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March 1996

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1. Introduction

The purpose of this paper is to analyze the effects of preference for children of a particular gender on fertility and children's welfare, with and without sex selection or "gender control." Impacts on investments in children's human capital, bequests and fertility are studied in a model based on parental altruism. Both *pure* sex preference, a feature of the parental utility function, and *indirect* preference, which arises as a result of price effects -- the two forms of sex preference originally distinguished by Ben-Porath and Welch (1976, 1980) -- are modeled.¹

Evidence of preference for male children has been observed in many societies. (See, e.g., Williamson, 1976; Cleland *et al.*, 1983; Zhang, 1990, and Seidl, 1995.) Recently, interest in sex preference has increased as a result of the spread, and increasing use, of a variety of techniques for sex selection (Bennett, 1982; Hrdy, 1988). The effects are sizeable. For example, the secondary sex ratio in South Korea (the number of boys born alive per 100 girls) reached 110.4 in 1985.² And the mortality of female infants in China, India, and several Moslem countries is about 15% higher than observed in both rich Western nations and other developing countries (Johansson and Nygren, 1991).

In the face of the already burgeoning use of sex selection, and the possibility of its future spread, the need for an adequate analysis of the social and economic implications of sex preference becomes more pressing. Most previous work on sex preference has taken gender as exogenous. Exceptions are provided by Leung (1994) and Seidl (1995). Seidl has examined the steady-state implications of sex selection for the sex ratio and the reproductive burden placed on women under varying marriage customs. Leung models the impact of sex selection

¹Ben-Porath and Welch referred to the two forms of sex preference as the *sex concern* and *price* effects respectively. Other related studies include, among others, Rosenzweig and Schultz (1982); Bloom and Grenier (1983); Behrman *et al.* (1986); and Leung (1988, 1991 and 1994).

²A figure between about 105 and 106 is normal. The ratios for higher parity births are much more extreme. Census figures for one city indicate a sex ratio of 3.05 for families having their third child and 5.82 for those having a fourth child. (*The Economist*, December 2, 1989, p. 42.)

on fertility for monogamous couples in an optimizing framework, but with exogenous expenditures on children. We add to Leung's insights by considering children's consumption, which is endogenously determined through human capital investments in, and bequests to, children. We find that the impact of sex preference on the treatment of sons vs. daughters may be very different when parents practice sex selection than when gender is exogenous, and that the effect of gender control on fertility may well be positive.

The model used in this paper assumes that parents care about the consumption of their sons and daughters as well as about the number of sons and daughters they have. Children's consumption is funded through earnings, which depend on human capital investments, and bequests. The entire investment in children's human capital is made by parents. Parents can choose their completed family size precisely. Initially, they are assumed to have no "gender control", that is sex selection techniques are unavailable. Later, we analyze what happens when, for simplicity, parents can cheaply exercise perfect control over the sex composition of offspring.

In both the gender control and no control cases our modeling of how the number of sons and daughters is determined in a family is deterministic. In the no control case boys and girls are born in fixed proportion in all families. Ignoring stochastic elements allows us to entertain a richer model in other respects, that is one which endogenizes investments in human capital and bequests. The results complement the stochastic analysis of Leung (1994).

The remainder of the paper is organized as follows. The next section sets out the basic model, and focuses on equilibrium without gender control. Section 3 then analyzes the effect of allowing such control. An example of the model is presented in section 4, and is used both to analyze the impact of differences in sex preference and to do comparative statics in the "no gender control" and "gender control" regimes. Section 5 concludes.

2. Sex Preference Without Gender Control

This section briefly reviews the properties of our model in the case where there is no gender control.³ The model assumes that there are only two periods, two generations, and a single all-purpose composite good.⁴ The first generation consists of identical individuals (parents) who are only present in the first period. Parents invest E_s and E_d units of the composite good in the education (human capital) of each son and daughter respectively, and bequeath B_s and B_d units of the composite good to each of their sons and daughters. For simplicity it is assumed that, in a particular family, all sons or daughters are identical, but genders differ, for example in earnings opportunities.

Each pair of parents is endowed with K units of the composite good. Each child supplies one unit of adult labor in the second period. There are η_s sons and η_d daughters in the young generation. Investing E_i in the upbringing and education of a child of gender i augments its effective human capital to $Y_i = w_i(\eta_s, \eta_d) f_i(E_i)$, $i = s, d$. Each $f_i(\cdot)$ is twice continuously differentiable and strictly concave. The functions f_s and f_d are allowed to differ, to pick up differences (which may be due in part to discrimination) across the sexes in the available technologies for human capital formation. Differences in the human capital rental

³For a detailed exploration of this model, see Davies and Zhang (1995).

⁴A similar model, but without sex preference, was used by Nerlove *et al.* (1984). One result of the two period formulation is that parents do not have a retirement period in which they might receive support from children. However, this feature does not rule out "reverse transfers" from children to parents. Negative bequests can take the form of children paying off parents' debts (perhaps incurred to pay for children's education), or a child making a transfer to a sibling whom the parent wished to leave a positive amount. Negative bequests can also readily occur in the presence of social security or rules forcing children to care for parents in old age. Even if parents transfer property to their children, under these arrangements the *net* bequest may well be negative. Thus, in the real world we may find societies where there is little effective constraint on the size of bequest.

rates, w_s and w_d , which we will refer to as wage rates, may be due to discrimination or to gender-specific differences in endowments of skills or abilities which are relevant for productivity in a given labor market. Following Samuelson (1985), we allow wages to depend on the number of male and female workers in the young generation. In the present section η_s is constrained to equal η_d , so this plays no role. But the possible dependence of w_s and w_d on η_s and η_d becomes important in the next section where we allow gender control. Sex i is said to "have better earnings opportunities" if $w_i f_i(E_i) > w_j f_j(E_j)$ and $w_i f'_i(E_i) > w_j f'_j(E_j)$ for any $E_i = E_j$, $i, j = s, d$ and $i \neq j$. If one gender has better opportunities, then there will be *indirect sex preference* for that gender.

When parents bequeath B_i units of the composite good, we assume the bequest is invested (in physical capital) and yields $B_i R$ units to the child as an adult in the second period, where $R \geq 1$ is the (fixed) interest factor. The bequest can be positive or negative (see note 3). Davies and Zhang (1995) analyze the effect of introducing a lower bound (which can be zero or negative) on the size of bequest.

In this section, while parents have perfect control over their number of children, they have no control over their sex. For analytical convenience, as in Ben-Porath and Welch (1976, 1980), Nerlove *et al.* (1984), and Becker and Barro (1988), fertility is treated as a continuous variable. The parents' utility function depends on their own consumption, C ; sons' and daughters' per capita consumption as adults in the second period, C_s and C_d respectively; and the number of sons and daughters, N_s and N_d :

$$(1) \quad U = U(C, C_s, C_d, N_s, N_d)$$

U is twice continuously differentiable and quasi concave. This is an extension of the function $U(C, N_s, N_d)$ used in Leung (1994).

If C_s and C_d enter symmetrically in (1), and the same is true for N_s and N_d , we say that parents have "no sex preference". If such symmetry does not hold, there are a variety of possibilities. The ones we consider display clear gender preference. Unless otherwise stated we assume that there is pure preference for sons, the definition being that the marginal utility of a son exceeds that of a daughter, that is $U_4 > U_5$, and the marginal utility of sons' consumption is greater than that of daughters, that is $U_2 > U_3$, whenever $C_s = C_d$ and $N_s = N_d$.⁵ Pure preference for daughters can be defined in a parallel manner.

The purpose of the present section, where there is no gender control, is to provide a benchmark against which results in later sections where gender control is allowed can be compared. In order to facilitate those comparison we isolate here the representative, or median, case where parents are endowed with an equal number of sons and daughters.⁶ That is $N_s = N_d = N/2$, and (1) may be re-written as:

$$(2) \quad U = V(C, C_s, C_d, N)$$

Parents maximize (2) through their choices of E_s, E_d, B_s, B_d and N , subject to

$$(3) \quad K \geq C + \frac{N}{2} (E_s + B_s + E_d + B_d),$$

$$(4) \quad C_s \leq w_s f_s(E_s) + R B_s,$$

$$(5) \quad C_d \leq w_d f_d(E_d) + R B_d$$

⁵An alternative definition might be that $U_2 > U_3$ or $U_4 > U_5$, whenever $C_s = C_d$ and $N_s = N_d$. This is also attractive, but leads to a multiplicity of cases. We have opted for the stated definition for the sake of a simpler exposition.

