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F. J. Anderson

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by

F.J. Anderson

Department of Economics Library

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Department of Economics

Social Science Centre

University of Western Ontario

London, Ontario, Canada

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Factor Returns, Comparative Advantage and Market Size: Some Core-Periphery Results

F.J. Anderson
Department of Economics
Lakehead University
June, 1993

Abstract

An increasing returns model with economies internal to firms along the lines of Venables (1985) is used to examine returns to factors and comparative advantage between two trading regions that differ in the sizes of their internal markets. The two regions are separated by transport costs. Their relative factor returns (real wages) are endogenously determined by a trade balance requirement. Both cross-hauling and one-way flows of IRS goods are examined. Real factor returns are always lower in the small-market region and the large-market region exhibits comparative advantage in the IRS good.
1. Introduction

The observation that manufacturing firms tend to be concentrated in densely-populated 'core' regions and export to less densely populated 'peripheral' regions strongly suggests interactions between increasing returns (IRS) in manufacturing activities and transport or other trade barriers that serve to (partially) separate regional markets of differing sizes. The relative concentration of IRS sectors in densely-populated regions could occur due to internal or external economies. The external economies argument, familiar to economic geographers in the form of 'agglomeration economies', is essentially a supply-side approach that does not depend on market separation (or, indeed, relative market sizes) at all: locationally specific external economies operate to reduce costs for individual manufacturing firms due to the presence of large numbers of other firms at the same location. An example of the external economies version is the 'fish and chips' model discussed in Krugman (1987). External economies are responsible for the concentration of (computer) chips in a single (core) location in this type of core-periphery model.

Internal economies models with partial market separation through transport costs come in two separate versions, those that stress effects of relative market size on product variety and those that
focus on the pro-competitive effects of relative market size.\footnote{Complete separation of the two versions is not found in presentations comparing autarkic markets with markets that are fully integrated (zero trade barriers). Krugman (1979) incorporates both product variety and procompetitive effects in such a model. The demand conditions required to obtain both effects for this case are examined in Anderson (1991). The two separate versions for full market integration versus autarky have also been set out in Dixit and Norman (1980, chapter 9).} In the first version, the role of a large (core) market as a superior generator of product variety relative to a small market is examined (Krugman 1980, 1991a, 1991b, Venables 1987). Firms are identical and equilibrium takes the Chamberlinian form under free entry/exit. In this version, demand conditions are deliberately chosen so that each firm's decision on optimal prices to charge in its own market and on export sales depends only on marginal cost and on a parameter in the demand function for aggregate IRS goods.\footnote{The details of the required demand structure can be found in Krugman (1980) and Venables (1987). The underlying per capita utility function is separable in the numeraire good and the goods comprising the IRS sector. The goods in the IRS sector exhibit constant elasticity of substitution (CES). Further, each firm takes prices and outputs of all other firms as parametric in its profit-maximization decision.} With transport cost separating a pair of markets, each firm charges a higher price in its export market than in its home market. Differences in market size lead to differences in the number of firms in each market with more firms in the larger market but have no effect on prices. Consumers located in the larger market therefore pay a lower average price for a diversified bundle of manufactures than consumers in the small market and so experience a higher level of welfare. There are net flows of differentiated products from the large-market region to the small-market region so the large region has comparative advantage in IRS goods (Venables 1987).
The second internal economies version with trade barriers suppresses product variety in the IRS sector and focuses on the effect of relative market size in generating pro-competitive effects. The structure of this model generalizes the Chamberlinian equilibrium in Dixit and Norman (1980, 267-73) to cases in which markets for homogeneous IRS goods are not completely separated by trade barriers (Venables 1985). Firms in each market have some incentive to export to the other market, giving rise to the potential for cross-hauling behaviour identified by Brander (1981), Brander and Krugman (1983) and Dixit (1983). As Venables demonstrates, partial market separation implies larger numbers of firms in the larger market under free entry with lower prices (higher welfare) for consumers located there due to pro-competitive behaviour. Comparative advantage results have not been derived for this version.

A full analysis of the effect of internal economies with partial market separation would integrate both the above versions to examine the prospects for product variety and pro-competitive effects simultaneously. The present paper has more modest objectives. It uses the pro-competitive version to extend the results in Venables (1985) to a fuller treatment of transport costs, trade balance requirements, and comparative advantage.

The model (section 2) consists of two sectors, a CRS sector producing in both regions, and an IRS sector producing in one or
both regions (initially in both regions). As usual in this literature, a single factor of production (labour) is assumed in order to remove relative factor endowment (Heckscher-Ohlin) effects. Ricardian effects are also eliminated by assuming identical productivities in both sectors in both regions. Differences in taste as a potential cause of trade are neutralized by assuming that per capita demand functions are the same in both regions. Regions differ by location and by market size, with the latter measured by the absolute amounts of each region's labour endowment. In contrast to existing treatments, interregional movements of CRS goods as well as movements of IRS goods incur transport costs. Thus the market separation assumption is generalized. The model explicitly incorporates a trade balance requirement.

Two types of results are derived for the cross-hauling cases examined in section 3. The factor pricing results generalize the results in Venables (1985) to the case in which transport costs are incurred on interregional movements of both goods with a trade balance requirement, confirming that pro-competitive effects operate to produce higher real factor returns (welfare) in the larger region. The comparative advantage result shows that (subject to a minor caveat) the larger region is the net exporter of IRS goods as in the product variety version. These results are extended in section 4 to cases in which the small region does not export IRS goods. Section 5 offers concluding observations.
2. The Model

The two trading regions in the model are designated as Region 1 and Region 2. Both regions use identical technologies. There are two goods, a composite CRS good (produced by both regions\(^3\)) and an IRS good (produced by one or both). One unit of labour is required to produce one unit of the CRS good. A firm producing IRS goods in either region requires one unit of labour per unit of the IRS good plus a fixed setup cost of \(F\) units of labour. The labour endowment in each region is fixed. Region 1 labour will be used as numeraire so that the wage rate and the price of CRS goods in Region 1 equals unity. For notational simplicity, variables that apply to Region 2 are distinguished from Region 1 variables by the use of an asterisk. Thus the Region 1 nominal wage (and marginal cost of IRS goods) is \(w=1\) and the Region 2 nominal wage (and marginal cost of IRS goods) is \(w^*\).

