Risk and Market Structure

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ABSTRACT

In a model of monopolistic competition under uncertainty, we explore the inter-relationship between risk and market structure (i.e., firm size, number of firms, and market power). Our results indicate that increasing uncertainty decreases the number of firms but has an ambiguous effect on firm sizes. Next, the Galbraith-Caves hypothesis that the firm's risk avoidance behavior increases with market power is shown to be, in general, not true. Thus, for instance, a monopolist may exhibit less risk avoidance compared to a competitive firm. Finally, a variety of circumstances are found to sufficiently depress the monopolist's risk such that the monopoly output exceeds that of competition—an interesting reversal of the prediction of the certainty model.
RISK AND MARKET STRUCTURE

I. INTRODUCTION

The purpose of this paper is to present a model of monopolistic competition under uncertainty and to analyze within such a model, the inter-relationship between risk and market structure. The market structure is taken here to consist of a list of elements like firm size, the number of firms in the industry, and degree of market power measured by both the Lerner measure of monopoly power and the measure of conjectural variation.\(^1\)

In particular, the issues that are raised are as follows. First, how does uncertainty affect the number of firms in an industry and their sizes? Second, how does the degree of market power affect the number of firms and their sizes? Third, what is the effect of market power on the risk avoidance of individual firms? Raising this issue, in fact, questions the validity of the Galbraith-Caves hypothesis which proposes that risk avoidance increases with market power.\(^2\) Taken seriously, the Galbraith-Caves hypothesis implies that a monopolist (relative to a competitive firm) would exhibit greater risk avoidance. As it turns out, this hypothesis is, in general, not true. Its possible invalidation raises a further interesting issue, namely, if risk avoidance of a monopolist (relative to a competitive firm) can be shown to be lower, then it stands to reason that monopoly output may in fact exceed that of a competitive industry!—a reverse of the prediction of a certainty model. We show that in a variety of circumstances, the monopoly risk could be sufficiently reduced such that the monopoly output exceed that of a competition.

The paper is organized as follows. In Section II, we present a model of monopolistic competition in an environment of demand uncertainty. In
Section III, we analyze the effects of uncertainty on the number of firms and their sizes. In Section IV, we pursue the question of how the degree of market power affects the risk avoidance behavior of firms. In Section V, we demonstrate the sufficient conditions that contribute to the interesting result of a greater (relative to a competitive industry) monopoly output. Finally, a conclusion is presented in Section VI.

II. A MODEL OF MONOPOLISTIC COMPETITION UNDER UNCERTAINTY

Consider a random market demand function

\[ p = p(q, \varepsilon), \quad \frac{\partial p}{\partial q} < 0, \quad \frac{\partial p}{\partial \varepsilon} > 0, \]

where \( p \) is price; \( \varepsilon \) is a random variable with probability density, \( d\Psi(\varepsilon) \);

\[ q = \sum_{i=1}^{N} q_i \]

is the aggregate industry output and \( q_i \) \((i=1, \ldots, N)\) is the output of the \( i \)th firm in the industry. We assume all firms to be identical in that they have an identical increasing concave utility function, \( U(\cdot) \), a cost function, \( C(\cdot) \), and a profit function, \( \pi(q_i) = p(q, \varepsilon) \cdot q_i - C(q_i) \). These assumptions imply \( q = Nq_i \).

As a simplification, we assume that the probability density function, \( d\Psi(\varepsilon) \), is sufficiently concentrated and compact so that we can approximate the expected utility function as

\[ E[U(\pi)] = U(E[\pi] - \frac{1}{2} R(E[\pi]) q_i^2 \sigma^2) \]

where \( E \) is an expectations operator; \( E[\pi] \) is the firm's expected profit; \( R(\cdot) = \frac{-U''(\cdot)}{U'(\cdot)} \) is the Arrow-Pratt measure of absolute risk aversion which is assumed to be decreasing in its argument so that \( R'(\cdot) < 0; \sigma^2 \) is the variance of the price level; and \( \frac{1}{2} R(E[\pi]) q_i^2 \sigma^2 \) is the risk premium. Likewise, as can be seen from (1), the variance of \( p \) is not constant but some function of \( q \). We,
therefore, write $\sigma^2 = \sigma^2(q, s^2)$ with $\frac{\partial \sigma^2}{\partial s^2} > 0$, where $s^2$ is the variance of the random variable $s$.

