Strategic Supplier Dynamics and Decision-making in Supply Chain Management: Exploring Market Segmentation, Copycatting, and Encroachment

Shobeir Amirnequiee, Western University

Supervisor: Pun, Hubert, The University of Western Ontario
Joint Supervisor: Naoum-Sawaya, Joe, The University of Western Ontario
Co-Supervisor: Zaric, Greg, The University of Western Ontario

A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Business
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Abstract

In this dissertation, we explore the intricate dynamics of supplier relationships and strategic decision-making within the realm of Operations Management, focusing on the critical aspects of supply chain management. The research consists of three papers, each offering unique insights into supplier dynamics and their implications for manufacturers and businesses.

The first paper presents a robust framework for joint learning of consumer preferences and market segmentation. Leveraging ideas from machine learning and mathematical programming, this framework efficiently segments the customer base and accurately learns preferences without compromising consumer privacy. By optimizing assortment decisions, this approach maximizes profits and offers superior prediction accuracy, enhancing marketing strategies in dynamic market scenarios.

The second paper delves into the pressing issue of supplier copycatting, where suppliers imitate original products, posing challenges to manufacturers and suppliers worldwide. Employing a game-theoretic approach, the research analyzes strategic responses of manufacturers and suppliers to cope with this emerging trend. The findings reveal the impacts of quality improvements and potential shifts in outsourcing decisions, providing valuable insights for managing supplier relationships and mitigating copycatting risks.

The third paper investigates the ramifications of supplier encroachment, as upstream suppliers venture into direct sales and compete with the buyers. Through a two-period game-theoretic model, the research examines optimal outsourcing strategies for buyers while considering the potential repercussions faced by encroaching suppliers. This comprehensive analysis sheds light on the dynamics of supplier-buyer collaborations, highlighting the importance of trust and commitment in maintaining successful partnerships.

Overall, this dissertation contributes valuable and comprehensive insights to the field of Operations Management. Employing a multi-method approach, we delve into supplier dynamics and decision-making, offering robust strategies and solutions to enhance supply chain efficiency and competitiveness. By addressing challenges such as consumer preference learning, supplier copycatting, and supplier encroachment, this research contributes to the growing body of knowledge in Operations Management and provides actionable guidance for businesses to thrive in the dynamic supply chain environments.

Keywords: Preference learning, market segmentation, copycatting, encroachment, game theory, machine learning.
Summary for Lay Audience

Picture a world where every choice you make as a consumer is a piece of a larger puzzle, revealing your preferences and shaping the products you encounter. In the intricate realm of supply chains, my research aims to decode these complexities, optimizing strategies that businesses employ to offer products tailored to your desires.

My exploration commences with understanding consumer preferences while valuing your privacy. Employing innovative techniques from machine learning and mathematical programming, my framework deciphers your past choices, helping businesses anticipate your needs without compromising your personal information.

In a competitive marketplace, the phenomenon of supplier copycatting poses a challenge. Imagine a strategic battle where businesses and suppliers respond to imitative maneuvers. By delving into the strategies employed by manufacturers and suppliers, my work reveals how quality improvements and calculated responses can safeguard businesses against these challenges.

Now, envision the dynamics of supplier relationships. Suppliers, often key allies, can transform into competitors, disrupting the traditional supply chain landscape. Employing game theory, I delve into this intriguing phenomenon, where mathematical models unveil how these shifts impact businesses’ strategic decisions, influencing outsourcing and overall success.

In summary, my research magnifies the inner workings of business decisions, from understanding your preferences ethically, to countering market challenges and deciphering supplier dynamics. This journey culminates in strategies that empower businesses to navigate complexities, ensuring that your shopping experience is meticulously orchestrated, and products on shelves are a result of well-informed, strategic decisions.

By unraveling these intricate threads, my research echoes beyond the academic realm, offering practical insights for businesses to thrive and consumers to enjoy products that resonate with their desires. In an era of evolving market dynamics, my work serves as a guiding light, enhancing the art and science of supply chain management, one strategic decision at a time.
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Sincerely,

Shobeir Amirnequiee
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Chapter 1

Robust Framework for the Joint Learning of Consumer Preferences and Market Segmentation
1.1 Introduction

Market segmentation and consumer preferences are critical parts of the assortment optimization problem, which helps a firm decide what assortment of products and to which segments of customers to offer in order to maximize revenue (e.g., Chen & Mišić 2022). Consumer preferences are integral to many product assortment decisions, including, helping devise a retailer’s pricing strategy (Cohen et al. 2020), shaping the design of a new product or service (Wang & Curry 2012), and determining what products or services should be delivered to a target market (Bertsimas & Mišić 2019). Accurate market segmentation is a necessity for the firm’s successful assortment strategy. Generally defined as the practice of finding homogeneous segments of consumers, segmentation has been shown to increase the firm’s expected profitability (Frank et al. 1972; DeSarbo & Grisaffe 1998; McDonald et al. 2003). As the online marketplace expands and global e-commerce sales rise, reaching $3,535 billion in 2019 from $1,548 billion in 2015 (Statista 2019), content and product personalization is becoming more pervasive than ever. This granular approach towards segmentation explains why firms offer varied versions of the same product, or exclusive products and services, to different segments of the market. For example, Apple is known to release the same generation of iPhones with varied specifications to European and American customers (Wakephone 2021). The same phenomenon exists across a variety of consumer product markets, including, the car industry (Voelk 2020), cosmetics (Milman 2019), and even Starbucks coffee (Floyd & Kersh 2021). From a firm’s perspective, developing an effective personalization framework involves vast collection and processing of choice and demographic data to elicit preferences. However, in this chapter, we propose a framework to simultaneously learn consumer segments and their preferences, in a robust and efficient way, using only the consumer choice data.

The large volumes of consumer data (e.g., clickstream, browsing and search history, eye-tracking data) are typically unstructured and thus highly noisy. A major source of noise in choice data is the “irrationality” of consumers in their decision-making. They may choose health insurance plans that cost them more money for the same (or
less) coverage, despite having access to all the relevant information (Bhargava et al., 2017), they fail to effectively compare cell-phone plans and might select one that does not match their usage pattern which increases their costs (Grubb & Osborne, 2015), they are willing to pay a threefold price for a branded drug over a less expensive alternative with the same dosage and same active ingredients (Bronnenberg et al., 2015), and they are particularly susceptible when faced a complex choice task or forced to choose promptly (McShane & Böckenholt, 2018), to name a few. Such irrationality contradicts with the underlying assumption—i.e., the random utility assumption—of certain discrete choice models such as multinomial logit (MNL) (e.g., Hu et al., 2022) and latent-class MNL (LC-MNL). Even though LC-MNL can jointly learn consumer segments and consumer utility functions (Train, 2009), it requires strong parametric assumptions on the unobserved. Besides, the MNL-based models suffer from the independence from irrelevant alternatives (IIA) property, which can generate unrealistic substitution patterns when different alternatives in the choice occasions have shared attributes (Ratliff et al., 2008; Van Ryzin & Vulcano, 2015). Irrationality can also further complicate the problem of learning consumer preferences for the businesses; for example, it can mislead the product developers to design a product that does not reflect consumer needs and wants, which may lead to a loss on the investment and can negatively affect the revenues.

The sheer volume of (noisy) data together with the methodological advances in machine learning (ML) and optimization, have contributed to the popularity of data-driven approaches and applications of data analytics in marketing research (Mišić & Perakis, 2020). Such methods have been applied to a variety of business problems, including inventory management (Bertsimas & Kallus, 2020), product line design (Bertsimas & Mišić, 2019), promotions (Cohen et al., 2017), and choice modelling (Farias et al., 2013). In particular, market segmentation (Liu et al., 2010) and targeted advertising (Bernstein et al., 2019) have significantly benefited from the recent advances. This explosive growth of information technology has provided online and brick-and-mortar firms with unprecedented ways to collect, purchase, and store extensive datasets on their customers’ demographics, past transactions, geo-locations, and so forth. The in-
formation offered by such datasets is often used for consumer profiling (Valletti & Wu, 2020) and targeted promotions (Andrews et al., 2016; Golrezaei et al., 2014) to improve the firms’ operations and profits. This fine-grained personalization framework induced by cutting-edge technology, however, gives rise to two major challenges for consumers, namely privacy and bias/discrimination.

With regards to privacy, it has been long believed that the operational success of the firm is strongly correlated with the specificity and the quantity of consumer data (Nowak & Phelps, 1995). Consequently, firms have been incentivized to collect larger volumes of highly detailed consumer data, which, given the lack of transparency, has hurt consumers (Cohen, 2018) and their trust (Liu et al., 2022). In one case, the names, addresses, and social security numbers of 43% of the US population were stolen from Equifax servers in 2017 (TechAdvisor, 2019). Beside the issues around consumer privacy, the collection and use of personal data have inadvertently led to analytics promoting bias and discrimination. Discriminatory analytics negatively affects people on different levels. Pricing algorithms employed by insurance companies, for example, have been shown to be biased against consumers born in the developing world (Fabris et al., 2021) or living in minority neighborhoods (Angwin et al., 2015). The penetration of algorithms into all aspects of our lives calls for the development of equitable methods that are unstained by prejudice against gender, ethnicity, age, and so forth. As such, the proposed learning model requires solely the availability of consumers’ past choices and extends existing models for preference learning to jointly learn the market segments without the need for any additional consumer data.

Methods of statistical learning theory and particularly Support Vector Machines (SVM) (Vapnik, 1998, 2013) have been successfully applied to preference elicitation (e.g., Evgeniou et al., 2005; Cui & Curry, 2005; Chapelle & Harchaoui, 2005) and their superior performance to the traditional methods such as hierarchical Bayesian (e.g., Arora et al., 1998), and other optimization-based approaches (e.g., Toubia et al., 2004) is established. However, SVM is shown to be sensitive to feature and label noise due to factors such as SVM’s poor handling of the outliers (Frénay & Verleysen, 2013), and the unboundedness of the loss function (Wu & Liu, 2007; Lin & Wang, 2002). An
1.1. Introduction

An extension to the SVM-based models used in preference learning is proposed in this work, which improves the existing models’ sensitivity to feature and label noise. The proposed models are consistent with the literature in that (1) they are based on the methods of statistical learning, and (2) the input data is collected from a choice-based conjoint analysis (CBC) experiment.

1.1.1 Contributions

First, building upon ideas from operations research (OR) and ML, we propose an extension to the existing SVM models in preference learning to learn individual consumer’s preferences and accurately predict their future choices. The framework, as presented in Section 1.3, relies on the idea that the past choices of a consumer are not equally important in predicting the future choices. Generally speaking, one defines the difficulty of a choice as the mental resources required to choose the optimal alternative. Given a user’s preference, the harder choices, intuitively, reveal more information about the user. However, (1) the difficulty of the choices are not known in advance, and (2) when there is higher difficulty, the user is more likely to make an inconsistent (i.e., wrong) choice. The existing SVM models for preference learning can handle wrong answers to difficult choice tasks (i.e., support vectors), but they are specifically sensitive to the outliers that are far from the SVM’s separating hyperplane (Frénay & Verleysen, 2013)—i.e., inconsistent answers to relatively easier choices. A distance-based approach in the vector space of the product differences is used in this chapter to identify and curb the negative effect of a wrongly-classified choice; where an easy choice is characterized as one that is further away from the center of the vectors of its class.

Second, as discussed earlier consumers make mistakes in their perception of a product and in their comparison of the existing alternatives. Across three models, an easy-to-adjust robust framework is proposed by allowing to control the degree of robustness. Our framework guarantees the robustness of the solution against the perturbations caused by consumer misconceptions, and handles response errors using a weight-
ing scheme that determines the relevance of each past choice in predicting the future choices. Besides, in all three models, the unobserved heterogeneity is controlled for using an intuitive pooling scheme, similar to that of Evgeniou et al. (2005).

Third, we present a model to simultaneously segment the customer base—using the observed choices only—and learn each segment’s preferences while preserving the robustness features of the individual-level model.

In summary, this chapter contributes to the Marketing literature by proposing an accurate and robust joint preference learning and market segmentation framework which also accounts for three types of noise in the consumer choice process: (1) inaccuracies in consumers’ perception of the product or service features, which may lead to consumers’ failure to objectively compare the different alternatives, (2) inconsistencies in consumer choices, which refers to consumers making accurate yet contradictory choices, and (3) response errors, that is, consumers effectively comparing the alternatives, given the available information, but failing to make the intended choice, which may or may not cause an inconsistency. For the individual-level model, we show across extensive experiments, with synthetic and empirical data, that the proposed approach consistently outperforms the existing ML-based model used in preference learning in a CBC task. We then show that the proposed framework accurately segments the market (according to the consumers observed choices) and simultaneously learns the preferences of each segment.

The rest of the chapter is structured as follows. Section 1.2 reviews the literature on optimization-based preference learning models. In Section 1.3 a robust ML framework to learn consumer preferences is introduced. We present the robust model for the joint learning of consumer preferences and market segmentation in Section 1.4. In Section 1.5 extensive empirical evidence, using Monte Carlo simulations and empirical choice data, is provided to support the use of our framework in practice. Finally, Section 1.6 concludes the chapter and highlights future research.
1.2 Literature Review

Market segmentation has been long established as an integral part of the firm’s marketing operations. The interconnectedness of the market segments and the pricing strategy has been explored in the early works of Mussa & Rosen (1978) and Moorthy (1984). While, Gabszewicz et al. (1986) and Desai (2001) study the impact of consumer characteristics (e.g., income and preferences) on the segmentation strategy. Methods of operations research and mathematical programming have been used to obtain optimal segmentation of the market; Liu et al. (2010) employ multi-objective evolutionary algorithms to approximate sets of Pareto-optimal segments, and DeSarbo & Grisaffe (1998) use combinatorial optimization and heuristics to find the optimal segmentation that satisfies resource constraints. Adopting a non-parametric approach, Van Ryzin & Vulcano (2015) and van Ryzin & Vulcano (2017) focus on the problem of demand forecast and propose a rank-based method of learning the segments. To address the issue of nonidentifiability caused by the number of segments (factorial in number of alternatives), they specify only a subset of customer types to be used in their choice model, which, in practice, is tantamount to the specification of structural forms used in the parametric models and has similar drawbacks. Their work differs from this chapter since their objective is to solve the problem of demand forecast with unknown product availability, and therefore, their methodology fits a different category of marketing problems. More recently, August et al. (2019) show how price-based segmentation in the software market can increase profitability and improve consumer welfare, whereas Wang et al. (2022) employ a combination of mixed integer programming and a MNL model to evaluate the benefits of a segmentation procedure based on price, quality, and service duration.

Preference modelling is a well-studied problem in the marketing literature. The preference elicitation problem has been traditionally approached through methods of conjoint analysis (Green & Rao, 1971; Green & Srinivasan, 1978; Rao et al., 2014). Since its introduction, conjoint analysis has been applied to a variety of business problems including selecting suppliers in a supply chain management context (Verma & Pullman, 1999).
Chapter 1. Preference Learning and Market Segmentation

1998), pricing and revenue maximization (Cohen et al., 2020; Dobson & Kalish, 1993), health-care management (Ryan & Farrar, 2000; Halme & Kallio, 2011), and product design and share-of-choice problem (Wang et al., 2009). Among the methods of CA, choice-based conjoint analysis (CBC) (Louviere et al., 2000) has gained remarkable popularity among practitioners and academics, mostly due to its ability to deal with the complexities of consumer choice and the ease of use (Green et al., 2001; Hauser & Toubia, 2005). Consistent with the literature, in this chapter we evaluate the performance of the proposed framework on simulated and real-world CBC datasets. Although, to the best of our knowledge, this chapter is the first attempt to perform simultaneous market segmentation and ensure non-discrimination, within body of literature on CBC.

Traditionally, methods of logistic regression (McFadden et al., 1973) and hierarchical Bayesian (Allenby & Rossi, 1998) have been used for conjoint estimation. Such models suffer from the “curse of dimensionality” and dependency on probabilistic assumptions (Evgeniou et al., 2005). Alternatively, preference learning problems can be formulated as optimization problems (Toubia et al., 2003; Cui & Curry, 2005; Bertsimas & O’Hair, 2013); which is the stream of research this chapter belongs to. A special class of optimization-based preference elicitation methods, referred to as CBC-SVM throughout this chapter, was introduced by Evgeniou et al. (2005) and Cui & Curry (2005). Evgeniou et al. (2005) study the standard problem of learning a utility function from a series of past choices made by an individual, and propose a framework to learn linear and non-linear (i.e., polynomial) utility functions. Assuming utility-maximizer consumers, if option \( x \) is selected over option \( y \), then the utility of option \( x \) (i.e., \( u(x) \)) must be larger than or equal to the utility of \( y \) (i.e., \( u(y) \)) to the consumer. Following that logic, an optimization problem is proposed, where every past choice is a constraint of the form \( u(x) \geq u(y) \). The optimal utility function is the one that satisfies all the constraints with the largest margin. The high accuracy and computational efficiency of the proposed approach is evaluated using simulated and real data. In a similar approach, Cui & Curry (2005) study the applications of SVM in marketing prediction and formulate the problem of preference learning using choice data as an SVM
1.2. Literature Review

The proposed CBC-SVM outperforms existing models, including logit and MNL, in predicting consumer choices. Even though Evgeniou et al. (2005) and Cui & Curry (2005) improve the accuracy of the existing models, their approach has the same shortcomings as SVM, and suffers from sensitivity to outliers and feature and label noise (Frénay & Verleysen, 2013; Wu & Liu, 2007; Lin & Wang, 2002). The framework proposed in this chapter guarantees the robustness of the solution against feature noise (i.e., the perturbations caused by consumer misconceptions), and handles label noise (i.e., response errors) using a weighting scheme that determines the relevance of each past choice in predicting the future choices. Beside, we extend CBC-SVM to account for market segmentation which enables the simultaneous learning of market segments and consumer preferences leading to higher predictive accuracy.

Building on CBC-SVM, Maldonado et al. (2015) focus on the identification of relevant attributes to the consumers in a CBC experiment. They extend CBC-SVM by adding feature selection, to better identify the attributes that are considered by consumers, and to improve the parsimony of the utility vectors. Unlike the common approach, which is to post-process the estimated utilities, they use a backward elimination of the attributes procedure, which is performed simultaneously to the utility function estimation. The results show that the proposed feature selection model for CBC outperforms CBC-SVM in terms of predictive performance for future consumer choices. In a similar work, Maldonado et al. (2017) propose a model to simultaneously learn the utility vectors and control for the heterogeneity, and improve the sparsity of the CBC-SVM by using the $\ell_1$-norm for complexity control in the objective function. In both papers, the identification of relevant attributes improves the interpretability and the accuracy of the model. This work is similar to Maldonado et al. (2015) and Maldonado et al. (2017) in that the third proposed model imposes $\ell_1$-norm minimization on the utility vectors, and therefore, improves the parsimony of the solution. Our framework, however, offers explicit robustness against the perturbations caused by consumer misconceptions and the response errors, and guarantees non-discrimination.

Since the introduction of optimization methods in preference learning, numerous operations research techniques have been applied to CA and conjoint estimation. For
examples, methods of linear programming (Srinivasan & Shocker, 1973), polyhedral optimization (Toubia et al., 2003, 2004), integer optimization (Bertsimas & O’Hair, 2013; Bertsimas & Mišić, 2019), and dynamic programming (Sauré & Vielma, 2019) have been used to elicit consumer preferences. Among the optimization approaches proposed for preference learning, Bertsimas & O’Hair (2013) use a combination of integer programming, maximin decision rule, and conditional value at risk (Rockafellar et al., 2000) to model consumer choice in a ranking task. Adaptive conjoint analysis is used to collect the choice data, and integer programming is used to model choice inconsistencies. In their approach, an upper-bound needs to be set for the number of inconsistencies in users’ choice data. In contrast, the framework proposed in this chapter does not make any assumptions about the extent of inaccuracies and inconsistencies in consumer choice, and it uses the standard CBC questions—that can contain any number of alternatives—without an indifference option, whereas Bertsimas & O’Hair (2013) limit the questionnaire to have two alternatives in each question.

1.3 The Individual-Level Robust Framework

We denote a consumer by superscript $i$, $i = 1, \ldots, N$. The data for each consumer contains $T$ choice occasions. The alternatives are denoted by $x_{kt} \in \mathbb{R}^J$, which is the vector of the attributes of the $k$-th alternative in the $t$-th choice task for consumer $i$. A consumer $i$, at choice occasion $t$, makes a choice from the $K$ alternatives in the choice set $M_i^t = \{x_{1t}, \ldots, x_{Kt}\}$, where similar to Maldonado et al. (2015), each alternative is the profile of a product or service described by $J$ attributes, with each attribute taking $n_j$ levels, $j = 1, \ldots, J$. The attributes describing a profile can be anything from the resolution of a digital camera (Abernethy et al., 2007) to the grace period of a credit card. A consumer is characterized by a vector $w^i \in \mathbb{R}^J$, $i = 1, \ldots, N$, (i.e., partworths vector) where the $j$th component of $w^i$ represents the value of the corresponding attribute level to the consumer. Finally, similar to Evgeniou et al. (2005) and Toubia et al. (2007), the decision rule is assumed to be the standard utility maximization, where the consumers’ preference is modelled by the linear utility function $u^i(x_{kt}) = x_{kt} \cdot w^i$. 
1.3. The Individual-Level Robust Framework

The choice data for consumer $i$ at choice task $t$ is of the form $\{\{x_{1t}, \ldots, x_{Kt}\}, x_{jt}\}$, where $x_{jt}$ denotes the selected alternative. It is assumed, without loss of generality, that the first alternative ($x_{1t}$) is always chosen (else, we can always shuffle the alternatives such that the first alternative is the selected one). Thus, all choice data will be of the form $\{\{x_{1t}, \ldots, x_{Kt}\}, x_{1t}\}$, or equivalently $\{x_{1t}, \ldots, x_{Kt}\}$, $t = 1, \ldots, T$. Learning user preferences can then be cast as a problem of learning the utility function, where the only unknown is the partworths vector $w^i$. Ideally, one would like to learn $w^i$ such that

$$
\forall k \in \{2, \ldots, K\}, \forall t \in \{1, \ldots, T\}, \text{ and } \forall i \in \{1, \ldots, N\}
$$

$$
u^i(x_{1t}) \geq u^i(x_{kt}) \iff w^i \cdot x_{1t} \geq w^i \cdot x_{kt} \iff w^i \cdot (x_{1t} - x_{kt}) \geq 0.
$$

(1.1)

The preference learning model is formulated in Evgeniou et al. (2005) and Maldonado et al. (2015) as

**CBC-SVM:** minimize $w^i, \xi_{kt}$

$$
\frac{1}{2}||w^i||^2 + C \sum_{t=1}^{T} \sum_{k=2}^{K} \xi_{kt}
$$

(1.2)

s.t. $w^i \cdot (x_{1t} - x_{kt}) \geq 1 - \xi_{kt}$ $\forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}$

(1.3)

$$
\xi_{kt} \geq 0 \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}.
$$

(1.4)

This approach has numerous advantages such as high accuracy (due to the complexity control), computationally efficiency (Cui & Curry, 2005; Evgeniou et al., 2005), and it can estimate a larger number of parameters (Chapelle & Harchaoui, 2005). However, SVM is shown to be very sensitive to label noise, because of the heavy reliance on support vectors. For instance, it is enough to have only 5% of the observations wrongly labelled for SVM performance to drop significantly (Frénay & Verleysen, 2013). In the preference learning context, label noise is characterized by response errors, i.e., when a consumer selects an alternative that does not maximize the utility. As discussed above, response errors occur very often and due to a myriad of reasons, which complicate the process of learning consumers’ true utilities. Wu & Liu (2007) discuss the drawbacks of
SVM for difficult learning problems and show that the performance is heavily affected by outliers (i.e., points that are located far away from the center of their class in the vector space).

