Patterned Heating Induced Propulsion

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Abstract

This study explores propulsion effects generated by patterned heating acting on smooth and corrugated surfaces. The model problem assumes that the upper plate moves freely, and the lower plate is stationary, equipped with grooves, and exposed to spatially distributed heating. Our findings identify two distinct propulsion effects: thermal streaming and thermal drift. Thermal streaming occurs when given sufficient heating intensity with net flow in the left or right direction characterized by a pitchfork bifurcation. The efficiency of this technique can be controlled using the wavelength of heating. Thermal drift represents a pattern interaction effect. Its strength depends on the relative positions of the heating and groove patterns and is most significant when the groove peaks and surface hot spots are quarter wavelengths apart. Changing the heating pattern position relative to grooves permits a change of direction of the propulsive effect. Strengths of propulsive effects increase with a reduction of Prandtl number and with the addition of a uniform heating component.

Keywords
Convection, Periodic heating, Spatial heating, Corrugations, Surface irregularities, Propulsion
Imagine two flat surfaces, one placed over the other. The top one can move, while the bottom one is fixed, has grooves, and is heated in a specific pattern. This study investigates how these heating and groove patterns can make the top plate move by itself. The movement of the top plate is influenced by the behaviour of the fluid between the plates when the bottom plate is heated. The first way this happens is through a process called thermal streaming. This process occurs when we heat the bottom plate intensely. The fluid between the plates starts to move, similar to how water circulates in a pot when it gets hot. This fluid movement, in turn, causes the top plate to move. The second process is known as thermal drift. It happens when we align the heating patterns with the groove patterns on the bottom plate. With proper alignment, the fluid moves. This movement then causes the top plate to move as well. By adjusting this alignment, we can change the direction of the top plate’s movement and its velocity. Furthermore, we discovered that the velocity of the top plate can be increased by using a better-conducting fluid and by adding uniform heating to the lower plate. In simple terms, this study demonstrates that by manipulating the form of heating and topography of the lower plate, we can generate and control the movement of the upper plate. This finding can be used for designing innovative propulsion processes with various applications.
Co-Authorship Statement

This thesis, structured in a monograph format, comprises Chapters 3 and 4 that reflect work based on two scholarly papers. One paper is published in the Journal of Fluid Mechanics, and the other is under review, both co-authored with Dr. Jerzy M. Floryan and Dr. Shoyon Panday. The entirety of the work in this thesis, including the content paralleling these papers, was executed by the author. Excluded from this thesis is the stability section found in the papers, as this segment was not part of the author's contributions.
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Lastly, my heart goes out in profound thanks to my parents, Dr. Salma Shirin and Dr. Ahmed Aman. Nurturing my passion for learning and discovery in the most organic and inspiring way, their sacrifices have been the foundation on which I have built my academic pursuits.
There are many more individuals who have contributed to my journey towards this thesis, each playing a significant role in their own right. I am immensely grateful for their contributions, however big or small.

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List of Abbreviations and Nomenclature

**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>RB</td>
<td>Rayleigh-Bénard</td>
</tr>
<tr>
<td>IB</td>
<td>Immersed Boundary</td>
</tr>
<tr>
<td>IBC</td>
<td>Immersed Boundary Conditions</td>
</tr>
</tbody>
</table>

**Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ra$</td>
<td>Rayleigh Number</td>
</tr>
<tr>
<td>$(x, y)$</td>
<td>Physical cartesian coordinate system</td>
</tr>
<tr>
<td>$h$</td>
<td>Half of the average channel height</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>$A$</td>
<td>Amplitude of the groove at the lower plate</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Wavenumber</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Phase difference between temperature and groove profiles</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Relative temperature</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Thermal diffusivity</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$c$</td>
<td>Specific heat</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Thermal expansion coefficient</td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity in the x-direction</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity in the y-direction</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>x-component of stress</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>y-component of stress</td>
</tr>
<tr>
<td>$n$</td>
<td>Normal unit vector</td>
</tr>
</tbody>
</table>
$F$  
Force

$F_x$  
x-component of force

$F_y$  
y-component of force

$U_{top}$  
Mean velocity of upper plate

$\psi$  
Stream function

$N_{vv}, N_{v\theta}$  
Nonlinear terms of momentum and energy equations

$\tilde{v}v, \tilde{u}u, \tilde{w}w, \tilde{v}\theta, \tilde{u}\theta$  
Velocity and temperature products in the physical space

$\tilde{v}v^{(n)}, \tilde{u}u^{(n)}, \tilde{w}w^{(n)}, \tilde{v}\theta^{(n)}, \tilde{u}\theta^{(n)}$  
Modal functions of the velocity and temperature products $\tilde{v}v, \tilde{u}u, \tilde{w}w, \tilde{v}\theta, \tilde{u}\theta$

$G_{u_k}^{(n)}$  
Chebyshev coefficients for the velocity in the x-direction

$G_{v_k}^{(n)}$  
Chebyshev coefficients for the velocity in the y-direction

$G_{p_k}^{(n)}$  
Chebyshev coefficients for the velocity in the x-direction
\( G\psi_k^{(n)} \) Chebyshev coefficients for stream function

\( G\theta_k^{(n)} \) Chebyshev coefficients for temperature

\( G\tilde{v}_k^{(n)}, G\tilde{u}_k^{(n)}, G\tilde{v}_k^{(n)}, G\tilde{\theta}_k^{(n)}, G\tilde{\theta}_k^{(n)} \) Coefficients of the Chebyshev expansions of the modal functions in the Fourier expansions representing velocity and temperature products

\( T_{k+1} \) \( k \)-th Chebyshev polynomial

\( \hat{y} \) Y coordinate in the computational domain

\( N_M \) Number of Fourier Modes

\( N_T \) Number of Chebyshev Polynomials

\( RF_{\phi} \) Residual for the momentum equation

\( RF_{\theta} \) Residual for the energy equation

\( \epsilon \) Phase difference between two wavenumbers

\( \beta, \gamma \) Multi-modal wavenumbers

\( H(x) \) Multi-modal shape function
\( K(x) \)  
Multi-modal temperature distribution

**Superscripts**

*  
Superscript denoting dimensional quantities

\( n \)  
Index for Fourier modes

**Subscripts**

\( L \)  
Subscript denoting lower plate

\( U \)  
Subscript denoting upper plate

\( uni \)  
Subscript denoting uniform value of temperature and Rayleigh number at the lower wall

\( per \)  
Subscript denoting periodic value of temperature and Rayleigh number at the lower wall

\( mean \)  
Subscript denoting mean value

\( crit \)  
Critical value

\( v \)  
Subscript denoting viscous component
\( p \) Subscript denoting pressure component
Chapter 1

1 Introduction

The concept of propulsion, the act of driving or pushing forward, is a fundamental aspect of fluid mechanics. It is the force that moves fluid through a conduit or propels a body through a fluid medium. Propulsion systems are integral to a wide range of applications, from the operation of analytical chemistry systems (Dasgupta & Liu, 1994) to the functioning of biological systems (Peerlinck et al., 2023).

Propulsion without moving parts is an area that remains unexplored; this is the focus of this research. Past literature has documented flow generated by wind and temperature variations as a means of ventilation using the directional buoyancy forces generated by the temperature differentials (Linden, 1999). Similar techniques previously used in drag reduction studies (Hossain & Floryan, 2020) will be evaluated for their latent potential to induce propulsion in this study. This research will explore how the interaction between a solid surface and the adjacent fluid can be manipulated through spatial temperature and geometrical modifications to create propulsion. This thesis will focus on two propulsion mechanisms: nonlinear thermal streaming and thermal drift.

Thermal streaming is a phenomenon where patterned heating is used to generate relative motion between parallel plates, while thermal drift involves the use of both heating and groove patterns to produce propulsion. The interaction between these patterns and their subsequent effects on the propulsive strength forms the focus of this investigation.
1.1 Motivation

The primary aim of this thesis is to conduct an in-depth exploration of a novel concept in propulsion technology: heating-driven propulsion. This research seeks to establish a fundamental understanding of the mechanisms and principles of heating-driven propulsion, a topic that has never been addressed in existing scientific literature. Central to this study is the examination of the fundamental physics that govern this innovative method of propulsion.

The core focus of the study is to build a comprehensive theoretical framework for heating-driven propulsion. This involves a detailed analysis of the fundamental physics underpinning this method. The aim is to elucidate and articulate the scientific concepts that define heating-driven propulsion, emphasizing the physical principles and mechanisms involved. This analysis forms the foundation for future theoretical and experimental investigations in this area.

Traditional propulsion systems are known to have certain limitations and drawbacks. In contrast, heating-driven propulsion systems potentially offer distinct advantages, such as scalability in smaller applications where traditional methods may be impractical, enhanced control mechanisms, absence of mechanical moving parts leading to reduced wear and tear, lower noise levels, and decreased risk of contamination in sensitive environments.

These compelling benefits, along with the exploration of the fundamental physics of heating-driven propulsion, present numerous opportunities for further exploration and development of these systems. This foundational research takes a significant step towards
unlocking the full potential of heating-driven propulsion techniques and paving the way for innovative applications.

1.2 Literature Review

The literature review focuses on the studies and findings relevant to the effects of heating patterns and surface topography modifications on fluid flows. This section provides a critical examination of past research and identifies the gaps in knowledge that this study seeks to explore.

