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by

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I. Introduction

There has recently been a resurgence of interest in the question of the consumer's optimal behavior under uncertain lifetime. As a result of the contribution of Levhari and Mirman (1977), building on the work of Yaari (1965), we now have a fairly complete analysis of the lifetime consumption plan for an isolated individual without bequest motive who suffers uncertain lifetime but does not have access to an annuity. Yaari had in fact derived the differential equations governing changes in consumption under a wider range of assumptions, investigating the effects of allowing a bequest motive and actuarially-fair annuities. So far, however, the only extension has been an analysis of the determinants of the initial level of consumption in the absence of a bequest motive or annuities by Levhari and Mirman. There is therefore considerable further work to be done in order to round out the analysis.

The first contribution of this paper is an investigation of the initial conditions governing the lifetime consumption plan in the absence of bequest motive but with actuarially-fair annuities. The conclusion is that, as Levhari and Mirman found without annuities, the introduction of uncertainty could either reduce or increase the initial level of consumption. In certain narrow cases one can predict the direction of the effect, but since the impact depends on the lifetime pattern of survival probabilities and earnings, as well as on taste parameters and the rate of interest, it cannot be signed a priori.

My second contribution is an application which demonstrates that despite the indeterminacy to which I have referred, the theory completed here has strong predictive power. The phenomenon on which I focus is that of the
differential explanatory value of the crude life cycle model at different age levels.\footnote{1} A remarkable feature of observed consumption is that while the middle-aged save at a moderate rate, as predicted by the naive life-cycle model, the elderly also usually save at a significant positive rate, contrary to the life-cycle model.\footnote{2} The question I ask is whether this contrast can be explained by uncertain lifetime. It is shown that, theoretically, such an observation \textit{could} be observed with a population of isolated individuals who had no bequest motive, with or without annuities.

The question of what behavior is \textit{likely}, given plausible assumptions on tastes, and in view of actual patterns of survival and earnings, is the final issue studied in this paper. Assuming that the rate of subjective discounting is less than the rate of interest, and that consumption plans are somewhat insensitive to changes in the rate of interest, the following results are obtained. First, the impact of uncertainty is usually to increase initial consumption when annuities are available, but to reduce it when they are not. Second, under annuities the elderly would dissave even more than in the crude life-cycle model, while in the absence of annuities their consumption would probably be far less than predicted by the naive life-cycle model, as appears to be observed.

The paper is organized as follows. Section II summarizes previous work on the impact of uncertain lifetime. Section III then analyzes the determinants of the initial level of consumption when actuarially-fair annuities are available. In Section IV I present the application of the theory to the question of the differential impact of uncertainty on the apparent observed propensity to consume at different ages. Finally, Section V explores the likely direction and magnitude of the consumption effects of uncertain
lifetime by means of a set of illustrative computations based on a combination of actual Canadian data for observables, and plausible alternative combinations of unobservable parameters.

II. Progress So Far

As mentioned, Yaari (1965) initiated the analysis of uncertain lifetime by deriving differential equations governing changes in consumption with age. He considered four cases defined by the presence or absence of a bequest motive and actuarially-fair annuities. Neglecting the bequest motive, he assumed that at time zero a consumer with probability $P(t)$ of surviving $t$ years, and a maximum attainable lifetime of $T$ years, would maximize the expected utility integral:

$$V = \int_{0}^{T} P(t) \alpha(t) U[C(t)]$$

(1)

where $C(t)$ is consumption at $t$, and $\alpha(t)$ is a subjective discount factor.4

Under (1), and suitable constraints, the following results emerge. In the absence of annuities, since the rate of time preference is higher than under certainty ($P(t) < 1$), it is likely that the planned rate of growth of consumption will be lower under uncertainty at each stage in the life cycle. However, an actuarially-fair annuity allows one to buy consumption in the future at a price of $P(t)(1+r)^{-t}$, where $r$ is the (constant) rate of interest, and therefore makes future consumption just cheap enough to offset completely the effect of higher time preference. The result is that with annuities consumers would likely plan for consumption to grow at much the same rate under uncertainty at different ages as under certainty.5
It is important to note that Yaari's conclusions concerned only the **shape** of the lifetime consumption path and not its level. For example, without annuities or bequest motive Yaari has shown that the consumption path will likely have a lower slope than under certainty. However, since under uncertainty it is necessary to stretch similar resources to provide for a longer possible lifetime, initial conditions differ and consumption may conceivably start at a lower level than under certainty. The only conclusions regarding the level of consumption which can be drawn directly from Yaari's article involve a comparison between the situation of a certain lifetime of length $T$ and that of an uncertain lifetime of maximum length $\bar{T}$. As Levhari and Mirman have pointed out, this comparison confuses the effects of uncertainty with those of a difference in life expectancy. Yaari himself did not draw attention to this comparison, perhaps expecting us to understand its limited interest.

Levhari and Mirman have extended Yaari's analysis by considering the determinants of the initial level of consumption in the case without annuities. In order to make the problem tractable they assume a constant rate of subjective discounting and "iso-elastic" instantaneous utility. In discrete time this gives:

$$V = \sum_{t=0}^{T} p_t (1+p)^{-t} \frac{c_t^{1-\gamma}}{1-\gamma}$$

Here $\gamma$ is both the elasticity of marginal utility with respect to the level of consumption, and the inverse of the elasticity of intertemporal substitution in consumption, $\sigma_c$.

