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by

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ABSTRACT

An atemporal linear model of stabilization is formulated for an economy which faces additive stochastic disturbances. The authorities wish to stabilize certain target variables through intervention in various markets in accordance with the movements of indicator variables. When there are superfluous markets: the weighting matrix is irrelevant because each variable is stabilized independently given the information available; optimal policy is such as to make the covariance between each target and each indicator zero; and the information from each indicator is efficiently utilized by intervening in accordance with its movement in a number of markets equal to the number of target variables. The assignment problem has a natural interpretation in this model. A comparison is made with the complete information case.
I. INTRODUCTION

In a deterministic setting, the theory of economic policy cast in terms of targets, instruments, and indicators is designed to treat the general case in which there can be a multiplicity of magnitudes of each kind. In contrast, this analysis in a stochastic framework has been applied predominantly to specific examples in which there exists a single variable in each category. This narrow scope limits severely the range of problems to which the theory can be applied, and does not permit an investigation of the consequences of there being different numbers of variables in the various categories.

This paper considers the case of multiple targets, indicators, and markets of intervention, in a stochastic framework when intervention is guided by a reaction function. In the conventional case of superfluous intervention markets, the well-known rules concerning optimal intervention can be stated in a simple fashion: the optimal stabilization regime is designed so that the covariance between every target and every indicator variable is equal to zero. This rule implies that, for the optimal regime, information obtained from the indicators does not permit any increase in the degree of stabilization since any systematic relationship between targets and indicators is eliminated.

The analysis presented here re-examines the importance of the relationship between the number of targets and the number of markets of intervention in a stochastic model. This is a relevant concern since it has been shown that, for a certain class of models, intervention should be undertaken in all markets in order to attain even a single target. Thus, it has been argued,
"Tinbergen's Principle" is valid only in deterministic models.

The model presented in this paper, with stochastic shocks entering in an additive fashion in the structural form, provides a counter example to reasoning of this sort. In that model, stabilization can be accomplished efficiently by limiting the number of markets of intervention to the number of target variables. No matter what the stabilization regime, it is shown that the authorities should monitor the movements of all (stochastically independent) indicator variables so as to gather all available information to guide their intervention. Specifically, in an optimized regime the information obtained from any indicator is used to guide intervention in a number of markets no smaller than the number of targets. On the question as to whether all indicators should be monitored by a limited number of intervention authorities (and hence specialization in intervention) or whether a large number of authorities should each be assigned different smaller sets of indicators to monitor (so that there is specialization in monitoring) this model is silent. But even to state the analysis in this manner sheds some light upon some neglected criteria in the "assignment problem."

II. THE MODEL

Consider an economy described by a linear model of the form:

$$A \cdot Y = u$$

(1)

where \(Y\) is a vector of the \(N\) endogenous variables, measured as their deviations from mean levels; \(u\) is a vector of the stochastic excess-supply disturbances, with zero means and known variance-covariance structure, impinging upon the \(N\) markets under analysis;\(^\text{7}\) and \(A\) is a matrix of coefficients indicating how movements in the endogenous variables influence the state of excess demand in these markets.
The first $T$ elements in vector $Y$ are the targets variables. These are magnitudes which have non-zero weights in the authorities' loss function. It is assumed that the authorities minimize a quadratic loss function, centered upon fixed values for these variables, by intervening in the markets at their disposal.

Intervention is made up of two parts. A portion is designed to keep the economy in its optimal position on average. This portion of the stabilization regime sets the levels of intervention independently of the period-by-period fluctuations in the economy so that this optimal position is attained on average. It is assumed that this part of the intervention policy has been carried out. For this reason, all variables are measured as deviations from mean levels. The other portion of the stabilization policy is designed in terms of a reaction function. Such endogenous intervention is functionally related to the deviations of the contemporaneously-observable variables from their means, and is designed to limit the movements of the target variables about their average levels.

