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Three Essays on Market Dynamics: Counterfeits, Tipping Policies, and Probabilistic Promotions

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A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Business

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Abstract

I study the complexities of market dynamics through the lens of investigating counterfeit goods, online platforms' tipping policies, and probabilistic price promotions. This dissertation consists of three essays that examine the strategic interactions between companies and consumers, aiming to offer insights into solutions companies can make to both better protect consumers' rights and enhance their profits.

In the first essay, I investigate the issue of counterfeits in online marketplaces, focusing on credence goods whose quality is, by definition, hard to ascertain even after consumption (e.g., nutritional supplements). I analyze a two-stage competition between genuine sellers and counterfeiters. I show that counterfeiters may utilize fake reviews to deceive consumers, and that reducing the prevalence of fake reviews can be achieved by platforms emphasizing the product-dependency of badges, ultimately advocating for consumer protection regulations.

In the second essay, I address the issue of "tip baiting" in online food delivery platforms, where some consumers may promise large tips before delivery for better service but reduce them post-delivery. Through a duopoly model, I investigate the effects of adjustable (or non-adjustable) tipping policies on platform profits, consumer behavior, and delivery performance. I discover that platforms favor adjustable tipping policies, which, while beneficial to them, disadvantage fair consumers and workers. This suggests a need for regulatory oversight to protect the rights of fair consumers and workers against the exploitative potential of the tip baiting practice.

In the third essay, I explore the design of probabilistic price promotions in a competitive setting, where firms offer consumers the chance to pay a promotional price or the full price through a lottery. The study focuses on the "zero-price effect," where free products are perceived to have higher intrinsic value. I establish that a simple lottery, wherein consumers either receive the product for free or are offered to pay the original list price, is more profitable than a complex lottery with many promotional prices.

Keywords: Game Theory, Credence Goods, Fake Reviews, Two-Sided Platforms, Probabilistic Price Promotions, Behavioral Operations

Summary for Lay Audience

This thesis examines how some of the online shopping and services we use every day can be improved for everyone involved. First, I study the impacts of fake credence goods (e.g., health supplements) on consumer surplus. This issue is even more challenging when some sellers also acquire fake product reviews to mislead consumers and boost sales. I find that online platforms can combat fake reviews by, for instance, clearly highlighting that badges are product-dependent.

I also study food delivery platforms' optimal tipping policy and whether they should allow consumers to adjust the tips after delivery has been completed. While placing orders, it is common for customers to specify a tip in order to receive faster delivery. However, some consumers would abuse platforms' tipping policy and intentionally renege on the tip they promised; a practice known as "tip baiting." My research suggests that platforms would always prefer to allow consumers to ex post adjust tips, even in the presence of tip baiting behavior; however, such a generous tipping policy would hurt consumers and workers. Hence, I advocate that third-party regulators need to intervene to safeguard consumers' and workers' rights, as platforms may lack the incentive to do so beyond a certain point.

Finally, I explore the optimal design of probabilistic promotions that give customers a chance to draw a lottery which determines the product price. I show that when properly designed, a simple lottery, where consumers either receive the product for free or are offered to pay the list price, is more profitable than a complex lottery with many promotional prices.

Co-Authorship Statement

I hereby declare that this thesis incorporates some material that is a result of joint research. Essay 1 was co-authored with Dr. Hubert Pun and Dr. Fredrik Odegaard. Essay 2 was co-authored with Dr. Hubert Pun. Essay 3 was co-authored with Dr. Fredrik Odegaard. As the first author, I was in charge of all aspects of the projects including formulating research questions, literature review, research design, model formulation and analysis, and preparing the first and the following complete of the manuscripts. With the above exceptions, I certify that this dissertation and the research to which it refers, is fully a product of my own work. Overall, this dissertation includes 3 original papers that are being currently under review in academic journals.

Essay 1 – Status: Under Review.

Lei, Y., Pun, H., Odegaard, F. (October 2021). Effect of Counterfeits and Fake Reviews in Markets for Credence Goods. Under 1st round review, *The International Journal of Management Science (Omega).*

Essay 2 – Status: Under Review.

Lei, Y., Pun, H. (August 2023). Two-Sided Platform Competition in the Presence of Tip Baiting. Reject and Resubmit for 2nd round review, *Production and Operations Management.*

Essay 3 – Status: Under Review.

Lei, Y., Odegaard, F. (August 2022). Probabilistic Price Promotions without Obligations. Revise and Resubmit for 2nd round review, *European Journal of Operational Research.*

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The completion of this dissertation would not have been possible without the collective efforts and encouragement of everyone mentioned above, along with many others who have helped me in various capacities. I extend my deepest thanks to all for your support during this challenging yet immensely rewarding journey.

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Chapter 1

1 Introduction

In this dissertation, I study the strategic interactions between consumers and firms in both online and offline markets. In the online platform setting (e.g., Amazon), counterfeits are a persistent problem and consumers often need to rely on external signals, such as product badges that are granted based on the fraction of positive reviews, to infer product quality. Indeed, Trustpilot (2019) finds that 77 percent of customers are more likely to purchase when products have badges. Therefore, the demand expansion effect of badges creates incentives to game the system. Specifically, unethical sellers may strategically acquire fake product reviews, which increase the chance of receiving badges, to mislead consumers. For example, AMZTigers claims that it can help sellers achieve Amazon's Choice badge for their products in less than two weeks (Walsh 2021).

Strategic interactions also exist in other online settings, such as on food delivery platforms, where consumers often specify a tip that they will give to delivery workers prior to delivery, in hopes of receiving a fast delivery. Upon completion of the delivery, consumers may be able to reduce their tip if the platform implements an adjustable tipping policy (e.g., Uber Eats). In this case, some consumers may strategically promise a large tip in order to expedite the delivery process, but deliberately reducing the tip to zero once the delivery is completed; a practice termed "tip baiting." For instance, an Uber Eats driver complained that a customer tip-baited her, as she delivered the food in a short time but her $13 tip was reduced to zero after delivery (Lenzen 2022).

Similarly, in the offline retail setting, firms are increasingly offering a new type of price promotions, probabilistic promotions, to take advantage of consumers' cognitive bias known as zero-price effect. In these promotions, firms strategically design the promotion parameters regarding the number of promotional prices, the chance of winning, and the list price. Supporting the case for this type of promotions, empirical research have found that compared to equivalent fixed price promotions, probabilistic price promotions can be more effective as they increase purchase likelihood (e.g., Choi and Kim 2007, Mazar et al. 2017, Hock et al. 2020).

The purpose of this thesis is to examine the strategic interactions among various stakeholders to provide managerial insights and better protect consumers' rights when firms may lack the incentive to do so.

1.1 Overview of Thesis and Specific Essays

In Chapter 2, I examine the issue of counterfeits in the presence of online fake product reviews. There are many studies on this research topic (e.g., Qian 2014, Cho et al. 2015, Gao et al. 2016, Pun and DeYong 2017, Shen et al. 2022). However, they either focus on non-deceptive counterfeits or study search/experience goods. My essay differs as I focus on deceptive counterfeits of credence goods, where consumers are unable to accurately assess product quality even after consumption.

I extend the literature by considering a market in which a genuine seller and a deceptive counterfeiter sell products under the same product listing. The products they offer are credence goods, meaning that their quality is difficult to determine even after purchase. Therefore, consumers rely on external signals, such as product badge, to infer product quality. Although prohibited by Amazon, the sellers may risk acquiring fake reviews, which increase the chance of obtaining such badges, to mislead consumers. Surprisingly, I find that the authentic seller would not acquire any fake reviews in equilibrium, whereas the counterfeiter may acquire some to mislead customers. Moreover, the units of acquired fake reviews decrease in the fraction of savvy consumers. This result offers important managerial insights that the operating platform can combat fake reviews by increasing the fraction of savvy customers. With regard to the impact of fake reviews, I show that having an option to acquire fake reviews may benefit both sellers, but always hurts consumers. Hence, the results advocate the necessity of regulating fake reviews to protect consumers' rights.

In Chapter 3, I investigate the impacts of platforms' tipping policies on consumers' ordering and tipping behaviors, delivery workers' efforts decisions, and platforms' profitability. Existing literature focuses on the case where workers' compensation solely come from the wage they receive from platforms (e.g., Rochet and Tirole 2003, Armstrong 2006, Bai et al. 2019, Zhang et al. 2022, Hu and Liu 2023). This essay contributes to this literature by also considering the impacts of consumers' tip on workers' total earnings and platforms' optimal wage and price decisions.

Since not all consumers intentionally reduce their tips after delivery, I consider a duopoly model consisting of two types of consumers: selfish consumers, who may engage in tip baiting, and fair consumers, who do not. I find that regardless of the fraction of selfish consumers, both platforms prefer the adjustable tipping policy over the non-adjustable one. This result has managerial implications that platforms with the non-adjustable policy can further enhance their profits by switching to the adjustable policy. Further, I show that platforms only have incentives to keep the fraction of selfish consumers sufficiently low, but not lower. However, the existence of selfish consumers always hurts those honest fair consumers and workers. Hence, I advocate that third-party regulators need to intervene to safeguard fair consumers' and workers' rights, as platforms may lack the incentive to do so beyond a certain point.

In Chapter 4, I study the optimal design of probabilistic price promotions where consumers through a lottery are either offered one of many promotional prices, including zero, or offered to purchase products at the original list price. There are many empirical research on probabilistic price promotions (e.g., Choi and Kim 2007, Mazar et al. 2017, Hock et al. 2020). However, very few analytical studies exist. Thus, this paper aims to fill the literature gap by offering an analytical framework of probabilistic promotions to offer insights into the optimal design of probabilistic promotions.

In the final Chapter 5, I present an overview of the main results and the related managerial insights.

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Chapter 2

2 Effect of Counterfeits and Fake Reviews in Markets for Credence Goods

Abstract: Counterfeits are a persistent problem in online marketplaces, in particular regarding credence goods (e.g., nutritional supplements), as their qualities are difficult or impossible to evaluate even after consumption. Concerned about product quality, customers frequently rely on external signals, such as product badges based on ratings. However, even product ratings are not foolproof as unethical sellers may acquire fake positive reviews to exploit product ratings and badge systems. To analyze the impact fake reviews have on credence goods, I consider a two-stage competition between an authentic seller and a deceptive counterfeiter. The market consists of two types of consumers: savvy customers, who understand that endorsement badges are product-dependent and not seller-dependent, and novice customers, who mistakenly believe product badges testify to a seller's authenticity. In the first stage, both sellers simultaneously decide on whether to acquire fake reviews, which partially influences if the product receives an endorsement badge. In the second stage, both sellers simultaneously set their prices and customers make purchasing decisions. The results indicate that, in equilibrium, the authentic seller does not acquire fake reviews, while the counterfeiter may do so to mislead customers. Moreover, the amount of fake reviews is decreasing in the fraction of savvy consumers, suggesting that online platforms can combat fake reviews by, for instance, clearly highlighting that badges are product-dependent. I also find that having the option to acquire fake reviews may benefit both sellers but always hurts consumers, emphasizing the need for regulation to protect consumers.

Key words: credence goods, deceptive counterfeits, fake reviews, product badges

2.1 Introduction

Online marketplaces have grown rapidly in recent years. In 2020, more than 256 million Americans were online consumers (Statista 2021). However, online consumers are plagued by counterfeits that are often sold by unscrupulous third-party sellers in e-commerce platforms. The U.S. Government Accountability Office (2018) finds that 20 out of 47 items purchased from third-party sellers across Amazon, Walmart, eBay, Sears Marketplace and Newegg are counterfeits.

Concerned about product quality, online customers may rely on external signals (e.g., product reviews and badges) to infer product quality prior to purchase. However, relying solely on product reviews can be time-consuming and overwhelming, as customers have to read through a large number of reviews to form an opinion about a product's quality. By contrast, because product badge (e.g., Amazon's Choice) is awarded to highly rated products, customers often regard the badge as a shortcut to quickly identify high-quality products. Indeed, Trustpilot (2019) finds that 77 percent of customers are more likely to purchase when products have badges.

Unfortunately, Amazon's Choice can be misleading. A product listing may be occupied by multiple sellers, with one seller listed as default seller and others as alternate sellers. Because Amazon's Choice is displayed only on the page of default seller, some customers might be misled to believe that the badge is awarded to the default seller. For example, Awesome Dynamic (2021) mistakenly states that "Amazon awards the Amazon's Choice badge to those *sellers* who constantly deliver stellar performance…" However, the badge is in fact awarded to a highly rated product (Amazon 2023). Additionally, unethical sellers may game the badge system by acquiring fake positive product reviews. For instance, sellers can use private Facebook groups to recruit reviewers to purchase their products and leave five-star reviews, and then these reviewers are reimbursed via PayPal (Proserpio et al. 2020). The five-star reviews boost the chance of obtaining Amazon's Choice badge (Simmonds 2020). In addition to directly acquiring fake reviews from customers, sellers can also resort to third-party companies specializing in "review campaigns." For example, AMZTigers claims that it can help sellers achieve Amazon's Choice badge for their products in less than two weeks (Walsh 2021). Because of fake reviews, even fake products can receive product badges. For example, Amazon's Choice was awarded to five fake cellphone chargers that claim Apple certification (Shifflett et al. 2019).

Fake reviews can be particularly damaging to certain products. For experience goods (e.g., branded shoes and luxury products), customers can often distinguish genuine products from fake products after purchasing (Qian 2014, Pun et al. 2021a). Even when customers are duped into buying counterfeits, they can report the counterfeits to online marketplaces in a timely manner and get refunded; the damage caused by counterfeits is thus limited. By contrast, credence goods (e.g., nutritional supplements and N95 masks) are products and services whose qualities are difficult to evaluate even after consumption (Darby and Karni 1973, Dulleck et al. 2011, Piccolo et al. 2018). As a result, customers cannot perfectly differentiate genuine products from counterfeits even after consumption, and the damage of counterfeiting is much more profound. For example, a customer did not know the Align (a Procter & Gamble brand) nutritional supplements she purchased from Amazon were fake until receiving an email notification from Amazon (Matsakis 2019). Similarly, Nutramax (2019) finds that an unscrupulous seller, BluTiger, sells counterfeit versions of Avmacol nutritional supplements on Amazon. In addition, 3M Company accuses a third-party seller on Amazon of selling counterfeit versions of its N95 masks, which are supposed to block 95 percent of small particles (Hufford 2020). Counterfeit masks are even more damaging during the pandemic, as customers may not realize their masks are counterfeits even after they get sick.

Although fake reviews exist in online marketplace, little is known about who acquires them and the quantity acquired. On one hand, acquiring fake reviews can be profitable, as a study involving 37,000 products on Amazon finds that on average, Amazon's Choice increases 17% in daily views and 25% in sales conversation rate of a product (Skrovan 2018). On the other hand, in addition to the monetary cost of fake reviews, the practice of acquiring fake reviews is strictly prohibited by Amazon, and legal actions and penalties would follow if sellers are caught acquiring fake reviews. Hence, it is unclear when it is profitable for sellers to acquire fake reviews. Online marketplaces may have incentives to under-regulate fake reviews too. He et al. (2021) empirically document that Amazon does not use all the potential policy levers to regulate fake reviews and takes over 100 days on average to delete them, suggesting that Amazon may benefit from fake reviews which increases its revenue by generating additional sales. Indeed, the U.S. Federal Trade Commission and the UK Competition and Markets Authority consider that Amazon is not taking sufficient actions to protect customers from fake reviews and warn the company of steep fines for misleading customers (Jackson 2021). Hence, for regulators to better combat fake reviews and protect consumers' rights, it is important to study the incentives and impacts of fake reviews.

In this paper, I consider a market in which a genuine seller and a deceptive counterfeiter sell products under the same product listing. The products they offer are credence goods, meaning that their quality is difficult to determine even after purchase. Motivated by customers' different perceptions of Amazon's Choice, I consider two types of consumers: savvy customers, who understand that Amazon's Choice is product-dependent and not seller-dependent, and novice customers, who mistakenly think the badge testifies a seller's authenticity. Although customers are uncertain about the authenticity of each seller, they have a prior belief about it. Observing the endorsement badge makes novice consumers adjust their beliefs. In the first stage, both sellers decide how many fake positive reviews to acquire simultaneously. Then, the public is informed about whether the product has received the badge or not. In the second stage, both sellers simultaneously set their prices. Later, the platform randomly selects one seller as the default seller and lists the other as an alternate seller. Finally, customers arrive and make purchasing decisions. With this model setup, I am interested in addressing the following research questions:

- Which seller acquires fake reviews?
- How many fake reviews are acquired?
- What are the impacts of having an option to acquire fake reviews on sellers and customers?

The results show that the authentic seller would not acquire any fake reviews in equilibrium, whereas the counterfeiter may acquire some to mislead customers. Moreover, the units of acquired fake reviews decrease in the fraction of savvy consumers. This result offers important managerial insights that the operating platform can combat fake reviews by increasing the fraction of savvy customers. For example, the platform can do so by providing more informative descriptions about the badge, making it clear that the badge is product-dependent and not seller-dependent. Moreover, though the counterfeiter acquires fewer fake reviews as the proportion of savvy customers increases, I find that fake reviews would not be eliminated unless the product badge has limited impact on expanding the market base. Thus, the results suggest that if platforms want to minimize fake reviews, they may consider re-positioning badge-winning products to less prominent locations, such as the middle of the search engine results. With regard to the impact of fake reviews, I show that having an option to acquire fake reviews may benefit both sellers, but always hurts consumers. Hence, the results advocate the necessity of regulating fake reviews to protect consumers' rights.

The rest of this paper is organized as follows. I review the related literature in Section 2.2. Then, I present the mathematical model in Section 2.3 and the equilibrium analysis in Section 2.4. Next, I examine the impact of having an option to acquire fake reviews on sellers and consumers in Section 2.5. Finally, Section 2.6 concludes the paper. While I focus on deceptive credence goods in the paper, I also present the analysis for other possible combinations of product characteristics in the Appendix A1.

2.2 Literature Review

This paper relates to the literature on counterfeits. There are multiple papers investigating the impact of *non-deceptive* counterfeits, e.g., Gao et al. 2016, Pun and DeYong 2017, Ghamat et al. 2021, Yi et al. 2022. This paper differs from these as my focus is on credence goods and hence on *deceptive* counterfeits. In the market for credence goods non-deceptive counterfeits would not have a viable value proposition.

Comparing to studies in the literature of deceptive counterfeits, this paper differs by focusing on credence goods, which are products that customers cannot perfectly differentiate between genuine and counterfeit even after purchasing. This contrasts with the other studies that examine products which can be distinguished either before purchasing (non-deceptive counterfeits) or after purchasing (deceptive counterfeits with experience goods). For example, Qian (2014) considers three brand-management strategies for competing against deceptive counterfeiters, including changing the quality of the authentic product, investing in enforcement efforts, and using non-price signals. Cho et al. (2015) investigate deceptive counterfeits in the context of a brand-name company, a counterfeiter, and a licit distributor, and explore multiple deterrence strategies. Pun et al. (2021a) and Pun et al. (2021b) examine the impact of adopting blockchain technology on deceptive counterfeiting. Shen et al. (2022) investigate the competition between brand name companies and copycats in a market setting with novice and expert customers. In contrast to these studies, this study uniquely contributes to the literature by identifying and developing targeted strategies specifically tailored for credence goods, thereby enhancing our understanding of consumer protection mechanisms in scenarios where product authenticity remains uncertain even post-purchase.

Next, I briefly review the literature on credence goods. Darby and Karni (1973) introduce the term 'credence goods' to define products and services with qualities that are difficult to judge even after consumption. Wolinsky (1993, 1995) cites a survey conducted by the U.S. Department of Transportation estimating that more than half of car repairs are unnecessary. Dulleck et al. (2011) study the role of liability, verifiability, reputation, and competition on credence goods markets. Zhou et al. (2022) consider a case of credence goods where healthcare service providers have informational advantage over patients, leading to incentives for overtreatment. Whereas the existing studies have almost exclusively focused on *service*, this paper contributes to this literature by studying *physical* credence goods. Piccolo et al. (2018) is the closest paper to mine. They consider a credence goods (physical product) market consisting of a high-quality seller and a low-quality seller who can deceptively advertise its product. This paper differs from theirs, as I adopt a two-period model where sellers may acquire fake product reviews in an earlier period, which in turn affects customers' purchasing decisions in a later period. By contrast, their paper studies advertising of credence goods, using a single-period model.

Lastly, this paper relates to the literature on consumer reviews. For instance, He and Chen (2018), Yang et al. (2021), and Guo et al. (2022) all investigate the impacts of consumer reviews on products' optimal price in the absence of fake reviews. This paper extends this literature by also considering the existence of fake reviews that may distort consumers' posterior beliefs about products' authenticity, which in turn affects products' prices. In the domain of fake reviews, Glazer et al. (2020) attempt to model the impact of fake reviews and find that fake reviews may benefit consumers and markets. Other studies have empirically documented the manipulation of rating through fake reviews (e.g., Anderson and Simester 2014, Luca and Zervas 2016, He et al. 2021). But these papers do not consider deceptive counterfeits or credence goods, which make it even harder for customers to evaluate product quality.

To the best of my knowledge, this paper is among the first to analyze the implications of acquiring fake reviews on deceptive credence goods in online marketplaces. Interestingly, I find that when the product is credence goods, the authentic seller would not acquire any fake reviews, whereas the counterfeiter may acquire some to mislead consumers. This result is new to the literature on counterfeits, where the impact of fake reviews has mostly been overlooked.

2.3 Model Setup

Consider an incumbent authentic brand seller (seller A) that sells credence goods (e.g., nutritional supplements/vitamins) to consumers on online platforms. Similar to Gao (2018), I define product quality as the benefits conveyed by consuming the product, and use $q > 0$ to denote the quality of authentic product. Furthermore, the product received an exogenous amount of $R > 0$ reviews from past sales. By the definition of credence goods, consumers are not able to accurately assess product quality even after consumption. Hence, some consumers may leave negative reviews even for products that in fact are authentic. For this reason, I assume that $0 < m < 1$ fraction of the product reviews are positive and the rest of reviews are negative.

In the beginning of stage 1, an unauthorized counterfeiter entrant (seller C) joins seller A's product listing and starts to offer (fake) identical-looking products under the same listing. The two sellers have the same marginal cost of production, which is normalized to zero. Due to the high fixed cost of research and development, the counterfeiter is incapable of providing genuine products. Because fake supplements/vitamins do not provide any benefits, the quality of fake product is zero.[1] As in Piccolo et al. (2018), I assume that each seller knows her own type and competitor's type (authentic or fake) from insider/specialty information. However, consumers only have a prior belief about each seller's type. Let $\phi \in (0,1)$ denote consumers' prior belief that each seller is authentic. On platforms where counterfeiting products are prevalent, ϕ tends to be small. Whereas on platforms that take serious actions against counterfeits, ϕ tends to be large.

In Figure 2.1, I attach a screenshot from Amazon to illustrate consumers' buying process. When consumers click on their desired product listing, they are directed to the page featuring a default seller (Curious Buy Nature, on the left of Figure 2.1) and can browse the product from an alternate seller (Vitamin King, whose page is shown on the right of Figure 2.1) under the box "Compare Offers on Amazon." Because the two sellers sell under the same product listing, they share the section of product reviews so that consumers cannot infer individual seller's type (authentic or fake) from the product review section (For example, both sellers share 457 product ratings).[2] However, consumers may update their beliefs upon observing the prominent "Amazon's Choice" badge. Because "Amazon's Choice" is only displayed on the page of default seller (the page of alternate seller does not display the badge, as shown in the right panel of Figure 2.1), some consumers might be misled to believe that the badge is awarded to the default seller, while in fact it is awarded to a highly rated product. See Awesome Dynamic (2021) and DataHawk (2022) for examples wherein people incorrectly state that "Amazon's Choice" is awarded to sellers, when it is actually awarded to products.

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[1] It is also possible that the fake product might be even detrimental to consumers' health, representing a negative product quality. However, this case will not qualitatively affect these results, as I can normalize the negative quality to zero.

[2] Consumers can see ratings/reviews for specific sellers by clicking on the seller ID. However, those reviews are for all the products sold by that seller. Therefore, it is almost impossible for consumers to trace specific reviews for the focal product from that particular seller.

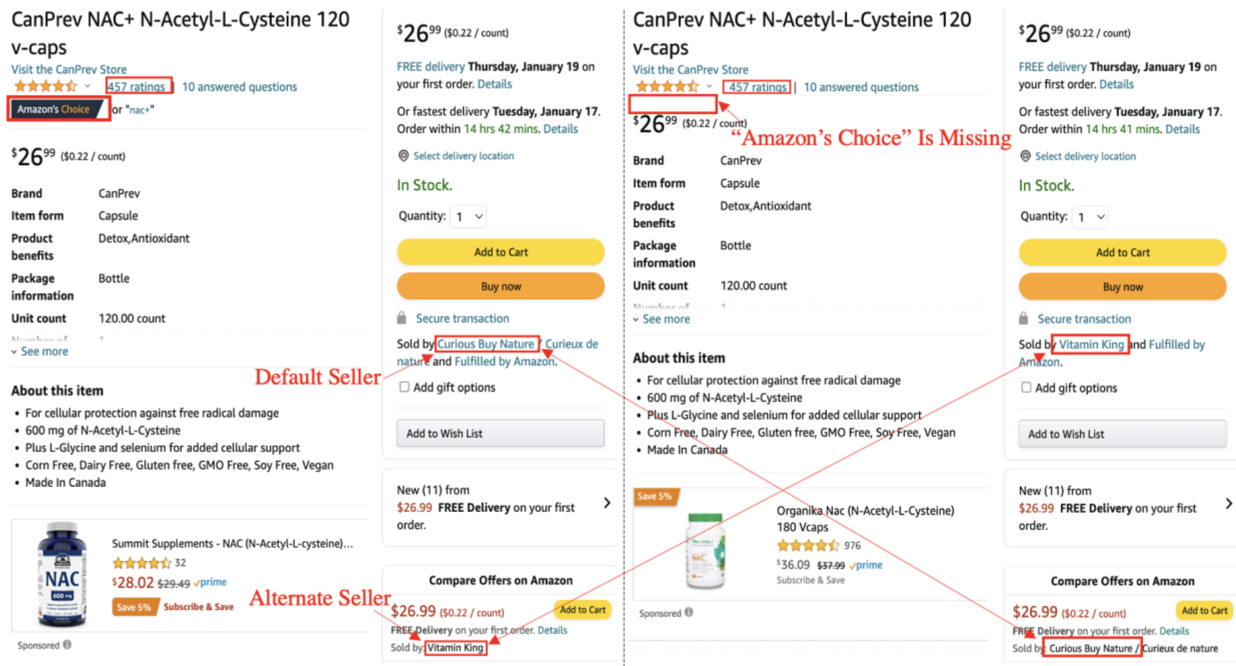

Figure 2.1. Two Sellers under the Same Product Listing

I capture this consumer heterogeneity by considering two types of consumers. A fraction $0 < \alpha < 1$ of consumers are savvy consumers who understand that Amazon’s Choice is product-dependent and not seller-dependent, hence observing the badge (if any) does not update their beliefs about an individual seller’s type. The other fraction $1 - \alpha$ of consumers are novice consumers who mistakenly think the badge testifies a seller’s authenticity, hence *incorrectly* updating their belief about the default seller’s authenticity and preferring the default seller (with badge) over the alternate seller (without badge).

Sellers have incentives to manipulate the product reviews so that the product has a higher chance of receiving the badge, which grants two benefits. First, a study involving 37,000 products on Amazon finds that on average, Amazon’s Choice increases 17% in daily views and 25% in sales conversation rate of a product (Skrovan 2018). Thus, I assume that if the product receives the badge, then the product’s market size is increased to $\tau \geq 1$. When $\tau = 1$, receiving the product badge does not affect the market size (the market size without the badge is normalized to 1). When $\tau > 1$, receiving the badge has a market expansion effect. Because the badge is awarded to a product rather than a particular seller, this market expansion effect is shared by both default seller and alternate seller. Second, novice consumers incorrectly update their beliefs about a seller’s authenticity upon observing the badge, enabling the seller to charge a higher price.

Both sellers simultaneously decide how many units of fake positive reviews to acquire. Empirical findings suggest that sellers pay a commission fee for every fake review acquired (He et al. 2021), and I use $\beta > 0$ to denote the unit cost of acquiring fake reviews. However, the practice of acquiring fake reviews is strictly prohibited by the operating platform (e.g., Amazon), and legal actions and penalties would follow if sellers are caught acquiring fake reviews. I use a parameter $\gamma_i \geq 0$ to indicate the expected sanction cost per fake review for seller $i, i \in \{A, C\}$. A larger γ_i may reflect either more severe sanctions or more intensive monitoring by the operating platform. Similar to Piccolo et al. (2018), I assume γ_i is a per unit cost parameter, because the expected sanction cost depends on the probability of getting caught, which increases in the quantities of fake reviews acquired. In addition, authentic sellers often face heavier expected sanction cost than counterfeiters do, as the latter usually reside in countries with more relaxed regulations and foreign lawsuits can be hard to pursue. Therefore, $\gamma_A > \gamma_C \geq 0$, and I normalize to have $\gamma_C = 0$. The total cost of acquiring fake reviews is thus $(\beta + \gamma_A)f_A$ for seller A and βf_C for seller C, where $f_i \geq 0$ represents the units of fake positive reviews acquired by seller i. Motivated by Amazon's Choice, which is a badge given to highly rated products, I assume that the probability of receiving the badge is positively related to the fraction of positive product reviews. Hence, the probability of a product receiving badge is $\frac{R*m+f_A+f_C}{R+f_A+f_C}$, where the numerator represents the amount of positive reviews and the denominator corresponds to the total number of reviews.

After acquiring fake reviews, the product may or may not receive the badge. In section 2.3.1, I discuss the case where the product receives the badge. In section 2.3.2, I consider the other case where the product does not receive the badge. In section 2.3.3, I summarize the game sequence.

2.3.1 Consumer Expectations With Product Badge

In the presence of product badge, the total market size becomes τ. Upon receiving the badge, both sellers simultaneously decide their respective prices in the beginning of second stage. In the model, the counterfeiter would earn zero demand if customers are able to identify her type, as no customers would knowingly buy fake supplements/vitamins that have zero quality. Therefore, the counterfeiter will always mimic the authentic seller's price, as any price difference will reveal the counterfeiter's type—that is, $p_i = p_{-i} = p^*$, which is the pooling equilibrium price and I provide detailed explanations in Section 2.4. This is also consistent with my example in Figure 2.1, where both sellers set the price to be $26.99. If the customers observe a nonequilibrium price from just one seller—that is, $p_i \neq p_{-i} = p^*$—they believe that seller i is a counterfeiter. Such belief about nonequilibrium price is standard when characterizing pooling equilibrium, see Moraga-González 2000, Gao 2018, Piccolo et al. 2018, Pun et al. 2021b. Note that a separating equilibrium, where sellers charge different prices, is not possible in my model, because it would reveal a counterfeiter's type and no customers would knowingly purchase products of zero quality (note that for a separating equilibrium to possibly sustain, the fake product would need to have a positive quality, though lower than that of the authentic product). Hence, the model only has a pooling equilibrium where both sellers charge the same price.

Once prices have been set, one of the sellers will become the default seller and the other becomes the alternate seller. According to Amazon, sellers can do a list of things to increase the chance of becoming the default seller, including pricing competitively, offering free shipping, and keeping enough stocks (Amazon 2022). Since I am not modelling product shipping and inventory in this paper, I assume that when the two sellers charge the same price, each seller has 1/2 chance to become the default seller.

Upon observing the prices of default seller (whose page displays the product badge) and of alternate seller (whose page does not display the badge), customers make purchasing decisions. Because customers are uncertain about seller's type, their purchasing decisions are based on the product's perceived quality, instead of the product's exact quality. Customers are heterogeneous in their valuation of a product, and the valuation (v) is uniformly distributed over [0,1].

For savvy customers, they understand that the badge is product-dependent and not seller-dependent, hence observing the badge (if any) does not update their beliefs about a seller's type. In addition, because both sellers would charge the same price in equilibrium, savvy customers remain indifferent between purchasing from the two sellers. Therefore, a savvy customer's expected utility of buying from seller i is $U_i^S = \phi q v - p_i$. For novice customers, they mistakenly believe that the default seller with badge must be authentic. Hence, a novice customer's perceived expected utility of buying from the default seller, say seller i, is $U_i^N = qv - p_i$. Moreover, a novice customer will never buy from the alternate seller, because (1) the default seller is perceived to be authentic whereas the alternate seller may be fake and (2) both sellers charge the same price.

2.3.2 Consumer Expectations Without Product Badge

In the absence of product badge, the total market size remains one. In this case, savvy consumers and novice consumers are effectively the same, as novice consumers do not update beliefs without observing the badge. Therefore, both types of consumers are indifferent between buying from the two sellers; the expected utility of buying a product from seller i is $U_i = U_i^S = \phi q v - p_i$. The rest of the model is the same as in section 2.3.1.

2.3.3 Summary of Game Sequence

Figure 2.2 summarizes the sequence of events. At the beginning of first stage, the counterfeiter enters the market by joining the product listing. Then, both sellers simultaneously decide how many units of fake positive reviews to acquire. Next, whether the product listing receives the badge or not is revealed to public. In the beginning of the second stage, both sellers simultaneously decide their prices, respectively. Later, one of the sellers is randomly chosen by the platform as the default seller, while the other is listed as an alternate seller. Finally, customers arrive and decide whether to buy from seller A, or seller C, or neither.

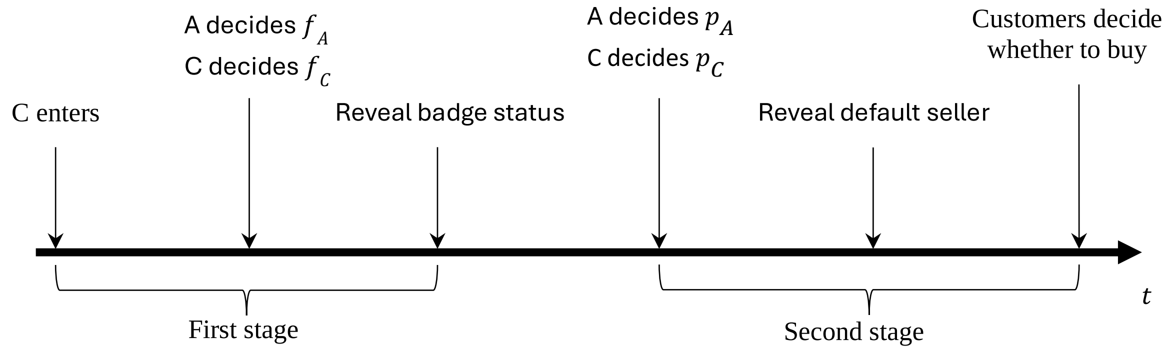

Figure 2.2 Sequence of Events

2.4 Equilibrium Analysis

As mentioned above that the model has only pooling equilibrium, as separating equilibrium would reveal a counterfeiter's type and no customers would knowingly buy fake supplements that have zero quality. Furthermore, I derive the perfect Bayesian equilibrium (PBE) and adopt the strongly undefeated perfect Bayesian equilibrium (SUPBE) to refine the pooling equilibrium (Mailath et al. 1993). As detailed in Miklós-Thal and Zhang (2013) and Gao (2018), the SUPBE yields the high-quality seller the highest profit among all possible pooling PBEs. Using this SUPBE refinement, I am able to derive a unique pooling PBE by solving seller A's profit maximization problem, given that seller C will always mimic seller A's price.

2.4.1 Equilibrium Price

The optimal pooling price depends on whether the product receives the badge or not. I use the superscript B and NB to denote the case where the product badge is and is not granted, respectively. I solve the two subgames using standard backward induction technique. Proofs of all formal results are given in the Appendix A2.

First consider the case where the product receives the badge (section 2.3.1). Let p^B denote the pooling price that sellers charge when the product receives badge. In the second period, knowing that seller C would follow seller A's price, I can express seller A's profit maximization problem as follows:

$$\max_{p^B \geq 0} \ \Pi_A^B(p^B) = p^B \left(\alpha \frac{\tau}{2} \int_{v:\phi q v - p^B \geq 0} dv + (1-\alpha) \frac{\tau}{2} \int_{v: qv - p^B \geq 0} dv \right). \tag{2.1}$$

The objective function states that for the α fraction of savvy consumers, they randomly with equal probability buy from a seller when purchasing the product provides non-negative utility; for the other $1 - \alpha$ fraction of novice consumers, they will only purchase from the default seller, whom each seller has 1/2 chance to become. Lemma 2.1 characterizes the optimal solution of problem (2.1).

Lemma 2.1. *When product receives the badge, the optimal pooling price is* $p^{B^*} = \frac{q\,\phi}{2(\alpha+(1-\alpha)\phi)}$.

Next, I analyze the case where the product does not receive the badge (section 2.3.2). Let p^{NB} denote the pooling price that sellers charge when the product does not receive badge. Knowing that seller C would follow seller A's price, I can express seller A's profit maximization problem as follows:

$$\max_{p^{NB}\geq 0} \; \Pi_A^{NB}(p^{NB}) = p^{NB}\frac{1}{2}\int_{v:\phi qv-p^{NB}\geq 0} dv. \qquad (2.2)$$

In the absence of badge, novice consumers do not receive additional information to infer a seller's type and effectively behave the same as savvy consumers do. Lemma 2.2 characterizes the optimal solution of problem (2.2).

Lemma 2.2. *When product does not receive the badge, the optimal pooling price is* $p^{NB^*} = \frac{q\,\phi}{2}$.

2.4.2 Optimal Amount of Fake Reviews

In stage 1, sellers simultaneously and individually decide how many units of fake reviews to acquire, maximizing their own expected profits before knowing if the product would receive the badge. Specifically, seller i's expected profit maximization problem can be expressed as:

$$\max_{f_i\geq 0} \; \mathbb{E}[\Pi_i(f_i)] = \frac{R*m+f_i+f_{-i}}{R+f_i+f_{-i}}\Pi_i^B\left(p^{B^*}\right) + \left(1-\frac{R*m+f_i+f_{-i}}{R+f_i+f_{-i}}\right)\Pi_i^{NB}\left(p^{NB^*}\right) - (\beta+\gamma_i)f_i \quad (2.3)$$

where $\frac{R*m+f_i+f_{-i}}{R+f_i+f_{-i}}$ represents the probability of receiving badge. Proposition 2.1 presents sellers' optimal acquiring quantities of fake reviews.

Proposition 2.1. *There exists a unique threshold* $\overline{\beta}$ *such that the optimal units of fake reviews to acquire are*

$$f_A^{\;*} = 0,$$

$$f_C^{\;*} = \begin{cases} \dfrac{\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}-4R\beta(\phi+(1-\phi)\alpha)}{4\beta(\alpha+(1-\alpha)\phi)}, & if\ \beta<\overline{\beta}, \\ 0, & if\ \beta\geq\overline{\beta}, \end{cases}$$

where $\overline{\beta} \equiv \frac{q\phi(1-m)(\tau-(\phi+(1-\phi)\alpha))}{8R(\alpha+(1-\alpha)\phi)}$.

Note that the authentic seller would not acquire any fake reviews in equilibrium ($f_A^{\;*}=0$), which is consistent with empirical findings that fake reviews are mostly acquired by sellers with low-quality products (He et al. 2021). When the unit cost of fake review is low ($\beta<\overline{\beta}$), acquiring fake reviews is a sound business as it increases the probability of receiving product badge, which in turn helps (1) increase the market size and (2) convince novice consumers of the default seller's authenticity. Hence, seller C would acquire a positive number of fake reviews. By contrast, the authentic seller would not acquire fake reviews for two reasons. First, due to the additional expected sanction cost, acquiring fake review is costly to seller A even though the unit cost of fake review is low. Second, because the badge is product-independent and not seller-independent, seller A is able to share the benefits of seller C acquiring fake reviews. When the unit cost of fake review is high ($\beta \geq \overline{\beta}$), the higher probability of receiving badge is not worth the investment, hence neither seller acquires fake reviews.

Next, I summarize in Proposition 2.2 the impact of two parameters, namely the fraction of savvy consumers (α) and customers' prior belief (ϕ), on sellers' optimal quantity of fake reviews.

Proposition 2.2. *Seller C's optimal quantity of fake reviews* ($f_C{}^*$) *has the following properties*:

a. *when* $\alpha < \overline{\alpha} \equiv \begin{cases} \frac{\phi(q(\tau(1-m)-\phi(1-m))-8R\beta)}{(1-\phi)(8R\beta+q\phi(1-m))}, & if\ \tau < \frac{8R\beta+q\phi(1-m)}{q\phi(1-m)} \\ 1, & if\ \tau \geq \frac{8R\beta+q\phi(1-m)}{q\phi(1-m)} \end{cases}$, $f_C{}^*$ *decreases in* α.

when $\alpha \geq \overline{\alpha}$, $f_C{}^* = 0$.

b. $f_C{}^*$ *increases in* ϕ *if and only if and only if* $\underline{\phi} < \phi < \overline{\phi}$, *where* $\overline{\phi} \equiv \frac{\sqrt{\alpha\tau}-\alpha}{1-\alpha}$ *and*

$$\underline{\phi} \equiv [\phi(8R\beta(\alpha-1) + q(\tau(1-m) + \alpha(m-1)) - \sqrt{(8R\beta(1-\alpha) + q(\alpha - m\alpha - \tau + m\tau))^2 - 32q\alpha R\beta(1-m-\alpha+m\alpha)})]/[2q(1-m)(1-\alpha)].$$

To derive further managerial insights, I illustrate Proposition 2.2a and 2.2b in the left and right panel of Figure 2.3, respectively; with $q = 2, \tau = 1, m = 0.6, R = 1, \beta = 0.01$, $\phi = 0.7$ (left panel only), $\alpha = 0.5$ (right panel only). Under this set of parameters, $\overline{\alpha} = 0.58$, $\underline{\phi} = 0.13$, and $\overline{\phi} = 0.41$. In Figure 2.3, the vertical axis corresponds to seller C's optimal purchasing quantity of fake reviews ($f_C{}^*$); the horizontal axis represents the fraction of savvy consumers (α) in the left panel, whereas it represents customers' prior belief about a seller's authenticity (ϕ) in the right panel.

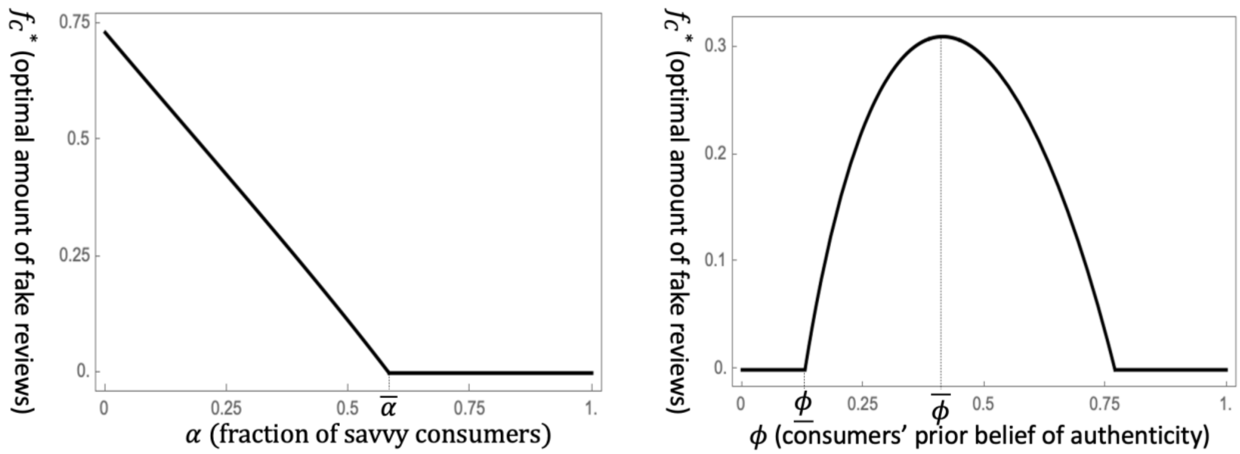

Figure 2.3. Impacts of α and ϕ on Acquiring Fake Reviews, Respectively

Proposition 2.2a has important managerial implication that the operating platform can combat fake reviews by increasing the fraction of savvy customers (shown in the left panel of Figure 2.3). For example, the platform can provide more informative and precise descriptions about the badge, making it clear that the badge is product-dependent and not seller-dependent. Doing so would convert novice customers, at least partially, to savvy customers who do not update beliefs about a seller's type upon observing the badge. Consequently, sellers' incentives to acquire fake reviews diminish, and we observe that when $\alpha < \overline{\alpha} = 0.58$, the quantity of fake reviews acquired ($f_C{}^*$) continues decreasing in α.

However, it remains unclear whether the quantity of fake reviews will keep decreasing such that it eventually reaches zero. Specifically, if the market expansion effect (τ) is sufficiently large, then seller C may choose to acquire a positive quantity of fake reviews even if all customers were savvy customers (i.e., $\alpha = 1$). It turns out that if the market expansion effect is larger than a threshold $\overline{\tau} \equiv \frac{8R\beta + q\phi(1-m)}{q\phi(1-m)}$, then seller C would always acquire positive units of fake reviews that are decreasing in α but never reach zero. By contrast, if the market expansion effect is less significant ($\tau < \overline{\tau}$), then seller C may completely stop acquiring fake reviews if the market has enough savvy customers ($\alpha > \overline{\alpha}$). A related managerial implication is that if combating fake reviews is the platform's top priority, then the platform can lower the market expansion effect of receiving the badge. Usually, products that receive the badge are exhibited at the top of the platform's search engine, hence receiving a lot of customer views. Thus, the platform can lower the market expansion effect by re-positioning these products to less prominent locations, such as the middle of the search engine results.

As shown in the right panel of Figure 2.3, there is a non-monotone change in $f_C{}^*$ as ϕ increases. We observe that seller C has no incentives to acquire fake reviews when the prior belief is either too low or too high. When ϕ is too low ($\phi \leq \underline{\phi}$), customers believe that the market is predominantly occupied by counterfeiters. In the presence of such pessimistic belief, sellers have to charge a low price, and the profit margin is low such that sellers have no incentives to acquire fake reviews. By contrast, when ϕ is too high ($\phi \geq \overline{\phi} = 0.77$), customers are optimistic about interacting with authentic sellers, who are then empowered to charge a high price because of products' perceived high quality. As a result, the appeal of fake reviews disappears.

When $\underline{\phi} < \phi < \overline{\phi}$, customers are skeptical about a seller's type. Therefore, the benefit of signaling authenticity to novice customers is large, and seller C acquires more fake reviews to increase the chance of receiving the badge. Interestingly, when ϕ is in the intermediate range ($\underline{\phi} < \phi < \overline{\phi}$), ${f_C}^*$ first increases and then decreases in ϕ. In contrast, when $\phi \geq \overline{\phi}$, customers are convinced of a seller's authenticity, and the appeal of fake reviews starts to decrease. Therefore, we observe a decrease in ${f_C}^*$ that eventually drops to zero.

2.4.3 Consumer Composition Effect on Equilibrium Pooling Price

Next, I summarize in Proposition 2.3 the impact of the fraction of savvy consumers (α) and customers' prior belief (ϕ), on the optimal pooling price.

Proposition 2.3. *The optimal pooling price has the following properties*:

a. when the product receives the badge, the optimal price ${p^B}^*$ decreases in α.

b. when the product does not receive the badge, the optimal price ${p^{NB}}^*$ increases in ϕ.

When the product receives the badge, novice customers updates beliefs and think that the default seller, whose page displays the badge, must be supplying authentic products. As a result, sellers are able to charge a high pooling price. However, savvy customers keep their prior beliefs that sellers may be counterfeiters, resulting in a low pooling price. Consequently, the larger the fraction of savvy customers, the lower the pooling price. When the product does not receive the badge, the fraction of savvy customers no longer affects the pooling price, as savvy customers and novice ones are effectively the same. In this case, the pooling price increases in customers' prior belief that a seller is authentic.

2.5 Equilibrium Outcome of Expected Profits, Consumer Surplus, and Social Welfare

2.5.1 Impact of Fake Reviews on Expected Profit

First, I examine the impact of unit cost of fake reviews (β) on sellers' expected profits ($\mathbb{E}[\Pi_A({f_A}^*)]$ and $\mathbb{E}[\Pi_C({f_C}^*)]$).

Proposition 2.4. *As the unit cost of fake reviews (β) increases,*

a. $\mathbb{E}[\Pi_A({f_A}^*)]$ and $\mathbb{E}[\Pi_C({f_C}^*)]$ weakly decrease.

b. *the gap between* $\mathbb{E}[\Pi_A(f_A{}^*)]$ *and* $\mathbb{E}[\Pi_C(f_C{}^*)]$ *increases if and only if* $\beta < \overline{\beta}/4$.

To give a visual illustration, I plot Proposition 2.4 in Figure 2.4 with $q = 2, \tau = 1, p = 0.6, R = 1, \phi = 0.7$, and $\alpha = 0.5$. Under this set of parameters, $\overline{\beta}/4 = 0.0031$. In Figure 2.4, the vertical axis corresponds to sellers' expected profits, and the horizontal axis represents the unit cost of fake reviews (β).