⁶It is not difficult, of course, to investigate cases where $N_s \neq N_d$ (see Davies and Zhang, 1995). The $N_s = N_d$ restriction is made only for expositional simplicity.

The parents' budget constraint is given by (3). It indicates that consumption plus investment in the human and physical capital of the children cannot exceed the parents' endowment. Note that any fixed costs of having children can be incorporated in E_s and E_d . Constraints (4) and (5) are the budget constraints facing each of the sons and daughters, respectively, in the second period.

Substituting the budget constraints in binding form into the utility function, parents maximize:

$$U = V[K - (1/2)N(E_s + B_s + E_d + B_d), w_s f_s(E_s) + RB_s, w_d f_d(E_d) + RB_d, N].$$

We focus on interior solutions. In the absence of gender control this is innocuous. If a couple desires to have children, then $C_s, C_d > 0$ is of course natural. And if no children are desired then the parents' consumption problem is trivial. The first-order conditions imply:

$$(6) \quad V_2 = V_3,$$

$$(7) \quad w_s f'_s(E_s) = w_d f'_d(E_d) = R.$$

$$(8i) \quad V_2/V_1 = (N/2)[w_s f'_s(E_s)]^{-1},$$

$$(8ii) \quad V_3/V_1 = (N/2)[w_d f'_d(E_d)]^{-1},$$

and

$$(9) \quad V_4/V_1 = (E_s + B_s + E_d + B_d)/2 = (\pi_s + \pi_d)/2.$$

The interpretation of (6)-(9) is straightforward: First, (6) says that the marginal utility of children's consumption to the parent must be the same for sons and daughters in equilibrium. Equation (7) indicates that parents optimally invest in child's human capital up to the point at which marginal yield is equal to the interest factor. The LHS of (8i) is the marginal rate of

substitution between own and sons' consumption. On the RHS, an investment of $[w_s f'_s(E_s)]^{-1}$ in son's human capital will produce a unit increase in each son's consumption. Since there are $N/2$ sons, sons' consumption price is $(N/2)[w_s f'_s(E_s)]^{-1}$, and (8i) says the *MRS* equals the price of sons' consumption relative to parents'. The interpretation of (8ii) is parallel. Finally, (9) says that the marginal rate of substitution between number of children and parental consumption must be equal to the "price" of an additional child - - here the average of the costs of boys and girls, $(\pi_s + \pi_d)/2$.

Under the assumption of pure sex preference, (6) implies that $C_s > C_d$ in equilibrium, that is the preferred sex ends up with greater consumption.⁷ If sons and daughters have the same earnings opportunities in the sense we defined earlier, i.e. $w_s f'_s(E_s) = w_d f'_d(E_d)$ and $w_s f'_s(E_s) = w_d f'_d(E_d)$ for any $E_s = E_d$, then (7) means that $E_s = E_d$ in equilibrium. Using $C_s > C_d$ and $E_s = E_d$ and the budget constraints (4) and (5), $B_s > B_d$.

3. Sex Preference With Gender Control

When techniques for sex selection become available parents must determine whether they would be better off practicing gender control or not. In other words, they decide whether to practice gender control by comparing U_{nc}^* , the highest level of U that can be achieved with the "no control" option, with U_{gc}^* , maximum utility with gender control. Although we do not explicitly analyse the determinants of the choice here, it is clear that parents are more likely to

⁷Under the pure sex preference assumption $V_2 > V_3$ when $C_s = C_d$ and (as is assumed here) $N_s = N_d$. The assumption of quasi-concave utility is sufficient to imply that $C_s > C_d$ is required for $V_2 = V_3$, as can be readily confirmed diagrammatically. In the real world $N_s \leq N_d$ is observed. All families with $N_s \leq N_d$ will have $C_s > C_d$. Thus while $C_s < C_d$ is certainly possible in families where $N_s > N_d$, $C_s > C_d$ would be observed in a clear majority of families.

choose gender control the cheaper it is to achieve such control, the greater their wealth, or the stronger their sex preference.⁸

If it is decided to pursue gender control, the parents' problem is to maximize (1), subject to the same constraints (3) - (5) as without gender control, except that the parents' budget constraint when binding is now:

$$(10) \quad K = C + N_s(E_s + B_s) + N_d(E_d + B_d) + G(N_s, N_d)$$

where $G(N_s, N_d)$ is the cost of achieving family composition (N_s, N_d) , which we assume to be increasing, concave, and twice continuously differentiable.⁹ We also assume that $G_1 = G_2$ when $N_s = N_d$. Substituting in the budget constraints once again, parents maximize

$$U = U[K - N_s(E_s + B_s) - N_d(E_d + B_d) - G(N_s, N_d), w_s f_s(E_s) + RB_s, w_d f_d(E_d) + RB_d, N_s, N_d],$$

With interior solutions we have the first-order conditions:

$$(11) \quad E_s: \quad -U_1 N_s + U_2 w_s f'_s(E_s) = 0,$$

$$(12) \quad B_s: \quad -U_1 N_s + U_2 R = 0,$$

$$(13) \quad E_d: \quad -U_1 N_d + U_3 w_d f'_d(E_d) = 0,$$

⁸Analysis of the choice to use gender control is not undertaken in this paper. We compare a regime where such control is impossible with one in which sex selection techniques are so cheap that they are employed by all parents. An investigation of the intermediate case, where some parents control gender but others do not, would be a worthwhile extension of our analysis.

⁹The assumption of concave G is motivated by some aspects of the real-world technology for sex selection. For illustration, assume that parents are unconcerned about birth order, and that the gender of an individual birth can be selected at constant cost. In order to achieve $N_i = N_j = 1$ with certainty sex selection must be used once. Gender can be left stochastic on the first birth, and the desired sex composition can be achieved on the second birth. Now, in order to achieve $N_i = N_j = 2$, sex selection may have to be used *either* once or twice. Starting off without using sex selection, if boys and girls are born alternately on the first two births, the sex selection technique only needs to be used for the final birth. Thus gender control exhibits increasing returns.

$$(14) \quad B_d: -U_I N_d + U_3 R = 0,$$

$$(15) \quad N_s: -U_I(E_s + B_s + G_1) + U_4 = 0,$$

$$(16) \quad N_d: -U_I(E_d + B_d + G_2) + U_5 = 0.$$

Note that parents continue to treat w_s and w_d as parametric, since although w_s and w_d depend on the total numbers of sons and daughters, i.e. $w_i = w_i(\eta_s, \eta_d)$, the gender selection of an individual family has a negligible effect on η_s and η_d .

As in the previous section, if parents decided not to have any children their decision problem would be trivial. If they set $N_i > 0$ then $C_i > 0$ follows under the usual assumption that marginal utility is infinite at zero consumption. So, aside from the case where N_s, C_s, N_d and C_d are all strictly positive, which is considered below, the only non-trivial possibility is that $N_i, C_i > 0$ but $N_j = C_j = 0, i \neq j$, which would leave only the first-order conditions for gender i holding with equality. Clearly, the greater the strength of gender preference, or the greater the imbalance in earnings opportunities across genders, the more likely it is that some families will be at this corner. Comparative statics for families at such a corner will be the same as in the standard model of fertility with intergenerational altruism but without the gender distinction, as analyzed for example by Becker and Tomes (1976).

Now, consider how some of the characteristics of equilibrium differ from the case without gender control. First, from (12) and (14):

$$(17) \quad U_2/U_3 = N_s/N_d$$

that is, the marginal utilities of sons' and daughters' per capita consumption are no longer equal. As we show below, with pure preference for sons we typically have $N_s > N_d$ under gender control. Thus (17) indicates the marginal utility of per capita consumption will generally be greater for sons than for daughters in equilibrium. It is possible that this will

require daughters to actually consume more than sons, despite pure preference for sons. Thus allowing parents the freedom to choose N_s/N_d could result in *an improvement* of daughters' welfare relative to sons'.