1.2.1 Heating Patterns and Fluid Flow

Convection can be influenced by the structure of the flow system and the externally applied thermal conditions. The effect of externally imposed heating patterns on a flow field has been a subject of interest for researchers for several decades. Studies have shown that localized heating can significantly impact fluid flow and heat transfer characteristics (Torrance & Rockett, 1969). Researchers have investigated various heating strategies, which can generally be divided into uniform and non-uniform heating categories. Given the vast array of possible heating patterns, the exploration of convection has mainly been focused on straightforward idealized cases.

A prominent example of such a reference problem is the thoroughly studied natural Rayleigh-Bénard (RB) convection. Observing this phenomenon involves a relatively simplistic setup – fluid sandwiched between two horizontal parallel smooth plates is
uniformly heated from the lower plate, inducing a vertical temperature gradient (Fig. 1-1). This gradient in temperature leads to a vertical density gradient in the fluid, as the fluid's density is inversely related to its temperature. In response to this density difference, buoyancy arises—a force that acts on volumes of fluid with varying densities due to a gravitational field. Gravity attempts to pull the cooler, denser fluid downward while the warmer, less dense fluid is pushed upward, creating a buoyant effect. The competition between this buoyancy-driven motion and the viscous and diffusive restoring forces is quantified by the Rayleigh number, $Ra$. As the temperature difference between the top and bottom plates increases, so do the buoyancy forces and the Rayleigh number. When the Rayleigh number surpasses a certain critical threshold, which is dependent on the boundary conditions of the system, buoyancy becomes the dominating force. This overcomes the stabilizing effects of viscosity and diffusion, leading to fluid instability and the onset of convective motion. RB convection can form structured cells with circular motion, as shown in Fig. 1-1. Since its first experimental observation in 1900 (Bénard, 1900), RB convection has undergone extensive investigation, evolving into a well-established phenomenon exploited in applications like polymerase chain reaction tests, as described in a study by Krishnan et al. (2002, 2004). However, as noted in the study, due to the required temperature difference needed to produce RB convection, the primary control parameter is forced to be the geometry.
Non-uniform heating patterns can induce convection at any heating intensity because of the spatial variation of temperature in the horizontal direction. This introduces horizontal temperature gradients, causing a forced response in the system and, thus, fluid movement. The temperature gradients cause modifications to the topology of the flow and cause convection (Fig. 1-2). This convection depends on the heating intensity and heating wavelength; therefore, it offers much more control over the structure of the flow topology. Due to the infinite number of patterns possible, the majority of research has concentrated on the most straightforward pattern, sinusoidal heating (Hossain & Floryan, 2013). The sinusoidal heating profile creates vertical and horizontal temperature gradients, resulting in motions referred to as horizontal convection.
The study of horizontal convection has also been explored as a technique for controlling flow in channels. The appropriate heating-cooling pattern induces a buoyancy force field, leading to the formation of separation bubbles, minimizing direct contact with the walls and modifying the Reynolds stress distribution, resulting in a propulsive force and a reduction in shear stress. This phenomenon is known as the super-thermo-hydrophobic effect (Hossain et al., 2012; Hossain & Floryan, 2016). Horizontal convection has also been studied for flow control in channels, where it has been found to reduce pressure losses (Inasawa et al., 2019) and the driving force in the case of relative motion between two parallel plates (Floryan et al., 2018).

1.2.2 Surface Topography Modifications

Surface topography modifications have also been widely studied for their ability to passively influence fluid flows. Research has shown that altering surface roughness or introducing structured patterns, such as riblets (Fig. 1-3), can significantly affect the flow characteristics and friction resistance (Bixler & Bhushan, 2015).
Surface roughness that affects flow behaviour is a classic topic in fluid dynamics. Traditional assumptions held that any roughness would increase the surface area and consequently elevate flow resistance due to friction (Hagen, 1854; Darcy, 1857). However, this notion was contested by Walsh (1980), who illustrated that specific surface geometries could mitigate drag in turbulent flows. Researchers found that the groove shape, spacing and height have strong influences on the groove’s drag reduction effectiveness on the flow (Choi et al., 1993; Bechert et al., 1997; Goldstein et al., 1998; Ao et al., 2021).

Mohammadi and Floryan (2012) delved into the mechanisms underlying drag formation by examining a channel with sinusoidal transverse grooves. Their investigation pinpointed three primary mechanisms: one hinged on wall shear stress, the second centred on the relationship between the mean pressure gradient and the surface’s structure, and the third emerged from the interplay between the periodic pressure field and the wall's geometry.

In separate research, Mohammadi and Floryan (2013a) assessed pressure loss in channels with grooves for laminar flows, discovering the possibility of reducing laminar drag by employing suitably designed grooves. Furthermore, their research explored the drag reduction capabilities of streamwise grooves, leveraging optimization strategies to
determine the most effective groove shapes for maximizing pressure loss reduction (Mohammadi & Floryan, 2013b).

1.2.3 Combined Effects of Heating Patterns and Surface Topography

While heating patterns and surface topography modifications have individually been the focus of numerous studies, the literature on their combined effects is relatively limited. Recent contributions to the literature have focused on thermal drift, a technique produced by the interaction of grooves and heating patterns. Abtahi and Floryan (2017, 2018) examined natural convection in a horizontal fluid layer subjected to heating and geometric variations. They took into account a periodic distribution for both the heating and lower plate geometry, measuring their relative positions using a phase difference. Their findings suggested that the combination of the heating and groove could generate a horizontal flow without the need for a mean pressure gradient. The direction of this flow can vary based on the phase difference between the heating and groove configurations.

Previous studies have shown that certain combinations of heating and groove patterns can significantly reduce flow pressure losses (Hossain & Floryan, 2020). The work of Inasawa et al. (2021) has also shown the potential of using a combination of heating and groove patterns for fluid transport.

1.2.4 Numerical Modelling

This research intends to scrutinize both smooth and grooved channel configurations. Given the incorporation of small-amplitude grooves in this study, an enhanced approach to
numerical modelling of the channel boundaries that is proficient in capturing the nuanced near-wall flow fields is imperative. Consider the Immersed Boundary (IB) method, which employs regular computational domains extending beyond the physical boundaries. One of its advantages is it eliminates the need for a grid that conforms to the geometry of the computational domain, thereby simplifying the grid generation process and enhancing its computational efficiency. Over time, various researchers have introduced numerous iterations of the IB method, as documented by Mittal (2005) and Peskin (2002). However, many IB methods suffer from limited spatial accuracy because they rely on lower-order techniques like finite difference, finite volume, or finite element, as highlighted by Mittal (2005).

The code used to conduct this research was developed and discussed by Panday and Floryan (2021). This code uses the Immersed Boundary Conditions (IBC) method, developed by Szumbarski and Floryan (1999). The IBC method amalgamates the efficiency intrinsic to the IB method with the elevated accuracy derived from spectral discretization (Husain & Floryan, 2008). When using the IBC method, the field equation gets discretized using a Fourier expansion in the horizontal direction when assuming a periodic flow. This transforms the governing partial differential equation into ordinary differential equations. The vertical direction is discretized by using Chebyshev polynomials, leading to algebraic equations.
1.2.5 Propulsion

The realm of fluid dynamics presents a vast array of propulsion mechanisms, yet a few stand out owing to their non-traditional nature of inducing propulsion through altering the interaction between the solid boundary and the adjacent fluid.

One of the earliest explorations into this non-conventional propulsion was by Hinch in his 1971 publication. Hinch's work demonstrated how moving thermal waves, when propagated through a fluid, can induce significant streaming effects (Hinch, 1971). Additional research demonstrated how a heated object in immersed fluid can propel itself. These observations reveal how techniques using thermal variations in fluid systems can lead to transport mechanisms (Mercier et al., 2014).

The phenomena associated with heating-induced propulsion are not limited to laboratory scales. In a study on planetary scales, Malkus (1970) discussed the "Hadley-Halley Circulation on Venus." Malkus explored the large-scale atmospheric circulations on the planet, primarily driven by thermal gradients on Venus. The work highlighted how thermal-induced convection could dominate the fluid dynamics in planetary atmospheres.

Vibrations emerge as another compelling external stimulus for propulsion. Haq's study explored how wall vibrations in fluid-filled channels can lead to propulsion. The findings suggest that under specific conditions and vibrational patterns, fluid flow can be effectively manipulated, leading to propulsion (Haq & Floryan, 2022).
Research on propulsion has become of importance in the burgeoning field of microfluidics, primarily because conventional pumping and flow-control techniques often have challenges when miniaturized.

Weigl et al. showcased the revolutionary potential of microfluidic devices in drug discovery and biomedical applications. They emphasized that to harness the full potential of these devices, effective propulsion and fluid control mechanisms are indispensable, noting the necessity of having controlled flow patterns in miniaturized lab setups, especially when dealing with high-throughput assays and precision measurements (Weigl et al., 2003). As explained by Ebbens and Howse (2010), when venturing into the smaller realm of nanofluidics, there are even more challenges of inducing propulsion at such a delicate scale.

In conclusion, the literature review has presented an extensive overview of previous studies on heating patterns, surface topography modifications, and their effects on fluid flow. It has explored the substantial work done to understand how individual aspects like heating patterns or surface modifications can significantly influence fluid flow characteristics. However, it has also recognized the gap in the current body of literature concerning the combined effects of these two factors. While the research in this domain is relatively limited, recent studies suggest a promising potential for employing these combined factors for optimized fluid transport systems. Therefore, the present study's focus on exploring these combined effects aims to bridge this gap and contribute novel insights to this area of research.
1.3 Objectives

The primary objective of this research is to investigate the effectiveness of propulsion strategies using heating patterns and surface topography modifications in order to achieve fluid transportation.

