A Quantitative Investigation of the Macroeconomic Effects of Capital Controls and the Stabilization of the Balance of Trade

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Citation of this paper:
A dynamic stochastic model of a small open economy is employed to numerically evaluate the macroeconomic effects of using capital controls to stabilize the balance of trade at some target level. The quantitative investigation shows that such a policy has small effects on both the equilibrium stochastic process of the economy and the level of welfare. These results are attributed to the relatively small magnitude of actual economic fluctuations. A fiscal strategy that would allow the government to successfully implement the policy is also studied in some detail.

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1. Introduction.

In the past few years, a series of intertemporal optimizing models that study the macroeconomic effects of economic policy in the context of the open economy have been developed. With the exception of empirical investigations like those undertaken by Ahmed (1986) and Hercowitz (1986), the majority of these studies have focused on the theoretical analysis of simplified dynamic prototypes. Capital controls, dual exchange rates and other issues of financial policy have been analyzed, using two-period models of perfect-foresight exchange economies, by Adams and Greenwood (1985) and Greenwood and Kimbrough (1987). The analysis of fiscal policy, in a two-period framework that incorporates endogenous production decisions, has been undertaken by Aschauer and Greenwood (1985), Greenwood and Kimbrough (1985) and Kimbrough (1986). In addition to this family of two-period models, a variety of overlapping-generations and infinite-horizon extensions have also been developed. Obstfeld (1981a) and (1981b) investigated exchange-rate dynamics and devaluations and Obstfeld (1986) and Frenkel and Razin (1986b) analyzed dual exchange rates in deterministic, infinite horizon models. Frenkel and Razin (1986a), (1986b) and (1987) studied fiscal policy and the dual exchange-rate regime using overlapping-generations prototypes.

In spite of the fact that the conclusions reached by the dynamic equilibrium literature mentioned above challenge the traditional wisdom of open-economy policy analysis based on the Mundell-Fleming model, the shortage of empirical work remains an obstacle in the process of translating the theory into specific policy recommendations. More recently, however, a series of developments in dynamic macroeconomic theory have made it possible to undertake quantitative studies of complex extensions of these dynamic models.
Thanks to the innovative work on real business cycles by Kydland and Prescott (1982) and Long and Plosser (1983), it is now possible to numerically calculate the equilibrium stochastic process that characterizes artificial economies where infinitely-lived individuals formulate optimal intertemporal plans. It would be an interesting development to make use of these new techniques to analyze quantitatively the macroeconomic effects of economic policy, which have been so thoroughly studied from a theoretical perspective.

The quantitative analysis of economic policy based on dynamic stochastic models is a very recent advancement even in closed-economy macroeconomics. The novel works by Cooley and Hansen (1987), Greenwood and Huffman (1988) and McGrattan (1988) constitute the first attempts to undertake such a task. This literature studies the effects of monetary and fiscal policies from the perspective of models where individuals are allowed to adjust their optimal state-contingent plans as the economic environment is altered. By proceeding in this manner, the analysis of economic policy complies with the standards set by Lucas (1976) and (1987). Thus, the agents' rules of behavior are not assumed to be invariant to policy changes and arguments about economic welfare are explicitly linked to lifetime utility-maximizing choices.

In an attempt to extend the line of research discussed in the previous paragraphs, the present paper undertakes a quantitative investigation of the macroeconomic effects of economic policy in the context of a small open economy. The essay studies a policy that stabilizes the balance of trade, at a certain target level, by imposing capital controls. These controls are enforced by introducing a tax, or subsidy, on foreign-interest income that is rebated to individuals in the form of a lump-sum transfer. The analysis utilizes a dynamic stochastic model, in combination with numerical simulation
techniques, to determine the effects of the policy on economic welfare and economic activity, and to calculate the state-contingent schedule of taxes that would allow the government to successfully implement the policy.

The paper is organized as follows: Section 2 describes a dynamic stochastic model of a small open economy which incorporates some theoretical developments made by Obstfeld (1981a) and Epstein (1983). The model considers optimal intertemporal planning in an environment where domestic capital and foreign financial assets can both be utilized as vehicles of savings, and where the rate of time preference is an increasing function of past consumption levels. In Section 3, the potential for the model to be a useful tool for policy analysis is numerically investigated by studying its ability to replicate the observed stylized facts of an actual economy. Section 4 studies the effects of introducing a policy that uses capital controls to stabilize the balance of trade at some desired level. The effect on economic welfare is determined with different measures of compensating variations in terms of constant-consumption paths, and the effect on economic activity is studied by comparing the statistical moments that characterize the equilibrium stochastic process of the policy-restricted version of the model with those of the unrestricted prototype. The same section also analyzes a fiscal strategy that the government could follow to implement the policy. A series of concluding remarks are included in the last section.

2. A Dynamic Stochastic Model of a Small Open Economy.

This section presents the artificial economy to be studied in the rest of the paper. In the first part of the section, the structure of preferences, technology and financial markets that characterizes the economy is described.
Since this paper deals with two situations, one where no capital controls or trade-balance targets are in place and one where such a policy is introduced, the financial structure of the artificial economy adopts two forms. In the unrestricted, or free-trade, version of the model, individuals have unlimited access to the world's capital market, whereas in the policy-restricted version of the model they are forced to accumulate a certain predetermined amount of foreign assets. This additional restriction is enforced by a government that levies the appropriate tax, or pays the appropriate subsidy, on foreign interest income. This schedule of taxes and subsidies is studied later in the paper. The second part of the section characterizes the equilibrium of the two versions of the model as the solution of discrete-time dynamic programming problems.

2.1 Preferences, Technology and the Financial Structure.

Preferences: All agents are infinitely-lived and identical, and preferences are described by the Stationary Cardinal Utility function formulated by Epstein (1983)\(^1\):

$$ E_0 \left[ \sum_{t=0}^{\infty} u(C_t - G(L_t)) \exp[-\Sigma_{r=0}^{t-1} v(C_r - G(L_r))] \right]. \quad (1) $$

where

$$ u(C_t - G(L_t)) = (1-\gamma)^{-1} [(C_t - L_t \omega/\omega)^{1-\gamma} - 1], \quad \omega > 1, \quad \gamma > 1, \quad (2) $$

and

$$ v(C_t - G(L_t)) = \beta \ln [1 + C_t - L_t \omega/\omega], \quad \beta > 0. \quad (3) $$

Here, \(C_t\) denotes private consumption and \(L_t\) are the productive services provided by labor.

