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PROFESSIONAL WORK-LEISURE DECISIONS UNDER ALTERNATIVE REMUNERATION METHODS: CHANGING THE SHAPE OF EDGORTH BOXES

Robert S. Woodward

April, 1976
I. Introduction

This paper develops the proposition that the method of remunerating workers is an important determinant of work-leisure decisions. The analysis is accomplished with a graphical model of an individual who maximizes utility from two goods, the outputs of his work and leisure activities, which he produces using his limited financial and time (own labor) resources. The paper focuses on the comparative outcomes of three different remuneration systems—fixed income, wages and piecework—for an individual with equal earnings in each alternative.

The model has five novel features. The first two, introducing work in the utility function and requiring personal expenditures to produce work outputs, distinguish this model from Becker's "Theory of the Allocation of Time." Our new assumptions may be intuitively understood by thinking of the work-leisure decisions of a self-employed professional, such as a physician, who gets direct satisfaction from his job and who produces a market good using his time and other market goods. Implications for individuals who get no utility from work are derived by considering lexicographic preferences for leisure goods. In this situation, Becker's single full wealth
constraint is contradicted and the conditions which collapse both this and Becker's analysis to the standard income-leisure models are derived.

The third feature treats income as a negative work expenditure along production isoquants. The first dollars of earned income offset the personal expenditures required to produce any work output. Where earnings are larger than work expenditures, the difference—net income—is available for leisure production. This treatment explicitly recognizes income's importance as part of the work production process and resolves the wage rate's dual role as a price of time and a determinant of market expenditures.¹

The fourth feature, using a graphic analysis, facilitates a comparison of outputs and resource allocations in each of the three separate income (and therefore model) specifications. The fifth feature, standardizing by income rather than utility, is necessary for strong graphic comparisons. This framework intuitively corresponds to the questions which an employer might have about the behavior of a staff member who could be paid a constant dollar income according to alternative remuneration methods.

This paper proceeds according to the following outline. In Section II a general equilibrium model of an individual's work-leisure decisions is presented. In Section III, the solutions for wage and for piecework remunerations are compared with that of fixed income. In Section IV, all three remuneration systems are contrasted under alternative assumptions about the utility and production functions.
II. The Model

In this section, we model the work-leisure decisions of an individual who derives utility from both work (V) and leisure (Z) outputs.

1) \[ U = U(V, Z) \]

Work output is assumed to be produced according to the function \( F \) using market goods \((x_v)\) and the individual's time \((t_v)\).

2) \[ V = F(x_v, t_v) \]

Similarly, leisure is produced according to the function \( G \) using different market goods \((x_z)\) and the individual's time \((t_z)\).

3) \[ Z = G(x_z, t_z) \]

The functions \( U, F \) and \( G \) are assumed to be homothetic, and \( F \) and \( G \) are assumed to have non-increasing returns to scale.\(^2\)

The production of leisure goods, and their inclusion in the utility function follows Becker. The assumptions that an individual gains utility from work output, and that he produces it using market goods purchased with his own money are less common. Scitovsky recognized the issue when he wrote:

"Artists, scientists, professional people, and businessmen often regard their work not merely as a means of earning income but as an important and interesting part of their lives. People who feel this way about their work will hardly let their working habits be influenced by changes in their income." (p. 87)

We include work output in the utility function as a more direct and flexible representation of such professional preferences than Scitovsky's vertical income-offer curve.

The professional's use of market goods is also easy to understand. Artists, scientists, physicians and lawyers have offices or workshops
suitably furnished with personnel and equipment. Even blue collar workers frequently own certain tools which are required in their job.

The individual's production of \( V \) and \( Z \) is assumed to be limited by both income and time constraints:

\[
4) \quad Y = (p_{xv} \cdot x_v) + (p_{xz} \cdot x_z) \quad \text{and}
\]

\[
5) \quad T = t_v + t_z
\]

where \( Y \) is total income,

\( p_{xv} \) is a fixed market price of \( x_v \),

\( p_{xz} \) is a fixed market price of \( x_z \), and

\( T \) is total time.