CBC-SVM provides a classifier in the space of differences of alternatives, that is $x_{1t} - x_{kt}$, $\forall t, k$. An outlier in this space can be, for example, a consumer’s inaccurate understanding of an attribute, lack of information with respect to certain attribute(s), or a flawed comparison of the alternatives.

Finally, central to the framework proposed in this chapter is the idea that the past choices of a consumer are not equally important. Choices can indeed vary widely in terms of how informative and how difficult they are. Therefore, to learn the most accurate utility functions, a preference elicitation framework should identify “meaningful” (“meaningless”) choices—characterized by the $J$-dimensional vector $(x_{1t} - x_{kt})$—from the rest and put higher (lower) weights on them. As shown by Lin & Wang (2002), the performance of CBC-SVM can be improved by incorporating an intuitive weighting scheme for the observations. However, CBC-SVM treats all observations (i.e., past choices) uniformly, and thus, is more susceptible to outliers and noise in data. Across the proposed models of this chapter, CBC-SVM is extended by introducing methods from machine learning that make possible the identification of “meaningful” (and “meaningless”) choices, and improve the robustness against feature noise using ideas from the robust optimization literature.

### 1.3.1 Handling Response Errors and Inconsistencies

In this section the focus is on the problem of learning preferences for a single consumer and thus the superscript $i$ is dropped. To further simplify the notation, it is assumed that each choice task is comprised of two alternatives (i.e., $K = 2$). At choice occasion $t$, the consumer observes $M_t = \{x_{1t}, x_{2t}\}$ and selects the alternative with the highest utility. A response error happens if the consumer chooses $x_{1t}$ while $\arg\max_{x \in M_t} w \cdot x = x_{2t}$. This “incorrect” response will lead to an inconsistency in the choice data, only if it violates transitivity. Else, it is still a noise that increases the difficulty of recovering an
1.3. The Individual-Level Robust Framework

accurate \( w \). In the context of CBC-SVM, (model (1.2)-(1.4)), a response error implies that the constraint \( w \cdot (x_{2t} - x_{1t}) \geq 1 - \xi_{1t} \) should be added to the model.

In that case, one should restrict the effect of choice \( t \) on the large-margin classifier. Such effect is exerted through penalizing the error term \( \xi_{2t} \) in the objective function. However, the erroneous choices are not known a priori. Consequently, a method, based on fuzzy and weighted SVM, is proposed to identify response errors and limit their effect on the resulting separating hyperplane.

The proposed approach is similar in essence to fuzzy and weighted SVM, where data points differ in the strength of their membership to a class (Lin & Wang, 2002). Based on the fuzzy memberships, a weighting scheme is defined, and training errors are penalized proportional to an observation’s weight. For example, an observation can partially belong to a class and be partially meaningless. In preference elicitation context, it translates to the following: a consumer choosing \( x_{1t} \) over \( x_{2t} \) does not necessarily imply that \( (x_{1t} - x_{kt}) \) fully belongs to the class of \((+1)\), instead, it might be the case that this choice 20% belongs to the \((+1)\) class and is 80% meaningless. One could interpret that as an 80% likelihood that the consumer is indifferent or \( x_{2t} \) generates a higher utility. The membership of a choice \( (x_{1t} - x_{kt}) \) to its corresponding class is denoted by \( s_{kt} \in [0, 1] \). Therefore, the contribution of \( (x_{1t} - x_{kt}) \) to the decision boundary learned by SVM should be adjusted according to \( s_{kt} \), since it is \((1 - s_{kt})\) meaningless. Parameter \( s_{kt} \) is defined using a distance-based measure that minimizes the effect of outliers. An incorrect choice (i.e., a vector) that is far away from the center of its class distorts the decision boundary. Given \( z_{kt} = x_{1t} - x_{kt} \), the center of a class in the vector space is denoted by \( \bar{z}_{kt} = \frac{1}{T(K-1)} \sum_{t=1}^{T} \sum_{k=2}^{K} z_{kt} \). Let \( D_{max} \) denote the Euclidean distance of the furthest \( z_{kt} \) from its centroid, such that \( D_{max} = \max_{z} \|z_{kt} - \bar{z}_{kt}\|^2 \), then, \( s_{kt} \) is given by

\[
s_{kt} = 1 - \frac{\|z_{kt} - \bar{z}_{kt}\|^2}{D_{max}}, \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}, \tag{1.5}
\]

where \( 0 \leq s_{kt} \leq 1 \), and the point that is furthest away from the centroid gets the lowest weight. An observation \( z_{kt} \) that is far away from the center of its class can heavily
distort the separating hyperplane produced by SVM if it is on the wrong side of the classifier. On the other hand, if such observation is located on the correct side of the classifier and far away from the margin, then it is not a support vector and would not affect the decision boundary. The distance to the center is thus used as a proxy to find outliers.

1.3.2 Handling Inaccuracies in Consumer Perception

The evidence from the literature illustrates that consumers may not accurately perceive product attributes, or may fail to accurately compare the alternatives. Thus, one should aim at learning preferences taking into consideration the uncertainties and perturbations in consumers’ perception of the alternatives. We employ methods of robust optimization (Ben-Tal et al., 2009; Trafalis & Gilbert, 2006) to achieve robustness against such perturbations. Consider a consumer evaluating alternatives $x_{1t}$ and $x_{2t}$ at choice occasion $t$. Misconceiving the first alternative’s attributes, for example, can be modelled by a perturbation in each attribute, that is $x_{1t} = \bar{x}_{1t} + e_{1t}, e_{1t} \in \mathbb{R}^J$, and an inaccurate comparison of the alternatives can be written as $z_{2t} = (\bar{x}_{1t} - \bar{x}_{2t}) + e_{2t}, e_{2t} \in \mathbb{R}^J$.

Following the literature (e.g., Bergmans, 1974; Hughes, 1991), we assume $e_{1t}, e_{2t},$ and $\epsilon_{2t}$ follow a Gaussian distribution (i.e., additive white Gaussian noise); even though the results of this paper are robust to changes in this assumption. Thus, both types of inaccuracies can be modelled, without loss of generality, by $z_{kt} = (\bar{x}_{1t} - \bar{x}_{kt}) + \epsilon_{kt}$, and equivalently $z_{kt} = \bar{z}_{kt} + \epsilon_{kt}, \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}$. The objective is to find the maximum-margin separating hyperplane that satisfies

$$w \cdot z_{kt} \geq 1 - \xi_{kt}, \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}, \quad (1.6)$$

which is equivalent to

$$w \cdot (\bar{z}_{kt} + \epsilon_{kt}) \geq 1 - \xi_{kt}$$

$$\implies w \cdot \bar{z}_{kt} + w \cdot \epsilon_{kt} \geq 1 - \xi_{kt}, \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}. \quad (1.7)$$
1.3. The Individual-Level Robust Framework

Even though $\epsilon_{kt}$ is unknown, we enforce a bound $\eta_{kt} \in \mathbb{R}$ on the norm of $\epsilon_{kt}$ (as a tuning parameter), such that

$$
\|\epsilon_{kt}\|_p \leq \eta_{kt}, \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}, 
$$

(1.8)

where $\|\cdot\|_p$ denotes the $p$-norm.

There is no guarantee that the solution of problem (1.2)-(1.4) with constraints (1.6), also satisfies constraints (1.7) and (1.8). A utility vector $w$ learned by CBC-SVM (1.2)-(1.4) is a robust feasible solution with respect to perturbations $\epsilon_{kt}$, if and only if for all

$$
\min [w \cdot z_{kt} + w \cdot \epsilon_{kt} + \xi_{kt}] \geq 1, \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}. 
$$

(1.9)

The minimum of $w \cdot \epsilon_{kt}$ is obtained by solving

$$
\begin{align*}
\text{minimize} & \quad w \cdot \epsilon_{kt} \\
\text{s.t.} & \quad \|\epsilon_{kt}\|_p \leq \eta_{kt} \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\},
\end{align*}
$$

(1.10)

(1.11)

where $\epsilon_{kt}$ is the only variable. Following Trafalis & Gilbert (2006), H"{o}lder’s inequality implies that the minimum of $w \cdot \epsilon_{kt}$ is $- \eta_{kt} \|w\|_q$, where $\|\cdot\|_q$ is the dual norm of $\|\cdot\|_p$. By replacing $w \cdot \epsilon_{kt}$ with $- \eta_{kt} \|w\|_q$ and introducing it into the standard formulation of CBC-SVM (1.2)-(1.4), the following robust CBC-SVM is obtained, which is robust against feature noise whose $\ell_p$-norm is bounded by $\eta_{kt}$ (but not against the label noise).

$$
\begin{align*}
\text{minimize} & \quad g_q(w) + C \sum_{t=1}^{T} \sum_{k=2}^{K} \xi_{kt} \\
\text{s.t.} & \quad w \cdot (\bar{x}_{1t} - \bar{x}_{kt}) - \eta_{kt} \|w\|_q \geq 1 - \xi_{kt} \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\} \\
\xi_{kt} & \geq 0 \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\},
\end{align*}
$$

(1.12)

(1.13)

(1.14)

where $g_q(w)$ is the complexity control term that depends on the choice of the norm and is a function of the $\ell_q$-norm of $w$. The program (1.12)-(1.14) is to show how we robus-
tatify SVM-CBC against inaccuracies in consumer perception. Therefore, the weighting scheme—which is used to robustify SVM-CBC against response errors—is not interacted with model (1.12)-(1.14). Our proposed models, robust against both feature and label noise, are presented in Section 1.3.3.

1.3.3 The Individual-Level Models

This section proposes three models to learn consumer preferences from choice data. The proposed models are compared to standard CBC-SVM \cite{Evgeniou2005} in Section 1.5.

Model I: Robust $\ell_1$-Norm with Weights

By taking the $\ell_1$-norm as the measure of distance in $\mathbb{R}^J$ (i.e., $p = 1$) within the robust framework presented in Section 1.3.2, the following constraint is imposed on the perturbations: $\|\epsilon_{kt}\|_1 \leq \eta$. Furthermore, by using the weighting scheme defined in equation (1.5), denoted by $s_{kt}$, the following optimization problem is obtained

\[
\begin{align*}
\text{minimize} \quad & \frac{1}{2}\|w\|_\infty + C \sum_{t=1}^{T} \sum_{k=2}^{K} \xi_{kt} s_{kt} \\
\text{subject to} \quad & w \cdot (x_{1t} - x_{kt}) - \eta \|w\|_\infty \geq 1 - \xi_{kt} \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\} \\
& \xi_{kt} \geq 0 \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}.
\end{align*}
\] (1.15)

Problem (1.15)-(1.17) can be rewritten as a linear program. Let the auxiliary variable $\mu \in \mathbb{R}^J$ be such that $\mu = \|w\|_\infty$. Also, let $w(j)$ denote the $j$-th element of $w$. Then $\mu = \|w\|_\infty$ can be imposed using three constraints: $\mu \geq 0$, $\mu \geq -w(j)$, and $\mu \geq w(j)$, for
1.3. The Individual-Level Robust Framework

all \( j \in \{1, \ldots, J\} \). The LP formulation is then given by

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{2} \mu + C \sum_{t=1}^{T} \sum_{k=2}^{K} \xi_{kt} \ s_{kt} \\
\text{s.t.} & \quad w \cdot (x_{1t} - x_{kt}) - \eta \mu \geq 1 - \xi_{kt} \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\} \\
& \quad \xi_{kt} \geq 0 \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\} \\
& \quad \mu \geq -w(j) \quad \forall j \in \{1, \ldots, J\} \\
& \quad \mu \geq w(j) \quad \forall j \in \{1, \ldots, J\} \\
& \quad \mu \geq 0 \quad \forall j \in \{1, \ldots, J\}.
\end{align*}
\]  

(1.18)

(1.19)

(1.20)

(1.21)

(1.22)

(1.23)

Model II: Robust \( \ell_2 \)-Norm with Weights

In model II, the \( \ell_2 \)-norm is taken as the measure of distance \((p = 2)\), which means \( \| \epsilon_{kt} \|_2 \leq \eta_{kt} \) is imposed on the perturbations caused by consumer misconception of the alternatives. The resulting problem is the following second-order cone program

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{2} ||w||^2 + C \sum_{t=1}^{T} \sum_{k=2}^{K} \xi_{kt} \ s_{kt} \\
\text{s.t.} & \quad w \cdot (x_{1t} - x_{kt}) - \eta ||w||_2 \geq 1 - \xi_{kt} \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\} \\
& \quad \xi_{kt} \geq 0 \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}. 
\end{align*}
\]  

(1.24)

(1.25)

(1.26)

Model III: Robust \( \ell_\infty \)-Norm with Weights

The third model that is presented considers the \( \ell_\infty \)-norm, i.e. \( p = \infty \). The resulting constraint \( \| \epsilon_{kt} \|_\infty \leq \eta \) implies that the maximum perturbation in the attributes should be less than \( \eta \) in absolute value, that is, \( \max\{|\epsilon_{kt}(i)| : i = 1, \ldots, J\} \leq \eta \). The resulting optimization problem is as follows

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{2} ||w||_1 + C \sum_{t=1}^{T} \sum_{k=2}^{K} \xi_{kt} \ s_{kt} \\
\text{s.t.} & \quad w \cdot (x_{1t} - x_{kt}) - \eta ||w||_1 \geq 1 - \xi_{kt} \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\} \\
& \quad \xi_{kt} \geq 0 \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}.
\end{align*}
\]  

(1.27)

(1.28)

(1.29)
The $\ell_1$ complexity control improves the sparseness of the solution, which is a desirable property of a preference learning model, as shown by Maldonado et al. (2015). Problem (1.27)-(1.29) can be formulated as a linear program by introducing the auxiliary variable $\lambda \in \mathbb{R}^J$, such that $\lambda(j) = |w(j)|$, where $w(j)$ is the $j$-th element of $w$. The resulting linear program is

$$\text{minimize}_{\mathbf{w}, \xi_{kt}} \quad \frac{1}{2} \sum_{j=1}^{J} \lambda(j) + C \sum_{t=1}^{T} \sum_{k=2}^{K} \xi_{kt} s_{kt}$$

$$\text{s.t.} \quad w \cdot (x_{1t} - x_{kt}) - \eta \sum_{j=1}^{J} \lambda(j) \geq 1 - \xi_{kt} \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}$$

$$\xi_{kt} \geq 0 \quad \forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}$$

$$\lambda(j) \geq -w(j) \quad \forall j \in \{1, \ldots, J\}$$

$$\lambda(j) \geq w(j) \quad \forall j \in \{1, \ldots, J\}$$

$$\lambda(j) \geq 0 \quad \forall j \in \{1, \ldots, J\}.$$  (1.35)

1.4 The Joint Learning of Consumer Preferences and Market Segments

In this section we present a framework to simultaneously segment the customer base—based on the observed choices only—and learn each segment’s preferences while preserving the robustness features of the individual-level models.

The input data ($\mathcal{M}$) for our proposed framework contains all the past choices of the individual consumers, such that $\mathcal{M} = \{M_1, \ldots, M_1^1, M_1^2, \ldots, M_2^T, \ldots, M_N^1, \ldots, M_N^N\}$, where the assignment of observed choices to consumers is arbitrary and there are no idiosyncratic features that would enforce the assignment of choice $M_i^j$ to a particular consumer. We denote by $d \in \{1, 2, \ldots, D\}$ a segment of the consumers. Let decision variable $u^{di} \in \{0, 1\}$ capture the assignment of a consumer to a segment, $L$ denote a large number, $w \in \mathbb{R}^{J \times D}$, and $\xi, s \in \mathbb{R}_+^{K \times T \times D}$. Then, the robust simultaneous learning
(RSL) model is as follows:

\[
\text{RSL: minimize}_{w, \xi, u} \quad \frac{1}{2} \sum_{d=1}^{D} \|w^d\|_q + C \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{d=1}^{D} \xi_{dkt}^s s_{dkt}^{s} \\
\text{s.t.} \quad w^d \cdot (x_{1t} - x_{kt}) = \eta^r \|w^d\|^p_{\ell_p} \geq 1 - \xi_{dkt}^s - (1 - u^{d})L \quad \forall k, \forall t, \forall d, \forall i. \\
\sum_{d=1}^{D} u^{d} = 1 \quad \forall i. \hspace{1cm} (1.37) \\
u^{d} \in \{0, 1\} \hspace{1cm} (1.38) \\
\xi_{dkt}^s \geq 0 \quad \forall k, \forall t, \forall d, \forall i. \hspace{1cm} (1.40)
\]

The RSL model (1.36)-(1.40) ensures that (1) consumers are divided into \(D\) segments with each consumer assigned to exactly one segment, and (2) the particular preferences of each segment are learned using the robust framework put forward in Section 1.3. Now, similar to Section 1.3, we present the reformulations of model (1.36)-(1.40), given different choices of the \(\ell_p\)-norm, in what follows.

### 1.4.1 RSL: \(\ell_1\)-norm

If we choose \(\ell_1\)-norm as the measure of distance in \(\mathbb{R}^J\), the perturbations will be bounded by: \(\|\xi_{dkt}^s\|_1 \leq \eta\). To reformulate the problem as a mixed-integer linear program, let the auxiliary variable \(\mu^d \in \mathbb{R}, \forall d \in \{1, \ldots, D\}\), be such that \(\mu^d = \|w^d\|_{\ell_\infty}\). Also, let \(w^d(j)\) denote the \(j\)-th element of \(w^d\). Then we impose equation \(\mu^d = \|w^d\|_{\ell_\infty}\) using three constraints: \(\mu^d \geq 0, \mu^d \geq -w^d(j), \text{ and } \mu^d \geq w^d(j)\), for all \(j\) and \(d\). The MILP formulation is
then given by

\[
\text{RSL}_{\ell_1}: \begin{array}{c}
\text{minimize} \\
\frac{1}{2} \sum_{d=1}^{D} \mu^d + C \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{k=2}^{K} \sum_{d=1}^{D} s^d s^d_{kt} \\
\text{s.t.} \\
\sum_{d=1}^{D} u^d = 1 \\
u^d \in \{0, 1\} \\
\mu^d \geq -w^d(j) \\
\mu^d \geq w^d(j) \\
\mu^d \geq 0 \\
\xi^d_{kt} \geq 0
\end{array}
\] (1.41)

\[
\begin{align*}
& w^d \cdot (x_{1t} - x_{kt}) - \eta \mu^d \geq 1 - \xi^d_{kt} - (1 - u^d) L & \forall k, \forall t, \forall d, \forall i \\
& \sum_{d=1}^{D} u^d = 1 & \forall i. \\
& u^d \in \{0, 1\} \\
& \xi^d_{kt} \geq 0 & \forall k, \forall t, \forall d, \forall i.
\end{align*}
\] (1.42)

1.4.2 RSL: \(\ell_2\)-norm

If \(\ell_2\)-norm is taken as the measure of distance (i.e., \(p = 2\)), the bound \(|\epsilon^d_{kt}|_2 \leq \eta_{kt}\) is imposed on the perturbations caused by the consumer misconceptions. The resulting problem is the following mixed integer second-order cone program.

\[
\text{RSL}_{\ell_2}: \begin{array}{c}
\text{minimize} \\
\frac{1}{2} \sum_{d=1}^{D} \|w^d\|_2^2 + C \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{k=2}^{K} \sum_{d=1}^{D} s^d s^d_{kt} \\
\text{s.t.} \\
\sum_{d=1}^{D} u^d = 1 \\
u^d \in \{0, 1\} \\
\mu^d \geq -w^d(j) \\
\mu^d \geq w^d(j) \\
\mu^d \geq 0 \\
\xi^d_{kt} \geq 0
\end{array}
\] (1.49)

\[
\begin{align*}
& w^d \cdot (x_{1t} - x_{kt}) - \eta \|w^d\|_2^2 \geq 1 - \xi^d_{kt} - (1 - u^d) L & \forall k, \forall t, \forall d, \forall i \\
& \sum_{d=1}^{D} u^d = 1 & \forall i. \\
& u^d \in \{0, 1\} \\
& \xi^d_{kt} \geq 0 & \forall k, \forall t, \forall d, \forall i.
\end{align*}
\] (1.50)

1.4.3 RSL: \(\ell_\infty\)-norm

Lastly, we consider the \(\ell_\infty\)-norm as the measure of distance (i.e., \(p = \infty\)), which implies \(|\epsilon^d_{kt}|_\infty \leq \eta\). That is, the maximum perturbation in the attributes should be less than
\eta in absolute value: \( \max \{ |\epsilon_{kt}(j)| : j = 1, \ldots, J \} \leq \eta \). Similar to Section 1.3.3, the resulting \( \ell_1 \) complexity control improves the sparseness of the solution and enables the analyst to identify the features that are worthwhile to different segments of the market. To present the robust problem as a mixed integer linear program, we introduce the auxiliary variable \( \lambda^d \in \mathbb{R}^J \), such that \( \lambda^d(j) = |w^d(j)| \), where \( w^d(j) \) is the \( j \)-th element of \( w^d \).

\[
\text{RSL}_\ell: \begin{array}{ll}
\text{minimize} & \frac{1}{2} \sum_{d=1}^{D} \sum_{j=1}^{J} \lambda^d(j) + C \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{k=2}^{K} \sum_{d=1}^{D} \xi_{kt}^d s_{kt}
\text{subject to} & w^d \cdot (x_{1t} - x_{kt}) - \eta \sum_{j=1}^{J} \lambda^d(j) \geq 1 - \xi_{kt}^d - (1 - u^d) L \quad \forall k, \forall t, \forall d, \forall i
\sum_{d=1}^{D} u^d = 1 \quad \forall i
u^d \in \{0, 1\}
\lambda^d(j) \geq -w^d(j) \quad \forall j, d
\lambda^d(j) \geq w^d(j) \quad \forall j, d
\lambda^d(j) \geq 0 \quad \forall j, d
\xi_{kt}^d \geq 0 \quad \forall k, \forall t, \forall d, \forall i.
\end{array}
\]

1.5 Empirical Evidence

The objective of this section is to illustrate the effectiveness and practicality of the proposed framework, using synthetic and real data. Section 1.5.1 describes the general simulation setup and Section 1.5.2 presents the simulation results for the individual-level preference elicitation models proposed in Section 1.3. In Section 1.5.3 we contrast the performance of the proposed framework with the benchmark using a choice-based conjoint data set from the literature. Lastly, in Section 1.5.4 we present the results for our robust simultanoues learning (RSL) approach toward learning consumer preferences.


