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ON THE CHOICE OF MECHANISM TO SELL TIMBER*

by

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February 1993

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On the Choice of Mechanism to Sell Timber

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Monitoring problems often constrain governments to sell timber using simple lump-sum or fixed stumpage rate mechanisms. With timber of homogeneous quality and no uncertainty concerning its value, lump-sum charges are more efficient than stumpage rates. When no uncertainty concerning value exists, but timber is of heterogeneous quality with timber prices reflecting this heterogeneity, then stumpage rates induce the harvesting of only the best grades of timber, while lump-sum charges result in the efficient choice of quality. Admitting uncertainty concerning the value of timber, implies that a mixture lump-sum payments and stumpage rates promotes efficient risk sharing and timber recovery.

1. Introduction

In North America significant volumes of timber are on government land, but the right to harvest this timber is sold to private firms. What is the appropriate mechanism for a government to sell timber? The answer to this question will depend to some extent upon the objective of a government, which is typically assumed to be rent maximization on behalf of the public, but to a greater extent upon the environment within which a government and firms interact. For a variety of reasons, most of which relate to the costs of monitoring the harvest, governments typically constrain themselves to either lump-sum payments or stumpage rates that are a fixed price per unit of timber harvested. Deriving the optimal mix of these two mechanisms within different environments is the purpose of this paper. For example, when timber is of homogeneous quality and no uncertainty concerning its value exists, then lump-sum charges are more efficient than stumpage rates. When no uncertainty concerning value exists, but timber is of heterogeneous quality and timber prices reflect this heterogeneity, then stumpage rates induce the harvesting of only the best grades of timber (also known as high-grading), while lump-sum charges result in the optimal choice of the minimum economic quality of timber. If uncertainty concerning the value of timber is admitted, then neither lump-sum payments nor stumpage rates alone need result in an efficient allocation. For while stumpage rates promote the efficient bearing of risk when a government is risk neutral and harvesters are risk averse, they also introduce moral hazard through high-grading.

In this paper, we develop a series of economic models designed to isolate factors important in the design of mechanisms to sell timber. We begin with a simple model due to Nautiyal and Love (1971) in which timber is implicitly assumed to be of homogeneous quality and where no uncertainty concerning its value exists. We discuss two different mechanisms, stumpage rates and lump-sum charges, and the effects these institutions have upon total timber recovery. We then proceed to introduce additional complications such as heterogeneous timber quality as well as uncertainty in the value of timber arising from price variability or measurement error in the timber cruise. The notation used throughout the paper is presented in Table 1.

2. The Model of Nautiyal and Love

Apparently, the first researchers to consider the problem of designing mechanisms to sell timber were Nautiyal and Love (1971). They considered an environment within which no uncertainty concerning the value of timber existed. Within their model, firms were assumed to maximize the returns from harvesting

$$R(Q) - C(Q)$$

by choice of total harvested volume Q , where the authors implicitly assumed that the first and second derivatives of $R(Q)$, the total revenue function, satisfied $R'(Q) > 0$ and $R''(Q) < 0$, while the first and second derivatives of $C(Q)$, the total cost function, satisfied $C'(Q) > 0$ and $C''(Q) > 0$. Nautiyal and Love conjectured that the shapes of $R(Q)$ and $C(Q)$ could arise because of poor timber quality at higher total harvested volumes Q , but did not explicitly introduce timber quality. Thus, a homogeneous Q was implicitly being assumed. In any case, in the absence of any government charges the rent maximizing Q is characterized by the first-order condition

$$R'(Q) = C'(Q). \tag{2.1}$$

This is depicted as point Q_E in Figure 1. (2.1) has a simple economic interpretation: marginal revenue equals marginal cost. When a fixed stumpage rate b per unit of volume harvested is introduced, the returns to harvesters become

$$R(Q) - C(Q) - bQ$$

and optimal Q is characterized by

$$R'(Q) = C'(Q) + b. \quad (2.2)$$

In Figure 1, this corresponds to Q_S . (2.2) can also be interpreted as marginal revenue equals marginal cost, but now the stumpage rate is part of the marginal harvesting cost. Nautiyal and Love go on to remark that were a lump-sum charge B used then the returns to harvesters would become

$$R(Q) - C(Q) - B$$

and the optimal Q would be characterized by (2.1) and would be depicted by Q_E in Figure 1.

The simplicity of the Nautiyal and Love model makes it compelling. There are several ways in which the model is incomplete. First, b and B are determined exogenously. In many jurisdictions, b and B are a market outcome determined by competitive bidding. Thus, a method for predicting the equilibrium b and B , and how they react to changes in the economic environment appears useful. Second, the model admits neither heterogeneity in timber quality nor uncertainty concerning the value or the volume of timber. Finally, investigating the optimal choice of mechanism under these conditions also appears useful. In the sections which follow, we investigate models in which timber can be of heterogeneous quality as well as uncertain value or volume. As a benchmark, we consider the case of perfect monitoring within an environment of heterogeneous timber but certain timber value and volume.

