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ABSTRACT

An optimal contract design problem is considered when contracts must be incomplete and simple. In those situations contracts may specify not only outcomes, but an allocation of the ex post decision rights, since there would be gains from an ex post revision or renegotiation. The objective of this paper is to investigate the extent to which constrained revisions can mitigate inefficiencies resulting from contractual incompleteness.

An optimal contract is characterized in two cases. First, when a contract is being used to facilitate trade between two risk neutral parties who must make relationship-specific investments, it is possible to implement the first best outcomes by a simple contract. Second, when a contract is being used to share risk, and there are no specific investments, it is generally not possible to implement the first best outcomes. When one party is risk neutral, however, it is possible to implement the first best outcomes by assigning all the ex post decision rights to that party.

JEL Classification : 026, 022

Key Words : incomplete contracts, renegotiation, risk sharing, specific investment
1. INTRODUCTION

Most contracts we observe in many trading relationships are incomplete and simple because of transaction costs and bounded rationality. When there are no binding constraints on the feasible contracts, contract terms are about allocations or outcomes (e.g., quantity, quality, compensation payment, effort, investment etc.). When contracts must be incomplete, however, the contracting parties may introduce other forms of contracts that can be used as substitutes for complete contracts.

One natural way of doing this is to introduce the possibility of renegotiation and/or revision of initial agreements as new information becomes available. Since the initial agreement on allocations is generally ex post inefficient, the parties can be made better off by agreeing to modify it in a mutually beneficial way. In order to determine a new allocation and to specify a clear division of the surplus from the revision, the parties need to include some revision or bargaining scheme in their contract. The parties can state explicitly each parties' ex post bargaining positions in the contract as a contractual term or they can implicitly embed these into their organizational mode.

The objective of this paper is to investigate the extent to which renegotiation and/or revision can mitigate inefficiencies resulting from the contractual incompleteness. In particular, the paper focuses on the issue of whether simple contracts can induce efficient relationship-specific investment, since a principal function of a long-term contract is to facilitate trade between two parties who must make relationship-specific investment.

I consider a variable quantity procurement model in which both the buyer's benefit and the seller's cost are uncertain, and both parties make relationship-specific investments. The source of incompleteness is the fact that the enforcement agency, the court, has only limited ability to enforce contracts. Specifically, it is assumed that investments and random variables are not verifiable.
To capture simplicity, I assume that the initial contract consists of two provisions - one on an allocation (a specification of quantity and transfer payment) and one on the revision scheme. The feasible revision schemes are assumed to be take-it-or-leave-it offers made by one of the parties. The one-sided bargaining assumption reflects some notion of hierarchy, social norm, etc., which we might see in practice. Given any contract of this type, the allocation specified in the contract will be enforced by the court unless both parties agree to change it to another allocation according to the prespecified revision scheme. Since the initial allocation (i.e. the allocation specified in a contract) determines the status quo point of the ex post bargaining process, the parties can constrain the revision game by appropriately choosing the initial allocation.

My concern is under what environments this simple contractual scheme could implement first-best outcomes. The main result of this paper is that non-verifiability does not prevent efficient relation-specific investment if agents are risk neutral. Specifically, when the parties are risk neutral, efficient investments and outcomes can be induced by a contract that specifies a quantity and a payment together with an ex post revision via a take-it-or-leave-it offer, regardless of the identity of the party who makes such an offer. This result deserves several comments.

First, the mechanism involved in this result can be described as follows: make one party (say the seller) the residual claimant of the relationship, and then choose the initial allocation so as to give the other party (the buyer) the right incentives to invest; in particular the initial allocation will involve a positive level of trade so as to make it costly for the buyer to underinvest (here, the assumption of variable quantity plays a major role). Given that the buyer is induced to make the efficient choice of investment, the seller who is the residual claimant will also invest efficiently. The first-best investment decisions and outcomes will therefore be implemented.

Second, this result is in sharp contrast to Hart and Moore (1988) in which inefficient investment are obtained. In their paper, the parties to a contract are allowed to make an
agreement on the prices corresponding to 'no trade' and 'trade' denoted by $p_0$ and $p_1$ respectively. The objective of the parties is to specify $(p_0, p_1)$ (i.e., choose two allocations $(0, p_0)$ and $(1, p_1)$), and design a revision game which can involve ex post exchanges of messages.