In the short run equilibrium, the number of firms, say $N$, is given, and the equilibrium firm size, $q_1$, is derived simply by maximizing expected utility (3). In the long run, however, where there is free entry, the number of firms and the size of each firm are endogenously determined as follows. Firstly, free entry leads to zero expected utility, $E[U(n)] = 0$, which after using (3) and the assumption that $U(0) = 0$ implies

\begin{equation}
\Delta^0 = E[n] - \frac{1}{2} R(E[n]) q_1^2 \sigma^2 = 0 \quad \forall i.
\end{equation}

This simply states that in the long run equilibrium, the expected profit of each firm must be equal to the risk premium. Secondly, in equilibrium the output, $q_1$, of each firm is optimally chosen to maximize expected utility (3), which yields after using (4) and some manipulation, the first order necessary condition

\begin{equation}
\Delta \equiv \{E[p] + \theta_i E[\delta]\} - \{C'(q_1) + c(q_1, q)\} = 0 \quad \forall i.
\end{equation}

The second order sufficient condition, of course, requires the partial derivative of $\Delta$,

\begin{equation}
\Delta_{q_1} < 0 \quad \forall i.
\end{equation}

The systems of equations (4) and (5), therefore, determine the number of firms, $N$, and the size of each firm, $q_1$.

Before proceeding to examine the implications of the model, we need to define the variables used in first order conditions (5). Firstly,

\begin{equation}
\delta = \frac{\partial p}{\partial q} q < 0.
\end{equation}

Secondly,

\begin{equation}
\theta_i = \frac{\partial q}{\partial q_i} q.
\end{equation}
is the ex ante or perceived elasticity of conjectural variation. For the sake of simplicity, we assume the conjectural variation, \( \frac{\partial q}{\partial q_i} = \beta \), is constant, so that the elasticity of conjectural variation is proportional to the firm's market share, i.e.,

\[
\theta_i = \beta \frac{q_i}{q} = \beta/N.
\]

(8)

Note that in the special case of Cournot behavior, \( \beta = 1 \) and the conjectural elasticity, \( \theta_i = q_i/q = \frac{1}{N} \), the market share. Furthermore, under perfectly competitive conditions, \( \theta_i = 0 \), and under pure monopoly, \( \theta_i = 1 \). Thirdly,

\[
\alpha(q_i, q) = \frac{R(E[\eta]) q_i [\sigma^2 + \theta_i \text{Cov}(\delta, p)]}{1 + \eta}
\]

(9)

is the firm's marginal risk premium where

\[
\eta = -\frac{R'(E[\eta]) E[\eta]}{R(E[\eta])}
\]

(10)

is the elasticity of absolute risk aversion, and \( \text{Cov}(\delta, p) \) is the covariance between \( \delta \) and \( p \). It is important to note that the marginal risk premium, \( \alpha \), is a function of both \( q_i \) and \( q \) so that changes in industry output, \( q \), will shift \( \alpha \). Furthermore, the shape of the \( \alpha \) function is not necessarily restricted by the second order condition (6) so that \( \alpha \) can be a general function of \( q_i \) for any given \( q \). Only in the special case of constant absolute risk aversion (in which case \( R'(\cdot) = 0 \) implying \( \eta = 0 \)) and a demand function which is additive in the random variable, \( \epsilon \), (in which case \( \text{Cov}(\delta, p) = 0 \)) will \( \alpha \) become a linear function of \( q_i \).

With the above definitions, the optimality conditions (5), therefore, say that each firm equates its perceived expected marginal revenue (the first L.H.S. term in (5)) with the effective marginal cost (the second L.H.S. term in (5)) made up of the marginal production cost, \( C'(q_i) \), plus the marginal risk premium, \( \alpha \). If the firm is risk neutral, then \( \alpha = 0 \), and the firm merely
equates perceived expected marginal revenue with marginal production cost. If in addition to risk neutrality, the firm is perfectly competitive, then the firm merely equates expected price with marginal production cost.