3. Heterogeneous Timber Quality with Costless Monitoring

Consider a representative harvester who faces fixed prices which depend upon a continuous measure of timber quality $t \in [0, 1]$. For a particular species assume that p , the price per unit of volume, is increasing in t ; *i.e.*, $p = p(t)$ where $p'(t) \geq 0$. Denote the volume of timber of quality t by $q(t)$. For expositional simplicity, consider the case of constant marginal harvesting costs c which are independent of timber quality, where $c < p(t)$ for some $t \in [0, 1]$.¹ Assume that the government can monitor

¹ The qualitative predictions are unaffected by the introduction of marginal harvesting costs c which depend upon t , provided $c'(t) < p'(t)$. The assumption of a continuous t can also be modified to include discrete mass points. Note too that the introduction of a fixed cost k does not alter our results qualitatively, nor does the presence of uncertainty concerning the value or the volume of timber.

timber quality as well as the harvest costlessly. In this case, the government charges a fee $b(t)$ for each unit of timber harvested which depends upon timber quality t . In particular,

$$b(t) = p(t) - c \quad b(t) \geq 0.$$

When monitoring is costless, the government can extract all of the rent, and does not distort timber recovery. Typically, however, it is very costly for a government to monitor timber quality as well as the harvest, and in these cases two alternative methods of payment are used: The first is a lump-sum payment B which is independent of the volume harvested, while the second is a fixed price stumpage rate per unit of timber harvested b where b is independent of timber quality t .

4. Heterogeneous Timber Quality with Costly Imperfect Monitoring

The model considered in this section is derivative of that presented in the empirical work of Paarsch (1993). There, as a benchmark case, the behaviour of an unregulated profit maximizing agent who harvests timber from his own land was considered first. When timber is privately owned, a profit maximizing harvester will choose a minimum (reservation) quality θ to maximize

$$\int_{\theta}^1 (p(t) - c)q(t) dt,$$

so

$$p(\theta_E) = c \tag{4.1}$$

defines the reservation quality θ_E . θ_E maximizes the rent from the resource and all of this rent accrues to the harvester. In Figure 2, we depict the equilibrium as the intersection of the $p(t)$ curve and the c curve. Obviously, (4.1) does not preclude some timber being left unused as slash, but not harvesting low quality timber is economically efficient in this case.²

Many governments auction the right to harvest timber on public land. The principal bidding variable at a majority of auctions is the stumpage rate b , which is the price per cubic unit of timber harvested. Although the stumpage rate usually varies by species, it is typically independent of timber quality. In many jurisdictions,

² We are, of course, abstracting from the *post messum* silviculture costs which may be important.

utilization standards which “fix” the minimum t at θ_G are used, with penalties for deviations from these standards. Suppose that $\theta_G = \theta_E$. Because monitoring is costly to perform π , the probability of getting caught deviating from θ_G , is typically less than one. Typically, the penalty for not complying with θ_G equals a simple multiple ρ of the stumpage for timber volumes of quality above θ_G that have been left behind.

Within the environment described above, calculating the equilibrium bid involves the following. Suppose initially that the winning harvester has bid $b(> 0)$ and consider the choice by a risk neutral harvester of θ to maximize

$$\psi(\theta) = \left(\int_{\theta}^1 (p(t) - b - c)q(t) dt \right) - \rho\pi b \int_{\theta_G}^{\theta} q(t) dt.$$

Here the term in parentheses on the left is the profit from choosing θ , while the term on the right is the expected value of the penalty for not setting $\theta = \theta_G$. Setting the derivative of $\psi(\theta)$ with respect to θ equal to zero implies that reservation quality θ_S is now determined by the marginal condition

$$p(\theta_S) = b + c - \rho\pi b \tag{4.2}$$

or

$$\theta_S = p^{-1}(b + c - \rho\pi b).$$

Note that if $\rho = 1/\pi$, then compliance with $\theta_G = \theta_E$ will be perfect. Typically, however, ρ is relatively small (less than 3) and often $\rho = 1$.

In competitive equilibrium zero profits will obtain, on average, implying that the equilibrium bid is defined implicitly by

$$b = \frac{\int_{p^{-1}(b+c-\rho\pi b)}^1 (p(t) - c)q(t) dt - \rho\pi b \int_{\theta_G}^{p^{-1}(b+c-\rho\pi b)} q(t) dt}{\int_{p^{-1}(b+c-\rho\pi b)}^1 q(t) dt} > 0.$$

Harvesters bid the average rent per unit of intra-marginally profitable timber where both the stumpage rate and the expected penalty for non-compliance are used to determine reservation quality.

When $\rho\pi < 1$, the solution to (4.2) is different from the solution to (4.1) because $p(t)$ is increasing in t and $b > 0$ which implies an increase in the reservation quality of timber, also known as high-grading. In Figure 2, the equilibrium obtains at the

intersection of the $p(t)$ curve and the $b + c$ curve.³ One implication of such a change is that the volume of timber harvested when stumpage rates are used is less than in the rent maximizing case

$$Q(\theta_S) = \int_{\theta_S}^1 q(t) dt < Q(\theta_E) = \int_{\theta_E}^1 q(t) dt.$$

Consider now the case of a lump-sum bid B which is independent of the volume of timber harvested. Because payment is independent of the harvested volume, (4.1) will determine reservation quality, so no high-grading occurs. Also, in competitive equilibrium zero profits will obtain, so harvesters choose B to satisfy

$$B = \int_{\theta_E}^1 (p(t) - c)q(t) dt.$$

Note that in this case the government extracts all of the resource rent, while in the stumpage rate case

$$\int_{\theta_E}^{\theta_S} (p(t) - c)q(t) dt > 0$$

of the rent is left in the forest.

5. Heterogeneous Quality, Imperfect Monitoring, and Risk Aversion

In this section, we consider the behaviour of a risk-averse harvester. Again, as a benchmark we first examine the behaviour of a profit maximizing harvester who owns the timber. We assume that the harvester has a von Neumann-Morgenstern utility function $U(y)$ with $U'(y) > 0$ and $U''(y) < 0$. Since the second derivative is negative, the harvester is risk averse. Without loss of generality, we assume that $U(0) = 0$. When the harvester owns the timber, he solves the following problem:

$$\max_{\langle \theta \rangle} U \left(\int_{\theta}^1 (p(t) - c)q(t) dt \right). \quad (5.1)$$

Note that because U is increasing, (5.1) is maximized when

$$\int_{\theta}^1 (p(t) - c)q(t) dt$$

³ (4.2) can be differentiated totally to find the effects of changes in c , ρ , and π upon the equilibrium θ_S and $Q(\theta_S)$.

is maximized. Thus, the harvester chooses

$$\theta = \theta_E.$$

Letting

$$R = \int_{\theta_E}^1 (p(t) - c)q(t) dt,$$

the harvester enjoys utility $U(R)$.