Maximizing (2) subject to the appropriate constraint one finds that the direction of the impact of uncertain lifetime is **a priori** indeterminate. In the special case where $p = r = 0$, initial consumption will be higher than
under certainty if $\gamma < 1$, and lower if $\gamma \geq 1$. When $\rho$ and $r$ are non-zero there are a number of distinct cases. Perhaps the most interesting is that where $\rho < r$, and, since there is considerable empirical evidence that $\sigma < 1$.\(^7\) In this case, unless $r$ or $\rho$ (or both) are very high, increased uncertainty will lead to a decline in the initial planned level of consumption.\(^8\)

Barro and Friedman (1977) have drawn attention to the implication of Yaari's analysis that under (2) the consumer with access to actuarially-fair annuities should plan for consumption to grow at the same rate under uncertain lifetime as under certainty. They have also pointed out that if earnings did not change with age, and $r$ and $\rho$ were zero, consumption would be constant throughout the lifetime at the same level under certainty and uncertainty. Unfortunately, this special result represents the only available analysis of the impact of uncertain lifetime on the level of consumption when annuities are available.

III. The Initial Level of Consumption With Actuarially-Fair Annuities

With actuarially-fair annuities (2) must be maximized subject to the constraint that current wealth plus the annuitized value of future earnings must exceed the annuitized value of the consumption plan:\(^9\)

$$W_0 + \sum_{t=0}^{T} \left( \frac{1+r}{1+p} \right)^t (E_t - C_t)(1+r)^{t-T} \geq 0$$

(3)

Maximizing (2) subject to (3), from the first order conditions one obtains:

$$C_t = \left( \frac{1+r}{1+p} \right)^{t/\gamma} C_0 \quad t=0, \ldots, T$$

(4)

The full consumption plan is found by substituting (4) in (3) to obtain:

$$C_0 = \frac{W_0 + H_0}{\sum_{t=0}^{T} \left( \frac{1+r}{1+p} \right)^t}$$

(5)

where $H_0 = \sum_{t=0}^{T} E_t (1+r)^{-t}$ and $1+g = \left( \frac{1+r}{1+p} \right)^{1/\gamma}$.\(^{10}\)
Since under both certainty and uncertainty planned consumption grows at the rate \( g \) we can determine the complete impact of uncertainty on lifetime consumption by analyzing its effect on \( C_o \) using (5). Note, first, that if \( W_o = 0 \) and \( E_t = E_o (1 + g)^t \), equation (5) yields \( C_o = E_o \) and the degree of uncertainty has no impact on the consumption plan. It was a case of this type—with \( r = g \)—that was considered by Barro and Friedman. Whenever earnings do not grow at the rate \( g \) throughout the lifetime, however, uncertainty will make a difference.

As pointed out by Levhari and Mirman, in comparing the consumption plan under alternative sets of \( P_t \)'s we must keep life expectancy constant. Since the expected remaining lifetime equals the sum of the \( P_t \)'s, in a comparison of \( P = (P_o, \ldots, P_T) \) and \( P' = (P'_o, \ldots, P'_T) \) we require:

\[
\sum_{0}^{T} P_t = \sum_{0}^{T} P'_t \tag{6}
\]

and for \( P' \) to embody more uncertainty than \( P \) we must have:

\[
\sum_{0}^{T} P'_t \leq \sum_{0}^{T} P_t \quad \tau = 1, \ldots, T \tag{7}
\]

with at least one inequality.

The task of determining the relation between \( C_o \) and the \( C'_o \) corresponding to \( P' \) can be simplified by noting that where (6) and (7) hold, \( P' \) can be derived from \( P \) by a series of "mean-preserving spreads." That is, \( P' \) can be obtained by a series of steps in which \( P_i \) is reduced by a small amount and some \( P_j \) is increased by the same amount, where \( P_i < P_j \). We can therefore determine the effect on \( C_o \) of increased uncertainty by analyzing the impact of such a small change.
It will help to assume for the present that \( W_0 = 0 \), and that the growth rate of earnings cannot be above the rate of interest throughout the entire lifetime. Both assumptions accord closely with what is observed. Under these assumptions there are two cases in which the impact of uncertainty is unambiguous and a third where it is instructive to consider why an ambiguous result arises. These three cases, which are non-exhaustive, are defined by relative values of \( r \), \( g \), and the growth rate of earnings, \( m_t \).

Case 1: \( m_t < g < r \quad t = 0, \ldots, T \)

The analysis of this and the following cases will be eased if we rewrite (5) as:

\[
C_0 = E_0 \frac{T \sum_{t=0}^{T} \frac{t}{(1+r)^t}}{T \sum_{t=0}^{T} \frac{(1+m_t)^t}{(1+r)^t}}
\]  

(5')

Note, first, that since \( \Pi (1+m_t) < (1+g)^t \) the numerator of the ratio on the right-hand side is less than the denominator and \( C_0 < E_0 \). Consider the impact on \( C_0 \) of a mean-preserving spread in \( P \). Since successive terms in the sum of the numerator decline more rapidly than those in the sum of the denominator, shifting the weights applied by \( P \) further into the future will have a larger absolute impact on the numerator. Combined with its smaller size this implies a greater proportional reduction in the numerator, and a decline in \( C_0 \).

Case 2: \( g < m_t < r \quad t = 0, \ldots, T \)

Where \( m_t \) is greater than \( g \) but the interest rate is higher than \( m_t \), the circumstances of case 1 are precisely reversed. The numerator in (5') will now exceed the denominator, giving \( C_0 > E_0 \), and by reducing some \( p_{ij} \) and increasing some \( p_{ij} \) by the same amount we will reduce the numerator by
a smaller absolute amount than the denominator. This implies that the proportional reduction in the numerator resulting from an increase in uncertainty is smaller than that in the denominator, with the result that $C_t$ will rise.

