APPENDIX A

SENSOR RESPONSE TIME TESTING THEORY
APPENDIX A

SENSOR RESPONSE TIME TESTING THEORY

This appendix contains a tutorial on sensor response time testing, including derivations of the equations used in arriving at the results presented in the body of this thesis.
A.1 Fundamentals of Dynamic Response

The dynamic response of a sensor or a system may be identified theoretically or experimentally. The theoretical approach usually requires a thorough knowledge of the design of the sensor, its construction details, the properties and geometries of the sensor's internal material as well as a knowledge of the properties of the medium surrounding the sensor. Since these properties are not known thoroughly, or may change under process operating or aging conditions, the theoretical approach alone can only provide approximate results. A remedy is to combine the theory with experiments that empirically determine the dynamic response.

Theory is used to determine the expected behavior of the sensor in terms of an equation called the "model," which relates the input and the output of the system. The system is then given an experimental input signal, and its output is measured and matched with the model. That is, the coefficients of the model are changed iteratively until the model matches the data within a predetermined convergence criterion. This process, performed on a digital computer, is referred to as "fitting". Once the fitting process is successfully completed, the coefficients of the model are identified and used to determine the response time of the sensor. However, if the sensor can be represented with a first-order model, fitting is not necessary because the response time can be determined directly from the output of the sensor.

The model for a sensor or a system may be expressed in terms of either a time domain or a frequency domain equation. The time domain model is usually a specific relationship that gives the transient output of the system for a given input signal such as a
step or a ramp signal. The frequency domain model is often represented as a general relationship called the “transfer function,” which includes the input and the output. If the transfer function is known, the system response can be obtained for any input. As such, the transfer function is often used in analyzing system dynamics.[A-1]

**A.2  Step and Ramp Response of a First-Order System**

Consider a thermocouple whose sensing section is assumed to be made of a homogeneous material represented by the mass "m" and specific heat capacity "c", as shown in Figure A1. The response of this system when it is suddenly exposed to a medium with temperature \( T_f \) may be derived theoretically using the energy balance equation that describes the system. Assuming that the thermal conductivity of the thermocouple material is infinite, we can write:

\[
mc \frac{dT}{dt} = hA(T_f - T)
\]  

(A.1)

where:

- \( h \) = heat transfer coefficient
- \( A \) = affected surface area
- \( T \) = response of the system as a function of time, \( t \).

Equation A.1 may be solved in the frequency domain by applying a Laplace transformation to both sides of the equation. This will allow us to express the solution in terms of a transfer function \( G \) of the following equation (A.2), which relates the Laplace transform of the output, \( T(s) \), to the Laplace transform of the input, \( T_f(s) \):

\[
G(s) = \frac{T(s)}{T_f(s)}
\]  

(A.2)

The Laplace transformation of Equation A.1 is:
Figure A.1  Step Response of a First-Order Thermal System
where \( p = hA/mc \), and \( s \) is the Laplace transform variable. Now, we can write Equation A.2 as follows, assuming that \( T(0) = 0 \):

\[
G(s) = \frac{T(s)}{p} = \frac{p}{s + p} \tag{A.4}
\]

In equation A.4, \( p \) is referred to as the pole of the transfer function. The reciprocal of \( p \) is expressed in the unit of time and is called the time constant (\( \tau \)) of the first-order system. Equation A.4 can be used to derive the response of the system to any input such as a step, a ramp, or a sinusoidal input. Proceeding to derive the step response, we substitute the Laplace transform of a step signal in Equation A.4 to arrive at the following expression (the Laplace transform of a step signal is \( \frac{a}{s} \)):

\[
T(s) = \frac{pa}{s(s + p)} = a \left[ \frac{1}{s} - \frac{1}{s + p} \right] \tag{A.5}
\]

where \( a \) is the step amplitude. The inverse Laplace transform of Equation A.5 will yield the step response of the system, as follows:

\[
O(t) = a(1 - e^{-\frac{t}{\tau}}) \tag{A.6}
\]

where \( \tau = 1/p \). At \( \tau = t \); the step response becomes \( O(t) = 0.632a \), and at \( t=\infty \), the step response becomes \( O(t)=a \). Thus, if we now perform an experiment in which the output of the system is measured for a step change in input, then the resulting data can be used to obtain the time constant (\( \tau \)) directly as shown in Figure A.2. That is, the time constant of
Figure A.2  Determination of Time Constant from Step Response of a First-Order System
the first-order system can be identified directly from the step response data by
determining the time required for the system output to reach 63.2 percent of its final
value.

The ramp response is obtained by substituting the Laplace transform of a ramp signal \( r/s^2 \)
for \( T_f(s) \) in Equation A.3:

\[
T(s) = \frac{rp}{s^2(s + p)}
\]

(A.7)

where \( rp \) is a constant that we denote as \( k \). An inverse Laplace transform of this equation
results in the ramp response (see Figure A.3):

\[
O(t) = \frac{k}{p} [t - \tau + \tau e^{-t/\tau}]
\]

(A.8)

Note that when \( t \gg \tau \), the exponential term will be negligible and we can write:

\[
O(t) = k(t - \tau)
\]

(A.9)

That is, the asymptotic response of the system which is delayed with respect to the input
by a value that is equal to the time constant (\( \tau \)) from the step response.

