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Essays on Credit Risks and Financial Frictions

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Abstract

This thesis consists of two main chapters studying the interplay between government policies and firms' financing, focusing on credit risks.

In Chapter 2, I study the effects of government-backed financing on aggregate productivity by exploiting an expansion of government loans to firms in Korea after 2017. I show that the borrowing cost decreased more for firms eligible for government loans relative to ineligible firms. Eligible firms with higher pre-policy borrowing costs had larger post-policy increases in investment than eligible firms with lower pre-policy borrowing costs. At the same time, the exit rate of low-productivity eligible firms decreased the most following the policy. To quantify the effect on aggregate productivity, I build a heterogeneous-firm model with endogenous entry and exit, borrowing cost, and investment. I find that an expansion of government loans to firms as large as the one observed in Korea decreases aggregate productivity by 0.3% over 10 years, explained by a 0.1% increase coming from higher investment by formally constrained firms and a 0.4% decrease attributed to the reduced exit rates among low-productive firms.

In Chapter 3, I study how the government's local debt financing influences firms' access to credit, in turn shaping the response of emerging economies to global financial conditions. Local currency debt allows emerging economies' governments to avoid currency mismatch, which is expected to insulate them from global financial fluctuation. Using data from 11 emerging economies, I document that this insulation is partial and the degree of the insulation depends on a country's financial development and debt level. I also find that banks in a country with low financial development relative to its debt level disrupt private credit more significantly when foreign capital exits from the local currency bond market. Low financial development relative to its debt level makes the local economy more exposed to external factors despite a seemingly lowered exposure of government debt, as government debt crowds out credit for firms. To better understand these patterns, I develop a sovereign default model with local currency bonds that can be held by local banks and a heterogeneous set of foreign investors. The model replicates key patterns observed in the data, related to the relationship between an economy's capacity to maintain private credit during capital outflows, credit risk, and external vulnerability.

Keywords

Credit risk, financial friction, government-backed financing, misallocation, zombie firms, emerging market, currency mismatch, capital flows, exchange rate

Summary for Lay Audience

This thesis explores the relationship between government policies and firms' financing focusing on the role of credit risks. I investigate how financial frictions stemming from credit risks interact with government interventions to shape firms' financing decisions. Specifically, the research examines two key aspects: the direct impact of government-backed financing on firms (Chapter 2), and the indirect effects of government debt financing policies on the financial markets firms use (Chapter 3).

In Chapter 2, I study the impact of government loans on small and mid-sized firms in South Korea. The newly elected government expanded government loans to these firms for inclusive economic growth. Using financial statement data on Korean firms, I find that firms eligible for government loans experienced reduced borrowing costs relative to ineligible firms, leading to increased investment by eligible firms facing high borrowing costs before this government loan expansion. However, this policy also prolonged the survival of less productive firms, resulting in a negative effect on overall productivity. While government support stimulated investment by constrained firms, positively impacting aggregate productivity with a better capital allocation across firms, it simultaneously dampened productivity by sustaining low-productivity firms. Using a heterogeneous firm dynamics model, I find that an expansion of government loans to firms as large as the one observed in Korea decreases aggregate productivity by 0.3% over 10 years, explained by a 0.1% increase coming from higher investment by formally constrained firms and a 0.4% decrease attributed to the reduced exit rates among low-productivity firms.

In Chapter 3, I explore how emerging governments' local currency debt financing affects firms' credit access and shapes the vulnerability of emerging economies to global financial conditions. Borrowing in local currency is expected to provide a shield for emerging economies from global financial market fluctuations because local currency debt is free from the currency mismatch problem. Using data from 11 emerging economies, I find this shield is only partial, dependent on the level of financial market development and government debt. In countries with underdeveloped financial markets and high government debt, local banks disrupt lending to firms when foreign investors withdraw from the local bond market. This leaves the local economy exposed to external shocks as government debt crowds out credit for firms, negatively impacting economic activity. To better understand these patterns, I develop a sovereign default model with local currency bonds that can be held by local banks and a heterogeneous set of foreign investors. The model replicates key patterns observed in the data, related to the relationship between an economy's capacity to maintain private credit during capital outflows, credit risk, and external vulnerability.

Dedication

To my husband Sungwook, my daughter Yuna and my mom

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Chapter 1

Introduction

My thesis consists of two main chapters. These two chapters explore the intricate interplay between government policies and firms' financing, with a particular focus on the role of credit risks.

Specifically in Chapter 2, I study the effects of government-backed financing on aggregate productivity using Korea's large expansion of government loans to firms. This expansion was aimed at small-mid-sized enterprises in response to the policy agenda of the newly elected government, which emphasized fostering a more favorable business environment for small-mid-sized enterprises and promoting inclusive economic growth. Combining data on financial statements of active and exiting firms, I show that borrowing costs decreased more for firms eligible for government loans relative to ineligible firms. Moreover, eligible firms with higher pre-policy borrowing costs had larger post-policy increases in investment than eligible firms with lower pre-policy borrowing costs, consistent with these firms having been financially constrained in their investment choices. At the same time, the exit rate of low-productivity eligible firms decreased the most following the policy.

I quantify the effect of government-backed financing on aggregate productivity using a heterogeneous firm model with endogenous borrowing costs. In the model, government loans enhance firms' credit access, helping financially constrained firms increase investment but also prolonging the survival of low-productivity firms, which captures the patterns documented with the data. These patterns summarize the main trade-off of government-backed financing on aggregate productivity. I find that an expansion of government loans to firms as large as the one observed in Korea decreases aggregate productivity by 0.3% over 10 years. This is explained by a 0.1% increase in productivity coming from higher investment by constrained firms and a 0.4% decrease attributed to the reduced exit rates among low-productive firms, which worsens the productivity composition of active firms in the economy.

In the following Chapter, I study how the way governments finance their debt determines firms' access to credit, in turn shaping the response of the economy to fluctuations in global financial conditions. In particular, I look at the effect of the government's local currency debt. Local currency debt helps emerging governments avoid currency mismatch, simplifying their debt management and insulating them from global financial fluctuation. However, this insulation is only partial, a phenomenon referred to as the "original sin redux". Using data from 11 emerging economies, I document that the degree of the insulation depends on a country's level of financial development and its debt level. Moreover, I also find that banks in a country with low financial development (relative to its debt level) disrupt private credit more significantly when foreign capital exits from the local currency bond market. Low financial development compared to its debt level makes the local economy more exposed to external factors despite a seemingly lowered exposure of government debt, as government debt crowds out credit for firms.

To better understand the cross-country patterns I document, I build a sovereign default model incorporating financial intermediaries and endogenous foreign investors' investment in local currency government bonds. In my model, foreign investors and domestic financial intermediaries hold local currency government bonds in equilibrium. Following a negative shock in the global financial market, emerging local currencies tend to depreciate, inducing foreign investors to reduce local currency bond holdings due to expected losses from their unhedged exposure to local currency. Consequently, domestic financial intermediaries increase their government bond holdings, crowding out the provision of private credit. This disruption adversely affects the economy, ultimately increasing the government default risk. This mechanism explains the key patterns observed in the data, related to the relationship between an economy's capacity to maintain private credit during capital outflows, credit risk, and external vulnerability.

Chapter 2

Government-Backed Financing and Aggregate Productivity

2.1 Introduction

Government-backed financing policies, typically in the form of loan guarantees, direct loans, and financial assistance programs, are implemented worldwide to promote firms' investments and growth.[1] However, their net impact on aggregate productivity remains uncertain. These policies have been successful in facilitating funding for financially constrained yet productive firms, potentially enhancing overall productivity (Stiglitz, 1993; Banerjee and Duflo, 2014; Jiménez, Peydró, Repullo, and Saurina Salas, 2018; Díez, Duval, and Maggi, 2022).[2] But, they also allow less productive firms to persist, potentially reducing aggregate productivity (Caballero, Hoshi, and Kashyap, 2008; Acharya, Lenzu, and Wang, 2021; Faria-e-Castro, Paul, and Sánchez, 2021).

I study the effects of a significant increase in government loans to firms using a novel dataset of Korean firms, addressing the trade-off between increased investment by productive but constrained firms and lower exit of low-productivity firms. The dataset of financial statements from Korean manufacturing firms includes data from both operating firms and exiting firms, allowing me to better measure the trade-off. The dataset covers 14,569 firms with assets over 9 millions USD, subject to external audits. Revenue of sample firms accounts for approximately 80 % of

[1] For example, Small Business Administration (SBA) Loans in the US, Canada Small Business Financing Program, Small and Medimum Enterprise Financing by Japan Finance Corporation in Japan.

[2] The role of financial constraints in distorting the allocation of capital has been widely studied and highlighted as a major factor in reducing aggregate productivity. See Buera, Kaboski, and Shin, 2011, Khan, Senga, and Thomas, 2014, Moll, 2014, Midrigan and Xu, 2014.

total sales. Among the sample firms, 88% are non-listed firms, while 86% are small-mid sized firms that are eligible for government loans.

I exploit an unprecedented increase in government loans brought by a change of government in 2017. The government loans to firms rose from 2.25 % of GDP before 2017 to 3.12 % by 2019, as shown in Figure 2.1. These loans are only available for small-mid sized firms, are extended at an interest rate below market rates, and also require less collateral than private loans.[3][4] The expansion of the government loan was not triggered by a crisis or recession episode. Instead, it responded to the political platform of the newly elected government, which included creating a more favorable business environment for small-mid sized enterprises and promoting inclusive growth.

I document the following patterns: after the policy shift, the borrowing cost, proxied by firm specific credit spreads, decreased for eligible firms relative to the one for non-eligible firms.[5] I take this as an indication of improved credit access for eligible firms. I also show that eligible firms increased investment more after the policy relative to ineligible firms. Moreover, eligible firms with higher pre-policy borrowing costs exhibited a 5-percentage-point greater increase in investment post-policy than eligible firms with lower pre-policy borrowing costs, consistent with the findings of Banerjee and Duflo (2014) and Jiménez, Peydró, Repullo, and Saurina Salas (2018).

I next document that the aggregate exit rates decreased, and the share of low-productivity firms increased after the policy, as depicted in Figure 2.1. I use the definition of zombie firms widely used by the literature to classify firms as low productive. These are older firms persistently incapable of servicing their debt with their operating profits.[6] Additionally, I find that the exit rate of low-productivity eligible firms decreased by 2.8 percentage points more than that of productive eligible firms. This finding aligns with the literature that studies the rise of zombie firms and credit misallocation resulting from government's subsidized loans (Acharya, Lenzu, and Wang, 2021), as well as some features of financial intermediation such as forbearance lending (Tracey,

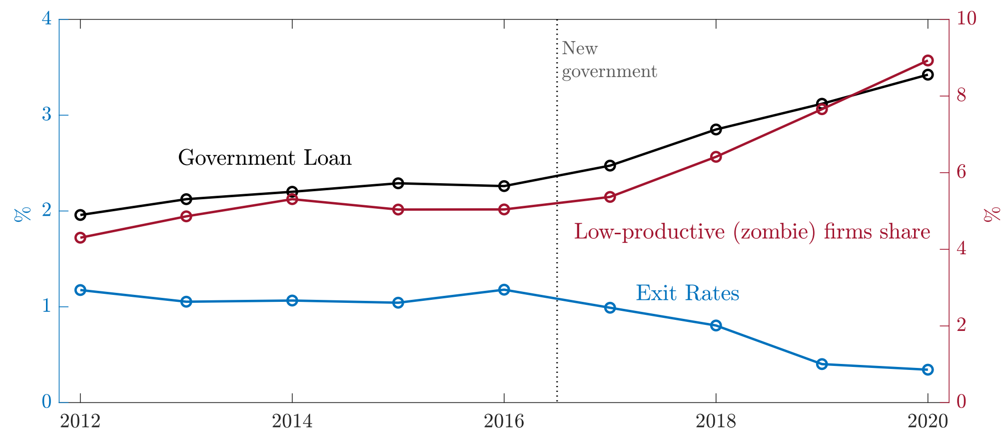
[3]To qualify as a small-mid sized enterprises, a firm's total assets must not exceed 380 million USD, and its three-year average annual sales should fall within the 60 to 120 million USD range, with sector-specific sales cutoffs.

[4]The government interest rate is, on average, 2.5% lower than the average interest rate on new loans offered by the bank, which stands at 3.7%, despite having a 1.2 percentage point higher delinquency rate than bank loans.

[5]I measure cost of funding based on credit spread calculated by the deviation of interest rates a specific firm pays from the Korea corporate bond yield (3yr, AA-).

[6]I follow McGowan, Andrews, and Millot (2017), R. Banerjee and Hofmann (2018), Hong, Igan, and Lee (2021), and define a firm as a zombie firm if its interest coverage ratio (i.e. the ratio of operating income to interest expenses, ICR) has been less than one or its operating profit is negative for at least three consecutive years and if it is at least 10 years old.

Figure 2.1: Government loans, exit rates and share of low-productivity firms



Notes: Government loans, which represent loans granted to private non-financial enterprises as a percentage of GDP, are indicated on the left-hand side. Exit, which indicates business closures, is also represented on the left-hand side. A firm is identified as a zombie firm if its interest coverage ratio (i.e., the ratio of operating income to interest expenses, ICR) has been less than one or its operating profit is negative for at least three consecutive years and if it is at least 10 years old. Zombie shares are represented on the right-hand side.
Sources: National Information & Credit Evaluation (NICE), Bank of Korea flow of funds statistics, Author's calculation

2019) and relationship lending (Faria-e-Castro, Paul, and Sánchez, 2021).

To quantify the aggregate effect of government-backed financing on productivity, I build a heterogeneous firm model with government-back financing. The model extends Arellano, Bai, and Kehoe (2019) and features heterogeneous intermediate goods firms that produce homogeneous product using capital as an input. They can borrow from private creditors and the government to finance capital accumulation. Firms are subject to idiosyncratic productivity shocks. Firms with insufficient cash-on-hand default and exit.

The presence of default risk introduces an endogenous financial constraint. Since the the interest rate on private loans is endogenous and compensates private creditors for the risk of default, firms with less cash-on-hand can borrow less and at higher interest rates. Due to this friction, firms with less cash-on-hand tend to invest less compared to firms with more cash-on-hand, even when other financial factors such as size and profitability are taken account, as in Khan, Senga, and Thomas (2014). Furthermore, these financial frictions heighten the vulnerability of firms with limited cash-on-hand, placing them at a greater risk of default and subsequent exit from the market.

In line with the program in Korea, I assume that government loans are subsidized, as they are provided at below market rates, and are extended in limited amounts. Government loans increase investment of financially constrained firms and also increase the survival rate of firms that would have defaulted and exited without those loans, thereby leading to a higher prevalence of low-productivity firms. This generates a general equilibrium effect similar to congestion externalities studied in Caballero, Hoshi, and Kashyap (2008), Acharya, Crosignani, Eisert, and Eufinger (2020), and Acharya, Crosignani, Eisert, and Steffen (2022).[7] [8] As the loans enable low-productivity firms to accumulate more capital and increase production, there is downward pressure on the equilibrium price of intermediate goods, reducing firms' profitability. This effect is not only damaging to the profitability of operating firms but also discourages potential entrants from entering the market.

I calibrate the model to match pre-policy aggregate key moments of firms' distribution in Korea. The calibrated model matches untargeted cross-sectional moments based on firms' net-income ratios. I introduce government loans into the calibrated model to replicate the observed decrease in exit rates over a three-year period following the policy shift in Korean data. For my main exercises, I simulate the transition of the economy after the introduction of government loans. I run the same regressions that I do in the empirical analysis using the paths of simulated firms.

The model effectively captures the firm-level heterogeneity in responses to government loans in terms of investment and exit, closely mirroring the patterns documented with the data. Firms initially characterized by higher pre-policy borrowing costs increase their investment by 4 percentage points more than firms with lower pre-policy borrowing cost in simulated data, while the data show a 5 percentage point higher increase. Additionally, the simulated data show that low-productivity firms' exit rate decreases by 2.3 percentage points more than high-productivity firms, whereas the data indicate a larger decrease of 2.8 percentage points for low-productivity firms compared to high-productivity firms. Indeed, the main trade-off of government loans on aggregate productivity through enhanced credit access is well summarized by these two heterogeneous responses.

The general equilibrium after the introduction of government loans features a reduced firms' profitability and an increase in the share of cash-strapped firms. Normal firms are more likely to transition into zombie firms, and zombie firms are more likely to remain as zombie firms.

[7] In my model, there is no explicit negative spillover effect resulting from a larger share of low-productivity (zombie) firms, as empirically documented by McGowan, Andrews, and Millot (2017), Gouveia and Osterhold (2018), R. Banerjee and Hofmann (2018). Instead, improved access to credit expands production capacity, both intensively and extensively, exerting downward pressure on the price through general equilibrium effects.

[8] Schivardi, Sette, and Tabellini (2020) argue that there is no causal relationship between the share of zombie firms and the relative performance of healthy firms. They posit that the relative performance of non-zombie firms worsens due to aggregate shocks, leading to a larger share of zombie firms.

These factors collectively lead to a greater share of zombie firms in the economy.

To quantify the aggregate effect on productivity, I decompose aggregate productivity into two components: capital allocation efficiency labelled as "intensive efficiency" and the composition of productivity labelled as "extensive efficiency".[9] As government loans assist firms with low cash-on-hand in increasing their investments, this leads to an improvement in intensive efficiency. However, the government's intervention changes the extensive margin and worsens the selection process, resulting in a decrease in extensive efficiency. The loss from extensive efficiency (0.4%) outweighs the gain from intensive efficiency (0.1%). Consequently, the economy experiences a decrease in productivity over 10 years by 0.3%.

Related literature My paper contributes to following strands of literature: Firstly, my paper contributes to the literature on firm dynamics and financial frictions, which studies the implications of firms' limited liability and endogenous borrowing constraints (Arellano, Bai, and Kehoe, 2019, Khan, Senga, and Thomas, 2014). My paper shares an emphasis on financial frictions arising from positive default probabilities, which can create a situation where firms with large internal funds have higher levels of investment than those with limited internal funds, even if firms with limited internal funds are more productive. This insufficient allocation of capital to firms with limited internal funds reduces aggregate productivity. In this context, I quantify the extent to which government fiscal support, in the form of lending, can enhance capital allocation by mitigating the frictions resulting from firms' limited liability. My work is also related to Ottonello and Perez (2019) in the sense that I study the aggregate effect of government loans, considering firm-level heterogeneous response to the policy and how this heterogeneity collectively impacts the aggregate level in a general equilibrium framework, while their work studies the aggregate effect of monetary policy.

Secondly, my research contributes to the literature on government-backed financing by empirically documenting the policy's impacts using newly constructed extensive firm-level data based on Korea's credit expansion to firms. This unique dataset includes both active and exiting firms, enabling a comprehensive study of the policy's impact on both investment and exit, which are closely related to the policy effects on aggregate productivity. Moreover, the Korean government's policy shift, which did not respond to economic shocks, provides a clear setting for studying the policy's effect. I find that the Korean government's credit expansion has facilitated increased investment among financially constrained firms, aligning with findings from studies

[9]The intensive efficiency equals 1 when capital is distributed across firms in a way that equalizes the marginal product of input across firms.

such as Banerjee and Duflo (2014), Jiménez, Peydró, Repullo, and Saurina Salas (2018), and Crouzet and Tourre (2021). Additionally, it has contributed to a reduction in firm exits, consistent with the results presented in Guerrieri, Lorenzoni, Straub, and Werning (2020).

Lastly, my work is related to the literature that studies credit misallocation in the context of zombie firms. In line with the literature that empirically documents the distortionary effect on the extensive margin arising from subsidized loans, such as government's subsidized loans (Acharya, Lenzu, and Wang, 2021), forbearance lending (Tracey, 2019), or relationship lending (Faria-e-Castro, Paul, and Sánchez, 2021), I empirically document that government loans with subsidized features distort the selection by decreasing exit rates among low-productivity firms the most. Furthermore, in my model, subsidized government loans expand production capacity both intensively and extensively, and generate general equilibrium effects. This is similar to congestion externalities studied in Caballero, Hoshi, and Kashyap (2008) and Acharya, Lenzu, and Wang (2021), but it differs from their models in that there is no explicit negative spillover effect resulting from a larger share of low-productive (zombie) firms. Instead, the expanded capacity due to improved credit access with subsidized loans imposes downward pressure on the price, as documented by Acharya, Crosignani, Eisert, and Eufinger (2020).

Layout The rest of the paper is organized as follows. Section 2.2 provides an overview of Korean government-backed financing and a description of the firm-level data, and section 2.3 documents the effects of government loans using Korean firms' data. In section 2.4, the model is introduced, and section 3.4 outlines the parametrization strategy and assesses the model's quantitative validity against the data. Section 2.7 investigates the aggregate implications of the policy, and section 3.5 concludes.

2.2 Korean Policy and Firm Level Data

2.2.1 Korean government-backed financing policy

The Korean government has long provided financial support to small and medium-sized enterprises (SMEs) primarily through loan guarantee programs and direct loans. In practice, various government ministries and agencies, as well as local governments, raise funds through budget allocation, borrowing from public funds, and bonds issuance to extend financial assistance to SMEs under favorable terms.

The change in government after the election in 2017 brought a significant policy change, resulting in an unprecedented increase in government loans. This is reflected in the increase in government loan amounts, as indicated in in Figure 2.1. Specifically, the government loan went from 2.25 % of GDP on average for 2014~16 to 2.85 % as of 2018, and 3.12 % as of 2019. The primary goal of the new government was to create a favorable business environment for SMEs and promote inclusive economic growth by leveling the playing field between large and small to medium-sized firms. Three major policy tasks, aimed at promoting SMEs, were included in the list of 100 key initiatives released by the new government in 2017. Additionally, in the same year, the Small and Medium Business Administration was elevated to ministry status, becoming the Ministry of SMEs and Startups.

Government loans target small-mid sized firms, whose status is determined by criteria defined by law.[10] To qualify as a SME, a firm's total assets must not exceed 380 million USD, and its three-year average annual sales should fall within the 60 to 120 million USD range, with sector-specific sales cutoffs. Moreover, a firm meeting the SME size benchmarks must maintain separation in ownership and management from entities known as Chaebols, such as Samsung or Hyundai. Firms exceeding size requirements can access government loans for three years after they exceed those requirements.

The government loans are commonly provided up to a specific limit at a fixed interest rate, lower than the market borrowing rates.[11] The interest rates for government loans are determined based on adjustments made around the benchmark interest rate presented in Figure 2.2. The adjustment depends on factors such as the credit rating of the company, the purpose of the funds, and the presence of collateral.[12]

2.2.2 Financial statements of Korean manufacturing firms

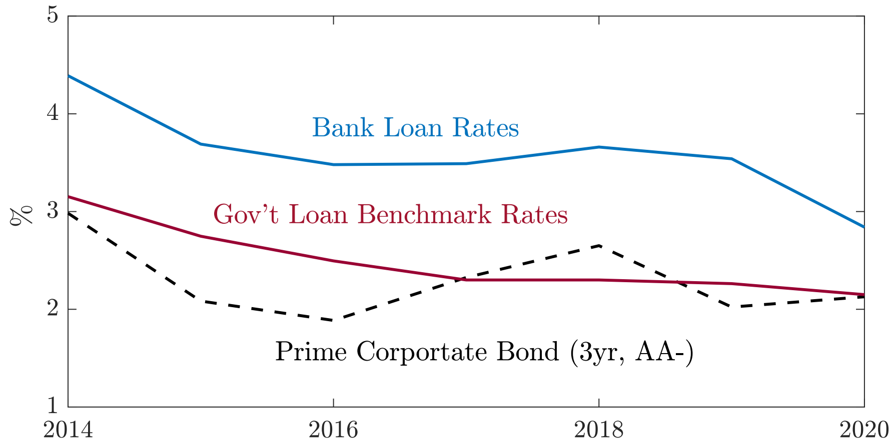
I construct a dataset of financial statements from Korean manufacturing firms, including operating and exiting firms. The inclusion of exiting firms allows me to observe their financial condition at the time of exit and provides a better understanding of the heterogeneous effects of the increase in government loans on firms' exit. Financial statements come from National In-

[10] Government loan programs encompass a diverse array of types, ranging from initiatives addressing management challenges like cash shortages to those bolstering innovation and promoting exports. These programs involve a range of institutions, including government ministries, agencies, and local governments. The specific eligibility criteria can differ based on the involved institutions and specific programs. However, a common prerequisite for eligibility is SME status.

[11] The government interest rate is, on average, lower than the average interest rate on new loans offered by the bank, despite having a higher delinquency rate than bank loans. There is no publicly available data on the delinquency rate of the government loans themselves. However, the government agency related to loan guarantee releases data on the delinquency rate for this loan guarantee, which is 3.7% lower than the delinquency rate for bank loans, which stands at 2.0%.

[12] According to the Survey of Small and Medium Industries' Financial Standing for 2019, 70% of SMEs pointed out lower interest rates as the main reason for applying for government loans.

Figure 2.2: Benchmark rates of the government loans



Notes: The interest rate for a loan is determined by making adjustments relative to the benchmark interest rate, considering factors such as the firm's credit rating, the intended use of the funds, and the presence of collateral. Bank loan rates represent the average interest rates applied to newly issued loans to firms. Prime corporate bond rates are the yields of corporate bonds with a maturity of 3 years and a credit rating above AA-.
Sources: Korea SMEs and Startups Agency, Bank of Korea

formation & Credit Evaluation (NICE). I construct the list of exiting firms based on information obtained from CRETOP, Korea Enterprise Data.

The data covers manufacturing firms with assets over 9 million USD, subject to external audits and required by law to release their balance sheet information to the Financial Supervisory Commission.[13] The sample consists of 12,976 active firms and 1,593 exiting firms, accounting for approximately 80% of total manufacturing sector sales. I categorize firms as non-listed and small-medium based on each firms' status in the year of 2020.[14] The indicator of small-mid sized firm is also subject to external audit. A firm classified as a small-mid sized firm is a firm which is officially confirmed to be eligible for government program for SMEs. The majority of sample firms are non-listed firms (88%) and are small-medium firms (86%) eligible for government loans.

Main financial information includes sales, net income, operating profit, interest expense, total debt, total and tangible assets. The key variables used for the analysis are credit spreads, investment, and an exit indicator. Credit spread is defined as the deviation of interest rates paid by a specific firm from the Korea corporate bond yield (3yr, AA-). The firm-specific interest

[13]Firms are allowed to enter, exit, and pause reporting for several years during the sample period when their assets go below the threshold.

[14]Usually, only a small portion of SMEs undergo the transition to become large firms. Specifically, on average, this transition rate amounts to just 0.004% for the period between 2017 and 2019.

rates are calculated using the total amount of debt and the total amount of interest expenses paid for a specific year. Tangible asset growth is employed for investment. The exit indicator denotes whether a firm has publicly announced its closure, excluding cases of merger with other firms.

2.3 Effects of Government-Backed Financing

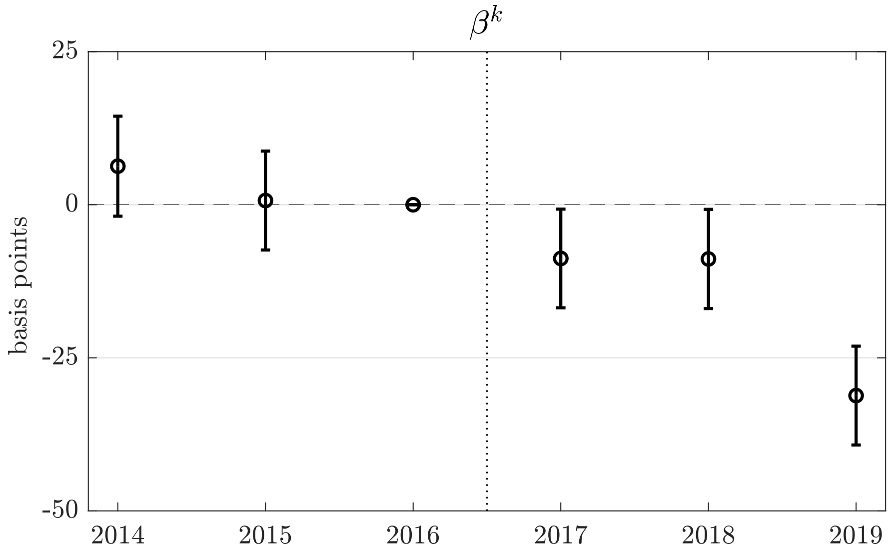
In this section, I document the effects of the Korean government's expansionary credit policy on firms' funding costs (subsection 2.3.1), investment (subsections 2.3.2), and exit (2.3.3). These findings help us understand the main trade-offs of the policy in terms of aggregate productivity through improved credit access. By examining the impact on funding costs, we can assess whether the policy improved firms' credit access. Analyzing the impact on investment allows us to determine whether firms that were financially constrained increased their investments, which can increase aggregate productivity by improving capital allocation across firms. Exploring the effect on firms' exit informs us about whether the policy led to the higher survival rates for low-productive firms, potentially decreasing aggregate productivity.

To conduct this analysis, I employ a difference-in-difference approach. I compare changes in these three key outcome variables between firms eligible and ineligible for government loans, and also consider how financial characteristics within eligible firms may impact these outcomes. Unlike some other programs like the US Treasury's PPP and Canada's CEBA, which responded to crises or unexpected shocks, Korean government-backed financing was influenced by the political agenda of the new government. This feature provides a clear framework for evaluating the policy's impact.

2.3.1 Effect on borrowing costs

I first investigate the effect of government loans on firms funding cost, measured as the credit spread between the interest rates a specific firm pays and the Korea corporate bond yield (3yr, AA-). The objective is to ascertain whether the increased government loans reduced borrowing cost for eligible firms. For this analysis, I estimate the following equation using data from 2014

Figure 2.3: Effect on credit spread by eligibility



Note: These plots show a difference in the spread gap between SMEs and large firms for specific years relative to year 2016 with 90% confidence intervals. Estimates from equation 2.1 represented in basis points.

to 2019:[15]

$$\text{Spread}_{ist} = \sum_{k \neq 2016} \beta^k \text{Year}_k D_{is}^{sme} + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist} \tag{2.1}$$

where Spread_{ist} is a firm i's credit spread in sector s for year t in basis points, Year_k is a dummy for year k, D_{is}^{sme} is an indicator for whether a firm is a SME, γ_{st} is sector-year interacted fixed effects, γ_i is a firm fixed effect, and X_{ist-1} is a vector of firm specific controls including equity to asset ratio, debt to asset ratio, cash to asset ratio, operational profit to asset ratio. I drop the dummy for the year 2016 (one year before the new government).

The results, which can be summarized with the coefficient β^k in Figure 2.3, show that following the policy change, the spread for the eligible firms decreased relative to the non-eligible firms. To be specific, the coefficient β^k represents the difference in the spread gap between SMEs and large firms for a given year relative to the year 2016. Therefore, the spread gap between the eligible and non-eligible firms decreased after the policy change.

[15]This sample period is used for all empirical analysis. This time frame was chosen in consideration of shifts in macroeconomic conditions within Korea. Years before 2014 were excluded due to substantial monetary easing measures that had already taken place in Korea. Furthermore, the year 2020 was omitted from the analysis due to the onset of the Covid-19 pandemic. During the 2014-2016 period characterized by a steady increase in government loans, the average key interest rate in Korea was 1.6%, whereas in the 2017-2019 period marked by a significant uptick in government loans under the new administration, the average key interest rate in Korea stood at 1.5%.

I next investigate whether the increased government loans change the sensitivity of credit spreads to firms' indebtedness, and this change depends on the eligibility. Specifically I estimate the following equation for two sub-periods, Before (2014-16) and After (2017-19):

$$\begin{aligned} \text{Spread}_{ist} = \; & \beta_0 \text{Debt Ratio}_{ist-1} + \beta_1 D_{is}^{sme} \text{Debt Ratio}_{ist-1} + \beta_2 \text{Debt Ratio}_{ist-1} \text{After}_t \\ & + \beta_3 D_{is}^{sme} \text{Debt Ratio}_{ist-1} \text{After}_t + \gamma_{st} + \gamma_i + \epsilon_{ist} \end{aligned} \tag{2.2}$$

where Debt ratio$_{ist}$ is firm i's debt to asset ratio in sector s for year t, D_{is}^{sme} is firms eligibility indicator as a SME, After$_t$ is a dummy after the policy, and all other specifications are same with equation 2.1.

The results indicate that the sensitivity of credit spread to the debt ratio decreased for eligible firms post-policy, while there was no change for non-eligible firms, as presented in Table A.3. Specifically, the estimate for coefficient β_0 indicates that a one-percentage-point increase in the debt ratio is associated with an average increase of 0.46 basis points in credit spread. β_1 indicates the difference in credit spread sensitivity to debt ratio of eligible firms, SMEs from non-eligible firms, large firms. β_2 indicates the change in credit spread sensitivity to the debt ratio for the non-eligible firms after the policy change, which is non significantly different from zero. This suggests that the policy did not have a significant impact on the sensitivity of credit spread to the debt ratio for non-eligible firms. On the other hand, the significantly negative β_3 indicates that the sensitivity of credit spread to the debt ratio decreased for eligible firms post-policy. Specifically, following the policy change, credit spreads for eligible firms increased less by 0.26 basis points for each one-percentage-point increase in the leverage ratio, while there was no change for large firms.

I interpret these empirical results as an indication of improved credit access for eligible firms following the policy change. Next, I explore questions related to how this improved credit access has influenced investment and exit.