In this section and the next it is assumed that IRS firms are present in both regions with free entry/exit ensuring that zero profits prevail in both IRS sectors. Denoting firm numbers in the two IRS sectors by \(n\geq 0\) and \(n^*\geq 0\), this assumption implies \(n>0\) and \(n^*>0\). IRS firms produce output for the domestic market, denoted by \(y\geq 0\) for Region 1 firms and \(y^*\geq 0\) for Region 2 firms, as well as for the market of the other region (\(x\geq 0, x^*\geq 0\)). With \(x>0\) and \(x^*>0\), there is cross-hauling of IRS goods between the two regions. The

\(^3\) The assumption that the CRS good is common to both regions is standard. See Markusen and Venables (1988), Krugman (1991a, 1991b), and Venables (1985, 1987) for example.
prices of the IRS good in the two regions are $p$, $p^*$. Interregional shipments of IRS goods incur transport cost per unit equal to $t$ in labour units. Each region is assumed to use its own labour inputs for outbound shipments of IRS goods so per unit (nominal) transport cost is $t$ in Region 1 and $w^*t$ in Region 2.

The zero-profit conditions are:

$$\pi = (p-1)y + (p^*-1-t)x - F = 0, \quad n>0. \quad (1)$$
$$\pi^* = (p^*-w^*)y^* + (p-w^*-w^*t)x^* - w^*F = 0, \quad n^*>0. \quad (2)$$

The first two terms in each of the zero-profit conditions represent revenue net of variable cost for output delivered to the home and foreign market respectively. Denoting the size of each region's market by its labour endowment $N$ and $N^*$, per capita demand for IRS goods is taken to be identical in both regions such that the regional (inverse) demand functions for IRS goods can be written as,

$$p = f(Q/N), \quad f' < 0 \quad (3)$$
$$p^* = w^*f^*(Q^*/N^*), \quad f^* < 0 \quad (4)$$

where $Q$ and $Q^*$ denote total consumption of IRS goods in the two regions and $f(\cdot)=f^*(\cdot)$ for $Q/N=Q^*/N^*$. The derivative conditions on (3) and (4) incorporate substitution and (normal) income effects. Firms in both regions are assumed to maximize profits under Cournot-Nash conjectures. Partial differentiation of (1) and (2)
under the Cournot-Nash assumption generates the following first-order profit maximization conditions:

\[(p-1) + yf'/N = 0, \quad y>0 \quad (5)\]
\[(p^*-1-t) + wxf'*/N* = 0, \quad x>0 \quad (6)\]
\[(p^*-w*) + wyf'*/N* = 0, \quad y*>0 \quad (7)\]
\[(p-w*-w*t) + xf'/N = 0, \quad x*>0 \quad (8)\]

Marginal conditions (5) and (6) apply to firms located in Region 1. Conditions (7) and (8) apply to firms in Region 2. Appropriate concavity conditions are assumed so that second-order conditions are met.

The marginal conditions are premised on the assumption that firms perceive that separate optimizing decisions are feasible for each of the two regional markets in the sense that consumers are unwilling or unable to shift purchases from one market to the other in response to observed price differentials. This segmented markets assumption remains in effect throughout the paper following Venables (1985).\(^4\)

The equilibrium numbers of firms in each region, \(n\) and \(n^*\), are obtained by assuming regional sales equal to regional consumption

\(^4\) If consumers are able to arbitrage utilizing the same transport technology as firms and the firms recognize this then marginal conditions (5) through (8) are subject to the restrictions \(p^*-p^*t\) for \(p^*>p\) and \(p-p^*t\) \(w^*t\) for \(p>p^*\). In the former case, firms take the two markets as integrated with \(p^*=p^*t\) when the inequality holds strongly. See Horstmann and Markusen (1986), Venables (1987) and Markusen and Venables (1988). See also the price discrimination discussion in section 4 of the present paper.
such that,

\[ Q/N = (ny + n^*x^*)/N = f^{-1}(p) = g(p) \]  \hspace{1cm} (9)

\[ Q^*/N^* = (n^*y^* + nx)/N^* = f^*^{-1}(p^*/w^*) = g^*(p^*/w^*) \]  \hspace{1cm} (10)

where \( f^{-1} = g \) and \( f^*^{-1} = g^* \) denote ordinary demand functions. As usual with such free entry/exit models, no attention is being paid to the requirement that \( n \) and \( n^* \) take on integer values. The solution to system (1) through (10), assuming one exists, can be obtained as follows.

For given market sizes \( N \) and \( N^* \), transport cost \( t \), set-up cost \( F \), and the Region 2 nominal wage \( w^* \), selection of the two regional prices \( p \) and \( p^* \) serve to determine per capita consumption in the two regions together with \( f' \) and \( f''^* \) from the demand conditions in (3) and (4). This information allows \( x, y, x^*, y^* \) to be obtained from the marginal conditions (5) through (8). Writing \( x, y, x^*, y^* \) as functions of the selected prices \( p \) and \( p^* \),

\[ y/N = Y(p), \quad Y'>0. \]  \hspace{1cm} (11)

\[ x/N^* = X(p^*), \quad X'>0. \]  \hspace{1cm} (12)

\[ y^*/N^* = Y^*(p^*), \quad Y''>0. \]  \hspace{1cm} (13)

\[ x^*/N = X^*(p), \quad X''>0. \]  \hspace{1cm} (14)

with the implicit dependence on \( w^*, F, \) and \( t \) suppressed from the notation. The market size variables \( N \) and \( N^* \) appear explicitly for
convenience in the subsequent discussion. The derivative conditions on (11) through (14) reflect standard stability requirements (Hahn 1962). For stability, each firm's marginal revenue is decreasing in the outputs of other firms in the same market. Thus an exogenous increase in (per capita) output in either regional market leads to a fall in price in that market via the demand equations which, in turn, causes firms to contract their optimal (per capita) outputs. Permitting the optimal values for $x, y, x^*, y^*$ to be notationally equivalent to the same variables in zero-profit equations (1) and (2) and substituting (11) through (14) into (1) and (2),

\[
\pi = (p-1)Y(p)N + (p^*-1-t)X(p^*)N^* - F = 0 \quad (15)
\]

\[
\pi^* = (p^*-w^*)Y^*(p^*)N^* + (p-w^*-w^*t)X^*(p)N - w^*F = 0 \quad (16)
\]

which must be solved for the equilibrium values of $p$ and $p^*$ conditional on $N, N^*, t, F, w^*$. The equilibrium firm numbers $n$ and $n^*$ are then obtained from (9) and (10).

