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by

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COMPUTATION AND EXISTENCE

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I. Introduction

Policies pursued by governments with the intention of meeting minimum or ceiling price targets for particular commodities are increasingly being recognized as potentially major distortions in modern economies, yet their quantitative impacts remain underexplored.¹ Price support policies, pricing policies of government marketing agencies, and more specific policies such as the Common Agricultural Policy are characteristic of these types of interventions. Since such policies will typically distort the economy away from a Pareto-optimal allocation of resources, quantification of social costs involved seems a worthwhile undertaking. Our aim in this paper is to present computational techniques for the evaluation of general equilibrium impacts of these policy interventions. Given the recent literature on 'fix price' equilibria in which rationing rules are used to allocate excess demands and supplies (such as in Malinvaud [1977]), we feel that it is important to make it quite clear that in this paper it's not our intention to consider minimum or ceiling prices supported by agent specific rationing rules through which supplies (demands) are allocated to specific agents if there is excess demand (excess supply). Instead we are concerned with non-discriminatory market mechanisms through which government policy objectives with the intent of fixing prices may be achieved. Price rigidities in our formulations are not institutional accidents but occur through deliberate government action and our concern is with equilibria in which all markets clear in the presence of these rigidities.

We consider three different formulations of government price intervention in the computationally oriented general equilibrium framework considered by Scarf [1973], Shoven and Whalley [1973], and others. We establish existence proofs and provide computational procedures which we feel will prove useful in

¹In some of the recent general equilibrium literature (Hansen and Manne [1974], Drèze [1975], Mathieson [1977]) some of the formal characteristics of equilibrium in the presence of constraints on equilibrium prices have been investigated, including existence. These pieces differ from analyses of price rigidities in the two-sector literature (Harris and Todaro [1970], Brecher [1971]) where the equilibrium processes which support constrained prices are more explicitly modelled but existence of equilibrium is not demonstrated.
future policy evaluation work which we plan with these formulations. Although
the majority of our discussion is in terms of Scarf's algorithm [1973, 1977],
later model specifications for analysis of particular policy issues can proceed
using Merrill's algorithm [1971], or more recent refinements of Scarf procedures
such as those due to van der Laan and Talman [1978], which economize on compu-
tational speed and storage. Alternative proofs of the existence theorems of
Section 1 using Kakutani's theorem are given by Nguyen [1979]. In a final section
of the paper we outline some of the policy issues we have in mind for eventual
application.

We first consider legislated minimum or ceiling prices where these
prices are defined in terms of a numeraire good. We later discuss extensions
of our analyses to include target prices denominated in nominal terms and we
indicate under what circumstances the traditional classical dichotomy between
real and monetary phenomena can break down. Legislated minimum prices
are enforced through government regulations requiring particular
agents to purchase commodities through a government marketing agency if
the market price is below the minimum price. With legislated ceiling prices
only selected agents can buy from the government agency at the ceiling price if
it is above the market price. The government agency acts as a monopolist
or monopsonist for some sets of agents and is used to enforce minimum or
ceiling price constraints. Rents which accrue from the operation of
minimum price constraints are returned in lump sum form to consumers;
losses from ceiling price operations are covered from taxation. We separately
consider minimum and ceiling producer prices and minimum consumer prices.
A difficulty with the infeasibility of meeting consumer demands given fixed
endowments at low ceiling consumer prices excludes an existence proof in this
case, although we believe this not to be a problem with realistic policy appli-
cations. Our existence proofs use the same price-revenue simplex adopted by Shoven
and Whalley [1973] in their analysis of general equilibrium with taxes.
Our analysis represents an extension of the Shoven-Whalley analysis where tax rates are endogenously determined to meet minimum (gross of tax) price constraints. The analysis of ceiling producer prices is comparable to a treatment of variable subsidies. Shoven and Whalley exclude subsidies in their discussion and we extend their analysis in this direction.

We next consider the operation of legislated minimum prices in a general equilibrium model with (ex post) segmented markets. In the spirit of the work following that of Harris and Todaro [1970] we consider minimum producer prices which apply to inputs in only a subset of activities in the economy. We adopt the equilibrium condition that the producer price in non-constrained activities equals the product of the higher price in constrained activities and the probability of employment in those activities. In the urban-rural migration literature this condition has been rationalized both on the basis of continually repeated drawings from those resources offered for sale at the higher price, and on the basis of an ex post market segmentation which prevents resources unemployed in the high price segment of the economy re-entering the full employment lower price segment. This latter rationalization is sometimes referred to in the literature on urban-rural migration as the 'one-way-ticket' assumption. In this formulation we allow activity specific minimum prices to be legislated as government policy and use the Harris-Todaro condition as a characterization of an equilibrium. We prove existence of an equilibrium, but unlike the case of economy-wide legislated minimum prices no government monopoly is needed to support such an equilibrium since the probabilistic equilibrium conditions are supported through unemployment.

In the third portion of our analysis we consider economy-wide minimum or ceiling prices supported through government market interventions.
We give government the ability to withdraw income from the private sector through a tax function and allow them to spend the proceeds on price maintenance programs. The government has no monopoly or monopsony power acquired through regulations requiring particular agents to transact only with a government market agency. Minimum or ceiling price objectives define trigger prices which authorize government market interventions to purchase or sell commodities according to given rules. This mechanism parallels agricultural price support programs and other price intervention schemes. We show existence of an equilibrium but the target prices may not be achieved in equilibrium, since it is clearly possible to construct a set of price objectives which for particular technologies would guarantee positive profits at any set of prices. However, if minimum price objectives are restricted to inputs, and ceiling price objectives restricted to outputs we are able to show that equilibrium prices will be constrained by the specified target values.

We first discuss the traditional general equilibrium model without minimum or ceiling price interventions and outline some of the fundamental concepts of Scarf's algorithm, stating the main theorem. We then apply this theorem in turn to our three formulations of price constrained equilibria. In a final section we discuss both applications and further extensions of the approach. We briefly outline our computational experience to date with these procedures.

II. General Equilibrium Without Minimum or Ceiling Price Interventions and Fundamental Concepts of Scarf's Algorithm

We consider an economy with \( n \) commodities, which has initial endowments of these commodities denoted by the vector \( W = (W_1, \ldots, W_n)' \); where \( W_i \geq 0 \) and is strictly positive for at least one \( i \). The demand side of the economy is represented by the market demands \( \xi_i(\Pi) \), \((i=1,\ldots,n)\) which are continuous, non-negative functions of the market prices \( \Pi \), \((\Pi=\Pi_1,\ldots,\Pi_n)\). These
functions are assumed to be homogeneous of degree zero and satisfy Walras' Law. The latter states that at any vector of prices the value of market demands equals the value of the economy's endowments. This can be interpreted as a condition that in aggregate the economy is on its budget constraint. The zero homogeneity of demands implies that only relative prices are of significance and any arbitrary normalization of commodity prices can be considered.

