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AN EMPIRICAL ANALYSIS

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Abstract

This paper investigates whether the poor empirical performance of intertemporal asset-pricing relationships, which has been documented in the literature, stems from the fact that they have been derived from barter-economy models. This question is addressed here by systematically estimating and testing the Euler equations governing asset choices that are derived from alternative barter- and monetary-economy models. The generalized-method-of-moments (GMM) estimation technique and monthly data on the US economy over the period 1959:02 - 1985:12 are employed. The estimation findings reaffirm the lack of support for the barter-economy specification embodying the conventional end-of-period-timing assumption in respect to consumption and investment choices. Furthermore, this finding is robust to a more conservative assumption concerning agents' information sets than has been made in prior work. With regard to the monetary models: the alternative money-in-the-utility function models considered find modest empirical support; the Lucas (1982) cash-in-advance model is rejected by the data; and the Lucas (1984)/Svensson (1985a) cash-in-advance models find considerable empirical support. These findings are suggestive of the importance of monetary considerations in the determination of asset prices. An alternative interpretation of the cash-in-advance asset-pricing relationships, in terms of a barter-economy asset-pricing specification embodying a start-of-period-timing convention for consumption and investment choices, is also suggested.

Predictions of real asset returns over the 1982:1 - 1986:8 period are generated from the Lucas (1984)/Svensson (1985a) model and the 'end-of-period' barter model, under a distributional assumption with respect to the
time-series properties of consumption, inflation and asset returns. Although the predictions from both models are quite poor, those of the former model come much closer to capturing the volatility of actual real asset returns than do those of the latter model.
I. Introduction

In recent years a great deal of attention has been devoted to the study of asset pricing. Of particular interest are the real, dynamic, general-equilibrium (GE) models of asset-pricing in the pure-exchange economies of Rubinstein (1976), Lucas (1978) and Breeden (1979). In these models, agents choose their consumption and investment plans so as to maximize an infinite discounted-sum of expected utility from consumption. The essential implication of this view is that the stochastic time-series properties of asset returns are intimately related to the stochastic time-series properties of consumption and to the degree of risk aversion of the investor.

This implication of the real, dynamic, pure-exchange GE models has been the subject of extensive empirical testing:-

Assuming a constant relative risk aversion (CRRA) utility function defined over a single (nondurable) consumption good, the estimation results of Hansen and Singleton (1982, 1983, 1984) reject the model.¹ In addition, the simulation results of Mehra and Prescott (1985) show that the model cannot generate equity premia even close to those observed.

Allowing for a nonseparable utility function of both nondurable- and durable-goods purchases, substantially modifies the dynamics of the consumption-asset return relationship. Following this approach, the model was tested by: Dunn and Singleton (1986) assuming a CRRA utility function and by Eichenbaum and Hansen (1987) assuming, alternatively, a quadratic and S-branch utility function; while Singleton (1985) tested a version of the model assuming a CRRA utility function against one with quadratic preferences. Along similar lines, Eichenbaum, Hansen and Singleton (1986) allow for a nonseparable CRRA utility function of both nondurable-goods purchases and
leisure choices. The empirical findings show little support for any of these models.

Epstein and Zin (1987) consider alternative specifications of preferences over a single (nondurable) consumption good which generalize the conventional time-additive expected utility function underlying the above studies. These generalizations permit a clear separation of behaviour with respect to risk and intertemporal substitution. The findings of this study show mixed support for the model.

Hansen and Singleton (1987) and Grossman, Melino and Shiller (1985) test a model assuming a CRRA utility function defined over a single (nondurable) consumption good, taking into account the temporal aggregation problem which arises when the agent's decision interval is shorter than the observation interval for measured consumption data. Once again little empirical support is found for the model.²

This paper investigates whether the aforementioned poor empirical performance of intertemporal asset pricing relationships stems from the fact that they have been derived from barter economy models. More specifically, the paper investigates, at an empirical level, whether the possible non-superneutrality of money affects the dynamics of consumption and asset-return relationships in a significant fashion. As Townsend (1987) points out: "Of course, the idea more generally is that the market imperfections or trading difficulties which give rise to money may well have implications for asset prices" (p. 222, Townsend 1987).³, ⁴

Currently, money is popularly introduced into dynamic GE models in one of the following ways: (i) by assuming real money balances enter as an argument in agents' utility functions; (ii) by imposing cash-in-advance constraints on agents' purchases of goods; and (iii) by allowing money as the sole means of
effecting intergenerational transactions (i.e. the overlapping-generations (OLG) model of money). Each of these methods has its advantages and disadvantages. Here we focus on monetary models which fall within the compass of (i) and (ii). The OLG model neglects the transactions demand for money - probably its main raison d'être - and is unlikely to serve as a useful basis for the econometric analysis of short-run time series data.

It is interesting to note that the potentially fundamental difference in the determination of asset prices and returns across barter and monetary economies has been highlighted in two recent theoretical studies. Abel (1987) uses the Lucas (1978) barter economy model to analyze, inter alia, the effects of changes in the conditional variance of the (exogenous) dividend process on asset prices. Giovannini (1987), on the other hand, uses the Svensson (1985a) cash-in-advance model and shows that changes in the conditional variance of the (exogenous) dividend process will have precisely the **opposite** effects on asset prices to those found in Abel (1987).

The remainder of the paper is organized as follows: Section II specifies and discusses the stochastic Euler equations linking consumption and asset returns, which serve as the basis for the empirical work. Section III discusses the estimation technique and the data. Section IV presents the estimation results. Section V contains a diagnostic and prediction analysis and Section VI concludes the paper.

II. Theoretical Background

In this section the stochastic Euler equations, governing agents' asset choices, which are derived alternatively from (a) a barter economy model, (b) money-in-the-utility function models and (c) cash-in-advance models are specified and discussed.
(a) A Barter Economy Model

Here we consider the barter economy model underlying the empirical analyses of Hansen and Singleton (1982, 1983, 1984). This model serves as a useful benchmark for comparison with the alternative monetary models.

In particular, consider an environment in which the representative agent has preferences defined over stochastic processes of consumption given by:

\[ (1) \quad E \sum_{t=0}^{\infty} \beta^t u(c_t) \]

where

\[ u(c_t) = \frac{c_t^\gamma}{\gamma} - 1 \quad \text{for } \gamma \neq 0 \]

or

\[ u(c_t) = \ln c_t \quad \text{for } \gamma = 0 \]

\[ 0 < \beta < 1, \quad \gamma < 1 \]

and where: \( E \) denotes the expectations operator conditioned on information at time 0, \( \beta \) is the subjective discount factor, \( u(\cdot) \) denotes the momentary utility function, \( c_t \) is consumption at time \( t \), \( \ln \) denotes the natural logarithm and \( \gamma \) is a preference parameter (\( [1-\gamma] \) is the coefficient of relative risk aversion). Assume that the agent maximizes (1) by choosing \( c_t \) and \( s_t \) subject to the budget constraint:

\[ (2) \quad c_t + \tilde{a}_t s_t = (\tilde{a}_t + \tilde{d}_t) s_{t-1} + y_t \]

where: \( s_t \) denotes shares (of assets) bought at time \( t \), \( \tilde{a}_t \) is the real share price at time \( t \), \( \tilde{d}_t \) is the real time-\( t \) dividend and \( y_t \) is endowment-income at time \( t \). This optimization problem implies the following stochastic Euler equation governing share choices:

\[ (3) \quad u_c(\cdot t) = \beta E \left[ u_c(\cdot t+1) \left( \frac{\tilde{a}_{t+1} + \tilde{d}_{t+1}}{\tilde{s}_t} \right) \right] \]
where: \( u_c(\cdot t) \equiv c_t^{y-1} \), is the marginal utility of consumption at time \( t \); \( E_t \) is the expectations operator conditioned on time \( t \) information. By acquiring an additional share at time \( t \) the agent foregoes time-\( t \) consumption, the marginal cost of which is measured by \( u_c(\cdot t) \tilde{a}_t \). The payoff from this share occurs at time \((t+1)\), permitting the agent to increase consumption at that time - the expected marginal benefit of which is measured by

\[
\beta E_t \left[ u_c(\cdot t+1) (\tilde{a}_{t+1} + d_{t+1}) \right].
\]

Equation (3), therefore, sets the marginal cost equal to the expected marginal benefit of acquiring an extra share.

Note that for empirical purposes (3) is rewritten as:-

(4) \( u_c(\cdot t) = \beta E_t \left[ u_c(\cdot t+1) \frac{p_t}{p_{t+1}} \left( \frac{a_{t+1} + d_{t+1}}{a_t} \right) \right] \)

where: \( p_t \) denotes the price level at time-\( t \), \( a_t \) is the nominal share price at time \( t \) and \( d_t \) is the nominal time-\( t \) dividend. In addition, prior to empirical work it is necessary to adopt a convention as to when agents consume and invest within each period. The empirical studies referred to in the introduction assume that agents consume and invest at the end of each period.