The lifetime utility function formulated in (1) is adopted from Epstein
(1983). It constitutes the stochastic analog of the utility function employed by Obstfeld (1981a) and (1981b). In Obstfeld's work, the existence of a well-behaved deterministic stationary equilibrium for the holdings of international assets is ensured by assuming that the rate of time preference is an increasing function of past consumption levels. Such a deterministic steady state is obtained when the world's real interest rate and the rate of time preference are equalized. Otherwise, as discussed in Helpman and Razin (1982) or Frenkel and Razin (1987), whenever the rate of interest is greater (smaller) than the rate of time preference, individuals will accumulate (deplete) foreign assets in order to finance an increasing (decreasing) consumption stream. Therefore, the constant-time-preference representation of lifetime utility cannot be utilized to study the process by which this long-run equilibrium is reached, and an alternative formulation such as the Stationary Cardinal Utility must be introduced.

The instantaneous-utility and time-preference functions defined in (2) and (3) simplify the analysis by specifying preferences in terms of the composite good \( G_t \cdot L_t^\omega/\omega \), thus making the intratemporal marginal rate of substitution between consumption and labor depend on the latter only. This allows the model to focus expressly on the interaction of domestic capital and foreign assets as alternative vehicles of savings. The cost, however, is that the wealth effect on labor supply is eliminated.

**Technology:** Domestic production is generated as follows:

\[
G(K_t, L_t, K_{t+1}) = \exp(e_t) K_t^{\alpha \omega} L_t^{1-\alpha} - (\phi/2)(K_{t+1} - K_t)^2, \quad (4)
\]

\[
0 < \alpha < 1, \quad \phi > 0.
\]

Here, \( e_t \) is a random shock to productivity or the terms of trade to be
discussed in more detail later, \( K_t^\alpha L_t^{1-\alpha} \) is a Cobb-Douglas production function, \( K_t \) is the domestic capital stock currently productive and \( (\Phi/2)(K_{t+1}-K_t)^2 \) is the cost of adjusting the capital stock as a function of net investment\(^4\). Next, the capital evolution equation is given by

\[
K_{t+1} = (1-\delta)K_t + I_g_t, \quad 0 \leq \delta \leq 1, \quad (5)
\]

where \( \delta \) is a constant rate of depreciation and \( I_g_t \) is gross investment.

**Financial Structure:** The financial structure of the economy adopts one of two forms. In the first case, free trade in foreign financial assets is allowed by the government. Here, agents have unrestricted access to a perfectly competitive, international capital market where foreign assets \( A_t \), paying or charging the non-random real rate of return \( r^* \), are exchanged with the rest of the world\(^5\). Thus, in the economy without capital controls, the holdings of foreign assets evolve according to

\[
A_{t+1} = TB_t + A_t(1+r^*), \quad (6)
\]

where \( TB_t \) is the balance of trade. In the second case the government permanently restricts foreign-asset accumulation to a predetermined target \( \bar{A} \). Here the evolution of foreign assets is described by

\[
A_{t+1} = \bar{A}, \quad (7)
\]

In this case, the balance of trade after the date in which the policy is implemented is given by

\[
TB_t = r^* \bar{A}.
\]

In both versions of the model, the combination of domestic production with foreign borrowing, or savings, results in the following resource constraint\(^6\):
\[ \text{C}_t + I_t + TB_t \leq \exp(e_t) K_t^\alpha L_t^{1-\alpha} - (\Phi/2)(K_{t+1} - K_t)^2. \]  

The cost of adjustment included in (4) and (8) indicates that the total cost of altering the capital stock increases with the size of the desired adjustment, and hence it implies that investment changes are to be undertaken in a gradual manner. This formulation of the technology also assumes that the domestic economy is a small participant in the world capital market, so that the interest rate \( r^* \) is regarded as an exogenous variable.

2.2 Stochastic Equilibrium and the Dynamic Programming Problem.

The Free-Trade Economy: The dynamic equilibrium of the unrestricted model is represented by a set of state-contingent decision rules for consumption, labor supply, capital accumulation and foreign-asset accumulation that maximize (1), given \( K_0, A_0 \) and \( e_0 \), subject to (4)-(6), (8), the intertemporal solvency restriction and the usual non-negativity restrictions on \( K, L \) and \( C \). The time-recursive structure of the Stationary Cardinal Utility function simplifies the analysis and solution of this intertemporal optimization problem because it implies that dynamic programming techniques can be applied. In each period, the state of the economy can be fully described once \( K_t, A_t \) and \( e_t \) are observed. Given this information, along with the knowledge of the stochastic process that governs the evolution of the disturbances, individuals choose \( K_{t+1}, A_{t+1}, C_t \) and \( L_t \) so as to solve the following dynamic programming problem:

\[ V(K_t, A_t, e^S_t) = \max \{ (1-\gamma)^{-1} \left[ (C_t - \frac{\hat{L}_t \omega}{\omega})^{1-\gamma} - 1 \right] + \left[ \sum_{r=1}^{2} \exp[-\beta \ln(1+C_t - \frac{\hat{L}_t \omega}{\omega})] \right] \}
\]
subject to
\[ C_t = \exp(e^s_t)K_t^\alpha L_t^{1-\alpha} - K_{t+1} + K_t(1-\delta) - (\delta/2)(K_{t+1}-K_t)^2 + (1+r^*)A_t - A_{t+1}, \]
where
\[ L_t^* = \arg\max_{L_t} \{ \exp(e^s_t)K_t^\alpha L_t^{1-\alpha} - L_t^{\omega/\omega} \}. \]

The symbol \( \pi_{sr} \) for \( s,r=1,2 \), denotes the one-step conditional transition probability of the next-period's technological or real-exchange-rate disturbance. These transition probabilities must satisfy the conditions that \( 0 \leq \pi_{sr} \leq 1 \) and \( \pi_{s1} + \pi_{s2} = 1 \) for \( s,r=1,2 \).

In order to preserve tractability, the stochastic structure of the shocks is simplified by introducing a two-point Markov process. Thus, at any date in time, the shocks can only take one of two values
\[ e_t \in E = \{ e^1, e^2 \}. \] (10)

Furthermore, it is also assumed that the transition probabilities and the shocks themselves are symmetric: \( \pi_{11} = \pi_{22} = \pi \) and \( e^1 = e^2 = e \). Therefore, the asymptotic standard deviation, \( \sigma_e \), and the first-order autocorrelation coefficient, \( \rho_e \), that characterize the stochastic process of the disturbances are determined by \( \sigma_e = \varepsilon \) and \( \rho_e = 2\pi - 1 \) respectively.