As noted above, a cross-hauling equilibrium implies $n>0$, $n^*>0$, $x>0$, $x^*>0$. But cross-hauling is not the only possible equilibrium. Under autarky, for example, $x=0$ and $x^*=0$. Marginal conditions (6) and (8) are replaced by the complementary slack conditions $(p^*-1-t) + w^*f^*/N^* \leq 0$ and $(p-w^*-w^*t) + x^*f^*/N \leq 0$. Autarky implies that the real wage is highest in the region with the larger market (Dixit and Norman 1980). Taking Region 1, for example, zero-profit
condition (1) becomes \( \pi = (p-1)y - F \). Substituting from marginal condition (5), \( \pi = -(p-1)^2N/f' - F \) for optimal choice of \( y \). Thus \( \partial \pi/\partial N = -(p-1)^2/f' > 0 \) and \( \partial \pi = y dp + (\partial \pi/\partial N) dN = 0 \) such that \( dp/dN = -(\partial \pi/\partial N)/y < 0 \) (Hotelling's Lemma). A similar argument from zero-profit condition (2) and marginal condition (7) establishes \( d(p^*/w^*)/dN^* < 0 \). Thus the larger region has the lower price and therefore the higher real wage under autarky (\( 1/p > w^*/p^* \)).

Other types of equilibria occur when one region, say Region 1, exports IRS goods to Region 2 but Region 2 does not export IRS goods to Region 1. This one-way movement of IRS goods can occur with \( n^* > 0 \) and \( x^* = 0 \) or with \( n^* = 0 \). In the case of \( n^* = 0 \), the complementary slack is obtained by replacing (2) by the condition \( \pi^* = 0 \). Section 3 will confine attention to cross-hauling equilibria while section 4 extends the analysis to one-way movements of IRS goods.

Equilibrium characteristics will also depend on the selection of \( w^* \) in Region 2. As discussed in section 3, \( w^* \) performs the function of ensuring overall trade balance between the two regions so the model can be extended to include \( w^* \) as an endogenous variable with trade balance requirements added to equations (9) through (16). First, though, it is useful to place restrictions on \( w^* \). If \( N = N^* \) equilibrium is obviously symmetrical and \( w^* = 1 \). In what follows it will be assumed that Region 1 has the larger market. With \( N > N^* \) and \( w^* = 1 \), Region 1 will turn out to be the net
exporter of IRS goods in the sense that Region 1's IRS trade surplus is \( S = p^*nx - pn^*x^* > 0 \) (see proposition 4, section 3). The IRS trade imbalance places downward pressure on Region 2's nominal wage such that \( dw^* < 0 \). Denoting real per unit transport cost on CRS goods as \( t_c \), Region 2's price of CRS goods equals Region 1's CRS price net of transport when \( w^* = 1 - w^*t_c \). Thus,

\[
(1 + t_c)^{-1} = w^{**} \leq w^* \leq 1
\]

(17)

where \( w^{**} \) denotes the CRS export point for Region 2. In the case of zero transport costs on CRS goods, \( w^* = 1 \) irrespective of trade flows.

As demonstrated by proposition 5 (section 3), a fall in \( w^* \) in Region 2 reduces the size of Region 1's trade balance in IRS goods, i.e. \( dS/dw^* > 0 \). The model can therefore be written with \( w^* \) endogenous by adding the following condition:

\[
S(w^*) = 0, \ S' > 0; \quad w^{**} < w^* < 1
\]

\[
S(w^*) \geq 0, \ S' > 0; \quad w^* = w^{**}
\]

(18)

where \( S = p^*nx - pn^*x^* \). The first part of (18) indicates that trade must be balanced in IRS goods if Region 2's wage is too high for it to export CRS goods. The second part indicates that Region 1 can run a surplus on its IRS account if Region 2's wage is low enough for it to export CRS goods in exchange.
It is useful at this point to show that \( N^* > N^* \) implies \( y^* > x \) and \( y > x^* \) in cross-hauling equilibrium (firms produce more output for their home region than is supplied by firms from the other region). Since \( w^* < 1 + t \), marginal conditions (6) and (7) imply \( y^* > x \). To establish \( y^* > x \), subtract zero-profit condition (1) from (2) such that 
\[
[(p^*-w^*)y^* - (p^*-1-t)x] + [(p-w^*-t)x^* - (p-1)y] \leq 0.
\]
Since \( (p^*-w^*) > (p^*-1-t) \) and \( y^* > x \) from (6) and (7), the first square-bracketed term in this expression is positive. So the second square-bracketed term must be negative. Multiplying (8) by \( x^* \) and (5) by \( y \) and subtracting, it is required that \( (p-w^*-t)x^* - (p-1)y = (y^2 - x^2) f'/N < 0 \) which is satisfied provided \( y^* > x \). Since \( y^* > x \) and \( y > x^* \) in cross-hauling equilibrium, the determinant property \( D = yy^*-xx^* > 0 \) emerges.

Cross-hauling equilibrium is shown in Figure 1 in price space. Satisfaction of the zero-profit conditions with optimal choice of \( x, y, x^*, y^* \) implies \( d\pi = ydp + xdp^* = 0 \) and \( d\pi^* = y^*dp^* + x^*dp = 0 \). In the neighbourhood of equilibrium the slopes of the zero-profit equilibrium loci \( \pi = 0 \) and \( \pi^* = 0 \) are measured by \( (dp/dp^*)_{\pi = 0} = -x/y \) and \( (dp/dp^*)_{\pi^* = 0} = -y^*/x^* \). With \( y^* > x \) and \( y > x^* \), the \( \pi^* = 0 \) locus cuts the \( \pi = 0 \) locus from above.