2.3.2 Effect on investment

I investigate whether the policy helps firms expand investment by reducing funding costs. To do this, I compare changes in investment among four groups categorized by eligibility and pre-policy credit spread. Firms with high pre-policy credit spreads, after controlling for other financial characteristics, may have faced higher borrowing costs for investments compared to firms with lower pre-policy credit spreads. Eligible firms with higher borrowing costs may have been constrained in their ability to invest, and thus would increase their investment more relative to firms with lower funding costs. Specifically, I estimate the following equation:

Table 2.1: Credit Spread Sensitivity to Debt Ratio

		Spread
β_0	Debt Ratio	0.46***
		(0.17)
β_1	Debt Ratio × eligible	-0.12
		(0.18)
β_2	Debt Ratio × After	-0.05
		(0.10)
β_3	Debt Ratio × eligible× After	-0.26***
		(0.09)
	Observations	57,625
	R^2	0.05

Notes: Eligible indicates the indicator a firms is SMEs. Estimates from equation 2.2 represented in basis points. Standard errors are in parentheses. *** $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

$$\begin{aligned}\text{Investment}_{ist} = \ & \beta_1 D^{sme}_{is} D^{High}_{is} \text{After}_t + \beta_2 D^{sme}_{is}(1 - D^{High}_{is})\text{After}_t \\ & + \beta_3 (1 - D^{sme}_{is}) D^{High}_{is} \text{After}_t + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist}\end{aligned} \tag{2.3}$$

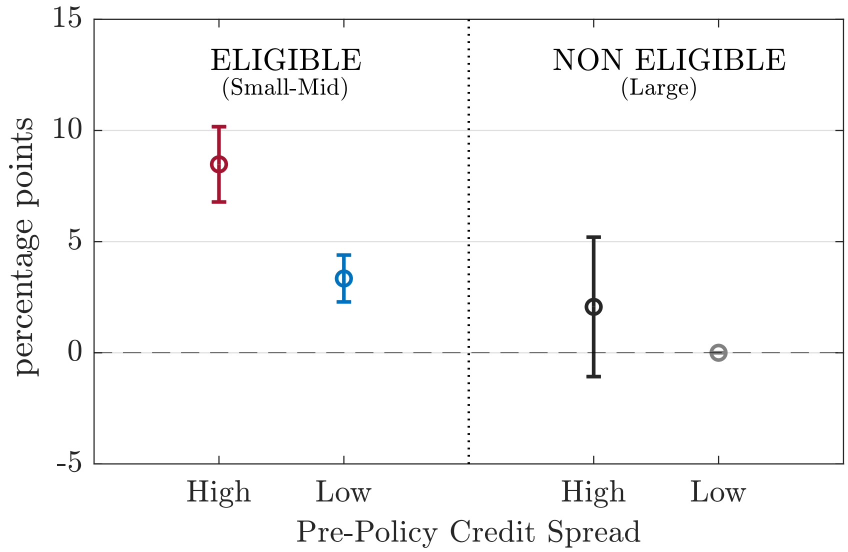
where D^{High}_{is} is an indicator of whether a firm's mean spread in the Before period is in the upper 10th percentile, and X_{ist} is a vector of firm specific controls including log of tangible asset, and operating profit to asset ratio to control for firms' marginal benefit to investment.

The eligible group exhibited an average increase in investment and this increase was more pronounced among firms with high pre-policy credit spread. Specifically, firms that initially paid higher credit spreads increased their investment by 5 percentage points more than firms with low pre-policy credit spreads.[16] On the contrary, I find no significant effect among ineligible firms. The results are illustrated in Figure 2.4.

To assess robustness, I conducted an event study using the same specifications applying year dummies separately for SMEs and large firms. Furthermore, the model was estimated using pre-policy credit spread values directly, without the use of dummy indicators. The outcomes

[16]This finding aligns with the findings of Banerjee and Duflo (2014), who demonstrate that constrained firms tend to use government credit to expand production, while unconstrained firms primarily use it as a substitute for other borrowing. In my analysis, the pre-policy credit spread serves as an approximation of firms' financial constraints and suggests that more constrained firms increased their investment to a greater extent.

Figure 2.4: Effect of policy on investment



Note: The figure plots the coefficient β_1, β_2, and β_3 respectively from the equation 2.3 with 90% confidence intervals. Eligible and non eligible groups are divided by the small-mid sized enterprises indicator. Firms in high group are firms whose credit spread in 'Before' preiod (2014-2016) was in 10th percentile.

remain consistent with the previously outlined results and are provided in Appendix A.1.

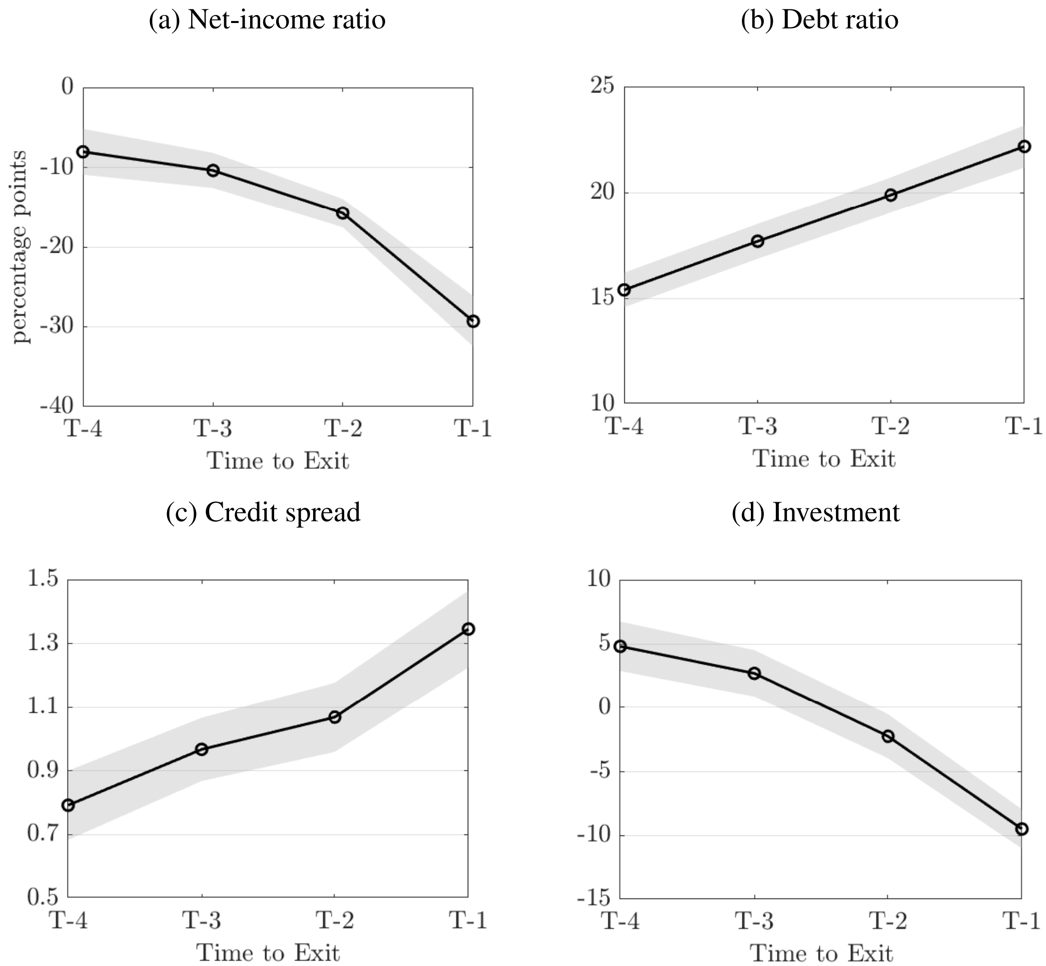
2.3.3 Effect on exit

Firms' exit rates decreased from 1.4% between 2014-2016 to 0.9% between 2017-2019, as presented in Figure 2.1. Importantly, the exit rates had remained stable during the periods preceding the policy change.[17] Before the exit, firms frequently undergo a cash shortage, an upsurge in debt ratio, increased credit spreads, decreased investment. This trend suggests that firms facing a sustained cash shortage face difficulties in obtaining financing, ultimately leading to their exit. See Figure 2.5.

Exit rates decreased particularly more among low-productive firms, which are often situated at the margin of potential exit. To classify these low-productive firms, I draw upon existing literature studying zombie firms: firms that have barely survived thanks to government financial assistance. Following McGowan, Andrews, and Millot (2017), R. Banerjee and Hofmann (2018), and Hong, Igan, and Lee (2021), I define zombie firms as those whose interest coverage ratio (ICR), i.e., the ratio of operating income to interest expenses, has remained below one, or

[17]This trend is consistent with the findings documented by Caballero, Hoshi, and Kashyap (2008), Acharya, Crosignani, Eisert, and Steffen (2022), and Faria-e-Castro, Paul, and Sánchez (2021), who show that improved credit access decreases firms' exits.

Figure 2.5: Financial state before exit



Notes: These plots show the relative financial state of firms with specific distance to exit. Specifically, those are series of coefficient of $y_i = \alpha + \sum_{k=1}^{4} \beta_k D_i^{T-k} + \epsilon_i$, where D_i^{T-k} is an indicator whether a specific firm i closes down and exits after k periods. The shaded area indicates the 90% confidence interval.

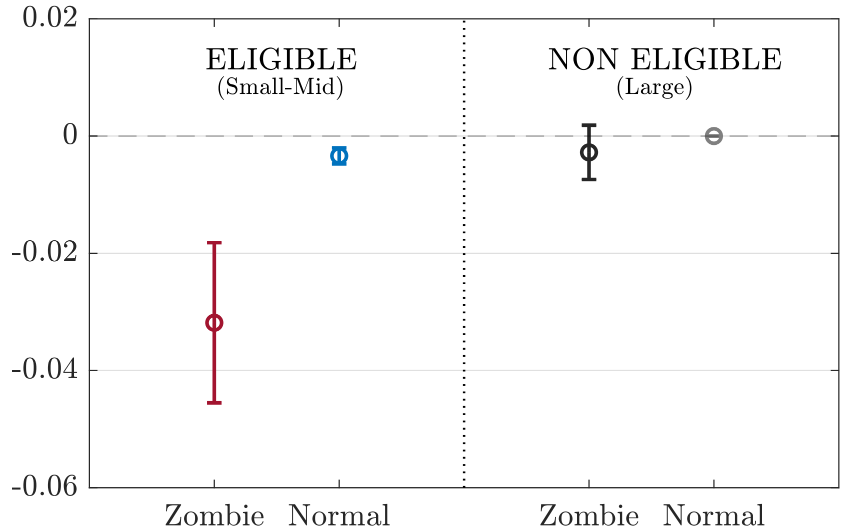
those that have sustained negative operating profits for a minimum of three consecutive years, provided they are at least 10-year old. The share of zombie firms increased strongly after 2017 as depicted in Figure 2.1.

I also estimate the additional regression to assess whether the decrease in exit rates was more pronounced among low-productive firms, using four groups categorized based on eligibility for the policy and a zombie indicator in the previous year:

$$\begin{aligned}\text{Exit}_{ist} = {} & \gamma_1 D_i^{sme} D_{it-1}^{Zombie} + \gamma_2 D_i^{sme}(1 - D_{it-1}^{Zombie}) + \gamma_3(1 - D_i^{sme}) D_{it-1}^{Zombie} + \gamma_t \\ & + \beta_1 D_i^{sme} D_{it-1}^{Zombie} \text{After}_t + \beta_2 D_i^{sme}(1 - D_{it-1}^{Zombie}) \text{After}_t + \beta_3 (1 - D_i^{sme}) D_{it-1}^{Zombie} \text{After}_t + \epsilon_{it}\end{aligned} \tag{2.4}$$

The eligible firms exhibited an average decrease in exit rates, and importantly, this decrease in

Figure 2.6: Effect of policy on exit probability



Note: The figure plots the coefficient β_1, β_2, and β_3 respectively from the equation 2.3 with 90% confidence intervals. Eligible and non eligible groups are divided by the small-mid sized enterprises indicator.

exit rates was particularly larger for low-productive firms. By contrast, there was no significant change in exit rates within the non-eligible firms. See the result in Figure 2.6.

Finally, I explored differences in exit rates conditional on firms' credit spreads. I categorized firms into four groups based on their eligibility and their lagged three-year mean credit spread.[18] This analysis is aimed to document how the policy influenced the exit threshold associated with credit spreads, not the treatment effect on a specific group. The result shows no discernible effect on the non-eligible group, a decrease in exit rates among the eligible group, and a particularly pronounced decrease in exit rates among those firms that historically paid high credit spreads. See Appendix A.2.

2.4 Model

I develop a heterogeneous firm model to interpret this cross-sectional evidence and study its aggregate implications, mainly based on Arellano, Bai, and Kehoe (2019).

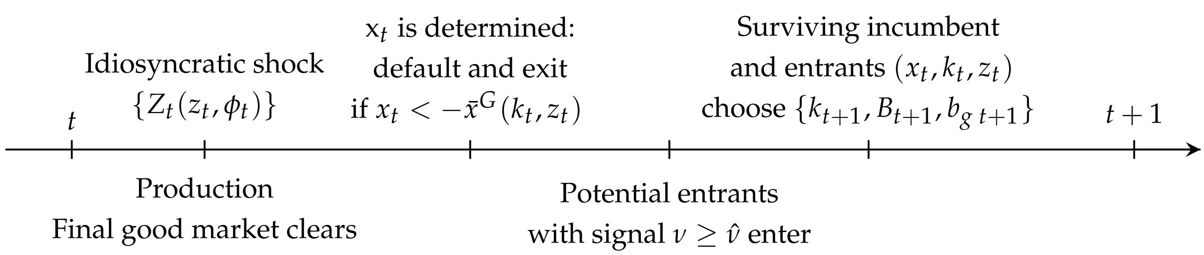
Time is discrete and infinite. There is no aggregate uncertainty, and in the following Sections,

[18]Firms with initially high pre-policy credit spreads were more likely to exit in the Before period (see Figure 2.5), which dampens the magnitude of change in their exit rates. To provide a more comprehensive analysis, I adopted a three-year average credit spread instead of pre-policy credit spread.

I study how the economy would respond to the introduction of the government loan. There are continuums of final goods firms, intermediate goods firms, private creditors, and the government. The final goods firms convert homogeneous intermediate goods into a final good and sell them at price 1. The intermediate goods firms are competitive and produce homogeneous products using capital as an input. They can borrow to finance investment and operating costs from private creditors and the government when the government loan is in place. In case it is infeasible for firms to pay the operating costs and debts, firms default and exit the market with zero value.

Before formally describing the economy, I provide a brief overview of the timeline. At the beginning of each period, intermediate goods firms are subject to two idiosyncratic shocks: persistent and i.i.d shock, which determine their production. Firms sell their outputs to final goods firms and the final goods market clears. The level of cash-on-hand is determined by revenue, operating costs and debt. Based on their cash-on-hand levels, the feasibility to continue operating is determined. At the end of the period, potential entrants receive a signal about their productivity in the following period. Surviving incumbent firms and entering firms then make decisions on borrowing and capital.

Figure 2.7: Timeline



2.4.1 Final Good Firms

The final good firms produce final goods y_F using Y as an input to maximize,

$$\max_{Y} \underbrace{\bar{z} Y^{\alpha_y}}_{y_F} - pY \tag{2.5}$$

where, p is the price of intermediate good and $\bar{z}$ is the average productivity of intermediate good firms, which are both endogenously determined by intermediate good firms' decisions. The total is determined not only by the quantity of intermediate good Y but also by the productivity composition of intermediate good firms $\bar{z}$, which I formally describe in subsection 2.4.4. First order condition gives the demand function for intermediate goods,

$$p = \bar{z}\alpha_y Y^{\alpha_y - 1}. \tag{2.6}$$

2.4.2 Intermediate Goods Firms

Environment Intermediate goods firm produce a homogeneous product y_t using capital k_t, and sell it to final good firms at price p. They face two types of idiosyncratic productivity shocks. One is persistent, z_t that follows $AR(1)$ process

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z \varepsilon_{z,t} \tag{2.7}$$

where the innovation $\varepsilon_{z,t} \sim N(0,1)$ are independent across firms, and independent of ϕ_t which is the other shock that is i.i.d. The productivity in period t is determined as $z_t \exp(\phi_t)$. The production also requires an operating cost, which consists of two components: a fixed cost f, and a cost proportional to the capital stock $f_k k_t$. As a result, firms' operating profit is:

$$\pi_t = p z_t \exp(\phi_t) k_t^{\alpha} - f - f_k k_t \tag{2.8}$$

Government loans and default rule Government loans have two distinct features. Firstly, the government extends loans to firms at risk-free rate, $q_g = \beta$, up to a specific limit. Secondly, the government's payment is contingent on firms' cash shortage. Cash shortage is determined as the sum of cash-on-hand after full repayment and the maximum funds a firm can raise. Specifically, when firms fully repay their debt, the cash-on-hand is determined as follows:

$$x_t^{FR}(k_t, \overbrace{B_t}^{b_t + b_{gt}}, z_t, \phi_t) = (1-\tau) p z_t \exp(\phi_t) k_t^{\alpha} - (f + f_k k_t) - b_t - b_{gt} + \tau(\delta k_t + r_f(b_t + b_{gt})) \tag{2.9}$$

The cash-on-hand with full repayment is calculated by starting with the after-tax revenue and subtracting the operating costs, and repayments on private and government loans. Additionally, the calculation takes into consideration the tax benefits associated with the depreciation of capital and debt.[19]

The maximum fund a firm can raise by borrowing from private lenders and the government, and

[19] The assumption of tax benefit of debt is common in the financial frictions literature. (See Covas and Haan, 2011, Jermann and Quadrini (2012), Begenau and Salomao (2019)) This feature makes debt more attractive and slows down the rate at which firms grow out of financial frictions. Here I subtract the risk-free rate for tractability reasons following Xiao (2020).

capital disposal is as follows:

$$\begin{aligned} \overline{x}^G(k,z) = &\max_{k_{t+1},b_{t+1},b_{gt+1}} q(k_{t+1},b_{t+1},b_{gt+1},z_t)b_{t+1} + q_g b_{gt+1} - \psi\,(k_t,k_{t+1}) \\ s.t. \quad & b_{g\,t+1} \leq \overline{b_g} \end{aligned} \tag{2.10}$$

where $\psi(k_t, k_{t+1})$ is capital investment and associated adjustment cost:

$$\psi(k_t,k_{t+1}) = \begin{cases} (k_{t+1} - (1-\delta)k_t) + p_k^+ \frac{(k_{t+1}-(1-\delta)k_t)^2}{2(1-\delta)k_t} & \text{if} \quad k_{t+1} - (1-\delta)k_t \geq 0 \\ \\ (k_{t+1} - (1-\delta)k_t) + p_k^- \frac{(k_{t+1}-(1-\delta)k_t)^2}{2(1-\delta)k_t} & \text{if} \quad k_{t+1} - (1-\delta)k_t < 0 \end{cases} \tag{2.11}$$

b_{t+1} is borrowing from private lenders, and q_t is the private debt price, which is discussed in the following subsection 2.4.3. b_{gt+1} is borrowing from the government, and q_g is the debt price of the government loan.

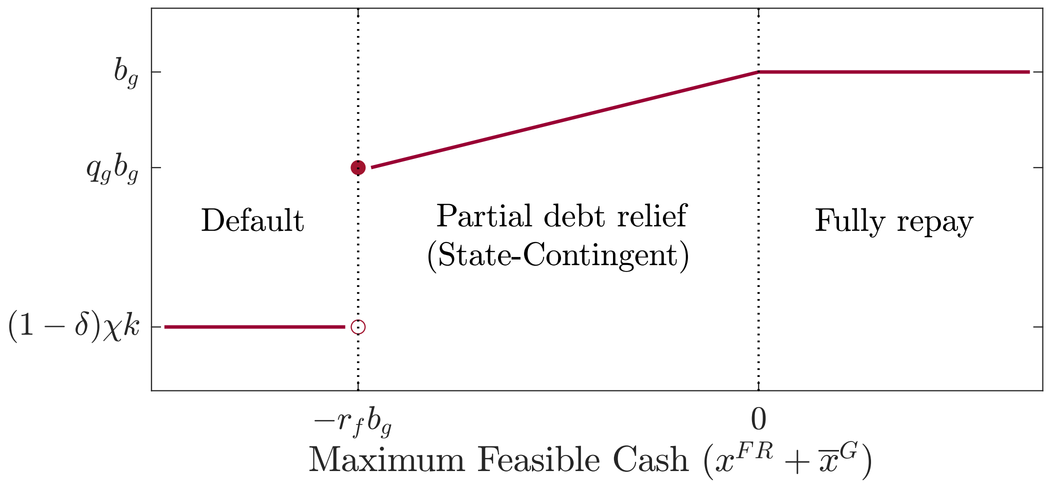
If $x_t^{FR} + \overline{x}^G(k,z) < 0$, then this firm experiences cash-shortage in period t. Given that a firm has borrowed b_g, the government payment from the firm depends on the cash shortage as follows:

$$\text{Payment} = \begin{cases} b_{gt} & \text{if} \quad x_t^{FR} + \overline{x}^G(k_t,z_t) \geq 0 \\ b_{gt} + x_t^{FR} + \overline{x}^G(k_t,z_t) & \text{if} \quad -\left(1-q_g\right)b_{g\,t} \leq x_t^{FR} + \overline{x}^G(k_t,z_t) < 0 \\ \max\left[b_{gt},(1-\chi)k_t\right] & \text{if} \quad x_t^{FR} + \overline{x}^G(k_t,z_t) < -\left(1-q_g\right)b_{g\,t} \end{cases} \tag{2.12}$$

If a firm does not experience any cash shortage as in the first case, the firm repays the government in full. If a firm's cash shortage is less than the interest payment on the government loan, as in the second case, the government alleviates the debt by an insufficient amount. The government receives less by the value of the cash shortage. If a firm's cash shortage exceeds the interest payment on the government loan, as in the third case, the firm defaults and exits. The government obtains priority for the seized capital after deducting the default cost, $\chi(1-\delta)k_{t+1}$. Firms' default set and contingent payment to the government are depicted in Figure 2.8.

Let's denote $\tilde{\phi}^G(k_t,B_t,z_t)$ the cutoff that determines the full repayment, such that $x_t^{FR}(k_t,B_t,z_t,\tilde{\phi}^G) + \overline{x}^G(k_t,z_t) = 0$, and $\hat{\phi}^G(k_t,B_t,z_t)$ the cutoff that determines defaults, such that $x_t^{FR}(k_t,B_t,z_t,\hat{\phi}^G) + \overline{x}^G(k_t,z_t) = -\left(1-q_g\right)b_g$. Given (k_t,B_t,z_t) the firm's cash-on-hand will vary by the realization

Figure 2.8: Payment to government



of ϕ as follows:

$$x_t(k_t, B_t, z_t, \phi_t) = \begin{cases} (1-\tau)pz_t \exp(\phi_t)\, k_t^\alpha - (f + f_k k_t) - B_t + \tau(\delta k_t + r_f B_t) & \text{if} \quad \tilde{\phi}^G \le \phi \\ (1-\tau)pz_t \exp\left(\tilde{\phi}^G\right) k_t^\alpha - (f + f_k k_t) - B_t + \tau(\delta k_t + r_f B_t) & \text{if} \quad \hat{\phi}^G \le \phi < \tilde{\phi}^G \\ \text{Default} & \text{if} \quad \phi < \hat{\phi}^G \end{cases} \tag{2.13}$$

Government loans decrease default sets by increasing the maximum amount of funds a firm can raise, $\overline{x}^G(k,z)$ in two ways: firstly it directly increases $\overline{x}^G(k,z)$ by lending at risk-free rate up to some limit and secondly, the subsidized nature of the government loan changes the private debt price schedule q, which also increases $\overline{x}^G(k,z)$. One of the key assumptions is that government loans are not available to potential entrants. A mass of actual entrants is not affected by government loans.

Recursive Problem The idiosyncratic state of a firm, (x,k,z), records its cash-on-hand x_t, the current capital stock k_t, the current persistent idiosyncratic shock z_t. The dynamic problem of surviving firm (x,k,z) consists of choosing total loan B', government loan b'_g, and next period's capital k'. Given the choice for total loan B' and government loans b'_g, the firm's choice for private loan is determined as $B' - b'_g$. The value of surviving firm (x,k,z) is as follows:

$$
\begin{aligned}
V(x,k,z) = \max_{k',B',b'_g} d + \beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' > \tilde{\phi}^G} V\left(x'\left(k',B',z',\phi'\right),k',z'\right) d\Phi\left(\phi'\right) \right] \\
\underbrace{+ \beta \sum_{z'} \pi(z' \mid z) \left[\left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G) \right) V\left(x'\left(k',B',z',\tilde{\phi}^G\right),k',z'\right) \right]}_{\text{Value from government's debt relief}} \\
\text{s.t} \quad d = x - \psi(k,k') + q\left(k',b',b'_g z\right)(B' - b'_g) + q_g b'_g \geq 0 \\
x(k',b',b'_g,z',\phi') = (1-\tau)pz' \exp(\phi') k'^{\alpha} - f_k k' - f - B' + \tau\left(\delta k + r_f B'\right) \\
\tilde{\phi}^G\left(k',B',b'_g,z'\right) = \log\left(\frac{-\overline{x}^G(k',z') + f + f_k k' + B' - \tau\left(\delta k + r_f B'\right)}{(1-\tau)pz'k'^{\alpha}} \right) \\
\hat{\phi}^G\left(k',B',b'_g,z'\right) = \log\left(\frac{-\overline{x}^G(k',z') + f + f_k k' + B' - \left(1 - q_g\right) b'_g - \tau\left(\delta k + r_f B'\right)}{(1-\tau)pz'k'^{\alpha}} \right) \\
b'_g \leq \overline{b_g}, \quad b'_g \leq B'
\end{aligned}
\tag{2.14}
$$

$\overline{x}^G(k,z)$ is defined in equation (2.10). The constraints in the last line indicate that the borrowing from the government capped by the limit $\overline{b_g}$, and the non-negative borrowing from private creditor $B' - b'_g \geq 0$ respectively. The value when the government loans is not in place is determined with $b_g = 0$.

Firm Entry I model firm entry in line with Clementi and Palazzo (2016). Every period there is a constant mass $M > 0$ of prospective entrants, each of which receives a signal ν about their productivity, with $\nu \sim Q(\nu)$. Conditional on entry, the distribution of the idiosyncratic shock z in the first period of operation is $G(z \mid \nu)$, strictly decreasing in ν. Firms have to pay an entry fee ($c_e > 0$) so not all firms find it optimal to enter.

Entrants only start operating in the period after the entry decision, but must make decision today on capital they want to start operating in the following period given starting capital k_e [20]. Entrants need to raise funds for capital investment and related adjust cost through issuing debt.

[20]Firm entry in my model is equivalent to a decision to grow its size and to be subject to external audits, to be consistent with the data. Therefore it is natural for entrants starting with some initial capital. I calibrate the parameters such that I match the relative average size of entrants to incumbents' average size in the data.

The value function of the potential entrant with signal v is

$$\begin{aligned} V^e(v) &= \max_{k',b'} \beta \sum_{z'} \int_{\phi'>\hat{\phi}} V\left(x'\left(k',b',z',\phi'\right),k',z'\right) d\Phi\left(\phi'\right) dG\left(z' \mid v\right) \\ \text{s.t} \quad & -\psi(k_e,k') + q^e(k',b',v)b' \geq 0 \\ & x(k',b',z',\phi') = (1-\tau)pz' \exp(\phi')k'^{\alpha} - f_k k' - f - b' + \tau\left(\delta k' + r_f b'\right) \\ & \hat{\phi}\left(k',b',z'\right) = \log\left(\frac{-\overline{x}\left(k',z'\right) + f + f_k k' + b' - \tau\left(\delta k' + r_f b'\right)}{(1-\tau)pz'k'^{\alpha}}\right) \end{aligned} \tag{2.15}$$

where $q^e(b',k',v)$ is debt price given debt b', capital k', signal v about productivity z'. Potential entrants make decision over private loan and capital since they cannot access to government loans. Furthermore the default and exit cutoff $\hat{\phi}$ is determined by the maximum fund without government loans,

$$\overline{x}(k,z) = \max_{k',b'} q(k',b',0,z)b' - \psi(k,k') \tag{2.16}$$

This indicates the government loans is accessible only after the potential entrants enter and survive. The surviving firms will have state (x,k,z), and then will be allowed to access to government loans.

An entrant invests and starts operating if and only if the value of entry exceeds the entry fee, i.e $V^e(v) \geq c_e$. Since an incumbent's value $V(x,k,z)$ is weakly increasing in the transitory productivity z and the conditional distribution $G(z \mid v)$ is strictly decreasing in v. Accordingly $V^e(v)$ is strictly increasing in the signal v. This means that there will be a threshold for the signal, denoted by $\hat{v}$, such that potential entrants will enter if and only if they receive a signal greater than or equal to $\hat{v}$,

$$V^e(\hat{v}) = c_e \tag{2.17}$$

2.4.3 Private creditor

The private creditor is perfectly competitive. The debt price adjusts to reflect the probability of default and is determined by equating the expected return from providing a loan to the lender's funding costs.

Incumbents The debt price of incumbent firms with capital k', total debt B', government loan b'_g, and productivity z is determined as follows:

$$q\left(k',B',b'_g,z\right) = \beta \sum_{z'} \left[\left(1-\Phi\left(\hat{\phi}^G\right)\right) + \Phi\left(\hat{\phi}^G\right) R^G(B',b'_g,k')\right] \pi(z' \mid z) \tag{2.18}$$

where,

$$\hat{\phi}^G\left(k', B', b'_g, z'\right)\right) = \log\left(\frac{-\overline{x}^G\left(k', z'\right) + f + f_k k' + B' - \left(1 - q_g\right) b'_g - \tau\left(\delta k + r_f B'\right)}{(1-\tau) p z' k'^{\alpha}}\right) \tag{2.19}$$

Upon default, the government takes the priority over the seized firm's capital after deducting default cost. The private lenders takes the remaining capital and should pay a fixed cost of the firm's default. Then the recuperation rate of private loan is as follows,

$$R^G(B', b'_g, k') = \min\left(1, \max\left(0, \frac{\chi(1-\delta)k' - b'_g - \eta}{B' - b'_g}\right)\right) \tag{2.20}$$

Debt price without the government loan is determined with $b_{g\,t+1} = 0$.

Entrants Similarly, the debt price of entering firms with capital k', debt b', and signal about the productivity ν is as follows:

$$q_e\left(k', b', \nu\right) = \beta \sum_{z'} \left[\left(1 - \Phi\left(\hat{\phi}\right)\right) + \Phi\left(\hat{\phi}\right) R\left(b', k'\right)\right] dG\left(z' \mid \nu\right) \tag{2.21}$$

where,

$$\hat{\phi}\left(k', b', z'\right) = \log\left(\frac{-\overline{x}\left(k', z'\right) + f + f_k k' + b' - \tau\left(\delta k' + r_f b'\right)}{(1-\tau) p z' k'^{\alpha}}\right) \tag{2.22}$$

$$R(b', k') = \min\left(1, \max\left(0, \chi\frac{(1-\delta)k'}{b'} - \eta\right)\right) \tag{2.23}$$

2.4.4 Stationary recursive equilibrium

The stationary recursive equilibrium for the economy consists of (i) policy and value functions of incumbent firms $\left\{B'(x,k,z), b'_g(x,k,z), k'(x,k,z), V(x,k,z)\right\}$; (ii) policy and value functions of entering firms $\{b'(\nu), k'(\nu), V(\nu)\}$; (iii) the bond price schedule $q^G\left(B', b'_g k', z\right)$, $q^e\left(b', k', \nu\right)$; (iv) price of final good p, demand for final good $y_f(p)$, average productivity of intermediate good firms $\overline{z}$, and mass of entrants ; (v) a stationary measure μ such that: (1) policy and value functions of intermediate goods firms solve firm's problem; (2) price of debt from private lenders is determined competitively; (3) final good market clears; (4) the cross-sectional distribution $\mu(x,k,z)$ is stationary.

Here I specify the equilibrium conditions.

Aggregate production of intermediate good satisfies

$$Y = \sum_z \int_\phi z \exp(\phi) \int_{x_{-1},k_{-1},z_{-1}} k(x_{-1}, k_{-1}, z_{-1})^\alpha \mu_{-1}\left(x_{-1}, k_{-1}, z_{-1}\right) d\Phi(\phi)\pi\left(z \mid z_{-1}\right) \tag{2.24}$$

This condition means that the outputs of defaulting firms are included in the total output, and the total production only depends on the previous distribution of firms. Accordingly, final good output satisfies $y_f = \bar{z}Y^{\alpha_y}$

Average productivity of intermediate good firms $\bar{z}$ is

$$\bar{z} = \sum_{z_i} z_i w(z_i) \tag{2.25}$$

where, $w(z_i)$ is a share of output produced by firms whose productivity is z_i:

$$w(z_i) = \int_\phi \frac{\int_{x_{-1},k_{-1},z_{-1}} z_i \exp(\phi) k\left(x_{-1}, k_{-1}, z_{-1}\right)^\alpha \mu_{-1}\left(x_{-1}, k_{-1}, z_{-1}\right) d\Phi(\phi)\pi\left(z_i \mid z_{-1}\right)}{Y} \tag{2.26}$$

Market clearing in the final goods market requires that total consumption equals to final good output, less the investment, the associated adjustment cost, and loss of resources from defaults:

$$\begin{aligned} C = y_f - \textstyle\int_{x,k,z} \psi\left(k, k'(x,k,z)\right) d\mu(x,k,z) \\ - \textstyle\int_{x_{-1},k_{-1},z_{-1}} \sum_z \int_{\phi<\hat{\phi}^G} \left[\eta - (\chi(1-\delta)k\left(x_{-1}, k_{-1}, z_{-1}\right)) d\Phi(\phi)\right] \pi\left(z \mid z_{-1}\right) d\mu_{-1}\left(x_{-1}, k_{-1}, z_{-1}\right) \end{aligned} \tag{2.27}$$

Specifically, the first term in equation (2.27) is final good output, and the second term is investment and related adjustment cost. The last term is related with firms' default. Firms with a previous state (x_{-1}, k_{-1}, z_{-1}) default given their choice for capital, debt and realized productivity z and ϕ. In this case the depreciated capital returns to the defaulting firm, and is used to repay to private lenders or the government after deducting cost related with default, $(1-\chi)\delta k + \eta$.

Finally, let $\mu(x, k, z)$ be the steady state distribution of firms with cash-on-hand x, capital k, and persistent productivity z. This distribution satisfies the following law of motion:

$$\mu(x', k', z') = \int \Lambda(x', k', z', x, k, z)\mu(x, k, z) + M \int_{\nu \geq \hat{\nu}} \Lambda^e(x', k', z', \nu) dQ(\nu) \tag{2.28}$$

The first term in the law of motion is determined by incumbent firms. To understand this term, we need to consider the probability that an incumbent firm with a particular state (x, k, z)

transitions to a different state (x', k', z'), which is denoted by $\Lambda(x', k', z', x, k, z)$. The transition probability $\Lambda(x', k', z', x, k, z) = \pi(z' \mid z)d\Phi(\phi')$ if, at that state (x, k, z), the decision rules $k' = k'(x, k, z)$ and $B' = B'(x, k, z)$ together with ϕ' produce the particular level of cash-on-hand x'. The determinants of x' is defined in equation (2.13). It is important to note that $\phi' \geq \hat{\phi}^G$ specified in equation (2.14), so that the firm does not default. If any of these conditions do not hold, then $\Lambda(x', k', z', x, k, z) = 0$.

The second term in the transition function comes from new entrants. Similar to the case of incumbent firms, conditional on receiving a signal about the productivity, with which their value of entering is greater than the entry cost, i.e. $\nu \geq \hat{\nu}$, where $V^e(\hat{\nu}) = c_e$, the probability that a new entrant with a signal ν transits to (x', k', z') is given by $\Lambda^e(x', k', z', \nu)$. The transition probability $\Lambda^e(x', k', z', \nu) = \pi(z' \mid \nu)d\Phi(\phi')$ if, given the signal ν, the decision rules $k' = k'(\nu)$ and $b' = b'(\nu)$ together with ϕ' produce the particular level of cash-on-hand x'. Here, $\phi' \geq \hat{\phi}$ specified in equation (2.15), so that the firm can survive. The default cutoff $\hat{\phi}$ is the cut-off without the government loans, because the government loans are available only after the potential entrants enter and survive and become a incumbent. If any of these conditions do not hold, then $\Lambda^e(x', k', z', \nu) = 0$.

