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RURAL-URBAN MIGRATION, SURPLUS LABOR,
AND INCOME DISTRIBUTION

by

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Abstract

This paper is a theoretical analysis of rural-urban migration in a less-developed country (LDC). Urban unemployment caused by a fixed minimum wage, and surplus labor and variable hours of work in rural areas are the main features of the model. Since these features characterize the labor scene in many LDC's, the model presents a more realistic analysis of rural-urban migration. It also provides a better link with the "surplus-labor" literature which has been a major strand of recent research in economic development. Income distribution effects of economic policies recommended to deal with urban unemployment are also discussed.
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Transfer of labour from agriculture to industry has been the cornerstone of many a model of economic development in recent years. In this context, the concept of "disguised unemployment" or "surplus labor" has figured prominently in the writings of Lewis (1954), Fei and Ranis (1964), Sen (1966), Zarembka (1970, 1972), and others. According to this concept, when some workers leave the farm, total output does not decrease because either the marginal product of labor is zero or, what is more likely, those who remain on the farm increase their labor input to keep total work-hours constant. Migration from rural to urban areas has been taking place on a large scale in many developing countries, often in spite of open unemployment, higher cost of living, and scarcity of housing and other facilities in the cities. Although the concept of surplus labor has received much attention in other development literature, most models which deal with this curious aspect of migration, or with rural-urban migration in general, assume no surplus labor, nor do they make any distinction between workers and hours of work which assumes that every worker puts in a fixed number of work-hours (see, for example, Todaro (1969), and Harris and Todaro (1970)). These assumptions simplify the analysis in most cases but they hardly characterize peasant economies where farming is more a way of life than just a means of livelihood. They are particularly unrealistic in a migration context because in many developing countries it
is quite common for a family to send one of its members in search of urban employment while the rest of them try to minimize the resulting loss of output by increasing their labor inputs. Much controversy prevails about the existence of redundant labor in such economies; nevertheless, there can be little doubt that work-hours in agriculture are variable and open to choice by a worker or the peasant family. ¹

In this paper we formulate a two-sector model of rural-urban migration which, among other things, allows for surplus labor and flexible hours of work in agriculture. As in Sen's study, the labor-leisure choice and the migration decision are integrated into a simple decision model of a peasant family. It has been noted in a number of surveys, for example by Caldwell (1969) in Ghana, that migration takes place mostly in search of "economic gain." Like Harris and Todaro, we assume that in the labor market, "economic gain" is measured by the differences in expected real wage in the two sectors. We also retain the assumption of a minimum wage in industry which leads to urban employment. The model, besides making more realistic assumptions, will provide a better link between the work on rural-urban migration and the labor-surplus literature which has been a major strand of recent research in economic development. It will also enable us to shed more light on the strange phenomenon, analyzed by Harris and Todaro as well, that migration continues despite the fact that new migrants face the grim prospect of immediate unemployment in the city. Finally, we shall consider the income-distribution effects of migration and various economic policies which have been recommended to deal with this problem. ² The main results are derived for a "small country" which treats the commodity-price ratio as exogenous. Effects of relaxing this assumption, however, are also discussed.
I. A Model of Rural-Urban Migration

We assume that the economy consists of two sectors—agriculture and industry. The agricultural sector comprises of identical peasant families, with a given stock of capital and land, \( N_A \) working members, and \( T \) total members (\( T \geq N_A \)). The agricultural production function is given by:

\[
X_A = q(F, K_A, L) \tag{1}
\]

where \( X_A \) is the output of the agricultural good, \( F \) and \( K_A \) are the fixed stocks of land and farm capital respectively, and \( L \) is total labor input in hours. The function is assumed to be smooth (twice differentiable throughout and subject to diminishing marginal productivity of labor, i.e., \( q'' < 0 \)). The working members contribute equal labor hours and income is divided equally among all family members.

\[
L = N_A \cdot \ell \tag{2}
\]

and

\[
X_A = T \cdot x_A \tag{3}
\]

where \( x_A \) and \( \ell \) respectively are output per capita and labor input per worker.

The manufacturing production function is given by:

\[
X_M = f(N_M, K_M), f' > 0, f'' < 0 \tag{4}
\]

where \( X_M \) is the output of the manufactured good, \( N_M \) is total labor, and \( K_M \) is the fixed capital stock in industry. Unlike agriculture, no
distinction is made between workers and hours of work in industry. It is assumed that everyone puts in the same, fixed hours of work. It is further assumed that workers are paid a real wage \( \bar{W}_M \) equal to the value of their marginal product, and there is a minimum wage \( \bar{W}_M \) in force.

\[
\bar{W}_M = f' = \bar{W}_M
\] (5)

The commodity-price ratio depends on the outputs of the two commodities:

\[
P = p(X_M / X_A), \quad p' > 0.
\] (6)

where \( P = P_A / P_m \) is the price of agricultural good in terms of the manufactured good. The value of marginal product in agriculture (real wage) is

\[
W_A = P \cdot q' \cdot \ell
\] (7)

Recall that \( q' \) is the marginal physical product per hour and \( \ell \) is the number of hours worked by each worker.