A common and convenient normalization is to consider vectors \( \Pi \) which lie on a unit simplex; i.e., all vectors \( \Pi \) such that \( \sum_{i=1}^{n} \Pi_i = 1 \).

The production side of the economy is characterized by an activity matrix \( A \), each column of which represents a feasible activity which can be operated at any non-negative level of intensity. We consider \( m \) activities, the notation \( a_{ij} \) denoting the output of good \( i \) in activity \( j \). Inputs are denoted by negative and outputs by positive entries in the matrix \( A \). Joint production is permissible, and it is also assumed that the matrix \( A \) includes \( n \) activities which represent the possibility of free disposal of any commodity; for convenience these are taken to be the first \( n \) activities. We assume that \( A \) satisfies the boundedness condition that the set of \( X \) such that \( AX + W \geq 0 \) are contained in a bounded set, where \( X \) denotes any vector of non-negative activity levels.

A competitive equilibrium in such a model is characterized by vectors \( (\Pi^*;X^*) \) such that two sets of conditions are satisfied

\[ g_i(\Pi) = W_i + \sum_{j=1}^{n} a_{ij} X_j^* \quad (i=1,\ldots,n) \]  

\[ (2.1) \]

\[ ^1 \text{Walras' Law is the condition that } \sum_{i=1}^{n} \Pi_i g_i(\Pi) = \sum_{i=1}^{n} \Pi_i W_i \text{ at any non-negative vector } \Pi. \]
(ii) no activity makes positive profit with those in use just
breaking even
\[ \sum_{i=1}^{n} \Pi_i^x a_{ij} \leq 0 \quad (=0 \text{ if } X_j^x > 0) \quad (j=1, \ldots, m) \] (2.2)

The existence of an equilibrium described by conditions (2.1) and (2.2)
can be shown using Scarf's algorithm [1973], [1977]. Extensions of this
approach due to Merrill [1971] and Eaves [1972] can be used to compute
equilibrium solutions for specific formulations in small amounts of execution
time. These algorithms are essentially search procedures across a unit
simplex which determine an equilibrium price vector and corresponding equi-
librium quantities. The no-cycling argument in these procedures due to
Lemke and Howson [1964] guarantees an approximation to a true equilibrium will
be found and this approximation becomes exact as a limiting process of search
over a dense grid of points in the unit simplex is considered.

For the search procedure we consider a simplicial subdivision of the
unit simplex \( S \). \(^1\) We consider \( V^1, \ldots, V^k \) as the vertices defining the simplices
in the simplicial subdivision where \( k > n \); each vertex \( V^j \) is an \( n \) dimensional
vector. Corresponding to each of these vertices we associate an \( n \) dimensional
vector label \( \lambda(V^j) \) according to the following rules

(1) We define \( \Pi_i^j = V_i^j \).

(2) If \( \Pi_i^j = 0 \), then \( \lambda(V^j) \) is the \( i^{th} \) unit vector where \( i \) is the
first coordinate of \( V^j \) equal to zero.

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\(^1\) An \( n \) dimensional simplex is defined as the convex hull of its \( n \)
vertices. A collection of simplices \( S^1, \ldots, S^k \) is termed a simplicial sub-
division of \( S \) if (1) \( S \) is contained in the union of the simplices \( S^1, \ldots, S^k \), and
(2) the intersection of any two simplices is either empty or a full face of
both. (See Scarf [1977].)
(3) If \( \Pi_i^j > 0 \) for all \( i \), we select that activity vector from matrix \( A \) which gives maximum per unit profit at the prices \( \Pi_i^j \).

Let \( (a_{i1}, \ldots, a_{in}) \) be that activity and let the maximum per unit profit be \( \gamma \);

(a) If \( \gamma > 0 \), then \( \lambda(v^j) \) is the negative of the above activity
(b) If \( \gamma \leq 0 \), \( \lambda(v^j) \) is \( (\xi_1(\Pi_i^j), \ldots, \xi_n(\Pi_i^j)) \).

Scarf's theorem states that there exists a simplex in the subdivision

with vertices \( V_1^j, \ldots, V_n^j \) such that

\[
\sum_{j} y_j \lambda(v^j) = W
\]

(2.3)

where \( W = W_1, \ldots, W_n \).

The theorem requires that the vector labels \( \lambda(v^j) \) satisfy the condition that

\[
\sum_{j} y_j \lambda(v^j) \leq 0, \quad y_j \geq 0 \text{ implies } y_j = 0 \quad (j = 1, \ldots, n)
\]

(2.4)

This follows from the boundedness assumption on the activity analysis matrix and the non-negativity of demands.

This theorem provides the basis for Scarf's algorithm which, when applied, determines a simplex with vertices \( V_1^j, \ldots, V_n^j \) whose vector labels form a feasible basis for a set of linear equalities involving all the labels corresponding to vertices in the subdivision. Some of the vertices will have vector labels which are market demands evaluated at the corresponding price vector, some vertices will have vector labels which are the negative of activity vectors, and other will be slack vectors.

The weights \( y_j \) in (2.3) can be redefined as \( X_j \) for weights on activity vectors, and as \( Z_j \) for weights on demand columns. The slack activities representing disposal of the \( n \) commodities appear as activity vectors with weights \( X_1, \ldots, X_n \).
Using this notation, the system of equations (2.3) can be written out explicitly as

\[ \sum_j z_j^i (\Pi^j) - \sum_j a_{ij} x_j = w_i \quad (i=1, \ldots, n) \]  

(2.5)

If simplicial subdivisions are considered with a finer and finer mesh, then a convergent subsequence of vertices of these subdivisions can be considered which tends to the vector \( V^* \). Associated with \( V^* \) is a value \( \Pi^* \).

As \( \Pi^j \) approaches \( \Pi^* \), \( \xi(\Pi^j) \) approaches \( \xi(\Pi^*) \), the \( z_j \) approach \( z_j^* \), and \( x_j \) approach

\[ \sum_j z_j^* (\Pi^*) - \sum_j a_{ij} x_j^* = w_i \quad (i=1, \ldots, n) \]  

(2.6)

In order to show that \( (\Pi^*, x^*) \) define a competitive equilibrium it is necessary to show that the properties of the equilibrium (2.1) and (2.2) are satisfied by this system of linear equalities.

This follows simply. If at least one \( z_j^* > 0 \) then from the construction of the vector labels the zero profit conditions (2.2) are satisfied. If all \( z_j^* \) are zero multiplying through (2.6) by \( \Pi^*_i \) and summing gives the negative of profits on the left-hand side and \( \sum_i \Pi^*_i w_i \) on the right-hand side which is a contradiction; hence at least one \( z_j^* > 0 \). Since at least one \( z_j^* > 0 \), multiplying through (2.6) by \( \Pi^*_i \) and adding implies that \( \sum z_j^* = 1 \) from Walras' Law.