In this case the relevant version of (4) is:

(5) \( u_c(\cdot t) = \beta E_t \left[ u_c(\cdot t+1) \frac{p_t}{p_{t+1}} \left( \frac{a^e_{t+1} + d^e_{t+1}}{a^e_t} \right) \right] \)

where: \( a^e_t \) is the nominal end-of-period-\( t \) share price and \( d^e_t \) is the nominal end-of-period-\( t \) dividend. Equation (5) will be referred to as the end-of-period barter model (or barter -e). Alternatively, if agents are assumed to consume and invest at the start of each period, the relevant version of (4) is:

(6) \( u_c(\cdot t) = \beta E_t \left[ u_c(\cdot t+1) \frac{p_t}{p_{t+1}} \left( \frac{a^s_{t+1} + d^s_{t+1}}{a^s_t} \right) \right] \)

or equivalently:
\( u_c(\cdot t) = \beta E \left[ u_c(\cdot t+1) \frac{P_t}{P_{t+1}} \left( \frac{a_t^e + d_t^e}{a_{t-1}} \right) \right] \)

where: \( a_t^s \) is the nominal start-of-period-\( t \) share price and \( d_t^s \) is the nominal start-of-period-\( t \) dividend and \( E \) denotes the expectations operator conditioned on information set \( \Theta_t \), which includes all information through time (\( t-1 \)), \( c_t \) and \( P_t \). Equation (6) or (6') will be referred to as the start-of-period barter model (or barter-s). It is interesting to estimate and test both (5) and (6'), since as will be indicated in subsection (c) below, (6') is in fact observationally equivalent to the stochastic Euler equation for share choices derived from the Lucas (1982) cash-in-advance model.

Finally, note that if one argues that agents receive information on \( c_t \) and \( P_t \) with a one-period lag - an argument possibly justified on grounds of measurement problems with this data - then instead of equations (5) and (6') we, respectively, get:

\[ E \left[ \frac{u_c(\cdot t)}{P_t} - \frac{\delta u_c(\cdot t+1)}{P_{t+1}} \left( \frac{a_{t+1}^e + d_{t+1}^e}{a_t^e} \right) \right] = 0 \]

where: \( E \) is the expectations operator conditioned on information set \( I_t \), which includes all information through to time (\( t-1 \)), \( a_t^e \) and \( d_t^e \).

\[ E \left[ \frac{u_c(\cdot t)}{P_t} - \frac{\delta u_c(\cdot t+1)}{P_{t+1}} \left( \frac{a_t^e + d_t^e}{a_{t-1}} \right) \right] = 0 \]

where: \( E \) is the expectations operator conditioned on time (\( t-1 \)) information. Equation (7) [(8)] will be referred to as the barter-e [barter-s] model with lagged information. It is also interesting to estimate and test both (7) and (8), since as will be shown in subsection (c) below, (8) is observationally equivalent to the stochastic Euler equation for share

(b) Money-In-The-Utility Function Models

Here we consider two alternative formulations of money-in-the-utility-function (MIUF) models. The first one is essentially based on Dixit and Goldman (1970), Fama and Farber (1979), LeRoy (1984a, b) and Stulz (1983) - in which it is assumed that agents' current choices of money currently yield direct utility. This model will be referred to as the contemporaneous - MIUF model. The second version is, in essence, based on Danthine and Donaldson (1986) - in which it is assumed that agents' current choices of money yield direct utility next period. This model will be referred to as the lagged - MIUF model. By including money in the utility function a rationalization of the transactions, precautionary and store-of-value demand for money is permitted.

First, in the contemporaneous - MIUF model, the representative agent has preferences defined over stochastic processes of consumption and real money balances given by:

\[
E \sum_{t=0}^{\infty} \beta^t u(c_t, M_t/P_t) \]

where, \( u(c_t, M_t/P_t) = \frac{\delta}{\gamma} \left( \frac{M_t}{P_t} \right)^{(1-\delta)} - 1 \) for \( \gamma \neq 0 \)

or \( u(c_t, M_t/P_t) = \delta \ln c_t + (1-\delta) \ln (M_t/P_t) \) for \( \gamma = 0 \)

\( 0 < \delta < 1, 0 < \beta < 1, \gamma < 1 \)
and where the new notation is: $M_t$ - money balances at time $t$, and $\delta$ is a preference parameter capturing the relative importance of consumption and real money balances in the utility function.\footnote{8} The agent is assumed to maximize (9) by choosing $c_t, s_t$ and $\bar{M}_t$ subject to the budget constraint:

\[(10) \quad \frac{p_t c_t}{p_t} + a_t s_t + \bar{M}_t = (a_t + d_t) s_{t-1} + \bar{M}_{t-1} + P_t y_t\]

where all notation has been previously defined. This optimization problem implies the following stochastic Euler equations, respectively, governing share and money choices:

\[(11) \quad u_c(\cdot t) = \beta E_{t+1} \left[ u_c(\cdot t+1) \frac{p_t}{p_{t+1}} \left( \frac{a_{t+1} + d_{t+1}}{a_t} \right) \right] \]

\[(12) \quad u_c(\cdot t) = u_{M/P}(\cdot t) + \beta E_{t} \left[ u_c(\cdot t+1) \frac{p_t}{p_{t+1}} \right] \]

where: $u_c(\cdot t) \equiv \delta \tilde{c}_t \frac{\tilde{M}_t}{P_t} (1-\delta) \gamma$

and $u_{M/P}(\cdot t) \equiv (1-\delta) \tilde{c}_t \left( \frac{\tilde{M}_t}{P_t} \right) (1-\delta) \gamma - 1$

i.e. the marginal utility of consumption and of real money balances at time $t$. Equation (11) has a similar interpretation as equation (3) and sets the marginal cost equal to the expected future marginal benefit of acquiring an additional share - with the marginal cost being measured by $u_c(\cdot t)(a_t/P_t)$ and the expected future marginal benefit by $\beta E_{t} \left[ u_c(\cdot t+1) \frac{a_{t+1} + d_{t+1}}{p_{t+1}} \right]$. With regard to money choices, notice that by acquiring an additional nominal money unit at time $t$, the agent foregoes time-$t$ consumption - the marginal cost of which is given by $u_c(\cdot t)/P_t$ - and increases time-$t$ money balances -
the marginal benefit of which is given by \( u_{M/P}(\cdot t)/P_t \). In addition, the extra nominal money unit is available to permit increased consumption at time \((t+1)\) - the expected future marginal utility of which is measured by

\[ \delta \mathbb{E} [u_c (\cdot t+1)/P_{t+1}] \].

Accordingly, equation (12) is seen to set the marginal cost equal to the sum of the current and expected future marginal benefit of acquiring an extra unit of money.

Turning to the lagged - MIUF model: in this case the representative agent is assumed to maximize:

\[
(13) \quad \mathbb{E} \sum_{t=0}^{\infty} \delta^t u \left( c_t, \frac{M_{t-1}}{P_t} \right)
\]

where, \( u \left( c_t, \frac{M_{t-1}}{P_t} \right) = \frac{\delta}{\gamma} \frac{c_t^{\gamma} (M_{t-1}/P_t)^{1-\gamma}}{\gamma} - 1 \quad \text{for } \gamma \neq 0 \)

or \( u \left( c_t, \frac{M_{t-1}}{P_t} \right) = 5 \ln c_t + (1-5) \ln \left( \frac{M_{t-1}}{P_t} \right) \quad \text{for } \gamma = 0 \).

by choosing \( c_t, z_t, \) and \( M_t \) subject to the budget constraint given by (10) above. This optimization problem implies the following stochastic Euler equations for share and money choices respectively:

\[
(14) \quad u_c (\cdot t) = \mathbb{E} \left[ u_c (\cdot t+1) \frac{P_t}{P_{t+1}} \frac{a_{t+1} + d_{t+1}}{a_t} \right]
\]

\[
(15) \quad u_c (\cdot t) = \mathbb{E} \left[ u_{M/P}(\cdot t+1) + u_c (\cdot t+1) \frac{P_t}{P_{t+1}} \right]
\]

where: \( u_c (\cdot t) = \delta c_t^{\delta - 1} \left( \frac{M_{t-1}}{P_t} \right)^{(1-5)\gamma} \)

and \( u_{M/P} (\cdot t+1) = (1-\delta) c_{t+1}^{\delta} \left( \frac{M_t}{P_{t+1}} \right)^{(1-5)\gamma-1} \)
i.e. the marginal utility of consumption at time $t$ and of real balances at time $(t+1)$. Equation (14) has the same interpretation as equation (11) - note though that $u_c(·t)$ differs across the equations. With respect to money choices, notice that acquiring an extra nominal money unit at time $t$ involves a reduction of time-$t$ consumption - the marginal cost of this is $u_c(·t)/P_t$. The increased time-$t$ money balances directly increases utility at time $(t+1)$ - in particular, the expected direct future marginal utility of the increase in $M_t$ is given by $BE_t\left[u_M/P(·t+1)/P_{t+1}\right]$. Also the additional money unit allows increased consumption at time $(t+1)$ - the expected future marginal utility of this is given by $BE_t\left[u_c(·t+1)/P_{t+1}\right]$. Thus, (15) sets the marginal cost equal to the expected future marginal benefit from holding an extra unit of money.