A reaction function represents an excess demand function on the part of the authorities. In optimally designing their reaction function, the authorities are choosing the best values for the parameters of their excess demand functions and, consequently, for those of the economy as a whole. The process is therefore one of setting appropriately the values of those elements of $A$ (the $a_{ij}$'s) which the authorities are able to influence. These elements can be identified by elimination. Clearly the authorities have no influence over the values of the elements in those rows of $A$ corresponding to markets in which they do not intervene. (This assumption is based upon the presumption that the private sector behavioral functions are independent of the stabilization regime pursued by the authorities.) In contrast, for
each row of $A$ describing a market of endogenous intervention, there is at least one element whose value can be set by the authorities. The number of markets of intervention is $M$ and, without loss of generality, the excess demands for these markets appear as the first $M$ rows of $A$.\textsuperscript{11}

Endogenous intervention obviously cannot be geared to variables whose values are not known within the period. This observation is represented algebraically by the elements of the columns of $A$ corresponding to these variables being treated as exogenous.\textsuperscript{12} In contrast, the varying weight which the authorities can put upon the guidance provided by those variables which can be monitored on a current basis is shown by the columns of $A$ corresponding to such variables having elements whose values can be chosen endogenously by the authorities. This paper calls all variables which are contemporaneously observable "indicators." The number of indicator variables is denoted by $I$.

This discussion concludes that only $I$ columns of $A$ have adjustable elements, and within these columns such elements lie in the first $M$ rows. Thus, the elements of $A$ whose values can be chosen by the authorities lie within this $M \times I$ submatrix, all of whose elements are assumed to be endogenous for the moment. In contrast, all elements outside this submatrix are taken as exogenous.

III. THE LOSS FUNCTION AND THE OPTIMAL REACTION FUNCTION

The loss function for the economy is of the conventional form:

$$L(Y) = Y' \cdot W \cdot Y$$

(2)

where $W$ is a positive semidefinite symmetric matrix of weights and a prime attached to a vector indicates its transpose. Only the first $T$ elements of $Y$ are target variables, so that all elements outside the $T \times T$ submatrix
in the northwest corner of W are zero.

The expected value of the loss function is dependent upon the elements of A because the response of Y to a particular shock depends upon them. The extrema of this function can be found by setting its partial derivatives with respect to each of the M x I adjustable elements of A to zero. This rule is applicable since the loss function is differentiable except for points at which the system determinant is zero. In general, these are points of discontinuity which can be ignored since they are not minima. Differentiation is carried out with respect to all \( a_{ij} \) for which \( 1 \leq i \leq M \), and \( j \) belongs to the set of subscripts corresponding to indicator variables.

The invertability of A assures that Y can be written as:

\[ Y = A^{-1}u \]

and the loss function expressed as:

\[ L = (A^{-1}u)' W A^{-1}u \]

Differentiation of this expression (using the rule that the transpose of a matrix product is the product of the transposes of the matrices with their order of multiplication reversed) yields

\[
\frac{\partial L}{\partial a_{ij}} = (u' \cdot \frac{\partial A^{-1}}{\partial a_{ij}}' \cdot W \cdot A^{-1}u) + (u'(A^{-1})' \cdot W \cdot \frac{\partial A^{-1}}{\partial a_{ij}} \cdot u)
\]

This equals

\[-(u'(A^{-1}1_i)(1_jA^{-1}))' \cdot W \cdot A^{-1}u \quad - \quad (u'(A^{-1})' \cdot W \cdot (A^{-1}1_i) \cdot (1_jA^{-1})u)\]

where \( 1_k \) is a column vector of zeros with a one in the \( k \)th row. By employing again the rule on the transpose of a matrix product, this can be written as

\[-(u'(A^{-1})'1_j1_i(A^{-1})' \cdot W \cdot A^{-1}u \quad - \quad (u'(A^{-1})' \cdot W \cdot (A^{-1}1_i)(1_jA^{-1})u)\]

Of course, \( u'(A^{-1})' = y' \) and \( A^{-1}u = y \) so that
\[
\frac{\partial L}{\partial a_{ij}} = -\{y_j' (A^{-1})' \cdot W \cdot y\} - \{y' \cdot W \cdot (A^{-1})^i_y \cdot y_j\}.
\]

Now, \(y_i' (A^{-1})'\) is a row vector made up of the \(i^{th}\) row of \((A^{-1})'\); this is equal to the \(i^{th}\) column of \(A^{-1}\) which can be written as \(A^{-1} y_i\). Thus, the two elements on the right-hand side of this equation are equal and the derivative can be written as:

\[
\frac{\partial L}{\partial a_{ij}} = -2y_i' (A^{-1})' \cdot W \cdot y \cdot y_j.
\]

For a minimum

\[
-2 E[y_i' (A^{-1})' \cdot W \cdot y \cdot y_j] = 0
\]

where \(E\) indicates expected value.