For a sinusoidal input, the response time is expressed in terms of the reciprocal of the
corner frequency of the frequency response plot (i.e., the break frequency of the Gain
portion of the Bode plot). If the corner frequency is denoted by the letter \( \omega \), we will show
in Equation A.10 that \( (1/\omega) \) is equal to the time constant (\( \tau \)) for a first-order system
(Figure A.4). Substituting \( j\omega \) for \( s \) in Equation A.4 and writing \( \tau \) for \( (1/p) \), we obtain:
Figure A.3  Ramp Response

Figure A.4  Frequency Response of a First-Order System and Calculation of Response Time from Break Frequency
\[ G(j\omega) = \frac{1}{j\omega \tau + 1} \]  
\[ \text{(A.10)} \]

where \( \omega \) is the frequency in radians per second and \( j = \sqrt{-1} \). The magnitude of \( G(j\omega) \) is:

\[ |G| = \left( \frac{1}{\omega^2 \tau^2 + 1} \right)^{\frac{1}{2}} \]
\[ \text{(A.11)} \]

The corner frequency is the frequency at which \( |G| = 0.707 \). Substituting this in Equation A.11 and solving for \( \tau \), we obtain \( \tau = \frac{1}{\omega} \).

A.3 Step and Ramp Responses of Higher-Order Systems

Although some systems, such as the simple thermal system discussed in section A.2, can be approximated with a first-order model, the transient behavior of most systems is generally written in terms of higher-order models that are represented by a transfer function of the following form:

\[ G(s) = \frac{T(s)}{T_f(s)} = \frac{1}{(s - p_1)(s - p_2)\cdots(s - p_n)} \]
\[ \text{(A.12)} \]

where \( p_1, p_2, \ldots, p_n \) are called the poles of the system transfer function. The reciprocal of these poles are denoted as \( \tau_1, \tau_2, \ldots, \tau_n \), which are called modal time constants. The following derivations (Equations A.13 through A.23) show that the overall time constant of a system is obtained by combining its modal time constants.

The response of a higher-order system to a step change in input is derived by substituting \( T_f(s) = 1/s \) in Equation A.12 and performing an inverse Laplace transform. This yields:
\[
T(t) = \frac{1}{(-p_1)(-p_2)\cdots(-p_n)} \left[ 1 + \frac{(-p_1)(-p_2)\cdots(-p_n)}{p_1(p_1 - p_2)\cdots(p_1 - p_n)} e^{p_1 t} + \cdots \right] 
\]

(A.13)

From this, we can write:

\[
T(\infty) = \frac{1}{(-p_1)(-p_2)\cdots(-p_n)} = \tau_1 \tau_2 \cdots \tau_n 
\]

(A.14)

Thus:

\[
\frac{T(t)}{T(\infty)} = 1 + \frac{1}{\tau_1 \tau_2 \cdots \tau_n} \left[ \frac{1}{-\tau_1} \left[ \frac{1}{-\tau_1} + \frac{1}{\tau_2} \right] \cdots \frac{1}{-\tau_1} + \frac{1}{\tau_n} \right] e^{-\frac{t}{\tau_1}} 
+ \frac{1}{\tau_1 \tau_2 \cdots \tau_n} \left[ \frac{1}{-\tau_2} \left[ \frac{1}{-\tau_2} + \frac{1}{\tau_1} \right] \cdots \frac{1}{-\tau_2} + \frac{1}{\tau_n} \right] e^{-\frac{t}{\tau_2}} + \cdots 
\]

(A.15)

Now we proceed to determine the expressions that yield the overall response time (\(\tau\)) of the system in terms of its modal time constants (\(\tau_1, \tau_2, \tau_3, \cdots\)). For typical process sensors, we can safely assume, based on experience with their dynamic response curves, that the values of the modal time constants rapidly decrease as we go from \(\tau_1\) to \(\tau_2\) to \(\cdots\) \(\tau_n\). Thus, if we let \(\tau_1\) be the slowest time constant (the largest in value) and evaluate the second exponential at \(\tau / \tau_1 = 1\), we obtain the following:

[A.2]
\[
\frac{\tau_1}{\tau_2} = e^{-t/\tau_2} \quad (at \ t = \tau_1)
\]

<table>
<thead>
<tr>
<th>(\tau_1/\tau_2)</th>
<th>2</th>
<th>0.135</th>
<th>3</th>
<th>0.050</th>
<th>4</th>
<th>0.018</th>
<th>5</th>
<th>0.007</th>
</tr>
</thead>
</table>