3. **Factor Prices and Trade: Cross-Hauling of IRS Goods**

For compactness, the effects of relative regional market size on relative real wages and trade flows will be examined for reductions in the size of Region 2's market size (\( dN^* < 0 \)) while
Figure 1
Equilibrium in Price Space

Figure 2
Effect of a Decrease in $N^*$
holding the size of Region 1's market constant (dN=0). Analogous results for variations in N will be mentioned in passing. In this section firms exist in both regions and are assumed to serve both regional markets (cross-hauling).

Proposition 1 (Factor Prices): For a given value of $w^*$ a decrease in Region 2's market size $N^*$ produces a decrease in Region 2's real wage ($w^*/p^*$) and an increase in Region 1's real wage ($1/p^*$). (Venables 1985)

Proof: The assumed decrease in $N^*$ shifts both the $\pi=0$ and $\pi^*=0$ loci upward as shown in Figure 2. Substituting marginal conditions (5) through (8) into (1) and (2),

\[
\pi = -N(p-1)^2/f' - N^*(p^*-1-t)^2/(w^*f^*)' - F
\]

\[
\pi^* = -N^*(p^*-w)^2/(w^*f^*)' - N(p-w^*-w^*t)^2/f' - w^*F
\]

Partially differentiating (19) and (20) with respect to $N^*$, $\partial \pi/\partial N^* = -(p^*-1-t)^2/(w^*f^*)' = -(x/N^*)^2 w^*f^*' > 0$, $\partial \pi^*/\partial N^* = -(p^*-w^*)^2/w^*f^*' = -(y^*/N^*)^2 w^*f^*' > 0$. Since $y^*>x$, the decrease in profit for $dN^*<0$ is largest for Region 2. Holding $d\pi=0$ and $d\pi^*=0$ and utilizing Hotelling's Lemma,

\[
d\pi = ydp + xdp^* - (x/N^*)^2 w^*f^*' dN^* = 0
\]
\[ \frac{dn^*}{dN^*} = x^*dp + y^*dp^* - (y^*/N^*)^2w^*f^{'N^*}dN^* = 0 \]  \hspace{1cm} (22)

Solving (21) and (22) for \(dp/dN^*\) and \(dp^*/dN^*\) and simplifying,

\[ \frac{dp}{dN^*} = w^*f^{'N^*}N^{x-y^*}(x-y^*)D^{-1} > 0 \]  \hspace{1cm} (23)

\[ \frac{dp^*}{dN^*} = w^*f^{'N^*}N^{-2(y^*2-x^*2)}D^{-1} < 0 \]  \hspace{1cm} (24)

where \(D = yy^*-xx^*>0\). Results (23) and (24) imply that a decrease in \(N^*\) lowers price in Region 1 such that Region 1’s real wage \((1/p)\) increases and raises \(p^*\) such that Region 2’s real wage \((w^*/p^*)\) decreases which proves Proposition 1. An alternative proof is provided by Venables (1985, 17-18 and appendix).

Intuitively, the differential effect on regional profit of \(dN^*<0\) produces a magnification effect on relative regional IRS prices required to maintain zero-profit equilibrium for firms in both regions. The same methodology can be used to establish that an increase in Region 1’s market size \((dN^*>0)\) produces the same qualitative effects on real wages.

---

*Proposition 2 (Firm Numbers):* For a given value of \(w^*\), a decrease in Region 2’s market size increases the number of firms in Region 1 and decreases the number of firms in Region 2.

---

*Proof:* Totally differentiating demand functions (3) and (4) with
the definitions of per capita consumption given by (9) and (10) and the derivatives of per capita outputs with respect to prices given by (11) through (14),

\[ dp = f' [nY' dp + (y/N)dn + nX' dp + (x*/N)dn*] \]  
(25)

\[ dp* = w*f'[nY* dp* + (y*/N*)dn* + nX' dp* + (x/N*)dn] \]  
(26)

Solving (25) and (26) for \( dn \) and \( dn* \):

\[ dn = N(N)(f'w*f')^{-1}[A*f'(y*/N*)dp - A*f'(x*/N)dp*]D^{-1} \]  
(27)

\[ dn* = N(N)(f'w*f')^{-1}[A*f'(y/N)dp* - A*f'(x/N*)dp]D^{-1} \]  
(28)

where \( A = 1 - f'(nY' + nX*) > 0 \) and \( A* = 1 - w*f'(nY*' + nX') > 0 \). Since \( dp < 0 \) and \( dp* > 0 \) for \( dN* < 0 \) (proposition 1), (27) and (28) imply \( dn/dN* < 0 \) and \( dn*/dN* > 0 \) which proves proposition 2.

The intuition here is that a decrease in the size of Region 2's market reduces profit by a larger amount in Region 2 than Region 1 giving rise to a bias in favour of exit from Region 2. A magnification effect on firm numbers is required to restore the zero-profit condition in both regions which raises firm numbers in Region 1. Since identical qualitative effects on regional prices accompany an increase in the size of Region's market, \( dN > 0 \) also implies an increase (decrease) in equilibrium firm numbers in Region 1 (Region 2).
· **Proposition 3A (Exports per Firm):** For a given \( w^* \), a decrease in Region 2's market size decreases exports per firm from Region 2 and increases or decreases exports per firm from Region 1.

· **Proposition 3B:** Proposition 3A holds with an increase in exports per firm from Region 1 provided the per capita demand function is not too convex (see Appendix).

---

**Proof:** The first part of proposition 3A is straightforward. Since \( dN=0 \), the impact of the decrease in Region 1's IRS price on \( x^* \) is given by the sign of \( X^{**} \) in equation (14). Since \( dp/dN^* > 0 \) (proposition 1) and \( dx^*/dp = NX^{*'} > 0, dx^*/dN^* > 0 \) which proves the first part.

The impact of \( dN^* < 0 \) on \( x \) is ambiguous since opposing effects are operating on the optimal choice of \( x \) by Region 1 firms. From equation (12), \( X^>' > 0 \) implies that \( d(x/N^*) > 0 \) for \( dp^* > 0 \) as a result of \( dN^* < 0 \). But \( d(x/N^*) > 0 \) with \( dN^* < 0 \) is consistent with an increase or decrease in \( x \). The Appendix shows that a restriction on the convexity of the per capita demand function is required to establish \( dx/dN^* < 0 \). Per capita demand functions that are linear or more concave (to the origin) than a linear function definitely satisfy the restriction as do demand functions that are not too convex relative to a linear function. Hence proposition 3B.\(^5\)

---

\(^5\) Note also that the qualitative content of propositions 3A and 3B would be reversed if \( dN > 0 \) instead of \( dN^* < 0 \); \( dx > 0 \) is now definite while \( dx^* < 0 \) requires the convexity condition in the appendix.
Proposition 4 (IRS Trade Balance and N*): For a given value of \( w^* \), a decrease in Region 2's market size normally leads to an improvement in Region 1's trade balance in IRS goods where Region 1's IRS trade balance is defined as \( S = p^*nx - pn^*x^* \).