For both the contemporaneous- and lagged- MIUF models it is assumed here that agents consume and invest at the end of each period. Accordingly, we set: $a_{t+1}^e = a_{t+1}^e$, $d_{t+1}^e = d_{t+1}^e$, $a_t = a_t^e$ and measure $M_t$, $M_{t-1}$ by end-of-period $t$ and $(t-1)$ money stocks, respectively. Finally, note that each pair of Euler equations for the MIUF models (i.e. equations (11) and (12); equations (14) and (15)) are jointly estimated and tested in this study.

\begin{c}\textbf{Cash-In-Advance Models}\end{c}

Here we sequentially consider three cash-in-advance (CIA) models which are based on: Lucas (1982), Lucas (1984) and Svensson (1985a). In Lucas (1982) each period is envisaged as comprising two subperiods. During the first subperiod only asset markets are open and during the second subperiod only goods markets are open. Agents trade money and assets during the asset market subperiod. Shares held at the beginning of this subperiod
entitle the owner to dividends from the sale of goods during the previous goods market subperiod - which is last period. Agents receive their endowment and trade goods during the goods market subperiod. Goods must be bought with money acquired in advance. Money which is not currently spent on goods will enter as a component of wealth at the beginning of the following period. This assumed sequencing of transactions is reflected in the representative agent's budget constraint:

\[(16) \quad a_t s_t + M_t = (a_t + d_{t-1}) s_{t-1} + M_{t-1} - P_{t-1} c_{t-1} + P_t v_t\]

The CIA constraint is:

\[(17) \quad M_t \geq P_t c_t\]

All notation is as previously defined. Assume next that the agent receives full-current information at the beginning of time \(t\) and maximizes (1) by choosing \(c_t, s_t, M_t\) subject to (16) and (17). This optimization problem implies the following stochastic Euler equation governing share choices:\(^{12}\)

\[(18) \quad u_c(\cdot t) = \mathbb{E}_t\left[u_c(\cdot t+1) \frac{P_t}{P_{t+1}} \left(\frac{a_{t+1} + d_t}{a_t}\right)\right]\]

where: \(u_c(\cdot t) \equiv c_t^{\gamma - 1}\), the marginal utility of consumption at time \(t\). By acquiring an additional share at the beginning of time \(t\), the agent foregoes money holdings at the start of time \(t\) and therefore foregoes consumption at the end of time \(t\) - the marginal cost is, thus, given by \(u_c(\cdot t)(a_t/P_t)\). The payoff from this share - \((a_{t+1} + d_t)\) - occurs at the beginning of time \((t+1)\), permitting the agent to increase money holdings at that time and hence consumption at the end of time \((t+1)\). The expected
future marginal benefit is measured by \( B \mathbb{E} \left[ u_c(t+1) \frac{a_{t+1} + d_t}{P_{t+1}} \right] \).

Equation (18), accordingly, is seen to set the marginal cost equal to the expected future marginal benefit of acquiring an additional share.

Given the timing convention adopted in Lucas (1982), it follows that the empirical formulation of (18), using our earlier notation, is:

\[
(19) \quad u_c(t) = B \mathbb{E} \left[ u_c(t+1) \frac{P_t}{P_{t+1}} \left( \frac{a_{t+1}^s + d_t^e}{a_t^s} \right) \right]
\]

or equivalently:

\[
(19') \quad u_c(t) = B \mathbb{E} \left[ u_c(t+1) \frac{P_t}{P_{t+1}} \left( \frac{a_t^e}{a_{t-1}^e} \right) \right]
\]

Notice that the equation (19') is observationally equivalent to equation (6'), i.e. the barter-s model.\(^{13}\)

Also note that the assumption of full-current information in Lucas (1982) results in a situation in which agents acquire, within each period, exactly the amount of cash they need to finance that period's consumption. In equilibrium it follows that the demand for money is given by the simple quantity theory equation.\(^{14}\) Equation (18) (or (19)) is thus consistent with a framework in which the income velocity of money is equal to unity and where there is only a transactions demand for money.

Lucas (1984) differs from Lucas (1982) in one fundamental respect, viz: in the former agents are not assumed to receive full current information at the beginning of time \( t \). Rather, agents are assumed to have only partial current information - specifically information on current asset prices - at the beginning of the time-\( t \) asset market subperiod, while full-current information is received at the beginning of the time-\( t \) goods market subperiod.
Under this informational assumption, the representative agent is viewed as maximizing (1) subject to (16) and (17). The implied stochastic Euler equation governing share choices is:

\[
(20) \quad E_t \left[ \frac{u_c(\cdot|t)}{P_t} - B \frac{u_c(\cdot|t+1)}{P_{t+1}} \left( \frac{a_{t+1} + d_t}{a_t} \right) \right] = 0
\]

where: $E_t$ is the expectations operator conditioned on information set $\Omega_t$, which includes all information through to time $(t-1)$ and $a_t$. Equation (20) has the same interpretation as equation (18) and sets the marginal cost equal to the future marginal benefit of acquiring an additional share — but (20) must be conditioned on the information set comprising $(t-1)$ and $a_t$ information, in contrast to the full current information set used in (18).

With the timing convention adopted in Lucas (1984), the empirical formulation of (20) is:

\[
(21) \quad E_t' \left[ \frac{u_c(\cdot|t)}{P_t} - B \frac{u_c(\cdot|t+1)}{P_{t+1}} \left( \frac{a_{t+1} + d_t}{a_t} \right) \right] = 0
\]

where $E_t'$ denotes the expectations operator conditioned on information set $\Omega_t'$, which includes all information through to time $(t-1)$ and $a_t^s$; or equivalently:

\[
(21') \quad E_{t-1} \left[ \frac{u_c(\cdot|t)}{P_t} - B \frac{u_c(\cdot|t+1)}{P_{t+1}} \left( \frac{a_{t+1} + d_t}{a_t} \right) \right] = 0
\]

It is interesting to note that equation (21') is observationally equivalent to equation (8) i.e. the barter-s model with lagged information.

The difference in the timing of information flows across Lucas (1982) and Lucas (1984) is an important one. In particular, since in Lucas (1984) agents must choose their money holding prior to knowing the current state and hence before they know their current consumption, they do not, in general, end up acquiring exactly the amount of money needed to finance that consumption.
This gives rise to a combined transactions, precautionary and store-of-value demand for money. Equation (20) (or (21)) is thus consistent with a framework which gives a more reasonable specification of the demand for money than does Lucas (1982).

In Svensson (1985a) each period also comprises two subperiods. The sequencing of markets within the period is precisely the reverse of that in Lucas (1982 and 1984). In particular, the goods market opens first and the asset market last within each period. Again, agents receive their endowment and trade goods during the goods market subperiod and goods can only be bought with money acquired in advance. During the asset market subperiod, agents trade money and assets. Shares held at the beginning of this subperiod entitle the owner to dividends from the sale of goods during the previous goods market subperiod - which lies within the current period in this model. Money balances chosen during the asset market subperiod enter as a component of wealth at the beginning of the following period. This sequencing of transactions gives rise to the budget constraint:

\[ p_t c_t + m_t + a_t s_t = (a_t + d_t) s_{t-1} + m_{t-1} + p_t y_t \]

and to the CIA constraint:

\[ m_{t-1} \geq p_t c_t \]

Assuming that the agent receives full-current information at the beginning of time \( t \), and maximizes (1) subject to (22) and (23) gives the following stochastic Euler equation governing share choices:

\[ E_t \left[ \frac{u_c (s+t+1)}{p_{t+1}} - B \frac{u_c (s+t+2)}{p_{t+2}} \frac{(a_{t+1} + d_{t+1})}{a_t} \right] = 0 \]

where: \( u_c (s+t) = c^{-1}_t \), the marginal utility of consumption at time \( t \). By acquiring an extra share at the end of time \( t \), the agent foregoes money holdings at that time and therefore foregoes consumption at the beginning of
time \((t+1)\) - the expected future marginal cost is, thus, given by \(a_t C_t \left[ u_c(\cdot t+1)/P_{t+1} \right] \). The payoff from this share - \((a_{t+1} + d_{t+1})\) - occurs at the end of time \((t+1)\), permitting the agent to augment money holdings at that time and hence increase consumption at the beginning of time \((t+2)\). The expected future marginal benefit is measured by \(BE_t \left[ u_c(\cdot t+2) \frac{a_t + d_t}{P_{t+2}} \right] \).

Thus, (24) sets the expected future marginal cost equal to the expected future marginal benefit from acquiring an extra share.

On first sight, (24) seems very different to (20) - the Lucas (1984) Euler equation. Thinking about the sequencing of transactions embedded in both reveals, however, that they capture exactly the same intertemporal trade-off when choosing shares. This is, perhaps, all the more evident when we note that the timing convention in Svensson (1985a) results in the following empirical formulation of (24):

\[
(25) \quad E_t \left[ \frac{u_c(\cdot t+1)}{P_{t+1}} - \beta \frac{u_c(\cdot t+2)}{P_{t+2}} \frac{(a^e_{t+1} + d^e_{t+1})}{a^e_t} \right] = 0
\]

Equation (25) is simply a one-period updated version of the Lucas (1984) empirical formulation i.e. equation (21').

Not surprisingly, equation (24) (or (25)) is also consistent with a framework which gives rise to a combined transactions, precautionary and store-of-value demand for money. Although agents are assumed to receive full-current information at the beginning of the period - as in Lucas (1982) and in contrast to Lucas (1984) - the sequencing of markets, as mentioned above, within the period is the reverse of that of Lucas (1982 and 1984). Hence, agents must choose their money holdings prior to knowing the consumption which these holdings can finance - the same story as Lucas (1984).