The values of the parameters \( \gamma \) (coefficient of relative risk aversion), \( \omega \) (1 plus the inverse of the intertemporal elasticity of substitution in labor supply), \( \alpha \) (capital's share in output), \( \delta \) (depreciation rate) and \( \beta \) (the consumption elasticity of the rate of time preference), are selected using long-run averages of actual data, the restrictions imposed by the deterministic steady-state equilibrium of the model and also by attempting to approximate some of the estimates obtained in the relevant empirical literature. These structural parameters are assigned the following values:
\( \alpha = 0.32, \ \beta = 0.11, \ \gamma = 1.6, \ \delta = 0.1, \ \omega = 1.455 \) and \( r^* = 0.04. \) \( \quad (11) \)

The model is calibrated by adjusting the value of the free parameters \( \phi, \ \epsilon \) and \( \pi \) to replicate a subset of the observed sample moments calculated with detrended post-war Canadian data. The first parameter is selected so as to mimic the volatility of private investment, and the second and third are determined so as to replicate the variability and first-order serial autocorrelation of GDP.

The Restricted-Trade Economy: The equilibrium of the restricted-trade model is characterized in a similar manner as in the free-trade model, except that foreign assets are not a choice variable and (7) replaces (6) because of the capital controls being implemented. The dynamic programming problem defined in (9) is modified to incorporate the restriction that \( A_{t+1} \) must equal \( A \) in every period. All other elements of the problem remain the same. In fact, this problem is equivalent to the one that characterizes a closed-economy real business cycle model, except that a constant equal to the trade-balance target is added into the resource constraint. Also, since the restricted model is used to evaluate the effects of imposing different trade-balance targets \( -r^*A, \) it adopts the same parameter values used to calibrate the unrestricted model.

3. The Unrestricted Economy: Free Trade.

In order to utilize the unrestricted model as a benchmark for the evaluation of the effects of the policy under discussion, it is important to establish first how useful is it as an accurate description of a real-world small open economy. According to Lucas (1976) and (1987), a useful model for policy-analysis purposes is one that fits historical data while it also
separates those elements of economic decision-making that are altered by the policy from those that remain unchanged. Thus, a useful model must replicate the properties of observed time series and must also capture the ability that agents have to modify their behavior as the environment changes. The latter property is obviously satisfied by the model studied here, since it allows individuals to reformulate their optimal intertemporal plans as policy changes are introduced. The task of this section is to enquire whether the first property is also satisfied. This is done by undertaking the calibration exercise mentioned earlier and by briefly discussing its results.

The numerical solution of the model follows the methodology suggested in Bertsekas (1976) and employed by Sargent (1980), Greenwood, Hercowitz and Huffman (1988) and, in the context of a similar model as the one studied in this section, by Mendoza (1989b). The procedure is based on performing iterations on the value function (9) using a discrete grid of admissible values for domestic capital and foreign assets, along with the two-point Markov structure of the shocks. Iterations are performed until convergence of the value function is achieved. The state-contingent decision rules for capital and foreign-asset accumulation, associated with the solution of the functional equation (9), are used to calculate the exact stationary joint probability distribution of the state variables of the artificial economy. This probability distribution is in turn utilized to calculate first and second order statistical moments that can be compared with those observed in the actual data, establishing in this manner the ability of the model to mimic the behavior of observed time series.

The unrestricted model of the small open economy is calibrated setting the parameters $\sigma_e=1.285\%$, $\rho_e=0.41$ and $\delta=0.023$. The statistical moments for
both this free-trade artificial economy and the actual Canadian data are listed in Table 1. A graph of the stationary marginal probability distribution of domestic capital and foreign assets is produced in Figure 1.

By comparing the moments of variability, persistence and comovement listed in Table 1, it transpires that the free-trade economy delivers a fairly accurate characterization of the observed stylized facts\(^9\). First, the model mimics the observed ranking of variability of all the aggregates listed, approximating closely the percentage standard deviation of each variable. Second, although the model exaggerates the GDP correlations of consumption, savings and labor, it reproduces a similar pattern of procyclical and countercyclical movements as that observed in the data, as well as the correlation between savings and investment\(^10\). Third, the model is also capable of partially approximating the ranking of the first-order serial autocorrelation coefficients, except that it underestimates the persistence of investment and the trade-balance-output ratio.

It is particularly important to notice that this benchmark model possesses the ability to replicate some of the moments that characterize foreign interest payments and the ratio of the trade balance to output. These results indicate that individuals in the artificial economy rationally choose to participate in the world's capital market in a manner such that their holdings of foreign financial assets exhibit 15.7% standard deviation, 0.97 first-order autocorrelation and -0.05 correlation with domestic output. These moments are consistent with, and indeed the result of, lifetime utility-maximizing decisions, and therefore they represent an economic environment in which welfare is maximized.

The rest of the paper studies what happens when the free-trade environment is modified by a government that dislikes the fluctuations in
Table 1

Statistical Moments: Canadian Data and Benchmark Economy\(^a\).

<table>
<thead>
<tr>
<th>Variables</th>
<th>(\text{Canadian Data 1946-1985 (I)})</th>
<th>(\text{Canadian Data 1946-1985 (II)})</th>
<th>(\text{Canadian Data 1946-1985 (III)})</th>
<th>(\text{Benchmark Economy Free Trade (I)})</th>
<th>(\text{Benchmark Economy Free Trade (II)})</th>
<th>(\text{Benchmark Economy Free Trade (III)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) GDP</td>
<td>2.810</td>
<td>0.615</td>
<td>1.000</td>
<td>2.807</td>
<td>0.614</td>
<td>1.000</td>
</tr>
<tr>
<td>2) GNP</td>
<td>2.950</td>
<td>0.643</td>
<td>0.995</td>
<td>2.864</td>
<td>0.616</td>
<td>0.994</td>
</tr>
<tr>
<td>3) C</td>
<td>2.460</td>
<td>0.701</td>
<td>0.586</td>
<td>2.140</td>
<td>0.688</td>
<td>0.943</td>
</tr>
<tr>
<td>4) S</td>
<td>7.306</td>
<td>0.542</td>
<td>0.662</td>
<td>5.635</td>
<td>0.602</td>
<td>0.923</td>
</tr>
<tr>
<td>5) I</td>
<td>9.820</td>
<td>0.314</td>
<td>0.639</td>
<td>10.028</td>
<td>-0.045</td>
<td>0.554</td>
</tr>
<tr>
<td>6) K</td>
<td>1.380</td>
<td>0.649</td>
<td>-0.384</td>
<td>1.364</td>
<td>0.705</td>
<td>0.594</td>
</tr>
<tr>
<td>7) L</td>
<td>2.020</td>
<td>0.541</td>
<td>0.799</td>
<td>1.929</td>
<td>0.614</td>
<td>1.000</td>
</tr>
<tr>
<td>8) (-r^A)</td>
<td>15.250</td>
<td>0.727</td>
<td>-0.175</td>
<td>15.672</td>
<td>0.971</td>
<td>-0.046</td>
</tr>
<tr>
<td>9) TB/Y</td>
<td>0.019</td>
<td>0.623</td>
<td>-0.129</td>
<td>0.019</td>
<td>0.018</td>
<td>-0.019</td>
</tr>
</tbody>
</table>

\[\text{CORR}(S,I) = 0.434\]

\[\text{CORR}(S,I) = 0.585\]

---

\(^a\)The data are measured in per-capita terms of the 15+ population, logged and detrended with a linear quadratic time trend. 1)-3), 5) and 8) are the totals from the national income accounts in 1981 dollars. 6) was obtained from the end-of-period net stocks of fixed non-residential capital in manufacturing and non-manufacturing industries, in 1981 prices. The labor data is an index of man hours worked by paid workers with 1981=100.0. Savings in 4) is generated as investment plus the trade balance surplus. The source of all is the CANSIM data retrieval.