The specific aims of this research include:

- Understanding thermal drift and nonlinear thermal streaming effects across various flow scenarios.
- Exploring the effects of various parameters on the propulsion. This includes analyzing the impact of different heating and groove patterns and their relative positions. An analysis of these variables will illuminate how they individually and collectively influence fluid transportation, thereby offering insights into optimizing propulsion strategies.
- Examining the conditions under which thermal streaming and thermal drift are active. This includes identifying the factors that influence the activation of these phenomena and how they interact with each other. Understanding the activation and interaction of these phenomena is pivotal for harnessing them effectively in practical applications.

By achieving these objectives, this research will contribute to the fundamental understanding of heating-driven propulsion and could lead to the development of innovative real-world propulsion techniques.
1.4 Overview of Present Work

This thesis is structured to facilitate a comprehensive exploration of the subject at hand. Chapter 1 serves as the introduction, outlining the scope of the research, detailing the objectives and motivations, and describing some foundational background concepts. This chapter aims to establish a contextual framework for the reader, setting the stage for the detailed exploration that follows. In Chapter 2, the focus shifts to modelling the problem. This section delineates the methods used to solve the flow, clearly explaining the research approach and methodology. Chapter 3 delves into an examination of the propulsion generated solely by heating. This part of the thesis explores the result of heating-induced propulsion. The exploration continues in Chapter 4, where the propulsion generated by the interplay of heating and grooves is examined. This chapter aims to uncover the combined effects of heating patterns and surface topography modifications, providing an analysis of their influence on propulsion. Concluding the thesis, Chapter 5 summarizes the principal findings and offers suggestions for future work based on this thesis’s reporting.
Chapter 2

2 Problem Formulation

2.1 Introduction

The focus of this thesis is to study the propulsive capabilities of thermal and geometry modulations on fluid. Due to the complexity of these modulations, this problem will be solved numerically by converting the problem into equations and boundary conditions and solving them using an algorithm described in the paper by Panday and Floryan (2021). Since a numerical approach will be used, approximate solutions will be calculated; however, these solutions will have very small error margins.

2.2 Geometry and Thermal Modulations

The system under consideration consists of two horizontal plates separated by a fluid layer, as depicted in Fig. 2-1. The fluid is incompressible and Newtonian.
Figure 2-1: Sketch of flow system

The upper plate is smooth, while the lower plate contains sinusoidal corrugations, leading to a gap geometry defined by the following non-dimensional equations:

\[ y_L^*(x^*) = -h^* + 0.5 A^* \cos(\alpha^* x^*) \]  
\[ y_U^*(x^*) = h^* \]  

(2.1.1a)  
(2.1.1b)

Where subscripts \( L \) and \( U \) describe the lower and upper plates, respectively, the \( ^* \) signifies dimensional quantities. The mean gap width is defined by \( 2h^* \). The gap extends for \( \pm \infty \) in the \( x^* \) direction. The gravitational acceleration \( g^* \) acts in the negative \( y \)-direction. The variable \( A^* \) is the groove amplitude, and the variable \( \alpha^* \) is the wavenumber, which modulates the wavelength \( \lambda^* = 2\pi/\alpha^* \). The upper plate is free to move in the \( x \)-direction, while the lower plate is fixed in space.
The lower plate is subject to patterned heating while the upper plate remains isothermal according to:

\[ T_L^*(x^*) = T_{uni}^* + 0.5T_{per}^* \cos(\alpha^*x^* + \Omega^*) \]  
\[ T_U^*(x^*) = T_U^* , \]  

(2.1.2a)

(2.1.2b)

where subscripts \textit{uni} and \textit{per} describe the mean and periodic heating components, respectively. The variable $\Omega^*$ refers to the phase shift between the groove period and the heating wavelength’s period. The relative temperature $\theta^*$ is described by $\theta^* = T^* - T_U^*$:

The relative temperatures along the upper and lower plates are described by the following equations

\[ \theta_L^*(x^*) = \theta_{uni}^* + 0.5\theta_{per}^* \cos(\alpha^*x^* + \Omega^*) \]  
\[ \theta_U^*(x^*) = 0. \]  

(2.1.3a)

(2.1.3b)

The fluid properties include thermal conductivity $k^*$, specific heat $c^*$, thermal diffusivity $\kappa^* = k^*/\rho^*c^*$, kinematic viscosity $\nu^*$, dynamic viscosity $\mu^*$, thermal expansion coefficient $\Gamma^*$, and variations in density $\rho^*$, are assumed to follow the Boussinesq approximation, which means that the algorithm only accounts for density variations, if they are multiplied by $g^*$, to ensure that this flow is buoyancy driven.

Using the mean gap height $h^*$ as the length scale and $\kappa^*\nu^*/(g^*\Gamma^*h^3)$ as the temperature scale, the expressions become,

\[ y_L(x) = -1 + 0.5A \cos(\alpha x) , \]  

(2.1.4a)
\[ y_U(x) = 1, \quad (2.1.4b) \]

\[ \theta_L(x) = Ra_{uni} + 0.5Ra_{per} \cos(\alpha x + \Omega), \quad (2.1.4c) \]

\[ \theta_U(x) = 0, \quad (2.1.4d) \]

where \( Ra_{uni} = g^* \Gamma^* h^3 T_{uni}/(\kappa^* \nu^*) \) and the \( Ra_{per} = g^* \Gamma^* h^3 T_{per}/(\kappa^* \nu^*) \). The Rayleigh number quantities describe the intensity of the respective heating components.

### 2.3 Governing Equations and Boundary Conditions

The field equations can be cast in the following dimensionless form,

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2.2.1a) \]

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \nabla^2 u, \quad (2.2.1b) \]

\[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \nabla^2 v + Pr^{-1} \theta, \quad (2.2.1c) \]

\[ u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = Pr^{-1} \nabla^2 \theta, \quad (2.2.1d) \]

where the velocities are scaled using the scaling \( U_v^* = v^*/h^* \), and the pressure is scaled using \( \rho^* U_v^{*2} \). The Prandtl number, \( Pr \), is defined by \( Pr = \nu^*/\kappa^* \).

The boundary conditions for the lower wall \((y = -1 + 0.5 A \cos(\alpha x))\) are:

\[ u = v = 0, \quad \theta = Ra_{uni} + 0.5 Ra_{per} \cos(\alpha x + \Omega). \quad (2.2.2a) \]
To ensure the upper wall is free to move, the boundary conditions for the upper wall \((y = 1)\) are:

\[
\left. \frac{\partial u}{\partial y} \right|_{\text{mean}} = 0, \quad u_{\text{per}} = 0, \quad v = 0, \quad \theta = 0. \tag{2.2.2b}
\]

where \(u\) and \(v\) are velocities in the \(x\) and \(y\) directions, respectively.

To ensure that only the thermal and geometric modulations are producing net fluid movement, there cannot be any pressure gradient influencing the flow. Therefore, the following constraint is imposed to establish no pressure differential in the horizontal direction:

\[
\left. \frac{\partial p}{\partial x} \right|_{\text{mean}} = 0 \tag{2.2.3}
\]

### 2.4 Plate Driving Force

To determine if the thermal and geometrical modifications can induce propulsion, the forces of the plates are examined. Identification of plate driving forces requires evaluating surface forces acting on the plates. The stress vector at the lower plate \(\tilde{\sigma}_L\) is expressed as:

\[
\tilde{\sigma}_L = \begin{bmatrix} \sigma_{x,L} & \sigma_{y,L} \\ n_{x,L} & n_{y,L} \end{bmatrix} = \begin{bmatrix} 2 \frac{\partial u}{\partial x} - p & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & 2 \frac{\partial v}{\partial y} - p \end{bmatrix} \tag{2.3.1}
\]
where \( \vec{n}_L = (n_{x,L}, n_{y,L}) = N_L \left( \frac{dy_L}{dx}, -1 \right) \), \( N_L = \left[ 1 + \left( \frac{dy_L}{dx} \right)^2 \right]^{-\frac{1}{2}} \), is the normal unit vector pointing outwards. The \( x \)-component of this vector has the form:

\[
\sigma_{x,L} = \sigma_{x,v,L} + \sigma_{x,p,L} = N_L \left[ 2 \frac{dy_L}{dx} \frac{\partial u}{\partial x} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] - N_L \frac{dy_L}{dx} p, \tag{2.3.2}
\]

where \( (\sigma_{x,v,L}, \sigma_{x,p,L}) \) denote the viscous and pressure contributions, respectively. The \( x \)-component of the total force \( F_{x,L} \) acting on the fluid at the lower plate per its unit length has the form:

\[
F_{x,L} = F_{x,v,L} + F_{x,p,L} = \lambda^{-1} \int_0^\lambda \left[ 2 \frac{dy_L}{dx} \frac{\partial u}{\partial x} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] dx - \lambda^{-1} \int_0^\lambda \frac{dy_L}{dx} p \ dx = 0 \tag{2.3.3}
\]

where \( F_{x,p,L} \) and \( F_{x,v,L} \) stand for the pressure and viscous contributions, which must be equal and opposite. Similar expressions for the upper plate have the form:

\[
\sigma_{x,U} = \sigma_{x,v,U} = \frac{du}{dy} \bigg|_{y=1}, \quad F_{x,U} = F_{x,v,U} = \lambda^{-1} \int_0^\lambda \frac{du}{dy} \bigg|_{y=1} \ dx = 0 \tag{2.3.4}
\]

and the total force must be null.