The model in this paper differs in several aspects from theirs. First, trading mechanisms differ in that in their paper, trade occurs if and only if both parties are willing to do so, while in this paper initial agreement is enforced unless both parties agree to revise it. Since, in this paper, the court will enforce the initial allocation if requested to do so by at least one party, the allocation puts a strict restriction on the revision stage. Therefore, it could affect the parties' investment decisions more effectively than the allocation in Hart and Moore (1988) does. Furthermore, in this paper, the parties can trade a variable quantity of the good, while in Hart and Moore (1988) the choice of quantities are limited to trade or no-trade. Since the level of quantity in the initial allocation affects the investment decisions, the fact that a variable quantity can be chosen further helps the implementation of efficient investment.

My third comment on the main result is to clarify the relation of this work with two other views on the issue of long-term contract versus integration. Williamson (1985) argues that when parties make specific investments, integration is a preferred mode of organization since incomplete contracts are subject to contractual hazards (e.g., opportunism). My result says that even when the parties make specific investments, an incomplete contract could induce efficient investments with a suitable choice of an ex post decision structure. This is not inconsistent with Williamson's view since both emphasize the importance of ex post governance structure. Grossman and Hart (1986) maintain that when integration is properly interpreted (as the integrating party having all the ex post decision rights), integration induces inefficient investments, but the allocation of the ex post decision rights has efficiency consequences, under the assumption that no ex ante contracts can be written. This contrasts with my result that if the parties can write a simple initial contract, efficient investments can be induced, regardless of the allocation of the ex post decision rights.
In the remaining part of this paper, I addresses the risk-sharing issue. The major result of this analysis is that the simple renegotiation mechanism constructed in the paper does not generally implement the efficient level of risk-sharing, except in particular cases (without investment, efficient risk-sharing is achieved only if at least one party is risk-neutral; in the case with investment, efficient risk-sharing is obtained in situations where at least one party does not face uncertainty or does not invest in the relationship). The reason for the lack of optimality with risk aversion is clearly the restriction on the ex post revision game. Note that the allocation of the ex post decision rights has efficiency consequences even without specific investments.

This paper is also related to several works in the Law and Economics literature which consider the efficiency of the specific performance remedy when renegotiation is feasible. The efficiency implication of the remedy for the risk allocation and for the specific investments (reliance in legal terms) has not been completely analyzed.

The paper is organized as follows. In Section 2, the formal model is introduced, and the set of feasible contracts and optimal contracts are defined. Section 3 considers the case in which the risk neutral parties make specific investments. Section 4 considers optimal risk sharing problem with and without investments. Section 5 contains concluding remarks.

2. MODEL

2a. Model

The basic procurement model involves two parties: a buyer (sponsor or principal) and a seller (producer or agent). For most part of the analysis, parties are assumed to be risk neutral. The buyer tries to procure some goods which will be produced at a variable quantity by the seller.

Let \( q \in R_+ \) denote the quantity of the goods to be procured and \( t \in R \) denote the monetary transfer from the buyer to the seller. Let \( \omega \) denote the random variable reflecting the exogenous uncertainty which will affect both the benefit to the buyer of having the goods and the cost to the
seller of producing the goods. It is assumed that the parties have a common prior distribution $F(\omega)$ which has a continuous density function on its support $\Omega$.

The parties make relationship specific investments ex ante (i.e., before $\omega$ is resolved). Let $\beta \in R_+$ denote the ex ante investment taken by the buyer to increase the benefit to the buyer of having the goods. Let $h_b(\beta)$ denote the direct cost to the buyer of making $\beta$ units of investment. It is a reliance expenditure in legal terms. Similarly, let $\sigma \in R_+$ denote the ex ante investment taken by the seller to reduce the cost to the seller of producing the goods. Let $h_s(\sigma)$ denote the direct cost to the seller of making $\sigma$ units of investment. It can be regarded as disutility of effort in monetary terms. It is assumed that both $h_b(\beta)$ and $h_s(\sigma)$ are increasing at increasing rate.

Let $v(q,\omega,\beta)$ denote the gross monetary benefit to the buyer from procuring $q$ units of the goods when $\beta$ is the level of her investment taken and $\omega$ is realized. Let $c(q,\omega,\sigma)$ denote the gross monetary cost to the seller of producing $q$ units of the goods when $\sigma$ is the level of his investment and $\omega$ is realized. Given the initial allocation $(q,t)$, investments $(\beta,\sigma)$, and realizations of $\omega$, the buyer's net payoff is $v(q,\omega,\beta) - t - h_b(\beta)$, and the seller's net payoff is $t - c(q,\omega,\sigma) - h_s(\sigma)$. It is assumed that both $v(q,\omega,\beta)$ and $-c(q,\omega,\sigma)$ are increasing in each of its arguments at decreasing rate.