We shall assume that there is no excess demand for labor in industry at the minimum wage so that the inequality sign can be dropped from equation (5) [\( \bar{W}_M = f' = \bar{W}_M \)]. The expected real wage in industry \( \bar{W}_M^{e} \) is

\[
\bar{W}_M^{e} = \bar{W}_M \frac{N_M}{N_u} \quad \frac{N_M}{N_u} \leq 1
\] (8)

where \( N_u \) is the total number of workers, both employed and unemployed, in the urban area, and \( N_M/N_u \) is the probability of being employed in industry. The labor constraint for the economy is

\[
N_u + N_A = \bar{N}
\] (9)
Workers will continue to migrate from agriculture to industry as long as the expected urban wage exceeds the real wage in agriculture. The equilibrium condition in the labor market, therefore, is

\[ W_A = W_M^e \]  \hspace{1cm} (10)

Equation (10) implies that leisure time in the city does not affect the migration decision of a peasant. Since an industrial job requires fixed hours, leisure time is largely beyond the control of an individual worker anyhow. If workers attach a different value to leisure in the two areas, migration equilibrium will be characterized by an inequality in (10). Similarly, it is possible that those who are lured by "city lights" might attach considerable value to just being in the city, so they will migrate even if \( W_A > W_u^e \). On the other hand, as in Keats' celebrated sonnet, one who has been long in city pent might move back to the country even if \( W_A < W_M \).

Replacing the equality in (10) by an inequality, however, complicates the algebra and makes it very difficult to derive any definite results.

**The Labor-Leisure Choice in Agriculture**

Following Sen (1966) we assume that every agricultural worker has a personal disutility function \( V \) related to his individual labour \( \lambda \), and each of the \( T \) individuals has a personal utility function \( U \) which relates his utility to income \( x_A \), or \( x_A/P \)--the amount of manufactured good which can be purchased with \( x_A \).

\[ U = U(x_A), \quad U' > 0, \quad \text{and } U'' \leq 0 \]  \hspace{1cm} (11)

\[ V = V(\lambda), \quad V' \geq 0, \quad \text{and } V''(\lambda) \geq 0 \]  \hspace{1cm} (12)

Peasants attach equal weights to everybody's utility or disutility, and
the family welfare $W$ is expressed as

$$W = \sum_{i=1}^{T} U_i - \sum_{i=1}^{N_A} V_i$$

$$= T \cdot U - N_A \cdot V \quad (13)$$

Each person’s labour input $\ell$ is so chosen (by himself or the head of the family) as to maximize $W$. Here we can distinguish between two cases: subsistence agriculture, and production for the market. 4

**Subsistence Agriculture**

If none of the agricultural output is marketed, the allocation rule for maximizing family welfare is

$$q' = \frac{V'(\ell)}{U'(x_A)} \quad (14')$$

The right-hand side is the rate of indifferent substitution between income and leisure for a worker. Individuals keep working up to the point where the marginal product of labor is just equal to the marginal rate of substitution between leisure and income. Whenever a worker leaves agriculture, the rest of the family has an option to adjust their hours of work or to leave them unchanged. It is highly likely that if a family decides to send one of its members to look for wage employment in industry, the family will try to minimize the resulting loss in agricultural output, especially if the migrant has to be supported for some time in the city. The work-hour decision in the face of migration is illustrated in Figure 1.

Income or output per capita ($x_A$) is measured on the $y$-axis, and leisure, which reaches a maximum at $Z$, is shown on the $x$-axis. The initial equilibrium is at $E$ where $(14')$ is satisfied: workers work for $ZH$ hours and enjoy $OH$ hours of leisure. When one worker moves to industry, the ratio of
workers to land in agriculture goes down. The output curve shifts out because, for each level of labor input, the marginal product of labor will now be higher. Three cases are distinguished in Figure 1. In 1(b), those who remain on the farm do not change their work hours, so total labor hours decline proportionately. In Figure 1(c), the indifference curves are parallel straight lines. The slopes of the output curves, and hence the marginal product of labor in the new and old equilibria, have to be equal. This is possible only if the total number of work hours is constant. Each worker, therefore, reduces leisure and increases work hours proportionately. The same result follows if the elasticity of substitution between land and labor is zero (Figure 1(d)). The latter two cases correspond to the classical description of labor-surplus agriculture in which the marginal product of a laborer is zero. At the other extreme is the first case where individuals do not change their leisure-labor choice in response to migration. These results will be very useful later in the paper when income distribution effects of different policies are discussed.