Equations (2.6) thus describe the demand supply equalities (2.1).

Thus, Scarf's algorithm guarantees finding a simplicial subdivision with vertices whose labels form a feasible basis for a set of linear equalities which, in the limit as these vertices converge to a common vector, guarantee that the conditions for a competitive equilibrium will be satisfied.

We now consider this same argument in the case of economies operating under minimum or ceiling price interventions of various forms.

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1 Strictly speaking this argument requires the strict positivity of all \( w_i \) but if any \( w_i = 0 \) a simple perturbation of both demands and the \( w \) vector in the application of Scarf's theorem can accommodate this situation.
III. Legislated Minimum and Ceiling Prices With Enforcement Through a Government Marketing Agency

In this section we consider cases where the government legislates a minimum or ceiling price as applying to a particular group of agents; we consider minimum and ceiling purchase prices for producers and minimum prices for consumers. The government enforces minimum price regulations by requiring the agents involved to purchase minimum priced commodities only from the government. Ceiling price regulations are enforced by allowing only the agents involved to buy from the government at the ceiling price. At a market price below the minimum price the government purchases the commodity at the market price and resells to a minimum price regulated agent at the higher price. Any proceeds the government acquires from these transactions are assumed to be redistributed to consumers. With ceiling prices the government buys at a higher market price and sells at the lower ceiling price; losses are covered by lump-sum taxation.¹

This formulation of minimum price interventions bears a strong similarity to the analysis of general equilibrium with taxation considered by Shoven and Whalley [1973]; ceiling prices operate in a manner similar to subsidies which are not considered by Shoven and Whalley but can be incorporated into their framework. In the Shoven-Whalley formulation producer and/or consumer tax rates are given and equilibrium involves not only prices but also an equilibrium level of tax collections redistributed to consumers. A conventional price simplex augmented by one additional dimension to reflect revenues is used both to compute an equilibrium and to prove existence. In the analysis of minimum price interventions tax rates are endogenously determined to support target minimum prices to groups of agents. Revenues accrue to government

¹The government only intervenes in the free market to purchase in these cases if agents wish to buy at the minimum or ceiling prices. Unlike the price support mechanism of Section V, the government neither accumulates nor holds stocks of commodities.
through the enforcement of minimum price regulations, and the same augmented price-revenue simplex can be used both to compute and prove existence of an equilibrium. With ceiling prices endogenous subsidy rates are involved instead of tax rates and revenues are needed by government to cover losses.

For convenience, we consider minimum and ceiling prices which are expressed in terms of an arbitrarily chosen numeraire good which we label the \( n \)th. We later discuss the analysis of these price interventions in nominal terms. Minimum or ceiling prices are thought of as policy objectives of government which involve the selection of policy parameters \( \lambda_i \geq 0, i=1,\ldots,n-1 \). These parameters define the minimum or ceiling prices \( \hat{P}_i \) for any given price of the numeraire good \( \hat{P}_n \). Clearly there cannot be a minimum or ceiling price for the numeraire good.

\[
\hat{P}_i = \lambda_i \hat{P}_n \quad (i=1,\ldots,n-1)
\]  

(3.1)

In discussion of minimum producer prices we restrict minimum prices to inputs. An input is defined as a commodity \( i \) for which \( a_{ij} \leq 0 \) for all \( j \); in these cases \( \lambda_i > 0 \) only if commodity \( i \) is an input. The reason for this limitation is that if minimum prices apply to outputs the existence proof no longer holds. This reflects the fact that with minimum output prices it is possible to choose a technology such that positive profits must be made at any set of prices for the \( \lambda_i \) specified. We exclude such cases here by only considering inputs but in policy evaluation work such a limitation should be unnecessary. A similar difficulty requires ceiling producer prices to be limited to outputs in proving existence.

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\(^1\)These policy parameters considered here are only differentiated by commodity. It is possible to extend our analysis so as to incorporate further differentiation. With minimum or ceiling producer prices \( \lambda_{ij} \) would give the policy parameter for commodity \( i \) in activity \( j \); with minimum or ceiling consumer prices \( \lambda_{iq} \) would give the parameter for commodity \( i \) when purchased by consumer \( q \).

Further differentiation of this form may be important in certain policy applications (e.g., minimum wages for regions, ceiling food prices for the poor) but is not pursued here to avoid notational complexity. This form of differentiation does appear in an alternative proof of the theorems of this section using Kakutani's theorem by Nguyen [1979].
Minimum Consumer Prices

We denote market prices by the vector $\Pi$ and regulated prices by the vector $\Pi^c$. Consumers are required to pay $\Pi^c$ for any purchases they make, producers pay $\Pi$, owners of endowments receive $\Pi$. For any commodity $i$, the consumer prices $\Pi^c_i$ are defined as $\max(\Pi_i, \lambda_n \Pi_n)$ where the vector $\Pi^c = (\Pi^c_1, \ldots, \Pi^c_n)$.

We define $R$ as the government proceeds from consumer price regulation. As these proceeds are redistributed to consumers, $R$ forms part of consumer incomes and, following Shoven and Whalley, we can define the market demand functions as non-negative, continuous, functions $\xi_i(\Pi, R)$ which are homogeneous of degree zero and satisfy Walras' Law which is stated as

$$\sum_{i=1}^{n} \Pi^c_i \xi_i(\Pi, R) = \sum_{i=1}^{n} \Pi_i W_i + R. \tag{3.2}$$

An equilibrium is defined as a vector of prices, activity levels and revenue $(\Pi^*, X^*, R^*)$ (with associated minimum prices $\Pi^c_*$) such that

1. Demands equal supplies (including disposals)

$$\xi_i(\Pi^*, R^*) = W_i + \sum_{j} a_{ij} X_j^* \quad i=1, \ldots, n. \tag{3.3}$$

2. No activity makes positive profits with those in use breaking even

$$\sum_{i} \Pi^*_i a_{ij} \leq 0 \quad (= 0 \text{ if } X^*_j > 0) \quad j=1, \ldots, m \tag{3.4}$$

A property of such an equilibrium which follows trivially from Walras' Law is that $R^* = \sum_{i=1}^{n} (\Pi^c_i - \Pi^*_i) \xi_i(\Pi, R)$.

To both compute such an equilibrium and prove existence, we consider a subdivision of an $(n+1)$ dimensional unit simplex. At any vertex $V_j$ the labelling rule $l(V_j)$ is as follows:

---

1This formulation assumes that any consumer initially owning commodities to which a minimum price regulation applies must sell all his endowment at prices $\Pi_i$ and repurchase for consumption purposes at prices $\Pi_i^c$. 