This highlights that the difference in defining periods is not important.
Rather, in Svensson's (1985b) words: "It is the timing of information which matters" (Svensson (1985b) p. 37). 15

III. Data and Estimation Technique

The sample period for the empirical analysis is February 1959 through December 1985. Monthly data on U.S. consumption, price, money and asset returns are employed. Consumption is measured, alternatively, as seasonally-adjusted, real, per-capita purchases of either nondurable goods or nondurable goods plus services. Correspondingly, the price level is measured by the implicit deflator for nondurable goods or that for nondurable goods plus services. The money supply is end-of-period, per-capita M1. Monthly, total population (age sixteen and over) is used in obtaining the per-capita measures of consumption and money. Nominal asset-returns are, alternatively, measured as the value-weighted or equally-weighted return series published by CRSP. A complete description of the data and its sources is provided in Appendix 1.

Estimates are obtained using the distribution-free, generalized-instrumental variables procedure outlined in Hansen and Singleton (1982) that is based on the methods described in Hansen (1982). This estimation strategy uses the stochastic Euler equations to define a family of orthogonality conditions, which are then used to construct a criterion function. When minimized, this criterion function provides consistent parameter estimates as well as a consistent estimate of the asymptotic covariance matrix. The asymptotic properties of this Generalized Method of Moments (GMM) estimator are described in Hansen (1982) while Tauchen (1986) examines its small sample properties.
In addition, Hansen (1982) proposes a J-test of the over-identifying restrictions implied by the orthogonality conditions of the model. Under the null hypothesis, this J-test statistic is distributed as Chi-squared with degrees of freedom equal to the sum of the number of instruments used in the estimation of each equation less the number of parameter estimates. This specification test will be undertaken below for each model.

To ensure accurate inference it is necessary to express the alternative stochastic Euler equations in stationary forms and to choose a set of stationary instruments from the relevant information set. The stationary forms and corresponding instrument sets are listed in Appendix 2 for each model used in the empirical analysis. As indicated, these equations and instruments involve real consumption and real money-balance growth rates, price inflation, nominal asset returns and ratios of real consumption to real money balances. Each of these variables or products thereof are assumed to be stationary throughout our empirical study. The instruments chosen are limited to one-period lagged values of the variables entering into the corresponding Euler equations and a constant. This choice follows on Tauchen's (1986) advocacy of a small number of instruments.

Finally notice here that, the disturbances implicit in the barter-e and barter-s models with lagged information (and thus also the Lucas (1984) and Svensson (1985a) models) follow an MA(1) process. This is taken into account in the estimation procedure.

IV. Estimation Results

The estimation results for the barter-e, MIUF and CIA/barter-s models are presented, respectively, in Tables 1-3.
The results presented in Table 1 confirm that the limited support for the barter-e model, conveyed by the original Hansen-Singleton estimates (for comparison see Hansen and Singleton (1982, 1984)) persists in our study, which uses an extended sample and revised data. The discount parameter estimate, \( \hat{\beta} \), is 0.99 and it is significantly different from zero. The estimated coefficient of relative risk aversion, \( (1-\hat{\gamma}) \), is in the concavity region - i.e. the region of parameter space for which the utility function is concave - but it is insignificantly different from zero for each measure of consumption and asset returns. Notice also that \( \hat{\gamma} \) is not significantly different from zero, so that preferences could be considered to be logarithmic in consumption. The J-test rejects the model when estimated with equally-weighted asset returns - just as it did in the original Hansen-Singleton (1982, 1984) sample. For the barter-e model with lagged information: similar estimates of \( \beta \) are obtained, \( (1-\hat{\gamma}) \) now lies outside of the concavity region but is again insignificantly different from zero for each consumption and asset return measure, \( \hat{\gamma} \) is generally insignificantly different from zero. However, the J-tests reveal no other conclusive evidence against this model.

The results for both MIUF models, presented in Table 2, show \( \hat{\beta} \) continues to equal 0.99 and to be highly significant. The point estimates of \( \delta \) and \( \gamma \) are within the concavity region for the utility function. The \( (1-\hat{\gamma}) \) estimates are, in general, within a considerably tighter range than was the case for the barter-e models but are insignificantly different from zero. \( \hat{\gamma} \) differs significantly from zero only for the model specifications employing the nondurables-consumption measure. Thus, the hypothesis of logarithmic separability of utility across consumption and real money balances is rejected (not rejected) for the cases involving nondurable- (nondurable plus service-)


consumption. The share parameter estimate, $\hat{\beta}$, ranges between 0.88 and 0.96 and is in all cases significantly different from unity. This suggests a significant role for real money balances in the representative agent's utility function - with real money balances taking on a 0.04 - 0.12 share, while consumption commands the remainder. The J-test fails to reject the chosen specification at the 5% level in seven of the eight cases examined. Despite the lack of precision in the estimates of $(1-\gamma)$, the significance of the real money balance share parameter and the general J-test acceptance of the models suggests that it may indeed be important to study asset pricing in monetary models.

Table 3 presents the results for the CIA models which, as indicated earlier, may also be interpreted as barter-s models. Turning first to the results for the Lucas (1982)/barter-s model: highly significant estimates of $\beta$ close to 0.99 are once again obtained. $(1-\hat{\gamma})$ is in all cases outside of the concavity region and is mostly insignificantly different from zero. $\hat{\gamma}$ differs significantly from zero, thereby rejecting the hypothesis of logarithmic utility. In addition, the J-test rejects the model at the 0.01 level in each case. Hence, this model is not at all supported here. For the Lucas (1984)/Svensson (1985a)/barter-s model with lagged information: significant and similar estimates of $\beta$, to those reported above, are obtained. The estimates of $(1-\gamma)$ are within the concavity region and range in magnitude from 2.55 to 4.81; they are significant at approximately the 7% level (one tail) in all cases and at the 2.5% level for the specifications using value-weighted asset returns. The relative precision of these estimates is considerably improved over the barter-s model specifications. Also note, $\hat{\gamma}$ is insignificantly differently from zero in all cases, supporting the hypothesis of logarithmic utility. Furthermore, the J-tests of the model are well within
the conventional acceptance regions in all cases. These results thus provide reasonably strong support for this model.

V. Diagnostic and Prediction Analysis

In continuing the investigation of the models that displayed considerable promise on the basis of the J-specification tests, the MIUF models and the Lucas (1984)/Svensson (1985a)/barter-s model with lagged information (which will henceforth, in this section, be referred to as the CIA model) were subjected to post-sample diagnostic tests. More specifically, the post-sample diagnostic test statistic, \( \tau \), developed by Hoffman and Pagan (1988), is used to test the validity of the orthogonality conditions implied by the models over the "out-of-sample" period: 1979:1 - 1986:8, based on the parameter estimates obtained over the 1959:2 - 1978:12 period. The latter period corresponds to the estimation period used in the Hansen and Singleton (1982, 1984) studies. In essence, this test allows us to examine whether the more recent period of highly volatile equity prices results in parameter estimates (and hence orthogonality conditions) that differ significantly across the 1959:2-1978:12 and 1979:1-1986:8 periods. Hoffman and Pagan (1988) provide the details for calculating \( \tau \) and show that it follows a chi-squared distribution with degrees of freedom equal to the sum of the number of instruments used in the estimation of each equation. The \( \tau \) values obtained in the present study are listed in Table 4.

For the MIUF models, the following results were obtained. First, when using the value-weighted asset return measure, the \( \tau \) values for both MIUF models are well within the conventional acceptance regions associated with the \( \chi^2 (10) \) distribution. Second, when using the equally-weighted asset return measure, the \( \tau \) values indicate the rejection of both models when compared to
the \( \chi^2 \) (10) distribution. These are consistent the fact that the parameter estimates, for both MIUF models, obtained over the 1959:2 - 1978:12 period were very similar to (quite different from) those obtained over the 1959:2 - 1985:12 period, reported in the previous section, when the value-weighted (equally-weighted) asset return measure was used.

For the CIA model, the \( t \) values are well within the conventional acceptance regions associated with the \( \chi^2 \) (6) distribution for each measure of consumption and asset returns. This result is consistent with the fact that the parameter estimates for the CIA model obtained over the 1959:2 - 1978:12 period were very similar to those obtained over the 1959:2 - 1985:12 period, reported above.

In view of the nonrejection of the CIA model on the basis of the post-sample diagnostic tests, this model is used to predict real asset returns. As a benchmark for comparison, the barter-e model is also used to give these predictions. One approach to generating empirically-tractable closed form predictions of real asset returns is to specify distribution functions. The specifications adopted and the resulting real asset return predictions are indicated as follows.