\(^b\)Percentage standard deviation (except 9)).

\(^c\)First-order autocorrelation coefficient.

\(^d\)Coefficient of correlation with GDP.
MARGINAL PROBABILITY DENSITY OF CAPITAL AND FOREIGN ASSETS
IN THE BENCHMARK ECONOMY
FIGURE 1
asset holdings and the balance of trade, and imposes capital controls in order
to eliminate them or to achieve larger trade surpluses. Since the free-trade
benchmark economy is not affected by distortions such as those introduced by
labor-income taxes, the theoretical results of Aschauer and Greenwood (1985)
follow, and hence capital controls and trade-balance targets will necessarily
reduce welfare.

4. Restricted Artificial Economies: Controlled Trade.

The previous section provided evidence illustrating that the unrestricted
model is a useful benchmark for dynamic policy analysis. The goal now is to
investigate the effects of introducing a policy that restricts foreign-asset
trading in order to stabilize the balance of trade at some target level. This
policy is enforced by imposing a tax, or subsidy, on foreign interest income.
The revenue from this tax is rebated to agents through lump-sum transfers.
This section begins by studying the effects of the policy on economic
activity, as expressed by the resulting changes in some of the statistical
moments of the main macro-aggregates. The welfare effect of the policy is
then analyzed by looking at alternative welfare measures, based on
compensating variations of stationary consumption paths that correspond to
lifetime utility-maximizing values. This section concludes by considering the
feasibility of the policy, which is studied by computing the tax-strategy that
the government must follow to successfully achieve its trade-balance target.

4.1 Capital Controls, Trade-Balance Targeting and Economic Activity.

The effects that a policy of imposing capital controls to stabilize the
balance of trade can cause on economic activity are uncovered by numerically
solving the policy-restricted version of the model. As discussed in Section 2, the policy takes the form of a restriction on foreign-asset accumulation according to which $A_{t+1} = A$ for all $t$. The solution method to be applied is the same, except that the domestic capital stock is the only choice variable and the model does not have to be calibrated. Note that, although the grid of possible initial values for the holdings of foreign assets includes the same elements as before, it collapses to the single point $A$ exactly one period after the policy is implemented. Alternatively, the target value of foreign assets could be allowed to depend upon the state-of-the-world, so that the same methodology could be used to study a somewhat less realistic situation in which the capital controls were state contingent.

In order to obtain a more complete evaluation of the effects of the policy on economic activity, four different strategies have been considered. In the first case the government simply picks $A$ so as to stabilize the balance of trade at its mean value observed in the benchmark economy. In the other three cases, $A$ is adjusted so as to deliver trade-balance surpluses approximately 12%, 30% and 60% higher than the average obtained in the free-trade economy. Recall that, once capital controls are introduced, balance-of-payments equilibrium implies that the value of $A$ is linked to the trade balance by the equality $TB_t = r^* A$. Thus, a large long-run trade surplus is associated with a high level of foreign debt because sufficient net exports must be generated to meet the debt commitment.

Panels A-E of Table 2 list the means, standard deviations and GDP correlations of the variables in the benchmark economy and in each of the restricted economies. The long-run effects of the policy on economic activity are analyzed by comparing the moments of the unrestricted economy
Table 2.
Statistical Moments for Alternative Artificial Economies.

<table>
<thead>
<tr>
<th></th>
<th>A Benchmark Economy Free Trade</th>
<th>B Restricted Economy 0% Trade-Balance Improvement</th>
<th>C Restricted Economy 12% Trade-Balance Improvement</th>
<th>D Restricted Economy 30% Trade-Balance Improvement</th>
<th>E Restricted Economy 60% Trade-Balance Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)&lt;sup&gt;a&lt;/sup&gt; (II)&lt;sup&gt;b&lt;/sup&gt; (III)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>(I)&lt;sup&gt;a&lt;/sup&gt; (II)&lt;sup&gt;b&lt;/sup&gt; (III)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>(I)&lt;sup&gt;a&lt;/sup&gt; (II)&lt;sup&gt;b&lt;/sup&gt; (III)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>(I)&lt;sup&gt;a&lt;/sup&gt; (II)&lt;sup&gt;b&lt;/sup&gt; (III)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>(I)&lt;sup&gt;a&lt;/sup&gt; (II)&lt;sup&gt;b&lt;/sup&gt; (III)&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>GDP</td>
<td>1.487 0.042 1.000</td>
<td>1.487 0.042 1.000</td>
<td>1.492 0.042 1.000</td>
<td>1.497 0.042 1.000</td>
<td>1.507 0.042 1.000</td>
</tr>
<tr>
<td>CNEP</td>
<td>1.459 0.042 0.994</td>
<td>1.458 0.042 1.000</td>
<td>1.460 0.042 1.000</td>
<td>1.460 0.042 1.000</td>
<td>1.461 0.042 1.000</td>
</tr>
<tr>
<td>C</td>
<td>1.119 0.024 0.943</td>
<td>1.119 0.027 0.976</td>
<td>1.118 0.027 0.976</td>
<td>1.116 0.027 0.977</td>
<td>1.114 0.027 0.977</td>
</tr>
<tr>
<td>S</td>
<td>0.368 0.021 0.923</td>
<td>0.368 0.017 0.939</td>
<td>0.373 0.017 0.939</td>
<td>0.381 0.017 0.940</td>
<td>0.393 0.017 0.941</td>
</tr>
<tr>
<td>I</td>
<td>0.340 0.034 0.554</td>
<td>0.340 0.017 0.939</td>
<td>0.342 0.017 0.939</td>
<td>0.343 0.017 0.940</td>
<td>0.347 0.017 0.941</td>
</tr>
<tr>
<td>K</td>
<td>3.399 0.046 0.594</td>
<td>3.397 0.064 0.544</td>
<td>3.415 0.064 0.543</td>
<td>3.434 0.064 0.542</td>
<td>3.472 0.064 0.539</td>
</tr>
<tr>
<td>L</td>
<td>1.008 0.019 1.000</td>
<td>1.008 0.020 1.000</td>
<td>1.010 0.020 1.000</td>
<td>1.012 0.020 1.000</td>
<td>1.017 0.020 1.000</td>
</tr>
<tr>
<td>-A</td>
<td>0.711 0.111 -0.046</td>
<td>0.708 0.000 0.000</td>
<td>0.795 0.000 0.000</td>
<td>0.926 0.000 0.000</td>
<td>1.142 0.000 0.000</td>
</tr>
<tr>
<td>TB</td>
<td>0.028 0.028 0.009</td>
<td>0.028 0.000 0.000</td>
<td>0.032 0.000 0.000</td>
<td>0.037 0.000 0.000</td>
<td>0.046 0.000 0.000</td>
</tr>
<tr>
<td>CORR(S,I) = 0.585</td>
<td>CORR(S,I) = 1.000</td>
<td>CORR(S,I) = 1.000</td>
<td>CORR(S,I) = 1.000</td>
<td>CORR(S,I) = 1.000</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>Mean.