Now consider the case where the government owns the timber. If government uses a lump-sum mechanism, and the industry is competitive, then a representative harvester will have zero rent in equilibrium.⁴ The equilibrium lump-sum bid B is

$$B = R = \int_{\theta_E}^1 (p(t) - c)q(t) dt.$$

In words, the government extracts all of the surplus, and the harvester enjoys the reservation utility; this is exactly the same as in the risk-neutral case.

Consider now the case of a fixed stumpage rate. Given b , the harvester solves the following problem:

$$\begin{aligned} \max_{\langle \theta \rangle} E[U] = \max_{\langle \theta \rangle} & (1 - \pi)U \left(\int_{\theta}^1 (p(t) - b - c)q(t) dt \right) + \\ & \pi U \left(\int_{\theta}^1 (p(t) - b - c)q(t) dt - \rho b \int_{\theta_G}^{\theta} q(t) dt \right). \end{aligned}$$

Since U is concave, $E[U]$ is also concave in θ , so the first-order condition characterizes the optimal θ . Note that

$$\frac{dE[U]}{d\theta} = -q(\theta) \{ (1 - \pi)U'(K(\theta))(p(\theta) - b - c) + \pi U'(K(\theta) - F(\theta))(p(\theta) - b - c + \rho b) \}$$

where

$$K(\theta) = \int_{\theta}^1 (p(t) - b - c)q(t) dt$$

⁴ Instead of considering a paradigm within which timber is sold at auction, one could also consider one wherein a principal-agent relationship exists between a government and an harvester, with the government making a take-it-or-leave-it offer. In this case, the principal has no incentive to allow positive profits for the agent.

and

$$F(\theta) = \rho b \int_{\theta_E}^{\theta} q(t) dt.$$

Now $dE[U]/d\theta = 0$ when

$$e(\theta) \equiv (1 - \pi)U'(K(\theta))(p(\theta) - b - c) + \pi U'(K(\theta) - F(\theta))(p(\theta) - b - c + \rho b) = 0.$$

Denote the solution to $e(\theta) = 0$ in terms of b by $\theta(b)$. Note that $\theta(0) = \theta_E$.

Consider the case of $b > 0$, then

Proposition 1: When $b > 0$, if $\rho \geq 1/\pi$, then $\theta(b) = \theta_E$, while if $\rho < 1/\pi$, then $\theta(b) > \theta_E$.

The proof of this proposition, like all of our proofs, is contained in the appendix. Note that when $\rho \geq 1/\pi$, the analysis becomes trivial. From now on we assume $\rho < 1/\pi$, so $\theta(b) > \theta_E$ with $b > 0$.

We want to compare $\theta(b)$ for a risk-averse harvester with that for a risk-neutral harvester. Denote $\theta^N(b)$ and $\theta^A(b)$ be the optimal θ s for risk-neutral and risk-averse harvesters, respectively. Note that $\theta^N(b)$ is implicitly defined by

$$p(\theta^N(b)) = b + c - \rho\pi b.$$

We show

Proposition 2: For all $b > 0$, the reservation timber quality for a risk-neutral harvester is higher than that for a risk-averse harvester; *i.e.*, $\theta^N(b) > \theta^A(b)$.

Next, we want to compare b^A and b^N , the equilibrium bids of risk-averse and risk-neutral harvesters respectively, when stumpage rates are used. We show

Proposition 3: When stumpage rates are used, the equilibrium bid of a risk-averse harvester is less than that of a risk-neutral harvester; *viz.*, $0 < b^A < b^N$.

Finally, we compare the revenue garnered by the government. Denote the revenue collected from the risk-neutral harvester by R^N which is

$$R^N = b^N \int_{\theta^N(b^N)}^1 q(t) dt + \rho\pi b^N \int_{\theta_E}^{\theta^N(b^N)} q(t) dt.$$

For a risk-averse harvester, the revenue collected by the government is

$$R^A = b^A \int_{\theta^A(b^A)}^1 q(t) dt + \rho\pi b^A \int_{\theta_E}^{\theta^A(b^A)} q(t) dt.$$

Note that $b^A < b^N$, but $\theta^A(b^A) < \theta^N(b^N)$, so R^N may be greater than or less than R^A .

Although we cannot compare R^N and R^A directly, we know that both are strictly less than R the rent garnered from the lump-sum bidding. Also, when we calculate R^N or R^A , we ignore the monitoring costs which can be quite significant. If monitoring is costly, then a government has no incentive to use stumpage rates. But stumpage rates are used by the majority of agencies which manage timber for governments. In order to explain why a government would use stumpage rates, we must introduce some uncertainty concerning the value or the volume of timber.

6. Timber of Uncertain Value and Risk Neutral Harvesters

The model considered here is identical to that of section 4 except now some uncertainty concerning the value or the volume of the harvest exists. This could arise because the profit margin on each grade is estimated with error or because the timber cruise is in error. To formalize this, we assume that instead of observing the exact volume, the harvester faces an uncertain volume

$$\tilde{q}(t) = \varepsilon q(t)$$

where ε is assumed to be a continuous random variable on the interval $[0, \infty)$ have the following two raw moments $E[\varepsilon] = 1$ and $E[\varepsilon^2] < \infty$. Assume that $\Phi(\varepsilon)$, the cumulative distribution function of ε , is known to all potential harvesters and that both the government and the harvesters are risk neutral.