Carrying this out for \(i=1,\ldots,M\) and for the \(I\) subscripts corresponding to indicators yields the equations

\[
E[(A^{-1}_M)' \cdot W \cdot y \cdot y_I] = 0_{MI}
\]

where \((A^{-1}_M)'\) is the first \(M\) rows of \(A^{-1}\), \(y_I\) is the vector of indicator variables only, and \(0_{MI}\) is an \(M \times I\) array of zeros. The matrices \((A^{-1}_M)'\) and \(W\) are not stochastic so this equation can be written in the form

\[
(A^{-1}_M)' \cdot W \cdot E[y \cdot y_I] = 0
\]

or

\[
(A^{-1}_M)' \cdot W \cdot \text{Cov}[y, y_I] = 0,
\]

where \(\text{Cov}\) denotes the variance-covariance matrix between \(y\) and \(y_I\).

The zeros in the weighting matrix permit us to concentrate our attention upon the first \(T\) elements of vector \(y\) in evaluating this product. Denote this abbreviated vector of target variables only by \(y_T\), and denote by \(W_T\) the \(N \times T\) matrix obtained by deleting the columns of zeros in \(W\). Then this equation can be written in the simpler fashion:
\[(A^{-1}_M)' \cdot W_T \cdot \text{Cov}[Y_T, Y_I] = 0_{MI}. \]

It should be noted that the ranks of \((A^{-1}_M)'\) and \(W_T\) are given by their subscripts, and that the rank of their product is the lesser of \(M\) and \(T\).

IV. NUMBER OF TARGETS EXCEEDS NUMBER OF INTERVENTION MARKETS

If the number of targets exceeds the number of intervention markets, then equation (3) cannot be simplified further and implies that the optimal intervention policy depends in a rather complicated way upon the market intervention structures and the weighting matrix, in addition to the covariance matrices between the target and indicator variables. This implication is consistent with the analysis of policymaking in a deterministic setting.

For example, a deficiency of markets of intervention in such a setting implies that not all the targets can be attained. Under these circumstances, an optimal policy is one in which the marginal rate of transformation between any two targets is equal to the marginal rate of substitution between them. The transformation rate is determined by the opportunities technically available to the authorities, and the rate of substitution is indicated by the weighting matrix, \(W\).

Equation (3) can be interpreted in this manner, so long as these rates are seen as referring to trade-offs between expected values. Thus with a small number of markets of intervention the expected discrepancy of the target variables from their desired levels is the important element determining the specific nature of the optimal stabilization policy. This rate of transformation, given by the elements of \((A^{-1}_M)'\), must be equated with the rate of substitution as indicated by the weighting matrix. If either of these matrices changes this has a direct influence on the form of the optimal policy and
thus on the expected squared deviations of the elements of $Y$.

V. NUMBER OF TARGETS IS NO GREATER THAN NUMBER OF INTERVENTION MARKETS

Consider next the case in which the number of markets of intervention is not less than the number of target variables, so that $T \leq M$. Under these circumstances equation (3) can be simplified substantially.

Multiply that equation by $(A_M^{-1} \cdot W_T)'$ to obtain the system

$$(A_M^{-1} \cdot W_T)' \cdot A_M^{-1} \cdot W_T \cdot \text{Cov}[Y_T', Y_I] = 0_{TI}$$

where $0_{TI}$ is a $T \times I$ matrix of zeros and $(A_M^{-1} \cdot W_T)' \cdot A_M^{-1} \cdot W_T$ is a square matrix of dimensions $T \times T$ and rank the lesser of $T$ and $M$. When $M$ is no less than $T$ this square matrix has a well-defined inverse. Multiplying through by this inverse yields the simple expression

$$\text{Cov}[Y_T', Y_I] = 0_{TI}. \quad (4)$$

This equation shows that with superfluous markets of intervention, their number or nature becomes irrelevant. Instead, the crucial test of optimality is whether there is zero covariance between every target and each indicator. In addition, the deterministic portion of the model and particularly the weighting matrix are not of prime concern in the design of optimal policy.