If convection generates mean shear stress at the upper plate, the free to move plate accelerates to a constant velocity, \( U_{\text{top}} \), in order to eliminate this stress and balance the forces acting on the plate. The \( U_{\text{top}} \) quantity is determined in postprocessing and measures the average velocity of the top plate over a period.
2.5 Numerical Solution

The system of equations must be solved numerically with sufficient precision to provide data with suitable quality for analyzing the effects of geometric irregularities and heating and their interactions. The spectral method is used for this purpose, as it provides both the necessary geometric flexibility, accuracy and the ability to handle many geometries with a minimum labour cost compared to traditional methods such as grid-based approaches.

The equations and boundary conditions expressed in equations 2.2.1-2.2.3 are expressed in terms of the stream function in order to eliminate the pressure terms. These equations are updated to take the form:

\[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \] (2.4.1)

Using this definition, the resulting system is governed by the following equations:

\[ \nabla^4 \psi - Pr^{-1} \frac{\partial \theta}{\partial x} = N_{\psi}, \quad \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = Pr N_{\psi \theta} \] (2.4.2)

where

\[ N_{\psi} = \frac{\partial}{\partial y} \left( \frac{\partial}{\partial x} \bar{u} \bar{u} + \frac{\partial}{\partial y} \bar{u} \bar{v} \right) - \frac{\partial}{\partial x} \left( \frac{\partial}{\partial x} \bar{u} \bar{v} + \frac{\partial}{\partial y} \bar{v} \bar{v} \right), \quad N_{\psi \theta} = \frac{\partial}{\partial x} \bar{u} \theta + \frac{\partial}{\partial y} \bar{v} \theta. \] (2.4.3)

The boundary conditions for the lower wall \((y = -1 + 0.5 A \cos(\alpha x))\) can be re-written as:

\[ \psi = \frac{\partial \psi}{\partial y} = 0, \theta = Ra_{uni} + 0.5 Ra_{per} \cos(\alpha x) \] (2.4.4)
This IBC concept allows the spectral method to accommodate the irregular shape of the lower plate without the need for a conforming mesh, streamlining the computational process and enhancing accuracy.

The boundary conditions for the upper wall ($y = 1$) can be re-written as:

$$\frac{\partial \psi}{\partial y} \bigg|_{per} = \psi \bigg|_{per} = 0, \frac{\partial \psi^2}{\partial y^2} \bigg|_{mean} = 0, \theta = 0 \quad (2.4.5)$$

and normalized with condition $\psi(y_L) = 0$. The x-dependencies of all unknowns and other required quantities were captured by expressing them as Fourier expansions.

$\left[u, v, p, \theta, \psi, \tilde{\nu}, \tilde{u}, \tilde{\nu}, \tilde{\psi}, \tilde{u} \theta \right](x, y) =$

$$\sum_{n=+N_m}^{n=-N_m}[u^{(n)}, v^{(n)}, p^{(n)}, \theta^{(n)}, \psi^{(n)}, \tilde{\nu}^{(n)}, \tilde{u}^{(n)}, \tilde{\nu}^{(n)}, \tilde{\psi}^{(n)}, \tilde{u} \theta^{(n)}] e^{in\alpha x} \quad (2.4.6)$$

where $u^{(n)}, v^{(n)}, p^{(n)}, \theta^{(n)}, \psi^{(n)}, \tilde{\nu}^{(n)}, \tilde{u}^{(n)}, \tilde{\nu}^{(n)}, \tilde{\psi}^{(n)}, \tilde{u} \theta^{(n)}$ represent modal functions. Expansions (2.4.6) were substituted into the field equations, and the Fourier modes were separated, resulting in the nonlinear, ordinary differential equations for the modal functions of the form

$$[\Gamma^4 D^4 - 2n^2 \alpha^2 \Gamma^2 D^2 + n^4 \alpha^4] \psi^{(n)}(\tilde{\gamma}) - i n \alpha Pr^{-1} \theta^{(n)}(\tilde{\gamma}) =$$

$$i n \alpha \Gamma D \tilde{u}^{(n)}(\tilde{\gamma}) + [\Gamma^2 D^2 + n^2 \alpha^2] \tilde{\nu}^{(n)}(\tilde{\gamma}) - i n \alpha \Gamma D \tilde{\psi}^{(n)}(\tilde{\gamma}) \quad (2.4.7a)$$

$$\Gamma^2 D^2 \theta^{(n)}(\tilde{\gamma}) - n^2 \alpha^2 \theta^{(n)}(\tilde{\gamma}) = Pr \Gamma \tilde{u} \tilde{\theta}^{(n)}(\tilde{\gamma}) + \Gamma D \tilde{\theta}^{(n)}(\tilde{\gamma}) \quad (2.4.7b)$$
The modal functions were expressed as Chebyshev expansions of the form

\[
\begin{bmatrix}
u^{(n)}, v^{(n)}, p^{(n)}, \theta^{(n)}, \psi^{(n)}, \nu \psi^{(n)}, \nu \theta^{(n)}, \nu \psi \theta^{(n)}
\end{bmatrix}(\hat{y}) \approx \sum_{k=0}^{N_T} \begin{bmatrix} G_u^{(n)}(k), G_v^{(n)}(k), G_p^{(n)}(k), G_\psi^{(n)}(k), G_\psi \theta^{(n)}(k), G_\psi \psi^{(n)}(k), G_\theta^{(n)}(k), G_\theta \psi^{(n)}(k), G_\theta \psi \theta^{(n)}(k) \end{bmatrix} T_{k+1}(\hat{y})
\] (2.4.7)

and the Galerkin projection method was employed to construct algebraic equations for the expansion coefficients. The discretization process provided spectral accuracy with the absolute error controlled by the number of Chebyshev polynomials \(N_T\) and the number of Fourier modes \(N_M\) used in the computations. Table 2-1 displays computed values of \(U_{top}\) for \(Ra_{per} = 1000, \alpha = 2\) and \(A = 0\) using various \(N_M\)’s and \(N_T\)’s.

<table>
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<tr>
<th>(N_T)</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>15</th>
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<td>3.4926</td>
</tr>
<tr>
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<td>3.4926</td>
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</tr>
<tr>
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</tr>
<tr>
<td>50</td>
<td>3.4915</td>
<td>3.4927</td>
<td>3.4927</td>
<td>3.4927</td>
<td>3.4927</td>
</tr>
</tbody>
</table>

Table 2-1. \(U_{top}\) at \(Ra_{per} = 1000, \alpha = 2\) and \(A = 0\) for various number of Fourier modes and Chebyshev polynomials. The red line identifies the accuracy limits.

Once the \(N_M\) and \(N_T\) have increased to where the results have consistent accuracy up to four decimal digits, the results are precise enough to be accepted. Table 2-1 shows calculated values of \(U_{top}\) for a single case using a range of \(N_M\) and \(N_T\). The red line identifies the range of \(N_M\) and \(N_T\) that produce acceptable results. Since there is a positive
correlation between the number of Fourier modes and Chebyshev polynomials used and the accuracy, this study used $N_M$ and $N_T$ beyond the accuracy limits shown in the table due to the wide variation of flow parameters used in this study, albeit at the expense of increased computing time. Generally, between 15 to 20 $N_M$ are used in the x-direction, and between 60 to 90 $N_T$ are used in the y-direction to achieve the desired accuracy.

Since the governing equations are nonlinear, they must be solved using an iteration process. The solution process begins by setting the non-linear terms in (2.4.2) to zero and solving the resulting system of equations. The flow quantities calculated in this manner are used to evaluate the non-linear terms which are then inserted into (2.4.2). This completes iteration zero. Iteration 1 starts by solving the complete equations using nonlinear terms which were evaluated in the previous iteration, computing new flow quantities as well as new values of the nonlinear terms. This process continues until the difference between solutions in two subsequent iterations becomes sufficiently small, i.e., the iteration process has converged.

There is no guarantee that the iteration process will converge. An under-relaxation is used to increase chances of convergence. The underrelaxation process for the momentum equation has the form

$$\psi_{n+1} = \psi_n + RF_\psi (\psi_{n+1,}\text{computed} - \psi_n) \quad (2.4.8)$$

where $RF_\psi$ is a relaxation factor for the momentum equations, and a value between 0 and 1, $\psi_n$ is the current stream function value, $\psi_{n+1,}\text{computed}$ is the new computed value, and $\psi_{n+1}$ is the new accepted value. This process for the energy equation has the form:

$$\theta_{n+1} = \theta_n + RF_\theta (\theta_{n+1,}\text{computed} - \theta_n), \quad (2.4.9)$$
where $RF_\theta$ is a relaxation factor for the energy equations, and a value between 0 and 1, $\theta_n$ is the current stream function value, $\theta_{n+1,\text{computed}}$ is the new computed value, and $\theta_{n+1}$ is the new accepted value. The relaxation factors used in the momentum and energy equations during this study were 0.025 and 0.0125, respectively.