2.4.5 Firm's decision

Here, I characterize firms' decisions as follows. I begin by analyzing firms' decision to borrow from the government. Next, I will characterize the decision rule as a function of their cash-on-hand, which is associated with a nonnegative equity payout constraint. Lastly, I will explain firms' optimal choices for capital and borrowing based on the first-order condition of Bellman equation (2.14).

Decision on borrowing from the government Proposition 2.4.1 characterizes the decision on how much to borrow from the government.

Proposition 2.4.1 *Given a choice for total debt and capital $\{B', k'\}$, if the total debt can be financed only by the government loan, $B' - \overline{b_g} \leq 0$, a firm will borrow only from the government $b'_g = B'$, and if the total debt cannot be financed only by the government loan due to the limit on the government loan, firm's borrowing from the government $b'_g = \overline{b_g}$*

Proof See appendix A.11.1

Intuitively, given total debt and capital, firms' value is strictly increasing by substituting private loans with government loans. By the Proposition 2.4.1, we can define firms' problem as a choice over total debt and capital, and the debt composition between a private loan and a government loan is determined by the level of total debt.

Decision rules associated with nonnegative equity payout constraint The firms' decision rules are characterized in more detail as a function of cash-on-hand as follows:

Proposition 2.4.2 *The optimal decision of a surviving firm with cash-on-hand x, persistent productivity z, and capital k is characterized by one of the following three cases:*

(1) ***Default*** *: there exists a threshold $\underline{x}(k,z)$ such that firms with $x < \underline{x}(k,z)$ default since it is infeasible for these firms to satisfy the non-negativity equity payout constraint.*

(2) ***Unconstrained*** *: there exists a threshold $\hat{x}(k,z)$ such that the firm is financially unconstrained if $x > \hat{x}(k,z)$, i.e., the nonnegative equity payout constraint is slack. The bond price, capital, and total borrowing do not vary with cash-on-hand, whereas equity payouts increase one for one with cash-on-hand.*

(3) ***Constrained*** *Firms with cash-on-hand $x \in [\underline{x}(k,z), \hat{x}(k,z)]$ are financially constrained, i.e., the nonnegative equity payout constraint is binding. The equity payout is zero.*

Proof See appendix A.11.2

Decision for capital and total borrowing Here I characterize firms' decisions mainly based on firms' first-order conditions for capital and debt. The first-order condition with respect to capital is:

$$\begin{aligned}
&\beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' \geq \tilde{\phi}^G} \frac{\partial V(x'(k', B', z', \phi'), k', z')}{\partial k'} d\Phi(\phi') + \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right) \frac{\partial V\left(x'(k', B', z', \tilde{\phi}^G), k', z'\right)}{\partial k'} \right] \\
&= (1 + \eta(x,k,z)) \left[\frac{\psi(k,k')}{\partial k'} - \frac{\partial q}{\partial k'} \left(B' - \overline{b_g}\right) \right] - \beta \sum_{z'} \pi(z' \mid z) \left(- \frac{\partial \hat{\phi}^G}{\partial k'} \right) \phi(\hat{\phi}^G) V\left(x'(k', B', z', \hat{\phi}^G), k', z'\right)
\end{aligned} \tag{2.29}$$

Where η is the multiplier associated with the nonnegative equity payout conditions and derivative of value function with respect to capital can be derived using the envelope condition:

$$\frac{\partial V(x', k', z')}{\partial k'} = (1 + \eta(x', k', z')) \left(p z' \exp(\phi') \alpha k'^{\alpha - 1} - f_k - \frac{\partial \psi(k', k''(x', k', z'))}{\partial k'} \right) \tag{2.30}$$

The optimal choice for capital is determined at which the expected marginal benefit is equated to the expected marginal cost. The expected marginal benefit of capital indicated in the left-hand side of equation (2.29), consists of two terms. The first term captures the marginal product in future states where the firm fully repays, and the second term captures the marginal product in future states where the firm gets partial debt relief from the government. The expected cost, given by the right-hand side of equation (2.29), equals the investment and related adjustment cost, which is captured as the first term, and a wedge, which is captured by the remaining terms in the right-hand side. The first term of the wedge comes from the increase in the bond price from investing an extra unit of capital. The second term of the wedge comes from the gain associated with a decrease in default risk with an additional unit of capital This term is proportional to the firm's future value evaluated at default cutoff $V\left(x'\left(k', B', z', \tilde{\phi}^G\right), k', z'\right)$, probability of the cutoff $\phi(\tilde{\phi}^G)$, and $-\frac{\partial \hat{\phi}^G}{\partial k'}$, which captures how the cutoff changes with capital. Since the default cutoff decreases with capital (higher probability to repay with a higher capital), the marginal cost of capital is the investment cost net of gains from increased repayment probability and debt price.

The first-order condition with respect to new borrowing is as follows:

$$\beta \sum_{z'} \pi(z' \mid z)\left[\int_{\phi' \geq \tilde{\phi}^G} (1 + \eta'(x'(k', B', z', \phi'), k', z'))\, d\Phi(\phi') + \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right)\left(1 + \eta'\left(x'(k', B', z', \tilde{\phi}^G), k', z'\right)\right)\right]$$
$$+ \beta \sum_{z'} \pi(z' \mid z)\left(\frac{\partial \hat{\phi}^G}{\partial B'}\right)\phi(\hat{\phi}^G) V\left(x'(k', B', z', \tilde{\phi}^G), k', z'\right) = (1 + \eta(x, k, z))\left[q + \frac{\partial q}{\partial B'}\left(B' - \overline{b_g}\right)\right] \tag{2.31}$$

The optimal level of new borrowing equates the marginal benefit of new borrowing to the expected marginal cost. Borrowing one more unit gives a direct increase in current resources of q and leads to a fall in the price of existing debt, giving a total change in current resources of $q + \frac{\partial q}{\partial B'}\left(B' - b_g\right)$. Notice that the fall in the debt price only applies to the debt from the private creditor $B' - b_g$ since the government loans do not require the compensation for default risks. These resources help relax the nonnegtive equity payout condition, hence are valued at the multiplier η. The marginal cost of borrowing, given by the left-hand side of the equation (2.31), consists of three terms. The first term reflects the cost of repaying full repayment states and the second term captures the cost of repaying in states with the government's partial debt relief. These terms are weighted by the shadow price of cash-on-hand in those states, $1 + \eta'$. The last term is the loss in value from the default.

2.4.6 Micro level policy effects: investment and exit

The effects of the introduction of government loans can be divided into two parts. Firstly, the policy affects the feasibility of individual firms to continue operating, as well as their decisions regarding leverage and investment. These responses generate the general equilibrium effect by changing the price at intermediate firms sell their products. Here, I am going to explain how the introduction of government loans change firms investment exit behavior given the price is fixed (no general equilibrium effect).

To illustrate the economic mechanisms through which government loans impact firms' investment decisions, let's consider a case where firms are constrained as in Proposition 2.4.2 and there is no capital adjustment cost. Since a firm is constrained, i.e. non negative dividend payout condition is binding, additional capital is associated with additional borrowing. Assume that the firm's total borrowing exceeds the government loan limit, and then the firm borrows up to the limit from the government, as shown in Proposition 2.4.1. By substituting equation (2.31) into equation (2.29), we can derive the optimality for capital.

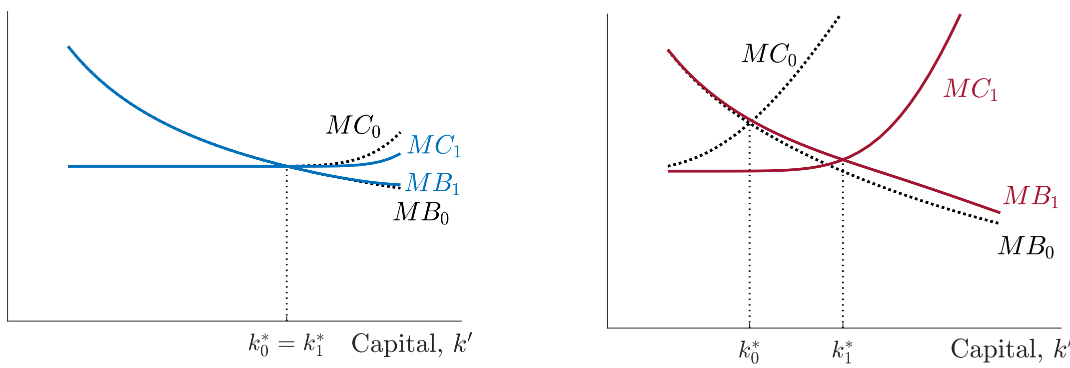
$$
\begin{aligned}
&\frac{\sum_{z'} \pi(z' \mid z)\left[\int_{\phi' > \tilde{\phi}^G} MPK(k', B', z', \phi') d\Phi(\phi') + \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right) MPK(k', B', z', \tilde{\phi}^G) + \left(-\frac{\partial \hat{\phi}_g}{\partial k'}\right)\phi(\hat{\phi}^G)\tilde{V}\right]}{\sum_{z'} \pi(z' \mid z)\left[\Delta + \frac{\partial \hat{\phi}^G}{\partial B'}\phi(\hat{\phi}^G)\tilde{V}\right]} \\
&= \frac{1 - \frac{\partial q}{\partial k'}\left(B'(x, k', z) - b_g\right)}{q(1 - \epsilon)} \\
&\text{where,} \quad \epsilon = -\frac{\partial q}{\partial B'}\frac{\left(B' - b_g\right)}{q} \\
&\qquad MPK(k', B', z', \phi') = (1 + \eta'(x', k', z'))\left[pz' \exp(\phi')\, \alpha k'^{\alpha - 1} - f_k + (1 - \delta)\right] \\
&\qquad \tilde{V} = V\left(x'\left(k', B', z', \tilde{\phi}^G\right), k', z'\right) \\
&\qquad \Delta = \int_{\phi' \geq \tilde{\phi}^G} (1 + \eta'(x'(k', B', z', \phi'), k', z') d\Phi(\phi') \\
&\qquad\quad + \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right)(1 + \eta'(x'(k', B', z', \tilde{\phi}^G), k', z') + \left(\frac{\partial \hat{\phi}^G}{\partial B'}\right)\phi(\hat{\phi}^G)\tilde{V}
\end{aligned} \tag{2.32}
$$

The marginal benefit curve, left hand of equation (2.32), is downward sloping due to diminishing returns to capital. The curve of marginal cost, right hand of equation (2.32), is flat at $\frac{1}{\beta}$ when capital can be financed without incurring default risk. However it becomes upward-sloping when the borrowing required to finance capital creates default risk, as debt price q decrease with borrowing and the debt price elasticity ϵ increases as well.

In Figure 2.9, I plot the marginal benefit and marginal cost schedules as a function of tomorrow's capital holding k' for two types of firms: those with high cash-on-hand in the left panel and those with low cash-on-hand in the right panel. These two types of firms share the same values for today's capital k and productivity z. The key distinction between these firms lies in the fact that low cash-on-hand firms need to resort to a high level of debt in order to maintain the same level of capital for tomorrow. Consequently, it can be inferred that low cash-on-hand firms are required to pay a higher interest rate to retain the same amount of capital compared to high cash-on-hand firms.

Figure 2.9: Marginal Benefit and Marginal Cost to Investment

(a) High cash-on-hand (risk free) (b) Low cash-on-hand (risky)



Notes: Responses of risk free and risky firms to government loans are presented as shifts of marginal benefit and marginal costs curves as a function of capital investment. Two types of firm share same level of productivity and capital. Left panel is for a firm with high cash-on-hand (risk free) and right panel is is firm with low cash-on-hand (risky). The black dashed lines plot the curves without government loans, and blue (risk free) and red (risky) solid lines plot the curves with government loans given the intermediate good price fixed.

In the initial equilibrium without government loans plotted with black dashed lines, high cash-on-hand firm's marginal cost and benefit curve intersects where the marginal cost curve is flat since the firm can finance their optimal level of capital without incurring default risk such that

$$\frac{1}{\beta} = \sum_{z'} \pi(z' \mid z) \int_{\phi'} \left[pz' \exp(\phi') \alpha k'^{\alpha-1} - f_k + (1-\delta) \right] d\Phi(\phi')$$

On the contrary, the marginal cost and benefit curve of the low cash-on-hand firms intersects where the marginal cost curve is upward sloping. Due to endogenous borrowing constraint arising from limited commitment, low cash-on-hand firms hold less capital than high cash-on-hand firms. In frictionless economy, the level of capital is not determined by the level of cash-on-hand.

When government loans are introduced with p fixed, which are indicated as solid lines, the marginal cost curve becomes flatten as firms are able to finance capital with less default risk. The marginal benefit, left hand of equation (2.32), increases as default risk becomes lower. In the region where the marginal cost curve is flat, the marginal benefit does not change with the government loans. In the region with positive default risk without government loans, the marginal benefit curve shifts up as the default probability given same choice is lower. Therefore, high cash-on-hand firm's new equilibrium stays same since the marginal cost curve was flat in state without government loans. Low cash-on-hand firm's new equilibrium is set at a higher capital investment. The risk free firms and risky firms only differ in their cash-on-hands, which implies that the risky firms are hit by bad shock, low ϕ, and end up having low cash-on-hand. The government loans help such firms with cash shortage not to reduce their investment much.

Turning our attention to the impact on the exit margin, Figure 2.8 shows that the firms rescued from the default and exit are precisely those with a cash shortage. Firms with limited cash-on-hand are more likely to experience this cash-shortage, and are more likely to be receive partial debt relieve and saved from exit thanks to the government loans.

In summary, the government loans help low cash-on-hand firms increase their investment and also help low-cash-on hand firms survive from defaulting. Cash shortages among firms can stem from diverse sources such as low capital size, low persistent productivity, or bad transitory shock. Depending on the underlying cause of a firm's low cash-on-hand, their investment response or change in exit margin will vary. This means that firms' response to the policy will vary depending on their specific characteristics, as we have observed in the data.

2.5 Model Calibration and Performance

In this section, I outline the solution method, describe how the model is parameterized, and present the model's performance. To overview the steps, I first solve the model without government loans and calibrate it in a way that the moments generated from the steady states of the economy without government loans match the pre-policy aggregate moments of Korean firm data from 2010 to 2016.[21] Next, I introduce government loans to the calibrated model. Beginning with the steady state without government loans, I first determine the new steady state of the economy with government loans. Then, I find the transition path between the two economies. Using the equilibrium price path and model solutions for policy functions, I simulate the economy over a 3-year period following the introduction of government loans to mimic the data. I construct panels of simulated firms based on this simulation. See Appendix A.7 for further details.

2.5.1 Functional forms and parameterization

Functional forms The i.i.d idiosyncratic productivity shock ϕ is log normally distributed, with mean 0 and and standard deviation σ_ϕ. The distribution of signals for the entrants is Pareto. I

[21] I calculated aggregate moments using sample periods from 2010 to 2016 to have more data on firms that exit and age 1 firms instead of using years from 2014 to 2016, which are used for empirical analysis in Section 2.3.

posit that $\nu \geq \underline{\nu} > 0$ and that $Q(\nu) = 1 - (\underline{\nu}/\nu)^{\xi}$, $\xi > 1$. The realization of the idiosyncratic productivity in the first period of operation follows the process $\log z = \rho_z \log \nu + \sigma_z \varepsilon_{z,t}$, where $\varepsilon_{z,t} \sim N(0,1)$. I set $\underline{\nu} = \exp\left(\frac{-4.5\sigma_z}{sqrt(1-\rho_z^2)}\right)$.

Parameterization I classify the parameters into two groups: those that are exogenously assigned and those that are chosen to match aggregate moments of Korean firm data. Each period reflects one year. Table 2.2 reports the parameter values.

There are 7 fixed parameters. The discount factor, β is set to be 0.97, so that the annual interest rate is 3%. The share of capital α is set to be 0.3, and the annual depreciation rate δ is set to be 10%. The tax rate τ is set to be 0.275 based on Korea's corporate tax rate. Following Xiao (2020), I set the recuperation rate of bond χ to be 0.47. Using the estimates of Foster, Haltiwanger, and Syverson (2008), I set the serial correlation of the firm-level productivity shock ρ_z to 0.9. The parameter that captures the return to scale of final good producer, α_y, is set to be 0.85, consistent with the range of estimates in Atkeson and Kehoe (2005). The mass of potential entrants M is normalized to 1.

The remaining 10 parameters are set to match pre-policy aggregate moments from 2010 to 2016. I calculate cross-sectional moments following Ocampo and Robinson (2023). The first five moments relate to incumbent firms, and include the mean investment of all incumbent firms, the mean investment of firms whose net-income asset ratio is above and below the median, mean credit spreads, exit rates. The next three moments pertain to entrants, including relative median size, relative TFP of entrants, age 1 firms' mean investment.[22] To maintain consistency with the data, entrants in the model are defined as firms that survive after experiencing transitory shocks. While relatively small, these firms have higher productivity than incumbent firms. [23] The last two moments cover firms' mean net-income asset ratio and relative TFP at exit. Table 2.3 presents the moments from data and model. All of relative moments are calculated with a moment relative to unconditional average or median.[24]

The model performs relatively well in matching key moments in the distribution of firms' financial states. It generates a similar mean investment level and effectively captures the heterogeneity of investment depending on firms' net-income ratios. Moreover, the model aligns well with

[22]I calculate firm TFP as the ratio of sales to average of current total asset size and previous total asset size. Age 1 firm does not have the previous total asset size, and I use mean of age 2 firms TFP as a target moment.

[23]Foster, Haltiwanger, and Syverson (2008, 2016) found that entrants, despite exhibiting similar levels of technical efficiency as incumbents, often faced lower demand schedules and charged lower prices. However, conditional on survival, entrants tended to display greater total factor productivity as demand schedules shifted outward.

[24]The detailed definition of the moments from data and model is presented in Appendix A.8.

Table 2.2: Parameterization

Description	*Parameter*	*Source*
Fixed parameters		
Discount rate	$\beta = 0.97$	Annual interest rate 3%
Share of capital	$\alpha = 0.3$	Standard business cycle models
Depreciation	$\delta = 0.1$	Standard business cycle models
Tax rate	$\tau = 0.275$	Korea's corporate tax rate
Bond recovery rate	$\chi_k = 0.47$	Xiao (2020)
Persistence of z	$\rho_z = 0.9$	Foster, Haltiwanger, and Syverson (2008)
Returns to scale	$\alpha_y = 0.85$	Atkeson and Kehoe (2005)
Fitted parameters from moment matching		
Volatility of z	$\sigma_z = 0.1$	
Volatility of ϕ	$\sigma_\phi = 0.13$	Internally calibrated
Invest adj cost	$p_k^+ = 1.8$	Internally calibrated
Dis-invest adj cost	$p_k^- = 2.8$	Internally calibrated
Fixed operating cost	$f = 0.52$	Internally calibrated
Proportional cost	$f_k = 0.07$	Internally calibrated
Default cost	$\chi = 0.2$	Internally calibrated
Entry cost	$c_e = 3.2$	Internally calibrated
Initial capital	$k_e = 0.2$	Internally calibrated
Pareto exponent	$\xi = 3.2$	Internally calibrated
Government loans	$\overline{b_g} = 0.134$	

the mean credit spread and exit rates observed in the data. The model also replicates moments related to entrants that are in line with empirical observations; entrants tend to be smaller, more productive, and invest more than average firms. Additionally, the model accurately reflects the fact that firms tend to have less cash and lower productivity at exit.

Lastly, the limit of government loan $\overline{b_g}$ is set to align with the change in exit rates over 3 years as observed in the data. Following the policy introduction, the model reflects a decrease in exit rates by 0.5 percentage points, whereas the data shows a decrease of 0.4 percentage points.

2.5.2 Model performance: pre-policy moments

Cross-sectional moments Table 2.4 presents the cross-sectional moments based on firms' net-income ratios, which were not explicitly targeted. I use cash-on-hand to capital ratio for the model moments and net-income to asset ratio for the data moments. The model performs relatively well in generating cross-sectional moments, with the exception of the credit spread. The model effectively captures firms' heterogeneity based on their net-income ratios. Firms with

Table 2.3: Targeted moments

Description	*Data*	*Model*
Incumbents		
Mean investment	0.11	0.11
Mean investment ($\frac{x}{k}$ < median)	0.06	0.07
Mean investment ($\frac{x}{k} \geq$ median)	0.15	0.14
Mean spread (%p)	1.46	1.61
Exit rates (%)	1.10	1.12
Entrants		
Median relative size at enter	0.16	0.17
Mean relative TFP at enter	1.81	1.55
Age 1 firms' mean investment	0.43	0.46
Firms that exit		
Mean net-income asset ratio at exit	-0.27	-0.30
Mean relative TFP at exit	0.61	0.59

higher net-income ratios tend to invest more, exhibit lower spreads, and have a lower likelihood of exiting. However, in the model, the dispersion of credit spreads is larger than that observed in the data. Firms with lower net-income ratios tend to be larger, as smaller firms struggle to survive with low net-income ratios. The variance in size is more pronounced for firms with net-income ratios below the first quartile and above the third quartile, as compared to firms within the interquartile net-income ratio range.

Zombie firms I define zombie firms in the model, as in the empirical analysis of section 2.3.3. In the model, firms are classified as zombie firms if their cash-on-hand is negative for three consecutive years and they are at least ten years old.[25] In Table 2.5, I present the properties of zombie firms based on both data and model simulations.

The properties of zombie firms observed in simulated firms are consistent with the data, and the model effectively captures the differences from normal firms, similar to what is observed in the data. In the data, the average share of zombie firms in the years before the policy shift was 5.1%, and it increased by 2.5 percentage points after the policy change. In the model, the

[25] Debt in the model is a one-period bond, and we cannot directly apply the concept of debt service from the data to the model. In the model, negative cash-on-hand indicates that firms are unable to cover their debt obligations solely from their operational profits, which corresponds to a similar definition used in the data. Furthermore, cash-on-hand in the data can be matched with net-income. On average, the net-income of firms with an interest coverage ratio less than 1 is negative in the data.

Table 2.4: Untargeted moments: Distribution by net-income asset ratio ($\frac{x}{k}$)

Moments	Net-income asset ratio [0,25]	[25,50]	[50,75]	[75,100]
Data				
Net-income asset ratio	-0.10	0.02	0.06	0.16
Investment	0.05	0.06	0.11	0.19
Spread	1.83	1.61	1.30	1.08
Exit rate (%)	3.49	0.84	0.23	0.09
Log size (Relative)	1.00	0.98	0.92	0.78
Std of log size (Relative)	1.00	0.85	0.95	1.09
Model				
Net-income asset ratio	-0.10	0.02	0.12	0.31
Investment	0.06	0.09	0.12	0.17
Spread	6.78	0.36	0.10	0.05
Exit rates (%)	4.66	0.33	0.08	0.05
Log size (Relative)	1.00	0.97	0.95	0.60
Std of log size (Relative)	1.00	0.71	0.61	1.14

Notes: Moments calculated based on firms policy functions and steady state distribution without government loans following Ocampo and Robinson (2023). For example, the exit rate of firms with first quartile net-income asset ratio is calculated as $E\left[\mathbf{1}(\text{exit}) \mid x^k \in Q_1\right] = \frac{\int_{x,k,z}\int_{\phi' \leq \hat{\phi}G(k'(x,k,z),B'(x,k,z),z')} \mathbb{1}\left(x^k \in Q_1\right)\pi(z'|z)\mu(x,k,z)}{\int_{x,k,z}\mathbb{1}\left(x^k \in Q_1\right)\mu(x,k,z)}$.

average share of zombie firms in the pre-policy steady state is 8.0%, and it increases by 4.1 percentage points over the three years after the introduction of government loans. I compare the relative mean differences between zombie firms and normal firms to validate my model. In the data, we observe that zombie firms are relatively larger, highly leveraged, less profitable, and invest much less compared to normal firms.

I also show that the model generates the negative correlation between age and investment as observed in the data, and the firms' financial state before exit, namely continued cash-shortage, higher leverage, higher spread, lower investment compared to firms that never exit or are far from the exit. See Appendix A.9.

2.6 Firm-Level Effect in the Model

This section explores firm-level effects in the model. I first present the changes in firms' policy functions related to investment and survival rates resulting from the introduction of government loans, within a partial equilibrium framework where the price p remains fixed. Next, I describe heterogeneity in firms' responses to government loans in a general equilibrium context. To

Table 2.5: Untargeted Moment Related with Zombie Firms

	Data	Model
Share of zombie firms	5.1	8.0
Δ zombie share*	2.5	4.1
Log Size+	115.2	111.0

	Data	Model
Debt to Asset Ratio*+	9.7	10.1
Profitability*+	-11.2	-15.5
Investment*+	-12.2	-7.1

Notes: All figures with a "*" symbol are measured in percentage points, while all figures without the symbol are measured in percentage. The variables denoted with a "+" symbol indicate the mean difference between zombie and non-zombie firms.

achieve this, I replicate the regression analysis performed in section 2.3, using the paths of simulated firms. I also provide transition probabilities for normal (non-zombie) and zombie firms, highlighting the changes in these transition probabilities following the policy shift.

2.6.1 Firms decision rules and credit spread schedules

Figure 2.10 plots policy functions for a firm with a median level of capital k and productivity z as a function of cash-on-hand levels. The left panel displays the survival probability and the right panel displays the optimal investment. The survival probability is obtained from firms' policy function as follows,

$$\sum_{z'} \left[1 - \Phi\left(\hat{\phi}^G\left(k'(x,k,z), B'(x,k,z), b'_g(x,k,z), z'\right)\right)\right] \pi(z' \mid z).$$

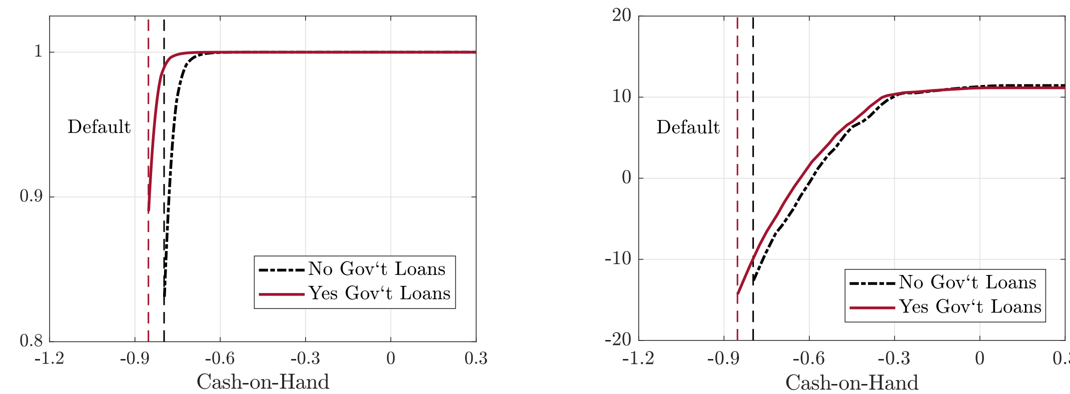
In each panel, I plot policy functions in economy with and without the government loans with p being fixed to pre-policy steady state level. The black dashed lines indicate policy functions for the economy without the government loans and the red solid lines indicate those for the economy with the government loans.

Let's first examine the survival probability. Firms with cash-on-hand x lower than $-\overline{x}^G$ should default, which is indicated by a vertical line. Firms with lower cash-on-hand x need to choose higher debt and, consequently, are less likely to survive. With the introduction of government loans, firms that would have defaulted without the government loans now survive. This is why the vertical line moves rightward with introduction of government loans, representing an increased repayment threshold. Furthermore, given the same level of cash-on-hand, firms are more likely to survive when government loans are available.

Turning to the policy function for investment, firms with a lower cash-on-hand tend to invest less, given the same level of capital and productivity. This is because lower cash-on-hand is associated with higher debt, higher default risk, and a higher marginal cost of investment. With

Figure 2.10: Survival Probability and Investment by Cash-on-Hand

(a) Survival Probability (b) Investment (%)



Notes: These plots display the survival probability and policy function for investment of firms with median capital k and productivity z with respect to the level of cash-on-hand x for the economy, both with and without government loans, while keeping p fixed at the steady-state level without government loans. Survival probability represents the likelihood of survival given firms' optimal choices regarding capital and debt., i.e. $\sum_{z'}\left[1-\Phi\left(\hat{\phi}^G\left(k'(x,k,z),B'(x,k,z),b_g'(x,k,z),z'\right)\right)\right]\pi(z'\mid z)$.

the introduction of government loans, firms can rely less on debt to finance the same level of investment. As a result, investment increases for firms with lower cash-on-hand. However, there is no change when cash-on-hand is high, as government loans do not alter their financing costs in this case.

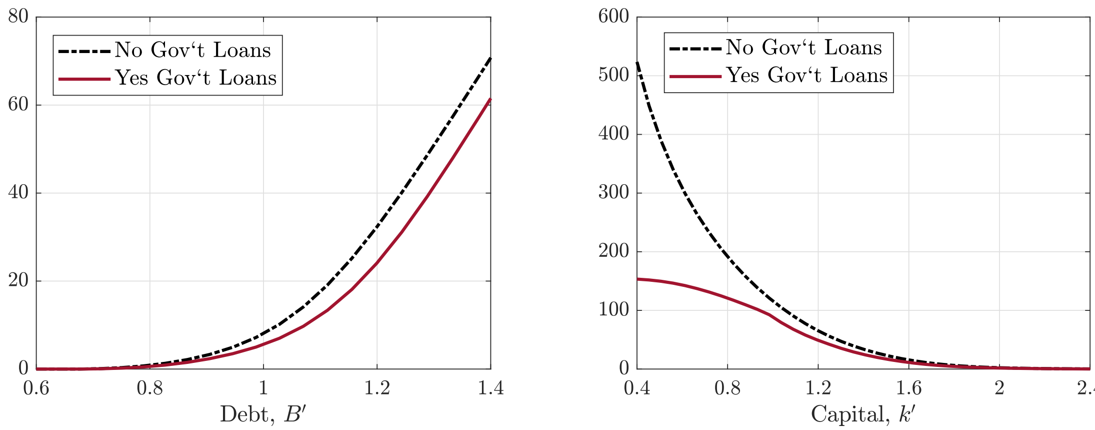
Figure 2.11 compares the credit spread between two economies, one with government loans and the other without. The left panel plots the credit spread schedule with respect to debt, given tomorrow's fixed capital choice and the same productivity. With the introduction of government loans, there is a reduction in the credit spread elasticity. In the right panel, the credit spread schedule is presented with respect to tomorrow's capital, assuming a fixed amount of debt and the same productivity. It's observed that, when firms hold more capital for tomorrow with the same debt and productivity, their credit spread decreases. However, with the introduction of government loans, firms holding less capital can borrow at a lower rate given the same amount of debt and same productivity.

2.6.2 Heterogeneity in firms' response to the policy: data vs model

Based on the calibrated model, my first step is to investigate whether the model generates predictions that align with the findings in the data regarding firm-level responses in terms of investment and exit to an increase in government loans. Using the panels of simulated firms, I replicate the specification outlined in the section 2.3.2 and 2.3.3 on the pooled sample.

Figure 2.11: Credit Spread Schedule

(a) Against debt (%) (b) Against capital (%)



Notes: These plots display credit spread schedule with persistent productivity to be fixed at average.

To compare the heterogeneous response of investment to government loans between the model and the data, I replicate the specification in equation 2.3. Specifically I regress the growth rates of capital on variables including a dummy variable indicating whether a firm's mean credit spread was high in the three years leading up to the introduction of government loans D_i, and the interaction of this high credit spread dummy with the period after the policy was implemented (specifically, three years after the introduction of government loans) D_i^{High} After $_t$. Additionally, I include lagged log capital size, lagged profitability (defined as the ratio of operational profit to capital) as in regression with data, and year fixed effects to control the general equilibrium effects. The specification is as follows,

$$\text{Investment}_{it} = \alpha_1 D_i^{\text{High}} \text{ After}_t + \gamma^x X_{it-1} + \gamma_t + \gamma^h D_i^{\text{High}} + \epsilon_{it} \tag{2.33}$$

where D_i^{High} is an indicator whether a firm's mean spread of Before period is the upper 10th percentiles. This specification aligns with the approach used in empirical findings.[26]

Firms initially characterized by higher pre-policy credit spreads increase their investment by 4 percentage points more than firms with lower pre-policy credit spreads in simulated data, while the data show a 5 percentage point higher increase. To be more precise, in equation 2.3, the

[26]The primary difference is that there is only 2 groups in the model specification while there are 4 groups in the model specification. This is because all firms are eligible for the government loans in the model, and there are two groups by pre-policy credit spreads. The other difference is the omission of firm fixed effects in the model. Furthermore, the model specification includes year fixed effects due to the one-industry nature of the model's economy, while the data specification includes year-sector fixed effects.

Table 2.6: Heterogeneity in firms response to the policy

(a) Investment

Δ Investment	
Data ($\beta_1 - \beta_2$)	Model (α_1)
5.14	4.02
[3.41 6.86]	

(b) Exit

Δ Probability to exit	
Data ($\beta_1 - \beta_2$)	Model (α_1)
-0.028	-0.023
[−0.012 − 0.045]	

Notes: The data estimates come from the equation 2.3 for investment and equation 2.4 for exit probability, with the 95% confidence interval presented in brackets. The model estimate are from equation 2.33 for investment and equation 2.34 for exit probability. Investment estimates are in percentage points, while exit probability estimates are in probability terms.

coefficient representing the differential impact of the policy on investment dependent on pre-policy credit spreads is denoted as $\beta_1 - \beta_2$ for the data and as α_1 for the model. To be specific, the heterogeneous response of investment with respect to pre-policy credit spread is captured by $\beta_1 - \beta_2$ in equation 2.3 for the data, and by α_1 in equation 2.33 for the model. You can find the results in Table 2.6a.