In place of (9), from (15) and (16) we have:

$$(18) \quad U_4/U_1 = E_s + B_s + G_1 = \pi_s,$$

and

$$(19) \quad U_5/U_1 = E_d + B_d + G_2 = \pi_d$$

That is, the marginal rates of substitution between boys or girls and parental consumption are now different, and equal to the marginal costs of boys and girls, π_s and π_d respectively. The additional costs, G_1 and G_2 , imply a new substitution effect between N_s and N_d . If $G_1 > G_2$ in equilibrium, as seems plausible¹⁰, this substitution effect will dampen the inequality between N_s and N_d under gender control.

Finally, the equality of rates of return on human and nonhuman investments in children continues to characterize equilibrium, but w_s and w_d now depend on η_s and η_d

$$(20) \quad w_i(\eta_s, \eta_d) f'_i(E_i) = R \quad i = s, d$$

Samuelson (1985) has drawn attention to the possible impact on gender selection of a dependence of w_s and w_d on η_s and η_d . As a simple case he looked at the situation where $w_i = w_i(\eta_s/\eta_d)$, that is where the wage rates depend on the sex ratio, but not on the absolute numbers of sons and daughters. This approach assumes that male and female workers are imperfect substitutes in production, and that the lack of substitutability is significant. (See

¹⁰This assumption reflects the fact that, in the real-world, if parents all desire $N_s > N_d$ (and are unconcerned about birth order), a majority will get the desired number of girls without the use of sex selection, but will have to use such a technique to "finish off" the family with the desired number of boys.

Seidl, 1995.) There does not appear to be any empirical evidence which would cast light on the realism of this assumption.

Samuelson's argument would suggest that if gender control becomes possible, the desire for additional sons will push up η_s/η_d , leading to a decline in w_s , but a rise in w_d . He goes on to suggest that the fall in w_s/w_d will dampen the tendency for η_s to rise, since the demand for sons declines if their wages fall. Sufficient conditions for this argument to go through here are for each N_i to be monotonically increasing in w_i and non-increasing in w_j , $i \neq j$. Although such monotonocity is not guaranteed in our model, it is found in the specific version considered in the next section (see Result 5), and appears to be the leading case.¹¹

It is difficult to characterize equilibria with gender control much more in the general model. Thus, we defer further discussion of behavior under gender control to the next section, which obtains comparative static results in a more narrowly specified version of the model.

4. Comparative Statics

In order to avoid ambiguous comparative statics results it is necessary to work with a more specialized model than considered above. The model considered below is similar to that employed by Becker and Barro (1988) in their study of aggregate fertility.

We simplify notation and write the family budget constraint with gender control as:

¹¹Consider the effects of a rise in w_s , to begin with holding constant C_s , which may be thought of as sons' "quality." Sons become cheaper. (Dollars spent on sons' human capital formation now produce higher returns. Sons' bequests can be reduced by more than efficient investments in their human capital rise.) Assuming sons are a normal good, both substitution and income effects tend to raise N_s . These effects will be dampened somewhat if we allow C_s to rise in reaction to parents' effective increase in real income, but they will tend to dominate unless the induced increase in C_s is extreme.

$$(21) \quad K + N_s Y_s^* + N_d Y_d^* = C + N_s C_s + N_d C_d$$

where Y_s^* and Y_d^* are the earnings of sons and daughters respectively *net* of efficient investments in their human capital, i.e., $Y_s^* = Y_s - E_s$ and $Y_d^* = Y_d - E_d$. Bequests to son and daughter are given by $B_s = (C_s - Y_s)/R$, and $B_d = (C_d - Y_d)/R$, respectively. For simplicity, we assume constant per unit gender control costs, G_1 and G_2 , and incorporate these as a further deduction from Y_s^* and Y_d^* . Thus, with gender control net earnings are given by

$$Y_{s_{gc}}^* = Y_s^* - G_1 \text{ and } Y_{d_{gc}}^* = Y_d^* - G_2.$$

The specific utility function studied here assumes that the child has a "threshold," or "subsistence" consumption level. The amount b_i , $i = s, d$ must be given to a child before the parent derives any utility from the child's consumption. Thus, rather than C_i , consumption net of subsistence costs, $c_i = C_i - b_i$, $i = s, d$, appears in the last two terms of the utility function. In the remainder of the paper, c_i will be referred to as "net consumption" to distinguish it from (gross) consumption, C_i . With this new notation the budget constraint can be re-stated as:

$$(22) \quad K - C - N_s(c_s + b'_s) + N_d(c_d + b'_d) = 0$$

where $b'_i = b_i - Y_i^*$, $i = s, d$. (In the gender control case $b'_i = b_i - Y_{i_{gc}}^*$.) Thus the cost of a child equals gross consumption minus net lifetime earnings.

In order to avoid generally ambiguous comparative static results it is necessary to restrict the utility function quite strongly. Here we use isoelastic forms for the components of the utility function, and hypothesize that the quality and quantity components for each gender

interact multiplicatively:¹²

$$(23) \quad U = \frac{C^{1-\gamma}}{1-\gamma} + \beta\delta \left[\frac{N_s^{1-\varepsilon}}{1-\varepsilon} \right] \left[\frac{c_s^{1-\theta}}{1-\theta} \right] + \beta(1-\delta) \left[\frac{N_d^{1-\varepsilon}}{1-\varepsilon} \right] \left[\frac{c_d^{1-\theta}}{1-\theta} \right] \quad 0 < \varepsilon < \theta < 1$$

An important feature of this specification is that, starting from a situation without pure sex preference ($\delta = 1/2$, and $b_s = b_d$ as explained below), it allows us to introduce pure preference for sons by raising δ , *without* altering the marginal utility of number of children. If we did not have the weight $(1 - \delta)$ on the daughters' term, raising δ would always increase the marginal utility of having children and result in an increase in fertility trivially.

Note that, for simplicity, the elasticities of marginal utility from both number of children (ε), and child's consumption (θ) are assumed the same for sons and daughters.¹³ Also note that the introduction of the subsistence consumption levels, b_s and b_d , necessitates a slight change in our definition of pure sex preference, in order to avoid confusion. To see why this is necessary note that if $b_s < b_d$ then with $\delta = 1/2$ marginal utility from sons' consumption will be less than from daughters' when $C_s = C_d$ (i.e. because $c_s > c_d$), and according to our original definition pure sex preference would exist. We thus modify the earlier definition to say that pure sex preference exists whenever $U_2 > U_3$ and $U_4 > U_5$ given that $c_s = c_d$ and $N_s = N_d$, that

¹²If $\varepsilon = 0$ were allowed parents would simply be taking a weighted average of their own "felicity" and that of their sons and daughters. While that might seem attractive at first, it is necessary to introduce concavity in N_s and N_d since otherwise the solution to the parents problem is to let c_s and c_d tend to zero (which produces very high child felicity per dollar spent on children) and N_s and N_d tend to infinity.

¹³Differences in the taste parameters ε and θ can be introduced for sons and daughters. However, the comparative statics results become very complicated. The assumption that both ε and θ are less than unity ensures that positive utility is obtained from having children, and is required for the marginal utilities of N , c_s , and c_d to be positive. The need for $\varepsilon < \theta$ becomes clear in (28) and (33) below.

is whenever $\delta > 1/2$. Differences between b_s and b_d then, like those between Y_s^* and Y_d^* , give rise to *indirect* sex preference.