If timber is privately owned, then an unregulated profit maximizing harvester will choose θ to maximize

$$\int_0^\infty \int_\theta^1 (p(t) - c)\varepsilon q(t) dt d\Phi(\varepsilon),$$

so

$$p(\theta_E) = c$$

again defines the reservation quality θ_E . To calculate the equilibrium bid when a stumpage rate is used, maximize

$$\int_0^\infty \left(\int_\theta^1 (p(t) - b - c) \varepsilon q(t) dt - \rho \pi b \int_{\theta_G}^\theta \varepsilon q(t) dt \right) d\Phi(\varepsilon)$$

with respect to θ , so reservation quality θ_S is again determined by the marginal condition

$$p(\theta_S) = b + c - \rho \pi b$$

implying that when $\rho \pi \leq 1$ the equilibrium bid is defined implicitly by

$$b = \frac{\int_0^\infty \int_{p^{-1}(b+c-\rho\pi b)}^1 (p(t) - c) \varepsilon q(t) dt - \rho \pi b \int_{\theta_G}^{p^{-1}(b+c-\rho\pi b)} \varepsilon q(t) dt d\Phi(\varepsilon)}{\int_0^\infty \int_{p^{-1}(b+c-\rho\pi b)}^1 \varepsilon q(t) dt d\Phi(\varepsilon)} \geq 0.$$

When a lump-sum bid B is used, (4.1) will determine reservation quality and harvesters will choose B to satisfy

$$B = \int_0^\infty \int_{\theta_E}^1 (p(t) - c) \varepsilon q(t) dt d\Phi(\varepsilon).$$

7. Timber of Uncertain Value and Risk Averse Harvesters

In this section, we assume that the government is risk neutral, but that harvesters are risk averse having von Neumann-Morgenstern utility functions $U(y)$ with $U'(y) > 0$ and $U''(y) < 0$. As in section 6, we introduce some uncertainty concerning the value or the volume of timber at each grade. Since harvesters are risk averse and the government is risk neutral, the optimal mechanism for risk sharing requires that the government bear all of the risk. When the government uses a lump-sum mechanism, harvesters bear all of the risk. When the government uses a stumpage rate mechanism, the government bears all of the risk, but a moral hazard problem exists in that harvesters have an incentive to high-grade. Thus, a mixture of lump-sum and stumpage rate mechanisms appears appropriate. In this section, we demonstrate this intuition.⁵ Denote any feasible stumpage rate/lump-sum bid combination by (b, B) .

⁵ When this problem is interpreted as a principal-agent problem, we show that the government uses a mechanism of the following form:

$$bQ + B$$

with $b > 0$.

In the context of an auction, we may think of B as an entry fee. The harvester's control variable is the stumpage rate b . If an harvester submits the winning bid b , he pays the government bQ ; the other harvesters have their entry fees refunded.

Consider a representative harvester's behaviour when the government uses (b, B) and he wins the auction. He will maximize

$$\max_{\langle \theta \rangle} E[(\theta; b, B)] = \max_{\langle \theta \rangle} E[(1 - \pi)U(\varepsilon K(\theta, b) - B) + \pi U(\varepsilon(K(\theta, b) - F(\theta, b)) - B)]$$

where

$$K(\theta, b) = \int_{\theta}^1 (p(t) - b - c)q(t) dt$$

and

$$F(\theta, b) = \rho b \int_{\theta_G}^{\theta} q(t) dt.$$

Here the expectation is taken with respect to ε . Since $E[U]$ is concave in θ , the optimal θ is characterized by the first-order condition

$$e(\theta) = E\{\varepsilon[(1 - \pi)U'(\varepsilon K(\theta, b) - B)(p(\theta) - b - c) + \pi U'(\varepsilon(K(\theta, b) - F(\theta, b)) - B)(p(\theta) - b - c + \rho b)]\} = 0.$$

We call this "the incentive constraint" and denote it (IC). Competition dictates that the winning harvester's expected utility satisfies

$$\max_{\langle \theta \rangle} E[(1 - \pi)U(\varepsilon K(\theta, b) - B) + \pi U(\varepsilon(K(\theta, b) - F(\theta, b)) - B)] = U(0) = 0.$$

That is, competition will drive the equilibrium $E[U]$ to the point where it is equal to that of not participating in the harvest. We call this "the participation constraint" and denote it (PC). The objective of the government is to maximize revenue garnered by choice of (b, B) subject to the incentive and participation constraints. Formally, the government maximizes the following:

$$\max_{\langle B, b, \theta \rangle} b \int_{\theta}^1 q(t) dt + \rho \pi b \int_{\theta_G}^{\theta} q(t) dt + B$$

subject to

$$E\{\varepsilon[(1 - \pi)U'(K(\theta, b) - B)(p(\theta) - b - c) + \pi U'(\varepsilon(K(\theta, b) - F(\theta, b)) - B)(p(\theta) - b - c + \rho b)]\} = 0$$

and

$$E[(1 - \pi)U(\varepsilon K(\theta, b) - B) + \pi U(\varepsilon(K(\theta, b) - F(\theta, b)) - B)] = U(0) = 0.$$

At this stage, we do not fully characterize the optimal (b, B) . Instead, we consider the conditions under which $b > 0$ for some $B < R$. The solution to the above problem will have the following properties:

Proposition 4: The optimal stumpage rate b is positive, and thus $R > B \geq 0$.

In other words, an optimal contract will have both a stumpage rate and a lump-sum fee which are positive. An implication of this proposition is that some high-grading will be optimal within this environment.