<sup>b</sup>Standard Deviation.

<sup>c</sup>Coefficient of correlation with GDP.
with the moments of the restricted economies. The general result obtained from this analysis is that capital controls, as a means of stabilizing the balance of trade at some target level, do not have substantial effects on various indicators of economic activity.

Consider first the average values listed in column I of each panel. As can be observed, the policy in question has minimal effects on the mean values of all non-controlled variables. Although the results indicate that the average of each aggregate, except consumption, increases slightly as the trade-balance target is raised, these increments are relatively small. Furthermore, the largest drop in average consumption that the policy can cause is only 0.5%, and that occurs when the trade-balance target is set 60% higher than the mean trade-balance of the benchmark economy. In fact, as column I in panels A and B shows, if the government concentrates only on stabilizing the balance of trade at the level it has in the free-trade economy, the average values of all aggregates are practically the same as in the benchmark economy.

The standard deviations listed in column II of each panel illustrate two findings. First, when capital controls are introduced to stabilize the balance of trade at its mean value in the benchmark economy, the only noticeable changes in the variability of the aggregates affect savings, investment and the domestic capital stock. Since in the restricted economy changes in savings are equivalent to changes in investment, the standard deviation of these two variables is identical. Restricting foreign-asset trading reduces the standard deviation of investment from 0.034 to 0.017. In contrast, the variability of the domestic capital stock increases from 0.046 to 0.064. These changes are consistent with the fact that, by controlling the option of using international trade as a means for the optimal intertemporal
allocation of consumption through the cycle, the government forces the dynamics of investment and the capital stock to behave as in a closed economy. Changes in $K$ must now respond to consumption-smoothing and consumption-substitution effects, instead of depending on fluctuations in the relative returns of capital and foreign assets. Therefore, movements in capital accumulation face an increasing supply price of the capital stock, and hence the variability of investment is smaller than in a small open economy -where the supply price of $K$ is a constant at the level of the world's interest rate.

The second finding that transpires from comparing the standard deviations is that increasing the trade-balance target, or reducing the value of $\bar{A}$, does not affect the variability of the aggregates, relative to the initial situation where trade is stabilized at its average value in the benchmark economy. Thus, the standard deviation of the variables appears to be independent of the size of the desired adjustment in the balance of trade.

The correlations with GDP listed in column III of panels A-E illustrate similar observations as the standard deviations. First, when controls are introduced to stabilize the balance of trade, the only noticeable change is that the correlation between investment and output increases substantially from 0.554 to 0.939. This is justified because, as discussed above, changes in savings have to be undertaken by changing investment, and also because the disturbances considered are not large and persistent enough to cause strong consumption-substitution effects. Thus, investment is essentially used as a means to smooth consumption, and hence it exhibits high positive correlation with domestic output. The second observation is that the GDP correlations also appear to be independent of the size of the trade-balance surplus target.

Since the only moments affected by the size of the trade-balance target
are the means, it is possible to conclude that the policy under consideration has the effect of shifting the stationary distribution of $K$ and $e$ without affecting its variance. This conclusion is reaffirmed by the graphs of the marginal limiting distribution of the capital stock produced in Figures 2-5. Increasing the trade-balance surplus target displaces this distribution rightwards, without noticeably affecting it in any other way. The direction of this displacement results from the fact that a higher trade-balance surplus reduces consumption in the deterministic stationary equilibrium, and hence it reduces the long-run rate of time preference. This in turn implies that the steady-state capital stock, consistent with an equality between the marginal productivity of capital and the rate of time preference, is rising.

To conclude, the analysis of Table 2 suggests that capital controls, used to attain some desired trade-balance target, do not have substantial effects on the equilibrium stochastic process of the economy. Although the dynamics of the system in the restricted-trade economy work in an entirely different manner, with investment instead of foreign assets being used to smooth consumption, the final result is that output, consumption and the supply of labor behave in almost the same manner. The shocks that enable the benchmark model to mimic Canadian business cycles are not large and persistent enough to allow capital controls to seriously harm the ability of individuals to smooth consumption through the cycle. The important question, however, is how much welfare is lost by imposing this apparently neutral policy. This issue is analyzed next.

4.2 Capital Controls, Trade-Balance Targeting and Economic Welfare.

A set of alternative measures have been formulated in an attempt to
MARGINAL PROBABILITY DENSITY OF THE DOMESTIC CAPITAL STOCK IN THE RESTRICTED ECONOMY (0% TRADE-BALANCE IMPROVEMENT)

MARGINAL PROBABILITY DENSITY OF THE DOMESTIC CAPITAL STOCK IN THE RESTRICTED ECONOMY (12% TRADE-BALANCE IMPROVEMENT)
MARGINAL PROBABILITY DENSITY OF THE DOMESTIC CAPITAL STOCK IN THE RESTRICTED ECONOMY (30% TRADE-BALANCE IMPROVEMENT) 

FIGURE 4

MARGINAL PROBABILITY DENSITY OF THE DOMESTIC CAPITAL STOCK IN THE RESTRICTED ECONOMY (60% TRADE-BALANCE IMPROVEMENT) 

FIGURE 5
determine the welfare effect of the policy under study. Following Lucas (1987), these welfare measures are based on compensating variations of constant-consumption paths. These compensating variations are calculated as follows. The solution of the dynamic programming problem for both the benchmark and the restricted economies includes a solution for the value function, $\mathcal{V}^u(K,A,e)$ and $\mathcal{V}^{r-*\bar{A}}(K,A,e)$ respectively. Solutions for these two value functions are calculated for each triple $(K,A,e)$ in the state space $K \times A \times \mathbb{E}$. Using the Stationary Cardinal Utility function presented in equation (1), it is possible to write a non-linear equation that spells out a constant-consumption path that yields the same lifetime utility expressed by each $\mathcal{V}^u(K,A,e)$ and $\mathcal{V}^{r-*\bar{A}}(K,A,e)$. Since each restricted economy must imply a reduced level of welfare, given that they constitute more constrained representations of a distortion-free environment, the level of constant consumption associated with each $\mathcal{V}^{r-*\bar{A}}(K,A,e)$ is lower than the one that represents each $\mathcal{V}^u(K,A,e)$. The percentage difference between these two consumption levels, for each triple in the state space and for each of the four values of $\bar{A}$ considered, is defined here as a compensating variation.