The whole structure, including the parties' utility functions, is assumed to be a common knowledge to the parties. The following assumptions are made throughout the paper.

Assumption 1. $v_{q\beta} > 0$, and $c_{q\sigma} < 0$.

Assumption 2. $v(0,\omega,\beta) = 0$, and $c(0,\omega,\sigma) = 0$.

Assumption 1 says that marginal benefit (cost) is increasing (decreasing) in investment. This is a key driving force in assuring that the choice of initial allocation could affect the investment decision of the party who will respond to the other party's take-it-or-leave-it offer. Assumption 2 says that the benefit to the buyer of having zero $q$ (or cost of producing zero $q$) does not
depend on the realization of uncertainty and the level of investment. Since there is no outside market for investments, this assumption suggests that the investment is relationship-specific.

The sequence of moves is illustrated in Figure 1.

< Figure 1 >

The parties at some initial date 0 design a long-term contract (The class of feasible contracts will be shortly described). Specific investments are made simultaneously and independently shortly after signing the contract, but before uncertainties are resolved. It is assumed that the level of investments are observable to both contracting parties right after they are made, but not verifiable to the court. Investments may be private efforts or it may be difficult to describe them in a verifiable way. It is also assumed that resolution of ω is observable (to the parties), but not verifiable (to the court). These assumptions rule out complete contingent contracts. It is further assumed that v and c are non-contractible.

Given these non-verifiability assumptions, the form of contracts on allocations will be in general simple. To focus on the role of revision schemes, only a specification of quantity and monetary transfer (i.e. (q^0,r^0)) is allowed and it is called the initial allocation. It is assumed that the initial allocation will be enforced by the court unless both parties agree to change it to another allocation. In this sense, the initial allocation is mutually binding. Since the initial allocation is generally ex post inefficient, the parties are willing to revise it into a new allocation in a mutually beneficial way. It is assumed that the parties cannot precommit not to renegotiate the initial allocation in a mutually beneficial way. The parties need to specify some revision rule or bargaining scheme to determine a new allocation and a division of the surplus from the revision.
There are many bargaining schemes the parties can conceive of, but the factors which make contracts on allocations incomplete may also constrain the enforceable bargaining schemes. In this paper only two special bargaining schemes - take-it-or-leave-it offers made by the buyer or the seller - are considered. These two schemes are very simple and analytically tractable. They are also useful in studying the issue of allocations of decision authority. The effects on the analysis of relaxing this simplicity of enforceable bargaining schemes are discussed in the concluding section.

Before defining the set of feasible contracts and optimal contracts, it is worthwhile to mention what kind of information the court has in the model. Recall that the court cannot observe $\beta$, $\sigma$, and $\omega$, and it cannot enforce complicated agreements and revision schemes. The court is assumed to be able to enforce the initial agreement on an allocation $(q^0, r^0)$ in the signed contract unless the parties voluntarily agree to replace this by another allocation. Hence, in a legal term, the court uses the specific performance remedy with a possibility of (voluntary) renegotiation.\(^5\)

The court is also assumed to be able to observe who has been assigned to have the ex post decision authority and to preserve the right of the party specified in the contract as the Stackelberg leader in the revision stage. Alternatively, if the allocation of the decision authority is too difficult to be enforced as an explicit contractual term, then it may be implicitly embedded in their organizational setting which is then supported by 'custom' or 'social norm'.\(^6\) It is assumed that in either way the parties can commit to a bargaining scheme they will use ex post.

Now the set of enforceable contracts can be defined. A contract is called enforceable if it has only two terms - an allocation and one of the take-it-or-leave-it offers. A typical enforceable contract has the form $( (q^0, r^0), i )$ for $i = b, s$. Depending on the bargaining scheme, there are two types of enforceable contracts which are defined up to the initial allocation. For each $i = s, b$, let the type $i$ contract denote the enforceable contract with the $i$-made take-it-or-leave-it offer as its revision scheme.
2b. **Optimal contracts**

The design of a Pareto optimal contract proceeds by maximizing one party's expected utility subject to the other party receiving a reservation utility level. Which party's utility level is taken as a constraint does not matter in the analysis. It is assumed that, as a Stackelberg leader, the buyer designs an incomplete contract to maximize her expected utility subject to the seller's ex ante participation constraint. Let $U^0$ denote the seller's ex ante reservation utility level which is determined by the ex ante market for contracts. It is assumed that the seller will sign a contract if he is indifferent between participating and not participating in the procurement relation determined by the contract.