Assuming that the conditional, joint distribution of the logarithms of consumption growth and real asset returns is normal with a time-varying mean and constant variance, the barter-e model (equation (5)) implies:

\[
(26) \quad E[t \ln (r_{t+1})] = -\alpha E[t \ln \left( \frac{c_{t+1}}{c_t} \right)] - \ln(\delta) - \frac{\sigma^2}{2}
\]

where the new notation is: \( r_{t+1} = \frac{(a_{t+1}^e + d_{t+1}^e) p_t}{a_t^e p_{t+1}} \), \( \alpha = \gamma -1 \) and \( \sigma^2 \) is the variance of \( \ln \left[ \left( \frac{c_{t+1}}{c_t} \right)^\alpha r_{t+1} \right] \). On the other hand, assuming that
the conditional joint distribution of the logarithms of consumption growth, real asset returns and price inflation is normal with a time-varying mean and constant variance, the CIA model (equation (25)) implies:

\[
E_t \ln (r_{t+1}) = -\alpha E_t \ln \left( \frac{c_{t+2}}{c_{t+1}} \right) \\
+ \sigma_1^2 + \ln (B) + \frac{\sigma_1^2}{2} - \frac{\sigma_2^2}{2}
\]

where the new notation is: \( \Delta \) is the first difference operator, \( \sigma_1^2 \) is the variance of \( \ln \left( \frac{c_{t+2}}{c_t} \right) \), \( \sigma_1^2 \) and \( \sigma_2^2 \) is the variance of \( \ln \left( \frac{c_{t+2}}{c_t} \frac{p_{t+1}}{p_t} r_{t+1} \right) \).

The details on the derivation of equations (26) and (27) are provided in Appendix 3. Notice how (26) and (27) differ—especially in the dating of consumption growth and by the inclusion of the expected acceleration of price inflation in (27). This latter difference highlights the non-superneutrality of the CIA model.

The predicted logarithm of real asset returns in (26) and (27) is operationalized by replacing: the conditional-expectations terms on the right hand side of the equations by the most recently observed consumption growth and acceleration of inflation in agents' information sets; \( \alpha \) and \( B \) by the corresponding model's GMM - parameter estimates and the variances by their sample counterparts.

It is worth recalling and emphasizing here that the GMM - estimation technique does not require or impose a distributional assumption on the time series properties of consumption, inflation and real asset returns. Accordingly, it does not restrict the time series properties of the conditional covariance of the levels of the intertemporal marginal rate of substitution of consumption and real asset returns; i.e. the time series nature of consumption risk is not restricted. By contrast, the lognormality
assumptions made above in deriving (26) and (27), while not ruling out time variation of consumption risk, place strong restrictions on its time series behavior - particularly by imposing constant variances on combinations of the logarithms of the intertemporal marginal rate of substitution of consumption, inflation and real asset returns.

Table 5 summarizes the prediction performance of the barter-e (equation (26)) and CIA (equation (27)) models. It lists the mean, minimum, maximum and standard deviation of the actual and predicted real asset returns as well as the correlation coefficients between the actual and predicted returns for the alternative measures of consumption and returns employed in the estimation. Actual real asset returns are constructed from the alternative nominal asset returns and price deflators of consumption. Perhaps the most striking feature of Table 5 is the comparative performance of the barter-e and CIA models in terms of the volatility of their predictions - in particular, the CIA model's predictions come much closer to capturing the volatility of actual real asset returns that do those of the barter-e model. However, the absolute prediction performance of both models is quite poor, as indicated by the correlation coefficients.

Figures 1-8 illustrate these findings over the extremely volatile 1982:1 - 1986:8 period. During this period the correlation between the actual and CIA - predicted real asset returns are approximately 0.03 (0.12) when consumption is measured by nondurable-goods-plus services (nondurable goods). The corresponding correlations between the actual and barter-e predicted real asset returns are -0.10 (0.045). The diagrams vividly illustrate the sharp difference in volatility between the predictions from the CIA and barter-e models. This is especially remarkable when consumption is measured by nondurable-goods-plus services - in these cases the estimated coefficient of
relative risk aversion is close to zero for the barter-e model. The CIA -model predictions capture some of the sharp movements in real stock returns in early 1982 and again in 1986 but, clearly, they also fail to capture a number of other significant swings.

These findings on forecasting performance reinforce: "The common conclusion, usually from monthly data, is that the predictable component of returns is a small part (less than 3%) of return variances" (Fama and French, 1988, p.1). In our study, the poor forecasting performance of the CIA model may be the result of inappropriate expectations proxies and misspecification of the distributional assumptions imposed on consumption, inflation and asset returns, rather than from the empirical failure of the model per se with time-additive, constant relative risk averse preferences. In particular, the restrictions placed on the time-series behavior of consumption risk, noted earlier, may, at least in part, be responsible for poor forecasting performance. We plan to explore these possibilities in further work.

VI. Summary and Conclusion

This paper investigates whether the poor empirical performance of intertemporal asset-pricing relationships, which has been documented in the literature, stems from the fact that they have been derived from barter-economy models. More specifically, the paper investigates, at an empirical level, whether money affects the dynamics of consumption and asset-return relationships in a significant fashion.

This question is addressed here by systematically estimating and testing the Euler equations governing asset choices that are derived from alternative barter- and monetary -economy models. The generalized-method-of-moments (GMM) estimation technique and monthly data on the US economy over the period
1959:02 - 1985:12 are employed. The barter-economy models include a model embodying the conventional end-of-period-timing assumption for consumption and investment choices; we also suggest and examine barter-economy models embodying a start-of-period-timing assumption for consumption and investment choices and two models - corresponding to each timing assumption - which embrace a one-period lag in information receipt on consumption and the price level. The monetary models include contemporaneous- and lagged-money formulations of the money-in-the-utility function model; the Lucas (1982) cash-in-advance model and the Lucas (1984)/Svensson (1985a) cash-in-advance models. We show the equivalence between the asset-pricing relationships derived from the Lucas (1984) and Svensson (1985a) models. In addition, we show the observational equivalence between: the asset-pricing relationships derived from the barter model embodying the 'start-of-period' timing assumption and the Lucas (1982) cash-in-advance model; and the asset-pricing relationships derived from the barter 'start-of-period' model with lagged information on consumption and the price level and the Lucas (1984)/Svensson (1985a) cash-in-advance models.

The key estimation findings are: First, the lack of support for the barter-economy specification, embodying the end-of-period-timing convention, is reaffirmed. Moreover, this finding is robust to the more conservative assumption concerning agents' information sets - i.e. one-period lagged information on consumption and the price level - than has been made in prior work. Second, with regard to the monetary/'start-of-period' barter models: the money-in-the-utility function models find modest empirical support; the Lucas (1982) cash-in-advance/'start-of-period' barter model is rejected by the data; and the Lucas (1984)/Svensson (1985a)'start-of-period' lagged-information barter model finds considerable empirical support.
Although the cash-in-advance models may, as indicated above, be interpreted as being consistent with barter-economy specifications, the above findings are also suggestive of the importance of monetary considerations in the determination of asset prices i.e. monetary non-supernaturalities may indeed significantly affect the dynamics of asset-pricing relationships. It is especially of interest, furthermore, to notice that it is precisely those monetary models which give rise to a combined transactions - precautionary and store-of-value demand for money that lead to asset-pricing relationships which are more consistent with the data.

In view of the support for the Lucas (1984)/Svensson (1985a)'/start-of-period' lagged-information barter model, this model is used to generate predictions of real asset returns over the 1982:1 - 1986:8 period. As a benchmark for comparison the 'end-of-period' barter model is also used to generate these predictions. Quite strikingly, the predictions of the former model come much closer to capturing the volatility of actual real asset returns than do those of the latter model. The prediction performance of both models is, however, quite poor. Poor forecasting performance of the Lucas (1984)/Svensson (1985a)'/start-of-period' lagged-information barter model may be due to inappropriate expectations proxies and misspecification of the distributional assumptions imposed on the time series behavior of consumption, inflation and asset returns rather than the empirical failure of the model per se. These possibilities will be explored in future work.
Footnotes

1. Also Ferson (1983) marginally rejects versions of the model which assume, alternatively, a constant relative and absolute risk aversion utility function defined over a single (nondurable) consumption good, using seasonally unadjusted quarterly data.

2. It is also interesting to note that Huffman (1986) undertakes a simulation study of asset pricing in a model that differs substantially from the models of Rubinstein (1976), Lucas (1978) and Breeden (1979), which underlies the empirical work mentioned in the text. In particular, Huffman considers an overlapping generations model of two-period lived agents in which physical capital accumulation is possible. This model is shown to be consistent with the apparent excess variability of asset prices relative to dividends, which has been observed in the data.

3. Assuming a CRRA utility function defined over the services provided by current and past acquisitions of durable and nondurable goods, Singleton (1985) tests the specification of the intertemporal asset-pricing relationship based on the barter-economy model against that based on a model similar to the Lucas (1982) cash-in-advance monetary model. The empirical results do not support either model. Below we consider the Lucas (1982) model, alternative cash-in-advance models and money-in-the-utility function models.

4. Poterba and Rotemberg (1987) estimate and test an intertemporal asset-pricing model assuming a CRRA utility function defined over a (nondurable) consumption good and liquidity services. The latter are assumed to depend on the levels of real money balances, real savings and time deposits and real holdings of short term government debt. Equity holdings enter the model as a numeraire asset in defining preferences. The model is reasonably well supported by the data. The (contemporaneous) money-in-the-utility function model studied in the present paper is a special case of the Poterba and Rotemberg model.