For a sensor such as an RTD, \(\tau_1/\tau_2\) is typically about 5 or greater. Therefore, the contribution of \(\tau_2\) is small by the time \(t = \tau_1\). Since \(\tau_1\) has the most important effect on \(\tau\), we can also assert that \(\tau_2\) and higher terms have a small influence when \(t = \tau\). Thus, we may write:

\[
\frac{T(t)}{T(\infty)} \approx 1 + \frac{1}{\frac{1}{\tau_1\tau_2\cdots\tau_n}} \left[ \frac{1}{-\tau_1} + \frac{1}{\tau_1} \right] e^{-t/\tau_1}
\]

(A.16)

Now, we can set \(\frac{T(t)}{T(\infty)} = 0.632\) and solve for \(\tau\) to obtain:

\[
e^{-t/\tau_1} = 0.368 \left(1 - \frac{\tau_2}{\tau_1}\right) \left(1 - \frac{\tau_3}{\tau_1}\right) \cdots \left(1 - \frac{\tau_n}{\tau_1}\right)
\]

(A.17)

Or:

\[
\tau = \tau_1 \left[ 1 - \ln \left(1 - \frac{\tau_2}{\tau_1}\right) - \ln \left(1 - \frac{\tau_3}{\tau_1}\right) - \cdots - \ln \left(1 - \frac{\tau_n}{\tau_1}\right) \right]
\]

(A.18)

For ramp response, we substitute \(\frac{k}{s^2}\) for \(T_f(s)\) in Equation A.12, where \(k\) is the ramp rate:
\[ T(s) = \frac{k}{s^2(s-p_1)(s-p_2)\cdots(s-p_n)} \]  

(A.19)

The sensor response may be evaluated by inverse Laplace transformation. The partial fraction method yields:

\[ O(s) = \frac{A_1}{s^2} + \frac{A_2}{s} + \frac{A_3}{s-p_1} + \frac{A_4}{s-p_2} + \cdots + \frac{A_n}{s-p_n} \]  

(A.20)

The arbitrary constants \( A_i \)'s must be evaluated if the complete response is required. However, we are interested only in determining the ramp time delay. Consequently, the exponential terms are of no interest, and we can concentrate on \( A_1 \) and \( A_2 \). These may be evaluated to give the following result:\(^{[A-3]}\)

\[ A_1 = k \]
\[ A_2 = -k \left[ \tau_1 + \tau_2 + \cdots + \tau_n \right] \]  

(A.21)

Therefore:

\[ O(\tau) \sim k \left[ \tau - (\tau_1 + \tau_2 + \cdots + \tau_n) \right] \]  

(A.22)

In this case, we obtain:

\[
\text{Ramp Time Delay} = \tau_1 + \tau_2 + \cdots + \tau_n
\]  

(A.23)

Equations A.18 and A.23 show that (a) the time constant of a first-order system is equal to the ramp time delay of the system and that (b) as the order of the system increases,
the time constant and the ramp time delay slowly depart from one another, with the ramp delay time greater than the time constant.
REFERENCES


APPENDIX B

PRESSURE SENSORS AND SENSING LINE DYNAMICS
APPENDIX B
PRESSURE SENSORS AND SENSING LINE DYNAMICS

This appendix contains the derivations of the formulas used in the body of this thesis to arrive at results that simulated the effects of length, blockages, and voids on the response time of a pressure sensing system (pressure transmitter and sensing line). In applying these formulas, the natural frequency and damping ratio of the system had to be calculated based on the physical properties (e.g., dimension, compliance, fluid density, bulk modulus, etc.) of the pressure sensing system. The resulting values were then used in the solutions to a second-order, underdamped linear differential equation to arrive at time domain and frequency domain results in terms of tables of numbers and PSD plots.
APPENDIX B
PRESSURE SENSORS AND SENSING LINE DYNAMICS

Figure B.1 illustrates a mechanical model of a pressure transmitter and its sensing line, which together constitute a pressure sensing system. The pressure transmitter is represented by a spring-mass viscous assembly that emulates a viscously damped single-degree-of-freedom system. As the process pressure $P_p$ changes, the subsequent pressure surge is transmitted through the sensing line and results in a volume change ($V_t$) in the pressure transmitter cavity. To describe the relationship between the volume change in the transmitter and the pressure required to induce this volume change, the term compliance $C_t$ is used. The compliance is defined as the change in transmitter volume that results from a change in pressure.

Figure B.1  Pressure Sensing System Model
\[ C_t = \frac{\Delta V_t}{\Delta P_s} \]  \hspace{1cm} (B.1)\\

where:

\[ C_t = \text{compliance, cm}^3/\text{bar} \]

\[ V_t = \text{volume change in pressure transmitter cavity, cm}^3 \]

\[ P_s = \text{pressure in sensing system, bar} \]

With no fluid in the system, the natural frequency of the transmitter is:

\[ \omega_n = \sqrt{\frac{k}{m}} \]  \hspace{1cm} (B.2)\\

where:

\[ \omega_n = \text{natural frequency, rad/sec} \]

\[ k = \text{spring constant, Newton/meter} \]

\[ m = \text{mass of the body, Kg} \]