The set of feasible contracts is defined as all enforceable contracts which satisfies the seller's ex ante participation constraint, i.e. which gives the seller the expected utility at least $U^0$. For each type of feasible contracts an appropriate initial allocation to maximize the buyer's expected utility can be calculated while taking into account the fact that it induces investments and it can be revised ex post. The optimal type $i$ contract is defined as a feasible contract with this maximizing initial allocation and $i$-led take-it-or-leave-it offer as its revision scheme.

Since the structure of model is symmetric, for concreteness, let us consider only the type $b$ contract - i.e., the buyer has all the ex post bargaining power. Let $\beta \in R_+$ and $\sigma \in R_+$ be given. For each $\omega$, let $(q(\beta, \sigma, \omega), r(\beta, \sigma, \omega))$ denote the final allocation (after the revision) from the type $b$ contract. The equilibrium investments $(\beta^e, \sigma^e)$ is defined as the Nash equilibrium of the subgame with payoff functions $(\int [v(q(\beta, \sigma, \omega), \omega, \beta) - h_b(\beta) dF(\omega), \int [t - c(q(\beta, \sigma, \omega), \omega, \sigma) - h_\sigma(\sigma) dF(\omega))].$

A contract is optimal if it is feasible and it maximizes the buyer's expected utility. Notice that an optimal contract in a given context may not induce first best outcomes. Consider the first best problem as a benchmark. If $\beta, \sigma,$ and $\omega$ were verifiable (and so contractible) and if there were no further constraints (e.g., simplicity requirement), then the buyer's contract design problem would be:
maximize \[ \int \left[ v(q, \omega, \beta) - t - h_b(\beta) \right] dF(\omega) \]
subject to \[ \int \left[ t - c(q, \omega, \sigma) - h_s(\sigma) \right] dF(\omega) \geq U^0 \]

For any given \((\beta, \sigma) \in R^2_+\), let \(\{(q^*(\beta, \sigma, \omega), t^*(\beta, \sigma, \omega) : \omega \in \Omega\}\) be the complete contingent solution of this first best problem conditional on \((\beta, \sigma)\). It is assumed that there are gains from the procurement for all states of nature so that \(q^*(\beta, \sigma, \omega) > 0\). Now the efficient investments \((\beta^*, \sigma^*)\) can be defined as follows.

\[
(\beta^*, \sigma^*) = \arg\max_{\beta, \sigma} \left[ v(q^*(\beta, \sigma, \omega, \beta) - c(q^*(\beta, \sigma, \omega, \sigma, \omega)) \right] dF(\omega) - h_b(\beta) - h_s(\sigma)
\]

The efficient investments maximize the difference between the total expected gain and the direct cost of investments. Since the parties have transferable money, it is a valid definition. It is assumed that both \(\beta^*\) and \(\sigma^*\) are positive. If \(\beta\) and \(\sigma\) were verifiable, then the parties would jointly choose \((\beta^*, \sigma^*)\) which maximizes the expected net gains from their relationship.

3. ANALYSIS

The objective of a contract in the procurement relation is to induce efficiencies in production (i.e., in the choice of \(q\)), and investments. Efficiency in production is first considered. Given any level of investments \((\beta, \sigma) \in R^2_+\), the ex post revision is conducted costlessly under symmetric information. Hence it can be expected that the final production or quantity is efficient conditional on investments \((\beta, \sigma)\) (i.e., given any \((\beta, \sigma)\), and each \(\omega\), the equilibrium quantity will be \(q^*(\beta, \sigma, \omega)\)). For otherwise the efficient production could give the parties bigger total gains and thus bigger shares regardless of the bargaining scheme. This suggests that there is no need for further revision or renegotiation. The problem of inducing efficient investments is now considered.
Proposition 1. Assume that both parties are risk neutral. Then,

(1) \((\beta^*, \sigma^*) = (\beta*, \sigma^*)\)

(2) The first-best outcome is obtained.

Proof. See the Appendix.