8. Some Solved Numerical Examples

In this section, we investigate the trade-offs which exist between stumpage rate and lump-sum payment schemes. We also investigate the optimal choice of b and B which maximize the revenues garnered from the harvest by the government. To get results we must be specific about the price function, the quality distribution, how averse harvesters are to risk, the distribution of measurement error, *etc.*. Thus, we consider solved numerical examples in which

$$p(t) = t \quad t \in [0, 1]$$

$$q(t) = 1 \quad t \in [0, 1]$$

$$U(y) = 1 - \exp(-\eta y) \quad \eta > 0$$

$$\Phi(\varepsilon) = 1 - \exp(-\varepsilon) \quad \varepsilon > 0.$$

Here, the price function is linear in quality; the quality distribution is uniform; utility comes from the family of von Neumann-Morgenstern utility functions for which the Arrow-Pratt measure of absolute risk aversion is a constant η ; and the distribution of the measurement error is exponential with mean and variance equal to one. With these assumptions

$$K(\theta, b) = 0.5(1 - \theta^2) - (1 - \theta)(b + c)$$

$$F(\theta, b) = \rho b(\theta - c),$$

so the expected utility of a harvester is

$$\begin{aligned} E[U] &= 1 - \frac{(1 - \pi) \exp(\eta B)}{1 + \eta K(\theta, b)} - \frac{\pi \exp(\eta B)}{1 + \eta K(\theta, b) - \eta F(\theta, b)} \\ &= 1 - \frac{(1 - \pi) \exp(\eta B)}{1 + \eta(0.5(1 - \theta^2) - (1 - \theta)(b + c))} - \\ &\quad \frac{\pi \exp(\eta B)}{1 + \eta(0.5(1 - \theta^2) - (1 - \theta)(b + c) - \rho b(\theta - c))}. \end{aligned}$$

The incentive constraint, which is the derivative of $E[U]$ with respect to θ set to zero, is

$$\begin{aligned} e(\theta, b) &= \frac{\eta(1 - \pi) \exp(\eta B) \partial K(\theta, b) / \partial \theta}{(1 + \eta K(\theta, b))^2} + \\ &\quad \frac{\eta \pi \exp(\eta B) (\partial K(\theta, b) / \partial \theta - \partial F(\theta, b) / \partial \theta)}{(1 + \eta K(\theta, b) - \eta F(\theta, b))^2} \\ &= \frac{\eta(1 - \pi) \exp(\eta B) (b + c - \theta)}{(1 + \eta(0.5(1 - \theta^2) - (1 - \theta)(b + c)))^2} + \\ &\quad \frac{\eta \pi \exp(\eta B) ((1 - \rho)b + c - \theta)}{(1 + \eta(0.5(1 - \theta^2) - (1 - \theta)(b + c) - \rho b(\theta - c)))^2} = 0. \end{aligned} \tag{8.1}$$

The participation constraint, which solves $E[U] = U(0) = 0$, is

$$\exp(-\eta B) = \frac{1 + \eta K(\theta, b) - (1 - \pi) \eta F(\theta, b)}{(1 + \eta K(\theta, b))(1 + \eta K(\theta, b) - \eta F(\theta, b))}. \tag{8.2}$$

The simultaneous solution of (8.1) and (8.2) defines the equilibrium b and θ for a given B .

In Table 2, we present simulation results for a variety of (B, c, π, ρ, η) tuples. The first row of the table gives the results for the risk neutral case when $B = 0$; hence, the “.” in place of η in column (1). The second row of the table gives the results for the risk neutral case when $B = R$; hence, the “.” in place of η in column (1) and $b = 0$ in column (7). The other rows for which there are “.”s denote simulation results for risk neutral harvesters who face different harvesting costs. In the rows of Table 2 which have asterisks on the tuples in column (1), we present the optimal choice of B , b , and θ for the above numerical example by maximizing with respect to B , b , and θ

$$\Omega = b(1 - \theta) + \rho \pi b(\theta - c) + B$$

subject to (8.1) and (8.2). Note that the sole use of stumpage rates can reduce the volume of timber recovered by over forty percent and reduce the total revenues garnered by the government by over thirty percent relative to the optimal contract. Compare row (3) column (2) with row (7) column (2) as well as row (10) column (2) with row (14) column (2).

Table 2 also indicates that the problem of high-grading is relatively greater when resource rents are high because b is high. High rents and high stumpage rates result in a greater disruption of the reservation quality. This can explain the apparent paradox of harvesters leaving large dimensioned stems, which would be economically useful in the absence of stumpage rates, as slash.

Note too that even a fifty percent detection rate ($\pi = 1/2$) when used in conjunction with a relatively low penalty of $\rho = 1$ can do a great deal for efficiency. In fact, if $\rho = 2$ when $\pi = 1/2$, then the efficient allocation will be mimicked. But the costs of monitoring can be substantial, and in many jurisdictions the use of large fines is politically unacceptable.

9. Conclusions

In this paper, we have investigated the choice of the appropriate mechanism for a government to sell timber in an imperfect world where monitoring the harvest is costly. Within different environments, we have examined which of simple lump-sum charges or stumpages rates is more efficient. These two mechanisms are examined because of their prevalence as policy instruments, and because monitoring costs typically prevent one from implementing other more sophisticated schemes such as those discussed in section 3. We find that, in the absence of uncertainty concerning the value or volume of timber on a site, lump-sum charges are more efficient than stumpage rates. Within such environments stumpage rates impede the efficient choice of timber quality, and reduce both total timber recovery and the resource rents collected by a government. When uncertainty concerning the value or volume of timber exists and harvesters are risk averse while the government is risk neutral, then a mix of stumpage rates and lump-sum charges promotes efficient timber recovery as well as efficient risk bearing. Of course, the precise structure of the mix will depend critically upon the environment within which the harvest is undertaken as the numerical examples in section 8 have illustrated.