Table 3 presents four alternative measures of the percentage welfare loss, based on the compensating variations, for each one of the restricted economies. First, since given a $\bar{A}$ there is a $\mathcal{V}^u$ and a $\mathcal{V}^{r-*\bar{A}}$ for each $(K,A,e)$, one can study the compensating variations for each state of nature. From these point-wise comparisons, the maximum and minimum percentage welfare losses are listed in columns I and II of the table. It is difficult, however, to establish an overall judgement of the long-run welfare effect of the policy by looking at these point-wise compensating variations, since they vary so much. In fact, maximum welfare losses are always associated with states of
Table 3.
Long-Run Welfare Effects of Stabilizing the Balance of Trade

<table>
<thead>
<tr>
<th>Percentage Change in the Balance of Trade(^a)</th>
<th>Percentage Welfare Loss</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I Maximum(^b)</td>
<td>II Minimum(^b)</td>
<td>III Ex Ante(^c)</td>
<td>IV Ex Post(^d)</td>
</tr>
<tr>
<td>0</td>
<td>35.00</td>
<td>0.006</td>
<td>0.019</td>
<td>0.008</td>
</tr>
<tr>
<td>12</td>
<td>7.00</td>
<td>0.006</td>
<td>0.022</td>
<td>0.009</td>
</tr>
<tr>
<td>30</td>
<td>2.15</td>
<td>0.009</td>
<td>0.072</td>
<td>0.015</td>
</tr>
<tr>
<td>60</td>
<td>3.29</td>
<td>0.016</td>
<td>0.386</td>
<td>0.038</td>
</tr>
</tbody>
</table>

\(^a\)As a percentage of the mean of the balance of trade in the benchmark economy.

\(^b\)Based on point-wise comparisons of the value function for each triple in the state space.

\(^c\)Expected percentage welfare loss calculated with the stationary probability distribution of the benchmark economy.

\(^d\)Expected percentage welfare loss calculated with the stationary probability distribution of the corresponding restricted economy.
nature that have zero long-run probability of occurrence. For example, in the case where the trade balance is stabilized at its average value in the benchmark economy, a 35% welfare loss occurs if the economy is at the lowest \( K \), the lowest \( A \) and the low value of \( e \) when the policy is implemented. But, according to the joint limiting distribution of the state variables in the free-trade economy, the long-run probability of starting out from such a situation is infinitesimal (see Figure 1). In general, the largest welfare losses occur when the initial \( A \), at the date the capital controls are introduced, is driven the longest way to reach \( \bar{A} \). But this implies that the initial \( A \) must have been located at one of the extremes of the foreign-asset grid, and thus the odds of having to implement the policy when the economy is in that particular situation are null.

More illustrative than maximum and minimum welfare losses are the welfare measures that condense the information provided by the list of all the compensating variations and their associated long-run probability. Two measures of this kind are provided in columns III and IV of Table 3. Column III presents an expected percentage welfare loss calculated using the long-run probability of occurrence of each triple \((K,A,e)\) in the free-trade benchmark economy. This measure is termed the \textit{ex ante} welfare loss because it considers the odds of introducing the policy when the economy is situated at some triple \((K,A,e)\) in the free-trade economy. In contrast, column IV lists an expected welfare loss calculated with the joint stationary probability distribution of the state variables in each restricted economy. This \textit{ex post} welfare loss considers the long-run probability of the restricted economy being at some state \((K,\bar{A},e)\) and compares the welfare level obtained in this environment with what similar triples would have provided if trade had not
been controlled. The *ex ante* welfare loss appears to be the most accurate to evaluate the policy because it captures the negative welfare effect that, on the average, is caused by moving the economy from a free-trade environment to a restricted-trade regime. In fact, the *ex post* welfare loss consistently underestimates the *ex ante* loss for each value of $\hat{\theta}$ studied.

Considering the *ex ante* measure, Table 3 suggests that the welfare losses associated to the policy under study are fairly small. If the government sets $\hat{\theta}$ so as to stabilize the trade balance at its mean value in the benchmark economy, the percentage welfare loss is only 0.019%. Even when the policy is designed so as to achieve a 60% trade-balance improvement, the loss in welfare measured in terms of constant consumption is only 0.386%.

The small welfare costs associated with the imposition of capital controls and the stabilization of the balance of trade are consistent with the findings of Lucas (1987) for the welfare cost of business cycles. He encountered that, when the risk-aversion parameter is set to 1 or 5 and the standard deviation of the log of consumption is set to 0.013 or 0.039, the largest cost of consumption instability is about 0.38%\textsuperscript{12}. The logic that explains why the welfare effects of the policy are so small is also consistent with the arguments of Lucas (1987). The disturbances that appear to be responsible for post-war Canadian business cycles are not large and persistent enough to allow capital controls to seriously reduce welfare. Risk-averse individuals wish to insure themselves against the risk of domestic shocks by participating in the world’s financial market, but since this risk is very small, not having unrestricted access to the world market does not hurt them very much. In fact, if the productivity or terms-of-trade disturbances are increased from 1.285% to 2.3%, so that business cycles of the order of 5.0%
standard deviation in GDP are generated, the ex ante welfare loss for the case of a 30% trade-balance improvement only rises from 0.072% to 0.166%. Thus, it appears that fairly large shocks and business cycles would be required to make a policy like the one studied here reduce welfare by a large amount. A similar result was also obtained by Cole and Obstfeld (1988). They encountered that, under certain specifications of tastes and technology, individuals can attain welfare-maximizing equilibria even when trade in international assets is forbidden.

It is important to mention, however, that this analysis focuses only on the role of international trade as a vehicle to optimally allocate consumption through the business cycle in a representative-agent small open economy. It does not consider other instances of international economic relations, such as the role of the transfer and development of technology in long-run growth and the gains from trade for heterogenous agents or multi-sector economies with traded and non-traded goods, in which depriving individuals or sectors from unrestricted access to world markets could have very harmful effects. Still, the investigation is a useful starting point because it illustrates that, without such considerations, the welfare costs associated to capital controls and the stabilization of the balance of trade are quite modest.