However, as discussed earlier, the viscous damping forces induced by the fluid in the sensing line and transmitter must also be included in the analysis in order to fully understand the dynamic behavior of the sensor system. As seen in Figure B-1, the transmitter volume change for a given pressure change must equal the equivalent piston volume change:\[ ^{[B-1]} \]

\[ P_s C_t = \frac{\pi^2 d_p^4 P_s}{16k} \text{ or } \frac{d_p^4}{k} = \frac{16C_t}{\pi^2} \]  \hspace{1cm} (B.3)\\

where:

\[ d_p = \text{diameter of the piston, cm} \]
As the process pressure $P_p$ increases or decreases, the kinetic energy of the fluid in the sensing line will change, depending on the mass and velocity of the fluid. The mass of the fluid is dependent on the volume of the fluid in the sensing line, and the velocity can be calculated by assuming a square velocity profile for the fluid in the sensing line. Based on this, the kinetic energy of the fluid can be shown to equal the following:

$$KE_F = \frac{\pi \rho L V_s^2 d_s^2}{8}$$  \hspace{1cm} (B.4)

where:

- $KE_F$ = kinetic energy of the fluid
- $\rho$ = Fluid density, g/cm$^3$
- $L$ = Length of the sensing line, cm
- $V_s$ = Average velocity of fluid in the sensing line, m/s
- $d_s$ = Inside diameter of the sensing line, cm

Noting that a rigid mass (representing the equivalent of a fluid mass attached to the piston mass $m$) must have the same kinetic energy as the fluid, the mass can be shown to equal:

$$M_e = \frac{\pi \rho L d_p^4}{4d_s^2}$$  \hspace{1cm} (B.5)

where:

- $M_e$ = rigid mass, g

The natural frequency of the pressure transmitting sensing line system then equals:
\[ \omega_n = \sqrt{\frac{k}{M_e + m}} \]  

(B.6)

For most cases \( M_e \gg m \), therefore, the natural frequency equation reduces to:

\[ \omega_n = \sqrt{\frac{k}{M_e}} \]  

(B.7)

Substituting Equations B.3 and B.5 into Equation B.7 provides the new equation (B.8) for the natural frequency of the system:

\[ \omega_n = \sqrt{\frac{\pi d_s^4}{4 \rho LC_s}} \]  

(B.8)

The damping ratio can also be calculated, as shown in Equation B.9. Note that the damping force is directly related to the pressure drop in the system due to the fluid viscosity (assuming that laminar conditions exist):

\[ \zeta = \frac{32 \mu}{\pi d_s^3} \sqrt{\frac{\pi LC_s}{\rho}} \]  

(B.9)

where

- \( \zeta \) = damping ratio
- \( \mu \) = dynamic viscosity of the fluid, Bar\cdotsecond

Equation B.9 can be further simplified by relating it to a nondimensional variable called the Stokes number:

\[ S_n = \frac{\omega_n d_s^2}{v} \]  

(B.10)
\[ S_n = \text{Stokes number} \]

\[ \nu = \text{kinematic viscosity of the fluid, Bar \cdot second} \]

Substituting the equation for the natural frequency (Equation B.8) into Equation B.10 results in the Stokes number at the natural frequency of the system:

\[ S_n = \frac{d_s^3 \rho}{\mu} \sqrt{\frac{\pi d_s^2}{4 \rho LC_t}} \]  

(B.11)

Solving for \( \mu \) yields:

\[ \mu = \frac{d_s^2 \rho}{S_n} \sqrt{\frac{\pi d_s^2}{4 \rho LC_t}} \]  

(B.12)

Substituting Equation B.12 into B.9 provides an equation that relates the Stokes number to the damping ratio at the natural frequency of the system:

\[ \zeta = \frac{16}{S_n} \quad \zeta = \frac{16 \nu}{\omega_n d_s^2} \]  

(B.13)

To account for other factors that are usually present in a pressure sensing system, the compliance should be modified to include: (1) the compliance of the fluid in the transmitter, (2) the compliance of the fluid in the sensing line, and (3) the compliance of any entrapped gas that might be present in the system.\[^{[B-2]}\] That is:

\[ C_s = C_t + C_{FT} + C_{FS} + C_B \]  

(B.14)

\[ C_s = \text{compliance of the total system, cm}^3/\text{bar} \]

\[ C_{FT} = \text{compliance of the fluid in the transmitter, cm}^3/\text{bar} \]

\[ C_{FS} = \text{compliance of the fluid in the sensing line, cm}^3/\text{bar} \]
\( C_B = \) compliance of any entrapped gas that might be present in the system, cm\(^3\)/bar

The remaining terms of this equation are as follows:

\[
C_{FT} = \frac{V_t}{B}, \quad C_{FS} = \frac{4V_{FS}}{\pi^2 B}, \quad \text{and} \quad C_B = \frac{V_b}{\gamma P_b} \quad \text{(B.15)}
\]

where:

\( V_{FS} = \) Volume of fluid in the sensing line, cm\(^3\)
\( V_b = \) Volume of gas bubble, cm\(^3\)
\( \gamma = \) Ratio of specific heats for gas bubble
\( P_b = \) Pressure applied to gas bubble, mbar
\( B = \) Bulk modulus for the fluid, bar

\[
C_s = \left( \frac{\Delta V_t}{\Delta P_s} \right) + \left( \frac{V_t}{B} \right) + \left( \frac{4V_{FS}}{\pi^2 B} \right) + \left( \frac{V_b}{\gamma P_b} \right) \quad \text{(B.16)}
\]

Therefore,

\[
C_s = \frac{4}{\pi^2 B} \left[ \frac{\pi^2}{4} \left( \frac{\Delta V_t}{\Delta P_s} \right) + \frac{\pi^2 V_t}{4} + V_{FS} + \frac{\pi^2 B V_b}{4\gamma P_b} \right] = \frac{4C'}{\pi^2 B} \quad \text{(B.17)}
\]

If the total system compliance \( C_s \) is considered instead of just \( C_t \), then substituting Equation B.17 into Equation B.7 yields:

\[
\omega_n = \sqrt{\frac{\pi^2 BA}{4p LC^1}} = \frac{\pi \sqrt{D}}{2L} \sqrt{\frac{V_{FS}}{C}} \quad \text{(B.18)}
\]
where:

\[ A = \text{cross-sectional area of the sensing line, cm}^2 \]

For infinitely stiff pipe walls, the acoustic velocity of the fluid in the sensing line can be approximated as follows:

\[ U_a \approx \frac{B}{\sqrt{\rho}} \]  

(B.19)

where:

\[ U_a = \text{acoustic velocity of the fluid, m/s} \]

\[ B = \text{bulk modulus for the fluid} \]

Therefore, using Equation B.17, the natural frequency of the system becomes:

\[
\omega_n = \frac{\pi U_a}{2L} \sqrt{\frac{V_{FS}}{\frac{\pi^2}{4} \left( BC_t + B \left( \frac{V_b}{\gamma P_b} \right) + V_l \right) + V_{FS}}} 
\]

(B.20)

The equation to calculate the natural frequency and damping ratio of the pressure sensing system enables us to describe the dynamic characteristics of the system in terms of step and ramp responses. To do so, we use the following relationships for linear second-order responses:
Step Response of Underdamped System

\[
x(t) = K \left\{ 1 - \frac{\omega_n}{\omega_d} e^{-\omega_d t} \sin \left[ \omega_d t + \arctan \left( \frac{\omega_n}{\alpha} \right) \right] \right\}
\]  \hspace{1cm} (B.21)

where:

\[ K = \text{the system gain} \]
\[ \omega_d = \text{the damped natural frequency} \left( \omega_n \sqrt{1 - \zeta^2} \right), \text{radian/s} \]
\[ \alpha = \omega_n \zeta \]

Ramp Response of Underdamped System

\[
x(t) = Kr \left[ t - \frac{2\alpha}{\omega_n^2} + \frac{1}{\omega_d} e^{-\omega_d t} \sin \left( \omega_d t + 2 \arctan \left( \frac{\omega_n}{\alpha} \right) \right) \right]
\]  \hspace{1cm} (B.22)

where:

\[ r = \text{ramp rate, bar/s} \]

Equation B.21 describes the response of the system to a step pressure input, and Equation B.22 describes the system response to a pressure ramp.
REFERENCES


APPENDIX C

CORRELATIONS BETWEEN RESPONSE TIME AND PROCESS CONDITIONS
APPENDIX C

RTD RESPONSE TIME VERSUS PROCESS CONDITIONS

In the body of this thesis, we stated that an RTD's response time depends on the process conditions in which the sensor is used. In this appendix, we develop correlations to quantify process condition effects by measuring response time in the laboratory at ambient conditions and extrapolating the results to process operating conditions. The correlations are useful to sensor designers, manufacturers, and users who seek to estimate the response time of a temperature sensor at process operating conditions based on data from laboratory measurements.

C.1 Response Time versus Flow Rate

The response time of an RTD or a thermocouple consists of an internal component and a surface component. The internal component depends predominantly on the thermal conductivity ($k$) of the material inside the sensor, while the surface component depends on the film’s heat-transfer coefficient ($h$). The internal component is independent of the process conditions, with the exception of the effect of temperature on material properties. The surface component is predominantly dependent on the process conditions, such as flow rate, temperature, and to a lesser extent, the process pressure. These parameters affect the film’s heat-transfer coefficient, which increases as the process parameters such as flow rate and temperature are increased. Figure C.1 illustrates how a temperature sensor’s response time may decrease as $h$ is increased. In this illustration, the effect of temperature on the material properties inside the sensor is ignored.
Figure C.1  Internal and Surface Components of Response Time as a Function of Heat-Transfer Coefficient
To derive the correlation between the fluid flow rate and response time ($\tau$), we recall the following equation for the response time ($\tau$) of a first-order thermal system:

$$\tau = \frac{mc}{UA} \quad \text{(C.1)}$$

In this equation, $m$ and $c$ are the mass and specific heat capacity of the sensing portion, respectively, and $U$ and $A$ are the overall heat transfer coefficient and the affected surface area of the temperature sensor, respectively. Note that we used the overall heat-transfer coefficient, $U$, as opposed to the film heat-transfer coefficient, $h$. The overall heat transfer coefficient accounts for the heat transfer resistance both inside the sensor and at the sensor surface. More specifically, we can write:

$$UA = \frac{1}{R_{tot}} = \frac{1}{R_{int} + R_{surf}} \quad \text{(C.2)}$$

where:

$R_{tot} = \text{total heat-transfer resistance}$

$R_{int} = \text{internal heat-transfer resistance}$

$R_{surf} = \text{surface heat-transfer resistance}$.