Proposition 1 says that with risk neutral parties, the simple contract (i.e., a choice of an allocation and ex post take-it-or-leave-it offer made by the buyer) can induce efficient investments, and thus can implement first-best outcomes. Note that since the structure is symmetric, there will be another contract with a take-it-or-leave-it offer made by the seller which induces the efficient investments.

The arguments behind Proposition 1 are as follows. Since the buyer (the party who has the decision authority) is the residual claimant of the surplus from the revision, she will choose the efficient level of her investment. To induce efficient investment by the seller (the party who delegates the decision authority), the initial allocation should be appropriately chosen since the initial allocation directly determines the payoff of the seller. Assumption 1 (on the sign of the cross derivative \(v_{\theta \beta}\)) guarantees that there exists a choice of quantity in the initial allocation which will induce the seller to make an efficient investment.

In the contract, the seller will choose his investment to minimize the expected cost of producing the prespecified quantity \(q^0\) in the initial allocation and the direct cost of investment, that is, \(\int [c(q^0, \omega, \sigma)] dF(\omega) + h_3(\sigma)\). Therefore the contract can specify \(q^0\) appropriately so that the seller has an incentive to choose the efficient investment \(\sigma^*\). Then they can set \(r^0\) so as to make the seller's ex ante participation constraint binding. In this way the type \(b\) contract can induce efficient investments and outcomes. Note that the variability of quantity plays an important role in making this argument.9

This result is in sharp contrast to Hart and Moore (1988) in which inefficient investment result is obtained. In their paper, the parties to a contract are allowed to make an agreement on the prices corresponding to 'no trade' and 'trade' denoted by \(p_0\) and \(p_1\) respectively. The
objective of the parties is to specify \( (p_0, p_1) \) (i.e., choose two allocations \((0, p_0)\) and \((1, p_1)\)), and design a revision game which can involve ex post exchanges of messages. The model in this paper differs in several aspects from theirs. First, trading mechanisms differ in that in their paper, trade occurs if and only if both parties are willing to do so, while in this paper initial agreement is enforced unless both parties agree to revise it. Since, in this paper, the court will enforce the initial allocation if requested to do so by at least one party, the allocation puts a strict restrictions on the revision stage. Therefore, it could affect the parties' investment decisions more effectively than the allocations in Hart and Moore (1988) do. Furthermore, in this paper, the parties can trade a variable quantity of the good, while in Hart and Moore (1988) the choice of quantities are limited to trade or no-trade. Since the level of quantity in the initial allocation affects the investment decisions, the fact that a variable quantity can be chosen further helps the implementation of efficient investment.

Rogerson (1984) argues, in a model with risk neutral parties, that when only one party makes an investment, that party should have all the ex post decision rights (i.e., all the ex post bargaining power) to induce an efficient investment. I show that even when the parties who does not invest has all the ex post decision rights, an efficient investment can be induced, in a variable quantity model, by an appropriate choice of the initial quantity requirement since the choice of investment by the investing party depends on the initially contracted quantity. Proposition 1 states that when both parties make investments, these two principles can be used to induce efficient investments.

Proposition 1 can also be interpreted as stating that the allocation of decision authority (or ownership structure) does not matter if both parties are risk neutral. This result contrasts to Grossman and Hart (1986), for they show that different allocation of decision authority matters for specific investments under the assumption that no ex ante contracts can be written. This contrasts with my result that if the parties can write a simple initial contract, efficient investments can be induced, regardless of the allocation of the ex post decision rights.
4. INCOMPLETE CONTRACTS AND RISK SHARING

In this section, risk sharing problem as well as inducing efficient investments is considered. The concern of the parties is how to divide the surplus from the ex post revision in order to induce a better risk sharing. Given that the class of enforceable revision schemes is restricted, only crude division of the surplus is expected, and thus first best outcomes cannot be in general implemented. In some cases, this does not matter (See Proposition 2). In general, however, the division of the surplus will be undesirable from a risk sharing point of view, and the loss from using an incomplete contract is likely to be large. Let $B(\cdot)$ and $S(\cdot)$ denote the von Neumann-Morgenstern utility functions of the buyer and the seller respectively. Let $S(w) = U^0$.