It is useful, in investigating the implications of this rule, to distinguish between two cases. Consider first a situation in which a contemporaneously observable variable has a non-zero weight in the loss function. This would characterize a model in which a particular magnitude serves as both an indicator and a target. In order to set to zero the covariance of this variable with every target variable, including itself, the policy must be one of pegging this variable at its target level. Thus, with a sufficiently large
number of markets of intervention, any variable with these characteristics is pegged at its desired level.

Consider next an argument in the loss function which is not contemporaneously observable, and therefore cannot be pegged. To minimize its contribution to the loss function, the optimal policy is to make its expected divergence from its desired level, given the information available from the indicator variables equal to zero. Now it can be shown that this expected divergence is a weighted sum of the deviations of the indicators from their mean values.\(^\text{16}\) The important element in these weights is the covariances of the targets with the indicators. In making these covariances zero, the optimal regime causes the expected values of the target variables to be at their desired levels no matter what the current values of the indicators.

It should be noted that the simple rule dictated by equation (4) does not permit the authorities to ignore the movements of any indicators. Indeed, the only way in which such a rule can be implemented is by the authorities monitoring all such stochastically independent variables\(^\text{17}\) so as to react to their movements appropriately. However, once this stabilizing reaction has been carried out, the resulting values of the indicators convey no information which would enable the authorities to do any better. The reason is that the process of implementing the information thereby conveyed causes the covariances to be reduced, and in the optimal regime forced to zero.

Finally, this analysis suggests that there is an important distinction between optimal stabilization in a model of limited information and one of complete information. If the sizes of all shocks are known with certainty in any period, then intervention in T markets geared to the movement of a single indicator enables the authorities to attain all targets.\(^\text{18}\) Monitoring
other indicators is not necessary since they contain no further information.

VI. REDUNDANT INSTRUMENTS

The optimality of the rule derived above (equation (4)) is dependent upon the number of markets of intervention being at least as great as the number of target variables. The fact that this equation is independent of the number of markets of intervention suggests that their number is irrelevant (so long as it is greater than T). This, in turn, raises the possibility that efficient stabilization can be carried out by intervening in a subset of the available markets. It is the purpose of this section to show that efficient stabilization can be effected by intervening in a number of markets no greater than the number of targets. That is, Tinbergen's Rule applies to this model even though it contains stochastic elements.

Let us verify this result by an example. Consider the case of a single target which is not contemporaneously observable. Assume further that there are two markets of intervention and two indicators. The ordering of endogenous variables places the two indicators as variables N - 1 and N. The object of the analysis is to choose $a_{l,N-1}$, $a_{l,N}$, $a_{2,N-1}$, and $a_{2,N}$ so that the expected squared deviation of the target variable is minimized.

Application of the first derivative rule with respect to the adjustable $a_{ij}$'s yields four equations in the four unknowns. In contrast, the rule above (equation (4)) on the covariance yields only two equations; (that is, the number of targets (one) multiplied by the number of indicators (two)). The difference between these numbers is due to the fact that with a single target the derivative of the expected value of the loss function with respect to $a_{2,N-1}$ is a multiple of the derivative with respect to $a_{l,N-1}$; similarly for $a_{2,N}$ and $a_{l,N}$. Thus there are only two functionally independent equations to
determine the optimum values of the four matrix elements. This implies that
two of these elements can be chosen arbitrarily with the other two set op-
timally and dependent upon these arbitrary values. Furthermore, the parameters
which are chosen arbitrarily must lie in different columns. The reason is
that it is the derivatives with respect to $a_{1,N-1}$ and $a_{2,N-1}$ which are
multiples of each other so that setting either to zero assures that the other
is. A similar argument applies to $a_{1,N}$ and $a_{2,N}$. If the two parameters
which are set arbitrarily lie in the same column, in general the derivatives
with respect to them will be different from zero.

Of the four original equations, the two which are functionally inde-
dendent can be written in the form:

$$a_{1,N-1} = \frac{A_1 \cdot (a_{1,N})^2 + B_1 \cdot a_{1,N} + C_1}{D_1 \cdot a_{1,N} + E_1}$$

and

$$a_{1,N} = \frac{A_2 \cdot (a_{1,N-1})^2 + B_2 \cdot a_{1,N-1} + C_2}{D_1 \cdot a_{1,N-1} + E_1}$$

(5)

The coefficients $A_1$, $A_2$, $B_1$, $B_2$, $C_1$, $C_2$, $D_1$, $D_2$, $E_1$ and $E_2$ are made up of the
other elements of the matrix $A$ and of the parameters describing the stochastic
structure of the model.