III. THE EFFECT OF UNCERTAINTY ON NUMBER AND SIZE OF FIRMS

Using the equilibrium conditions (4) and (5) which determine the number, \( N \), and size, \( q_1 \), of firms, we now examine how these variables are affected by changing uncertainty. Differentiating (5) and (4) respectively with respect to \( s^2 \) yields

\[
\begin{bmatrix}
\Delta_{q_1} & \Delta_N \\
\Delta_{q_1}^o & \Delta_N^o
\end{bmatrix}
\begin{bmatrix}
\frac{dq_1}{ds^2} \\
\frac{dN}{ds^2}
\end{bmatrix}
= \begin{bmatrix}
\frac{Rq_1}{1+\eta} \frac{\partial \sigma}{\partial s^2} \\
\frac{1}{2} Rq_1^2 \frac{\partial \sigma^2}{\partial s^2}
\end{bmatrix}
\]

where \( \Delta_{q_1}, \Delta_N \) and \( \Delta_{q_1}^o, \Delta_N^o \) are the partial derivatives of \( \Delta \) and \( \Delta^o \) (defined by (5) and (4)) with respect to the corresponding variables. The solution to (11) gives us the effects on \( q_1 \) and \( N \) of an expected utility preserving increase in uncertainty, i.e., holding expected utility fixed at zero, its equilibrium level.

Applying the Routh theorem, the stability of the equilibrium system requires

\[
(12) \quad -(\Delta_{q_1} + \Delta_N^o) > 0
\]

and

\[
(13) \quad -(\Delta_{q_1} + \Delta_N^o) \Delta^* > 0
\]

where

\[
\Delta^* = \Delta_{q_1} \Delta_N^o - \Delta_{q_1}^o \Delta_N
\]

is the determinant of the L.H.S. matrix of (11). Now note that from (4) and (5),
\[ \Delta_0 = \Delta = 0, \text{ and from the second order condition (6), } \Delta q_1 < 0. \text{ Thus, from (12) and (13), the stability of the equilibrium system implies} \]

\[ \Delta^* > 0, \quad \Delta_N^0 < 0, \quad \Delta q_1 < 0. \]

Given these conditions, the comparative static results are as follows:

\[ \frac{dq_1}{ds} = R q_1 \frac{\partial^2}{\partial s^2} \left[ \frac{\Delta_N^0}{1+\eta} - \frac{1}{2} q_1 \Delta_N^0 \right] / \Delta^* \]

is unsigned since sign \( \Delta_N \) is ambiguous. \(^7\)

\[ \frac{dN}{ds} = \frac{1}{2} R q_1^2 \frac{\partial^2}{\partial s^2} \Delta q_1 / \Delta^* < 0 \]

since \( \frac{\partial^2}{\partial s^2} > 0 \). Thus we conclude that in a non-competitive market with a constant conjectural variation, an increase in uncertainty reduces the number of firms, but the effect on the size of firms is ambiguous. It can also be readily verified that the same conclusion applies to a perfectly competitive model.

IV. THE EFFECT OF MARKET POWER ON RISK BEHAVIOR

Another interesting question that issues from the model is how the degree of market power affects the firm's risk avoidance behavior. To appropriately define the concept of market power, we rewrite the first order conditions (5) as

\[ \Delta = E[p] \{1 - \theta_1 \gamma\} - \{C'(q_1) + \alpha(q_1, q)\} = 0 \quad \forall q \]

where \( \gamma \) is the Lerner measure of monopoly power \(^8\) which is the inverse of the elasticity of expected demand and \( \theta_1 \) is the elasticity of conjectural variation previously defined. The measure of the \( i^{th} \) firm's market power is therefore \( \theta_1 \gamma \). In what follows, we assume that the elasticity of expected demand and, therefore, \( \gamma \), is a constant parameter. A change in market power can thus be
affected by changing either $\gamma$, the Lerner measure of monopoly power, or changing $\theta_i$ which can be affected by changing $\beta$, the conjectural variation parameter.