4.3 Capital Controls and the Government’s Fiscal Strategy.

The numerical methods applied in this paper can also be utilized to study a fiscal strategy that the government could follow to successfully implement the policy under consideration. By charging the appropriate tax on foreign interest income, and at the same time rebating it in the form of a lump-sum transfer, the government can force individuals to always hold the target level
of foreign assets $\tilde{A}$. Under this fiscal regime, the dynamic programming problem that characterizes the free-trade economy would include a period-by-period constraint of the following form:

$$
C_t = \exp(e^s_t K_t^{2\ell_t} - K_{t+1} + K_t(1-\delta) - (\Phi/2)(K_{t+1} - K_t)^2 + [1+r^*(1-r_t)]A_t - A_{t+1} + T_t.
$$

(12)

Where $r_t$ is the percentage tax on foreign interest income and $T_t$ is a lump-sum transfer. The government sets $T_t = r^* r_t A_t$, but individuals take both the tax and the transfer as exogenously given.

The procedure utilized to calculate the schedule of taxes and transfers that enables the government to set $A_{t+1} = \tilde{A}$, starting from any initial triple $(K_t, A_t, e_t)$, is the following. The solution of the dynamic programming problem of the policy-restricted version of the model delivers an optimal, state-contingent decision rule for domestic capital accumulation that is used to calculate the consumption-based rate of return of a risk-free asset $r_t$:

$$
U^{r^* A}_C(t)/(\exp(-v(t)) E[U^{r^* A}_C(t+1)]) = 1 + r_t.
$$

(13)

Here, $U^{r^* A}_C(t)$ is the lifetime marginal utility of consumption at date $t$ in the restricted economy. This marginal utility includes not only the instantaneous marginal utility, but also the marginal change in the rate of time preference and its effect on the valuation of expected future consumption benefits\textsuperscript{13}. Next, when interest-income taxes are introduced into the free-trade economy, optimizing individuals will allocate consumption so as to equate the marginal rate of substitution between $C_t$ and $C_{t+1}$ with its effective intertemporal relative price $1+r^*(1-r_t)$. Therefore, considering that (13) determines the intertemporal relative price of consumption that must
prevail in the free-trade environment if $A_{t+1}$ is to be set at $A$, it follows that when the government sets $r_t$ so as to make $1+r^*(1-r_t)-1+r_t$, it will succeed in implementing the desired policy. This implies that the government's fiscal strategy is to set $r_t=1-r_t/r^*$ and $T^t=r^*A_t$. By rebating the proceeds of the tax in the form of a lump-sum transfer, this fiscal strategy ensures that no alterations in the value of initial wealth are caused, and hence that no other changes in the behavior of the aggregates than the ones discussed in 4.1 will take place.

Clearly, since $r_t$ depends on the initial state $(K_t, A_t, e_t)$, the tax or subsidy that needs to be charged or paid is also state contingent. Thus, although the individuals take the tax rate as given, this is in fact a random variable. Table 4 presents some of the statistical moments that characterize the stochastic process of the schedule of taxes on foreign interest income for each $A$ target. Graphs illustrating the complete tax schedule for the cases of 30% and 60% trade-balance improvements under favorable and unfavorable disturbances are produced in Figures 6-9.

The mean values listed in the first row of Table 4 suggest that the average tax rate increases with the size of the desired improvement in the trade-balance surplus. If the goal is to stabilize the balance of trade at its average value in the benchmark economy, so that in fact $A$ is at the center of the marginal limiting distribution of foreign assets in the free-trade economy, the average tax rate is close to 0%. As the capital controls are tightened, the mean tax rises gradually to reach about 3% in the case that a 60% trade-balance improvement is desired. Thus, on the average, the taxes or subsidies required to enforce capital controls and stabilize the balance of trade appear to be small, regardless of the desired trade-balance goal.
Table 4.
Statistical Moments that Characterize the Stochastic Process of the Foreign Interest Income Tax.

<table>
<thead>
<tr>
<th>Statistical Moments</th>
<th>Percentage Change in the Balance of Trade&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0002</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.1672</td>
</tr>
<tr>
<td>First-Order Autocorrelation</td>
<td>-0.0691</td>
</tr>
<tr>
<td>Correlation with GDP</td>
<td>-0.1403</td>
</tr>
</tbody>
</table>

<sup>a</sup>As a percentage of the mean of the balance of trade in the benchmark economy.
TAX RATES WITH FAVORABLE SHOCK
FOR A 60% INCREMENT IN TRADE SURPLUS

TAX RATES WITH UNFAVORABLE SHOCK
FOR A 60% INCREMENT IN TRADE SURPLUS
The analysis of the average tax rates contrasts with the tax rates that Figures 6-9 illustrate. The reason is that these graphs depict the required tax, or subsidy, that needs to be imposed to implement the policy starting from every possible initial state of nature contained in the state space. For example, if in the case that a 60% trade-balance improvement is desired the economy starts at $K=3.65$, $A=-0.23$ and experiences a favorable productivity shock, Figure 6 shows that approximately a 63% tax rate must be imposed. However, these large tax rates need only be imposed for one period, because the next date the economy starts at $A=A^*=1.14$ and will never deviate from this coordinate in the foreign-asset-holding grid. In the long run, only points where $A=A^*$ have non-zero probability of occurrence, so that the one-time large taxes or subsidies have no effect on the statistical moments reported in Table 4. In fact, as the four graphs illustrate, around the area where $A=A^*$, the schedule of taxes is always relatively flat and close to zero.

The standard deviations and first-order autocorrelation coefficients reported in Table 4 indicate that the limiting distribution of the foreign interest-income tax approximately preserves its variability and persistence as the trade-balance target is raised. There is, however, a tendency for the standard deviation to increase by a very small amount and for the first-order autocorrelation coefficient to fall slightly. In general, the standard deviation appears somewhat large compared with that of the macro-aggregates listed in Table 2, but the serial autocorrelation is very close to zero in all four cases reported.

The output-correlation coefficients listed in the last row of Table 4 indicate that the tax exhibits weakly countercyclical behavior. This result appears to follow from the weak countercyclical time-path of the trade balance.
in the benchmark economy (see Table 1). The negative correlation between \( r \) and GDP increases from -0.14, in the case where no trade-balance improvement is planned, to -0.09 in the case where a 60% improvement is the goal.