(1) \( \Pi_i^j = V_i^j \quad (i=1, \ldots, n) \), \( R_i^j = V_{n+1}^j \); \( \Pi_{i n}^j = \max (\Pi_1^j, \lambda_i^j, \Pi_n^j) \).

(2) If \( V_i^j = 0 \quad (i=1, \ldots, n+1) \), then \( \lambda(V_i^j) \) is the \( i^{th} \) unit vector where \( i \) is the first coordinate of \( V_i^j \) equal to zero.

(3) If \( V_i^j > 0 \quad (i=1, \ldots, n+1) \), select that activity giving maximum per unit profit at prices \( \Pi_i^j \). Define this maximum profit as \( \gamma \).

(a) If \( \gamma > 0 \) then \( \lambda(V_i^j) \) is the negative of the activity generating \( \gamma \) with a zero as the \((n+1)^{st}\) element of the vector label.

(b) If \( \gamma \leq 0 \) then

\[
\lambda(V_i^j) \text{ is } (\xi_1^j(\Pi_i^j, R_i^j) + \theta, \ldots, \xi_n^j(\Pi_i^j, R_i^j) + \theta, (-R_i^j + \sum_{i=1}^{n} (\Pi_i^j - \Pi_i^j) \xi_i^j(\Pi_i^j, R_i^j) + \Delta))
\]

where \( \theta \) and \( \Delta \) are positive constants. As in Shoven-Whalley, \( \theta \) is a perturbation term relaxing any requirement of the strict positivity of the \( \hat{w}_i \), while \( \Delta \) is a constant whose value is chosen to exclude \( R_i^* = 1 \), \( \Pi_i^* = 0 \quad (i=1, \ldots, n) \), as a possible equilibrium solution.

Applying Scarf's Theorem and considering the limiting process of taking a finer and finer mesh of subdivisions allows the set of equations

\[
\sum_j y_j \lambda(V_j^i) = \hat{w} \text{ where } \hat{w} = (\hat{w}_1 + \theta, \ldots, \hat{w}_n + \theta, \Delta)
\]

to be written out explicitly as

\[
\sum_j Z_j^i(\xi_j(\Pi^*, R^*) + \theta) - \sum_j a_{ij} X_j^* = \hat{w}_i + \theta \quad \quad \quad (3.5)
\]

\[
r_{n+1}^* + \sum_j Z_j^i(-R^* + \sum_{i=1}^{n} (\Pi_i^* - \Pi_i^*) \xi_i(\Pi_i^*, R_i^*) + \Delta) = \Delta \quad \quad \quad (3.6)
\]

where the same notation is used for \( Z_j^i \) and \( X_j^* \) as in (2.5) and (2.6). The term \( r_{n+1}^* \) refers to the weight on the \((n+1)^{st}\) unit vector.
The set of linear equalities (3.5) and (3.6) is similar to that considered by Shoven and Whalley in their analysis of taxation and the same approach to the proof of existence applies. (3.5) establishes that at least one $Z^*_j > 0$ and the zero profit conditions are satisfied. If $R^* = 0$, (3.5) establishes $\sum_j Z^*_j \geq 1$ and (3.6) that $\sum_j Z^*_j \leq 1$ which implies \( \sum_j Z^*_j = 1 \). This implies $r^*_{n+1} = 0$ and a competitive equilibrium is established in this case. If $R^* > 0$, $r^*_{n+1} = 0$ and $\sum_j Z^*_j = 1$ and the competitive equilibrium conditions hold once again. The limit $R^* = 1$, $\Pi^* = 0$ is excluded by a choice of $0 < \Delta < 1$ in the same way as in Shoven and Whalley.

This formulation of minimum prices is thus a generalization of the Shoven-Whalley analysis of taxes to the case where the consumer tax rates required to achieve a target minimum price are unknown ex ante.

**Minimum Producer Prices**

The same structure as above can also be applied to legislated minimum producer prices. In this case consumers pay $\Pi$, owners of endowments receive $\Pi$, but producers are required to pay $\Pi^P$. For any commodity $i$, $\Pi^P_i = \max(\Pi_i, \lambda_i \Pi_n)$. Demand functions are $\xi(\Pi, R)$ and satisfy Walras' Law written as

\[
\sum_{i=1}^{n} \Pi_i \xi_i(\Pi, R) = \sum_{i=1}^{n} \Pi_i W_i + R.
\]

Equilibrium involves demand supply equalities as in (3.3) and zero profit conditions as in (3.4) but with profits evaluated at $\Pi^P$ rather than $\Pi^*$. To prove existence in this case we restrict minimum prices to inputs only for the reasons stated above.
To apply Scarf's theorem, the labelling rules above are changed in two ways. In 3(a) the \((n+1)^{st}\) element becomes \(\sum_{i} (\Pi_{i}^{1} - \Pi_{i}^{p})a_{i,s}^{*}\). Since minimum prices apply only to inputs this term is non-negative. In 3(b) the \((n+1)^{st}\) element becomes \((-R^{j} + \Delta)\). Defining \(\hat{\lambda}\) as above and writing out the set of linear equalities \(\sum_{j} \lambda_{j}(v_{j}^{i}) = \hat{\lambda}\) in the limit as the mesh of each subdivision becomes increasingly fine gives

\[
\sum_{j} Z_{j}^{*} (\xi_{i}(\Pi_{i}^{*}, R^{*}) + \theta) - \sum_{j} a_{i,j}^{*} x_{j}^{*} = \lambda_{i} + \theta
\]

\[
r_{n+1}^{*} + \sum_{j} Z_{j}^{*} (-R^{*} + \Delta) + \sum_{i,j} (\Pi_{i}^{*} - \Pi_{i}^{p}) a_{i,j}^{*} x_{j}^{*} = \Delta.
\]

Again the same arguments as in Shoven and Whalley can be used to show that \(Z_{j}^{*} > 0, r_{n+1}^{*} = 0, \sum_{j} \lambda_{j}^{*} = 1;\) while \(0 < \Delta < 1\) excludes \(R^{*} = 0, \Pi^{*} = 0\).

A similar analysis goes through when legislated minimum consumer and producer prices are simultaneously considered.

**Ceiling Producer Prices**

We once again denote market prices by the vector \(\Pi\) and regulated ceiling producer prices by the vector \(\Pi^{p}\), consumers pay \(\Pi\), owners of endowments receive \(\Pi\). For any commodity \(\Pi_{i}^{p} = \min (\Pi_{i}, \lambda_{i}^{*} \Pi_{i})\). For a proof of existence in this case it is necessary to limit ceiling prices to outputs.