As it turns out, the effects of both $\gamma$ and $\beta$ on the firm size and the risk avoidance behavior of firms are ambiguous. A change in $\beta$ has, however, a known effect on the number of firms in the industry, whereas a change in $\gamma$ does not.

Differentiating the system of equations (5) and (4) with respect to $\gamma$ yields, after noting $\Delta_{q_i}^o = 0$,

\[
\begin{bmatrix}
\Delta_{q_i} & \Delta_N \\
0 & \Delta_N^o
\end{bmatrix}
\begin{bmatrix}
\frac{dq_i}{d\gamma} \\
\frac{dN}{d\gamma}
\end{bmatrix}
= 
\begin{bmatrix}
-\Delta_Y \\
-\Delta_Y^o
\end{bmatrix}
\]

(18)

The effects of $\gamma$ on $N$ and $q_i$ are

\[
\frac{dq_i}{d\gamma} = \frac{-\Delta_Y \Delta_N^o + \Delta_N \Delta_Y^o}{\Delta^*}
\]

(19)

and

\[
\frac{dN}{d\gamma} = \frac{-\Delta_{q_i} \Delta_Y^o}{\Delta^*}
\]

(20)

Using the stability conditions defined in (14), $\Delta^* > 0$, $\Delta_{q_i} < 0$ and $\Delta_N^o < 0$. However, it is easily verified that $\Delta_Y$ and $\Delta_Y^o$ are in general unsigned. Thus it is impossible to predict the effects of a change in the Lerner measure of monopoly power, $\gamma$, on the number and size of firms.

The effect of a change in $\gamma$ on risk avoidance behavior can now be examined by determining the effect of $\gamma$ on the marginal risk premium, $\alpha$. Note that $\alpha$ is an appropriate measure of the effect of uncertainty on the firm's risk avoidance behavior. For example, if the firm is risk neutral,
then \( \alpha = 0 \), and the firm shows no risk avoidance behavior. As the marginal risk premium \( \alpha \) increases, the firm's risk avoidance also increases. From the definition of \( \alpha \) in (9), differentiating it with respect to \( \gamma \) yields

\[
\frac{d\alpha}{d\gamma} = \frac{\partial \alpha}{\partial \gamma} + \frac{\partial \alpha}{\partial q_i} \frac{dq_i}{d\gamma} + \frac{\partial \alpha}{\partial q} \frac{dN}{d\gamma}.
\]

Now since the effects of \( \gamma \) on \( q_i \) and \( N \) are ambiguous as shown by (19) and (20), the effect of \( \gamma \) on \( \alpha \) is, not surprisingly, also ambiguous.

For completeness of discussion, we now analyze the effects of a change in the conjectural variation, \( \beta \), on the number, size and risk avoidance behavior of firms. Analogous to the preceding analysis, differentiating the system (4) and (5) with respect to \( \beta \) yields the following comparative static results,

\[
\frac{dq_i}{d\beta} = -\frac{\Delta_{\beta} \Delta_{N} + \Delta_{N} \Delta_{\beta}}{\Delta^*} \cdot
\]

\[
\frac{dN}{d\beta} = -\frac{\Delta_{q_i} \Delta_{\beta}}{\Delta^*} = 0.
\]

It can be easily verified that \( \Delta_{\beta} = 0 \) and hence \( \frac{dN}{d\beta} = 0 \) but \( \Delta_{\beta} \) cannot be unambiguously signed and hence sign \( \frac{dq_i}{d\beta} \) is ambiguous. It therefore follows again that the effect of \( \beta \) on risk avoidance, given by

\[
\frac{d\alpha}{d\beta} = \frac{\partial \alpha}{\partial \beta} + \frac{\partial \alpha}{\partial q_i} \frac{dq_i}{d\beta} + \frac{\partial \alpha}{\partial q} \frac{dN}{d\beta} = \frac{\partial \alpha}{\partial \beta} + \frac{\partial \alpha}{\partial q_i} \frac{dq_i}{d\beta}.
\]

is ambiguous because while \( \frac{dN}{d\beta} = 0 \), sign \( \frac{dq_i}{d\beta} \) is ambiguous.