Case 3: $r < m_t < g$ \hspace{1cm} $t = 0, \ldots, T$

In this case the numerator in (5') is smaller than the denominator, as in case 1. However the terms in the sums of both numerator and denominator are increasing with time, and the terms in the denominator are increasing more rapidly than those in the numerator. Reducing $P_j$ and increasing $P_j$ therefore will have a larger absolute effect on the denominator. This larger absolute impact could combine with the larger absolute size of the denominator to give a percentage change either greater or less than in the numerator. The impact of increased uncertainty on the scale of lifetime consumption in this case is thus indeterminate a priori.

Before moving on we should ask what difference it would make if we allowed non-zero $W_0$. Since this would enlarge the numerator in equation (5), in each case it would reduce the percentage change in the numerator resulting from rising uncertainty. This means that $W_0 > 0$ would reinforce the positive impact of uncertainty on consumption in the second case, weaken or reverse the negative impact in case 1, and leave case 3 ambiguous.

It must be noted that it is not only under case 3 that results are indeterminate. Whenever we do not have a pure example of cases 1 or 2 the impact of uncertainty cannot be predicted a priori. This is unfortunate since the typical lifetime growth of earnings does not satisfy the requirements of either case 1 or 2. Actual growth rates start at a high level (perhaps above $r$) and beyond the age of about 30 decline more or less continuously to rates little above zero or negative as age increases. Hence the only way to assess the likely lifetime impact is to choose plausible combinations of $g$ and $r$ and combine them with observed $P'$s and $E'$s to compute hypothetical lifetime consumption plans under uncertainty, as is done in Section V below.
IV. **An Application**

As pointed out in the introduction, it is striking that the elderly have often been observed to be net accumulators (when wealth is defined to omit annuitized pensions). Whereas the consumption of the young and middle-aged is roughly what would be predicted by the crude life-cycle model assuming certainty, this is therefore not the case for the elderly.

It does not appear that the contrast between the consumption behavior of the "young" and "old" can plausibly be explained by a bequest motive. In the first place, a number of surveys have explicitly asked respondents their reasons for saving, and very few, even among the elderly, have listed the desire to leave a bequest.16 Secondly, due to the secular growth of earnings, despite any regression toward the mean in earnings capacities, most children will have greater lifetime wealth than their parents without material transfers in adulthood. Unless parents are very highly altruistic towards their children, in most cases one would expect gifts or bequests to be viewed as unnecessary. (It is even possible that reverse intergenerational transfers, say on the part of children to support aged parents, may be as frequent as intentional transfers to a succeeding generation.)

If the bequest motive is only relevant for a small minority of families, can the contrast between the consumption behavior of the young and that of the old be explained by uncertain lifetime? This section applies the theory developed by Yaari, Levhari and Mirman, and the present paper to address this question.

It might be thought that it would be straightforward to determine from the analysis completed in the previous section whether the "low consumption" of the old can be explained by uncertain lifetime. It could be pointed out,
for example, that in the case without annuities, although initial consumption may be higher than under certainty, consumption grows at a lower rate and will become much lower than under certainty by the time retirement is reached. Does this explain what is observed? The answer is no, for the following reason. When we say that the consumption of an old person seems "low" we do not mean that it is low in relation to what a person who started out at age zero with the same opportunities but lived a life under certain lifetime would consume. What we have in mind is that it is low in relation to what would be consumed by a person with certain lifetime but otherwise all the same characteristics as the old person in question \textit{at the time of observation}. While it is unambiguous, without annuities, that under uncertainty an old person would consume much less than one who had started life with the same resources but had lived under certainty, it is not true in general that of two old people with identical resources and characteristics one subject to uncertain lifetime would consume less than one subject to certainty.

(1) \textbf{Definitions}

In order to carry out the analysis we require some further definitions. I will use the term "resources" to refer to the stock constraining lifetime consumption at age \( t \). When annuities are available, for example, resources at age \( t \) would be:

\[
R_t = W_t + \sum_{\tau=1}^{T} \mathbb{E}[(1+r)^{t-\tau} = W_t + H_t
\]

(Note that since we are looking at ages other than zero we must adjust the probabilities of being alive at future stages for the fact that survival to \( t \) has already occurred.) With this definition it is natural to refer to the "true propensity to consume" at age \( t \) as:

\[
PC_t = \frac{C_t}{R_t}
\]
The ratio of this propensity under uncertainty to that under certainty will be written:

\[ \chi_t = \frac{PC_t^u}{PC_t^c} \]

(10)

When one observes that people consume less in old age than would be expected on the basis of the crude life-cycle model one is not, in fact, observing something about \( \chi_t \) directly. This is because \( R_t^u \) differs from \( R_t^c \). The ratio of actual consumption under uncertainty to that which would occur under certainty, the "apparent ratio of propensities to consume" is related to the true ratio as follows:

\[ \chi_t^A = \frac{C_t^u}{C_t^c} = \frac{PC_t^u}{PC_t^c} \cdot \frac{R_t^u}{R_t^c} = \chi_t \frac{R_t^u}{R_t^c} \]

(11)

This "apparent" ratio is the variable whose time path we would like to chart in order to see how, at different ages, the consumption of a person of given characteristics at the age in question would differ under certainty and uncertainty.

(ii) Determinants of \( \chi_t^A \) When Annuities Are Available

I will discuss the determinants of \( \chi_t^A \) in two steps. Changes in the ratio \( R_t^u/R_t^c \) with \( t \) will be examined first; those in the ratio of true propensities to consume, \( \chi_t \), second.