5. See e.g. the discussion in Kareken and Wallace (1980) and McCallum (1982).

6. The intuition behind these results is as follows. When the coefficient of relative risk aversion exceeds unity, an increase in dividend risk increases the individual's desire to postpone consumption to the future. In the barter economy this leads to an increased demand for assets and thus to higher asset prices. In contrast, in the cash-in-advance economy, assets are less liquid than money and cannot be used to provide for next period's consumption. Therefore, in order to postpone consumption to next period, agents shift their wealth out of stocks and into money. This results in lower asset prices in the cash-in-advance economy.

7. Note that of studies listed above only LeRoy (1984a,b) is a fully - GE study in the sense the stochastic processes of all prices and interest rates are endogenous.
8. Note that \((1-\gamma)\) is the coefficient of relative risk aversion with respect to the 'composite' good: 
\[
c^0_t (M_t / P_t)^{(1-\delta)}
\]

9. \((1-\gamma)\) is now the coefficient of relative risk aversion with respect to the 'composite' good: 
\[
c^0_t (M_{t-1} / P_t)^{(1-\delta)}
\]

10. One could also consider a start-of-period-timing-convention formulation of both MIUF models and the lagged-information (with respect to \(c_t\) and \(P_t\)) counterparts of both their start- and end-of-period versions.

11. Note that Townsend (1987) presents a CIA analytical framework which is very similar to that of Svensson (1985a). This is highlighted by the similarity of the first order condition for physical capital holdings in Townsend's model with the first order condition for equity holdings in Svensson's model. Notice also that Townsend derives a standard (i.e. equivalent to a barter model specification) asset-pricing relationship for market securities (when expressed relative to the marginal costs and benefits of the CIA constrained-good), while Svensson derives a nonstandard one. This apparent inconsistency is reconciled by noting that, in deriving the standard asset-pricing relationship for securities, Townsend implicitly assumes a different timing structure for transactions that is embodied in his main, explicit analytical framework.

12. For the CIA models, separate, observable Euler equations for money and share choices cannot be derived i.e. in order to eliminate the budget-constraint and CIA-constraint multipliers it is necessary to substitute the Euler equation for consumption and money choices into that for share choices.

13. Notice that in going from (19) to (19') the information set has also been restricted to exclude \(d^e_t\). We do this because information on \(d^e_t\) is of no use in constructing instruments for testing the orthogonality condition implied by (19'). In particular, the most recently observed rate of return will be used as an instrument - i.e. \(a_{t-1} + d^e_{t-1}\) - and this does not involve \(d^e_t\) at all.

14. This result assumes a positive rate of return on a nonmonetary asset.

15. It is interesting to recall that the barter-e and barter-s models only (and fundamentally) differ in their timing conventions for consumption and investment decisions. More specifically, the former (latter) assumes that these decisions are made at the end (start) of each period. Both consumption and investment decisions are made simultaneously in each of these models and the associated intertemporal asset-pricing relationships differ. Lucas (1984) and Svensson (1985a) also only (and fundamentally) differ in their timing conventions for consumption and investment decisions. The former (latter) assumes consumption takes place at the end (start) of the period. But consumption and investment decisions at made at different points of time in both models. Thus, the former
(latter) assumes investment takes place at the start (end) of each period. The associated intertemporal asset-pricing relationships are equivalent.

16. We also estimated and tested some of the Euler equations in which the variables entering into the equations and instrument set involved real asset returns and ratios of price inflation instead of nominal asset returns and levels of price inflation. The results were not significantly influenced by this modification.

17. Notice that ratios of real consumption to real money balances enter both KIUF models. Due of the nonlinearity of these models, it is important to express real consumption and real money balances in the same units. This was achieved by dividing the annualized nominal consumption series by twelve.
References


Appendix 1

The data set consists of monthly observations from 1958:01 through 1985:12. Actual estimation started with observation 1959:02. Data definitions and sources are discussed as follows:

Consumption

Two measures of real consumption expenditure are used:—real purchases of nondurable goods and real purchases of nondurable goods plus services. Theses data are taken from the CITIBASE data tape. The former series is listed under GCN82 and the latter series under GCS82. Both series are based on constant 1982 prices.

Prices

Prices are defined as the implicit deflators of the two consumption series. These deflators are calculated from the real and corresponding nominal consumption measures. Nominal purchases of nondurables and nondurables plus services are listed, respectively, under GCN and G on the CITIBASE data tape.

Population

The total population (age sixteen and over) series is the CITIBASE POP variable.

Money Supply

The money supply is defined as the seasonally adjusted value of M1 and is from the CITIBANK data tape (Series FM1).

Asset Returns

Asset returns were obtained from the Center for Research in Security Prices (CRSP) monthly tape. The returns are alternatively measured as a value-weighted (VWRD) and equally-weighted (EWRD) return on stocks traded on the New York Stock Exchange.
Appendix 2

This appendix lists the stationary form of each stochastic Euler equation and the associated instrument set chosen in the empirical analysis.

(a) Barter-e Model [eq. (5)]

\[ 1 = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\gamma-1} \frac{p_t}{p_{t+1}} \left( \frac{a_t^{e} + d_t^{e}}{a_t^{e}} \right) \right] \]

- which is obtained from equation (5) by dividing it by \( u_c(\cdot^t) \) and noting the definition of the latter.

\[ z = \begin{cases} 1, & \frac{c_{t-1}}{c_{t-2}} - 1, \frac{p_{t-1}}{p_{t-2}} \left( \frac{a_t^{e} + d_t^{e}}{a_t^{e}} \right) - 1 \end{cases} \]

where, \( z \) denotes the instrument set.

(b) Barter-e Model With Lagged Information [ eq. (7) ]

\[ E_t \left[ \left( \frac{c_t}{c_{t-1}} \right)^{\gamma-1} \frac{p_{t-1}}{p_t} - \beta \left( \frac{c_{t+1}}{c_{t-1}} \right)^{\gamma-1} \frac{p_{t-1}}{p_{t+1}} \left( \frac{a_{t+1}^{e} + d_{t+1}^{e}}{a_t^{e}} \right) \right] = 0 \]

- which is obtained from equation (7) by dividing it by \( u_c(\cdot^{t-1})/p_{t-1} \) and noting the definition of the latter.

\[ z = \begin{cases} 1, & \frac{c_{t-1}}{c_{t-2}} - 1, \frac{c_{t-1}}{c_{t-3}} - 1, \frac{p_{t-2}}{p_{t-1}} - 1, \frac{a_t^{e} + d_t^{e}}{a_{t-1}^{e}} - 1 \end{cases} \]

(c) Contemporaneous - MIUF Model [eqs. (11) and (12)]

\[ 1 = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\delta \gamma-1} \left( \frac{M_{t+1}/p_{t+1}}{M_t/p_t} \right)^{(1-\delta)\gamma} \frac{p_t}{p_{t+1}} \left( \frac{a_t^{e} + d_t^{e}}{a_t^{e}} \right) \right] \]

\[ 1 = \frac{(1-\delta)}{\delta} \frac{c_t}{(M_t/p_t)} + \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\delta \gamma-1} \left( \frac{M_{t+1}/p_{t+1}}{M_t/p_t} \right)^{(1-\delta)\gamma} \frac{p_t}{p_{t+1}} \right] \]
-which are, respectively, obtained by dividing equations (11) and (12) through by $u_c(\cdot t)$ and noting the definition of both the $u_c(\cdot t)$ and $u_{M/P}(\cdot t)$.

$$z = \left\{ 1, \frac{c_t}{c_{t-1}} - 1, \frac{M_t/P_t}{M_{t-1}/P_{t-1}} - 1, \frac{P_t - 1}{P_t} - 1, \frac{(a^e_t + d^e_t)}{a^e_{t-1}} - 1 \right\}$$

(d) Lagged - MIUF Model [ eqs. (14) and (15) ]

$$1 = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\delta - 1} \left( \frac{M_t/P_{t+1}}{M_{t-1}/P_t} \right)^{(1-\delta)\gamma} \frac{P_t}{P_{t+1}} \left( \frac{a^e_{t+1} + d^e_{t+1}}{a^e_t} \right) \right]$$

$$1 = \beta E_t \left[ \frac{P_t}{P_{t+1}} \left( \frac{c_{t+1}}{c_t} \right)^{\delta - 1} \left( \frac{M_t/P_{t+1}}{M_{t-1}/P_t} \right)^{(1-\delta)\gamma} \left\{ 1 + \frac{(1-\delta)}{\delta} \frac{c_{t+1}}{M_t/P_{t+1}} \right\} \right]$$

-which are, respectively, obtained by dividing equations (14) and (15) through by $u_c(\cdot t)$ and $u_{M/P}(\cdot t+1)$.

$$z = \left\{ 1, \frac{c_t}{c_{t-1}} - 1, \frac{M_t-1/P_t}{M_{t-2}/P_{t-1}} - 1, \frac{P_t - 1}{P_t} - 1, \frac{(a^e_t + d^e_t)}{a^e_{t-1}} - 1 \right\}$$

(e) Lucas (1982)/Barter's Model [ eq. (19')/eq. (6') ]

$$1 = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\gamma - 1} \frac{P_t}{P_{t+1}} \left( \frac{a^e_t + d^e_t}{a^e_{t-1}} \right) \right]$$

- which is obtained from equation (19') or (6') by dividing by $u_c(\cdot t)$ and taking note of the definition of the latter.