4a. The case without investments

Consider the case without investments to focus on the issue of risk sharing. If both parties are risk neutral, then there is nothing to solve in the pure risk sharing problem. Any revision scheme will result in efficient production and the initial allocation will determine the ex ante distribution of the gains so that the seller's participation constraint is binding. Now, consider the case in which one party (say, the buyer) is risk neutral and the other party (say, the seller) is risk averse. In this case the efficient risk sharing requires that the buyer will assume all the risks as a residual claimant of the ex post surplus. Therefore, the type $b$ contract with $(q^0, r^0) = (0, w)$ can implement first best outcomes since, if $(q^0, r^0) = (0, w)$, then the seller always gets $w$. In other words, when one party is risk neutral and the other party is risk averse, the risk neutral party should have all the ex post bargaining power to induce efficient risk sharing.

If both parties are risk averse, then it can be expected from the above discussion that no feasible contract can implement first best outcomes in general. Even though both types of contracts allow some sharing of risks with $(q^0, r^0)$ where $q^0 \neq 0$, they may implement first best outcomes in very special cases. At this point, one may ask what the second best optimal contract is in this general case. The answer to this question depends on the specific structure of the
model, especially the exact form of the parties' utility functions and the nature of the probability
distribution functions.

The above results can be interpreted in the context of decision authority relationship. In
particular, it says that allocation of the ex post decision authority (or ownership structure) has
efficiency consequences. This result differs from the existing literature in which allocation of the
decision authority matters through its effect on specific investments.\textsuperscript{10}

4b. The case with investments

In this subsection, the optimal contract design problem is considered in the context of both risk
sharing and specific investment. The analysis focuses on only the case in which the buyer is
risk neutral and the seller is risk averse, since the case in which the buyer is risk averse and the
seller is risk neutral can be analyzed in a symmetric way. It has been already mentioned that
when both parties are risk averse even without investments, no feasible contract can implement
first best outcomes. When the problem is more complicated by introducing specific investments,
the situation generally becomes worse. Hence the most general case is not examined. I hope that
analyzing other cases provides for some insights to this general case.

In this subsection, it is assumed that the buyer is risk neutral and the seller is risk averse. It
turns out that, even in this case, neither types of contracts can implement first best outcomes in
general (this is a necessary consequence of the contractual constraints in the model, in
particular, the restriction on the ex post revision game). The following proposition identifies
two special cases in which first best outcomes can be implemented.

\textbf{Proposition 2.} Assume that the buyer is risk neutral and the seller is risk averse. The
type b contract can implement first best outcomes if one of the two following conditions
is satisfied.

(1) $c(q,\omega,\sigma)$ is independent in $\omega$.

(2) $c_\sigma = 0$ (or $\sigma^* = 0$).
Proposition 2 says that when one party is risk neutral and the other party is risk averse, the contract which assigns the decision authority to the risk neutral party can implement first best outcomes \textit{either} when the risk averse party does not face direct uncertainty \textit{or} when the risk averse party does not need to make investment. Before giving intuition behind these results, recall that when the buyer is risk neutral and the seller is risk averse, efficient risk sharing requires that the buyer assumes all risks and the seller gets constant utility for all realizations of \( \omega \).

Consider the first case - no payoff uncertainty in the seller's cost function which is now denoted by \( c(q, \sigma) \). The type \( b \) contract with its initial allocation \( (q^0, r^0) \) gives the seller ex post utility of \( S[ t - c(q^0, \sigma) - h_z(\sigma) ] \) for all \( \omega \in \Omega \). Thus, by choosing \( q^0 \) appropriately, the type \( b \) contract can induce the seller to choose \( \sigma^* \). Then choose \( r^0 \) such that the seller's participation constraint is binding. Now the seller's ex post utility is constant for all \( \omega \in \Omega \) and the buyer assumes all the risks, which satisfies the requirement for optimal risk sharing. The buyer will choose efficient investment \( \beta^* \) since she is risk neutral and she is the residual claimant of the revision process. Therefore the type \( b \) contract can implement first best outcomes when the seller does not face the direct payoff uncertainty.

In the second case in which there is no investment for the seller, the only problem is risk sharing since the buyer will make an efficient investment as a residual claimant. As it has been shown in Proposition 1, by choosing \( (q^0, r^0) = (0, \omega) \) in the type \( b \) contract, the seller gets constant ex post utility \( S(\omega) \) for all \( \omega \) and the buyer assumes all the risks, which is required for the optimal risk sharing.