For a homogeneous cylindrical sheath, the internal and surface heat transfer resistances may be written as follows for a single-section lumped model:

$$R_{int} = \frac{\ln(r_o / r_i)}{2\pi kL} \quad \text{(C.3)}$$
\[ R_{\text{surf}} = \frac{1}{2\pi h L r_0} \]  

where:

\( r_o \) = outside radius of sensor  
\( r_i \) = radius at which the sensing tip is located  
\( k \) = thermal conductivity of sensor material  
\( L \) = effective heat-transfer length  
\( h \) = film’s heat-transfer coefficient

Substituting Equation C.3 and C.4 in Equation C.1 and C.2 yields:

\[ \tau = \frac{mc}{UA} = mc \left( \ln\left(\frac{r_o}{r_i}\right) + \frac{1}{2\pi k L r_0} \right) \]  

where:

\[ \rho \] is the density of the material in the sensor.

Writing Equation C.6 in terms of two constants \( C_1 \) and \( C_2 \), we will obtain:

\[ \tau = C_1 + C_2 / h \]  

where:

\[ C_1 = \frac{\rho c r_0^2}{2k} \ln\left(\frac{r_o}{r_i}\right) \]  

\[ C_2 = \frac{\rho c r_0}{2} \]
We can use Equation C.7 to estimate the response time of a temperature sensor at process operating conditions based on the response time measurements made in a laboratory. The procedure is to make laboratory response time measurements in at least two different heat-transfer media (with different values of $h$) in order to identify $C_1$ and $C_2$. Once we have identified $C_1$ and $C_2$, we can use Equation C.7 to estimate the temperature sensor’s response time in process media for which we can calculate the value of $h$ based on the type of media and its temperature, pressure, and flow conditions.

A useful application of Equation C.7 is to estimate the response time of a temperature sensor at a given process flow rate based on response time measurements made in a laboratory setup. More specifically, we can derive a correlation for response time-versus-fluid flow rate by determining the relationship between the heat-transfer coefficient ($h$) in Equation C.7 and the fluid flow rate ($u$). We obtain the heat-transfer coefficient by using general heat-transfer correlations that involve the Reynolds number, Prandtl number, and Nusselt number, which have the following relationships:

$$ Nu = f(Re, Pr) \quad \text{(C.10)} $$

In Equation C.10, $Nu = hD/k$ is the Nusselt number, $Re = Du\rho/\mu$ is the Reynolds number, and $Pr = c\mu/k$ is the Prandtl number. These heat-transfer numbers are all dimensionless, and their parameters are defined as follows:

- $h$ = film’s heat-transfer coefficient
- $D$ = sensor’s diameter
- $k$ = thermal conductivity of process fluid
For the correlation of Equation C.10, the literature provides several options for flow passing a single cylinder. Two of the most common correlations are that of Rohsenow and Choi\textsuperscript{[C-2]} and that of Perkins and Leppert.\textsuperscript{[C-3]} The Rohsenow and Choi correlation is as follows:

\[
Nu = 0.26 \, Re^{0.6} \, Pr^{0.3} \text{ for } 1,000 < Re < 50,000
\]  \hspace{1cm} (C.11)

The Perkins and Leppert correlation is the following:

\[
Nu = 0.26 \, Re^{0.5} \, Pr^{1/3} \text{ for } 40 < Re < 10^5
\]  \hspace{1cm} (C.12)

The Perkins and Leppert correlation covers a wider range of Reynolds numbers and is probably better suited for air, while the first correlation is more suited for water. Substituting Equation C.11 or C.12 in Equation C.10 will yield the following:

\[
h = C_1' \, u^{0.6} \text{ or } h = C_2' \, u^{0.5}
\]  \hspace{1cm} (C.13)

where \( C_1' \) and \( C_2' \) are constants and \( u \) is the fluid flow rate. Substituting the relations given by C.13 into Equation C.7, we will obtain the correlation between the response time and fluid flow rate:

\[
\tau = C_1 + C_2 \, u^{-0.6} \text{ for water}
\]  \hspace{1cm} (C.14)

or:
Using either of the two Equations C.14 or C.15, one can identify the two constants of the response-versus-flow correlation for a given sensor by making measurements at two or more flow rates in water or in other convenient media in a laboratory. Once these constants are identified, they can be used to estimate the sensor’s response time in other media for which the flow rate \( (u) \) is known.