Once the differences between the current values and next iteration values,

$$\psi_{n+1,\text{computed}} - \psi_n, \theta_{n+1,\text{computed}} - \theta_n,$$  \hspace{10mm} (2.4.10)

decrease below a certain threshold, the solution is assumed to be converged. The convergence error, or the accepted difference between solution in two subsequent iterations, is selected by the user. All results presented in this study were obtained with convergence criterion of $10^{-4}$. 
Chapter 3

3 Smooth Plate with Patterned Heating

3.1 Introduction

This chapter describes thermal streaming, an effect capable of creating propulsion that relies solely on patterned heating, which activates nonlinear thermal streaming. The effectiveness of this technique for inducing propulsion will be examined and quantified using the $U_{top}$ value. The configuration will consist of two parallel smooth plates without any geometry modifications and only thermal modulations applied at the lower plate. Eq. 2.1.4a, then becomes $y_L(x) = -1$.

3.2 Sinusoidal Heating at the Bottom Plate

One would assume that the application of a sinusoidal heating pattern at the bottom plate would only result in the convection of the fluid. However, the following results will

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1 A version of this chapter has been published as-

demonstrate how a symmetric heating profile can produce net movement of the fluid and upper plate.

The flow topologies over two periods are showcased in Fig. 3-1. Figure 3-1A illustrates flow and temperature patterns for $Ra_{per} = 790$. Due to the density variations and buoyancy forces, fluid above hot spots moves upward, forming heated plumes and drawing fluid along the lower plate from its left and right sides, resulting in the formation of counter-rotating rolls. The rolls form discrete cells which do not interact with each other and have left/right symmetry, thus producing zero mean shear stress at the upper plate, which results in no movement of this plate at this low heating intensity. Fig. 3-1B illustrates the flow and temperature fields for $Ra_{per} = 855$; the flow similarly remains stationary at this increased heating intensity and has the same topology. However, the heat plumes have noticeably grown in the vertical direction.

Fig. 3-1C shows the flow and temperature patterns for $Ra_{per} = 900$. This increase in heating intensity produces asymmetrical movement in the fluid. Instead of the rolls situated in discrete cells, there is now a stream tube carrying fluid meandering between the rolls. This stream tube flows in the x-direction, which is the same direction the heat plumes are tilting. This movement produces mean shear, pulling the upper plate in the direction of fluid moving in the stream tube. The rolls aid in propelling the stream tube in the net positive x-direction. Previous studies on patterned convection with the no-slip condition applied at a fixed upper plate observed similar convection and heat plume growth (Hossain & Floryan, 2013). However, the plumes would remain vertical in all cases due to the upper
plate holding the plumes and restricting any tilting. This study replaced the upper plate’s no-slip condition with zero-mean shear at the upper plate (2.2.2b). Due to this condition, the plumes have the freedom to tilt.

Fig. 3-1D shows the field and temperature fields for $Ra_{per} = 1100$. The heat plumes growth and tilting are more visible. The size of the counter-rotating rolls is dictated by the heating intensity, as shown in the difference in size of the upper rolls in Fig. 3-1C and Fig. 3-1D. This shrinkage of the upper rolls allows for the increase in the width of the stream tube weaving between the rolls. The volume of the stream tube, which makes up all the fluid being transported, can be associated with the magnitude of $U_{top}$. 
Figure 3-1. Flow and temperature fields for $Ra_{unt} = 0$, $Pr = 0.71$, $\alpha = 2$ and $Ra_{per}$ (A) 790, (B) 855, (C) 900, (D) 1100. Dashed lines mark stream tubes carrying fluid in the horizontal direction. The background colour illustrates the temperature field scaled with its maximum.
Now that the inducing mechanism for thermal streaming is found to be the intensity of heating, the flow’s response as the heating intensity is changed must be investigated. The change in the character of fluid motion as the heating intensity varies is best represented by plotting variations of $U_{top}$ as a function of $Ra_{per}$. Figure 3-2 demonstrates that the upper plate has no movement until a critical heating value is reached, where the system can respond in two ways, with net fluid movement in either the positive or negative x-direction; this net movement will be referred to as nonlinear thermal streaming. The system’s transition from a state without upper plate movement to two possible moving states is characterized as supercritical pitchfork bifurcation. As shown in Fig. 3-4, the heating wavelength can affect the critical value required for bifurcation. The bifurcation points are $Ra_{per, cr} = 884.5, 1151.22, 1120$ for $\alpha = 2, 2.8, 1.2$, respectively. This correlates with the previously demonstrated contour plots, which had $\alpha = 2$, where it is shown that flow is only induced when heating intensity exceeds $Ra_{per} = 884.5$. 
A relatively large heating intensity produces stronger plumes, which may either tilt to the right or left. Plume tilting results in a loss of the flow symmetry, leading to the formation of a stream tube carrying the fluid either to the left or right. Points D and F in Fig. 3-2 identify two types of flows that may occur for the same flow conditions with their topologies illustrated in Fig. 3-3. These figures show large red heat plumes tilting towards the left and right directions, thus influencing the direction of the stream tube.
Figure 3-3. Flow and temperature fields corresponding with points D and F from Fig. 3-2 at $Ra_{uni} = 0$, $\alpha = 2$, $Pr = 0.71$ and $Ra_{per} = 1000$. Dashed lines mark stream tubes carrying fluid in the horizontal direction. The background colour illustrates the temperature field scaled with its maximum.

As previously discovered, the critical conditions leading to bifurcation depend on the heating pattern quantified by the heating wavenumber, as illustrated in Fig. 3-2. This problem can be viewed as an interaction of thermal plumes forming a spatial pattern. The most effective pattern producing bifurcation at the lowest $Ra_{per,cr}$, occurs for intermediate wavenumbers, at around $\alpha \approx 2$, as demonstrated in Fig. 3-4. Plumes which are either too far from each other or too close do not produce bifurcation in the range of $Ra_{per}$ used in this analysis.
Figure 3-4. Variations of the critical periodic Rayleigh number $Ra_{per,cr}$ resulting in the movement of the upper plate as a function of the wavenumber $\alpha$ for $Pr = 0.71$ and $Ra_{uni} = 0$.

Since the growth of heat plumes is essential for thermal streaming, the propagation of heat in the fluid becomes an important parameter. Therefore, this technique’s dependence on the fluid medium must be studied. The Prandtl number is a property of a fluid which quantifies the momentum diffusivity in relation to the thermal diffusivity. In Fig. 3-5, the bifurcation points are $Ra_{per,cr} = 224, 884.5, 1204, 1755$ for $Pr = 0.15, 0.71, 1, 1.5$, respectively. Demonstrating how $U_{top}$ strongly depends on the Prandtl number, whose reduction decreases the minimum heating intensity required to cause bifurcation.
Figure 3-5. Variation of the upper plate velocity $U_{top}$ as a function of $Ra_{per}$ for $\alpha = 2$, $Ra_{uni} = 0$ and $Pr = 0.15, 0.71, 1, 1.5$.

This demonstrates the relevance of the horizontal temperature gradients, which decrease with an increase of $Pr$ everywhere except in the thermal boundary layer near the lower plate. These gradients, displayed in Fig. 3-6, directly measure the horizontal gradient of buoyancy force that drives the fluid movement. They must increase significantly with an increase of $Pr$ to produce the same $U_{top}$, as illustrated in Fig. 3-5.
Figure 3-6. Flow fields and horizontal temperature gradients for $Ra_{uni} = 0$, $\alpha = 2$ and (a) $Pr = 0.15$, $Ra_{per} = 261$, (b) $Pr = 0.71$, $Ra_{per} = 1140$ and (c) $Pr = 1$, $Ra_{per} = 1685$. Dashed lines mark stream tubes carrying fluid in the horizontal direction. Background colours illustrate the horizontal gradients of the buoyancy force scaled with $Ra_{per}$. 
The previous cases have looked at scenarios where the mean temperature at the lower plate is equal to the temperature of the upper. Varying the mean relative temperature of the lower plate affects the thermal streaming and the critical conditions to induce it.

Data in Fig. 3-7 illustrate the effects of uniform heating added or removed to the lower plate. A modest increase in such heating results in a significant reduction of $Ra_{per,cr}$, i.e., the use of $Ra_{uni} = 100$ reduces $Ra_{per,cr}$ from 882 to 605. Uniform cooling has the opposite effect by increasing $Ra_{per,cr}$ from 882 at $Ra_{uni} = 0$ to 1170 at $Ra_{uni} = -100$.

Figure 3-7. Variations of the upper plate velocity $U_{top}$ as a function of $Ra_{per}$ for $\alpha = 2$, $Pr = 0.71$ and $Ra_{uni} = 100, 50, 0, -50, -100$. Bifurcation points are $Ra_{per,cr} = 605, 745, 884.5, 1027, 1170$ for $Ra_{uni} = 100, 50, 0, -50, -1000$, respectively.
3.3 Multi-modal Heating Patterns

Now consider more complex temperature distributions, which require using several Fourier modes for their characterization. All temperature distributions can be presented as

$$\theta_L(x) = Ra_{per}H(x)$$  \hspace{1cm} (3.3.1)

where $H$ is the known shape function describing their spatial distributions satisfying condition, which maintains $Ra_{per}$ as a measure of the heating intensity. Because adding modes can quickly increase the complexity of the study, the focus will be on shapes involving just two wave numbers, i.e., $\beta$ and $\gamma$, to illustrate the complexity and unpredictability of possible flow responses. The shape function has the form

$$H(x) = K(x)/\{maxK(x) - minK(x)\}$$  \hspace{1cm} (3.3.2a)

$$K(x) = \cos(\beta x) + \cos(\gamma x + \varepsilon)$$  \hspace{1cm} (3.3.2b)

where each component has equal amplitudes, and $\varepsilon$ is the phase difference between them.