We conclude in this section that in general, the effect of market power on risk avoidance is ambiguous. The Galbraith-Caves hypothesis that risk avoidance increases with market power is, in general, not true. At best, it is a very weak conjecture requiring perhaps very stringent restrictions. For example, even allowing a constant absolute risk aversion (so that \( \eta = 0 \)) and a demand function with an additive random term (so that \( \sigma^2 \) is a constant and
Cov(\delta, \ p) = 0), it is still impossible to unambiguously generate the Galbraith-Caves hypothesis. A direct consequence of this is that a monopolist may, in fact, exhibit less risk avoidance relative to a competitive firm.

As is shown in the next section, it is precisely the ambiguity of the effect of market power on risk avoidance that raises an interesting implication regarding the comparative effect of uncertainty on monopoly and competitive equilibrium output. An immediate corollary of this ambiguity result is that the standard prediction of the certainty model that output (price) of monopoly is always less (greater) than that of competitive equilibrium does not carry over to an uncertainty model. There exists a variety of circumstances, all operating toward minimizing the risk of a monopolist, that may result in a monopoly output (expected price) greater (smaller) than that of competitive equilibrium. The following section precisely analyzes this issue.

V. MONOPOLY VERSUS COMPETITION UNDER UNCERTAINTY

A. Competition

In a perfectly competitive industry, the long-run equilibrium conditions are given by (5) and (4) but with \( \theta_i = 0 \) for all \( i \). For the purpose of this section, we need only to rewrite the first order conditions (5) as

\[
(25) \quad E[p(q_c, s)] = c'(q_i) + \alpha_c(q_i, q_c) \quad \forall i
\]

where

\[
(26) \quad \alpha_c(q_i, q_c) = R(E[\pi]) q_i^2 c_c^2/(1 + \eta_c)
\]

is the \( i \)th competitive firm's marginal risk premium; \( q_c = Nq_i \) is the competitive equilibrium output; \( N \) is the equilibrium number of firms; \( c_c^2 \) is the variance of price facing a competitive firm in equilibrium; and \( \eta_c \) is the competitive
firm's elasticity of absolute risk aversion.

Now, in order that the comparison between monopoly and competitive equilibrium is meaningful, it is necessary that the cost function of the monopoly situation be appropriately defined. In the certainty model, the prediction that monopoly output is always less than that of competitive equilibrium stems from the crucial assumption that the marginal cost curve of the monopolist is identically the horizontal summation of the marginal cost curves of the competitive firms in equilibrium (or the aggregate supply of the competitive industry). In what follows, a similar definition is used.

The individual firm's cost function has been previously defined to be $C(q_i)$ for all $i$. Since all firms are assumed identical, so that the aggregate output $q_c = Nq_i$, the total cost of the competitive industry producing the aggregate output $q_c$ and with each firm producing $q_i$ can be written as

$$C_c(q_c) = NC(q_i).$$

Differentiating the above with respect to $q_i$ yields

$$C'_c(q_c) = C'(q_i)$$

where $C'(q_i)$ is the marginal cost of an individual firm producing $q_i$ whereas $C'_c(q_c)$ is the 'horizontal summation' of the firms' marginal costs. Note that (28) implies $C''(q_c) = \frac{1}{N} C''(q_i)$ which says that the slope of the 'horizontal summation' of individual marginal costs is $\frac{1}{N}$ of the slope of the marginal cost of each individual firm. With this definition, we now proceed to analyze the monopoly problem.
B. Monopoly

Let \( q_m \) be the monopoly equilibrium output; \( \pi_m \) the monopoly profit and \( C_c(q_m) \) the monopoly cost. The monopoly equilibrium, of course, is characterized by the first order condition (5) with \( \theta_i \) replaced by \( \theta_i = 1 \). It is, therefore, rewritten as

\[
\Delta^m = E[p(q_m, \varepsilon)] + E[\delta(q_m, \varepsilon)] - \{C_c'(q_m) + \alpha_m(q_m)\} = 0
\]

where

\[
\alpha_m(q_m) = R(E[\pi_m]) q_m [\sigma_m^2 + Cov(\delta(q_m, \varepsilon), p(q_m, \varepsilon))]/(1 + \eta_m)
\]

is the monopolist's marginal risk premium; \( \eta_m \) is the monopolist's elasticity of absolute risk aversion; and \( \sigma_m^2 \) is the variance of price faced by the monopolist in equilibrium. The second order condition for the monopoly optimum requires the derivative of \( \Delta^m \),

\[
\Delta^m_{q_m} < 0.
\]