Production for the Market

If all agricultural output is sold on the market in exchange for the manufactured good, individual's utility depends on $x_M$, the amount of manufactured good that can be purchased with $x_A$. Since there are many peasant families, we assume that each of them believes that its decisions would not affect the commodity-price ratio. $P$ can therefore be treated as exogenous to the individual's decision making. In such a case family welfare is maximized when

$$q' = \frac{v'(q)}{U'(x_M)} \cdot \frac{1}{P}$$

(14)

which is similar to (14') except that now the commodity-price ratio has to be explicitly taken into account.
The specification of the model is now complete. It consists of equations (1)–(13) and either (14) or (14'). The endogenous variables are the two outputs \((X_A, X_M)\) and the commodity-price ratio \(P\), number of workers \((N_A)\), work hours \((\ell_1\) and \(L)\), wage rate \((W_A)\), output per worker \((x_A)\), family welfare \((W)\) and individual utilities and disutilities \((U\) and \(V)\) in agriculture, and employment \((N_M)\), unemployment \((N_u)\), and the actual and expected wage rates \((W_M, W_M^E)\) in the manufacturing sector. A given minimum wage determines marginal product of labor in industry, industrial employment, and hence output of the manufactured good. Peasants decide the number of workers in agriculture and their work hours, taking into account the wage rate and the employment situation in industry, and once the outputs of the two commodities are known, the commodity-price ratio is determined. Equilibrium value of all the endogenous variables can thus be determined for any given minimum wage. Here it should be noted that the model as set out above has one degree of freedom which is used up by specifying the minimum wage. If this wage rate was allowed to be endogenous, one of the other endogenous variables such as \(P, N_M, N_A\), etc. could be institutionally determined.

**Income Distribution in the Model**

There are five factors of production in this model: land, labor, and capital in agriculture; and labor and capital in manufacturing. Since labor is the only mobile factor in the model, and other factors remain fixed in each sector, it is useful to distinguish between labor and non-labor income. The functional distribution of income, which is the only one discussed here, thus will be described by the incomes of labor and capital in manufacturing, and those of labor, land, and capital in agriculture. This classification is consistent with a broad range of institutional arrangements.
in both sectors. For example, if we assume that peasants own all land and capital in agriculture, the entire rural sector can be treated as one big family. On the other hand, if peasants merely supply labor, income of labor can be estimated separately, and different estimates can be made according to whether the workers are paid their marginal value product or something else.

The distribution of factor incomes in this framework depends mainly on employment and output in the two sectors, the institutional arrangements in use, and the commodity-price ratio. Real labor income in industry (i.e., income expressed in terms of the manufacturing good) is $\bar{w}_{M}N_{M}$. Real income of capital in industry is $X_{M} - \bar{w}_{M}N_{M}$. The total real income in agriculture is $FX_{A}$, and if workers are paid the marginal value product, the real wage bill is $Pq'\cdot L$, and $P(X_{A} - q'\cdot L)$ will be the income of land and capital.

The critical decision variable in this model is work hours in agriculture. Whenever migration takes place, those remaining on the farm reconsider their labor-leisure choice. If they let total work hours decline, agricultural output will fall. However, the commodity-price ratio will rise, other things being equal. Real income of farmers, thus, could increase. In making their migration decisions, farmers will consider the marginal utility of income, marginal disutility of work, employment conditions in industry and the commodity-price ratio.
Figure 1: Individual Worker's Labor-Leisure Choice: Subsistence Farming
II. The Small Country Case

Consider a small country for which the commodity-price ratio is determined on the world market. Treating $P$ as exogenous, let us analyze the effects of changing the urban minimum wage. Differentiate equilibrium conditions (5), (10), and (14) with respect to $\bar{W}_M$ to obtain:

$$f'' \frac{dN_M}{d\bar{W}_M} = 1 \tag{15}$$

$$\left( Pq'' \frac{q'}{N_A} + \frac{Pq'}{N_A} \right) \frac{dL}{d\bar{W}_M} - \frac{dN_A}{d\bar{W}_M} \left[ \frac{\bar{W}_M N_M}{(N-N_A)^2} + \frac{Pq'L}{N_A^2} \right] = \frac{N_M}{N-N_A} \left( 1 + \frac{\bar{W}_M}{f''N_M} \right) \tag{16}$$

$$\left( Pq'' - \frac{V''}{U} \frac{1}{N_A} + \frac{Pq'}{T} \frac{V'}{U'} \frac{U''}{U'} \right) \frac{dL}{d\bar{W}_M} + \left( \frac{V''}{U} \frac{L}{N_A^2} - \frac{V''}{U'} \frac{PfAX}{T^2} \frac{k}{U'} \right) \frac{dN_A}{d\bar{W}_M} = 0 \tag{17}$$

Now define the following elasticities:

$y$ - the (absolute value of) elasticity of marginal utility of income with respect to individual income \( y = -\frac{U''(x_M)}{U'(x_M) x_M} \);

$h$ - the elasticity of marginal disutility of labor w.r.t. hours of work \( h = \frac{y''(L)}{y'(L)} \cdot L \);

$m$ - the (absolute value of) elasticity of marginal product of labor w.r.t. work-hours in agriculture \( m = -\frac{q''(L)}{q'(L) L} \); and

$r$ - the elasticity of output-price ratio w.r.t. the output ratio \( r = \frac{P'}{P} \cdot \frac{X_M}{X_A} \);