No Gender Control

When gender control is impossible, parents maximize (23) subject to the additional constraint that $N_s = N_d = N/2$, as in section 2. Thus we work again with a function of the type V. The first-order conditions are:

$$(24) \quad \frac{\partial V}{\partial c_s} = V_2 - \frac{N}{2}V_1 = \frac{N}{2} \left[\beta \left(\frac{N}{2} \right)^{-\epsilon} \delta c_s^{-\theta} - C^{-\gamma} \right] = 0$$

$$(25) \quad \frac{\partial V}{\partial c_d} = V_3 - \frac{N}{2}V_1 = \frac{N}{2} \left[\beta \left(\frac{N}{2} \right)^{-\epsilon} (1-\delta) c_d^{-\theta} - C^{-\gamma} \right] = 0$$

$$(26) \quad \frac{\partial V}{\partial N} = V_4 - \frac{V_1}{2} \left[(c_s + b'_s) + (c_d + b'_d) \right] = 0$$

$$= \frac{\beta}{2} \left[\delta \left(\frac{c_s}{1-\theta} \right)^{1-\theta} + (1-\delta) \left(\frac{c_d}{1-\theta} \right)^{1-\theta} \right] \left(\frac{N}{2} \right)^{-\epsilon} - \frac{C^{-\gamma}}{2} \left[(c_s + b'_s) + (c_d + b'_d) \right] = 0.$$

From the first two of these conditions we obtain:

$$(27) \quad \frac{c_s}{c_d} = \left[\frac{\delta}{1-\delta} \right]^{1/\theta}.$$

This relationship indicates that there are two possible sources of inequality in sons' and daughters' consumption here. First, suppose that sons and daughters have equal subsistence requirements, that is, $b_s = b_d$. Then, bequests will be made so that sons' per capita consumption exceeds daughters' whenever $\delta > 1/2$ (i.e. whenever there is pure sex preference

for males). But inequality can also result from $b_s \neq b_d$. If boys' subsistence is cheaper than girls', that is $b_s < b_d$, then although $c_s = c_d$ with $\delta = 1/2$, we have $C_s < C_d$. And, of course, if boys' subsistence needs are greater than girls' we have $C_s > C_d$.

Substituting (24) and (25) into (26), denoting children's mean net consumption $\bar{c}$, and letting $b' = b'_s + b'_d$, we have the striking relationship:

$$(28) \quad \frac{c_s + c_d}{2} = \bar{c} = \left[\frac{1-\theta}{\theta-\epsilon} \right] \left[\frac{b'_s + b'_d}{2} \right] = \left[\frac{1-\theta}{\theta-\epsilon} \right] \frac{b'}{2}$$

which, together with (27) implies that c_s and c_d are independent of parental wealth¹⁴ and that children's mean consumption is not affected by the degree of pure sex preference. Note that the proportionality of $\bar{c}$ to $\frac{b'}{2}$ implies that b_i must, on average, exceed children's net earning power for a sensible equilibrium.¹⁵

On the basis of (27) and (28) we can state:

Result 1: In the absence of gender control, consumption of both sons and daughters responds to pure sex preference but is unrelated to parental wealth. The average consumption of sons and daughters is unrelated to the degree of sex preference.

The lack of impact of sex preference on mean consumption, and of parental wealth on C_s and C_d is very interesting. As δ goes up (we look at more and more discriminatory

¹⁴Becker and Barro (1988, p. 10) obtain a similar result. See their equation (16).

¹⁵From (28) there would appear to be two possibilities for ensuring $c_s + c_d > 0$. If $b'_s + b'_d > 0$, then $\theta - \epsilon > 0$ is sufficient. But, it might also appear that a combination of $b'_s + b'_d < 0$ and $\theta - \epsilon < 0$ would be acceptable. The problem with the latter possibility is that it makes having children profitable in pecuniary terms, so that the parent will desire unbounded fertility. Becker and Barro (1988, p. 9) argue it is realistic to assume that the b_i s exceed the present value of children's lifetime earnings *net of parental investments in children's human capital*, so that $b' > 0$.

parents), although sons' consumption rises and daughters' falls, overall consumption per child does not change. The lack of response to K here reflects the fact that, for given values of ϵ , θ , δ , b_s , and b_d , there is a unique optimal scale at which to "operate" a child of given gender. It is efficient to give the child the fixed optimal C_s or C_d and, if K changes, to make adjustments only on the C or N margins. It is possible to remove this restrictive feature by using a different utility function, but only at the cost of making (28), and the resulting analysis, more cumbersome. Note, however, that this feature of the model is not necessarily "unrealistic". In the real world there may be a strong positive correlation between K and b . Thus the fact that the crude correlation between parental wealth and children's consumption is positive in the real world is consistent with the model.¹⁶

In order to derive more comparative statics on children's consumption we can substitute (27) into (28) to get:

$$(29) \quad c_i = \left[\frac{1-\theta}{\theta-\epsilon} \right] (1 + \alpha_i)^{-1} b', \quad i = s, d$$

where

$$\alpha_s = \left[\frac{1-\delta}{\delta} \right]^{1/\theta}, \quad \alpha_d = \left[\frac{\delta}{1-\delta} \right]^{1/\theta}.$$

From (29) we see that both c_s and c_d are increasing in b'_s and b'_d . (Recall that $b' = b'_s + b'_d$.)

This provides us with:

¹⁶The parameter b_i represents the cost of achieving the minimum standard of living parents regard as acceptable for gender i children. (If children's consumption fell below this level, parents would get *less* utility with kids than without. No doubt such thresholds exist.) It seems plausible that subsistence consumption levels should be positively associated with K . Even if they were not, in wealthier societies the cost of achieving the minimum level will likely be greater, due, e.g., to higher labour costs. Within a society, it would be more expensive to achieve a given standard of living in higher wage families if the importance of own time costs were taken into account.

Result 2: In the absence of gender control, sons' and daughters' net consumption will rise equiproportionally if (i) sons' *or* daughters' subsistence needs, b_i , increase, or if (ii) sons' *or* daughters' net earnings, Y_i^* decline.¹⁷

Closed form solutions for C and N cannot be obtained. The remaining comparative statics are done by substituting (28) into the budget constraint, and by using (27) and (28) to eliminate c_s and c_d from (26). This gives two equations in C and N . The first can be thought of as a budget constraint governing feasible choices of C and N , given the consumption standard for children in (28). The second relationship says that the marginal rate of substitution between C and N must equal the effective price ratio.¹⁸

The complete comparative statics results for the no gender control case are set out in Table 1. Those not already set out earlier are summarized in:

Result 3: In the absence of gender control, increases in family wealth raise C and N unambiguously. A drop in sons' or daughters' subsistence needs, b_i , or a rise in their net earnings, Y_i^* , also causes an increase in desired fertility, N , but reduces parental consumption, C . Finally, an increase in the sex preference parameter, δ , *must increase total fertility, N , and reduce C .*

As we see below, the unambiguous prediction that pure sex preference raises fertility even extends to the case where parents have control over children's gender.

¹⁷Note that while the child's net earnings, Y_i , are endogenous, their efficient level, Y_i^* , is determined exogenously by the earnings function and the interest rate. This is what allows us to analyze comparative static reactions to changes in Y_i^* .

¹⁸All of the comparative statics are derived in the appendix to Davies and Zhang (1996), copies of which are available from the authors on request.

Gender Control Case

With gender control, parents maximize (23) by their choice of c_s , c_d , N_s , and N_d . The first-order conditions for c_s and c_d correspond to (24) and (25), except that $N/2$ is replaced by N_s or N_d . The first-order conditions for N_s and N_d are:

$$(30) \quad \frac{\partial U}{\partial N_s} = U_4 - U_1(c_s + b'_s) = \beta \delta N_s^\epsilon \left[\frac{c_s^{1-\theta}}{1-\theta} \right] - C'(c_s + b'_s) = 0,$$

$$(31) \quad \frac{\partial U}{\partial N_d} = U_5 - U_1(c_d + b'_d) = \beta(1-\delta) N_d^\epsilon \left[\frac{c_d^{1-\theta}}{1-\theta} \right] - C'(c_d + b'_d) = 0.$$

From the first-order conditions for c_s and c_d corresponding to (27) in the case without gender control, we have:

$$(32) \quad \left[\frac{c_s}{c_d} \right]^\theta \left[\frac{N_s}{N_d} \right]^\epsilon = \frac{\delta}{1-\delta}$$

from which at least one of the ratios c_s/c_d or N_s/N_d (the "sex ratio") must exceed unity if there is pure sex preference. From all the first-order conditions together we have:

$$(33) \quad c_s = \left[\frac{1-\theta}{\theta-\epsilon} \right] b'_s, \quad c_d = \left[\frac{1-\theta}{\theta-\epsilon} \right] b'_d$$

implying that, in contrast to the case without gender control, *pure sex preference does not lead to inequality in consumption between son and daughter when there is gender control*. The entire impact of such sex preference is felt on the sex ratio N_s/N_d . For instance, if $\delta > 1/2$ (pure sex preference) and $b'_s = b'_d$, then according to (31), $c_s = c_d$. It follows from (32) that $N_s/N_d > 1$, and that the excess of sons over daughters rises with the degree of pure sex preference, as is intuitive. These results indicate that a side effect of allowing parents to

practice gender control could be an improvement in welfare for daughters, as we found earlier in the general model.