Given the precise form of the coefficients, it can be shown that there
exist two solutions to the quadratic equations. It is necessary to evaluate
the expected loss function at both these points to determine which is the
global minimum. It is not necessary to take higher order derivatives in
order to establish whether these points are relative minima rather than
maxima or saddle points. This should become apparent upon evaluation of
the loss function at these points. Furthermore, the set of points for which
the loss function becomes very large is well-defined. These are the points
which lie close to those for which the system determinant is equal to zero. These are points of discontinuity in the loss function, and therefore are points which the first derivative rule avoids. Thus, the procedure here identifies a set of points among which must be the global minimum.

The extension of this verbal analysis to the case of a general number of targets, \( T \) (but less than the number of markets) and more indicators is straightforward. The derivative of the loss function with respect to the elements in any column of \( A \) yields \( T \) functionally independent equations. The other \( M - T \) equations which result from the differentiation are functionally dependent upon those \( T \) equations. This is exemplified in system (4) above by there being only \( T \) equations for each indicator variable. Thus, \( M - T \) elements in each column can be set arbitrarily with the other elements chosen optimally given those arbitrary values.

The introduction of further indicator variables creates more equations to determine the optimal values of the relevant elements of \( A \). Since each of these equations is a quadratic expression in the choice variables, the number of relative minima increases along with the number of indicators and number of equations. The problem of multiple extrema is handled by evaluation of the expected loss function at these points.

These results demonstrate that the assumption employed above, that the authorities can set the values of all the \( M \times I \) adjustable parameters of \( A \), is too restrictive. What is necessary is that only \( T \) parameters in each of the \( I \) columns be chosen optimally in light of the (possibly arbitrary) values of the other \( (M - T) \times I \) adjustable parameters (and the values of the elements in the rest of the matrix). If all these parameters lie within the same \( T \) rows of matrix \( A \), the authorities are intervening in only \( T \) markets. Since this intervention policy is optimal, and the number of markets of
intervention is equal to the number of targets, Tinbergen's Rule is valid for this model.

VII. STABILIZATION AND THE DIVISION OF RESPONSIBILITY

It was demonstrated in the previous section that the authorities need to choose optimally the values of only a subset of parameters in order to stabilize the target variables if there are superfluous markets. The restriction on this choice is only that in each column of $A$ corresponding to an indicator at least $T$ parameters must be chosen optimally in order to utilize fully the information which such a variable provides. A question which arises immediately is whether that information should be employed in all markets or in merely $T$ markets.

This point can be put succinctly by considering the single target case. For this case only one element in each indicator column of $A$ must be chosen optimally. It is a matter of indifference whether these elements are all from the same row in the matrix or each is from a different row. In economic terms this means that efficient stabilization can be carried out by intervention in a single market, where this intervention is guided by the movements in all indicator variables. Alternatively, it can be done by intervening in a multiplicity of markets, where intervention in each one is guided by a single variable.

In the single target case, the problem is that of determining whether efficiency gains from specialization in information-gathering are greater than those from centralizing intervention policy. Use of a single market clearly centralizes intervention policy but this necessitates that the market authorities have access to information about the movements in all indicator variables. In contrast, intervention in many markets by many authorities permits specialization in information-gathering since each authority's
intervention needs to be guided by only a single indicator.

It is tempting to make a comparison between this analysis and that usually designated by the term "the assignment problem." The problems are clearly analogous and the conclusions here provide some justification for the assumption that each authority monitors a single indicator variable, an assumption which seemed arbitrary in the earlier literature. However, the question of specialization in information-gathering versus specialization in intervention arises in its starkest form only for the single target case. With multiple targets, multi-market intervention is required. What this model does show is that in the atemporal, stochastic framework used here, there is no assignment problem as that expression is usually interpreted. It is a matter of indifference how indicators are assigned to market authorities to guide their intervention. What is of some interest is how many indicators should be assigned to each authority since this determines whether specialization occurs in intervention or in monitoring. In order to answer this question, however, these costs need to be introduced explicitly. This is an exercise which would take us beyond the scope of this model.