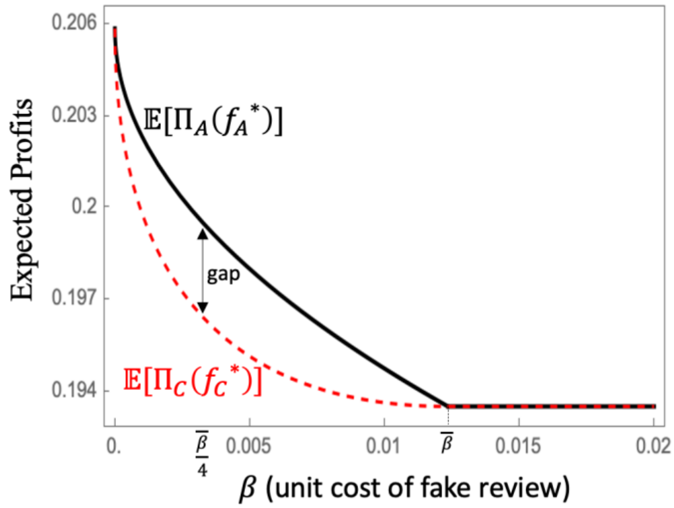

Figure 2.4. Impact of β on Sellers' Expected Profits

When $\beta < \overline{\beta}$, seller C acquires fake reviews and her expected profit decreases as fake reviews becomes more expensive. Although seller A does not directly acquire fake reviews, her profit is also affected by the unit cost of fake reviews, as a higher unit cost leads to less fake reviews acquired by seller C and a lower chance of receiving the badge. Because both sellers charge the same price and have equal chance of being the default seller, they have the same expected revenue from selling products. However, seller C incurs additional cost due to acquiring fake reviews. As a result, seller A always has a higher expected profit than seller C does. When $\beta \geq \overline{\beta}$, the unit cost of fake reviews is so high that seller C completely stops acquiring fake reviews. Therefore, sellers' profits are no longer affected by β.

An interesting observation is that the gap between $\mathbb{E}[\Pi_A(f_A{}^*)]$ and $\mathbb{E}[\Pi_C(f_C{}^*)]$ first increases and then decreases in β. When $\beta < \overline{\beta}/4$, the unit cost of fake reviews is low so that seller C acquires a large quantity of them. As β increases, seller C pays more for fake reviews while enjoying the same expected revenue of seller A. Hence, the profit gap increases. However, as β continues to increase, acquiring fake reviews becomes more expensive such that seller C acquires less of them, resulting in a gradual convergence of the two sellers' profits. Eventually, when $\beta \geq \overline{\beta}$, seller C completely stops acquiring fake reviews, and the gap between profits closes.

An interesting issue is whether the sellers would be better or worse off without the opportunity to acquire fake reviews. To assess this issue I next consider a benchmark where sellers do *not* have the option to acquire fake reviews. For notational convenience, I use the superscript Λ to denote solutions to the benchmark model where fake reviews are not available.

When acquiring fake review is not an option, sellers' expected profits are equivalent to those in the main model by fixing f_i and f_{-i} to 0. That is, $\mathbb{E}[\Pi_i^\Lambda] = \mathbb{E}[\Pi_i(f_A = 0, f_C = 0)]$. Then, Proposition 2.5 demonstrates the impact of the fake review option on sellers' optimal expected profits. For ease of exposition, I numerically illustrate Proposition 2.5 in Figure 2.5 with $q = 2, \tau = 1, m = 0.6, R = 1, \phi = 0.7$, and $\alpha = 0.5$.

Proposition 2.5. *Both sellers are better off when acquiring fake reviews is an option if and only if $\beta < \overline{\beta}$.*

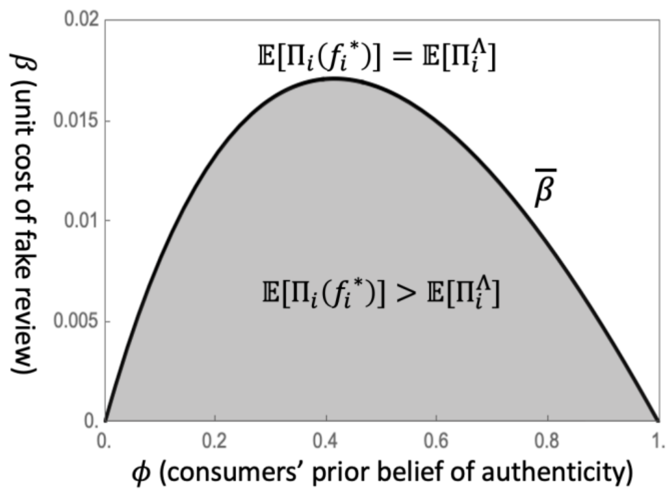

Figure 2.5. Comparison between Sellers' Expected Profits in the Main Model and That in the Benchmark

I show that having the option to acquire fake reviews weakly benefits both sellers, as their action space is enlarged. In the benchmark where acquire fake reviews is not an option, sellers' expected profits are the same as seller C chooses to not acquire any fake reviews ($\beta \geq \overline{\beta}$) in the main model. When $\beta < \overline{\beta}$, seller C chooses to acquire positive units of fake reviews, from which we can infer that acquiring positive units of fake reviews must provide a higher profit than that in the benchmark, otherwise she would have not acquired fake reviews. Interestingly, seller A is also better offer when acquiring fake reviews is an option, even though she never actually acquires any fake reviews. Essentially, seller A becomes better off because she can free ride on seller C's investment in fake reviews. Since I have set $\tau = 1$ in Figure 2.5, the market expansion effect does not exist, and seller A becomes better off solely because fake reviews increase the probability of receiving the badge that enables sellers to charge a higher price.

2.5.2 Impact of Fake Reviews on Consumer Surplus

In this section, I examine the impact fake reviews have on consumer surplus. From the perspective of a social planner, I am able to observe customers' realized surplus, which is non-observable to customers themselves due to the nature of credence goods. Consequently, the definition of consumer surplus is similar to the typical definition of realized consumer surplus, except that the former is not observable to customers. Specifically, I can express consumer surplus as:

$$CS^* = \left(\frac{R*m + f_A^{\ *} + f_C^{\ *}}{R + f_A^{\ *} + f_C^{\ *}}\right)\left[\alpha \int_{v:\phi qv - p^{B^*} \geq 0} \left(\frac{qv}{2} - p^{B^*}\right) dv + (1-\alpha) \int_{v:qv - p^{B^*} \geq 0} \left(\frac{qv}{2} - p^{B^*}\right) dv\right]$$
$$+ \left(1 - \frac{R*m + f_A^{\ *} + f_C^{\ *}}{R + f_A^{\ *} + f_C^{\ *}}\right) \int_{v:\phi qv - p^{NB^*} \geq 0} \left(\frac{qv}{2} - p^{NB^*}\right) dv. \tag{2.4}$$

Equation (2.4) states that conditional on receiving the badge (with probability $\frac{R*m+f_i+f_{-i}}{R+f_i+f_{-i}}$), savvy consumers randomly purchase from either seller with equal probability, and half of them obtain benefit q from consuming the authentic product, whereas the other half obtain zero benefit from consuming the fake product. For the other $1-\alpha$ fraction of novice consumers, they are convinced of the default seller's authenticity and exclusively buy from the default seller, who coincides with the authentic seller with probability 1/2. By contrast, conditional on not receiving the badge (with probability $1 - \frac{R*m+f_i+f_{-i}}{R+f_i+f_{-i}}$), novice consumers behave the same as savvy ones, and randomly purchase from either seller with equal probability.

Similar to the discussion in Section 2.5.1 above, I next consider consumer surplus in a benchmark model where fake reviews are not available; as above, I denote the benchmark model consumer surplus with a superscript Λ, CS^{Λ^*}, which are obtained by fixing the units of fake reviews ($f_A^{\ *}$ and $f_C^{\ *}$) to zero in equation (2.4). In order to quantify the damage of fake reviews, I further define the detrimental effect of fake review (H_f) as the gap between consumer surplus in the benchmark and that in the main model,

$$H_f \coloneqq CS^{\Lambda^*} - CS^*.$$

Proposition 2.6 compares consumer surplus in the main model with that in the benchmark.

Proposition 2.6. *Consumer surplus has the following properties*:

a. consumer surplus is always weakly lower when acquiring fake reviews is an option, $CS^* \leq CS^{\Lambda^*}$.

b. the harm of fake review (H_f) *increases in product quality* q *if and only if* $q > \overline{q}$,

where $\overline{q} \equiv \frac{8R\beta(\alpha+(1-\alpha)\phi)}{\phi(1-m)(\tau-(\phi+(1-\phi)\alpha))}$.

When the product quality is low ($q \leq \overline{q}$), the product is not highly valued by consumers, as it cannot provide much benefits. Hence, the pooling price will be low, and the market is unattractive such that neither seller has incentives to acquire fake reviews. As a result, the main model coincides with the benchmark, and the harm of fake review is zero. Interestingly, when the product quality is high ($q > \overline{q}$), the harm of fake review can increase in product quality. As the quality increases, the product becomes more valuable to consumers, who are willing to pay a higher price for it. However, the pooling price will be limited unless consumers are convinced that their intended product is authentic, not fake. Therefore, receiving product badge, which convinces novice consumers of the default seller's authenticity, is critical. As a result, seller C would acquire fake reviews to increase the probability of receiving badge. The higher the product quality, the stronger the incentive to acquire fake reviews, and the larger the harm of fake review.

2.5.3 Impact of Fake Reviews on Social Welfare

Lastly, I examine the impact of fake reviews on the society. To do so, I define social welfare (SW) as the sum of seller A's expected profit, seller C's expected profit, and consumer surplus. Proposition 2.7 compares social welfare in the main model (SW^*) with that in the benchmark (SW^{Λ^*}).

Proposition 2.7. *There exist unique thresholds $\tilde{\tau}$, $\tilde{\tilde{\tau}}$ and $\underline{\beta}$ such that*

a. *when $\beta \geq \overline{\beta}$, $SW^* = SW^{\Lambda^*}$.*
b. *when $\underline{\beta} \leq \beta < \overline{\beta}$, $SW^* \leq SW^{\Lambda^*}$.*
c. *when $\beta < \underline{\beta}$, $SW^* > SW^{\Lambda^*}$.*

$$\text{where } \underline{\beta} \equiv \begin{cases} \overline{\beta}, & \text{if } \tau \geq \tilde{\tilde{\tau}} \\ \dfrac{q(1-m)\left[(1-\phi)^2\alpha^2 - \left((4\tau-3)\phi^2 + (2-4\tau)\phi + 1\right)\alpha + 4\phi^2(\tau-1)\right]^2}{32R\phi(\alpha+(1-\alpha)\phi)^3\left(\tau - (\phi+(1-\phi)\alpha)\right)}, & \text{if } \tilde{\tau} < \tau < \tilde{\tilde{\tau}} \\ 0, & \text{if } 1 \leq \tau \leq \tilde{\tau} \end{cases}$$

$$\tilde{\tau} \equiv \frac{(1-\phi)^2\alpha^2 + (3\phi^2 - 2\phi - 1)\alpha - 4\phi^2}{-4\phi(\alpha+(1-\alpha)\phi)} \text{ and } \tilde{\tilde{\tau}} \equiv \frac{(2\phi^3 - 3\phi^2 + 1)\alpha^2 - (4\phi^3 - 7\phi^2 + 2\phi + 1)\alpha + 2\phi^3 - 4\phi^2}{-2\phi(\alpha+(1-\alpha)\phi)}.$$

To help exposition, I present Proposition 2.7 in Figure 2.6 with $q = 2, m = 0.6, R = 1, \phi = 0.7$, and $\alpha = 0.5$. Under this set of parameters, $\tilde{\tau} = 1.01$ and $\tilde{\tilde{\tau}} = 1.17$. The horizontal axis represents the magnitude of market expansion effect (τ), and the vertical axis corresponds to the unit cost of fake review (β).

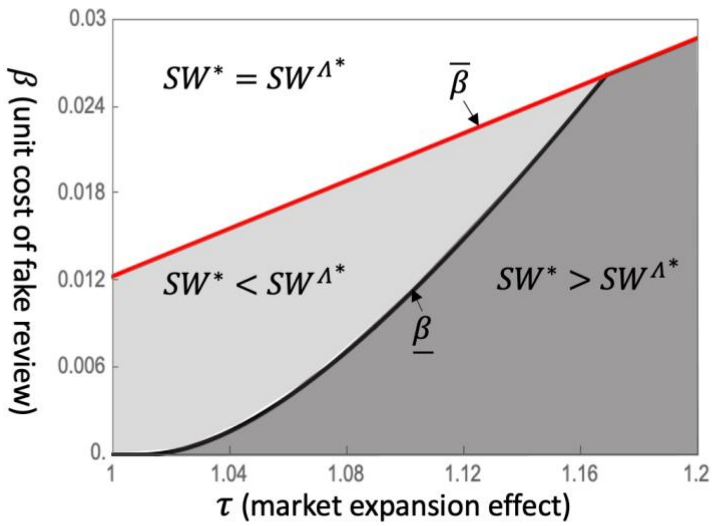

Figure 2.6. Comparison between Social Welfare in the Main Model and That in the Benchmark

When fake reviews are overly expensive ($\beta \geq \overline{\beta}$), neither seller can afford the costly fake reviews. As a result, the main model coincides with the benchmark, and social welfare is the same in these two scenarios ($SW^* = SW^{\Lambda^*}$).

However, when fake reviews are affordable ($\beta < \overline{\beta}$), the comparison between social welfare becomes more nuanced. Recall that social welfare is defined to be the sum of sellers' expected profits and consumer surplus. From discussions in Section 2.5.1 and 2.5.2 we see that on one hand, sellers benefit from having the option to acquire fake reviews. On the other hand, consumers are worse off from fake reviews because novice consumers are more likely to be deceived into purchasing from the counterfeiter. Therefore, social welfare, affected by the two opposite effects, can be either larger or smaller than its counterpart in the benchmark. When acquiring fake reviews is sufficiently cheap ($\beta < \underline{\beta}$), having the option to acquire fake reviews would greatly enhance sellers' profits such that the profit gain outweighs consumers' loss. Consequently, social welfare in the main model is larger than that in the benchmark ($SW^* > SW^{\Lambda^*}$). By contrast, when the unit cost of fake review is at median level ($\underline{\beta} \leq \beta < \overline{\beta}$), acquiring fake reviews would incur a substantial cost such that the profit gain is insufficient to compensate consumers' loss. Therefore, social welfare in the main model is less than that in the benchmark ($SW^* < SW^{\Lambda^*}$).

2.6 Conclusion

This paper considers a market where an authentic seller and a deceptive counterfeiter sell under the same product listing, offering credence goods whose qualities are hard to evaluate even after consumption. Customers do not know each seller's type (authentic or fake) but have a prior belief about it. Furthermore, customers may update their belief if the product receives an endorsement badge, which is more likely to be given to a highly rated product. In the first stage, both sellers simultaneously decide how many units of fake positive reviews to acquire. Next, whether the product receives a badge or not is revealed to the public. In the beginning of the second stage, both sellers simultaneously decide their prices. Later, one of the sellers is randomly chosen by the platform as the default seller, while the other is listed as an alternate seller. Finally, customers arrive and make purchasing decisions.

2.6.1 Results and Managerial Implications to Supply Chain and Operations Managers

The results show that the authentic seller would not acquire any fake reviews in equilibrium, whereas the counterfeiter may acquire some to mislead customers. Moreover, the units of acquired fake reviews decrease in the fraction of savvy consumers. This result offers important managerial insights that the operating platform can combat fake reviews by increasing the fraction of savvy customers. For instance, supply chain and operations managers can leverage this insight by investing in consumer education programs, thereby enhancing the ability of customers to discern authentic reviews, which can directly impact the demand-supply dynamics. The platform can do so by providing more informative descriptions about the badge, making it clear that the badge is product-dependent and not seller-dependent, thereby reinforcing supply chain transparency and consumer trust.

Although the counterfeiter acquires fewer fake reviews as more customers become savvy, I find that fake reviews would not be eliminated unless the market expansion effect of receiving a badge is sufficiently small. Usually, products that receive the badge are exhibited at the top of the platform's search engine, hence receiving a lot of customer views (large market expansion effect). Therefore, platform operations managers could use this finding to reassess the placement of badge-winning products, potentially re-positioning them to less prominent locations, such as the middle of the search engine results. This strategic re-positioning could mitigate the counterfeiter's incentives to acquire fake reviews, thus preserving the integrity of the supply chain ecosystem.

I also investigate the impact of having an option to acquire fake reviews on sellers and consumers. In particular, I find that when acquiring fake reviews is feasible, both sellers might be better off but consumers are always worse off. Hence, the results highlight the necessity of regulating fake reviews to protect consumers' rights. Moreover, I show that the presence of fake reviews can lower social welfare when the unit cost of fake reviews is intermediate. In this case, acquiring fake reviews would incur a substantial cost such that sellers' gain in profit is insufficient to compensate consumers' loss, resulting in a net loss in social welfare. From an operational perspective, this implies that the implementation of robust verification systems and the allocation of resources towards counterfeit detection can be justified not only ethically but also economically.

2.6.2 Limitations and Future Research Directions

To keep the model mathematically tractable, I have made several assumptions, and relaxing some of the assumptions can lead to interesting future work. First, my model does not consider the marketplace platform as a decision maker. Since sellers' fake review decisions depend on the market expansion effect, it would be interesting to include a platform that determines the magnitude of market expansion effect (e.g., where to place badge-winning products in the search results). Placing badge-winning products at the top of search results would increase sellers' revenue, which in turn increases the platform's revenue that is often a share of sellers' revenue. However, large market expansion effect may incentivize sellers to acquire more fake reviews, and the platform's reputation would suffer. The dynamics among the platform, the authentic seller, and the counterfeiter may provide additional managerial insights. Second, for simplicity I have assumed that product rating is the only deciding factor of receiving product badge. In practice, other factors, such as free shipping of product and inventory level, can also affect the chance of receiving product badge. Incorporating these additional factors might be challenging yet fruitful for future studies. Other possible extensions include studying the impact of using verifiable tools (e.g., blockchain) on credence goods.

Chapter 3
3 Two-Sided Platform Competition in the Presence of Tip Baiting

Abstract: When ordering food from online platforms, consumers often specify a tip that they will give to delivery workers, in hopes of receiving a fast delivery. Upon completion of the delivery, consumers may be able to reduce their tip if the platform implements an adjustable tipping policy (e.g., Uber Eats). In this case, some consumers may trick workers by promising a large tip before delivery but deliberately reducing the tip to zero once the delivery is completed; a practice termed "tip baiting." By contrast, consumers are not allowed to *ex post* reduce the tip in the presence of a non-adjustable tipping policy (e.g., DoorDash). In this paper, I develop a multi-stage duopoly model to examine the impacts of different tipping policies (adjustable or not) on platforms' profits, consumers' tipping behavior, and workers' delivery performance. There are two types of consumers: *selfish consumers*, who may engage in tip baiting, and *fair consumers*, who do not. I find that regardless of the fraction of selfish consumers, both platforms prefer the adjustable tipping policy over the non-adjustable one. This result has managerial implications that platforms with the non-adjustable policy can further enhance their profits by switching to the adjustable policy. Further, I show that platforms only have incentives to keep the fraction of selfish consumers sufficiently low, but not lower. However, the existence of selfish consumers always hurts those honest fair consumers and workers. Hence, I advocate that third-party regulators need to intervene to safeguard fair consumers' and workers' rights, as platforms may lack the incentive to do so beyond a certain point. Interestingly, I also find that platforms' price and wage can be negative under certain conditions. Moreover, in a setting exclusively involving fair consumers who do not engage in tip baiting, my analysis reveals an unexpected downward adjustment in both price and wage as consumers become more stringent in their tipping practices.

Key words: two-sided competition, tip baiting, negative wage and price, signaling game

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3.1 Introduction

Two-sided platforms match consumers who demand certain products/service with their providers. For example, consumers can order food on platforms, such as DoorDash and Uber Eats, who then have the food delivered by independent gig workers. To coordinate matching between consumers and workers, platforms charge consumers a price for using the platforms and pay workers a wage in exchange for their delivery service. Moreover, when placing orders, consumers often specify the amount of tip that they will give to workers, in hopes of getting a fast delivery because food is best served while it is still hot. Consumers are free to not leave any tips on their orders, but doing so may result in long delivery time. For instance, because of not tipping, a McDonald's order took more than an hour to be delivered, and the food arrived cold (Laurinavičius and Baliūnaitė 2022). By contrast, research shows that offering a tip usually reduces delivery time (Castillo et al. 2022).

After food has been delivered, consumers may or may not be able to reduce the tip they promised, subject to platforms' specific tipping policies. For instance, DoorDash implements a non-adjustable tipping policy that makes it very difficult for consumers to reduce the tip after delivery (Gebel and Delfino 2022). On the other hand, consumers are free to reduce their tip (even to zero if they wish) after delivery on Uber Eats, which adopts an adjustable tipping policy (Antonelli 2022).

Unfortunately, there exist consumers who abuse Uber Eats' adjustable tipping policy by *ex ante* promising a large tip to incentivize workers to deliver their orders quickly, but then deliberately reducing the tip to zero after the delivery is completed; a practice termed "tip baiting." For instance, an Uber Eats driver complained that a customer tip-baited her, as she delivered the food in a short time but her $13 tip was reduced to zero after delivery (Lenzen 2022). This tip baiting practice also applies to other industries where consumers are allowed to *ex post* reduce their tip. For example, in the grocery delivery industry, an Instacart shopper was promised a large tip of $55 before accepting an order, and the tip was completely removed after delivery (O'Brien and Yurieff 2020).

Due to the potential risk of tip baiting, tipping policy has a significant impact on workers' choice of platforms. Moreover, in the presence of the adjustable tipping policy, consumers may pay a different tip upfront because they know they can reduce the tip later, compared to the case of the non-adjustable policy. Consequently, tipping policy also affects consumers' willingness to use the platform. Hence, several questions arise:

- Should platforms adopt the adjustable or non-adjustable tipping policy in the presence of tip baiting?
- What are the impacts of different tipping policies on platforms' optimal prices and wages?
- How does tip baiting affect workers' welfare and platforms' profits?

To address these questions, I consider a duopoly where two platforms compete on both the demand side (consumers) and the supply side (workers) by setting their respective tipping policies, prices, and wages. To capture the practice of tip baiting, I consider two types of consumers: selfish consumers and fair ones. In the presence of an adjustable tipping policy, selfish consumers may renege on their promised tip after delivery, regardless of the realized delivery time. On the other hand, fair consumers pay their promised tip in full if the delivery arrives in a short time, and reduce some of the tip if the delivery takes long. When the non-adjustable tipping policy is selected, all consumers have to stick to their promised tip and cannot reduce it after the delivery is completed.

Next, I provide a summary of my findings. First, I find that regardless of the fraction of selfish consumers, both platforms prefer the adjustable tipping policy over the non-adjustable one, because they are able to pay a lower wage to workers and attract more consumers with the adjustable policy. This result has important managerial implications that for platforms who are currently offering a non-adjustable tipping policy (e.g., DoorDash), they can be better off by switching to the adjustable policy, like their competitor does (e.g., Uber Eats).

Second, I show that platforms only have incentives to keep the fraction of selfish consumers sufficiently low, but not lower. However, the existence of selfish consumers always hurts those honest fair consumers and workers. Therefore, my findings offer policy implications that third-party regulators need to intervene to safeguard fair consumers' and workers' rights, as platforms may lack the incentive to do so beyond a certain point. For example, regulators may require platforms to have a rating system for consumers, where workers can share their experience with each other if they had been tip-baited by a particular consumer. As workers gradually learn from their collective experience, it will be increasingly difficult for selfish consumers to engage in tip baiting.

Third, I demonstrate that platforms' optimal price and wage can both be negative. Under certain conditions, workers are willing to accept a negative wage, which implies that workers need to pay platforms (e.g., a participation fee or a royalty) for the delivery opportunities, because they anticipate to be compensated by receiving large tips from consumers. Similarly, there are situations wherein platforms' price can be negative, which implies that platforms subsidize consumers for placing orders. For example, DoorDash offers $20 sign up bonus to new consumers to stimulate demand (Sabatier 2023).

Fourth, in an environment that includes only fair consumers who do not tip bait workers, my study uncovers a counterintuitive result that platform-set prices and wages in tipping scenarios may unexpectedly decrease below those in non-tipping scenarios as consumer become stricter with their tipping practices. This insight delineates the impact of consumer tipping behaviors on platforms' price and wage decisions, highlighting the intricate balance of interests that platforms must navigate in the gig economy.

The rest of this paper is organized as follows. In section 3.2, I review the related literature. In section 3.3, I present the model setup. In section 3.4, I analyze consumers' and workers' optimal decisions regarding tipping and delivery effort, respectively. In section 3.5, I characterize the competition between platforms and examine their equilibrium solutions. In section 3.6, I discuss two related policy implications of the tipping policy. Lastly, I conclude with managerial implications and directions for future research in section 3.7.

3.2 Literature Review

Our paper is closely related to the literature of two-sided markets (e.g., Rochet and Tirole 2003; Armstrong 2006; Bai et al. 2019; Zhang et al. 2022a; Hu and Liu 2023). For example, Armstrong (2006) considers pricing strategies in two-sided market competition with three different market structures and finds that equilibrium prices critically depend on the relative sizes of cross-group externalities, fee structures, and the choice of single-homing or multi-homing. Bai et al. (2019) study a monopoly platform's optimal price and wage decisions when consumers are both time- and price-sensitive. They find that the platform should offer a higher payout ratio as consumers become more impatient with waiting, capacity decreases, or demand increases. Both Zhang et al. (2022a) and Hu and Liu (2023) explore the impact of wage schemes on two-sided platform competition and characterize the conditions under which a particular wage scheme is the most profitable. Additionally, Hu and Liu (2023) extend these insights to the scenario of uncertain demand.

In contrast to the above studies where workers' earnings solely come from the wage paid by platforms, my paper contributes to this literature by also considering the impacts of consumers' tip on workers' total earnings and platforms' wage and price decisions. In the presence of consumers' tip, I find interesting results that platforms' price can be negative under certain conditions. This result is similar to that in DellaVigna and Malmendier (2004), where they find with a two-part tariff contract, investment goods should be priced below marginal cost. Then, if the marginal cost is normalized to zero, their findings suggest a negative unit price. In contrast to their paper that studies a monopolist's optimal contract in response to consumer bias, my paper focuses on a duopoly in the context of two-sided market competition. In addition, I show that in some cases, platforms' wage to workers can be negative too. This finding has been briefly noted in Tan et al. (2020), where the authors mention that two-sided platforms may charge the supply side either a positive participation fee (which is equivalent to a negative wage in my paper) or a negative fee. However, my paper is different because I also consider the strategic interactions between consumers and workers, and investigate how would the interplay affect the platforms' optimal structure of tipping policy.

Another related literature is the nascent stream on platform regulations that aim for maximizing labor welfare and/or social welfare. Tang et al. (2021) characterize the conditions under which a hybrid matching system with a "female-only" option can be a win-win outcome for safety-concerned female customers and a ride-hailing platform. Benjaafar et al. (2022) find that to improve gig workers' welfare, regulators should impose a floor on the effective wage rather than on the nominal wage. Hu et al. (2022) investigate the implications of worker classification (employee or contractor) by considering a queueing model with a service platform and both full-time and part-time workers. The authors show that a uniform classification may not be welfare-maximizing and propose a hybrid mode where full-time workers are classified as employees and part-time workers are treated as contractors. Zhang et al. (2022b) consider a model where a welfare-maximizing government interacts with a profit-maximizing platform that matches consumers to food delivery drivers. They show that to curb reckless driving, governments should impose a higher traffic-related incident penalty on the platform, and not on the drivers. My paper is similar to these studies, as I find that third-party regulators need to intervene to safeguard consumers' and workers' rights under certain conditions wherein platforms do not have incentives to do so. However, my paper differs because I focus on two competing platforms, whereas there is only a monopoly platform being considered in the studies above. In the duopoly setting, while Hu et al. (2024) concentrate on the selection of corporate social responsibility strategies by platforms, my research focuses on how these platforms leverage tipping policies as a means of competitive advantage.

Moreover, my paper contributes to the literature on consumer tipping. Chen et al. (2023) empirically show that when consumers are asked to evaluate workers' service prior to tipping, they subsequently reduce their tipping amount, as giving feedback and giving tips are perceived as substitutes. Other empirical research focus on the impact of tip recommendations on consumers' tipping behavior and find that large tip recommendations are likely to be considered unreasonable and may elicit negative consumer reactions, such as lower likelihood of tipping (e.g., Fitzsimons and Lehmann 2004; Carr 2007; Haggag and Paci 2014). By contrast, Alexander et al. (2020) find that large tip suggestions increase the amount of tips received, but have no impact on consumer satisfaction, repatronage, or spending. In the context of delivery service, Castillo et al. (2022) conduct a scenario-based experiment and find that consumer tipping usually reduces delivery time and mitigates uncertainty in the crowdsourced delivery fleet.

In contrast to the above empirical studies, my paper contributes to the consumer tipping literature by offering an analytical model to study the impact of consumers' tipping behavior on workers' delivery effort and platforms' decisions regarding the tipping policy, wage, and price. Shy (2014) is a closely related paper to ours. In a theoretical study, Shy (2014) finds that tipping is mostly effective when firms pay minimum wage; however, it is ineffective under full employment, where firms would reduce workers' hourly wage in response to the increase in tipping. My paper share similar findings that in the presence of consumer tipping, firms pay less wage to workers. However, Shy (2014) assumes that tipping does not affect workers' service quality, whereas my paper considers that large tip motivates workers to exert high level of delivery effort, which in turn is more likely to result in short delivery time. In addition, the tipping rate is an exogenously given parameter in Shy (2014), whereas consumers endogenously decide how much tip to offer and whether to reduce the tip after delivery in my paper.

3.3 Model Setup

Consider a two-sided market competition in which two platforms (e.g., DoorDash and Uber Eats) compete for both consumers who use the platforms to order food and workers who deliver food to consumers' places. To facilitate matching between consumers and workers, platform i ($i = 1, 2$) offers wage w_i to workers in exchange for their service and charges consumers price p_i for the delivery service. The food price is set by restaurants and is the same on both platforms. Therefore, I normalize the food price to zero, as it does not affect consumers' decisions about choosing between platforms.

3.3.1 Workers

After a matching has been established, consumers wait for their food to be delivered. The waiting time for using platform i's delivery service (W_i) is stochastic and can be either high (W_H) or low (W_L). Specifically,

$$W_i = \begin{cases} W_L & \text{with probability } \theta_{E_i}, \\ W_H & \text{with probability } 1 - \theta_{E_i}, \end{cases}$$

where $W_H > W_L \geq 0$. Without loss of generality, I normalize $W_L = 0$. Moreover, the waiting time W_i is partially determined by worker's delivery effort E_i, where $E_i \in \{E_H, E_L\}$. Although the delivery time is also influenced by factors that are out of workers' control (e.g., road congestion and food preparation time by restaurants), I assume that ceteris paribus, exerting a high level of delivery effort (E_H) is more likely to result in a fast delivery than a low effort (E_L) does. That is, $1 > \theta_{E_H} > \theta_{E_L} > 0$.

Exerting delivery effort is costly to workers, and a high level of effort costs more than low effort does. For example, when exerting high effort, workers can choose to not take any breaks in hopes of achieving a fast delivery. Hence, I assume that exerting high effort incurs a positive cost $c_e > 0$, and without loss of generality exerting low effort is cost-free.

3.3.2 Consumers

Consumers prefer the food to be delivered sooner than later, as food is best served while it is still hot. Following Bai et al. (2019) and Chen et al. (2022), I assume that consumers incur linear waiting cost $c_w W_i$ for food delivery, where $c_w > 0$ represents the unit cost of waiting time. In addition, when ordering food on platforms such as DoorDash and Uber Eats, consumers are required to specify the tip amount they will give to workers prior to food delivery.

After food has been delivered, consumers may or may not be able to reduce the tip that they had specified, depending on platform's tipping policy. For instance, DoorDash adopts a non-adjustable policy where consumers cannot *ex post* reduce the tip. By contrast, Uber Eats implements an adjustable tipping policy that allows consumers to freely reduce tip post food delivery. With an adjustable tipping policy, consumers can use tip to motivate workers for a fast delivery. Specifically, consumers *ex ante* give a tip in hopes of receiving a fast delivery (W_L), and they may *ex post* reduce the tip if the food arrives cold (W_H). Unfortunately, there also exist selfish consumers who abuse platforms' adjustable tipping policy by *ex ante* promising a large tip to motivate a fast delivery but *ex post* deliberately reducing the tip to zero, even if workers successfully delivered food in a short time.

I then consider two types of consumers, $X \in \{F, S\}$. A fraction $0 < \alpha < 1$ of consumers are fair consumers ($X = F$), who do not *ex post* reduce tip if the delivery is indeed fast ($W_i = W_L$) but will deduct $\lambda \in (0, 1]$ fraction of the promised tip if the delivery arrives late ($W_i = W_H$). Here, I assume $\lambda \neq 0$ to avoid the case where workers' final tip is not affected by delivery time so that they would never exert high delivery effort. The remaining fraction $1 - \alpha$ of consumers are selfish consumers ($X = S$), who would deliberately reduce tip to zero post food delivery, regardless of the realized delivery time. Consumers' type is their private information and not observed by workers or platforms.

3.3.3 Game Sequence

The game sequence is as follows: In the first stage, both platforms individually choose their tipping policy (adjustable or not). In practice, both DoorDash and Uber Eats set workers' wage first and then choose service price later (Zhang et al. 2022a). I follow this setting such that in the second stage, platforms first decide wage w_i and w_j. After wages have been set, platforms decide their respective price p_i and p_j in the third stage. In the fourth stage, consumers decide how much tip to offer while placing food orders. In the fifth stage, upon observing an order's earning (wage plus tip), workers then decide whether to exert a high effort or low effort to deliver the food. Lastly, the delivery is completed and consumers decide whether to reduce the previous tip, if an adjustable tipping policy is adopted.

3.4 Analysis of Consumers' and Workers' Decisions

Because workers *ex ante* do not know consumers' type, the model belongs to incomplete information games, and I use perfect Bayesian equilibrium (PBE) as my solution concept. As PBE does not impose any constraints on off-equilibrium beliefs, it often suffers from a plethora of equilibria; I then refine the equilibria with the divinity criterion (D1-Criterion), which is a standard refinement method used in signaling games (e.g., Banks and Sobel 1987; Cho and Kreps 1987). All proofs are provided in the Appendix B.

In the fifth stage, workers decide whether to exert high or low delivery effort based on the expected tip amount, which in turn depends on platforms' tipping policy. Let $S_i \in \{N, A\}$ denote platform i's tipping policy, where $S_i = N$ represents a non-adjustable policy in which consumers cannot *ex post* reduce the tip (e.g., DoorDash), and $S_i = A$ indicates an adjustable tipping policy where consumers can reduce tip after food delivery (e.g,. Uber Eats).

When platform i adopts the non-adjustable tipping policy ($S_i = N$), fair consumers and selfish consumers are essentially the same, because selfish consumers now cannot renege on their tip and have to pay the amount they specified prior to food delivery. Knowing that consumers cannot *ex post* reduce their tip, workers would not exert high delivery effort, as their final earning remains the same regardless of the delivery effort.

By contrast, when platform i selects the adjustable tipping policy ($S_i = A$), consumers may incentivize workers to exert high effort by offering a large tip. Let $T_i(X)$ denote the tip that type-X consumers promise to offer to platform i's worker before food delivery.[3] If there exists a pooling equilibrium where workers observe a uniform tip on all orders from the platform i ($T_i(S) = T_i(F) = T_i$), then observing the tip does not help identify whether a food order is from selfish or fair consumers. Nonetheless, workers know that with probability α, the order is from a fair consumer who will later reduce λ fraction of the tip T_i if the delivery

[3] On platforms such as Instacart, workers can directly observe the tip amount before accepting an order (Burgess 2022). On other platforms such as DoorDash, workers may only see the total earnings (base pay plus tip) before accepting an order. Nevertheless, workers are usually able to infer the tip before delivery, because they know the typical base pay is \$2.5 for DoorDash so that a total earning of \$5.5 would indicate a tip of \$3 (EntreCourier 2022).

takes long, which happens with probability $1-\theta_{E_i}$. On the other hand, with probability $1-\alpha$, the order is from a selfish consumer who will always reduce the tip to zero after delivery, regardless of the realized delivery time. Hence, platform i's workers choose to exert a high delivery effort if and only if

$$\underbrace{\alpha\Big(T_i-(1-\theta_{E_H})\lambda T_i\Big)}_{\text{Expected Tip When Exert High Effort}} - \underbrace{c_e}_{\text{Cost of High Effort}} \geq \underbrace{\alpha\Big(T_i-(1-\theta_{E_L})\lambda T_i\Big)}_{\text{Expected Tip When Exert low Effort}}, \tag{3.1}$$

which simplifies to $T_i \geq \widehat{T}_i \equiv \frac{c_e}{\alpha\lambda(\theta_{E_H}-\theta_{E_L})}$. That is, workers only exert a high delivery effort if the tip before delivery is large enough to compensate their delivery cost.

In contrast to the pooling equilibrium, if there exists a separating equilibrium where workers observe different tips ($T_i(S) \neq T_i(F)$), then upon observing the tip they are able to distinguish between selfish and fair consumers. Due to the single crossing property, workers would attribute the larger tip to selfish consumers in equilibrium. This is because workers know it is costless for selfish consumers to promise a large tip, as they always *ex post* reduce the tip to zero and do not bear any cost. On the other hand, it is more costly for fair consumers to promise a large tip, because after delivery they do end up paying the tip in full or partially.

Hence, for an order with the larger tip, workers know it is from selfish consumers and choose to exert low effort, as they know the tip, no matter how large it appears, will eventually be reduced to zero. For an order with the smaller tip, workers know it is from fair consumers and choose to exert a high delivery effort if and only if

$$\underbrace{T_i(F)-(1-\theta_{E_H})\lambda T_i(F)}_{\text{Expected Tip When Exert High Effort}} - \underbrace{c_e}_{\text{Cost of High Effort}} \geq \underbrace{T_i(F)-(1-\theta_{E_L})\lambda T_i(F)}_{\text{Expected Tip When Exert low Effort}}. \tag{3.2}$$

In the fourth stage, consumers decide the tip amount before food delivery, anticipating workers' effort decision. Lemma 3.1 states that in equilibrium, selfish consumers will mimic fair consumers' tip amount.

Lemma 3.1. *When placing orders on platform i, selfish consumers and fair consumers commit the same amount of tip T_i^* in equilibrium, which is*

$$T_i^* = \begin{cases} \widehat{T}_i & \text{if } S_i = A \text{ and } c_w > \overline{c_w}, \\ 0 & \text{otherwise}, \end{cases}$$

where $\overline{c_w} \equiv \frac{c_e(1-\lambda(1-\theta_{E_H}))}{\alpha\lambda W_H(\theta_{E_H}-\theta_{E_L})^2}$. Given the optimal tip, $E_i^* = E_H$ if and only if $T_i^* = \widehat{T}_i$.

The intuition behind Lemma 3.1 is that a separating equilibrium, where different types of consumers commit different amounts of tip, would reveal whether a food order is from fair consumers or selfish consumers. Because no workers will knowingly exert high effort to deliver selfish consumers' orders, selfish consumers will pretend to be fair consumers by committing the same tip so that they may also enjoy the benefits of high delivery effort. Fair consumers do not have incentives to deviate from the tip as well, because paying more or less than necessary are suboptimal. Therefore, workers would only exert high effort when the pooling tip is large enough, and consumers are only willing to pay a large tip to expedite the delivery when they are impatient enough with waiting ($c_w > \overline{c_w}$).

3.5 Analysis of Platform Competition

In this section, I first introduce platforms' demand and supply functions, which depend on their tipping structure. Next, I characterize the competition between platforms in each of three cases where both platforms offer the adjustable tipping policy (in section 3.5.1), both platforms adopt the non-adjustable tipping policy (in section 3.5.2), one platform offers the adjustable policy while the other adopts the non-adjustable policy (in section 3.5.3), respectively. Then, I compare platforms' profits under each of the three cases and offer insights into platforms' optimal tipping policy in section 3.5.4.

Demand/Supply Functions. Similar to Zhang et al. (2022a) and Hu and Liu (2023), I use a linear model of competition where platform i's demand decreases in her own price p_i and increases in her competitor's price p_j ($i, j = 1, 2$ and $i \neq j$). Moreover, platform i's demand is also affected by consumers' expected waiting cost and their tip, which would depend on the platform's tipping policy. Knowing from Lemma 3.1 that both types of consumers will commit the same tip T_i^* and workers exert effort E_i^* accordingly, I can formulate platform i's total demand from all consumers as

$$d_i = \Big[M - \Big(p_i + \underbrace{T_i^*}_{\text{Tip before Delivery}} - \underbrace{\mathbb{1}_{S_i=A}(\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*)}_{\text{Expected Reduction in Tip after Delivery}} + \underbrace{(1-\theta_{E_i^*})c_w W_H}_{\text{Expected Waiting Cost}}\Big)$$
$$+\gamma\Big(p_j + T_j^* - \mathbb{1}_{S_j=A}(\alpha(1-\theta_{E_j^*})\lambda T_j^* + (1-\alpha)T_j^*) + (1-\theta_{E_j^*})c_w W_H\Big)\Big]^+, \tag{3.3}$$

where $M > 0$ represents the market size and $\gamma \in (0, 1)$ measures the level of competition on the demand side. The larger the γ is, the fiercer the competition becomes. Moreover, when platform i adopts the adjustable tipping policy (i.e., the indicator function $\mathbb{1}_{S_i=A}$), fair consumers, which accounts for α fraction of all consumers, will reduce λ fraction of the committed tip if the delivery takes long, which happens with probability $1-\theta_{E_i^*}$. The rest $1-\alpha$ fraction are selfish consumers, who always completely remove the tip after delivery. On the other hand, when the non-adjustable tipping policy is selected, all consumers (fair or selfish) need to pay the committed tip in full.

The demand function in equation (3.3) can also be derived from consumers' microeconomic utility functions. For example, following procedures in the literature (e.g., Kwark et al. 2014 and Zhang et al. 2022a), I can assume that some consumers are single-homing, meaning that they either use their desired platform for food delivery service or not use the service at all. By contrast, other consumers multi-home between the two platforms, and will choose the platform that offers them a higher utility. By aggregating consumers' choices of platforms, I can derive the demand function in equation (3.3) (The derivation procedures are detailed in the Appendix B).

Again as in Zhang et al. (2022a) and Hu and Liu (2023), I use a supply function where platform i's supply of workers (s_i) increases in her wage w_i and decreases in the competing platform's wage w_j. Moreover, workers also receive tips from consumers and incur delivery effort cost for delivering food orders from platform i. Hence, I modify the supply function as

$$s_i = \Big[w_i + \underbrace{T_i^* - \mathbb{1}_{S_i=A}(\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*)}_{\text{Expected Tip from Consumers}} - \underbrace{\mathbb{1}_{E_i^*=E_H} c_e}_{\text{Delivery Effort Cost}}$$
$$-\beta\Big(w_j + T_j^* - \mathbb{1}_{S_j=A}(\alpha(1-\theta_{E_j^*})\lambda T_j^* + (1-\alpha)T_j^*) - \mathbb{1}_{E_j^*=E_H} c_e\Big)\Big]^+, \tag{3.4}$$

where $\beta \in (0,1)$ measures the level of competition on the supply side. Additionally, platform i's workers would only incur the delivery cost c_e if they choose to exert a high effort (i.e., the indicator function $\mathbb{1}_{E_i^*=E_H}$).

Similar to the demand function, the supply function in equation (3.4) can also be derived based on a microeconomic foundation by assuming single-homing workers, who only work for their desired platform, and multi-homing workers, who are registered on both platforms and sell their service to the platform that offers them a better pay (base wage plus expected tips from consumers).

3.5.1 Symmetric Adjustable Tipping Policy

In this section, I consider the case where both platforms adopt the adjustable tipping policy (i.e., $S_i = S_j = A$). In the third stage, platforms decide their prices, which in turn affect platforms' demand. In addition, a transaction happens only when a consumer is paired with a worker. Therefore, the volume of transactions on platform i is determined by the lesser of demand and supply on the platform. That is, the transaction volume is $\min\{d_i, s_i\}$, where d_i and s_i are given in equations (3.3) and (3.4) by plugging in $S_i = S_j = A$. Because the profit margin from a transaction is $p_i - w_i$, I can then formulate each platform's profit maximization problem in the third stage as

$$\max_{p_i} \quad \pi_i^{AA} = (p_i - w_i)\min\{d_i, s_i\}, \tag{3.5}$$

where the superscript AA denotes that both platforms select the adjustable tipping policy.

Similarly, platforms choose their respective wages paid to workers in the second stage, and I can formulate each platform's profit maximization problem as

$$\max_{w_i} \quad \pi_i^{AA} = (p_i - w_i)\min\{d_i, s_i\}. \tag{3.6}$$

Proposition 3.1 presents platforms' equilibrium solutions.

Proposition 3.1. *When both platforms select the adjustable tipping policy, consumers pay tips T^* and workers exert effort E^* on both platforms as stated in Lemma 3.1: if $c_w \le \overline{c_w}$, $T^* = 0$ and $E^* = E_L$; otherwise, $T^* = \widehat{T_i}$ and $E^* = E_H$. In addition, both platforms charge price p^* and offer wage w^* in equilibrium:*

(i) *when $c_w \le \overline{c_w}$ and $M \ge \overline{M}$,*

$$w^* = \frac{(1+\gamma)(M + c_w W_H(1-\gamma)(\theta_{E_L}-1))}{(\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4},$$
$$p^* = \frac{((\beta-2)\gamma^2 + (\beta-1)\beta\gamma - 2\beta + 3)\,(M + c_w W_H(1-\gamma)(\theta_{E_L}-1))}{(1-\gamma)\,((\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4)};$$

(ii) *when* $c_w \leq \overline{c_w}$ *and* $M < \overline{M}$,

$$w^* = 0,$$
$$p^* = \frac{M}{1-\gamma} - c_w W_H (1-\theta_{E_L});$$

(iii) *when* $c_w > \overline{c_w}$ *and* $M \geq \widehat{M}$,

$$\begin{aligned}
w^* =& \Big[\lambda(\theta_{E_H} - \theta_{E_L})(1+\gamma)(M + c_w W_H (1-\gamma)(\theta_{E_H} - 1)) - c_e\Big(\left(2\gamma^2 - \gamma - 4\right)(\lambda - 1) \\
&+ \beta(\gamma - 3)(1+\gamma)(1 + \lambda(\theta_{E_L} - 1)) + \lambda\left(3 + \gamma - \gamma^2\right)\theta_{E_L} + \beta^2\gamma(1 + \lambda(\theta_{E_L} - 1)) \\
&+ \lambda\left(1 - \gamma^2\right)\theta_{E_H}\Big)\Big] / [\lambda(\theta_{E_H} - \theta_{E_L})\left((\beta - 2)\gamma^2 + (1-\beta)^2\gamma + 4 - 3\beta\right)], \\
p^* =& [c_e(2 - \beta - \gamma) - \lambda(\theta_{E_H} - \theta_{E_L})(M + w^*(\beta - 1) - c_w W_H (\theta_{E_H} - 1)(\gamma - 1)) \\
&+ c_e\lambda(\theta_{E_H}(1-\gamma) + \theta_{E_L}(1-\beta) + \beta + \gamma - 2)] / [\lambda(\theta_{E_H} - \theta_{E_L})(\gamma - 1)];
\end{aligned}$$

(iv) *when* $c_w > \overline{c_w}$ *and* $M < \widehat{M}$,

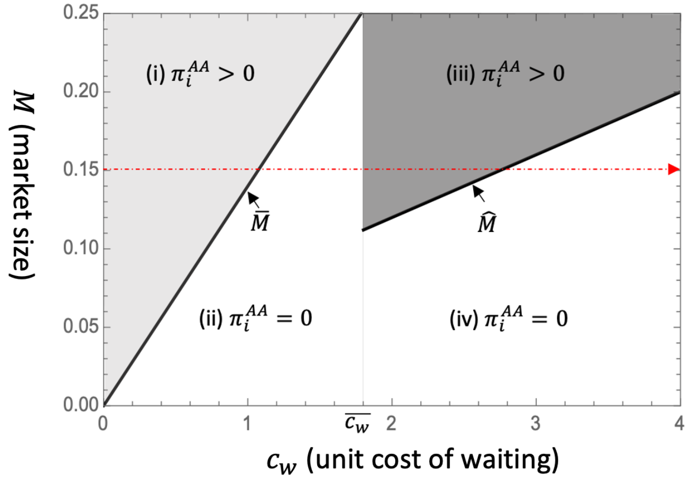
$$w^* = \frac{c_e(\lambda(1-\theta_{E_L}) - 1)}{\lambda(\theta_{E_H} - \theta_{E_L})},$$
$$p^* = c_w W_H(\theta_{E_H} - 1) + \frac{c_e(\lambda(1-\theta_{E_H}) - 1)}{\lambda(\theta_{E_H} - \theta_{E_L})} + \frac{M}{1-\gamma},$$

where $\widehat{M} \equiv (c_w W_H(1-\theta_{E_H}) + c_e)(1-\gamma)$, $\overline{M} \equiv c_w W_H(1-\theta_{E_L})(1-\gamma)$, *and* $\overline{c_w}$ *is defined in Lemma 3.1. Moreover, with these optimal solutions, the transaction volume* $min\{d_i, s_i\}$ *and profit* π_i^{AA} *are positive in regions* (i) *and* (iii)*, but are equal to zero in regions* (ii) *and* (iv).

I find that demand and supply are perfectly matched for each platform in equilibrium. Essentially, if a platform's demand exceeds its supply, the total number of transactions is equal to the number of workers on the platform. If the platform, while maintaining the same wage, marginally increases its price, it could enhance its profit margin and become better off. Thus, demand cannot surpass supply in equilibrium. By the same logic, it is also not feasible for supply to outdo demand in equilibrium, as in that case the platform could increase profits by lowering the wage.

To derive further insights, Proposition 3.1 is shown in Figure 3.1 with $\alpha = 0.8$, $\lambda = 0.5$, $\gamma = 0.8$, $c_e = 0.2$, $W_H = 1$, $\theta_{E_H} = 0.8$, and $\theta_{E_L} = 0.3$. Under this set of parameters, $\overline{c_w} = 1.8$. The horizontal axis is consumers' unit cost of waiting time (c_w), and the vertical axis represents the market size (M).