Similarly, I compare the heterogeneous response of exit to government loans between the model and the data. Specifically, I estimate the following regression based on simulated firms,

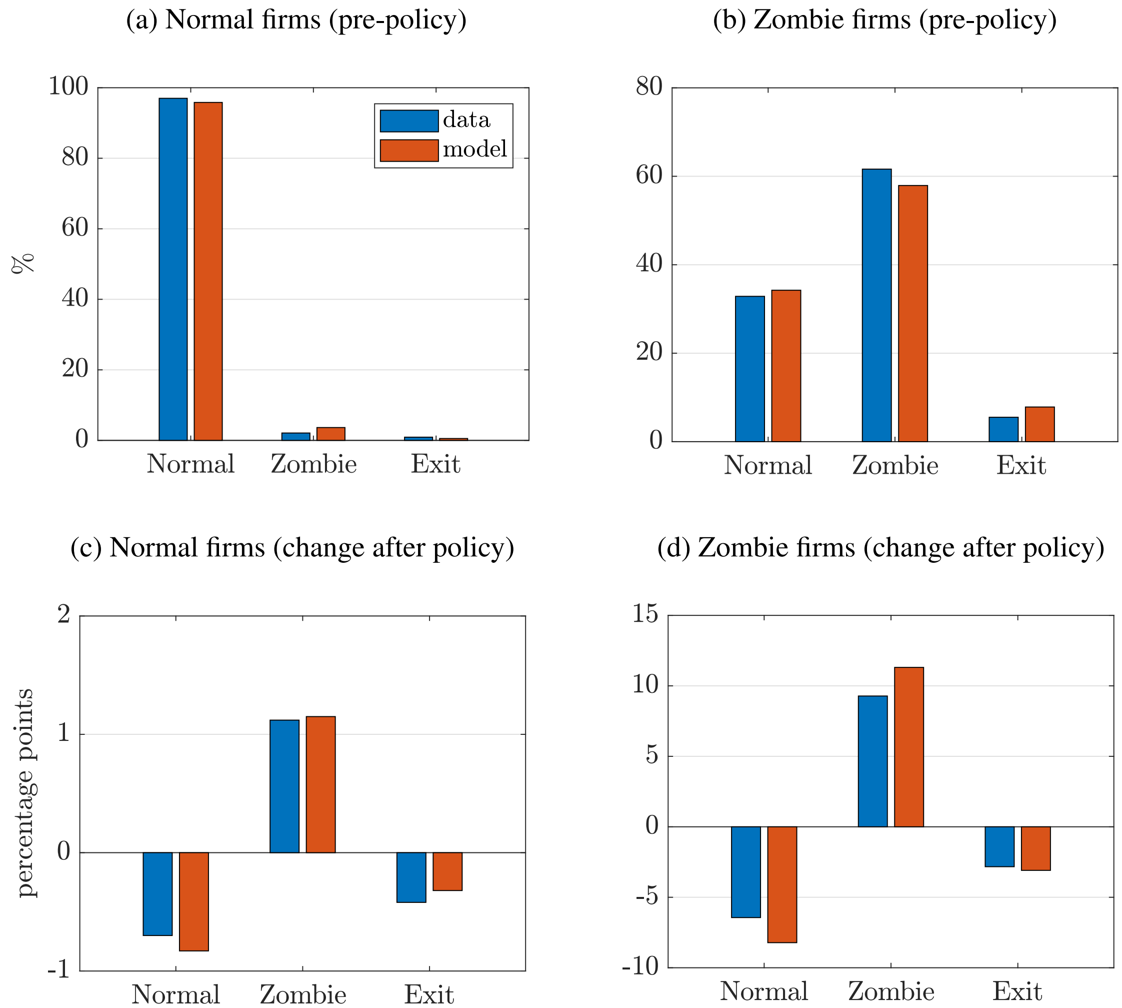
$$\text{Exit}_{\,it} = \alpha_1 D_{it-1}^{\text{Zombie}} \ \text{After}_{\,t} + \gamma_z D_{it-1}^{\text{Zombie}} + \gamma_t + \epsilon_{it} \tag{2.34}$$

In the model, the exit rate of low-productive firms decreases more by 2.3 percentage points compared to productive firms, whereas in the data, it decreases by a greater margin of 2.8 percentage points. Specifically, the heterogeneity in terms of change in exit rates based on the indicator for low-productive (zombie) firms is captured by $\beta_1 - \beta_2$ in equation 2.4 for the data, and by α_1 in equation 2.34 for the model. You can find the results in Table 2.6b.

I also analyze the transition probabilities of firms' statuses, which can be categorized as either zombie firms or normal firms (non-zombie firms). In the following year, a firm can transition to being a zombie or normal firm, or it can exit. Figure 2.12 compares these transition probabilities for the years preceding the government loans between the model and the data. The calibrated model accurately captures the observed transition probabilities in the data, even though it was not explicitly targeted during calibration.

To see the changes in transition probabilities, following the policy introduction, normal firms are more likely to transition into zombie firms, and zombie firms are more likely to remain as zombie firms. Furthermore, both zombie firms and normal firms exhibit reduced exit rates,

Figure 2.12: Transition Probability



Notes: The pre-policy transition probability is calculated using data from 2014 to 2016, and the model's pre-policy probability is based on steady state without government loans. The change in transition probabilities for the data is the difference between the Before (2014-2016) and After (2017-2019), while the change in transition probabilities for the model is over a three-year period following the introduction of government loans.

but the decline in exit rates is more pronounced for zombie firms. These observed patterns are effectively captured by the model, both qualitatively and quantitatively, presented in the lower panels in Figure 2.12. Normal firms are more likely to become zombie firms because a greater number of firms are in operation, leading to congestion, which adversely affects normal firms' profitability. Zombie firms are more likely to persist in their status due to reduced exit rates and continued low profitability.

2.7 Effect on Aggregate Productivity

My empirical and quantitative results suggest that government loans play a dual role. They help financially constrained firms increase their investments, thereby enhancing aggregate productivity through more efficient capital allocation. Simultaneously, these loans help low-productive firms in surviving, worsening the composition of active firms' productivity. To quantify these two offsetting effects, I decompose aggregate productivity into two components: capital allocation efficiency, as in Hsieh and Klenow (2009), and a composition of productivity.

First, I define the efficient level of output that a planner could achieve by reallocating fixed quantities of factors across a fixed mass of firm as follows,

Proposition 2.7.1 *In an economy where a planner can freely reallocate capital across firms to maximize production, for a given mass of firms,* $M = \int d\mu(x_{-1}, k_{-1}, z_{-1})$*, and total capital,* $K = \int k(x_{-1}, k_{-1}, z_{-1}) d\mu(x_{-1}, k_{-1}, z_{-1})$*, aggregate production is given by* $Y^* = M^{1-\alpha}\mathbf{E}\left[\tilde{z}^{\frac{1}{1-\alpha}}\right]^{1-\alpha} K^\alpha$*, where,* $\tilde{z} = \sum_z z\pi(z \mid z_{-1})$ *.*

Proof See appendix A.11.3.

As a direct corollary of the result, the output in the decentralized economy can be decomposed as follows,

$$Y = \underbrace{M^{1-\alpha}}_{\text{Size effect}} \times \overbrace{\underbrace{\mathbf{E}\left[\tilde{z}^{\frac{1}{1-\alpha}}\right]^{1-\alpha}}_{\text{Ext. efficiency}} \times \underbrace{\frac{Y}{Y^*}}_{\text{Int. efficiency}}}^{\text{Aggregate Productivity}} \times \underbrace{K^\alpha}_{\text{Capital qtys.}} \tag{2.35}$$

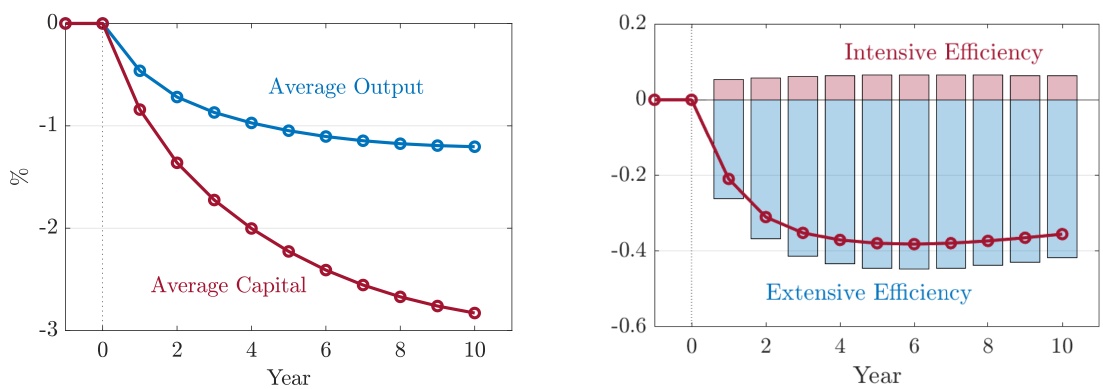
The aggregate (average) TFP depends on two components. The first term reflects the composition of productivity across active firms, shaped by selection along the extensive (exit and entry) margin. The second term represents capital allocation efficiency, in line with Hsieh and Klenow (2009), Hopenhayn (2014). This term equals 1 when capital is distributed across firms in a way that equalizes the marginal product of input across firms. I label the first term as "extensive efficiency" and the second term as "intensive efficiency." [27]

Figure 2.13 illustrates the 10-year transition path following the introduction of government

[27] The calibrated model suggests the intensive inefficiency is 1.1% in steady state of economy without government loans. This finding aligns with Midrigan and Xu (2014), who observed that the Korean manufacturing sector's TFP losses due to intensive inefficiency (marginal product of capital dispersion) ranged from 0.3% to 2.1% based on data from the years 1991 to 1999. 1 minus intensive efficiency is the intensive inefficiency that indicates how the economy is far from the efficient level of output.

Figure 2.13: Transition path over 10 years

(a) Average Firm Size (b) Aggregate Productivity



Notes: The figures indicate the percentage deviation from the steady state without government loans after the introduction of government loans in year 0 over 10 years.In the right panel, the red line represents the sum of changes in intensive and extensive efficiency, which is the net change in aggregate productivity

loans. The left panel displays the average output and capital.[28] Firms' size decreases on average due to the general equilibrium effect.

The right panel of Figure 2.13 shows the aggregate productivity decomposed into intensive and extensive efficiency. Government loans assist firms with low cash-on-hand in increasing their investments, improving intensive efficiency. However, the government's intervention also alters the exit and entry decision of firms and worsens the composition of active firms, resulting in a decrease in extensive efficiency. My results indicate that the loss from extensive efficiency (-0.4%) outweighs the gain from intensive efficiency (0.1%), leading to a decreases in aggregate productivity of 0.3% over the 10-year period.

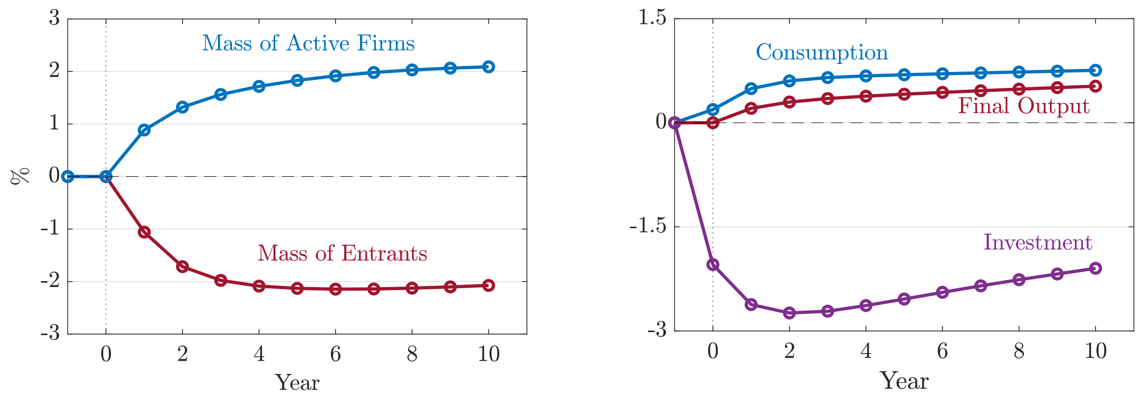
Figure 2.14 illustrates the transition path of aggregate variables. The left panel displays the mass of active firms and entrants. The lowered prices, due to general equilibrium effects, discourage potential entrants from entering the market. Exit rates of incumbent firms decrease because the impact of government loans outweighs the general equilibrium effect, leading to a larger mass of active firms in the economy.

The right panel shows the paths of consumption, final output, and investment. In period 0, firms reduce their investment because they anticipate lower prices in the following years, while the level of final output remains the same because it is determined in the previous year. Furthermore,

[28] Average output is calculated as $\frac{\int_{x_{-1},k_{-1},z_{-1}} \sum_z \int_\phi z \exp(\phi) k(x_{-1},k_{-1},z_{-1})^\alpha \pi(z|z_{-1}) d\mu_{-1}(x_{-1},k_{-1},z_{-1})}{\int_{x_{-1},k_{-1},z_{-1}} d\mu_{-1}(x_{-1},k_{-1},z_{-1})}$, and average capital is calculated $\frac{\int_{x_{-1},k_{-1},z_{-1}} k(x_{-1},k_{-1},z_{-1}) d\mu_{-1}(x_{-1},k_{-1},z_{-1})}{\int_{x_{-1},k_{-1},z_{-1}} d\mu_{-1}(x_{-1},k_{-1},z_{-1})}$.

Figure 2.14: Transition path over 10 years

(a) Mass of Firms and Entrants (b) Investment, Output, Consumption



Notes: The figures indicate the percentage deviation from the steady state without government loans after the introduction of government loans in year 0 over 10 years.

Table 2.7: Steady State Comparison

	Δ		Δ		Δ
Productivity	-0.3	Active Firms	+2.6	Capital	-0.4
(Intensive)	+0.1	Entrants	-2.2	Final output	+1.1
(Extensive)	-0.3			Consumption	+1.3

Notes: The percentage changes from the steady state without government loans to the new steady state with government loans.

as fewer firms exit and default costs decrease. Therefore, consumption increases in period 0. Over the course of 10 years, final output increases due to a larger mass of operating firms, even though per-firm production is lower. This increase in final output, coupled with reduced investment, leads to higher consumption.

Table 2.7 presents the percentage changes between steady states. Aggregate productivity decreases by 0.3%. Entrants decrease by 2.2%, but the mass of operating firms increases by 2.6% due to lowered exit rates. The increase in the mass of operating firms results in higher final output by 1.1% but lower investment leads to lower capital. The combination of higher final output, lower exit rates, and reduced capital levels leads to a 1.2% increase in consumption.

2.8 Policy Experiment

The gain from government-backed financing on aggregate productivity comes from improved capital allocation across firms, primarily benefiting young firms. These young firms, while

Table 2.8: Allowing Potential Entrans' Access to Government Loans

Δ	Only incumbents	Allow to entrants
Productivity	-0.3	-0.1
(Capital allocation)	+0.1	+0.1
(Composition)	-0.3	-0.1

Notes: Only incumbents indicate the percentage changes from the steady state without government loans to the new steady state with government loans accessible only to incumbent firms and Allow to entrants indicate those from the steady state without government loans to the new steady state with government loans accessible both to incumbent and potential entrants.

small, exhibit high productivity and often require increased borrowing for expansion. However, their small size renders them more constrained in accessing credit due to limited collateral, making them vulnerable to transient shocks. Consequently, capital misallocations among young firms tend to be higher compared to older firms. Furthermore, the gain from government loans in terms of improved capital allocation is predominantly concentrated among young firms.[29]

Given that the gain is mostly concentrated on young firms, we can consider an alternative policy to allow potential entrants access to government loans. In Table 2.8, I compare the change in aggregate productivity between the steady state without government loans and the steady state with government loans excluding potential entrants, denoted as Only incumbents, and between the steady state without government loans and the steady state with government loans including potential entrants, denoted as Allow to entrants. Allowing potential entrants access to government loans changes the entry margin significantly, leading to a larger mass of potential entrants entering the market. This generates general equilibrium effects, crowding out low-productivity incumbent firms. As a result, the loss from compositional productivity is limited compared to the case allowing only for incumbent firms.

2.9 Conclusion

I study the effect of government-backed financing policy on aggregate productivity, addressing the trade-off of the policy. I exploit an extensive panel dataset of Korean manufacturing firms and a policy shift by the Korean government, which significantly increase in government loans after 2017.

[29]Capital misallocation, measured by the dispersion of the average revenue product of capital, is notably higher among young firms in the data, a phenomenon effectively captured by the model. Additionally, the gain in terms of improved capital allocation is mostly concentrated among young firms. For further details, refer to Appendix A.10.

The credit spread of the firms eligible for government loans decreased more than that of non-eligible firms, suggesting improved credit access among eligible firms. Moreover, eligible firms with higher pre-policy credit spreads exhibited greater post-policy increases in investment. However, the exit rate of low-productive eligible firms decreased most following the policy. These findings capture the main trade-off of government loans on aggregate productivity through enhanced firms' credit access.

To quantify these two off-setting effect, I build a heterogeneous-firm model incorporating both government and private loans. The calibrated model generates heterogeneous responses to government loans in terms of investment and exit, consistent with the data. Over a span of 10 years, aggregate productivity experiences a decrease of 0.3%. The gain resulting from increased investment by constrained firms is 0.1%, while the loss due to a decreased exit rate among low-productive firms is 0.4%.

Chapter 3

Sovereign Local Currency Debt and Original Sin Redux

3.1 Introduction

A substantial body of literature has extensively documented that currency mismatch leaves emerging economies (EMEs) vulnerable to global financial conditions. The currency mismatch arises in emerging economies for which it is too costly to borrow abroad in their local currency (LC). Eichengreen, Hausmann, and Panizza (2005) used the term “original sin” to describe the inability to borrow in local currency. The original sin phenomenon has been pointed out as a critical factor contributing to the difficulties emerging economies face in managing their debt levels, leading to a higher degree of debt intolerance. [1]

However, since the mid-2000s, the share of emerging government debt issued in foreign currencies (FC) has fallen(Du, Pflueger, and Schreger, 2020, Ottonello and Perez, 2019). Contrary to the expectation, higher borrowing in domestic currency has not insulated EME from the vicissitudes of global financial markets. Hofmann, Shim, Shin, et al. (2020) refer to this phenomenon as the “original sin redux”. As an example, during the financial market turbulence amid the Covid-19 pandemic, EMEs experienced an average 8 % currency depreciation against the dollar, capital outflows resulting in a 3.7 percentage point decrease in foreign holdings of local currency (LC) bonds, and an increase in local currency bond yields of 154 basis points.

[1] Debt intolerance indicates the relationship between a country's credit rating (credit risk) and its external debt. It is reported that credit risk tends to increase more rapidly with respect to debt in emerging markets than in advanced countries as if the former have less debt management capacity. See Eichengreen, Hausmann, and Panizza (2007), Reinhart, Rogoff, and Savastano (2003)

It has been argued that the original sin redux arises from the following negative feedback loop through foreign investors' currency mismatch, as presented by Carstens and Shin (2019), Hofmann, Shim, Shin, et al. (2020). During a global financial market turmoil, emerging currencies tend to depreciate against the dollar, leading foreign investors to incur capital losses denominated in their own currency and trigger a sell-off of EME's local currency bonds. The consequence is an increase in local currency bond yields. When debt is denominated in local currency, the currency mismatch problem is shifted from sovereigns to foreign investors with unhedged positions.[2]

Using data from 11 emerging economies, I document empirical patterns consistent with the original sin redux and find significant heterogeneity across countries.[3] Following Du and Schreger (2016), I decompose the local currency sovereign spread into two parts: currency risk and default risk. Currency risk arises because foreign investors are concerned about returns in their own currency, not in the currency of issuance. Default risk arises from the possibility of an outright default by the issuer.

Specifically, I document the following empirical patterns: firstly, the increase in the share of local currency debt has not decoupled default risk from global financial shocks. Furthermore, I find the sensitivity of local currency default risk to global financial shocks is more elevated in EMEs with a lower degree of financial development relative to their debt levels, and a higher sensitivity of local currency default risk to global financial shocks is associated with higher default risk.

To explore cross-country heterogeneity in more depth, I find that in EMEs with lower financial development relative to their debt levels, there is a tendency for domestic banks' credit to local firms to be more adversely affected by foreign capital outflows. This finding suggests a possible mechanism linking capital outflows to the rise in LC bond yields. When an adverse global financial shock induces foreign investors to unwind their positions in LC bonds, it places pressure on domestic banks to absorb the excess supply of LC bonds. The government debt takes up the credit that could have been provided to firms, which in turn adversely affects economic activity and leads to a higher sovereign default risk. [4] To put it another way, a high level of government debt crowds out the level of financial development, making the economy more vulnerable to

[2]According to the BIS survey result presented in Cantú, Chui, et al. (2020), about half of the central banks do not have information on whether foreign investors have hedged their LC government bond FX exposures or not. Central banks with the information report that only a small portion of foreign investors' FX exposures are hedged.

[3]Sample countries are the ones that borrow abroad in their local currencies, including Brazil, Colombia, Hungary, Indonesia, Malaysia, Mexico, the Philippines, Poland, South Africa, Thailand, and Turkey.

[4]This pattern has been widely documented in the literature. See Gennaioli, Martin, and Rossi (2014), Perez (2015), Sosa-Padilla (2018), Farhi and Tirole (2018).

external shocks, even if the government debt exposure seems lower due to borrowing in the local currency.

I interpret the above empirical findings documented through the lens of a sovereign default model in which the government issues bonds to local banks and a heterogeneous set of foreign investors. As in Gertler and Kiyotaki (2010), banks receive deposits from domestic households, invest in domestic firms, and purchase government bonds. However, collateral constraints impose limitations on banks' access to household savings. For simplicity, I assume all government bonds are denominated in local currency. Foreign investors differ in the fee they need to pay to buy local currency government bond. The fee can be interpreted as a stand-in for the degree of risk aversion of an individual investor. The marginal foreign investor is the one who prices bonds. The government can default on its debt. A default is followed by decreased aggregate productivity and a utility loss. As observed in the data, the local currency, treated as an exogenous and stochastic process in the model, tends to depreciate and exhibit higher volatility when there is an adverse global financial shock.

I provide a numerical example in which the model can mimic the documented empirical patterns. The mechanism is that given an adverse global financial shock, foreign investors anticipate a decrease in the expected return on LC bonds (due to the expected depreciation) and an increase in the return volatility on LC bonds (due to the higher exchange rate volatility). This induces foreign investors to reduce local currency bond holdings. As a result, domestic financial intermediaries increase their government bond holdings, which leads to a reduction in private credit due to collateral constraints. This disruption adversely affects the economy, ultimately increasing the government default risk.

The level of financial development, captured by the degree of collateral constraint, relative to the debt level plays a significant role in determining the intensity of interaction between capital outflow from the local currency government bond market and private credit disruption. This intensity, in turn, determines the extent to which shocks in the global financial market lead to higher default risks on local currency government bonds. This mechanism generated by the model helps explain the key cross-sectional patterns observed in the data, particularly with regard to the relationship between an economy's ability to maintain private credit during capital outflows, credit risk, and external vulnerability.

Related literature This paper builds on the literature based on the standard sovereign default models such as Arellano (2008), Aguiar and Gopinath (2006), incorporating a banking sector along the line with Gertler and Kiyotaki (2010). Particularly, the paper contributes to three strands of the literature on open macro emerging economies.

The paper is related to literature that links sovereign risk, the banking sector's fragility, and economic activity. In the sovereign debt literature, several papers study the linkage between sovereign defaults and banking crises characterized by large private credit contraction. Gennaioli, Martin, and Rossi (2014), Perez (2015), Sosa-Padilla (2018), and Farhi and Tirole (2018) propose a model in which banks holding the government bond are impeded from providing credits to firms conditional on a government default. They show that such a mechanism can generate substantial output costs of a sovereign default. Different from the above papers, I focus more on periods characterized by rising sovereign LC spreads and significant capital outflows but no actual default, mainly driven by shifts of global financial conditions. In that sense, my work is also closer to Arellano, Bai, and Bocola (2017) that show the increase in sovereign credit spreads tightens leverage constraint deteriorating financial intermediaries' balance sheets and constrains credit supply to firms and output. My paper shares the emphasis on financial intermediation with these papers. But my work departs from their works by explicitly modeling foreign investors' behaviors, motivated by the significance of foreign investors' impact on EMEs LC bond markets as pointed out by Ho (2019), Carrera, Aguirre, Raffin, et al. (2020).

My research is complementary to theirs: I consider currency risks borne by foreign investors holding the LC bonds and study interactions between foreign investors' decisions and their impacts on EMEs through domestic banks. Foreign capital outflow from the LC bond markets, triggered by the shifts of global financial conditions, has a recessionary effect on EMEs because domestic banks need to hold more government bonds in such periods. And this leads to disruption of private credit and an increase in default risks. The foreign investors' behavior and its impact on EMEs is the novel key mechanism in this paper.

My paper also complements the literature that studies EMEs issuing sovereign debts internationally in LC. Methodologically, I follow Du and Schreger (2016) to measure the default risk on LC sovereign debt separately from currency risk. Recent work paying more attention to benefits from LC debts, such as Du, Pflueger, and Schreger (2020), Ottonello and Perez (2019), mainly studies the government's currency composition problem. These papers study the implication of monetary credibility in currency composition dynamics with focusing on the hedging benefit of LC debt. Meanwhile, policy papers including Ho (2019), Carstens and Shin (2019), Hofmann, Shim, Shin, et al. (2020) study the phenomena that borrowing in the LC has not eliminated the external vulnerability EMEs suffered with their debt mainly denominated in FC. My work studies the determinants of the differential degree of the external vulnerability with foreign holdings of LC bonds as in Du, Pflueger, and Schreger (2020). Risk-averse foreign investors in Du, Pflueger, and Schreger (2020) require a higher risk premium for holding bonds whose dollar returns are more procyclical. In my model, foreign investors reduce the LC bonds' investment when the expected return in dollar terms is low, leading to foreign capital outflows from the LC bond market. Foreign investors solely hold the LC government bond in Du, Pflueger, and Schreger (2020). Domestic banks also hold the bond in my paper, which generates the interaction between foreign capital movements in the LC bond markets and domestic banks' private credit supply.

Finally, the paper also contributes to the literature that studies the impacts of the global financial cycle on emerging economies (Rey, 2015, Bruno and Shin, 2015). I show that the degree of global financial states' impact on developing countries is associated with the financial development and its debt service ratio. Financial development relative to the debt level determines the domestic banking sector's capability to continue providing private credit when a global financial condition is tightened. In this regard, my work is in line with literature that empirically studies interactions between global financial cycles and domestic credit market (Di Giovanni, Kalemli-Ozcan, Ulu, and Baskaya, 2017), and also related with literature that works on the determinant of the external vulnerability (Iacoviello and Navarro, 2019, Gonzalez-Aguado, 2018). I take the effects of global financial states' changes in reduced form: decline in productivity and currency depreciation with higher uncertainty in FX market. I study how domestic banks' private credit supply responds to foreign capital movement triggered by a shift in global financial states and associate the responses to the degree of external vulnerability. The paper is also close to literature that establishes a significant fraction of sovereign spreads volatility is accounted for by the global risk premium volatility (Bianchi, Hatchondo, and Martinez, 2018, Longstaff, Pan, Pedersen, and Singleton, 2011).

Layout The rest of the paper is organized as follows. Section 3.2 presents empirical evidence regarding the effects of global financial shocks on LC debt market. I lay out the setup of of the model in section 3.3, and perform a quantitative evaluation of the model to see how the model explains the empirical facts in section 3.4. Section 3.5 concludes.

3.2 Empirical Motivating Evidence

This section presents empirical evidence regarding the effects of global financial shocks on emerging LC debt market based on cross-country comparison. Subsection 3.2.1 describes the construction of variables of interest that are used for analysis and the sources of the data, and subsection 3.2.2 presents empirical evidence that will be mainly studied with the model in the following section.

3.2.1 Data

There are 11 emerging economies in the sample ranging from 2007 to June 2020.[5] In this section, I present the detailed construction process of key variables. See appendix B.2 for the detailed sources of variables.

LC bond yield spread The nominal spread on LC bond can be decomposed into a risk-free rate and a default risk (credit risk). With assuming of a frictionless financial market, I construct LC risk free rate by swapping the dollar cash flows from a default-free U.S. Treasury bond into the LC using a cross-currency swap (CCS) following Du and Schreger (2016). The risk free rate is compensation for changes in value paid to investor induced by exchange rate fluctuations added to investors' borrowing cost, the U.S. Treasury bond yield. Then in the absence of financial market frictions, the LC spread over the LC risk free rate is positive only if there is default risks on the debt. Specifically, I construct an implied long-term forward premium between emerging economies' currencies and the US dollars (ρ_t) using the fixed-for-floating CCS and the US dollar interest rate swap, and define the premium as currency risk. The currency risk added to U.S. risk free rate (r_t^*) is defined as a risk-free rate on LC bond, and the spread of LC bond over the risk-free rate on LC bond, or the deviation from covered interest parity (CIP) between the LC bond and U.S. treasury bond is defined as a credit risk. Then the nominal LC bond yield (y_t^{LC}) is as follows:

$$y_t^{LC} = \underbrace{r_t^* + \rho_t}_{\text{LC risk free rate}} + CS_t.$$

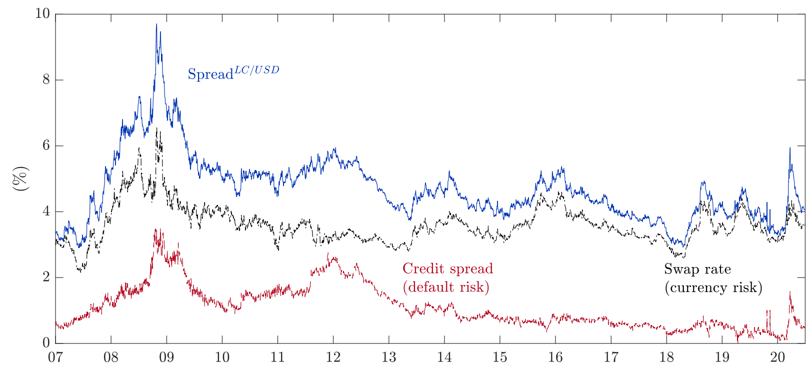
I construct the series of currency risks and credit spreads. The average LC yields, currency risks, and credit risks of the sample countries are depicted in Figure 3.1. The summary statistics for the series of the sample countries are reported in Appendix B.4. On average, the credit spread is 1.1% and around 77% of the nominal spread is composed of currency risk and the remaining 23% is composed of credit spread.[6]

Foreign holdings of LC sovereign debt securities Foreign holdings of LC sovereign debt se-

[5]Sample countries include Brazil, Colombia, Hungary, Indonesia, Malaysia, Mexico, Philippines, Poland, South Africa, Thailand, and Turkey. The nominal LC yield spreads are constructed from 2007 to June 2020 while the variables used for regression are constructed quarterly up to Q1 2020 due to data availability.

[6]Du and Schreger (2016) reported that 75% of the nominal spread is composed of currency risk and 25% is composed of credit spread based on 13 EMEs (sample countries with South Korea, Peru) from 2005 to 2013.

Figure 3.1: EME's average nominal LC yield spread, swap rate and credit spread



Note: Average of 11 EMEs (Brazil, Colombia, Hungary, Indonesia, Malaysia Mexico, Philippines, Poland,South Africa, Thailand, Turkey)

Sources: Bloomberg, St. Louis Fed, Author's calculation

curities are calculated LC government debt held by foreign investors as the percentage of total outstanding LC government debt. The data is sourced from Arslanalp and Tsuda (2014) and the Institute of International Finance (IIF). Arslanalp and Tsuda (2014) constructed 24 emerging economies' government debt held by foreign investors in local and hard currency from 2004 to 2019 quarterly. IIF quarterly releases related data of 17 emerging economies. Most of data comes from IIF. Philippines' data is sourced from Arslanalp and Tsuda (2014) and South Africa's data from 2007 to 2010 is also sourced from Arslanalp and Tsuda (2014) due to the availability of data released from IIF. In the first Table in the appendix, the average foreign holding of LC government debt and change over the sample periods are reported. In the following Figure, the series of 11 sample countries' foreign holdings are plotted. Over the sample periods, all the sample countries excluding Hungary experienced an increase in foreign participants in the LC sovereign debt market. Participation of foreign investors decreased temporarily during the period of the financial crisis in 2008. LC sovereign debt held by foreigners significantly increased from 2009 to 2014 as foreign investors chased for yields amid continuing monetary easing of advanced countries. Then foreign participants gradually decreased with the Fed's tapering and the following concerns over emerging economies' currency risks.

Banks' exposure to government and private sector Banking sectors' holdings of government debt are measured as banks' net claims on the domestic governments (central and local government and public non-financial sector) as a share of the banking sector's total assets, following Gennaioli, Martin, and Rossi (2014), Kumhof and Tanner (2005). Claims on private sectors are measured as claims on non financial private sectors. Data is sourced from IFS. Claims by banking sectors (other depository corporations) are considered here because of data limitations.[7]

Financial development indicator I used the ratio of liquid liabilities to GDP as an indicator for financial development, which is sourced from the World Bank. This has been one of the main indicators used for financial development in the literature including King and Levine (1993), Rousseau and Wachtel (2011). The liquid liabilities are known as broad money, which includes currency and deposits in the central bank, and deposits at financial intermediaries.

3.2.2 The effects of global shocks on LC sovereign bond market

This section describes the cross-country difference in impacts of global shocks on LC sovereign debt market. I investigate factors that determines the degree of global shocks pass-throughs. Firstly I document that higher reliance on foreign capital leads to more vulnerability using movements of LC yields and credit spread during recent financial market turbulence amid the Covid-19 pandemic, as presented in Carstens and Shin (2019) and Hofmann, Shim, Shin, et al. (2020). Secondly, I link the level of financial development to the vulnerability to global shocks and find a country with low financial market depth shows a higher vulnerability to the shocks.

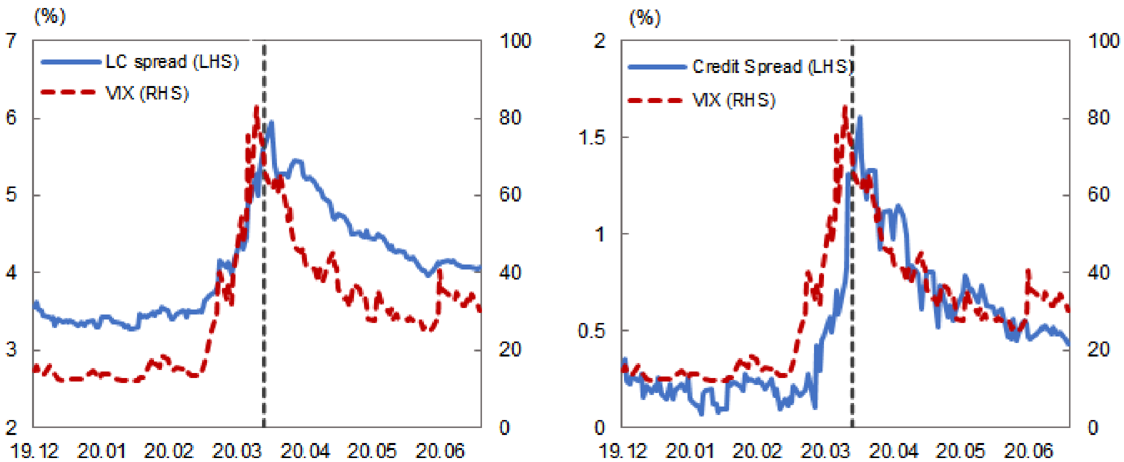
Original sin redux during the COVID-19 pandemic

What was observed in the global financial market during the Covid-19 pandemic shows that borrowing in LC was not sufficient for insulating EMEs from changes in global financial conditions. Some countries, especially countries heavily relying on foreign capital, still show high debt intolerance dependent on global financial conditions. Hofmann, Shim, Shin, et al. (2020) lays out the key mechanisms of such original sin redux phenomena focusing on interactions of currency fluctuations and financial market outcomes in EMEs. EMEs' currencies tend to decline significantly with tightened global financial conditions. The currency decline leads to capital outflows from EMEs' LC bond market as foreign investors evaluate gains and losses in terms of dollars (or other advanced currency), and increase in the LC bond yield. Thus, reliance on foreign capital leads to a greater vulnerability to global financial shocks.