A. Appendix

In this appendix, we present the proofs of the propositions contained in the text of the paper.

Proof of Proposition 1: Evaluate $e(\theta)$ at $\theta = \theta_E$. Using the fact that $p(\theta_E) = c$, we have

$$e(\theta_E) = bU'(K(\theta_E))(1 - \rho\pi).$$

If $\rho \geq 1/\pi$, then $e(\theta_E) \geq 0$. Note too that $dE[U](\theta_E)/d\theta = -q(\theta_E)e(\theta_E)$, so that we have $dE[U](\theta_E)/d\theta \leq 0$. Since θ_E is the lower bound for θ , the harvester chooses $\theta(b) = \theta_E$. If $\rho < 1/\pi$, then $dE[U](\theta_E)/d\theta > 0$, so the harvester chooses $\theta(b) > \theta_E$.

Proof of Proposition 2: Evaluate $e(\theta)$ at $\theta = \theta^N(b)$.

$$\begin{aligned} e(\theta^N(b)) &= (1 - \pi)U'(K(\theta^N(b)))(p(\theta^N(b)) - b - c) + \\ &\quad \pi U'(K(\theta^N(b)) - F(\theta^N(b)))(p(\theta^N(b)) - b - c + \rho b). \end{aligned}$$

Note that when $b > 0$, $\theta^N(b) > \theta_E$, so $F(\theta^N(b)) > 0$, since U' is strictly decreasing, $U'(K(\theta^N(b))) < U'(K(\theta^N(b)) - F(\theta^N(b)))$. Hence, we have

$$\begin{aligned} e(\theta^N(b)) &> (1 - \pi)U'(K(\theta^N(b)))(p(\theta^N(b)) - b - c) + \\ &\quad \pi U'(K(\theta^N(b)))(p(\theta^N(b)) - b - c + \rho b) \\ &= U'(K(\theta^N(b)))(p(\theta^N(b)) - b - c + \pi\rho b) = 0. \end{aligned}$$

Thus, we have $e(\theta^N(b)) > 0$. Therefore, $dE[U](\theta^N(b))/d\theta < 0$. This proves that $\theta^N(b) > \theta^A(b)$.

Proof of Proposition 3: Assuming perfect competition in the industry, b^A is characterized by the zero profit condition

$$\begin{aligned} (1 - \pi)U \left(\int_{\theta^A(b^A)}^1 (p(t) - b^A - c)q(t) dt \right) + \\ + \pi U \left(\int_{\theta^A(b^A)}^1 (p(t) - b^A - c)q(t) dt - \rho b^A \int_{\theta_G}^{\theta^A(b^A)} q(t) dt \right) = 0. \end{aligned}$$

By Jensen's Inequality, since U is strictly concave,

$$U\left(\int_{\theta^A(b^A)}^1 (p(t)-b^A-c)q(t) dt - \rho\pi b^A \int_{\theta_G}^{\theta^A(b^A)} q(t) dt\right) > \\ (1-\pi)U\left(\int_{\theta^A(b^A)}^1 (p(t)-b^A-c)q(t) dt\right) + \\ \pi U\left(\int_{\theta^A(b^A)}^1 (p(t)-b^A-c)q(t) dt - \rho b^A \int_{\theta_G}^{\theta^A(b^A)} q(t) dt\right) = 0.$$

Because U is increasing,

$$\int_{\theta^A(b^A)}^1 (p(t)-b^A-c)q(t) dt - \rho\pi b^A \int_{\theta_G}^{\theta^A(b^A)} q(t) dt > 0.$$

Note that the above equation is the function which the risk-neutral harvester maximizes. For the risk-neutral harvester, the optimal choice of θ is $\theta^N(b^A)$, so we have

$$\int_{\theta^N(b^A)}^1 (p(t)-b^A-c)q(t) dt - \rho\pi b^A \int_{\theta_G}^{\theta^N(b^A)} q(t) dt > \\ \int_{\theta^A(b^A)}^1 (p(t)-b^A-c)q(t) dt - \rho\pi b^A \int_{\theta_G}^{\theta^A(b^A)} q(t) dt > 0.$$

Since b^N is characterized by

$$\int_{\theta^N(b^N)}^1 (p(t)-b^N-c)q(t) dt - \rho\pi b^N \int_{\theta_G}^{\theta^N(b^N)} q(t) dt = 0$$

and $\int_{\theta^N(b)}^1 (p(t)-b-c)q(t) dt - \rho\pi b \int_{\theta_G}^{\theta^N(b)} q(t) dt$ is decreasing in b , we have $b^N > b^A$. Of course, $b^A > 0$, otherwise, the harvester enjoys the positive profit which violates the zero profit condition.

