0 \) and produce only money. As time passes, goods are consumed or depreciated and are not replaced causing the equilibrium fraction of agents with goods to decline. Goods inventories will continue to decline until \( g_e = g^* \) at some time \( t \). At that point agents will observe that \( V_m^* \).
\(c_m = V_g - c_g\) and will choose \(0 < \delta^* < 1\) to maintain \(g^*\). The economy will continue to operate in this steady state forever if there is no change in the parameters. If \(g_0 < g^*\) initially, then \(V_m - c_m < V_g - c_g\) and goods inventories will rise until the steady state level of \(g^*\) is achieved.

The commodity called gold must have higher per unit production costs than the consumption goods.\(^4\) Gold may also have a higher depreciation rate and higher storage costs than consumption goods and still be used as a medium of exchange. The service that money provides as a faster method of trading than barter can outweigh the extra costs associated with its production and use. However, if any of the costs becomes too high then the equilibrium level of the money supply in equation (6) is driven down to zero. Given the return to accepting money is fixed by the production costs in equilibrium, a commodity money can be driven out of the economy (say through increased storage costs) by forcing its equilibrium supply to zero even though it is generally accepted by agents (\(\pi = 1\)) and its return is positive.

The boundaries for each parameter can easily be calculated as functions of the other parameters using population fraction equations in (6). The expression in (8) gives the possible range for the production cost of money. As \(c_m\) approaches \(c_m\) from below \(m^*\) is forced to zero. As \(c_m\) approaches \(c_g\) from above, the equilibrium return to accepting money goes to zero and the monetary equilibrium ceases to exist. If \(c_g \leq c_m\) and \(c_m\) declines too far then \(g^*\) goes to zero and there is again no monetary equilibrium.

If either the depreciation rate on money or the storage cost for money increases, then both the upper and lower bounds on \(c_m\) will decline. In other words, a medium of exchange must balance high costs of one type with low costs of another type if it is to circulate in a monetary equilibrium. The response of \(\overline{c_m}\) to changes in the parameters \(\beta\) or \(x\) was ambiguous. However, for low values of \(x\) the upper boundary on \(c_m\) tended to increase with \(x\) and decline with \(\beta\) (the opposite effects were true for high \(x\) values). Since

\(^4\) If agents were able to produce unrestricted and divisible quantities of goods then money would be valued only if the total production cost of money was greater than the total production cost of goods. Agents would produce until \(V_m - c_m = V_g (g^*) - c_g g^*\) where \(g^*\) is the levels of goods produced by each agent in equilibrium.
\( c_m \in \{ (c_m, \bar{c}_m) \mid s_m - s_m + (\gamma_m - \gamma_m)c_m \leq 0 \} \)
\( \{ (c_m, \bar{c}_m) \mid s_m - s_m + (\gamma_m - \gamma_m)c_m > 0 \} \)

where:
\[
\bar{c}_m = \frac{\beta x(\pi - x)(u - e) + (r + \beta x^2 + \gamma_m)c_m + s_m - s_m}{r + \beta x^2 + \gamma_m}
\]
\[
c_m = \frac{s_m - s_m + (r + \beta x^2 + \gamma_m)c_m}{r + \beta x^2 + \gamma_m}
\]

increases in \( x \) make money relatively more attractive than barter for low \( x \), \( \bar{c}_m \) increases to signal that types of money which were previously inferior and unacceptable could then circulate as media of exchange. For high \( x \) values the rate of money trades is still faster than barter trades but the relative gap is smaller. As such, better quality monies are needed (ie. \( \bar{c}_m \) falls) as \( x \) rises close to its maximum.

High storage costs or depreciation rates for money can also cause agents to stop using the medium of exchange in equilibrium. The upper boundaries on \( s_m \) and \( \gamma_m \) fall as \( x \) rises so that better quality commodities are required for media of exchange when there are more trading partners. Increases in the arrival rate of trading partners, \( \beta \), have an ambiguous affect on the range of \( s_m \) and \( \gamma_m \). More costly types of money (higher \( s_m \) and \( \gamma_m \)) become acceptable as \( \beta \) rises when \( x \) is low. But when there is a greater number of potential trading partners, monies with high storage and depreciation costs become unacceptable following increases in \( \beta \). The basic conclusion is that the more difficult is trade using barter the wider is the range of acceptable media of exchange.