The financial shock triggered by the Covid-19 pandemic provides a vivid illustration of original sin redux. During the Covid-19 pandemic, all of sample EMEs LC bond markets experienced

[7] The role of non-bank financial companies in the financial market is sizable and increasing, however, only the banking sectors are considered for analysis because the data collection for non-bank financial companies is at an early stage and limited.

Figure 3.2: EME's average nominal LC yield spread (5-yr) amid the Covid-19 pandemic



(a) LC Yield Spread (b) Credit Spread

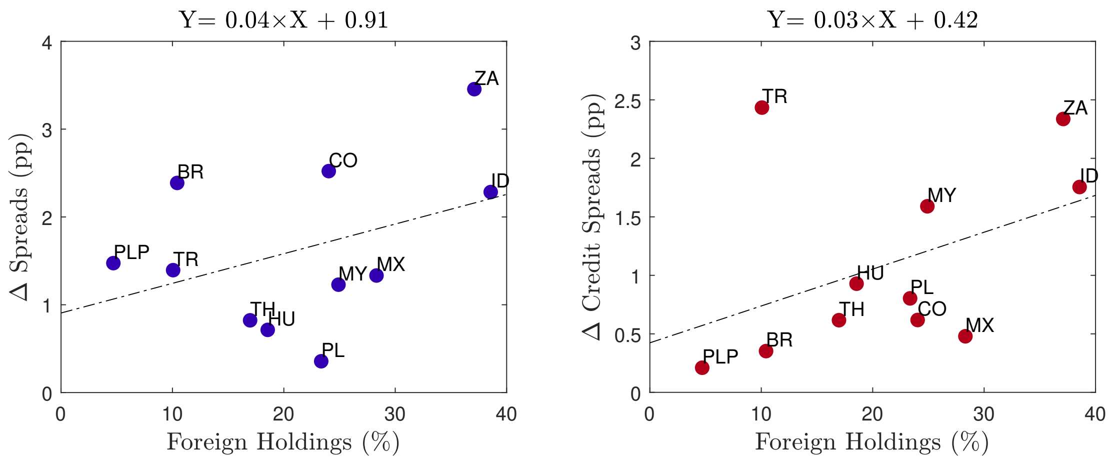
Notes: (1) Average of sample 11 EMEs.
(2) Black dashes line indicates the period when the Fed announced the unlimited bond purchases (March 23, 2020).

Sources: Bloomberg, St. Louis Fed, Author's calculation

massive bond portfolio outflows, sharp exchange rate depreciation, and surges in bond yields. By late March 2020, EME currencies had depreciated by around 8% against the dollar on average compared to their levels before the outbreak of Covid-19 pandemic. The share of foreign holdings by the end of March 2020 was 18.4% on average, decreased by 2.2pp compared to the end of 2020, and decreased by 3.7pp compared to the end of March 2019. Figure 3.2 shows the EME's average nominal LC yield spreads over the U.S. treasury yields, credit spreads and VIX index during the periods of financial turbulence amid Covid-19 pandemic. As the Covid-19 pandemic has sparked widespread, EME's LC spreads and credit spreads reacted sensitively to global risks. Black dashed line in Figure 3.2 indicates the day when the Fed announced the unlimited bond purchases. LC spreads and credit spreads dropped after the announcement as the global financial shocks decreased.

In Figure 3.3, it is demonstrated that when governments rely more heavily on foreign finance, their sovereign local currency (LC) bond markets become more susceptible to global financial shocks. EMEs with higher proportions of foreign ownership in their LC bond markets witnessed notably larger spikes in their LC bond spreads and credit spreads during the turbulent periods of the Covid-19 pandemic. This is consistent with Hofmann, Shim, Shin, et al. (2020) that documents a larger reliance on foreign capital leads to a greater vulnerability to global financial shocks with an emphasis on interactions between currency fluctuations and EMEs LC bond

Figure 3.3: Changes in LC Yield Spreads (5-yr) & the Level of Foreign Holdings



(a) Changes in LC spread over US spread (b) Change in Credit spread

Notes: (1) Foreign holdings/Total outstanding of LC sovereign bond (%, as of end of 2019).
(2) Change in spreads between the last week of February and the third week of March 2020, before the Fed announces the unlimited bond purchases (March 23, 2020).

Sources: Arslanalp and Tsuda (2014), IIF, Bloomberg

market.

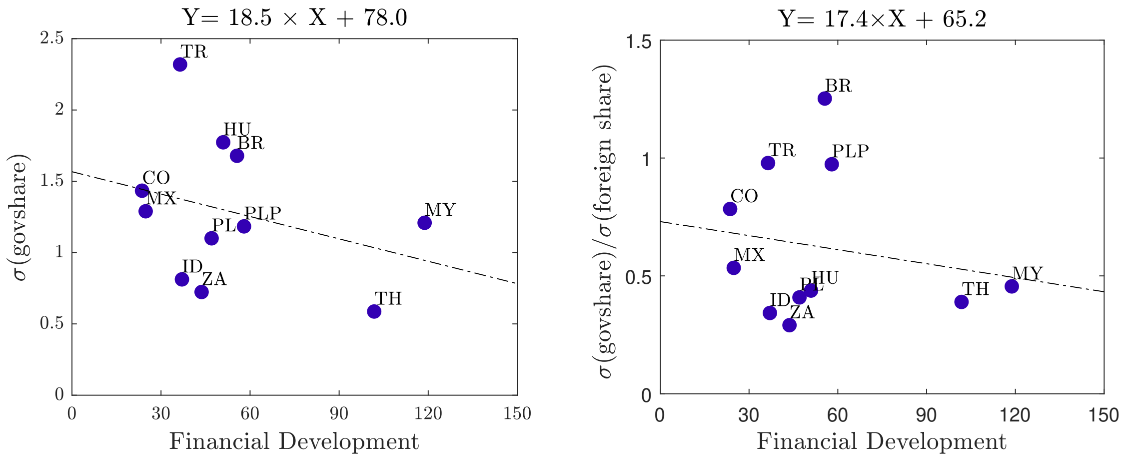
Financial development and vulnerability to global shocks

Financial development and banks' balance sheet (B/S) composition volatility In the data there is a negative relationship between the level of financial development of a country and the domestic banks' balance sheet (B/S) composition volatility as depicted in the left panel of Figure 3.4. The negative relationship indicates that banks in a less financially developed country are more likely to adjust their B/S composition on a greater scale. Scaled by the volatility of foreign holdings of LC bonds, we also see a negative relationship. Considering the negative relationship between foreign holdings and banks' claims on government, banks in a less financially developed country tend to increase their government claims (decrease private credit) to a greater degree when foreign capital outflows from the LC bond market. [8]

Credit channel vulnerability I measure the vulnerability of credit channels and investigate the relationship between the level of financial development and the vulnerability. The result shows that private credit tends to be more adversely affected by foreign capital outflows from the LC bond market (higher credit channel vulnerability) in a less financially developed country. Such

[8] See Appendix B.4.

Figure 3.4: Financial Development[1] & Banks' B/S composition Volatility



(a) σ(govshare) (b) σ(govshare)/σ(foreign share)

Notes: (1) Liquid liabilities to GDP (%, average between 1997~ 2017).
(2) St.dev of domestic banks' net claims on government as a share of total claim.
(3) St.dev of foreign holding as a share of total outstanding LC government bond.
(4) St.dev is calculated based on hp-filtered series.

Sources: Arslanalp and Tsuda (2014), IIF, IFS, World Bank

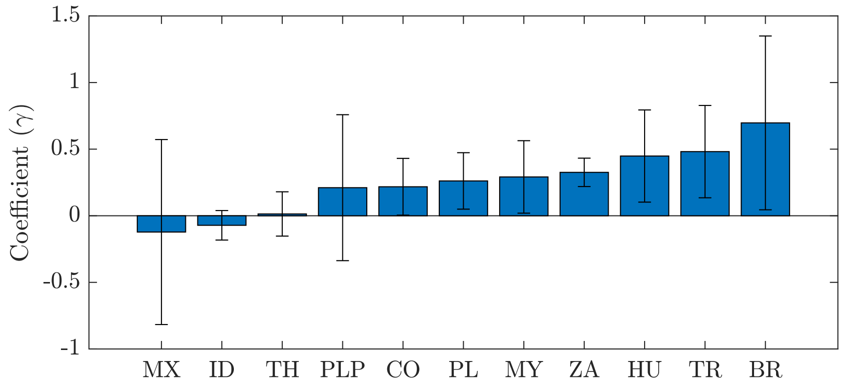
an economy shows a higher credit risk and also a higher vulnerability to global financial shocks. Specifically, I measure the credit channel vulnerability regressing the change in private credit on the change in foreign holdings of LC debt for each country as follows:

$$\Delta\text{Private Credit}_t = \gamma\Delta\text{Foreign Holding}_t + \beta_l \text{X}_{t-1} + \beta_g \text{Global}_t + \epsilon_t \tag{3.1}$$

where ΔForeignholding$_t$ is a change in the foreign holdings of LC bond, $X_{i,t}$ is a vector of control variables including change in nominal exchange rate, change in volatility of exchange rate, debt to GDP ratio, claims on the government as the share of total claim, inflation rate, real GDP growth rate, and Global$_t$ is a vector of global control variables, including the VIX index, the BBB-Treasury spread, the 10-Year Treasury yield, the TED spread, and the US Federal Funds Rate considered following Du, Pflueger, and Schreger (2020). For dependent variable ΔPrivate Credit$_t$, I use the growth of banks claims on private sector net of total claim growth inspired by Gennaioli, Martin, and Rossi (2018). The private credit growth net of total claims growth gives information on the change of the banking sector's B/S composition. A lower private credit growth net of total claims growth indicates the expansion of the balance sheet is mainly driven by the increase in claims on the government.

The coefficient of interest is γ, which indicates that a one percent point increase in foreign

Figure 3.5: Coefficient γ in equation (3.1)



Note: (1) γ is the regression coefficient of the growth of banks claim on private sector net of the growth of banks' total claim (pp) on changes in foreign holdings of LC government bond (pp)
(2) The line through the bar indicates 95% confidence interval of each coefficient.

holdings is related to γ percent point higher increase in private credit than total credit. Higher γ represents that banks exhibit a larger decrease in private credit supply when foreign capitals exit from the LC bond market. I define higher γ as a higher credit channel vulnerability. The coefficient is positive and significant for all sample countries except for Mexico, Indonesia, Thailand, Philippines.

I examine the relationship between credit channel vulnerability and the level of financial development, while also considering the debt-to-GDP ratio as an additional factor. Credit vulnerability measures the extent to which domestic banks restrict private credit when they are required to hold more government bonds, necessitating consideration of the debt level. In an economy with low financial development and a low debt-to-GDP ratio, the impact on domestic banks' ability to provide private credit would likely be limited even with low financial development.

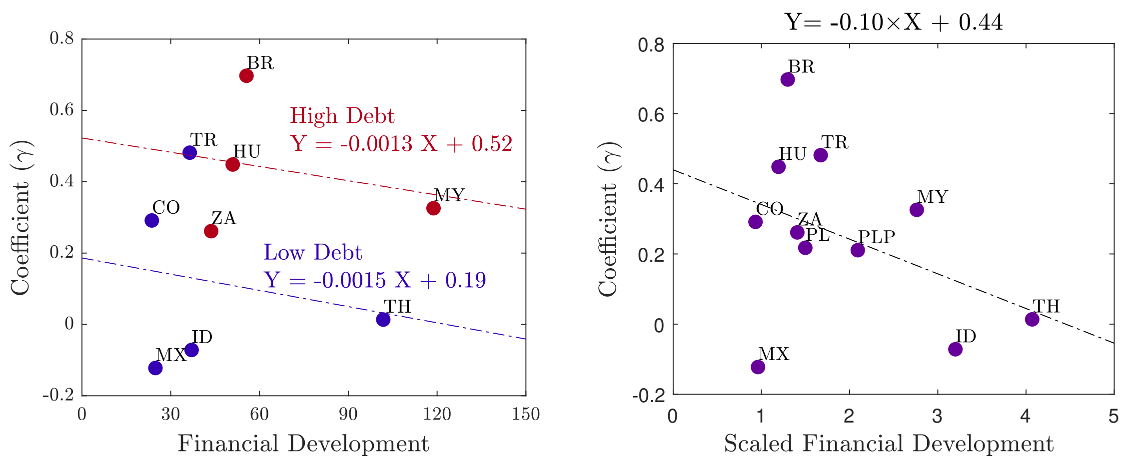
In Table 3.1, I categorize the sample countries based on their debt level and financial development. It is observed that the debt level tends to be higher in countries with more developed financial markets. This tendency shows that it leads to misinterpretation to link credit vulnerability only to financial development. For instance, Thailand possesses a developed financial market, yet its government debt level is relatively low, resulting in low credit channel vulnerability. This cannot solely be attributed to financial development, as the low debt level also plays a significant role. Therefore, to accurately interpret the empirical patterns, it is essential to consider the relative level of financial development compared to the debt level. This entails examining how developed the local financial market is relative to government debt.

Table 3.1: Sample Countries Profile by the Level of Debt[1] and Financial Development

	Low debt	High debt
Low financial development	Colombia, Indonesia, Mexico, Turkey	South Africa
High financial development	Thailand	Brazil, Hungary, Malaysia

Notes: (1) Debt to GDP
(2) ”Low” and ”high” is determined by the median level.

Figure 3.6: Financial Development & Credit Channel Vulnerability



(a) (b)

Notes: (1) Coefficient γ in equation (3.1) for each country.
(2) ”Low” and ”high” is determined by the median level.
(3) Relative financial development is the level of financial development scaled by the debt to GDP ratio.

Figure 3.6a shows the relationship of ”high debt group” and ”low debt group” between the level of financial development and credit vulnerability. In both groups, I find that a level of financial development is negatively related to credit vulnerability. Thus I use the level of financial development scaled by the debt-to-GDP ratio to gauge the relationship. I find a negative relationship between the scaled level of financial development and the credit channel vulnerability, which is depicted in Figure 3.6b. The average credit channel vulnerability of countries with high-scaled financial development is 0.19 while that of countries with low-scaled financial development is 0.32.

To assess robustness, I conducted a pooled regression analysis with interaction terms. Specifically, I included interaction terms between the change in foreign holdings and financial development, as well as interaction terms between the change in foreign holdings, financial development, and an indicator of whether a specific country's mean debt-to-GDP ratio is higher than the sample countries' median. In this regression, credit vulnerability is modelled as a function of financial development and the interaction terms between financial development and debt-to-GDP ratio. The results indicate that credit vulnerability decreases as financial development increases. However, the impact of financial development diminishes with higher debt-to-GDP ratios. In essence, higher levels of financial development reduce credit vulnerability, but the benefits derived from financial market development are less pronounced in economies with higher debt levels. For further details, please refer to the Appendix B.5.

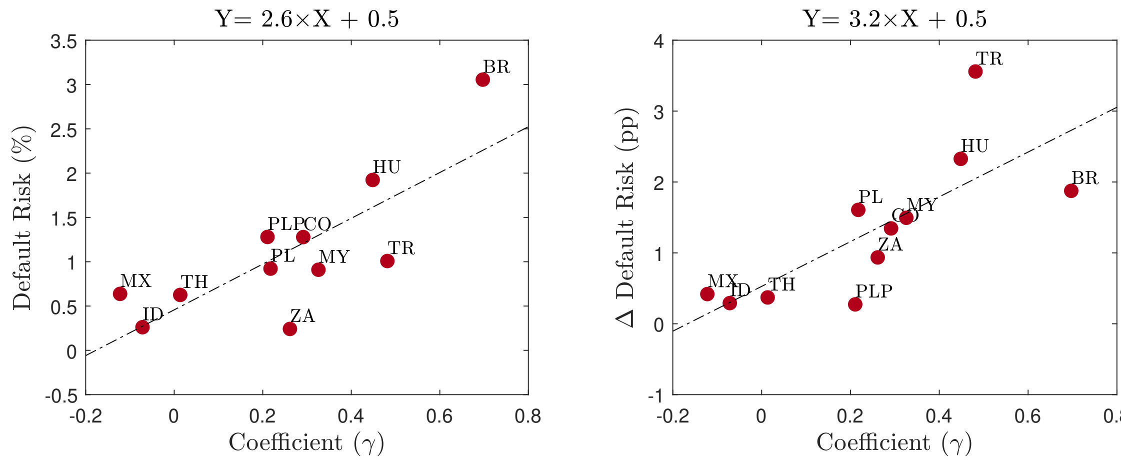
Figure 3.7 illustrates the relationship between credit channel vulnerability and default risks. In Figure 3.7a, we observe that economies with higher credit channel vulnerability tend to experience higher default risk. Additionally, Figure 3.7b indicates that these economies are more susceptible to global shocks, as default risk reacts more sensitively to changes in global financial conditions. Lower credit channel vulnerability implies that banks maintain private credit supply during periods of capital outflows from the local currency bond market when they are required to hold more government bonds. This resilience to external shocks results in a lesser increase in credit risks during periods of high global financial stress.

Let's link financial development relative to government debt level to the relationship between credit channel vulnerability, default risk, and external vulnerability. In countries with more developed financial markets relative to government debt level, banks do not significantly increase their claims on the government relative to their total claims during foreign capital outflows from the local currency bond market following global financial shocks. This reflects their ability to continue supplying private credit during sudden stops, thereby reducing the economy's sensitivity to global financial conditions. Conversely, in less financially developed countries, banks adjust their balance sheets by allocating a larger share to the government when capital outflows occur from the local currency debt market. This leads to a significant decline in private credit and an increase in default risks. To support this relationship, I present regression results in Appendix B.7, which shows the connection between financial development relative to government debt level, credit risk, and external vulnerability.

3.3 Model

In this section, I build a three-period small open economy model to study the empirical features presented in the previous section. The model incorporates a banking sector along the lines of Gertler and Kiyotaki (2010) into a sovereign default model where the government bond is

Figure 3.7: Credit Channel Vulnerability & Default Risks



(a) (b)

Notes: (1) Coefficient γ in equation (3.1) for each country.
(2) Difference between average default risk in low global financial risk periods and that in high global financial risk periods. Periods of high global financial risk is the period when the VIX index is above the average + 1.5 times of the st.dev.

to be purchased by both domestic and foreign investors. I extend the model along two key dimensions. First the foreign investors' investments in the government bond is endogenously determined instead being determined by the government's decision as in Erce and Mallucci (2018), Gonzalez-Aguado (2018). Second I allow losses (haircuts) from the government's default to be different by whether the bond is held by domestic or foreign investors. The extension enables to capture interactions between foreign investor's investment decision depending on the state of global financial risk and its impacts on EMEs economy.

There is a small open economy that lasts for three periods $t = 0, 1, 2$. The economy is populated by a representative household whose members randomly switch between being workers and bankers, firms, foreign investors, and a government. Households sell labor to firms and lend to banks as a form of deposit. Firms produce consumption goods with capital borrowed from banks and labor. Bankers take deposits from households and lend to firms and the government, but do not have access to international markets. The government issues one-period local currency bond to finance its expenditures. Foreign investors invest in LC government bonds.

3.3.1 Exogenous states

In the model, the exogenous state is given by $\Lambda_t = (z_t, S_t, x_t)$, where z_t is total factor productivity, S_t is the nominal exchange rate and x_t is an indicator whether global financial risk is high at time t or not.

The productivity process is as follows:

$$\log(z_t) = \mu_z + \rho_z \log(z_{t-1}) + \varepsilon_{z,t} - \phi_z x_t, \tag{3.2}$$

where $|\rho_z| < 1$ and $\varepsilon_{z,t} \sim N\left(0, \sigma_z^2\right)$. Note that the economy's productivity is assumed to decline by ϕ_z in high global financial risk periods.

The nominal exchange rate changes dollar returns of foreign investors holding LC bond and the investor's level of investment, which is as follows :

$$\log(S_t) = \mu_s + \rho_S \log(S_{t-1}) + \varepsilon_{S,t} + \phi_S x_t, \tag{3.3}$$

where $|\rho_S| < 1$, $\varepsilon_{S,t} \mid x_t = 0 \sim N\left(0, \sigma_{s0}^2\right)$, $\varepsilon_{S,t} \mid x_t = 1 \sim N\left(0, \sigma_{s1}^2\right)$, and $\sigma_{s1} = \sigma_{s0}(1+\eta)$. It is assumed that the nominal exchange rate depreciates ($\phi_s > 0$) and the variance of shocks to the exchange rate increases ($\eta > 0$) in high risk aversion periods.

The process of states of global financial risk follows a two-state Markov process, where $x_t = 0$ indicates a normal time and $x_t = 1$ a period of high global financial risk. Transition probabilities are π_{01}, π_{10}, where π_{01} is the probability from state state 0 to 1. In high global financial risks periods, the emerging economy is assumed to experience decline in productivity and currency depreciation with higher volatility.

3.3.2 Government

The government finances an exogenous level of public spending $\overline{g}$ in period 0 with LC government bond. The bond issued in period 0 is non-defaultable [9]. In period 1 and 2, the government finances its expenditure G_1, G_2. The instrument that the government can access in period 1 includes proportional taxes on labor income constant across states τ and debt that the government can default. The government only can access to labor income taxes in period 2. The government bond is held by foreign investors and domestic banks.

[9] The argument behind non-defaultable debt in period 0 is that the main focus is on how the default risk of bond issued in period 1 increases depending on the state of global financial risk.

3.3.3 Private sector

Households There is a representative household composed of a measure 1 of workers and a measure 1 of bankers. Workers starts with endowment $\overline{n}_0^h$ in period 0 and choose how much to deposit (a_1) at price q_0^a and consume (c_1) out of endowment. In period 1, a measure λ of workers become new bankers and workers transfer $\overline{N}$ to newly born bankers. The λ of bankers cease to operate transferring the net worth to household. In period 1 workers decide on the level of deposit and labor supply, and they consume after tax labor income ($(1-\tau)w_1 l_1$), net worth transferred from exiting bankers, and deposit paid by banks net of savings for period 2. In period 2 they consumes after tax labor income based on their labor supply decision, net-worth transferred from domestic banks (N_2) and saving deposited in period 1. Lifetime utility of workers in household is as follows:

$$\max_{\left[c_{t=0,1,2}, l_{t=1,2}, a_{t=1,2}\right]} c_0 + \mathbb{E}_0 \left[\sum_{t=1}^{t=2} \beta^t \left(c_t - \frac{l_t^{1+\frac{1}{\zeta}}}{1+\frac{1}{\zeta}} \right) \right] \tag{3.4}$$

$$\begin{aligned} \text{s.t.} \quad & c_0 + q_0^a a_1 = \overline{n}_0^h \\ & c_1 + q_1^a a_2 = (1-\tau)\, w_1 l_1 + a_1 + \lambda(N_1 - \overline{N}) \\ & c_2 = (1-\tau)\, w_2 l_2 + a_2 + N_2. \end{aligned} \tag{3.5}$$

Preferences over consumption are assumed to be linear as in Arellano, Bai, and Bocola (2017) and Chari, Dovis, and Kehoe (2020) and decreasing and convex over labor, with $\zeta > 0$ being the Frisch elasticity of labor supply. The linearity of preference over consumption ensures $q_t^a = \beta$ because the household would not be willing to supply a deposit to the bank unless the price of the deposit is at least as large as the rate at which they discount the future. Labor supply satisfies the following conditions:

$$(1-\tau) w_t = l_t^{\frac{1}{\zeta}} \tag{3.6}$$

Firms A representative firm produce consumption goods in period 1, 2. The firm rents capital from banks at rate $R_{k,t}$, and hires workers at wage w_t. In period 1 and2, the firm maximizes the following objective function:

$$\max_{k_t, l_t} z_t k_t^{\alpha} l_t^{1-\alpha} - r_{k,t} k_t - w_t l_t \tag{3.7}$$

The first order conditions are:

$$r_{k,t} = z_t \alpha k_t^{\alpha-1} l_t^{1-\alpha} \tag{3.8}$$

$$w_t = z_t(1-\alpha) k_t^{\alpha} l_t^{-\alpha}. \tag{3.9}$$

Domestic banks At the beginning, a unit mass of risk neutral bankers endowed with N_0 start the business. In period 1, the bankers cease to operate with a probability λ and transfer the net worth to households, and go back to households as workers.

In period 0, the banks choose the level of investment in capital k_1, which depreciates at rate δ and default-free LC government bond b_1. Capital investment brings a return $R_{k,1}$ period 1, and the government bond bring 1 unit of consumption in period 1 with paying price q_0 in period 0. Then the banks net worth in period 1 is as follows:

$$N_1 = \underbrace{(r_{k,1} + (1-\delta))}_{R_{k,1}} k_1 + b_1 - a_1. \tag{3.10}$$

In period 1, the banks choose capital investment and defaultable government bonds. Capital investment gives a return of $R_{k,2}$ in period 2. With investment in the government bond with a price q_1 in period 1, they receive 1 unit of consumption goods next period if the government repays ($D_{t+1} = 0$), and receive $\psi_d < 1$ unit of consumption goods if the government defaults ($D_{t+1} = 1$). The banks net worth in period 2 is as follows:

$$N_2 = \underbrace{(r_{k,2} + (1-\delta))}_{R_{k,2}} k_2 + b_2(1-D_2) + b_2\psi_d D_2 - a_2. \tag{3.11}$$

In period 0 and 1, the banks also decide on how much to borrow from the households as a form of deposit a_{t+1} at price q_t^a, which will be used as the resource for the investment along with the banks' net worth N_t. The budget constraint for banks is then,

$$k_{t+1} + q_t b_{t+1} \leq N_t + q_t^a a_{t+1}, \text{ for } t = 0, 1. \tag{3.12}$$

Banks are also constrained on how much they can borrow using deposits. In particular, they face the following collateral constraint:

$$a_{t+1} \leq \chi N_t \text{ for } t = 0, 1. \tag{3.13}$$

The constraint indicates that the amount the banks can borrow from households cannot exceed a certain fraction $\chi \in (0,1)$ of the banks net worth.

The value of bankers can be defined using one state variable, net worth. The value in period 1, $V_1^B(N_1)$ is as follows:

$$V_1^B(N_1) = \max_{[a_2,k_2,b_2]} \beta \mathbb{E}_1 [N_2] \tag{3.14}$$

which is subject to the law of motion for net worth (3.11), the collateral constraint (3.13), and the budget constraint (3.12). Given that the budget constraint, we can substitute $a_2 = \frac{k_2+q_1b_2-N_1}{q_1^a}$ into the collateral constraint:

$$k_2 + q_1 b_2 \leq (q_1^a \chi + 1) N_1. \tag{3.15}$$

The first order conditions are as follows:

$$\begin{aligned} b_2 &: q_1(1+\mu_1) = \beta \mathbb{E}_1((1-D_2)+D_2\psi_d) \\ k_2 &: (1+\mu_1) = \beta \mathbb{E}_1(R_{k,2}) \end{aligned} \tag{3.16}$$

where μ_1 is the Lagrangian multiplier of the collateral constraint in period 1. Notice that the expected interest rate the firm needs to pay is higher than $\frac{1}{\beta}$ with the collateral constraint being bind. Combining two equations in (3.16) brings the following condition:

$$\mathbb{E}_1(R_{k,2}) = \frac{\mathbb{E}_1((1-D_2)+D_2\psi_d)}{q_1}, \tag{3.17}$$

Substituting the law of motion for net worth (3.11) and the budget constraint (3.12) into the banks' value function (3.14),

$$V_1^B(N_1) = \beta \left(\mathbb{E}_1 \left[R_{k,2} \right] (q_1^a a_2 + N_1 - q_1 b_2) + \mathbb{E}_1 \left[((1-D_2)+D_2\psi_d \right] b_2 - a_2 \right), \tag{3.18}$$

and using the banks' optimization condition (3.17), we can derive the value function as follows:

$$V_1^B(N_1) = \beta(\mathbb{E}_1 \left[R_{k,2} \right] (q_1^a a_2 + N_1) - a_2) - \beta \underbrace{(\mathbb{E}_1 \left[R_{k,2} \right] q_1 - \mathbb{E}_1 \left[((1-D_2)+D_2\psi_d \right])}_{=0} b_2. \tag{3.19}$$

Given that the collateral constraint binds such as $a_2 = \chi N_1$ and $q_1^a = \beta$

$$V_1^B(N_1) = (\beta\mathbb{E}_1\left[R_{k,2}\right] + \beta\chi(\mathbb{E}_1\left[R_{k,2}\right]\beta - 1))N_1. \tag{3.20}$$

The value in period 0, $V_0^B(N_0)$ is as follows:

$$V_0^B(N_0) = \max_{[a_1,k_1,b_1]} \beta\mathbb{E}_0\left[\lambda N_1 + (1-\lambda)V_1^B(N_1)\right] \tag{3.21}$$

which is subject to the law of motion for net worth (3.10), the budget constraint (3.12), the collateral constraint (3.13) and the value function in period 1 (3.20).

The first order conditions are as follows:

$$\begin{aligned} b_1 &: q_0(1+\mu_0) = \beta\mathbb{E}_0[W(\Lambda_1)] \\ k_1 &: (1+\mu_0) = \beta\mathbb{E}_0[W(\Lambda_1)R_{k,1}] \end{aligned} \tag{3.22}$$

where μ_0 is the Lagrangian multiplier of the collateral constraint in period 0, and $W(\Lambda_1)$ is the marginal value of an additional unit of net worth as follows,

$$W(\Lambda_1) = \lambda + (1-\lambda)(\beta\mathbb{E}_1\left[R_{k,2}\right] + \beta\chi(\mathbb{E}_1\left[R_{k,2}\right]\beta - 1)). \tag{3.23}$$

Note that $W(\Lambda_1) = 1$ when the collateral constraint in period 1 does not bind. Combining two equations in (3.22), we have a following condition:

$$\frac{\mathbb{E}_0[W(\Lambda_1)]}{q_0} = \mathbb{E}_0[W(\Lambda_1)R_{k,1}] \tag{3.24}$$

The banks optimality conditions in (3.17), (3.24) indicates that banks have to be indifferent between investing in the government bonds and and in capital for the banks to be willing to hold the government debt.

Foreign investor I assume there exists a unit mass of foreign investors labeled by $i \in [0,1]$, which can invest in the emerging government's LC bonds in period 0 and 1. Foreign investors have access to an international risk free asset. I follow Alvarez, Atkeson, and Kehoe (2009), Fanelli and Straub (2020) in assuming that foreign investors face heterogeneous participation costs. In particular, each investor i is obligated to pay a participation cost of i per dollar invested.

Denote by $\tilde{R}_{i,t}$ the return on the LC bond in dollar terms when investor i purchases the bond in period t:

$$1 + \tilde{R}_{i,t} \equiv \frac{\frac{1}{(1+r^*)}\left(\left[(1 - D_{t+1}) + D_{t+1}\psi\right]/S_{t+1}\right)}{\left[q_t(1+i)\right]/S_t} \tag{3.25}$$

where D_t is the government's decision to default, $D_t = 1$ if it defaults and $D_t = 0$ if it repays; $\psi \in (0, 1)$ is the fraction foreign investors can receive as a compensation for holding defaulted debt, which is assumed $\psi \leq \psi_D$; q_t is the bond price; r^* is risk free rate. When the foreign investor i buys the government bond in period t, the investor needs to pay $\frac{q_t(1+i)}{S_t}$ dollars in period t. The denominator of the term in right side of equation (3.25) indicates the cost paid by the investor in period t, which is denominated in dollar terms. The investor are going to be paid 1 unit of domestic consumption goods when the government repays ($D_{t+1} = 0$), and be paid ψ unit of domestic consumption goods when the government defaults ($D_{t+1} = 1$) in period t+1. $\frac{\left[(1-D_{t+1})+D_{t+1}\psi\right]}{S_{t+1}}$ denotes the dollar return paid to foreign investors in period t+1. The numerator indicates the discounted return which is converted into dollar terms. The return on the bond free from default issued in period 0 is the return with $D_{t+1} = 0$.

And denote the log return as $\tilde{r}_i \equiv \ln(1 + \tilde{R}_i)$, which is as follows:

$$\tilde{r}_{i,t} = \underbrace{\ln((1 - D_{t+1}) + D_{t+1}\psi) + \ln(S_t) - \ln(S_{t+1}) - \ln(q_t) - r^*}_{\tilde{r}_t} - i \tag{3.26}$$

The log return of investor i can be expressed as the return which does not depend on the type of investor $\tilde{r}_t$ net of i. Then the expected log return on an investor i's investment $E_t(\tilde{r}_{i,t})$ equals to $E_t(\tilde{r}_t) - i$ and the variance $\text{Var}_t(\tilde{r}_{i,t})$ equals to $\text{Var}_t(\tilde{r}_t)$.