$e$ - the (absolute value of) wage elasticity of employment \( -\frac{dN_M}{d\bar{W}_M} \cdot \frac{\bar{W}_M}{N_M} \).
From (15) we know that \( \frac{dN_M}{d\bar{W}_M} = 1/f'' \). Using this result and the elasticities defined above, equations (16) and (17) can be rewritten as

\[
\begin{bmatrix}
\frac{P_M}{N} (1-m) \\
- \frac{P_M}{L} (m+b) \\
\end{bmatrix} \begin{bmatrix}
\frac{dL}{d\bar{W}_M} \\
\frac{dN_A}{d\bar{W}_M} \\
\end{bmatrix} = \begin{bmatrix}
\frac{N_M}{N-N_A} \left(1 + \frac{f'}{f''} \right) \\
0 \\
\end{bmatrix}
\]  
(18)

or \( D \cdot B = A \)

where \( w = \frac{\bar{W}_M}{W_A} \) is the ratio of manufacturing wage to agricultural wage, \( D \) denotes the 2 x 2 matrix on the L.H.S. and \( B \) and \( A \) represent the column vectors on the left- and right-hand sides respectively, of (18).6 Expansion of \( |D| \) yields

\[
|D|^2 = \left( \frac{w_A}{L} \right)^2 [y(I-m-b) - m(I+h) - (m+h+by)w_NN_A/(N-N_A)^2]
\]  
(19)

If \( |m+b| \geq 1, |D| < 0 \). On the other hand, if \( |m+b| < 1 \), which implies that \( m \) will be very small because \( 0 < b < 1 \), \( |D| \) can be non-negative. In the surplus labor case, with \( y = h = 0 \),

\[
|D| = -\left( \frac{w_A}{L} \right)^2 \left[ 1 + wNN_A/(N-N_A)^2 \right] < 0.
\]

In what follows, we shall consider cases of both positive and negative values for \( |D| \).

Using Cramer's rule we obtain:

\[
\frac{dL}{d\bar{W}_M} = \frac{1}{|D|} \frac{w_A}{L} (y+h) \frac{N_M}{(N-N_A)} (1-e)
\]  
(20)

If \( |D| < 0 \), \( dL/d\bar{W}_M \leq 0 \) as \( e \leq 1 \). Note, however, that in the surplus labor case also \( |D| < 0 \), but \( y = h = 0 \), so \( dL/d\bar{W}_M = 0 \) regardless of the magnitude.
of e. If \(|D| > 0\), \(\frac{dN_A}{d\bar{w}_M} \leq 0\) as \(e \geq 1\). Using the same procedure once again we get:

\[
\frac{dN_A}{d\bar{w}_M} = \frac{1}{|D|} \frac{Pq'}{L} (m+h+by) \frac{N_M}{N-N_A} (1-e)
\]  

(21)

If \(|D| < 0\), \(\frac{dN_A}{d\bar{w}_M} \leq 0\) as \(e \leq 1\). When \(|D| > 0\), the inequalities are reversed, and \(\frac{dN_A}{d\bar{w}_M} \leq 0\) as \(e \geq 1\). Observe from equation (21) that in the surplus labor case, although \(|D| < 0\), \(\frac{dN_A}{d\bar{w}_M}\) is not independent of e. This is in direct contrast with the result derived above for \(\frac{dL}{d\bar{w}_M}\).

These results are easily explained because e determines how expected urban wage responds to changes in \(\bar{w}_M\). If \(e = 1\), both the wage bill and expected wage in the urban sector remain constant. There is thus no incentive for migration in either direction. Nevertheless, if \(e < 1\), expected urban wage increases when the minimum wage is raised. Consequently, more workers move to the city and \(N_A\) goes down (\(dN_A/d\bar{w}_M < 0\)). If \(|D| > 0\), \(\frac{dL}{d\bar{w}_M}\) and \(dN_A/d\bar{w}_M > 0\): there is reverse migration to the rural sector and total labor input goes up. Although expected manufacturing wage increases when \(\bar{w}_M\) is raised, peasants increase their labor input so much that \(Pq' \lambda > \bar{w}^e_M\). We know that \(dL/d\bar{w}_M = \frac{1}{N_A} \left( \frac{dL}{d\bar{w}_M} - \frac{L}{N_A^2} \frac{dN_A}{d\bar{w}_M} \right)\). By substituting from (20) and (21) we get:

\[
\frac{dL}{d\bar{w}_M} = \frac{N_M}{N-N_A} \frac{1}{N_A} \frac{1}{|D|} \frac{W_A}{L} (1-e) [y(1-b) - m]
\]

As long as \(y > |by+m|\), a condition necessary to make \(|D| > 0\), \(dL/d\bar{w}_M > 0\) for \(e < 1\).
Migration and the Labor Market Equilibrium

We know that the marginal value product in agriculture equals $Pq'x$, and $\bar{w}_M$ is equal to $f'$. Then, using (8) and (9), the equilibrium condition in the labor market, (10), can be written as

$$E = p \left( \frac{x_M}{x_A} \right) q'x' - \frac{f'N_M}{N-N_A} = 0 \quad (10')$$

To analyze the effects of rural-urban migration, let us first consider how changes in $N_M$ and $N_A$ affect $E$. Differentiating (10') partially with respect to $N_A$ we get:

$$\frac{\partial E}{\partial N_A} = \frac{w_A}{N_A} n (1-m) - \frac{w_A}{N_A} - \frac{f'N_M}{(N-N_A)^2} \quad (22)$$

where $n = \frac{dL}{dn} \frac{A}{L}$ is the elasticity of total work-hours in agriculture with respect to the number of farm workers. Clearly, the sign of $\frac{\partial E}{\partial N_A}$ depends on $n$ and $m$. It can be verified from (21) that $\frac{dL}{dn} A$, and hence $n$, is equal to zero in the labor surplus case and greater than zero in general. Therefore, if $m > 1$, and in the labor surplus case, $\frac{\partial E}{\partial N_A} < 0$. The sign of (22) can be positive if $m < 1$.