### C.2 Response Time versus Temperature

Unlike flow, the effect of temperature on a temperature sensor’s response time cannot be estimated with great confidence. This is because temperature can either increase or decrease a temperature sensor’s response time. Temperature affects both the internal and the surface components of the response time. Its effect on the surface component is similar to that of the flow. That is, as temperature is increased, the film’s heat-transfer coefficient \( (h) \) generally increases and causes the surface component of response time to decrease. However, temperature’s effect on the internal component of response time is more subtle. High temperatures can cause the internal component of response time to either increase or decrease depending on how temperature affects the properties and on the geometry of the material inside the sensor. Because of differences in the thermal coefficient of expansion of the material inside the sensor and the sheath, the insulation material inside the sensor may become either more or less compact at higher temperatures. Consequently, the sensor material’s thermal conductivity, and therefore the internal response time, can either increase or decrease. Furthermore, voids such as gaps and cracks in the sensor’s construction material can either expand or contract at
high temperatures. This causes the internal response time to either increase or decrease depending on the size, orientation, and location of the void. At high temperatures, the sheath sometimes expands so much that an air gap is created at the interface between the sheath and the insulation material inside the sensor. In this case, the response time can increase significantly with temperature. It is possible, however, to account for temperature’s effect on the surface component of response time. In the following paragraph we will use equation C.14 to demonstrate this point for a sensor in water.

Neglecting temperature’s effect on the internal component of response time, the term $C_1$ in Equation C.14 will be unchanged. Therefore, we only need to account for temperature’s effect on the second term of Equation C.14. For a given reference flow rate, it can be shown that the second term of Equation C.14 is affected by temperature as follows: \[ h(T) = C_3(T) \frac{h(T_1)}{h(T_2)} \] (C.16)

Therefore, if we know the value of constant $C_3$ at room temperature (approximately 21°C), we can find its value at temperature ($T_2$) if we know $h(21°C)/h(T_2)$. Based on Equation C.12 (Rohsenow and Choi correlation), we can write:

\[ \frac{h(21°C)}{h(T)} = (4.3612)K(T)^{-0.7} \mu(T)^{0.3} \rho(T)^{-0.6} C_p(T)^{-0.3} \] (C.17)

From the Perkins and Leppert correlation, we have:

\[ \frac{h(21°C)}{h(T)} = (3.3603)K(T)^{2/3} \mu(T)^{1/6} \rho(T)^{-0.50} C_p(T)^{-1.3} \] (C.18)
A plot of Equations C.17 and C.18 for water is shown in Figure C.2. The data in this figure are for a pressure of approximately 140 bars (about 2000 psi). However, since the properties of water are not strongly dependent on pressure, the data should hold for pressures of up to about ±30 percent of 140 bars. Note that there is a large difference between the two curves in Figure C.2. This arises from the fact that two different heat-transfer correlations are used.

The data in Figure C.2 can be used to identify the heat transfer ratio that is needed in Equation C.16 to calculate $C_3$ at a given temperature based on measurements made at room temperature. This $C_3$ is then used in Equation C.14 to determine the response time-versus-flow curve of a sensor at any given temperature. Figure C.3 shows response time-versus-flow rate results at two different temperatures for an RTD. These results are derived from plunge tests of the RTD in room temperature water at different flow rates. The response time results from these plunge tests were used together with the corrections developed in this appendix and the Rohsenow and Choi correlation data of Figure C.2 to arrive at the results presented in Figure C.3.
From Rohsenow and Choi Correlation

From Perkins and Leppert Correlation

Water Temperature (°C)

Figure C.2  Correlations for Determining the Effect of Temperature on Response Time of Temperature Sensors
Figure C.3  RTD Response-versus-Flow Results at Two Different Temperatures

\[ \tau_{(20^\circ C)} = 2.21 + 1.098U^{-0.6} \]

\[ \tau_{(300^\circ C)} = 2.21 + 0.613U^{-0.6} \]
REFERENCES


APPENDIX D

SELF-HEATING INDEX AND ITS CORRELATION WITH RTD RESPONSE TIME
This appendix develops the correlation between the self-heating index (SHI) of an RTD and its response time and presents the procedure for measuring SHI. Like the LCSR test, self-heating measurements can be performed remotely on RTDs as installed in an operating plant. The results are used for two purposes: (1) to calculate the self-heating error in steady-state temperature measurement with RTDs, and (2) to trend SHI as a way of monitoring for gross degradation of RTD response time. (The self-heating error is an inherent phenomenon in RTDs that arises from the use of an electronic current that must be applied to the RTD to measure its resistance.) Typically, both self-heating measurements and LCSR tests are performed together in nuclear power plants to provide a complete picture of the dynamic health of an RTD.

### D.1 SHI Measurement

A Wheatstone bridge is used to perform a self-heating test on an RTD and to measure its SHI. The measurement may be made in a laboratory to provide baseline data, or while the RTD is installed in a plant to provide a trendable value with which to monitor for response time degradation.