An agent's overall welfare (W) is defined as the weighted average of the value an agent receives in each state where the weights are the fractions of time spent in the states. Substituting the value functions into this expression yields the result in equation (9).

\[
RW = (\beta g^2 x^2 + \beta g m x^2)(u - e) - gs_g - ms_m
\]
\[
[\beta g^2 x^2 + \beta g m x^2 + g \gamma_g + m \gamma_m](6c_m + (1 - \theta)c_m)
\]
The welfare maximizing level of money can then be found by differentiating with respect to \( m \).

Solving the resulting equation gives the optimal values for \( g \) and \( m \) in equation (10). The maximizing level of the money supply will be positive if and only if the numerator is positive which is not guaranteed.

If \( m^{**} \) in equation (10) is negative (i.e. the numerator is less than zero), then redefine the optimal money supply to be zero. The higher is \( x \) the greater is the likelihood that the welfare maximizing level of \( m \) is zero. For low \( x \), the benefit of faster trade provided by money outweighs the loss from greater production costs and permits \( m^{**}>0 \). However, the opposite is true for high \( x \) when the added benefit of faster trade than barter is small but the production costs are still high. There is also nothing to ensure that the economy would operate at or near this optimal level of the money supply. The actual value of \( m \) given by equation (6) could in fact be very different from its optimal supply. If money costs are smaller (except for production costs) or not too much larger than goods costs, then there will be an overproduction of money by the agents so that \( m^* > m^{**} \).

In the preceding paragraph we showed that under certain circumstances the optimal supply of commodity money is non-zero. However, when do the agents themselves choose an equilibrium supply of commodity money that yields higher welfare than a pure barter equilibrium? It is straightforward to show that a commodity money equilibrium will provide higher welfare than a pure barter equilibrium if and only if \(-\beta x^2(u-e-c_g) + (r+\beta x\pi)(c_m-c_g) > 0 \). In other words, commodity money can be welfare enhancing if \( x \) and \( (u-e-c_g) \) are small enough compared to \( (c_m-c_g) \). The easiest interpretation is that the double coincidence of wants problem must be sufficiently difficult (i.e. \( x \) small) and the equilibrium money
supply cannot be too large (i.e. $u-e-c_g$ too small and $c_m-c_g$ too large) in order for money to be welfare enhancing.

See Table 1 for a summary of the comparative statics for the equilibrium and optimal population fractions. The optimal supply of money declines as the production, depreciation, and storage costs of money become larger. When $x$ is low, $m^{**}$ increases with $x$ because the velocity of money rises relatively faster than the velocity of barter. This makes money more attractive. However, for high $x$ values, the optimal money supply falls with $x$ so that it reaches zero before $x$ reaches one. When there is an ample supply of trading partners (high $x$) then there is not the need for money and agents would actually be better off by producing goods instead.

<table>
<thead>
<tr>
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<th>$g^*$</th>
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The goods output of this economy can be defined by the rate at which goods are produced. For a monetary equilibrium this would be defined by $Y_m = \beta g^2 x^2 + \beta g m x \pi + \gamma_g g$ while for a barter equilibrium it would be $Y_b = \beta x^2 + \gamma_g$. The output of a monetary economy will be larger, $Y_m > Y_b$, if and only if $g^* > (\beta x^2 + \gamma_g) / (\beta x (\pi - x))$. Whether this is true depends on the parameters of the model. In particular, $Y_m > Y_b$ only if $u - e - c_g$ is sufficiently small. The addition of money to the economy may or may not increase the rate of goods production. Similarly, it can be shown that welfare in a monetary equilibrium can be larger than that of a barter equilibrium if $x$ is small enough (barter is sufficiently difficult), $u - e - c_g$ is small
enough, or if $c_n - c_g$ is large enough (high return to trading for money). Only if the double coincidence of wants problem is sufficiently difficult will money production be welfare increasing.

Consider now the response of output to a change in the parameters of a monetary equilibrium. Output, $Y_m$, may increase or decrease after a change in the goods inventories of the agents. The higher are goods inventories, $g$, the more likely is $\frac{\partial Y_m}{\partial g} < 0$. If this is true then from the results of Table 1 we can see that the economy's output will decline following increases in the costs associated with the medium of exchange. In these circumstances a negative shock to money will decrease output. If, however, $\frac{\partial Y_m}{\partial g} > 0$, then negative shocks to money will increase output. Therefore, the money supply and output may be either positively or negatively correlated. The two variables tend to be positively correlated if the initial level of goods inventories was sufficiently high.