Assuming that the system wavelength is $\lambda_s$, then the system must contain an integer number of wavelengths of each component, i.e., $\lambda_s = m\lambda_\beta = n\lambda_\gamma$. Once $\beta$ and $\gamma$ have been selected, the temperature distribution $H(x)$ depends only on $\varepsilon$, which varies in the range $0 \leq \varepsilon \leq 2\pi$. 
Figure 3-8. Variations of the upper plate velocity $U_{\text{top}}$ as a function of the phase difference $\varepsilon$ for $Ra_{\text{per}} = 1200$, $Ra_{\text{uni}} = 0$, $Pr = 0.71$ and $(\beta, \gamma) = (2,6), (2,4)$. The dotted lines show the reference case of $U_{\text{top}}$ achieved using heating with a single wavenumber $\alpha = 2$.

Responses of the flow system to heating involving two wavenumbers are illustrated in Fig. 3-8 for two cases, i.e., $(\beta, \gamma) = (2,6), (2,4)$, which lead to the same system wavelengths $\lambda_s = \pi$. The magnitude of $U_{\text{top}}$ is significantly smaller when compared with the one-mode heating, and its direction changes as a function of $\varepsilon$ in an antisymmetric manner with respect to $\varepsilon = \pi$. The phase difference between the modes acts as a control parameter, changing both the magnitude and the direction of $U_{\text{top}}$. 
Figure 3-9. Flow and horizontal temperature gradients for $Ra_{per} = 1200$, $Ra_{uni} = 0$, $Pr = 0.71$, $\varepsilon = \frac{\pi}{2}$ for $(\beta, \gamma) = (A) (2,0), (B) (2,4), (C) (2,6)$. Dashed lines mark stream tubes carrying fluid in the horizontal direction. Background color illustrates the horizontal temperature gradient scaled with $Ra_{per}$. 
Horizontal temperature gradients determine the horizontal gradients of buoyancy force driving convection. As shown in Fig. 3-9A, when heating involves one Fourier mode, these gradients change direction only once per wavelength. Figures 3-9B and 3-9C illustrate how the use of two Fourier modes doubles the number of direction changes, producing weaker convection. The use of still more complex temperature distributions requiring multiple Fourier modes further increases the frequency of direction changes of the driving force, thus reducing the effectiveness of convection. One may conclude that the best propulsion is achieved when all heating energy is placed in one Fourier mode or, at the least, temperature distributions leading to multiple changes of direction of the horizontal temperature gradient are avoided.

These limited results demonstrate that a wide range of possible behaviours of the flow system arises when it is subject to heating governed by multiple wavenumbers. It is not straightforward to infer the likely properties of the flow under such conditions without the use of a combination of analysis and computations.

### 3.4 Heating of the Upper Plate

The next consideration is whether applying periodic heating at the upper plate instead of the lower plate changes the propulsion effects. To study this new state, boundary conditions in 2.2.2 need to be replaced with

\[
y = -1: \quad \frac{\partial u}{\partial y}_{\text{mean}} = 0, \quad u_{\text{per}} = 0, \quad v = 0, \quad \theta = 0,
\]

(3.3.1a)
\[ y = 1: \ u = v = 0, \ \theta = 0.5 Ra_{per} \cos(\alpha x). \] (3.3.1b)

Making the upper plate stationary and allowing the lower plate to move.

It is relatively simple to show that the governing systems for the two problems are closely related. If the heating is moved to the upper plate and the following transformations made 
\[ u \rightarrow -U, \ v \rightarrow -V, \ p \rightarrow P, \ \theta \rightarrow -\theta, \ x \rightarrow -X + \pi, \ y \rightarrow -Y, \] the underlying equations are unchanged, but the thermal boundary conditions are reversed in sign. Given this relationship between the two cases, there is no need to dwell further on the stationary heated upper plate and moving lower plate, as all the interesting properties can be inferred directly from the results when the lower plate is stationary and heated. The same relative movement of the plates can be created by heating the lower or the upper plate.

### 3.5 Summary

The chapter delves into a novel propulsion mechanism solely based on patterned heating, which initiates the nonlinear thermal streaming effect. This propulsion mechanism is unique because it does not involve any geometric alterations to the bottom plate and only requires stationary heating.

The setup consists of two parallel horizontal plates. The upper plate is free to move, with no external constraints restricting its motion. In contrast, the bottom plate is restricted in space and experiences sinusoidal heating. As a consequence of this heating, convective currents form, causing a shear force on the upper plate. This shear propels the upper plate
into motion, accelerating it until the average shear force is neutralized; this phenomenon is labeled "thermal streaming."

The speed of motion of the upper plate is contingent on the intensity and spatial distribution of the heating applied to the lower plate. A diagram depicting the upper plate's speed in relation to the heating intensity shows that a supercritical pitchfork bifurcation characterizes the relationship.

Further observations from the study indicate an optimal range of wavenumbers for achieving optimal propulsion through this technique. Specifically, heating wavelengths that are either excessively long or extremely short diminish the effectiveness of thermal streaming. Additionally, a reduced Prandtl number augments the efficiency of the propulsion, a scenario that mirrors the addition of uniform heating to the plate.

One of the findings from the study is the optimal deployment of heating energy. The most effective propulsion is obtained when the heating energy is consolidated into a singular Fourier mode. Another takeaway is that the occurrence of the thermal streaming effect is independent of which plate is subjected to heating. This discovery suggests broader applications and adaptability of the principle in various setups and conditions.
Chapter 4

4 Grooved Plate with Patterned Heating

4.1 Introduction

This chapter describes another effect capable of creating propulsion, the thermal drift effect, which relies on heating and geometrical non-uniformities. This case has both sinusoidal heating and grooves imposed at the lower plate, Eq. 2.1.4a remains $y_L(x) = -1 + 0.5A \cos(\alpha x)$. Once propulsion due to the thermal drift effect is studied, the combined effects of the thermal drift and thermal streaming are examined.

4.2 Thermal Drift Effect

The discussion begins with a description of the thermal drift effect. This effect, previously examined in studies (Abtahi & Floryan, 2017, 2018; Inasawa et al., 2021), was found to drive fluid transport due to the pattern interaction effect (Floryan & Inasawa, 2021). This effect can also propel the upper plate if it encounters no external resistance. Viscous shear

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2 A version of this chapter has been submitted to the Journal of Fluid Mechanics

associated with convective motion may accelerate the plate until it reaches a velocity, $U_{top}$, at which point the mean shear becomes zero.

Velocity and temperature fields are illustrated in Fig. 4-1 for $\alpha = 2$ and a heating intensity, $Ra_{per} = 500$, well below the critical heating intensity needed to produce thermal streaming. In Fig. 4-1A, where $\Omega = 0$, when the hot spots are located at the groove crests, the flow topology is made of counter-rotating pairs of rolls with the fluid moving upwards above the hot spots; symmetry is conserved in this case, and no propulsion is demonstrated. The flow field has a similar structure in Fig. 4-1B, where $\Omega = \pi$ when the hot spots are located at the groove troughs. These positions both preserve the symmetry of the fluid. All other positions of the hot spots relative to the grooves result in the formation of a net horizontal flow directed, either to the right when $0 < \Omega < \pi$ or to the left when $\pi < \Omega < 2\pi$, as illustrated in Figs. 4-1B, 4-1D, respectively.
Figure 4-1. Flow and temperature fields for $\alpha = 2, A = 0.025$, $Ra_{uni} = 0$, $Ra_{per} = 500$ and (A)-(D) corresponding to $\Omega = 0, \pi, 3\pi/2$, respectively. Dashed lines identify stream tubes carrying fluid in the horizontal direction.
This is further proved in Fig. 4-2, where variations of $U_{top}$ as a function of the phase shift $\Omega$ are illustrated. When hot spots are located at the groove crests or troughs, at $\Omega = 0$ and $\Omega = \pi$, respectively, no $U_{top}$ is generated. All other values of $\Omega$ produce a $U_{top}$, which is positive for the range $0 < \Omega < \pi$ and negative for the range $\pi < \Omega < 2\pi$. Positions where hot spots are either at $\Omega \approx \frac{\pi}{2}$ or at $\Omega \approx \frac{3\pi}{2}$ produces the fastest plate movement.

Figure 4-2. Variation of the upper plate velocity $U_{top}$ as a function of the phase shift $\Omega$ and for $\alpha = 2$, $Pr = 0.71$, $Ra_{per} = 500$, $Ra_{uni} = 0$ and

$A = 0.01, 0.025, 0.05, 0.075, 0.1$.

Figure 4-2 shows that a larger groove amplitude produces a larger $U_{top}$ at all $\Omega$ positions. This is supported in Fig. 4-3, which shows that the propulsion speed increases proportionally to the groove amplitude.
Figure 4-3. Variations of the upper plate velocity $U_{top}$ as a function of the groove amplitude $A$ for $\alpha = 2$, $Pr = 0.71$, $Ra_{per} = 500$, $Ra_{uni} = 0$.