Let \( \hat{t} \) denote the age at which earnings become zero. Beyond this age, that is in retirement, resources consist entirely of non-human wealth and are therefore the same irrespective of whether the lifetime is uncertain.\(^{17}\)

Before \( \hat{t} \), it can be shown, it is likely that \( R_t^u/R_t^c < 1 \). The reason is that, typically, \( \hat{t} < \bar{t} \), where \( \bar{t} \) is the initially expected age of death. In the 1970-71 Canadian data used in Section V we find, for example, that \( \hat{t} = 67 \) years,
while $\bar{t} = 72$ years. This means that $H_t^u$ and $H_t^c$ are the same except that each term in $H_t^u$ is weighted by $P_t / P_{\bar{t}}$, which is less than one. Hence $H_t^u / H_t^c \ll 1$ and $R_t^u / R_t^c \ll 1$ during the working lifetime. The direction of change of $R_t^u / R_t^c$ during the working years is indeterminate because $H_t^u / H_t^c$ may increase or decrease at different points. There are two systematic tendencies at work however. One is simply that $H_t^u / H_t^c$ must rise at the end of the working years since in year $\bar{t}$ itself it rises to one (see equation (8)). The other is that over most of the working life the relative importance of $W_t$ must be increasing, since its size is growing while that of $H_t^u$ and $H_t^c$ is declining due to the reduction in remaining working years. The illustrative computations of Section V show that, typically, these tendencies may cause a mild increase in $R_t^u / R_t^c$ over the working years.

How are changes in $R_t^u / R_t^c$ modified by those in $P_t^u / P_t^c$? Using continuous time we may write:

$$
\chi_t = \frac{\int_{\bar{t}}^{t} e^{(g-r)(\tau-t)} d\tau}{\int_{\bar{t}}^{t} \frac{P(\tau)}{P(t)} e^{(g-r)(\tau-t)} d\tau} = \frac{\phi_t^c}{\phi_t^u}
$$

(13)

where $\phi_t^i = 1 / P_t^i$. In order to analyze changes in $\chi_t$ with age, consider the relationship between $\phi_t^c$ and $\phi_{t+1}^c$, on the one hand, and between $\phi_t^u$ and $\phi_{t+1}^u$, on the other.

First we have:

$$
\phi_{t+1}^c = \left[ \phi_t^c - \int_{\bar{t}}^{t} e^{(g-r)(\tau-t)} d\tau + \int_{\bar{t}}^{\bar{t}+1} e^{(g-r)(\tau-t)} d\tau \right] e^{-g}
$$

(14)
It is observed that over much of the lifetime $\tau_{t+1} - \tau_t$ is small. Since in addition it is likely that $g < r$, the third term within the brackets will be small in relation to the others for most of the lifetime, and we have:

$$\frac{\phi_{t+1}^c}{\phi_t^c} \approx \left[ 1 - \frac{\int_\tau^{\tau_{t+1}} e^{(g-r)(\tau-t)}d\tau}{\phi_t^c} \right] e^{r-g}$$

(15)

Turning to $\phi_t^u$:

$$\phi_{t+1}^u = \left[ \phi_t^u - \int_\tau^{\tau_{t+1}} \frac{P(\tau)}{P(t)} e^{(g-r)(\tau-t)} d\tau \right] \left[ \frac{P(t)}{P(t+1)} \right] e^{r-g}$$

(16)

or

$$\frac{\phi_{t+1}^u}{\phi_t^u} = \left[ 1 - \frac{\int_\tau^{\tau_{t+1}} \frac{P(\tau)}{P(t)} e^{(g-r)(\tau-t)} d\tau}{\phi_t^u} \right] \left[ \frac{P(t)}{P(t+1)} \right] e^{r-g}$$

(17)

It is straightforward to compare the percentage rates of change given by equations 15 and 17. We may identify three cases:

**Case 1:** $\phi_t^c = \phi_t^u$ (PC$^c_t = PC^u_t$)

Since in general $\phi_t^c \neq \phi_t^u$ (except where $r = g$ and $\phi_t^c = \phi_t^u$ by definition), this case is considered for its instructive quality only.

Note, first, that as $t$ rises both $\phi_t^c$ and $\phi_t^u$ fall. However, the percentage rate at which $\phi_t^u$ declines is the lower for two reasons. First, $\frac{P(t)}{P(t+1)} > 1$, and the bracketed expression in equation (17) is therefore multiplied by a
larger constant than that in (15). Second, \( \frac{P(t)}{P(t)} < 1 \), and the expression within brackets in equation (17) is therefore larger than that in (15). We may conclude that if at any \( t \) \( \phi^C_t = \phi^U_t \), as age rises beyond \( t \) the true propensity to consume under uncertainty will fall relative to that under certainty.

**Case 2:** \( \phi^C_t < \phi^U_t \) (\( PC^C_t > PC^U_t \))

Here the propensity to consume is initially lower under uncertainty than under certainty. The same argument for case 1 applies a fortiori since the expression within brackets in (17) will now tend to be larger than that in equation (15) merely because \( \phi^U_t \) is larger than \( \phi^C_t \). Hence, if we start from a situation in which \( PC^U_t < PC^C_t \), \( PC^U_t/PC^C_t \) will again decline continuously as age rises.