The issue of multiple media of exchange has been considered by Kiyotaki and Wright (1989, 1990) in a one country model and by Matsuyama, Kiyotaki, and Matsui (1992) in a two country international model. In their first paper, Kiyotaki and Wright (1989) showed that a random matching model could generate equilibria with multiple commodity monies or equilibria with a commodity money existing alongside fiat money. Aiyagari and Wallace (1991) also considered this in a more general framework with an arbitrary number of goods and mixed strategies. Subsequently, Kiyotaki and Wright (1990) provided an example in which two types of fiat currency circulated as media of exchange but with differing rates of acceptance. Even a fiat money that was dominated in terms of its flow yield of utility could circulate with a greater probability of acceptance if the agents believed that that would occur.

This paper's model can also be set up to include two types of money, say gold and silver, for which the supplies are endogenously determined. Agents may endogenously choose to produce goods and each of the two types of money. For both monies to have value the agents would have to believe that everyone would use them both as media of exchange. It is also necessary that the production, storage, and depreciation costs of the two monies not be too dissimilar. This ensures that the population fractions
of agents are within the unit interval.

For the economic agents to produce goods and two types of money, they must be indifferent among the three possibilities. This implies once again that the return to accepting money in equilibrium is the savings in production costs when trading for money instead of producing it yourself. An equilibrium with valued currency will hence require money production costs to be greater than goods production costs.

The first important result to note is that there will be no equilibrium with two media of exchange if one type of money has strictly superior characteristics. A potential medium of exchange with strictly higher costs of use would not have a positive supply in any steady state equilibrium. Therefore, when agents use multiple media of exchange they are balancing high costs of one type with low costs of another type so that overall they are indifferent between the two monies. As would be expected, the supply of a medium of exchange will respond negatively to increases in its own costs but positively to increases in the costs associated with other monies in circulation. The response of the money supply to other changes was basically ambiguous.

The model will quite readily generate equilibria with multiple types of media of exchange. There cannot however be a commodity that circulates as a medium of exchange and has strictly greater costs (production, storage, and depreciation) than another commodity which agents will also use as money. With two media of exchange, each money must have the lowest costs in at least one but not all categories. If any of the costs for a particular money rose high enough then that medium of exchange would be driven out of the economy in equilibrium. No steady state equilibrium could be maintained with positive amounts of that money being produced. Conversely, if the costs of one type of money fell far enough then it could drive the supply of the other medium of exchange to zero.

From this model we can see the importance of the properties of a commodity in determining whether or not agents will believe in its value as a medium of exchange. Production costs of money must
be sufficiently large for agents to value it. This can be interpreted as requiring a commodity that is sufficiently scarce to serve as a medium of exchange. Money must have low enough depreciation rates and storage costs as well. It is quite possible however to have a commodity serving as a medium of exchange that has higher costs of production, storage, and depreciation than other objects in the economy. The benefit of being a generally acceptable medium of exchange can outweigh the greater costs.

**III. Government Intervention:**

If there were a government that could influence certain parameters of the model then they could potentially act to improve the overall welfare of the agents. Assume that the government can impose a subsidy or tax or some other form of legal restriction on the production of gold so as to change the fixed cost of money production. Since production costs are in terms of utility, a tax could be thought of as, say, the introduction of a licensing procedure or legal restriction on the production of money that annoyed producers and increased their disutility of production. Both the optimal and equilibrium levels of m decline as c_m increases although the equilibrium level falls faster. Suppose that \( m^* > m^{**} > 0 \) were the original equilibrium and optimal money supplies, respectively. The government can raise c_m to lower the equilibrium money supply to \( m^{**} \). However, the higher production costs will also lower the optimal money supply to some \( m^{**} < m^{**} \). Since the equilibrium money supply falls faster than the optimal value there will, however, be a point at which \( m^* = m^{**} < m^{**} \). Let \( c_m \) represent the production cost for this point at which the equilibrium and optimum coincide. However, it is not guaranteed that this point can be reached with a positive money supply. The government may find that in chasing after the optimal money supply they only manage to drive money out of the economy even though there originally was a positive optimal money supply. This was more likely to occur the greater was the number of trading partners (x) in the economy since the added benefit of money is smaller the more trading partners there are.

When the government first decides to change c_m in order to improve the welfare of the agents it
will appear that it is possible to do so. However, changing \( c_m \) alters the equilibrium and the environment such that there is no guarantee that the new equilibrium will even provide a greater welfare level than the initial equilibrium. This is an application of the Lucas critique because the government action to improve welfare will change the economic environment and may actually decrease overall welfare. This was more likely to occur for small \( x \) values. A rational government with foresight, however, would take into account the effect it has on the economy. Such a government would select the optimal level for \( c_m \) to maximize welfare and should not fall into the trap of chasing after further illusory potential increases.