Differentiating (10') with respect to $N_M$ yields:

$$\frac{\partial E}{\partial N_M} = \frac{f''N_M}{N-N_A} (1-e) \left[ \frac{w_A}{L} \right]^2 \frac{(1-m)(y+h)}{|D|} - 1 \quad (23)$$

If $e = 1$, $\frac{\partial E}{\partial N_M}$ is zero; otherwise its sign depends on $e$, $m$, and $|D|$. Here it is interesting to point out that $e < 1$ is sufficient to make $\frac{\partial E}{\partial N_M} > 0$ in the Harris-Todaro model. In our model, this result follows only if, in addition, the terms within square brackets in (23) are negative, for example, when $m = 1$, or $(1-m)/|D| < 0$. In the surplus labor case, $m$ does not matter.
Since \( y = h = 0 \),

\[
\frac{\partial E}{\partial N_M} = \frac{-f''}{N-N_A} (1-e)
\]

which is \( \geq 0 \) as \( e \leq 1 \).

The slope of the line depicting the equilibrium condition \( E = 0 \) in \( N_A, N_M \) space is given by \(-\left( \frac{\partial E}{\partial N_M} / \frac{\partial E}{\partial N_A} \right)\). By combining (22) and (23) it is clear that this slope is not unique and it can be negative just as easily as it can be positive. Further, there is no unique way of getting the same slope, positive or negative. Consider, for example, the labor surplus case. We know that \( \partial E/\partial N_A < 0 \). The slope of the \( E = 0 \) line is \( \leq 0 \) as \( e \leq 1 \). In general, however, even if \( e > 1 \), this slope can be positive, when \( m > 1 \) and \( |p| > 0 \), for instance.

Three different cases are presented in Figure 2. In all cases only the minimum wage at point \( M \) is consistent with full employment throughout the economy. Starting at \( M \), an increase in minimum wage will lower manufacturing employment. At the new equilibrium \( Q \), however, agricultural labor force is larger than before in Figure (2a), smaller than before in (2b), but it remains unchanged in Figure (2c) where \( e = 1 \). Output of the manufactured good does go down in all cases, but how work-hours and output in agriculture respond depends on \( e \). In all cases in which \( e = 1 \), equations (20) and (21) tell us that agricultural work force, total labor hours and hence farm output, remain constant. if \( e \neq 1 \), agricultural output can increase or decrease, except in the labor surplus case when it remains unchanged.

Income distribution effects of changing the industrial minimum wage can now be determined. Whenever \( \tilde{W}_M \) is raised, manufacturing employment and output decline. The urban wage bill will remain unchanged if \( e = 1 \). If \( e < 1 \), there will be fewer jobs but workers will have higher real incomes than before.
Figure (2a)

Figure (2b)

Figure (2c)
Since \( P \) is unchanged, real income of capital in manufacturing will decline (\( e < 1 \)) remain constant (\( e = 1 \)), or increase (\( e > 1 \)). In agriculture, as long as there is surplus labour, output remains constant. If some workers move to the city, those who remain on the farm shall earn a higher average real income. However, the opposite result will follow if \( \frac{dN_A}{d\bar{w}_M} \) is positive. As noted above, in all cases in which \( e = 1, N_A, L \), farm output, and hence real incomes of peasants will not respond to any changes in the urban minimum wage.

III. The Case with Endogenous \( P \)

In this section we try to derive the type of results reported above for the small country case using the full model set out in Section I. The commodity-price ratio is endogenously determined. As before, by differentiating (5), (10), and (14) with respect to \( \bar{w}_M \) we obtain after simplification:

\[
\begin{align*}
\left[ \begin{array}{c}
\frac{Pq'}{N_A} \frac{(1 - m - rb)}{N_A} \\
- \frac{Pq'}{L} (m + h + by(1 - r) + br) \\
\frac{Pq'}{N_A} (h + y)
\end{array} \right] &= \frac{dL}{d\bar{w}_M} = \left[ \begin{array}{c}
\frac{N_M}{N - N_A} (1 - e) + re \frac{N_M}{N_A} \\
\frac{N_M}{N - N_A} \\
\frac{br N_M (1 - y) e}{L}
\end{array} \right]
\end{align*}
\]

or \( D' \cdot B = A' \)

Expansion of \( |D'| \) yields

\[
\left( \frac{W_A}{L} \right)^2 \left[ y(1 - m - b) - h(m + rb) - (m + br) - (m + h + by(1 - r) + br) w_N N_M \left[ \frac{(N - N_A)^2}{N_A} \right] \right]
\]

(25)

If \( |m + b| \geq 1, \) and \( r \) or \( y \leq 1, |D'| < 0. \) In the surplus labor case,

\[
|D'| = - \left( \frac{W_A}{L} \right)^2 \left[ (m + br)(1 + w_N N_M/(N - N_A)^2) \right] < 0
\]

(26)
A comparison of these two results with (18) and (19) suggests that, in
general, the sign of the 2 x 2 determinant now depends also on r, the elastic-
ity of P with respect to relative outputs. However, if there is surplus
labor, it does not matter if P is endogenous or not; the determinant is
unambiguously negative in either case.