An investor i maximises the following quadratic objective function by choosing $b^*_{i,t}$,

$$\left(E_t(\tilde{r}_t) - i\right) b^*_{i,t} - \frac{\Gamma}{2}\text{Var}_t(\tilde{r}_t) b^{*2}_{i,t} \tag{3.27}$$

where $\Gamma > 0$ is preference parameter that measures the level of risk aversion. $E_t(\cdot)$ and $\text{Var}_t(\cdot)$ indicates the expectation and variance taken with respect to the information set at date t. The investor i's bond holding then satisfies:

$$b^*_{i,t} = \frac{\left(E_t(\tilde{r}_t) - i\right)}{\Gamma\,\text{Var}_t(\tilde{r}_t)} \tag{3.28}$$

Let's denote $\hat{i}_t \in [0, 1]$ the marginal foreign investors who participate in the bond market in period t:

$$\hat{i}_t = E_t(\tilde{r}_t). \tag{3.29}$$

Thus, investing is optimal for all investors $i \in [0, \hat{i}_t]$. Foreign holdings of the government bonds b_t^* is determined by integrating equation (3.28):

$$\int_{i=0}^{i=\hat{i}_t} b_{i,t}^* di = \frac{1}{\Gamma \operatorname{Var}_t(\tilde{r}_t)} \int_{i=0}^{i=\hat{i}_t} (E_t(\tilde{r}_t) - i) \ di \tag{3.30}$$

Using the condition for the marginal foreign investor (3.29) and the following equilibrium condition:

$$\int_{i=0}^{i=\hat{i}_t} b_{i,t}^* di = b_t^*,$$

we can derive foreign holdings b_t^* as follows:

$$b_t^* \quad = \quad \frac{E_t(\tilde{r}_t)^2}{2\Gamma \operatorname{Var}_t(\tilde{r}_t)} \tag{3.31}$$

The foreign investment is determined by the expected log return and the variance of the return. I present how the foreign investment is determined in period 1, particularly when the government issues the defaultable bond. The expected log return equals to the sum of the expected return when the government defaults and the expected return when the government repays by the law of total expectation:

$$E_t(\tilde{r}_t) = E_t(\tilde{r}_t \mid D_{t+1} = 1) \overbrace{Pr(D_{t+1} = 1)}^{\Delta_{t+1}} + E_t(\tilde{r}_t \mid D_{t+1} = 0) \overbrace{Pr(D_{t+1} = 0)}^{1-\Delta_{t+1}}. \tag{3.32}$$

Using the process of the nominal exchange rate in equation (3.3) and the definition of $\tilde{r}_t$ in equation (3.26), we can have following equations for the conditional expectation:

$$\begin{aligned} E_t(\tilde{r}_t \mid D_{t+1} = 0) &= (1 - \rho_s) \ln S_t - \phi_s E_t(x_{t+1}) - \ln(q_t) - r^* \\ E_t(\tilde{r}_t \mid D_{t+1} = 1) &= \underbrace{\ln(\psi)}_{<0} + (1 - \rho_s) \ln S_t - \phi_s E_t(x_{t+1}) - \ln(q_t) - r^* \end{aligned}$$

With denoting $Pr(D_{t+1} = 1)$ by Δ_{t+1} and substituting the banks' optimality condition (3.17), we can decompose the expected log return into default risk, currency risk, and compensation for these risks:

$$E_t(\tilde{r}_t) = \underbrace{\ln(\psi)\Delta_{t+1} - \ln(\Delta_{t+1}\psi_D + (1-\Delta_{t+1}))}_{\text{Default risk}} + \underbrace{(1-\rho_s)\ln S_t - \phi_s E_t(x_{t+1})}_{\text{Currency risk}} + \underbrace{\mathbb{E}_t(R_{k,t+1}-1) - r^*}_{\text{Compensation for risk}} \quad (3.33)$$

The first two terms in (3.33) indicate default risk, which is decreasing in the default probability, Δ_{t+1}. The next two term measure currency risk, associated with the expected currency depreciation. Note that currency risk increases if it is more probable that a state of high global financial risk is realized. The last two terms are the compensation for these risks.

The variance of the log return is as follows based on the total law of variance:

$$\begin{aligned}\text{Var}_t(\tilde{r}_t) = \;& \sigma_s^2(1+\eta E_t(x_{t+1})) + \phi_s^2 E_t(x_{t+1})(1-E_t(x_{t+1})) \\ &+ \underbrace{[E_t(\tilde{r}_i \mid D_{t+1}=1) - E_t(\tilde{r}_i \mid D_{t+1}=0)]^2}_{(\ln(\psi))^2} \Delta_{t+1}(1-\Delta_{t+1})\end{aligned} \quad (3.34)$$

The variance of the return is decomposed into two parts, one related to the nominal exchange rate and the other related to the default risk.

All else equal, foreign LC bond holdings increase when (1) a default probability Δ_{t+1} is low, (2) the compensation rate for holding defaulted debt ψ is high, (3) it is more likely that a global financial state is realized as normal, low $E_t(x_{t+1})$, (4) the expected return of domestic banks' capital investment $E_t(R_{k,t+1})$ is high. With a higher default probability, the expected log return decreases with the condition that $\psi \leq \psi_D < 1$, and the variance increases unless the default risk is too high. Higher ψ is associated with a higher expected return and a smaller variance, which leads the investors to increase investment. A higher probability that the global financial risk is high decreases the expected return because the local currency is likely to be more depreciated. The variance of log return increases with a higher risk in local currency market. Domestic banks' higher expected return on capital investment enlarges a deviation from UIRP and increases foreign investor's expected return on the bond.

3.3.4 Competitive equilibrium

Definition 1. A competitive equilibrium given government policies is allocations $\{c_t\}_{t=1,2,3}$ $\{a_t, l_t, k_t, b_t, b_t^*\}_{t=1,2}$ and prices $\{r_t, w_t\}_{t=1,2}$ $\{q_t^a\}_{t=0,1}$ such that given sovereign bond prices $\{q_t\}_{t=0,1}$ government policies $\{D_2, B_1, B_2, G_1, G_2\}$ exogenous state $\{\Lambda_t\}_{t=0,1,2}$ and initial values $\overline{n}_0^h$, N_0, the following holds:

1. $\{c_t\}_{t=1,2,3}$ $\{a_t, l_t\}_{t=1,2}$ solve the household's problem in (3.4) ~ (3.5).

2. $\{l_t, k_t\}_{t=1,2}$ solve the firm's problem in (3.7).

3. $\{b_t^*\}_{t=1,2}$ satisfies equation (3.31).

4. $\{a_t, k_t, b_t\}_{t=1,2}$ solve the financial intermediaries' problem in (3.10) ~ (3.14), (3.21) .

5. Capital, labor and deposit markets clear, and the government bond market clears: $b_t + b_t^* = B_t$

6. Resource constraint holds: $c_t + G_t + k_{t+1} - (1-\delta)k_t = q_t b_{t+1}^* - b_t^*(1 - D_t) + z_t F(n_t, k_t)$

3.3.5 Government's problem

The following section describes the government's problem in each period. $\{b, b^*, k\}$ is a set of endogenous state variables. The government chooses the bond issuance B and the share of bond held by bank and foreign investors is endogenously determined. I redefine the endogenous state variable as $\{B, f, k\}$, where f is the foreign holdings of LC debt, $\frac{b^*}{B}$.

Government's problem in period 2

Let $V_2(B_2, f_2, k_2, \Lambda_2)$ be the value with the option to default in period 2 such that

$$V_2(B_2, f_2, k_2, \Lambda_2) = \max_{D=\{0,1\}} \{(1-D)V_2^R(B_2, f_2, k_2, \Lambda_2) + D[V_2^D(B_2, f_2, k_2, \Lambda_2) - v]\} \tag{3.35}$$

where V_2^R is the value from repaying debt, and V_2^D is the value from defaulting.

$$\begin{aligned} V_2^R(B_2, f_2, k_2, \Lambda_2) &= U\left(\tau w_2 n_2 - B_2 + W_2\right) \\ V_2^D(B_2, f_2, k_2, \Lambda_2) &= U\left(\tau w_2 n_2 - B_2\psi_d + B_2 f_2(\psi_d - \psi) + W_2\right) \end{aligned}$$

If the government chooses to default, the government pays only a fraction of the debt. Specifically, the government pays ψ_d fraction of the debt to domestic banks and pays ψ fraction to foreign investors with $\psi \leq \psi_d < 1$. With the government's default, productivity is reduced, and the government suffers the utility cost v.

It is convenient to write the government's default default decision as a cutoff rule based on the default cost v. Given that default costs v are i.i.d., the default decision $D(B_2, f_2, k_2, \lambda_2)$ can be characterized by a cutoff cost $v^*(B_2, f_2, k_2, \lambda_2)$ where the value of repaying equals to the value of defaulting on debt such that,

$$v^*(B_2, f_2, k_2, \Lambda_2) = V_2^D(B_2, f_2, k_2, \Lambda_2) - V_2^R(B_2, f_2, k_2, \Lambda_2) \tag{3.36}$$

Then $D(B_2, f_2, k_2, \lambda_2) = 1$, whenever $v \leq v^*(B_2, f_2, k_2, \Lambda_2)$ and $D(B_2, f_2, k_2, \lambda_2) = 0$ otherwise. Let Φ be the cumulative distribution of v, then default probability given $(B_2, f_2, k_2, \Lambda_2)$ is equal to $\Phi(v^*(B_2, f_2, k_2, \Lambda_2))$.

Government's problem in period 1 and 0

$V_1(B_1, f_1, k_1, \Lambda_1)$ is the value in period 1 such that

$$V_1(B_1, f_1, k_1, \Lambda_1) = \max_{B_2} U(G_1) + \beta_g \mathbb{E}_1 \left[V_2(B_2, f_2, k_2, \Lambda_2) \right] \tag{3.37}$$

$$\begin{aligned} s.t. \quad G_1 &= q_1 B_2 - B_1 + \tau w_1 n_1 \\ G_2 &= \tau w_2 n_2 - (1-D)B_2 - D(B_2 f_2 \psi + B_2(1-f_2)\psi_d) + W_2 \end{aligned} \tag{3.38}$$

where, W_2 is the government's endowment in period 2.

V_0 is the value in period 0 such that

$$\begin{aligned} V_0 \quad &= U(\overline{g}) + \beta_g \mathbb{E}_0 \left[V_1(B_1, f_1, k_1, \Lambda_1) \right] \\ \text{s.t.} \quad \overline{g} &= q_0 B_1 \end{aligned}$$

3.4 Quantitative Analysis

This section performs a quantitative evaluation of our model to study how global states affect EMEs LC bond market, and how the effects vary depending on the level of financial development. I first discuss the calibration strategy in subsection 3.4.1, and illustrate model mechanisms in the following subsection. In subsection 3.4.3, I perform a quantitative exercise with the model to see how the level of financial development explains the vulnerability of global shocks as discussed in the previous section.

3.4.1 Functional form and parameterization

I start with some functional forms. The preferences of the government are given by the standard utility function $U(G) = \frac{G^{1-\sigma}-1}{1-\sigma}$, where σ is the risk aversion parameter. I assume that the government's default incurs two types of cost, productivity decline and a disutility cost, v. Productivity shocks z_t are assumed to follow an AR(1) process as in equation (3.2). Following Chatterjee and Eyigungor (2012), I assume that productivity suffers a convex penalty $\max\left\{0, \lambda_0 z + \lambda_1 z^2\right\}$ with $\lambda_0 \leq 0 \leq \lambda_1$ with government's default as follows,

$$z(D) = \begin{cases} z & \text{if } D = 0 \\ z - \max\left\{0, \lambda_0 z + \lambda_1 z^2\right\} & \text{otherwise.} \end{cases} \tag{3.39}$$

The disutility cost v is assumed to follow a logistic distribution with location λ_d and scale σ_D as in Arellano, Bai, and Mihalache (2020).

I first choose a subset of parameters values that can be directly pinned down from the data or that have standard values from the literature. I estimated the process of TFP and nominal exchange rate for each sample country, and use the average value for the parameters. The transition probabilities are calculated using the VIX index with defining high global financial risk periods ($x_t = 1$) as the periods with the VIX index above the average plus 1.5 times of its st.deviation. The set of parameters, assigned directly, includes risk aversion parameter set to a standard value, $\sigma = 2$, capital share to $\alpha = 0.33$. I choose capital depreciation rate (δ) to be 0.1, the Frisch elasticity (ζ) to be 0.33, risk free rate (r^*) to be 0.5% following Arellano, Bai, and Mihalache (2020). Tax rate is set to be 28% as in Wu (2020). The discount rate of households are set to be 0.96 and the discount rate of the government is set to be 0.92. The productivity decline in high global financial risk periods, ϕ_z, is set to be 0.03, and the increase in nominal exchange rate and increase in st.dev of shocks to the nominal exchange rate are set to be 0.1 respectively. The parameters of the default cost function λ_0 and λ_1 are set to be -0.17 and 0.21. Compensation rate of holding defaulted debt for domestic banks is set to be 0.1 while that for foreign investors is set to be 0.05. The scale parameter for disutility cost of default is set to be 0.01.

The second set of parameters $\{\chi, \Gamma, \lambda_d, \overline{g}\}$ is chosen to match four key moments of sample EMEs data. The moments are (1) the average LC debt to GDP ratio, 29.0 % (2) the average foreign holdings, 20.8% (3) the average default risk, 1.1%, (4) the average increase in default risk with change in states of global financial risk 1.3pp. The model lasts for three periods, and the economy starts with the exogenous variables held at the mean level. In period 1, economic agents decides optimizing the objective function given the exogenous states realized in the beginning of the period and state variables decides in period 0, bank's net worth, capital stock, government debt level, and foreign holdings. I compute the moments in period 1, and use these moments to choose the parameters. Banks' leverage constraint parameter χ is set to be 0.352 and foreign investors' risk preference parameter Γ is to be 5.85. Distuility cost of default is set to be 1.247 and exogenous government spending is set 0.205. Table 3.2 summarizes all values for the pa-

Table 3.2: Parameter Values

Parameters	**Description**	**Value**
Parameters from the data		
ρ_z	Autocorrelation of TFP	0.93
σ_z	Std. dev of TFP shocks	0.025
ρ_s	Autocorrelation of nominal exchange rate	0.95
σ_s	Std. dev of nominal exchange rate shocks	0.06
π_{01}, π_{10}	Transition probability	0.045, 0.78
Parameters assigned		
σ	Risk aversion	2.0
α	Capital share	0.33
δ	Capital depreciation rate	0.1
ζ	Frisch elasticity	0.33
β	Private discount rate	0.96
β_g	Government discount rate	0.92
r^*	Risk free rate	0.005
τ	Tax rate on labor income	0.28
W_2	Government endowment in t=2	0.42
ϕ_z	Productivity decline	0.03
ϕ_s	Nominal exchange rate increase	0.1
η	increase in std.dev of nominal exchange rate shocks	0.1
λ_0	Productivity in default	-0.17
λ_1	Productivity in default	0.21
ψ_D	Compensation rate for domestic banks	0.1
ψ	Compensation rate for foreign investors	0.05
σ_D	Enforcement shock	0.01
Parameters from moment matching		
χ	Leverage constraint	0.352
Γ	Preference parameter of foreign investors	5.85
λ_d	Disutility cost of default	1.247
$\overline{g}$	exogenous government spending	0.205

rameters. Table 3.3 reports the target moments in the model and the data. Overall the model reproduces the targeted main features of the data.

3.4.2 Model mechanisms

I examine the model mechanisms based on the government's decision in period 1.

Table 3.3: Model Fit

	Data	**Model**
mean (LC debt/y, %)	29.0	29.1
mean (foreign holding, %)	20.8	20.8
mean (default risk, %)	1.1	3.1
mean (increase in default risk, pp)	1.3	1.3

Incentives to issue debt The government's decision on the debt issuance B_2 in period 1 mainly depends on the effect of bond issuance on banks and resultant tax revenue. The first order condition with respect to the debt issuance is as follows:

$$\begin{aligned} &\overbrace{\left[q_1 + \frac{\partial q_1}{\partial B_2} B_2\right]}^{\text{revenue effect}} U'(G_1) + \beta_g \mathbb{E}_1 [U'(G_2) \overbrace{\left(\frac{\partial TR_2}{\partial B_2}\right)}^{\text{crowding-out}}] \\ &= \beta_g \underbrace{\mathbb{E}_1 \left[U'(G_2) \mid D = 0\right]}_{\text{mg. cost in repayment states}} + \beta_g \underbrace{\mathbb{E}_1 \left[U'(G_2) \left(\psi_d - (\psi_d - \psi)\left(f_2 + B_2 \frac{\partial f_2}{\partial B_2}\right)\right) \mid D = 1\right]}_{\text{mg. cost in default states}} \end{aligned} \tag{3.40}$$

where TR_2 is tax revenue in period 2 such that $TR_2 = \tau w_2 n_2$. The government condition for issuing additional bond equates the revenue from the additional unit of debt net of its crowding-out effect to the cost of repaying in in the next period as in equation (3.40). Issuing an additional unit of bond increases total revenues by q_1 net of effects from bond price declines with increasing debt, which is denoted as revenue effect in the equation. The government also takes into account that issuing additional debt constrains banks' investment for capital unless banks collateral constraint does not bind, as less capital investment is associated with lower tax revenue. Given that the collateral constraint binds, the amount of banks investment in capital and the government bond is bounded by the level of net worth as in equation (3.15). The more the banks hold more government bond the less the next period's capital stock invested. With the collateral constraint binding, the government's issuance of additional debt has an impact on the banks' investment in capital as follows:

$$-\frac{\partial k_2}{\partial B_2} = \frac{\partial (q_1 B_2 (1 - f_2))}{\partial B_2} = q_1 (1 - f_2) \left[1 + \frac{B_2}{q_1} \frac{\partial q_1}{\partial B_2} + \frac{B_2}{(1 - f_2)} \frac{\partial (1 - f_2)}{\partial B_2}\right] \tag{3.41}$$

I focus the channel through which the government's debt issuance crowds out capital investment via foreign holdings, which is captured by the elasticity of domestic banks' bond holding with respect to the government's debt issuance, $(\frac{B_2}{(1-f_2)} \frac{\partial(1-f_2)}{\partial B_2})$. The government's debt issuance has an impact on capital investment through foreign investors in three ways. First it increases the banks' government bond holding and decreases capital investment overall. Second it increase the banks' expected return of capital investment $E_1(R_{k,2})$ with constraining banks' capital investment, which induces more foreign capital. This allows the domestic banks to invest more capital. Third higher debt decreases the probability of repayment, which leads foreign investors reduce investing in the government debt and banks to hold more government bond and to reduce capital investment. Taken together, the crowding-out effect depends on the elasticity of banks bond holding with respect to the government's debt issuance. A higher elasticity leads the banks to hold more government bonds with the government's additional issuance of debt, and this crowds out more capital investment and reduces tax revenue to a greater extent. If the government repays the debt, it costs one unit of the government expenditure. If it defaults, the cost varies by how much of debt is held by banks and foreign investors because compensation rate for holding defaulted debt is different by whether the debt is held by banks or foreign investors.

Incentives to default The default decision is characterized by a cutoff cost $v^* = V_2^D - V_2^R$ in equation (3.36). The value of repayment V_2^R and default V_2^D is as follows:

$$\begin{aligned} V_2^R(B_2, f_2, k_2, \Lambda_2) &= U\left(TR_2^R - B_2\right) \\ V_2^D(B_2, f_2, k_2, \Lambda_2) &= U\left(TR_2^D - B_2\psi_d + B_2 f_2(\psi_d - \psi)\right) \end{aligned} \tag{3.42}$$

where TR_2^R is tax revenue when the government repays and $TR_2^D$is tax revenue when the government defaults[10]. The default probability increases in v^*, and thus, there are four variables that affect the default probability: the level of debt B_2, capital k_2, foreign holdings, f_2, and productivity z_2. Note that the nominal exchange rate does not affect the government value. It changes foreign investors realized return for holding LC bonds, but has no impact on the government value and default decision. A higher level of debts and a lower level of capital increases the default probability, as it decrease the government's tolerance to debt with raising debt burden and lowering tax revenue. Given that $\psi_d > \psi$, higher foreign holdings makes the government be more likely to default. With a larger share of debt being held by foreign investors, the government's cost of compensating defaulted debt holders become smaller, which increases the government's incentive to default. The government tends to default when productivity is low because the default cost related to productivity is marginal when the productivity is low as assumed in equation (3.39).

[10]With the government's default, the productivity declines as in equation (3.39), and therefore the outputs and tax revenues in default and repayment states are not same even with the same level of capital

Default risk conditional on global states I investigate how to explain the documented empirical findings using a model. Specifically, in Emerging Market Economies with lower financial development relative to government debt levels, both local currency default risk and the sensitivity of such risk to global financial conditions are observed to be higher. Based on what was observed in the data, the level of credit vulnerability is considered the main factor that explains the phenomena.

To explain the empirical findings using a model, we need to see how the realization of a ”high-risk state” in period 1 changes the default risk and its change in default risk. With a realization of a high state in period 1, it is more likely that the next period state is also high given that $\pi_{01} < \pi_{11}$. Then foreign investors anticipate a decrease in the expected return on LC bonds (due to the expected depreciation) and an increase in the return volatility on LC bonds (due to the higher exchange rate volatility). This induces foreign investors to reduce local currency bond holdings.

Consequently, domestic financial intermediaries increase their government bond holdings compared to low-risk states. This has three effects on a default probability. First foreign holding of the debt is lower in high states and this decreases the government's incentive to default, leading to lower default risks. Second, the government's debt issuance constrains banks' capital investment more in high states, which is associated with a higher default probability. Lastly, this decreases the government's incentive to issue debt because the crowding-out effect is more significant in high states, which is associated with a lower default probability. With a high state realization, foreign holdings decrease, capital investment decreases, and the government's debt issuance falls when other state variables are equal. To summarize, lower foreign holdings and lower debt issuance reduce default probability, while lower capital investment increases default probability. The consequent effects on default risk from shifts in states of global financial risk depend on what effects are dominant.

The level of financial development, captured by the degree of collateral constraint relative to the debt level, plays a significant role in determining the intensity of interaction between capital outflow from the local currency government bond market and private credit disruption. Ultimately, this intensity determines the extent to which shocks in the global financial market lead to higher default risks on local currency government bonds.

Policy rules Figure 3.8 presents policy rules as a function of government debt B_2 in a high global financial risk state and a low state, relative to the mean level of productivity and the nominal exchange rate. The figures in the first row display how default probability and bond price varies with bond issuance. The left figure in the second row plots how foreign holdings changes with bond issuance and the right figure plots how capital investment change with bond issuance. Foreign holdings decrease with the government bond issuance. Capital investment falls with bond issuance when the debt level is not too high, as the government bond issuance constrain banks from investing in capital. There are no capital inflows to the government bond market, with the debt level being above a certain level due to high default probability. With such a high level of debt, the government's revenue decreases with additional debt issuance because the decrease of revenue from price drop with issuing more debt outweighs the increase of revenue from issuing more debts. To see the policy rules by the states of global financial risks, foreign holdings decrease, and capital investment drops in high states as discussed in the previous part. The default probability is higher and bond price is lower in high states than in low states given the same level of debt issuance. This is mainly because the effect of fall in capital investment outweighs the effect of decrease in foreign holdings.

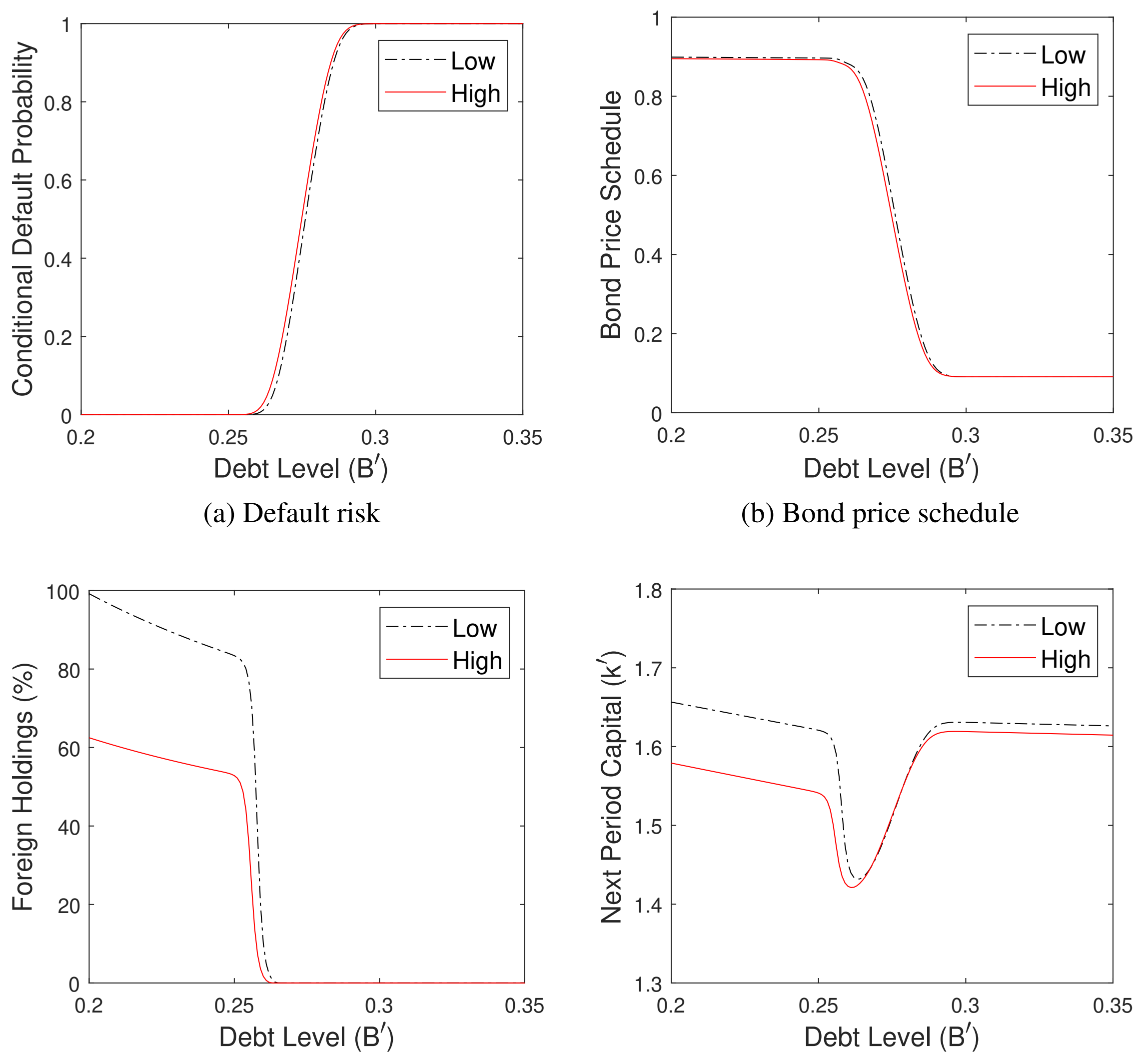
3.4.3 Financial development and external vulnerability

In this section I perform a quantitative exercise with the model and see how the model generates the empirical features presented in the previous section. The empirical features includes higher banks' B/S composition volatility, higher vulnerability of credit channel and LC bond market with less financial development.

In the model the level of financial development of a country is controlled by the parameter χ in banks collateral constraint (3.13). The higher χ is associated with the lower friction in financial sector and allows banks to extend investment in capital and the government bonds more, which I associate with a higher financial development. I perform the following quantitative exercise to see the implication of the model with regard to the relationship between the level of financial development and the vulnerability of LC bond market to global financial risks. I vary the value for parameter χ to differ the level of financial development. I compare the scaled financial development, credit channel vulnerability, default risk and external vulnerability of the economy with different level of financial development. I only change the value of the parameter χ for this exercise, and I keep the other parameter fixed at their level as in Table 3.2.

Table 3.4 presents selected moments of data, the benchmark model, models with lower and higher value for χ. The model captures the difference in banks' B/S composition volatility with the level of financial development. Banks in a less financially developed economy tends to adjust their B/S in a greater scaled. With regressing the growth rate of capital net of total claim growth on the change in foreign holdings, I calculated the coefficient γ that measures the credit channel vulnerability as in 3.2.2. The model generates the empirical pattern that a country with

Figure 3.8: Policy Rules



(a) Default risk

(b) Bond price schedule

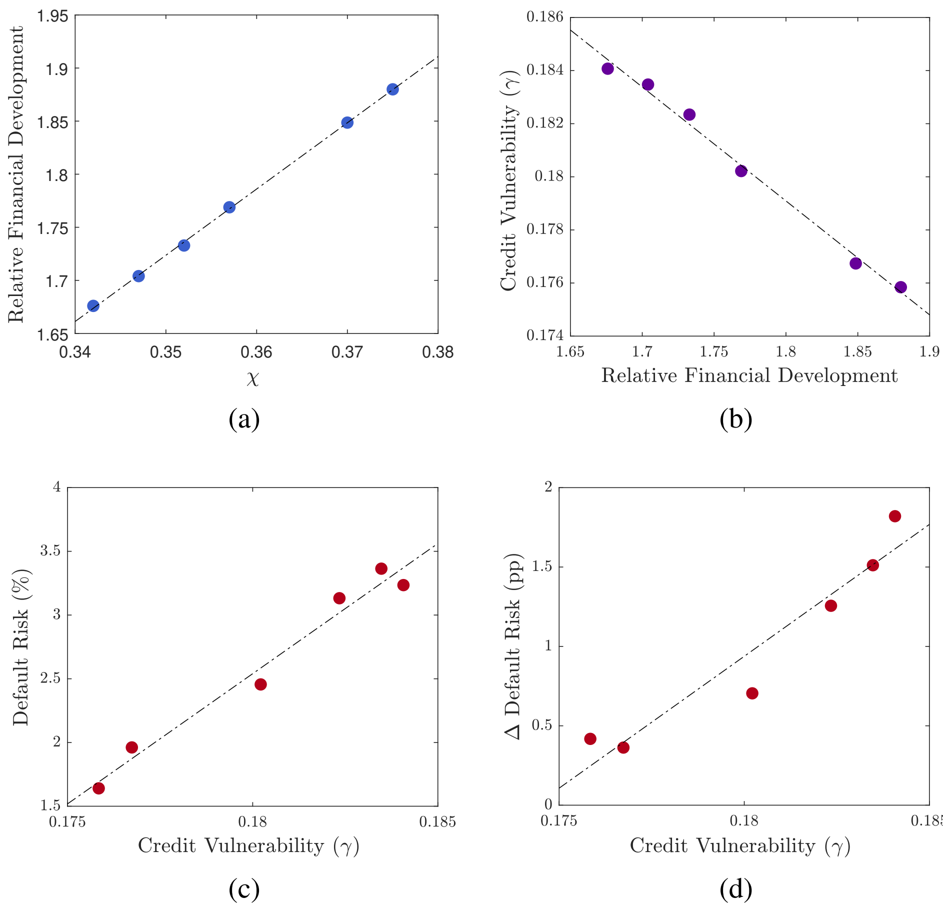
(c) Foreign holdings

(d) Capital

Note: The figures plot the conditional default probability, bond price schedule, foreign holdings, and next period capital as the government's bond issuance B_2 varies given the TFP and the nominal exchange rate held at their mean level with the state of global financial risk being "high" and "low".

low scaled financial development shows a higher credit channel vulnerability. In other words, the economy with lower χ reduces capital investment to a greater extent when foreign capital outflows from the LC bond market and banks need to hold more government debt. The mean default risk in the economy with lower χ is higher and the LC bond market's vulnerability to global financial risks also higher with larger increase in default probability with shifts in global financial conditions.

Figure 3.9: Selected Moments with Different Parameter Value for χ



Note: The figures plot ralative financial development: financial development (deposit to GDP ratio) scaled by the debt to GDP ratio, credit vulnerability (γ): coefficient of change in private credit on the change in foreign holdings, average default risk, and average increase in default risk to global back shock

In Figure 3.9, I show the level of scaled of financial development, credit vulnerability, and external vulnerability with varying the value for χ. The feature generated by the model is consistent with what is found in data: a less financially developed economy shows higher credit vulnerability, and also higher vulnerability to global financial conditions.

Table 3.4: Selected Moments: Data, Benchmark and Alternative Economies

	Data	**Benchmark**	**Low**	**High**
χ	-	0.352	0.342	0.37
mean (financial development[1], %)	54.3	50.4	48.9	53.4
mean (scaled financial development[2])	1.92	1.73	1.68	1.85
σ (govshare)	1.28	1.83	1.91	1.67
σ (govshare) / σ (foreign share)	0.623	0.161	0.163	0.157
γ(Δprivate credit, Δforeignholding)	0.214	0.182	0.184	0.177
mean (default risk, %)	1.1	3.1	3.2	2.0
mean (increase in default risk, pp)	1.3	1.3	1.8	0.4

Notes: (1) Deposit to GDP (%)
(2) Financial development scaled by debt to GDP ratio

3.5 Conclusion

This paper investigates the key factors determining the cross-country difference in impacts of global shocks on the LC sovereign debt market. I mainly study the phenomenon where EMEs are vulnerable to global financial conditions even with their significant share of debt being denominated in LC, which was termed as ”original sin redux” by Carstens and Shin (2019). I fill gaps in the literature by linking financial development, credit channel vulnerability, and vulnerability to global financial conditions.

I illustrate LC yields and credit spread movement during recent financial market turbulence amid the Covid-19 pandemic to show that borrowing in LC has not insulated EMEs from changes in global financial conditions. I document that higher reliance on foreign capital leads to more vulnerability during the periods, as presented in Carstens and Shin (2019) and Hofmann, Shim, Shin, et al. (2020). I link the level of financial development to the vulnerability to global shocks and find a country with low financial development shows a higher vulnerability to the shocks. I empirically show that the private credit tends to be more adversely affected by foreign capital outflows from the LC bond market (higher credit channel vulnerability) in a less financially developed country. Such an economy shows a higher credit risk and also a higher vulnerability to global financial shocks.

I develop a model consistent with all these empirical features. I extend a standard sovereign default model incorporated with the financial intermediation sector by allowing foreign investor's decisions to be endogenously determined and losses to be different by whether domestic or foreign investors hold the bond. I capture interactions between foreign investor's investment decisions depending on the state of global financial risk and its impacts on EMEs with these extensions. I perform a quantitative exercise with varying a parameter governing friction in the financial sector to see how the credit vulnerability and vulnerability to global financial risks vary by financial development level. The model generates the main features in data that a less financially developed economy shows higher credit vulnerability and also higher vulnerability to global financial conditions.