Another important feature indicated by (33) is that differences in b_i or Y_i^* between sons and daughters give $b'_s \neq b'_d$, and therefore $c_s \neq c_d$. An interesting aspect is that $Y_s^* > Y_d^*$ tends to give $c_s < c_d$. That is, if sons become higher earning relative to daughters, *ceteris paribus* they will consume less -- a perhaps counterintuitive result in an analysis of the results of preference for males.

Note from (33) that, as in the case without gender control, a child's consumption is increasing in his/her subsistence cost and decreasing in his/her net earnings. This interesting result was also obtained by Becker and Barro (1988), who comment:

"...each person is endowed with a high level of consumption when people are more costly to produce. In effect, it pays to raise the 'utilization rate' of each descendant (in the sense of a higher c_i) when their costs of production are greater." (Becker and Barro, 1988, p. 10)

The efficient utilization rate for a descendant is the c_i level which maximizes the utility per dollar generated by a child. It is efficient to spend just the right amount on a child to maximize this "bang per buck." If it is desired to raise or lower the total utility being produced by having children the efficient way to do so is by adjusting the number of children, not expenditure per child.

Referring to (32) we can see that gaps between c_s and c_d caused by differences in b_i or Y_i^* will lead to further differences in N_s and N_d . If, for example, $b_s > b_d$, we have a tendency for $c_s > c_d$ which will reduce N_s/N_d , according to (32). On the other hand, by tending toward $c_s < c_d$ Y_s^* increases N_s/N_d . Hence these different aspects of indirect sex preference lead to

directly opposed results, and create a situation where if sons tend to be favoured in consumption they will be disfavoured in terms of numbers, and vice versa.

The results so far are summarized in:

Result 4: With gender control, sons' and daughters' net consumption is unrelated to the degree of sex preference, δ , or to parental wealth. Sons' or daughters' consumption varies directly with own subsistence needs and inversely with own net earnings. A child's consumption is not affected by the subsistence needs or net earnings of children of the other gender. The sex ratio rises with the degree of pure sex preference, falls as sons' subsistence needs rise relative to daughters', and rises as sons' net earnings increase relative to daughters'.

As we found in the case without gender control, it is not possible to obtain closed form solutions for parents' consumption, C , or fertility, N . Comparative statics are, however, shown in Table 2, and are summarized in:¹⁹

Result 5: With gender control, increases in family wealth raise C , N_s , and N_d unambiguously. A rise in a child's subsistence needs, or a drop in the child's net earnings, increases C and the number of children of the opposite sex; and reduces the number of same-sex children and

¹⁹The comparative statics are set out in the appendix to Davies and Zhang (1996), copies of which are available from the authors. They are performed as follows for C and N_s . Equations (32) and (33) are used to rewrite the budget constraint in terms of C and N_s only. Then, (33) is substituted into (30) to get a condition stating the equality of the marginal rate of substitution between C and N_s and the relevant price ratio, given that all other variables are being selected optimally. This provides two equations in C and N_s (which cannot be solved explicitly). In order to do comparative statics on N_d , a similar procedure is followed to get two equations in C and N_d . Results for N can be inferred from those for the other variables, by keeping in mind the requirements of budget balance.

overall fertility. An increase in the sex preference parameter, δ , reduces N_d and C , and increases N_s and total fertility, N .

Explanation of Comparative Static Results

What is the explanation for the above results on the impact of pure sex preference on children's consumption and desired fertility? Take c_s and c_d first. In the no gender control case, from (24) - (26), the marginal rates of substitution between "number" (of children) and "quality" of sons and daughters (c_s and c_d) must take on the following values:

$$(34) \quad \frac{V_4}{V_2} = \frac{V_4}{V_3} = \frac{(c_s + b'_s) + (c_d + b'_d)}{N}$$

where $[(c_s + b'_s) + (c_d + b'_d)]/2$ is the price of number, and $N/2$ is the price of quality of children. Now, when δ rises there is little direct effect on V_4 . (Starting from $\delta = 1/2$, since $c_s = c_d$ there is initially *no* direct effect.) However, when δ increases, V_2 rises and V_3 falls. The increase in c_s and reduction in c_d called for by the comparative static results reduce V_2 and increase V_3 respectively to restore equilibrium.

What happens in the gender control case is quite different. In place of (34), from the first-order conditions we have:

$$(35) \quad \frac{U_4}{U_2} = \frac{c_s + b'_s}{N_s}; \quad \frac{U_5}{U_3} = \frac{c_d + b'_d}{N_d}.$$

Here δ has no direct effect on the marginal rates of substitution between number and quality since it raises U_4 and U_2 equi-proportionately, and reduces U_5 and U_3 by the same fraction.

Thus, there is no need for c_s or c_d to change in response to δ .²⁰

Turning to fertility, in the no gender control case from (26) we have:

$$(36) \quad \frac{V_4}{V_1} = \frac{(c_s + b'_s) + (c_d + b'_d)}{2}.$$

As noted earlier, a striking feature of this case is that the mean consumption of sons and daughters does not change with δ . Thus the RHS of (36) is invariant. V_1 is not affected directly by δ , but it is not difficult to show that V_4 increases with δ if $\delta > 1/2$. The increase in N which occurs as a result of a rise in δ in the no gender control case re-equilibrates (36) by reducing V_4 , but also by forcing a decline in C , which raises V_1 .

Finally, with gender control, from (30) and (31) we have:

$$(37) \quad \frac{U_4}{U_1} = c_s + b'_s \quad ; \quad \frac{U_5}{U_1} = c_d + b'_d.$$

Again the RHSs are invariant with respect to δ . On the LHSs, U_4 rises with δ and U_5 falls. Hence an increase in N_s and a decline in N_d tend to restore equilibrium.

Effect of Gender Control on Fertility

It has sometimes been presumed that the likely effect of gender control on fertility is negative, since this is predicted by models with a mechanical stopping rule. If parents have children until they reach fixed target numbers of boys and girls and then stop, gender control *will* reduce fertility. However, this crude intuition neglects important non-stochastic aspects of the economic impact of gender control which become clear in the present model.

²⁰Note that δ has an *indirect* effect on the marginal rates of substitution, due to its impact on N_s and N_d . This does not alter the interpretation. From the first order conditions, U_4/U_2 and U_5/U_3 contain the factors $1/N_s$ and $1/N_d$ respectively. Thus the changes in N_s and N_d induced by the change in δ do not upset the equalities in (35).

As shown in the appendix (available from the authors)²¹ we have:

Result 6: With the specific preferences assumed here, provided $b'_s \leq b'_d$, fertility is always higher with gender control than without if $\delta > 1/2$.

The same conclusion was obtained by Leung (1994) in a stochastic model where costs of children are exogenous. Our result complements Leung's since the cost of a child here depends on how much parents decide to spend on investments in children's human capital and bequests. The latter elements are exogenous in Leung (1994).

Another way of putting this result is that, provided daughters are not cheaper than sons, introducing gender control will raise fertility when there is pure preference for sons.²² The reason is that the extra degree of freedom allowed by gender choice allows parents to get more "quality" from an equal expenditure on children. In a sense explained below, the marginal utility from having children goes up relative to that from own consumption, and parents respond by having more children.

Consider this result more carefully. Assume, for simplicity, that $b'_s = b'_d$. Consider the family which has no gender control. It has N offspring, whose net consumption is, on average, $\bar{c}$. Now introduce sex selection. Our results indicate that *both* sons and daughters will then consume c_i equal to the same $\bar{c}$ as without gender control. [See (28) and (33).] Thus the relative cost of parental consumption, C , and number of children, N , is unaffected by allowing

²¹The appendix analyzes the $b'_s = b'_d$ case. As discussed below, the extension to $b'_s < b'_d$ is straightforward.