$$z = \left\{ 1, \frac{c_t}{c_{t-1}} - 1, \frac{P_t - 1}{P_t} - 1, \frac{(a^e_{t-1} + d^e_{t-1})}{a^e_{t-2}} - 1 \right\}$$
(f) Lucas (1984)/Svensson (1985a)/Barter-R Model With Lagged Information

\[ t^{-1} \left[ \left( \frac{c_t}{c_{t-1}} \right)^{Y-1} \frac{P_{t-1}}{P_t} - \beta \left( \frac{c_{t+1}}{c_{t-1}} \right)^{Y-1} \frac{P_{t-1}}{P_{t+1}} \left( \frac{a_e}{a_t} + \frac{d_e}{a_t} \right) \right] = 0 \]

- which is obtained from equation (21') or (25) lagged one period or (8)

by dividing by \( u_c(t) / P_{t-1} \) and noting the definition of the latter.

\[ z = \left\{ 1, \frac{c_{t-1}}{c_{t-2}} - 1, \frac{P_{t-2}}{P_{t-1}} - 1, \frac{c_{t-1}}{c_{t-3}} - 1, \frac{P_{t-3}}{P_{t-1}} - 1 \left( \frac{a_e}{a_{t-1}} + \frac{d_e}{a_{t-1}} \right) - 1 \right\} \]
Appendix 3

Derivation of Equation (26):

Noting the definition of marginal utility, equation (5) may be rearranged to give:

(A1) \[ E \left[ \frac{\left( \frac{c_{t+1}}{c_t} \right)^\alpha}{r_{t+1}} \right] = \frac{1}{\beta} \]

where \( \alpha = \gamma - 1 \), \( r_{t+1} = \frac{a_t^e + d_{t+1}^e}{a_t^e} \frac{P_t}{P_{t+1}} \). Define \( V_{t+1} = \ln \left[ \frac{\left( \frac{c_{t+1}}{c_t} \right)^\alpha}{r_{t+1}} \right] \)

and assuming that the distribution of \( V_{t+1} \), conditioned on time-\( t \) information, is normal with mean \( \eta_t \) and variance \( \sigma^2 \); it follows that:

(A2) \[ E \left[ \frac{\left( \frac{c_{t+1}}{c_t} \right)^\alpha}{r_{t+1}} \right] = \exp \left[ \eta_t + \sigma^2 / 2 \right] \]

Equating the right-hand sides of (A1) and (A2) and solving form \( \eta_t \) gives:

(A3) \[ \eta_t = -\ln(\beta) - \sigma^2 / 2 \]

Next note:

(A4) \[ V_{t+1} - \eta_t = \alpha \ln \left( \frac{c_{t+1}}{c_t} \right) + \ln(r_{t+1}) + \ln(\beta) + \frac{\sigma^2}{2} \]

Taking the time-\( t \) conditioned expectation of (A4) and rearranging gives:

(A5) \[ E \left[ \ln \left( \frac{r_{t+1}}{c_{t+1}/c_t} \right) \right] = \alpha E \left[ \ln \left( \frac{c_{t+1}}{c_t} \right) \right] - \ln(\beta) - \frac{\sigma^2}{2} \]

i.e. equation (26) in the text.

Derivation of Equation (27):

Noting the definition of marginal utility and dividing equation (25) by \( u_c' \) gives:

(A6) \[ E \left[ \frac{\left( \frac{c_{t+1}}{c_t} \right)^\alpha}{P_t} - \beta \frac{\left( \frac{c_{t+2}}{c_t} \right)^\alpha}{P_{t+2}} \frac{P_t}{P_{t+1}} \frac{a_t^e + d_{t+1}^e}{a_t^e} \right] = 0 \]

or, equivalently:

(A7) \[ E \left[ \frac{\left( \frac{c_{t+1}}{c_t} \right)^\alpha}{P_t} - \beta \frac{\left( \frac{c_{t+2}}{c_t} \right)^\alpha}{P_{t+2}} \frac{P_{t+1}}{P_{t+2}} r_{t+1} \right] = 0 \]
Defining $V_{t+1} = \ln \left( \frac{c_{t+1}}{c_t} \right)^{\alpha} \frac{p_t}{p_{t+1}}$,

$V_{t+2} = \ln \left( \frac{c_{t+2}}{c_t} \right)^{\alpha} \frac{p_{t+1}}{p_{t+2}} \cdot r_{t+1}$

assuming that the distribution of $V_{t+1}$ ($V_{t+2}$), conditioned on time-$t$ information, is normal with mean $\eta_1$ ($\eta_2$) and variance $\sigma_1^2$ ($\sigma_2^2$); it follows that:

$E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha} \frac{p_t}{p_{t+1}} - \beta \left( \frac{c_{t+2}}{c_t} \right)^{\alpha} \frac{p_{t+1}}{p_{t+2}} r_{t+1} \right]$

$= \exp \left[ \eta_1 - \beta \sigma_1^2/2 \right] - \beta \exp \left[ \eta_2 - \beta \sigma_2^2/2 \right]$

Equating the right-hand sides of (A7) and (A8) and solving for $(\eta_1 - \eta_2)$

$E_t \left[ \ln \left( \frac{c_{t+1}}{c_t} \right) + \ln \left( \frac{p_t}{p_{t+1}} \right) - \beta \ln \left( \frac{c_{t+2}}{c_t} \right) - \ln \left( \frac{p_{t+1}}{p_{t+2}} \right) - \ln(\beta) \right]$

$= - \frac{1}{2} \ln(\beta) - \sigma_1^2/2 - \sigma_2^2/2$

Taking the time-$t$ conditioned expectation of (A10) and rearranging gives:

$E_t \left[ \ln(\tau_{t+1}) \right] = - \alpha E_t \left[ \ln \left( \frac{c_{t+2}}{c_{t+1}} \right) \right] + E_t \left[ \Delta \ln \left( \frac{p_{t+2}}{p_{t+1}} \right) \right]$

$- \ln(\beta) - \sigma_1^2/2 - \sigma_2^2/2$

where, $\Delta$ denotes the first difference operator. A(11) is equation (27) in the text.
Table 1
Estimation Results for the Barter-e Models
(1959:2 - 1985:12)

<table>
<thead>
<tr>
<th>Consumption</th>
<th>Return</th>
<th>$\hat{\alpha} = \hat{\gamma} - 1$</th>
<th>SE($\hat{\alpha}$)</th>
<th>$\hat{\beta}$</th>
<th>SE($\hat{\beta}$)</th>
<th>$\chi^2$</th>
<th>df</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDS</td>
<td>EWR</td>
<td>-0.43</td>
<td>2.215</td>
<td>0.990</td>
<td>0.005</td>
<td>5.38*</td>
<td>1</td>
<td>.980</td>
</tr>
<tr>
<td>NDS</td>
<td>VWR</td>
<td>-0.054</td>
<td>1.637</td>
<td>0.995</td>
<td>0.004</td>
<td>1.01</td>
<td>1</td>
<td>.684</td>
</tr>
<tr>
<td>ND</td>
<td>EWR</td>
<td>-1.361</td>
<td>0.934</td>
<td>0.991</td>
<td>0.003</td>
<td>6.48*</td>
<td>1</td>
<td>.993</td>
</tr>
<tr>
<td>ND</td>
<td>VWR</td>
<td>-0.737</td>
<td>0.696</td>
<td>0.995</td>
<td>0.002</td>
<td>1.26</td>
<td>1</td>
<td>.761</td>
</tr>
</tbody>
</table>

A. Barter-e Model [ eq. (5) ]

B. Barter-e Model with Lagged Information [eq. (7)]

<table>
<thead>
<tr>
<th>Consumption</th>
<th>Return</th>
<th>$\hat{\alpha} = \hat{\gamma} - 1$</th>
<th>SE($\hat{\alpha}$)</th>
<th>$\hat{\beta}$</th>
<th>SE($\hat{\beta}$)</th>
<th>$\chi^2$</th>
<th>df</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDS</td>
<td>EWR</td>
<td>2.881</td>
<td>3.072</td>
<td>0.99</td>
<td>0.007</td>
<td>5.56</td>
<td>4</td>
<td>.76</td>
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<tr>
<td>NDS</td>
<td>VWR</td>
<td>1.511</td>
<td>2.261</td>
<td>0.99</td>
<td>0.005</td>
<td>1.99</td>
<td>4</td>
<td>.26</td>
</tr>
<tr>
<td>ND</td>
<td>EWR</td>
<td>2.106</td>
<td>1.434</td>
<td>0.99</td>
<td>0.004</td>
<td>4.80</td>
<td>4</td>
<td>.69</td>
</tr>
<tr>
<td>ND</td>
<td>VWR</td>
<td>0.821</td>
<td>1.017</td>
<td>0.99</td>
<td>0.003</td>
<td>2.35</td>
<td>4</td>
<td>.33</td>
</tr>
</tbody>
</table>

Notes: (i) SE denotes the standard error of the corresponding parameter estimate; df and Prob denote the degrees of freedom and probability value of the $\chi^2$ statistic.

(ii) NDS (ND) denotes the nondurable goods-plus-services (nondurable goods-) measure of consumption; EWR (VWR) denotes the equally- (value-) weighted asset return measure.