We now redefine \(R\) to be the lump sum tax collected by government to cover losses involved in administering the ceiling price programme. Walras' Law in this case becomes

\[
\sum_{i=1}^{n} \Pi_{i}^{*} e_{i}^{*} (\Pi_{i}, R) = \sum_{i=1}^{n} \Pi_{i}^{*} w_{i} - R \quad \text{where} \quad R \leq \sum_{i=1}^{n} \Pi_{i}^{*} w_{i}
\]  

(3.8)

As we exclude an equilibrium solution where \(\Pi_{i}^{*} = 0\), for all \(i\), by allowing \(R\) to be negative over a small region of the simplex and showing that a negative
value of $R$ involves a contradiction, Walras' Law can be stated in a slightly more general form as

$$\sum_{i=1}^{n} \Pi_i (\Pi_i R) = \min \left( \sum_{i=1}^{n} \Pi_i W_i, \sum_{i=1}^{n} \Pi_i W_i - R \right) \text{ where } R \leq \sum_{i=1}^{n} \Pi_i W_i$$  \hspace{1cm} (3.9)

As before we consider a subdivision of an $(n+1)$ dimensional unit simplex and we associate an $(n+1)$ dimensional vector of prices and revenue with any vertex $V^j$ according to the following rules

(1) $\Pi_i^j = \min(V_i^j, \lambda_i V_i^j)$ \hspace{1cm} i = 1, \ldots, n  \hspace{1cm} (3.10)$

(2) $R^j = f(V^j)$ where $f$ is a continuous linear homogeneous function\(^1\) with the properties that $R^j = 0$ if $V_{n+1}^j = 0$, $R^j = -1$ if $V_{n+1}^j = 1$, $-1 \leq R^j \leq \sum_{i=1}^{n} \Pi_i^j W_i$, and $R^j = \sum_{i=1}^{n} \Pi_i^j W_i$ for some $0 < V_{n+1}^j < 1$.

The value of $R^j$ can be thought of as defining the relative incomes of the private and public sectors. We choose a continuous function which allows the full range of possible values of $R^j$ but also allows us to exclude an equilibrium in which all prices equal zero.

Our vector labelling rules are the same as those for minimum consumer prices except that in 3(a) the $(n+1)^{st}$ element becomes $\sum_{i=1}^{n} (\Pi_i^j - \Pi_i) a_{i\delta}$.

Applying Scarf's theorem and considering the limiting process with an increasingly fine mesh in successive subdivisions allows the set of equations

\[
\begin{align*}
V_i^j & = \min \left( \sum_{i=1}^{n} \Pi_i W_i, \frac{V_i^j}{n+1} (\sum_{i=1}^{n} V_i^j - V_{n+1}^j) \right) \sum_{i=1}^{n} V_i^j \\text{ where } \delta > 0 \text{ and is chosen large enough to ensure } R_i^j = \sum_{i=1}^{n} \Pi_i^j W_i \text{ for some } 0 \leq V_{n+1}^j \leq 1.
\end{align*}
\]

\(^1\)A function which fits this description is $R^j = \min(\sum_{i=1}^{n} \Pi_i W_i, \sum_{i=1}^{n} \Pi_i W_i - R)$.
\[ \sum_{j} y_j \delta(V^j) = \hat{w} \] to be written out as
\[ \sum_{j} Z_j^* (\xi_i^* (R_i^*, R^*) + \Theta) - \sum_{j} a_{ij} X_j^* = W_i + \Theta \quad (i=1, \ldots, n) \quad (3.11) \]

\[ r_{n+1}^* + \sum_{j} Z_j^* (R_{n+1}^* + \Delta) + \sum_{i} (\Pi_i^p - \Pi_i^*) a_{ij} X_j^* = \Delta \quad (3.12) \]

Using the condition that ceiling prices apply only to outputs, it follows that one \( Z_j^* > 0; r_{n+1}^* = 0 \) if \( R^* \geq 0; \sum Z_j^* = 1 \) if \( R^* > 0; \)
and the conditions for a competitive equilibrium are satisfied. If \( R^* < 0 \) a contradiction is involved provided one \( \Pi_i^* > 0 \); the case where \( \Pi_i^* = 0 \) for all \( i \) and \( v_{n+1}^* = 1 \) is excluded by a restriction \( 0 < \Delta < 1 \).

This formulation extends the Shoven-Whalley treatment of taxes by incorporating production subsidies and changing the mapping from the \((n+1)^{st}\) element of the simplex to \( R \) to obtain an existence proof. In this case we can compute producer subsidy rates necessary to achieve target ceiling producer prices.

A difficulty exists in constructing a similar analysis for ceiling consumer prices since ceiling prices could be so low that private sector demands are infeasible given the economy's resource endowment. Low producer ceiling prices on outputs would simply result in activities shutting down rather than creating difficulties of infeasibility. We anticipate that in most practical applications problems of infeasibility of demands under ceiling prices will not occur.
IV. *Legislated Sector Specific Minimum Prices*

Our analysis of Section III can be extended to the case where inputs have sector specific minimum prices. Unlike our analysis of Section III no government monopoly agency is necessary to sustain equilibrium. An equilibrium probability of employment in the minimum price sector supports equilibrium in place of the government market agency.

Our formulation follows that of Harris and Todaro [1970] in their analysis of urban-rural migration. In this case some or all of the inputs are subject to minimum price restrictions only if these are employed in a particular portion of the economy (the urban sector; for instance); otherwise their prices are flexible. Decisions of owners of inputs to sell in particular sectors may not be able to be freely changed after decisions have been made. Asset owners will equate the probability of employment in the minimum price sector times the higher minimum price to the free market price. With repeated drawings for the right to be employed at the higher price from the pool of input owners wishing to sell, such an equilibrium is sustainable over time. Alternatively one can think of input owners receiving one way tickets to the higher priced sectors of the economy which prevent their returning unemployed inputs to the free market sector. The probabilistic equilibrium condition would be satisfied in this case in an ex ante sense, with no ex post return possible should their particular inputs be unemployed.

Following Section III the input prices in the minimum price sectors are defined as follows:

\[ \hat{\Pi}_i = \max(\Pi_i, \lambda_i \Pi_n) \quad (i=1,\ldots,n) \quad (4.1) \]

where \( \lambda_i \) describe minimum price policy parameters.
We assume the division of the economy into minimum price and non-minimum price sectors is described by a simple partition of the set of available activities and is fixed.

We treat owners of resources as expected income maximizers so that in equilibrium the free market price \( \Pi_i \) equals the expected price received in the minimum price sector

\[
\Pi_i = P(\hat{\Pi}_i, \Pi_i) \cdot \hat{\Pi}_i \quad (i=1,\ldots,n)
\] (4.2)

where \( P(\hat{\Pi}_i, \Pi_i) \) defines the probability of the \( i^{th} \) input being employed in the minimum-price sector. We define this probability as \( (1-s_i) \),

where \( s_i = (1 - \frac{\Pi_i}{\hat{\Pi}_i}) \). If \( \Pi_i = \hat{\Pi}_i \), \( s_i = 0 \) implying full employment of the \( i^{th} \) input. \( s_i \) can be interpreted as a fraction of the \( i^{th} \) input remaining unemployed due to a binding minimum price in the minimum price portion of the economy.