Furthermore, graphical convenience is gained and generality is not lost by defining the units of \( x_v \) and \( x_z \) so that

\[
6) \quad p_{xv} = p_{xz}.
\]

The model considers the implications of three alternative remuneration systems, or specifications of \( Y \). These are fixed income, wages, and piecework.\(^3\) Fixed income represents an amount of money paid to the individual regardless of his work or leisure activities,

\[
7) \quad Y = \bar{Y}
\]

where \( \bar{Y} \) is exogeneously determined. Real life examples of a truly fixed income are hard to find. Even tenured professors' income is affected if they do no work over sufficient time.

Wages define an income which is proportional to time spent working,

\[
8) \quad Y = w \cdot t_v
\]

where \( w \) is an exogeneous wage rate. Piecework income is defined as

\[
9) \quad Y = p_v \cdot V
\]

where \( p_v \) is an exogeneous price paid per unit of \( V \) produced by the worker.
III. Comparing Fixed Income with Wage, and with Piecework Remunerations

In this section, we develop the pairwise comparisons of the effects of fixed income with wage, and with piecework remunerations. The specific endogeneous variables considered are the production of work and leisure outputs, the allocation of time and financial resources, and net income. As a point of departure, it is useful to start with the graphically familiar setting of a fixed income. We then introduce wage and piecework incomes.

A. Fixed Income

The model for an individual with fixed incomes $\bar{Y}$ corresponds exactly to well-known Edgeworth Box analyses and needs only a brief summary. By definition, the income $\bar{Y}$ is available for the purchase of $x^*_v$ and $x^*_z$ regardless of work or leisure activities. Thus the boundaries of the set of feasible input combinations are defined by the time $T$ and by the maximum units of work market inputs, $x^*_v$, and leisure market inputs, $x^*_z$, which $\bar{Y}$ will buy. Since we defined the units of $x^*_v$ and $x^*_z$ so that their prices are equal, $x^*_v=x^*_z$. Thus the set of feasible input combinations is a rectangle or Edgeworth Box.

In Figure 1, the production isoquants $V_1$, $V_2$ and $V_3$ represent the technologies of the production function $F$. Similarly $Z_1$, $Z_2$ and $Z_3$ represent the technology of $G$. If both $F$ and $G$ are subject to decreasing returns to scale, the production possibility curve ($ZfVF$ in Figure 2) derived from the contract curve has the normal shape, and the second order conditions of the utility maximizing equilibrium (point D in Figure 2) are satisfied. Additionally, point D in Figure 2 corresponds to some point, such as D, on the contract curve in Figure 1.
FIGURE 1

The Feasible Input Combinations
And The Contract Curve for Fixed Income
FIGURE 2

Optimum Outputs for Alternative Remunerations
A zero fixed income provides an important starting point in the graphical development of the wage and the piecework income equilibria. As might be anticipated, the model of an individual with no income is an Edgeworth Box with a zero vertical dimension (Figure 3). While the interpretation of each of the Z and V isoguants is unchanged, the contract curve is now restricted to the time axis (RoU in Figure 3). If the production isoguants intersect the time axis, a production possibility frontier Z0Vo may be derived. As with a positive fixed income, the utility maximizing combination of outputs, indicated by point A in Figure 2, also corresponds to some point A in Figure 3.

B. Wage Income

There are two reasons why the solutions for wage income are different than those of fixed income. First if a wage earner spends no time at work, he earns no income and cannot spend any money on either work or leisure activities. The set of feasible input combinations which show market resources as proportion to work time must be a triangle rather than a rectangle.

Second and fundamental to the analysis of both wage and piecework incomes is the formation of "activity", as opposed to technological, production isoguants. Each of the technological isoguants, such as the Vi in Figures 1,3 and 4, reflect a set of alternative combinations of labor and market goods required to produce some output level. We define an "activity" isoguant as the set of alternative combinations of labor and market goods required to produce Vi where earnings offset market expenditures.
FIGURE 3
Production Isoquants for an Individual with No Financial Resources
For example, suppose a lawyer earning $50 per hour can write a will using either one hour of his time and $75 of secretarial assistance, or two hours of his time and $35 of secretarial assistance. After being reduced by income, net expenditures are $25 in the first alternative. In the second, net expenditures are -$65 making a net income of $65 available for leisure production.

For each point on any initial production isoquant V, a point on a "activity" isoquant can be derived which reflects the same time spent working and a net purchase of market goods \[xv = (Y/p_xv)\]. The starting points are the initial technological production isoquants for V and a wage rate, indicated by the slope of the line RwS in Figure 4. If we assume \[p_xv = 1\], wage income for any particular time spent working is measured by the vertical distance between the horizontal axis RwU and the line RwS. Both net income and net work expenditures along any "activity" isoquant are measured as the vertical distance between it and the line RwU. When the "activity" isoquant falls below RwU, net income is positive and net work expenditures are negative.