1.5.1 Simulation Design

We follow the literature by using the standard simulation design in the prior work (Evgeniou et al., 2005; Maldonado et al., 2015). The objective of the simulation is to generate a data set of \( T \) past choices for each consumer \( i = 1, \ldots, N \), of the form \( S^i = \{ M^i_1, \ldots, M^i_T \} \), where \( M^i_t = \{ x_{1t}, \ldots, x_{Kt} \} \), \( t = 1, \ldots, T \). In each experiment, different data sets are simulated with varied levels of noise. Similar to Maldonado et al. (2015), data sets across all individual-level experiments contain choice data for \( N = 100 \) users on \( T = 12 \) choice occasions, which are randomly divided into a training set of size \( T_{\text{training}} = 10 \) and a testing set with \( T_{\text{testing}} = 2 \). Each choice occasion is comprised of \( K = 4 \) alternatives, where the alternatives are generated using a random design with four attributes \( (J = 4) \), and \( n_j = 4 \) levels, \( j = 1, \ldots, J \). Random design is selected since it provides a more accurate simulation of consumer data that is not generated under controlled conditions. These data sets are used to estimate the utility functions using the three proposed models as well as CBC-SVM, which is proposed by Evgeniou et al. (2005). The predictive performance of the models is compared in an out-of-sample task. Consistent with the literature (Evgeniou et al., 2005; Toubia et al., 2007; Maldonado et al., 2015), to simulate each user, a random partworths vector is drawn for each attribute from a Gaussian distribution with mean \( (-\rho, -\rho/3, \rho/3, \rho) \) and covariance matrix \( \Sigma = \sigma I \), where \( I \) is the \( 4 \times 4 \) identity matrix.

The two parameters \( (C, \eta) \) for the proposed models and CBC-SVM are calibrated as done in Evgeniou et al. (2005) and Maldonado et al. (2015), using grid search with leave-one-out cross-validation. The grid search uses the following sets to find the best combination of parameters

\[
C \in \{2^{-6}, \ldots, 2^{-1}, 2^0, 2^1, \ldots, 2^6\}\quad (1.62)
\]
\[
\eta \in \{10^{-2}, 10^{-1}, 10^0, 10^1, 10^2\}\quad (1.63)
\]

Given a training set of length \( T_{\text{training}} \), this procedure tunes the parameters by iteratively leaving one past choice out, and estimating the models using the remaining \( T_{\text{training}} - 1 \) choice occasions. Then the individual partworths obtained from \( T_{\text{training}} - 1 \)
observations are used to predict the consumer choice in the left-out choice occasion (i.e., validation set). After repeating this procedure for each observation ($T_{training}$ times), the model parameters are set to the values—from parameter sets (1.62)-(1.63)—that maximize the predictive performance measured by the hit-rate. Finally, when all parameters are fixed, the partworths vectors $w_i$, $i = 1, \ldots, N$, are obtained through resolving the models with the fixed parameters and all $T_{training}$ choice occasions in the training set. These partworths vectors are then used in an out-of-sample prediction task.

1.5.2 Experiment 1: Simulation for the Individual-Level models

Since the experiment designs of Evgeniou et al. (2005) and Maldonado et al. (2015) do not allow for explicit modelling of the response errors and consumer misconceptions, the experiment slightly diverges from their standard simulation exercise. To model consumer misconceptions of the alternatives or their failure to compare the alternatives objectively, an additive noise is introduced. Let $z_{kt} = (\bar{x}_{1t} - \bar{x}_{kt}) + \epsilon_{kt}$, $\forall k \in \{2, \ldots, K\}, \forall t \in \{1, 2, \ldots, T\}$. It is assumed that the nominal vectors (i.e., $\bar{x}_{kt}$) are known, and for each comparison, a normally distributed random noise $\epsilon_{kt}$ with $\epsilon_{kt} \sim N(0, \sigma_0)$ is added. To model the response errors, the probability of error $\phi \in [0, 1]$ is defined, which indicates the probability of a consumer choosing an alternative that does not maximize her utility. Therefore, $\sigma_0$ and $\phi$ are parameters that can vary across individuals and choice occasions. We refer to $\sigma_0$ and $\phi$ as the misconception error and the response error parameters, respectively. Table 1.1 summarizes the four cases studied in this experiment, where the most difficult case is the one with high response error and high misconception error. Note that in this experiment, the partworths vectors are generated with $\rho = 0.5$ and $\sigma = 3$ (similar to Evgeniou et al. (2005)).

<table>
<thead>
<tr>
<th>Response Error: Low</th>
<th>Misconception Error: Low</th>
<th>Misconception Error: High</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.05, \sigma_0 = 0.2$</td>
<td>$\phi = 0.05, \sigma_0 = 0.4$</td>
<td></td>
</tr>
<tr>
<td>$\phi = 0.3, \sigma_0 = 0.2$</td>
<td>$\phi = 0.3, \sigma_0 = 0.4$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.1: Levels of response error probability ($\phi$) and misconception error ($\epsilon_{kt} \sim N(0, \sigma_0)$)
To compare the performance of the proposed models with CBC-SVM, 100 random choice occasions are generated for each individual under each noise condition. Then the mean and variance of hit-rates are used to compare the models. Table 1.2 summarizes the results in terms of mean hit-rate, where each noise condition is denoted by a pair; LH for example, indicates the case with Low misconception error and High response error.

<table>
<thead>
<tr>
<th>Noise condition</th>
<th>The model</th>
<th>No Noise</th>
<th>LL</th>
<th>LH</th>
<th>HL</th>
<th>HH</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Noise</td>
<td>SVM-CBC</td>
<td>0.82</td>
<td>0.58</td>
<td>0.48</td>
<td>0.53</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>Model I: Robust (\ell_1)-Norm with Weights</td>
<td>0.78</td>
<td>0.56</td>
<td>0.46</td>
<td>0.51</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>Model II: Robust (\ell_2)-Norm with Weights</td>
<td>0.80</td>
<td>0.59</td>
<td>0.48</td>
<td>0.53</td>
<td>0.41*</td>
</tr>
<tr>
<td></td>
<td>Model III: Robust (\ell_\infty)-Norm with Weights</td>
<td>0.81</td>
<td>0.61*</td>
<td>0.49*</td>
<td>0.54*</td>
<td>0.42*</td>
</tr>
</tbody>
</table>

Table 1.2: Average out-of-sample hit-rate for 100 consumers, 100 choice occasions each. In each setting, boldface indicates the highest. “*” denotes hit-rates that are significantly better than the benchmark.

As illustrated in Table 1.2 with the exception of the no noise case, model III consistently outperforms the benchmark, the \(\ell_1\)-norm, and the \(\ell_2\)-norm models. The superior performance of the \(\ell_\infty\) model can be attributed to (1) the improved sparseness of the utility vectors learned by this model, and (2) the fact that the \(\ell_\infty\)-norm bounds the maximum perturbation (as opposed to the sum of the perturbations) in the individual utility vectors. Model II, on the other hand, never performs worse than the SVM-CBC benchmark (except for the no noise setting), while it always performs better than model I and always worse than model III. This result shows that bounding the \(\ell_2\)-norm of the perturbations provides higher robustness against consumer misconceptions, compared to a bound on the summation of the perturbation vector elements (i.e., the \(\ell_1\)-norm). Even though model I cannot outperform the benchmark model in terms of the hit-rate, it offers a more consistent prediction by having a smaller hit-rate variance. Table 1.3 summarizes the variance of hit-rate under the five noise settings. Similar to the hit-rates, model III consistently outperforms the SVM-CBC model. However, model I shows a better performance compared to the benchmark in terms of the variance of the prediction. The results for model II are mixed; while this model’s pre-
dictions never shows a higher variability compared to the other robust models, it outperforms SVM-CBC model under LL and HL settings. This result shows that while the $\ell_2$-norm bound on the perturbation vector can generate a competitive hit-rate, it does not provide the lowest variance.

<table>
<thead>
<tr>
<th>The model</th>
<th>Noise condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Noise</td>
</tr>
<tr>
<td>SVM-CBC</td>
<td>0.11</td>
</tr>
<tr>
<td>Model I: Robust $\ell_1$-Norm with Weights</td>
<td>0.11</td>
</tr>
<tr>
<td>Model II: Robust $\ell_2$-Norm with Weights</td>
<td>0.11</td>
</tr>
<tr>
<td>Model III: Robust $\ell_\infty$-Norm with Weights</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 1.3: Variance of out-of-sample hit-rate for 100 consumers, 100 choice occasions each. In each setting, boldface indicates the lowest.

In summary, as expected, in the unrealistic case of perfectly rational consumer preferences (no noise, no errors), the SVM-CBC performs the best. The results show that if the consumers’ misconception of the alternatives and their response errors are disregarded, SVM-CBC would be the superior model. On the other hand, in the more realistic situation where the consumers’ misjudgments (e.g., irrationality) are taken into account, the proposed robust models predict consumers’ future choices with higher accuracy and a lower variance.

1.5.3 Experiment 2: Real-World Data for the Individual-Level Models

This section evaluates the performance of the proposed robust models using the real-world credit card conjoint study dataset collected by [Allenby & Ginter (1995)] and made available by [Rossi et al. (2012)]. Obtained from a bank wanting to offer credit cards to its out-of-state customers, this conjoint study includes the choice data of 946 respondents each providing responses for 14 to 17 paired-comparison choice tasks (i.e., $T \in \{14, \ldots, 17\}$ and $K = 2$). Each credit card is described by seven attributes ($J = 7$), where the attributes take two, three, or four levels, that is, $n_j \in \{2, 3, 4\}, \forall j$. Table 1.4 summarizes the attributes and the attributes levels.
Chapter 1. Preference Learning and Market Segmentation

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>High, Medium, Low fixed, Medium variable.</td>
</tr>
<tr>
<td>Rewards</td>
<td>Four reward programs consisting annual fee waivers and interest rebate reductions.</td>
</tr>
<tr>
<td>Annual fee</td>
<td>High, Medium, Low.</td>
</tr>
<tr>
<td>Rebate</td>
<td>Low, Medium, High.</td>
</tr>
<tr>
<td>Credit line</td>
<td>Low, High.</td>
</tr>
<tr>
<td>Grace period</td>
<td>Short, Long.</td>
</tr>
</tbody>
</table>

Table 1.4: Description of the attributes (Allenby & Ginter, 1995).

To have the same level of information on each respondent’s utility function, we randomly remove one, two, and three choice task(s) from the respondents who responded to 15, 16, and 17 choice tasks, respectively. This way we obtain choice tasks of length $T = 14$ for every respondent, then we split the choice sets into $T_{training} = 10$ and $T_{testing} = 4$. Similar to Experiment 1, we find the optimal parameters using grid search with leave-one-out cross-validation, and compare the performance of the three proposed robust models against SVM-CBC.

Table 1.5 summarizes the mean and the variance of the hit-rate across the four models discussed in this chapter. Consistent with Experiment 1, when consumer misconceptions and irrationalities exist, model III outperforms SVM-CBC and the other two proposed robust models. In this case, models I and II provide a higher mean hit-rate compared to SVM-CBC as well. In terms of the variance, the models show identical performance.

<table>
<thead>
<tr>
<th>The model</th>
<th>Mean hit-rate</th>
<th>Hit-rate variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVM-CBC</td>
<td>0.57</td>
<td>0.27</td>
</tr>
<tr>
<td>Model I: Robust $\ell_1$-Norm with Weights</td>
<td>0.57</td>
<td>0.27</td>
</tr>
<tr>
<td>Model II: Robust $\ell_2$-Norm with Weights</td>
<td>0.59*</td>
<td>0.27</td>
</tr>
<tr>
<td>Model III: Robust $\ell_\infty$-Norm with Weights</td>
<td>0.61*</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Table 1.5: Mean and variance of out-of-sample hit-rate over the 946 customers’ choice data in the credit card dataset. In each column, boldface indicates the best. “*” denotes hit-rates that are significantly better than the benchmark.
1.5.4 Experiment 3: Simulation for the RSL Market Segmentation Models

While an individual consumer’s choice, $M_i$, is simulated in this experiment for $T = 24$ choices (i.e., randomly drawn $T_{\text{training}} = 20$ and $T_{\text{testing}} = 4$) per consumer, the input data for the segmentation model ($M = \{M_1^1, \ldots, M_T^1, M_1^2, \ldots, M_T^2, \ldots, M_1^N, \ldots, M_T^N\}$) contains all the past choices of all individual consumers. We denote by $d \in \{1, 2, \ldots, D\}$ a segment of the consumers, and we set $D = 2$ throughout this section. We are interested in efficient segmentation of the $N$ consumers in the market into $D$ segments, and in accurately predicting the future choices of each segment. We generate choice data for $N = 100$ instances, where each choice occasion is comprised of $K = 4$ alternatives, and the alternatives are generated using a random design with four attributes ($J = 4$), and $n_j = 4$ levels, $j = 1, \ldots, J$. The parameters for the proposed models and the benchmark are calibrated using the same procedure as above. The error parameters, $\sigma_0$ and $\phi$, are set to three different levels each (i.e., low, medium, and high), leading to the nine combinations of error levels described in Table 1.6; where the most difficult case is the one with high response error and high misconception error.

<table>
<thead>
<tr>
<th>Response Error</th>
<th>Misconception Error</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>$\phi = 0.1, \sigma_0 = 0.1$</td>
<td>$\phi = 0.1, \sigma_0 = 0.25$</td>
<td>$\phi = 0.1, \sigma_0 = 0.4$</td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td>$\phi = 0.25, \sigma_0 = 0.1$</td>
<td>$\phi = 0.25, \sigma_0 = 0.25$</td>
<td>$\phi = 0.25, \sigma_0 = 0.4$</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>$\phi = 0.4, \sigma_0 = 0.1$</td>
<td>$\phi = 0.4, \sigma_0 = 0.25$</td>
<td>$\phi = 0.4, \sigma_0 = 0.4$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.6: Levels of response error probability ($\phi$) and misconception error ($\epsilon_{kt} \sim N(0, \sigma_0)$) for the RSL model.

As illustrated in the experiments above (Sections 1.5.2 and 1.5.3), the $\ell_{\infty}$-norm bound on the perturbations shows the best performance in predicting future consumers choices. Hence, in this section, we choose the $\ell_{\infty}$ version of RSL—i.e., model (1.54)-(1.61)—to evaluate against the five benchmarks presented below.

1. **Full Information**: in order to respect consumers’ privacy, we discard the part of the data that shows which consumer has made a given choice, and then we
use the RSL model to segment the market and learn the segments’ preferences. However, under this benchmark, we train SVM-CBC with full information and perfect assignments; such that, the model uses the data on individual consumers and can trace back a given choice to the consumer who made the choice. In other words, under this benchmark, each consumer forms a one-person segment and we have $D = N$. This approach is impractical and it violates consumer privacy, yet it is valuable to know how much prediction accuracy would be lost to ensure consumer privacy. We expect this model to have the highest prediction accuracy, since it uses some user data that is not made available to either RSL or the other four benchmarks.

2. **SVM-CBC-S**: we change the SVM-CBC formulation and introduce a segment assignment variable ($u_{di}$) and the segmentation constraint. In essence, SVM-CBC-S is similar to non-robust RSL, since this model is not robustified against perturbations and response errors, and the robustness terms $s_{kt}^{di}$ (in the objective function) and $\eta ||w^d||_p$ (in the constraints) are not present here. Model (1.64)-(1.68) presents the SVM-CBC-S formulation.

**SVM-CBC-S**: minimize $\min_{w,\xi,u} \frac{1}{2} \sum_{d=1}^{D} ||w^d||_2^2 + C \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{k=2}^{K} \sum_{d=1}^{D} \xi_{kt}^{di}$ \hspace{1cm} (1.64)

subject to $w^d \cdot (x_{1t} - x_{kt}) \geq 1 - \xi_{kt}^{di} - (1 - u_{di}) L \hspace{1cm} \forall k, \forall t, \forall d, \forall i \hspace{1cm} (1.65)$

$\sum_{d=1}^{D} u_{di} = 1 \hspace{2cm} \forall i \hspace{1cm} (1.66)$

$u_{di} \in \{0, 1\} \hspace{1cm} (1.67)$

$\xi_{kt}^{di} \geq 0 \hspace{2cm} \forall k, \forall t, \forall d, \forall i \hspace{1cm} (1.68)$

3. **Naive**: under this benchmark, the SVM-CBC model is used to learn the preferences assuming all consumers form exactly one segment. Therefore, this method learns one preference vector for all consumers, and then the learned preference
1.5. Empirical Evidence

<table>
<thead>
<tr>
<th>Noise condition</th>
<th>LL</th>
<th>LM</th>
<th>LH</th>
<th>ML</th>
<th>MM</th>
<th>MH</th>
<th>HL</th>
<th>HM</th>
<th>HH</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSL_{\infty}</td>
<td>0.67*</td>
<td>0.64*</td>
<td>0.59*</td>
<td>0.67*</td>
<td>0.63*</td>
<td>0.59*</td>
<td>0.61*</td>
<td>0.61*</td>
<td>0.55*</td>
</tr>
<tr>
<td>SVM-CBC-S</td>
<td>0.61</td>
<td>0.52</td>
<td>0.44</td>
<td>0.58</td>
<td>0.51</td>
<td>0.41</td>
<td>0.51</td>
<td>0.48</td>
<td>0.38</td>
</tr>
<tr>
<td>Full Information</td>
<td>0.74</td>
<td>0.71</td>
<td>0.65</td>
<td>0.72</td>
<td>0.64</td>
<td>0.61</td>
<td>0.69</td>
<td>0.64</td>
<td>0.56</td>
</tr>
<tr>
<td>Naive</td>
<td>0.65</td>
<td>0.59</td>
<td>0.55</td>
<td>0.64</td>
<td>0.61</td>
<td>0.53</td>
<td>0.61*</td>
<td>0.60</td>
<td>0.51</td>
</tr>
<tr>
<td>Random</td>
<td>0.64</td>
<td>0.56</td>
<td>0.54</td>
<td>0.63</td>
<td>0.59</td>
<td>0.51</td>
<td>0.61*</td>
<td>0.58</td>
<td>0.51</td>
</tr>
<tr>
<td>Clustering</td>
<td>0.56</td>
<td>0.53</td>
<td>0.52</td>
<td>0.53</td>
<td>0.52</td>
<td>0.49</td>
<td>0.53</td>
<td>0.52</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Table 1.7: Average out-of-sample prediction accuracy in terms of hitrate, for 100 instances. "*" denotes the second-best hitrate in each column.

vector is used to predict future choices of the single segment present in the market.

4. Random: we randomly divide the consumers into $D$ non-empty segments, and then use SVM-CBC to learn each segment’s preferences. The learned preferences are then used to predict the segments’ future choices.

5. Clustering: lastly, under this benchmark we use K-means clustering with $D$ centroids to obtain $D$ segments of the market according to the consumers’ observed choices. After clusters (segments) are formed, SVM-CBC is used to learn each segment’s preferences, and the future choices of each segment are predicted.

Table 1.7 summarizes the mean hit-rates of RSL and the five benchmarks, where each noise condition is denoted by a pair; ML for example, indicates the case with Medium misconception error and Low response error.

We offer several insights based on Table 1.7. Our proposed robust simultaneous learning framework, RSL, outperforms all the benchmarks that do not require data on individual consumer profiles. As mentioned earlier, the Full Information benchmark is highly granular and assumes $D = N$, which is impractical. Nonetheless, by comparing the prediction accuracy of RSL and the Full Information benchmark we can demonstrate the price of privacy; that is, how much the prediction accuracy is affected when a privacy-preserving data collection policy is imposed.\footnote{“The problem with data protection laws is that it presumes the data collection was ok” (Edward Snowden, 2019).} We observe that at the low-
est level of error the model would lose 17% of its prediction accuracy, but this loss of accuracy is reduced to merely 1.8% at the highest error level. This can be encouraging news for practitioners who are concerned about the effects of equitable practices on profitability. Furthermore, from a choice prediction point of view, we show that certain seemingly intuitive segmentation strategies can harm the firm’s profitability. Segmenting the market using K-means clustering, for example, offers a worse choice prediction accuracy than Naive and Random benchmarks. Meaning that, assuming all consumers form a single segment or arbitrary segments can be better strategies than using K-means clustering, as far as future choice prediction accuracy is concerned. Our analysis lends more credence to Jiang & Tuzhilin (2006) and Liu et al. (2010) who argue segmenting the market through clustering techniques may not be useful when the objective is to predict future consumer responses. Nonetheless, the Clustering benchmark has the lowest-variance predictive performance across different error levels, which signals a possible advantage over Naive, Random, and Full Information benchmarks. Lastly, from a practitioner’s point of view, the error levels for the choices made by a group of customers are unobserved. Unless there exists evidence of how error-prone customers in a particular market are, practitioners may want to maximize the prediction accuracy on average. In this case, RSL offers a more consistent prediction accuracy; while the Full Information benchmark loses 24% of its accuracy from LL to HH, compared to 18% for RSL.

1.6 Conclusions and future Research

In this chapter, we present a robust framework to simultaneously segments the customer base—based on the observed choices only—and learn each segment’s preferences while requiring only the consumers’ past choices as the input data. The proposed framework guarantees the robustness of the solution against the perturbations caused by consumer misconceptions and handles response errors using a weighting scheme that determines the relevance of each past choice in predicting future choices. In particular, the proposed models account for the three typical types of errors, (1) in-
accuracies in consumers’ perception of the product or service features, which may lead to consumers’ failure to objectively compare different alternatives available to them, (2) inconsistencies in consumer choices, which refers to consumers making contradictory choices, and (3) response errors, that is, consumers effectively comparing the alternatives, given the available information, but failing to make the intended choice, which may or may not cause an inconsistency. In three experiments, using simulated and real-world choice data, we establish the superior performance of the proposed models—compared to the SVM-CBC model and five market segmentation methodology benchmarks. We show that our robust simultaneous learning framework outperforms the SVM-CBC model and benchmarks of the market segmentation strategy, both in terms of the prediction accuracy of future choices and the variability of the predictions.

Future work can be carried out in different directions. First, the efficacy of the proposed framework in predicting consumer responsiveness to the firm’s pricing strategy (e.g., a rise or drop in the prices of the assortment) can be studied. Second, the proposed framework can be coupled with a dynamic questionnaire design algorithm to take advantage of the higher accuracy of the proposed models in forming the most informative choice tasks. Third, the proposed models can be applied to other types of conjoint analysis applications (such as menu-based choice) to investigate how using such robust models to capture consumer misconceptions can improve the quality of the market segmentation and the learned utility functions attributed to each segment, especially when consumers have the freedom to build their own preferred option. Finally, different weighting schemes and identification methods can be formulated to identify and mitigate the negative effect of an inconsistent choice. For example, the prediction performance of robust models with non-linear weights and bounded penalties can be compared with the models proposed in this chapter.
Chapter 2

Outsourcing Decision in the Presence of Supplier Copycatting
2.1 Introduction

Counterfeits and pirated goods pose an unprecedented threat to businesses worldwide. This problem has intensified to staggering levels in recent years. The Organisation for Economic Cooperation and Development (OECD) reports a 154% growth in the global trade of counterfeits, from $200 billion in 2005 to $509 billion in 2016, which represents 3.3% of world trade (OECD, 2018). A report by the U.S. Government Accountability Office sheds more light on the prevalence of counterfeit products: after investigating 47 consumer goods purchased from third-party retailers (including Amazon, Walmart, eBay, Sears Marketplace, and Newegg), the government agency discovered that 20 of the items, including travel accessories and cosmetics, were counterfeit (United States Government Accountability Office, 2018).