We can therefore rewrite (4.2) as

\[
\Pi_i = (1-s_i)\hat{\Pi}_i
\] (4.3)

The production side, as before, is characterized by an activity analysis matrix \( A \), with \( M \) activities, but these activities are partitioned. Activities \( 1,\ldots,M_1 \) characterize the non-minimum price sector and activities \( M_1+1,\ldots,M \) the minimum price sector.

The demand side of the model is represented, as before, by a set of market demand functions which are non-negative, continuous, and homogeneous of degree zero in an \( (n+1) \) vector \((\Pi, T)\) where \( \Pi \) defines the vector of free

---

\(^1\) In the Harris-Todaro formulation of labour market migration this probability is defined as the ratio of employed (NE) to total labour (NT) in the minimum wage sector:

\[
p = \frac{\text{NE}}{\text{NT}}; \quad \phi' > 0.
\]

But \( \frac{\text{NE}}{\text{NT}} = \psi(\frac{\Pi_i}{\hat{\Pi}_i}); \psi' > 0 \) in their formulation and therefore \( (1-s_i) \) has a similar interpretation to \( p \) in Harris and Todaro.
market prices and $T$ is an income correction term which is homogeneous of
degree one in $\Pi$.

We consider a total income correction $R$ to the term $\sum_{i=1}^{n} \Pi_i W_i$ to
comprise two portions

$$R = \sum_{i=1}^{n} s_i \Pi_i W_i - \sum_{i=1}^{n} \sum_{j=M_1+1}^{M} (\Pi_i - \Pi^*_i) a_{ij} X_j$$

(4.5)

$$= \sum_{i=1}^{n} s_i \Pi_i W_i - T$$

The first term of (4.5) gives the loss of income evaluated at prices
$\Pi$ due to the unemployed resources from positive $s_i$ terms. The second term $T$
is the extra income generated from employment at higher prices in the minimum
price sector. $T$ depends not only on the intersectoral price differential
but also on the activity levels $X_j$; $j=M_1+1, \ldots, M$. $T$ is of unambiguous
sign due to our limitation of minimum prices to inputs.

Walras' Law in this framework can be written as

$$\sum_{i=1}^{n} \Pi_i = \sum_{i=1}^{n} (1-s_i) \Pi_i W_i + T$$

(4.6)

An equilibrium under this formulation is defined by the vectors
$(\Pi^*, X^*, T^*)$ such that

(1) $\sum_{i=1}^{n} s_i (\Pi^*_i, T^*) = \sum_{i=1}^{M} a_{ij} X^*_j + (1-s_i) W_i, \quad (i=1, \ldots, n)$

(4.7)

(2) Zero profit conditions hold for both sectors:

(a) $\sum_{i=1}^{n} a_{ij} \leq 0 \quad (=0 \text{ if } X^*_j > 0) \quad j=1, \ldots, M_1$

(b) $\sum_{i=1}^{n} a_{ij} \leq 0 \quad (=0 \text{ if } X^*_j > 0) \quad j=M_1+1, \ldots, M$

(4.8)

where $s^*_i$ and $\Pi^*_i$ follow from the definitions above.
In order to obtain an equilibrium we search on a \((n+1)\) dimensional unit simplex \((\Pi, T)\) and apply Scarf's algorithm as before.

The vector labelling rules in this case are as follows:

1. \(\Pi^j_i = V^j_i\) \((i=1, \ldots, n)\), \(T^j = V^j_{n+1}\)

2. If \(\Pi^j_i = 0\), \(\lambda(V^j)\) is the \(i^{th}\) unit vector where \(i\) is the first coordinate of \(V^j\) equal to zero.

3. Evaluate the profit of each of the activities in the minimum price sector by using \(\Pi^j\) and calculate the profitability of all activities in the non-minimum price sector using \(\Pi^j\). Find that activity which gives the maximum per unit profit, \(\gamma\). Suppose the \(\lambda^{th}\) activity gives maximum profit.

   (a) If \(\gamma > 0\), \(\lambda(V^j)\) is the negative of activity \(\lambda\) with zero as the \((n+1)^{st}\) element if that activity belongs to the non-minimum price sector, and \(\Sigma_{i} (\Pi^j_i \cdot \Pi^j_i - \Pi^j_i) a_{i\lambda}\) as the \((n+1)^{st}\) element if the \(\lambda^{th}\) activity is in the minimum price sector.

   (b) If \(\gamma \leq 0\), \(\lambda(V^j)\) is the augmented market vector

\[
\{(\xi_1(\Pi^j_1, T^j_1) + s_{11}W^j + \theta, \ldots, \xi_n(\Pi^j_n, T^j_n) + s_{nn}W^j + \theta), (-T + \Delta)\}
\]

where \(\theta\) and \(\Delta\) are positive constants which fill the same role as in Section III.

Given these vector labelling rules, we can apply Scarf's algorithm as before.

Using similar arguments to those in Shoven and Whalley we can show that at least one \(Z^*_j > 0\), \(R^*_{n+1} = 0\), \(\Sigma^*_j = 1\) and we can exclude the situation where \(R^* = 1\), \(\Pi^*_i = 0\) for all \(i\) by a restriction on \(\Delta\) such that \(0 < \Delta < 1\).

The need to restrict minimum prices to inputs appears in the demonstration that one \(Z^*_j > 0\) since if minimum price constrained activities make positive profits at the minimum prices they must also make positive profits at the free prices.
V. Equilibrium Under Market Interventions Controlled by Target Minimum Or Ceiling Prices

The minimum price interventions described in the previous two sections allow legislated minimum or ceiling prices for either consumers or producers to be achieved through government regulation. In Section III minimum price regulations require the price constrained agents to purchase from government at the minimum prices. Any proceeds accruing to government through purchase at free market prices and sale at minimum prices are assumed to be redistributed to consumers. In Section IV segmented markets allow sector specific minimum prices to be supported through probabilistic equilibrium conditions connecting the sectors.

These forms of minimum price intervention by government do not correspond to the more familiar idea of minimum or ceiling price supports common in much of the partial equilibrium literature. In partial equilibrium analysis an economy-wide minimum price is often viewed as a constraint which involves an excess supply at a binding minimum price. Similarly, a ceiling price is one involving excess demand. A minimum price is supported in this literature either by a rationing rule allocating buyers to sellers, or by government intervention to purchase the excess supply. The latter formulation parallels government price support policies common with agricultural producers.