C. Monopoly Versus Competition

The comparison between monopoly and competition is now conveniently facilitated by rewriting (29) and (25) respectively as

\[
E[p(q_m, \varepsilon)] = - E[\delta(q_m, \varepsilon)] + C_c'(q_m) + \alpha_m(q_m)
\]

and

\[
E[p(q_c, \varepsilon)] = C_c'(q_c) + \alpha_c(q_c, q_c)
\]

since from (28), \( C'(q_i) = C'_c(q_c) \). We shall now attempt to demonstrate that there exists a variety of circumstances, all working to minimize the risk of the monopolist, which are sufficient to yield a monopoly output (expected price) greater (smaller) than that of a competitive industry. To do this, observe that the optimal monopoly equilibrium condition (32) holds only for equilibrium output \( q_m^* \). For any other output \( q_m^0 \neq q_m \) (where \( q_m^0 \) is in the
neighbourhood of \( q_m \), the second order condition (31) implies

\[
E[p(q^0_m, \epsilon)] > E[\delta(q^0_m, \epsilon)] + C'_c(q^0_m) + \alpha_m(q^0_m) \text{ iff } q^0_m > q_m
\]

Now if we let \( q_c = q^0_m \), then

\[
E[p(q_c, \epsilon)] > E[\delta(q_c, \epsilon)] + C'_c(q_c) + \alpha_m(q_c) \text{ iff } q_c > q_m
\]

and using (33),

\[
\alpha_c(q_i, q_c) > E[\delta(q_c, \epsilon)] + \alpha_m(q_c) \text{ iff } q_c < q_m
\]

In what follows, our motive is to present sufficient conditions such that

\[
\alpha_c(q_i, q_c) - \alpha_m(q_c) + E[\delta(q_c, \epsilon)] > 0 \Rightarrow q_c < q_m. \tag{34}
\]

Since \( E[\delta] < 0 \), demonstrating the above is equivalent to requiring the marginal risk premium (or avoidance) of the monopolist to be sufficiently smaller than that of a competitive firm—a requirement which contradicts the Galbraith-Caves hypothesis.

Using the definitions of \( \alpha_c(q_i, q_c) \) and \( \alpha_m(q_m) \) and substituting \( q_c \) for \( q_m \), condition (34) is rewritten as

\[
q_i \frac{R(E[\pi_i]) \sigma_{C_c}^2}{1 + \eta_c} - q_c \frac{R(E[\pi_m(q_c)])}{1 + \eta_m} \left( c^2 + \text{Cov}(\delta, p) \right) + E[\delta] > 0 \Rightarrow q_c < q_m. \tag{35}
\]

To evaluate the above, a few more details need to be considered. First, noting that \( q_c = Nq_i \) and \( C_c(q_c) = NC(q_i) \),

\[
E[\pi_m(q_c)] = E[p(q_c, \epsilon)] q_c - C_c(q_c)
\]

\[
= N \left[ E[p(q_c, \epsilon)] q_i - C(q_i) \right]
\]

\[
= N E[\pi_i]
\]

i.e., the expected profit of a monopolist is \( N \) times that of a competitive firm.