A common form of counterfeiting is what investigators call the “third shift” (also called the “midnight shift” or “ghost shift”), which occurs when firms outsource their production to suppliers overseas, and the suppliers activate a “third shift” to produce a counterfeit product using the firm’s intellectual property (Parloff, 2006). The pervasiveness of the supplier copycatting challenge has made it a pressing concern for businesses (Lorenzetti, 2016; Ramli & Chen, 2016). As an example, Umbra, a Canadian producer of home decor products, has been troubled by suppliers that use “third shifts” on several occasions. The firm’s overseas suppliers have successfully used this strategy to encroach into the market with counterfeit goods (Smith, 2015). Other examples include Joyme, a major Chinese supplier for IKEA that copied IKEA’s designs and encroached into the market with counterfeit products (Slater, 2014). More recently, Brilliance Motors, a Chinese supplier of BMW, manufactured its own car, Brilliance V5, which pundits believe to be a copy of BMW X1 (Akre, 2019).

Effective IP agreements can be prohibitively costly for manufacturers to design, negotiate, and enforce (Slater, 2014; EURObiz, 2016; Ghamat et al., 2021). Specifically, if an IP dispute cannot be settled by the parties out of court, it often takes years—in some cases nearly a decade (Schindler, 2021; Olijnyk, 2021)—to obtain the court’s final judgment (Heer et al., 2020). In the United States, for example, the costs associated with
Chapter 2. Supplier Copycatting

the IP litigation process commonly exceed USD 1 million (World Intellectual Property Organization, 2018). This reality often forces the outsourcing manufacturer to cut ties with the supplier. In other words, in a supplier copycatting setting, the suppliers face future repercussions (e.g., losing the manufacturer’s business) for copying the manufacturer’s product and encroaching into the market. Therefore, prior to making a market entry decision, the supplier must evaluate the costs associated with encroachment. At the same time, the manufacturer has alternative outsourcing opportunities to consider when dealing with a supplier/copycat. The existence of future repercussions for the supplier and future outsourcing alternatives for the manufacturer necessitates a two-period approach to study the supplier copycatting phenomenon.

In this chapter, we adopt a two-period game-theoretic approach and consider a setting where an original manufacturer sells its product to customers over two periods. At the beginning of the game, this manufacturer outsources the production to a high-quality supplier that has the capability to produce a copycat product and encroach into the market. After the manufacturer launches the product and coordinates marketing efforts to create demand, the supplier decides whether to enter the market with a copycat product. Upon deciding to enter the market independently, however, the supplier faces the consequences of copycatting and loses the manufacturer’s wholesale business in the subsequent period. In which case, the manufacturer will be outsourcing the production to an alternative supplier with a lower quality.

Our research studies the effects of the existence of supplier’s copycatting capability on the optimal strategy. We focus on four research questions: Can the quality of the low-quality supplier’s production regulate the high-quality supplier’s decision to encroach and the manufacturer’s profit? How does the quality of the copycat product affect the supplier’s optimal pricing and encroachment strategies? What is the effect of the market entry costs on encroachment? How are the manufacturer’s and the supplier’s optimal strategies affected when incorporating a time discount factor to model the higher net present value of the first-period profits?

We find the following answers to the above research questions. First, one would expect that since higher production quality improves customers’ utility, the manufacturer
should be better off upon an increase in the quality of the low-quality supplier’s production. However, we find conditions where the opposite is true. Moreover, we show that as the low-quality supplier’s quality is improved, the copycatting (high-quality) supplier can become more likely to encroach into the market. Second, we show that under certain conditions, upon an improvement in the quality of the copycat product, the copycatting supplier’s profit is reduced. Furthermore, an increase in the quality of the copycat product can convince the supplier against encroachment. Third, an increase in the copycatting cost can improve the copycatting supplier’s profit. In other words, under certain conditions, the supplier would prefer a costlier market entry. And fourth, our analysis suggests that the high-quality supplier’s tendency to encroach is reduced upon a decrease in the net present value of the future profits.

The rest of this chapter is organized as follows. In the next section, we review the extant literature on supplier encroachment and copycatting. Then, we present in Section 2.3 the two-period supplier copycatting game. The optimal solution of the mathematical model is discussed in Section 2.4 where we examine the manufacturer’s outsourcing decisions in the presence of supplier copycatting over two periods. We conduct sensitivity analyses in Section 2.5 and explore an extension of our mathematical model in Section 2.6. Finally, Section 2.7 contains our concluding remarks and discussion of future research.

2.2 Literature Review

This paper is related to two research streams within the field of supply chain management. First, we build upon the supplier encroachment literature, beginning with the work of Arya et al. (2007). These authors study the supply chain dynamics when the supplier encroaches into the market, demonstrating that the manufacturer may benefit from the supplier’s encroachment. Ha et al. (2016) then extend this work by making the product quality endogenous. They show that, contrary to the exogenous case, the manufacturer will be worse off if the supplier decides to encroach. Similarly, Guan et al. (2019) investigate the combined effect of the manufacturer’s strategic inventory
withholding and the supplier’s encroachment using a two-period model, and find situa-
tions where a combined use of the strategies will improve both the manufacturer’s and the supplier’s profits. Wang et al. (2013) consider a setting where an original man-
ufacturer outsources to competitive and non-competitive suppliers, studying the three basic Cournot competition games between the manufacturer and the competitive sup-
plier. Chen et al. (2019) examine a case where manufacturers of substitutive products choose between competition and coopetition, and show that the optimal coopetition strategy heavily depends on the degree of product substitution. Likewise, Cui (2019) investigates a supply chain with a high-quality manufacturer and an encroaching low-quality supplier, such that the supplier free rides on the manufacturer’s quality improvement investment. In doing so, they demonstrate that without the manufacturer’s investment in quality, the supplier will always have incentives to encroach. Furthermore, limited supply can drastically affect the dynamics of the supplier encroachment phenomenon. As such, Ghamat et al. (2018) and Yang et al. (2018) explore the optimal outsourcing and distribution strategies of a supplier under a capacity constraint. Another factor that can significantly influence the effects of supplier encroachment is the information structure. For this reason, Li et al. (2014), Huang et al. (2018), Guan et al. (2020), and Gao et al. (2021) study the implications of information asymmetry for supply chains that are subject to supplier encroachment.

Our paper differs from previous work on supplier encroachment in that we investi-
gate this phenomenon’s implications for a supply chain when the encroaching supplier is also a copycat who enters the market with a copied product. Notably, in the aforemen-
tioned studies, the supplier’s market encroachment is not an illicit practice. However, in supplier copycatting, the supplier’s market entry is unlawful in nature, as it occurs by means of a copycat product. Hence, the supplier will face future repercussions (e.g., losing the manufacturer’s business).

The second stream of research that this work builds on concerns copycatting and counterfeiting in a supply chain. It is important to note that we consider a case of non-deceptive counterfeits—that is, customers can differentiate the counterfeit from the original product prior to making a purchase decision. Within this stream, Gao
et al. (2017a) design a two-firm, two-period game to investigate the economic implications of a counterfeiter’s entry into the market, and to analyze the dynamics of the competition between the counterfeiter and the incumbent. They show that the counterfeiter’s entry improves consumer welfare, since it pushes the incumbent to lower the price, and identify the conditions for the successful entry of the counterfeiter. Using a similar two-period model, Pun & DeYong (2017) study the competition between an authentic manufacturer and a copycat firm for strategic consumers, over two periods. In their framework, the copycat makes a market entry decision after the first period, while strategic consumers anticipate the future prices and might postpone their purchase to the second period. The authors find that a lower level of quality from the original manufacturer can improve the manufacturer’s prices and profits, while a lower density of strategic consumers may decrease the manufacturer’s profits. At the same time, Gao et al. (2017b) show a different result with copycats’ encroachment in the luxury goods market, demonstrating that a higher quality can harm a copycat’s chances of a successful market entry. Hou et al. (2020) also focus on counterfeit-deterrence strategies in the luxury goods market, and find that the manufacturer can discourage the copycat from entering the market by launching a fighter brand. Yi et al. (2020) investigate the impacts of counterfeiting on global supply chains, and show that in a setting with a manufacturer and a retailer, the retailer is better positioned to enforce anti-counterfeiting strategies.

In contrast to the body of work in this second stream, we examine the case of supplier copycatting, in which the copycat is not some third-party counterfeiter but a supplier to the original manufacturer. Compared to third-party copycatting, supplier copycatting poses a unique challenge to the manufacturer’s operations. While a third-party copycat can influence the manufacturer’s bottom line only through market competition, a supplier copycat affects the manufacturer’s profit through competition and the wholesale price. This situation puts the encroaching supplier in the delicate position of having to simultaneously manage both market competition and future partnership with the manufacturer.

Among the few scholarly works on supplier copycatting, Ghamat et al. (2021) and
Pun & Hou (2021) are closely relevant to this chapter. Ghamat et al. (2021) consider a setting with an original manufacturer, a copycatting supplier, and a third-party copycat; and study the effectiveness of an IP agreement against supplier copycatting. From a practical point of view, however, IP agreements tend to be expensive and challenging to enforce (EURObiz, 2016; Smith, 2015; Harris, 2015). As noted in the introduction, using IP agreements and relying on the legal systems to enforce them can be prohibitively costly and laborious. Consequently, manufacturers cannot solely depend on the efficacy of IP agreements against the challenge of supplier copycatting, and instead consider ending current partnerships if copycatting occurs. Moreover, Pun & Hou (2021) consider a manufacturer that sells a product that requires the completion of multiple tasks using a single-period model. Their focus is on whether the government or the manufacturer should be responsible for protecting the IP rights. In contrast, we examine the future of manufacturer-supplier partnerships and the impact of copycatting on the optimal market strategies.

2.3 The Mathematical Model

We consider a manufacturer, $M$, that outsources the production of its product to a supplier over two periods, $t \in \{1, 2\}$. There are two suppliers with different levels of qualities; such that the consumers derive utility from the two components of quality: process quality and brand value. Supplier $A$ produces a high-quality product for the manufacturer (with total quality normalized to one), but it has copycatting capabilities. Supplier $B$ produces a low-quality product (with quality $b \in (0, 1)$), but it does not have copycatting capabilities. In the first period, the manufacturer outsources the production to supplier $A$. If supplier $A$ produces a copycat product (with quality $q \in (0, 1)$), then in the second period, the manufacturer outsources the production to supplier $B$. Otherwise, the manufacturer continues to outsource to supplier $A$ in the second period.

At the beginning of the game, the manufacturer decides on the level of marketing investment $I$ (e.g., promotions and campaigns), which in turn determines the size of
the market: \( \Psi (I) = \beta \sqrt{I} \); where \( \beta > 0 \) denotes the efficiency of the manufacturer’s marketing efforts, and there are diminishing marginal returns on this marketing investment. Manufacturers often make massive investments to build a market for their products, whereas copycats barely invest in marketing and opt for free-riding on the original manufacturer’s investment (Pun & DeYong, 2017). Zara, for example, who is famous for producing fast fashion, is a major free-rider in the fashion industry for advertising. It invests a mere 0.3% of its sales on advertising, while the industry averages between 3% and 5% (Kumar et al., 2007). Thus, the manufacturer’s marketing investment \( I \) sets the total size of the market, \( \Psi (I) \), for both the original and the copycat products.

Then, at \( t = 1 \) the manufacturer outsources the production to supplier \( A \), and subsequently the supplier decides on whether or not to encroach into the market with a copy of the manufacturer’s original product; a decision denoted by \( E \in \{ Y, N \} \). The manufacturer’s outsourcing decision at \( t = 2 \) depends on the supplier \( A \)’s encroachment decision in the first period. If the supplier does not encroach into the market \( (E = N) \), the manufacturer retains the relationship with the supplier and outsources the production to the supplier in \( t = 2 \) as well. On the other hand, if supplier \( A \) encroaches into the market with a copycat product in the first period \( (E = Y) \), in the second period, the manufacturer breaks off the business relationship with the supplier. Instead, the manufacturer outsources the production to supplier \( B \). When the outsourcing and encroachment decisions are realized, the wholesale price \( w_{jt}, j \in \{ A, B \} \), the manufacturer’s market price \( p_{Mt} \), and the copycatting supplier’s market price \( p_{At} \) (if applicable) are set.

Producing a copycat product and encroaching into the market entails certain costs for supplier \( A \). The supplier needs to make a market entry investment \( F > 0 \) (e.g., to establish the distribution channel). We assume that upon making the market entry investment and establishing a distribution channel, the supplier can utilize the channel in the subsequent period as well, making \( F \) a one-time investment cost.

Consumers have heterogeneous valuations of the (quality of the) products, such that the consumer type \( x \), follows a uniform distribution with density \( \Psi (I) \) (market
size). The consumers’ decision in each period is between buying the manufacturer’s original product, buying the supplier’s copycat product (if applicable), or making no purchase at all. A type-$x$ consumer derives utility $U_{Mt}(x, p_{Mt}) = x - p_{Mt}$ from the manufacturer’s product (if outsourced to $A$) and $U_{Mt}(x, p_{Mt}) = bx - p_{Mt}$ (if outsourced to $B$), and $U_{At}(x, p_{At}) = qx - p_{At}$ from supplier $A$’s copycat product. For notational convenience, let $U_{At}(x, p_{At}) = 0$ refer to the case when the supplier does not produce a copycat product. Hence, the manufacturer’s and supplier’s demands in period $t$ are, respectively, $D_{Mt}(I, p_{Mt}) = \Psi(I) \int_{\Omega} dx$ and, if applicable, $D_{At}(I, p_{At}) = \Psi(I) \int_{\Gamma} dx$, where $\Omega = \{x : U_{Mt}(x, p_{Mt}) > \max[0, U_{At}(x, p_{At})]\}$ and $\Gamma = \{x : U_{At}(x, p_{At}) > \max[0, U_{Mt}(x, p_{Mt})]\}$.

In Figure 2.1, we illustrate the demand functions in period $t$ when supplier $A$ encroaches. We denote the indifferent customer types by $x_{Mt}$ and $x_{At}$, which represent the types of customers who are indifferent between the manufacturer’s product and the copycat product, and the copied product and no purchase, respectively. The manufacturer’s and the copycat’s demands are therefore $D_{Mt} = (1 - x_{Mt}) \Psi(I)$ and $D_{At} = (x_{Mt} - x_{At}) \Psi(I)$, respectively. Note that (1) we always have $x_{Mt} \geq x_{At}$, and (2) when supplier $A$ does not encroach, the grey area disappears.

![Customer valuation, market size, and demand.](image)

Following the literature on multi-period games (e.g., Biyalogorsky & Koenigsberg, 2014; Abbey et al., 2017), we use $\theta$ to denote the time discount factor for profits obtained in the second period. Hence, the manufacturer’s profits, $\pi_{Mt}$, in $t = 1$ is $\pi_{M1} =$
(\(p_{M1} - w_{A1}\)) \(D_{M1} - I\) and in \(t = 2\), the manufacturer’s profit under no supplier encroachment (outsourcing to A) and encroachment (outsourcing to B), respectively are, \(\pi_{M2} = \theta (p_{M2} - w_{A2}) D_{M2}\) and \(\pi_{M2} = \theta (p_{M2} - w_{B2}) D_{M2}\).

As to the suppliers’ profits, we have assumed zero production cost for the suppliers (e.g., Cho et al., 2015; Gao et al., 2017a; Pun & DeYong, 2017). When supplier A decides against encroachment \((E = N)\), its profit will contain only the manufacturer’s purchase orders; hence we have \(\pi_{A1} = w_{A1} D_{M1}\) and \(\pi_{A2} = \theta w_{A2} D_{M2}\). Under encroachment \((E = Y)\), the supplier will have to make a one-time market entry investment cost \(F\), but it will benefit from the extra revenues generated by selling the copied product directly to the end customers. In that case, the supplier’s profit in \(t = 1\) and \(t = 2\) respectively are \(\pi_{A1} = p_{A1} D_{A1} + w_{A1} D_{M1} - F\) and \(\pi_{A2} = \theta p_{A2} D_{A2}\). The first term of \(\pi_{A1}\) is the revenue from selling copycat, and the second term of \(\pi_{A1}\) is the revenue from the wholesale market. As to supplier B, it has the second-period profit function of \(\pi_{B2} = \theta w_{B2} D_{M2}\).

It is worth reminding that supplier B will never be chosen by the manufacturer in the first period, and hence no first-period profit function.

The sequence of the events in the proposed game is explained below. The first seven stages occur in period one, and the remaining stages occur in period two.

1. As the Stackelberg leader, the manufacturer decides on the market size \(\Psi(I)\) through setting the marketing investment \(I\).

2. The manufacturer outsources the production to supplier A.

3. Supplier A decides on encroachment \(E = \{Y, N\}\).

4. Supplier A decides on the wholesale price \(w_{A1}\).

5. The manufacturer decides on the market price \(p_{M1}\).

6. If supplier A decides to encroach, \(E = Y\), then it sets the market price for the copycat product \(p_{A1}\).

7. Customers make the purchase decision.
8. At the beginning of $t = 2$, if there has been no encroachment in the first period ($E = N$) supplier $A$ decides on the wholesale price $w_{A2}$. Otherwise, supplier $B$ decides on the wholesale price $w_{B2}$.

9. The manufacturer decides on the market price $p_{M2}$.

10. If there has been encroachment in the first period ($E = Y$) supplier $A$ decides on the market price $p_{A2}$.

11. Customers make the purchase decision.

2.4 Optimal Solution

We obtain the optimal solution by applying the standard technique for solving a vertically differentiated model and backward induction. We consider a feasible space where all prices, demands, and profits are positive, i.e., $F < \bar{F}$. To ensure mathematical tractability, we assume $\theta = \frac{1}{2}$ in this section, and we demonstrate the robustness of our result in Section 6 when $\theta$ takes on different values. Besides, we assume that the quality of the manufacturer’s product is higher than the quality of the copycat product. In terms of notations, we use the following to denote the two subgames: (1) Subgame $N$, where supplier $A$ does not encroach in the first period ($E = N$); and (2) Subgame $Y$, where supplier $A$ encroaches in the first period ($E = Y$).

Recall that the manufacturer decides on the level of marketing investment at the beginning of the game. Following backward induction, the last step of the solution is the manufacturer’s maximization of the profits by setting the optimal levels of marketing investment ($I$). Since the marketing investment determines the market size, it directly affects supplier $A$’s encroachment decision. Therefore, two different optimization problems shall be solved:

\[
\max_I \pi^N_M (I) \quad s.t. \quad \pi^N_A (I) \geq \pi^Y_A (I) \tag{2.1}
\]

\[
\max_I \pi^Y_M (I) \quad s.t. \quad \pi^N_A (I) \leq \pi^Y_A (I) \tag{2.2}
\]
2.4. Optimal Solution

Equation (2.1) describes the manufacturer’s optimization problem under no encroachment (i.e., subgame $N$), and Equation (2.2) describes the manufacturer’s problem under encroachment (i.e., subgame $Y$). In each problem, the corresponding constraint is either non-binding (interior solution) or binding. For ease of exposition, we summarize the notation using subscripts $\{\text{int}, \text{bin}\}$ to refer to the aforementioned cases. Therefore, within each of the two subgames $N$ and $Y$, the manufacturer’s optimal level of marketing investment ($I^*$) can either be an interior ($I^*_{\text{int}}$) or a binding ($I^*_{\text{bin}}$) solution. It is noteworthy that since the manufacturer’s profit functions are both strictly concave and continuous, when the interior solution is feasible it is always optimal as well.

Proposition 2.4.1 provides the classification of the potential subgame perfect equilibria and the corresponding conditions in our proposed game. The corresponding thresholds are presented in Appendix A. Figure 2.2 illustrates this proposition using $b = \frac{9}{10}$, where the x-axis is the copycat’s quality $q$ (spanning from $q = 0$ to $q = 1$) and the y-axis is the market entry investment $F$ (spanning from $F = 0$ to $F = \frac{7}{10000}$). The structural insights are robust to a wide array of parameter settings.

**Proposition 2.4.1** The following enumerates the necessary and sufficient conditions for the potential subgame perfect equilibria:

(a) $Y_{\text{int}}$ is the equilibrium if and only if one of the following sets of conditions hold: $[b_0 < b \leq b_1, q_0 < q < q_1, F < F_3]$, $[b_1 < b < 1, q_0 < q < b, F < F_5]$.  

(b) $N_{\text{int}}$ is the equilibrium if and only if one of the following sets of conditions hold: $[b \leq b_0]$, $[b_0 < b \leq b_1, q \leq q_0]$, $[b_0 < b \leq b_1, q_0 < q < q_1, F > F_4]$, $[b_0 < b \leq b_1, q \geq q_1]$, $[b_1 < b < 1, q \leq q_0]$, $[b_1 \leq b < 1, q > q_0, F > F_6]$.  

(c) Otherwise, $N_{\text{bin}}$ is the equilibrium.

On the one extreme, when the copycat product is of low quality (blue region), customers are not willing to pay for supplier $A$’s copycat product. The additional revenue that supplier $A$ receives from selling the copycat product cannot justify the loss of the wholesale business in the second period. Therefore, the supplier would not encroach
into the market even when the market entry investment is negligible ($F = 0$). The manufacturer can make an optimal market investment decision without worrying about the supplier’s encroachment incentives (i.e., an interior solution).

On the other extreme, when the copycat product has high quality (yellow region), the competition between the manufacturer’s product and the copycat is intense. Hence, supplier $A$ cannot charge a high price for its copycat product in either period. At the same time, the supplier must set a low wholesale price in the first period due to the intense competition between the two products. Our result shows that the manufacturer can reduce the market size (i.e., binding solution) to discourage the supplier from entering the market. Even though the market size is smaller, the manufacturer holds a monopoly in the market. Without selling a copycat, supplier $A$ can charge a higher wholesale price in both periods.

Supplier $A$ would encroach into the market with a copycat product only when $q$ is sufficiently high to make encroachment profitable, but not too high that an intense market competition damages the supplier’s bottom line. This is the setting where the customers can derive relatively significant utility from consuming the copycat product, while the competition with the manufacturer’s product is not intense. Specifically,
when the entrance cost $F$ is small (the green region), the supplier has a strong incentive to encroach into the market. It is too costly for the manufacturer to manipulate the supplier’s encroachment decision with a binding marketing investment. Therefore, the manufacturer would make an interior marketing investment in anticipation of a market with two products.

We derive further managerial implications in Corollary 2.4.2 and 2.4.3.

**Corollary 2.4.2** If $q \geq q_2$ and $F = F_5$ then an increase in the quality of the copycat product will make the supplier switch from encroachment ($E = Y$) to no encroachment ($E = N$).

One might expect that the supplier is more likely to encroach into the market when the quality of its copied product improves. However, this intuition is not true when $q \geq q_2$ and $F = F_5$ (the right side of the boundary between regions $Y_{in}$ and $N_{in}$). This is because, an increase in copycat product’s quality intensifies the market competition with the manufacturer’s product, and so the products’ market prices are reduced. At this boundary point, the extra revenue generated from selling copycats cannot justify the entry cost and loss of revenue from the wholesale market in the second period. Therefore, the supplier would no longer encroach.

**Corollary 2.4.3** $Y_{in}$ is never the equilibrium.

If $Y_{in}$ could be the equilibrium, that would imply under certain conditions the manufacturer would reduce the marketing investment and shrink the market in order to provide an incentive for supplier $A$ to encroach. However, losing market monopoly and the shrinking market size are a double blow to the manufacturer who can avoid this negative situation. If the combination of the copycat product’s quality ($q$) and the market entry cost ($F$) are ideal for encroachment, the manufacturer opts for a larger market size to reap the benefits of outsourcing the production to supplier $B$ in the second period ($Y_{in}$). On the other hand, a binding market size would be too small for the copycat to cover the cost of encroachment. Therefore, $Y_{in}$ can never be the equilibrium.