Proof of Proposition 4: For this problem, we form the following Lagrangean:

$$\max_{\langle B, b, \theta \rangle} \mathcal{L} = \max_{\langle B, b, \theta \rangle} b \int_{\theta}^1 q(t) dt + \rho\pi b \int_{\theta_G}^{\theta} q(t) dt + B - \lambda_1 IC - \lambda_2 PC.$$

Suppose that $b = 0$ is optimal. The incentive constraint implies that $\theta = \theta_E$ and the participation constraint implies that B solves

$$E[U(\varepsilon K(\theta_E, 0) - B^e)] = U(0) = 0.$$

Thus, by IC and PC, if $b = 0$ is optimal, then $(0, B^e)$ with $\theta = \theta_E$ is the unique solution. If $(0, B^e)$ with $\theta = \theta_E$ is indeed optimal, it should satisfy the following necessary conditions:

$$\frac{\partial \mathcal{L}(0, B^e, \theta_E)}{\partial B} = 0, \quad \frac{\partial \mathcal{L}(0, B^e, \theta_E)}{\partial b} = 0, \quad \text{and} \quad \frac{\partial \mathcal{L}(0, B^e, \theta_E)}{\partial \theta} \leq 0.$$

But

$$\begin{aligned} \frac{\partial \mathcal{L}(0, B^e, \theta_E)}{\partial B} &= 1 + \lambda_2 \mathbb{E}[U'(\varepsilon K(\theta_E, 0) - B^e)] \\ \frac{\partial \mathcal{L}(0, B^e, \theta_E)}{\partial b} &= (1 - \rho\pi)\lambda_1 \mathbb{E}[\varepsilon U'(\varepsilon K(\theta_E, 0) - B^e)] + \\ &\quad \int_{\theta_E}^1 q(t) dt \{1 + \lambda_2 \mathbb{E}[\varepsilon U'(\varepsilon K(\theta_E, 0) - B^e)]\} \\ \frac{\partial \mathcal{L}(0, B^e, \theta_E)}{\partial \theta} &= -\lambda_1 p'(\theta_E) \mathbb{E}[\varepsilon U'(\varepsilon K(\theta_E, 0) - B^e)]. \end{aligned}$$

Imposing $\partial \mathcal{L} / \partial \theta \leq 0$, implies that $(1 - \rho\pi)\lambda_1 \mathbb{E}[\varepsilon U'(\varepsilon K(\theta_E, 0) - B^e)]$ must be non-negative. This reduces to a comparison of the expression $\mathbb{E}[U'(\varepsilon K(\theta_E, 0) - B^e)]$ with the expression $\mathbb{E}[\varepsilon U'(\varepsilon K(\theta_E, 0) - B^e)]$. Note that by setting $\partial \mathcal{L} / \partial B = 0$, we have $\lambda_2 < 0$, so it suffices to show that

$$\begin{aligned} \mathbb{E}[U'(\varepsilon K(\theta_E, 0) - B^e)] &= \int_0^\infty U'(\varepsilon K(\theta_E, 0) - B^e) d\Phi(\varepsilon) > \\ \mathbb{E}[\varepsilon U'(\varepsilon K(\theta_E, 0) - B^e)] &= \int_0^\infty \varepsilon U'(\varepsilon K(\theta_E, 0) - B^e) d\Phi(\varepsilon). \end{aligned}$$

Let

$$D = \mathbb{E}[\varepsilon U'(\varepsilon K(\theta_E, 0) - B^e)] - \mathbb{E}[U'(\varepsilon K(\theta_E, 0) - B^e)],$$

then

$$\begin{aligned} D &= \int_0^\infty (\varepsilon - 1) U'(\varepsilon K - B^e) d\Phi(\varepsilon) \\ &= \int_0^1 (\varepsilon - 1) U'(\varepsilon K - B^e) d\Phi(\varepsilon) + \int_1^\infty (\varepsilon - 1) U'(\varepsilon K - B^e) d\Phi(\varepsilon) \\ &< U'(K - B^e) \int_0^1 (\varepsilon - 1) d\Phi(\varepsilon) + U'(K - B^e) \int_1^\infty (\varepsilon - 1) d\Phi(\varepsilon) \\ &= U'(K - B^e) \int_0^\infty (\varepsilon - 1) d\Phi(\varepsilon) = 0. \end{aligned}$$

The inequality comes from the fact that $U'(y)$ is strictly decreasing and $K > 0$. If $\varepsilon > 1$, then $U'(\varepsilon K - B^e) < U'(K - B^e)$, and if $\varepsilon < 1$, then $U'(\varepsilon K - B^e) > U'(K - B^e)$. The equality comes from the fact that the expectation of ε equals one. Thus, $b = 0$ cannot be optimal when uncertainty exists concerning either the value or the volume of timber.

B. Bibliography

Nautiyal, J. and D. Love. "Some Economic Implications of Methods of Charging Stumpage," *Forestry Chronicle*, 47 (1971), 25-28.

Paarsch, H.. "The Effect of Stumpages Rates on Timber Recovery." *Canadian Journal of Economics*, 26 (1993), 107-120.