Second, we assume \( R" = 0 \), so that
(37) \[ R(N \, E[\pi(q_i)]) = R(E[\pi(q_i)]) + R'(E[\pi(q_i)]) \cdot (N-1) \cdot E[\pi(q_i)] \]

This assumption is not too restrictive since \( R^p \) involves the fourth derivative of the utility function on which neither intuition nor economic theory has much to say. Third, using (36) and (37), it is easily verified that

(38) \[ \eta_m = N \cdot \eta_c / [1 + \eta_c (1-N)] \]

Note that \( \eta_m = \eta_c \) when \( N = 1 \) and \( \eta_m \) increases with both \( N \) and \( \eta_c \). Based on economic intuition and theoretical grounds Arrow (1965) proposed that the restrictions placed on attitude to risk should satisfy the conditions that absolute risk aversion be decreasing but the relative risk aversion be increasing. These restrictions imply the elasticities of absolute risk aversion for the monopolist and the competitive firm should satisfy

(39) \[ 0 < \eta_m < 1 \Rightarrow 0 < \eta_c < \frac{1}{2N-1} \]

Using (36), (37) and (38), condition (35) can now be rewritten as

(40) \[ \frac{q_i \cdot R(E[\pi(q_i)])}{1 + \eta_c} \cdot \left[ \sigma_c^2 - S(N)(\sigma_m^2 + \text{Cov}(\delta, p)) \right] + E[\delta] > 0 \Rightarrow q_c < q_m \]

where \( S(N) \equiv N(1 + \eta_c)^2 / (1 + \eta_m)^2 \). Observe that \( S(1) = 1 \), and \( S'(N) = (1 + \eta_c)(1 - \eta_c(3N-1))/(1 + \eta_m) < 0 \) given restriction (39).

In the case of risk neutrality where \( R = 0 \) or in the absence of uncertainty where \( \sigma_c^2 = \sigma_m^2 = \text{Cov}(\delta, p) = 0 \), it is easily verified that condition (40) can never hold since \( E[\delta] < 0 \). However, under uncertainty and risk aversion, there exists at least three effects, which involve the minimization of terms like \( S(N) \), \( \text{Cov}(\delta, p) \) and \( \sigma_m^2 \), that may sufficiently reduce the risk of the monopolist such that condition (40) will hold and hence yielding a greater monopoly output relative to that of a competitive industry.
The first effect is due to the risk reducing scale factor $S(N)$ which arises from the assumption of decreasing absolute risk aversion. When $N = 1$, $S(1) = 1$ which says that the risk reducing scale factor is non-operative and hence the monopolist's attitude to risk is identical to that of a competitive firm. However, $S'(N) < 0$, which says that as $N$ increases, the monopolist's risk preference is scaled down. The intuition of this effect is as follows. As the number of firms in a competitive industry increases, so does the difference in expected profit between the monopolist and a competitive firm. Thus, because of decreasing absolute risk aversion, the monopolist risk attitude relative to that of a competitive firm is scaled downward by $S(N)$.

The second effect is generated by the sign of Cov($\delta, p$) which describes the covariance between the price level and the slope of the demand curve. In general, Cov($\delta, p$) could take any sign depending on the specification of the random demand function. The tendency of our condition (40) to hold is of course intensified by a negative Cov($\delta, p$). In fact, a simple example of a demand function in which the random term, $\epsilon$, enters multiplicatively in the following manner, $p(q, \epsilon) = p(q)\epsilon$ with $\frac{\partial p}{\partial q} < 0$, would give us Cov($\delta, p$) < 0.

The third effect is due to our conjecture that a monopolist is at least informationally superior to a competitive firm. In light of condition (40), this is equivalent to arguing that the unexplained price variance $\sigma_m^2 \leq \sigma_c^2$ for any given output level. The reasoning is as follows. A competitive firm has information only on its past market prices whose variations were unaffected by the firm's individual output levels since an individual's competitive firm is assumed to be sufficiently small. Let the unexplained price variance estimated by a competitive firm be $\sigma_c^2$. The monopolist, however, has an informational advantage in that its past output levels were observed to affect
prices. This relationship could be used to predict demand. Thus, part of the total price variance can be explained precisely due to the relationship between output levels and prices, leaving an unexplained variance $\sigma_m^2$. Because of the monopolist's informational superiority, it is, in general, true that $\sigma_m^2 \leq \sigma_c^2$. It can, of course, be argued that such informational advantage cannot persist since information can generally be acquired even by a competitive firm. This argument, however, does not nullify our conclusion that $\sigma_m^2 \leq \sigma_c^2$ because acquiring information is costly; and at least in our example where the crucial information is the relationship between output levels and prices, the information acquiring cost of a competitive firm is at least as high as that of a monopolist.