## 2.5 Sensitivity Analysis

In this section, we investigate the impact of the model parameters on the optimal profits: supplier B’s quality \( (b) \), supplier A’s market entry investment cost \( (F) \), and quality of the copycat product sold by supplier A \( (q) \). Proposition 2.5.1 shows that the manufacturer can be worse off upon an increase in the quality of the product supplied by supplier B.

**Proposition 2.5.1** If \([q_0 < q < q_1, F_5 < F < F_6]\) and \( b = b_1 \), then an increase in \( b \) (supplier B’s quality) reduces the manufacturer’s profit.

Parameter \( b \) determines the quality of the product that supplier B can provide to the manufacturer, so one would expect that since a higher quality improves customers’ utility, the manufacturer should be better off upon an increase in \( b \). However, we find the conditions where the opposite is true. For exposition, Figure 2.3 shows the manufacturer’s profit at different values of \( b \) (spanning from \( b = \frac{1}{2} \) to \( b = 1 \)) under the parameter setting \( q = \frac{1}{2} \) and \( F = \frac{3}{100000} \).

![Figure 2.3: The manufacturer’s optimal profit for different values of \( b \).](image)

In Figure 2.3, at the lowest values of \( b \) (\( N_{int} \) region), the quality of supplier A’s copycat product \( q \) would be very close to the quality of the manufacturer’s second-period product \( b \). The fierce second-period competition forces supplier A to keep the selling price too low to cover the costs of encroachment. Since encroachment would generate a loss for supplier A, there would not be a threat of copycatting, and the manufacturer can make the optimal marketing investment \( I_{int} \) to maximize its profit.
As $b$ increases, so does the quality gap between the manufacturer’s and the copycat’s products in the second period; and since the two products are now less of substitutes, supplier $A$ will be able to charge higher prices for its copycat product. As mentioned in Section 2.3, supplier $A$ free rides on the manufacturer’s marketing investment, and due to its inferior brand value (compared to the manufacturer), the quality of its copycat product is lower than the manufacturer’s original product. In the $N_{bin}$ region, we show that the manufacturer’s profit falls as $b$ increases; this is in fact because $I_{bin}$ is decreasing in $b$. In other words, the manufacturer too would be worse off upon a reduction in the market size. Within this region, the threat of copycatting is strong enough that to maintain its monopoly, the manufacturer shrinks the market by making a marketing investment equal to $I_{bin}$, and deters the supplier’s encroachment.

The drop in the manufacturer’s profit occurs after the equilibrium strategy switches from $N_{int}$ to $N_{bin}$. This is where the manufacturer can prevent supplier $A$’s encroachment by shrinking the market. As the manufacturer slashes its marketing investment, its demand and consequently the profits drop. And as $b$ increases (within the $N_{bin}$ region), the second-period competition cools off and preventing supplier $A$ from encroachment becomes costlier, which further reduces the manufacturer’s profit.

At the other extreme, where $b$ is large ($Y_{int}$ region), the copycat product’s quality differs sufficiently from the manufacturer’s product quality. Encroachment is a lucrative opportunity for the supplier in this scenario; such that even a reduction in the market size cannot deter the copycat. Therefore, the manufacturer is forced to accept the competitor in the market and lose its monopoly.

**Proposition 2.5.2** If $[b_1 < b < 1, q_0 < q, F_5 < F < F_6]$ then an increase in $F$ (supplier $A$’s market entry investment cost) improves the supplier’s profit.

Parameter $F$ captures the cost of encroachment incurred by supplier $A$, thus an increase in $F$ makes encroachment more expensive and less appealing. So, intuition suggests that supplier $A$ should be worse off upon an increase in $F$, since this would prohibit supplier $A$’s encroachment into the market, and it would help the manufacturer maintain a monopoly. However, we find the conditions under which the contrary holds. We
illustrate Proposition 2.5.2 in Figure 2.4 (under parameter setting $b = \frac{9}{10}$, $q = \frac{6}{10}$), where the x-axis spans from $F = 0$ to $F = \frac{1}{1000}$.

![Graph showing Supplier A's optimal profit for different values of $F$.]

Figure 2.4: Supplier A’s optimal profit for different values of $F$.

When $F$ is small, encroachment is a profitable alternative for supplier A, such that the manufacturer cannot deter the copycat by reducing marketing investment and shrinking the market. Therefore, $Y_{int}$ will be the equilibrium. Within the $Y_{int}$ region, supplier A’s profit linearly decreases in $F$; since, the supplier’s optimal strategy is to encroach, and it must pay the price of encroachment.

At the other extreme, when $F$ is large, encroachment does not pay off and so the manufacturer does not need to shrink the market to deter competition. This makes $N_{int}$ the equilibrium.

The improvement in supplier A’s profit happens in the intermediate region of $N_{bin}$, where $\pi_A$ is increasing in $F$. In this region, the marketing investment is $I_{bin}$, meaning that the manufacturer is exerting control over the market size to maintain its monopoly, while encroachment gets pricier as $F$ increases. As the price of encroachment rises, the severity of the copycatting risks facing the manufacturer drops. Therefore, the manufacturer can gradually expand the market by increasing the investment from $I_{bin}$ to $I_{int}$. An expansion of the market under the $N_{bin}$ region implies higher demand for the manufacturer and supplier A without a price competition; and thus, increasing profits for both firms.

**Proposition 2.5.3** If $[b_1 < b < 1, q_0 < q, F = F_0]$ then an increase in $q$ (the quality of the copycat’s product) reduces supplier A’s optimal profit.
As the quality of the copycat’s product improves, the consumers’ utility derived from the copycat product increases, which enables the supplier to raise its market price and earn higher profits. However, we find the conditions where (even free) improvements in the supplier’s quality harm its profit. The rationale behind Proposition 2.5.3 is illustrated in Figure 2.5 (where we have $b = \frac{9}{10}$ and $F = \frac{4}{100000}$), and the x-axis spans from $q = 0$ to $q = \frac{9}{10}$.

Figure 2.5: Supplier A’s optimal profit for different values of $q$.

At low levels of $q$ ($N_{int}$ region), supplier A cannot earn enough from encroachment to outweigh the associated costs, so the manufacturer can maintain its monopoly by making a marketing investment of $I_{int}^N$.

When $q$ rises beyond the $N_{int}$ and into the $N_{bin}$ region, there is a fall in the supplier’s profits, which is due to the heightened levels of copycatting risk facing the manufacturer, and the manufacturer’s response by lowering the marketing investment and reducing the market size. Within both $N_{bin}$ regions, the increasing $q$ threatens the manufacturer with the prospects of tighter competition, persuading the manufacturer to gradually reduce the market size (i.e., $I_{bin}$ is decreasing in $q$) which lowers the supplier’s (and the manufacturer’s) profits.

In the $Y_{int}$ region, the manufacturer cannot dissuade the supplier from encroachment by shrinking the market, so a price competition occurs where the supplier is forced to lower its prices since an increasing $q$ tightens the competition.

In contrast, at the highest levels of $q$ ($N_{bin}$ region), the manufacturer opts to shrink the market by spending $I_{bin}$ on marketing, because, here in this region, it is possible to
deter the supplier’s encroachment by manipulating the market size.

### 2.6 Extensions

In Section 2.4 we ensured mathematical tractability by setting the discount parameter \( \theta = \frac{1}{2} \). Parameter \( \theta \) is crucial to supplier A’s encroachment decision. Since, encroachment deprives the copycat of the second-period revenue earned from the wholesale contract, and the weight of the second-period profits in total profits can determine whether supplier A is willing to forgo its second-period wholesale profit to maximize first-period profits through encroachment. In this section we show the robustness of the results for the cases where second-period profits are worth lower and higher compared to what we presented in Section 2.4, that is, here we set \( \theta = \frac{1}{4} \) and \( \theta = \frac{3}{4} \). (Note that the analytical results for the equilibrium under various \( \theta \) values can be obtained from the authors upon request.) In particular, we present Figure 2.6 with three panels to explain the impacts of an increase in \( \theta \) on the equilibrium. For all plots, we use \( b = \frac{9}{10} \) (the same as Figure 2.2).

Figure 2.6 illustrates the robustness of our results under different values of \( \theta \). The rise in \( \theta \) increases the relative importance of the second-period profits in the total profit. Meaning that, supplier A will be losing a more valuable source of income if it decides to encroach. Therefore, at a higher \( \theta \) supplier A will be less aggressive with respect to encroachment, reducing the threat of copycatting to the manufacturer. This explains
the shrinkage in the $Y_{\text{int}}$ and $N_{\text{bin}}$ regions as $\theta$ rises (from the left panel to the right). When supplier $A$ is less likely to encroach, then (1) the green encroachment region ($Y_{\text{int}}$) reduces in size, and (2) the manufacturer does not need to shrink the market at higher values of $F$ to maintain its monopoly, and hence the yellow $N_{\text{bin}}$ region declines in size as well.

2.7 Conclusion

In this paper, we adopt a two-period game-theoretic approach to investigate the phenomenon of supplier copycatting. We consider an original manufacturer that outsources the production of its product to a high-quality supplier with the capability to encroach into the market with a copycat product; where the manufacturer will end its relationship with the high-quality supplier and outsource to an alternative supplier if copycatting occurs. Our model incorporates the future outsourcing alternatives available to the manufacturer and the future repercussions for the copycatting supplier, that is, the reality that the manufacturer discovers the copycatting supplier’s “third shift” practice and cuts ties with the supplier.

We discuss the dynamics of the two-period game and highlight the nuances that would be otherwise neglected in a single-period approach. More specifically, we find the following answers to the four research questions posed in Section 2.1. First, one would expect that since higher production quality improves customers’ utility, the manufacturer should be better off upon an increase in the quality of the low-quality supplier’s production. However, we find conditions where the opposite is true. Moreover, we show that as the low-quality supplier’s quality is improved, the copycatting supplier can become more likely to encroach into the market. Second, we show that under certain conditions, upon an improvement in the quality of the copycat product, the high-quality supplier’s profit is reduced. Furthermore, an increase in the quality of the copycat product can convince the supplier against encroachment. Third, an increase in the costs associated with the high-quality supplier’s market encroachment can improve the supplier’s profit. In other words, under certain conditions, the sup-
plier would prefer a costlier market entry. And fourth, our analysis suggests that the high-quality supplier’s tendency to encroach is reduced upon a decrease in the net present value of the future profits.

2.7.1 Managerial Implications

The results of this paper have significant implications for manufacturers that outsource production, as well as the suppliers to such manufacturers. From the manufacturer’s perspective, it is critical to gauge the outsourcing risks before contracting a supplier. Our results suggest that manufacturers should carefully evaluate (1) the market-entry barriers and costs for a prospective copycat, (2) the quality gap between the high- and low-quality suppliers, (3) the net present value of their future profits, and (4) their marketing investment which translates into market size and demand. For example, since Umbra’s products (i.e., household items) can be easily sold over the internet and the chosen suppliers were located in countries where governments do not impose heavy fines on copycats, the relative market-entry costs were low. Furthermore, Umbra has made major marketing investments to expand the market for household items of superior design. As our results show, a supplier with copycatting capabilities would likely encroach when the ratio of the market size to market-entry cost is sufficiently large. Hence, we conclude that the decision makers at Umbra overlooked the three main criteria (as suggested by the results) to gauge the outsourcing risks, and unfortunately, had to deal with supplier copycatting.

On the other hand, when Brilliance Auto decided to take advantage of BMW’s IP and encroached into the market, it incurred a significant market-entry cost due to the more complicated nature of the product and the increased difficulty of market distribution for a copycat car. The magnitude of the auto industry market size and the quality of the copied product outweighed the costs of encroachment, and hence, as our results suggest, BMW faced supplier copycatting.
2.7.2 Limitations and Future Research Directions

There are several possible extensions to our framework that can lead to interesting future work. First, we assume that upon encroachment it is certain that the manufacturer will catch the copycatting supplier. Instead, one could imagine a likelihood for the event of the manufacturer catching the copycatting supplier. This probability can be defined as a function of the manufacturer’s market surveillance investment, such that at the beginning of the game, the manufacturer makes an investment in market surveillance technologies (e.g., blockchain technology), which in turn determines the manufacturer’s chances of catching a copycatting supplier. Future researchers could take this approach and experiment with different surveillance investment specifications. Furthermore, since we illustrate the significance of the copied product’s quality in the manufacturer’s optimal strategy and optimal profit, future work could make the copycat’s product quality endogenous (i.e., the supplier decides on the quality of its counterfeit product after deciding on market entry). This extension may tip the scales in the copycat’s favor, whereas the setting that we study is more likely to reinforce the manufacturer’s position. Finally, this work could be extended to an infinite horizon game, where the manufacturer and the supplier repeatedly make the pricing and encroachment decisions. We hope that these findings will assist researchers in addressing the rising concerns around supplier copycatting involved in outsourcing production.
Chapter 3

Navigating Supplier Encroachment: Game-Theoretic Insights for Outsourcing Strategies
3.1 Introduction

It has become a common practice for upstream suppliers to invest in direct sales channels while they earn revenues from their existing wholesale contracts with downstream buyers. Often labelled as supplier encroachment, the possible benefits of this phenomenon have been extensively studied—e.g., reduced double marginalization (Arya et al., 2007)—and the limited external validity of such results have been demonstrated as well (e.g., Liu et al., 2021). In practice, however, 47% of supplier-buyer collaborations fail, where the most commonly identified culprit is “lack of trust and commitment” (Webb, 2017a,b). A supplier’s encroachment into the market signals (1) the supplier’s intention and readiness to compete against the buyer, and (2) the supplier’s divided attention between its role as a supplier to the buyer and direct selling. Therefore, supplier encroachment can indicate the supplier’s inadequacy in terms of trust and commitment, respectively. Furthermore, it is a widespread belief that a supplier’s reach for the end consumers poses a major threat to its wholesale customers, which explains why some downstream firms respond to the upstream firms’ encroachment by threatening to break off the relationship with the encroaching supplier (Tedeschi, 2000; Yoon, 2016). An example is the case of Stacks and Stacks (a midsize office supply company) versus its supplier, FedEx; when the latter encroached into the office supply market by acquiring Kinko (a chain of office supply stores, currently doing business as FedEx Office), the former recognized it as a threat to its business and the company’s president reacted to FedEx’s encroachment: “Why would I want to do business with a company that openly competes with me?” (Tedeschi, 2005)

This phenomenon does not affect only the mid- to large-sized firms. Smaller buyer, and even online sellers have to deal with the negative consequences of supplier encroachment. In one case in 2021, a supplier encroached into an online seller’s market and then sued the seller for violation of intellectual property (IP) laws. Developing an enforceable and legally-binding contract between the supplier and the buyer seems to be the solution to this concern. However, the associated costs of such contracts may
render them infeasible for smaller firms (EURObiz, 2016). For example, the IP litigation process in the United States can cost upwards of USD 1 million (World Intellectual Property Organization, 2018). This major limitation often forces the outsourcing buyer to abandon the encroaching supplier and seek new partnerships. Meaning that, in a supplier encroachment setting, the buyer has future outsourcing opportunities, and the encroaching supplier might face repercussions for encroaching on the market (e.g., losing the buyer’s business). The existence of future outsourcing opportunities (for the buyer) and the future repercussions (for the supplier) makes it imperative to consider a two-period modelling approach to investigate the supplier encroachment phenomenon.

In this chapter, using a two-period game-theoretic approach, we consider a setting where a buyer of an original product sells its product to the customers over two time periods. The buyer outsources the production to its preferred supplier(s), from a pool of suppliers characterized by a low process-quality and a high process-quality supplier. Similar to Chen & Lee (2017) and Chen et al. (2020), our notion of process quality concerns the processes of sourcing and producing a product (e.g., sustainability, durability, and safety standards). The high (process) quality supplier has the capability to produce its own product and encroach into the market. If selected by the buyer, the high-quality supplier will decide whether to encroach into the market under its own brand name. If it decides to enter the market independently, however, the high-quality (encroaching) supplier faces the consequences of betraying the buyer’s trust, that is, the possibility of losing the buyer’s business in the subsequent period.

Our research focuses on four central questions: (1) How does the existence of future outsourcing opportunities shape the buyer’s optimal outsourcing and pricing strategies? (2) How is the high-quality supplier’s optimal strategy affected if encroaching into the market leads to repercussions? (3) What is the effect of the quality of the high-quality supplier’s independent product on both the buyer’s and the high-quality supplier’s profits? (4) How does the quality gap between high- and low-quality supplier influence the buyer’s profit?

We find the answers listed below to the above research questions. Most importantly,
these results are not observed in the single-period game and only exist when considering the buyer’s future outsourcing opportunities and the supplier’s future repercussions. (1) We show that under certain conditions, (a) an increase in the quality of the high-quality supplier’s product would convince the buyer to switch from the low-quality supplier to the encroaching supplier, and (b) after a raise in the quality of the non-encroaching supplier’s production, the buyer would abandon this supplier and outsource to the encroaching supplier. (2) The encroaching supplier may discontinue the encroachment (i.e., the market presence) upon (a) an increase in the quality of its own brand product; or (b) a rise in the quality of the non-encroaching supplier’s production. (3) An increase in the encroaching supplier’s product quality can improve the buyer’s profits and harm the encroaching supplier’s profits, even in the absence of quality enhancement cost. Finally, (4) the non-encroaching supplier can be worse off from an increase in its production quality, even when the cost of improving the quality is negligible.

The rest of this chapter is organized as follows. In the next section, we review the extant literature on supplier encroachment. Then, to delineate the impacts of the existence of future opportunities and future repercussions on the dynamics of the game, we study the one-period benchmark, introduce the mathematical model, and present the one-period optimal solution in Section 3.3. In Section 3.4, we extend the mathematical model to capture the multi-period nature of the game, and examine the buyer’s outsourcing decisions in the presence of supplier encroachment over two periods. The optimal solution of the two-period mathematical model is discussed and compared with the single-period benchmark in Section 3.5. We conduct sensitivity analyses in Section 3.6 and explore several extensions of our mathematical model in Section 3.7. Finally, Section 3.8 contains our concluding remarks and discussion of future research.

3.2 Literature Review

In this chapter we build upon the supplier encroachment literature, beginning with the work of Arya et al. (2007), where they suggest that a supplier should decide to en-
croach into the market as long as the extra revenue generated by encroachment covers the cost of entry. They also find that the upstream firm can benefit from the downstream firm’s encroachment into the retail market. Li et al. (2014) investigate the same phenomenon under the assumption of asymmetric information, that is, the buyer will have more information on demand and the market, which changes the dynamics of the supplier-buyer relationship such that both firms could be worse off upon the supplier’s decision to encroach. Likewise, Huang et al. (2018), Guan et al. (2020), and Gao et al. (2021) also study the influence of information asymmetry on supplier encroachment. In a recent work, Ha et al. (2022) investigate the implications of information sharing decisions in the supply chains where encroachment occurs through online retail platforms only, and they show that encroachment and information sharing can complement one another within the firm’s strategy.

Ha et al. (2016) extend the work of Arya et al. (2007) and endogenize the product quality. Similar to Li et al. (2014), they show that the buyer cannot be better off after supplier’s encroachment if the product quality is endogenous. Taking a more nuanced approach, Wang et al. (2013) investigate a setting with three firms in the market: the buyer of an original product, a competitive supplier, and a non-competitive supplier. They study the three Cournot equilibria for the competition between the buyer and the competitive supplier. In one of the few two-period game-theoretic models of supplier encroachment, Guan et al. (2019) investigate the impacts of a buyer’s strategic inventory withholding on supplier encroachment. They identify conditions under which both the buyer and the supplier will be better off if the supplier encroaches and the buyer holds strategic inventory. Cui (2019) proposes a two-firm game (including an encroaching low-quality supplier and a high-quality buyer) to investigate a case where the supplier free-rides on the quality improvement investments made by the buyer. They find that the low-quality supplier will always have incentives to encroach into the market if the buyer decides against investing in quality improvements.

To model the production process more realistically, Ghamat et al. (2018) and Yang et al. (2018) study the optimal strategies of firms in supply chains bounded by capacity constraints. They show that unlike the case of unconstrained capacity, the buyer, the
supplier, and end consumers can all be better off upon the introduction of capacity constraints on the supplier. Liu et al. (2021) further challenge the setting and assumptions of Arya et al. (2007); they show that the results of Arya et al. (2007) may not hold even if the condition of information symmetry is satisfied. They demonstrate that when there are multiple retailers in the supplier’s distribution channel, encroachment is significantly less profitable. In particular, if a given supplier sells its product through various retailers, it may be worse off upon encroachment. In a recent work, Shi et al. (2023) study the effects of supplier’s organizational structure on encroachment. They endogenize the supplier’s choice between an integrated structure (i.e., both the indirect and direct sales are handled through the same division in the organization) and a decentralized structure (i.e., there is a separate division handling the encroachment), and show that a decentralized structure could facilitate the supplier’s encroachment, but it would intensify the market competition nevertheless.

This work is different from the previous work on supplier encroachment in that we investigate the implications of the existence of future outsourcing opportunities for the buyer, and the possibility of future repercussions for the supplier if it loses the buyer’s trust. We propose a two-period game-theoretic approach that contains two suppliers with different capabilities and different qualities, and find the analytical conditions for encroachment to be beneficial/detrimental to the encroaching supplier and the buyer.

3.3 The One-Period Benchmark

In this section, we focus on the one-period supplier encroachment game—that is, one in which there are no future outsourcing opportunities for the buyer and the encroaching supplier faces no repercussions after betraying the buyer’s trust. To preserve consistency, we employ a naming scheme similar to that of Chapter 2 throughout this chapter. Consider a buyer (denoted by $M$) that outsources the production over one period. At the beginning of the game, the buyer chooses a supplier from a pool of suppliers characterized by supplier $A$ with high process quality and encroaching capabilities, and supplier $B$ with a lower process quality. Later, in Section 3.7.1, we demonstrate the
robustness of our results to the case where dual sourcing is available to the buyer.

Without loss of generality, we normalize supplier $A$’s process quality to one, and denote by $b \in (0,1)$ the process quality of supplier $B$. The buyer outsources the production to supplier $j \in \{A, B\}$, and then, to sell to the buyer, the selected supplier sets the wholesale price $w_j$. Subsequently, the buyer sets the market price, $p_M$, for the sale of the original product to the end customers. We define the total quality of a product in the market as the multiplication of the product’s process quality and its brand value. The buyer’s original products have their brand values normalized to 1. The high-quality supplier ($A$) is capable of producing its own product with a lower brand value $\lambda \in (0,1)$ and entering the market independently. Table 3.1 summarizes the definition of quality, comprised of process quality and brand value.

<table>
<thead>
<tr>
<th>The Product</th>
<th>Process Quality</th>
<th>Brand Value</th>
<th>Total Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>M’s Original Product Outsourced to A</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>M’s Original Product Outsourced to B</td>
<td>$b$</td>
<td>1</td>
<td>$b$</td>
</tr>
<tr>
<td>A’s Independent Product</td>
<td>1</td>
<td>$\lambda$</td>
<td>$\lambda$</td>
</tr>
</tbody>
</table>

Table 3.1: The definition of quality.

If the buyer outsources to supplier $A$, the supplier can decide whether to encroach; we denote this decision by the binary variable $E \in \{0, 1\}$. Further, in order to encroach, supplier $A$ needs to make a market entry investment of $F > 0$. Lastly, if supplier $A$ decides to encroach, then it will also have to decide on the market price for its own product, $p_A$.