Figure 3.1 Equilibrium Regions When Both Platforms Choose the Adjustable Tipping Policy



I illustrate Figure 3.1 by considering a special case where the market size M is fixed at 0.15, and the waiting cost changes from low to high (the red dashed line). Recall from the demand functions that a platform's demand is negatively affected by consumers' expected waiting cost. In equilibrium region (i), a low waiting cost results in high demand for placing orders on platforms. As a result, platforms can charge a high service price and can afford to pay a decent wage that motivates workers to deliver orders, gaining a positive profit.

As consumers' waiting cost continues to increase (in equilibrium region (ii)), fewer consumers place orders on platforms, who then have to lower their service price. In addition, as consumers are still insensitive to waiting time ($c_w \leq \overline{c_w}$), they have no incentives to pay tips for a fast delivery. In this case, workers' compensation only comes from the wage they receive from platforms, but platforms cannot afford to pay a satisfying wage as their profit margin is already thin due to the low service price. Weighing the trade-offs, platforms choose price and wage such that no transactions occur, resulting in zero profits.

When consumers become impatient with waiting ($c_w > \overline{c_w}$), they are now willing to offer a large tip in hopes of receiving a fast delivery, and the equilibrium solutions switch to region (iii). Even though the tip and higher waiting cost make the demand even smaller, platform's profit margin has widened, as workers now receive extra compensation from consumers' tip, which enables platforms to pay a lower wage while still incentivizing workers to deliver. As a result, platforms start to generate positive profits again. Interestingly, due to the extra tip from consumers, platforms' wage can even be negative under certain situations, while still ensuring a positive earning (wage plus tip) for workers. A negative wage implies that instead of paying workers for their delivery service, workers actually need to pay platforms for the delivery opportunities, where they will be compensated by receiving large tips from consumers. I will formally establish this finding in a later Proposition 3.8 and provide more detailed discussions there.

Finally, the equilibrium solutions move to region (iv), as consumers' waiting cost increases further. In this region, platforms' situation is very similar to that in region (ii), where they are unable to pay a satisfying wage, even with the help of consumer tip, because their price is limited by the low demand caused by the high waiting cost. Once again, platforms set price and wage such that both demand and supply are equal to zero, leading to zero profits.

3.5.2 Symmetric Non-Adjustable Tipping Policy

In this section, I consider the case where both platforms select the non-adjustable tipping policy (i.e., $S_i = S_j = N$). In this case, fair consumers and selfish consumers are essentially the same, because selfish consumers now cannot renege on their tip and have to pay the amount they specified prior to food delivery. Knowing that consumers cannot *ex post* reduce their tip, workers would not exert high delivery effort, as their final earning remains the same regardless of the delivery effort. Consequently, consumers would not give any tips because they know workers will not be motivated to exert high effort.

Similar to the previous section 3.5.1, I can obtain platforms' demand and supply by plugging $S_i = S_j = N$ into equations (3.3) and (3.4). Next, following the same steps in section 3.5.1, I solve the platforms' optimal decisions with regard to price and wage. Proposition 3.2 summarizes the equilibrium solutions.

Proposition 3.2. *When both platforms select the non-adjustable tipping policy, $T^* = 0$ and $E^* = E_L$ on both platforms as stated in Lemma 3.1. In addition, both platforms charge price p^* and offer wage w^* in equilibrium*:

(i) *when $M \geq \overline{M}$,*

$$w^* = \frac{(1+\gamma)(M + c_w W_H(1-\gamma)(\theta_{E_L} - 1))}{(\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4},$$
$$p^* = \frac{((\beta-2)\gamma^2 + (\beta-1)\beta\gamma - 2\beta + 3)\,(M + c_w W_H(1-\gamma)(\theta_{E_L} - 1))}{(1-\gamma)\,((\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4)};$$

(ii) *when $M < \overline{M}$,*

$$w^* = 0,$$
$$p^* = \frac{M}{1-\gamma} - c_w W_H(1-\theta_{E_L}).$$

With these optimal solutions, the transaction volume min$\{d_i, s_i\}$ and platforms' profits are positive in region (i) *but equal to zero in region* (ii).

When both platforms adopt the non-adjustable tipping policy, the optimal wages and prices turn out to be a subset of the equilibrium solutions presented in Proposition 3.1. Recall that when consumers' waiting cost is low ($c_w \leq \overline{c_w}$), they would not pay any tips when placing orders. Because there is no tip offered in the first place, whether the tipping policy is adjustable or not becomes irrelevant to platforms' subsequent decisions on wage and price, thus the equilibrium solutions in Proposition 3.2 coincide with the first two equilibrium regions in Proposition 3.1. On the other hand, even when consumers are impatient with waiting ($c_w > \overline{c_w}$), they would still not give any tips, which are futile to motivate high effort in the case of a non-adjustable policy. As a result, the equilibrium regions that are caused by positive tips (regions (iii) and (iv)) are unique in Proposition 3.1 but not feasible in the current Proposition 3.2. The rest interpretation of Proposition 3.2 is already covered in the discussion following Figure 3.1, and hence omitted here.

3.5.3 Asymmetric Tipping Policy

Having discussed the symmetric tipping policies above, I now consider the asymmetric case where one platform offers the adjustable tipping policy, while the other adopts the non-adjustable policy; without loss of generality, let platform i be the former and platform j be the latter (i.e., $S_i = A$ and $S_j = N$).

When transactions happen on platform i, consumers' and workers' behaviors are the same as in the adjustable policy section 3.5.1, whereas consumers' and workers' behaviors are consistent with those in the non-adjustable policy section 3.5.2 for matching on platform j. That is, platform i's consumers give the tip T_i^* as presented in Lemma 3.1, and workers only exert high effort if they receive the positive tip (i.e., $T_i^* = \widehat{T_i}$). On the other hand, due to the non-adjustable policy, platform j's consumers would not give any tips and workers always exert low effort (i.e., $T_j^* = 0$).

For each platform, I can obtain its demand and supply by plugging $S_i = A$ and $S_j = N$ into the demand function (3.3) and supply function (3.4). Then, I can formulate each platform's profit maximization problems by following similar steps in section 3.5.1. Proposition 3.3 summarizes these equilibrium solutions when platforms choose different tipping policies.

Proposition 3.3. *When platforms adopt different tipping policies* ($S_i = A$ *and* $S_j = N$), *tipping and effort decisions are as stated in Lemma 3.1*: *if* $c_w \le \overline{c_w}$, $T_i^* = 0$ *and* $E_i^* = E_L$; *otherwise,* $T_i^* = \widehat{T_i}$ *and* $E_i^* = E_H$. *On the other hand,* $T_j^* = 0$ *and* $E_j^* = E_L$ *always hold. In addition, platforms' optimal prices and wages are*:

(i) *when* $c_w \le \overline{c_w}$ *and* $M \ge \overline{M}$,

$$w_i^* = w_j^* = \frac{(1+\gamma)(M + c_w W_H(1-\gamma)(\theta_{E_L}-1))}{(\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4},$$
$$p_i^* = p_j^* = \frac{((\beta-2)\gamma^2 + (\beta-1)\beta\gamma - 2\beta + 3)\,(M + c_w W_H(1-\gamma)(\theta_{E_L}-1))}{(1-\gamma)\,((\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4)};$$

(ii) *when* $c_w \le \overline{c_w}$ *and* $M < \overline{M}$,

$$w_i^* = w_j^* = 0,$$
$$p_i^* = p_j^* = \frac{M}{1-\gamma} - c_w W_H(1-\theta_{E_L});$$

with the above optimal solutions, both platforms' transaction volume and profits are positive in region (i) *but equal to zero in region* (ii).

(iii) *when $c_w > \overline{c_w}$, the optimal prices are different and*

$$p_i^* = \Big[c_e(2+\lambda(\theta_{E_H}+\theta_{E_L}-2)-\gamma(\beta+\gamma+(\theta_{E_L}-1)\beta\lambda+(\theta_{E_H}-1)\gamma\lambda))-\Big(\lambda(\theta_{E_H}-\theta_{E_L})$$
$$(w_j^*(\beta-\gamma)+M(1+\gamma)+w_i^*(\beta\gamma-1)-c_wW_H(\theta_{E_H}-1)(\gamma^2-1))\Big)\Big]/[\lambda(\theta_{E_H}-\theta_{E_L})(\gamma^2-1)],$$
$$p_j^* = [c_e(\beta-\gamma)(1+(\theta_{E_L}-1)\lambda)+\lambda(\theta_{E_H}-\theta_{E_L})(w_i^*(\beta-\gamma)$$
$$+M(1+\gamma)+w_j^*(\beta\gamma-1)-c_wW_H(\theta_{E_L}-1)(\gamma^2-1))]/[\lambda(\theta_{E_H}-\theta_{E_L})\left(1-\gamma^2\right)],$$

and the optimal wages w_i^* and w_j^* are given by

$$w_i^* = \begin{cases} \frac{c_e(\lambda(1-\theta_{E_L})-1)}{\lambda(\theta_{E_H}-\theta_{E_L})} & \text{if } M < \widehat{M}, \\ w_{im} & \text{if } \widehat{M} \le M < \widetilde{M}, \\ w_{il} & \text{if } M \ge \widetilde{M} \end{cases}$$
$$w_j^* = \begin{cases} 0 & \text{if } M < \widehat{M}, \\ \frac{\beta(1+\gamma)(M+c_e(\gamma-1)+c_wW_H(1-\gamma)(\theta_{E_H}-1))}{\left(\beta^2-2\right)\gamma^2+\beta\left(\beta^2-1\right)\gamma+4-3\beta^2} & \text{if } \widehat{M} \le M < \widetilde{M}, \\ w_{jl} & \text{if } M \ge \widetilde{M}, \end{cases}$$

where the unique thresholds $\widetilde{M}$, w_{im}, w_{il}, w_{jl} are defined in the Appendix B. With these optimal solutions in region (iii)*, when $M < \widehat{M}$, both platforms' transaction volumes and profits are zero*; *when $\widehat{M} \le M < \widetilde{M}$, platform i's transaction volume and profit are positive, whereas those of platform j are zero*; *when $M \ge \widetilde{M}$, both platforms' transaction volumes and profits are positive.*

Once again, when consumers' waiting cost is low ($c_w \le \overline{c_w}$), they would not pay any tips in the first place so that the asymmetric tipping policies are irrelevant. Thus, both platforms set the same price and wage, and the first two equilibrium solutions in Proposition 3.3 are identical to the first two in Proposition 3.1. However, when consumers become impatient with waiting ($c_w > \overline{c_w}$), they commit a positive tip $\widehat{T}_i$ on platform *i* but zero tip on platform *j*, which in turn results in distinct demands/supplies and causes platforms to charge different optimal prices and wages.

3.5.4 Optimal Tipping Policy

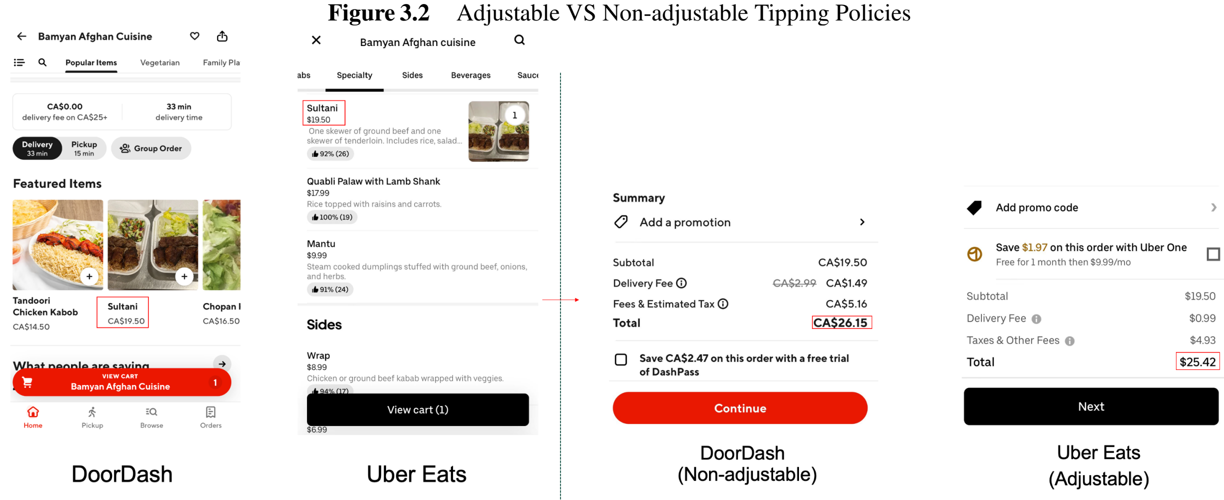
Based on these equilibrium solutions under the above three cases, I can now compare platforms' profits to offer insights into the optimal design of tipping policy. Comparisons of results below are in a weak sense. Proposition 3.4 states that in equilibrium, offering the adjustable tipping policy is the dominant strategy for both platforms.

Proposition 3.4. *Regardless of the fraction of selfish consumers, both platforms prefer the adjustable tipping policy over the non-adjustable policy in equilibrium.*

Platforms prefer the adjustable tipping policy for two reasons. First, with the adjustable tipping policy, consumers have a choice to reduce the expected waiting cost by offering a tip to motivate a fast delivery. Because platforms' demand is negatively affected by consumers' expected waiting cost, platforms' demand under the adjustable policy is higher than that under the non-adjustable policy, all else equal. Second, in the presence of the adjustable policy, workers receive compensation from both platforms (wage) and consumers (tip). Thus, platforms are able to offer a lower wage while keeping workers' total earnings unchanged, compared to the non-adjustable policy wherein worker's total earnings solely come from platforms. Because platforms' profit margin is larger (due to the lower wage paid to workers) and demand is higher (due to the lower expected waiting cost), their profits are higher under the adjustable tipping policy than under the non-adjustable policy. This result has important managerial implications that for platforms who are currently adopting a non-adjustable tipping policy (e.g., DoorDash), they can be better off by switching to the adjustable policy.

Next, I provide a real-world example in Figure 3.2 to support my recommendation for favoring the adjustable tipping policy. As shown in the left panel of Figure 3.2, consumers can order the dish "Sultani" from Bamyan Afghan Cuisine on both DoorDash and Uber Eats, where the food price is the same ($19.50). However, after incorporating additional fees charged by the platforms, the final price before tipping becomes different: DoorDash charges $26.15 that is higher than the total price on Uber Eats ($25.42). Since the food is the same but the total price is cheaper on Uber Eats, it is a logical conclusion that consumers would prefer Uber Eats over DoorDash for this order. And I believe that my results of tipping policy can help explain the price difference: In contrast to Uber Eats with the adjustable tipping policy, DoorDash needs to pay a higher wage to workers because their current non-adjustable policy discourages consumers from paying tips, as workers do not have incentives to exert high effort when they know consumers cannot take tips back after delivery. Due to the higher wage expense, DoorDash then needs to charge consumers a higher price to maintain their profit margin, which puts them at a disadvantage when competing with Uber Eats. To stay competitive, I believe that DoorDash can switch to the adjustable tipping policy, allowing them to pay lower wages to workers and therefore, charge more affordable prices to consumers.

Figure 3.2 Adjustable VS Non-adjustable Tipping Policies



3.6 Policy Implications

In this section, I discuss several policy implications of the adjustable tipping policy on consumers, workers, and platforms. Analyses below are based on the optimal solutions in Propositions 3.1 and 3.4.

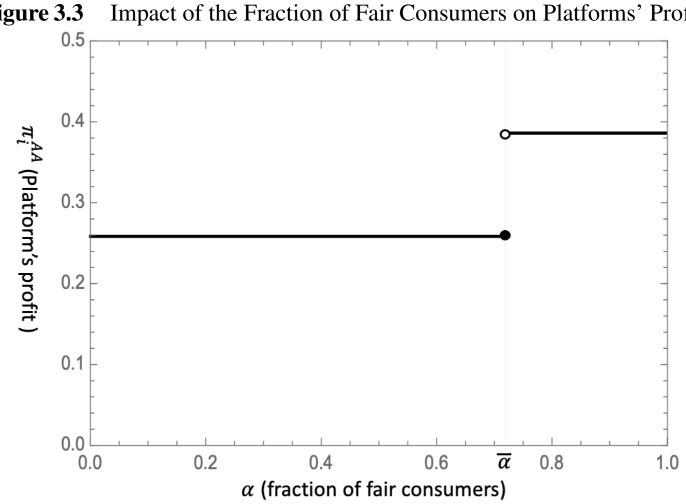
3.6.1 Need of Third-Party Regulator

I first analyze the impact of selfish consumers on platforms' profits. In particular, my results indicate that platforms have incentives to keep the fraction of fair consumers above a certain threshold, but they become indifferent once the proportion of fair consumers is already sufficiently high. Hence, the existence of selfish consumers does not bother platforms beyond a certain point; a result that I formally establish in Proposition 3.5.

Proposition 3.5. *Platforms have incentives to keep the fraction of fair consumers sufficiently high, but not higher. That is, (1) if $c_w > \widehat{c_w}$ and $M \geq \widehat{M}$, π_i^{AA} is the highest when $\alpha > \overline{\alpha}$, and $\frac{\partial \pi_i^{AA}}{\partial \alpha} = 0$ for all $\alpha \neq \overline{\alpha}$; (2) if either $c_w \leq \widehat{c_w}$ or $M < \widehat{M}$, $\frac{\partial \pi_i^{AA}}{\partial \alpha} = 0$ for all α, where $\widehat{c_w} \equiv \frac{c_e(1-\lambda(1-\theta_{E_H}))}{\lambda W_H(\theta_{E_H}-\theta_{E_L})^2}$ and $\overline{\alpha} \equiv \frac{c_e(1-\lambda(1-\theta_{E_H}))}{c_w \lambda W_H(\theta_{E_H}-\theta_{E_L})^2}$.*

To help exposition, I present Proposition 3.5 in Figure 3.3 with $M = 1$, $\lambda = 0.5$, $\gamma = 0.8$, $c_e = 0.2$, $c_w = 2$, $\beta = 0.7$, $W_H = 1$, $\theta_{E_H} = 0.8$, and $\theta_{E_L} = 0.3$. Under this set of parameters, $\widehat{M} = 0.12$, $\widehat{c_w} = 1.44$ and $\overline{\alpha} = 0.72$. The horizontal axis is the fraction of fair consumers (α), and the vertical axis is platforms' profits under optimal solutions (π_i^{AA}).

Figure 3.3 Impact of the Fraction of Fair Consumers on Platforms' Profits



When the unit waiting cost is high ($c_w > \widehat{c_w}$) and the market is not too small ($M \geq \widehat{M}$), consumers are impatient with waiting and willing to pay tips if doing so is not too costly. When the fraction of fair consumers is sufficiently high ($\alpha > \overline{\alpha}$), workers suspect that a food order is likely coming from fair consumers. Therefore, a relatively small tip is enough to induce high delivery effort, and consumers are willing to pay the small tip to expedite delivery. Consequently, platforms are able to pay a lower wage, leading to the leap in profits.

Moreover, platforms' profits do not depend on the fraction of fair consumers, except at the discontinuity point ($\alpha = \overline{\alpha}$) where consumers switch from not paying tips to paying positive tips. This is because platforms' transaction volume turns out unaffected by the proportion of fair consumers. Recall that workers base their effort decisions on the expected tip: The larger the fraction of fair consumers, the smaller the *ex ante* tip needs to be. These two cancel out and result in the same amount of expected tip, which in turn results in the same supply of workers. Because demand and supply are perfectly matched in equilibrium, transaction volume is thus unaffected by the fraction of fair consumers. In addition, the optimal solutions in Proposition 3.1 suggest that the fraction of fair consumers has no impact on the optimal wage and price. Because both transaction volume and profit margin are independent from the share of fair consumers, platforms generate the same profits and do not have incentives to further eliminate selfish consumers beyond a certain point.

When either condition ($c_w > \widehat{c_w}$ or $M \geq \widehat{M}$) is violated, platforms set price and wage such that either no transactions occur and receive zero profits, or generate positive profits but consumers never pay tips, in which the fraction of fair consumers does not matter because they do not pay tips anyway. Thus, platforms' profit is a constant that does not change in the fraction of fair consumers.

Impact of Selfish Consumers on Fair Consumers and Workers. Having analyzed the impact of selfish consumers on platforms, I now investigate their impact on fair consumers and workers. To do so, I consider a benchmark where all consumers are fair ($\alpha = 1$) and define the harm of selfish consumers as the difference in utilities, between in the benchmark and those in the main model. As detailed in the Appendix B, I use a Hotelling model to illustrate the market competition. Specifically, I use v to denote consumers' valuation of using platforms due to the convenience for food delivery, and use t_c to denote the unit misfit cost between consumers' preference and platforms. Then, I can express fair consumer's utility of using platform i as:

$$U_F \equiv v - t_c x - \Big(\underbrace{p^*}_{\text{Price to Platform}} + \underbrace{T_i^* - (1-\theta_{E_i^*})\lambda T_i^*}_{\text{Expected Tip}} + \underbrace{(1-\theta_{E_i^*})c_w W_H}_{\text{Expected Waiting Cost}} \Big), \tag{3.7}$$

where x represents the consumer's location on the Hotelling line.

Similarly, I can define a worker's utility of working for platform i in the main model as her expected earnings minus all cost:

$$U_W \equiv \underbrace{w^*}_{\text{Wage from Platform}} + \underbrace{T_i^* - (\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*)}_{\text{Expected Tip from Delivering an Order}} - \underbrace{\mathbb{1}_{E_i^*=E_H} c_e}_{\text{Delivery Effort Cost}} - t_w x - g, \tag{3.8}$$

where t_w represents the unit misfit cost between workers' preference and platforms, and g represents the cost incurred for the delivery service (e.g., fuel expenses for their vehicles). Note that the case for platform j can be similarly derived. And in the case where no transaction occurs, $U_F = U_W = 0$.

For notational convenience, I use the superscript Δ to denote the benchmark case. Fair consumer's utility and worker's utility in the benchmark are obtained by setting $\alpha = 1$ in equations (3.7) and (3.8), respectively. Hence, the harm of selfish consumers to a fair consumer (or worker) is the difference between fair consumer's (or worker's) utility in the benchmark and that in the main model:

$$H_F \equiv U_F^{\Delta} - U_F, \tag{3.9}$$

$$H_W \equiv U_W^{\Delta} - U_W. \tag{3.10}$$

I find that the existence of selfish consumers always harms those honest fair consumers and workers, which are summarized in Proposition 3.6.

Proposition 3.6. *In the presence of selfish consumers, fair consumers and workers are worse off than they are in the benchmark where all consumers are fair. That is,* $H_F \geq 0$ *and* $H_W \geq 0$*.*

In the presence of selfish consumers, workers suspect that an order has a substantial risk of coming from selfish consumers who will later completely remove the tip. Hence, they demand a larger tip to compensate the risk, which causes consumers to be less likely to pay the large tip in the main model than in the benchmark where all consumers are fair. Consequently, both fair consumers and workers are worse off in the main model, because without paying tips fair consumers are unable to expedite the delivery and have to endure the high expected waiting cost and workers lose the extra earnings of receiving tips from fair consumers.

Together, the findings in Propositions 3.5 and 3.6 have important policy implications that third-party regulators need to intervene to safeguard fair consumers' and workers' rights, as platforms may lack the incentive to do so beyond a certain point (recall from Proposition 3.5 that platforms only have incentives to keep the fraction of fair consumers sufficiently high, but not higher). For example, the regulator may require platforms to have a rating system for consumers, where workers can share their experience with each other if they had been tip-baited by a particular consumer. By doing so, workers would have more information about whether an order is likely from selfish consumers who tip-baited other workers in the past. Due to the rating system, selfish consumers will find it increasingly hard to tip-bait workers who gradually learn from their collective experience, and the fraction of selfish consumers will then decrease.

3.6.2 All Consumers Are Fair

Building on the above benchmark case where all consumers are fair ($\alpha = 1$), I further explore the dynamics among platforms, workers, and fair consumers to offer deeper insights. Because fair consumers may *ex post* deduct a certain percentage of the promised tip if food delivery takes long, I am particularly interested in understanding the impact of consumers' leniency regarding tip reduction on platforms' prices, wages, and profits. Interestingly, I find that the additional price charged by platforms when consumers pay tips can be either higher or lower than the price without tips, depending on how lenient consumers are.

Proposition 3.7. *There exist unique thresholds $\bar{\lambda}$, λ_p, and λ_w such that while transactions occur,*

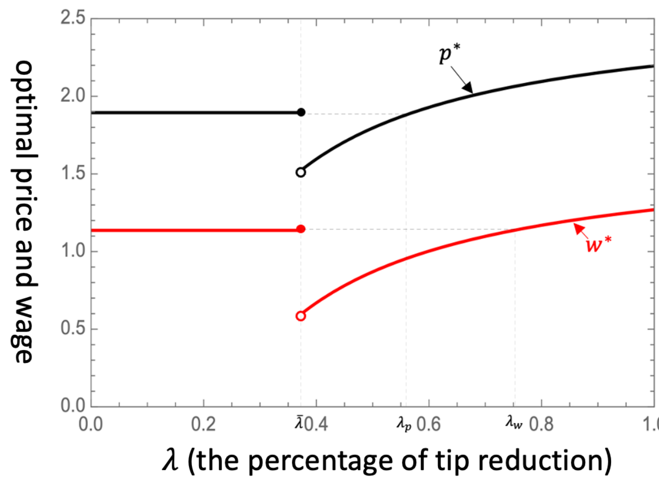
(i) *when tips are paid ($\lambda > \bar{\lambda}$), platforms charge fair consumers a higher price than in the scenario without tips if and only if $\lambda > \lambda_p$;*

(ii) *when tips are paid ($\lambda > \bar{\lambda}$), platforms pay workers a higher wage than in the scenario without tips if and only if $\lambda > \lambda_w$,*

where $\bar{\lambda}$, λ_p, and λ_w are defined in the Appendix B.

Figure 3.4 showcases Proposition 3.7 with $\alpha = 1$, $\gamma = 0.8$, $c_e = 0.2$, $c_w = 2$, $\beta = 0.7$, $W_H = 1$, $\theta_{E_H} = 0.8$, and $\theta_{E_L} = 0.3$. Under this set of parameters, $\bar{\lambda} = 0.37$, $\lambda_p = 0.57$, and $\lambda_w = 0.75$. The horizontal axis represents the percentage of tip reduction λ (it is equivalent to say that the smaller λ is, the more lenient consumers are with tipping), and the vertical axis is the optimal price and wage.

Figure 3.4 Impact of Consumer Leniency on Optimal Price and Wage



Initially when fair consumers are lenient with tipping ($\lambda \leq \bar{\lambda}$), they barely *ex post* reduce tips even if the delivery takes long. Anticipating consumers' lenient behaviors, workers would have no incentives to exert high effort. Therefore, fair consumers would not offer tips in the first place, as tipping does not reduce delivery time.

However, when customers are getting more harsh in punishing late delivery by withholding tips (increasing λ), one might expect that platforms need to increase wage, as workers now face a significant risk of losing tips so that platforms need to offer a higher wage to insurance against the potential loss from consumer tips. Because platforms need to pay a higher wage, one might also expect they should pass on the increased wage expense to consumers by charging a higher price.

Yet, a counterintuitive phenomenon occurs: instead of observing an increase in wage and price, there is a downward adjustment in both when fair consumers started to become less lenient ($\lambda > \bar{\lambda}$). During this transition, tipping becomes an effective mechanism to induce high effort from workers, as the final tip is now more outcome-based. Since workers receive extra compensation from tips, platforms are enabled to pay a lower wage while still ensuring a competitive total package to workers ($\lambda \leq \lambda_w$). Consequently, they transfer some of the benefits from lower wage expense to consumers by charging a lower service price ($\lambda \leq \lambda_p$), contradicting what one might expect.

It appears that the scenario where prices and wages increase due to tipping leniency only manifests when consumers are sufficiently harsh and would take back most of the promised tip if the delivery takes long. In such instances, tips become an unreliable supplement to wages, leading platforms to secure worker participation by providing higher wages ($\lambda > \lambda_w$). As a result, platforms charge consumers a higher price to offset the increased wage expense ($\lambda > \lambda_p$).

3.6.3 Negative Price and Wage

Studies in the literature of two-sided market competition typically find that price and wage need to be positive to ensure profitable matching for platforms (e.g., Zhang et al. 2022a; Hu and Liu 2023). However, I find that in the presence of consumer tips, negative price and wage are possible under certain conditions, which I summarize in the following proposition.

Proposition 3.8. *There exist unique thresholds M_p and M_w such that*

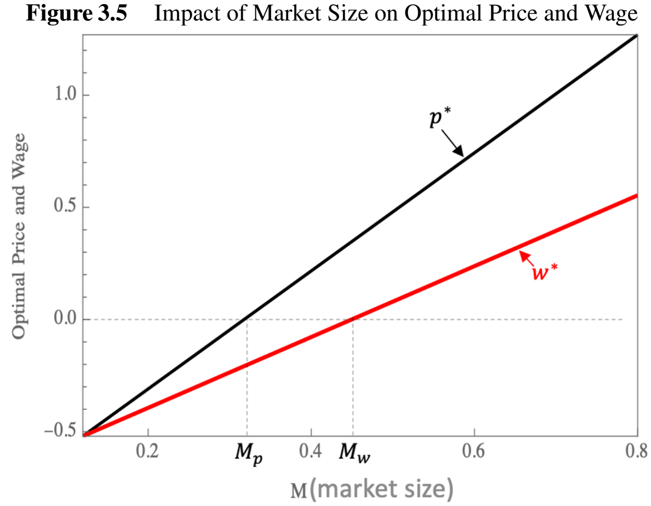
(i) *when $c_w \leq \overline{c_w}$, $p^* < 0$ if and only if $M < \overline{M}$, $w^* \geq 0$ always holds;*

(ii) *when $c_w > \overline{c_w}$, $p^* < 0$ if and only if $M < M_p$, $w^* < 0$ if and only if $M < M_w$,*

where M_p and M_w are defined in the Appendix B.

To derive further insights, I plot Proposition 3.8 in Figure 3.5 with $\alpha = 0.8$, $\lambda = 0.5$, $\gamma = 0.8$, $c_e = 0.2$, $c_w = 2$, $\beta = 0.7$, $W_H = 1$, $\theta_{E_H} = 0.8$, and $\theta_{E_L} = 0.3$. Under this set of parameters, $\overline{c_w} = 1.8$, $M_p = 0.32$, and $M_w = 0.45$. The horizontal axis represents the market size (M), and the vertical axis is the optimal price and wage.

Figure 3.5 Impact of Market Size on Optimal Price and Wage



Interestingly, I observe that both price and wage are negative when the market size is small ($M < M_p$). A negative price indicates that instead of charging consumers a price for using platforms, platforms would subsidize consumers for placing orders. For example, this could be that platforms provide consumers with large coupons to stimulate demand. In contrast to the conventional case where platforms make profits from consumers, platforms actually make profits from workers in this case (note that a negative wage implies for every transaction, workers need to pay platforms for the delivery opportunities). In particular, workers are willing to accept the deal because they anticipate to be compensated by receiving large tips from consumers, which exceed the cost they pay to platforms and still result in a positive earning for them.

The negative wage and price is an interesting result that has mostly been overlooked in the literature, where platforms' wage is positive because wage is the only source of income for workers. And because wage needs to be positive, the price must also be positive to sustain a positive profit margin (price minus wage) for platforms. However, in practice workers also receive tips from delivering food, in addition to the wage they receive from platforms. Hence, my paper complements the two-sided market competition literature, as I find that in the presence of consumer tips, positive wage and price are not always guaranteed and could be negative under certain conditions.

3.7 Concluding Remarks

This paper studies two-sided platform competition where consumers promise a tip while placing food orders and may or may not be able to reduce the tip after delivery is completed, depending on the platforms' tipping policy. Specifically, platforms can either offer an adjustable tipping policy where consumers can *ex post* reduce their tip (e.g., Uber Eats), or offer a non-adjustable policy where they cannot (e.g., DoorDash). In addition, the adjustable tipping policy may be abused by some selfish consumers, who initially promise a large tip to motivate a fast delivery but will completely remove the tip after delivery, regardless of the delivery time. In contrast to selfish consumers, fair consumers stick to their promise and pay the tip in full if the delivery is indeed fast, and would partially reduce the tip if the delivery turns out slow.

To understand the interplay among platforms, workers, and consumers, I consider a stylized model where two platforms first decide their respective tipping policy (adjustable or not). Later, platforms decide how much wage to offer to workers in exchange for their delivery service, and how much price to charge consumers for using the platforms. Consumers then decide which platform to use and how much tip they promise to pay while placing orders. Once a matching between consumer and worker has been established, workers decide whether to exert high or low delivery effort. Finally, the delivery is completed and consumers decide whether to reduce the tip they promised, if an adjustable tipping policy is selected.

3.7.1 Results and Managerial Insights

Our results indicate that regardless of the fraction of selfish consumers, adopting the adjustable tipping policy is each platform's dominant strategy. This result has important managerial implications that for platforms who are currently offering a non-adjustable tipping policy (e.g., DoorDash), they can be better off by switching to the adjustable policy, like their competitor does (e.g., Uber Eats). The adjustable tipping policy is better because of two reasons. First, it enables consumers to effectively reduce the expected waiting time through tipping, which in turn increases platforms' demand; while in the non-adjustable policy, workers do not have incentives to work hard if their final tip is unaffected by delivery time. Second, with the adjustable tipping policy platforms are able to pay a lower wage, as workers receive extra income from consumers' tip.

I also find that platforms are indifferent about selfish consumers under certain conditions. However, the existence of selfish consumers always harms those honest fair consumers and workers. Hence, my findings offer policy implications that third-party regulators need to intervene to safeguard fair consumers' and workers' rights, as platforms may lack the incentive to do so beyond a certain point. For instance, the regulator may require platforms to have a rating system about consumers, where workers can share with each other about their past experience with a particular consumer. With the rating system, it will be increasingly hard for selfish consumers to tip-bait workers.

Furthermore, my investigation into a scenario solely populated by fair consumers sheds light on the complexity of platforms' pricing and wage strategies. I show that the price set by platforms when consumers pay tips does not have a uniform relationship with the price in non-tipping scenarios. Instead, it can be either higher or lower, contingent upon the degree of generosity that consumers apply to their tipping practices. This highlights that even in the absence of selfish consumers, the equilibrium price and wage set by platforms are sensitive to the nuances of consumers' tipping leniency.

Finally, I find that platforms' price and wage can be negative under certain conditions. In the presence of the adjustable tipping policy, consumers have incentives to pay tips to motivate fast delivery, thus workers' total earning comes from both platforms (wage) and consumers (tip). Under certain conditions, workers are willing to accept a negative wage, which implies that workers need to pay platforms for the delivery opportunities, because they expect to be compensated by receiving large tips from consumers. There are also situations where platforms' price can be negative, which implies that platforms subsidize consumers for placing orders. For example, platforms may provide consumers with large coupons to stimulate demand.

3.7.2 Limitations and Future Research Directions

I conclude the paper by discussing some limitations about my model and other directions for future research in the area. First, I have assumed that consumers would calculate the optimal tip to give when placing orders. However, in practice consumers may simply follow the tip suggestion by platforms, such as giving 10%, 15%, or 20% tips, because calculating the optimal tip requires some cognitive resources and may be cumbersome to some consumers. Thus, future research may also investigate the optimal design of tip recommendation by platforms. For example, what is the optimal percentage of tips to recommend and how many tipping suggestions should platforms make. Second, my model does not consider the possible batched orders, where workers combine multiple orders from different consumers into a single delivery route. In this case, food delivery time depends on not only workers' delivery effort but also the proximity to locations of other orders in the batch. Therefore, consumers may give different tips if their order is being stacked with others. It would be interesting to study the impact of batched orders on platforms' tipping policy and subsequent wage and price decisions.

Chapter 4
4 Probabilistic Price Promotions Without Obligations

Abstract: This paper studies the design of probabilistic price promotions where consumers through a lottery are either offered one of many promotional prices, including zero, or offered to purchase products at the original list price. Upon observing the realized promotion price, consumers are, however, not obligated to purchase. Based on the cognitive bias *zero-price effect*, where consumers attach a special value to free products, I derive the optimal promotion parameters regarding the number of promotional prices, the chance of winning, and the list price. I consider a duopoly where a firm operating the probabilistic price promotion competes against a firm operating a standard fixed price promotion. The market consists of two types of consumers, *trusting consumers*, who form unbiased expectations based on the lottery parameters, and *skeptical ones*, who form skewed expectations. I find that a simple lottery, wherein consumers either receive the product for free or are offered to pay the original list price, is more profitable than a complex lottery with many promotional prices. Moreover, firms should only offer probabilistic price promotions when the zero-price effect is larger than a threshold, which decreases in the fraction of trusting consumers. These findings offer key managerial implications: firms with excellent reputations, resulting in a large fraction of trusting consumers, should offer the simple lottery to capitalize on the zero-price effect. Conversely, those with mediocre reputations, drawing a high fraction of skeptical consumers, should prioritize fixed price promotions to eliminate any uncertainty in the final price.

Key words: probabilistic price promotion, zero-price effect, consumer skepticism, game theory

4.1 Introduction

Discounts and promotions represent more than 25% of revenue for typical consumer goods (Rapperport, 2015). Hence, designing promotions right is critical for retailers to thrive. In contrast to the conventional fixed price promotion where the discount is fixed, probabilistic price promotion, where the price or discount is *ex ante* uncertain to consumers, is quickly gaining popularity among retailers. For example, the Canadian home supply company Reno-Depot has run advertising campaigns where the hidden discount ranges from 10% off to 100% off but only revealed if consumers come to the store to scratch a card, see Figure 4.1 (Reno-Depot, 2019). Many other retailers have announced similar campaigns, explicitly stating the corresponding probabilities of receiving different possible discounts and not requiring consumers to buy if they are unsatisfied with the final discount (see, for example, Home Hardware, 2020; PartSource, 2022). Supporting the case for this type of promotions, empirical research have found that compared to equivalent fixed price promotions, probabilistic price promotions can be more effective as they increase purchase likelihood (Choi & Kim, 2007; Mazar, Shampanier, & Ariely, 2017; Hock, Bagchi, & Anderson, 2020).

One explanation behind the attractiveness of probabilistic price promotions is the well-established psychological phenomenon known as the *Zero-Price Effect*, where products received for free are perceived as

Figure 4.1 Probabilistic Price Promotion

Discount Percentage	Number of Cards with Discount (based on 1,000 cards)	Odds of obtaining the Discount
10 %	900	90 %
15 %	55	5,5 %
20 %	34	3,4 %
25 %	7	0,7 %
50 %	3	0,3 %
100 %	1	0,1 %

having higher intrinsic value. In particular, Shampanier et al. (2007) empirically show that when a product becomes free, consumers attach extra positive utilities to it, and that the effect vanishes with any other price offers (e.g., \$0.01). Although probabilistic price promotion only involves a chance to receive, but does not guarantee, free products, Dallas and Morwitz (2018) have shown through experiments that consumers are just as likely to evaluate *pseudo-free offers*, which require some cognitive tasks, as truly free offers. For example, pseudo-free offers can require consumers to complete a survey or, as in my paper, to evaluate a probabilistic price promotion. Consequently, when a firm offers probabilistic price promotion that includes free products, consumers may be more likely to purchase from the firm due to zero-price effect.

However, it is unclear whether the increased purchase likelihood leads to increased profits. First, if consumers' chance of winning is "too high," then "too many" products are sold at promotional prices, which may not be maximizing the firm's profit. Furthermore, consumers are often not obligated to proceed with purchasing if they are unsatisfied with the final price, and have the option to switch to competitors. Thus, the firm needs to strategically design the promotional prices. Finally, when a firm offers probabilistic price promotion involving a chance to receive products for free, some consumers might be skeptical and convinced that the promotion is merely a trick used to fool them. After all, receiving products for free sounds too good to be true. Indeed, previous research have shown that some consumers are skeptical about extreme price discounts, see Urbany et al. (1988), Ford et al. (1990), Kim and Kramer (2006). Hence, several questions arise:

- What is the optimal design of probabilistic price promotion in the presence of skeptical consumers?
- How would zero-price effect affect firms' pricing decisions and revenues?
- When is it more profitable to offer probabilistic price promotion than fixed price promotion?

To address these questions I consider a duopoly market and analyze the competing price promotions through a game-theoretic model. The market consists of a Hotelling linear city where consumers are uniformly distributed over $[0,1]$. The two firms are located at the two extreme points and sell identical homogeneous products. Firm B offers a conventional fixed price promotion, while firm A offers a probabilistic price promotion such that her product discount is *ex ante* uncertain to consumers. Specifically, firm A's uncertain discount is designed such that consumers will through an individual lottery either receive one of n promotional prices, including a zero price (i.e., 100% discount), or are offered to buy at the original list price. Although the probabilities of receiving each price discount are publicly announced by firm A, consumers need to visit firm A and draw the lottery to finalize their individual discount. If consumers are unsatisfied with the final discount/price, they are not obligated to proceed with the purchase and may switch to the competing firm.

Because consumers cannot verify whether the true probabilities of receiving promotional prices are indeed the claimed ones, some consumers might be skeptical and convinced that the promotion is merely a trick used to fool them. I then consider two types of consumers: trusting consumers and skeptical consumers. Trusting consumers view firm A's probabilistic price promotion in good faith and believe the lottery outcomes will be truthfully determined according to the announced probabilities. On the other hand, skeptical consumers consider the lottery essentially rigged and the uncertain price will always turn out to be the highest one in their draws. The game sequence is as follows: First, firm A designs the lottery and firm B decides her price, simultaneously. Then, consumers decide which firm to visit. For those consumers who visited firm A, they draw a lottery, which determines the final price, and decide whether to purchase from firm A or switch to firm B.

A summary of my findings and contributions are as follows: (1.) I show that instead of providing a complex lottery that includes many different promotional prices, firms would be better off by offering a simple lottery through which consumers either receive the product for free or are offered to pay the original list price. Moreover, firms should only offer probabilistic price promotion when the zero-price effect is larger than a given threshold, such that the probabilistic price promotion significantly boosts demand. Furthermore, the given threshold is decreasing in the fraction of trusting consumers. These results have important managerial implications that for a fixed level of zero-price effect, firms with excellent reputation, and thus higher fraction of trusting consumers, should implement probabilistic price promotions. By contrast, firms with mediocre or bad reputation, and thus more likely to have skeptical consumers regarding an lottery-based promotion, should stick to conventional fixed price promotions. In addition, when designing a probabilistic price promotion, firms should strive to keep the promotion simple, as a complex promotion is both unnecessary and less profitable.

(2.) I find that as consumers increasingly value free products (larger zero-price effect), firms should set a higher chance of receiving promotional prices to satisfy consumers' desire for free products. However, the original list price exhibits non-monotone pattern and may either increase or decrease in the zero-price effect. The non-monotone change in list price is a result of the two firms engaging in a price war that gets fiercer as consumers add more value to free products.

(3.) Motivated by the fact that probabilistic price promotions often have very short promotion periods (e.g., Kent's probabilistic promotion lasted for only 1 day[4]) such that the two firms are unlikely to offer probabilistic price promotion at the same time, I focus my attention on asymmetric promotion strategies in the main model. In an extension, I consider the case where both firms implement probabilistic price promotion (symmetric promotion strategies) and show that my main insights remain unchanged. In another extension, I consider the scenario where firms can acquire government certification that verifies the credibility of probabilistic price promotion. In the presence of government certification, skeptical consumers no longer question the legitimacy of the promotion and are converted to trusting consumers. Surprisingly my results include that under certain conditions, firms may not want to acquire government certification even if it is free.

(4.) In general, I propose a novel framework that investigates the optimal design of probabilistic price promotion in the presence of two behavioral biases, i.e., the zero-price effect and consumer skepticism. Probabilistic price promotion is quickly gaining popularity, but there exist very few theoretical studies to guide the optimal design of such promotion. Hence, my study fills the literature gap, helps understand the increasingly popular practice of promotion, and offers insights into its optimal design.

The rest of this paper is organized as follows. In section 2, I review the related literature. In section 3, I present the model setup. In section 4, I analyze the competition between firms and compare my main model results with those in a benchmark to derive further insights. In section 5, I explore the main model with two extensions to demonstrate robustness of results. Lastly, I conclude with managerial implications and directions for future research in section 6. Throughout the paper, I use "increase" and "decrease" in a weak sense, unless otherwise specified. All proofs are provided in the Appendix C.

4.2 Literature Review

The underlying business problem and proposed model formulation are similar to the mechanism of probabilistic selling and opaque products. Probabilistic selling refers to the selling strategy in which sellers *ex ante* hide one or more product attributes (e.g., color of shirt or specific hotel), and only *ex post* reveal to consumers the exact product with all specific attributes. For example, Fay and Xie (2008) study probabilistic selling in a monopoly setting and find that probabilistic selling can be beneficial as it lessens the negative effect of demand uncertainty and mismatch to capacity is reduced. Anderson et al. (2015), who through both an empirical data analysis and theoretical exposition, consider the revenue impact and optimal price thresholds in *Name-Your-Own-Price* sales channels. Elmachtoub and Hamilton (2021) show the power of probabilistic selling in a setting where risk-neutral consumers and pessimistic consumers coexist, and characterise conditions under which probabilistic selling outperforms discriminatory pricing. In the above

[4] https://twitter.com/DartmouthXing/status/1020034457053605891.

studies, consumers pay the same price but may receive different products. This paper studies the opposite case where consumers receive the same product but may pay different prices that are *ex ante* uncertain. Furthermore, in probabilistic selling, consumers' uncertainty is not resolved until after the sales transaction, while in the proposed probabilistic promotion, the consumers' uncertainty is resolved prior to the sales transaction.

Another related literature is the nascent stream on randomized or markovian pricing, e.g. Wu et al. (2014), Moon et al. (2018), Wu et al. (2020), Chen et al. (2023). Both Wu et al. (2014) and Wu et al. (2020) center on a monopolist who, over an infinite time horizon, randomly switches the price between a high and low level according to a Markov process, but differ in specific assumptions regarding the consumer behavior and firm strategy space. Wu et al. (2014) show that a randomized pricing strategy outperforms a fixed or flat pricing strategy, while Wu et al. (2020) extend the insights in the presence of price guarantees. Different from Wu et al. (2014) and Wu et al. (2020) wherein the firm's random price is binary distributed, my paper studies a more general case where firm's random price is determined through a lottery that has many outcomes. Chen et al. (2023) also consider a more general random price distribution, and conditions under which a binary price distribution is optimal are derived. However, their paper considers a monopolist setting, while I focus on a duopoly setting where consumers may switch to a competing firm if they are unsatisfied with the promotional prices determined through a lottery.

Moreover, this paper contributes to the predominant empirical literature within marketing that suggests probabilistic price promotions can be more effective than fixed price promotions. For instance, Choi and Kim (2007) conduct two laboratory experiments and find that probabilistic price promotion can be an effective promotional tool to improve consumers' perceptions about products and increase their shopping intention. Goldsmith and Amir (2010) use three experiments to demonstrate that an uncertain prize may be almost as effective as offering the best possible outcome with certainty. Mazar et al. (2017) conduct experiments of probabilistic price promotion where consumers may receive product for free. They find that consumers are more likely to purchase in a uncertain pricing promotion than in a fixed price promotion, even though they offer the same amount of expected discount. Similarly, Hock et al. (2020) show that comparing to a fixed price promotion, probabilistic price promotion can lead to higher purchase likelihood and increased average overall spending.

In contrast to the above abundant empirical studies of probabilistic price promotion, very few analytical studies exist. Lei (2022) is the closest paper to ours. He considers a market where firms do not announce the corresponding probabilities of different discounts in a probabilistic price promotion. Hence, consumers have to form their own judgements about the expected discount. Our study differs from his study in two aspects. First, I incorporate the psychological phenomenon, zero-price effect, into my model and investigate how would such effect impact firms' pricing decisions. Second, consumers in my study know the distribution of different discounts, therefore firms endogenously determine the perceived value of a probabilistic price promotion.

Finally, this paper contributes to the growing literature on pricing and revenue management in the presence of various behavioral biases and consumer considerations.For reviews in this area see Shen and Su (2007) and Özer and Zheng (2012), see also Strauss et al. (2018) for a review on the tangential choice-based modeling. For a specific behavioral analysis related to pricing strategies, see Özer and Zheng (2016). A traditional classification regarding consumer consideration centers on whether consumers are *myopic* or *strategic* in their purchase decision making. For an example relevant to my model framework, see Mantin et al. (2011). The myopic/strategic distinction is mainly relevant to dynamic pricing models over multiple periods, but can be generalized to include consumer decision evaluations within single periods. On the behavioral aspect, the most established paradigm within economic modeling is that of *Prospect Theory*. Stemming from the seminal work of Daniel Kahneman and Amos Tversky in the late 1970s, the general tenet within Prospect Theory is that consumer evaluations are highly context dependent and that choices involving equal financial consequences may nevertheless produce starkly different decisions. For an analysis involving both theoretical and empirical exposition, see Wang (2018). In my paper, consumers are strategic in the sense that they evaluate consequences beyond the immediate pay-off. Furthermore, I incorporate two novel behavioral considerations: (1) consumers' outlook regarding the integrity of firms and their promotions, and (2) the established cognitive bias zero-price effect.

4.3 Model Setup

Consider a Hotelling linear city where consumers are uniformly distributed over $[0,1]$. Two firms, A and B, selling homogeneous products, are located at opposite ends; firm A is located at 0, and firm B at 1. The two firms have the same fixed marginal cost of production, which is normalized to zero, and compete for market share through individual price promotions.

Firm B operates a standard fixed price promotion by announcing a fixed price p_B. By contrast, firm A offers a probabilistic price promotion through a lottery such that consumers are *ex ante* uncertain about the final price p_A. The lottery is designed such that the final price is,

$$\begin{cases} p_A \in \{0, \frac{1}{n}\overline{p}, \frac{2}{n}\overline{p}, \dots, \frac{n-1}{n}\overline{p}\} & \text{with probability } \gamma, \\ p_A = \overline{p} & \text{with probability } 1-\gamma, \end{cases}$$

where $0 \leq \gamma \leq 1$. That is, with probability γ, consumers win the lottery and receive a promotional price from a set of n promotion outcomes $\{0, \frac{1}{n}\overline{p}, \frac{2}{n}\overline{p}, \dots, \frac{n-1}{n}\overline{p}\}$, where each promotion price outcome is equally likely to be selected, i.e., $p_A = \frac{i}{n}\overline{p}$ with probability $\frac{\gamma}{n}$, $i = 0, 1, \dots, n-1$. Otherwise, consumers lose the lottery and are offered to pay the list price $\overline{p}$. Firm A decides and publicly advertises the list price $\overline{p}$, the probability of winning lottery γ, and the number of promotion outcomes n, where $n \in \mathbb{Z}$ and $n \geq 1$. Nevertheless, consumers must arrive at firm A's store to draw and observe their realized individual outcome of the lottery.