Figure 4-4A illustrates the distribution of the x-component of the pressure force $\sigma_{xp,L}$ acting on the fluid at the lower plate that drives the fluid in the horizontal direction. The form of the pressure field is dictated by the convection pattern, whose structure, in turn, is dictated by the heating pattern. As this pattern moves relative to the grooves, the projection of the pressure force onto the surface topography changes, producing $\sigma_{xp,L}$ of different magnitudes and directions – this projection is responsible for horizontal fluid movement. As shown in Fig. 4-4B, the convective motion generates an x-component of shear stress $\sigma_{xv,L}$ at the grooved plate with large periodic x-variations. The mean value of the shear stress $F_{xv,L}/\lambda$ must be equal to the mean value of the pressure force $F_{xp,L}/\lambda$ as the upper plate does not support any forces.
Figure 4-4. Distributions of (A) the $x$-component of the pressure force $\sigma_{xp,L}$ and (B) the $x$-component of the shear force $\sigma_{xv,L}$ acting on the fluid at the lower plate for $\alpha = 2$, $A = 0.025$, $Ra_{per} = 500$, $Ra_{uni} = 0$, $Pr = 0.71$, and $\Omega = 0, \frac{\pi}{2}, \pi, 3\pi/2$. The total forces are $F_{xp,L} = 0, 1.49, 0, -1.49$, $F_{xv,L} = 0, -1.49, 0, 1.49$ for $\Omega = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, respectively.

Uniform heating applied to the lower plate increases the plate velocity as the intensity of convection increases, while uniform cooling produces the opposite effect, as illustrated in Fig. 4-5A. Reduction of $Pr$ increases the strength of convection as mixing associated with convective motion is less effective in reducing horizontal temperature gradients, leading to an increase of $U_{top}$ while its reduction has the opposite effect, as illustrated in Fig. 4-5B.
Figure 4-5. Variations of the upper plate velocity $U_{top}$ as a function of the phase shift $\Omega$

for (A) $Pr = 0.71, Ra_{uni} = -100, -50, 0, 50, 100,$ and for (B) $Ra_{uni} = 0, Pr = 0.5, 0.71, 7.$ In both cases, $\alpha = 2, A = 0.05$ and $Ra_{per} = 500.$ Thick dashed lines in A shows variations of the maximum of $|U_{top}|$ as a function of $\Omega$.

4.3 Thermal Streaming and Thermal Drift Combined

When the thermal drift and streaming effects are active, the combined effect occurs for a sufficiently high heating intensity. It is possible to estimate the minimum heating required to activate the streaming for the grooved plate by placing hot spots either at the groove peaks or at the groove troughs, as this eliminates thermal drift.

Results in Fig. 4-6 demonstrate relatively minor changes in critical conditions to those found in the case of the smooth plate when using $\Omega = 0$ and $\pi.$ The same results also illustrate different responses when the hot spots are placed at the groove troughs and the groove peaks, inducing the thermal drift effect.
Figure 4-6. Variations of the upper plate velocity $U_{top}$ as a function of the heating intensity $Ra_{per}$ for $\Omega = 0, \pi/2, \pi, 3\pi/2, A = 0.025, Pr = 0.71, Ra_{uni} = 0, \alpha = 2$. Dotted lines provide plate velocity achieved with smooth plates. The critical conditions are $Ra_{per,cr} = 910,855$ at $\Omega = 0, \pi$ for a grooved plate, respectively, and $Ra_{per,cr} = 880$ for a smooth plate.

The combined system response depends on the relative position of the heating and groove patterns, as illustrated in Fig. 4-7. $U_{top}$ varies as a function of phase difference $\Omega$ in a regular, nearly sinusoidal manner for $Ra_{per} = 500$. Increasing heating intensity to $Ra_{per} = 800$, i.e., an increase by a factor of 1.6, results in a rise in the maximum possible $U_{top}$ by a factor of 4.7 while retaining its sinusoidal variations with $\Omega$. When heating intensity is further increased to $Ra_{per} = 900$, variations of $U_{top}$ remain regular except when the phase difference is near $\Omega = \pi$, corresponding to hot spots near the groove troughs. Under such conditions, limit points appear in variations of $U_{top}$ as a function of $\Omega$ – for such conditions, a slight change of $\Omega$ can change the direction of plate movement.
System response shows hysteresis - when $\Omega$ increases towards $\Omega = \pi$ and crosses the value of 3.23, $U_{\text{top}}$ jumps from $U_{\text{top}} = 1.53$ to $U_{\text{top}} = -2.46$. When $\Omega$ decreases back, $U_{\text{top}}$ does not jump back until the phase shift reaches a value of $\Omega = 3.06$ when it jumps from $U_{\text{top}} = -1.67$ to $U_{\text{top}} = 2.43$. The hysteresis loop is relatively narrow, i.e., it occurs between $\Omega = 3.06$ and $\Omega = 3.23$, with $U_{\text{top}}$ being positive at the upper side of the loop and negative at the lower side. Increasing the heating intensity to $Ra_{\text{per}} = 1000$ results in the formation of additional limit points near $\Omega = 0$, which corresponds to hot spots near the groove peaks with another hysteresis loop forming there and a widening of the hysteresis loop near $\Omega = \pi$.

Figure 4-7. Variations of the upper plate velocity $U_{\text{top}}$ as a function of the phase shift $\Omega$ for $\alpha = 2, A = 0.025, Pr = 0.71, Ra_{\text{uni}} = 0$, and $Ra_{\text{per}} = 500, 800, 900, 1000$. Circles identify limit points. The dotted lines provide plate velocity achieved with smooth plates.
The evolution of $U_{\text{top}}$ as a function of $Ra_{\text{per}}$ near $\Omega = 0$, which corresponds to hot spots near the groove peaks, is illustrated in Fig. 4-8A. Using $\Omega = 0$ results in a symmetric pitchfork bifurcation, i.e., the upper plate moves either to the right or to the left in the same manner. Placement of the hot spot very close to the groove peak and on its left side at $\Omega = 0.08$ produces plate movement to the right with $U_{\text{top}}$ increasing monotonically with $Ra_{\text{per}}$, i.e., no bifurcation occurs. A separate solution branch corresponding to the plate moving to the left appears when $Ra_{\text{per}} > 850$. This branch does not connect to the branch describing plate movement to the right; it ends at a limit point at $Ra_{\text{per}} = 956$ with $U_{\text{top}} = -1.49$. Moving the hot spot further away to the left to $\Omega = 0.52$ produces a similar response but with the $U_{\text{top}}$ varying with $Ra_{\text{per}}$ more regularly and with a new solution branch forming at a larger $Ra_{\text{per}}$, i.e., the limit point of the new branch is at $Ra_{\text{per}} = 1056$ with $U_{\text{top}} = -2.71$. Placing a hot spot on the right side of the groove peak at $\Omega = -0.08$ forces plate movement to the right with a new solution branch corresponding to negative $U_{\text{top}}$ appearing at a limit point $Ra_{\text{per}} = 956$ and $U_{\text{top}} = 1.49$. Moving the hot spot further away to $\Omega = -0.52$ moves the limit point to $Ra_{\text{per}} = 956$ and $U_{\text{top}} = -1.49$.

As illustrated in Fig. 48-B, variations of $U_{\text{top}}$ as a function of $Ra_{\text{per}}$ near $\Omega = \pi$, which corresponds to hot spots located near the groove troughs, are qualitatively similar to those observed for $\Omega$ near 0. Using $\Omega = \pi$ leads to a symmetric pitchfork bifurcation. Moving the hot spot to the left of the trough, to $\Omega = 0.08$, produces a negative $U_{\text{top}}$, which is a monotonic function of $Ra_{\text{per}}$. A second branch corresponding to positive $U_{\text{top}}$ appears at $Ra_{\text{per}} = 896$ with $U_{\text{top}} = 1.39$. Moving the hot spots further away to the left, to $\Omega = 0.52$, produces more regular variations of $U_{\text{top}}$ as a function of $Ra_{\text{per}}$, with the second branch
originating at $Ra_{\text{per}} = 1000$ with $U_{\text{top}} = 2.7$. Moving the hot spot to the right produces a near mirror image of the flow response with limit points at $Ra_{\text{per}} = 896$ and $U_{\text{top}} = -1.39$ for $\Omega = -0.08$ and $Ra_{\text{per}} = 1000$ and $U_{\text{top}} = -2.7$ for $\Omega = -0.52$.

Comparisons of data displayed in Figs. 4-6 to Figs. 4-8 show regular variations of $U_{\text{top}}$ as a function of $Ra_{\text{per}}$ for $Ra_{\text{per}} < \sim 800$ where thermal drift dominates and regular variations for $Ra_{\text{per}} > \sim 1100$ where, depending on $\Omega$, either nonlinear streaming or thermal drift dominates, producing a nearly identical system response. The system response exhibits somewhat unpredictable but significant changes of $U_{\text{top}}$, including direction changes for $Ra_{\text{per}}$ varying between these limits.

Figure 4-8. Variations of the upper plate velocity $U_{\text{top}}$ as a function of the heating intensity $Ra_{\text{per}}$ for (A) $\Omega$ near 0, i.e., $\Omega = -0.52, -0.08, 0, 0.08, 0.52$, and (B) $\Omega$ near $\pi$, respectively.
i.e., $\Omega = \pi - 0.52, \pi - 0.08, \pi + 0.08, \pi + 0.52$. All results are for $A = 0.025$, $\alpha = 2$, $Pr = 0.71$, $Ra_{uni} = 0$. Circles identify limit points. The dotted lines provide plate velocity achieved with smooth plates. The critical conditions are $Ra_{per,cr} = 910,855$ for $\Omega = 0, \pi$ for a grooved plate, respectively, and $Ra_{per,cr} = 880$ for the smooth plate. Dashed lines provide data for $\Omega = \pi/2$ (red) and $\Omega = 3\pi/2$ (blue).