Expressions for \( \frac{dL}{d\bar{w}_M} \) and \( \frac{dN_A}{d\bar{w}_M} \) are somewhat more complicated now.

Using Cramer's rule we get:

\[
\frac{dL}{d\bar{w}_M} = \frac{1}{|\bar{D}|} \left[ \left( \frac{N_M}{N-N_A} (1-e) + \frac{e b r N_M}{N_A} b \right) \frac{Pq'}{N_A} (h+y) + \frac{e b r N_M}{L} (1-y) \frac{W_A}{N_A} \left( 1 + w \frac{N_A N_M}{(N-N_A)^2} \right) \right]
\]

(27)

\[
\frac{dN_A}{d\bar{w}_M} = \frac{1}{|\bar{D}|} \left[ \frac{Pq'}{N_A} (1-m-rb) \frac{e b r N_M (1-y)}{L} + \frac{Pq'}{L} (m+h+by(1-r)+br) \left( \frac{N_M}{N-N_A} (1-e) + \frac{e b r N_M}{N_A} b \right) \right]
\]

(28)

The sign of \( \frac{dL}{d\bar{w}_M} \) depends on \( |\bar{D}|, e \) and \( y \), and \( |\bar{D}|, m, r, b \), and \( y \) will deter-
mine the sign of \( \frac{dN_A}{d\bar{w}_M} \). These signs are obviously not unique.

The Surplus Labor Case

More definite results can be derived in this case because \( y = h = 0 \)
and \( |\bar{D}| < 0 \).

\[
\frac{dL}{d\bar{w}_M} = \frac{1}{|\bar{D}|} \frac{e b r N_M}{L} \frac{W_A}{N_A} \left( 1 + w \frac{N_A N_M}{(N-N_A)^2} \right)
\]

(29)

\[
\frac{dN_A}{d\bar{w}_M} = \frac{1}{|\bar{D}|} \frac{Pq'}{L} \left( \frac{N_M b r}{N_A} + \frac{N_M (m+br) (1-e)}{N-N_A} \right)
\]

(30)

\( \frac{dN_A}{d\bar{w}_M} < 0 \) if \( e \leq 1 \), otherwise its sign is ambiguous. Both (29) and (30) are
different from the corresponding expressions for the small country case.
Consider a situation in which \( e = 1 \) so that any change in \( \overline{w}_M \) does not affect the expected urban wage. In the small country case, total work-hours in agriculture do not change (\( dL/d\overline{w}_M = 0 \)), hence \( q' \) stays the same and no new migration takes place. In the present case, however, the higher urban wage disturbs the initial equilibrium given by (10'): the expected urban wage is unchanged but \( P \) falls. Equilibrium is restored as more workers move to the city (\( dN_A/d\overline{w}_M < 0 \)), which increases unemployment and lowers expected wage. In agriculture, \( L \) declines (\( dL/d\overline{w}_M < 0 \)), so \( q' \) rises, and farm output falls to stem the decline in the commodity-price ratio.

**Migration and the Labor Market Equilibrium Once Again**

As in the small country case, to analyze the effects of migration more fully, let us differentiate (10') partially w.r.t. to \( N_M \) and \( N_A \). We get:

\[
\frac{\partial E}{\partial N_M} = \frac{w_A}{L} (1 - m - br) \frac{dL}{dN_M} - \frac{f'' N_M}{N - N_A} (1 - e) + \frac{f' br}{N_A} \quad \text{for } N_A \text{ const.} \tag{31}
\]

\[
\frac{\partial E}{\partial N_A} = \frac{w_A}{L} (1 - m - rb) \frac{dL}{dN_A} - \frac{w_A}{N_A} - \frac{f' N_M}{(N - N_A)^2} \quad \text{for } N_M \text{ const.} \tag{32}
\]

It is difficult to determine the signs of (31) and (32) unambiguously.

\( dL/dN_M = f'' dL/d\overline{w}_M \), and \( f'' < 0 \), but as discussed above, in general, \( dL/d\overline{w}_M \) has an indeterminate sign. Similarly, \( dL/dN_A = \frac{dL}{d\overline{w}_M} \left( \frac{dN_A}{d\overline{w}_M} \right) \) and this ratio also has no unique sign in the general case. In the surplus labor case, we know that \( dL/dN_M > 0 \). Therefore, if \( e \leq 1 \), and \( |m + rb| \leq 1 \), \( \frac{\partial E}{\partial N_M} > 0 \).