For any realized outcome price p_A, consumers are not obligated to purchase at firm A, and can choose to visit and buy from firm B instead.

Because consumers cannot verify whether the true probability of winning lottery is indeed γ, some consumers might be skeptical and convinced that the promotion is merely a trick used to fool them. I then consider two types of consumers. A fraction of $\beta \in (0,1)$ consumers are *trusting* consumers, who view firm A in good faith and believe the lottery outcomes will be truthfully determined according to the claimed probabilities. By contrast, the remaining $1-\beta$ fraction of consumers are *skeptical* consumers, who think the lottery is merely a marketing trick and that the uncertain price will always turn out to be the highest outcome $\overline{p}$ (i.e., in essence skeptical consumers assign $\gamma = 0$). All consumers have unit demand and homogeneous valuation v for the product. Following a standard assumption in the literature, I assume v is sufficiently high such that the market is fully covered, see Fudenberg and Tirole (2000), Shin and Sudhir (2010), Li and Jain (2016), and Bae et al. (2022). If the market was not fully covered, then by definition, consumers in the middle of the Hotelling line would not purchase from either firm. In this case, the two firms act as local monopolists and do not compete with each other.

Let u_A^T and u_A^S denote a trusting and skeptical consumer's utility of visiting firm A, respectively. A trusting consumer perceives firm A's price lottery in good faith and form expected utility of buying from firm A. Empirical research has shown that when a product becomes free, consumers attach a special value to it and derive extra positive utility of α. This phenomena has been labeled the *zero-price effect* (Shampanier et al., 2007). Thus, a trusting consumer's expected utility of visiting firm A is,

$$u_A^T(x) = v + \frac{\gamma}{n}\alpha - \Big(\frac{\gamma}{n}\sum_{i=0}^{n-1}\min\{\frac{i}{n}\overline{p}, p_B + t\} + (1-\gamma)\min\{\overline{p}, p_B + t\}\Big) - tx^2. \tag{4.1}$$

For any realized price p_A, consumers compare it with their outside option of switching to firm B, which would incur the entire travel cost t (since consumers are at firm A to draw the lottery and the distance between firm A and firm B is 1) plus paying price P_B. In addition, the quadratic cost is governed by a transportation cost parameter $t > 0$ and the consumer's location x.

In contrast to the trusting consumer, the skeptical consumer, who assumes firm A's "uncertain" price will always be $p_A = \overline{p}$, has the following (biased) "expected" utility of visiting firm A,

$$u_A^S(x) = v - \overline{p} - tx^2. \tag{4.2}$$

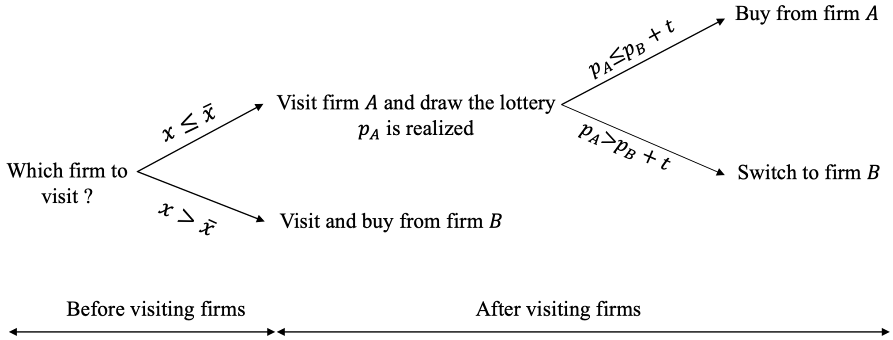
Since firm B's price is fixed and transparent, both trusting and skeptical consumers derive the same utility of purchasing from firm B,

$$u_B(x) = v - p_B - t(1-x)^2. \tag{4.3}$$

Figure 4.2 summarizes consumers' decision-making process. For each type of consumers, define the consumer located at $\bar{x}$ as being indifferent between visiting firm A and firm B,

$$\begin{cases} u_A^T(\bar{x}) = u_B(\bar{x}) & \text{if consumer is trusting,} \\ u_A^S(\bar{x}) = u_B(\bar{x}) & \text{if consumer is skeptical.} \end{cases} \tag{4.4}$$

Figure 4.2 Consumers' Decision-Making Process



For each consumer type, equation (4.4) solves uniquely for,

$$\overline{x} = \begin{cases} \frac{n(p_B+t)+\alpha\gamma-n(\frac{\gamma}{n}\sum_{i=0}^{n-1}\min\{\frac{i}{n}\overline{p},p_B+t\}+(1-\gamma)\min\{\overline{p},p_B+t\})}{2tn} & \text{if consumer is trusting,} \\ \frac{p_B+t-\overline{p}}{2t} & \text{if consumer is skeptical.} \end{cases} \tag{4.5}$$

That is, a consumer will only visit firm A if her distance to firm A is less than or equal to $\overline{x}$. Consequently, $\overline{x}$ also represents the market share or demand for firm A. Note that $\overline{x}$ is different for each type of consumers, and that consumers' type only impacts their initial decisions on which firm to visit. However, once they have arrived at either firm, their type becomes irrelevant. For skeptical consumers who draw the lottery, their chance of winning the lottery is the same as that of trusting consumers, and they compare the realized price with $p_B + t$ to make a final purchasing decision.

The sequence of events is as follows: First, firm A designs the lottery ($\overline{p}$, γ, and n) and firm B decides her price (p_B), simultaneously. Then, consumers decide which firm to visit. Later, for the consumers who arrived at firm A, they draw the lottery and decide whether to buy from firm A or switch to firm B.

4.4 Price Promotion Strategies

In this section, I first investigate the optimal number of promotion outcomes, and then characterise the competition between firms and study the impact of zero-price effect on firms' pricing and profits. Lastly, I compare my main model results with those in a benchmark where both firms implement fixed price promotion (the standard Hotelling model) to derive further insights.

4.4.1 Optimal Number of Promotion Outcomes

After drawing the lottery, consumers choose to either buy at the realized price at firm A or switch to firm B. I consider two cases.

Case 1: For any potential price p_A, consumers would choose to continue purchasing at firm A, i.e., $\overline{p} \leq p_B + t$. In this case, for trusting consumer, the expected transaction price from visiting firm A is given by,

$$\mathbb{E}[p_1] = \frac{\gamma}{n}\sum_{i=0}^{n-1}\frac{i}{n}\overline{p} + (1-\gamma)\overline{p}$$

$$= \frac{\overline{p}(2n-(1+n)\gamma)}{2n}. \tag{4.6}$$

Hence, the demand expression (4.5) can be simplified to,

$$\bar{x} = \begin{cases} \frac{n(p_B+t)+\alpha\gamma-\mathbb{E}[p_1]n}{2tn} & \text{if consumer is trusting,} \\ \frac{p_B+t-\overline{p}}{2t} & \text{if consumer is skeptical.} \end{cases} \tag{4.7}$$

Given the above expressions, firm A's expected profit maximization problem can be expressed as

$$\begin{aligned} \max_{\overline{p},\gamma,n} \quad & \Pi_{A1}(\overline{p},\gamma,n) = \mathbb{E}[p_1]\left[\beta(\frac{n(p_B+t)+\alpha\gamma-\mathbb{E}[p_1]n}{2tn}) + (1-\beta)(\frac{p_B+t-\overline{p}}{2t})\right] \\ \text{s.t.} \quad & \overline{p} \in [0, p_B+t], \\ & \gamma \in [0,1], \\ & n \geq 1. \end{aligned} \tag{4.8}$$

Case 2: Consumers choose to continue purchasing at firm A only if the realized price $p_A \leq \frac{k}{n}\overline{p}$, where $k \in \mathbb{Z}$ and $0 \leq k \leq n-1$. The expected transaction price for trusting consumers from visiting firm A is then given by,

$$\begin{aligned} \mathbb{E}[p_2] &= \frac{\gamma}{n}[\sum_{i=0}^{k}\frac{i}{n}\overline{p} + (n-k-1)(p_B+t)] + (1-\gamma)(p_B+t) \\ &= \frac{\gamma}{n}[\frac{\overline{p}k(k+1)}{2n} + (n-k-1)(p_B+t)] + (1-\gamma)(p_B+t). \end{aligned} \tag{4.9}$$

Note that $\mathbb{E}[p_2]$ is not the expected price that trusting consumers will pay to firm A, but rather the expected transaction price paid to either firm A or B if they were to draw the lottery. In (4.9), $\frac{\gamma}{n}\frac{\overline{p}k(k+1)}{2n}$ is the expected price that trusting consumers pay to the firm A; when the realized price is greater than $\frac{k}{n}\overline{p}$, trusting consumers switch to firm B, which entails cost p_B+t.

In addition, with $\frac{k}{n}\overline{p}$ being the highest acceptable price, it implies $\overline{p} > p_B+t$ and $\frac{k}{n}\overline{p} \leq p_B+t$. In this case, firm A's demand from skeptical consumers is 0, since $\frac{p_B+t-\overline{p}}{2t} < 0$. That is, the list price is too high and skeptical consumers will not visit firm A, as they (incorrectly) assume the realized price will be $\overline{p}$ for sure. Hence, the demand expression (4.5) can be simplified to,

$$\bar{x} = \begin{cases} \frac{n(p_B+t)+\alpha\gamma-\mathbb{E}[p_2]n}{2tn} & \text{if consumer is trusting,} \\ 0 & \text{if consumer is skeptical.} \end{cases} \tag{4.10}$$

Given the above expressions, firm A's expected profit maximization problem can be expressed as

$$\begin{aligned} \max_{\overline{p},\gamma,n} \quad & \Pi_{A2}(\overline{p},\gamma,n) = \frac{\gamma}{n}\frac{\overline{p}k(k+1)}{2n}\beta(\frac{n(p_B+t)+\alpha\gamma-\mathbb{E}[p_2]n}{2tn}) \\ \text{s.t.} \quad & \overline{p} > p_B+t, \\ & \frac{k}{n}\overline{p} \leq p_B+t, \\ & \gamma \in [0,1], \\ & n \geq 1. \end{aligned} \tag{4.11}$$

Having discussed firm A's profits in both cases, I am now ready to establish a insightful result that for any lottery with $n \geq 2$ promotion outcomes, there exists a dominant lottery with $n = 1$.

Proposition 4.1. *The optimal number of promotion outcomes is $n^* = 1$.*

Technical details are provided in the Appendix C, but the intuition behind Proposition 4.1 are as follows. In the first case where consumers never switch for any potential price, firm A can construct a simple lottery that has the same expected price as the original lottery, but contains only one promotion outcome, namely zero price. Compared to the original lottery, the simple lottery is more profitable because with consumers having a higher chance to receive the product for free, and due to zero-price effect, more consumers will visit firm A. In the second case where consumers may switch to firm B for some realized prices, firm A can improve their expected profit by constructing a refined lottery that eliminates all the realized prices at which consumers would switch to firm B (i.e., eliminate prices that are higher than $p_B + t$). Therefore, by construction, consumers would not switch for any realized price in the refined lottery, and firm A is back to the first case and there exists a dominant simple lottery with only the zero price promotion outcome.

4.4.2 Competing with Simple Lottery

Since I have shown in Proposition 4.1 that the optimal number of promotion outcomes is $n^* = 1$, hereafter I base my analysis on the case of the simple lottery wherein consumers either pay the list price $\overline{p}$, or receive the promotional price zero. That is, the final price at firm A is,

$$p_A = \begin{cases} 0 & \text{with probability } \gamma, \\ \overline{p} & \text{with probability } 1-\gamma. \end{cases}$$

In such a lottery, I can deduce that in equilibrium, firm A's price $\overline{p}$ must not exceed $p_B + t$, otherwise firm A would certainly earn zero profit. The reason being, if firm A's in-store consumers win the lottery, they obtain the product for free, and if they lose the lottery they simply switch to buy from firm B instead. Therefore, firm A's demand in equation (4.5) can be simplified as,

$$\overline{x} = \begin{cases} \frac{p_B - \overline{p}(1-\gamma) + t + \alpha\gamma}{2t} & \text{if consumer is trusting,} \\ \frac{p_B + t - \overline{p}}{2t} & \text{if consumer is skeptical.} \end{cases} \tag{4.12}$$

Given the above demand expression, firm A's expected profit maximization problem is given by,

$$\begin{aligned} \max_{\overline{p},\gamma} \quad & \Pi_A(\overline{p},\gamma) = (1-\gamma)\overline{p}\left(\frac{\beta(p_B - \overline{p}(1-\gamma) + t + \alpha\gamma) + (1-\beta)(p_B - \overline{p} + t)}{2t}\right) \\ \text{s.t.} \quad & \overline{p} \in [0, p_B + t], \\ & \gamma \in [0,1]. \end{aligned} \tag{4.13}$$

The objective function in equation (4.13) indicates that firm A only earns profit if consumers lose the lottery and receive price $\overline{p}$, which happens with probability $1-\gamma$. In addition, β fraction of consumers are trusting consumers and the remaining $1-\beta$ fraction are skeptical consumers, whose demands are shown in expression (4.12), respectively.

Similarly, since firm B's demand is $1-\bar{x}$, her profit maximization problem is given by,

$$\begin{aligned} \max_{p_B} \quad & \Pi_B(p_B) = p_B\left(\frac{\beta(\overline{p}(1-\gamma)+t-p_B-\alpha\gamma)+(1-\beta)(\overline{p}-p_B+t)}{2t}\right) \\ \text{s.t.} \quad & p_B \geq 0. \end{aligned} \tag{4.14}$$

Although the optimal solutions to (4.13) and (4.14) depend on the parameters α, β, and t, in order to simplify the notation I denote the optimal solutions by $\overline{p}^*$ and γ^*, and p_B^*, respectively, with corresponding optimal values $\Pi_A(\overline{p}^*,\gamma^*)$ and $\Pi_B(p_B^*)$. Note in particular that if the zero-price effect α is "too large," the appeal of free products is so strong such that all trusting consumers would visit firm A to draw the lottery, resulting in zero demand for firm B. Hence, for the purpose of this paper I restrict the analysis on the non-trivial cases where both firms have positive demands and compete for trusting consumers. This implies that I impose the condition $\alpha < \frac{\overline{p}(1-\gamma)-p_B+t}{\gamma}$. Depending on the strength of the zero-price effect, there are three different levels of the probabilistic price promotion, which I summarize through the following formal equilibria of the game.

Proposition 4.2. *The game has three equilibria as follows*:

(i) *for all* β*, there exists a unique lower bound threshold* $\underline{\alpha} \equiv \frac{t(1-\beta)}{\beta}$ *such that if* $\alpha < \underline{\alpha}$*, then it is optimal to not offer probabilistic price promotion*: $\gamma^* = 0$, $\overline{p}^* = t$, $p_B^* = t$;

(ii) *there exist* $\hat{\beta} \equiv \frac{11-\sqrt{73}}{4} \approx 0.61$ *and an upper bound threshold* $\overline{\alpha}$ *given by*

$$\overline{\alpha} \equiv \begin{cases} \frac{t\left(\beta^3+3\beta^2-13\beta+3+(1+\beta)\sqrt{\beta^4+4\beta^3-26\beta^2+12\beta+9}\right)}{2\beta\left(\beta^2-5\beta+3\right)} & \text{if } \beta \leq \hat{\beta} \\ \frac{12t(1-\beta)}{\beta(4-\beta)} & \text{if } \beta > \hat{\beta} \end{cases}$$

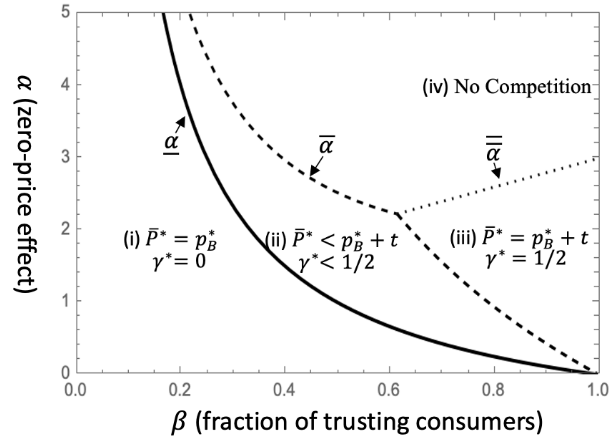
such that if $\underline{\alpha} \leq \alpha < \overline{\alpha}$*, then it is optimal to offer partial probabilistic price promotion*:

$$\begin{aligned} \gamma^* &= \frac{\alpha(2+\beta)-\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{3\alpha\beta}, \\ \overline{p}^* &= \frac{-2\alpha(1-\beta)+\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{3(1-\beta)}, \\ p_B^* &= \frac{9t-\alpha(2+\beta)+\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{9}; \end{aligned}$$

(iii) *for* $\beta > \hat{\beta}$*, there exists an adjusted upper bound* $\overline{\overline{\alpha}} \equiv t(1+2\beta)$ *such that if* $\overline{\alpha} \leq \alpha < \overline{\overline{\alpha}}$*, then it is optimal to offer maximal probabilistic price promotion*:

$$\begin{aligned} \gamma^* &= 1/2, \\ \overline{p}^* &= \frac{6t-\alpha\beta}{2+\beta}, \\ p_B^* &= \frac{t(4-\beta)-\alpha\beta}{2+\beta}. \end{aligned}$$

Figure 4.3 Equilibrium Regions



To showcase the above results and provide further intuition, Figure 4.3 presents a numerical illustration of Proposition 4.2 with $t = 1$. The horizontal axis represents the fraction of trusting consumers (β), and the vertical axis represents the boost in utility from zero-price effect (α). Region (iv) represents the case where the zero-price effect is overly large such that all trusting consumers visit firm *A* (i.e., where condition $\alpha < \frac{\overline{p}(1-\gamma)-p_B+t}{\gamma}$ is violated).

Probabilistic price promotion is a double-edged sword to firm *A*. On one hand, the possibility of winning products for free has great appeal to consumers, many of whom would be attracted to visiting firm *A* and trying their luck. Hence, the demand of firm *A* increases with probabilistic price promotion (market expansion effect). On the other hand, probabilistic price promotion is risky as consumers may indeed be very lucky and win products for free, in which case firm *A* would not generate any revenue (loss of revenue effect). The trade-off between these two effects results in the different regions in Figure 4.3.

For the purpose of discussion, consider the case when $\beta = 0.8$. When the boost in utility is small ($\alpha < \underline{\alpha}$), it is optimal for firm *A* to not offer probabilistic price promotion. From firm *A*'s perspective, weak zero-price effect does not justify the risk of giving products for free, as it cannot significantly boost demand. Therefore, firm *A* is better off with announcing zero probability of receiving free products. Both firms would charge price of t, and each serves half of the market. The model then reduces to the standard Hotelling model.

When the boost in utility is intermediate ($\underline{\alpha} \leq \alpha < \overline{\alpha}$), it is optimal for firm *A* to offer partial probabilistic price promotion, where consumers have positive probability of receiving the free products. In this region, consumers get more excited about free products, and even some of the distant consumers are willing to visit firm *A* and draw their lottery, if given such an option. Hence, tailoring to the desire of consumers, firm *A* offers probabilistic price promotion such that consumers may receive products for free with positive possibility. Upon arriving at firm *A*, some consumers indeed get lucky and receive free products. The other less lucky consumers receive the price $\overline{p}^*$, and they need to re-evaluate their options: buy the product from firm *A* at price $\overline{p}^*$ or switch to firm *B* and pay p_B^*. Although firm *A*'s price $\overline{p}^*$ is more expensive than her competitor's offer p_B^*, the consumers are reluctant to switch to firm B due to the high transportation cost. After all, if they switch to firm B, they need to travel the whole street and incur costly transportation cost. If they buy at firm A, they only need to pay the price $\overline{p}^*$, as the transportation cost to firm A has already incurred in the past and becomes a sunk cost now. Weighing the trade-offs, all these less lucky consumers would just buy from firm A despite the higher price. And the induced "sunk cost" is one of the key pillars of the probabilistic price promotion strategy.

When the boost in utility is large ($\overline{\alpha} \leq \alpha < \overline{\overline{\alpha}}$), it is optimal for firm A to offer maximal probabilistic price promotion, where consumers receive free products with probability 1/2. In this region, the appeal of free products is tempting enough such that firm A's sets her price $\overline{p}^* = p_B^* + t$, which is exactly the switching cost for firm A's in-store consumers to switch to firm B. Note that the switching cost $p_B^* + t$ is also the maximum price that firm A can charge upon consumers' arrival. For any price higher than the maximum price $p_B^* + t$, the in-store consumers would leave firm A without purchasing and switch to firm B, despite the transportation cost. Moreover, firm A is only able to set the maximum price when free products are attractive enough ($\overline{\alpha} \leq \alpha < \overline{\overline{\alpha}}$). Had the boost in utility been intermediate, firm A would have set her price $\overline{p}^*$ to be less than $p_B^* + t$, otherwise the high price ($p_B^* + t$) would have discouraged many consumers from visiting firm A in the first place. Next, I present a straightforward corollary of Proposition 4.2 that offers important managerial insights into selecting the right promotion.

Corollary 4.1. *The threshold of offering probabilistic promotion $\underline{\alpha}$ is strictly decreasing in β.*

Corollary 4.1 can be observed in Figure 4.3, and implies that firms with excellent reputation, resulting in a high fraction of trusting consumers, should implement probabilistic price promotion to take advantage of the zero-price effect. By contrast, firms with mediocre or bad reputation, resulting in a high fraction of skeptical consumers questioning the credibility of such promotion, should not offer probabilistic price promotion. Instead, firms with mediocre reputation should implement the conventional fixed price promotion to eliminate any uncertainty in the final price.

Next, Proposition 4.3 examines the impact of α on firms' prices.

Proposition 4.3. *The optimal price of each firm has the following properties*:

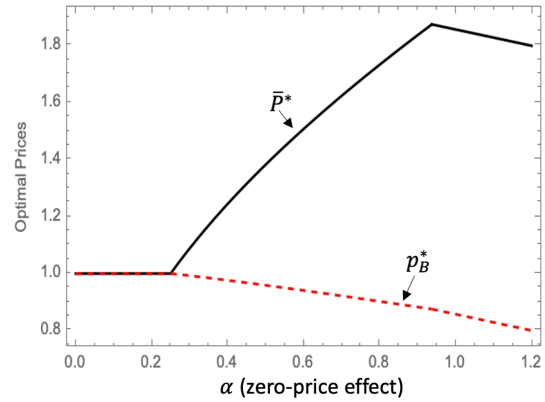
(i) *firm A's optimal price $\overline{p}^*$ is strictly increasing in α if and only if $\underline{\alpha} \leq \alpha < \overline{\alpha}$;*

(ii) *firm B's optimal price p_B^* is decreasing in α.*

To provide further intuition, Proposition 4.3 is numerically illustrated in Figure 4.4 with $\beta = 0.8$ and $t = 1$; under this set of parameters, $\underline{\alpha} = 0.25$ and $\overline{\alpha} = 0.94$. The horizontal axis represents the zero-price effect (α), and the vertical axis corresponds to the optimal prices $\overline{p}^*$ and p_B^* for firms A and B, respectively.

When the boost in utility from zero-price effect is small ($\alpha < 0.25$), firm A would not implement probabilistic price promotion as giving out products for free cannot stimulate enough demands to compensate the loss of revenue effect. Hence, the model reduces to the standard Hotelling model, in which α does not affect firms' prices.

Figure 4.4 Impact of the Boost in Utility From Zero-Price Effect on Firms' Prices



When the boost in utility from zero-price effect is intermediate ($0.25 \leq \alpha < 0.94$), firm A starts implementing probabilistic price promotion and I observe that firm A's price increases in α. There is no free lunch, and free products are not so "free." In particular, consumers accept a high premium price $\overline{p}$ in exchange for the opportunity of receiving free products. The more consumers value free products, the higher the premium price is. On the other hand, firm B's price p_B^* starts to decrease in α because she has to lower her price to compete for consumers, who increasingly appreciate the opportunity of receiving free products from firm A.

When the boost in utility from zero-price effect is large ($\alpha \geq 0.94$), perhaps surprisingly, the opposite happens—the more consumers value free products, the lower the premium price is. In this range of α, firm A's price $\overline{p}^*$ reaches its upper bound $p_B^* + t$. As firm B's price p_B^* continues to decrease, firm A has to decrease her price as well because any price higher than $p_B^* + t$ would incentivize the in-store consumers to leave firm A and switch to firm B. The two firms engage in a price war that gets fiercer as consumers add more value to free products.

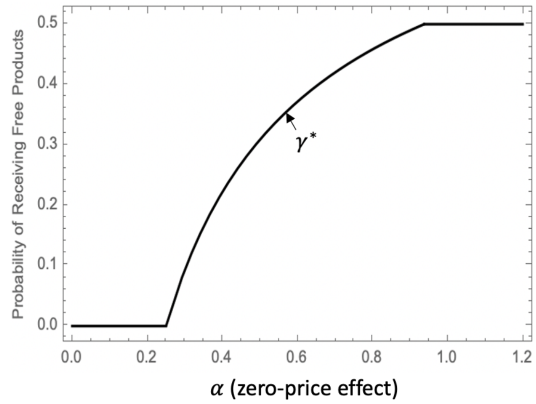
Next, Proposition 4.4 presents the impact of α on the probability of receiving free products (γ^*), and is illustrated in Figure 4.5 with $\beta = 0.8$ and $t = 1$.

Proposition 4.4. *The probability of receiving free products γ^* is increasing in α.*

Recall that probabilistic price promotion is a double-edged sword such that the market expansion effect and the loss of revenue effect co-exist. When the boost in utility from zero-price effect is small, the loss of revenue effect dominates the market expansion effect such that firm A announces zero probability of receiving free products.

As consumers increasingly appreciate free products, firm A starts to announce positive probability of receiving products for free. By increasing the probability of receiving free products, firm A attracts more consumers coming to her store and drawing the lottery. Although some of the consumers may receive the product for free (with probability γ^*), the other less lucky consumers would pay the high premium $\overline{p}^*$. Overall, the extra revenue from these less lucky consumers outweighs the loss of revenue from those lucky

Figure 4.5 Impact of the Boost in Utility from Zero-Price Effect on the Probability of Receiving Free Products



consumers. To leverage on the market expansion effect, firm A sets a higher probability of receiving free products (γ^*) as α increases.

Intuitively, γ^* cannot always increase in α, otherwise firm A would be giving out free products for the most of the time, and loss of revenue effect would be severe. Proposition 4.2 and Proposition 4.4 together state that firm A would at most set γ^* to 0.5, which happens when firm A's price reaches the upper bound $p_B^* + t$. Note that probabilistic price promotion works because the less lucky consumers would pay a high premium $\overline{p}^*$ that covers the loss of revenue effect. Increasing γ^* to above 0.5 would require more revenue from the less lucky consumers to compensate the loss of revenue attributed to lucky consumers. But since the premium price $\overline{p}^*$ now decreases in α due to the price war, firm A receives less revenue from the paying consumers, not more; and so her optimal strategy is to maintain $\gamma^* = 0.5$, not more.

Next, Proposition 4.5 presents the impact of α on firms' profits.

Proposition 4.5. *There exist unique thresholds α_l and α_u such that*:

(i) *firm A's profit $\Pi_A(\overline{p}^*, \gamma^*)$ is strictly increasing in α if and only if $\alpha_l < \alpha < \alpha_u$*;

(ii) *firm B's profit $\Pi_B(p_B^*)$ is decreasing in α,*

where

$$\alpha_l \equiv \min\left\{\overline{\alpha}, \frac{3t\left(\beta^2 - 11\beta + 4 + \sqrt{\beta^4 + 3\beta^3 + 4\beta^2 + 12\beta + 16}\right)}{4\beta(4-\beta)}\right\}$$

and

$$\alpha_u \equiv \begin{cases} \overline{\alpha} & \text{if } \beta \le \hat{\beta}, \\ \overline{\overline{\alpha}} & \text{if } \hat{\beta} < \beta \le \frac{3}{4}, \\ \frac{3t(2-\beta)}{2\beta} & \text{if } \frac{3}{4} < \beta. \end{cases}$$

To again provide better intuition I provide a numerical illustration of Proposition 4.5 in Figure 4.6. In the left panel the regions defined by α_l and α_u have been amended to Figure 4.3 to give a visual presentation of their relative positions. The middle and right panels numerically illustrate Proposition 4.5 with $\beta = 0.8$ and $t = 1$. Under this set of parameters, $\alpha_l = 0.39$ and $\alpha_u = 2.25$.

Figure 4.6 Impact of the Boost in Utility from Zero-Price Effect on Firms' Profits

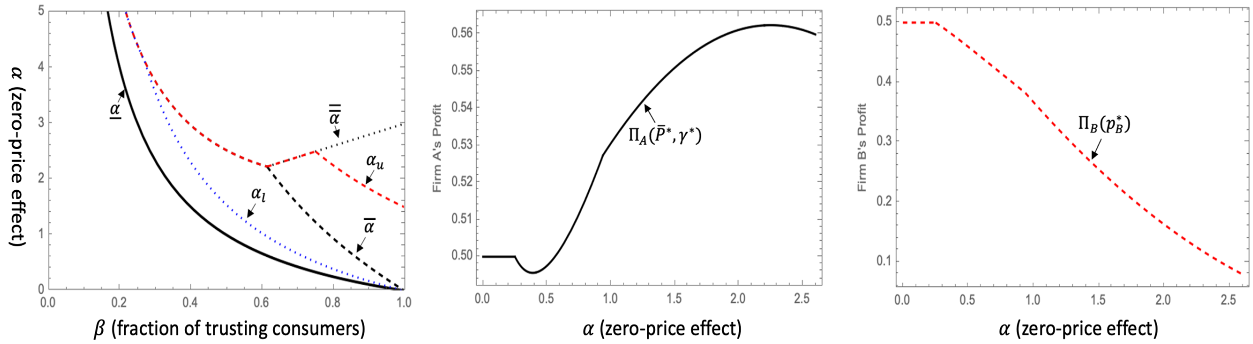


Figure 4.6 shows that firm A's profit $\Pi_A(\overline{p}^*, \gamma^*)$ first decreases, then increases, and then decreases again. It is perhaps surprising that firm A's profit would decrease when she switches from assigning zero probability to assigning positive probability (γ) of receiving free products. After all, it is at firm A's discretion to set γ such that her profit is maximized. So, why would firm A set a positive γ that results in a lower profit? It turns out that when α is non-trivial ($0.25 < \alpha \leq 0.39$), the two firms are trapped in prisoner's dilemma: Each firm chooses to either maintain their current price t or change prices. If they maintain their current price t, then they stay in the standard Hotelling model and each earns profit 0.5. However, changing price is firm A's dominant strategy regardless of firm B's action: If firm B maintains price t and firm A switches to assigning positive probability to receiving free products, then consumers will favor firm A because they appreciate free products (because α is non-trivial). As a result, firm A should increase price $\overline{p}$ to take advantage of the zero-price effect; if firm B changes her price, then firm A is also better off changing her price than maintaining price t due to the zero-price effect. Knowing that changing price is a dominant strategy to firm A, firm B is better off changing her price as well. So, the equilibrium is such that both firms change prices: firm A increases $\overline{p}$ and assigns positive probability of receiving free products to leverage on consumers' increased valuation of free products, and firm B lowers her price to compete, though both firms would be worse off by making such changes.

As consumers add more value to free products ($0.39 < \alpha < 2.25$), firm A is empowered to set a higher premium price, which results in higher revenue from those less lucky consumers. As a result, firm A's profit starts to increase in α. From firm B's perspective, an increase in α entails a further decrease in her price p_B in order to stay competitive in a market where consumers increasingly appreciate free products. Consequently firm B's profit continues decreasing.

Recall that when $\alpha > 0.94$, the two firms starts engaging in a price war that gets fiercer as α increases. When $\alpha \geq 2.25$, the price war is so intense such that firm A's premium price is no longer high enough to compensate the loss of revenue effect. Hence, firm A's profit starts to decrease again.

4.4.3 Benchmark Against Fixed Price Promotion

In this section, I benchmark my main model results with those in a benchmark where both firms implement fixed price promotion (i.e. the standard Hotelling model). I first identify the conditions under which implementing probabilistic price promotion is more profitable than implementing a fixed price promotion. Next, I examine the magnitude of potential gain from probabilistic price promotion.

In the Hotelling model with quadratic transportation cost, both firms charge price t and each serves half of the market. Let Π_A^H denote firm A's optimal expected profit in the Hotelling benchmark, then $\Pi_A^H = t/2$. Proposition 4.6 compares firm A's optimal expected profits under probabilistic price promotion vis-à-vis fixed price promotion.

Proposition 4.6. *There exist unique thresholds $\tilde{\beta}$ and α_b such that probabilistic price promotion is more profitable than fixed price promotion if and only if $\beta > \tilde{\beta}$ and $\alpha > \alpha_b$, where $\tilde{\beta}$ is implicitly given as the point at which $\Pi_A(\overline{p}^*, \gamma^*) = \Pi_A^H$ for $\alpha = \overline{\alpha}$, and*

$$\alpha_b \equiv \frac{A^2 + t^2\beta^2(4\beta^2 - 149\beta + 181) - 2At\beta(5+\beta)}{3A\beta^2}$$

and

$$A \equiv t\beta\left(\frac{27\sqrt{96\beta^5 + 513\beta^4 + 6\beta^3 - 351\beta^2 + 5352\beta - 5616} - 16\beta^3 - 1293\beta^2 + 6171\beta - 4430}{2}\right)^{1/3}.$$

Proposition 4.6 states that probabilistic price promotion is more profitable than fixed price promotion when (1) the fraction of trusting consumers are sufficiently high ($\beta > \tilde{\beta}$) and (2) free products are sufficiently appealing to consumers ($\alpha > \alpha_b$). If any of the two conditions is violated, then firm A either does not offer probabilistic price promotion at all or is trapped in the prisoner's dilemma in which offering partial probabilistic price promotion is even worse than not offering it at all. The detailed explanations are already introduced in the discussion of Figure 4.6 and therefore omitted here.

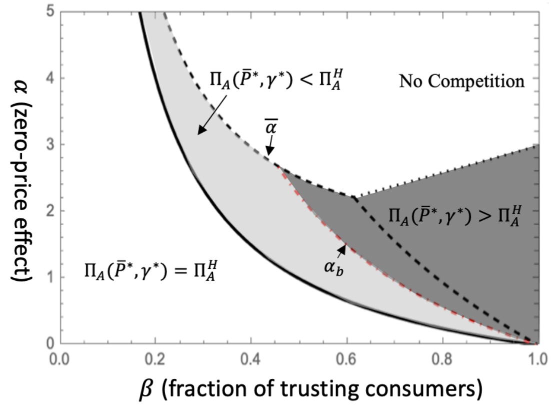
To provide a graphical reference and illustration of Proposition 4.6 I amend Figure 4.3 by including the threshold α_b; see Figure 4.7. As before the horizontal axis represents β and the vertical axis corresponds to α (with $t = 1$ as before). The red line represents α_b. Note that $\alpha_b < \overline{\alpha}$ when $\beta > \tilde{\beta} \approx 0.45$.

We have identified conditions under which probabilistic price promotion is more profitable than fixed price promotion. Next, I answer the question that compared with fixed price promotion, how much more profits can probabilistic price promotion generate?

Proposition 4.7. *The upper bound of firm A's optimal expected profit from probabilistic price promotion is 112.5% of that from fixed price promotion*: $\Pi_A(\overline{p}^*, \gamma^*) \leq 1.125\Pi_A^H$.

The upper bound of profit is achieved when $\beta > \frac{3}{4}$ and $\alpha = \alpha_u$ (which is defined in Proposition 4.5). Intuitively, firm A achieves the highest profit when the proportion of trusting consumers is large. However, a stronger zero-price effect is not necessarily beneficial to firm A. On one hand, a larger α empowers firm A

Figure 4.7 Comparing Firm A's Profit in the Main Model with That in the Benchmark



to charge a higher price that those less lucky consumers would eventually pay. On the other hand, the two firms engage in the price war that becomes fiercer as α increases. Therefore, firm A's profit is a constant balance between the two opposite effects, where the maximal profit is obtained when $\alpha = \alpha_u$.

4.5 Model Extensions

In this section, I examine the robustness of results with two extensions. The first extension considers the consequences if there are no skeptical consumers, while the second extension analyzes the implications if firm B offers a probabilistic price promotion similar to that of firm A.

4.5.1 Reassurance Through Government Certification

In the main model, I assumed that a fraction of $1-\beta$ consumers are skeptical about the announced probabilities of lottery outcomes. To eliminate consumers' doubts, firms may signal their commitment to the announced probabilities by acquiring government approval or certification to validate the promotion campaign. For example, Home Hardware launched a probabilistic price promotion and stated that the promotion was governed by Canadian law (Home Hardware, 2020). In the presence of government certification or oversight, skeptical consumers may no longer question the legitimacy of the promotion and are converted to trusting consumers.

In this extension, I consider the scenario where firm A can convert all skeptical consumers to trusting consumers (i.e., $\beta = 1$) by acquiring government certification that incurs a fixed cost c. The game sequence is almost identical to that in the main model, except that firm A now decides whether to acquire the certification in the beginning of the game (prior to both firms announcing their respective promotions). I use $\mathbb{S} = \{Y, N\}$ to denote firm A's certification strategy. When firm A acquires the certification, $\mathbb{S} = Y$, the expected profit functions are denoted by $\Pi_A^Y(\overline{p},\gamma)$ and $\Pi_B^Y(p_B)$, and when firm A does not acquire the certification, $\mathbb{S} = N$, the expected profit functions are $\Pi_A^N(\overline{p},\gamma) \equiv \Pi_A(\overline{p},\gamma)$ and $\Pi_B^N(p_B) \equiv \Pi_B(p_B)$, as in the main model.

With government certification, firm A's profit maximization problem is given by,

$$\begin{aligned} \max_{\overline{p},\gamma} \quad & \Pi_A^Y(\overline{p},\gamma) = (1-\gamma)\overline{p}\left(\frac{p_B - \overline{p}(1-\gamma) + t + \alpha\gamma}{2t}\right) - c \\ \text{s.t.} \quad & \overline{p} \in [0, p_B + t], \\ & \gamma \in [0,1]. \end{aligned} \tag{4.15}$$

Similarly, Firm B's profit maximization problem is given by,

$$\begin{aligned} \max_{p_B} \quad & \Pi_B^Y(p_B) = p_B\left(\frac{\overline{p}(1-\gamma) + t - p_B - \alpha\gamma}{2t}\right) \\ \text{s.t.} \quad & p_B \geq 0. \end{aligned} \tag{4.16}$$

Problems (4.15) and (4.16) are very similar to (4.13) and (4.14), except that β is now updated to 1 and firm A incurs a fixed cost c for acquiring the certification.

In the beginning of the game, firm A decides whether to acquire the certification and her overall profit maximization problem can be expressed as,

$$\Pi_A^{\mathbb{S}} = \max\{\Pi_A^N(\overline{p}^*,\gamma^*), \Pi_A^Y(\overline{p}^*,\gamma^*)\}, \tag{4.17}$$

where the optimal solutions $\overline{p}^*$ and γ^* implicitly depend on the certification strategy and are given by the solutions to (4.13) and (4.15).

Proposition 4.8 states that firm A will acquire the government certification if and only if it is sufficiently cheap.

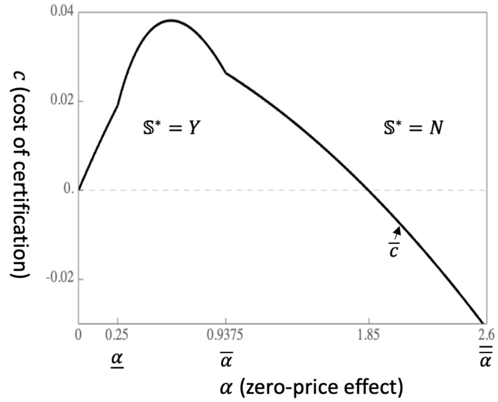
Proposition 4.8. *Signalling commitment is profitable*, $\mathbb{S}^* = Y$, *if and only if* $c < \overline{c}$, *where*

$$\overline{c} \equiv \begin{cases} \frac{\alpha(3t-\alpha)}{36t} & \text{if } \alpha < \underline{\alpha}, \\ \frac{162t^2(\beta-1)+t\alpha(171\beta-252)+\alpha^2\left(2\beta^2+35\beta-64\right)+(90t+32\alpha-2\alpha\beta)\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{324t\beta} & \text{if } \underline{\alpha} \leq \alpha < \overline{\alpha}, \\ \frac{(3t+\alpha)(1-\beta)(12t-2\alpha-3t\beta-4\alpha\beta)}{18t(2+\beta)^2} & \text{if } \overline{\alpha} \leq \alpha < \overline{\overline{\alpha}} \wedge \beta > \hat{\beta}. \end{cases}$$

For ease of exposition, I numerically illustrates Proposition 4.8 in Figure 4.8 with $\beta = 0.8$ and $t = 1$. The horizontal axis represents the boost in utility from zero-price effect (α), and the vertical axis is the fixed cost required for government certification (c).

How much firm A is willing to pay for government certification depends on how much extra revenue she can gain from converting skeptical consumers to trusting consumers. However, government certification can hurt firm A when zero-price effect is sufficiently strong. On the positive side, government certification converts skeptical consumers to trusting consumers who appreciate the probabilistic price promotion. As a result, firm A is able to attract more consumers to her store. On the negative side, government certification intensifies competition between the two firms such that prices would drop. In the absence of government certification, skeptical consumers function as a buffer between the two firms because they tend to purchase

Figure 4.8 Comparing Firm *A*'s Profits with and without Government Certification



from firm B so that firm B faces less pressure to engage in the price war. In the presence of government certification, all consumers become trusting consumers who appreciate firm A's probabilistic price promotion. Hence, firm B has to aggressively cut her price to compete for consumers, and the two firms engage in the price war. The stronger the zero-price effect is, the fiercer the price war is. Therefore, I observe that the maximal cost firm A is willing to pay for certification ($\overline{c}$) first increases and then decreases.

It is interesting to note that firm A may not want to acquire government certification, even if it is free. When $\alpha > 1.85$, I observe that $\overline{c} < 0$. In this region, zero-price effect is very strong such that consumers strongly favor firm A's probabilistic price promotion. In order to compete for these consumers, firm B has to tremendously lower her price, and the price war becomes very intense. In this case, firm A would rather not have the certification, even if it is free, so that the skeptical consumers can be used as a buffer against the price war.

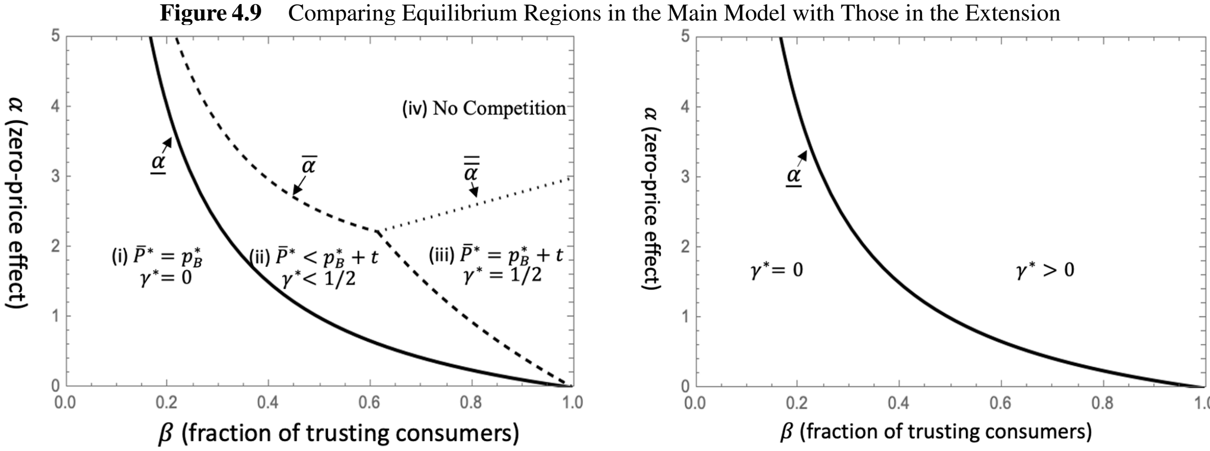
4.5.2 Symmetric Probabilistic Price Promotions

In the main model I assumed that only firm A implements probabilistic price promotion and firm B implements fixed price promotion. In this extension, I relax this assumption and consider the case where firm B can also implement probabilistic price promotion if she wishes. The game sequence here is almost identical to the one in the main model, except that instead of deciding p_B, firm B now simultaneously decides her probability of offering free products γ_B and her alternate price $\overline{p_B}$.

When both firms decide to offer probabilistic price promotion, I assume that consumers can at most visit each firm once. They cannot continuously switching between the two firms hoping for the free products. If firm A's in-store consumers did not receive free products and switch to firm B to draw the lottery again, I would expect the same action from firm B's in-store consumers as the two firms are symmetric. As a result, one firm's incoming consumers and exiting consumers cancel out such that consumers effectively do not switch. Then, firm A's profit maximization problem is,

$$\max_{\overline{p},\gamma} \quad \Pi_A(\overline{p},\gamma) = (1-\gamma)\overline{p}\left(\beta(\frac{\overline{p_B}\,(1-\gamma_B) - \overline{p}(1-\gamma) + \alpha\,(\gamma-\gamma_B) + t}{2t}) + (1-\beta)(\frac{\overline{p_B} - \overline{p} + t}{2t})\right)$$

Figure 4.9 Comparing Equilibrium Regions in the Main Model with Those in the Extension



(a) Equilibrium Regions in the Main Model

(b) Equilibrium Regions in the Extension

$$
\begin{aligned}
\text{s.t.}\quad & \overline{p} \geq 0, \\
& \gamma \in [0,1].
\end{aligned} \tag{4.18}
$$

Similarly, firm B's profit maximization problem is,

$$
\begin{aligned}
\max_{\overline{p_B},\gamma_B}\quad & \Pi_B(\overline{p_B},\gamma_B) = (1-\gamma_B)\overline{p_B}\left(\beta\Big(\frac{\overline{p}(1-\gamma)-\overline{p_B}\,(1-\gamma_B)+\alpha\,(\gamma_B-\gamma)+t}{2t}\Big)+(1-\beta)\Big(\frac{\overline{p}-\overline{p_B}+t}{2t}\Big)\right) \\
\text{s.t.}\quad & \overline{p_B} \geq 0, \\
& \gamma_B \in [0,1].
\end{aligned} \tag{4.19}
$$

Proposition 4.9 presents the equilibria in this extension.

Proposition 4.9. *The game has two symmetric equilibria as follows*:

(i) *when* $\alpha < \underline{\alpha}$, *it is optimal to not offer probabilistic price promotion*:

$$
\begin{aligned}
\gamma^* = \gamma_B^* = 0, \\
\overline{p}^* = \overline{p_B}^* = t;
\end{aligned}
$$

(ii) *when* $\alpha \geq \underline{\alpha}$, *it is optimal to offer probabilistic price promotion*:

$$
\begin{aligned}
\gamma^* = \gamma_B^* &= \frac{\alpha(1+\beta) - \sqrt{\alpha(1-\beta)(4t+\alpha-\alpha\beta)}}{2\alpha\beta}, \\
\overline{p}^* = \overline{p_B}^* &= \frac{\alpha(\beta-1) + \sqrt{\alpha(1-\beta)(4t+\alpha-\alpha\beta)}}{2(1-\beta)}.
\end{aligned}
$$

In Figure 4.9, I compare the equilibrium regions in the main model with those in this extension, with $t = 1$. As before, the horizontal axis is the fraction of trusting consumers β, and the vertical axis represents the boost in utility from zero-price effect α.

We observe that my results are robust: Firm A would offer probabilistic price promotion when zero-price effect is sufficiently large ($\alpha \geq \underline{\alpha}$), regardless of firm B's promotion strategy. That is, offering probabilistic price promotion is a dominant strategy when consumers appreciate and feel excited about free products. Because the two firms now appear symmetric, consumers would buy from the most efficient firm—those consumers located in $[0, 0.5]$ would buy from firm *A*, and the other consumers would buy from firm *B*. This purchasing pattern resembles that in the standard Hotelling model. In addition, regions (ii), (iii) and (iv) in the left panel merge into one region ($\gamma^* > 0$ in the right panel) because of the symmetric equilibrium.

4.6 Concluding Remarks

This paper presents a model to study the optimal design of probabilistic price promotion in the presence of behavioral biases, specifically the cognitive zero-price effect and the general consumer skepticism regarding lottery-based promotions. There are two firms (*A* and *B*) selling a homogeneous product to consumers. Firm *B* offers the conventional fixed price promotion where the discount/price is fixed. On the other hand, firm *A* implements the novel probabilistic price promotion in which the discount is *ex ante* uncertain to consumers and will be determined by drawing a lottery at firm *A*'s store. Upon revealing the final discount, firm *A*'s in-store consumers are not obligated to proceed with purchasing and can switch to firm *B* if they wish. There are two types of consumers, trusting ones, who believe the lottery will be drawn according to the claimed probabilities, and skeptical ones, who assume the lottery is a mere gimmick used to fool consumers and and the uncertain price will always turn out to be the highest one.