²²Recall that the cost of a child of type i equals $C_i - Y_i^* = c_i + b'_i$. Using (33), this cost equals $[(1-\epsilon)/(\theta-\epsilon)]b'_i$.

gender control. What *is* affected is that parents get more utility from a given number of, and expenditure on, children since they can select to have more children of the preferred sex.

What happens when there is gender control if sons and daughters do not have equal b'_i ? It could be, for example, that while subsistence needs are the same ($b_s = b_d$), sons' net earnings are higher ($Y_s^* > Y_d^*$). In that case $b'_s < b'_d$, and parents will optimally set $c_s < c_d$. [See (33).] From (37), the sex ratio (N_s/N_d) now exceeds unity. This means that the cost of N declines with the introduction of gender control. This decline introduces a substitution effect toward a further increase in N . If girls earned more than boys there would of course be a substitution effect in the opposite direction, making the impact of gender control on desired fertility ambiguous.

Our analysis thus predicts that in societies where male-female differences in earnings opportunities are greater, the impact of gender control on total fertility would be larger than elsewhere. Hence where labor market discrimination against women and the degree of pure sex preference are both high, allowing the use of sex selection technologies would have a doubly large impact on fertility.

The tendency toward higher fertility under gender control found here is not an artifact of the specific utility function. It can be obtained with a range of other specifications. The character of the result can be better appreciated by considering a "stripped-down" version of the general utility function (1), which has often been used in the literature. Suppose that parents' utility only depends on own consumption, N_s , and N_d , and does so additively. Then, sufficient conditions for the introduction of gender control to increase total fertility are that (a) girls are not cheaper than boys, and (b) the elasticity of marginal utility from N_s is less than unity and no greater than the corresponding elasticity for N_d .

Finally, note that our analysis is robust to differences between families in the propensity to produce boys. (Empirically, while there is evidence of statistically significant variation in different couples' probabilities of producing a boy, the variation is small. See, e.g., Ben-Porath and Welch, 1976, pp. 299-301.) While the analysis remains deterministic we can readily introduce "endowed" differences in sex ratios across families. As long as some couples desire a sex ratio that differs from their endowed sex ratio, then with boys and girls equally expensive, allowing gender control will increase fertility. The same result is achieved under plausible assumptions with pure sex preference as long as girls are not cheaper than boys.²³

5. Conclusion

This paper has analyzed the impact of both pure and indirect sex preference for sons on sons' and daughters' consumption, and on fertility. Two regimes have been investigated, one in which parents control the gender of offspring, and one in which they do not.

The analysis has generated some interesting, and perhaps surprising predictions for the case where sex selection technology becomes widely available. Crude intuition suggests that if parents could cheaply practice gender control they would have smaller families (as a result of avoiding unwanted "wrong sex" births). Our analysis shows that with isoelastic utility and pure preference for sons, parents who can practice gender control cheaply will have larger families than if sex selection techniques are unavailable. This effect is magnified if sons'

²³Suppose that all families desire a sex ratio equal to $\eta > 1/2$. Then, with boys cheaper than girls, all those families which have endowed sex ratios less than η will desire greater fertility if gender control is introduced. The majority of families would have sex ratios below η in the absence of gender control, so most parents would have more children if gender control were introduced. This would translate into an aggregate increase in fertility with suitable restrictions on the distribution of endowed sex ratios and on the parameters of the utility functions.

earnings opportunities are better than daughters'. These results complement the insights of Leung (1994) since although they are obtained in a deterministic framework we are able to study the determination of children's consumption which takes place via bequests and investments in children's human capital.

We have also shown that the impacts of higher gender bias differ between the control and no-control regimes. When there is no gender control we find that if parents have stronger sex preference the impact is felt in smaller consumption for daughters. In contrast, when there is gender control the impact of greater sex preference can come out in part in the sex ratio. Indeed, in the extended example considered here the effect occurs entirely via the sex ratio and individual daughters may consume as much, or even more than individual sons.

The discussion in this paper has examined two generations in isolation. It therefore abstracts from evolutionary considerations affecting the sex ratio which have been investigated, e.g., by Fisher (1930), Samuelson (1985), and Seidl (1995). Fisher argued that the sex ratio will tend to 1:1 in the long-run, since if the ratio differs from unity individuals who are genetically predisposed to produce more of the scarce gendered offspring (who mate more frequently) have a higher expected number of descendants. Samuelson suggested that this argument might have some relevance when differences in the propensity to produce sons vs. daughters are based on preferences rather than genes. In the same way that parents with *genes* for unequal production of sons and daughters vanish as a proportion of the population in Fisher's model, parents with similarly skewed *preferences* should be eclipsed numerically in the long-run. While this is an interesting argument, its relevance is reduced if there is a very scarce initial supply of parents with no pure sex preference, and by the fact that the mechanism may take many generations to have a substantial effect.

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Table 1
Comparative Statics - No Gender Control

Effect on ...	Effect of...					
	K	δ	b_s, b'_s	b_d, b'_d	Y_s^*	Y_d^*
C	+	—	+	+	—	—
c_s	0	+	+	+	—	—
c_d	0	—	+	+	—	—
$c_s + c_d$	0	0	+	+	—	—
N	+	+	—	—	+	+

Notes: a) Comparative statics for b_i and Y_i^* , $i = s, d$, are derived from those for b'_i discussed in the text, using the relationship $b'_i = b_i - Y_i^*$.
b) The effects of δ on C and N indicated here are obtained on the assumption that $b'_s \leq b'_d$.

Table 2
Comparative Statics - Gender Control Case

Effect on ...	Effect of...					
	K	δ	b_s, b'_s	b_d, b'_d	Y_s^*	Y_d^*
C	+	—	+	+	—	—
c_s	0	0	+	0	—	0
c_d	0	0	0	+	0	—
N_s	+	+	—	+	+	—
N_d	+	—	+	—	—	+
N	+	+	—	—	+	+

Note: Comparative statics for b_i and Y_i^* , $i = s, d$, are derived from those for b'_i discussed in the text, using the relationship $b'_i = b_i - Y_i^*$.

The Effects of Gender Control on Fertility and Children's Consumption

Appendix

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Appendix

This appendix provides the comparative statics discussed in subsections 4.b-4.d of the paper. It also proves Result 6 of subsection 4.e.

Comparative Statics: No Gender Control

First, substitute (28) into the budget constraint, (21), to obtain:

$$(A1) \quad C - K + \frac{N}{2} \left[\frac{1-\varepsilon}{\theta-\varepsilon} \right] (b'_s + b'_d) = 0$$

or

$$C - K + \frac{N}{2} H b' = 0$$

where $H = \frac{1-\varepsilon}{\theta-\varepsilon}$ and $b' = b'_s + b'_d$. As explained in the text, (A1) gives the budget constraint for the (C, N) choice, conditional on c_s and c_d being selected optimally. (A1) is the first of two equations in C and N which form the basis of the comparative statics.