We denote by $\psi$ the type of a customer in the retail market where $\psi$ follows a uniform distribution with density $\Psi$ (total demand). We summarize a customer’s utility function in Table 3.2.

Customers in the retail market make the purchase decision based on the utility-maximization criterion. Consequently, the realized demand for the original buyer and
3.3. The One-Period Benchmark

<table>
<thead>
<tr>
<th>Customer’s Purchase Decision</th>
<th>Customer’s Utility Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>M’s Original Product Outsourced to A</td>
<td>$U_M(\psi, p_M) = \psi - p_M$</td>
</tr>
<tr>
<td>M’s Original Product Outsourced to B</td>
<td>$U_M(\psi, p_M) = b\psi - p_M$</td>
</tr>
<tr>
<td>A’s Independent Product (if $E = 1$)</td>
<td>$U_A(\psi, p_A) = \lambda\psi - p_A$</td>
</tr>
<tr>
<td>No Purchase</td>
<td>$U(\psi, p) = 0$</td>
</tr>
</tbody>
</table>

Table 3.2: A customer’s utility function.

the encroaching supplier will be as follows:

$$D_M(p_M) = \Psi \int_{\Upsilon} d\psi$$

$$D_A(p_A, E) = E \Psi \int_{\Xi} d\psi,$$

where

$$\Upsilon = \{\psi : U_M(\psi, p_M) > \max[0, U_A(\psi, p_A)]\}$$

$$\Xi = \{\psi : U_A(\psi, p_A) > \max[0, U_M(\psi, p_M)]\}.$$  

Figure 3.1 visualizes the realized demand for the original buyer and the high-quality supplier under $E = 1$. The demands for $M$ and $A$ can therefore be rewritten as:

$$D_M = (1 - \psi_M)\Psi$$

$$D_A = (\psi_M - \psi_A)\Psi,$$

where $\psi_M$ is the type of a customer who derives the same utility from the buyer’s original product outsourced to supplier $A$ and the supplier $A$’s independent product, and $\psi_A$ is the customer type who derives equal utility from the supplier $A$’s independent product and making no purchase at all.

The buyer earns revenue from the sale of its product to the consumers in the retail market, and pays the wholesale price ($w_j, j \in \{A, B\}$) to the chosen supplier. Therefore, the buyer’s profit is $\pi_M = D_M(p_M - w_j)$.

To ensure tractability, we make the assumption that both suppliers have the pro-
Chapter 3. Supplier Encroachment

Figure 3.1: Illustration of the demand.

duction cost of zero (similar to Cho et al., 2015; Pun & DeYong, 2017). Later, in Section 3.7.2, we show the robustness of our results to the case where suppliers have different and non-zero production costs. A supplier earns a profit of zero when it is not outsourced to. Hence, if chosen, supplier A’s profit, without market encroachment ($E = 0$), is equal to the wholesale revenue of selling the original product to the buyer, i.e., $\pi_A = D_M w_A$. However, if supplier A decides to encroach into the market ($E = 1$), its profit will include the additional revenue of selling its independent product to customers less the market entry investment, $F$. Thus, supplier A’s profit function in case of encroachment is $\pi_A = D_M w_A + D_A p_A - F$. Finally, if supplier B is chosen, its profit is given by $\pi_B = D_M w_B$.

Figure 3.2: The sequence of decisions in the one-period benchmark.

As illustrated in Figure 3.2, the sequence of the decisions in the proposed supplier encroachment game is as follows. First, the buyer, who is the Stackelberg leader, chooses a supplier $j \in \{A, B\}$ to which it will outsource the production. If the cho-

\[\text{Enter} \quad \text{Not Enter}\]

\[[w_A, p_M, p_A] \quad [w_A, p_M] \quad [w_B, p_M]\]

2In Appendix B.4 we show the robustness of our results to the case where the encroaching supplier needs one period to establish a distribution network and establish its own brand.
sen supplier is \( A \), it decides on whether to encroach into the market, \( E \in \{0, 1\} \). Next, the chosen supplier decides on the wholesale price, \( w_j \), and the buyer sets the market price, \( p_M \). Lastly, if \( E = 1 \), supplier \( A \) decides on the market price for its independent product, \( p_A \), and then the customers make the purchase decisions.

### 3.3.1 Optimal Solution

We use backward induction to solve our proposed vertically differentiated model. Where all consumers would prefer the product supplied by supplier \( A \), given equal prices for the products supplied by \( A \) and \( B \). In the one-period case, there are three subgames:

1. Subgame \( A \), when \( M \) outsources to \( A \) but no market encroachment occurs;
2. Subgame \( A^E \), when \( M \) outsources to \( A \) and \( A \) encroaches on the market with its own independent product;
3. Subgame \( B \), when \( M \) outsources to \( B \) and hence no encroachment occurs.

The classification of the potential subgame perfect equilibria and the corresponding conditions for the single-period game are presented in Proposition 3.3.1. All the thresholds introduced in this proposition are defined in Appendix B.1.

**Proposition 3.3.1** The potential subgame perfect equilibria and the corresponding conditions are as follows:

(a) Subgame \( A \) is the equilibrium if and only if \( F > F_1 \).

(b) Subgame \( A^E \) is the equilibrium if and only if \( F \leq F_1 \) and \( b \leq b_1 \).

(c) Subgame \( B \) is the equilibrium if and only if \( F \leq F_1 \) and \( b > b_1 \).

Figure 3.3 presents the subgame perfect equilibria region-plot for the single-period case \( (F = \frac{1}{200}) \). Supplier \( A \) would consider encroaching into the retail market only if the quality of its independent product \( (\lambda) \) is high enough to outweigh the cost of encroachment. Therefore, the \( A^E \) region cannot occur at lower values of \( \lambda \). Region \( A \), where the buyer outsources to supplier \( A \) but the supplier does not encroach, is capped by the \( F = F_1 \) threshold. In particular, if supplier \( A \)'s product quality is too low to
cover the cost of encroachment (lower part of Figure 3.3), the buyer’s decision will be to outsource to the supplier (regardless of supplier B’s quality), knowing that no encroachment will occur. That is why, \( F_1 \) only depends on \( \lambda \) and is independent of \( b \), making \( F = F_1 \) a horizontal line.

![Figure 3.3: Subgame perfect equilibria for the one-period benchmark.](image)

On the other hand, when supplier A’s independent product has a high quality (the top part of Figure 3.3), the buyer’s choice is to either outsource to the encroaching supplier (i.e., the \( A^E \) region) or to the lower-quality supplier (i.e., the \( B \) region). The boundary between the \( A^E \) and the \( B \) regions is given by the \( b = b_1 \) threshold. A high brand value \( \lambda \) prompts supplier A to encroach, and thus the buyer would consider outsourcing to supplier B, since the competition between the buyer’s product and supplier A’s independent product would be intense if \( \lambda \) is close to 1 (similarly, the competition is mild when the supplier’s brand value is low, i.e., \( \lambda \) is closer to 0). Within this region, if supplier B’s quality is relatively high (the top right region), the buyer outsources to supplier B to avoid possible competition with the encroaching supplier. However, if supplier B’s quality (\( b \)) is low (the top left region), the buyer’s outsourcing strategy would depend on both \( \lambda \) and supplier B’s quality. That is, the threshold \( b_1 \) is a function of \( \lambda \), such that, an increase in the quality of supplier B makes strategy B
the equilibrium.

The existence of future outsourcing opportunities for the buyer and future repercussions for the encroaching supplier transforms the dynamics of the supplier encroachment game. Next, we illustrate the differences in the dynamics of the one-period game outlined in this section versus the two-period game.

### 3.4 The Two-Period Game: Mathematical Model

In the two-period case, we investigate a buyer’s outsourcing strategy over two time periods, $t \in \{1, 2\}$, where the alternatives available to the buyer are suppliers $A$ (with process quality normalized to one) and $B$ (with process quality $b \in (0, 1)$). At the beginning of period $t$, the buyer outsources the production to its preferred supplier $j \in \{A, B\}$ and then the wholesale price $w_{jt}$ and the market price $p_{Mt}$ are set. The game is such that the firms’ second-period decisions are to maximize $\pi_k^2$, $k \in \{A, B, M\}$, while the firms’ first-period decisions aim at maximizing the total profit, that is, $\pi_k = \pi_k^1 + \pi_k^2$.

Product modification is a common practice within a product’s life cycle (Iyer & Soberman, 2000; Sadders, 2021). In our model the buyer implements product modifications at the beginning of period two. The outcome of this process is a product that is not the same as what the suppliers would produce in the first period. It is a modified product, with new design and technical details. Hence, in order to produce its own independent product which could be a substitute to the buyer’s product, supplier $A$ must have access to the technical details and the blueprints of the product in that period; which would happen only if the buyer outsources the production to the supplier. Furthermore, buyers periodically evaluate and assess their suppliers (Carr & Pearson, 1999; Brown, 2010). In our model, at the beginning of period two, the buyer performs a market investigation and evaluates the performance of the supplier of the first period. It is based on the market investigation that the buyer decides on its second-period sup-

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3. Later, in Section 3.7.1 we demonstrate the robustness of our results to the case where dual sourcing is available to the buyer.

4. In Appendix B.5 we show the robustness of the results under long-term outsourcing contracts; where the buyer outsources the production under a two-period contract, as long as the suppliers avoid encroachment.
Chapter 3. Supplier Encroachment

 Particularly, this is where the buyer decides whether to continue outsourcing to a supplier that has breached its trust.\footnote{Later, in Section 3.7.3 we demonstrate the robustness of the results to the case where the encroaching supplier retains its access to the market and continues to sell a substitutable product even after the buyer drops the supplier.}

Producing its own independent product and encroaching into the market in the two-period game entails costs and risks for supplier $A$. Regarding the capital investment cost ($F > 0$), we assume that upon making the market entry investment and establishing a distribution channel, supplier $A$ can utilize the channel in the subsequent period as well, making $F$ a one-time investment cost. As to the risks, we denote by $\phi \in [0, 1]$ the probability that the buyer has to end the wholesale business with the encroaching supplier (which would only happen if $E_1 = 1$). From a modeling perspective, $\phi$ represents move by nature. Where it captures the unforeseen circumstances under which the buyer cannot outsource the second-period production to the encroaching supplier. For example, even under $E_1 = 1$ the buyer might still be interested in outsourcing the second-period production to the encroaching supplier, but bankruptcy or operational failures may render the supplier unavailable to the buyer. Note that, since the second period is the last period of the game, if supplier $A$ encroaches in the second period only, there will be no future repercussions, and the supplier only incurs the cost of encroachment, $F$. The introduction of a probabilistic future repercussion for supplier $A$ complicates the supplier’s market entry decision. Supplier $A$ now must evaluate the revenue from both the wholesale contract with the buyer and selling its own independent product to customers in period $t = 1$ against the market entry investment $F$ (i.e., the cost) and the likelihood of losing the buyer’s wholesale contract in $t = 2$ (i.e., the risks).

Since the buyer implements product modification at the beginning of period two, we assume that all consumers are present at the beginning of both periods, and there would be a market size of $\Psi$ in each $t \in \{1, 2\}$. As in the single-period case, we denote by $\psi$ the type of a customer in the retail market where $\psi$ follows a uniform distribution with density $\Psi$ (total demand). Besides, customers in the retail market make the purchase decision based on the utility-maximization criterion. We summarize a cus-
3.4. The Two-Period Game: Mathematical Model

tomer’s utility function for the two-period case in Table 3.3.

<table>
<thead>
<tr>
<th>Customer's Purchase Decision</th>
<th>Customer's Utility Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>M’s Original Product Outsourced to A</td>
<td>( U_{M1}(\psi, p_{M1}) = \psi - p_{M1} )</td>
</tr>
<tr>
<td>M’s Original Product Outsourced to B</td>
<td>( U_{M1}(\psi, p_{M1}) = b\psi - p_{M1} )</td>
</tr>
<tr>
<td>A’s Independent Product (if ( E = 1 ))</td>
<td>( U_{A1}(\psi, p_{A1}) = \lambda\psi - p_{A1} )</td>
</tr>
<tr>
<td>No Purchase</td>
<td>( U_t(\psi, p_t) = 0 )</td>
</tr>
</tbody>
</table>

Table 3.3: A customer’s utility function under the two-period game.

Therefore, the realized demand for the original buyer and the encroaching supplier in the two-period case will be as follows:

\[
D_{M1}(p_{M1}) = \Psi \int_{\Upsilon} d\psi \tag{3.7}
\]
\[
D_{A1}(p_{A1}, E) = E\Psi \int_{\Xi} d\psi, \tag{3.8}
\]

where

\[
\Upsilon = \{ \psi : U_{M1}(\psi, p_{M1}) > \max\{0, U_{A1}(\psi, p_{A1})\}\}\tag{3.9}
\]
\[
\Xi = \{ \psi : U_{A1}(\psi, p_{A1}) > \max\{0, U_{M1}(\psi, p_{M1})\}\}. \tag{3.10}
\]

Therefore, the buyer’s profits, \( \pi_{Mt} \), in \( t = 1 \) and \( t = 2 \) are, respectively,

\[
\pi_{M1} = D_{M1}(p_{M1} - w_{j1}), \tag{3.11}
\]
\[
\pi_{M2} = D_{M2}(p_{M2} - w_{j2}). \tag{3.12}
\]

Analogous to Section 3.3, the production costs for both suppliers are assumed to be zero (e.g., Gao et al., 2017a; Pun & DeYong, 2017). Consequently, if supplier \( A \) does not encroach on the market in the first period (\( E_1 = 0 \)), its profit in period one is \( \pi_{A1} = D_{M1} w_{A1} \). However, if supplier \( A \) decides to encroach on the market in period one (\( E_1 = 1 \)), its profit will be affected by the revenue of selling its own independent

\footnote{Later, in Section 3.7.2, we demonstrate the robustness of our results to the case where suppliers have different and non-zero production costs.}
product to customers less the market entry investment, \( F \). Thus, supplier \( A \)'s profit function in \( t = 1 \) in case of encroachment is formulated as

\[
\pi_{A1} = D_{M1} w_{A1} + D_{A1} p_{A1} - F.
\]  
(3.13)

As to the supplier \( A \)'s profit function in \( t = 2 \), if the supplier has encroached in the first period and made the investment, \( F \), then a market entry in \( t = 2 \) (i.e., \( E_1 = 1 \) and then \( E_2 = 1 \)) will be free. In contrast, market encroachment in \( t = 1 \) creates the risk of losing the buyer’s wholesale business in \( t = 2 \). That is, there is a probability \( \phi \) that supplier \( A \)'s profit in the second period will be 0, and a probability \( 1 - \phi \) that supplier \( A \) will earn revenue from both the wholesale contract with the buyer and selling its independent product to the retail customers. Table 3.4 shows supplier \( A \)'s profit function in \( t = 2 \) under different scenarios. The first four rows of the table show \( A \)'s profit when the production has been outsourced to supplier \( B \) in \( t = 2 \). Rows 5 and 6 describe the scenario where supplier \( A \) is selected in \( t = 2 \), and the firm decides not to encroach on

<table>
<thead>
<tr>
<th>Supplier B</th>
<th>( M )'s Choice ( in t = 2 )</th>
<th>( A )'s Decision ( in t = 1 )</th>
<th>( A )'s Decision ( in t = 2 )</th>
<th>Profit Function ( in t = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not enter</td>
<td>Not enter</td>
<td>( \pi_{A2} = 0 )</td>
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<td>Enter</td>
<td>Not enter</td>
<td>( \pi_{A2} = 0 )</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Supplier A</td>
<td>Not enter</td>
<td>Not enter</td>
<td>( \pi_{A2} = D_{M2} w_{A2} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Enter</td>
<td>( \pi_{A2} = D_{M2} w_{A2} + D_{A2} p_{A2} - F )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Enter</td>
<td>Not enter</td>
<td>( \pi_{A2} = (1 - \phi) \left[ D_{M2} w_{A2} \right] )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Enter</td>
<td>( \pi_{A2} = (1 - \phi) \left[ D_{M2} w_{A2} + D_{A2} p_{A2} \right] )</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: Supplier \( A \)'s profit in \( t = 2 \).
the market in the first period. Supplier $A$’s profit in $t = 2$ with a previous encroachment on the market in $t = 1$ is tabulated in rows 7 and 8.

Finally, the profit function for supplier $B$ in period $t$ is given by

$$\pi_{Bt} = D_{Mt} w_{Bt}. \quad (3.14)$$

As illustrated in Figure 3.4, the sequence of the decisions in the proposed two-period supplier encroachment game is as follows. The first six decisions occur in the first period, and the remaining decisions are made in period two.

1. The buyer (i.e., the Stackelberg leader), chooses its preferred supplier $j \in \{A, B\}$ to outsource the first-period production to.
2. If the buyer chooses supplier $A$, the supplier decides on encroachment ($E_1$).
3. The preferred supplier sets the wholesale price, $w_{j1}$.
4. The buyer decides on the market price, $p_{M1}$.
5. If $E_1 = 1$, supplier $A$ decides on the retail price of its independent product $p_{A1}$.
6. Customers make the purchase decision.
7. The buyer conducts market investigation and chooses its preferred supplier to outsource the second-period production to. If $E_1 = 1$ there is a probability $\phi$ that the buyer drops the encroaching supplier and outsources to supplier $B$.
8. If supplier $A$ is selected in $t = 2$, then it makes the second-period encroachment decision ($E_2$).
9. The preferred supplier decides the wholesale price for period two, $w_{j2}$.
10. The buyer sets the market price $p_{M2}$.
11. Then, if $E_2 = 1$, supplier $A$ decides on $p_{A2}$.
12. Customers make the purchase decision.
3.5 The Two-Period Game: Optimal Solution

As in the one-period game, we use backward induction to solve for the equilibrium. We introduce the notation $XY$ to describe outcomes of the game where $X \in \{A, B\}$ and $Y \in \{A, B\}$ denote the suppliers selected by the buyer in time periods $t = 1$ and $t = 2$, respectively. In this notation, we show supplier $A$’s encroachment decision in the corresponding period by superscript $E$. Subgame $BA^E$, for example, describes the case where $M$ outsources the production to supplier $B$ in the first period and supplier $A$ in the second period, and supplier $A$ decides to launch its own product and encroach.

As illustrated in Figure 3.4 a key difference between the one-period and the two-period game is the impact of the probabilistic node—that is, after $M$ conducts the market investigation and decides whether to continue outsourcing to the encroaching supplier. This node can affect the outcome of the game only if supplier $A$ has been outsourced to in $t = 1$ and it has decided to encroach in the first period ($E_1 = 1$). Therefore, the probabilistic nature of the game alters the outcomes $XY$ with $X = A^E$. 
When $A^E$ is the outcome of the first period, regardless of what the buyer’s optimal choice of supplier in the second period may be, there will always be a chance—with probability $\phi \in [0, 1]$—that the encroached supplier is dropped by $M$, making supplier $B$ the second-period supplier (i.e., $A^E B$). Thus, if $A^E$ is realized in the first period, the second-period profits of the three firms $(A, B, M)$ will be in expected form, such that the prospective profit to be earned in the $A^E B$ outcome gets the weight $\phi$ and the other possible outcome ($A^E A^E, A^E A$, or $A^E B$) gets the weight of $1 - \phi$. Specifically, instead of $A^E A^E$ and $A^E A$ as possible outcomes of the game, we have $A^E A^E/A^E B$ and $A^E A^E/A^E B$, respectively. To simplify the notation, however, we drop the reference to the probabilistic node (i.e., $A^E B$) and shorten the name of such outcomes to $A^E A^E$ and $A^E A$, respectively.

Proposition 3.5.1 provides the analytical classification of the potential subgame perfect equilibria and the corresponding conditions for the two-period game. The corresponding thresholds are presented in Appendix B.1.

**Proposition 3.5.1** The following enumerates the sufficient and necessary conditions for each subgame perfect equilibrium:

(a) $A^E A^E$ is the equilibrium if and only if $[F \leq F_1$ and $b \leq b_1$ and $\phi \leq \phi_1]$ or $[F > 0$ and $b \leq b_1$ and $F \leq F_2$ and $b \leq b_2]$.

(b) $AA^E$ is the equilibrium if and only if $[F \leq F_1$ and $b \leq b_1$ and $\phi > \phi_1]$.

(c) $AA$ is the equilibrium if and only if $[F > F_1$ and $b \leq b_1$ and $F > F_2]$ or $[F > F_1$ and $b > b_1]$.

(d) $BA$ is the equilibrium if and only if $[F_1 < F \leq F_2$ and $b_2 < b \leq b_1]$.

(e) $BB$ is the equilibrium if and only if $[F \leq F_1$ and $b > b_1]$.

This proposition presents the necessary and sufficient conditions under which the buyer and the suppliers exhibit different equilibrium behavior from one period to the next (e.g., $BA$ and $AA^E$). Such decision-making dynamics—which are further discussed in Propositions 3.5.2 and 3.5.3—cannot be identified under the single-period setting.
Proposition 3.5.1 also shows that the following four outcomes are never the equilibrium: $A^E A$, $A^E B$, $AB$, and $BA^E$. First, $A^E A$ is never the equilibrium because if supplier $A$ makes the one-time market entry investment $F$ and encroaches in the first period, then it would always prefer to encroach in the second period as well, so that it can maximize the returns on investment. If the buyer selects supplier $A$ in the first period (which could potentially lead to a market competition with the supplier), then it is suboptimal for $M$ to switch to supplier $B$ in the second period (unless the encroaching supplier is dropped at the beginning of $t = 2$). Second, outcomes $A^E B$ and $AB$ are both seen as $AB$ by the buyer. Given the higher quality of supplier $A$, there would be no incentive for the buyer to abandon supplier $A$ after the first period and outsource to supplier $B$ in $t = 2$. Third, to encroach in only one period (either $t = 1$ or $t = 2$) is a costly decision for supplier $A$. This strategy can be an equilibrium only if supplier $A$ is the sole supplier to the buyer (i.e., the supplier earns revenues from wholesale contracts in both periods), and the probability of getting dropped by the buyer ($\phi$) is high (i.e., it is a high-risk strategy to encroach in $t = 1$). Thus, $BA^E$ is less profitable than $BA$ and hence, never the equilibrium.

We illustrate the equilibrium strategies of the two-period game in Figure 3.5b, using the parameter setting $\phi = \frac{1}{10}$ and $F = \frac{1}{200}$. Figure 3.5a shows the equilibrium strategies in the single-period case (cf. Figure 3.3; we copy that figure here for ease of discussion).

We discuss the underlying intuition of the Proposition 3.5.1 using Figure 3.5b. At high values of $\lambda$, if supplier $B$’s quality is low, the buyer’s strategy would be to outsource the production to supplier $A$ in both periods (and accept the risk of competing with its own supplier), and if supplier $B$’s quality is sufficiently high, to outsource to supplier $B$ in both periods (and avoid the competition with supplier $A$). Since these equilibrium strategies are repeated at each period, the $F = F_1$ and $b = b_1$ thresholds are the same as the one-period case (cf. Proposition 3.3.1).

Next, consider the case when the quality of supplier $A$’s independent product ($\lambda$) is intermediate. The equilibrium of the game depends on supplier $B$’s quality. (1) If $b$ is sufficiently low, the buyer outsources the production to supplier $A$ and gives supplier $A$
3.5. The Two-Period Game: Optimal Solution

(a) One-period benchmark.  
(b) Two-period game.