Now, if \( e = 1 \), we know from (30) that \( dN_A/d\overline{w}_M < 0 \). Since \( dL/d\overline{w}_M < 0 \) (equation 29), \( dL/dN_A > 0 \). Now if \( |m + rb| \geq 1 \), \( \frac{\partial E}{\partial N_A} < 0 \). In this very
special case, the slope of the \( E = 0 \) line in \( N_A, N_M \) space will be positive. However, if \( m + rb = 1 \), and \( e > 1 \), both this slope and \( \partial E/\partial N_M \) can be negative. 9

In summary, the above discussion shows that, as in the small country case, the \( E = 0 \) line does not have a unique slope in the \( N_A, N_M \) space. Many more parameters are involved when \( P \) is allowed to be endogenous. Consequently, even in case of surplus labor, several additional restrictions are required to derive any definite results about the effects of changing urban minimum wage and rural-urban migration in general.

Analysis of income distribution effects is also more complicated now. To illustrate, consider the labor surplus labor case depicted in Figure (2b). When \( P \) is exogenous, the migration equilibrium \( (E = 0) \) line is horizontal. The wage elasticity of urban employment equals unity. An increase in minimum wage reduces urban employment proportionately, and the urban wage bill remains constant. Since \( P \) is assumed to remain constant, real earnings of capital in manufacturing decline. In agriculture, the number of workers, work hours, output and real incomes remain unchanged. When \( P \) is allowed to fluctuate (the full model), however, \( e = 1 \) is not sufficient to ensure a unique slope for the \( E = 0 \) line in the labor surplus case. In addition, \(|m + rb|\) has to equal one, and then this line has a positive slope. An increase in minimum wage, thus, will reduce \( N_A \) and more peasants will move to the city. As in the small country case, manufacturing output will decline and the real urban wage bill will remain unchanged. Nevertheless, in agriculture, equation (29) tells us, total work hours will decline. Farm output will therefore be lower. Since output of both goods is falling, it is not clear what the commodity-price ratio would be in the new equilibrium. If the original \( P \) is restored, the real value of farm output will still be lower than before, but the effect on real earnings of land and labor will be uncertain. 10
IV. Employment Policies and Income Distribution

It was noted in the last section that a minimum wage above the
market-clearing level causes unemployment and loss of output in the
economy. In this connection, several first-and second-best policies have
been recommended in the literature to restore full employment, or to at
least improve the economy's welfare. This section deals with the effects
of such policies on the functional distribution of income. For brevity, we
discuss only the case in which $P$ is endogenous, and all farm output is
marketed.

Wage Subsidy in Manufacturing

It is recommended by Bhagwati and Srinivasan (1974) and others that
since the unemployment is in the urban areas, a wage subsidy to manufac-
turing will improve welfare. The equilibrium, in the presence of a wage
subsidy $S$, will be given by:

\[ f'_M = \bar{W}_M - S \quad (33) \]

\[ Pq'(L)L = \bar{W}_M \frac{N_M}{N - N_A} \quad (34) \]

As the subsidy is increased, more workers will be employed in manufacturing,
the expected wage rate will go up and migration out of the rural areas will
increase. Effects of these happenings on income distribution can be easily
determined. Total earnings of labor in industry will increase. Since
manufacturing output is increasing, the earnings of capital, expressed in
terms of the manufactured good, will also increase. The effect on agricul-
tural output and income will depend on how hours of work in agriculture
respond. If total work-hours remain unchanged, agricultural output stays
constant. However, because of a higher $X_M'$, the commodity-price ratio rises, so the market value of agricultural output will increase. Moreover, a higher $P$ and constant $L$ suggest that real wage bill in agriculture will also go up. If work-hours and output in agriculture go down, $P$ will rise more than before, but agricultural output is lower. Since both $P$ and $q'$ have risen, the real income of labor in agriculture will also increase unless there is a more than offsetting reduction in hours of work, $L$.

**Employment Policies in Agriculture**

Since workers move out of agriculture, the unemployment problem could be ameliorated by giving incentives to workers to stay on in agriculture. These incentives can take the form of a wage or a production subsidy to agriculture (the two are equivalent in terms of this model). If a production subsidy is introduced in the rural sector, the equilibrium conditions will be:

$$f' = \bar{w}_M$$

(35)

$$P^* q' = \bar{w}_M \frac{N_M}{N - N_A}$$

(36)

where $P^* = (P_A + t)/P_M$ and $t$ is the subsidy per unit of output in agriculture. The commodity-price ratio facing producers in agriculture is higher than before. If the labor market was in equilibrium before the production subsidy was introduced, now the real farm wage will exceed the expected wage in industry, causing workers to migrate back to the rural areas. Agricultural output will expand. There will be no change in
employment, output, or real factor incomes in manufacturing as long as the minimum wage is unaltered. Both P and L are higher. Real labor income in agriculture, therefore, will rise unless $m$, the elasticity of marginal product of labor with respect to labor hours is substantially greater than 1.