The following procedure is used:

1. Connect the RTD to a Wheatstone bridge (Figure D-1). The same Wheatstone bridge arrangement that is used for the LCSR test can also be for a self-heating test.
2. Adjust the bridge power supply to run a small current (1 to 2 mA) through the RTD.
3. Balance the bridge and record the first self-heating data point, which consists of the RTD resistance ($R$) and the bridge of the current ($I$). The RTD resistance is the same as the value that is shown on the decade box when the bridge is balanced. The current may be measured with a multimeter across one of the fixed resistors in the bridge.
Figure D-1  Wheatstone Bridge Arrangement for Self-Heating Test
4. Calculate the power input to the RTD using the \( P = I^2R \) equation and fill the result in a table such as Table D-1.

5. Increase the bridge current to about 10 mA, wait for steady state, balance the bridge, record the next self-heating data point in table D-1, and calculate the new value of power.

6. Increase the current by about 5 mA, wait for steady state, balance the bridge, record the new values of R and I, and calculate the new value of power. Repeat this step until the current is 40 to 60 mA depending on the amount of current that is allowed for the RTD.

7. Plot the resistance data \((R)\) versus power \((P)\). This is called the self-heating curve, which is typically a straight line for a normal RTD (Figure D-2).

8. Calculate the slope of the self-heating curve. This is referred to as the self-heating index or SHI, and its value is expressed in terms of ohms/watt \((\Omega/w)\).

---

Table D-1  Self-Heating Data for an RTD from Testing in a Nuclear Power Plant

<table>
<thead>
<tr>
<th>Resistance (ohms)</th>
<th>Current (ma)</th>
<th>Power (mw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>423.4</td>
<td>11.5</td>
<td>55.7</td>
</tr>
<tr>
<td>424.0</td>
<td>20.7</td>
<td>181.2</td>
</tr>
<tr>
<td>425.1</td>
<td>32.1</td>
<td>438.8</td>
</tr>
<tr>
<td>426.4</td>
<td>41.4</td>
<td>732.2</td>
</tr>
<tr>
<td>428.1</td>
<td>51.2</td>
<td>1123.1</td>
</tr>
</tbody>
</table>

**Results**

Self Heating Index: 4.4

(ohms/watt)
Figure D-2   Self-Heating Curve of an RTD Being Tested in a Nuclear Power Plant
D.2 Correlation between SHI and Response Time

The steady-state relation between temperature and $I^2R$ heating generated in an RTD is given by:

$$Q = UA(T - \theta) \quad \text{(D.1)}$$

where:

- $Q$ = Joule heating generated in the RTD by applying $I^2R$ heating
- $U$ = overall heat-transfer coefficient at the RTD’s sensing tip (includes both internal and surface heat transfer)
- $A$ = heat-transfer area
- $T$ = RTD temperature
- $\theta$ = temperature of fluid in which the RTD is installed

For constant fluid temperature, Equation D.1 may be written as follows:

$$\Delta Q = UA\Delta T \quad \text{(D.2)}$$

Therefore, the temperature rise per unit power generated in the RTD is:

$$\frac{\Delta T}{\Delta Q} = \frac{1}{UA} \quad \text{(D.3)}$$

The resistance of the RTD’s platinum element is approximately proportional to its temperature (i.e., $\Delta R = \alpha \Delta T$ where $\alpha$ is the temperature coefficient of resistance). Thus:

$$\frac{\Delta R}{\Delta Q} = \frac{\text{Constant}}{UA} \quad \text{(D.4)}$$
On the other hand, an RTD’s response time is approximately given by the following equation, assuming that the RTD is a first-order system:

$$\tau \approx \frac{mc}{UA}$$  \hspace{1cm} (D.5)

where:

- $m$ = mass of the sensing tip of the sensor
- $c$ = specific heat capacity of the sensor material

If the heat capacity $c$ remains constant, then:

$$\tau = \frac{\text{Constant}}{UA}$$  \hspace{1cm} (D.6)

Comparing equations D.6 and D.4 leads to the conclusion that $\tau$ is proportional to $\frac{\Delta R}{\Delta Q}$

That is:

$$\tau \propto \frac{\Delta R}{\Delta Q} \quad \text{or} \quad \tau \propto SHI$$  \hspace{1cm} (D.7)

where $\frac{\Delta R}{\Delta Q}$ is equal to SHI, and $\alpha$ represents proportionality. Equation D.7 shows that a change in an RTD’s response time can be identified from a change in its SHI.

**D.3 Temperature Rise in the LCSR Test**

In this section, we calculate the temperature rise in an RTD for a self-heating value of 4.4 $\Omega$ /W and 40 mA of current. If the RTD is a 200-ohm sensor, at a PWR temperature of 300°C, its resistance will be about 424 ohms. With 40 mA of current, 0.68 watts of power will be generated in the RTD. With a self-heating index of 4.4 $\Omega$/W, this will cause a 3.0 ohm increase in resistance. Using a temperature coefficient of resistance of 0.8 $\Omega$/°C for the 200 ohm RTD, this corresponds to about 3.75°C. That is, in this RTD, a LCSR current of 40 mA will increase the RTD temperature by 3.75°C above the process temperature.