The optimal production cost is given by \( c_m^* \):

\[
c_m^* = (s_g - s_m)(r + \beta x^m) + \beta x(u - e - c_g) \left[ \pi(r + \beta x^m + x\gamma_m) + (r + \beta x^m)c_g \right] \frac{2(r + \beta x^m + \gamma_m)}{2(r + \beta x^m)r + \beta x^m + \gamma_m} \tag{11}
\]

The effect of government intervention is illustrated in Figure 1, assuming specific values for the parameters. This graph plots welfare as a function of the fraction of goods traders in the economy. Points A and C mark the equilibrium outcomes for money production costs of \( c_{m1} \) and \( c_m^* \), respectively. The curve ACE joins points of equilibrium welfare for increasing levels of \( c_m \). Points B and D are the maximum welfare levels for production cost levels of \( c_{m1} \) and \( c_m^* \), respectively. The curve BDE joins points representing the maximum welfare for higher and higher production costs. Note how agents are overproducing money, or underproduction goods, so that there is potential for welfare to be improved by increasing goods production (assuming \( c_m < c_m^* \) initially). At E, with production costs of \( c_m \), the equilibrium outcome occurs where welfare is maximized. A naive government starting at point A will see welfare level B as its target. However, in chasing after this maximum welfare they will end up at point E which, although higher than the initial welfare, is much less than the original target. An economy starting at point C that tries to get to the optimum at point D will end up lowering its welfare level since outcome E is below the initial equilibrium.

A forward looking government starting at point A would realize the effects it was having on the economic environment and would stop raising \( c_m \) at \( c_m^* \) with an equilibrium at C instead of trying to
achieve the seemingly possible further welfare increase to point D. If the initial \( c_m \) is below \( c_m^* \), then the optimal government intervention is a tax to raise gold production costs so that the inventories of goods increase while the supply of money declines. However, if the initial \( c_m \) is above \( c_m^* \) then the optimal government policy is a subsidy to lower costs and promote the production of money.

There is a role in this model for a rational government. They can impose an optimal tax or subsidy so that money production costs are \( c_m^* \) and the overall welfare of agents is maximized. If there are a large enough number of trading partners for each agent then the optimal policy may be to use taxes to drive money out of the economy.

**IV. Fiat Money:**

Now consider the model if there is an endogenous producible money called gold and a fiat currency offered in fixed supply by the government. As with goods and gold, agents may only hold one unit of the fiat currency at any given time. The fiat currency is assumed not to depreciate as does the producible money although it does possibly have a non-zero storage cost. Let \( V_f \) represent the value function for an agent with one unit of fiat currency in storage, \( f \) the supply of fiat money, and \( \pi_f \) and \( \pi_m \) the probabilities with which agents accept fiat and commodity money, respectively. The value functions for this form of the model can then be written as follows:

\[
\begin{align*}
\bar{r}_g &= \beta g \bar{x}_{g}[ u - e + \max_{\theta} \{ \theta(V_g - c_g) + (1 - \theta)(V_m - c_m) \} - V_g ] \\
&
+ \beta_{\max}^{\pi_m} \pi_m (V_m - V_g) + \beta_{\max}^{\pi_f} \pi_f (V_f - V_g) \\
&
+ \gamma_g [ \max_{\theta} \{ \theta(V_g - c_g) + (1 - \theta)(V_m - c_m) \} - V_g ] - s_g 
\end{align*}
\]

\[
\begin{align*}
\bar{r}_f &= \beta g \bar{x}_{f}[ u - e + \max_{\theta} \{ \theta(V_g - c_g) + (1 - \theta)(V_m - c_m) \} - V_f ] - s_f
\end{align*}
\]

After consuming an agent can choose to produce either his production good with probability \( \theta \) or
\[ rV_m = \beta gx \Pi_m[ u - e + \max_\theta (\theta(V^g - c^g) + (1-\theta)(V_m - c_m) ) - V_m ] \]

\[ + \gamma_m[ \max_\theta (\theta(V^g - c^g) + (1-\theta)(V_m - c_m) ) - V_m ] - s_m \]

a unit of the commodity money with probability 1-\theta. Agents will be indifferent between the two production opportunities in equilibrium so that \( V^g - c_g = V_m - c_m \). This still implies that a commodity money must have higher production costs than goods if it is to be valued in equilibrium. The value of accepting the fiat currency will be positive (\( V_f > V_g \)) in a monetary equilibrium but it is uncertain whether the value of having the fiat currency is greater or smaller than the value of a commodity money. A sufficient condition for \( V_f > V_m \) is that the commodity money have higher storage costs which would be a reasonable assumption. Agents will not trade fiat money for commodity money. The only trading opportunity open to money holders is to find a suitable goods trader. Trading potential is basically identical for fiat and commodity money holders but commodity money traders face the risk of having their money depreciate while in storage.