Proof: Proposition 4 follows from propositions 1 through 3. For \( dN^*<0, \ dp<0, \ dp^*>0 \) (proposition 1) and \( dn>0, \ dn^*<0 \) (proposition 2). If proposition 3A applies and \( dx<0 \) with \( dx^*<0 \) then proposition 4 holds with the modest qualification that the combined effects of the five variables tending to improve Region 1's IRS trade balance outweigh the opposing impact of \( dx<0 \). If the per capita demand convexity condition referred to in proposition 3B holds then proposition 4 is unqualified. An analogous and equally strong presumption that \( dS/dN > 0 \) applies to an increase in Region 1's market size.

The impact of proposition 4 is to create a virtual certainty that the region with the larger market runs a positive trade balance on IRS goods with \( w^*=1 \). Suppose both regions begin with markets of equal size (\( N=N^* \)). The cross-hauling solution is symmetrical at \( w^*=1 \) such that \( S=0 \) (IRS trade balance). Reducing the size of Region 2 leads to \( p^*nx-pn^*x^*>0 \) (proposition 4). This in turn places downward pressure on Region 2's nominal wage as money flows out of Region 2 or its exchange rate depreciates. The intuitive
expectation is that the decline in $w^*$ improves Region 2's competitive position and thus acts to eliminate its trade deficit. The following proposition supports the expectation that Region 1's IRS trade surplus is reduced when $dw^*<0$.

---

Proposition 5 (IRS Trade Balance and $w^*$): For given market sizes in the two regions, a decline in Region 2's nominal wage leads to a deterioration in Region 1's IRS trade balance.

---

Proof: Holding market sizes constant and totally differentiating the zero-profit equations (1) and (2) with respect to prices and $w^*$ using Hotelling's Lemma,

\[
ydp + xdp^* = 0 \\
x^*dp + y^*dp^* = [y^*+(1+t)x^*+F]dw^* \tag{29}
\]

Solving (29) and (30), $dp/dw^* = -x[y^*+(1+t)x^*+F]D^{-1} < 0$ and $dp^*/dw^* = y[y^*+(1+t)x^*+F]D^{-1} > 0$. Dividing (2) by $w^*$ it is also true that $y^*d(p^*/w^*) + x^*d(p/w^*) = 0$. Since $dp/dw^*<0$ this implies $d(p^*/w^*)/dw^* < 0$ such that $d(p^*/w^*)/dw^* > 0$. So, for $dw^*<0$, the real wage decreases in Region 1 and increases in Region 2. Equations (11) through (14) imply $dx/dw^* > 0$ and $dx^*/dw^* < 0$. So $dw^*<0$ increases each Region 2 firm's exports to Region 1 and reduces each Region 1 firm's exports to Region 2.

Turning to firm numbers, the demand equations (3) and (4) can be
totally differentiated using the definitions of per capita consumption in (9) and (10) such that

\[ f'(y/N)dn + f'(x*/N)dn* = B \]  \hspace{1cm} \text{(31)}

\[ f^{**}(x/N*)dn + f^{**}(y*/N*)dn* = B* \]  \hspace{1cm} \text{(32)}

where \( B = dp - f'[nd(y/N) + n^*d(x*/N)] \) and \( B* = d(p*/w*) - f^{**}[n^*d(y*/N*) + nd(x/N*)] \). For \( dw^*<0, \ dp>0 \) and \( d(p*/w*)<0 \) as shown above. For \( dp>0 \) and \( dp^*<0 \), we have \( d(y/N)>0, \ d(x*/N)>0, \ d(y*/N*)<0, \ d(x/N*)<0 \) from (11) through (14). Thus \( B>0 \) and \( B^*<0 \). Solving for \( dn \) and \( dn^* \),

\[ dn = NN^*(f'f^{**})^{-1}[Bf^{**}(y*/N*) - B*f'(x*/N)]D^{-1} < 0 \]  \hspace{1cm} \text{(33)}

\[ dn^* = NN^*(f'f^{**})^{-1}[B*f'(y/N) - B^{*}f^{**}(x/N*)]D^{-1} > 0 \]  \hspace{1cm} \text{(34)}

Thus, price effects, firm numbers effects and exports per firm effects all combine in the same direction to ensure \( dS/dx^* > 0 \) as required for proposition 5.

Taken together, propositions 4 and 5 show that the disequilibrium on the IRS trade account between the two regions induced by \( N^*<N \) (with \( w^*=1 \)) is corrected by downward pressure on \( w^* \). The new equilibrium at \( N^*<N \) and \( w^*<1 \) takes one of two forms (see conditions 18). If the IRS trade account is balanced at \( w^{**} < w^* < 1 \) then only intra-industry trade in IRS goods takes place. If \( S > 0 \) at \( w^*=w^{**} \) then Region 2 balances its trade deficit on IRS goods
with a trade surplus on CRS goods and both intraindustry and interindustry trade occur in equilibrium. If the interindustry result is observed the following proposition can be stated.

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Proposition 6 (Comparative Advantage): If interindustry trade is required to achieve trade balance between the regions, then the interindustry trade pattern reveals that the large region has a comparative advantage in IRS goods.

No direct proof of proposition 6 is offered. The constraint $w^* \geq w^{**}$ ensures that Region 2’s nominal wage may not be capable of falling far enough to produce $S=0$ for $N^*>N$. This is obviously true if $t_c=0$ such that $w^*=w^{**}=1$, for example. The Region 1 net trade surplus in IRS goods is offset by imports of CRS goods.