Next, as noted in the text, we can solve (27) and (28) for c_s and c_d obtaining:

$$(A2) \quad \begin{aligned} (i) \quad c_s &= \frac{\left[\frac{1-\theta}{\theta-\varepsilon} \right] b'}{1 + \left[\frac{1-\delta}{\delta} \right]^{1/\theta}} = A_s b' \\ (ii) \quad c_d &= \frac{\left[\frac{1-\theta}{\theta-\varepsilon} \right] b'}{1 + \left[\frac{\delta}{1-\delta} \right]^{1/\theta}} = A_d b' \end{aligned}$$

which is the same as (29). (The notation A_s, A_d is introduced for convenience.) Note that:

$$(A3) \quad \begin{aligned} (i) \quad & \frac{\partial A_s}{\partial \delta}, \frac{\partial c_s}{\partial \delta} > 0; \quad \frac{\partial c_s}{\partial b'} > 0 \\ (ii) \quad & \frac{\partial A_d}{\partial \delta}, \frac{\partial c_d}{\partial \delta} > 0; \quad \frac{\partial c_d}{\partial b'} > 0 \\ (iii) \quad & c_s + c_d + b'_s + b'_d = c_s + c_d + b' = \left[\frac{1-\varepsilon}{\theta-\varepsilon} \right] b'. \end{aligned}$$

Substituting (A2) into (26), and using A3(iii), we obtain our second equation in C and N :

$$(A4) \quad \frac{\partial V}{\partial N} = \chi = G b'^{1-\theta} \left[\frac{N}{2} \right]^{-\varepsilon} - H b' C^{-\gamma} = 0$$

where χ is introduced for convenience, and

$$G = \frac{\beta}{2(1-\theta)} [\delta A_s^{1-\theta} + (1-\delta)A_d^{1-\theta}] > 0.$$

In order to sign results below it is important to investigate $\frac{\partial \chi}{\partial \delta}$ and $\frac{\partial \chi}{\partial b'}$. We have:

$$\frac{\partial \chi}{\partial \delta} = b'^{1-\theta} \left[\frac{N}{2} \right]^{-\varepsilon} \frac{\partial G}{\partial \delta}$$

and:

$$(A5) \quad \frac{\partial G}{\partial \delta} = \frac{\beta}{2(1-\theta)} \left[A_s^{1-\theta} + \delta(1-\theta)A_s^{-\theta} \frac{\partial A_s}{\partial \delta} - A_d^{1-\theta} + (1-\delta)(1-\theta)A_d^{-\theta} \frac{\partial A_d}{\partial \delta} \right].$$

From (A2) and (28), $A_s + A_d = \left[\frac{1-\theta}{\theta-\varepsilon} \right]$. Hence $\frac{\partial A_s}{\partial \delta} = -\frac{\partial A_d}{\partial \delta}$. Also, using (A2) and (27), $\delta A_s^{-\theta} = (1-\delta)A_d^{-\theta}$. Using these facts, (A5) becomes:

$$(A6) \quad \frac{\partial G}{\partial \delta} = \frac{\beta}{2(1-\theta)} (A_s^{1-\theta} - A_d^{1-\theta})$$

Since $\theta < 1$, and in equilibrium in the no gender control case, $A_s > A_d$, we have $\frac{\partial G}{\partial \delta} > 0$, and $\frac{\partial \chi}{\partial \delta} > 0$. Next,

$$(A7) \quad \frac{\partial \chi}{\partial b'} = G(1-\theta) \left[\frac{N}{2} \right]^{-\varepsilon} b'^{-\theta} - HC^{\gamma}$$

Now, note that dividing (A4) by b' yields:

$$(A8) \quad Gb'^{-\theta} \left[\frac{N}{2} \right]^{-\varepsilon} - HC^{\gamma} = 0$$

which differs from (A7) only in dividing the first term of (A7) (which is positive) by $(1-\theta) < 1$. Hence $\frac{\partial \chi}{\partial b'} < 0$.

We now take the total derivatives of our two equations in C and N . Reversing order, from (A4) and (A1) respectively we have:

$$(A9) \quad \gamma C^{-\gamma-1} H b' dC - \varepsilon b'^{1-\theta} G \left[\frac{N}{2} \right]^{-\varepsilon-1} dN = -\frac{\partial \chi}{\partial \delta} \partial \delta - \frac{\partial \chi}{\partial b'} (db'_s + db'_d)$$

and

$$(A10) \quad dC + H b' dN = dK - \frac{N}{2} H (db'_s + db'_d).$$

The Hessian determinant is:

$$(A11) \quad (D) = \begin{vmatrix} \gamma C^{\gamma-1} H b' & -\epsilon b'^{1-\theta} G\left[\frac{N}{2}\right]^{-\epsilon-1} \\ 1 & H b' \end{vmatrix} = (+)(+) - (+)(-) > 0$$

which satisfies the second order condition.

We proceed, indicating only the signs of the relevant elements where that is sufficient. First,

$$(A12) \quad \frac{\partial C}{\partial K} = \frac{\begin{vmatrix} 0 & - \\ 1 & + \end{vmatrix}}{|D|} = \frac{-(+)(-)}{(+)} > 0$$

$$(A13) \quad \frac{\partial C}{\partial \delta} = \frac{\begin{vmatrix} - & - \\ 0 & + \end{vmatrix}}{|D|} = \frac{(-)(+)}{(+)} < 0$$

$$(A14) \quad \frac{\partial C}{\partial b'_s} = \frac{\partial C}{\partial b'_d} = \frac{\begin{vmatrix} -\frac{\gamma \chi}{\partial b'} & -\epsilon b'^{1-\theta} G\left[\frac{N}{2}\right]^{-\epsilon-1} \\ -\frac{N}{2} H & H b' \end{vmatrix}}{|D|}$$

$$= \frac{H}{|D|} \left[-b' \frac{\partial \chi}{\partial b'} - \frac{N}{2} \epsilon b'^{1-\theta} G\left[\frac{N}{2}\right]^{-\epsilon-1} \right] = \frac{H \psi}{|D|} = \frac{(+)(?) }{(+)}$$

where ψ is introduced for convenience. Now, substituting in for $\frac{\partial \chi}{\partial b'}$ from (A7),

$$\psi = C^{\gamma} b' H - G(1 - \theta + \epsilon) b'^{1-\theta} \left[\frac{N}{2}\right]^{-\epsilon}$$

and, using (A4), this simplifies to

$$\psi = G b'^{1-\theta} \left[\frac{N}{2}\right]^{-\epsilon} (\theta - \epsilon).$$

Since we have assumed $\theta - \epsilon > 0$, we have $\psi > 0$, $\frac{\partial C}{\partial b'_s} > 0$, and $\frac{\partial C}{\partial b'_d} > 0$. Next,

$$(A15) \quad \frac{\partial N}{\partial K} = \frac{\begin{vmatrix} + & 0 \\ 1 & 1 \end{vmatrix}}{|D|} = \frac{(+)(+)}{(+)} > 0,$$

$$(A16) \quad \frac{\partial N}{\partial \delta} = \frac{\begin{vmatrix} + & - \\ 1 & 0 \end{vmatrix}}{|D|} = \frac{-(+)(-)}{(+)} > 0,$$

and

$$(A17) \quad \frac{\partial N}{\partial b'_s} = \frac{\partial N}{\partial b'_d} = \frac{\begin{vmatrix} + & + \\ 1 & - \end{vmatrix}}{|D|} = \frac{(+)(-) - (+)(+)}{(+)} < 0.$$

Comparative Statics: Gender Control

Here we do comparative statics on C and N_s . Most of the results for N_d can be inferred. The rest can be obtained by a procedure similar to that shown here for N_s . (When all of these results are obtained, those for N follow from budget balance.) Substituting (33) into (30) we get the first of two equations in C and N_s :

$$(A18) \quad \frac{\partial U}{\partial N_s} = \chi_s = \delta M N_s^{-\epsilon} - H b'_s C^{-\gamma} = 0$$

where $M = \frac{\beta}{1-\theta} \left[\frac{(1-\theta)b'_s}{\theta-\epsilon} \right]^{1-\theta}$. Next, using (32) and (33), from the budget constraint we have:

$$(A19) \quad C - K + N_s H P = 0$$

where $P = b'_s + b'_s \frac{1/\theta}{b'_d} \frac{\theta-1}{\theta} \left[\frac{1-\delta}{\delta} \right]^{1/\epsilon}$. In preparation for the exercise below, note that:

$$(A20) \quad \frac{\partial \chi_s}{\partial b'_s} = -C^{-\gamma} H + \beta \delta N_s^{-\epsilon} \left[\frac{1-\theta}{\theta-\epsilon} \right]^{1-\theta} b'_s^{-\theta} < 0$$

where the sign is determined by an argument similar to that used for $\frac{\partial \chi}{\partial b'}$ earlier.