VI. CONCLUSION

In summary, our results indicate that the prediction on the inter-relationship between risk and market structure, is, in general, not so straightforward. More specifically, we have first shown that increasing demand uncertainty decreases the number of firms but the effect on firm size is ambiguous. Second, we have also found that increasing market power, whether measured by Lerner's monopoly power measure ($\gamma$) or the conjectural variation parameter ($\beta$), has an ambiguous effect on firm size. The number of firms in the industry, however, is unaffected by $\beta$ but is ambiguously affected by $\gamma$. Third, the effect of market power on the risk avoidance is, in general, ambiguous. The Galbraith-Caves hypothesis that risk avoidance increases with market power, is, in general, not true. Had the Galbraith-Caves hypothesis been true, it would suggest that relative to a competitive firm, a monopolist would choose to avoid more risk, thus, strengthening the standard prediction that a competitive industry's output would always exceed
that of a monopolist. Having shown that the Galbraith-Caves hypothesis to be at best a very weak proposition, we finally raise the interesting implication that the monopoly output may exceed that of a competitive industry.

In fact, there exists at least three reasons, all working toward minimizing the monopoly risk, which may yield the interesting result of a greater (relative to competition) monopoly output. The first effect is the economies of scale in risk arising from the decreasing absolute risk aversion assumption. The second effect is the negativeness of covariance between the price level and the slope of the demand curve. The last effect is the monopolist's informational advantage over a competitive firm in predicting the market demand.
1. The Lerner measure of monopoly power is defined in Section IV and the measure of conjectural variation is defined in Section II.

2. See Galbraith (1967) and Caves (1970). See also Edwards and Heggestad (1973) for a formalization and the test of the Galbraith-Caves hypothesis.

3. See Pratt (1964). See also Samuelson (1970) for a defense of local approximations when the probability distribution is compact and concentrated.

4. From now on, all elasticity measures are defined in terms of absolute values.

5. See Diamond and Stiglitz (1974) for a detailed discussion on the question of the appropriate measure of increasing risk when there is risk aversion.


7. \[
\Delta N = \frac{\partial \bar{E}(p)}{\partial q} + \theta \frac{\partial \bar{E}(\delta)}{\partial q} + \frac{\partial \bar{\epsilon}}{\partial q}
\]

8. See Lerner (1934).

9. \[
\Delta^o \gamma = (1+\eta) \frac{\partial \bar{E}(\pi)}{\partial \gamma} - \frac{\bar{E}(\pi)}{\sigma^2} \frac{\partial^2}{\partial \gamma} \text{ and } \Delta \gamma = \frac{\partial \bar{E}(p)}{\partial \gamma} + \theta \frac{\partial \bar{E}(\delta)}{\partial \gamma} + \frac{\partial \bar{\epsilon}}{\partial \gamma}.
\]

10. Since \( \beta \) does not appear in (4) it is clear that \( \Delta^o \beta = 0 \) and

\[
\Delta \beta = \frac{1}{\bar{N}} [E(\delta) + \beta q_\delta \text{ cov}(\delta,p)/(1+\eta)] \text{ which cannot be signed since } \text{ cov}(\delta,p) \text{ cannot be signed.}
\]

11. \( \sigma_m^2 = \sigma^2(q_m^2) \) and \( \sigma_c^2 = \sigma^2(q_c^2) \). In general, they differ if \( q_m \neq q_c \). Furthermore, even if \( q_m = q_c \), it is still possible for \( \sigma_m^2 \) to differ
from $\sigma^2_c$ if it can be argued that the variance of the error term, $\sigma^2$, is not identical for the monopolist and competitive firm for some extraneous reasons like asymmetrical information.

12 Let $R(x)$ be the absolute risk aversion. Then the relative risk aversion $R^*(x) \equiv R(x)/x$. Now decreasing absolute risk aversion means $R'(x) < 0$, and increasing relative risk aversion means $0 < R^*(x) = R'(x)x + R(x) = 1 - \frac{R'(x)x}{R(x)} \equiv \eta > 0$. 
References