Figure 3.5: The equilibrium strategies under the one- and the two-period games.

the chance to launch a competing product. Supplier $A$ considers $\lambda$ to be large enough to outweigh the costs of encroachment and thus the supplier encroaches in both periods; and $A^E A^E$ is the equilibrium strategy. (2) As the process quality of supplier $B$ increases, the supplier becomes more appealing to the buyer. The buyer outsources to supplier $B$ in the first period in response to the risk posed by the encroaching supplier. For supplier $A$, the revenues of a one-period encroachment are insufficient to cover the costs of encroachment and thus supplier $A$ does not encroach in the second period; hence, $B A$ is the equilibrium strategy. (3) As $b$ further increases, supplier $A$ loses its relative quality advantage, and since a one-period encroachment is not affordable at this level of supplier $A$’s independent product quality ($\lambda$), the supplier avoids encroachment altogether. Therefore, the buyer outsources the production in both periods to supplier $A$ and the equilibrium strategy $A A$ is obtained.

The $A^E A^E$ and the $B A$ regions are separated by the $b = b_2$ threshold. When the quality of supplier $A$’s independent product ($\lambda$) is large enough to prompt supplier $A$ to encroach, the buyer would consider outsourcing to supplier $B$. Within the range of medium $\lambda$, if supplier $B$’s quality is relatively high, the buyer outsources the first-period production to supplier $B$, but if supplier $B$’s quality ($b$) is low, the buyer’s out-
sourcing strategy will depend on both the encroaching supplier and supplier $B$’s quality. Therefore, $b_2$ is a function of both $\phi$ and $\lambda$, and $b = b_2$ has a downward slope, such that, an increase in the quality of supplier $B$ makes strategy $BA$ the equilibrium. We also have the $BA$ and the $AA$ regions separated by the $b = b_1$ threshold. Since the intuition is already laid out above, we omit the discussion to avoid repetition.

Both $AE$ and $BA$ regions are separated from the equilibrium strategy region of $AA$ by the $F = F_2$ threshold. When the quality of supplier $A$’s independent product is too poor to cover the encroachment costs, the buyer will choose to outsource to this supplier (regardless of supplier $B$’s quality), knowing that supplier $A$ will not encroach (Region $AA$). Therefore, $F_2$ depends on $\lambda$ and is independent of $b$, which makes $F = F_2$ a horizontal line.

Furthermore, $F_1$ and $\phi_1$ are measures of supplier $A$’s likelihood of encroachment and facing repercussions for breaching the buyer’s trust. Consistent with intuition, supplier $A$ would consider market entry if the quality of its product ($\lambda$) and the market size ($\Psi$) are sufficiently large, and if establishing a distribution channel is affordable (i.e., low $F$). Therefore, the value of $F_1$ is associated with supplier $A$’s likelihood of encroachment. As the likelihood of facing repercussions ($\phi$) increases, supplier $A$ will be less likely to encroach. Hence, $\phi_1$ has an inverse association with the likelihood of encroachment. Moreover, $F_2$ captures the favorability of supplier $A$ to the buyer. If the buyer operates in a highly-competitive market where establishing a distribution network is cheap, then outsourcing to a supplier with the capability to launch its own product is a less favorable alternative to the buyer. Lastly, $b_1$ and $b_2$ represent the balancing factors at play in the buyer’s outsourcing decision. The increase in the market competition intensity and the increase in the quality of supplier $B$, both improve the appeal of supplier $B$ to the buyer.

To derive additional managerial insight, we consider in Propositions 3.5.2 and 3.5.3, respectively, the effects of the quality of supplier $A$’s independent product ($\lambda$) and the quality of supplier $B$’s product on the equilibrium of the game.

**Proposition 3.5.2** Take the two-period model of the supplier encroachment game:
(a) If \([F = F_1, b \leq b_1, \phi \leq \phi_1, F \leq F_2, \text{ and } b > b_2]\), then the buyer would switch from supplier \(B\) to supplier \(A\) if the quality of supplier \(A\)'s independent product is improved.

(b) If \([F > F_1, b \leq b_1, F \leq F_2, \text{ and } b = b_2]\), then supplier \(A\)'s encroachment decision would change from \(E_2 = 1\) to no entry \((E_2 = 0)\) if the quality of supplier \(A\)'s independent product is improved.

Figure 3.5b provides a nuanced picture of the supplier encroachment phenomenon. Where low quality of supplier \(B\)'s product \((b)\) coincides with low quality of supplier \(A\)'s independent product \((\lambda)\), the alternative of supplier \(B\) is not appealing to the buyer, while supplier \(A\) would not be interested in the alternative of encroachment. Therefore, the buyer outsources the production to supplier \(A\) in both periods (i.e., \(AA\)) with no concerns regarding the supplier’s market entry and a possible competition. The case with low \(b\) and high \(\lambda\), however, is more complicated. Intuitively speaking, as the quality of supplier \(A\)'s independent product \((\lambda)\) increases, we would expect that (1) the encroachment alternative becomes more attractive to supplier \(A\), and (2) due to the possibility of a more intense competition, the buyer becomes more interested in the alternative of supplier \(B\). In contrast to this expectation, we observe that the equilibrium switches three times as \(\lambda\) grows above the \(AA\) region: from \(AEAE\) to \(BA\), from \(BA\) to \(AEAE\), and from \(AEAE\) to \(BB\). As for the lower \(AEAE\) region, when \(\lambda\) grows above the \(AA\) region, the buyer has the choice to keep outsourcing to supplier \(A\) in both periods or to outsource to the alternative supplier, which would threaten supplier \(A\)'s position as the sole supplier. But, since \(\lambda\) and \(b\) are sufficiently small, the market competition is not fierce enough to justify switching to a strategy that involves (the inferior-quality) supplier \(B\). In other words, even though the buyer has to share the market with supplier \(A\), \(b\) and \(\lambda\) are small enough to persuade the buyer to forgo the alternative supplier in favor of supplier \(A\). From supplier \(A\)'s perspective, \(\lambda\) is sufficiently large, and \(F\) and \(\phi\) are sufficiently small, to make the alternative of encroachment into the market in both time periods cost-effective. Hence, \(AEAE\) is the equilibrium strategy.

As the quality of supplier \(A\)'s independent product \((\lambda)\) rises above the lower \(AEAE\)
into the $BA$ region, (1) the market competition is intensified, since $\lambda$ approaches 1, but also, (2) the value of $\lambda$ is not high enough yet to justify supplier $A$’s encroachment in one of two periods only. These circumstances mean that while the buyer is losing revenue due to the market competition, supplier $A$ still needs the prospect of encroaching into the market in both periods to afford any encroachment at all. Thus, the buyer has the opportunity to select the alternative supplier in $t = 1$, which would scare off supplier $A$ from encroaching and deter the competition altogether, giving rise to the $BA$ region. Subsequently, $\lambda$ increases further into the upper $AEAE$ region, where the competition is escalated. But unlike in the lower $AEAE$ and $BA$ regions, $\lambda$ is sufficiently high to justify a one-period encroachment for supplier $A$; for the buyer, this means that outsourcing to supplier $B$ in $t = 1$ will not make supplier $A$ less aggressive (in $t = 2$) nor deter the competition anymore. Therefore, since the quality gap between what the suppliers can offer to the buyer under a wholesale contract ($1 - b$) is sufficiently large, the buyer will avoid the inferior outcomes of $BAE$ and $BB$, and select supplier $A$ in both time periods, which generates the upper $AEAE$ region.

Note that $\phi$ shapes the equilibrium outcome of the game. When the probability of getting dropped by the buyer ($\phi$) increases, it becomes more risky and less profitable (in expected terms) for supplier $A$ to encroach in the first period. Therefore, as $\phi$ approaches 1, the lower and the upper $AEAE$ regions shrink. The same logic applies to the $BA$ region: at a higher $\phi$, the buyer is aware that the alternative of encroachment in $t = 1$ is less appealing to supplier $A$. Hence, choosing the alternative supplier becomes less interesting and the $BA$ region contracts, while the $AA$ strategy becomes more favorable and the corresponding region expands. Finally, as $\lambda$ approaches 1, the intensity of the market competition reaches its maximum and the buyer becomes interested solely in the alternative of supplier $B$ to ward off the competition at all costs. Thus, subgame $BB$ is the equilibrium.

**Proposition 3.5.3** In the proposed two-period model of supplier encroachment:

(a) If $[F > F_1, b = b_1, F \leq F_2, \text{ and } b > b_2]$, then the buyer would switch from supplier $B$ to supplier $A$ if supplier $B$’s product quality ($b$) is improved.
(b) If \( F > F_1, b \leq b_1, F \leq F_2, \) and \( b = b_2 \), then supplier A’s encroachment decision would change from \( E_2 = 1 \) to no entry \( (E_2 = 0) \) if supplier B’s product quality \( (b) \) is improved.

Now we focus on the equilibrium for medium values of the quality of supplier A’s independent product \( (\lambda) \). For low values of \( b \) within this range, we obtain the equilibrium strategy \( A^E A^E \), since the alternative of supplier B is not profitable enough for \( M \) \( (b \ll 1) \), and \( \lambda \) is sufficiently large to marginally cover the costs and the risks of encroachment in two periods for the encroaching supplier. As \( b \) increases, the threat of supplier B becomes more imminent to supplier A. In addition, within this range of \( \lambda \), the quality of supplier A’s independent product is not high enough to make encroachment in both periods an obvious choice for supplier A. In fact, supplier A would only encroach if it can do so in both periods and if the alternative of supplier B is not appealing enough to the buyer. Based on this rationale, \( M \) chooses supplier B in the first period in order to send a signal to the encroaching supplier and ensure that supplier A does not encroach, which gives rise to the \( BA \) region.

When supplier B’s quality \( (b) \) further grows into the \( AA \) region, the competition between suppliers A and B reaches maximum intensity, while the buyer has previously signaled the seriousness of the threat of the alternative supplier (i.e., region \( BA \)) to supplier A. Given that a medium \( \lambda \) does not incentivize an aggressive market entry strategy, supplier A adopts a cautious approach to avoid an inferior equilibrium (such as \( A^E B \)) and opts for no market entry \( (E_1 = E_2 = 0) \). It is worth mentioning that from the buyer’s perspective, an \( AA \) equilibrium is always superior to \( BB \). However, if supplier B is relatively appealing (i.e., \( b \) is not too small), \( M \) would only choose supplier A in both periods if it is certain that supplier A will not attempt to encroach.

### 3.6 Sensitivity Analysis

In this section, we investigate the dynamic impact of the mathematical model parameters on the firms’ profits. The results in this section are only available under the two-period approach—that is, the insights presented below cannot be obtained under the single-period setting. In particular, we formalize the impact of the following parame-
ters on profits in two propositions: quality of supplier A’s independent product ($\lambda$) in Proposition 3.6.1; and quality of the low-quality supplier ($b$) in Proposition 3.6.2.

**Proposition 3.6.1** Under the two-period model of supplier encroachment:

(a) If $[F > F_1, b \leq b_1, F \leq F_2, \text{and } b = b_2]$, then the buyer benefits from an increase in $\lambda$.

(b) If $[F > F_1, b \leq b_1, F \leq F_2, \text{and } b = b_2]$ or $[F \leq F_1, b = b_1, \text{and } \phi \leq \phi_1]$, then supplier A’s profit plummets as $\lambda$ increases.

The presence of a competing product in the market is a threat to the buyer’s bottom line. Therefore, M’s profit is expected to plunge as $\lambda$ increases (i.e., as the likelihood of supplier encroachment intensifies). Additionally, since supplier A is capable of producing its own independent product and selling it to the customers directly, we would expect that the supplier is always better off at a higher $\lambda$.

![Figure 3.6: Impact of supplier A’s independent product quality ($\lambda$) on the optimal profit.](image)

(a) The buyer’s optimal profit. (b) Supplier A’s optimal profit.

Using the parameter settings $\phi = \frac{1}{10}$, $F = \frac{1}{10}$, and $b = \frac{4}{10}$, Proposition 3.6.1 is illustrated in Figure 3.6. Note that as the quality of supplier A’s independent product increases, the equilibrium strategy changes four times—captured by the breakpoints. The second equilibrium strategy shift (denoted by $b = b_2$ in both subfigures) is from $A^E A^E$ to $BA$, which gives M uncontested access to the whole market (since $A$ does not encroach) in both periods, and thus improves its profit (Figure 3.6a). The
same shift in the equilibrium strategy means that supplier $A$ loses the wholesale contract in the first period and will no longer encroach on the market in $t = 2$; consequently, supplier $A$ would be worse off upon an increase in $\lambda$ (Figure 3.6b). Following the logic of Proposition 3.6.1, there is another drop in supplier $A$’s profit in Figure 3.6b, which occurs when the equilibrium strategy changes from $AEAE$ to $BB$ (denoted by $b = b_1$). While $AEAE$ allows supplier $A$ to earn revenues from both the wholesale contract with the buyer and selling directly to the customers, the outcome $BB$ implies zero revenue for the supplier. Hence, the shift from $AEAE$ to $BB$ brings about a major reduction in the supplier’s profit.

Next, Proposition 3.6.2 shows the interesting result that a boost in the quality of supplier $B$ ($b$) can negatively affect this supplier’s profits.

**Proposition 3.6.2** In the two-period model of supplier encroachment, if $[F > F_1, \ b = b_1, \ F \leq F_2, \ and \ b > b_2]$, then an increase in the quality of the second supplier ($b$) would worsen supplier $B$’s profit.

We illustrate Proposition 3.6.2 in Figure 3.7, where the parameters are set as follows: $\phi = \frac{1}{10}$, $F = \frac{1}{200}$, and $\lambda = \frac{6}{10}$. As $b$ increases, supplier $B$ becomes a more appealing alternative from the buyer’s perspective, since supplier $B$—unlike supplier $A$—cannot encroach on the market independently and compete with the buyer. Thus, supplier $B$ would be expected to earn higher profits as $b$ rises. However, if the conditions of Proposition 3.6.2 hold true, then an increase in $b$ would force a shift in the equilibrium strategy from $BA$ to $AA$, which reduces supplier $B$’s profits to zero, causing a sharp drop in supplier $B$’s profit (denoted by $b = b_1$).

### 3.7 Extensions

#### 3.7.1 Dual Sourcing

The buyer has, thus far, selected its preferred supplier at the beginning of period $t \in \{1, 2\}$—meaning that all production in period $t$ is outsourced to one selected supplier,
while the other supplier earns no revenue. In this extension, we replace the buyer’s binary decision with a continuous one: $M$ can partially outsource the production to the suppliers $A$ and $B$, and potentially use both suppliers’ capacity in a period. We introduce decision variable $\gamma \in [0, 1]$ to denote the portion of the production that is outsourced to supplier $A$, and the rest of the production (i.e., $1 - \gamma$) is outsourced to supplier $B$. Therefore, the dual sourcing variable $\gamma$ is such that $\gamma = 1$ is equivalent to outsourcing the whole production to supplier $A$, and $\gamma = 0$ is equivalent to outsourcing the whole production to supplier $B$.

Proposition 3.7.1 shows that even if the buyer had the option to dual source, it would never exercise that option; in short, we always have $\gamma = 0$ or $\gamma = 1$. Simply put, this is because (1) neither of the suppliers in the supplier encroachment game are restricted by capacity constraints and therefore both suppliers $A$ and $B$ can deliver their contractual obligations to the buyer regardless of the order size, and (2) any level of dual sourcing (i.e., $\gamma > 0$) enables supplier $A$ to launch its own product and compete with the buyer. Hence, if the buyer is accepting the risk of having its supplier encroach into the same market, it would want to take full advantage (i.e., $\gamma = 1$) of the higher process quality of supplier $A$, otherwise the buyer would be incurring the risk without reaping the reward.

**Proposition 3.7.1** Dual sourcing is never the equilibrium.
3.7.2 Different Production Costs

Previously, to ensure mathematical tractability, we assumed that both suppliers have the same production cost of zero. In this extension we study the case where suppliers $A$ and $B$ have different per-unit production costs. Normalizing the lower-quality supplier $B$’s production cost to zero, we introduce the production cost of $c > 0$ for supplier $A$. We numerically show that the results presented in Section 3.5 hold with a strictly positive production cost as well. Figure 3.8 shows the impacts of non-zero production cost $c$ on the equilibria, for different values of $\lambda$ and $b$, with $\phi = \frac{1}{10}$, $F = \frac{1}{200}$, and the production cost $c = \frac{8}{100}$. To make the comparison easier, we put Figure 3.5b from Section 3.5 as Figure 3.8a, beside the equilibrium strategy region-plot of the two-period game with $c > 0$ (Figure 3.8b). We observe that three of the four equilibrium boundaries are shifted up due to the introduction of $c > 0$; such that both $A^E A^E$ regions and the $BB$ region are shrunken, whereas the $AA$ region is expanded. With a non-zero production cost, supplier $A$’s per-unit encroachment profit is reduced from $p_{At}$ to $p_{At} - c$. Therefore, supplier $A$ would need to sell more to cover the one-time investment cost ($F$), which makes encroachment a less profitable alternative. Since supplier $A$ has less incentives to encroach, the two $A^E A^E$ regions shrink and the $AA$ region expands. Similarly, it becomes safer for the buyer to outsource to supplier $A$, and thus, the $BB$ region gets smaller. Even though the decision boundaries are shifted in the case of $c > 0$, the same shifts in the equilibrium strategy are observed as $\lambda$ increases (i.e., $AA$, to $A^E A^E$, to $BA$, back to $A^E A^E$, and to $BB$) and as $b$ increases (i.e., $A^E A^E$, to $BA$, to $AA$).

3.7.3 Supplier A Retains Access to the Market after Being Dropped

In the main model, Section 3.4 we discussed the consequences that supplier $A$ could face if it decides to encroach into the market with its independent product and breach the buyer’s trust. In particular, we assumed that the encroaching supplier loses the access to the market and cannot continue selling its own independent product to the customers if it is dropped by the buyer. In this extension we relax that assumption and
allow the encroaching supplier to retain its access to the market after it is dropped by the buyer; and we demonstrate the robustness of the main results (Section 3.5) analytically. Specifically, we find the necessary and sufficient conditions for the classification of the potential subgame perfect equilibria under the new assumption and show that all subgame-perfect equilibria of Proposition 3.5.1 continue to hold. Furthermore, our main results presented in Propositions 3.5.2 and 3.5.3 hold true under this relaxation. The analytical results are presented in Appendix B.3.

Figure 3.9 illustrates how allowing the encroaching supplier to retain its access to the market impact the equilibrium, for different values of $\lambda$ and $b$. Figure 3.9a corresponds to the main model, and consistent with other extensions we set $\phi = \frac{1}{10}$, and $F = \frac{1}{200}$ to obtain Figure 3.9b. We observe that the significant shifts in the buyer’s and supplier A’s optimal strategies remain unaltered. Namely, (1.) an increase in the quality of supplier A’s independent product ($\lambda$) can make M switch from supplier B to supplier A, (2.) an increase in the quality of supplier A’s independent product ($\lambda$) can make supplier A switch from market-entry ($E_2 = 1$) to no-entry ($E_2 = 0$), and (3.) an increase in the quality of supplier B’s product ($b$) can make M switch from sup-
3.7. Extensions

(a) $A$ might lose its access to market in $t = 2$. (b) $A$ always retains its access to market in $t = 2$.

Figure 3.9: The impact of allowing the encroaching supplier to retain its access to the market in $t = 2$ on the equilibrium strategy.

plier $B$ to supplier $A$. It can also be observed that all the equilibrium regions of the main model are found in Figure 3.9b as well.

The gravity of the encroachment risk grows with $\lambda$, therefore, for low and medium values of $\lambda$ the equilibria are perfectly aligned with the main model (Figure 3.9a). For high values of $\lambda$, however, new regions appear. Buyer knows that it cannot prevent supplier $A$ from encroachment in $t = 2$ if $E_1 = 1$, but it can still penalize the encroaching supplier for breaching the trust by dropping the supplier. To show the viability of supplier $B$’s threat to the encroaching supplier, the buyer outsources the production to supplier $B$ in $t = 2$; which makes encroachment a less lucrative alternative to supplier $A$, and gives rise to the regions $AB$ and $A^E B$. In the $AA^E$ region, supplier $A$ knows that it will lose the bigger share of its second-period revenues (i.e., wholesale contract income) if it is dropped by the buyer. Therefore, to mitigate the risks of encroachment, the supplier chooses a more conservative strategy and encroaches on the market only in the second period and only if $\lambda$ is sufficiently large.
3.8 Conclusion

We propose a two-period model to study the realities of supplier encroachment; that is, the outsourcing opportunities available to the buyer and the consequences of losing the buyer’s trust for the encroaching supplier. In our model, the original buyer outsources the production to a supplier pool characterized by two suppliers, which are separated by their process quality and their capability to encroach on the market with their own independent product.

We obtain insights from contrasting our proposed model with a one-period model that is the standard in the literature; and answer the research questions as follows: (1) The existence of future outsourcing opportunities strengthens the buyer’s leading role in the competition. We find necessary and sufficient conditions under which the buyer would switch suppliers from one period to another. In particular, we show that under certain conditions, (a) a threatening boost in the quality of the encroaching supplier’s independent product would convince the buyer to drop the non-encroaching supplier so that it redirects its business to the encroaching supplier, and (b) a boost in the quality of the non-encroaching supplier may persuade the buyer to drop the non-encroaching supplier and outsource to the encroaching supplier instead. In addition, we demonstrate that it is never optimal for the buyer to initially outsource to the high-quality supplier (in $t = 1$) and later (in $t = 2$) switch to the low-quality supplier. (2) Due to the possibility of future consequences, the high-quality supplier must evaluate the revenues generated by encroachment against the cost and the risks of encroachment (e.g., losing the buyer’s trust). We illustrate the conditions under which the high-quality supplier would discontinue its encroachment upon (a) an increase in the quality of its own independent product, and (b) a rise in the quality of the low-quality supplier. Moreover, we show that the high-quality supplier would either encroach in the second period or in both periods. (3) We find that an increase in the quality of the high-quality supplier’s independent product can improve the buyer’s profits and decrease the encroaching supplier’s profits. (4) We establish that the non-encroaching supplier can be worse off upon an increase in its quality.
3.8. Conclusion

3.8.1 Managerial Implications

Both the buyers of original products and the upstream suppliers (either with or without encroachment capabilities) can apply the insights offered in this paper into their practices. A downstream buyer should investigate the encroachment capabilities of its prospective suppliers, compare the prospective suppliers in terms of their process qualities, and evaluate the quality of an independent product produced by the upstream suppliers. Such measures can be used as proxies to gauge the trustworthiness of prospective suppliers; since, as mentioned earlier, 47% of supplier-buyer relationships collapse, and the main culprit is “lack of trust and commitment” (Webb 2017a,b).

In the case of Stacks and Stacks, for example, FedEx had significant encroachment capabilities due to its access to enormous resources, it was a major player with a high quality in the office supply market, and could offer an independent product of quality to the market. These provided FedEx, the upstream firm, with an opportunity to encroach into the market and sell office supplies directly to the end-customers.