There is an important by-product from proposition 5. The decline in $w^*$ required to re-establish trade balance following a decrease in $N^*$ (or an increase in $N$) leads to an increase in Region 2’s real wage and a decrease in Region 1’s real wage as noted above. A question that immediately arises concerns the net effect of a fall in $N^*$ accompanied by the equilibrating fall in $w^*$. With $w^*$

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8 This behaviour is consistent with the results obtained by Venables (1985, 13-15) on the effects of border taxes under cross-hauling (see also comments by Horstmann and Markusen 1986, 243 and the range of results in Markusen and Venables 1988). A tariff placed on IRS imports by Region 2 or a subsidy by Region 2 on its IRS exports reduces profits for Region 1 firms, induces exit in Region 1 and entry in Region 2, and thereby increases (decreases) welfare measured by the real wage in Region 2 (Region 1). A fall in $w^*$ required to equilibrate the balance of trade has similar effects as discussed above.
constant, proposition 1 showed that the decline in $N^*$ reduces the real wage in Region 2 and raises it in Region 1. Assuming that trade was balanced prior to the decline in $N^*$, proposition 3 implies that $w^*$ must then decline to rebalance the trade account. But the fall in $w^*$ tends to reverse the real wage changes caused by the original decline in $N^*$. Is it possible for the equilibrating fall in $w^*$ to raise the real wage in Region 1 above the real wage in Region 1 even though $N^*<N$? The following two propositions address this issue.

**Proposition 7 (Factor Prices Conditional on $N^*$ with Zero Interindustry Trade):** If Region 1’s market is larger than Region 2’s market and trade is balanced such that $w**<w^*<1$, the real wage in Region 1 exceeds the real wage in Region 2.

**Proof:** The proof is by contradiction. Assume that $N^*<N$ and $w^*<1$ with real wages equal in both regions such that $w^*p=p^*$. Clearly $p^*<p$. With relative prices equal in both regions, $f'=f^{**}$. Substituting $w^*p$ for $p^*$ in marginal condition (6), conditions (6) and (8) can be written as

\begin{align}
 p + xf'/N^* &= (1+t)/w^* \\
p + x*f'/N &= w^*(1+t)
\end{align}

Subtracting (36) from (35), $x/N^* - x^*/N = (w^{*-1}-w^*)(1+t)f^{-1} < 0$. Since $x/N^* < x^*/N$ and $N^*<N$, $x<x^*$. Turning to firm numbers in the
two regions, equal per capita consumption in both regions implies
\[
(ny+n^*x^*)/N = (n^*y^*+nx)/N^* \quad \text{or,}
\]
\[
n/n^* = (y^*/N^* - x^*/N)/(y/N - x/N)
\]
(37)

Marginal conditions (5) and (7) can be written as:

\[
(p-1) + yf'/N = 0
\]
(38)

\[
[(p*/w*) - 1] + ysf'/N^* = 0
\]
(39)

for \(f' = f^*\). With \(p = p*/w^*\), (38) and (39) imply \(y/N = y^*/N^*\). From marginal conditions (5) and (6), \(y/N = -(p-1)/f'\) and \(x/N^* = -(p*1-t)/(w* f') = -[(p-(1+t)w*-1)]/f'\). Since \((1+t)w^{-1} > 1\), \(y/N > x/N^*\). From (7) and (8), \(y^*/N^* = -(p*-w*)/(w* f') = -(p-1)/f'\) (for \(p = p*/w^*\)) and \(x^*/N = -[(p-(1+t)w*)]/f'\). Since \((1+t)w^* > 1\) (as shown in section 2), \(y^*/N^* > x^*/N\). Combining results, \(n/n^* < 1\) in equation (37). With \(p^* < p\), \(x < x^*\), and \(n < n^*\), the assumption of equal real wages in both regions \((w^*p = p^*)\) has the effect of generating a trade surplus for Region 2 in IRS goods, i.e., \(pn^*x^* > p^*x_n\). Region 2's IRS trade surplus cannot be balanced by CRS exports from Region 1 since \(w^* < 1\). Thus \(w^*\) must increase which leads to a fall in the real wage in Region 2 and a rise in the real wage in Region 1 (see proposition 5 proof). This invalidates the assumption that real wages are equal in both regions and proves proposition 7.
Proposition 8 (Factor Prices Conditional on N*, Positive Interindustry Trade): If Region 1's market is larger than Region 2's and Region 2 is exporting CRS goods, a decrease in the size of Region 2 increases (reduces) the real wage in Region 1 (2).

In the light of proposition 1, no proof of proposition 8 is needed. When Region 2 exports CRS goods, w*=w**. A fall in the size of Region 2 increases its trade deficit in IRS goods in accordance with proposition 3. The increased IRS trade deficit is offset by increased exports of CRS goods with no decline in w*. Proposition 8 is stronger than proposition 7 since proposition 7 does not imply that a monotonic relationship exists between regional market size variables and real wage variables, only that the large-market region has a higher real wage in intratrade trade equilibrium.

The factor pricing and trade equilibria described in this section have been deliberately restricted to cases in which IRS goods are involved in intratrade trade (with or without interindustry trade). It has been shown that the region with the larger market generates the higher real wage allowing for the endogenous adjustment in relative regional wages required for overall trade equilibrium. When the trade equilibrium involves interindustry trade, the large-market region is expected to have comparative advantage in IRS goods. The 'home market' effect is significant both in the sense that the large market location generates
superior factor returns and in the sense that it tends to export IRS goods to the small-market region when interindustry trade is present.

4. One-way Flows of IRS Goods

Cases in which trade takes place without intraindustry trade in IRS goods and the implications of this for factor prices are the subject of the present section. It is clear from the previous section that one-way flows of IRS goods cannot involve IRS exports from the small-market to the large-market region provided proposition 4 holds. Since positive transport costs ensure $w^*<1$ for $N^*<N$, the large region cannot be exporting CRS goods and cannot, therefore, be a net importer of IRS goods. Thus one-way movement of IRS goods implies IRS imports by the small-market region ($w^*=w^{**}$).

The results of the previous section imply that one-way trade cases are expected to emerge when Region 2's market size is relatively small. Continued decreases in $N^*$ under cross-hauling equilibrium lead to $dn^*<0$ and $dx^*<0$ once interindustry trade is established.