The effectiveness of thermal drift increases proportionally to the groove amplitude, as illustrated in Fig. 4-2. When thermal streaming is active, variations of $U_{top}$ with $A$ become complex and change with $\Omega$ and $Ra_{per}$, as illustrated in Fig. 4-9. Figure 4-9A shows the system response for $Ra_{per} = 900$, which is just above the critical heating of $Ra_{per} = 880$ for a smooth plate. No bifurcation occurs at $A = 0.0004$, but one occurs at $A = 0.0006$, producing wide hysteresis loops. Further increase of the amplitude to $A = 0.005$ dramatically narrows down the hysteresis loop. Still, a further increase to $A = 0.02$ appears to change the magnitude of $U_{top}$ but does not drastically affect the size of the hysteresis loop. Figure 4-9B illustrates what happens when heating corresponds to $Ra_{per} = 1000$. No bifurcation occurs for $A = 0.005$ and $A = 0.0093$, but one is formed when the amplitude reaches $A = 0.0094$. The results show that the amplitude must increase by a factor of nearly 10 to produce bifurcation when heating increases from $Ra_{per} = 900$ to $Ra_{per} = 1000$. The hysteresis loop is very narrow at $A = 0.0094$ but significantly widens when the amplitude increases to $A = 0.025$, demonstrating a trend opposite to that observed for $Ra_{per} = 900$. 
Figure 4-9. Variations of the upper plate velocity $U_{top}$ as a function of the phase shift $\Omega$ for (A) $Ra_{per} = 900$, $A = 0, 0.001, 0.005, 0.02$, and (B) for $Ra_{per} = 1000$, $A = 0, 0.005, 0.0093, 0.0094, 0.025$. All results are for $\alpha = 2$, $Ra_{uni} = 0$, and $Pr = 0.71$. Circles identify limit points. Black dashed lines provide plate velocity achieved with smooth plates.

As previously seen in Fig. 4-2A, uniform heating strengthens thermal drift and reduces the critical Rayleigh number required for the onset of thermal streaming, as shown in Fig. 3-2. The effects of adding uniform heating when both effects are active are illustrated in Fig. 4-10. In general, uniform cooling reduces $U_{top}$ and eliminates bifurcations that would occur with periodic heating. Heating has the opposite effect, lowering the critical Rayleigh number and increasing the achievable $U_{top}$. 
Figure 4-10. Variations of the upper plate velocity $U_{\text{top}}$ as a function of the phase shift $\Omega$ for $\alpha = 2, A = 0.025, Ra_{\text{per}} = 1000, Pr = 0.71$ and $Ra_{\text{uni}} = -100, -50, 0, 50, 100$. Circles identify limit points. Dashed lines provide plate velocity achieved with smooth plates.

The type of fluid strongly affects the magnitude of $U_{\text{top}}$. The propulsive effect relies on horizontal temperature gradients, which diminish with an increase of $Pr$ as this leads to more intense convection, resulting in a reduction of these gradients. The thermal streaming and drift are affected similarly, as illustrated in Figs 4-2B and 3-7. The cumulative effect is shown in Fig. 4-11. Reduction of $Pr$ reduces the critical Rayleigh number and increases the magnitude of $U_{\text{top}}$. A slight increase of the Prandtl number from $Pr = 0.71$ to $Pr = 1$ eliminates bifurcations and significantly reduces $U_{\text{top}}$. 
Figure 4-11. Variations of the upper plate velocity $U_{\text{top}}$ as a function of the phase shift $\Omega$ for $\alpha = 2, A = 0.025, Ra_{\text{uni}} = 0, Ra_{\text{per}} = 1000$ for $Pr = 0.55, 0.71, 1, 1.5$. Circles identify limit points. Dashed lines provide plate velocity achieved with smooth plates.

4.4 Summary

The current chapter delves into an additional mechanism for generating propulsion, the thermal drift effect. The scenario under examination involves a lower plate enhanced with both sinusoidal heating patterns and grooves. The central focus of the chapter revolves around studying the thermal drift effect, its synergistic relationship with thermal streaming, and how altering flow parameters impacts the efficacy of propulsion.

In the study's model, there are parallel horizontal plates. The upper plate is allowed motion in the x-direction, while the lower plate is stationary and possesses grooves and periodic heating. The grooves and heating patterns are harmoniously tuned to the same wave number.
Two distinct effects are responsible for propulsion in this setup: firstly, the thermal drift effect. This effect emerges from the interplay between the groove and heating patterns. The system's response is forced, causing thermal drift. The potency of this effect is intricately tied to the relative positioning of the groove and the heating patterns. Optimal movement of the upper plate is observed when hot spots are strategically positioned midway between the peaks and troughs of the grooves. Interestingly, the motion's direction can vary based on whether the hot spot is situated to the right or left of the groove's peak. However, it is important to note that the thermal drift effect vanishes when hot spots align perfectly with either the groove peaks or troughs.

The nonlinear thermal streaming effect, which was explored in the prior chapter, needs an adequate heating intensity to become activated. Representing system bifurcation, it can coexist with thermal drift, collectively propelling the plate irrespective of the relative positioning of the heating and grooved patterns.

When these two effects are combined, the peak velocity is observed when hot spots lie halfway between the groove's peaks and troughs. When hot spots are closer to the groove peaks or troughs, the movement is slightly subdued. A minuscule shift in their relative positioning can lead to significant alterations in both direction and speed.

Additionally, the study observed phenomena like the emergence of limit points and hysteresis loops when the relative positions of the heating pattern and grooves change. Another observation was the amplification in propulsion as the Prandtl number decreased and an escalation when a uniform heating component was introduced at the lower plate.
Chapter 5

5 Conclusions and Future Work

5.1 Conclusion

This thesis explores the intricate propulsion effects generated from sinusoidal heating—both in isolation and in tandem with surface corrugations—on the lower plate of horizontal parallel plate configurations. The physical setup is delineated, with the lower plate being static, while the upper plate has the liberty to move, representing a free surface. To ensure precision and robustness in the data acquisition process, a previously formulated algorithm deploying the IBC method was used. The combination of Fourier expansions and Chebyshev polynomials facilitated the spatial discretization of the horizontal and vertical dimensions, respectively.

Initially, the investigation explores scenarios where periodic heating is the sole influence, and both plates maintain smooth surfaces. It was observed that as the periodic heating surpassed a critical value, it produced the phenomenon of thermal streaming. Consequently, the upper plate exhibited motion, either in the positive or negative x-direction. The outset of thermal streaming is not arbitrary but contingent upon the wavenumber associated with the heating period and the Prandtl number intrinsic to the fluid in question. For instance, wavenumbers oscillating around $\alpha \sim 2$ exhibit the minimal required heating intensity for propulsion. Any deviations from this range, either increase
or decrease, necessitate an increase in heating intensity to produce propulsion. Furthermore, diminishing the Prandtl number effectively reduces the required heating intensity.

As the study transitions to its second phase, the focus shifts to the composite effects caused by the amalgamation of periodic heating and grooves on the lower plate. At heating intensities marginally below the critical benchmark, the well-documented thermal drift effect manifests, especially when there is a shift in the positioning of the heating period relative to the grooves. As the heating intensity escalates beyond the critical value, a synergy of thermal streaming and thermal drift becomes active. Under these high intensities, even a slight adjustment in the relative position of the heating can cause changes in both the direction and velocity of the upper plate's motion.

5.2 Future Work

A numerical analysis was conducted for this fundamental flow problem, and the important parameters were identified. The results found in this thesis require experimental validation. It is suggested that a liquid with a low Prandtl number, such as Galinstan, be used. By placing a light sheet on its top surface, its movement can be tracked. Using a combination of periodic and uniform heating, a sufficient velocity of the floating sheet can be achieved, allowing for precise measurement. An estimate based on periodic heating with amplitude $A^* = 10^9 \degree C$, $Ra_{per} = 500$, $g^* = 9.81 \, m/s^2$, $k^* = 16.5 \, Wm^{-1}^\circ C$, $\rho^* = 6.44 \times 10^3 \, kg/m^3$, $c^* = 296 \, J/kg^{\circ}C$, $\kappa^* = 8.66 \times 10^{-6} \, m^2A^{-1}$, $\nu^* =$
3.73 \times 10^{-7} \text{ m s}^{-2}, \mu^* = 0.0024 \text{ Pa s} \text{ and } \Gamma^* = 18.3 \times 10^{-6} \text{ C}^{-1}, \text{ the anticipated sheet velocity should be in the } \text{mm s}^{-1} \text{ range. However, it is essential to note that Galinstan comprises various components, so its properties could vary based on its specific composition.}

Another interesting pathway to explore would be to study the transient properties of these phenomena. The steady-state cases have been outlined in this thesis. However, the onset of plate movement and the acceleration to a constant velocity would be important factors to investigate for the practical use of these propulsion mechanisms. These factors would be seen during experimental analysis; however, for a deeper understanding of the mechanisms’ time dependence, a transient algorithm is required.

Finally, the scope of this work has been focused on horizontal channels with sinusoidal heating profiles and grooves. A partial analysis of two-mode heating was conducted. However, there are many more complex cases to be studied. Future work could delve into such complex cases where heating profiles can follow more complex functions, and groove shapes following a triangular or rectangular profile can be analyzed.
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