To capture this type of minimum price analysis we once again consider minimum prices defined in terms of a numeraire good (the n\textsuperscript{th}) but treat them as target minimum prices which control government purchase interventions in the market. We consider specific rules which determine these government purchase interventions and our formulation of Walras Law allows income to be transferred from the public to the private sector to finance price support intervention.
Given that economy-wide minimum prices are involved which refer to activities as well as the rest of the economy we once again need to restrict target minimum prices to inputs to guarantee that these prices are achievable in equilibrium since the minimum price constraints could otherwise be such that positive profits necessarily occur at any prices.  

We consider target minimum prices \( \hat{\Pi}_i \), as before, defined in terms of the numeraire good

\[
\hat{\Pi}_i = \lambda_i \Pi_n \quad (i=1, \ldots, n-1)
\]

These control government price support interventions which are defined by the quantity purchases \( G_i \) which are functions of the vector \( \Pi \). Below, we consider a specific function for the \( G_i \) which, in the case where target minimum prices are restricted to inputs, guarantees that a market equilibrium exists in which market prices are greater than or equal to target minimum prices.

The \( G_i \) are assumed to satisfy the restriction that

\[
\sum_{i=1}^{n} \Pi_i G_i \leq \sum_{i=1}^{n} \Pi_i W_i
\]

so that at any price vector sufficient income can be withdrawn from the private sector to finance the planned government purchases. The private sector is assumed to be left with the balance of the economy's income,

\[
(\sum_{i=1}^{n} \Pi_i W_i - \sum_{i=1}^{n} \Pi_i G_i),
\]

which finances private sector demands. From (5.2) this income term is non-negative. This leads to a statement of Walras Law that

\[
\sum_{i=1}^{n} \Pi_i (\xi_i(\Pi) + G_i) = \sum_{i=1}^{n} \Pi_i W_i
\]

---

1 Realistic policy applications will hopefully confront situations where computed equilibria fulfill the desired minimum price constraints even where the minimum prices apply to non-inputs, although an important issue with minimum price constraints is that of the attainability of minimum price targets and this can also be investigated numerically.
With this formulation we can consider an n dimensional unit simplex defined in terms of the \( \Pi_i \) with vector labelling rules as follows:

1. \( \Pi_i^j = V_i^j \) \( (i=1,\ldots,n) \).

2. If \( \Pi_i^j = 0 \), then \( \mathcal{L}(V^j) \) is the \( i \)th unit vector where \( i \) is the first coordinate of \( \Pi_i \) equal to zero.

3. If \( \Pi_i^j > 0 \), select that activity giving maximum per unit profit at prices \( \Pi_i^j \); define this maximum profit as \( \gamma \).
   
   (a) If \( \gamma > 0 \), then \( \mathcal{L}(V^j) \) is the negative of the activity generating \( \gamma \).
   
   (b) If \( \gamma \leq 0 \), then \( \mathcal{L}(V^j) \) is the vector \( (\xi_1(\Pi)+G_1,\ldots,\xi_n(\Pi)+G_n) \).

Applying Scarf's Theorem and considering the limiting process of earlier sections allows the set of equations \( \sum_{j} y_i^j \mathcal{L}(V^j) = W \) where \( W = (W_1,\ldots,W_n) \) associated with the limit to be written out explicitly as

\[
\sum_{j} Z_j^* (\xi_i(\Pi^*)+G_i^*) - \sum_{j} a_{ij} X_j^* = W_i \quad (i=1,\ldots,n) \tag{5.4}
\]

The arguments of Section II can be used to argue both that one \( Z_j^* > 0 \) and that the \( \sum Z_j^* = 1 \). There is, however, no argument that the equilibrium described by equations (5.4) will meet the constraints represented by the target minimum prices. Given the ability to specify minimum price constraints which guarantee positive profits for a given technology, it is hardly surprising that only under certain conditions can the \( G_i \) functions guarantee that minimum price objectives will be attained in equilibrium.

If we consider target minimum prices to apply only to inputs and if \( W_n > 0 \), then we can consider the following function for the determination of the \( G_i \).
\( G_1 = 0 \quad \text{if } \lambda_1 = 0 \)

\( G_1 = W_1 + \varepsilon \cdot [\max(\frac{\hat{\Pi}_1}{\Pi_1}, K)] \quad \text{if } \lambda_1 > 0 \) and \( \hat{\Pi}_1 > \Pi_1 \) \hspace{1cm} (5.5)

\( G_1 = 0 \quad \text{if } \lambda_1 > 0 \) and \( \hat{\Pi}_1 < \Pi_1 \)

\( K \) is an arbitrarily chosen, large but finite, positive constant. \( \varepsilon \) is chosen such that \( \varepsilon \leq \lambda_i < W_i \). Since the \( \lambda_i \) can only be positive for the goods 1, \ldots, n+1, this last condition guarantees that \( \sum \Pi_i G_i \leq \sum \Pi_i W_i \), so that Walras' Law is satisfied. The term \( K \) ensures that the \( G_i \) are bounded.

The \( G_i \) functions are thus constructed, along with the private sector demands \( \xi_i (\Pi) \), to satisfy Walras Law and are strictly larger than the corresponding \( W_i \). A technical difficulty with the functions (5.5) is that they are not continuous at \( \hat{\Pi}_1 \) and this can be accommodated by defining terms \( \hat{\Pi}_1 > \Pi_1 \), where \( \hat{\Pi}_1 = \lambda' \Pi_1 \) with \( \lambda' > \lambda_1 \) and modifying (5.5)(c) to

\( G_1 = \frac{\hat{\Pi}_1 - \Pi_1}{\hat{\Pi}_1 - \Pi_1} \cdot (W_1 + \varepsilon \cdot [\max(\frac{\hat{\Pi}_1}{\Pi_1}, K)]) \quad \text{where } \lambda_1 > 0 \) and \( \hat{\Pi}_1 < \Pi_1 \leq \hat{\Pi}_1 \) \hspace{1cm} (5.6)

\( G_1 = 0 \quad \text{when } \Pi_1 > \hat{\Pi}_1 \)

Using (5.5) (a) and (b) incorporating the modifications in (5.6) (c) and (d) to represent \( G_i \), the market demand functions \( (\xi_i (\Pi) + G_i) \) are non-negative, continuous, homogeneous of degree zero in \( \Pi \), and satisfy Walras Law (5.3). From the equations (5.4) since \( \sum Z^\ast_j = 1 \), and the fact that \( a_{ij} \leq 0 \) if \( \lambda_1 > 0 \) (positive target minimum prices apply only to inputs), \( \Pi_1 \leq \hat{\Pi}_1 \) involves a contradiction since \( G_i > W_i \). In this case, therefore, the equilibrium prices \( \Pi^\ast \) must exceed the target prices \( \hat{\Pi}_1 \).