For the sake of clarity, I have illustrated the model in the context of brick-and-mortar stores. However, this model can be applied to online retail settings as well, where the Hotelling line would represent consumers' heterogeneous preferences toward the two firms. Online stores operating probabilistic price promotion typically require consumers to create an account or register personal information (e.g., names and contact information) before drawing the lottery. After consumers have invested time and efforts in creating accounts, they are more reluctant to switch to another firm, where they may have to go through the process again.

4.6.1 Results and Managerial Insights

This paper finds that a simple lottery, wherein consumers either receive the product for free or are offered to pay the original list price, is more profitable than a complex lottery that includes many different promotional prices. In addition, firms should only offer probabilistic price promotion when the zero-price effect is larger than a threshold. The larger the fraction of trusting consumers are, the smaller the threshold is. These findings provide important managerial insights that firms with excellent reputation, which results in a large fraction of trusting consumers, should implement probabilistic price promotion, and a simple promotion is better than a complex one. By contrast, firms with mediocre or bad reputation are better off by implementing the conventional fixed price promotion to eliminate consumer skepticism. This paper also finds that a larger zero-price effect would incentivize firms to set a higher chance of winning free products. However, the price, which consumers receive if they lose the lottery, can either increase or decrease in zero-price effect. The non-monotone change in price is explained by a price war between the two competing firms.

4.6.2 Future Research Direction

Probabilistic price promotion is quickly gaining popularity among retailers, and this paper aims to better understand and provide insights into the optimal design of such promotion. I have made several assumptions to keep the mathematical model tractable, and relaxing these assumptions could result in interesting directions for future research. First, I have assumed that the fraction of trusting consumers are exogenous. Future research can consider the case where the fraction of trusting consumers are endogenous, and firms can choose how much to invest in building a good reputation which determines the fraction of trusting consumers. Second, it is possible that other behavioral biases, in addition to zero-price effect, also contribute to the popularity of probabilistic price promotion. For example, consumers may be over-confident and think they have a better chance winning the lottery than they really have. It might be fruitful for future research to explore these additional behavioral aspects of designing probabilistic price promotions.

Chapter 5

5 Conclusion

In this dissertation, I delved into the intricate dynamics of market behavior, examining the strategic interplay between consumers and firms across online and offline marketplaces. Each of the three essays presents a unique perspective on how to better consumer rights and bolster company profits amid the prevailing market complexities.

5.1 Summaries of Essays and Contributions

Chapter 2 examines the counterfeit issue in online marketplaces, shedding light on the complexity of fake credence goods. By exploring a duopoly model of genuine sellers and counterfeiters, this essay finds that in equilibrium, the authentic seller does not acquire fake reviews, while the counterfeiter may do so to mislead customers. Moreover, the number of fake reviews is decreasing in the fraction of savvy consumers, suggesting that online platforms can combat fake reviews by, for instance, clearly highlighting that badges are product-dependent. I also find that having the option to acquire fake reviews may benefit both sellers but always hurts consumers, emphasizing the need for regulation to protect consumers.

Chapter 3 studies the optimal pricing and wage decisions of food delivery platforms, in the presence of "tip baiting" where consumers ex ante promise a large tip to expedite the delivery but intentionally reduce the tip to zero after delivery is completed. I find that regardless of the fraction of selfish consumers who may engage in tip baiting, both platforms prefer the adjustable tipping policy over the non-adjustable one. This result has managerial implications that platforms with the non-adjustable policy can further enhance their profits by switching to the adjustable policy. Further, I show that platforms only have incentives to keep the fraction of selfish consumers sufficiently low, but not lower. However, the existence of selfish consumers always hurts those honest fair consumers and workers. Hence, I advocate that third-party regulators need to intervene to safeguard fair consumers' and workers' rights, as platforms may lack the incentive to do so beyond a certain point.

Chapter 4 focuses on the optimal design of probabilistic promotions, an increasingly popular practice that is under-explored in the literature. The study sheds light on the optimal promotion parameters regarding the number of promotional prices, the chance of winning, and the list price. Interestingly, I find that in the presence of zero price effect, a simple lottery, wherein consumers either receive the product for free or are offered to pay the original list price, is more profitable than a complex lottery with many promotional prices. Moreover, my results provide managerial implications that firms with excellent reputations should offer the simple lottery to take advantage of the zero-price effect. Conversely, the less established firms might be better off by sticking with the traditional fixed price promotions to eliminate any uncertainty in the final price.

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Appendix A1. Other Combinations of Product Characteristics

In the paper, I have focused on the case where the focal product is credence goods and deceptive in nature. To better understand the uniqueness of deceptive credence goods, I now explore other possible combinations of product characteristics in Appendix A1. Specifically, I examine three scenarios where the product is non-credence goods and deceptive, or credence goods and non-deceptive, or non-credence goods and non-deceptive. For notational convenience, I use the subscript s_j, $j \in \{1, 2, 3\}$, to denote solutions in the three cases. Proofs of Lemmas and Propositions in the main paper and in the Appendix A1 are presented in the Appendix A2.

Case 1 Non-credence Goods and Deceptive in Nature

By the definition of non-credence goods, consumers are able to assess product quality after consumption. Thus, for the consumers who had purchased from the (authentic) incumbent seller before the first stage, they would be able to confirm that the product is indeed authentic and leave positive product reviews correspondingly (In reality, consumers may leave negative reviews even for authentic products due to many reasons. For example, the package might be damaged during the shipping process, or consumers later realized that the product, even of high quality, was a mismatch. However, these possible reasons are beyond the scope of this paper, and I assume that consumers leave positive (or negative) reviews if the product is authentic (or fake)). Consequently, the fraction of positive reviews is now 1, instead of $0 < m < 1$ as in the main model. The rest of the model setup is identical to that in the main model.

Due to the exclusive positive reviews, the product listing will receive the badge for sure. As a result, novice consumers will only purchase from the default seller, as they do in the main model. For savvy consumers, although observing the badge does not update their belief about a seller's authenticity, observing a perfect product rating does. In particular, savvy consumers can infer that at least one seller (i.e., the incumbent seller) is authentic, otherwise the product rating would have not been perfect. However, savvy consumers cannot tell who the incumbent seller was, given that the product review section is now shared by both sellers (as explained in the discussion of Figure 1). Therefore, after observing the perfect product rating, savvy consumers' posterior belief about the authenticity of seller i becomes

$$\tilde{\phi} \coloneqq \Pr(seller\ i\ is\ real \mid at\ least\ one\ seller\ is\ real) = \frac{\phi}{\phi + \phi(1-\phi)}.$$

It is noteworthy that the posterior belief ($\tilde{\phi}$) is different from that in the main model because of the perfect rating. From savvy consumers' perspective, it is possible that both sellers are counterfeiters in the main model. By contrast, the possibility that both sellers are counterfeiters is eliminated due to the perfect product rating in this case, which leads to a different posterior belief.

Similar to problem (1), seller i's profit maximization problem in the second stage can be expressed as follows:

$$\max_{p_i \geq 0} \ \Pi_{i,s1}^{B}(p_i) = p_i\left(\alpha\frac{\tau}{2}\int_{v:\tilde{\phi}qv-p_i\geq 0} dv + (1-\alpha)\frac{\tau}{2}\int_{v:qv-p_i\geq 0} dv\right). \tag{2.5}$$

Next, I present the optimal solution to problem (2.5) in Lemma 2.3.

Lemma 2.3. *When the product is non-credence goods and deceptive in nature, the optimal pooling price is* ${p_{s1}^{B}}^{*} = \frac{q}{2(1+\alpha(1-\phi))}$, *which is higher than the optimal pooling price* ${p^{B}}^{*}$ *in the main model.*

Observing a perfect rating indicates that at least one of the sellers is authentic, resulting in a greater posterior belief $\tilde{\phi}$ than that in the main model. Therefore, savvy consumers are willing to pay a higher price for the product, which leads to a higher pooling price than that in the main model.

Having analyzed the optimal pooling price, I now examine sellers' fake review decisions in the first stage. Similar to problem (3), seller i's expected profit maximization problem in the first stage can be expressed as:

$$\max_{f_i \geq 0} \ \mathbb{E}[\Pi_{i,s1}(f_i)] = \frac{R+f_i+f_{-i}}{R+f_i+f_{-i}}\Pi_{i,s1}^{B}\left({p_{s1}^{B}}^{*}\right) - (\beta+\gamma_i)f_i.$$

Lemma 2.4 states that neither seller would acquire fake reviews in this case. The intuition here is straightforward: Since the product will receive badge for sure, there is no scope for fake reviews.

Lemma 2.4. *When the product is non-credence goods and deceptive in nature,* ${f_{A,s1}}^{*} = {f_{C,s1}}^{*} = 0$.

Case 2 Credence Goods and Non-deceptive in Nature

By the definition of non-deceptive counterfeits, consumers are able to ex ante distinguish authentic products from fake ones. In this case, the authentic seller is essentially a monopoly, as no consumers would knowingly buy fake supplements/vitamins. Conditional on the product receives badge, seller A's profit maximization problem in the second stage can be expressed as follows:

$$\max_{p_A \geq 0} \ \Pi_{A,s2}^{B}(p_A) = \tau\, p_A \int_{v:qv-p_A\geq 0} dv. \tag{2.6}$$

Similarly, conditional on the product does not receive badge, seller A's profit maximization problem in the second stage can be expressed as follows:

$$\max_{p_A \geq 0} \ \Pi_{A,s2}^{NB}(p_A) = p_A \int_{v:qv-p_A\geq 0} dv. \tag{2.7}$$

It is obvious that the optimal solutions to problem (2.6) and (2.7) are the same, as summarized in Lemma 2.5.

Lemma 2.5. *When the product is credence goods and non-deceptive in nature, the optimal price is*

$p_{s2}{}^* = \frac{q}{2}$, *which is higher than the optimal pooling price* p^{B^*} *in the main model.*

When the product is non-deceptive in nature, consumers are certain about its quality, enabling seller A to charge a high price that perfectly matches her high-quality product. Regarding seller A's fake review decision, her expected profit maximization problem in the first stage can be expressed as:

$$\max_{f_A \geq 0} \ \mathbb{E}[\Pi_{A,s2}(f_A)] = \frac{R*m+f_A}{R+f_A}\Pi_{A,s2}^{B}(p_{s2}{}^*) + \left(1-\frac{R*m+f_A}{R+f_A}\right)\Pi_{A,s2}^{NB}(p_{s2}{}^*) - (\beta+\gamma_A)f_A.$$

Lemma 2.6 presents seller A's optimal quantity of fake reviews when the product is non-deceptive credence goods.

Lemma 2.6. *There exists a unique threshold* $\hat{\tau}$ *such that the optimal units of fake reviews to acquire are*

$$f_{A,s2}{}^* = \begin{cases} \dfrac{\sqrt{q\,R(\gamma_A+\beta)(1-m)(\tau-1)} - 2R(\gamma_A+\beta)}{2(\gamma_A+\beta)}, & if\ \tau > \hat{\tau}, \\ 0, & if\ \tau \leq \hat{\tau}, \end{cases}$$

where $\hat{\tau} \equiv \frac{(1-m)q+4R(\gamma_A+\beta)}{(1-m)q}$.

Interestingly, when the product is non-deceptive credence goods, the authentic may acquire fake reviews, as opposed to in the main model where seller A would never acquire any fake reviews. The stark contradiction stems from seller A's free-riding behavior. In the main model when the product is deceptive, consumers cannot distinguish the sellers so that the authentic seller and the counterfeiter would equally split the market. Moreover, because the authentic seller bears additional reputation cost of acquiring fake reviews, it is the counterfeiter who would acquire fake reviews, if any, in equilibrium. By contrast, when the product is non-deceptive as in this case, the counterfeiter no longer has incentives to acquire fake reviews because consumers know its product is fake. Consequently, the authentic seller has to rely on its own for fake reviews.

The product badge now loses its appeal to signal product quality, which is already ex ante known to consumers, and the only benefit of receiving the badge comes from the market expansion effect. Therefore, seller A would only acquire fake reviews to increase the probability of receiving the badge, if it can help attract significantly more consumers (i.e., $\tau > \hat{\tau}$).

Case 3 Non-credence Goods and Non-deceptive in Nature

As discussed in the previous Case 2, seller A is effectively a monopoly when the focal product is non-deceptive. Hence, seller A's optimal price is the same as stated in Lemma 2.5 (i.e., $p_{s3}{}^* = p_{s2}{}^* = \frac{q}{2}$). Furthermore, when the product is non-credence goods, the fraction of positive product reviews would be 1, as explained in case 1. Therefore, the product would receive badge with certainty, and seller A does not acquire any fake reviews (i.e., $f_{A,s3}{}^* = f_{A,s1}{}^* = 0$).

Appendix A2. Proofs of Results

Proof of Lemma 2.1:

Let p^B denote the pooling price that sellers charge when the product receives badge. Knowing that seller C would follow seller A's price, I can express seller A's profit maximization problem as follows:

$$\max_{p^B \geq 0} \; \Pi_A^B(p^B) = p^B \left(\alpha \frac{\tau}{2} \int_{v:\phi q v - p^B \geq 0} dv + (1-\alpha) \frac{\tau}{2} \int_{v:qv - p^B \geq 0} dv \right).$$

$\frac{\partial^2 \Pi_A^B(p^B)}{\partial {p^B}^2} = \frac{-\tau(\alpha(1-\phi)+\phi)}{q\phi} < 0 \implies \Pi_A^B(p^B)$ is strictly concave in p^B. Solve

$$\frac{\partial \Pi_A^B(p^B)}{\partial p^B} = \frac{\tau(2p^B(-\alpha-\phi+\alpha\phi)+q\phi)}{2q\phi} = 0,$$

I have that ${p^B}^* = \frac{q\,\phi}{2(\alpha+(1-\alpha)\phi)} > 0$, which is an interior solution. Moreover, the binding solution $(p^B = 0)$ can never be optimal, as it generates zero profit for seller A, who can easily deviate by charging a positive price, such as ${p^B}^* = \frac{q\,\phi}{2(\alpha+(1-\alpha)\phi)}$, to obtain positive profit. Hence, I conclude that the unique pooling price is ${p^B}^* = \frac{q\,\phi}{2(\alpha+(1-\alpha)\phi)}$. □

Proof of Lemma 2.2:

Let p^{NB} denote the pooling price that sellers charge when the product does not receive badge. Knowing that seller C would follow seller A's price, I can express seller A's profit maximization problem as follows:

$$\max_{p^{NB} \geq 0} \; \Pi_A^{NB}(p^{NB}) = p^{NB} \frac{1}{2} \int_{v:\phi q v - p^{NB} \geq 0} dv.$$

$\frac{\partial^2 \Pi_A^{NB}(p^{NB})}{\partial {p^{NB}}^2} = -\frac{1}{q\phi} < 0 \implies \Pi_A^{NB}(p^{NB})$ is strictly concave in p^{NB}. Solve

$$\frac{\partial \Pi_A^{NB}(p^{NB})}{\partial p^{NB}} = \frac{q\phi - 2p^{NB}}{2q\phi} = 0,$$

I have that ${p^{NB}}^* = \frac{q\,\phi}{2} > 0$, which is an interior solution. Moreover, the binding solution $(p^{NB} = 0)$ can never be optimal, as it generates zero profit for seller A, who can easily deviate by charging a positive price, such as ${p^{NB}}^* = \frac{q\,\phi}{2}$, to obtain positive profit. Hence, I conclude that the unique pooling price is ${p^{NB}}^* = \frac{q\,\phi}{2}$.

□

Proof of Proposition 2.1:

Note that seller C generates the same expected revenue as seller A does, because they charge the same price. That is, $\Pi_C^B({p^B}^*) = \Pi_A^B({p^B}^*)$ and $\Pi_C^{NB}({p^{NB}}^*) = \Pi_A^{NB}({p^{NB}}^*)$. Regarding fake reviews decision, seller A's expected profit maximization problem can be expressed as:

$$\max_{f_A \geq 0} \ \mathbb{E}[\Pi_A(f_A)] = \frac{R*m+f_A+f_C}{R+f_A+f_C}\Pi_A^B\left(p^{B^*}\right) + \left(1 - \frac{R*m+f_A+f_C}{R+f_A+f_C}\right)\Pi_A^{NB}\left(p^{NB^*}\right) - (\beta+\gamma_A)f_A \qquad \text{(EC. 2.1)}$$

Similarly, seller C's expected profit maximization problem can be expressed as:

$$\max_{f_C \geq 0} \ \mathbb{E}[\Pi_C(f_C)] = \frac{R*m+f_A+f_C}{R+f_A+f_C}\Pi_C^B\left(p^{B^*}\right) + \left(1 - \frac{R*m+f_A+f_C}{R+f_A+f_C}\right)\Pi_C^{NB}\left(p^{NB^*}\right) - \beta f_C. \qquad \text{(EC. 2.2)}$$

I apply the Karush-Kuhn-Tucker (KKT) conditions to solve EC.2.1 and EC.2.2. $\frac{\partial^2\mathbb{E}[\Pi_A(f_A)]}{\partial {f_A}^2} = \frac{(1-m)q\phi R(\tau-(\phi+(1-\phi)\alpha))}{4(f_A+f_C+R)^3(\alpha(\phi-1)-\phi)} < 0 \Longrightarrow \mathbb{E}[\Pi_A(f_A)]$ is strictly concave in f_A. And $\frac{\partial^2\mathbb{E}[\Pi_C(f_C)]}{\partial {f_C}^2} = \frac{(1-m)q\phi R(\tau-(\phi+(1-\phi)\alpha))}{4(f_A+f_C+R)^3(\alpha(\phi-1)-\phi)} < 0 \Longrightarrow \mathbb{E}[\Pi_C(f_C)]$ is strictly concave in f_C. The KKT solutions to EC.2.1 are given by the standard equations:

$$\begin{aligned} \frac{d\mathbb{E}[\Pi_A(f_A)]}{df_A} - \lambda\left(\frac{d(-f_A)}{df_A}\right) &= 0, \\ \lambda &\geq 0, \qquad \text{(EC. 2.3)} \\ \lambda(-f_A) &= 0. \end{aligned}$$

Similarly, the KKT solutions to EC.2.2 are given by the standard equations:

$$\begin{aligned} \frac{d\mathbb{E}[\Pi_C(f_C)]}{df_C} - \mu\left(\frac{d(-f_C)}{df_C}\right) &= 0, \\ \mu &\geq 0, \qquad \text{(EC. 2.4)} \\ \mu(-f_C) &= 0. \end{aligned}$$

Solving EC.2.3 and EC.2.4 leads to Proposition 2.1:

$${f_A}^* = 0,$$

$${f_C}^* = \begin{cases} \dfrac{\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))} - 4R\beta(\phi+(1-\phi)\alpha)}{4\beta(\alpha+(1-\alpha)\phi)}, & if\ \beta < \overline{\beta}, \\ 0, & if\ \beta \geq \overline{\beta}, \end{cases}$$

$where\ \overline{\beta} \equiv \frac{q\phi(1-m)(\tau-(\phi+(1-\phi)\alpha))}{8R(\alpha+(1-\alpha)\phi)}$. □

Proof of Proposition 2.2:

I first prove Proposition 2.2a. I can re-arrange the threshold of acquiring fake reviews in terms of α. That is, $\beta < \overline{\beta} \Leftrightarrow \alpha < \alpha_1 \equiv \frac{\phi(q(\tau(1-m)-\phi(1-m))-8R\beta)}{(1-\phi)(8R\beta+q\phi(1-m))}$. Moreover, I can easily obtain that

$$\alpha_1 < 1 \Leftrightarrow \tau < \overline{\tau} \equiv \frac{8R\beta+q\phi(1-m)}{q\phi(1-m)}.$$

Next, I consider two cases based on the value of α.

Case a: When $\tau \geq \overline{\tau}$, $\alpha < 1 \leq \alpha_1$ always holds. Hence, seller C acquires positive units of fake reviews, and ${f_C}^* = \frac{\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}-4R\beta(\phi+(1-\phi)\alpha)}{4\beta(\alpha+(1-\alpha)\phi)}$.

Therefore, $\frac{\partial {f_C}^*}{\partial \alpha} = \frac{(1-m)qR\tau(1-\phi)\phi}{-4(\alpha+\phi-\alpha\phi)\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}} < 0$.

Case b: When $\tau < \overline{\tau}$, I have that $\alpha_1 < 1$. And

$$
{f_C}^* = \begin{cases} \frac{\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}-4R\beta(\phi+(1-\phi)\alpha)}{4\beta(\alpha+(1-\alpha)\phi)}, & if\ \alpha < \alpha_1, \\ 0, & if\ \alpha \geq \alpha_1, \end{cases}
$$

If $\alpha < \alpha_1$, I have shown that $\frac{\partial {f_C}^*}{\partial \alpha} = \frac{(1-m)qR\tau(1-\phi)\phi}{-4(\alpha+\phi-\alpha\phi)\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}} < 0$.

If $\alpha \geq \alpha_1$, then seller C does not acquire fake reviews and $\frac{\partial {f_C}^*}{\partial \alpha} = 0$.

Define $\overline{\alpha} \equiv \begin{cases} \alpha_1, & if\ \tau < \overline{\tau} \\ 1, & if\ \tau \geq \overline{\tau} \end{cases}$, then I have proven Proposition 2.2a.

Next, I prove Proposition 2.2b. Similar to proof of Proposition 2.2a, I first re-arrange the threshold of acquiring fake reviews with respect to ϕ. Solve

$$
\beta = \overline{\beta} \Leftrightarrow \frac{\big(q(m(1-\alpha)+\alpha-1)\big)\phi^2 + (-q\alpha + mq\alpha - 8R\beta + 8R\alpha\beta + q\tau - mq\tau)\phi - 8R\alpha\beta}{8R(\alpha+\phi-\alpha\phi)} = 0
$$

with respect to ϕ, I obtain two solutions. Let

$$
\phi_1 \equiv [\phi(8R\beta(\alpha-1) + q(\tau(1-m)+\alpha(m-1)) - \sqrt{(8R\beta(1-\alpha)+q(\alpha-m\alpha-\tau+m\tau))^2 - 32q\alpha R\beta(1-m-\alpha+m\alpha)})]/[2q(1-m)(1-\alpha)]
$$

denote the smaller solution, and

$$
\phi_2 \equiv [\phi(8R\beta(\alpha-1) + q\big(\tau(1-m)+\alpha(m-1)\big) + \sqrt{(8R\beta(1-\alpha)+q(\alpha-m\alpha-\tau+m\tau))^2 - 32q\alpha R\beta(1-m-\alpha+m\alpha)})]/[2q(1-m)(1-\alpha)]
$$

denote the larger solution.

I observe that in the term $\big(q(m(1-\alpha)+\alpha-1)\big)\phi^2$, $m(1-\alpha)+\alpha$ is a convex combination between m and 1. Therefore, the coefficient of ϕ^2 is negative, and I conclude that $\beta < \overline{\beta} \Leftrightarrow \phi_1 < \phi < \phi_2$.

Next, I consider two cases based on the value of ϕ.

Case 1: when $\phi \leq \phi_1$ or $\phi \geq \phi_2$, then ${f_C}^* = 0$ and $\frac{\partial {f_C}^*}{\partial \phi} = 0$.

Case 2: when $\phi_1 < \phi < \phi_2$, ${f_C}^* = \frac{\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}-4R\beta(\phi+(1-\phi)\alpha)}{4\beta(\alpha+(1-\alpha)\phi)}$.

Solve $\frac{\partial f_C{}^*}{\partial \phi} = \frac{-(1-m)qR[(1-\alpha)^2\phi^2+(2\alpha-2\alpha^2)\phi+\alpha^2-\alpha\tau]}{4(\alpha+\phi-\alpha\phi)\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}} = 0$ with respect to ϕ, I obtain two solutions. Let $\phi_3 \equiv \frac{-\sqrt{\alpha\tau}-\alpha}{1-\alpha}$ denote the smaller solution, and $\phi_4 \equiv \frac{\sqrt{\alpha\tau}-\alpha}{1-\alpha}$ denote the larger solution. It is obvious that $\phi_3 < 0$, and I conclude that $\frac{\partial f_C{}^*}{\partial \phi} > 0 \Leftrightarrow \phi < \phi_4$.

Then, I define $\underline{\phi} \equiv \phi_1$ and $\overline{\phi} \equiv \phi_4$. Regarding Proposition 2.2b, I have proven the direction that $\underline{\phi} < \phi < \overline{\phi} \Rightarrow \frac{\partial f_C{}^*}{\partial \phi} > 0$, as shown above. Next, I prove the other direction that $\frac{\partial f_C{}^*}{\partial \phi} > 0 \Rightarrow \underline{\phi} < \phi < \overline{\phi}$. I use proof by contrapositive here so that proving $\frac{\partial f_C{}^*}{\partial \phi} > 0 \Rightarrow \underline{\phi} < \phi < \overline{\phi}$ is equivalent to proving $\left(\phi \leq \underline{\phi} \vee \phi \geq \overline{\phi}\right) \Rightarrow \frac{\partial f_C{}^*}{\partial \phi} \leq 0$. And I have shown this above, as (1) when $\phi \leq \underline{\phi} \equiv \phi_1$, I have $f_C{}^* = 0$ and $\frac{\partial f_C{}^*}{\partial \phi} = 0$, and (2) when $\phi \geq \overline{\phi} \equiv \phi_4$, I have $\frac{\partial f_C{}^*}{\partial \phi} \leq 0$. Since I have proven both directions, I conclude proof of Proposition 2.2b. □

Proof of Proposition 2.3:

Proof of Proposition 2.3a: since $p^{B^*} = \frac{q\,\phi}{2(\alpha+(1-\alpha)\phi)}$, I have $\frac{\partial p^{B^*}}{\partial \alpha} = \frac{-q\phi(1-\phi)}{2(-\alpha-\phi+\alpha\phi)^2} < 0$.

Proof of Proposition 2.3b: since $p^{NB^*} = \frac{q\,\phi}{2}$, I have $\frac{\partial p^{NB^*}}{\partial \phi} = \frac{q}{2} > 0$. □

Proof of Proposition 2.4:

I first prove Proposition 2.4a. Consider two cases depending on the value of β.

Case 1: when $\beta \geq \overline{\beta}$, $f_A{}^* = f_C{}^* = 0$. Hence, seller A and seller C have the same expected profits, and $\mathbb{E}[\Pi_i(f_i{}^*)] = p\Pi_i^B\left(p^{B^*}\right) + (1-p)\Pi_i^{NB}\left(p^{NB^*}\right),\ i \in \{A, C\}$. Because $\Pi_i^B\left(p^{B^*}\right)$ and $\Pi_i^{NB}\left(p^{NB^*}\right)$ do not contain β when sellers do not acquire fake reviews, I conclude that $\frac{\partial \mathbb{E}[\Pi_A(f_A{}^*)]}{\partial \beta} = \frac{\partial \mathbb{E}[\Pi_C(f_C{}^*)]}{\partial \beta} = 0$.

Case 2: when $\beta < \overline{\beta}$, $f_A{}^* = 0$ and $f_C{}^* = \frac{\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}-4R\beta(\phi+(1-\phi)\alpha)}{4\beta(\alpha+(1-\alpha)\phi)}$.

$\mathbb{E}[\Pi_A(f_A{}^*)] = (\frac{R*m+f_C{}^*}{R+f_C{}^*})\Pi_A^B\left(p^{B^*}\right) + \left(1 - \frac{R*m+f_C{}^*}{R+f_C{}^*}\right)\Pi_A^{NB}\left(p^{NB^*}\right)$, and I have $\frac{\partial \mathbb{E}[\Pi_A(f_A{}^*)]}{\partial \beta} = \frac{(1-m)^2q^2R^2\beta\phi^2(-\alpha-\phi+\alpha\phi)(-\alpha+\tau-\phi+\alpha\phi)^2}{4\sqrt{2}*\sqrt{q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}^3} < 0$. Regarding seller C's expected profit, I have $\mathbb{E}[\Pi_C(f_C{}^*)] = \left(\frac{R*m+f_C{}^*}{R+f_C{}^*}\right)\Pi_C^B\left(p^{B^*}\right) + \left(1 - \frac{R*m+f_C{}^*}{R+f_C{}^*}\right)\Pi_C^{NB}\left(p^{NB^*}\right) - \beta\, f_C{}^* = \mathbb{E}[\Pi_A(f_A{}^*)] - \beta\, f_C{}^*$, and $\frac{\partial \mathbb{E}[\Pi_C(f_C{}^*)]}{\partial \beta} = \frac{\partial \mathbb{E}[\Pi_A(f_A{}^*)]}{\partial \beta} - f_C{}^*$. Because $\frac{\partial \mathbb{E}[\Pi_A(f_A{}^*)]}{\partial \beta} < 0$ and $f_C{}^* > 0$, I conclude that $\frac{\partial \mathbb{E}[\Pi_C(f_C{}^*)]}{\partial \beta} < 0$.

Since I have shown that $\frac{\partial \mathbb{E}[\Pi_A({f_A}^*)]}{\partial \beta} \leq 0$ and $\frac{\partial \mathbb{E}[\Pi_C({f_C}^*)]}{\partial \beta} \leq 0$ in both cases, I conclude that $\mathbb{E}[\Pi_A({f_A}^*)]$ and $\mathbb{E}[\Pi_C({f_C}^*)]$ weakly decrease in β.

Next, I prove Proposition 2.4b. Again, I consider the above two cases based on the value of β.

Case 1: when $\beta \geq \overline{\beta}$, ${f_A}^* = {f_C}^* = 0$. Hence, seller A and seller C have the same expected profits, and the gap between their profits is zero. Therefore, $\frac{\partial(\mathbb{E}[\Pi_A({f_A}^*)] - \mathbb{E}[\Pi_C({f_C}^*)])}{\partial \beta} = 0$.

Case 2: when $\beta < \overline{\beta}$, ${f_A}^* = 0$ and ${f_C}^* = \frac{\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))} - 4R\beta(\phi+(1-\phi)\alpha)}{4\beta(\alpha+(1-\alpha)\phi)}$.

The gap between sellers' expected profits is $\mathbb{E}[\Pi_A({f_A}^*)] - \mathbb{E}[\Pi_C({f_C}^*)] = \beta {f_C}^*$. Hence, I have

$$\begin{aligned}
\frac{\partial(\mathbb{E}[\Pi_A({f_A}^*)] - \mathbb{E}[\Pi_C({f_C}^*)])}{\partial \beta} &= \frac{\partial(\beta\, {f_C}^*)}{\partial \beta} \\
&= {f_C}^* + \beta * \frac{(-1+m)qR\phi(-\alpha+\tau-\phi+\alpha\phi)}{4\beta\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}} > 0 \\
&\Leftrightarrow \beta < \frac{q\phi(1-m)(\tau-(\phi+(1-\phi)\alpha))}{32R(\alpha+(1-\alpha)\phi)} \\
&\Leftrightarrow \beta < \overline{\beta}/4.
\end{aligned}$$

So far, I have proven the direction that $\beta < \overline{\beta}/4 \Rightarrow \frac{\partial(\mathbb{E}[\Pi_A({f_A}^*)] - \mathbb{E}[\Pi_C({f_C}^*)])}{\partial \beta} > 0$. Next, I prove the other direction that $\frac{\partial(\mathbb{E}[\Pi_A({f_A}^*)] - \mathbb{E}[\Pi_C({f_C}^*)])}{\partial \beta} > 0 \Rightarrow \beta < \overline{\beta}/4$. I use proof by contrapositive here so that proving $\frac{\partial(\mathbb{E}[\Pi_A({f_A}^*)] - \mathbb{E}[\Pi_C({f_C}^*)])}{\partial \beta} > 0 \Rightarrow \beta < \overline{\beta}/4$ is equivalent to proving $\beta \geq \overline{\beta}/4 \Rightarrow \frac{\partial(\mathbb{E}[\Pi_A({f_A}^*)] - \mathbb{E}[\Pi_C({f_C}^*)])}{\partial \beta} \leq 0$. This is also true, because (1) if $\frac{\overline{\beta}}{4} \leq \beta < \overline{\beta}$, then $\frac{\partial(\mathbb{E}[\Pi_A({f_A}^*)] - \mathbb{E}[\Pi_C({f_C}^*)])}{\partial \beta} \leq 0$, where the equality occurs at $\beta = \overline{\beta}/4$, and (2) if $\beta \geq \overline{\beta}$, then $\frac{\partial(\mathbb{E}[\Pi_A({f_A}^*)] - \mathbb{E}[\Pi_C({f_C}^*)])}{\partial \beta} = 0$. Since I have shown both directions, I conclude that the gap between $\mathbb{E}[\Pi_A({f_A}^*)]$ and $\mathbb{E}[\Pi_C({f_C}^*)]$ increases if and only if $\beta < \overline{\beta}/4$. □

Proof of Proposition 2.5:

When acquiring fake review is not an option, which I denote by the superscript Λ, sellers' expected profits are equivalent to those by fixing f_A and f_c to 0 in the main model. Then, consider two cases depending on the value of β.

Case 1: when $\beta \geq \overline{\beta}$, ${f_A}^* = {f_C}^* = 0$. Then, the main model coincides with that in the case where acquiring fake review is not an option. Hence, $\mathbb{E}[\Pi_i({f_i}^*)] = \mathbb{E}[\Pi_i^\Lambda],\ i \in \{A, C\}$.

Case 2: when $\beta < \overline{\beta}$, ${f_A}^* = 0$ and ${f_C}^* = \frac{\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}-4R\beta(\phi+(1-\phi)\alpha)}{4\beta(\alpha+(1-\alpha)\phi)} > 0$.

Since ${f_C}^* = \underset{f_C \geq 0}{\operatorname{argmax}}\, \mathbb{E}[\Pi_C(f_C)]$, it must be that $\mathbb{E}[\Pi_C({f_C}^*)] > \mathbb{E}[\Pi_C(f_c = 0)] = \mathbb{E}[\Pi_C^\Lambda]$.

Regarding seller A's expected profits, I have that

$$\begin{aligned}\mathbb{E}[\Pi_A({f_A}^*)] &= \mathbb{E}[\Pi_C({f_C}^*)] + \beta {f_C}^* \\ &> \mathbb{E}[\Pi_C(f_c = 0)] = \mathbb{E}[\Pi_A(f_A = 0)] = \mathbb{E}[\Pi_A^\Lambda].\end{aligned}$$

The inequality follows because $\mathbb{E}[\Pi_C({f_C}^*)] > \mathbb{E}[\Pi_C(f_c = 0)]$ and $\beta {f_C}^* > 0$.

Hence, I have proven the direction that $\beta < \overline{\beta} \Rightarrow \mathbb{E}[\Pi_i({f_i}^*)] > \mathbb{E}[\Pi_i^\Lambda]$. By proof of contrapositive, proving the other direction that $\mathbb{E}[\Pi_i({f_i}^*)] > \mathbb{E}[\Pi_i^\Lambda] \Rightarrow \beta < \overline{\beta}$ is equivalent to proving $\beta \geq \overline{\beta} \Rightarrow \mathbb{E}[\Pi_i({f_i}^*)] \leq \mathbb{E}[\Pi_i^\Lambda]$, which I have shown in case 1 that $\mathbb{E}[\Pi_i({f_i}^*)] = \mathbb{E}[\Pi_i^\Lambda]$ when when $\beta \geq \overline{\beta}$. Since I have shown both directions, I conclude that Both sellers are better off, $\mathbb{E}[\Pi_i({f_i}^*)] > \mathbb{E}[\Pi_i^\Lambda]$, when acquiring fake reviews is an option if and only if $\beta < \overline{\beta}$. □

Proof of Proposition 2.6:

Consumer surplus in the main model can be expressed as

$$\begin{aligned}CS^* = \left(\frac{R*m + {f_A}^* + {f_C}^*}{R + {f_A}^* + {f_C}^*}\right)\left[\alpha \int_{v:\phi qv - p^{B^*} \geq 0} \left(\frac{qv}{2} - p^{B^*}\right) dv + (1-\alpha) \int_{v:qv - p^{B^*} \geq 0} \left(\frac{qv}{2} - p^{B^*}\right) dv\right] \\ + \left(1 - \frac{R*m + {f_A}^* + {f_C}^*}{R + {f_A}^* + {f_C}^*}\right) \int_{v:\phi qv - p^{NB^*} \geq 0} \left(\frac{qv}{2} - p^{NB^*}\right) dv.\end{aligned}$$

While consumer surplus in the benchmark where sellers do not have an option to acquire fake reviews is

$$\begin{aligned}CS^{\Lambda^*} = m\left[\alpha \int_{v:\phi qv - p^{B^*} \geq 0} \left(\frac{qv}{2} - p^{B^*}\right) dv + (1-\alpha) \int_{v:qv - p^{B^*} \geq 0} \left(\frac{qv}{2} - p^{B^*}\right) dv\right] \\ +(1-m) \int_{v:\phi qv - p^{NB^*} \geq 0} \left(\frac{qv}{2} - p^{NB^*}\right) dv.\end{aligned}$$

Define the harm of fake review (H_f) as the gap between consumer surplus in the benchmark and that in the main model. That is,

$$\begin{aligned}H_f &\coloneqq CS^{\Lambda^*} - CS^* \\ &= \left(m - \frac{R*m + {f_A}^* + {f_C}^*}{R + {f_A}^* + {f_C}^*}\right)\left[\alpha \int_{v:\phi qv - p^{B^*} \geq 0} \left(\frac{qv}{2} - p^{B^*}\right) dv + (1-\alpha) \int_{v:qv - p^{B^*} \geq 0} \left(\frac{qv}{2} - p^{B^*}\right) dv\right]\end{aligned}$$

$$+\left(\frac{R*m+{f_A}^*+{f_C}^*}{R+{f_A}^*+{f_C}^*}-m\right)\int_{v:\phi qv-p^{NB^*}\geq 0}\left(\frac{qv}{2}-p^{NB^*}\right)dv$$

$$=\left(m-\frac{R*m+{f_A}^*+{f_C}^*}{R+{f_A}^*+{f_C}^*}\right)[\alpha\int_{v:\phi qv-p^{B^*}\geq 0}\left(\frac{qv}{2}-p^{B^*}\right)dv+(1-\alpha)\int_{v:qv-p^{B^*}\geq 0}\left(\frac{qv}{2}-p^{B^*}\right)dv$$

$$-\int_{v:\phi qv-p^{NB^*}\geq 0}\left(\frac{qv}{2}-p^{NB^*}\right)dv]$$

$$=\left(m-\frac{R*m+{f_A}^*+{f_C}^*}{R+{f_A}^*+{f_C}^*}\right)\left(\frac{q(1-\alpha)(1-\phi)(-4(1-\alpha)\phi^2-3\alpha\phi-\alpha)}{16(-\alpha-\phi+\alpha\phi)^2}\right) \qquad \text{(EC. 2.5)}$$

The second term in EC.2.5 is negative (i.e., $\frac{q(1-\alpha)(1-\phi)\left(-4(1-\alpha)\phi^2-3\alpha\phi-\alpha\right)}{16(-\alpha-\phi+\alpha\phi)^2}<0$). Then, consider two cases depending on the value of β.

Case 1: when $\beta\geq\overline{\beta}$, ${f_A}^*={f_C}^*=0$. In this case, $H_f=0*\left(\frac{q(1-\alpha)(1-\phi)\left(-4(1-\alpha)\phi^2-3\alpha\phi-\alpha\right)}{16(-\alpha-\phi+\alpha\phi)^2}\right)=0$.

Case 2: when $\beta<\overline{\beta}$, ${f_A}^*=0$ and ${f_C}^*>0$. In this case, $m-\frac{R*m+{f_C}^*}{R+{f_C}^*}<0$. Hence, $H_f>0$.

Because I have shown that $H_f\geq 0$ in both cases, I have proven Proposition 2.6a that $CS^*\leq CS^{A^*}$.

Next, I prove Proposition 2.6b. I first re-arrange the threshold of acquiring fake reviews in terms of q. That is,

$$\beta<\overline{\beta}\Leftrightarrow q>\overline{q}\equiv\frac{8R\beta(\alpha+(1-\alpha)\phi)}{\phi(1-m)(\tau-(\phi+(1-\phi)\alpha))}$$

Then, I consider two cases based on the value of q.

Case 1: when $q\leq\overline{q}$, ${f_A}^*={f_C}^*=0$. In this case, $H_f=0$ and $\frac{\partial H_f}{\partial q}=0$.

Case 2: when $q>\overline{q}$, ${f_A}^*=0$ and ${f_C}^*=\frac{\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}-4R\beta(\phi+(1-\phi)\alpha)}{4\beta(\alpha+(1-\alpha)\phi)}>0$.

In this case,

$H_f=\left(m-\frac{R*m+{f_C}^*}{R+{f_C}^*}\right)\left(\frac{q(1-\alpha)(1-\phi)\left(-4(1-\alpha)\phi^2-3\alpha\phi-\alpha\right)}{16(-\alpha-\phi+\alpha\phi)^2}\right)$, and $\frac{\partial H_f}{\partial q}>0\Leftrightarrow q>\frac{2R\beta(\alpha+(1-\alpha)\phi)}{\phi(1-m)\left(\tau-(\phi+(1-\phi)\alpha)\right)}=\overline{q}/4$.

Therefore, in case 2, the condition $q>\overline{q}/4$ is always satisfied, and I have $\frac{\partial H_f}{\partial q}>0$ always holds.

Then, I have proven the direction that $q > \overline{q} \Rightarrow \frac{\partial H_f}{\partial q} > 0$. Next, I prove the other direction that $\frac{\partial H_f}{\partial q} > 0 \Rightarrow q > \overline{q}$. I use proof by contrapositive here so that proving $\frac{\partial H_f}{\partial q} > 0 \Rightarrow q > \overline{q}$ is equivalent to proving $q \leq \overline{q} \ \Rightarrow \frac{\partial H_f}{\partial q} \leq 0$. This is also true, as I have shown in case 1 that when $q \leq \overline{q}$, $\frac{\partial H_f}{\partial q} = 0$. □

Proof of Proposition 2.7:

Social welfare (SW^*) is defined to be the sum of seller A's expected profit, seller C's expected profit, and consumer surplus. Let SW^{Λ^*} denote social welfare in the benchmark where acquiring fake reviews is not an option. Define $\Delta_{sw} \equiv SW^{\Lambda^*} - SW^*$. Then, I can express Δ_{sw} as

$$\Delta_{sw} = \left(m - \frac{R * m + f_A{}^* + f_C{}^*}{R + f_A{}^* + f_C{}^*}\right)\left[\Pi_A^B\left(p^{B^*}\right) + \Pi_C^B\left(p^{B^*}\right)\right] + \left(\frac{R * m + f_A{}^* + f_C{}^*}{R + f_A{}^* + f_C{}^*} - p\right)\left[\Pi_A^{NB}\left(p^{NB^*}\right)\right.$$
$$\left.+\Pi_C^{NB}\left(p^{NB^*}\right)\right] + (\beta + \gamma_A)f_A + \beta f_C + H_f.$$

Then, consider two cases depending on the value of β.

Case 1: when $\beta \geq \overline{\beta}$, $f_A{}^* = f_C{}^* = 0$. In this case, the benchmark is the same as the main model. Hence, $\Delta_{sw} = 0$.

Case 2: when $\beta < \overline{\beta}$, $f_A{}^* = 0$ and $f_C{}^* = \frac{\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}-4R\beta(\phi+(1-\phi)\alpha)}{4\beta(\alpha+(1-\alpha)\phi)} > 0$. Plug $f_C{}^*$ into Δ_{sw}, I have that

$$\Delta_{sw} \geq 0 \Leftrightarrow \left[\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)\big(\tau-(\phi+(1-\phi)\alpha)\big)} - 4R\beta(\phi+(1-\phi)\alpha)\right] *$$
$$\Big[4\sqrt{2}(\alpha(1-\phi)+\phi)\sqrt{(-1+m)qR\beta(\alpha(-1+\phi)-\phi)(\tau+\alpha(-1+\phi)-\phi)\phi} + (1-m)q(\alpha-\alpha^2$$
$$+ 2\alpha(1+\alpha-2\tau)\phi - (-1+\alpha)(4+\alpha-4\tau)\phi^2)\Big] \geq 0.$$

Moreover, I know that the first square bracket is positive because the numerator in $f_C{}^* = \frac{\sqrt{2q\phi R\beta(1-m)(\alpha+(1-\alpha)\phi)(\tau-(\phi+(1-\phi)\alpha))}-4R\beta(\phi+(1-\phi)\alpha)}{4\beta(\alpha+(1-\alpha)\phi)}$ is positive. Hence, I have that

$$\Delta_{sw} \geq 0 \Leftrightarrow 4\sqrt{2}(\alpha(1-\phi)+\phi)\sqrt{(-1+m)qR\beta(\alpha(-1+\phi)-\phi)(\tau+\alpha(-1+\phi)-\phi)\phi} +$$
$$(1-m)q(\alpha-\alpha^2+2\alpha(1+\alpha-2\tau)\phi-(-1+\alpha)(4+\alpha-4\tau)\phi^2) \geq 0. \qquad \text{(EC. 2.6)}$$

Next, solve $(1-m)q(\alpha-\alpha^2+2\alpha(1+\alpha-2\tau)\phi-(-1+\alpha)(4+\alpha-4\tau)\phi^2) = 0$ with respect to τ, I have $(1-m)q(\alpha-\alpha^2+2\alpha(1+\alpha-2\tau)\phi-(-1+\alpha)(4+\alpha-4\tau)\phi^2) \geq 0 \Leftrightarrow$

$$\tau \leq \tilde{\tau} \equiv \frac{(1-\phi)^2\alpha^2+\left(3\phi^2-2\phi-1\right)\alpha-4\phi^2}{-4\phi(\alpha+(1-\alpha)\phi)}.$$

In addition, $\tilde{\tau} > 1$ always holds. Then, I further consider two subcases, depending on the value of τ.

Case 2a: when $1 \leq \tau \leq \tilde{\tau}$, $\Delta_{sw} > 0$ is true for all β, as EC.2.6 can be re-arranged as $X\sqrt{Y\beta} + Z > 0$, where $X > 0,\ Y > 0,\ Z \geq 0$.

Case 2b: when $\tau > \tilde{\tau}$, I have $(1-m)q(\alpha - \alpha^2 + 2\alpha(1+\alpha-2\tau)\phi - (-1+\alpha)(4+\alpha-4\tau)\phi^2) < 0$, and $\Delta_{sw} \geq 0 \iff \beta \geq \beta_1 \equiv \frac{q(1-m)\left[(1-\phi)^2\alpha^2 - \left((4\tau-3)\phi^2+(2-4\tau)\phi+1\right)\alpha+4\phi^2(\tau-1)\right]^2}{32R\phi(\alpha+(1-\alpha)\phi)^3\left(\tau-(\phi+(1-\phi)\alpha)\right)}$.

Moreover, I have that $\beta_1 < \overline{\beta} \iff \tau < \tilde{\tilde{\tau}} \equiv \frac{(2\phi^3-3\phi^2+1)\alpha^2-(4\phi^3-7\phi^2+2\phi+1)\alpha+2\phi^3-4\phi^2}{-2\phi(\alpha+(1-\alpha)\phi)}$. And it is easy to verify that $\tilde{\tilde{\tau}} > \tilde{\tau}$ holds. If $\tau \geq \tilde{\tilde{\tau}}$, then $\beta_1 \geq \overline{\beta}$. In this case, the condition $\beta < \beta_1$ is always true because I are in case 2 where $\beta < \overline{\beta}$ by definition. Consequently, $\Delta_{sw} < 0$ follows. Next, define

$$\underline{\beta} \equiv \begin{cases} \overline{\beta}, & if\ \tau \geq \tilde{\tilde{\tau}} \\ \beta_1, & if\ \tilde{\tau} < \tau < \tilde{\tilde{\tau}} \cdot \\ 0, & if\ 1 \leq \tau \leq \tilde{\tau} \end{cases}$$

I then conclude our proof:

1. when $\tau \geq \tilde{\tilde{\tau}}$, $\underline{\beta} = \overline{\beta}$. In this case, Proposition 2.7 is reduced to Proposition 2.7a and 2.7c, which have been proven above.
2. when $\tilde{\tau} < \tau < \tilde{\tilde{\tau}}$, $\underline{\beta} = \beta_1$. In this case, Proposition 2.7 contains Proposition 2.7a, 2.7b, and 2.7c, which have been proven above.
3. when $1 \leq \tau \leq \tilde{\tau}$ ($\tau \geq 1$ by definition), $\underline{\beta} = 0$. In this case, Proposition 2.7 contains Proposition 2.7a and 2.7b, which have been proven above. □

Proof of Lemma 2.3:

Let p_{s1}^B denote the pooling price when the product is non-credence goods and deceptive in nature. Knowing that seller C would follow seller A's price, I can express seller A's profit maximization problem as follows:

$$\max_{p_{s1}^B \geq 0} \ \Pi_{A,s1}^B(p_{s1}^B) = p_{s1}^B\left(\alpha\frac{\tau}{2}\int_{v:\tilde{\phi}qv-p_{s1}^B\geq 0} dv + (1-\alpha)\frac{\tau}{2}\int_{v:qv-p_{s1}^B\geq 0} dv\right).$$

$\frac{\partial^2\Pi_{A,s1}^B(p_{s1}^B)}{\partial {p_{s1}^B}^2} = \frac{-\tau(1+\alpha(1-\phi))}{q} < 0 \implies \Pi_{A,s1}^B(p_{s1}^B)$ is strictly concave in p_{s1}^B. Solve

$$\frac{\partial\Pi_{A,s1}^B(p_{s1}^B)}{\partial p_{s1}^B} = \frac{\tau(q-2p_{s1}^B(1+\alpha(1-\phi)))}{2q} = 0,$$

I have that ${p_{s1}^B}^* = \frac{q}{2(1+\alpha(1-\phi))} > 0$, which is an interior solution. Moreover, the binding solution ($p_{s1}^B = 0$) can never be optimal, as it generates zero profit for seller A, who can easily deviate by charging a positive price, such as ${p_{s1}^{B}}^{*} = \frac{q}{2(1+\alpha(1-\phi))}$, to obtain positive profit. Hence, I conclude that the unique pooling price is ${p_{s1}^{B}}^{*} = \frac{q}{2(1+\alpha(1-\phi))}$. In addition, ${p_{s1}^{B}}^{*} = \frac{q}{2(1+\alpha(1-\phi))} > \frac{q\,\phi}{2(\alpha+(1-\alpha)\phi)} = p^{B^*}$. □

Proof of Lemma 2.4:

Seller i's fake review decision is to choose f_i that maximize her expected profits, as expressed below

$$\max_{f_i \geq 0} \ \mathbb{E}[\Pi_{i,s1}(f_i)] = \frac{R + f_i + f_{-i}}{R + f_i + f_{-i}} \Pi_{i,s1}^{B}\left({p_{s1}^{B}}^{*}\right) - (\beta + \gamma_i) f_i.$$

It is obvious that the optimal unit of fake review is 0, as

$\mathbb{E}[\Pi_{i,s1}(f_i = 0)] = \Pi_{i,s1}^{B}\left({p_{s1}^{B}}^{*}\right) \geq \Pi_{i,s1}^{B}\left({p_{s1}^{B}}^{*}\right) - (\beta + \gamma_i) f_i = \mathbb{E}[\Pi_{i,s1}(f_i)]$, for $f_i \geq 0$. □

Proof of Lemma 2.5:

Conditional on the product receives badge, seller A's profit maximization problem in the second period can be expressed as follows:

$$\max_{p_A \geq 0} \ \Pi_{A,s2}^{B}(p_A) = p_A \frac{\tau}{2} \int_{v: qv - p_A \geq 0} dv. \qquad (EC.2.7)$$

Similarly, conditional on the product does not receive badge, seller A's profit maximization problem in the second period can be expressed as follows:

$$\max_{p_A \geq 0} \ \Pi_{A,s2}^{NB}(p_A) = p_A \frac{1}{2} \int_{v: qv - p_A \geq 0} dv. \qquad (EC.2.8)$$

It is obvious that the optimal solutions to problem (EC.2.7) and (EC.2.8) are the same, as τ is a constant. Next, let p_{s2} denote the optimal solution to problem (EC.2.7) and (EC.2.8).