This analysis of the government's fiscal strategy suggests that, in the long run, the authorities can achieve their goal of restricting foreign-asset trading to stabilize the balance of trade in a relatively easy manner. Although some high taxes or subsidies may be needed initially, in the stochastic steady-state the tax on foreign interest income has a low mean, a constant variance, is almost serially uncorrelated and exhibits weakly countercyclical behavior.

5. Concluding Remarks.

This paper analyzed, within the context of a dynamic stochastic model of a small open economy, the macroeconomic effects of a policy that utilizes capital controls to stabilize the balance of trade. The effects of this policy on economic activity and economic welfare were determined by employing numerical techniques recently applied in closed-economy dynamic macroeconomic models. The same numerical methods were also used to study a fiscal strategy that would allow the government to successfully implement its policy.

The quantitative results indicated that the policy considered has almost no effect on the equilibrium stochastic processes that describe output, consumption and the supply of labor. The dynamic processes of savings, investment and the capital stock are significantly altered, however, because once foreign-asset trading is restricted, the domestic capital stock is the only vehicle that can be used to reallocate consumption intertemporally. The analysis also determined that the policy has minimal welfare effects, measured
in terms of percentage changes in constant-consumption paths. These results suggest that the kind of productivity or terms-of-trade disturbances that explain observed business cycles in post-war Canada, according to the model, are too small and transitory to allow the policy to cause large changes in both the evolution of some of the macro-aggregates and the level of welfare. The analysis of a feasible fiscal strategy that forces individuals to hold the target level of foreign assets suggested that the policy under consideration can be easily implemented.

These results proved to be robust to changes in the desired target levels of foreign-asset holdings or the surplus in the balance of trade, and they also appeared to remain relatively unchanged for business cycles in which output is almost twice as variable as in the actual data. Thus, it is concluded that much larger disturbances and business cycles would be required for capital controls and trade-balance targeting to cause large effects on economic activity and very harmful effects on economic welfare.
1. The information set needed to formulate the rational expectation defined in (1) is discussed later in the text.

2. Frenkel and Razin (1987) study how a well-defined steady-state equilibrium can also be obtained by assuming that individuals face a positive probability of dying each period.

3. The functions (2) and (3) were formulated so as to satisfy the conditions from Theorem 5 in Epstein (1983). This theorem proves that, under the appropriate conditions, the Stationary Cardinal Utility satisfies the requirements of dynamic programming and makes consumption in any period behave as a normal good. Since consumption in the present model can be redefined in terms of the composite C-G(L), it is easy to show that Theorem 5 continues to hold. Also, Theorems 3 and 4 by the same author established that the same conditions, added to either a neoclassical production function or a linear technology, are sufficient to guarantee the existence of a stationary limiting distribution of the state variables.

4. The relevance of capital-adjustment-costs in small open-economy real business cycle models was explored by Mendoza (1989b). If capital accumulation does not bear explicit adjustment costs, the desire of individuals to equalize the expected marginal-utility-weighted rates of return paid on domestic capital and foreign assets causes investment in the artificial economy to exhibit too much variability and too little comovement and persistence, relative to actual moments.

5. The assumption that \( r^* \) is non-random is introduced for simplicity, but is not a trivial one. A fluctuating interest rate introduces additional consumption-substitution and consumption-smoothing effects. However, interest-rate shocks with less than 5% standard deviation did not have a significant impact in the moments that characterize the equilibrium process of the unrestricted economy. This is because, as formally shown in Mendoza (1988), the income and substitution effects caused by shocks to \( r^* \) are very weak as long as these disturbances are small and stationary, the interest rate is low and foreign-interest income is a small component of total output. In the present case, the interest rate is set to 4% and Canadian foreign-interest payments are only 2% of GDP.

6. This specification of the financial structure ignores the role of international trade in contingent claims as a form of risk sharing. However, the introduction of a risk-less international asset still allows individuals to ensure themselves against the risk of fluctuations in domestic productivity. Furthermore, recent findings by Cole and Obstfeld (1988) suggest that, for certain specifications of preferences and technology, the competitive allocations are independent of the completeness of international financial markets.

7. Although Chamberlain and Wilson (1984) showed that solvency restrictions may take complicated forms in stochastic models, in the numerical investigation performed here the conditions from Theorem 4 in Epstein...
(1983) were successfully utilized to enforce long-run solvency. In this case, these conditions are:

\[ \exp[\nu(0)] < 1+r^* \quad \text{and} \quad \lim_{\gamma \to \infty} \exp[\nu(C-G(L))] > 1+r^*. \]

Long-run solvency has been verified by noting that the stationary probability of setting foreign-asset holdings below -1.14 is infinitesimal. Thus, the usual solvency requirement that \( \lim_{t \to \infty} [A_t/(1+r^*)^t] = 0 \) holds for every \( A_t \) that has non-zero long-run probability of being reached from any initial triple \((K_0, A_0, e_0)\).

8. The parameter \( \alpha \) is determined with the long-run average of the ratio of labor income to net national income at factor prices. The value of \( \delta \) is the one commonly used in the real business cycle literature, with it the model mimics the actual long-run average of the investment-output ratio. \( T_{\eta \epsilon} a_{\nu \epsilon} \gamma \omega \) is in the range of the estimates of the intertemporal elasticity of substitution in labor supply \((1/\omega-1)\) obtained by Macurdy (1981) and Heckman and Macurdy (1980, 1982). The interest rate \( r^* \) is set to the annual equivalent of the value suggested by Kydland and Prescott (1982) for the U.S. economy. The risk-aversion parameter \( \gamma \) is in the range of the estimates obtained by Hansen and Singleton (1983) and Friend and Blume (1975). The value of \( \beta \) is determined using the long-run average of the GNP/GDP ratio and the other parameter values, so as to ensure that in the deterministic steady-state the rate of time preference equals \( r^* \).

9. A detailed analysis of the ability of the free-trade model to mimic and explain Canadian business cycles is carried out in Mendoza (1989a) and (1989b).

10. The model predicts almost perfect positive correlation between \( C, S \) and GDP because the small open-economy assumption, along with the non-random interest rate, eliminates the intertemporal consumption-substitution effect. Also, \( L \) exhibits the same persistence and output correlation as GDP because of the Cobb-Douglas structure of the production function and the instantaneous-utility and time-preference functions defined in (2) and (3).

11. Standard deviations, instead of percentage standard deviations, have been considered in order to abstract from the effects of changes in the mean of each variable that do not affect their variance.

12. The analysis by Lucas (1987) was performed by imposing a stochastic consumption stream, with trend and cycle components, to an isoelastic, time-separable utility function.

13. Since the percentage standard deviation of the subjective discount factor in the four restricted-trade economies is less than 0.01%, the impatience-effect caused by changes in the rate of time preference is likely to be very small.
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