As an example of how the "activity" isoquants are derived, consider the initial technological isoquant Vi in Figure 4. At point M, Vi is produced using only market goods. No time is required and no wage income is earned. The zero income leaves the use of market goods unaffected. At point N, the same Vi is produced using both market goods and time. Net purchases of xv equal the initial purchases indicated at N minus expenditures recovered from earned income. Since N happens to be selected as the point where wage income exactly equals initial expenditures on xv, net income and
FIGURE 4

The Feasible Input Combinations
And Contract Curve for Wage Income
net expenditures on $x_v$ both equal zero at $N_w$. At point $Q$, the same $V_i$ is produced using only time. Since no expenditures are initially required, net income at $Q_w$ equals wage earnings. Performing a similar operation on each point on $V_i$, and on all other isoquants $V_j$, we derive a family of "activity" isoquants $V_{wi}$ and $V_{wj}$.

Although we considered points, such as $M$, where an individual's net income was negative in order to derive the "activity" isoquants, the set of feasible input combinations must be limited to those points along the net $V$ isoquants where net income is non-negative. These combinations are contained in the triangle $R_wS_U$.

The contract curve resulting from the tangencies between the $V_w$ and $Z$ isoquants is indicated by a line such as $R_wK_EB_U$ in Figure 4. Because wage income reduces the market good requirement of the "activity" isoquants, the contract curve between $U$ and $E$ indicates a greater good intensity in leisure production and time intensity in work production than evidenced in the contract curve for fixed income.

The shape and position of the resulting production possibility frontier (PPF) depends upon the substitutability of $x_z$ for $t_z$ in the production of $Z$ and upon the amount of $t_v$ required as a substitute for the first unit of $x_v$. The PPF $Z_wW_w$ in Figure 2 is drawn with the assumptions, which are implicit in the drawings, of substantial flexibility in the production of both $V$ and $Z$. The slope of the $Z_wW_w$ has no kink at the point which corresponds to point $E$ on the contract curve. At another extreme, if $G$ is assumed to be a Leontief production function and if both $F$ and $G$ are homogeneous of degree one, then the production possibility frontier is a triangle as indicated in Figure 5.
FIGURE 5

Production Possibility Frontier for Wage Income
Where Leisure Uses Time and Goods in Fixed Proportions
Regardless of the shape of the PPF, the utility maximizing equilibrium for the wage earner, B in Figure 2, indicates the optimum income as well as the optimum combination of Z and V produced. Point B in Figure 2 corresponds to point B in Figure 4 which uniquely determines an amount of time allocated to the production of V, and therefore wage income.

The exact shape of the wage PPF is particularly important to the individual who gets no utility from work output. If we assume lexicographic preferences for leisure, this wage model becomes similar to Becker's "Theory of the Allocation of Time" with a single exception. In both analyses, utility is maximized where the production of leisure goods is the greatest, such as points K in Figures 2 and 4, or at point E in Figure 5.

But our graphical analysis does not support the use of a single "full wealth" constraint suggested by Becker. Along the segment of the contract curve [RwE] where net income equals wage earnings, the shadow price of time and market goods are undefined. Along [EBU], the shadow prices are defined but that of time is unlikely to be the wage rate. For a single constraint to be valid, the shadow price of time would have to be defined and equal to the wage rate.

Despite some similarities, our wage model does not further reduce to the well-known model of income and leisure. Where pxv=1, the line RwS does correspond exactly to the income-leisure budget constraint. But we replace preferences for income and leisure with production isoquants.

The difference is important. Unless preferences for income and leisure are assumed to incorporate information about production technologies, utility maximization on the income-leisure budget constraint is unlikely
to maximize Z or use resources efficiently. Along the interval [RwK] of the contract curve, the individual is on a section of the PPF with a positive slope. Along the interval (ES), he is using resources inefficiently and produces below the PPF. The equilibrium of the standard income-leisure model will maximize leisure production only if preferences happen to maximize utility at point K. The individual will use resources efficiently only if preferences happen to maximize utility along the interval [KE].