$\frac{\partial^2 \Pi_{A,s2}^{B}(p_{s2})}{\partial {p_{s2}}^2} = \frac{-2\,\tau}{q} < 0 \implies \Pi_{A,s2}^{B}(p_{s2})$ is strictly concave in p_{s2}. Solve

$$\frac{\partial \Pi_{A,s2}^{B}(p_{s2})}{\partial p_{s2}} = \frac{(q - 2p_{s2})\tau}{q} = 0,$$

I have that ${p_{s2}}^{*} = \frac{q}{2} > 0$, which is an interior solution. Moreover, the binding solution ($p_{s2} = 0$) can never be optimal, as it generates zero profit for seller A, who can easily deviate by charging a positive price, such as ${p_{s2}}^{*} = \frac{q}{2}$, to obtain positive profit. Hence, I conclude that the unique pooling price is ${p_{s2}}^{*} = \frac{q}{2}$. In addition, ${p_{s2}}^{*} = \frac{q}{2} > \frac{q\,\phi}{2(\alpha+(1-\alpha)\phi)} = p^{B^*}$. □

Proof of Lemma 2.6:

Seller A's expected profit maximization problem can be expressed as:

$$\max_{f_A \geq 0} \ \mathbb{E}[\Pi_{A,s2}(f_A)] = \frac{R * m + f_A}{R + f_A} \Pi_{A,s2}^{B}({p_{s2}}^{*}) + \left(1 - \frac{R * m + f_A}{R + f_A}\right) \Pi_{A,s2}^{NB}({p_{s2}}^{*}) - (\beta + \gamma_A) f_A. \qquad (\text{EC}.2.9)$$

Let $f_{A,s2}$ denote the optimal solution to EC.2.9. Because the product is non-deceptive in nature, consumers are able to ex ante distinguish authentic products from fake ones. Hence, seller A will only acquire fake reviews, which increase the probability of receiving badge, if the badge can attract additional consumers (i.e., $\tau > 1$). When the product badge does not exhibit the market expansion effect (i.e., $\tau = 1$), it is obvious that the optimal solution to EC.2.9 is ${f_{A,s2}}^* = 0$.

I now base our following analysis on the condition that $\tau > 1$. In particular, I apply the Karush-Kuhn-Tucker (KKT) conditions to solve EC.2.9. $\frac{\partial^2 \mathbb{E}[\Pi_{A,s2}(f_{A,s2})]}{\partial {f_{A,s2}}^2} = \frac{q\, R(1-m)(1-\tau)}{2(R+f_{A,s2})^3} < 0 \Longrightarrow \mathbb{E}[\Pi_{A,s2}(f_{A,s2})]$ is strictly concave in $f_{A,s2}$. The KKT solutions to EC.2.9 are given by the standard equations:

$$\begin{aligned} \frac{d\mathbb{E}[\Pi_{A,s2}(f_A)]}{df_{A,s2}} - \lambda\left(\frac{d(-f_{A,s2})}{df_{A,s2}}\right) &= 0, \\ \lambda &\geq 0, \\ \lambda(-f_{A,s2}) &= 0. \end{aligned} \tag{EC.2.10}$$

Solving EC.2.10 leads to Lemma 2.6:

$${f_{A,s2}}^* = \begin{cases} \dfrac{\sqrt{q\, R(\gamma_A+\beta)(1-m)(\tau-1)} - 2R(\gamma_A+\beta)}{2(\gamma_A+\beta)}, & if\ \tau > \hat{\tau}, \\ 0, & if\ \tau \leq \hat{\tau}, \end{cases}$$

$where\ \hat{\tau} \equiv \frac{(1-m)q+4R(\gamma_A+\beta)}{(1-m)q}$. □

Appendix. B Microeconomic Foundations and Proofs of Lemmas and Propositions

Microeconomic Foundations of the Demand Function

I consider a Hotelling model where two platforms are located at the opposite end points of a line of length 1 (Without loss of generality, say platform i is at 0 and platform j is at 1). Consumers have a unit demand. In addition, some consumers are single-homing, meaning that they either use their desired platform for food delivery service or not use the service at all. By contrast, other consumers multi-home between the two platforms, and will choose the platform that offers them a higher utility. Let v denote consumers' valuation of using platforms for food delivery due to the convenience. In addition, the unit cost of misfit between consumers' preferences and platforms is denoted by t_c.

Each of the two platforms has l_s number of single-homing consumers who only use their desired platform for delivery service. Hence, for a fair consumer at location x and who is single-homing on platform i, her respective utilities of using platform i and j are

$$\begin{cases} U_i^{FS} = v - t_c x - \Big(p_i + T_i^* - \mathbb{1}_{S_i=A}(1-\theta_{E_i^*})\lambda T_i^* + (1-\theta_{E_i^*})c_w W_H\Big) \\ U_j^{FS} = 0, \end{cases}$$

where T_i^* is the tip that consumers will commit to offer prior to delivery and $(1-\theta_{E_i^*})c_w W_H$ represents the expected waiting cost for placing orders on platform i. To simplify notation, let $k_i \equiv p_i + T_i^* - \mathbb{1}_{S_i=A}(1-\theta_{E_i^*})\lambda T_i^* + (1-\theta_{E_i^*})c_w W_H$. Then equating $U_i^{FS} = U_j^{FS}$, I solve for platform i's demand from fair, single-homing consumers is $d_i^{FS} = \frac{v}{t_c} - \frac{1}{t_c}k_i$. Because demand also needs to be non-negative, I use the positive part function so that demand becomes $d_i^{FS} = \Big[\frac{v}{t_c} - \frac{1}{t_c}k_i\Big]^+$.

For selfish consumers, let $z_i \equiv p_i + T_i^* - \mathbb{1}_{S_i=A}T_i^* + (1-\theta_{E_i^*})c_w W_H$ denote the cost they incur when using platform i's service. Then a selfish consumer who single-homes on platform i has the following utilities of using platforms

$$\begin{cases} U_i^{SS} = v - t_c x - z_i \\ U_j^{SS} = 0. \end{cases}$$

Equating $U_i^{SS} = U_j^{SS}$, I solve for platform i's demand from selfish, single-homing consumers is $d_i^{FS} = \Big[\frac{v}{t_c} - \frac{1}{t_c}z_i\Big]^+$.

Next, let's discuss the case for multi-homing consumers. I assume the size of multi-homing consumers l_m. Then a fair, multi-homing consumer at location x has the following utilities of buying from platform i and j, respectively

$$\begin{cases} U_i^{FM} = v - t_c x - k_i \\ U_j^{FM} = v - t_c(1-x) - k_j. \end{cases}$$

Then equating $U_i^{FM} = U_j^{FM}$, I solve for platform i's demand from fair, multi-homing consumers is $d_i^{FM} = \Big[\frac{-k_i+k_j+t}{2t_c}\Big]^+$.

Similarly, a selfish, multi-homing consumer at location x has the following utilities of buying from platform i and j, respectively

$$\begin{cases} U_i^{SM} = v - t_c x - z_i \\ U_j^{SM} = v - t_c(1-x) - z_j. \end{cases}$$

Then equating $U_i^{SM} = U_j^{SM}$, I solve for platform i's demand from selfish, multi-homing consumers is $d_i^{FM} = \left[\frac{-z_i+z_j+t}{2t_c}\right]^+$. Combining these segments together, platform i's aggregate demand from all consumers can then be represented as

$$\begin{aligned} d_i &= \Big[l_s(\alpha d_i^{FS} + (1-\alpha)d_i^{SS}) + l_m(\alpha d_i^{FM} + (1-\alpha)d_i^{SM})\Big]^+ \\ &= \Big[\frac{l_m t_c + 2l_s v}{2t_c} - \frac{(l_m+2l_s)(\alpha k_i + (1-\alpha)z_i)}{2t_c} + \frac{l_m(\alpha k_j + (1-\alpha)z_j)}{2t_c}\Big]^+ \end{aligned}$$

If I normalize $\frac{l_m+2l_s}{2t_c}$ to 1, then $\frac{l_m}{2t_c}$ becomes less than 1, and I denote $\gamma \equiv \frac{l_m}{2t_c}$. I also let $M \equiv \frac{l_m t_c + 2l_s v}{2t_c}$. Hence, the total demand becomes $d_i = \Big[M - (\alpha k_i + (1-\alpha)z_i) + \gamma(\alpha k_j + (1-\alpha)z_j)\Big]^+$. Then, plugging back $k_i = p_i + T_i^* - \mathbb{1}_{S_i=A}(1-\theta_{E_i^*})\lambda T_i^* + (1-\theta_{E_i^*})c_w W_H$ and $z_i = p_i + T_i^* - \mathbb{1}_{S_i=A}T_i^* + (1-\theta_{E_i^*})c_w W_H$ into d_i, I have

$$\begin{aligned} d_i &= \Big[M - (\alpha k_i + (1-\alpha)z_i) + \gamma(\alpha k_j + (1-\alpha)z_j)\Big]^+ \\ &= \Big[M - (\alpha(p_i + T_i^* - \mathbb{1}_{S_i=A}(1-\theta_{E_i^*})\lambda T_i^* + (1-\theta_{E_i^*})c_w W_H) + (1-\alpha)(p_i + T_i^* - \mathbb{1}_{S_i=A}T_i^* + (1-\theta_{E_i^*})c_w W_H)) \\ &\quad + \gamma(\alpha(p_j + T_j^* - \mathbb{1}_{S_j=A}(1-\theta_{E_j^*})\lambda T_j^* + (1-\theta_{E_j^*})c_w W_H) + (1-\alpha)(p_j + T_j^* - \mathbb{1}_{S_j=A}T_j^* + (1-\theta_{E_j^*})c_w W_H))\Big]^+ \\ &= \Big[M - \Big(p_i + T_i^* - \mathbb{1}_{S_i=A}(\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) + (1-\theta_{E_i^*})c_w W_H\Big) \\ &\quad + \gamma\Big(p_j + T_j^* - \mathbb{1}_{S_j=A}(\alpha(1-\theta_{E_j^*})\lambda T_j^* + (1-\alpha)T_j^*) + (1-\theta_{E_j^*})c_w W_H\Big)\Big]^+, \end{aligned}$$

which is the demand function I presented in equation (3.3).

Microeconomic Foundations of the Supply Function

The microeconomic foundations of the supply function can be similarly derived following the same procedure above. I assume that some workers are single-homing, meaning that they either work for their desired platform or not work at all. By contrast, other workers are multi-homing and registered on both platforms and work for the platform that offers them a better pay (base wage plus expected tips from consumers).

Each of the two platforms has m_s size of single-homing workers. Let the unit cost of misfit between workers' preferences and platforms is denoted by t_w. When workers deliver, they receive a base wage plus expected tips from consumers. Moreover, they also incur a cost for their delivery service (e.g., fuel expenses for their vehicles), which I denote as g. If they decide to exert high delivery effort (i.e., the indicator function $\mathbb{1}_{E_i^*=E_H}$), they incur an additional effort cost c_e.

Hence, for platform i's single-homing worker with location x, her respective utilities of delivering for platform i and j are

$$\begin{cases} U_i^{WS} = w_i + T_i^* - \mathbb{1}_{S_i=A}(\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) - \mathbb{1}_{E_i^*=E_H}c_e - t_w x - g \\ U_j^{WS} = 0. \end{cases}$$

Equating $U_i^{WS} = U_j^{WS}$, I solve for platform i's supply from single-homing workers is

$$s_i^{WS} = \Big[\frac{w_i + T_i^* - \mathbb{1}_{S_i=A}(\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) - \mathbb{1}_{E_i^*=E_H} c_e - g}{t_w}\Big]^+.$$

Next, let's discuss the case for multi-homing workers. I assume the size of multi-homing consumers m_m. Then a multi-homing consumer at location x has the following utilities of working for platform i and j, respectively:

$$\begin{cases} U_i^{WM} = w_i + T_i^* - \mathbb{1}_{S_i=A}(\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) - \mathbb{1}_{E_i^*=E_H} c_e - t_w x - g \\ U_j^{WM} = w_j + T_j^* - \mathbb{1}_{S_j=A}(\alpha(1-\theta_{E_j^*})\lambda T_j^* + (1-\alpha)T_j^*) - \mathbb{1}_{E_j^*=E_H} c_e - t_w(1-x) - g. \end{cases}$$

Then equating $U_i^{WM} = U_j^{WM}$, I solve for platform i's supply from multi-homing workers is

$$\begin{aligned} s_i^{WM} = [&t_w + w_i + T_i^* - \mathbb{1}_{S_i=A}(\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) - \mathbb{1}_{E_i^*=E_H} c_e \\ &-\Big(w_j + T_j^* - \mathbb{1}_{S_j=A}(\alpha(1-\theta_{E_j^*})\lambda T_j^* + (1-\alpha)T_j^*) - \mathbb{1}_{E_j^*=E_H} c_e\Big)]/2t_w. \end{aligned}$$

Combining these two segments together, platform i's aggregate supply from all workers can then be represented as

$$\begin{aligned} s_i &= m_s s_i^{WS} + m_m s_i^{WM} \\ &= \frac{m_m t_w - 2g m_s}{2t_w} + \frac{(m_m + 2m_s)(w_i + T_i^* - \mathbb{1}_{S_i=A}(\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) - \mathbb{1}_{E_i^*=E_H} c_e)}{2t_w} \\ &- \frac{m_m(w_j + T_j^* - \mathbb{1}_{S_j=A}(\alpha(1-\theta_{E_j^*})\lambda T_j^* + (1-\alpha)T_j^*) - \mathbb{1}_{E_j^*=E_H} c_e)}{2t_w} \end{aligned}$$

If I normalize $\frac{m_m+2m_s}{2t_w}$ to 1, then $\frac{m_m}{2t_w}$ is obviously less than 1, and I denote $\beta \equiv \frac{m_m}{2t_w}$. To focus on the impact of tipping policy on supply of workers, I normalize the constant term ($\frac{m_m t_w - 2g m_s}{2t_w}$) in the supply function to 0, which is equivalent to normalizing the base supply to zero. Hence, the supply function becomes

$$\begin{aligned} s_i = \Big[&w_i + T_i^* - \mathbb{1}_{S_i=A}(\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) - \mathbb{1}_{E_i^*=E_H} c_e \\ &-\beta\Big(w_j + T_j^* - \mathbb{1}_{S_j=A}(\alpha(1-\theta_{E_j^*})\lambda T_j^* + (1-\alpha)T_j^*) - \mathbb{1}_{E_j^*=E_H} c_e\Big)\Big]^+, \end{aligned}$$

which is the supply function I presented in equation (3.4).

Proof of Lemma 3.1.

When platform i selects the non-adjustable tipping policy ($S_i = N$), consumers cannot reduce tip after delivery. Knowing that consumers cannot *ex post* reduce their tip, workers would not exert high delivery effort, as their final earning (wage plus tip) remains the same regardless of delivery effort. Knowing that workers would not exert high effort regardless of tip, all consumers would give zero tip.

I then discuss the case when platform i selects the adjustable tipping policy ($S_i = A$). I first show that the game does not have a separating equilibrium. Suppose for contradiction that there exists a separating equilibrium where selfish and fair consumers commit different tips on platform i (i.e., $T_i^*(S) \neq T_i^*(F)$). Then by the definition of separating equilibrium, workers are able to identify consumers' type based on the tipping amount. Because workers know selfish consumers will always renege on the tip, they will exert only low effort for selfish consumers' orders, as high effort is costly and workers eventually do not receive any tip regardless of effort. Hence, selfish consumers' utility would be $U_S(T_i^*(S)) = v - t_c x - p_i - (1-\theta_{E_L})c_w W_H$.

However, selfish consumers can unilaterally deviate by committing the same tip as fair consumer does ($T_i^*(F)$). In this case, workers are unable to distinguish between consumers and exert either high or low effort. Hence, consumers' utility of committing $T_i^*(F)$ would be $U_S(T_i^*(F)) = v - t_c x - p_i - (1-\theta_{E_i})c_w W_H \geq U_S(T_i^*(S))$, where $E_i \in \{E_H, E_L\}$. That is, when the tip $T_i^*(F)$ is large enough to motivate high effort ($E_i = E_H$), $U_S(T_i^*(F)) = v - t_c x - p_i - (1-\theta_{E_H})c_w W_H > U_S(T_i^*(S))$. When the tip $T_i^*(F)$ is not large enough and only motivates low effort ($E_i = E_L$), $U_S(T_i^*(F)) = v - t_c x - p_i - (1-\theta_{E_L})c_w W_H = U_S(T_i^*(S))$. Hence, I reach a contradiction and conclude there does not exist a separating equilibrium, because selfish consumers have incentives to deviate to pay $T_i^*(F)$.

Next, I derive the pooling equilibrium solutions. Let T_i denote the tip that both fair and selfish consumers commit before delivery. Hence, platform i's workers choose to exert a high delivery effort if and only if

$$\underbrace{\alpha\Big(T_i - (1-\theta_{E_H})\lambda T_i\Big)}_{\text{Expected Tip When Exert High Effort}} - \underbrace{c_e}_{\text{Cost of High Effort}} \geq \underbrace{\alpha\Big(T_i - (1-\theta_{E_L})\lambda T_i\Big)}_{\text{Expected Tip When Exert low Effort}}, \tag{EC.3.1}$$

which simplifies to $T_i \geq \widehat{T}_i \equiv \frac{c_e}{\alpha\lambda(\theta_{E_H}-\theta_{E_L})}$. Therefore, if consumers want to induce high effort, they would commit $\widehat{T}_i$; otherwise, they pay zero tip to induce low effort. Consumers compare the utility of inducing high effort with that of low effort, and decide to pay $T_i = 0$ to induce low effort if and only if

$$\underbrace{(1-\theta_{E_L})c_w W_H}_{\substack{\text{Expected Waiting Cost}\\ \text{with Low Effort}}} \leq \underbrace{(1-\theta_{E_H})c_w W_H}_{\substack{\text{Expected Waiting Cost}\\ \text{with High Effort}}} + \underbrace{\frac{c_e}{\alpha\lambda(\theta_{E_H}-\theta_{E_L})} - (1-\theta_{E_H})\lambda\Big(\frac{c_e}{\alpha\lambda(\theta_{E_H}-\theta_{E_L})}\Big)}_{\text{Expected Tip Spent to Induce High Effort}}, \tag{EC.3.2}$$

which simplifies to $c_w \leq \overline{c_w} \equiv \frac{c_e(1-\lambda(1-\theta_{E_H}))}{\alpha\lambda W_H(\theta_{E_H}-\theta_{E_L})^2}$. Hence, combining both situations when $S_i = N$ and $S_i = A$, I conclude that the optimal tip is

$$T_i^* = \begin{cases} \widehat{T}_i & \text{if } S_i = A \text{ and } c_w > \overline{c_w}, \\ 0 & \text{otherwise.} \end{cases}$$

To eliminate any out-of-equilibrium, I apply the D1-Criterion to refine the set of perfect Bayesian equilibria. Denote $\Omega(X,\hat{\chi},T_i') \equiv \bigcup_{\mu:\mu(\hat{\chi}|T_i')=1}\{a \in MBR(\mu,T_i')|U_i^*(X) < U_i(T_i',a,X)\}$ to be the set of mixed best responses (MBR) of the workers for which the X-type of consumer is strictly better off deviating toward giving tip T_i' than giving their equilibrium tip T_i^*. Note that $\mu(\hat{\chi}|T_i') = 1$ represents that the worker believes that tip T_i' only comes from types in the subset $\hat{\chi} \in \chi$.

Next, I specify workers' off-equilibrium beliefs as follows: When they observe a uniform tip not equal to T_i^*, they believe that it is from selfish consumers. When workers observe two different tips, where one is equal to T_i^*, they believe the order with T_i^* is from fair consumers and the other order is from selfish consumers. When workers observe any other two different tips, they believe the orders are from selfish consumers. Next, I show neither type of consumer has incentive to deviate from the pooling equilibrium tip. Consider two cases:

Case 1: when $S_i = A$ and $c_w > \overline{c_w}$, $T_i^* = \widehat{T}_i$, with which workers exert high effort. Suppose fair consumers deviate to $T_i' \neq T_i^*$, then workers believe the order is from selfish consumers and thus exert low delivery effort. Consequently, their utility would be $U_F(T_i') = v - t_c x - (p_i + T_i' - (1-\theta_{E_L})\lambda T_i' + (1-\theta_{E_L})c_w W_H) < \max_{T_i} U_F(T_i) = U_F(T_i^*) = v - t_c x - (p_i + T_i^* - (1-\theta_{E_H})\lambda T_i^* + (1-\theta_{E_H})c_w W_H)$. Therefore, $\Omega(F,\hat{\chi},T_i') \subset \Omega(S,\hat{\chi},T_i')$, where $\Omega(F,\hat{\chi},T_i')$ is an empty set, and hence $\chi(T_i') = \{S\}$. Repeating this process for any off-the-equilibrium tip, workers' beliefs are restricted to $\chi(T_i') = \{S\}$. Next, I show that selfish consumers do not deviate as well. Suppose selfish consumers deviate to $T_i' \neq T_i^*$, then their utility would be $U_S(T_i') = v - t_c x - p_i - (1-\theta_{E_L})c_w W_H < U_S(T_i^*) = v - t_c x - p_i - (1-\theta_{E_H})c_w W_H$. Hence, selfish consumers do not have incentive to deviate as well.

Case 2: when ($S_i = A$ and $c_w \leq \overline{c_w}$) or ($S_i = N$), $T_i^* = 0$, with which workers exert low effort. Suppose fair consumers deviate to $T_i' \neq T_i^*$, then their utility would be $U_F(T_i') = v - t_c x - (p_i + T_i' - (1-\theta_{E_L})\lambda T_i' + (1-\theta_{E_L})c_w W_H) < U_F(T_i^*) = v - t_c x - p_i - (1-\theta_{E_L})c_w W_H$. Therefore, $\Omega(F,\hat{\chi},T_i') \subset \Omega(S,\hat{\chi},T_i')$, where $\Omega(F,\hat{\chi},T_i')$ is an empty set, and hence $\chi(T_i') = \{S\}$. Repeating this process for any off-the-equilibrium tip, workers' beliefs are restricted to $\chi(T_i') = \{S\}$. Next, I show that selfish consumers do not deviate as well. Suppose selfish consumers deviate to $T_i' \neq T_i^*$, then their utility would be $U_S(T_i') = v - t_c x - p_i - (1-\theta_{E_L})c_w W_H = U_S(T_i^*)$. Hence, selfish consumers cannot profitably deviate as well.

Because I have shown in both cases that fair and selfish consumers do not deviate, I conclude that the pooling equilibrium of tip is unique and survives D1-Criterion. Moreover, $E_i^* = E_H$ if and only if $T_i^* = \widehat{T}_i$ follows directly from EC.3.1 and other analyses discussed above. ∎

Proof of Proposition 3.1.

When both platforms select the adjustable tipping policy, platform i's demand can be obtained by plugging $S_i = S_j = A$ into the demand function:

$$d_i = \Big[M - \Big(p_i + \underbrace{T_i^*}_{\text{Tip before Delivery}} - \underbrace{(\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*)}_{\substack{\text{Expected Reduction in Tip}\\ \text{after Delivery}}} + \underbrace{(1-\theta_{E_i^*})c_w W_H}_{\text{Expected Waiting Cost}}\Big)$$
$$+\gamma\Big(p_j + T_j^* - (\alpha(1-\theta_{E_j^*})\lambda T_j^* + (1-\alpha)T_j^*) + (1-\theta_{E_j^*})c_w W_H\Big)\Big]^+.$$

Similarly, platform i's supply is

$$s_i = \Big[w_i + T_i^* - (\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) - \mathbb{1}_{E_i^*=E_H}c_e$$
$$-\beta\Big(w_j + T_j^* - (\alpha(1-\theta_{E_j^*})\lambda T_j^* + (1-\alpha)T_j^*) - \mathbb{1}_{E_j^*=E_H}c_e\Big)\Big]^+.$$

Next, I show that in equilibrium, platform i's demand and supply must be perfectly matched ($d_i = s_i$). Suppose for contradiction that $d_i \neq s_i$ with equilibrium price p_i^* and wage w_i^*, then I have two cases.

Case 1: $d_i > s_i$. In this case, platform i's profit is $\pi_i^{AA} = (p_i^* - w_i^*)\min\{d_i, s_i\} = (p_i^* - w_i^*)s_i$. However, by deviating toward a slightly higher price $p_i > p_i^*$, platform i's new profit becomes larger than π_i^{AA}. Hence, platform i has incentives to deviate and case 1 is not an equilibrium.

Case 2: $d_i < s_i$. In this case, platform i's profit is $\pi_i^{AA} = (p_i^* - w_i^*)\min\{d_i, s_i\} = (p_i^* - w_i^*)d_i$. However, by deviating toward a slightly lower wage $w_i < w_i^*$, platform i's new profit becomes larger than π_i^{AA}. Hence, platform i has incentives to deviate and case 2 is not an equilibrium.

Therefore, I reach a contradiction and conclude in equilibrium, platform i's demand and supply must be perfectly matched ($d_i = s_i$). Platform j's case is the same and can be obtained by changing the subscript from i to j.

In the third stage, wage is already set in stage two and from Lemma 3.1 I have

$$T_i^* = \begin{cases} \frac{c_e}{\alpha\lambda(\theta_{E_H}-\theta_{E_L})} & \text{if } c_w > \overline{c_w}, \\ 0 & \text{otherwise,} \end{cases}$$

where $\overline{c_w} \equiv \frac{c_e(1-\lambda(1-\theta_{E_H}))}{\alpha\lambda W_H(\theta_{E_H}-\theta_{E_L})^2}$. Given the optimal tip, $E_i^* = E_H$ if and only if $T_i^* = \frac{c_e}{\alpha\lambda(\theta_{E_H}-\theta_{E_L})}$. Therefore, supply s_i, which is affected by T_i^*, E_i^*, w_i, is realized at this point. Since in equilibrium demand and supply are perfectly matched for each platform, the optimal price p_i and p_j in the third stage are obtained from solving $d_i = s_i$ and $d_j = s_j$, respectively, and I have that

(a) when $c_w > \overline{c_w}$, consumers pay the positive tip and I have

$$p_i^* = \frac{\frac{w_j(\beta-\gamma)+M(1+\gamma)+w_i(-1+\beta\gamma)-c_w W_H\left(-1+\gamma^2\right)(-1+\theta_{E_H})}{1+\gamma} + \frac{c_e(-2+\beta+\gamma+\gamma(-1+\theta_{E_H})\lambda+\beta(-1+\theta_{E_L})\lambda-(-2+\theta_{E_H}+\theta_{E_L})\lambda)}{(\theta_{E_H}-\theta_{E_L})\lambda}}{1-\gamma},$$

$$p_j^* = \frac{\frac{w_i(\beta-\gamma)+M(1+\gamma)+w_j(-1+\beta\gamma)-c_w W_H\left(-1+\gamma^2\right)(-1+\theta_{E_H})}{1+\gamma} + \frac{c_e(-2+\beta+\gamma+\gamma(-1+\theta_{E_H})\lambda+\beta(-1+\theta_{E_L})\lambda-(-2+\theta_{E_H}+\theta_{E_L})\lambda)}{(\theta_{E_H}-\theta_{E_L})\lambda}}{1-\gamma};$$

(b) when $c_w \leq \overline{c_w}$, consumers do not pay any tip and I have

$$p_i^* = \frac{w_i - w_i\beta\gamma - M(1+\gamma) + w_j(-\beta+\gamma) + c_w W_H\left(-1+\gamma^2\right)\left(-1+\theta_{E_L}\right)}{\gamma^2-1},$$
$$p_j^* = \frac{w_j - w_j\beta\gamma - M(1+\gamma) + w_i(-\beta+\gamma) + c_w W_H\left(-1+\gamma^2\right)\left(-1+\theta_{E_L}\right)}{\gamma^2-1}.$$

In addition, because supply is defined using the positive part function, e.g.,

$$s_i = \Big[\underbrace{w_i + T_i^* - (\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) - \mathbb{1}_{E_i^*=E_H} c_e - \beta\Big(w_j + T_j^* - (\alpha(1-\theta_{E_j^*})\lambda T_j^* + (1-\alpha)T_j^*) - \mathbb{1}_{E_j^*=E_H} c_e\Big)}_{\text{The Inside Part, } s_i^I}\Big]^+$$

the inside part s_i^I can be negative, which still causes $s_i = 0$. This implies that in the cases where transactions do not occur (i.e., $s_i = 0$), there can be infinitely many optimal wages, as long as they satisfy $s_i^I \leq 0$, which result in $\pi_i^{AA} = 0$. For the sake of equilibrium analysis, I choose the wage that makes s_i^I exactly equal to zero as the optimal solution in the case of $\pi_i^{AA} = 0$ (Choosing a different wage that satisfies $s_i^I \leq 0$ will not affect any result, because they all lead to $\pi_i^{AA} = 0$).

Because demand and supply are perfectly matched in equilibrium, then I can formulate platform i's profit maximization problem in the second stage as

$$\begin{aligned} \max_{w_i} \quad & \pi_i^{AA} = (p_i^* - w_i)s_i \\ \text{s.t.} \quad & s_i^I \geq 0. \end{aligned} \tag{EC.3.3}$$

Similarly, platform j's profit maximization problem can be formulated as

$$\begin{aligned} \max_{w_j} \quad & \pi_j^{AA} = (p_j^* - w_j)s_j \\ \text{s.t.} \quad & s_j^I \geq 0. \end{aligned} \tag{EC.3.4}$$

I apply the Karush-Kuhn-Tucker (KKT) conditions to solve EC.3.3 and EC.3.4. In both (a) when $c_w > \overline{c_w}$ and (b) when $c_w \leq \overline{c_w}$, I have $\frac{\partial^2 \pi_i^{AA}}{\partial {w_i}^2} = \frac{2\left(\beta\gamma+\gamma^2-2\right)}{(1-\gamma)(1+\gamma)} < 0 \implies \pi_i^{AA}$ is strictly concave in w_i. And $\frac{\partial^2 \pi_j^{AA}}{\partial {w_j}^2} = \frac{2\left(\beta\gamma+\gamma^2-2\right)}{(1-\gamma)(1+\gamma)} < 0 \implies \pi_j^{AA}$ is strictly concave in w_j. The KKT solutions to EC.3.3 are given by the standard equations:

$$\begin{aligned} \frac{\partial \pi_i^{AA}}{\partial w_i} - \mu\left(\frac{\partial - s_i^I}{\partial w_i}\right) &= 0, \\ \mu &\geq 0, \\ \mu(-s_i^I) &= 0. \end{aligned} \tag{EC.3.5}$$

Similarly, the KKT solutions to EC.3.4 are given by the standard equations:

$$\begin{aligned} \frac{\partial \pi_j^{AA}}{\partial w_j} - h\left(\frac{\partial - s_j^I}{\partial w_j}\right) &= 0, \\ h &\geq 0, \\ h(-s_j^I) &= 0. \end{aligned} \tag{EC.3.6}$$

Solving EC.3.5 and EC.3.6 leads to Proposition 3.1:

(i) when $c_w \le \overline{c_w}$ and $M \ge \overline{M}$,

$$w_i^* = w_j^* = \frac{(1+\gamma)(M + c_w W_H(1-\gamma)(\theta_{E_L}-1))}{(\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4},$$

$$p_i^* = p_j^* = \frac{((\beta-2)\gamma^2 + (\beta-1)\beta\gamma - 2\beta + 3)\,(M + c_w W_H(1-\gamma)(\theta_{E_L}-1))}{(1-\gamma)\,((\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4)};$$

(ii) when $c_w \le \overline{c_w}$ and $M < \overline{M}$,

$$w_i^* = w_j^* = 0,$$

$$p_i^* = p_j^* = \frac{M}{1-\gamma} - c_w W_H(1-\theta_{E_L});$$

(iii) when $c_w > \overline{c_w}$ and $M \ge \widehat{M}$,

$$\begin{aligned} w_i^* = w_j^* = &\Big[\lambda(\theta_{E_H}-\theta_{E_L})(1+\gamma)(M + c_w W_H(1-\gamma)(\theta_{E_H}-1)) - c_e\Big(\left(2\gamma^2-\gamma-4\right)(\lambda-1) \\ &+ \beta(\gamma-3)(1+\gamma)(1+\lambda(\theta_{E_L}-1)) + \lambda\left(3+\gamma-\gamma^2\right)\theta_{E_L} + \beta^2\gamma(1+\lambda(\theta_{E_L}-1)) \\ &+ \lambda\left(1-\gamma^2\right)\theta_{E_H}\Big)\Big] / [\lambda(\theta_{E_H}-\theta_{E_L})\left((\beta-2)\gamma^2 + (1-\beta)^2\gamma + 4 - 3\beta\right)], \\ p_i^* = p_j^* = &[c_e(2-\beta-\gamma) - \lambda(\theta_{E_H}-\theta_{E_L})(M + w_i^*(\beta-1) - c_w W_H(\theta_{E_H}-1)(\gamma-1)) \\ &+ c_e\lambda(\theta_{E_H}(1-\gamma) + \theta_{E_L}(1-\beta) + \beta + \gamma - 2)] / [\lambda(\theta_{E_H}-\theta_{E_L})(\gamma-1)]; \end{aligned}$$

(iv) when $c_w > \overline{c_w}$ and $M < \widehat{M}$,

$$w_i^* = w_j^* = \frac{c_e(\lambda(1-\theta_{E_L})-1)}{\lambda(\theta_{E_H}-\theta_{E_L})},$$

$$p_i^* = p_j^* = c_w W_H(\theta_{E_H}-1) + \frac{c_e(\lambda(1-\theta_{E_H})-1)}{\lambda(\theta_{E_H}-\theta_{E_L})} + \frac{M}{1-\gamma},$$

where $\widehat{M} \equiv (c_w W_H(1-\theta_{E_H}) + c_e)(1-\gamma)$, $\overline{M} \equiv c_w W_H(1-\theta_{E_L})(1-\gamma)$, and $\overline{c_w}$ is defined in Lemma 3.1. Moreover, with these optimal solutions, transaction volume $\min\{d_i, s_i\}$ and profit π_i^{AA} are positive in regions (i) and (iii), but are equal to zero in regions (ii) and (iv). ■

Proof of Proposition 3.2.

When both platforms select the non-adjustable tipping policy, platform i's demand can be obtained by plugging $S_i = S_j = N$ into the demand function:

$$d_i = \Big[M - \Big(p_i + T_i^* + (1-\theta_{E_i^*})c_w W_H\Big) + \gamma\Big(p_j + T_j^* + (1-\theta_{E_j^*})c_w W_H\Big)\Big]^+.$$

Similarly, platform i's supply is

$$s_i = \Big[w_i + T_i^* - \mathbb{1}_{E_i^*=E_H}c_e - \beta\Big(w_j + T_j^* - \mathbb{1}_{E_j^*=E_H}c_e\Big)\Big]^+.$$

From Lemma 3.1 I know that when $S_i = S_j = N$, $T_i^* = 0$ and $E_i^* = E_L$, regardless of c_w. In addition, note that this case is the same as in (b) when $c_w \le \overline{c_w}$ in Proof of Proposition 3.1, where consumers do not pay tip. Therefore, the rest of proof is already covered in Proof of Proposition 3.1 and the optimal solutions are

(i) when $M \geq \overline{M}$,

$$w_i^* = w_j^* = \frac{(1+\gamma)(M + c_w W_H(1-\gamma)(\theta_{E_L}-1))}{(\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4},$$
$$p_i^* = p_j^* = \frac{((\beta-2)\gamma^2 + (\beta-1)\beta\gamma - 2\beta + 3)\,(M + c_w W_H(1-\gamma)(\theta_{E_L}-1))}{(1-\gamma)\,((\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4)};$$

(ii) when $M < \overline{M}$,

$$w_i^* = w_j^* = 0,$$
$$p_i^* = p_j^* = \frac{M}{1-\gamma} - c_w W_H (1-\theta_{E_L}).$$

With these optimal solutions, transaction volume $\min\{d_i, s_i\}$ and platforms' profits are positive in region (i) but equal to zero in region (ii). ■

Proof of Proposition 3.3.

In the asymmetric case where one platform offers the adjustable tipping policy, while the other adopts the non-adjustable policy; without loss of generality, let platform i be the former and platform j be the latter (i.e., $S_i = A$ and $S_j = N$). Hence, platform i's demand is

$$d_i = \Big[M - \Big(p_i + T_i^* - (\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) + (1-\theta_{E_i^*})c_w W_H\Big) + \gamma\Big(p_j + T_j^* + (1-\theta_{E_j^*})c_w W_H\Big)\Big]^+.$$

and platform i's supply is

$$s_i = \Big[w_i + T_i^* - (\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) - \mathbb{1}_{E_i^*=E_H} c_e - \beta\Big(w_j + T_j^* - \mathbb{1}_{E_j^*=E_H} c_e\Big)\Big]^+.$$

On the other hand, platform j's demand is

$$d_j = \Big[M - \Big(p_j + T_j^* + (1-\theta_{E_j^*})c_w W_H\Big) + \gamma\Big(p_i + T_i^* - (\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) + (1-\theta_{E_i^*})c_w W_H\Big)\Big]^+.$$

and platform j's supply is

$$s_j = \Big[w_j + T_j^* - \mathbb{1}_{E_j^*=E_H} c_e - \beta\Big(w_i + T_i^* - (\alpha(1-\theta_{E_i^*})\lambda T_i^* + (1-\alpha)T_i^*) - \mathbb{1}_{E_i^*=E_H} c_e\Big)\Big]^+.$$

From Lemma 3.1 I know that $T_i^* = \begin{cases} \widehat{T}_i & \text{if } c_w > \overline{c_w}, \\ 0 & \text{if } c_w \leq \overline{c_w}, \end{cases}$ and $E_i^* = E_H$ if and only if $T_i^* = \widehat{T}_i$; $T_j^* = 0$ and $E_j^* = E_L$.

In addition, by the same rationale presented in Proof of Proposition 3.1, demand and supply must be perfectly matched for each platform in equilibrium. Hence, the optimal price p_i and p_j in the third stage are obtained from solving $d_i = s_i$ and $d_j = s_j$, respectively, and I have that

(a) when $c_w > \overline{c_w}$:

$$\begin{aligned}
p_i^* =& \Big[c_e(2+\lambda(\theta_{E_H}+\theta_{E_L}-2)-\gamma(\beta+\gamma+(\theta_{E_L}-1)\beta\lambda+(\theta_{E_H}-1)\gamma\lambda))-\Big(\lambda(\theta_{E_H}-\theta_{E_L}) \\
& (w_j(\beta-\gamma)+M(1+\gamma)+w_i(\beta\gamma-1)-c_wW_H(\theta_{E_H}-1)(\gamma^2-1))\Big)\Big]/[\lambda(\theta_{E_H}-\theta_{E_L})(\gamma^2-1)], \\
p_j^* =& [c_e(\beta-\gamma)(1+(\theta_{E_L}-1)\lambda)+\lambda(\theta_{E_H}-\theta_{E_L})(w_i(\beta-\gamma) \\
& +M(1+\gamma)+w_j(\beta\gamma-1)-c_wW_H(\theta_{E_L}-1)(\gamma^2-1))]/[\lambda(\theta_{E_H}-\theta_{E_L})\left(1-\gamma^2\right)],
\end{aligned}$$

and (b) when $c_w \leq \overline{c_w}$:

$$\begin{aligned}
p_i^* &= \frac{w_i - w_i\beta\gamma - M(1+\gamma) + w_j(-\beta+\gamma) + c_wW_H\left(-1+\gamma^2\right)\left(-1+\theta_{E_L}\right)}{\gamma^2-1}, \\
p_j^* &= \frac{w_j - w_j\beta\gamma - M(1+\gamma) + w_i(-\beta+\gamma) + c_wW_H\left(-1+\gamma^2\right)\left(-1+\theta_{E_L}\right)}{\gamma^2-1}.
\end{aligned}$$

Similar to Proof of Proposition 3.1, I let s_i^I denote the inside part of s_i. Because demand and supply are perfectly matched in equilibrium, then I can formulate platform i's profit maximization problem in the second stage as

$$\begin{aligned}
\max_{w_i} \quad & \pi_i^{AN} = (p_i^* - w_i)s_i \\
\text{s.t.} \quad & s_i^I \geq 0.
\end{aligned} \tag{EC.3.7}$$

Similarly, platform j's profit maximization problem can be formulated as

$$\begin{aligned}
\max_{w_j} \quad & \pi_j^{AN} = (p_j^* - w_j)s_j \\
\text{s.t.} \quad & s_j^I \geq 0.
\end{aligned} \tag{EC.3.8}$$

I apply the Karush-Kuhn-Tucker (KKT) conditions to solve EC.3.7 and EC.3.8. In both (a) when $c_w > \overline{c_w}$ and (b) when $c_w \leq \overline{c_w}$, I have $\frac{\partial^2\pi_i^{AN}}{\partial {w_i}^2} = \frac{2\left(\beta\gamma+\gamma^2-2\right)}{(1-\gamma)(1+\gamma)} < 0 \implies \pi_i^{AN}$ is strictly concave in w_i. And $\frac{\partial^2\pi_j^{AN}}{\partial {w_j}^2} = \frac{2\left(\beta\gamma+\gamma^2-2\right)}{(1-\gamma)(1+\gamma)} < 0 \implies \pi_j^{AN}$ is strictly concave in w_j. The KKT solutions to EC.3.7 are given by the standard equations:

$$\begin{aligned}
\frac{\partial\pi_i^{AN}}{\partial w_i} - \mu(\frac{\partial - s_i^I}{\partial w_i}) &= 0, \\
\mu &\geq 0, \\
\mu(-s_i^I) &= 0.
\end{aligned} \tag{EC.3.9}$$

Similarly, the KKT solutions to EC.3.8 are given by the standard equations:

$$\begin{aligned}
\frac{\partial\pi_j^{AN}}{\partial w_j} - h(\frac{\partial - s_j^I}{\partial w_j}) &= 0, \\
h &\geq 0, \\
h(-s_j^I) &= 0.
\end{aligned} \tag{EC.3.10}$$

Solving EC.3.9 and EC.3.10 leads to Proposition 3:

(i) when $c_w \le \overline{c_w}$ and $M \ge \overline{M}$,

$$w_i^* = w_j^* = \frac{(1+\gamma)(M + c_w W_H(1-\gamma)(\theta_{E_L}-1))}{(\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4},$$
$$p_i^* = p_j^* = \frac{((\beta-2)\gamma^2 + (\beta-1)\beta\gamma - 2\beta + 3)\,(M + c_w W_H(1-\gamma)(\theta_{E_L}-1))}{(1-\gamma)\,((\beta-2)\gamma^2 + (\beta-1)^2\gamma - 3\beta + 4)};$$

(ii) when $c_w \le \overline{c_w}$ and $M < \overline{M}$,

$$w_i^* = w_j^* = 0,$$
$$p_i^* = p_j^* = \frac{M}{1-\gamma} - c_w W_H(1-\theta_{E_L});$$

with the above optimal solutions, both platforms' transaction volume and profits are positive in region (i) but equal to zero in region (ii).

(iii) when $c_w > \overline{c_w}$, the optimal prices are different and

$$\begin{aligned}
p_i^* =& \Big[c_e(2 + \lambda(\theta_{E_H} + \theta_{E_L} - 2) - \gamma(\beta + \gamma + (\theta_{E_L} - 1)\beta\lambda + (\theta_{E_H} - 1)\gamma\lambda)) - \Big(\lambda(\theta_{E_H} - \theta_{E_L}) \\
&(w_j^*(\beta - \gamma) + M(1+\gamma) + w_i^*(\beta\gamma - 1) - c_w W_H(\theta_{E_H} - 1)(\gamma^2 - 1))\Big)\Big] / [\lambda(\theta_{E_H} - \theta_{E_L})(\gamma^2 - 1)], \\
p_j^* =& [c_e(\beta - \gamma)(1 + (\theta_{E_L} - 1)\lambda) + \lambda(\theta_{E_H} - \theta_{E_L})(w_i^*(\beta - \gamma) \\
&+ M(1+\gamma) + w_j^*(\beta\gamma - 1) - c_w W_H(\theta_{E_L} - 1)(\gamma^2 - 1))] / [\lambda(\theta_{E_H} - \theta_{E_L})\left(1 - \gamma^2\right)],
\end{aligned}$$

and the optimal wages w_i^* and w_j^* are given by

$$w_i^* = \begin{cases} \frac{c_e(\lambda(1-\theta_{E_L})-1)}{\lambda(\theta_{E_H}-\theta_{E_L})} & \text{if } M < \widehat{M}, \\ w_{im} & \text{if } \widehat{M} \le M < \widetilde{M}, \\ w_{il} & \text{if } M \ge \widetilde{M} \end{cases}$$
$$w_j^* = \begin{cases} 0 & \text{if } M < \widehat{M}, \\ \frac{\beta(1+\gamma)(M + c_e(\gamma-1) + c_w W_H(1-\gamma)(\theta_{E_H}-1))}{\left(\beta^2-2\right)\gamma^2 + \beta\left(\beta^2-1\right)\gamma + 4 - 3\beta^2} & \text{if } \widehat{M} \le M < \widetilde{M}, \\ w_{jl} & \text{if } M \ge \widetilde{M}, \end{cases}$$

and the unique thresholds $\widetilde{M}$, w_{im}, w_{il}, w_{jl} are defined below

$$\begin{aligned}
\widetilde{M} \equiv& (-1+\gamma)\Big(-c_e(\beta+\gamma)(-1+\beta\gamma) + c_w W_H(-4 + \gamma - \gamma(\theta_{E_H} + 2\gamma(-1+\theta_{E_L})) \\
&+ \beta^2(3 + \gamma(-1 + \theta_{E_H} + \gamma(-1+\theta_{E_L})) - 3\theta_{E_L}) + \beta^3\gamma(-1+\theta_{E_L}) + 4\theta_{E_L} + \beta(1+\gamma \\
&+ \gamma^2(-1+\theta_{E_H}) - \theta_{E_H} - \gamma\theta_{E_L}))\Big) / [(-1+\beta)\left(-4 - 3\beta + (1+\beta)^2\gamma + (2+\beta)\gamma^2\right)],
\end{aligned}$$

$$\begin{aligned}
w_{im} \equiv& \Big[-((1+\gamma)(-M + c_w W_H(-1+\gamma)(-1+\theta_{E_H}))(\theta_{E_H} - \theta_{E_L})\lambda) + c_e(-4 - (-4 + \theta_{E_H} + 3\theta_{E_L})\lambda \\
&+ \beta\gamma(1 + (-1+\theta_{E_L})\lambda) - \beta^2\left(-3+\gamma^2\right)(1 + (-1+\theta_{E_L})\lambda) + \beta^3\gamma(-1 + \lambda - \theta_{E_L}\lambda) \\
&+ \gamma^2(2 + (-2 + \theta_{E_H} + \theta_{E_L})\lambda))\Big] / [\left(4 - 3\beta^2 + \beta\left(-1+\beta^2\right)\gamma + \left(-2+\beta^2\right)\gamma^2\right)(\theta_{E_H} - \theta_{E_L})\lambda],
\end{aligned}$$

$$w_{il} \equiv \Big[(1+\gamma)(\theta_{E_H}-\theta_{E_L})(M(-4-3\beta+(1+\beta)^2\gamma+(2+\beta)\gamma^2)-c_wW_H(-1+\gamma)(4-4\theta_{E_H}+\beta(3+2\gamma(-1 + \theta_{E_H})+\gamma^2(-1+\theta_{E_L})-3\theta_{E_L})+\beta^2\gamma(-1+\theta_{E_L})+\gamma(-1+2\gamma(-1+\theta_{E_H})+\theta_{E_L})))\lambda-c_e(-16 - 4(-4+\theta_{E_H}+3\theta_{E_L})\lambda+\beta^4\gamma^2(1+(-1+\theta_{E_L})\lambda)+2\beta^3\gamma(-3+\gamma^2)(1+(-1+\theta_{E_L})\lambda)+\beta^2(9-8\gamma^2+\gamma^4) (1+(-1+\theta_{E_L})\lambda)-2\gamma^4(2+(-2+\theta_{E_H}+\theta_{E_L})\lambda)+\gamma^2(17+(-17+6\theta_{E_H}+11\theta_{E_L})\lambda) + 2\beta\gamma(5+(-5+\theta_{E_H}+4\theta_{E_L})\lambda-\gamma^2(3+(-3+\theta_{E_H}+2\theta_{E_L})\lambda)))\Big]/\Big[\left(4-3\beta+(-1+\beta)^2\gamma+(-2+\beta)\gamma^2\right) \left(-4-3\beta+(1+\beta)^2\gamma+(2+\beta)\gamma^2\right)(\theta_{E_H}-\theta_{E_L})\lambda\Big],$$

$$w_{jl} \equiv (1+\gamma)\Big[M(-4-3\beta+(1+\beta)^2\gamma+(2+\beta)\gamma^2)+(\gamma-1)(c_e(\gamma+\beta(-3+\gamma(\beta+\gamma))) + c_wW_H(-4+\gamma+\beta^2(\gamma-\gamma\theta_{E_H})+\beta(3(-1+\theta_{E_H})+\gamma(2+\gamma-\gamma\theta_{E_H}-2\theta_{E_L}))-\gamma(\theta_{E_H}+2\gamma(-1+\theta_{E_L})) + 4\theta_{E_L}))\Big]/\Big[\left(4-3\beta+(-1+\beta)^2\gamma+(-2+\beta)\gamma^2\right)\left(-4-3\beta+(1+\beta)^2\gamma+(2+\beta)\gamma^2\right)\Big].$$

With these optimal solutions in region (iii), when $M < \widehat{M}$, both platforms' transaction volumes and profits are zero; when $\widehat{M} \leq M < \widetilde{M}$, platform i's transaction volume and profit are positive, whereas those of platform j are zero; when $M \geq \widetilde{M}$, both platforms' transaction volumes and profits are positive. ■

Proof of Proposition 3.4.