Taking total derivatives of (A18) and (A19) respectively we have:

$$(A21) \quad \gamma C^{\gamma-1} H b'_s dC - \varepsilon N_s^{-\varepsilon-1} \delta M dN_s = -M N_s^{\varepsilon} - \frac{\partial \chi_s}{\partial \delta} d\delta - \frac{\partial \chi_s}{\partial b'_s} db'_s$$

and

$$(A22) \quad dC + H P dN_s = dK - N_s H \left[\frac{\partial P}{\partial \delta} d\delta + \frac{\partial P}{\partial b'_s} db'_s + \frac{\partial P}{\partial b'_d} db'_d \right]$$

where

$$\frac{\partial P}{\partial \delta} = b'_s{}^{1/\theta} b'_d{}^{\frac{\theta-1}{\theta}} \frac{1}{\varepsilon \delta^2} \left[\frac{1-\delta}{\delta} \right]^{\frac{1-\varepsilon}{\varepsilon}} > 0,$$

$$\frac{\partial P}{\partial b'_s} = 1 + \frac{1}{\theta} \left[\frac{b'_d}{b'_s} \right]^{\frac{\theta-1}{\theta}} \left[\frac{1-\delta}{\delta} \right]^{1/\varepsilon} > 0,$$

$$\frac{\partial P}{\partial b'_d} = \left[\frac{\theta-1}{\theta} \right] \left[\frac{b'_s}{b'_d} \right]^{1/\theta} \left[\frac{1-\delta}{\delta} \right]^{1/\varepsilon} < 0,$$

and the last sign follows from $0 < \theta < 1$. From (A21) and (A22) we have:

$$(A23) \quad |D| = \begin{vmatrix} + & - \\ 1 & + \end{vmatrix} = (+)(+)-(+)(-) > 0$$

satisfying the s.o.c. Next,

$$(A24) \quad \frac{\partial C}{\partial K} = \frac{\begin{vmatrix} 0 & - \\ 1 & + \end{vmatrix}}{|D|} = \frac{-(+)(-)}{(+)} > 0,$$

$$(A25) \quad \frac{\partial C}{\partial b'_s} = \frac{\begin{vmatrix} + & - \\ - & + \end{vmatrix}}{|D|} = \frac{(+)(+)-(-)(-)}{(+)} \geq 0$$

$$(A26) \quad \frac{\partial C}{\partial b'_d} = \frac{\begin{vmatrix} 0 & - \\ + & + \end{vmatrix}}{|D|} = \frac{-(+)(-)}{(+)} > 0$$

$$(A27) \quad \frac{\partial C}{\partial \delta} = \frac{\begin{vmatrix} -MN_s^{-\varepsilon} & -\varepsilon N_s^{-\varepsilon-1} \delta M \\ \frac{N_s H}{\varepsilon \delta^2} b_s^{1/\theta} b_d^{1-\theta} \left[\frac{1-\delta}{\delta} \right]^{\frac{1-\varepsilon}{\varepsilon}} & HP \end{vmatrix}}{|D|}$$

$$= MN_s^{-\varepsilon} H \left\{ -P + \frac{b_s^{1/\theta} b_d^{1-\theta}}{\delta} \left[\frac{1-\delta}{\delta} \right]^{\frac{1-\varepsilon}{\varepsilon}} \right\} = MN_s^{-\varepsilon} HZ$$

and Z simplifies to

$$Z = b'_s \left[-1 + \left(\frac{b'_s}{b'_d} \right)^{\frac{1-\theta}{\theta}} \left[\frac{1-\delta}{\delta} \right]^{\frac{1-\varepsilon}{\varepsilon}} \right]$$

Sufficient conditions for $Z < 0$, and therefore $\frac{\partial C}{\partial \delta} < 0$, are (i) $\left[\frac{1-\delta}{\delta} \right] \leq 1$ (pure sex preference), and (ii) $b'_s \leq b'_d$ (indirect sex preference), with at least one strict inequality. (Recall that $0 < \varepsilon < \theta < 1$). Turning to N_s ,

$$(A28) \quad \frac{\partial N_s}{\partial K} = \frac{\begin{vmatrix} + & 0 \\ 1 & 1 \end{vmatrix}}{|D|} = \frac{(+)(+)}{(+)} > 0$$

$$(A29) \quad \frac{\partial N_s}{\partial b'_s} = \frac{\begin{vmatrix} + & + \\ 1 & - \end{vmatrix}}{|D|} = \frac{(+)(-)-(+)(+)}{(+)} < 0$$

$$(A30) \quad \frac{\partial N_s}{\partial b'_d} = \frac{\begin{vmatrix} + & 0 \\ 1 & + \end{vmatrix}}{|D|} = \frac{(+)(+) }{(+)} > 0$$

$$(A31) \quad \frac{\partial N_s}{\partial \delta} = \frac{\begin{vmatrix} + & - \\ 1 & + \end{vmatrix}}{|D|} = \frac{(+)(+) - (+)(-)}{(+)} > 0$$

As indicated earlier, some of the comparative static results for N_d and N follow immediately. For example, since a change in δ has no effect on c_s and c_d , from $\frac{\partial C}{\partial \delta} < 0$, which holds under the conditions indicated, we have $\frac{\partial N}{\partial \delta} > 0$. For N_d , using (32) and (33), (A28) implies $\frac{\partial N_d}{\partial K} < 0$. The remaining results for N_d can be derived by repeating the comparative static exercise reported here for C and N_d .

Effect of Gender Control on Fertility

The following analysis assumes $b'_s = b'_d$. (The text discusses the impact of relaxing this assumption.) Comparing (28) and (33), and using $b'_s = b'_d$, we get the useful result:

$$(A32) \quad \left. \frac{c_s + c_d}{2} \right|_{\text{no control}} = \left[\frac{1-\varepsilon}{\theta-\varepsilon} \right] \frac{b'}{2} = c_i \left|_{\text{control}} = \left[\frac{1-\varepsilon}{\theta-\varepsilon} \right] b'_i = \bar{c}, \quad i=s,d.$$

This means that the total cost of children equals $N(\bar{c} + b'/2)$ whether or not there is gender control. In other words, the (C, N) budget constraint does not change when we introduce gender control. The question then is just whether parents locate at a higher or lower point on this constraint when gender control is allowed.

With no gender control, from (27) and (A32), and noting $\delta > 1/2$, we derive:

$$(A33) \quad c_s = \frac{2\bar{c}}{1 + \left[\frac{1-\delta}{\delta} \right]^{1/\theta}} > \bar{c}.$$

Next, from (24)

$$(A34) \quad N = 2 \left[\frac{\beta \delta c_s^{-\theta}}{1-\varepsilon} \right]^{1/\varepsilon} C^{1/\varepsilon} = S C^{1/\varepsilon}.$$

which tells us where parents will locate on the C, N budget constraint, given optimal choice of c_s .

If we introduce gender control, from (32) and (33) we have:

$$(A35) \quad N_s = \frac{N}{1 + \left[\frac{1-\delta}{\delta} \right]^{1/\varepsilon}}$$

From the first order condition for c_s , noting that $c_s = \bar{c}$,

$$N_s = \left[\frac{\beta \delta \bar{c}^{-\theta}}{1-\varepsilon} \right]^{1/\varepsilon} C^{\gamma \varepsilon}.$$

Substituting in (A35), we derive:

$$(A36) \quad N = \left[1 + \left[\frac{1-\delta}{\delta} \right]^{1/\theta} \right] \left[\frac{\beta \delta \bar{c}^{-\theta}}{1-\varepsilon} \right]^{1/\varepsilon} C^{\gamma \varepsilon} = TC^{\gamma \varepsilon}$$

Comparing (A36) with (A34) we see that fertility will be higher under gender control if:

$$\frac{T}{S} = \left[\frac{1 + \left[\frac{1-\delta}{\delta} \right]^{1/\theta}}{2} \right] \left[\frac{c_s}{\bar{c}} \right]^{\theta/\varepsilon} > 1.$$

Substituting in (A33) we need:

$$(A37) \quad \frac{T}{S} = \left[\frac{2}{1 + \left[\frac{1-\delta}{\delta} \right]^{1/\delta}} \right]^{\frac{\theta}{\varepsilon} - 1} > 1$$

The inequality follows from the facts that $\frac{\theta}{\varepsilon} - 1 > 0$ and that the expression in brackets exceeds unity for $\delta > \frac{1}{2}$.