This analysis suggests that if neither of the two conditions in Proposition 2 is satisfied, then the initial quantity \( q^0 \) will be chosen to compromise two conflicting objectives: optimal risk sharing requires small \( q^0 \), but efficient investment requires large \( q^0 \). In this general case, the type \( b \) contract cannot implement first best outcomes, and thus it is a second best contract.
5. CONCLUSION

In this paper, a situation in which two contracting parties are forced to write an incomplete contract has been studied. The paper has investigated the extent to which the parties can mitigate this incompleteness by building into their contract a scheme for revising ex post the terms of trade. For the case where the parties are risk neutral and must make specific investments, it has been shown that it is possible to induce efficient investments and efficient outcomes regardless of the allocation of the ex post decision authority. For the case where the parties are risk averse, but where there are no specific investments, it has been shown that it is not generally possible to implement first best outcomes.

It is often argued that the explicit agreements observed in practice are relatively simple contrary to the complicated contracts prescribed by contract theory. The analysis has showed that the simple explicit agreement together with the allocation of the ex post decision authority can perform well and can even implement first best outcomes in a number of interesting cases in spite of the stringent contractual constraints.

One may ask what other kind of contracts can induce first best outcomes when simple contracts in the model fail to implement first best outcomes. There are at least two ways to relax the assumed simplicity of contracts in the model without changing the non-verifiability of investments and realizations of uncertainty.\textsuperscript{11}

First, the assumed simplicity of agreement on allocations can be relaxed. If the non-verifiability were the only constraint, then a more complicated mechanism or game form could be constructed to determine the allocation which is enforced unless both parties agree to revise it. A mechanism or game form consists of strategy spaces for each party and an outcome function which maps strategies to an allocation. It is of course required that the strategies must be verifiable. It is an open question whether allowing this type of mechanism can improve the risk sharing performance of contracts in the model of this paper.\textsuperscript{12}
Second, more general bargaining schemes might be allowed without changing the simplicity of agreement on allocations. Recall that revision schemes have been modelled by take-it-or-leave-it offers in part because they are simple and analytically tractable and in part because they represent some of institutional factors such as constrained authority relationships. Alternative bargaining schemes may be considered as long as they are not contingent upon non-verifiable variables and they can be committed to by the parties. For example, alternative sequential bargaining model (based on the infinite horizon model of Rubinstein (1982)) may be considered as a feasible revision scheme. It is not clear at this point whether such a scheme can provide correct incentives for efficient investments and optimal risk sharing.

I conclude this paper by mentioning additional future research directions. First, recall that it has been assumed that the revisions are costless. This is a strong assumption which has to be relaxed in order to provide insights on real contractual situations. Second, the parties in the model have symmetric information throughout their relationship. Since asymmetric information can be useful in understanding constrained authority relationships, it may be interesting to consider a model which incorporate both asymmetric information and incomplete contracts. Third, to explain the sustainability of a specific organizational mode, the model has to be extended to a dynamic or repeated setting. In this attempt, reputation will play an important role. Fourth, in the model of this paper the parties have clear ideas on the possible states of nature and have prior distributions on them. In real contractual situations, one unresolved problem is how to deal with unforeseen contingencies. The framework of constrained authority relationship I have used in the paper may be a useful way to tackle those kinds of problems.
APPENDIX

Proof of Proposition 1. Recall that efficient investments are defined by

\[
(\beta^*, \sigma^*) \equiv \arg\max_{\beta, \sigma} \int [\nu(q^*(\beta, \sigma, \omega), \omega, \beta) - c(q^*(\beta, \sigma, \omega), \omega, \sigma)] \, dF(\omega) - h_b(\beta) - h_s(\sigma) \quad (A.1)
\]

To show \((\beta^c, \sigma^c) = (\beta^*, \sigma^*)\), let \((q^0, t^0) \in R^+ \times R^2\), \((\beta, \sigma) \in R^+_2\) and \(\omega \in \Omega\) be given. Then the buyer solves ex post:

\[
M(\beta, \sigma, \omega) \equiv \max_{q, t} \nu(q, \omega, \beta) - t - h_b(\beta)
\]

subject to \(t - c(q, \omega, \sigma) - h_s(\sigma) \geq t^0 - c(q^0, \omega, \sigma) - h_s(\sigma) \) \quad (A.2)

It can be easily shown that

\[
M(\beta, \sigma, \omega) = \nu(q^*(\beta, \sigma, \omega), \omega, \beta) - c(q^*(\beta, \sigma, \omega), \omega, \sigma) - h_b(\beta) - (t^0 - c(q^0, \omega, \sigma)) \quad (A.3)
\]