(iii) *represents significance at .05 level (one tail)
Table 2
Estimation Results for the MIUF Models
(1959:2 - 1985:12)

<table>
<thead>
<tr>
<th>Consumption</th>
<th>Return</th>
<th>( \hat{\alpha} = \hat{\gamma} - 1 )</th>
<th>( SE(\hat{\alpha}) )</th>
<th>( \hat{\beta} )</th>
<th>( SE(\hat{\beta}) )</th>
<th>( \delta )</th>
<th>( SE(\delta) )</th>
<th>( \chi^2 )</th>
<th>df</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Contemporaneous - MIUF Model [eqs. (11) and (12)]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDS</td>
<td>EWR</td>
<td>-0.376</td>
<td>2.418</td>
<td>0.990</td>
<td>0.006</td>
<td>0.943</td>
<td>0.010</td>
<td>6.65</td>
<td>7</td>
<td>.337</td>
</tr>
<tr>
<td>NDS</td>
<td>VWR</td>
<td>-0.347</td>
<td>0.738</td>
<td>0.994</td>
<td>0.003</td>
<td>0.959</td>
<td>0.008</td>
<td>6.01</td>
<td>7</td>
<td>.262</td>
</tr>
<tr>
<td>ND</td>
<td>EWR</td>
<td>-0.129</td>
<td>0.103</td>
<td>0.991</td>
<td>0.002</td>
<td>0.894</td>
<td>0.016</td>
<td>21.79</td>
<td>7</td>
<td>.991</td>
</tr>
<tr>
<td>ND</td>
<td>VWR</td>
<td>-0.236</td>
<td>0.250</td>
<td>0.994</td>
<td>0.002</td>
<td>0.915</td>
<td>0.015</td>
<td>12.68</td>
<td>7</td>
<td>.823</td>
</tr>
<tr>
<td>B. Lagged - MIUF Model [eqs. (14) and (15)]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>NDS</td>
<td>EWR</td>
<td>-0.751</td>
<td>3.040</td>
<td>0.990</td>
<td>0.007</td>
<td>0.941</td>
<td>0.010</td>
<td>6.50</td>
<td>7</td>
<td>.311</td>
</tr>
<tr>
<td>NDS</td>
<td>VWR</td>
<td>-0.409</td>
<td>0.882</td>
<td>0.994</td>
<td>0.003</td>
<td>0.959</td>
<td>0.008</td>
<td>7.00</td>
<td>7</td>
<td>.363</td>
</tr>
<tr>
<td>ND</td>
<td>EWR</td>
<td>-0.017</td>
<td>0.302</td>
<td>0.989</td>
<td>0.003</td>
<td>0.884</td>
<td>0.018</td>
<td>11.16</td>
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<td>.735</td>
</tr>
<tr>
<td>ND</td>
<td>VWR</td>
<td>-0.208</td>
<td>0.234</td>
<td>0.993</td>
<td>0.002</td>
<td>0.914</td>
<td>0.016</td>
<td>13.85</td>
<td>7</td>
<td>.873</td>
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</tbody>
</table>

Notes: As for Table 1
Table 3
Estimation Results for the CIA and Barter-s Models (1959:2 - 1985:12)

<table>
<thead>
<tr>
<th>Consumption</th>
<th>Return</th>
<th>( \hat{\gamma} )</th>
<th>( \hat{\theta} )</th>
<th>( \text{SE}(\hat{\theta}) )</th>
<th>( \chi^2 )</th>
<th>df</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Lucas (1982)/Barter-s Model [eq. (19')/eq. (6')]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>NDS</td>
<td>EWR</td>
<td>4.658</td>
<td>1.861</td>
<td>0.98</td>
<td>0.004</td>
<td>10.03</td>
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</tr>
<tr>
<td>NDS</td>
<td>VWR</td>
<td>2.634</td>
<td>1.463</td>
<td>0.99</td>
<td>0.003</td>
<td>9.44</td>
<td>2</td>
</tr>
<tr>
<td>ND</td>
<td>EWR</td>
<td>0.492</td>
<td>0.662</td>
<td>0.99</td>
<td>0.002</td>
<td>9.02</td>
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<tr>
<td>ND</td>
<td>VWR</td>
<td>1.60</td>
<td>0.858</td>
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<td>0.003</td>
<td>10.41</td>
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<td>NDS</td>
<td>EWR</td>
<td>-4.517</td>
<td>3.164</td>
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<td>0.006</td>
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<td>NDS</td>
<td>VWR</td>
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<td>0.004</td>
<td>2.63</td>
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</tr>
<tr>
<td>ND</td>
<td>EWR</td>
<td>-2.545</td>
<td>1.662</td>
<td>0.99</td>
<td>0.003</td>
<td>5.95</td>
<td>4</td>
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<td>VWR</td>
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</table>

Notes: As for Table 1
Table 4

$t$ values for the "out-of-sample" period, 1979:1 - 1986:8, based on the parameter estimates obtained over the 1959:2 - 1978:12 period

A. Contemporaneous - MIUF Model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$t$</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDS</td>
<td>EWR</td>
<td>21.81*</td>
<td>10</td>
</tr>
<tr>
<td>NDS</td>
<td>VWR</td>
<td>4.53</td>
<td>10</td>
</tr>
<tr>
<td>ND</td>
<td>EWR</td>
<td>20.38*</td>
<td>10</td>
</tr>
<tr>
<td>ND</td>
<td>VWR</td>
<td>7.38</td>
<td>10</td>
</tr>
</tbody>
</table>

B. Lagged - MIUF Model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$t$</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDS</td>
<td>EWR</td>
<td>0.65</td>
<td>10</td>
</tr>
<tr>
<td>NDS</td>
<td>VWR</td>
<td>10.24</td>
<td>10</td>
</tr>
<tr>
<td>ND</td>
<td>EWR</td>
<td>1.37</td>
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<tr>
<td>ND</td>
<td>VWR</td>
<td>12.47</td>
<td>10</td>
</tr>
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</table>

C. CIA Model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$t$</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDS</td>
<td>EWR</td>
<td>5.22</td>
<td>6</td>
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<td>NDS</td>
<td>VWR</td>
<td>2.89</td>
<td>6</td>
</tr>
<tr>
<td>ND</td>
<td>EWR</td>
<td>2.15</td>
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<tr>
<td>ND</td>
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<td>3.15</td>
<td>6</td>
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Notes: As for Table 1 and: (i) df denotes degrees of freedom (ii)* denotes significance at the 5% level.
Table 5

Actual and Predicted Real Asset Returns
(1959:2 - 1986:8)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>St. Dev.</th>
<th>Correlation Coefficient</th>
</tr>
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<tr>
<td><strong>EWR and NDS</strong></td>
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</tr>
<tr>
<td>Actual</td>
<td>0.0063</td>
<td>0.2540</td>
<td>-0.1959</td>
<td>0.0528</td>
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</tr>
<tr>
<td>Barter-e Prediction</td>
<td>0.0019</td>
<td>0.0158</td>
<td>0.0036</td>
<td>0.0019</td>
<td>0.036</td>
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<tr>
<td>CIA Prediction</td>
<td>0.0165</td>
<td>0.0819</td>
<td>-0.0437</td>
<td>0.0198</td>
<td>-0.007</td>
</tr>
<tr>
<td><strong>VWR and NDS</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Actual</td>
<td>0.0040</td>
<td>0.1512</td>
<td>-0.1332</td>
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<tr>
<td>Barter-e Prediction</td>
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<td>0.0035</td>
<td>0.0002</td>
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</tr>
<tr>
<td>CIA Prediction</td>
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<td>0.0742</td>
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<td>0.011</td>
</tr>
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<td><strong>EWR and ND</strong></td>
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<tr>
<td>Actual</td>
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</tr>
<tr>
<td>Barter-e Prediction</td>
<td>0.0091</td>
<td>0.0457</td>
<td>-0.0272</td>
<td>0.0109</td>
<td>-0.033</td>
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<tr>
<td>CIA Prediction</td>
<td>-0.0029</td>
<td>0.1096</td>
<td>-0.1106</td>
<td>0.0280</td>
<td>0.089</td>
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<tr>
<td><strong>VWR and ND</strong></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Actual</td>
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<td>0.1517</td>
<td>-0.1387</td>
<td>0.0429</td>
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<td>Barter-e Prediction</td>
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<td>-0.0145</td>
<td>0.0059</td>
<td>0.037</td>
</tr>
<tr>
<td>CIA Prediction</td>
<td>0.0050</td>
<td>0.0709</td>
<td>-0.0663</td>
<td>0.0206</td>
<td>0.058</td>
</tr>
</tbody>
</table>

Notes: As for Table 1 and:
(i) St. Dev. denotes the standard deviation
(ii) The correlation coefficient measures the correlation between actual and predicted real asset returns.
Figure 1
CIA Model -- VWR Asset Return, NDS Consumption

Figure 2
Barter-e Model -- VWR Asset Return, NDS Consumption
Figure 3
CIA Model -- EWR Asset Return, NDS Consumption

Figure 4
Barter-e Model -- EWR Asset Return, NDS Consumption
Figure 5
CIA Model -- VWR Asset Return, ND Consumption

Figure 6
Barter-e Model -- VWR Asset Return, ND Consumption
Figure 7
CIA Model -- EWR Asset Return, ND Consumption

Act.  --  Pred.  

Figure 8
Barter-e Model -- EWR Asset Return, ND Consumption

Act.  --  Pred.  