**Case 3:** \( \phi^C_t > \phi^U_t \) (\( PC^C_t < PC^U_t \))

This is the case in which the propensity to consume is initially higher under uncertainty than under certainty. Here the ratio \( \phi^C_t/\phi^U_t \) will increase with age if \( \phi^C_t \) is sufficiently greater than \( \phi^U_t \) to outweigh the influence of the terms involving \( P(t) \) explored above. In fact the propensity to consume under uncertainty will increase in relation to that under certainty if:

\[
\frac{\phi^C_t}{\phi^U_t} > \frac{\frac{t+1}{t} \int_{t}^{t+1} e^{(x-g)(\tau-t)} d\tau}{\left[ \frac{P(t)}{P(t+1)} \right]^{\frac{1}{Y}} \int_{t}^{t+1} \left[ \frac{P(\tau)}{P(t)} \right]^{1/Y} e^{(x-g)(\tau-t)} d\tau}
\]

Since the expression on the right-hand side will not be much greater than unity, with typical \( P(\tau) \), this is quite a weak condition.
What conclusions can be drawn from the above analysis? We know that, in the main, $R^u_t/R^c_t$ will increase over the lifetime, tending to make the apparent propensity to consume under uncertainty rise relative to that under certainty with age. However, the influence of the true propensity to consume is ambiguous. If $PC^u_o < PC^c_o$, $PC^u_t/PC^c_t$ will fall throughout the lifetime and $\chi^A_t$ may rise, fall, or behave differently at different points in the life cycle. On the other hand, if $PC^u_o/PC^c_o$ is sufficiently greater than unity, $\chi^A_t$ will increase throughout the lifetime, reinforcing the influence of rising $R^u_t/R^c_t$ and giving an almost monotonic increase in the apparent propensity to consume under uncertainty relative to that under certainty throughout the lifetime. Which of the two situations, if either, is "typical" is a question addressed in Section V.

(iii) Determinants of $\chi^A_t$ When Annuities Are Unavailable

Consider the continuous form of (3) already implicit in the previous subsection:

$$v(t) = \int_t^T \frac{f(t)}{P(t)} \frac{C(t)^{1-\gamma}}{1-\gamma} e^{\rho(t-\tau)} d\tau$$

(9)

Assuming that the institutional structure in the absence of annuities would require wealth on death to be non-negative with probability one, we have the constraint:

$$W(0)e^{rt} + \int_0^t [E(\tau) - C(\tau)] e^{-\rho(t-\tau)} d\tau \geq 0 \quad t=0, \ldots, T$$

(20)

In view of the concavity of the typical age profile of earnings it seems likely, with realistic parameters, that maximizing (19) subject to (20) would give a solution with two distinct segments. For the first several years...
the constraint would probably be binding, but at some point, \( t^* \), desired consumption would fall below current earnings. Beyond \( t^* \) planned consumption would become the same as if we had set up (19) and (20) using \( t^* \) as the initial period and had maximized subject to the requirement that (20) should hold only for \( t = T \). Over this latter segment, assuming (20) never again to bind before the maximum-attainable age, we would have:

\[
\dot{C} = \frac{1}{\gamma} (r - \rho + \frac{P}{P})
\]

with the solution:

\[
C(t) = \left[ \frac{P(t)}{P(t^*)} \right]^{1/\gamma} e^{g(t-t^*)} C(t^*)
\]

where

\[
g = \frac{r-\rho}{\gamma}
\]

Substituting in equation (19) we obtain:

\[
C(t) = \frac{W(t) + \int_t^T E(\tau)e^{r(t-\tau)}d\tau}{\int_t^T \left[ \frac{P(t)}{P(t^*)} \right]^{1/\gamma} e^{(g-r)(t-\tau)}d\tau}
\]

As in the case with annuities, this optimal plan is intertemporally consistent in the sense of Strotz (1955).

Consider the determinants of changes in \( \chi_t^A \) with \( t \). Since earnings typically become zero before the expected age of death, in contrast to the situation with insurance we have:

\[
R^u(t) = R^c(t) = W(t) + \int_t^\xi (t) e^{r(t-\tau)}d\tau \quad \hat{\xi} = \xi
\]

This means that the "true" and "apparent" ratios of propensities to consume are the same. Furthermore we have:
\[
\chi_t = \frac{\int_{T}^{t} e^{(g-r)(\tau-t)} d\tau}{\int_{\frac{P(t)}{P(t+1)}} \frac{1}{\gamma} e^{(g-r)(\tau-t)} d\tau} = \frac{\phi_t^c}{\phi_t^u} \quad (25)
\]

which leads to equation (15) once again for \( \frac{\phi_{t+1}^c}{\phi_t^c} \) and to

\[
\frac{\phi_t^u}{\phi_t^u} = \frac{\left[1 - \int_{T}^{t} \frac{P(t)}{P(t+1)} \frac{1}{\gamma} e^{(g-r)(\tau-t)} d\tau \right]}{\frac{P(t)}{P(t+1)} e^{r-g}} \quad (26)
\]

As long as \( \gamma > 0 \) the behavior of \( \frac{\phi_{t+1}^u}{\phi_t^u} \) and therefore of \( \chi_t \) is qualitatively the same as in the case with annuities.

Clearly the analysis of changes in the apparent propensity to consume under uncertain lifetime relative to that under certainty is simpler in the case without, than in that with, annuities. If, under uncertainty, one would initially consume less than, as much as, or slightly more than under certainty, the propensity to consume will fall monotonically in relation to that under certainty as age increases. On the other hand, if initial consumption is significantly greater than in the case of certainty, the propensity to consume under uncertainty will rise as age increases relative to that under certainty. It is thus not obvious that uncertain lifetime can explain the low observed consumption of the aged. This depends on the type of consumption plan that is generated by actual patterns of P, E, and W, the interest rate, and typical values of the behavioral parameters.

V. The Quantitative Magnitude of Consumption Effects Caused by Uncertain Lifetime

The previous sections have provided a variety of theoretical results, but do not indicate the likely direction or magnitude of the consumption effects of uncertain lifetime. As pointed out above these effects depend
on the actual lifetime patterns of P, E, and W, behavioral parameters, and
the rate of interest. This section combines actual Canadian data on observables
and plausible values of the unobservable parameters to perform illustrative
computations of lifetime consumption behavior which help to answer these
questions.