From Propositions 1 and 2, I know that when the tipping policy is symmetric, both platforms set the same price and wage, and enjoy the same profit. Then, let π_i^{AA} denote platforms' profit when both platforms adopt the adjustable policy, π_i^{NN} denote platforms' profit when both platforms adopt the non-adjustable policy. In the asymmetric case, one platform offers the adjustable tipping policy, while the other adopts the non-adjustable policy; without loss of generality, let platform i be the former and platform j be the latter (i.e., $S_i = A$ and $S_j = N$). Then, let π_i^{AN} denote platform i's profit and π_j^{AN} denote platform j's profit in the asymmetric case. Next, I consider two cases:

Case 1: when $c_w \leq \overline{c_w}$. In this case, consumers are insensitive to long waiting time and do not pay any tip. As a result, whether the tipping policy is adjustable or not does not affect platforms' consequent decisions on wage and price (when $c_w \leq \overline{c_w}$, the solutions in Proposition 3.1, 2, 3 are the same). Hence, $\pi_i^{AA} = \pi_i^{NN} = \pi_i^{AN} = \pi_j^{AN}$.

Case 2: when $c_w > \overline{c_w}$. In this case, I have the following ranking of thresholds: $\widetilde{M} > \overline{M} > \widehat{M}$. Thus, I consider 4 subcases, depending on the value of M.

Case 2a: when $M \geq \widetilde{M}$. In this case π_i^{AA} is determined with optimal price and wage in Proposition 3.1 region (iii), π_i^{NN} is determined with optimal price and wage in Proposition 3.2 region (i), π_i^{AN} and π_j^{AN} are determined with optimal price and wage in Proposition 3.3 region (iii) when $M \geq \widetilde{M}$. Plugging in these optimal prices and wages, I have

$$\pi_i^{AA} = \frac{(-1+\beta)^2(1+\gamma)(-2+\gamma(\beta+\gamma))(M+c_e(-1+\gamma)+c_wW_H(-1+\gamma+\theta_{E_H}-\gamma\theta_{E_H}))^2}{(-1+\gamma)(4-3\beta+(-1+\beta)^2\gamma+(-2+\beta)\gamma^2)^2}, \quad \text{(EC.3.11)}$$

$$\pi_i^{NN} = \frac{(-1+\beta)^2(1+\gamma)(-2+\gamma(\beta+\gamma))(M+c_wW_H(-1+\gamma+\theta_{E_L}-\gamma\theta_{E_L}))^2}{(-1+\gamma)(4-3\beta+(-1+\beta)^2\gamma+(-2+\beta)\gamma^2)^2}, \tag{EC.3.12}$$

$$\begin{aligned}
\pi_i^{AN} = &\Big[(1+\gamma)(-2+\gamma(\beta+\gamma))(M(-1+\beta)(-4-3\beta+(1+\beta)^2\gamma+(2+\beta)\gamma^2)+c_e(-1+\gamma)(4-3\beta^2\\
&+\beta(-1+\beta^2)\gamma+(-2+\beta^2)\gamma^2)-c_wW_H(-1+\gamma)(-4+\gamma+\beta^3\gamma(-1+\theta_{E_H})+4\theta_{E_H}+\beta(1+\gamma-\gamma\theta_{E_H}\\
&+\gamma^2(-1+\theta_{E_L})-\theta_{E_L})-\gamma(2\gamma(-1+\theta_{E_H})+\theta_{E_L})+\beta^2(3-3\theta_{E_H}+\gamma(-1+\gamma(-1+\theta_{E_H})+\theta_{E_L}))))^2\Big]\\
&/\Big[(-1+\gamma)\left(4-3\beta+(-1+\beta)^2\gamma+(-2+\beta)\gamma^2\right)^2\left(-4-3\beta+(1+\beta)^2\gamma+(2+\beta)\gamma^2\right)^2\Big],
\end{aligned} \tag{EC.3.13}$$

$$\begin{aligned}
\pi_j^{AN} = &\Big[(1+\gamma)(-2+\gamma(\beta+\gamma))(M(-1+\beta)(-4-3\beta+(1+\beta)^2\gamma+(2+\beta)\gamma^2)-(-1+\gamma)\\
&(-c_e(\beta+\gamma)(-1+\beta\gamma)+c_wW_H(-4+\gamma-\gamma(\theta_{E_H}+2\gamma(-1+\theta_{E_L}))+\beta^2(3+\gamma(-1+\theta_{E_H}+\gamma(-1+\theta_{E_L}))\\
&-3\theta_{E_L})+\beta^3\gamma(-1+\theta_{E_L})+4\theta_{E_L}+\beta(1+\gamma+\gamma^2(-1+\theta_{E_H})-\theta_{E_H}-\gamma\theta_{E_L}))))^2\Big]/\Big[(-1+\gamma)\\
&(4-3\beta+(-1+\beta)^2\gamma+(-2+\beta)\gamma^2)^2(-4-3\beta+(1+\beta)^2\gamma+(2+\beta)\gamma^2)^2\Big].
\end{aligned} \tag{EC.3.14}$$

Comparing profits, I have that (1) $\pi_i^{AA} > \pi_i^{NN}$ always holds; (2) $\pi_i^{AN} > \pi_i^{AA}$ and $\pi_j^{AN} < \pi_i^{AA}$ always hold.

Case 2b: when $\overline{M} \le M < \widetilde{M}$. In this case π_i^{AA} is determined with optimal price and wage in Proposition 3.1 region (iii), π_i^{NN} is determined with optimal price and wage in Proposition 3.2 region (i), π_i^{AN} and π_j^{AN} are determined with optimal price and wage in Proposition 3.3 region (iii) when $\widehat{M} \le M < \widetilde{M}$. Plugging in these optimal prices and wages, I have π_i^{AA} is that in EC.3.11, π_i^{NN} is that in EC.3.12, $\pi_j^{AN} = 0$, and $\pi_i^{AN} = \frac{\left(-1+\beta^2\right)^2(1+\gamma)(-2+\gamma(\beta+\gamma))(M+c_e(-1+\gamma)+c_wW_H(-1+\gamma+\theta_{E_H}-\gamma\theta_{E_H}))^2}{(-1+\gamma)\left(4-3\beta^2+\beta\left(-1+\beta^2\right)\gamma+\left(-2+\beta^2\right)\gamma^2\right)^2}$. Comparing profits, I have that (1) $\pi_i^{AA} > \pi_i^{NN}$ always holds; (2) $\pi_i^{AN} > \pi_i^{AA}$ and $0 = \pi_j^{AN} < \pi_i^{AA}$ always hold.

Case 2c: when $\widehat{M} \le M < \overline{M}$. In this case π_i^{AA} is determined with optimal price and wage in Proposition 3.1 region (iii), π_i^{NN} is determined with optimal price and wage in Proposition 3.2 region (ii), π_i^{AN} and π_j^{AN} are determined with optimal price and wage in Proposition 3.3 region (iii) when $\widehat{M} \le M < \widetilde{M}$. Plugging in these optimal prices and wages, I have π_i^{AA} is that in EC.3.11, $\pi_i^{NN} = 0$, $\pi_j^{AN} = 0$, and $\pi_i^{AN} = \frac{\left(-1+\beta^2\right)^2(1+\gamma)(-2+\gamma(\beta+\gamma))(M+c_e(-1+\gamma)+c_wW_H(-1+\gamma+\theta_{E_H}-\gamma\theta_{E_H}))^2}{(-1+\gamma)\left(4-3\beta^2+\beta\left(-1+\beta^2\right)\gamma+\left(-2+\beta^2\right)\gamma^2\right)^2}$. Comparing profits, I have that (1) $\pi_i^{AA} > \pi_i^{NN} = 0$ always holds; (2) $0 = \pi_j^{AN} < \pi_i^{AA}$ always hold; (3) $\pi_i^{AN} \ge \pi_i^{AA}$ always hold, where the equality takes place when $M = \widehat{M}$.

Case 2d: when $M < \widehat{M}$. In this case π_i^{AA} is determined with optimal price and wage in Proposition 3.1 region (iv), π_i^{NN} is determined with optimal price and wage in Proposition 3.2 region (ii), π_i^{AN} and π_j^{AN} are determined with optimal price and wage in Proposition 3.3 region (iii) when $M < \widehat{M}$. Plugging in these optimal prices and wages, I have $\pi_i^{AA} = \pi_i^{NN} = \pi_i^{AN} = \pi_j^{AN} = 0$.

Combining case 1 and the above four subcases of case 2, I have established that the following results: (1) $\pi_i^{AA} \ge \pi_i^{NN}$, (2) $\pi_i^{AN} \ge \pi_i^{AA}$, and (3) $\pi_j^{AN} \le \pi_i^{AA}$. Therefore, if platform i chooses $S_i = A$, platform j should choose $S_j = A$ as well, because $\pi_j^{AA} = \pi_i^{AA} \ge \pi_j^{AN}$. On the other hand, if platform i chooses $S_i = N$, platform j should still choose $S_j = A$, because $\pi_j^{NA} = \pi_i^{AN} \ge \pi_j^{NN}$ (note that π_i^{AN} represents the profit of the platform with adjustable policy in the asymmetric case; so when $S_i = N$ and $S_j = A$, platform j's profit is equal to

π_i^{AN}). Because I have shown that regardless of platform i's tipping policy, platform j is weakly better off by choosing $S_j = A$, I conclude that the adjustable policy is platform j's weakly dominant strategy. By the same logic, I can also show that the adjustable policy is platform i's weakly dominant strategy. In the main paper, I compare tipping policies in a weak sense such that the adjustable policy is each platform's dominant strategy, instead of saying weakly dominant strategy. ■

Proof of Proposition 3.5.

I first rewrite consumer's condition of paying tip (EC.3.1) in terms of α: $c_w > \overline{c_w} \iff \alpha > \overline{\alpha} \equiv \frac{c_e(1-\lambda(1-\theta_{E_H}))}{c_w\lambda W_H(\theta_{E_H}-\theta_{E_L})^2}$. Then, I re-organize Proposition 3.1 by replacing the condition $c_w > \overline{c_w}$ with the equivalent one $\alpha > \overline{\alpha}$. For discussions below, I refer to Proposition 3.1 as the new one where the condition $c_w \geq \overline{c_w}$ is replaced with $\alpha > \overline{\alpha}$. That is, Proposition 3.1 equilibrium region (i) occurs when $\alpha \leq \overline{\alpha}$ and $M \geq \overline{M}$; region (ii) occurs when $\alpha \leq \overline{\alpha}$ and $M < \overline{M}$; region (iii) occurs when $\alpha > \overline{\alpha}$ and $M \geq \widehat{M}$; region (iv) occurs when $\alpha > \overline{\alpha}$ and $M < \widehat{M}$. In addition, it holds that $0 < \overline{\alpha} < 1 \iff c_w > \widehat{c_w} \equiv \frac{c_e(1-\lambda(1-\theta_{E_H}))}{\lambda W_H(\theta_{E_H}-\theta_{E_L})^2}$. Then, I discuss two cases based on the value of c_w.

Case 1: $c_w > \widehat{c_w}$. In this case, $\overline{M} > \widehat{M}$. Also note that both M-thresholds do not contain α. I further discuss three subcases based on the value of M.

Case 1a: when $M < \widehat{M}$. In this case,

$$\pi_i^{AA} = \begin{cases} 0 & \text{if } \alpha > \overline{\alpha}, \\ 0 & \text{if } \alpha \leq \overline{\alpha}, \end{cases}$$

where the first zero is obtained in region (iv) and the second profit is in region (ii). Hence, $\frac{\partial \pi_i^{AA}}{\partial \alpha} = 0$ for all α.

Case 1b: when $\overline{M} > M \geq \widehat{M}$. In this case,

$$\pi_i^{AA} = \begin{cases} \frac{(-1+\beta)^2(1+\gamma)(-2+\gamma(\beta+\gamma))(M+c_e(-1+\gamma)+c_wW_H(-1+\gamma+\theta_{E_H}-\gamma\theta_{E_H}))^2}{(-1+\gamma)(4-3\beta+(-1+\beta)^2\gamma+(-2+\beta)\gamma^2)^2} & \text{if } \alpha > \overline{\alpha}, \\ 0 & \text{if } \alpha \leq \overline{\alpha}, \end{cases}$$

where the positive profit is obtained in region (iii) and the zero profit is in region (ii). Hence, when either $\alpha > \overline{\alpha}$ or $\alpha < \overline{\alpha}$, I have $\frac{\partial \pi_i^{AA}}{\partial \alpha} = 0$. And π_i^{AA} is not differentiable at the discontinuity point $\alpha = \overline{\alpha}$. In addition, π_i^{AA} is obviously the highest if $\alpha > \overline{\alpha}$.

Case 1c: when $M \geq \overline{M}$. In this case,

$$\pi_i^{AA} = \begin{cases} \frac{(-1+\beta)^2(1+\gamma)(-2+\gamma(\beta+\gamma))(M+c_e(-1+\gamma)+c_wW_H(-1+\gamma+\theta_{E_H}-\gamma\theta_{E_H}))^2}{(-1+\gamma)(4-3\beta+(-1+\beta)^2\gamma+(-2+\beta)\gamma^2)^2} & \text{if } \alpha > \overline{\alpha}, \\ \frac{(-1+\beta)^2(1+\gamma)(-2+\gamma(\beta+\gamma))(M+c_wW_H(-1+\gamma+\theta_{E_L}-\gamma\theta_{E_L}))^2}{(-1+\gamma)(4-3\beta+(-1+\beta)^2\gamma+(-2+\beta)\gamma^2)^2} & \text{if } \alpha \leq \overline{\alpha}, \end{cases}$$

where the first profit is obtained in region (iii) and the second profit is in region (i). Hence, when either $\alpha > \overline{\alpha}$ or $\alpha < \overline{\alpha}$, I have $\frac{\partial \pi_i^{AA}}{\partial \alpha} = 0$. And π_i^{AA} is not differentiable at the discontinuity point $\alpha = \overline{\alpha}$. In addition, I have established that the first profit (π_i^{AA} when $\alpha > \overline{\alpha}$) is strictly larger than the second profit (π_i^{AA} when $\alpha \le \overline{\alpha}$) in case 2a and 2b from the Proof of Proposition 3.4.

Case 2: $c_w \le \widehat{c_w}$. In this case, $\overline{\alpha} \ge 1$ so that the condition $\alpha > \overline{\alpha}$ never holds and the model only has two equilibrium regions where consumers do not pay tip (region (i) and (ii)). For all α, the model is in either region (i) or (ii), but not both. Hence, the profit function is no longer a piece-wise function and thus $\frac{\partial \pi_i^{AA}}{\partial \alpha} = 0$ for all α.

Combining case 1 and 2, I have proven Proposition 3.5 (1) when $c_w > \widehat{c_w}$ and $M \ge \widehat{M}$, π_i^{AA} is the highest if $\alpha > \overline{\alpha}$, and $\frac{\partial \pi_i^{AA}}{\partial \alpha} = 0$ for all $\alpha \neq \overline{\alpha}$; and (2) when either $c_w \le \widehat{c_w}$ or $M < \widehat{M}$, $\frac{\partial \pi_i^{AA}}{\partial \alpha} = 0$ for all α. ■

Proof of Proposition 3.6.

In the benchmark where $\alpha = 1$, platforms' optimal solutions are almost identical to that in Proposition 3.1, except the condition in the main model $c_w > \overline{c_w} = \frac{c_e(1-\lambda(1-\theta_{E_H}))}{\alpha\lambda W_H(\theta_{E_H}-\theta_{E_L})^2}$ is replaced by $c_w > \widehat{c_w} \equiv \frac{c_e(1-\lambda(1-\theta_{E_H}))}{\lambda W_H(\theta_{E_H}-\theta_{E_L})^2}$ ($\widehat{c_w}$ equals to $\overline{c_w}$ when $\alpha = 1$). Hence, $\overline{c_w} > \widehat{c_w}$. In addition, the expressions of p^*, w^*, $\overline{M}$, and $\widehat{M}$ do not contain α and are unaffected. For notational convenience, I use the subscript k, $k \in \{1,2,3,4\}$, to denote the equilibrium region number in Proposition 3.1. For example, p_1^* denote that the optimal price is in region (i) in Proposition 3.1. Moreover, it is easy to show that $\overline{M} > \widehat{M}$ when $c_w > \widehat{c_w}$.

I then discuss several cases, using platform i as example. Note that the case for platform j is the same.

Case 1: when $c_w \le \widehat{c_w}$ and $M < \overline{M}$. In this case, the optimal solutions in both the benchmark and the main model are in region (ii), where no transactions occur. Hence, $U_F = U_F^{\Delta} = 0$ and $H_F = 0$, and $U_W = U_W^{\Delta} = 0$ and $H_W = 0$.

Case 2: when $c_w \le \widehat{c_w}$ and $M \ge \overline{M}$. In this case, the optimal solutions in both the benchmark and the main model are in region (i). Hence, $U_F = U_F^{\Delta} = v - t_c x - \left(p^* + (1-\theta_{E_L})c_w W_H\right)$, and therefore $H_F = U_F^{\Delta} - U_F = 0$. Similarly, a worker's utility in the main model and in the benchmark are $U_W = U_W^{\Delta} = w^* - t_w x - g$. Hence, $H_W = U_W^{\Delta} - U_W = 0$.

Case 3: when $\widehat{c_w} < c_w \le \overline{c_w}$ and $M < \widehat{M}$. In this case, the optimal solutions in the benchmark are in region (iv) where no transactions occur, and the optimal solutions in the main model are in region (ii) where no transactions occur. Hence, $H_F = H_W = 0$.

Case 4: when $\widehat{c_w} < c_w \le \overline{c_w}$ and $\widehat{M} \le M < \overline{M}$. In this case, the optimal solutions in the benchmark are in region (iii), and the optimal solutions in the main model are in region (ii) where no transactions occur. Hence, $H_F = U_F^{\Delta} - 0 = v - t_c x - \left(p_3^* + \widehat{T_i^{\Delta}} - (1-\theta_{E_H})\lambda\widehat{T_i^{\Delta}} + (1-\theta_{E_H})c_w W_H\right)$. Because consumers choose to order food on platforms, it must be that $U_F^{\Delta} \ge 0$, otherwise consumers would have not participated. Hence, $H_F = U_F^{\Delta} \ge 0$. Regarding workers, $H_W = U_W^{\Delta} - 0 = w_3^* + \widehat{T_i^{\Delta}} - (1-\theta_{E_H})\lambda\widehat{T_i^{\Delta}} - c_e - t_w x - g \ge 0$ (transaction occurs means that workers must have non-negative utilities, otherwise they would have not participated).

Case 5: when $\widehat{c_w} < c_w \leq \overline{c_w}$ and $M \geq \overline{M}$. In this case, the optimal solutions in the benchmark are in region (iii), and the optimal solutions in the main model are in region (i). Therefore, I have

$$\begin{aligned} H_F &= \underbrace{v - t_c x - \left(p_3^* + \widehat{T_i^\Delta} - (1-\theta_{E_H})\lambda \widehat{T_i^\Delta} + (1-\theta_{E_H}) c_w W_H\right)}_{U_F^\Delta} - \Big(\underbrace{v - t_c x - \left(p_1^* + (1-\theta_{E_L}) c_w W_H\right)}_{U_F}\Big) \\ &= \frac{(1-\beta)(1+\gamma)(-c_e + c_w W_H \theta_{E_H} - c_w W_H \theta_{E_L})}{4 - 3\beta + \gamma - 2\beta\gamma + \beta^2\gamma - 2\gamma^2 + \beta\gamma^2} \end{aligned}$$

where $\widehat{T_i^\Delta} \equiv \frac{c_e}{\lambda(\theta_{E_H} - \theta_{E_L})}$ is obtained by setting $\alpha = 1$ in $\widehat{T_i}$ in the main model. In addition, $H_F > 0$ always holds in this case where $c_w > \widehat{c_w}$. Similarly, I have

$$\begin{aligned} H_W &= w_3^* + \widehat{T_i^\Delta} - (1-\theta_{E_H})\lambda \widehat{T_i^\Delta} - c_e - w_1^* \\ &= \frac{(-1+\gamma^2)\left(c_e + c_w W_H(-\theta_{E_H} + \theta_{E_L})\right)}{4 - 3\beta + (-1+\beta)^2\gamma + (-2+\beta)\gamma^2}. \end{aligned}$$

In addition, $H_W > 0$ always holds in this case where $c_w > \widehat{c_w}$.

Case 6: when $c_w > \overline{c_w}$ and $M < \widehat{M}$. In this case, the optimal solutions in both the benchmark and the main model are in region (iv), where no transactions occur. Hence, $H_F = H_W = 0$.

Case 7: when $c_w > \overline{c_w}$ and $M \geq \widehat{M}$. In this case, the optimal solutions in both the benchmark and the main model are in region (iii). Therefore, I have

$$\begin{aligned} H_F &= \underbrace{v - t_c x - \left(p_3^* + \widehat{T_i^\Delta} - (1-\theta_{E_H})\lambda \widehat{T_i^\Delta} + (1-\theta_{E_H}) c_w W_H\right)}_{U_F^\Delta} - \Big(\underbrace{v - t_c x - \left(p_3^* + \widehat{T_i} - (1-\theta_{E_H})\lambda \widehat{T_i} + (1-\theta_{E_H}) c_w W_H\right)}_{U_F}\Big) \\ &= (\widehat{T_i} - \widehat{T_i^\Delta})(1 - (1-\theta_{E_H})\lambda) > 0, \end{aligned}$$

where the "$>$" sign follows because $\widehat{T_i} > \widehat{T_i^\Delta}$. Similarly, I have

$$\begin{aligned} H_W &= w_3^* + \widehat{T_i^\Delta} - (1-\theta_{E_H})\lambda \widehat{T_i^\Delta} - c_e - t_w x - g - (w_3^* + \widehat{T_i} - (\alpha(1-\theta_{E_H})\lambda \widehat{T_i} + (1-\alpha)\widehat{T_i}) - c_e - t_w x - g) \\ &= (\widehat{T_i^\Delta} - \alpha * \widehat{T_i})(1 - (1-\theta_{E_H})\lambda) \\ &= 0. \end{aligned}$$

Because I have exhausted all cases, I conclude that $H_F \geq 0$ and $H_W \geq 0$ always hold. ■

Proof of Proposition 3.7.

This proof can be similarly derived as in Proof of Proposition 3.5. I first rewrite consumer's condition of paying tip (EC.3.1) in terms of α: $c_w > \overline{c_w} \iff \lambda > \overline{\lambda} \equiv \frac{c_e}{c_e(1-\theta_{E_H}) + c_w W_H(\theta_{E_H} - \theta_{E_l})^2}$. Then, I re-organize Proposition 3.1 by replacing the condition $c_w > \overline{c_w}$ with the equivalent one $\lambda > \overline{\lambda}$. For discussions below, I refer to Proposition 3.1 as the new one where the condition $c_w \geq \overline{c_w}$ is replaced with $\lambda > \overline{\lambda}$. That is, Proposition 3.1 equilibrium region (i) occurs when $\lambda \leq \overline{\lambda}$ and $M \geq \overline{M}$; region (ii) occurs when $\lambda \leq \overline{\lambda}$ and

$M < \overline{M}$; region (iii) occurs when $\lambda > \overline{\lambda}$ and $M \geq \widehat{M}$; region (iv) occurs when $\lambda > \overline{\lambda}$ and $M < \widehat{M}$. As I aim to prove this proposition while transactions occur, I focus below on region (i) and region (iii) where transactions volume is positive (recall that no transactions happen in regions (ii) and (iv)).

For notational convenience, I use the subscript k, $k \in \{1,2,3,4\}$, to denote the equilibrium region number in Proposition 3.1. For example, p_1^* denote that the optimal price is in region (i) in Proposition 3.1. Hence, when tips are paid ($\lambda > \overline{\lambda}$), the equilibrium solution is in region (iii) with optimal price p_3^* and wage w_3^*. In the scenario where transactions happen without tips, the equilibrium region is in region (i) with optimal price p_1^* and wage w_1^*. Hence, I have

$$p_3^* > p_1^* \iff \lambda > \lambda_p \equiv c_e(3\beta-\gamma+2\beta\gamma-\beta^2\gamma+2\gamma^2-\beta\gamma^2-4)/\Big[c_wW_H(-3\theta_{E_H}{}^2+2\beta\theta_{E_H}{}^2+\beta\gamma\theta_{E_H}{}^2$$
$$-\beta^2\gamma\theta_{E_H}{}^2+2\gamma^2\theta_{E_H}{}^2-\beta\gamma^2\theta_{E_H}{}^2+6\theta_{E_H}\theta_{E_L}-4\beta\theta_{E_H}\theta_{E_L}-2\beta\gamma\theta_{E_H}\theta_{E_L}+2\beta^2\gamma\theta_{E_H}\theta_{E_L}-4\gamma^2\theta_{E_H}\theta_{E_L}+2\beta\gamma^2\theta_{E_H}\theta_{E_L}$$
$$-3\theta_{E_L}{}^2+2\beta\theta_{E_L}{}^2+\beta\gamma\theta_{E_L}{}^2-\beta^2\gamma\theta_{E_L}{}^2+2\gamma^2\theta_{E_L}{}^2-\beta\gamma^2\theta_{E_L}{}^2)+c_e(-4+3\beta-\gamma+2\beta\gamma-\beta^2\gamma+2\gamma^2-\beta\gamma^2+3\theta_{E_H}$$
$$-2\beta\theta_{E_H}-\beta\gamma\theta_{E_H}+\beta^2\gamma\theta_{E_H}-2\gamma^2\theta_{E_H}+\beta\gamma^2\theta_{E_H}+\theta_{E_L}-\beta\theta_{E_L}+\gamma\theta_{E_L}-\beta\gamma\theta_{E_L})\Big]\, and$$

$$w_3^* > w_1^* \iff \lambda > \lambda_w \equiv c_e(3\beta-\gamma+2\beta\gamma-\beta^2\gamma+2\gamma^2-\beta\gamma^2-4)/\Big[c_e(-4+3\beta-\gamma+2\beta\gamma-\beta^2\gamma+2\gamma^2-\beta\gamma^2$$
$$+\theta_{E_H}-\gamma^2\theta_{E_H}+3\theta_{E_L}-3\beta\theta_{E_L}+\gamma\theta_{E_L}-2\beta\gamma\theta_{E_L}+\beta^2\gamma\theta_{E_L}-\gamma^2\theta_{E_L}+\beta\gamma^2\theta_{E_L})+c_wW_H(-\theta_{E_H}{}^2+\gamma^2\theta_{E_H}{}^2$$
$$+2\theta_{E_H}\theta_{E_L}-2\gamma^2\theta_{E_H}\theta_{E_L}-\theta_{E_L}{}^2+\gamma^2\theta_{E_L}{}^2)\Big].\blacksquare$$

Proof of Proposition 3.8.

I first prove Proposition 3.8 (i) that when $c_w \leq \overline{c_w}$, $p^* < 0$ if and only if $M < \overline{M}$, $w^* \geq 0$ always holds. Depending on the value of M, the equilibrium solutions are in either region (i) or (ii) of Proposition 3.1.

Case 1a: when $M \geq \overline{M}$. In this case, the equilibrium solutions are in region (i), where

$$w^* = \frac{(1+\gamma)(M+c_wW_H(1-\gamma)(\theta_{E_L}-1))}{(\beta-2)\gamma^2+(\beta-1)^2\gamma-3\beta+4},$$
$$p^* = \frac{((\beta-2)\gamma^2+(\beta-1)\beta\gamma-2\beta+3)\,(M+c_wW_H(1-\gamma)(\theta_{E_L}-1))}{(1-\gamma)\,((\beta-2)\gamma^2+(\beta-1)^2\gamma-3\beta+4)}.$$

In this case where $M \geq \overline{M}$, $p^* > 0$ and $w^* > 0$ always hold.

Case 1b: when $M < \overline{M}$. In this case, the equilibrium solutions are in region (ii), where

$$w^* = 0,$$
$$p^* = \frac{M}{1-\gamma} - c_wW_H(1-\theta_{E_L}).$$

In this case where $M < \overline{M}$, $p^* < 0$ and $w^* = 0$ always hold.

Hence, I have shown that when $c_w \leq \overline{c_w}$, $w^* \geq 0$ always holds. Regarding the price part, the direction that $M < \overline{M} \implies p^* < 0$ is already shown in case 1b. According to proof by contrapositive, proving the other direction that $p^* < 0 \implies M < \overline{M}$ is equivalent to proving $M \geq \overline{M} \implies p^* \geq 0$, which is true as I have shown in case 1a. Hence, Proposition 3.8 (i) is true.

Next, I prove Proposition 3.8 (ii) that when $c_w > \overline{c_w}$, $p^* < 0$ if and only if $M < M_p$, $w^* < 0$ if and only if $M < M_w$. Depending on the value of M, the equilibrium solutions are in either region (iii) or (iv) of Proposition 3.1.

Case 2a: when $M \geq \widehat{M}$. In this case, the equilibrium solutions are in region (iii), where

$$\begin{aligned} w^* =& \Big[\lambda(\theta_{E_H} - \theta_{E_L})(1+\gamma)(M + c_w W_H(1-\gamma)(\theta_{E_H} - 1)) - c_e\Big(\left(2\gamma^2 - \gamma - 4\right)(\lambda - 1) \\ &+ \beta(\gamma - 3)(1+\gamma)(1 + \lambda(\theta_{E_L} - 1)) + \lambda\left(3 + \gamma - \gamma^2\right)\theta_{E_L} + \beta^2\gamma(1 + \lambda(\theta_{E_L} - 1)) \\ &+ \lambda\left(1 - \gamma^2\right)\theta_{E_H}\Big)\Big] / [\lambda(\theta_{E_H} - \theta_{E_L})\left((\beta - 2)\gamma^2 + (1-\beta)^2\gamma + 4 - 3\beta\right)], \\ p^* =& [c_e(2 - \beta - \gamma) - \lambda(\theta_{E_H} - \theta_{E_L})(M + w^*(\beta - 1) - c_w W_H(\theta_{E_H} - 1)(\gamma - 1)) \\ &+ c_e\lambda(\theta_{E_H}(1-\gamma) + \theta_{E_L}(1-\beta) + \beta + \gamma - 2)] / [\lambda(\theta_{E_H} - \theta_{E_L})(\gamma - 1)], \text{ and} \end{aligned}$$

$$\begin{aligned} &p^* < 0 \iff M < M_p \equiv \Big[(-1+\gamma)(c_w W_H(3 - 2\beta + (-1+\beta)\beta\gamma + (-2+\beta)\gamma^2)(-1+\theta_{E_H})(\theta_{E_H} - \theta_{E_L})\lambda \\ &- c_e(4 + \gamma - 2\gamma^2 + (-4 + 3\theta_{E_H} + \theta_{E_L} + \gamma(-1 - 2\gamma(-1+\theta_{E_H}) + \theta_{E_L}))\lambda + \beta^2\gamma(1 + (-1+\theta_{E_H})\lambda) \\ &+ \beta(1+\gamma)(-3 + \gamma + \gamma(-1+\theta_{E_H})\lambda - (-3 + 2\theta_{E_H} + \theta_{E_L})\lambda)))\Big] / [(3 - 2\beta + (-1+\beta)\beta\gamma + (-2+\beta)\gamma^2)(\theta_{E_H} - \theta_{E_L})\lambda]. \end{aligned}$$

Similarly,

$$\begin{aligned} &w^* < 0 \iff M < M_w \equiv \frac{c_w W_H(-1+\gamma^2)(-1+\theta_{E_H})}{1+\gamma} + \Big[c_e(4 + \gamma - 2\gamma^2 + (-4 + \theta_{E_H} + 3\theta_{E_L} + \gamma(-1 + \theta_{E_L} \\ &- \gamma(-2 + \theta_{E_H} + \theta_{E_L})))\lambda + \beta^2\gamma(1 + (-1+\theta_{E_L})\lambda) + \beta(-3+\gamma)(1+\gamma)(1 + (-1+\theta_{E_L})\lambda))\Big] / [(1+\gamma)(\theta_{E_H} - \theta_{E_L})\lambda]. \end{aligned}$$

In addition, $M_p > \widehat{M}$ and $M_w > \widehat{M}$ always hold.

Case 2b: when $M < \widehat{M}$. In this case, the equilibrium solutions are in region (iv), where

$$\begin{aligned} w^* &= \frac{c_e(\lambda(1 - \theta_{E_L}) - 1)}{\lambda(\theta_{E_H} - \theta_{E_L})}, \\ p^* &= c_w W_H(\theta_{E_H} - 1) + \frac{c_e(\lambda(1 - \theta_{E_H}) - 1)}{\lambda(\theta_{E_H} - \theta_{E_L})} + \frac{M}{1-\gamma}. \end{aligned}$$

In this case where $M < \widehat{M}$, $p^* < 0$ and $w^* < 0$ always hold.

Combining case 2a and 2b, I have proven Proposition 3.8 (ii) that when $c_w > \overline{c_w}$, $p^* < 0$ if and only if $M < M_p$, $w^* < 0$ if and only if $M < M_w$. ■

Appendix. C Proofs of Propositions

Proof of Proposition 4.1

Let n^*, $\overline{p}^*$, γ^* denote firm A's optimal number of promotion outcomes, optimal list price, and optimal probability of winning lottery, respectively. Also let p_B^* denote firm B's optimal price. Suppose for contradiction that $n^* \geq 2$.

After drawing the lottery at firm A, consumers can either purchase at the realized price at firm A or switch to firm B. I then discuss two cases separately:

Case 1: for any realized price, consumers choose to continue purchasing at firm A. In this case, consumers will not switch even if the realized price is the highest one $\overline{p}^*$. The expected transaction price from visiting firm A is

$$\begin{aligned}
\mathbb{E}[p_1] &= \frac{\gamma^*}{n^*}\sum_{i=0}^{n^*-1}\frac{i}{n^*}\overline{p}^* + (1-\gamma^*)\overline{p}^* \\
&= \frac{\gamma^*}{n^*}\frac{\overline{p}^*}{n^*}\frac{n(n-1)}{2} + (1-\gamma^*)\overline{p}^* \\
&= \frac{\overline{p}^*(2n^* - (1+n^*)\gamma^*)}{2n^*}.
\end{aligned}$$

From problem (4.8) in the paper, I have that firm A's profit is

$$\Pi_A(\overline{p}^*, \gamma^*, n^*) = \mathbb{E}[p_1][\beta(\frac{n^*(p_B^*+t) + \alpha\gamma^* - \mathbb{E}[p_1]n^*}{2tn^*}) + (1-\beta)(\frac{p_B^*+t-\overline{p}^*}{2t})].$$

Now, I construct a simple lottery that has the same expected price as the original lottery, but contains only one promotion outcome—zero price. Let $\overline{p}_s$ and γ_s denote the alternate price and probability of winning from the simple lottery. Then by construction I have

$$(1-\gamma_s)\overline{p}_s = \mathbb{E}[p_1] = \frac{\overline{p}^*(2n^* - (1+n^*)\gamma^*)}{2n^*}. \tag{EC.4.1}$$

Let $\overline{p}_s = \overline{p}^*$, then I can solve for $\gamma_s = \frac{1+n^*}{2n^*}\gamma^*$. The profit from offering this simple lottery is

$$\begin{aligned}
\Pi_A(\overline{p}_s, \gamma_s) &= (1-\gamma_s)\overline{p}_s\left[\beta(\frac{p_B^*+t+\alpha\gamma_s - (1-\gamma_s)\overline{p}_s}{2t}) + (1-\beta)(\frac{p_B^* - \overline{p}_s + t}{2t})\right] &\text{(EC.4.2)}\\
&= \mathbb{E}[p_1]\left[\beta(\frac{p_B^*+t+\alpha\gamma_s - \mathbb{E}[p_1]}{2t}) + (1-\beta)(\frac{p_B^*+t-\overline{p}^*}{2t})\right] &\text{(EC.4.3)}
\end{aligned}$$

Hence, the difference in profits between these two lotteries is that

$$\begin{aligned}
\Pi_A(\overline{p}_s, \gamma_s) - \Pi_A(\overline{p}^*, \gamma^*, n^*) &= \mathbb{E}[p_1]\beta(\frac{p_B^*+t+\alpha\gamma_s - \mathbb{E}[p_1]}{2t} - \frac{n^*(p_B^*+t)+\alpha\gamma^* - \mathbb{E}[p_1]n^*}{2tn^*}) \\
&= \mathbb{E}[p_1]\beta(\frac{p_B^*+t+\alpha(\frac{1+n^*}{2n^*}\gamma^*) - \mathbb{E}[p_1]}{2t} - \frac{(p_B^*+t)+\alpha\frac{\gamma^*}{n^*} - \mathbb{E}[p_1]}{2t}) \\
&= \mathbb{E}[p_1]\beta(\frac{\alpha\gamma^*(\frac{n^*-1}{2n^*})}{2t}) \\
&> 0.
\end{aligned}$$

Therefore, the original lottery with $n \geq 2$ is not optimal, as firm A can profitably deviate by offering this constructed simple lottery with $n = 1$. I then derive contradiction and conclude that $n^* = 1$.

Case 2: Consumers choose to continue purchasing at firm A only if the realized price $p_A \leq \frac{k}{n^*}\overline{p}^*$, where $k \in \mathbb{Z}$ and $0 \leq k \leq n-1$. The expected transaction price from visiting firm A is

$$\begin{aligned}\mathbb{E}[p_2] &= \frac{\gamma^*}{n^*}[\sum_{i=0}^{k}\frac{i}{n^*}\overline{p}^* + (n^* - k - 1)(p_B^* + t)] + (1-\gamma^*)(p_B^* + t) \\ &= \frac{\gamma^*}{n^*}[\frac{\overline{p}^* k(k+1)}{2n^*} + (n^* - k - 1)(p_B^* + t)] + (1-\gamma^*)(p_B^* + t).\end{aligned} \tag{EC.4.4}$$

Note that $\mathbb{E}[p_2]$ is not the expected price that consumers will pay to firm A, but rather the expected transaction price (pay to either firm A or B) conditional on visiting firm A. In EC.4.4, $\frac{\gamma^*}{n^*}\frac{\overline{p}^* k(k+1)}{2n^*}$ is the expected price that consumers pay to the firm A; when the realized price is greater than $\frac{k}{n^*}\overline{p}^*$, consumers switch to firm B, which entails cost $p_B^* + t$. Now I show that $k \neq 0$ in equilibrium.

Suppose $k = 0$ for contradiction. This suggests that consumers only purchase at firm A if the realized price is zero, so that firm A would earn zero profit. However, this cannot sustain in equilibrium, as firm A can deviate to earn a positive profit by setting the alternate price $\overline{p}^* = p_B^*$. By doing so, firm A would be able to attract some consumers to draw the lottery at firm A, since the worst outcome is to receive the highest price p_B^* (same as the competitor's fixed price). After drawing the lottery, consumers who receive the lottery price $\frac{\overline{p}^*}{n}$ would buy at firm A, since the paying price at firm A, $\frac{\overline{p}^*}{n} = \frac{p_B^*}{n}$, is lower than that of switching to firm B—$p_B^* + t$. And this entails that $k \geq 1$ (because there exists at least one positive price, $\frac{\overline{p}^*}{n}$, consumers will accept). Hence I derive contradiction and conclude $k \geq 1$.

Now I have concluded that $1 \leq k \leq n-1$. From discussion in the main paper, I have that in case 2, $\frac{k}{n^*}\overline{p}^*$ being the highest acceptable price implies $\overline{p}^* > p_B^* + t$ and $\frac{k}{n^*}\overline{p}^* \leq p_B^* + t$, which is then equivalent to $p_B^* + t < \overline{p}^* \leq \frac{n(p_B^*+t)}{k}$. In this case, the skeptical consumers who will visit firm A is denoted as $[\frac{p_B^*+t-\overline{p}^*}{2t}]^+ = 0$. That is, the alternate price is too high and skeptical consumers will not visit firm A as they assume to receive this alternate price for sure. From demand expression (4.10) I have that firm A's demand is

$$\bar{x} = \begin{cases} \frac{n^*(p_B^*+t)+\alpha\gamma^* - \mathbb{E}[p_2]n^*}{2tn^*} & \text{if consumer is trusting,} \\ 0 & \text{if consumer is skeptical.} \end{cases} \tag{EC.4.5}$$

Therefore, firm A's profit from trusting consumers is

$$\begin{aligned}\Pi_A^T(\overline{p}^*, \gamma^*, n^*) &= \frac{\gamma^*}{n^*}\frac{\overline{p}^* k(k+1)}{2n^*}\beta(\frac{n^*(p_B^*+t) + \alpha\gamma^* - \mathbb{E}[p_2]n^*}{2tn^*}) \\ &= \frac{\gamma^*}{n^*}\frac{\overline{p}^* k(k+1)}{2n^*}\beta(\frac{n^*(p_B^*+t) + \alpha\gamma^* - (\frac{\gamma^*}{n^*}[\frac{\overline{p}^* k(k+1)}{2n^*} + (n^* - k - 1)(p_B^* + t)] + (1-\gamma^*)(p_B^* + t))n^*}{2tn^*}),\end{aligned}$$

and firm A's profit from skeptical consumers is

$$\Pi_A^S(\overline{p}^*, \gamma^*, n^*) = 0. \tag{EC.4.6}$$

Now, I construct a refined lottery that eliminates all the lottery outcomes at which consumers would not pay. That is, with probability 1, consumers win the refined lottery whose set of outcomes is $\{0, \frac{\overline{p}^*}{n^*}, \frac{2\overline{p}^*}{n^*}, \ldots, \frac{k\overline{p}^*}{n^*}\}$ (since there are $k+1$ outcomes in this set, the realized price $p_A = \frac{i\overline{p}^*}{n^*}$ with probability $\frac{1}{k+1}$, where $i \in \{0, 1, \ldots, k\}$). The expected price of this refined lottery is

$$\frac{\overline{p}^*}{(k+1)n^*}\sum_{i=0}^{k} i = \frac{\overline{p}^* k}{2n^*}.$$

When offering this refined lottery, a trusting consumer's expected utility of visiting firm A is

$$u_{AR}^{T}(x) = v - \frac{\overline{p}^* k}{2n^*} + \frac{1}{k+1}\alpha - tx^2. \tag{EC.4.7}$$

A skeptical consumer assumes the worst outcome and has the following "expected" utility of visiting from firm A offering the refined lottery:

$$u_{AR}^{S}(x) = v - \frac{k\overline{p}^*}{n^*} - tx^2. \tag{EC.4.8}$$

Recall that because firm B's price is fixed and transparent, both trusting and skeptical consumers derive the same utility of purchasing from firm B:

$$u_B(x) = v - p_B^* - t(1-x)^2. \tag{EC.4.9}$$

Then, define the consumer located at $\overline{x}^R$ as the consumer indifferent between visiting firm A and visiting firm B when offered the refined lottery:

$$\begin{cases} u_{AR}^{T}(\overline{x}^R) = u_B(\overline{x}^R) & \text{if consumer is trusting,} \\ u_{AS}^{S}(\overline{x}^R) = u_B(\overline{x}^R) & \text{if consumer is skeptical.} \end{cases} \tag{EC.4.10}$$

Equation (4.10) solves uniquely for

$$\overline{x}^R = \begin{cases} \frac{n^*(2p_B^*(1+k)+2t(1+k)+2\alpha)-k\overline{p}^*(1+k)}{4(1+k)tn^*} & \text{if consumer is trusting,} \\ \frac{(p_B^*+t)n^*-k\overline{p}^*}{2tn^*} & \text{if consumer is skeptical.} \end{cases} \tag{EC.4.11}$$

Therefore, firm A's profit from trusting consumers when offering this refined lottery is

$$\Pi_{AR}^{T}(\overline{p}^*, n^*) = \frac{\overline{p}^* k}{2n^*}\beta\left(\frac{n^*(2p_B^*(1+k)+2t(1+k)+2\alpha)-k\overline{p}^*(1+k)}{4(1+k)tn^*}\right)$$

and firm A's profit from skeptical consumers when offering this refined lottery is

$$\Pi_{AR}^{S}(\overline{p}^*, n^*) = \frac{\overline{p}^* k}{2n^*}(1-\beta)\frac{(p_B^*+t)n^*-k\overline{p}^*}{2tn^*}.$$

Next, the difference in trusting consumers' profits from the refined lottery and that from the original lottery is

$$\begin{aligned}
&\Pi_{AR}^{T}(\overline{p}^*, n^*) - \Pi_{A}^{T}(\overline{p}^*, \gamma^*, n^*) \\
=&\frac{\overline{p}^* k}{2n^*}\beta\left(\frac{n^*(2p_B^*(1+k)+2t(1+k)+2\alpha)-k\overline{p}^*(1+k)}{4(1+k)tn^*}\right) \\
&-\frac{\gamma^*}{n^*}\frac{\overline{p}^* k(k+1)}{2n^*}\beta\left(\frac{n^*(p_B^*+t)+\alpha\gamma^*-\left(\frac{\gamma^*}{n^*}\left[\frac{\overline{p}^* k(k+1)}{2n^*}+(n^*-k-1)(p_B^*+t)\right]+(1-\gamma^*)(p_B^*+t)\right)n^*}{2tn^*}\right).
\end{aligned}$$

And I have that $\Pi_{AR}^T(\overline{p}^*, n^*) - \Pi_A^T(\overline{p}^*, \gamma^*, n^*) > 0 \iff \alpha > \alpha_R \equiv \frac{(1+k)(k\overline{p}^* - 2n^*(p_B^*+t))}{2n^*}$. Note that $\frac{k}{n^*}\overline{p}^* \le p_B^* + t \iff k\overline{p}^* \le n^*(p_B^* + t)$. Therefore, I have $k\overline{p}^* < 2n^*(p_B^* + t)$ and I can conclude $\alpha_R < 0$. Hence, $\alpha > \alpha_R$ always holds and I conclude $\Pi_{AR}^T(\overline{p}^*, n^*) > \Pi_A^T(\overline{p}^*, \gamma^*, n^*)$.

Similarly, the difference in skeptical consumers' profits from the refined lottery and that from the original lottery is

$$\Pi_{AR}^S(\overline{p}^*, n^*) - \Pi_A^S(\overline{p}^*, \gamma^*, n^*) = \Pi_{AR}^S(\overline{p}^*, n^*) > 0,$$

as I know from EC.4.6 that $\Pi_A^S(\overline{p}^*, \gamma^*, n^*) = 0$. Moreover, because the refined lottery also generates more profits from trusting consumers, I have shown that this refined lottery generates more profits than the original lottery does:

$$\begin{aligned}\Pi_{AR}(\overline{p}^*, n^*) &= \Pi_{AR}^T(\overline{p}^*, n^*) + \Pi_{AR}^S(\overline{p}^*, n^*)\\ &> \Pi_A^T(\overline{p}^*, \gamma^*, n^*) + \Pi_A^S(\overline{p}^*, \gamma^*, n^*)\\ &= \Pi_A(\overline{p}^*, \gamma^*, n^*)\end{aligned}$$

Finally, as I did in Case 1, I now construct a simple lottery that has the same expected price as the refined lottery, but contains only one promotion outcome—zero price. Let $\overline{p}_s$ and γ_s denote the alternate price and probability of winning from the simple lottery. Then by construction I have

$$(1-\gamma_s)\overline{p}_s = \frac{\overline{p}^* k}{2n^*}. \tag{EC.4.12}$$

Let $\gamma_s = \frac{1}{k+1}$, then I can solve for $\overline{p}_s = \frac{(1+k)\overline{p}^*}{2n^*}$. The profit from offering this simple lottery is

$$\begin{aligned}\Pi_A(\overline{p}_s, \gamma_s) &= (1-\gamma_s)\overline{p}_s[\beta(\frac{p_B^* + t + \alpha\gamma_s - (1-\gamma_s)\overline{p}_s}{2t}) + (1-\beta)(\frac{p_B^* - \overline{p}_s + t}{2t})] &\text{(EC.4.13)}\\ &= \frac{\overline{p}^* k}{2n^*}[\beta(\frac{n^*(2p_B^*(1+k) + 2t(1+k) + 2\alpha) - k\overline{p}^*(1+k)}{4(1+k)tn^*}) + (1-\beta)(\frac{p_B^* - \overline{p}_s + t}{2t})] &\text{(EC.4.14)}\end{aligned}$$

Hence, the difference in profits between the simple lottery and the refined lottery is that

$$\begin{aligned}\Pi_A(\overline{p}_s, \gamma_s) - \Pi_{AR}(\overline{p}^*, n^*) &= \frac{\overline{p}^* k}{2n^*}(1-\beta)(\frac{p_B^* - \overline{p}_s + t}{2t} - \frac{(p_B^* + t)n^* - k\overline{p}^*}{2tn^*})\\ &= \frac{\overline{p}^* k}{2n^*}(1-\beta)(\frac{p_B^* + t - \frac{(1+k)\overline{p}^*}{2n^*}}{2t} - \frac{(p_B^* + t) - \frac{k}{n^*}\overline{p}^*}{2t})\\ &> 0.\end{aligned}$$

In conclusion, I am able to construct a simple lottery that generates higher profit than the refined lottery, which in turn generates higher profit than the original lottery. Therefore, the original lottery with $n \ge 2$ is not optimal, as firm A can profitably deviate by offering this constructed simple lottery with $n = 1$. I then derive contradiction and conclude that $n^* = 1$. Because I have shown in both Case 1 and Case 2 that $n^* = 1$, I have completed the proof for Proposition 4.1 that the optimal number of promotion outcomes is $n^* = 1$. ■

Proof of Proposition 4.2

Firm A's profit maximization problem is

$$\begin{aligned}\max_{\overline{p},\gamma}\quad & \Pi_A(\overline{p},\gamma)=(1-\gamma)\overline{p}\left(\beta(\frac{p_B-\overline{p}(1-\gamma)+t+\alpha\gamma}{2t})+(1-\beta)(\frac{p_B-\overline{p}+t}{2t})\right)\\ \text{s.t.}\quad & \overline{p}\in[0,p_B+t],\\ & \gamma\in[0,1].\end{aligned}\tag{EC.4.15}$$

Similarly, Firm B's profit maximization problem is

$$\begin{aligned}\max_{p_B}\quad & \Pi_B(p_B)=p_B\left(\beta(\frac{\overline{p}(1-\gamma)+t-p_B-\alpha\gamma}{2t})+(1-\beta)(\frac{\overline{p}-p_B+t}{2t})\right)\\ \text{s.t.}\quad & p_B\geq 0.\end{aligned}\tag{EC.4.16}$$

We prove Proposition 4.2 in two steps: (1) I apply the Karush-Kuhn-Tucker (KKT) conditions to solve EC.4.15 and EC.4.16; (2) I show that the solutions from step 1 are global optimal.