Once again using the fact that agents are indifferent with respect to their production decisions, an expression defining the equilibrium fraction of goods traders can be derived from the value functions. Remembering that \( m = 1 - g - f \), the steady state equilibrium condition for the number of goods traders is given by the following:

\[ (\beta gx(1-x)(u - e) + s_g - \beta mx c_m + (\beta gx^2 + \gamma_g + \beta mx)c_g ) \frac{r + \beta gx}{r + \beta x(g+f)} \]

\[ + (s_f + \beta gx c_g ) \frac{\beta fx}{r + \beta x(g+f)} - s_m + r c_g - [r + \beta gx + \gamma_m] c_m = 0 \]

The response of the number of goods traders to a change in the supply of fiat currency is uncertain. However, when fiat currency is first introduced to an economy operating at an equilibrium with a commodity money and population fractions defined by equation (6) then the following is true:

A sufficient condition for this derivative to be positive is that commodity money storage and depreciation costs be greater than those costs for goods and fiat money. If this is true then the fraction of goods traders
\[
\frac{dg}{df} \bigg|_{f=0} = \frac{\gamma_m c_m + s_m - s_f}{r(1-x)(u-\epsilon-c_g) + (r+\beta x)(c_m-c_g) + \gamma_m c_m - \gamma_s c_g + s_m - s_g}
\]

in the model will initially rise as a fiat currency is introduced. The fiat currency will initially only crowd out the commodity money. Because of the inventory restrictions on the storage of money and goods, however, further increases in the supply of fiat currency will eventually crowd out both goods and the commodity money. The introduction of fiat money tended to drive out commodity money much faster than goods. Agents have a certain level of need for media of exchange, whether in fiat or commodity form, so that more fiat money implies less commodity money.

Now consider the question of the effect of the fiat currency on the overall welfare of the agents. Aggregate welfare is defined again as the weighted average of the value functions for each state where the weights are the fractions of agents in each state. For a given level of the fiat currency the agents’ optimal choice of goods inventories (g) is unchanged from the results for the model without fiat currency given in equation (10). The optimum is defined by the same level of goods inventories with and without fiat currency. The supply of commodity money is simply reduced by the amount of the fiat money. If the supply of government fiat currency becomes too high then optimal supply of the commodity money is driven to zero.

If the equilibrium level of g is determined from equation (15) and substituted into the welfare equation then we have welfare as a function of the supply of fiat money. Differentiating the resulting expression gives the following derivative:

\[
\frac{dW}{df} = \frac{\left[ (r(1-x)+\beta x)(u-\epsilon-c_g) - (r+\beta x)(c_m-c_g) \right] \left[ \gamma_m c_m + s_m - s_f \right]}{\Psi(f)}
\]

where \(\Psi(f)\) is a complex polynomial in the supply of fiat money. Obviously there is no f which can set the derivative to zero and maximize welfare. Welfare will always be either increasing or decreasing in the supply of fiat money. However, evaluating the derivative at \(f=0\) yields equation (18).
\[
\frac{drW}{df} \bigg|_{f=0} = \frac{\left[ (r(1-x) + \beta x)(u-e-c_g) - (r+\beta x)(c_m-c_g) \right][\gamma_m c_m + s_m - s_f]}{r(1-x)(u-e-c_g) + (r+\beta x)(c_m-c_g) + \gamma_sc_s + s_m - s_g}
\] (18)

The change in welfare when money is first introduced can be shown to be positive provided \(\gamma_m c_m + s_m - s_f > 0\) which is likely true since one would expect it to cost more to store, the commodity money than the fiat money. Using equation (15), we see that the expression \(\left[ (1-x) + \beta x \right](u-e-c_g) - (r+\beta x)(c_m-c_g)\) and the denominator in equation (18) must be positive or else the equilibrium goods inventories will be outside the unit interval. When welfare is increasing in the supply of fiat money then the government will want to raise that supply at least until that point at which the supply of the commodity money is driven to zero. This is likely to happen before the fiat money supply reaches its maximum. The responsiveness of welfare to fiat money will change at that point once the economy moves into a non-commodity money equilibrium.