Two one-way trade sub-cases are possible: either $n^*>0$, $x^*=0$ or $n^*=0$. In the first case, positive numbers of IRS firms exist in both regions but the Region 2 firms do not export. The marginal condition on $x^*$ in equation (8) is replaced by the complementary slack $(p-w^*-w^*t) + x^*f'/N \leq 0$ for $x^*=0$. In the second case, zero-
profit equilibrium in both regions would imply $n^* < 0$ with the zero-profit condition for Region 2 replaced by the complementary slack $\pi^* \leq 0$ for $n^* = 0$.

The following two propositions address the factor pricing implications in these two sub-cases.

Proposition 9 ($n^* > 0, x^* = 0$): When IRS firms in Region 2 do not export, the real wage in Region 1 exceeds the real wage in Region 2. A fall in Region 2's market size lowers the real wage in Region 2 and increases the real wage in Region 1.

A relatively informal discussion is sufficient here. Consider an autarky equilibrium with $N > N^*$ such that $p < p^*/w^*$. Further assume that $p^* > (1+t)$ and $p < w^*(1+t)$. Referring to marginal conditions (6) and (8), these assumptions are sufficient to ensure that $x > 0$ and $x^* = 0$ when trade is opened. Further assume that $n^* > 0$ in trade equilibrium ($n^* = 0$ is dealt with in the next proposition). Since Region 2's IRS firms serve only their own market, the $\pi^* = 0$ and relevant marginal conditions are simplified. Equation (2) becomes $\pi^* = (p^* - w^*)y^* - w^*F = 0$ and marginal condition (7) is $(p^* - w^*) + w^*y^*f/\pi^* = 0$. These two equations serve to determine $p^*/w^*$ and $y^*$ and are equivalent to the autarky result for Region 2 described in section 2. A decrease in Region 2's market size ($dN^* < 0$) decreases firm size ($dy^* < 0$) and raises price ($dp^* > 0$).

Along the same lines, if Region 1 did not export to Region 2
(autarky), its larger market size would ensure $p < p^*/w^*$. Since $x > 0$, exports to Region 2 serve to 'subsidize' firms in Region 1 and Region 1's domestic price is therefore even lower in free entry/exit equilibrium than under autarky. Further, since $dp^*/dN^* < 0$, the size of the 'subsidy' increases with $dN^* < 0$ such that $dp/dN^* > 0$. These observations establish proposition 9.

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Proposition 10 ($n^* = 0$): If equilibrium implies IRS firms located in Region 1 only, Region 1’s real wage is higher than Region 2’s real wage. A decrease in the size of Region 2 need not worsen the real wage differential.

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Proof: The relevant zero-profit and marginal conditions are now equations (1), (5), and (6) applying to Region 1 firms. Since no firms exist in Region 2 and Region 2 is exporting CRS goods ($w^* = w^{**}$), the demand equations are $p = f(ny/N)$ and $p^*/w^{**} = f^*(nx/N^*)$ or, equivalently, $ny/N = f^{-1}(p) = g(p)$ and $nx/N^* = f^{-1}(p^*/w^{**}) = g(p^*/w^{**})$. Substituting $g(p)/n = y/N$ and $g^*(p^*/w^{**})/n = x/N^*$ into marginal conditions (5) and (6),

\[
p + g(p)/[ng'(p)] = 1 \tag{40}
\]

\[
p^*/w^{**} + g^*(p^*/w^{**})/[ng^*(p^*/w^{**})] = (1+t)/w^* \tag{41}
\]

where (40) is the marginal condition for sales to Region 1 and (41) is the marginal condition for sales to Region 2. Suppose (40) holds and further suppose that firms set per capita output in
Region 2 equal to per capita output in Region 1. In this case, g=g*, g'=g** and \( p = p*/w** \). By inspection of (40) and (41), the left-hand side of (41) is less than the right-hand side since \((1+t)/w** < 1 \) (\( t > 0 \) and \( w** < 1 \)). Profit-maximizing firms reduce per capita output in Region 2 relative to Region 1 such that \( p*/w** > p \). This establishes the first part of proposition 10.

The precise relationship between real wages in the two regions can be further examined by drawing on results from the literature on spatial price discrimination. Benson (1984) has proposed a useful functional form for (per capita) demand which uses a single demand parameter to categorize spatial price behaviour when a single firm serves both home and foreign markets. This functional form can be extended to \( n > 1 \) in order to illustrate the issues involved in the present case. For convenience, denote the relative price of IRS goods in either region by \( P \) (equal to \( p \) or \( p*/w** \)) and per capita consumption in either region by \( nV \) where, as before, \( n \) is the number of Region 1 firms, and now \( V \) stands for \( y/N \) or \( x/N^* \). The deflated marginal cost terms \([1 \) and \( (1+t)/w^*] \) on the right-hand sides of (40) and (41) are denoted by \( C \). Following Benson, the form of the per capita demand function is:

\[
P(nV) = a - zb(nV)^2, \quad a > 0, \ b > 0, \ z > -1
\]

\( (42) \)

such that \( P' = dP/d(nV) = -z^2b(nV)^{z-1}; P'' = d^2P/d(nV)^2 = -z^2(z-1)b(nV)^{z-2} \)

with \( a, b, \) and \( z \) as parameters. Marginal conditions (5) and (6)
become $P + VP' = C$. Totally differentiating, $dP + VP''d(nV) + P'dV = dC$. Dividing through by $dP$: $1 + VP''d(nV)/dP + P'dV/dP = dC/dP$.

Noting that $dV = (1/n)d(nV)$, $1 + VP''/P' + 1/n = dC/dP$.

Substituting for $P'$ and $P''$ from (42), $dC/dP = 1 + z/n$ or,

$$\frac{dP}{dC} = [1 + z/n]^{-1} \quad (43)$$

Equation (43) shows the response of the real price of IRS goods to an increase in deflated marginal cost. For $z > 0$, the per capita demand function is strictly concave from below. If $z = 1$ it is linear. For $0 < z < 1$ and $-1 < z < 0$ the demand curve is strictly convex from below. At $z = 0$, the per capita demand curve is perfectly elastic. Moving from Region 1 to Region 2, deflated marginal cost increases from 1 to $(1 + t)/w^{**}$. For a given value of $n$, the associated increase in the real price of IRS goods depends on the convexity of per capita demand. For positive values of $z$, $0 < dP/dC < 1$ indicating price discrimination against the near (Region 1) consumers (freight absorption). For negative values of $z$, $dP/dC > 1$, indicating discrimination against the distant (Region 2) buyers (phantom freight). At $z = 0$, $dP/dC = 0$, indicating absence of spatial price discrimination (delivered-cost pricing).