Table 1
Definition of Symbols

Q = total volume of timber recovered

$R(Q)$ = total revenues from harvesting Q

$C(Q)$ = total costs from harvesting Q

b = stumpage rate paid per unit of volume harvested

B = lump-sum charge for all timber

c = marginal cost of harvesting one unit of timber

t = timber quality which is contained on the interval $[0,1]$

$p(t)$ = price per unit of volume of timber of quality t

$q(t)$ = volume of timber of quality t

θ_E = efficient minimum reservation timber quality

θ_G = government's regulated minimum reservation quality

θ_S = stumpage rate reservation timber quality

π = probability of detection for deviations from θ_G

ρ = multiple of stumpage rate charged as a fine for deviating from θ_G

$U(y)$ = von Neumann-Morgenstern utility function of the argument y

$\tilde{q}(t)$ = realized volume of timber of quality t

ε = proportional error in $\tilde{q}(t)$

$\Phi(\varepsilon)$ = cumulative distribution function of ε

Table 2
Equilibrium Solutions for Several (B, c, π, ρ, η) Tuples

	(B, c, π, ρ, η)	Ω	θ_E	θ_S	$Q(\theta_E)$	$Q(\theta_S)$	b
(1)	(0.0000,0.10,0.1,1, ·)	0.2451	0.10	0.66	0.90	0.34	0.63
(2)	(0.8100,0.10,0.1,1, ·)	0.8100	0.10	0.25	0.90	0.90	0.00
(3)	(0.0000,0.10,0.1,1,1)	0.2737	0.10	0.59	0.90	0.41	0.60
(4)	(0.1250,0.10,0.1,1,1)	0.3460	0.10	0.42	0.90	0.58	0.36
(5)	(0.0000,0.10,0.5,1,1)	0.3793	0.10	0.31	0.90	0.69	0.48
(6)	(0.0000,0.10,0.1,1,1.2)	0.2793	0.10	0.58	0.90	0.42	0.59
(7)	(0.2284,0.10,0.1,1,1)*	0.3639	0.10	0.26	0.90	0.74	0.18
(8)	(0.0000,0.25,0.1,1, ·)	0.1708	0.25	0.72	0.75	0.28	0.52
(9)	(0.5625,0.25,0.1,1, ·)	0.5625	0.25	0.25	0.75	0.75	0.00
(10)	(0.0000,0.25,0.1,1,1)	0.1840	0.25	0.68	0.75	0.32	0.51
(11)	(0.1250,0.25,0.1,1,1)	0.2529	0.25	0.45	0.75	0.55	0.22
(12)	(0.0000,0.25,0.5,1,1)	0.2627	0.25	0.44	0.75	0.56	0.40
(13)	(0.0000,0.25,0.1,1,1.2)	0.1868	0.25	0.67	0.75	0.33	0.50
(14)	(0.1803,0.25,0.1,1,1)*	0.2583	0.25	0.36	0.75	0.64	0.12
	(1)	(2)	(3)	(4)	(5)	(6)	(7)

Figure 1: Equilibrium in the Nautiyal and Love Model

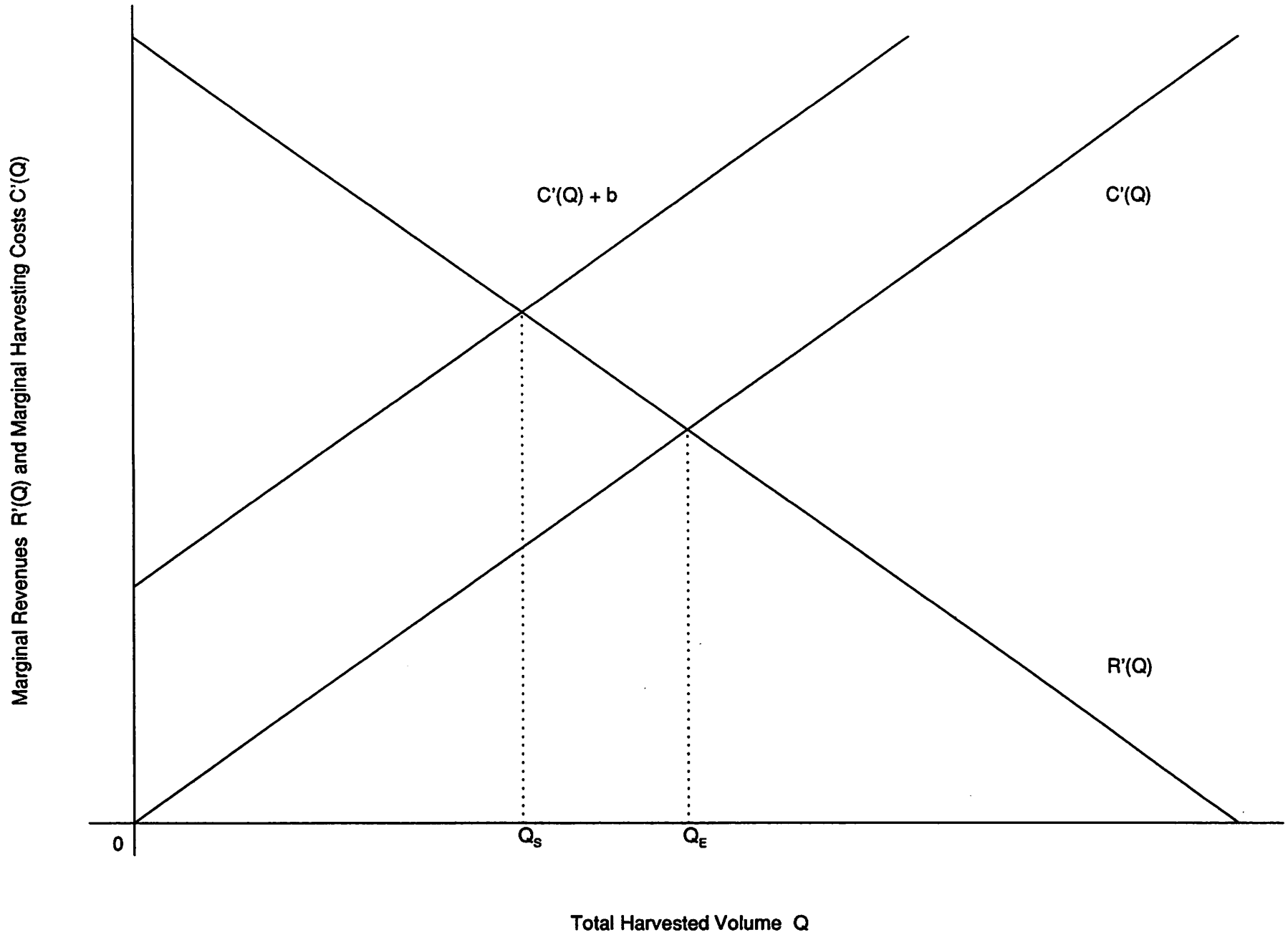


Figure 2: Equilibrium in a Model with Heterogeneous Timber Quality