C. Comparing Wage and Fixed Income

Turning to the comparison of the equilibria of wage and fixed income, we derive the major conclusions of this section by analysing the position of the wage PPF relative to the PPF for fixed income. The derivation of the Z intercept of the wage PPF is straightforward. If a wage earner does not work at all, his income is zero and his maximum possible production of Z is identical to the individual with no income. Thus Zw equals Zo in Figure 2.

We accomplish the remaining comparison by assuming that Y is set at the optimum income for wage w and by superimposing an Edgeworth Box for this fixed income over the triangle RwSU, Figure 6. The most important features of this superimposition are the fixed income and the wage isoquants, Vwi and Vwi respectively. By definition, Vwi indicates the work production at the utility maximizing point B on the contract curve RwKEBU.

Vfi is the isoquant for the same level of output but for a fixed income. Since the fixed income is assumed equal to the wage income at B, and since Vwi and Vfi are both defined as points on the isoquant Vi minus
FIGURE 6

Superimposed Edgeworth Box and Wage Triangle
income, $V_f$ also passes through $B$ in Figure 6.

At $B$, the slopes of $V_f$ and $V_w$ are unequal. Thus the fixed income contract curve passes not through $B$ but through points such as $H$ and $I$. At $H$ the fixed income earner produces the same work output, $V_f$, but more leisure, $Z_j$. At $I$, he produces the same leisure, $Z_i$, but more work, $V_f$. Between $H$ and $I$ he is able to produce more of both $Z$ and $V$. It therefore follows that the segment $HI$ of the PPF for a fixed income equal to the wage income at $B$ will dominate, or pass northeast of, any such point $B$. The position of the intercept $V_f$ relative to $V_w$ in Figure 6 is indeterminant and unimportant.

The implications of these comparative equilibria are as follows.

1) Since $Z_fV_f$ always dominates point $B$, an individual earning wages is always at a lower utility level than if he receives the same amount as a fixed income. 2) Since $Z_fV_f$ passes to the northeast of point $B$ but with a slope favoring the substitution of leisure for work, the utility maximizing combination of $Z$ and $V$ for fixed income—point $D$ in Figure 2—necessarily indicates more $Z$, but may or may not indicate more $V$. Point $D$ in Figure 6 must lie along a limited section of the contract curve $IHR_f$. We must therefore conclude that wages, when compared with fixed income, cause the individual to spend a larger amount of time at work. 3) If the individual with fixed income also happens to produce more $V$, $D$ in Figure 6 will fall between $H$ and $I$. In this situation, the wage earner uses fewer market goods for work production than he would if he receives a fixed income. This smaller expenditure on $x_V$ indicates that expenditures on $x_Z$, or net income, is greater than for fixed income.
D. Piecework and Fixed Incomes

A third remuneration method, piecework, pays the individual an income proportional to his work output. This section first describes how income affects the production isoquants and then compares the resulting equilibria with those of a fixed income.

As with wages, piecework income is assumed to reduce net expenditures on work inputs $xv$ along each production isoquant. Since piecework income is proportional to output rather than to working time, the "activity" isoquants are formed by shifting down all points on each technological $V$ isoquant by $(pv \cdot Vi)/pxv$. For example, let isoquants $V1$, $V2$ and $V3$ indicate $n+1$, $n+2$, and $n+3$ units of output respectively, Figure 7. Adjusting for income, the net isoquants appear to be shifted down in proportion to their output as indicated by the isoquants $Vp1$, $Vp2$ and $Vp3$.

If $F$ and $G$ are subject to diminishing returns to scale, there is at least a section of a contract curve, illustrated by $EpCU$, which corresponds to a negatively sloped section of a production possibility frontier, such as $ZpVp$. Let $C$ in Figures 2 and 7 represent the utility maximizing combinations of $Z$ and $V$.