When the seller chooses \(\sigma\), he solves

\[
\sigma^c \equiv \arg\max_{\sigma} \int (t^0 - c(q^0, \omega, \sigma) - h_s(\sigma)) \, dF(\omega)
\]

By the implicit function theorem, the interior maximum \(\sigma^c\) as a continuous function of \(q^0\) satisfies

\[
- \int c_{\sigma}(q^0, \omega, \sigma^c(q^0)) \, dF(\omega) - h_s'(\sigma^c(q^0)) \equiv 0 \quad (A.4)
\]

Note that \(\sigma^c(q^0)\) is increasing since \(d\sigma^c/dq^0 = \int c_{\sigma}(q^0) \, dF(\omega) / \Delta > 0\) where \(\Delta \equiv - \int c_{\sigma}(q^0) \, dF(\omega) - h_s'<0\) from the second order condition and \(c_{\sigma q} < 0\) from Assumption 1. Also observe that \(\sigma^c(q^0) = 0\) when \(q^0 = 0\), and that \(\sigma^c(q^0) > \sigma^*\) when \(q^0 = \arg\max_\omega q^*(\beta^*, \sigma^*, \omega)\). Therefore, there exists a \(q^0\) such that \(\sigma^c(q^0) = \sigma^*\) by the intermediate value theorem. Hence we have \(\sigma^c = \sigma^*\). Now the buyer's investment problem is
\[ \beta^* \equiv \arg\max_\beta \int M(\beta, \sigma^*, \omega) \, dF(\omega) \]
\[ = \beta^* \quad \text{by (A.1) and (A.3).} \]

Now in order to satisfy the seller's participation constraint, pick \( \Phi^0 \) such that

\[ \int [\Phi^0 - c(\Phi^0, \omega, \sigma^*)] \, dF(\omega) - h_3(\sigma^*) = U^0. \quad \text{(A.5)} \]

Then the (type b) contract with the initial allocation \((\Phi^0, \Phi^0)\) can induce efficient investments and first best outcomes. \[\text{Q.E.D.}\]
Figure 1
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NOTES


2. After the earlier version of this paper Chung (1988) was completed, several works studying the role of renegotiation in contracts have been available to the author. In particular, Aghion-Dewatripont-Rey (1989), Malcomson and MacLeod (1989), and Rubinstein and Wolinsky (1990) are closely related to this paper.


4. An exception is Rogerson (1984). Rogerson argues, in a model with risk neutral parties, that when only one party makes an investment, that party should have all the ex post decision rights (i.e., all the ex post bargaining power) to induce an efficient investment. This argument is compared with the result of this paper in Section 3.

5. This can be also regarded as 'expectation damage remedy' if the bargaining schemes are restricted to the take-it-or-leave-it offers as in this paper. To show this, consider a type b contract. When the buyer proposes a new allocation ex post (i.e. the buyer breaches the initial agreement on an allocation), the seller can recover from the buyer a level of utility that puts the seller in the same position he would have been in had the initial allocation been completed. If we allow more general bargaining schemes which give positive shares on both parties, this interpretation is not valid.
6. When the allocation of the ex post decision authority is embedded in the organizational mode, the parties need some informal enforcement mechanism to preserve the authority relationship. Custom or social norm may play that role.

7. In the section 4 in which risk sharing is considered, both types of contracts are examined.

8. Since the total gain, \( v(q, \omega, \beta) - c(q, \omega, \sigma) \), is strictly concave in \( q \), \( q^* \) is uniquely defined for each \( (\beta, \sigma, \omega) \).

9. Let \( (q^0, \sigma^0) \) denote the initial allocation which induces efficient outcomes in the type \( b \) contract. Observe that \( (q^0, \sigma^0) \neq (0, U^0) \). To check this claim, consider \( (0, U^0) \) as the initial allocation in a type \( b \) contract, then the seller's ex post utility for any level of \( e \) made is \( U^0 - h_2(\sigma) \).

Therefore the seller will choose \( \sigma^e = 0 \) so that \( \sigma^e < \sigma^* \) since \( \sigma^* \) is assumed to be positive. In other words, to induce efficient investment, the optimal contract requires \( q^0 > 0 \).


11. Rogerson (1988) considers the issue of designing an optimal contract to induce efficient investments when complex contracts can be written.

12. Hart and Moore (1988) consider the case in which sending a message can be verified. The authors show that in that case optimal risk sharing can be achieved when there are no investments.
REFERENCES