A similar analysis can be used where target ceiling prices are involved. However a general existence proof is not possible since the same difficulties of infeasibility discussed with ceiling consumer prices also arise. In this case
the \( G_i \) functions describe government sales rather than government purchase policies. Using the \( G_i \) functions

\[
\begin{align*}
(a) \quad G_i &= -\xi_i(\Pi) \quad \text{if } \lambda_i > 0 \text{ and } \Pi_i \geq \hat{\Pi}_i \\
&= \frac{\hat{\Pi}_i - \Pi_i}{\hat{\Pi}_i - \hat{\Pi}_i} (-\xi_i(\Pi)) \quad \text{if } \lambda_i > 0 \text{ and } \hat{\Pi}_i \leq \Pi_i \leq \hat{\Pi}_i \\
(c) \quad G_i &= 0 \quad \text{otherwise}
\end{align*}
\]

and the same labelling rules as above, can be used although the argument that ceiling prices must be met in the equilibrium no longer goes through. It remains to be determined by numerical investigation whether rules of the above type will achieve target minimum or ceiling prices in practical policy analyses.
VI. Specifying Minimum or Ceiling Prices in Monetary Terms

In this section we examine the cases discussed in earlier sections but where minimum or ceiling prices are defined in monetary terms rather than in terms of a numeraire good. In these formulations we remove the traditional neutrality of money in general equilibrium models in the sense that real characteristics of an equilibrium may be affected by a change in the money stock.

In conventional general equilibrium models money is neutral in the sense that its presence does not affect real characteristics of an equilibrium. Through an equation of exchange an equilibrium price level is determined for a given money stock. A change in the money stock will change the price level without changing relative commodity prices. Equilibrium relative commodity prices of goods $\Pi_i$ (normalized to sum to any constant) can be obtained such that the demand supply conditions (2.5) for our information of a traditional general equilibrium model hold

$$E_i(\Pi^*) - \Sigma a_{ij} X_j^* = W_i \quad (i=1, \ldots, n). \quad (6.1)$$

A specified stock of money, $M$, will then serve to determine the price level if money acts simply as a medium of exchange. In the case where no minimum or ceiling price interventions occur, a competitive equilibrium in a monetary economy will involve an equilibrium price level $P_{n+1}^*$ for the given money stock $M$. If the value of transactions in barter terms is defined in terms of incomes, then from the Cambridge\(^1\) equation of exchange,

$$M = P_{n+1}^* k \Sigma_{i=1}^{n} \Pi_i^* W_i \quad (6.2)$$

or

$$P_{n+1}^* = \frac{M}{k \Sigma_{i=1}^{n} \Pi_i^* W_i} \quad (6.3)$$

where $k$ defines the velocity of circulation. $\Pi_i^*$ is not affected by $M$, since

\(^1\)We use the Cambridge version of the equation of exchange as we take income, instead of consumer expenditures, to define transactions.
changes proportionality with $M$.

In analyzing an economy with minimum or ceiling prices defined in

terms of a numeraire good we have been able to represent minimum or ceiling

price interventions in terms of the scalars $\lambda_i$. With minimum or ceiling

prices defined in nominal terms, however, the policy interventions for the

minimum price case are specified in the form

$$p_{n+1}^* \Pi_i \geq U_i \quad (i=1, \ldots, n) \quad (6.4)$$

where $p_{n+1}^*$ is the equilibrium price level and $U_i$ is a specified number of

units of account. In such cases, when the money stock changes the minimum

price constraints will change. The neutrality of money thus disappears in

such cases although computation of equilibria using our earlier formulations

is still possible. The additional equation (6.2) appears in our equilibrium

conditions. We thus search on an $(n+2)$ dimensional simplex with the $(n+1)^{st}$

element defining the price level $p_{n+1}$ and the $(n+2)^{nd}$ element our additional

dimension from earlier sections. The equation of exchange thus serves as

an additional demand supply equality for the additional 'commodity' money. 

Through the policy interventions (6.4) changes in the money stock can affect the

relative prices of the $n$ 'real' commodities thus departing from traditional

neutrality propositions.

VII. Possible Future Applications and Extensions

At this stage we have not applied our computational techniques to

any real world situations although this remains the objective of our work.

In this section we outline our current thinking as to possible fruitful

areas for applied policy evaluation work using the techniques presented in

earlier sections.
A number of minimum or ceiling price interventions fit naturally within
the formulations we have developed earlier, while for a number of others the
application is less obvious. A characteristic of our analysis, as we have stressed
earlier, is that we do not explicitly consider rationing as an accompaniment
of minimum or ceiling price policies. Some extension of our analysis would
therefore be necessary to directly consider wage and price controls as
currently operating in many countries or the minimum wage problem as usually
formulated. In these cases excess demands cause supplies to be allocated
to specific agents on the demand side of the economy through queues,
bureaucratic, or alternative quasilegal mechanisms and these we have not modelled.

At a more micro level than that of economy-wide wage and price controls
there are some clear areas of application for our techniques. Our ceiling
price analysis can, with modifications, be applied to the McKinnon (1973) and
Shaw (1973) analysis of financial market failures in less developed countries
where government monopolies of the form we analyze result in the market failure.
Energy pricing also seems a natural area of investigation. Other ceiling price
policies frequently occur (usually outside the U.S.) with food price ceilings,
transportation price regulation, housing mortgage rate ceilings and other
policies which involve government covering the losses. A frequent justification
for such policies is that they help the poor and, the distributional effects
of such policies could be investigated using our techniques.

The application of our sector specific minimum price formulation
to urban-rural migration problems is clearly natural since that area
provides the motivation for the Harris-Todaro literature. A further fruitful
area, however, would seem to be that of a general equilibrium modelling of occupational choice where similar probabilistic equilibrium conditions are involved between (ex post) segmented labour markets.

Lastly, our government price support intervention formulation would seem to fit well for an analysis of agricultural support policies, and this would seem especially useful in an analysis of the EEC common agricultural policy. A further area of application would be international trade interventions such as the recent steel 'trigger price' restrictions introduced in the U.S.

As yet our computational experience with these procedures is somewhat limited but is sufficiently encouraging that development towards practical policy evaluation is something we envisage in the near future.
VIII. Conclusion

In this paper we have presented three formulations of general equilibrium under government policy interventions designed to achieve minimum or ceiling target prices. In the first, minimum prices are legislated and regulated agents are required to buy commodities from a government monopoly agency. With ceiling prices, the government only allows certain agents to buy from the government agency at the ceiling price and covers the losses involved. These formulations are similar to the general equilibrium analysis of taxes by Shoven and Whalley [1973] but here tax or subsidy rates are variable.

In our second formulation we follow Harris and Todaro [1970] and consider sector specific minimum prices where a probability of unemployment in the high price region of the economy supports an equilibrium. In our final formulation we consider direct government market intervention designed to achieve target minimum or ceiling prices. We provide procedures for the numerical determination of such equilibria based on Scarf's algorithm and we are also able to obtain existence proofs in a number of cases. In a final section we discuss future policy applications of our formulations.
References