In comparing the effects of fixed and piecework incomes, the position of the production possibility frontier $ZpVp$ relative to $ZfVf$ is important. As with wage income when $tv=0$, if the individual with a piecework remuneration produces no work output, he earns no income. Thus $Zp = Z0 = Zw$ in Figure 2. Also in a manner parallel to wage income, the utility maximizing point on the PPF determines the optimal quantity of $V$ and thus piecework income.
FIGURE 7
Feasible Input Combinations and Contract Curve
For Piecework Income
Since we again want to compare equilibria for identical income, we set fixed income equal to optimal piecework income and superimpose the resulting Edgeworth Box on Figure 7. In the resulting superimposition of Figure 8, the most important features are the net V isoquants $V_{pi}$ and $V_{fi}$. $V_{pi}$ is the "activity" isoquant for the optimum work output indicated by C. By definition, each point on $V_{pi}$ equals a point on an initial technological isoquant $V_1$ with the net purchases of $x_V$ reduced by the optimum piecework income. Since we set fixed income equal to piecework earnings at C, the "activity" isoquant for the same output level, $V_{fi}$, maps onto $V_{pi}$ exactly. The two isoquants are identical.

Because of this equality, $V_{fi}$ and $V_{pi}$ are both tangent to the same Z isoquant at point C. Thus the contract curves for piecework, CLU, and for fixed income, KCKU, and the two production possibility frontiers $Z_{pVp}$ and $Z_{fVf}$ intersect at point C. In some of the following analysis, we simplify by drawing C on the contract curve of fixed income.

For larger work outputs such as $V_{j}$, income is higher for piecework than for fixed income and $V_{pj}$ falls below $V_{fj}$ in Figure 8. Accordingly, less leisure production is sacrificed to produce additional units of work outputs. $V_{pj}$ is tangent to Z2 at L, and $V_{fj}$ is tangent to Z1 at K. As a result, at point C in Figure 2 the absolute slope of $Z_{fVf}$ is greater than that of $Z_{pVp}$.

The comparative equilibria depend critically upon the substitutability of V for Z in the utility function. If utility is gained from Z and V in fixed proportions, C and D are equal and all the equilibria characteristics of piecework and fixed incomes are identical. But if Z and V are utility substitutes, then a number of differences appear. First, utility is greater
FIGURE 8

Contract Curves for Piecework
And Fixed Incomes
for fixed income because of the greater slope of $zfVf$. Second, work output and the use of time and market inputs in producing $V$ are greater for piecework as a result of the production incentive of the piecework remuneration method. Because of the larger work outputs, leisure production and the use of time and market inputs are less for piecework. Finally, the larger work expenditures lead to a lower net income for piecework.

IV. A Three-Way Comparison between Wage, Piecework and Fixed Incomes

This section summarizes the analysis by comparing the results of the three remuneration systems simultaneously. The importance of the elasticity of substitution of $Z$ for $V$ in the utility function is highlighted by considering two alternatives, small and substantial substitutability. Where there is no substitutability, the equilibria for piecework and fixed incomes are identical and the comparison reduces to that of wage and fixed incomes summarized in Section III C.

The strongest comparative results occur when there is limited substitutability. We define limited substitutability as occurring when the individual both produces more $Z$ when paid a piecework income than he does for a fixed income and produces more $V$ when paid with a fixed income than he does with wages. By definition, these conditions indicate that both C and D fall between H and I on the fixed income contract curve. From above, we know that C falls to the northeast of D, exemplified by Cl and Dl in Figure 9.

Knowing the relative position of points B, C and D in both input and output spaces, we can derive relative outputs produced and inputs used by an individual paid the same income according to the three alternative remuneration methods, Table 1. First, utility is greatest for fixed income,
FIGURE 9

Alternative Positions of Wage, Piecework and Fixed Income Equilibria In Input Space
intermediate for piecework, and least for wages. Work outputs are greatest for piecework, intermediate for fixed income, and least for wages. Leisure outputs are greatest for fixed income, intermediate for wages, and least for piecework. Time at work is greatest for wages, intermediate for piecework, and least for fixed income. Leisure time is greatest for fixed income, intermediate for piecework, and least for wages. Work expenditures are greatest for piecework, intermediate for fixed income and least for wages. Leisure expenditures and net income, both defined as earnings minus work expenditures, are greatest for wages, intermediate for fixed income, and least for piecework.

These results may be intuitively understood by recognizing first that wage income induces large time allocations to work and leaves large market resources to produce leisure. The effect of this input distortion is to reduce both work and leisure outputs. Second, piecework induces a large work output effect which increases, relative to fixed income, the required work inputs of time and market expenditures. Third, the input distortion of wages appears to reduce utility more than the output effect of piecework.