VIII. CONCLUSION

This paper investigates the problem of optimal stabilization policy when private sector expectations are naive, and when the structure of the economy is known with certainty. Additive stochastic terms are assumed to accurately represent the influence of disturbances upon the economy. The principle of certainty equivalence is known to hold in such a framework. This principle establishes that there is a direct correspondence between stabilization policies when the authorities have full information about the economy and when they do not have current readings on certain variables.
This analogy suggests a number of theorems which this paper has proven. First, if there are $T$ targets, then efficient stabilization requires intervention in no more than $T$ markets. It is a matter of indifference which markets are left out of the intervention regime. Furthermore, in this case the effect of optimal stabilization is to make the correlation between the targets and the indicators equal to zero. This conclusion is obvious once it is noted that if this were not true the movement of the indicator variables would provide information about the current values of the target variables and thereby create an incentive for a change in the intervention pattern. In addition, it is shown that all indicators (up to the number of stochastically independent shocks) are monitored in the optimal system. It is here that the analogy with the full information case breaks down. Optimal stabilization in that setting can be achieved by gearing intervention to a single indicator. Finally, in the stochastic setting the movement in each indicator is used to guide the intervention in at least $T$ markets, but clearly there is no need for these $T$ markets to be the same for all indicators.

The last observation makes apparent that the assignment problem needs to be phrased in a slightly different manner within the framework used in this paper. The question is whether these $T$ markets should all be the same (so that each market intervention authority has to monitor all indicators) or whether responsibility for monitoring should be dispersed by assigning a smaller number to each authority, with the total number of authorities being greater in this latter case. That is, the assignment problem is one of determining how many indicators should be assigned to an authority. The solution to this problem depends upon the costs of monitoring different indicators versus the costs of decentralization of intervention policy. These costs have not been captured in the present model. As to the problem of
how indicators should be assigned to intervention authorities, this model has a definite answer. Optimal stabilization can be accomplished no matter how this assignment is made.
FOOTNOTES

1 Early development of the theory of economic policy and coinage of these phrases is due to Tinbergen (1956). Thiel (1957, 1964) initiated the analysis within a stochastic setting. See Mundell (1968) for a discussion, and an extension to some dynamic principles to guide intervention policy.

2 Henderson (1979) provides an example, in a stochastic setting, of a multi-target, multi-indicator problem. The work of Brainard (1967) and Poole (1970), is typical of the single target analysis. Friedman (1975) provides a thorough discussion of this field, and critically surveys a long list of references.

Kareken, Muench, and Wallace (1973) investigate intertemporal aspects of intervention policy. This has not been done here because recent developments in expectations formation mechanisms make clear that feedback rules require an analysis which is different from that for built-in stabilizers.

3 The stabilization policy of the authorities is one of allowing the settings of all contemporaneously observable variables to adjust. Thus, the distinction between instruments and indicators (which is based upon the exogeneity of the former) is not relevant here. It is for this reason that the expression "market of intervention" is used in this paper.

4 Poole (1970), Friedman (1977), and Boyer (1978) investigate optimal policy. Poole's attention is concentrated mainly upon policies which keep certain indicator variables constant. Friedman's model is formulated so that each stabilization period is divided into two parts: an observation stage and a stabilization stage. This division tends to obscure the fact that the optimal policy conforms to the rule stated here.
5 It is worth noting that this sort of stabilization model is very similar to the typical rational expectations problem (Sargent and Wallace (1975)). The distinction is that in the one case private sector behavior is exogenous whereas the authorities provide stabilizing behavior; in the other the authorities' behavior is predetermined and the private sector seeks unbiased forecasts of future values of the endogenous variables. This paper shows that optimal stabilization can be achieved by either sector no matter what the behavior of the other. A fuller discussion would contain a game-theoretic solution to this problem.

6 Tinbergen's Principle is traditionally derived from a model in which intervention is an exogenous response to a persistent disturbance of known size. Brainard (1967) has dealt with such a model in a stochastic framework with multiplicative uncertainty and shown that all policies should be employed so as to attain even a single target.