Step 1: Firm A optimizes $\Pi_A(\overline{p},\gamma)$ over a convex set of linear constraints as shown in EC.4.15, namely $g_n\leq 0, n\in N=\{1,2,3,4\}$, where $g_1(\overline{p},\gamma)=-\overline{p},\ g_2(\overline{p},\gamma)=\overline{p}-(p_B+t),\ g_3(\overline{p},\gamma)=-\gamma,\ g_4(\overline{p},\gamma)=\gamma-1$. The KKT solutions to EC.4.15 are given by the standard equations

$$\begin{aligned}\nabla\Pi_A(\overline{p},\gamma)-\sum_{n\in N}\mu_n\nabla g_n(\overline{p},\gamma)&=0,\\ \mu_n&\geq 0,\ \forall n\in N,\\ \mu_n g_n(\overline{p},\gamma)&=0,\ \forall n\in N.\end{aligned}\tag{EC.4.17}$$

Similarly, the KKT solutions to EC.4.16 are given by the standard equations

$$\begin{aligned}\nabla\Pi_B(p_B)-\lambda(\frac{d(-p_B)}{dp_B})&=0,\\ \lambda&\geq 0,\\ \lambda(-p_B)&=0.\end{aligned}\tag{EC.4.18}$$

Solving EC.4.17 and EC.4.18 with the constraint $\alpha<\frac{\overline{p}(1-\gamma)-p_B+t}{\gamma}$ gives

(i) when $\alpha<\frac{t(1-\beta)}{\beta}$:

$$\gamma^*=0,$$
$$\overline{p}^*=t,$$
$$p_B^*=t;$$

(ii) when $(\beta>\frac{11-\sqrt{73}}{4}\wedge\frac{t(1-\beta)}{\beta}\leq\alpha<\frac{12t(1-\beta)}{\beta(4-\beta)})$ or $(\beta\leq\frac{11-\sqrt{73}}{4}\wedge\frac{t(1-\beta)}{\beta}\leq\alpha<\frac{t\left(\beta^3+3\beta^2-13\beta+3+(1+\beta)\sqrt{\beta^4+4\beta^3-26\beta^2+12\beta+9}\right)}{2\beta\left(\beta^2-5\beta+3\right)})$:

$$\gamma^* = \frac{\alpha(2+\beta) - \sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{3\alpha\beta},$$
$$\overline{p}^* = \frac{-2\alpha(1-\beta) + \sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{3(1-\beta)},$$
$$p_B^* = \frac{9t - \alpha(2+\beta) + \sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{9};$$

(iii) when $\beta > \frac{11-\sqrt{73}}{4}$ and $\frac{12t(1-\beta)}{\beta(4-\beta)} \leq \alpha < t(1+2\beta)$:

$$\gamma^* = 1/2,$$
$$\overline{p}^* = \frac{6t-\alpha\beta}{2+\beta},$$
$$p_B^* = \frac{t(4-\beta)-\alpha\beta}{2+\beta}.$$

For the convenience of notation, I denote

$$\underline{\alpha} \equiv \frac{t(1-\beta)}{\beta},$$
$$\hat{\beta} \equiv \frac{11-\sqrt{73}}{4},$$
$$\overline{\alpha} \equiv \begin{cases} \frac{t\left(\beta^3+3\beta^2-13\beta+3+(1+\beta)\sqrt{\beta^4+4\beta^3-26\beta^2+12\beta+9}\right)}{2\beta\left(\beta^2-5\beta+3\right)} & \text{if } \beta \leq \hat{\beta}, \\ \frac{12t(1-\beta)}{\beta(4-\beta)} & \text{if } \beta > \hat{\beta}, \end{cases}$$
$$\overline{\overline{\alpha}} \equiv t(1+2\beta).$$

Step 2: The solutions I obtained from step 1 would be global optimal if in EC.4.15 and EC.4.16: (a) the constraints are convex and (b) the objective functions are concave. (a) can be directly verified from EC.4.15 and EC.4.16. I now show that (b) is also true.

$\frac{d^2\Pi_B(p_B)}{dp_B{}^2} = \frac{-1}{t} < 0 \Rightarrow \Pi_B(p_B)$ is strictly concave in p_B. Because $\frac{\partial^2\Pi_A(\overline{p},\gamma)}{\partial\overline{p}^2} = \frac{(1-\gamma)(\beta\gamma-1)}{t} < 0$ and $\frac{\partial^2\Pi_A(\overline{p},\gamma)}{\partial\gamma^2} = -\frac{\overline{p}(\overline{p}+\alpha)\beta}{t} < 0$, $\Pi_A(\overline{p},\gamma)$ is jointly strictly concave in $(\overline{p},\gamma)$ if

$$\begin{aligned} D &= \frac{\partial^2\Pi_A(\overline{p},\gamma)}{\partial\overline{p}^2}\frac{\partial^2\Pi_A(\overline{p},\gamma)}{\partial\gamma^2} - (\frac{\partial^2\Pi_A(\overline{p},\gamma)}{\partial\overline{p}\partial\gamma})^2 \\ &= [\overline{p}^2\left(-4-4\beta-4\beta^2+12\beta\gamma+12\beta^2\gamma-12\beta^2\gamma^2\right) \\ &+ \overline{p}\left(4p_B+4t+4p_B\beta+4t\beta-4\alpha\beta^2-8p_B\beta\gamma-8t\beta\gamma+4\alpha\beta\gamma+12\alpha\beta^2\gamma-12\alpha\beta^2\gamma^2\right) \\ &- p_B^2-2p_Bt-t^2+2p_B\alpha\beta+2t\alpha\beta-\alpha^2\beta^2-4p_B\alpha\beta\gamma-4t\alpha\beta\gamma+4\alpha^2\beta^2\gamma-4\alpha^2\beta^2\gamma^2]/(4t^2) \\ &> 0. \end{aligned}$$

(i) when $\alpha < \underline{\alpha}$, $D = \frac{\beta\left(4t^2+4t\alpha-4t^2\beta-4t\alpha\beta-\alpha^2\beta\right)}{4t^2} > 0$.

(ii) when $\underline{\alpha} \leq \alpha < \overline{\alpha}$, $D = \frac{\left(2\alpha-2\alpha\beta-\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}\right)\left(-36t-14\alpha+2\alpha\beta+7\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}\right)}{108t^2} > 0$.

(iii) when $\beta > \hat{\beta}$ and $\overline{\alpha} \leq \alpha < \overline{\overline{\alpha}}$, $D = \frac{(\alpha\beta-6t)\left(6t-12t\beta-5\alpha\beta+6t\beta^2+2\alpha\beta^2\right)}{4t^2(2+\beta)^2} > 0$.

Since both (a) and (b) are satisfied, the solutions from step 1 are global optimal. ■

Proof of Corollary 4.1.

Since $\underline{\alpha} \equiv \frac{t(1-\beta)}{\beta}$, I have $\frac{\partial\alpha}{\partial\beta} = \frac{-t}{\beta^2} < 0$. ■

Proof of Proposition 4.3.

We first characterise $\frac{\partial \overline{p}^*}{\partial\alpha}$ and $\frac{\partial p_B^*}{\partial\alpha}$ in each region identified in Proposition 4.2, then prove Proposition 4.3.

(i) When $\alpha < \underline{\alpha}$: $\gamma^* = 0,\ \overline{p}^* = t,\ p_B^* = t$. $\frac{\partial \overline{p}^*}{\partial\alpha} = 0$, and $\frac{\partial p_B^*}{\partial\alpha} = 0$.

(ii) When $\underline{\alpha} \le \alpha < \overline{\alpha}$: $\gamma^* = \frac{\alpha(2+\beta)-\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{3\alpha\beta}$, $\overline{p}^* = \frac{-2\alpha(1-\beta)+\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{3(1-\beta)}$, $p_B^* = \frac{9t-\alpha(2+\beta)+\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{9}$.

$$\frac{\partial \overline{p}^*}{\partial\alpha} = \frac{9t+8\alpha-2\alpha\beta-4\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{6\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}} > 0,$$

and

$$\frac{\partial p_B^*}{\partial\alpha} = \frac{9t+8\alpha+\beta(2\alpha\beta-10\alpha-9t)-(4+2\beta)\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{18\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}} < 0.$$

(iii) When $\beta > \hat{\beta}$ and $\overline{\alpha} \le \alpha < \overline{\overline{\alpha}}$: $\gamma^* = 1/2,\ \overline{p}^* = \frac{6t-\alpha\beta}{2+\beta},\ p_B^* = \frac{t(4-\beta)-\alpha\beta}{2+\beta}$. $\frac{\partial \overline{p}^*}{\partial\alpha} = -\frac{\beta}{2+\beta} < 0$, and $\frac{\partial p_B^*}{\partial\alpha} = -\frac{\beta}{2+\beta} < 0$.

We now prove Proposition 4.3(i). The direction that $\underline{\alpha} \le \alpha < \overline{\alpha} \implies \frac{\partial \overline{p}^*}{\partial\alpha} > 0$ is shown above. I now prove the other direction that $\frac{\partial \overline{p}^*}{\partial\alpha} > 0 \implies \underline{\alpha} \le \alpha < \overline{\alpha}$. Using proof by contrapositive, it is equivalent to prove that $\alpha < \underline{\alpha} \vee \alpha \ge \overline{\alpha} \implies \frac{\partial \overline{p}^*}{\partial\alpha} \le 0$. I have shown that (i) $\alpha < \underline{\alpha} \implies \frac{\partial \overline{p}^*}{\partial\alpha} = 0$ and (iii) $\beta > \hat{\beta} \wedge \overline{\alpha} \le \alpha < \overline{\overline{\alpha}} \implies \frac{\partial \overline{p}^*}{\partial\alpha} = -\frac{\beta}{2+\beta} < 0$. So, the remaining cases to show are (a) $\beta \le \hat{\beta} \wedge \alpha \ge \overline{\alpha} \implies \frac{\partial \overline{p}^*}{\partial\alpha} \le 0$ and (b) $\beta > \hat{\beta} \wedge \alpha \ge \overline{\overline{\alpha}} \implies \frac{\partial \overline{p}^*}{\partial\alpha} \le 0$. But α cannot be in these two cases, otherwise the constraint $\alpha < \frac{\overline{p}(1-\gamma)-p_B+t}{\gamma}$ is violated. Therefore, cases (a) and (b) do not exist and I have proven Proposition 4.3(i).

Lastly, Proposition 4.3(ii) follows directly because $\frac{\partial p_B^*}{\partial\alpha} \le 0$ in all the three regions. ■

Proof of Proposition 4.4.

In region (i) and (iii), γ^* is constant so that $\frac{\partial\gamma^*}{\partial\alpha} = 0$. Hence, this proof reduces to show that $\frac{\partial\gamma^*}{\partial\alpha} \ge 0$ in region (ii) where $\underline{\alpha} \le \alpha < \overline{\alpha}$. In this region, $\gamma^* = \frac{\alpha(2+\beta)-\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{3\alpha\beta}$, and $\frac{\partial\gamma^*}{\partial\alpha} = \frac{3t(1-\beta)}{2\alpha\beta\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}} > 0$. ■

Proof of Proposition 4.5.

I first characterise $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha}$ and $\frac{\partial\Pi_B(p_B^*)}{\partial\alpha}$ in each region, then prove the Proposition.

(i) When $\alpha < \underline{\alpha}$: $\gamma^* = 0,\ \overline{p}^* = t,\ p_B^* = t$. $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha} = 0$ and $\frac{\partial\Pi_B(p_B^*)}{\partial\alpha} = 0$.

(ii) When $\underline{\alpha} \le \alpha < \overline{\alpha}$: $\gamma^* = \frac{\alpha(2+\beta)-\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{3\alpha\beta}$, $\overline{p}^* = \frac{-2\alpha(1-\beta)+\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{3(1-\beta)}$, $p_B^* = \frac{9t-\alpha(2+\beta)+\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{9}$. $\frac{\partial^2\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha^2} > 0 \implies \frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha}$ is monotonically increasing. In addition, $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha}|_{\alpha=\underline{\alpha}} < 0$. Next, I solve the equation $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha} = 0$ with respect to α.

$$\begin{aligned}\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha} = 0 \iff 9t\left(6\alpha(-3+\beta)+5\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}\right)\\ +4\alpha\left(-\alpha(-4+\beta)^2+\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}(8+\beta)\right) = 0.\end{aligned}$$

Solving above gives roots $\alpha_1 = 0$, $\alpha_2 = -\frac{3t}{\beta}$, $\alpha_3 = \frac{3t\left(\beta^2-11\beta+4-\sqrt{\beta^4+3\beta^3+4\beta^2+12\beta+16}\right)}{4\beta(4-\beta)}$, and $\alpha_4 = \frac{3t\left(\beta^2-11\beta+4+\sqrt{\beta^4+3\beta^3+4\beta^2+12\beta+16}\right)}{4\beta(4-\beta)}$. Roots α_1, α_2, α_3 are all non-positive, and α_4 is the only root that is greater than $\underline{\alpha}$. Because $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha}$ is monotonically increasing, I conclude that $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha} > 0 \iff \alpha > \alpha_4$. I now discuss whether α_4 lies in $[\underline{\alpha},\overline{\alpha})$ and divide the discussion into two cases: (a) $\beta > \hat{\beta}$ and (b) $\beta \le \hat{\beta}$. Note that because $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha}$ is monotonically increasing and $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha}|_{\alpha=\underline{\alpha}} < 0$, it must be that $\alpha_4 > \underline{\alpha}$. So my discussion reduces to whether $\alpha_4 < \overline{\alpha}$.

In case (a) $\beta > \hat{\beta}$, because (1) $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha}$ is monotonically increasing, (2) $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha}|_{\alpha=\underline{\alpha}} < 0$, and (3) $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha}|_{\alpha=\overline{\alpha}} > 0$, α_4, at which $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha}|_{\alpha=\alpha_4} = 0$, must lie in $[\underline{\alpha},\overline{\alpha})$.

In case (b) $\beta \le \hat{\beta}$, $\frac{\partial\overline{\alpha}}{\partial\beta} = \frac{t\left(-9+30\beta-65\beta^2+32\beta^3-8\beta^4\right)}{2\beta^2\left(3-5\beta+\beta^2\right)^2} + \frac{t\left(-27+72\beta+126\beta^2-220\beta^3+65\beta^4+16\beta^5-8\beta^6\right)}{2\beta^2\left(3-5\beta+\beta^2\right)^2\sqrt{9+12\beta-26\beta^2+4\beta^3+\beta^4}} < 0$, and $\frac{\partial\alpha_4}{\partial\beta} = \frac{3\left(-128t+16t\beta+36t\beta^2+20t\beta^3+11t\beta^4\right)}{8\sqrt{\beta^2-\beta+4}(-4+\beta)^2\beta^2(2+\beta)} + \frac{3\left(-32t(2+\beta)+16t\beta(2+\beta)-14t\beta^2(2+\beta)\right)}{8(-4+\beta)^2\beta^2(2+\beta)} < 0$.

Moreover, with out loss of generality pick $\beta = 0.2$, I have $\overline{\alpha}(\beta = 0.2) = 5.39t < 6.07t = \alpha_4(\beta = 0.2)$. Then pick $\beta = \hat{\beta}$, I have $\overline{\alpha}(\beta = \hat{\beta}) = 2.23t > 0.97t = \alpha_4(\beta = \hat{\beta})$. Combine this with the fact that $\overline{\alpha}$ and α_4 monotonically decrease in β, it must be the case that $\overline{\alpha}$ and α_4 cross once at some $\beta \in (0.2,\hat{\beta})$. I denote $\underline{\beta}$ to be such β and solve for $\underline{\beta} \approx 0.27$. In sum, α_4 lies in $[\underline{\alpha},\overline{\alpha})$ when $\beta > \underline{\beta}$.

Regarding $\frac{\partial\Pi_B(p_B^*)}{\partial\alpha}$, it always holds that $\underline{\alpha} \le \alpha < \overline{\alpha} \implies \frac{\partial\Pi_B(p_B^*)}{\partial\alpha} < 0$.

(iii) When $\beta > \hat{\beta}$ and $\overline{\alpha} \le \alpha < \overline{\overline{\alpha}}$: $\gamma^* = 1/2$, $\overline{p}^* = \frac{6t-\alpha\beta}{2+\beta}$, $p_B^* = \frac{t(4-\beta)-\alpha\beta}{2+\beta}$. $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha} = \frac{\beta(6t-3t\beta-2\alpha\beta)}{4t(2+\beta)^2} > 0 \iff \alpha < \frac{3t(2-\beta)}{2\beta}$. And $\beta > \frac{3}{4} \implies \frac{3t(2-\beta)}{2\beta} < \overline{\overline{\alpha}}$. Moreover, $\frac{\partial\Pi_B(p_B^*)}{\partial\alpha} = \frac{\beta(t\beta+\alpha\beta-4t)}{t(2+\beta)^2} < 0$.

Let

$$\alpha_u \equiv \begin{cases}\overline{\alpha} & \text{if } \beta \le \hat{\beta},\\ \overline{\overline{\alpha}} & \text{if } \hat{\beta} < \beta \le \frac{3}{4},\\ \frac{3t(2-\beta)}{2\beta} & \text{if } \frac{3}{4} < \beta,\end{cases}$$

and

$$\alpha_l \equiv \begin{cases}\overline{\alpha} & \text{if } \beta \le \underline{\beta}\\ \alpha_4 & \text{if } \beta > \underline{\beta}\end{cases} = \min\{\overline{\alpha},\alpha_4\}.$$

Then I have shown the direction that $\alpha_l < \alpha < \alpha_u \implies \frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha} > 0$. Next, I prove the other direction that $\frac{\partial\Pi_A(\overline{p}^*,\gamma^*)}{\partial\alpha} > 0 \implies \alpha_l < \alpha < \alpha_u$. Using proof by contrapositive, it is equivalent to prove that $\alpha \le \alpha_l \vee \alpha \ge$

$\alpha_u \implies \frac{\partial \Pi_A(\overline{p}^*,\gamma^*)}{\partial \alpha} \leq 0$, which is true as I have shown that (1) $\alpha < \underline{\alpha} \implies \frac{\partial \Pi_A(\overline{p}^*,\gamma^*)}{\partial \alpha} = 0$, (2) $\underline{\alpha} \leq \alpha < \alpha_l \implies \frac{\partial \Pi_A(\overline{p}^*,\gamma^*)}{\partial \alpha} < 0$, (3) $\beta > \frac{3}{4} \wedge \alpha \geq \alpha_u \implies \frac{\partial \Pi_A(\overline{p}^*,\gamma^*)}{\partial \alpha} \leq 0$, and (4) when $\beta \leq \frac{3}{4}$, $\alpha \geq \alpha_u$ cannot be true, otherwise the constraint $\alpha < \frac{\overline{p}(1-\gamma)-p_B+t}{\gamma}$ is violated.

So I have proven Proposition 4.5(i).

Lastly, Proposition 4.5(ii) follows directly because $\frac{\partial \Pi_B(p_B^*)}{\partial \alpha} \leq 0$ in all the three regions. ■

Proof of Proposition 4.6.

We compare $\Pi_A(\overline{p}^*,\gamma^*)$ with $\Pi_A^H = t/2$ in each region.

(i) When $\alpha < \underline{\alpha}$: $\gamma^* = 0,\ \overline{p}^* = t,\ p_B^* = t$. $\Pi_A(\overline{p}^*,\gamma^*) = t/2 = \Pi_A^H$.

(ii) When $\underline{\alpha} \leq \alpha < \overline{\alpha}$: $\gamma^* = \frac{\alpha(2+\beta)-\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{3\alpha\beta}$, $\overline{p}^* = \frac{-2\alpha(1-\beta)+\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{3(1-\beta)}$, $p_B^* = \frac{9t-\alpha(2+\beta)+\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{9}$.

When $\alpha = \underline{\alpha}$, $\Pi_A(\overline{p}^*,\gamma^*) = t/2$. And I first discuss three cases based on the value of β. Case (1) $\beta \leq \underline{\beta} \approx 0.27$, case (2) $\underline{\beta} < \beta \leq \hat{\beta}$, and case (3) $\beta > \hat{\beta}$.

In case (1) $\beta \leq \underline{\beta}$, from proof of Proposition 4.5 I know that $\alpha_4 \geq \overline{\alpha}$. Hence, $\Pi_A(\overline{p}^*,\gamma^*)$ is monotonically decreasing in α and $\Pi_A(\overline{p}^*,\gamma^*) \leq t/2$ always holds in region (ii).

In case (2) $\underline{\beta} < \beta \leq \hat{\beta}$, from proof of Proposition 4.5 I know that $\Pi_A(\overline{p}^*,\gamma^*)$ first decreases then increases in region (ii). Next, I further separate my proof in two possible cases that when $\alpha = \overline{\alpha}$, (2a) $\Pi_A(\overline{p}^*,\gamma^*) > t/2$, or (2b) $\Pi_A(\overline{p}^*,\gamma^*) \leq t/2$. And I first solve for the condition in which $\Pi_A(\overline{p}^*,\gamma^*) = t/2$.

First pick $\beta = \underline{\beta}$, when $\alpha = \overline{\alpha}$ I have $\Pi_A(\overline{p}^*,\gamma^*) = 0.48t < t/2$. Then pick $\beta = \hat{\beta}$, when $\alpha = \overline{\alpha}$ I have $\Pi_A(\overline{p}^*,\gamma^*) = 0.54t > t/2$. In addition, when $\underline{\beta} < \beta < \hat{\beta} \wedge \alpha = \overline{\alpha}$, $\frac{\partial \Pi_A(\overline{p}^*,\gamma^*)}{\partial \beta} > 0$ always holds. Hence, there must exist a $\beta \in (\underline{\beta},\hat{\beta})$ at which $\Pi_A(\overline{p}^*,\gamma^*) = t/2$ when $\alpha = \overline{\alpha}$. I denote $\tilde{\beta}$ to be such β and solve for $\tilde{\beta} \approx 0.45$.

When $\underline{\beta} < \beta \leq \tilde{\beta}$, we are in case (2b) where when $\alpha = \overline{\alpha}$, $\Pi_A(\overline{p}^*,\gamma^*) \leq t/2$. Hence, $\Pi_A(\overline{p}^*,\gamma^*) \leq t/2$ holds for all $\alpha \in [\underline{\alpha},\overline{\alpha})$.

When $\tilde{\beta} < \beta \leq \hat{\beta}$, we are in case (2a) where when $\alpha = \overline{\alpha}$, $\Pi_A(\overline{p}^*,\gamma^*) > t/2$. Because $\Pi_A(\overline{p}^*,\gamma^*) < t/2$ at α_l, $\Pi_A(\overline{p}^*,\gamma^*) > t/2$ at $\overline{\alpha}$, and $\Pi_A(\overline{p}^*,\gamma^*)$ is monotonically increasing in α when $\alpha_l < \alpha < \overline{\alpha}$, there must exist an $\alpha \in (\alpha_l,\overline{\alpha})$ at which $\Pi_A(\overline{p}^*,\gamma^*) = t/2$. I denote α_b to be such α and solves for

$$\begin{aligned}\alpha_b \equiv\ & \frac{t\beta(27\sqrt{96\beta^5+513\beta^4+6\beta^3-351\beta^2+5352\beta-5616}-16\beta^3-1293\beta^2+6171\beta-4430)^{1/3}}{2^{1/3}3\beta^2}\\ & -\frac{2^{1/3}t^2\left(-4\beta^2+149\beta-181\right)}{3t\beta(27\sqrt{96\beta^5+513\beta^4+6\beta^3-351\beta^2+5352\beta-5616}-16\beta^3-1293\beta^2+6171\beta-4430)^{1/3}}\\ & -\frac{2t(5+\beta)}{3\beta}.\end{aligned}$$

Hence, $\alpha > \alpha_b \iff \Pi_A(\overline{p}^*,\gamma^*) > t/2$. For notational convenience, let

$$A \equiv t\beta\left(\frac{27\sqrt{96\beta^5+513\beta^4+6\beta^3-351\beta^2+5352\beta-5616}-16\beta^3-1293\beta^2+6171\beta-4430}{2}\right)^{1/3}$$

and I re-write $\alpha_b \equiv \frac{A^2+t^2\beta^2(4\beta^2-149\beta+181)-2At\beta(5+\beta)}{3A\beta^2}$.

In case (3) $\beta > \hat{\beta}$, from proof of Proposition 4.5 I know that $\Pi_A(\overline{p}^*, \gamma^*)$ is monotonically increasing in α when $\alpha_l < \alpha < \overline{\alpha}$. In addition, I always have that $\Pi_A(\overline{p}^*, \gamma^*) > t/2$ at $\overline{\alpha}$. Hence, there must exist an $\alpha \in (\alpha_l, \overline{\alpha})$ at which $\Pi_A(\overline{p}^*, \gamma^*) = t/2$, and α_b is such α. And I conclude that $\alpha > \alpha_b \iff \Pi_A(\overline{p}^*, \gamma^*) > t/2$.

(iii) When $\beta > \hat{\beta}$ and $\overline{\alpha} \leq \alpha < \overline{\overline{\alpha}}$: $\gamma^* = 1/2,\ \overline{p}^* = \frac{6t-\alpha\beta}{2+\beta},\ p_B^* = \frac{t(4-\beta)-\alpha\beta}{2+\beta}$.

$\Pi_A(\overline{p}^*, \gamma^*) = \frac{\beta(3t+\alpha)(6t-\alpha\beta)}{4t(2+\beta)^2} > t/2$.

Regarding Proposition 4.6, the direction that $\beta > \tilde{\beta} \wedge \alpha > \alpha_b \implies \Pi_A(\overline{p}^*, \gamma^*) > t/2 = \Pi_A^H$ is shown above. I now prove the other direction that $\Pi_A(\overline{p}^*, \gamma^*) > t/2 \implies \beta > \tilde{\beta} \wedge \alpha > \alpha_b$. Using proof by contrapositive, it is equivalent to prove that $\beta \leq \tilde{\beta} \vee (\beta > \tilde{\beta} \wedge \alpha \leq \alpha_b) \implies \Pi_A(\overline{p}^*, \gamma^*) \leq t/2$. And it is true because:

(1) when $\beta \leq \tilde{\beta}$, α is either in region (i) ($\alpha < \underline{\alpha}$), in which case $\Pi_A(\overline{p}^*, \gamma^*) = t/2$, or in region (ii) ($\underline{\alpha} \leq \alpha < \overline{\alpha}$), in which case $\Pi_A(\overline{p}^*, \gamma^*) \leq t/2$ always holds.

(2) when $\beta > \tilde{\beta} \wedge \alpha \leq \alpha_b$, α is either in region (i) ($\alpha < \underline{\alpha}$), in which case $\Pi_A(\overline{p}^*, \gamma^*) = t/2$, or in region (ii) ($\underline{\alpha} \leq \alpha < \overline{\alpha}$), in which case $\Pi_A(\overline{p}^*, \gamma^*) \leq t/2$ always holds because $\alpha \leq \alpha_b$. ■

Proof of Proposition 4.7.

From Proposition 4.6 I know that $\Pi_A(\overline{p}^*, \gamma^*) > t/2 \iff \beta > \tilde{\beta} \wedge \alpha > \alpha_b$. Therefore, the upper bound of $\Pi_A(\overline{p}^*, \gamma^*)$ lies in $\beta > \tilde{\beta} \wedge \alpha > \alpha_b$. Next, I prove Proposition 4.7 by comparing the maximal profits in three cases based on the value of β: case (1) $\tilde{\beta} < \beta \leq \hat{\beta}$, case (2) $\hat{\beta} < \beta \leq \frac{3}{4}$, and case (3) $\beta > \frac{3}{4}$.

In case (1) $\tilde{\beta} < \beta \leq \hat{\beta}$, From Proposition 4.6 I know that for a fixed β, the maximum of $\Pi_A(\overline{p}^*, \gamma^*)$ is obtained at $\alpha = \overline{\alpha}$. Moreover, from Proof of Proposition 4.6 I know that when $\underline{\beta} < \beta < \hat{\beta} \wedge \alpha = \overline{\alpha}$, $\frac{\partial \Pi_A(\overline{p}^*, \gamma^*)}{\partial \beta} > 0$ always holds. Therefore, the maximal profit is obtained when $\beta = \hat{\beta}$ and $\alpha = \overline{\alpha}$. I use $\Pi_{A1}(\overline{p}^*, \gamma^*)$ to denote such maximal profit.

In case (2) $\hat{\beta} < \beta \leq \frac{3}{4}$, from Proposition 4.6 I know that for a fixed β, the maximum of $\Pi_A(\overline{p}^*, \gamma^*)$ is obtained when $\alpha = \alpha_u = \overline{\overline{\alpha}}$. At $\alpha = \alpha_u$, $\frac{\partial \Pi_A(\overline{p}^*, \gamma^*)}{\partial \beta} = \frac{t(3-4\beta)}{2} > 0$. Therefore, the maximal profit is obtained when $\beta = \frac{3}{4}$ and $\alpha = \alpha_u$. I use $\Pi_A^2(\overline{p}^*, \gamma^*)$ to denote such maximal profit.

In case (3) $\beta > \frac{3}{4}$, from Proposition 4.6 I know that for a fixed β, the maximum of $\Pi_A(\overline{p}^*, \gamma^*)$ is obtained when $\alpha = \alpha_u = \frac{3t(2-\beta)}{2\beta}$. At $\alpha = \alpha_u$, $\frac{\partial \Pi_A(\overline{p}^*, \gamma^*)}{\partial \beta} = 0$. Therefore, the maximal profit is obtained when $\alpha = \alpha_u$ with any $\beta > \frac{3}{4}$. I use $\Pi_A^3(\overline{p}^*, \gamma^*)$ to denote such maximal profit.

Based on my analysis above, it is obvious that $\Pi_A^3(\overline{p}^*, \gamma^*) = \Pi_A^2(\overline{p}^*, \gamma^*) > \Pi_{A1}(\overline{p}^*, \gamma^*)$. Pick any $\beta \in (\frac{3}{4}, 1)$ and $\alpha = \alpha_u$, I have

$$\Pi_A^3(\overline{p}^*, \gamma^*) = (1-\gamma)\overline{p}\left(\beta(\frac{p_B - \overline{p}(1-\gamma) + t + (\frac{3t(2-\beta)}{2\beta})\gamma}{2t}) + (1-\beta)(\frac{p_B - \overline{p} + t}{2t})\right)$$

$$= (1-1/2)(\frac{6t - (\frac{3t(2-\beta)}{2\beta})\beta}{2+\beta})[\beta(\frac{\frac{t(4-\beta)-(\frac{3t(2-\beta)}{2\beta})\beta}{2+\beta} - (\frac{6t-(\frac{3t(2-\beta)}{2\beta})\beta}{2+\beta})(1-1/2) + t + (\frac{3t(2-\beta)}{2\beta})/2}{2t})$$

$$+(1-\beta)(\frac{\frac{t(4-\beta)-(\frac{3t(2-\beta)}{2\beta})\beta}{2+\beta}-(\frac{6t-(\frac{3t(2-\beta)}{2\beta})\beta}{2+\beta})+t}{2t})]$$
$$=1.125(t/2)$$
$$=1.125\Pi_A^H.\blacksquare$$

Proof of Proposition 4.8.

We prove this proposition in two steps. First, I solve the equilibrium solution when firm A acquires the government certification ($\mathbb{S}=Y$). Let $\Pi_A^Y(\overline{p},\gamma)$ denote firm A's profit with government certification. Second, I compare firm A's profits with and without government certification to determine her government certification strategy.

Step 1: This step is similar to proof of Proposition 4.2. I only consider the region where α is not overly large so that the two firms are competing: $\alpha<\frac{\overline{p}(1-\gamma)-p_B+t}{\gamma}$.

With government certification, firm A's profit maximization problem is

$$\begin{aligned}\max_{\overline{p},\gamma}\quad & \Pi_A^Y(\overline{p},\gamma)=(1-\gamma)\overline{p}(\frac{p_B-\overline{p}(1-\gamma)+t+\alpha\gamma}{2t})-c\\ \text{s.t.}\quad & \overline{p}\in[0,p_B+t],\\ & \gamma\in[0,1].\end{aligned}\tag{EC.4.19}$$

Similarly, Firm B's profit maximization problem is

$$\begin{aligned}\max_{p_B}\quad & \Pi_B^Y(p_B)=p_B(\frac{\overline{p}(1-\gamma)+t-p_B-\alpha\gamma}{2t})\\ \text{s.t.}\quad & p_B\geq 0.\end{aligned}\tag{EC.4.20}$$

In step 1a, I apply the Karush-Kuhn-Tucker (KKT) conditions to solve EC.4.19 and EC.4.20. Then, in step 1b, I show that the solutions from step 1a are global optimal.

Step 1a: Firm A optimizes $\Pi_A^Y(\overline{p},\gamma)$ over a convex set of linear constraints as shown in EC.4.19, namely $g_n\leq 0, n\in N=\{1,2,3,4\}$, where $g_1(\overline{p},\gamma)=-\overline{p},\ g_2(\overline{p},\gamma)=\overline{p}-(p_B+t),\ g_3(\overline{p},\gamma)=-\gamma,\ g_4(\overline{p},\gamma)=\gamma-1$. The KKT solutions to EC.4.19 are given by the standard equations

$$\begin{aligned}\nabla\Pi_A(\overline{p},\gamma)-\sum_{n\in N}\mu_n\nabla g_n(\overline{p},\gamma)=0,\\ \mu_n\geq 0,\ \forall n\in N,\\ \mu_n g_n(\overline{p},\gamma)=0,\ \forall n\in N.\end{aligned}\tag{EC.4.21}$$

Similarly, the KKT solutions to EC.4.20 are given by the standard equations

$$\begin{aligned}\nabla\Pi_B^Y(p_B)-\lambda(\frac{d(-p_B)}{dp_B})=0,\\ \lambda\geq 0,\\ \lambda(-p_B)=0.\end{aligned}\tag{EC.4.22}$$

Solving EC.4.21 and EC.4.22 with the constraint $\alpha < \frac{\overline{p}(1-\gamma)-p_B+t}{\gamma}$ gives

$$\gamma^* = 1/2,$$
$$\overline{p}^* = \frac{6t-\alpha}{3},$$
$$p_B^* = \frac{3t-\alpha}{3}.$$

Hence, $\Pi_A^Y(\overline{p}^*,\gamma^*) = \frac{18t^2+(3\alpha-36c)t-\alpha^2}{36t}$.

Step 1b: The solutions I obtained from step 1a would be global optimal if in EC.4.19 and EC.4.20: (a) the constraints are convex and (b) the objective functions are concave. (a) can be directly verified from EC.4.19 and EC.4.20. I now show that (b) is also true.

$\frac{d^2\Pi_B^Y(p_B)}{dp_B{}^2} = \frac{-1}{t} < 0 \Rightarrow \Pi_B^Y(p_B)$ is strictly concave in p_B. Because $\frac{\partial^2\Pi_A^Y(\overline{p},\gamma)}{\partial\overline{p}^2} = \frac{(1-\gamma)(\gamma-1)}{t} = \frac{-1}{4t} < 0$ and $\frac{\partial^2\Pi_A^Y(\overline{p},\gamma)}{\partial\gamma^2} = -\frac{\overline{p}(\overline{p}+\alpha)}{t} = \frac{2(\alpha-6t)(3t+\alpha)}{9t} < 0$, $\Pi_A^Y(\overline{p},\gamma)$ is jointly strictly concave in $(\overline{p},\gamma)$ if

$$\begin{aligned}D &= \frac{\partial^2\Pi_A^Y(\overline{p},\gamma)}{\partial\overline{p}^2}\frac{\partial^2\Pi_A^Y(\overline{p},\gamma)}{\partial\gamma^2} - (\frac{\partial^2\Pi_A^Y(\overline{p},\gamma)}{\partial\overline{p}\partial\gamma})^2 \\ &= [\overline{p}^2(-12\gamma^2+24\gamma-12)+\overline{p}\left(8p_B+8t-4\alpha-8p_B\gamma-8t\gamma+16\alpha\gamma-12\alpha\gamma^2\right) \\ &\quad - p_B^2-2p_Bt-t^2+2p_B\alpha+2t\alpha-\alpha^2-4p_B\alpha\gamma-4t\alpha\gamma+4\alpha^2\gamma-4\alpha^2\gamma^2]/(4t^2) \\ &> 0.\end{aligned}$$

At the equilibrium solution obtained from step 1a, I have $D = \frac{(6t-\alpha)\alpha}{12t^2} > 0$. Since both (a) and (b) are satisfied, the solution from step 1a is global optimal.

Step 2: I now compare $\Pi_A^Y(\overline{p}^*,\gamma^*)$ with firm A's optimal profit when she does not acquire the certification ($\Pi_A^N(\overline{p}^*,\gamma^*) \equiv \Pi_A(\overline{p}^*,\gamma^*)$, as in the main model). From proof of Proposition 4.6 we have that:

(i) When $\alpha < \underline{\alpha}$: $\Pi_A(\overline{p}^*,\gamma^*) = t/2$. Then,

$$\Pi_A^Y(\overline{p}^*,\gamma^*) > \Pi_A(\overline{p}^*,\gamma^*) \iff \frac{18t^2+(3\alpha-36c)t-\alpha^2}{36t} > t/2 \iff c < \frac{\alpha(3t-\alpha)}{36t}$$

(ii) When $\underline{\alpha} \le \alpha < \overline{\alpha}$:

$$\Pi_A(\overline{p}^*,\gamma^*) = \frac{\left(2\alpha\beta-2\alpha+\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}\right)^2\left(9t+2\alpha+\alpha\beta-\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}\right)}{162t\alpha(1-\beta)\beta}.$$

Then, I have that

$$\Pi_A^Y(\overline{p}^*,\gamma^*) > \Pi_A(\overline{p}^*,\gamma^*) \iff \frac{18t^2+(3\alpha-36c)t-\alpha^2}{36t} >$$
$$\frac{\left(2\alpha\beta-2\alpha+\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}\right)^2\left(9t+2\alpha+\alpha\beta-\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}\right)}{162t\alpha(1-\beta)\beta} \iff$$
$$c < \frac{162t^2(\beta-1)+t\alpha(171\beta-252)+\alpha^2\left(2\beta^2+35\beta-64\right)+(90t+32\alpha-2\alpha\beta)\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{324t\beta}.$$

(iii) When $\beta > \hat{\beta}$ and $\overline{\alpha} \leq \alpha < \overline{\overline{\alpha}}$: $\Pi_A(\overline{p}^*, \gamma^*) = \frac{\beta(3t+\alpha)(6t-\alpha\beta)}{4t(2+\beta)^2}$. Then,

$$\Pi_A^Y(\overline{p}^*, \gamma^*) > \Pi_A(\overline{p}^*, \gamma^*) \iff \frac{18t^2 + (3\alpha - 36c)t - \alpha^2}{36t} > \frac{\beta(3t+\alpha)(6t-\alpha\beta)}{4t(2+\beta)^2}$$
$$\iff c < \frac{(3t+\alpha)(1-\beta)(12t - 2\alpha - 3t\beta - 4\alpha\beta)}{18t(2+\beta)^2}.$$

Let

$$\overline{c} \equiv \begin{cases} \frac{\alpha(3t-\alpha)}{36t} & \text{if } \alpha < \underline{\alpha}, \\ \frac{162t^2(\beta-1)+t\alpha(171\beta-252)+\alpha^2\left(2\beta^2+35\beta-64\right)+(90t+32\alpha-2\alpha\beta)\sqrt{\alpha(1-\beta)(9t+4\alpha-\alpha\beta)}}{324t\beta} & \text{if } \underline{\alpha} \leq \alpha < \overline{\alpha}, \\ \frac{(3t+\alpha)(1-\beta)(12t-2\alpha-3t\beta-4\alpha\beta)}{18t(2+\beta)^2} & \text{if } \overline{\alpha} \leq \alpha < \overline{\overline{\alpha}} \wedge \beta > \hat{\beta}. \blacksquare \end{cases}$$

Proof of Proposition 4.9.

The proof is very similar to Proof of Proposition 4.2.

Firm A's profit maximization problem is

$$\begin{aligned} \max_{\overline{p},\gamma} \quad & \Pi_A(\overline{p}, \gamma) = (1-\gamma)\overline{p}\left(\beta\left(\frac{\overline{p_B}\,(1-\gamma_B) - \overline{p}(1-\gamma) + \alpha\,(\gamma - \gamma_B) + t}{2t}\right) + (1-\beta)\left(\frac{\overline{p_B} - \overline{p} + t}{2t}\right)\right) \\ \text{s.t.} \quad & \overline{p} \geq 0, \\ & \gamma \in [0,1]. \end{aligned} \tag{EC.4.23}$$

Similarly, firm B's profit maximization problem is

$$\begin{aligned} \max_{\overline{p_B},\gamma_B} \quad & \Pi_B(\overline{p_B}, \gamma_B) = (1-\gamma_B)\overline{p_B}\left(\beta\left(\frac{\overline{p}(1-\gamma) - \overline{p_B}\,(1-\gamma_B) + \alpha\,(\gamma_B - \gamma) + t}{2t}\right) + (1-\beta)\left(\frac{\overline{p} - \overline{p_B} + t}{2t}\right)\right) \\ \text{s.t.} \quad & \overline{p_B} \geq 0, \\ & \gamma_B \in [0,1]. \end{aligned} \tag{EC.4.24}$$

We prove Proposition 4.9 in two steps: (1) I apply the Karush-Kuhn-Tucker (KKT) conditions to solve EC.4.23 and EC.4.24; (2) I show that the solutions from step 1 are global optimal.

Step 1: Firm A optimizes $\Pi_A(\overline{p}, \gamma)$ over a convex set of linear constraints as shown in EC.4.23, namely $g_n \leq 0, n \in N = \{1,2,3\}$, where $g_1(\overline{p}, \gamma) = -\overline{p}$, $g_2(\overline{p}, \gamma) = -\gamma$, $g_3(\overline{p}, \gamma) = \gamma - 1$. The KKT solutions to EC.4.23 are given by the standard equations

$$\begin{aligned} \nabla\Pi_A(\overline{p}, \gamma) - \sum_{n \in N} \mu_n \nabla g_n(\overline{p}, \gamma) = 0, \\ \mu_n \geq 0, \ \forall n \in N, \\ \mu_n g_n(\overline{p}, \gamma) = 0, \ \forall n \in N. \end{aligned} \tag{EC.4.25}$$

Similarly, firm B optimizes $\Pi_B(\overline{p_B}, \gamma_B)$ over a convex set of linear constraints as shown in EC.4.24, namely $h_n \leq 0, n \in N = \{1,2,3\}$, where $h_1(\overline{p_B}, \gamma_B) = -\overline{p_B},\ h_2(\overline{p_B}, \gamma_B) = -\gamma_B,\ h_3(\overline{p_B}, \gamma_B) = \gamma_B - 1$. The KKT solutions to EC.4.24 are given by the standard equations

$$\begin{aligned} \nabla \Pi_B(\overline{p_B}, \gamma_B) - \sum_{n \in N} \lambda_n \nabla h_n(\overline{p_B}, \gamma_B) = 0, \\ \lambda_n \geq 0,\ \forall n \in N, \\ \lambda_n h_n(\overline{p_B}, \gamma_B) = 0,\ \forall n \in N. \end{aligned} \tag{EC.4.26}$$

Solving EC.4.25 and EC.4.26 gives

(i) when $\alpha < \underline{\alpha} \equiv \frac{t(1-\beta)}{\beta}$,

$$\begin{aligned} \gamma^* = \gamma_B^* = 0, \\ \overline{p}^* = \overline{p_B}^* = t; \end{aligned}$$

(ii) when $\alpha \geq \underline{\alpha}$,

$$\begin{aligned} \gamma^* = \gamma_B^* &= \frac{\alpha(1+\beta) - \sqrt{\alpha(1-\beta)(4t+\alpha-\alpha\beta)}}{2\alpha\beta}, \\ \overline{p}^* = \overline{p_B}^* &= \frac{\alpha(\beta-1) + \sqrt{\alpha(1-\beta)(4t+\alpha-\alpha\beta)}}{2(1-\beta)}. \end{aligned}$$

Step 2: The solutions I obtained from step 1 would be global optimal if in EC.4.23 and EC.4.24: (a) the constraints are convex and (b) the objective functions are concave. (a) can be directly verified from EC.4.23 and EC.4.24. I now show that (b) is also true.

Because the two firms are symmetric, I only need to show (b) is true for firm A—that is, the objective function is concave in EC.4.23. The case for firm B would be identical. Because $\frac{\partial^2 \Pi_A(\overline{p},\gamma)}{\partial \overline{p}^2} = \frac{(1-\gamma)(\beta\gamma-1)}{t} < 0$ and $\frac{\partial^2 \Pi_A(\overline{p},\gamma)}{\partial \gamma^2} = -\frac{\overline{p}(\overline{p}+\alpha)\beta}{t} < 0$, $\Pi_A(\overline{p}, \gamma)$ is jointly strictly concave in $(\overline{p}, \gamma)$ if

$$\begin{aligned} D = &\frac{\partial^2 \Pi_A(\overline{p},\gamma)}{\partial \overline{p}^2} \frac{\partial^2 \Pi_A(\overline{p},\gamma)}{\partial \gamma^2} - (\frac{\partial^2 \Pi_A(\overline{p},\gamma)}{\partial \overline{p} \partial \gamma})^2 \\ &= [\overline{p}^2(-4 - 4\beta - 4\beta^2 + 12\beta\gamma + 12\beta^2\gamma - 12\beta^2\gamma^2) + \overline{p}(4t + 4t\beta - 4\alpha\beta^2 - 8t\beta\gamma + 4\alpha\beta\gamma + 12\alpha\beta^2\gamma - 12\alpha\beta^2\gamma^2 \\ &+ 4p_B + 4\beta p_B - 8\beta\gamma p_B - 4\alpha\beta\gamma_B - 4\alpha\beta^2\gamma_B + 8\alpha\beta^2\gamma\gamma_B - 4\beta p_B\gamma_B - 4\beta^2 p_B\gamma_B + 8\beta^2\gamma p_B\gamma_B) - t^2 + 2t\alpha\beta \\ &- \alpha^2\beta^2 - 4t\alpha\beta\gamma + 4\alpha^2\beta^2\gamma - 4\alpha^2\beta^2\gamma^2 - 2tp_B + 2\alpha\beta p_B - 4\alpha\beta\gamma p_B - p_B^2 + 2t\alpha\beta\gamma_B - 2\alpha^2\beta^2\gamma_B + 4\alpha^2\beta^2\gamma\gamma_B \\ &+ 2t\beta p_B\gamma_B + 2\alpha\beta p_B\gamma_B - 2\alpha\beta^2 p_B\gamma_B + 4\alpha\beta^2\gamma p_B\gamma_B + 2\beta p_B^2\gamma_B - \alpha^2\beta^2\gamma_B^2 - 2\alpha\beta^2 p_B\gamma_B^2 - \beta^2 p_B^2\gamma_B^2]/(4t^2) \\ &> 0. \end{aligned}$$

(i) when $\alpha < \underline{\alpha}$, $D = \frac{\beta\left(4t^2+4t\alpha-4t^2\beta-4t\alpha\beta-\alpha^2\beta\right)}{4t^2} > 0.$

(ii) when $\alpha \geq \underline{\alpha}$, $D = \frac{\left(8t+\alpha-\alpha\beta-\sqrt{\alpha(1-\beta)(4t+\alpha-\alpha\beta)}\right)\left(-\alpha+\alpha\beta+\sqrt{\alpha(1-\beta)(4t+\alpha-\alpha\beta)}\right)}{16t^2} > 0$

Since both (a) and (b) are satisfied, the solutions from step 1 are global optimal. ■

Curriculum Vitae

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