These strong results are weakened in the more general case of large substitutability, defined as occurring when points C and D fall outside of H and I in their respective directions. For example, see C2 and D2 in Figure 9. Although we define this case by the greater leisure production of wage relative to piecework income, and the greater work production of wage relative to fixed income, no strong three-way conclusions about input allocations follow, Table 2.
TABLE 1
Comparative Characteristics of Equilibria of Three Remuneration Systems
Assuming Limited Substitution in the Utility Function

<table>
<thead>
<tr>
<th>Variable</th>
<th>Remuneration System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed Income</td>
</tr>
<tr>
<td>Utility</td>
<td>Greatest</td>
</tr>
<tr>
<td>Work Production</td>
<td>Intermediate</td>
</tr>
<tr>
<td>Leisure Production</td>
<td>Greatest</td>
</tr>
<tr>
<td>Time at Work</td>
<td>Least</td>
</tr>
<tr>
<td>Time at Leisure</td>
<td>Greatest</td>
</tr>
<tr>
<td>Work Expenditures</td>
<td>Intermediate</td>
</tr>
<tr>
<td>Leisure Expenditures or Net Income</td>
<td>Intermediate</td>
</tr>
<tr>
<td>or Net Income (gross income minus work expenditure)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Gross earnings are equal in all remuneration systems. See the text for an explanation of how this equality is accomplished.
TABLE 2

Comparative Characteristics of Equilibria of Three Remuneration Systems
Assuming Substantial Substitution in the Utility Function

<table>
<thead>
<tr>
<th>Variable</th>
<th>Remuneration System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed Income</td>
</tr>
<tr>
<td>Utility</td>
<td>Greatest</td>
</tr>
<tr>
<td>Work Production</td>
<td>Least</td>
</tr>
<tr>
<td>Leisure Production</td>
<td>Greatest</td>
</tr>
<tr>
<td>Time at Work</td>
<td>Less</td>
</tr>
<tr>
<td>Time at Leisure</td>
<td>Greater</td>
</tr>
<tr>
<td>Work Expenditures</td>
<td>Less</td>
</tr>
<tr>
<td>Leisure Expenditures</td>
<td>Greater</td>
</tr>
<tr>
<td>and Net Income</td>
<td></td>
</tr>
</tbody>
</table>

Note: Gross earnings are equal in all remuneration systems. See the text for an explanation of how this is accomplished.
V. Conclusion

This paper demonstrates that the method of remunerating professionals
does affect their allocation of time and market resources and their produc-
tion of work and leisure outputs. Not surprisingly, wages cause a larger
allocation of time to work activity than equal money paid according to piece-
work or fixed income; fixed incomes leave individuals at the highest utility
level; and piecework causes the largest work output. Less acceptable intui-
tively, the input allocation distortion caused by wages reduces utility more
than the output price incentive of piecework income.

Finally, the model allows three conclusions about input productivity.
From the information in Table 1, we conclude that the work productivity of
time, \( V/\tau_v \), is less for wages than for fixed income. Second, the leisure
productivity of time, \( Z/\tau_z \), is greater for wages than for piecework. Third,
the leisure productivity of expenditures, \( Z/x_z \), is greater for fixed income
than for wages. No conclusions about the work productivity of expenditures
are possible because the rank of work outputs and work expenditures are the
same for all remuneration systems.
FOOTNOTES

* Brookings Economic Policy Fellow assigned to DASPE/Health, Department of Health Education and Welfare. On leave from the University of Western Ontario. The paper is for discussion purposes only and does not represent the official policy of any organization. Helpful comments from Dave Burgess, Mark Frankena, David Greenberg, Howard Tuckman and Jeffrey Williams are gratefully acknowledged.

1. See Pollack and Wachter's footnote 14 for an explicit statement of the wage dilemma.

2. Pollak and Wachter argue that the existence of a shadow price for Z requires that F and G be homogeneous of degree one. In the context of this model, the shadow price of the single non-market commodity may be calculated as the value of foregone production of the market good (V) at the utility maximizing point on the production possibility frontier (PPF). This calculation only requires the weaker assumption of non-increasing returns to scale.

3. A fourth alternative remuneration, a mark-up on work expenditures, yields no determinant solution in the context of this model. The indeterminancy occurs because the specification effectively removes the production possibility frontier as a constraint.

4. Nothing is being said here about efficiency or optimality conditions.

5. As with wages, the PPF may at first slope upward from Zp. Moreover in the case of piecework income, the PPF may be discontinuous in such an upward sloping portion.
REFERENCES