Here it should be noted that the employment subsidy in manufacturing or the production subsidy to agriculture, introduced by itself is a second-best policy. The former introduces a distortion in the labor market, while the latter causes the producer-price ratio to diverge from the marginal rate of transformation in production. Without knowing all the parameters of the model, therefore, the two policies cannot be ranked in terms of their welfare effects. This difficulty with ranking policies has received much attention in the literature on the "second-best," and it has been raised by Harris-Todaro, and Bhagwati and Srinivasan in the context of rural-urban migration. As the above discussion shows, the income distribution effects of these policies are quite different. A wage subsidy in manufacturing will increase the real earnings of both labor and capital in manufacturing. Real labor income in agriculture could go down but, by varying their hours of work, farmers will prevent it. A production subsidy in agriculture, on the otherhand, will leave real incomes in manufacturing unchanged. The real value of farm output, however, will be higher and real earnings of farm labor are also likely to increase. The two policies could thus be ranked in terms of their income distribution effects.11

V. Conclusions and Summary

In this paper we have presented a theoretical model of rural-urban migration. The main features of the model are urban unemployment, an industrial minimum wage, and variable hours of work in agriculture. The
model provides an analytical framework for examining the distributional effects of migration and various economic policies designed to reduce or eliminate urban unemployment. Different concepts of surplus labor in agriculture are also considered. The analysis shows that the work-hour decision in agriculture, after some workers move to the city, holds the key to the income distribution effects of various employment policies. Many conclusions, derived by Harris and Todaro on the implicit assumptions of fixed individual work-hours, thus are significantly altered. The model sets out the main variables which govern the work-hour decision in agriculture, and identifies other parameters which also affect the functional income distribution.
Footnotes

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1There is a vast theoretical and empirical literature on surplus labor. See, for example, Lewis (1954), Jorgenson (1967) and Zarembka (1972). Kao, Anschel and Eicher provide a useful survey, and some empirical evidence is presented by Cho. On the question of variability of work-hours, see Sen (1966).

2For ease of exposition we shall use the terms farm, agriculture, and rural areas interchangeably. Since most of the employment available in towns is nonagricultural, we shall not distinguish among the terms industry, towns, and urban areas.

3Some readers of an earlier version of this paper have suggested that all workers, especially the new migrants, may not have the same probability of finding a job in industry. Also, what matters in the migration decision is not the actual but the subjective probability of employment in industry—employment prospects as foreseen by each worker, and these will vary from one worker to the next, and from time to time. Harberger (1973), while dealing with the Panamanian case, illustrates the point by assuming that the migrant in effect buys a series of lottery tickets—say a 40 per cent chance of a job after two months, a 60 per cent chance after six months, and so on. A probability distribution will then replace $N_M/N_u$ in (8).

Along the same lines, workers' attitude towards risk could also be introduced. These points will certainly make the discussion more realistic, but they would not add much to the analysis of variable hours of work and surplus labor which are the main concerns of the paper.

4Sen (1966) derives some of these results algebraically by totally differentiating (14) with respect to $N_A$, the number of workers on the farm. See his equations (27)-(37) and the discussion thereafter. The extreme case of surplus labor is characterized by flat regions in the marginal utility and marginal disutility schedules, which correspond to straight-line indifference curves in Figure 1. Figure 1 is adapted from Zarembka (1972) who derives these results algebraically also.

5Besides the assumptions already stated, we assume that non-labor resources are fully divisible and can be reallocated after some workers move to the city. Since there is a large number of peasant families, we shall treat $T$ and $N_A$ as continuously divisible. Finally, we assume that both $U'$ and $V' > 0$. 
All the results derived here relate to the case in which all farm output is marketed. The only change required for subsistence farming is to drop $P$ from the second row of $|D|$. 

Since the model has only one degree of freedom, there is no stipulation that the minimum wage is being held constant when $N_M$ or $N_A$ changes.

The case in which $|e| < 1$ is of interest because Harris-Todaro cite some empirical evidence by Katz (1968), and Erickson (1969) that wage elasticity of employment is indeed $< 1$.

As in the last section, all results relate to commercial agriculture. Results for the subsistence farming case, although not presented in the text, are very similar. In general, $\frac{dN_A}{dW_M}$ and $\frac{dL}{dW_M}$ have ambiguous signs. Therefore, the $E = 0$ line does not have a unique slope.

From (6) we obtain:

$$\frac{dP}{dW_M} = \frac{P'}{X_A} \left( \frac{f'}{f} - \frac{X_M}{X_A} q' \frac{dL}{dW_M} \right)$$

$$= - \frac{rP}{W_M} \left( \frac{s}{e + bz} \right)$$

where $s = \frac{W_N}{X_M}$ is the share of labor in manufacturing, $b$, as defined above, is the labor's share in agriculture, and $z = \frac{W}{L} \frac{dL}{dW_M}$. Clearly, $\frac{dP}{dW_M}$ can be $\geq 0$.

We know that in the new equilibrium $L$ is lower than before, so $q'$ is higher. Since the sign of $\frac{dP}{dW_M}$ is ambiguous, it is not clear how $Pq' \cdot L$ will change.

It has been assumed throughout the above discussion that the subsidy in either sector can be financed by lump-sum taxation. Taxation, however, can itself be used to modify the income distribution.
References