7 Equilibrium in a multi-sectoral, multi-market model requires market-clearing of each market in each sector. If the excess demand functions for a number of sectors are combined together, then equilibrium in this market is necessary but not sufficient for equilibrium in each sector individually.

8 It can be shown that such a two-step procedure minimizes the loss function. The proof is the same as that used to establish that the expected squared deviation of a stochastic variable from any value is equal to its variance plus the squared difference between the mean of the variable and that value. See Thiel (1971, p. 92).

9 It is assumed that the authorities have at their disposal a sufficiently large number of markets so that they are able to accomplish this. By Tinbergen's Principle this number must be no less than the number of targets.
The literature on rational expectations has emphasized that the expectations-generating mechanism of the private sector and therefore its excess demand functions may depend directly on the authorities' policy regime. See Sargent and Wallace (1975), and Lucas (1976). Expectations are taken to be naive in the present model.

The modifier "independent" has, for simplicity, been left out of the expression "market of intervention." Any dependence between intervention in different markets reduces the value of \( M \) correspondingly.

Information available from previous periods and relevant to the present period due to serial dependence of the \( u \)'s is assumed to have been used already such that the adjusted error terms have the characteristics presumed above.

This paper concentrates its attention upon the question of observability for convenience, although the important characteristic of an indicator variable is that the dependence of the excess demand in at least one market of intervention can be chosen by the authorities. As McCallum and Whitaker (1977) have emphasized, contemporaneous observability is not necessary for this to be true.

See Thiel (1971) for the differentiation of an inverse matrix with respect to an element in the original matrix.

This matrix indicates the changes in all endogenous variables caused by a (differential) unit change in the excess demand in the \( i \)th market. In light of this, equation (3) can be interpreted (as in a deterministic model) as arguing that the differential changes in the endogenous variables vector must be perpendicular (in an expected value sense) to the endogenous variable vector itself.
In a single target, single market of intervention setting, this principle is well-known. It was important during the policy discussions in the sixties in the US concerning the influence of monetary policy upon the level of employment in a setting where the authorities were attempting to stabilize the latter.

See Graybill [1961, p. 63].

Since the covariance between the targets and every indicator is zero, it is clear that all indicators should be monitored and that optimal intervention will be guided by their movements. The only case when this is not true is when the indicators are not stochastically independent in which case zero covariance for any indicator is guaranteed by the covariances of the other indicators being zero. With stochastic independence, Friedman [1975] has pointed out that this rule indicates that the authorities should "look at everything' in a particular way."

The result can be shown by noting that the target variables and the shocks are exogenous in this case. The unknowns then become any indicator and the elements of the matrix A relating its movement to the excess demands in the T markets. There are then sufficiently numerous unknowns to guarantee a solution. This provides an interesting comparison with Brainard's [1967] conclusion that all markets of intervention should be employed in a stochastic model. That result is dependent, as this paper demonstrates, upon the precise form of the model. The result derived here, that all indicators should be monitored, is likely to have much wider application.

The demonstration can be described diagrammatically as follows. Picture the equations in \((a_{1,N-1}, a_{1,N})\) space. The denominators of these expressions assure that the \(a_{1,N-1}\) locus has a vertical asymptote at \(a_{1,N} = -E_1/D_1\) and
the \( a_{1,N} \) locus has a horizontal asymptote at \( a_{1,N-1} = \frac{-E_2}{D_2} \). The two expressions asymptote to the same line as \( a_{1,N-1} \) and \( a_{1,N} \) become very large. These loci each have two branches. One of the loci has branches whose asymptotes form an angle of less than \( \frac{\pi}{2} \) radians. For the other locus the branches have asymptotes which form an angle of greater than \( \frac{\pi}{2} \) radians. Viewing the graph so that each locus is a function of the variable on the abscissa, the slope of one branch of one locus is everywhere (for the same value of the abscissa variable) steeper than the other locus.

20 The "assignment problem," as the reader is aware, arose in the context of a disequilibrium model of macroeconomic stabilization. The problem is to design market intervention schemes based on differential control techniques so as to achieve certain targets. The present model differs from this in many regards. Nonetheless, as the text shows, an analogous problem arises in the present framework.

21 This aspect of the Mundell [1962] analysis has been criticized by Cooper [1968] and Roper [1972].
REFERENCES


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