Figures 1 and 2 outline the time paths of P and E on which the computa-
tions reported below are based. It is evident from Figure 1 that the path
of P(t) for Canadian males in 1971 approximates an inverse logistic curve.
In fact

$$P(t) = \frac{(1+a)e^{-bt}}{1+a e^{ct}} \quad 0 < b, c < 1$$  (27)

fits the data well, giving $R^2$ of .999. The fitted curve is therefore
used below to make possible calculations in continuous time. Figure 2 shows
the concave shape of the age profile of earnings for adult males in Canada
in 1970. In the computations reported below this is approximated by a
fourth order polynomial ($R^2 = .994$). It is assumed throughout that secular
economic growth would shift the entire profile upward at a constant rate of
2% per year.

(i) Impact of Uncertain Lifetime on the Consumption Plan
of An Individual

The analysis developed by Yaari (1965), Levhari and Mirman (1977), and
section III of this paper implies that an individual will plan his consumption
to grow at the same rate under uncertain lifetime as under certainty if
annuities are available, but at a lower rate if they are not. Whether or not
annuities are available the initial level of consumption may start at a higher
or lower level depending on taste parameters, survival probabilities, the interest
rate, and earnings.
Table 1 shows the ratio of consumption under uncertainty to that under certainty that would hold throughout the lifetime for a person who could take advantage of actuarially-fair annuities. The combinations of \( g \) and \( r \) chosen were selected as follows. There is considerable empirical evidence that \( \sigma^c < 1 \), and it therefore seems attractive to assume that \( \gamma > 1 \). Since \( g = \frac{r - \rho}{\gamma} \), a negative rate of subjective discounting would be necessary to give \( g > r \). Hence both Table 1 and subsequent tables show only situations where \( g \leq r \). (In addition we assume throughout that \( \rho \leq r \) since \( g < 0 \) would likely be atypical.) Table 1 indicates that with this restriction consumption under uncertainty will almost certainly be less than under certainty, unless \( r \) is unrealistically high.

Table 2 charts differences in the consumption plans that would be followed under certainty and uncertainty when annuities are not available. Once again, in most cases the initial level of consumption is lower under uncertainty. As age rises, however, consumption under uncertainty now falls relative to that under certainty, and at an increasing rate, becoming very low by the time retirement is reached.

(11) Impact of Uncertain Lifetime on the Apparent Propensity to Consume at Different Ages

The question of whether the low consumption of the elderly relative to that predicted by a naive life-cycle model can be explained by uncertain lifetime was partially answered in Section IV. The analysis there showed that it was theoretically possible for uncertain lifetime to explain this phenomenon either with or without annuities. Here we determine whether the theory would in practice predict "low consumption" for the elderly in the two regimes, as well as whether the magnitude of the effect is significant.
In order to compute $\chi^A_t$ we need $W(t)$ in addition to $P(t)$ and $E(t)$. One approach would be to generate values by simulating lifetime behavior either with or without annuities. I have taken a different approach. On the argument that what is required is simply a set of realistic values for non-human wealth at different ages, I have used the observed age profile of median non-human wealth in Canada in 1970 to provide $W(t)$. Figure 3 shows this profile on a family basis. In the illustrative computations it is assumed that median wealth per male would be 80% of that for families at all ages and that secular growth would shift this profile upwards at the same rate as that of earnings.

Table 3 shows changes in $\chi^A_t$ with age and their determinants, when annuities are available. There is a fairly smooth increase with age in resources under uncertainty relative to those under certainty with both $r = .03$ and $r = .06$. The tendency this creates for the apparent propensity to consume under uncertainty relative to that under certainty to increase with age is reinforced (except with $g = r$ where as previously noted $PC^U_t = PC^C_t$ by definition) by a smooth increase in $PC^U_t/PC^C_t$ as well. The net effect is shown in the final column. In all cases $\chi^A = APC^U_t/APC^C_t$ rises steadily with age. Hence with annuities one would expect the consumption of the old to seem somewhat "high", relative to that predicted by the life-cycle model under certainty.

As noted previously it is much easier to analyze changes in $\chi^A_t$ without, rather than with, annuities. This is because as long as earnings become zero before the end of the (initially) expected lifespan, "resources" are the same under certainty and uncertainty, and there is therefore a correspondence between "true" and "apparent" propensities to consume. Hence Table 4, which presents the result of illustrative computations in the case without annuities,
shows only the ratio of the true propensities to consume out of resources at
different ages under uncertainty and certainty. Note that, like Table 3, it
confirms the theoretical prediction that $\chi_t$ will fall monotonically with $t$ if
initially less than unity, and rise monotonically if sufficiently greater than
unity.

Given the present data and the assumptions on parameter values which
have been made, Table 4 shows that it is only with a low value of $\gamma$ ($\gamma=1$)
that there is a tendency for $\chi_t$ to increase with age. With $\gamma$ significantly
above unity it would appear that the usual pattern must be one of a declining
$\chi_t$ over the lifetime.

Tables 3 and 4 point to an important contrast between likely consumption
behavior under uncertain lifetime in the two regimes defined by the presence
or absence of annuities. Whereas with annuities systematic changes in both
resources and the true propensity to consume would typically make $\chi_t^A$ rise
throughout the lifetime, such changes would work in just the opposite direction
when annuities are absent. This conclusion implies that the "low consumption"
of the elderly would likely not be observed if actuarially-fair annuities
were available, but can indeed be explained if such annuities are not available.