An increase in the number of consumers in either region increases

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7 Price discrimination against distant buyers assumes market segmentation. See note 2. Price discrimination against distant buyers is built in to some models (e.g. Venables 1987).
profit since each new consumer is served in accordance with (40) or (41). With free entry/exit the increase in profit leads to \( dn > 0 \). The difference between regional wages rises or falls depending on the impact of changes in \( n \) on \( dP/dC \) in equation (43). In the case of freight absorption \( (z > 0) \), (43) shows that a decline in the size of Region 2 \( (dn < 0) \) reduces price discrimination against Region 2 consumers while the reverse is true for \( z < 0 \). This establishes the second part of proposition 10.

Spatial price discrimination disappears if Region 1 is very large \( (z/n \approx 0) \) irrespective of the convexity properties of demand (Lim and Anderson 1988). In this case, Region 2 consumers pay transport in both directions in the sense that \( p^* = p(1+t)/w^{**} = p(1+t)(1+t_o) \) (Anderson 1982).

In summary, the corner cases in which the small-market region does not export IRS goods produce results similar in spirit to the cross-hauling (intraindustry trade) results described in the previous section. Comparative advantage in IRS goods obviously lies with the region with the larger market since the small-market region is not exporting IRS goods and real wages are highest in the large market as well. If some IRS firms are present in the small-market region, a fall in its market size worsens the real wage gap. But if firms are absent from the small-market region, a fall in its market size need not worsen the real wage gap since the impact of changes in the size of either region on the gap depends on the form of spatial pricing practiced by the firms in
the large-market region.

5. Conclusions

The purpose of the present paper has been to establish a set of factor pricing and comparative advantage propositions based on the existence of spatially separated markets of differing size where one industry experiences increasing returns to scale in the form of internal economies. The assumptions of the model neutralize all other rationales for factor price differences and interregional trade. Unlike Venables (1985), transport costs separate the markets for both IRS and CRS goods. The cross-hauling results (section 3) demonstrate that real wages are highest in the large-market region allowing for the small-market region's cost structure to be endogenously determined by a trade balance requirement. When both intraindustry trade (cross-hauling) and interindustries trade are present, it is the larger region that normally generates (net) exports of the IRS good and which can, therefore, be said to have comparative advantage in IRS production. Extending the results to cases of one-way movements of IRS goods (section 4), similar real wage and comparative advantage results hold when IRS firms in the small-market region serve only their own consumers and when IRS firms in the small-market region are entirely crowded out. In the latter case, the results are consistent with the spatial price discrimination literature.
These results contribute to understanding regional manufacturing concentration and the stylized facts of a centre-periphery structure. Interregional trade equilibrium based on internal economies generates a 'home-market' effect in the form of higher real wages in concentrated markets. If labour is interregionally mobile, a positive feedback process between real wage differences and labour endowments produces concentration of manufacturing in the large-market region along the lines suggested by Arthur (1989). The small-market region tends to disappear in favour of a spatially efficient concentration of factors. If small-market regions are to survive in the long-run, the migration process would have to be weak or blocked, leaving a real wage disparity against the small-market region. Alternatively, the model itself could be modified. The small-market region could be assumed to possess compensating advantages, such as natural resources endowments, that permit it to pay long-run real wages high enough to attract mobile labour away from the concentrated manufacturing location.

Appendix

The validity of proposition 4 is dependent on \( dN^* < 0 \) generating an improvement in Region 1's IRS trade balance, i.e. \( d(p^nx - pn^*x^*)/dN^* < 0 \). Changes in prices and firm numbers in both regions and the decrease in exports per firm in the small-market region tend to improve the trade balance while the effect on exports per
firm in the large-market region is ambiguous. Using marginal condition (6), denote a large-market firm's marginal revenue from exports by \(\text{MR} = p^* + w^*x^*/N^* = 1+t\). Totally differentiating \(\text{MR}\) with \(w^*\) and \(t\) constant,

\[
d(\text{MR}) = dp^* + w^*f'^d(x/N^*) + (x/N^*)w^*f''d(Q^*/N^*)
\]

\[
= [1+(x/N^*)f''f'^{−1}]dp^* + w^*f'^d(x/N^*) = 0 \quad (A1)
\]

The standard stability condition requires \([\cdot]>0\) in (A1) such that an increase in price as a result of reduced rival outputs leads to \(dx>0\) at any specified market size \(N^*\). Expanding (A1),

\[
d(\text{MR}) = [\cdot]dp^* + (w^*f'^{N^*-1})dx - (xw^*f'^{N^*-2})dN^* = 0 \quad (A2)
\]

or

\[
dx/dN^* = \left(N^*/w^*f'^\right)\{xw^*f'^{N^*-2} \left[\cdot \right]dp^*/dN^* \right) \quad (A3)
\]

The sign of \(dx/dN^*\) depends on the sign of \([\cdot]\). Since \(dp^*/dN^* < 0\), the result is ambiguous as noted in proposition 3. Specifically, for a small enough \([\cdot]\), \(dx/dN^* > 0\), which operates against the other five variables in its effect on the IRS trade balance. If the per capita demand function is linear, then \(f''=0\) and \([\cdot] = 1\).

Further, using equation (24), \(dp^*/dN^* = w^*f'^{N^*-2}(y^*x^* - x^2x^*)d^{-1}\.

Substituting into (A3) with \([\cdot]=1\) and simplifying:
\[
\frac{dx}{dN^*} = N^{* -1}(xy^* - yy^*)D^{-1} \\
= N^{* -1}yy^*(x-y^*)D^{-1} < 0
\] (A4)

Linearity is, therefore, a strong condition for the Region 1 firm export variable to reinforce the effects of the other variables on the trade balance. Demand functions with \(f^{**} > 0\) that are not 'too convex' to the origin (relative to a linear function) will also satisfy \(dx/dN^* < 0\) as will all demand functions that are concave to the origin \((f^{**} < 0)\).

References