The welfare maximizing level of fiat money in an equilibrium without commodity money will likely be less than that level which drives the commodity money out of the economy. In the presence of a commodity money, the optimal government strategy may be to raise the supply of fiat currency until the private money is driven out and then remove some of the fiat money until welfare is maximized. However, for large values of \(x\) implying less of a need for money because trade is easier, it will be optimal to completely remove the fiat money as well once agents have set \(\pi_m=0\) and will no longer accept the commodity money as a medium of exchange. For smaller numbers of trading partners it will be optimal to leave a certain quantity of fiat money in circulation.

Suppose the government outlaws the use of private commodity money (ie. increases \(c_m\) so it is prohibitively high to produce) and then introduces a fiat money to replace it. If the supply of fiat money injected is equal to the commodity money previously circulating in equilibrium then the agents' welfare will increase provided \(\gamma_m c_m + s_m - s_f > 0\) which is expected to be true. Obviously, setting the fiat money supply at its welfare maximizing level will improve the agents' situation even further. If, however, no
fiat money or only a small amount of it replaces the commodity money then it is possible that aggregate welfare will be reduced. This occurred if the commodity money equilibrium provided higher welfare than the pure barter equilibrium. Therefore, agents benefit from the removal of the commodity money only if it is replaced with sufficiently high quantities of the fiat currency. If \( x \) was high and the commodity money actually decreased welfare then the government need not replace the commodity money with a fiat money in order to increase welfare.

The introduction of a fiat money will not stop agents from producing and using their old commodity money provided of course that not too much fiat currency is introduced. The commodity money can be driven out of the economy if sufficiently large levels of fiat currency are injected into circulation. The overall welfare of the agents will be increased by the introduction of the fiat money because it replaces a commodity money which is costly to produce. If the commodity money is removed from circulation by a law then welfare will be increased only if sufficient quantities of fiat money are injected to serve as a new medium of exchange.

V. Conclusion:

Given the opportunity to produce their own money the agents in this economy chose to overproduce money (for a wide set of parameter values) to such an extent that it could substantially lower their overall welfare. The producers fail to account for the externality that their behaviour while producing affects their future returns once they became goods or money traders. The production of money was too high and goods too low for the agents to be able to maximize their overall welfare. The supply of money produced by the agents increased as the properties of the commodity money improved, or in other words, as the costs of its use declined.

A commodity must be sufficiently scarce or sufficiently difficult to produce if it is to be valued and serve as a medium of exchange in this environment. Conversely, the production, depreciation, and
storage costs of money also cannot be too high or else no agent would willingly use it as a medium of exchange. It is possible however to have a commodity money circulating as a generally accepted medium of exchange in equilibrium even if it has strictly greater costs than the consumption goods. The benefits of faster trade can outweigh the higher user costs. The optimal level of commodity money was positive under certain conditions, a significant double coincidence of wants problem. An equilibrium with commodity money could also yield greater aggregate welfare than a pure barter equilibrium.

The model was able to generate equilibria with multiple types of money being used by the agents. However, there could be no endogenous medium of exchange which had strictly greater costs of each type than another medium of exchange. Agents would choose not to use such a commodity as a money. If the characteristics of one type of money were shocked so that its depreciation rate or storage cost decreased by a sufficient amount, then all other types of private money could be driven out of the economy. Similarly, if the production costs for one type of money became sufficiently small relative to the production costs for goods or other monies then the agents might cease to use the other money in their trading.

The introduction of an exogenous government fiat money into the economy could only drive out the commodity money if there was a sufficiently large supply of the fiat currency. This was true even, if the fiat currency had strictly lower costs than any commodity money. The introduction of some fiat currency would, however, increase the overall welfare of the agents. The government could increase the agents’ welfare by outlawing the production of commodity money and replacing it with sufficiently large quantities of fiat money. The agents still had use of a generally accepted medium of exchange but were no longer required to pay a production cost to acquire it.

The money supply and goods output may be positively or negatively correlated in response to changes in the parameters of the model. The correlation was positive only if the initial level of goods inventories was sufficiently high.
References:


Welfare as a Function of Money Production Costs and Goods Traders

Figure 1

\( cm1 < cm^* < \tilde{c}m \)