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Appendix A

Appendix to Chapter 2

A.1 Credit spread and investment

Continuous pre-policy credit spread The following equation shares the same specifications as equation 2.3, with the only difference being the use of the pre-policy credit spread, represented by the mean credit spread before the policy change, instead of a dummy indicator. The outcomes are presented in Table A.1, consistently suggesting that investments increased more significantly among firms with higher pre-policy credit spreads within the eligible group. Conversely, no significant effects were observed among non-eligible firms.

$$\text{Investment}_{ist} = \beta_1 D_{is}^{sme}\text{Before CR}_{is}\text{After}_t + \beta_2\text{Before CR}_{is}\text{After}_t + \gamma^x X_{ist-1} + \gamma_{st} + \gamma_i + \epsilon_{ist} \quad \text{(A.1)}$$

Table A.1: Heterogeneous response of investment by pre-policy credit spread

	Investment(pp)
Before CR × SME× After (β_1)	1.33*** (0.28)
Before CR × After (β_2)	0.05 (0.26)

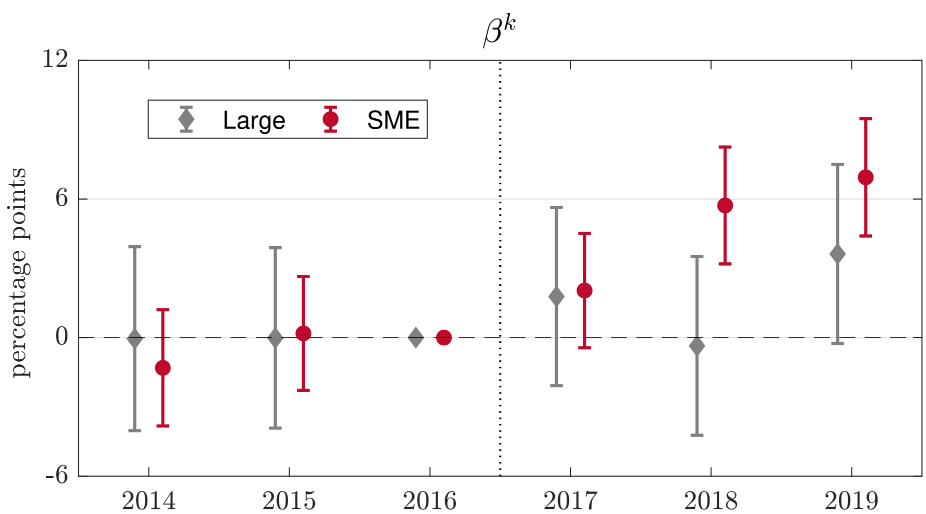
Event study I conducted event study analysis based on the same specification as equation 2.3, with the only difference being the use of year dummy variable instead of After dummy. The specific specification is as follows:

$$\text{Investment}_{ist} = \sum_{k \neq 2016} \beta^k \text{Year}_k D_i^{\text{High}} + \gamma^x X_{\text{ist}-1} + \gamma_{st} + \gamma_i + \epsilon_{\text{ist}} \tag{A.2}$$

I conducted separate estimations of equation A.2 for both the eligible group (SMEs) and the non-eligible group (large firms). The coefficient β_k, depicted in the plotted figures, represents the difference in investment between groups with low and high pre-policy credit spreads, relative to the year 2016. In the figures, grey diamonds represent large firms and red circles represent SMEs.

For the non-eligible group, there was no discernible shift in the investment difference between low and high pre-policy credit spread groups. Conversely, within the eligible group, there existed no significant difference in investment between these groups before the policy alteration. However, following the policy changes, firms with high pre-policy credit spreads exhibited a significant increase in investment.

Figure A.1: Investment response by pre-policy credit spread

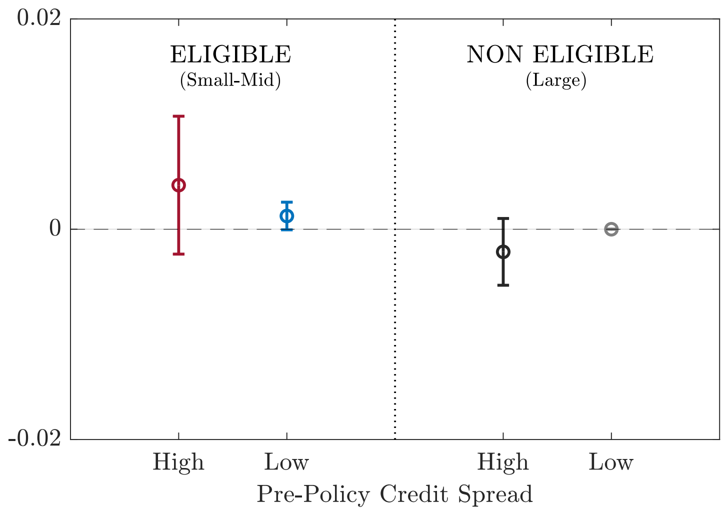


Notes: These plots show a difference in the investment between high pre-policy credit spread and low pre-policy credit spread firms for specific years relative to year 2016, separately for SMEs and large firms. 90% confidence intervals are plotted.

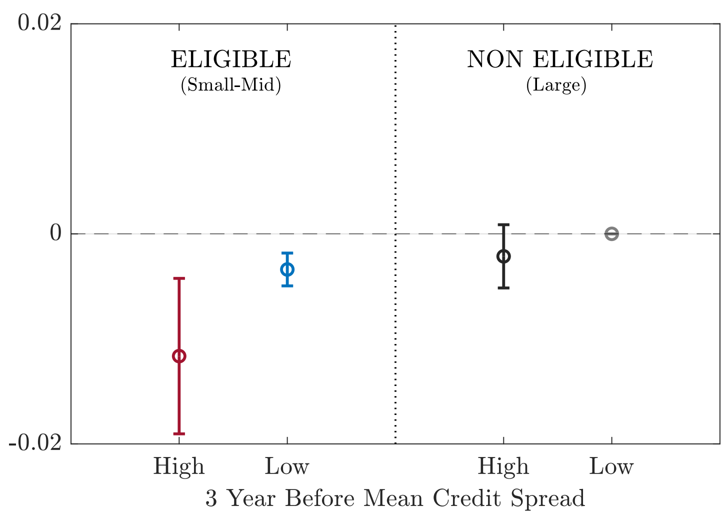
A.2 Credit spread and exit rates

Firms with initially high pre-policy credit spreads were more likely to exit in the 'Before' period, thereby dampening the magnitude of change in their exit rates. To delve into this phenomenon further, I replicated the analysis using the three-year before average credit spread. This approach enables a closer examination of how the policy influenced the exit threshold concerning credit spreads, rather than treatment impact on a specific group. The outcomes, as illustrated in the following figure, are as follows: no discernible effect on the non-eligible group (large firms), while firms that, on average, maintained higher credit spreads experienced more pronounced decrease in terms of exit rates.

(a) Change in exit probability by pre-policy credit spread



(b) Change in exit probability by 3 years before mean credit spread



A.3 Effect of government loans on exit rates and zombie share

$$Y_{it} = \alpha + \sum_{k \neq 2016} \beta^k \text{Year}_k D_i^{sme} + \gamma_t + \epsilon_{it} \tag{A.3}$$

Figure A.3: Exit Rates

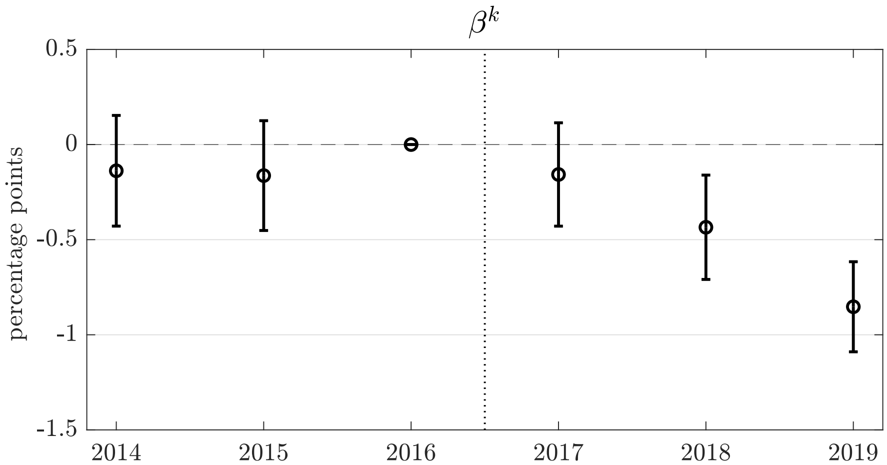
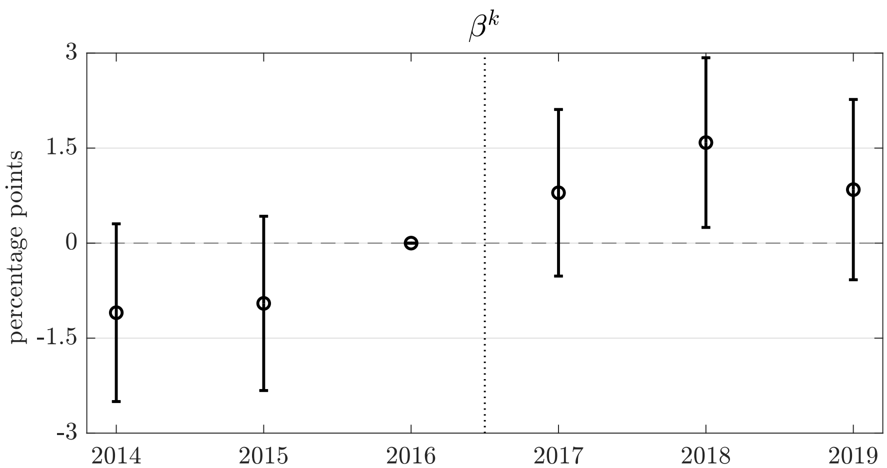


Figure A.4: Zombie Share



A.4 Detailed coefficients estimates tables

Table A.2: Estimates of equation 2.3

		Investment
β_1	Prehigh × eligible × After	8.48*** (1.03)
β_2	Prelow × eligible× After	3.34*** (0.64)
β_3	Prehigh × ineligible × After	2.06 (1.91)
γ_1^x	Lagged log tangibles asset	-27.22*** (0.26)
γ_2^x	Lagged profit to asset ratio	0.41*** (0.02)
	Observations	57,382
	R^2	0.21

Notes: Eligible indicates whether a firm is an SME, while Prehigh and Prelow indicate that a firm's mean spread in the Before period is in the upper 10th percentile or the lower 90th percentile, respectively. Estimates from Equation 2.3 are presented in percentage points. Standard errors are shown in parentheses. *** $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table A.3: Estimates of equation 2.4

		Exit
γ_1	Zombie × eligible	0.070*** (0.007)
γ_2	Normal × eligible	0.010*** (0.001)
γ_3	Zombie × ineligible	0.003 (0.003)
β_1	Zombie × eligible × After	-0.032*** (0.008)
β_2	Normal × eligible× After	-0.003*** (0.001)
β_3	Zombie × ineligible × After	-0.003 (0.003)
	Observations	70,463
	R^2	0.01

Notes: Eligible indicates whether a firm is an SME, while Zombie and Normal indicate that a firm's status in $t-1$ was either a zombie firm or a normal (non-zombie) firm, respectively. Estimates from Equation 2.4 are presented as changes in probabilities. Standard errors are shown in parentheses. *** $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

A.5 Exposure analysis: Bartik (1991)

To validate the model prediction, I performed the sector (industry) level regression using regional data based on Bartik exposure analysis. Given government loans in period t, sector s is assumed to have a higher exposure to the policy if a sector s had a higher share of small-mid enterprises in region r whose output share was relatively higher before the policy shift. Specifically, the exposure to the policy is calculated as follow,

$$\text{Exposure to Gov' Loan}_{st} = \sum_{r=1}^{13} \underbrace{\frac{\text{number of SMEs}_{sr}}{\text{number of firms}_{sr}}}_{\text{SMEs share in } r \text{ region } s \text{ industry}} \times \overbrace{\underbrace{\frac{\text{total output}_r}{\text{total output}}}_{\text{output share in region } r} \times Gov_t}^{Shock} \tag{A.4}$$

Using the exposure, I conduct a following panel regression:

$$y_{st} = \beta \text{Exposure to Gov' Loan}_{st} + \gamma_t + \gamma_s + \epsilon_{st} \tag{A.5}$$

The result shows that the increase in government loans decrease firms' exit and investment but increase the share of zombie firms, which are consistent with the model prediction.

Table A.4: Aggregate Effects with a Reduced Form

	Exit rates	Investment	Zombie shares	Δ log TFP
β	-0.009**	-0.065**	0.027*	-0.002*
	(0.003)	(0.021)	(0.013)	(0.001)

A.6 Stationary Recursive Equilibrium of the Economy with Government Loans

The stationary recursive equilibrium for this economy consists of (i) policy and value functions of incumbent intermediate goods firms $\{B'(x,k,z), k'(x,k,z), V(x,k,z)\}$; (ii) policy and value functions of entering firms $\{b'(v), k'(v), V(v)\}$; (iii) the bond price schedule $q(B',k',z)$, $q^e(b',k',v)$; (iv) price of final good p, demand for final good $y_f(p)$, average productivity of intermediate good firms $\overline{z}$, and mass of entrants ; (v) a stationary measure μ such that:

(1) Given p, the function of $B'(x,k,z)$, $k'(x,k,z)$ solve the problem of incumbent firms, and $V(x,k,z)$ is the associated value function,

$$V(x,k,z) = \max_{k',B',b'_g} d + \beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' > \tilde{\phi}^G} V(x'(k',B',z',\phi'), k', z') \, d\Phi(\phi') \right] + \beta \sum_{z'} \pi(z' \mid z) \left[\left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G \right) V\left(x'\left(k',B',z',\tilde{\phi}^G\right), k', z' \right) \right] \tag{A.6}$$

subject to (A.7) - (A.11)

$$d = x - c(k,k') + q\left(k',B',b'_g,z\right)(B' - b'_g) + q_g b'_g \geq 0 \tag{A.7}$$

$$x(k',B',z',\phi') = (1-\tau)pz'\exp(\phi')k'^{\alpha} - f_k k' - f - B' + \tau\left(\delta k + r_f B'\right) \tag{A.8}$$

$$\tilde{\phi}^G\left(k',B',b'_g,z'\right) = \log\left(\frac{-\overline{x}^G(k',z') + f + f_k k' + B' - \tau\left(\delta k + r_f B'\right)}{(1-\tau)pz'k'^{\alpha}} \right) \tag{A.9}$$

$$\hat{\phi}^G\left(k',B',b'_g,z'\right) = \log\left(\frac{-\overline{x}^G(k',z') + f + f_k k' + B' - \left(1 - q_g\right)b'_g - \tau\left(\delta k + r_f B'\right)}{(1-\tau)pz'k'^{\alpha}} \right) \tag{A.10}$$

$$\overline{x}^G(k,z) = \max_{k',B',b'_g} q(k',B',b'_g,z)(B' - b'_g) + q_g b'_g - c(k,k') \tag{A.11}$$

$$b'_g \leq \overline{b_g}, \quad b'_g \leq B' \tag{A.12}$$

(2) Given p, the function of $b'(v)$, $k'(v)$ solve the problem of entering firms, and $V(v)$ is the associated value function,

$$V^e(v) = \max_{k',b'} d + \beta \sum_{z'} \int_{\phi' > \hat{\phi}} V(x'(k',b',z',\phi'), k', z') \, d\Phi(\phi') \, dG(z' \mid v) \tag{A.13}$$

subject to (A.14) - (A.17)

$$d = -\psi(k_e, k') + q^e(k', b', \nu)b' \geq 0 \tag{A.14}$$

$$x(k', b', z', \phi') = (1-\tau)pz' \exp(\phi') k'^{\alpha} - f_k k' - f - b' + \tau\left(\delta k' + r_f b'\right) \tag{A.15}$$

$$\hat{\phi}(k', b', z') = \log\left(\frac{-\bar{x}(k', z') + f + f_k k' + b' - \tau\left(\delta k' + r_f b'\right)}{(1-\tau)pz'k'^{\alpha}}\right) \tag{A.16}$$

$$\overline{x}(k, z) = \max_{k', b'} q(k', b', 0, z)b' - \psi(k, k') \tag{A.17}$$

(3) The bond price schedule ensures that lenders break even,

$$q\left(k', B', b'_g, z\right) = \beta \sum_{z'} \left[\left(1 - \Phi\left(\hat{\phi}^G\right)\right) + \Phi\left(\hat{\phi}^G\right) R^G\left(B', b'_g, k'\right)\right] \pi(z' \mid z) \tag{A.18}$$

where,

$$\hat{\phi}^G\left(k', B', b'_g, z')\right) = \log\left(\frac{-\overline{x}^G(k', z') + f + f_k k' + B' - \left(1 - q_g\right) b'_g - \tau\left(\delta k + r_f B'\right)}{(1-\tau)pz'k'^{\alpha}}\right) \tag{A.19}$$

$$R^G(B', b'_g, k') = \min\left(1, \max\left(0, \frac{\chi(1-\delta)k' - b'_g - \eta}{B' - b'_g}\right)\right) \tag{A.20}$$

$$q_e(k', b', \nu) = \beta \sum_{z'} \left[\left(1 - \Phi\left(\hat{\phi}\right)\right) + \Phi\left(\hat{\phi}\right) R(b', k')\right] dG(z' \mid \nu) \tag{A.21}$$

where,

$$\hat{\phi}(k', b', z') = \log\left(\frac{-\overline{x}(k', z') + f + f_k k' + b' - \tau\left(\delta k' + r_f b'\right)}{(1-\tau)pz'k'^{\alpha}}\right) \tag{A.22}$$

$$R(b', k') = \min\left(1, \max\left(0, \chi\frac{(1-\delta)k'}{b'} - \eta\right)\right) \tag{A.23}$$

(4) The aggregate production of intermediate good satisfies

$$Y = \sum_{z} \int_{\phi} z \exp(\phi) \int_{x_{-1}, k_{-1}, z_{-1}} k(x_{-1}, k_{-1}, z_{-1})^{\alpha} \mu_{-1}(x_{-1}, k_{-1}, z_{-1})\, d\Phi(\phi) \pi(z \mid z_{-1}) \tag{A.24}$$

(5) Average productivity of intermediate good firms $\bar{z}$ is

$$\bar{z} = \sum_{z_i} z_i w(z_i) \tag{A.25}$$

where, $w(z_i)$ is a share of output produced by firms whose productivity is z_i:

$$w(z_i) = \int_{\phi} \frac{\int_{x_{-1},k_{-1},z_{-1}} z_i \exp(\phi) k\left(x_{-1}, k_{-1}, z_{-1}\right)^{\alpha} \mu_{-1}\left(x_{-1}, k_{-1}, z_{-1}\right) d\Phi(\phi) \pi\left(z_i \mid z_{-1}\right)}{Y} \tag{A.26}$$

(6) p clears final good market.

$$y_f(p) = \bar{z} Y^{\alpha_y} \tag{A.27}$$

(7) The cross-sectional distribution of μ is a stationary measure of firms consistent with the firms decision rules and the law of motion for the stochastic variable.

A.7 Solution Algorithm

I first discretize the idiosyncratic productivity shock z using Rouwenhorst method. The discretized shocks consist of 11 productivity points, and associated transition matrices $\pi(z' \mid z)$. The idiosyncratic state x is discretized into 15 endogenous grids that depend on the firm's state $\{k, z\}$. I use 50 points for capital and 100 points for borrowing. The state space for the firm's problem has $\#x \times \#k \times \#z \times = 8,250$ grid points. The resulting array for bond price schedule, $q(k', b', z)$ has $\#k' \times \#b' \times \#z \times = 55,000$ grid points. I also discretize the i.i.d productivity shock ϕ into 101 points using Gaussian quadrature method and use it to evaluate the integrals in the debt price and the firm's continuation value.

I solve the model with two loops: an inner and an outer loop. In the inner loop, there are two separate procedures. Taking as given the price , p, I first find the default cut-off of cash-on-hand and associated debt price schedules. Next, given the found default cut-off and debt price schedules, I find the value function and related policy functions by iteratively solving each firms optimization problem until the value function converges. In the outer loop, taking as given the converged decisions from the inner loop, I start with a distribution of firms $\mu(x, k, s)$ and iterate until the distribution converges. Using a bisection search, I determine the price that clears the final good market.

A.7.1 Debt price schedules

Given price p, I first construct maximum level of fund that firm (k, z) can raise, $\overline{x}(k, z)$ and bond price schedule $q(k', b', z)$. I start with an initial guess of $\overline{x}^0(k, z)$. Given $\overline{x}^0(k, z, z_\mu, S)$ I construct the associated default cut-off,

$$\hat{\phi}_0^G(k', B', z') = \log\left(\frac{-\overline{x}_0^G(k', z') + f + f_k k' + B' - \left(1 - q_g\right)\overline{b_g} - \tau\left(\delta k + r_f B'\right)}{(1-\tau)pz'k'^{\alpha}}\right) \tag{A.28}$$

and the associated full repayment cut-off,

$$\tilde{\phi}_0^G(k', B', z') = \log\left(\frac{-\overline{x}_0^G(k', z') + f + f_k k' + B' - \tau\left(\delta k + r_f B'\right)}{(1-\tau)pz'k'^{\alpha}}\right) \tag{A.29}$$

and the associated bond price schedule,

$$q_0(k', B', z) = \beta \sum_{z'} \left[\left(1 - \Phi\left(\tilde{\phi}_0^G\right)\right) + \Phi\left(\hat{\phi}_0^G\right) R^G(B', k')\right] \pi(z' \mid z) \tag{A.30}$$

Here the debt price can be determined only if $B' > \overline{b_g}$. Otherwise the firm will finance their debt only via the government loan by the Proposition 2.4.1.

In the first step of the iteration, I update the $\bar{x}^1(k,z)$ using

$$\overline{x}_1^G(k,z) = \max_{k',B'} q_0\left(k', B', \overline{b_g}, z\right)\left(B' - \overline{b_g}\right) + q_g\overline{b_g} - \psi\left(k, k'\right)$$

Using updated $\overline{x}^1\left(k,z\right)$, I construct the associated default cutoff of productivity shock $\hat{\phi}_1^G\left(k', B', z'\right)$ and bond price schedule $q_1\left(k', B', z\right)$ using the analogs of A.28, A.30.

I continue this process iteratively until the constructed sequence of $\overline{x}_n^G(k,z)$ converge. I then record the associated array of default cutoff $\hat{\phi}_n^G\left(k', B', z'\right)$, full repayment cutoff $\tilde{\phi}_n^G\left(k', B', z'\right)$ and bond price schedule $q_n\left(k', B', z\right)$, which I hold fixed during each iteration of the firm decision rules. Using the bond price schedule, I construct the price schedule with respect to total debt as follows:

$$Q(k', B', z) = q_g\frac{\overline{b_g}}{B'} + q(k', B', z)\frac{B' - \overline{b_g}}{B'}$$

If $B' < \overline{b_g}$, the debt price equals to q_g.

A.7.2 Inner Loops: Firm decisions rules

Given price p, I solve for the decision rules iterating over value functions. I iterate on a set of arrays of grid $\{X(k,z)\}$ that varies with (k,z)

$$X(k,z) = \{x_1, \ldots, x_N\}$$

(1) Given an initial guess for value function $V^0(x,k,z)$,$V_{nb}^0(k,z)$, and for the set of arrays of grids $\left\{X^0(k,z)\right\}$, I solve for the cutoff $\hat{x}_1(k,z)$ by solving for $\hat{k}'(k,z)$ and $\hat{B}'(k,z)$, which is firms optimal decision when the nonnegative equity payout constraint does not bind, following Arellano, Bai, and Kehoe (2019). Specifically, I find $\hat{x}_1(k,z)$ by first solving a "relaxed" version of the firm's problem, where I drop the non-negative equity payout condition for the current period only. The associated decision for capital and borrowing is denoted by $\hat{k}(k,z)$ and $\hat{B}(k,z)$, which can be obtained by solving the following problem,

$$\max_{k',B'} -c(k,k') + Q(k',B',z)B' + \beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' > \tilde{\phi}^G} V_0 \left(x'\left(k',B',z',\phi'\right), k', z' \right) d\Phi(\phi') \right]$$
$$+\beta \sum_{z'} \pi(z' \mid z) \left[\left(\Phi\left(\bar{\phi}^G\right) - \Phi\left(\hat{\phi}^G\right) V_0 \left(x'\left(k',B',z',\bar{\phi}^G\right), k', z' \right) \right] \tag{A.31}$$

Then the level of cash-on-hands, where the nonnegative equity payout constraint does not bind is

$$\hat{x}^{n+1}(k,z) = \psi(k, \hat{k}'(k,z)) - Q(\hat{k}'(k,z), \hat{B}'(k,z), z)\hat{B}'(k,z) \tag{A.32}$$

Construct the grid $\left\{X^{n+1}(k,z)\right\} = \left\{x_1^{n+1}, x_2^{n+1}, \ldots, x_N^{n+1}\right\}$ by setting

$$x_1^{n+1} = -\bar{x}(k,z) \quad \text{and} \quad x_N^{n+1} = \hat{x}^1(k,z)$$

That is, we know that if the cash-on-hand x is so low, $x + \bar{x}(k,z) < 0$, even with the maximum funds raised by borrowing and disposing of capital, the associated dividends $d = x + \bar{x}(k,z)$ is negative and the firm will default. We also know that if the cash-on hand x is sufficiently high, so that $x \geq \hat{x}(k,z)$, the optimal decisions will be given by the nonbinding level of capital $\hat{k}(k,z)$ and borrowing $\hat{b}(k,z)$ because the decision is not affected by nonnegative equity payout condition. I then choose a set of intermediate points $\{x_2, \ldots, x_{N-1}\}$. Therefore, along with the value function $V(x,k,z)$ the endogenous grid $X = \{x_1, \ldots, x_N\}$ is updated in each iteration of the loop. Here's the specific steps.

a. First I construct the value for each choice $\{k', B'\}$ over the grid points such that

$$W_0(k',B',z) = \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' > \tilde{\phi}^G} V_0 \left(x'\left(k',B',z',\phi'\right), k', z' \right) d\Phi(\phi') \right]$$
$$+ \sum_{z'} \pi(z' \mid z) \left[\left(\Phi\left(\bar{\phi}^G\right) - \Phi\left(\hat{\phi}^G\right) V_0 \left(x'\left(k',B',z',\bar{\phi}^G\right), k', z' \right) \right] \tag{A.33}$$

The value off the grid over x is calculated using linear interpolation.

b. Then I find the optimal options over the grids that maximize

$$-\psi(k,k') + Q(k',B',z)B' + W_0(k',B',z)$$

c. Using the solution as a initial guess, I solve the optimization problem using Powell's method.[1]

For calculation of default and full repayment cutoff, $\overline{x}^G$ off the grid points over k were calculated using linear interpolation. Value off the grid points for k', and x' are also calculated using linear interpolation. For example let's assume $k' \in [k_{i-1}, k_i]$. Then we can calculate $\overline{x}^G(k,z)$ which is off grid of capital using linear interpolation, and accordingly we can calculate $\tilde{\phi}^G(k', B', z')$ and $\hat{\phi}^G(k', B', z')$. For given shocks z' and ϕ', $x'(k', B', z', z')$ can be calculated using equation A.8. Using this, I calculate $V_0(x', k_{i-1}, z')$ and $V_0(x', k_i, z')$ by interpolating between x grid points based on the grid $\left\{X^0(k,z)\right\}$, respectively for k_{i-1} and k_i. Then I interpolate between two capital grid points. Furthermore when $x > x^0_N$, $V_0(x', k_i, z') = x' + V^{nb}_0(k', z')$.

(2) Solve for decisions at the intermediate points and find policy function, $\{k'(x,k,z), b'(x,k,z)\}$. At these nodes, since the non equity payout constraint is binding, for each (x,k,z) I can solve for b' off-grid given k' from the following equation,

$$x - \psi(k, k') + Q(k', B', z) B' = 0$$

Then using the condition, I can find optimal choice for capital and borrowing in a previously outlined way.

(3) I update the value function to V^{n+1} using

$$V^{n+1}(x,k,z) = x - \psi(k,k') + Q(k',B',z)B' + \underbrace{\beta \sum_{z'} \int_{\phi' > \hat{\phi}(k',b'z')} V^n(x'(k',B',z',\phi'),k',z')}_{W(k',B',z)} \quad \text{(A.34)}$$

- Policy functions for firms with binding NEP : $k' = k'(x,k,z)$, $B' = B'(x,k,z)$
- Policy functions for firms with non-binding NEP : $k' = \hat{k}'(k,z)$, $B' = \hat{B}'(k,z)$

(4) Iterate until the value functions $W^n(k', b', z)$ on grid for capital, debt, and productivity converge.

[1]Specifically I use the fminsearch subroutine from the source codes accompanying the book Fehr, H. & Kindermann, F. (2018). Introduction to Computational Economics using Fortran. Oxford: Oxford University Press.

A.7.3 Outer loop: Stationary distribution and equilibrium price

In an outer loop, I update the price p based on stationary cross-sectional distribution.

I use a histogram-based approach to tracking the cross-sectional distribution following Young (2010). I use grids for net-income ratio, $\frac{x}{k}$ which is denoted by x_k, rather than cash-on-hand, x, itself. I use 101 grids point for net-income ratio, ranged from -2 to 2. I use denser grids for k, such that $n_k^o = 80$.

I simulate firms on a discretion grid of $\#x^k \times \#k \times \#z \times = 88,880$. Since there is a finite number of grid points for x^k and k, first I need to allocate the mass of any x' and k' to points on the x^k-grid and k-grid. Specifically, I allocate the mass of firms with any x' and k' to the bracketing interval $\left[x_{i-1}^k, x_i^k\right]$ on the x^k-grid in proportion to how close $x^k = \frac{x'}{k'}$ is to each side of the interval. Specifically, let $\omega_{xk}\left(x_i^k, x^k\right)$ be the probability that the choice of x^k is assigned to x_i :

$$\omega_{xk}\left(x_i^k, x^k\right) = \frac{x^k - x_{i-1}^k}{x_i^k - x_{i-1}^k} \quad \text{and} \quad \omega_{xk}\left(x_{i-1}^k, x^k\right) = 1 - \omega_x\left(x_i^k, x^k\right)$$

and $\omega_{xk}\left(x_i^k, x^k\right) = 0$ if $x^k \notin \left[x_{i-1}^k, x_i^k\right]$.

The same idea goes for k'. Let $\omega_k\left(k_j, k'\right)$ be the probability that the choice of k' is assigned to k_j :

$$\omega_k\left(k_j, k'\right) = \frac{k' - k_{j-1}}{k_j - k_{j-1}} \quad \text{and} \quad \omega_k\left(k_{j-1}, k'\right) = 1 - \omega_k\left(k_j, k'\right)$$

and $\omega_k\left(k_j, k'\right) = 0$ if $k' \notin \left[k_{j-1}, k_j\right]$.

I update the distribution as follows until the distribution $\mu(x, k, z)$ converges.

$$\begin{aligned}
&\mu'(x_i^k, k_j, z') = \\
&\sum_{x_i^k, k_j, z} \int_{\phi' \geq \tilde{\phi}^G} \omega_{xk}\left(x_i, x^k\left(k'(x,k,z), B'(x,k,z), z', \phi'\right)\right) \omega_k\left(k_j, k'(x,k,z)\right) d\Phi(\phi')\pi(z' \mid z)\mu(x_i^k, k_j, z) \\
&+ \sum_{x_i^k, k_j, z} \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G\right) \omega_{xk}\left(x_i, x^k\left(k'(x,k,z), b'(x,k,z), z', \tilde{\phi}^G\right)\right) \omega_k\left(k_j, k'(x,k,z)\right) \pi(z' \mid z)\mu(x_i^k, k_j, z) \\
&+ M \int_{\nu \geq \hat{\nu}} \int_{\phi' \geq \hat{\phi}(k', b', z')} \omega_x\left(x_i, x'\left(k'(\nu), b'(\nu), z', \phi'\right)\right) \omega_k\left(k_j, k'(\nu)\right) d\Phi(\phi') H(z' \mid \nu) dG(\nu)
\end{aligned} \tag{A.35}$$

where,

$$x^k = \frac{x'(k'(x,k,z), B'(x,k,z), z', \phi')}{k'(x,k,z)}$$

$$x'(k', B', z', \phi') = (1-\tau)pz' \exp(\phi')k'^{\alpha} - f_k k' - f - B' + \tau\left(\delta k + r_f B'\right)$$

$$\tilde{\phi}^G\left(k', B', b_g', z'\right)\right) = \log\left(\frac{-\overline{x}^G(k', z') + f + f_k k' + B' - \tau\left(\delta k + r_f B'\right)}{(1-\tau)pz'k'^{\alpha}}\right)$$

$$\hat{\phi}^G\left(k', B', b_g', z'\right)\right) = \log\left(\frac{-\overline{x}^G(k', z') + f + f_k k' + B' - \left(1 - q_g\right)b_g' - \tau\left(\delta k + r_f B'\right)}{(1-\tau)pz'k'^{\alpha}}\right)$$

Given the converged distribution $\mu(x^k, k, z)$, I calculate the excess demand $ED(p)$,

$$ED(p;\mu) = \left[\frac{\overline{z}(\mu)\alpha_y}{p}\right]^{\frac{1}{1-\alpha_y}} - Y(\mu)$$

where,

$$Y(\mu) = \sum_{z'} \int_{\phi} \sum_{x_i^k, k_j, z} z' \exp(\phi)k'\left(x_i^k k_j, k_j, z\right)^{\alpha} \mu\left(x_i^k, k_j, z\right) d\Phi(\phi)\pi\left(z' \mid z\right)$$

$$\overline{z}(\mu) = \sum_{z_i} z_i \underbrace{\frac{\int_{\phi} \sum_{x_i^k, k_j, z} z_i \exp(\phi)k\left(x_i^k k_j, k_j, z\right)^{\alpha} \mu\left(x_i^k, k_j, z\right) d\Phi(\phi)\pi\left(z_i \mid z\right)}{Y}}_{\text{Share of output produced by firms with } z_i}$$

I use a bisection search to determine the price that clears the final good market. Specifically, I choose two prices, p_l and p_h, such that excess demand $ED(p_l) > 0$ and $ED(p_h) < 0$. Set $p^1 = \frac{p_l + p_h}{2}$. If $ED(p^1) < 0$ then update price as $p^2 = \frac{p_l + p^1}{2}$, and if If $ED(p^1) > 0$ then update price as $p^2 = \frac{p^1 + p^h}{2}$. I iterate the procedure until the price p^n converges.

A.7.4 Transition path

I use the following algorithm to compute the transition path between two states with and without government loans.

(1) Calculate the stationary equilibrium with and without government loans. Save the associated policy functions and value functions of firms, the equilibrium price in two economies, and the distribution of the steady state in the economy without government loans.

(2) Assume that the economy transitions to the new steady state over a period of $T = 200$

years. Government loans are introduced at the beginning of period 1, specifically before decisions on default and firms' decisions regarding capital and borrowing, but after the realization of persistent and transitory productivity shocks. Guess the path for price $\{\mathbb{P}\}^0$, which is the vector of price from period 1 to T.