Finally, it should be emphasized that the above conclusions depend
crucially on the assumptions (a) that the rate of subjective discounting is
less than the rate of interest, and (b) that the elasticity of intertemporal
substitution in consumption is less than unity. As has been pointed out
there is fairly strong empirical justification for these assumptions. Some
confidence in what is judged "likely" or "typical" in this section is therefore justified.
VI. Conclusion

This paper has made three distinct contributions to our understanding of consumer behavior under uncertain lifetime. First it has completed the analysis begun by Yaari (1965) of the optimal lifetime consumption path of an isolated individual without a bequest motive when actuarially-fair annuities are available. This is the counterpart in the case of annuities to the contribution of Levhari and Mirman (1977) for the case without annuities. Adding an analysis of initial conditions to Yaari's study of the shape of the consumption path, I have shown that the impact of uncertain lifetime when annuities are available is indeterminate a priori except in two narrow cases.

The second contribution of the paper has been the application of the theory here completed to the explanation of the observed low consumption of the elderly relative to that predicted for them by a naive life-cycle model under certainty. It has been demonstrated that with or without annuities the influences governing changes in the apparent propensity to consume under uncertainty relative to that under certainty are quite simple. The analysis shows that an explanation of the low consumption of the elderly is possible with or without annuities.

The final contribution of the paper has been to explore the likely direction and magnitude of the consumption effects of uncertain lifetime by means of a set of illustrative computations. Using actual Canadian data for observables, and drawing on available empirical evidence on unobservables, both the question of differences in the optimal consumption path over the lifetime for an individual under certainty and uncertainty, and that of the explanation of the low consumption of the elderly have been addressed. The individual lifetime consumption plan has been demonstrated likely to call for lower consumption throughout the lifetime under uncertainty whether or not annuities
are available. With regard to the second question, it has been shown that the low consumption of the elderly can easily be simulated under a wide range of plausible parameter values when annuities are not available, but not when actuarially-fair annuities can be obtained. Hence although theoretically uncertain lifetime could explain the low consumption of the old with or without annuities, with actual data on observables and plausible assumptions on unobservables the explanation is only forthcoming when annuities are absent.
NOTES

1 By "crude life-cycle model" I refer to a model which neglects both uncertainty and the bequest motive, but may take into account non-zero rates of interest and time preference. Such models are also designated by the term "naive life-cycle model" below.

2 For a summary of the empirical evidence see Mirer (1979, p. 435). The bulk of the evidence suggests that the retired either, on average, continue to accumulate wealth, or do not draw down their wealth significantly, when wealth is defined to exclude the annuitized value of pensions. Even those who argue in qualified support of the naive life-cycle model provide evidence that the rate of dissaving is extremely low among the retired. See Lydall (1955, p. 147) and Projector (1968, p. 14).

Clearly, if one corrected the measure of wealth to include annuitized pensions, "net worth" would decline in retirement. In the framework of the simple life-cycle model it would remain unexplained, however, why the elderly should not run down their financial and tangible assets, as well as annuitized pensions.

3 Yaari's treatment of the bequest motive was not very satisfactory. The utility from bequest was made independent of characteristics of heirs. If there is a systematic relationship between parents' resources and those of their children in the absence of transfers (perhaps via "regression toward the mean" in inherent ability), very different conclusions regarding bequest behavior would be reached if utility was allowed to depend on children's independent resources. See, e.g., Bevan (forthcoming), or Davies (1979a, Ch. 13).
The rate of time preference for consumption at time $t$ rather than at time zero is $1 - P(t)\alpha(t)$. (Note that time preference will not in general be constant even if, say, $\alpha(t) = e^{-\rho t}$ where $\rho$ is constant.) We must therefore refer to $1 - \alpha(t)$ by another term. Accordingly this latter rate is referred to as the "rate of subjective discounting" throughout the paper.

If $\alpha(t) = e^{-\rho t}$, $\rho$ constant, and $U[C(t)] = \frac{t}{1-\gamma}$, as in Levhari and Mirman (1977) as well as the present paper, we can express these results less guardedly. Without annuities consumption will grow less quickly under uncertain lifetime than uncertainty, while with annuities consumption will grow at the same rate in the two cases.

Levhari and Mirman (1977, p. 266).

The argument that this is the most plausible set of values for these parameters is set out in Section V.

Levhari and Mirman (1977, p. 275) show that if $x = \left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\gamma}}(1+r)^{-1}$ is considerably less than unity consumption could rise. Since with $r = \rho = 0$, $x = 1$, and $\frac{dx}{dr}, \frac{dx}{d\rho} < 0$, low values of $x$ can be obtained with high values of $r$ or $\rho$, or both. What values of $r$ and $\rho$ would be high enough to make $x$ sufficiently low for consumption to rise depends on the time path of $P(t)$.

The example provided by Levhari and Mirman, and my own illustrative computations suggest, however, that the required values are unrealistically high.

Yaari (1965, pp. 145 and 146).

Note that the plan given by (4) and (5) is "intertemporally consistent" in the sense of Strotz (1955). Intertemporal consistency requires that the optimal consumption plan should remain unchanged as the consumer ages. Here the consumer will always plan for consumption to grow at $g$, and resources in subsequent periods will always be as initially predicted, giving intertemporal consistency.
11. See Levhari and Mirman (1977, p. 272). Note that although \( P' \) is more risky than \( P \) in an unambiguous statistical sense (stochastic dominance) the consumer may be better off under \( P' \). This reflects the fact that lifetime utility is not concave in the length of life.

12. This term was introduced by Rothschild and Stiglitz (1970, pp. 227-229).

13. While it is true that most people start their working lives with low net worth, it could be argued that discounted capital transfers (gifts and inheritances) should be included in "initial wealth". Aside from the fact that such transfers are highly uncertain, evidence from both sample surveys and simulation suggests that inheritance is small compared with lifetime earnings for the large majority. (See Blinder 1974, pp. 90-91, and Davies 1979a, Ch. 13.)

14. Note that the parameters \( \rho \) and \( \gamma \) have no effect other than their impact on \( g \) and their values therefore need not be separately specified.

15. Although in cross-section, after peak earnings are passed, the growth rate of earnings could become sizeably negative, we are concerned here with changes in earnings for an individual as age rises. Hence it is not certain that earnings will actually decline at any age. This depends on the rate of secular growth of earnings.


17. Recall that we are studying the situation of a person observed at age \( t \). Of course if we considered someone with given \( W \) at age zero by the time we reached age \( t \) his \( W \) would differ under certainty and uncertainty. This is not, however, the situation being examined, as has been explained above.
From (8) we have
\[
\frac{H_{t}^{u}}{H_{t}^{u}} = (1 - \frac{F_{t}}{F_{t}^{u}})(1 + r)\left(\frac{1}{P_{t}} - \frac{1}{P_{t+1}}\right)
\]
while
\[
\frac{H_{t}^{c}}{H_{t}^{c}} = (1 - \frac{F_{t}}{F_{t}^{c}})(1 + r)
\]

Which rate of change will be larger is indeterminate since although \(H_{t}^{u} < H_{t}^{c}\) and the first term is therefore smaller for the case of uncertainty, \(\frac{P_{t}}{P_{t+1}} > 1\).

In the data for Canadian males used in Section V, \(\tilde{\tau}_{t+1} - \tilde{\tau}\) rises slowly from about 0.1 at age 20 to 0.4 at 65, and then increases rapidly to 0.6 at 80, and 0.8 at 90.

The plausibility of \(g < r\) is discussed in Section V. Note that since \(\tilde{\tau}_{t+1} - \tilde{\tau}\) is growing with \(t\) (see previous note), and \(\tilde{\tau} - t\) (expected remaining lifespan) is falling with \(t\), the approximation becomes questionable at the most advanced ages. This accounts for a slight difference in behavior beyond about age 70 from that predicted here in the illustrative computations of Section V.

Refer to (13). If \(g = r\), \(\varphi_{t}^{c} = \tilde{\tau} - t\) and \(\varphi_{t}^{u} = \int_{0}^{T} [P(\tau)/P(t)] d\tau\) which also equals \(\tilde{\tau} - t\).

Yaari (1965, p. 139) points out that this is an extreme assumption. Allowing this probability to lie below unity would, however, involve a great loss of simplicity. Note that the present approach differs from that of Levhari and Mirman (1977) by explicitly recognizing the importance of earnings.

See, e.g., Irvine (1978, p. 304).
The curve

\[ P(t) = \frac{(1.00031) \exp(-0.002114t)}{1 + (0.00031) \exp(0.14651t)} \]

was fitted by least squares using numerical methods.

The curve

\[ E(t) = -43,963.8 + 4541.4t - 153.458t^2 + 2.3567t^3 - 0.0139t^4 \]

was again fitted by least squares.

The elasticity \( \sigma^c \) is a measure of the sensitivity of the consumption plan to changes in the net real rate of interest. The fact that most consumption studies find little effect of interest rates on consumption (von Furstenberg and Malkiel, 1977, pp. 840-842) argues that \( \sigma^c < 1 \) and \( \gamma > 1 \), accordingly. It is interesting to note that the upper bound estimate for \( \sigma^c \) obtained with an explicit model of life-cycle behavior by Ghez and Becker (1975, p. 137) was 0.28, implying a lower bound for \( \gamma \) of 3.6. Blinder (1976, p. 632) refers to the results of three separate studies which all imply \( \gamma \approx 1.5 \). These studies estimated \( \gamma \) in a static context, however, as the elasticity of the marginal utility of income.

The rate \( r = .06 \) was singled out as Blinder (1974, pp. 98-99) suggested this as a "best guess" at the long-run real rate of return in the U.S., on the basis of empirical work by Nordhaus (1973). It seems advisable to experiment with a lower rate since real rates of return on financial assets have evidently fallen since the early 1960's when Nordhaus made his estimate.

Note that in three cases \( x_t \) actually falls slightly from age 70 to age 80. This reflects the fact that the approximation embodied in equation (15) becomes less tenable at the most advanced ages, as discussed above in note 20.
References


Figure 1

Probabilities of Survival to Given Ages, Males: Canada, 1971

Figure 2

Median Male Earnings by Age: Canada, 1970

Note: The population includes non-earners.

Source: Derived from Statistics Canada (1971a, Table 7) and (1971b, Table 11).
Figure 3

Median Net Worth by Age, Family Units: Canada, 1970

Note: Families consist of people related by blood or marriage sharing a dwelling unit. Net worth is the sum of tangible and financial assets less all debts. Equity in insurance and pension plans is excluded.

Source: Adjusted estimates from the 1970 Survey of Consumer Finance described in Davies (1979b).
Table 1

Consumption Under Uncertain Lifetime as a Ratio to that Under Certainty for an Individual of Given Initial Resources, With Annuities - Illustrative Computations

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Table 2

Consumption Under Uncertain Lifetime as a Ratio to that Under Certainty for an Individual of Given Initial Resources, Without Annuities - Illustrative Computations

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Apparent Propensity to Consume Under Uncertain Lifetime
Relative to that Under Certainty, With Annuities -
Illustrative Computations

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### Table 4

Apparent Propensity to Consume Under Uncertain Lifetime Relative to that Under Certainty, Without Annuities - Illustrative Computations

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