(3) Taking the paths of price as given, I calculate the full transition path by iterating the following steps:

a. Solve for policy and value functions over the transition for $t = T-1, T-2, \ldots, 1$ by iterating backward. Specifically, I derive the policy and value functions for period t by using the value functions derived from period $t+1$:

$$\begin{aligned} V^t(x,k,z) = \max_{k',B',b'_g} d + \beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' > \tilde{\phi}^G} V^{t+1}\left(x'\left(k',B',z',\phi'\right),k',z'\right) d\Phi\left(\phi'\right) \right] \\ + \beta \sum_{z'} \pi(z' \mid z) \left[\left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G \right) V^{t+1}\left(x'\left(k',B',z',\tilde{\phi}^G\right),k',z'\right) \right] \end{aligned} \tag{A.36}$$

b. Compute the evolution of firms distribution over the transition for $t = 2, 3, \ldots, T$ by iterating forward. Specifically, I update the firms distribution for period t from the firms distribution for period $t-1$:

$$\begin{aligned} &\mu'_t(x_i^k, k_j, z') = \\ &\sum_{x_i^k, k_j, z} \int_{\phi' \geq \tilde{\phi}^G} \omega_{xk}\left(x_i, x^k\left(k'(x,k,z), B'(x,k,z), z', \phi'\right)\right) \omega_k\left(k_j, k'(x,k,z)\right) d\Phi(\phi') \pi(z' \mid z) \mu_{t-1}(x_i^k, k_j, z) \\ &+ \sum_{x_i^k, k_j, z} \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G\right) \omega_{xk}\left(x_i, x^k\left(k'(x,k,z), b'(x,k,z), z', \tilde{\phi}^G\right)\right) \omega_k\left(k_j, k'(x,k,z)\right) \pi(z' \mid z) \mu_{t-1}(x_i^k, k_j, z) \\ &+ M \int_{\nu \geq \hat{\nu}} \int_{\phi' \geq \hat{\phi}(k',b',z')} \omega_x\left(x_i, x'\left(k'(\nu), b'(\nu), z', \phi'\right)\right) \omega_k\left(k_j, k'(\nu)\right) d\Phi(\phi') H(z' \mid \nu) dG(\nu) \end{aligned} \tag{A.37}$$

Here, note that the cash-on-hand cutoff for default and being unconstrained varies with the equilibrium price, and the cutoff for potential entrants' signal to enter also varies with the equilibrium price.

c. Given the firms' distribution in each period t, I calculate the excess demand for each period as follows,

$$ED_t = \left[\frac{\bar{z}_t \alpha_y}{p_t} \right]^{\frac{1}{1-\alpha_y}} - Y_t$$

where,

$$Y_{t+1} = \sum_{z_{t+1}} \int_{\phi} \sum_{x_i^k, k_j, z} z_{t+1} \exp(\phi) k_{t+1} \left(x_i^k k_j, k_j, z\right)^{\alpha} \mu_t\left(x_i^k, k_j, z\right) d\Phi(\phi) \pi\left(z_{t+1} \mid z\right)$$

$$\bar{z}_{t+1} = \sum_{z_i} z_i \underbrace{\frac{\int_\phi \sum_{x_i^k, k_j, z} z_i \exp(\phi) k_{t+1} \left(x_i^k k_j, k_j, z\right)^\alpha \mu_t \left(x_i^k, k_j, z\right) d\Phi(\phi) \pi \left(z_i \mid z\right)}{Y_{t+1}}}_{\text{Share of output produced by firms with } z_i}$$

c. Calculate the vector of market-clearing price path $\{P\}^*$ such that $p_t^* = Y_t^{\alpha_y - 1} \bar{z} \alpha_y$. If the difference between the vector of market-clearing price path and $\{P\}^n$ is smaller than a pre-specified tolerance then stop. Otherwise I update the price vector $\{P\}^{n+1} = \lambda \{P\}^n + (1 - \lambda) \{P\}^*$, and go back to the step a.

(4) Once the market-clearing price is found, I calculate the firms policy and value functions, and distributions over the transitions.

A.7.5 Simulation and replication procedure

The simulation procedure aims to compare the heterogeneous responses of firms simulated from the model over the three years following the introduction of government loans and data. I simulate the economy for $T = 500$ periods and introduce government loans at $t = T - 2$. Policy functions and the equilibrium price remain the same from period 1 to T-3, while these policy functions and equilibrium price will transition from $t = T - 2$ to $t = T$. I start with $N = 5000$ firms, and every period, some firms exit, while the surviving firms make choices regarding capital and debt. Additionally, every period, firms enter based on the signals they receive from a pool of $N = 5000$ potential entrants. I construct the panel data of firms by discarding the first 450 years and replicate the estimation based on data using the constructed panel.

A.8 Definition of moments: Data and Model

(1) Net-income ratio

- Data : $\frac{\text{Net income}}{\text{Total asset}}$
- Model : $\frac{\mathrm{x}_t}{\mathrm{k}_t}$

(2) Investment

- Data: $\frac{2\times\left[\text{Tangible Asset}_{t+1}-\text{Tangible Asset}_t\right]}{\text{Tangible Asset}_{t+1}+\text{Tangible Asset}_t}$
- Model: $\frac{2\times[k_{t+1}-(1-\delta)k_t]}{k_{t+1}+(1-\delta)k_t}$

(3) Spread

- Data: $\frac{2\times\text{Interest expense}_t}{\text{Total debt}_t+\text{Total debt}_{t-1}}\times 100$ – Korean corporate bond rate (AA- 3yr)
- Model: $\left(\frac{1}{q_t}-\frac{1}{\beta}\right)\times 100$

(4) Size

- Data: Tangible asset size
- Model: Capital size

(5) Profitability

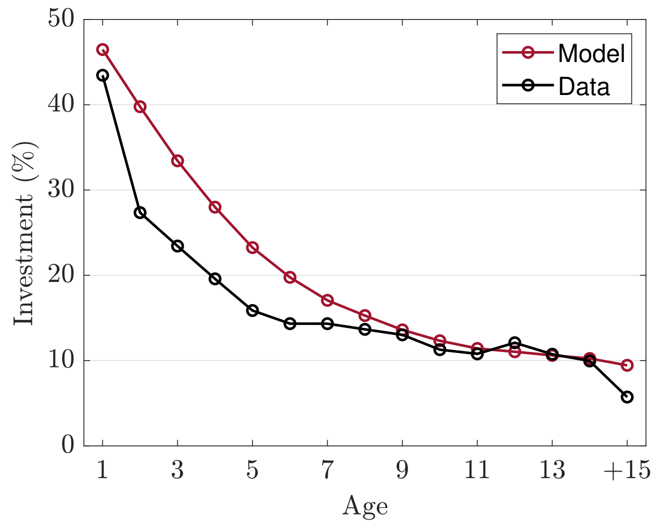
- Data: Operational profit / Total asset
- Model : $\frac{pz_t\exp(\phi_t)k_t^{\alpha}-f-f_k k_t}{k_t}$

(6) Individual firm's TFP (revenue-based)

- Data: $\frac{\text{Sales}_t\times 2}{\text{Total asset}_{t-1}+\text{Total asset}_t}$
- Model: $z_t\exp(\phi_t)$

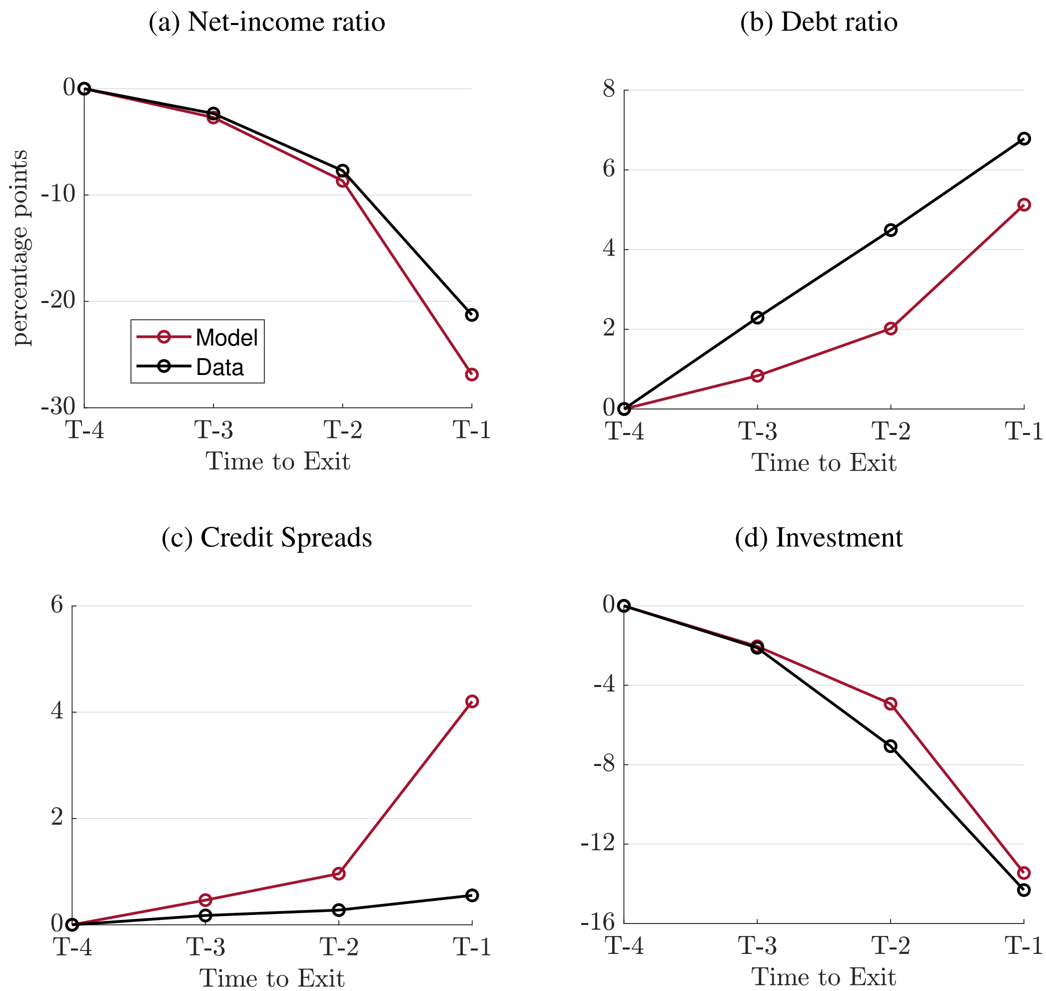
A.9 Model performance: Untargeted

Figure A.5: Investment by Age: Data vs Model



Notes: The figure shows investment by firms age.

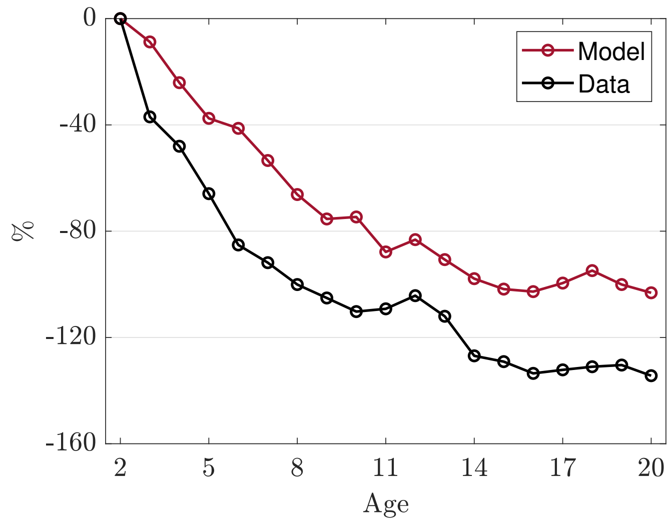
Figure A.6: Financial States Before Firm Exits: Data vs Model



Notes: These plots show the relative financial state of firms with specific distance to exit based on data and simulated firms.

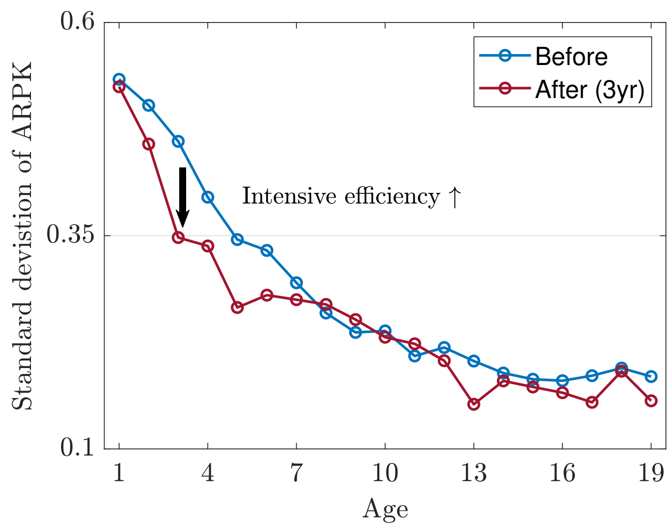
A.10 Capital misallocaion: dispersion of average revenue product of capital

Figure A.7: Capital misallocation by age



Notes: The figure presents the percentage change in the standard dispersion of the revenue-asset ratio for specific firm ages relative to firms aged 2. For the data, it is calculated as the standard deviation of $\frac{\text{sales}\times 2}{\text{previous year's asset+current year's asset}}$, while for the model, it is calculated as the standard deviation of the sales-to-capital ratio.

Figure A.8: Improvement in capital allocation with the policy



Notes: The figure presents the standard deviation of the sales-to-capital ratio (average revenue product of capital) for firms in specific years during the period without government loans and during periods spanning over three years after the introduction of government loans, based on model simulation.

A.11 Proofs

A.11.1 Proof of Proposition 2.4.1

The proposition can be verified by confirming that firms' value strictly increases when substituting private loans with government loans given the same total debt amount. Given a choice for a total debt, denoted as B', a firm will borrow $b'_g = \overline{b_g}$ from the government if the selected total debt exceeds the government limit; otherwise, the firm will solely borrow from the government.

The derivative of firms value function in equation 2.14 with respect to b'_g given B' stays same,

$$\frac{\partial V(x,k,z)}{\partial b'_g} = \underbrace{(q_g - q) + \frac{\partial q}{\partial b'_g}(B' - b'_g)}_{\zeta \geq 0} + \beta \sum_{z'} \pi(z' \mid z) \underbrace{\left(-\frac{\hat{\phi}^G}{b'_g}\right)}_{>0} V\left(x'\left(k', B', z', \tilde{\phi}^G\right), k', z'\right) \tag{A.38}$$

The value of substituting private loans with government loans comes from two aspects: one arises from an increase in debt price, given the same borrowed debt amount (higher funding capacity), which is captured by the first two terms in equation A.38 (denoted as ζ). The other comes from a reduced default probability, given the same borrowed debt amount, which is captured by the last term in equation A.38.

Using the following conditions,

$$q\left(k', B', b'_g, z\right) = \beta \sum_{z'} \left[\left(1 - \Phi\left(\hat{\phi}^G\right)\right) + \Phi\left(\hat{\phi}^G\right) R^G(B', b'_g, k')\right] \pi(z' \mid z)$$

where,

$$\hat{\phi}^G\left(k', B', b'_g, z'\right) = \log\left(\frac{-\bar{x}^G(k', z') + f + f_k k' + B' - (1-\beta)\, b'_g}{p z' k'^{\alpha}}\right)$$

$$R^G(B', b'_g, k') = \min\left(1, \max\left(0, \frac{\chi(1-\delta)k' - b'_g - \eta}{B' - b'_g}\right)\right)$$

We can derive,

$$\frac{\partial q}{\partial b'_g} = \beta \sum_{z'} \left[\left(-\frac{\hat{\phi}^G}{\partial b'_g}\right) \Phi'(\hat{\phi}^G)\left(1 - R^G\right) + \Phi(\hat{\phi}^G) \frac{\partial R^G}{\partial b'_g}\right] \pi(z' \mid z)$$

Now we have,

$$\zeta = \underbrace{\beta \sum_{z'} \left[\Phi\left(\hat{\phi}^G\right)(1 - R^G)\right] \pi(z' \mid z)}_{q_g - q} + \beta \sum_{z'} \left[\left(-\frac{\hat{\phi}^G}{\partial b'_g}\right) \Phi'(\hat{\phi}^G)\left(1 - R^G\right)(B' - b_{g'}) - \Phi(\hat{\phi}^G)(1 - R^G)\right] \pi(z' \mid z)$$

$$= \beta \sum_{z'} \left[\left(-\frac{\hat{\phi}^G}{\partial b'_g}\right) \Phi'(\hat{\phi}^G)\left(1 - R^G\right)(B' - b_{g'})\right] \pi(z' \mid z) \geq 0 \tag{A.39}$$

Therefore, $\frac{V(x,k,z)}{b'_g} > 0$.

A.11.2 Proof of Proposition 2.4.2

Default: Firms only default if there is no feasible set that satisfies the non-negative dividends payout condition, i.e.,

$$\nexists (k', B') \text{ such that } x - \psi(k, k') + q\left(k', b', b'_g z\right)\left(B' - b'_g\right) + q_g b'_g \geq 0$$

Then we can define the default threshold on x such that $x + \overline{x}^G(k,z) < 0$, where $\overline{x}^G(k,z) = \max_{k',B'} q\left(k', B', \overline{b_g}, z\right)(B' - \overline{b_g}) + q_g \overline{b_g} - \psi(k, k')$, which indicates the maximum level of fund a firm can raise with debt financing and capital disposal. If $x + \overline{x}^G(k,z) < 0$, then there is no a feasible set for a firm to satisfy the non-negative dividends payout condition. This is because a firm cannot avoid a negative dividend level even after maximizing their fund. Therefore if $x < \underline{x}(k,z) = -\overline{x}^G(k,z)$, firms default.

Unconstrained: For $x > \underline{x}(k,z) = -\overline{x}^G(k,z)$, we can construct a threshold $\hat{x}(k,z)$ such that the firms' choice for (k', B') does not depend on firms level of cash-on-hand, x, by solving a relaxed version of the firm's problem following Arellano, Bai, and Kehoe (2019), Ottonello and Winberry (2020). Specifically, we can solve the relaxed version of the probelm by dropping the nonnegative equity payout constraint for the current period only as follows,

$$\begin{gathered} \max_{k',B'} -\psi(k, k') + q\left(k', b', b'_g z\right)\left(B' - b'_g\right) + q_g b'_g + \\ \sum_{z'} \pi(z' \mid z)\left(\left[\int_{\phi' > \tilde{\phi}^G} V\left(x'\left(k', B', z', \phi'\right), k', z'\right) d\Phi(\phi')\right] + \left[\Phi\left(\tilde{\phi}^G\right) - \Phi\left(\hat{\phi}^G\right)\right] \tilde{V}\right) \end{gathered} \tag{A.40}$$

where $\tilde{V} = \left(x'\left(k', B', z', \tilde{\phi}^G\right), k', z'\right)$

Then we can construct the threshold

$$\hat{x}(k,z) = \psi\left(k, \hat{k}'(k,z)\right) - q\left(\hat{k}'(k,z), \hat{B}'(k,z), z\right)\left(\hat{B}'(k,z) - \overline{b_g}\right) + q_g \overline{b_g}$$

where $(\hat{k}, \hat{B}')$ is the solution for the problem in equation A.40. Note that cash-on-hand x enters simply as an additive constant in the objective function (the value of constrained firms is the sum of value in equation A.40 and cash-on-hand x) and not in any constraint. Therefore, the solution for the relaxed problem does not depend on the level of cash-on-hand. If firms' cash-on-hand is above the threshold, then the dividends increases one for one with the cash-on-hand and the choice for capital and borrowing does not vary with the cash-on-hand.

Constrained: I will show that firms with $x \in [\underline{x}](k,z), \hat{x}(k,z)$ pay zero dividends, $d = 0$. The optimality condition for B' is as follows,

$$\beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' \geq \tilde{\phi}^G} (1 + \eta'(x'(k', B', z', \phi'), k', z'))\, d\Phi(\phi') + \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right)\left(1 + \eta'\left(x'(k', B', z', \tilde{\phi}^G), k', z'\right)\right) \right]$$
$$+ \beta \sum_{z'} \pi(z' \mid z) \left(\frac{\partial \hat{\phi}^G}{\partial B'} \right) \phi(\hat{\phi}^G) V\left(x'(k', B', z', \tilde{\phi}^G), k', z'\right) = (1 + \eta(x,k,z)) \left[q + \frac{\partial q}{\partial B'} \left(B' - \overline{b_g}\right) \right] \tag{A.41}$$

I will show that if a constrained firm pays a positive dividend, i.e., $\eta(x,k,z) = 0$ (the non-negative dividends payout condition slacks), then this leads to a contradiction.

Let's first consider a case of a firm with a zero probability of default in the next period. In this case, we can write the condition in equation A.41 as follows,

$$\beta + \beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi'} \eta'(x'(k', B', z', \phi'), k', z') d\Phi(\phi') \right] = \beta$$

Since the firm is constrained, which implies a positive debt, $\eta'(x'(k', B', z', \phi'), k', z') > 0$ for some positive mass of realizations of z' and ϕ', which results in a contradiction.

Next, let's consider a case of a firm with a positive probability to default in the next period. In this case, using the following condition,

$$q + \frac{\partial q}{\partial B'}\left(B' - \overline{b_g}\right) = \beta \sum_{z'} \left\{ \left[1 - \Phi\left(\hat{\phi}^G\right)\right] + \left[\left(-\frac{\partial \hat{\phi}^G}{\partial B'} \right) \phi(\hat{\phi}^G) \left(B' - \overline{b_g}\right)\left(1 - R^G\right) \right] \right\} \pi(z' \mid z)$$

we can write the condition in equation A.41 as follows,

$$\beta \sum_{z'} \pi(z' \mid z) \left[\int_{\phi' \geq \tilde{\phi}^G} \eta' \left(x'(k', B', z', \phi'), k', z'\right) d\Phi(\phi') + \left(\Phi(\tilde{\phi}^G) - \Phi(\hat{\phi}^G)\right) \eta' \left(x'(k', B', z', \tilde{\phi}^G), k', z'\right) \right]$$
$$+ \beta \sum_{z'} \pi(z' \mid z) \left(\frac{\partial \hat{\phi}^G}{\partial B'}\right) \phi(\hat{\phi}^G) \left[\left(B' - \overline{b_g}\right)\left(1 - R^G\right) + V\left(x'(k', B', z', \tilde{\phi}^G), k', z'\right) \right] = 0$$

Since the firm is constrained $\eta'(x'(k', B', z', \phi'), k', z') > 0$ for some positive mass of realizations of z' and ϕ' and $\left[\left(B' - \overline{b_g}\right)\left(1 - R^G\right) + V\left(x'(k', B', z', \tilde{\phi}^G), k', z'\right)\right] > 0$, which results in a contradiction.

A.11.3 Proof of Proposition 2.7.1

Given capital stock K, and a measure of firms $\mu(x, k, z)$ with mass M, the planner's problem can be defined using equation 2.24,

$$\max_{k(x_{-1}, k_{-1}, z_{-1})} \int_{x_{-1}, k_{-1}, z_{-1}} \sum_z zk\left(x_{-1}, k_{-1}, z_{-1}\right)^\alpha \pi\left(z \mid z_{-1}\right) d\mu_{-1}\left(x_{-1}, k_{-1}, z_{-1}\right)$$
$$\text{s.t. } \int k(x_{-1}, k_{-1}, z_{-1})\, \mathrm{d}\mu(x_{-1}, k_{-1}, z_{-1}) \leq K.$$

The first order condition is given by

$$k(x_{-1}, k_{-1}, z_{-1}) = \left(\frac{\alpha}{\lambda_k}\right)^{\frac{1}{1-\alpha}} \tilde{z}^{\frac{1}{1-\alpha}}$$

where, $\tilde{z} = \sum_z z\pi\left(z \mid z_{-1}\right)$ is the conditional expected productivity given today's productivity, and λ_k is the Lagrangian multiplier for the resource constraint.

Integrating the equation, we have

$$\underbrace{\int k(x_{-1}, k_{-1}, z_{-1}) d\mu(x_{-1}, k_{-1}, z_{-1})}_{K} = \left(\frac{\alpha}{\lambda_k}\right)^{\frac{1}{1-\alpha}} \int \tilde{z}^{\frac{1}{1-\alpha}} d\mu(x_{-1}, k_{-1}, z_{-1})$$

Then the solution to the planner's problem is as follows,

$$k(x_{-1}, k_{-1}, z_{-1}) = \frac{\tilde{z}^{\frac{1}{1-\alpha}}}{\int \tilde{z}^{\frac{1}{1-\alpha}} d\mu(x_{-1}, k_{-1}, z_{-1})} K$$

Notice that the planner's allocation does not depend on the level of cash-on-hands x, and the planner equates the marginal product of capital across all firms.

The maximum output that can be achieved with reallocation resource across firms, Y^* is defined as in Proposition 2.4.2,

$$Y^* = K^\alpha \left[\int \tilde{z}^{\frac{1}{1-\alpha}} d\mu(x_{-1}, k_{-1}, z_{-1}) \right]^{1-\alpha}$$

Accordingly, we can write the output as follows,

$$Y = M^{1-\alpha} \times K^\alpha \times \left[\frac{\int \tilde{z}^{\frac{1}{1-\alpha}} d\mu(x_{-1}, k_{-1}, z_{-1})}{M} \right]^{1-\alpha} \times \frac{Y}{Y^*}$$

where, $M = \int d\mu(x_{-1}, k_{-1}, z_{-1})$

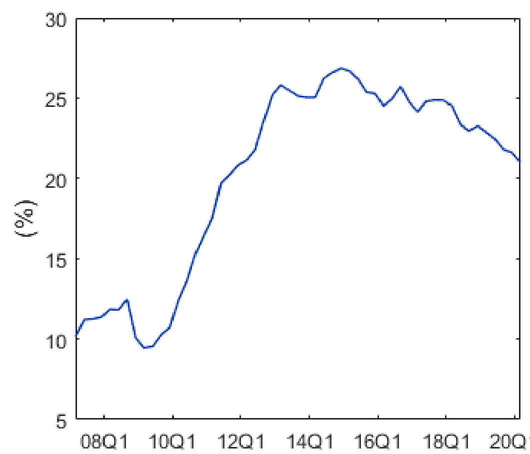
Appendix B

Appendix to Chapter 3

B.1 Foreign Holdings of LC Sovereign Debt Securities

	Average	Change (07Q1~20Q1)
Brazil	12.13	10.29
Colombia	11.58	9.38
Hungary	26.30	-2.64
Indonesia	30.84	16.34
Malaysia	24.04	15.37
Mexico	26.50	18.48
Philippines	5.00	4.64
Poland	28.69	6.87
South Africa	5.85	5.61
Thailand	11.64	10.35
Turkey	15.95	1.53
(Mean[2])	20.17	10.01

(a) By country



(b) Average of sample countries

Notes: (1) Table reports the average LC debt held by foreign investors as % of total outstanding.
(2) The equal weighted mean of the 11 country means.

Sources: Arslanalp and Tsuda (2014), IIF

B.2 Data description and source

	Description	Sources
Nominal LC yield spread	Unhedged 5-year zero-coupon LC government yield over US treasury yield.	Bloomberg
Swap rate	5-year implicit forward premium of LC (Calculated by spot rate from fixed LC for US Libor, cross currency swap, less spot rate from fixed US for Libor interest swap)	Bloomberg
Credit spread	Swapped 5-yr zero-coupon LC sovereign yield over US treasury yield. (Nominal LC yield spread less swap rate)	Author's calculation (Du & Schreger)
Foreign holdings of LC sovereign bond (level)	Outstanding central government debt securities denominated in local currency held by foreign investors.	Arslanalp and Tsuda
Foreign holdings of LC sovereign bond (share)	Share of Foreign holdings of LC sovereign bond as a percentage of total outstanding LC government bonds.	Arslanalp and Tsuda IIF
Exchange rate	Local currency units relative to US dollar	Bloomberg
Exchange rate volatility	Estimated exchange rate volatility with Garch (1,1)	Author's estimation
Real GDP growth rate	Percentage change in real GDP corresponding to the quarter of the previous year.	IFS
$\frac{\text{Government Debt}}{GDP}$	Debt owed by country's general government sector as a percentage of nominal GDP	National Institutes of each country
Banks' claims on government	Sum of net claims on (central government, local government, public nonfinancial) / Total claims	IFS
Bank's claims on private sector	Banks' claims on the non-financial private sector.	IFS
Vix	30 day implied volatility of the S&P,	FRED (St.Louis Fed)
Ted Spread	the spread between 3-month dollar Libor and the 3-Month Treasury Bill	FRED (St.Louis Fed)
Fed Funds Rate	the effective overnight Federal Funds Rate	FRED (St.Louis Fed)
BBB-Treasury Spread	the option-adjusted spread of the Bank of America Merrill Lynch US Corporate BBB Index over US Treasuries	FRED (St.Louis Fed)
10-Year Treasury Spread	10-yr Treasury constant maturity rate	FRED (St.Louis Fed)

B.3 Sovereign LC Bond Yield Spread Decomposition

(%)

	Yield Spread	Swap rate (Currency risk)	Credit spread
Brazil[2]	9.64	6.59	3.05
Colombia[3]	5.19	3.93	1.29
Hungary	3.20	1.28	1.92
Indonesia	5.82	5.56	0.26
Malaysia	1.58	0.67	0.91
Mexico	4.50	3.86	0.64
Philippines	3.14	1.86	1.28
Poland	1.88	0.96	0.92
South Africa	5.85	5.61	0.24
Thailand	0.85	0.23	0.63
Turkey	10.13	9.13	1.01
(Mean)	4.71	3.61	1.10

Notes: This table reports the average daily nominal yield spread (LC over US treasury bond), cross currency swap rate (currency risk) and credit spreads from Jan.2007 to June 2020. Brazil data starts from Apr.2007 and Columbia data ends at Nov.2019 due to data availability. The figures indicated with mean is the equally weighted mean of the 11 countries.
Sources: Bloomberg, Author's calculation

B.4 Foreign Holdings of LC Bonds and Banks Balance Sheet Compositions

	Corr(ΔFH, ΔPrivate)	Corr(ΔFH, ΔGov)
Brazil	0.73	-0.77
Colombia	0.14	-0.40
Hungary	0.41	-0.40
Indonesia	0.11	-0.41
Malaysia	0.68	0.29
Mexico	-0.20	-0.17
Philippines	-0.26	0.01
Poland	0.24	-0.39
South Africa	0.19	-0.04
Thailand	-0.02	0.14
Turkey	0.62	-0.69
(Mean)	0.24	-0.26

Notes: ΔFH is %p change in the share of LC debt held by foreign investors (yoy). ΔPrivate is the growth of banks' private claims net of the growth of total claims. ΔGov is an annual growth of banks' claims on the government. Mean is the equally weighted mean of the 11 countries' correlations.
Sources: Arslanalp and Tsuda (2014), Intitute of International Finance, IFS

B.5 Credit Vulnerability and Financial Development

$$\Delta \text{ Private Credit }_{i,t} = \Delta \overbrace{\text{Foreign Holding }_{i,t}}^{\text{FH}_{i,t}} \overbrace{(\gamma_0 + \gamma_1 \text{ FD}_i + \gamma_2 \text{ FD}_i \times \text{High Debt}_i)}^{\gamma\text{: Credit channel vulenrability}} + \beta \text{Controls}_{i,t} + \epsilon_{i,t} \quad \text{(B.1)}$$

Table B.1: Estimates of equation B.1

		Dependent variable: ΔPrivate Credit		
γ_0	$\Delta \text{FH}_{i,t}$	0.314***	0.338***	0.269***
		(0.088)	(0.091)	(0.103)
γ_1	$\Delta \text{FH}_{i,t} \times \text{FD}_i$	-0.005**	-0.004**	-0.005**
		(0.002)	(0.002)	(0.002)
γ_2	$\Delta \text{FH}_{i,t} \times \text{FD}_i \times \text{High Debt}_i$	0.004***	0.003**	0.004**
		(0.002)	(0.002)	(0.002)
	Observations	570	570	570
	Local Controls	Y	Y	Y
	Global Controls	N	Y	N
	Time FE	N	N	Y
	R-squared	0.17	0.20	0.24

Notes: FD_i indicates country i's mean ratio of liquid liabilities to GDP, and High Debt_i is an indicator that country i's debt to GDP is higher than sample EMEs median. Standard errors are shown in parentheses. *** $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

B.6 Credit Vulnerability and Relative Financial Development

$$\Delta \text{ Private Credit }_{i,t} = \Delta \overbrace{\text{Foreign Holding }_{i,t}}^{\text{FH}_{i,t}} \underbrace{(\gamma_0 + \gamma_1 \text{ RF}_i)}_{\gamma\text{: Credit channel vulenrability}} + \beta \text{Controls}_{i,t} + \epsilon_{i,t} \qquad \text{(B.2)}$$

Table B.2: Estimates of equation B.2

		Dependent variable: ΔPrivate Credit	
γ_0	$\Delta \text{FH}_{i,t}$	0.314*** (0.059)	0.380*** (0.109)
γ_1	$\Delta \text{FH}_{i,t} \times \text{RF}_i$	-0.138*** (0.039)	-0.125*** (0.040)
Observations		570	570
Local Controls		Y	Y
Global Controls		N	Y
Time FE		Y	N
R-squared		0.21	0.20

Notes: RF_i indicates country i's mean ratio of liquid liabilities to debt as a percentage. Standard errors are shown in parentheses. *** $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

$$\Delta \text{ Private Credit }_{i,t} = \Delta \overbrace{\text{Foreign Holding }_{i,t}}^{\text{FH}_{i,t}} \underbrace{(\gamma_0 + \gamma_1 \text{ High}_i)}_{\gamma\text{: Credit channel vulenrability}} + \beta \text{Controls}_{i,t} + \epsilon_{i,t} \qquad \text{(B.3)}$$

Table B.3: Estimates of equation B.3

		Dependent variable: ΔPrivate Credit	
γ_0	$\Delta FH_{i,t}$	0.469*** (0.099)	0.242*** (0.072)
γ_1	$\Delta FH_{i,t} \times High_i$	-0.316*** (0.082)	-0.282*** (0.084)
Observations		570	570
Local Controls		Y	Y
Global Controls		N	Y
Time FE		Y	N
R-squared		0.20	0.24

Notes: $High_i$ is an indicator that country *i*'s mean ratio of liquid liabilities to debt is higher than sample countries' median. Standard errors are shown in parentheses. *** $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

B.7 Relative Financial Development, Default Risk and External Vulnerability

$$\underbrace{\text{Credit Spread}_{i,t}}_{\text{Default risk}_{i,t}} = \text{High Risk}_t \underbrace{(\alpha_0 + \alpha_1 \text{RF}_i)}_{\alpha\text{: External vulenrability}} + \beta_f \text{RF}_i + \beta \text{Controls}_{i,t} + \epsilon_{i,t} \tag{B.4}$$

Table B.4: Estimates of equation B.4

		Dependent variable: Credit Spread (bp)	
β_f	RF_i	-23.96*** (3.18)	-26.45*** (3.23)
α_0	High Risk_t	172.20 *** (69.16)	63.99*** (38.05)
α_1	$\text{High Risk}_t \times \text{RF}_i$	-25.14* (13.53)	-25.79* (14.38)
	Observations	581	581
	Local Controls	Y	Y
	Global Controls	N	Y
	Time FE	Y	N
	R-squared	0.37	0.26

Notes: High Risk_t is an indicator that the VIX index is above the average of 1.5 times of the St.Dev. RF_i is country i's mean ratio of liquid liabilities to debt. Standard errors are shown in parentheses. *** $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Curriculum Vitae

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