3.8.2 Future Research

This work can be extended in a number of ways. First, we assume that the probability of the buyer dropping the encroaching supplier is exogenously set. However, this probability can be defined endogenously based on a contract between the buyer and the supplier. Such that, a stronger (and costlier) contract between the firms can provide a more solid basis for trust. Furthermore, the buyer can be modelled to sell more than one product, to be able to target different customer segments. For example, the manufacture can launch a new, lower-quality product to compete with the supplier’s independent product in the second period. This can allow the scholars to investigate the impacts of market segmentation and targeting strategies on supplier encroachment. Finally, as illustrated in this paper, the suppliers’ process qualities and the quality of the encroaching supplier’s independent product play a pivotal role in determining the firms’ optimal strategies. Future work can investigate the consequences of endogenous quality for the firms. Such that, quality is a function of the firm’s research and devel-
Opment investment; and a higher investment can allow the firm to target customers with higher valuations.


BIBLIOGRAPHY


Appendix A

Companion for Outsourcing Decision in the Presence of Supplier Copycatting

A.1 Solution Method and Mathematical Proofs

In this appendix we present the solution method and the mathematical proofs. We have carefully followed the literature on multi-period game to solve the math model (e.g., [Lim & Tan, 2010], [Ferrer & Swaminathan, 2006], [Pun & DeYong, 2017]). Specifically, in the second period, each firm makes decisions that maximize its profit in the second period. In the first period, each firm makes decisions that maximize its profit in both periods. We start with the preliminaries—which includes introduction of notations and a summary of the game. We then proceed with the details of the solution method and the proofs of propositions. In particular, to present the proofs we follow three steps: (1) backward induction and using first- and second-order conditions to solve for market and wholesale prices, (2) supplier A’s encroachment decision through solving system of inequalities for the supplier, and (3) using KKT conditions to solve for the optimal marketing investment (manufacturer’s decision).
A.1.1 Preliminaries

As illustrated in Figure [A.1], there are two cases in the extensive form of the game. Each case is comprised of a sequence of decisions starting with the manufacturer making the appropriate marketing investment \( I \). As mentioned earlier, we use the standard solution method for vertically differentiated games (i.e., backward induction) to solve the game. Using backward induction means that first the firms’ second-period profit functions \( \pi_{k2}, k \in \{M, A, B\} \), are optimized, and then their total profits \( \pi_k \). We denote by \( \pi_k \) (no time-period subscript) the firm \( k \)’s total profit; for example, \( \pi_M = \pi_{M1} + \pi_{M2} \).

Consumers have heterogeneous valuations of the (quality of the) products, such that the consumer type \( x \), follows an even distribution with density \( \Psi(I) \) (market size). The consumers’ decision in each period is between buying the manufacturer’s original product, buying supplier’s copycat product (if applicable), or making no purchase at all. A type-\( x \) consumer derives utility \( U_{Mt}(x, p_{Mt}) = x - p_{Mt} \) from the manufacturer’s product produced by supplier \( A \) and \( U_{Mt}(x, p_{Mt}) = bx - p_{Mt} \) if produced by supplier \( B \) (note that supplier \( B \) is a second-period option only). Similarly, consumers derive \( U_{At}(x, p_{At}) = qx - p_{At} \) from supplier \( A \)’s copycat product. Such that, the manufacturer’s and the copycat’s demands are \( D_{Mt} = (1 - x_{Mt}) \Psi(I) \) and \( D_{At} = (x_{Mt} - x_{At}) \Psi(I) \), respectively.

The manufacturer’s profits, \( \pi_{Mt} \), in \( t = 1 \) is \( \pi_{M1} = D_{M1}(p_{M1} - w_{A1}) - I \) and in \( t = 2 \), the manufacturer’s profit under \( E = N \) and \( E = Y \), respectively are, \( \pi_{M2}^N = \theta D_{M2}(p_{M2} - w_{A2}) \) and \( \pi_{M2}^Y = \theta D_{M2}(p_{M2} - w_{B2}) \). When the supplier decides against encroachment (\( E = N \)), we have \( \pi_{A1}^N = w_{A1}D_{M1} \) and \( \pi_{A2}^N = \theta w_{A2}D_{M2} \). Under encroachment (\( E = Y \)), the supplier’s profit in \( t = 1 \) and \( t = 2 \) respectively are \( \pi_{A1}^Y = p_{A1}D_{A1} + w_{A1}D_{M1} - F \) and \( \pi_{A2}^Y = \theta p_{A2}D_{A2} \).

Following backward induction, the last step of the solution is the manufacturer’s maximization of the profits by setting the optimal levels of marketing investment \( I \).
Therefore, two different optimization problems shall be solved:

\[
\max_I \pi^N_M (I) \quad \text{s.t.} \quad \pi^N_A (I) \geq \pi^Y_A (I) \tag{A.1}
\]

\[
\max_I \pi^Y_M (I) \quad \text{s.t.} \quad \pi^N_A (I) \leq \pi^Y_A (I) \tag{A.2}
\]

Equation\(\text{A.1}\) describes the manufacturer’s optimization problem under no encroachment, and Equation\(\text{A.2}\) describes the manufacturer’s problem under encroachment. In each problem, the corresponding constraint is either non-binding (interior solution) or binding. For ease of exposition, we summarize the notation using subscripts \{\text{int}, \text{bin}\} to refer to the aforementioned cases. Therefore, within each of the two subgames N and Y, the manufacturer’s optimal level of marketing investment \((I^*)\) can either be an interior \((I^*_\text{int})\) or a binding \((I^*_\text{bin})\) solution. It is noteworthy that since the manufacturer’s profit functions are both strictly concave and continuous, when the interior solution is feasible it is always optimal as well.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figureA1.png}
\caption{The proposed two-period copycatting game in extensive form.}
\end{figure}
A.1.2 Backward Induction

We illustrate this process by solving the case of no encroachment; where the only available product in the market is the manufacturer’s original product (i.e., no copycats). We start by using the customer utility function at $t = 2$

$$U_{M2}(x, p_{M2}) = x - p_{M2} \quad (A.3)$$

to solve for the indifferent consumer $x_{M2}$. Where we get $x_{M2} = p_{M2}$, so we have for the demand function

$$D_{M2} = (1 - x_{M2}) \Psi(I)$$
$$= (1 - p_{M2}) \beta \sqrt{I} \quad (A.4)$$

The second-period profit functions will therefore be:

$$\pi^N_{M2} = \theta D_{M2} (p_{M2} - w_{A2})$$
$$= \theta (1 - p_{M2}) \beta \sqrt{I} (p_{M2} - w_{A2}), \quad (A.6)$$

for the manufacturer, and

$$\pi^N_{A2} = \theta w_{A2} D_{M2}$$
$$= \theta w_{A2} (1 - p_{M2}) \beta \sqrt{I}, \quad (A.8)$$

for the supplier. Following the backward induction, we start with the last decision of the second period, that is, manufacturer’s market price $p_{M2}$. We check the second-order conditions for $\pi^N_{M2}$, and since

$$\frac{\partial^2 \pi^N_{M2}}{\partial p_{M2}^2} = -2 \beta \theta \sqrt{I} < 0, \quad (A.10)$$

we proceed to use first-order conditions to solve for the last decision under the no
encroachment case, i.e., $p_{M2}$, where we set

$$\frac{\partial \pi^N_{M2}}{\partial p_{M2}} = 0 \implies (A.11)$$

$$\sqrt{T} \beta \theta (-2p_{M2} + w_{A2} + 1) = 0 \implies (A.12)$$

$$p^*_{M2} = \frac{w_{A2} + 1}{2}. \quad (A.13)$$

Next, we plug $p^*_{M2}$ into $\pi^N_{A2}$ and we solve for the penultimate decision of the no encroachment case, the suppliers wholesale price $w_{A2}$. We have

$$\pi^N_{A2} = \theta w_{A2}(1 - p_{M2}) \beta \sqrt{T} \quad (A.14)$$

$$= \theta w_{A2}(1 - \frac{w_{A2} + 1}{2}) \beta \sqrt{T} \quad (A.15)$$

$$= - \frac{1}{2} \sqrt{T} \beta \theta (w_{A2} - 1) w_{A2}, \quad (A.16)$$

such that

$$\frac{\partial^2 \pi^N_{A2}}{\partial w_{A2}^2} = -\beta \theta \sqrt{T} < 0. \quad (A.17)$$

Then to solve for the optimal wholesale price, we use the first-order condition and we set

$$\frac{\partial \pi^N_{A2}}{\partial w_{A2}} = 0 \implies (A.18)$$

$$\frac{1}{2} \sqrt{T} \beta \theta (1 - 2w_{A2}) = 0 \implies (A.19)$$

$$w^*_{A2} = \frac{1}{2}, \quad (A.20)$$

and we plug $w^*_{A2}$ back into $p^*_{M2}$ and obtain $p^*_{M2} = \frac{3}{4}$. Therefore, we get the following optimal second-period profits

$$\pi^N_{M2} = \frac{1}{16} \beta \theta \sqrt{T}, \quad (A.21)$$

$$\pi^N_{A2} = \frac{1}{8} \beta \theta \sqrt{T}. \quad (A.22)$$
Following backward induction, we move on to the first period. Similar to above, we use the customer utility function to find the indifferent customer, where we have

$$U_{M1}(x, p_{M1}) = x - p_{M1}, \quad (A.23)$$

and therefore $x_{M1} = p_{M1}$, so we have for the demand function

$$D_{M1} = (1 - x_{M1}) \Psi(I)$$

$$= (1 - p_{M1}) \beta \sqrt{I}. \quad (A.24)$$

In the first period, the firms optimize their total profits. Where the total profit functions for the case of no encroachment are

$$\pi^N_M = \pi^N_{M1} + \pi^N_{M2} = [D_{M1}(p_{M1} - w_{A1}) - I] + \left[\frac{1}{16} \beta \theta \sqrt{I}\right] \quad (A.26)$$

$$= [(1 - p_{M1}) \beta \sqrt{I}(p_{M1} - w_{A1}) - I] + \left[\frac{1}{16} \beta \theta \sqrt{I}\right], \quad (A.27)$$

for the manufacturer, and

$$\pi^N_A = \pi^N_{A1} + \pi^N_{A2} = [D_{M1}w_{A1}] + \left[\frac{1}{8} \beta \theta \sqrt{I}\right] \quad (A.29)$$

$$= [(1 - p_{M1}) \beta \sqrt{I}w_{A1}] + \left[\frac{1}{8} \beta \theta \sqrt{I}\right] \quad (A.30)$$

for the supplier.

Following backward induction, the last decision of the first period is the manufacturer’s market price $p_{M1}$. We check the second-order conditions for $\pi^N_M$, and since

$$\frac{\partial^2 \pi^N_M}{\partial p_{M1}^2} = -2 \beta \sqrt{I} < 0, \quad (A.32)$$

we proceed to use first-order conditions to solve for the last decision in the first period.
under the no encroachment case, i.e., $p_{M1}$, where we set

$$\frac{\partial \pi^N_M}{\partial p_{M1}} = 0 \implies (A.33)$$

$$\beta \sqrt{I} (-2p_{M1} + w_{A1} + 1) = 0 \implies (A.34)$$

$$p^*_{M1} = \frac{w_{A1} + 1}{2}. \quad (A.35)$$

Next, we plug $p^*_{M1}$ into $\pi^N_A$ and we solve for the penultimate decision of the first period under no encroachment case, the supplier’s wholesale price $w_{A1}$. We have

$$\pi^N_A = [(1 - p_{M1}) \beta \sqrt{I} w_{A1}] + \left[ \frac{1}{8} \beta \theta \sqrt{I} \right]$$

$$= [(1 - \frac{w_{A1} + 1}{2}) \beta \sqrt{I} w_{A1}] + \left[ \frac{1}{8} \beta \theta \sqrt{I} \right]$$

$$= \frac{1}{8} \sqrt{I} \beta (\theta - 4(w_{A1} - 1)w_{A1}), \quad (A.38)$$

such that

$$\frac{\partial^2 \pi^N_A}{\partial w_{A1}^2} = -\beta \sqrt{I} < 0. \quad (A.39)$$

Then to solve for the optimal wholesale price, we use the first-order condition and we set

$$\frac{\partial \pi^N_A}{\partial w_{A1}} = 0 \implies (A.40)$$

$$\frac{1}{2} \beta \sqrt{I} (1 - 2w_{A1}) = 0 \implies (A.41)$$

$$w^*_{A2} = \frac{1}{2}, \quad (A.42)$$

and we plug $w^*_{A1}$ back into $p^*_{M1}$ and obtain $p^*_{M1} = \frac{3}{4}$. Therefore, we get the following optimal total profits

$$\pi^N_M = \frac{1}{16} \beta \sqrt{I}(\theta + 1) - I, \quad (A.43)$$

$$\pi^N_A = \frac{1}{8} \beta \sqrt{I}(\theta + 1). \quad (A.44)$$
Following the exact same procedure laid out above, we solve the game under the encroachment case, i.e., $E = Y$. In this case, supplier $A$ produces a copycat product and sells its product in both periods; where we get the following optimal second-period profits

\[
\begin{align*}
\pi^Y_{M2} &= \frac{b\sqrt{I}\beta(b-q)}{16b-8q} \\
\pi^Y_{A2} &= \frac{9b\sqrt{I}\beta q(b-q)}{16(q-2b)^2} \\
\pi^Y_{B2} &= \frac{b\sqrt{I}\beta(b-q)}{8b-4q}
\end{align*}
\] (A.45, A.46, A.47)

and the following optimal total profits

\[
\begin{align*}
\pi^Y_{M} &= \frac{1}{8}\sqrt{I}\beta\left(\frac{b\theta(q-b)}{q-2b} + \frac{16(q-2)(q-1)}{((q-5)q+8)^2}\right) - I \\
\pi^Y_{A} &= \frac{1}{16}\sqrt{I}\beta\left(\frac{9bq\theta(b-q)}{(q-2b)^2} + \frac{16}{(q-5)q+8}\right) - F.
\end{align*}
\] (A.48, A.49)

**A.1.3 Supplier’s Encroachment Decision**

The next step in the backward induction is the supplier $A$’s choice between no encroachment ($E = N$) and encroachment ($E = Y$). At this stage, the supplier compares its total profit under $E = N$ against the total profit under $E = Y$ to make a decision. Where we have

\[
\begin{align*}
\pi^N_{A} &= \frac{1}{8}\beta\sqrt{I} \left(\theta + 1\right) \\
\pi^Y_{A} &= \frac{1}{16}\sqrt{I}\beta\left(\frac{9bq\theta(b-q)}{(q-2b)^2} + \frac{16}{(q-5)q+8}\right) - F,
\end{align*}
\] (A.50, A.51)

and the parameters feasibility bounds are as follows

\[
\{ I > 0, 0 < q < 1, 0 < \beta < 1, 0 < \theta < 1, q < b < 1, 0 < F < \bar{F}\},
\] (A.52)
where

\[
\bar{F} = \frac{1}{16} \left( \frac{1}{(q^2 - 5q + 8)^2} \left( \frac{81}{4} b^4 I q^6 \theta^2 - \frac{405}{2} b^4 I q^5 \theta^2 + \frac{3321}{4} b^4 I q^4 \theta^2 - 1620 b^4 I q^3 \theta^2 + \frac{288}{127} b^4 I q^3 \theta^2 + 1296 b^4 I q^2 \theta^2 - 1440 b^4 I q^2 \theta^2 + 2304 b^4 I q\theta + 1024 b^4 I - \frac{81}{2} b^3 I q^7 \theta^2 + 405 b^3 I q^6 \theta^2 - 3321 b^3 I q^5 \theta^2 + 3240 b^3 I q^4 \theta^2 - 576 b^3 I q^4 \theta - 2592 b^3 I q^3 \theta + 2880 b^3 I q^3 \theta - 360 b^3 I q^3 \theta + 1536 b^3 I q^2 \theta - 72 b^3 I q^2 \theta + 360 b^3 I q^2 \theta - 576 b^3 I q^2 \theta + 512 b^3 I q^2 \theta + 64 I q^2 \right) \right) ^{\frac{1}{2}}. 
\]

We solve a system of inequalities comprised of \( \{ \pi_A^N > \pi_A^Y \} \) and the parameter bounds, and obtain the necessary and sufficient conditions for \( E = N \) to be the equilibrium. Remember that to ensure mathematical tractability, we set \( \beta = \theta = \frac{1}{2} \).

**Corollary A.1.1** The supplier decides for encroachment \((E = Y)\) if and only if one of the following holds true:

\[
\left( b_0 < b \leq b_1, q_0 < q < q_1, I > I_0 \right), \text{ or } \left( b_1 < b < 1, q_0 < q, I > I_0 \right). 
\]

Where

\[
q_0 = -\frac{1}{2} \left[ \frac{\kappa_2}{9(3b+2)^2 \sqrt[4]{\kappa_3 + \sqrt{\kappa_4}}} + \frac{\sqrt[4]{\kappa_4 + \kappa_5}}{\kappa_7} \right] - \frac{\sqrt[4]{\kappa_4 + \kappa_5}}{\kappa_7} \frac{\kappa_2}{9(3b+2)^2 \sqrt[4]{\kappa_3 + \sqrt{\kappa_4}}} + \frac{\sqrt[4]{\kappa_4 + \kappa_5}}{\kappa_7} \left( \frac{\kappa_2}{9(3b+2)^2 \sqrt[4]{\kappa_3 + \sqrt{\kappa_4}}} + \frac{\sqrt[4]{\kappa_4 + \kappa_5}}{\kappa_7} \right) \right] ^{\frac{1}{2}} 
\]
\[ q_1 = \frac{1}{2} \sqrt{-\frac{\kappa_6}{4 \sqrt{\frac{\kappa_2}{9(3b+2)^2}}}} + \frac{\kappa_2}{9(3b+2)^2 \sqrt[3]{\kappa_3 + \sqrt{\kappa_4}}} - \frac{9(3b+2)^2 \sqrt{\kappa_3 + \sqrt{\kappa_4}}}{\kappa_7} \tag{A.56} \]

\[ I_0 = \frac{\kappa_8}{\kappa_9} \tag{A.57} \]

\[ b_0 = 0.569089 \tag{A.58} \]

\[ b_1 = \frac{1}{6} \left( 15 - \sqrt{129} \right) \tag{A.59} \]

\[ \kappa_1 = -\frac{-3b^2 - 23b - 10}{4(3b+2)} \tag{A.60} \]

\[ \kappa_2 = \frac{(-3b^2 - 23b - 10)^2}{4(3b+2)^2} - \frac{2(69b^2 + 192b + 16)}{9(3b+2)} + \frac{3}{2} \left( -423b^4 - 8064b^3 + 13152b^2 + 384b + 256 \right) \sqrt[3]{27b^3 + 54b^2 + 36b + 8} \tag{A.61} \]

\[ \kappa_3 = -276102b^6 + 2156544b^5 - 3503520b^4 + 2090880b^3 + 638208b^2 + 18432b + 8192 \tag{A.62} \]

\[ \kappa_4 = 76535062272b^{12} - 1173537621504b^{11} + 6887183106816b^{10} - 15245558664192b^9 + 11523422112768b^8 + 4560708354048b^7 - 8312477810688b^6 + 2444828147712b^5 - 108307611648b^4 + 32869711872b^3 \tag{A.63} \]

\[ \kappa_5 = -276102b^6 + 2156544b^5 - 3503520b^4 + 2090880b^3 + 638208b^2 + 18432b + 8192 \tag{A.64} \]

\[ \kappa_6 = -\frac{(3b^2 - 23b - 10)^3}{(3b+2)^3} + \frac{4(69b^2 + 192b + 16)(-3b^2 - 23b - 10)}{3(3b+2)^2} + \frac{512(3b^2 + b)}{3(3b+2)} \tag{A.65} \]

\[ \kappa_7 = 9\sqrt[3]{2} \sqrt{27b^3 + 54b^2 + 36b + 8} \tag{A.66} \]
\[ \kappa_8 = 4096b^4 F^2 q^4 - 40960b^4 F^2 q^3 + 167936b^4 F^2 q^2 - 327680b^4 F^2 q + 262144b^4 F^2 \quad (A.67) \]

\[ 8192b^3 F^2 q^5 + 81920b^3 F^2 q^4 - 335872b^3 F^2 q^3 + 655360b^3 F^2 q^2 - 524288b^3 F^2 q^+ \]

\[ 6144b^2 F^2 q^6 - 61440b^2 F^2 q^5 + 251904b^2 F^2 q^4 - 491520b^2 F^2 q^3 + 393216b^2 F^2 q^2 - \]

\[ 2048b^2 q^7 + 20480b^2 F^2 q^6 - 83968b^2 F^2 q^5 + 163840b^2 F^2 q^4 - 131072b^2 F^2 q^3 + 256F^2 q^8 - \]

\[ 2560F^2 q^7 + 10496F^2 q^6 - 20480F^2 q^5 + 16384F^2 q^4 \]

\[ \kappa_9 = \frac{81b^4 q^6}{16} - \frac{621b^4 q^5}{8} + \frac{8217b^4 q^4}{16} - 1728b^4 q^3 + 2856b^4 q^2 - 1536b^4 q + 256b^4 - \frac{81b^3 q^7}{8} + \]

\[ \frac{621b^3 q^6}{4} - \frac{8217b^3 q^5}{8} + 3456b^3 q^4 - 5712b^3 q^3 + 3072b^3 q^2 - 512b^3 q + \frac{81b^2 q^8}{16} - \frac{675b^2 q^7}{8} + \]

\[ \frac{9585b^2 q^6}{16} - \frac{8595b^2 q^5}{4} + 3762b^2 q^4 - 2160b^2 q^3 + 384b^2 q^2 + \frac{27bq^8}{4} - \frac{171bq^7}{2} + \frac{1683bq^6}{4} - \]

\[ 906bq^5 + 624bq^4 - 128bq^3 + \frac{9q^8}{4} - \frac{45q^7}{2} + \frac{273q^6}{4} - 60q^5 + 16q^4. \]

**Corollary A.1.2** *The copycatting supplier always decides against encroachment if \( 0 < I < I_0 \).*

**A.1.4 Manufacturer’s Decision on Marketing Investment (Endogenous \( I \))**

Finally, the last step in solving the model is the manufacturer’s choice of optimal marketing investment, which in turn determines the size of the market. The manufacturer optimization problems are tabulated below:

\[ \max_I \pi^N_M (I) \quad \text{s.t.} \quad \pi^N_A (I) \geq \pi^Y_A (I) \quad (A.69) \]

\[ \max_I \pi^Y_M (I) \quad \text{s.t.} \quad \pi^N_A (I) \leq \pi^Y_A (I). \quad (A.70) \]
The feasible sets of the optimization problems \[\text{A.69}\] and \[\text{A.70}\] are defined as follows:

\[
S^N = \{(b \leq b_0) \lor (b_0 < b \leq b_1, q \geq q_0) \lor (b_0 < b \leq b_1, q_0 < q < q_1, I < I_0) \lor (b_0 < b \leq b_1, q \geq q_1) \lor (b_1 < b < 1, q \leq q_0) \lor (b_1 < b < 1, q > q_0, I < I_0)\}
\]

\[
S^Y = \{(b_0 < b \leq b_1, q_0 < q < q_1, I > I_0) \lor (b_1 < b < 1, q_0 < q, I > I_0)\}.
\]

Such that for the manufacturer’s optimization problem under \(E = N\) we have

\[
\max_I \pi^N_M (I) \text{ s.t. } \pi^N_A (I) \geq \pi^Y_A (I)
\]

\[\equiv\]

\[
\max_I \pi^N_M (I) \text{ s.t. } I, b, q, F \in S^N,
\]

and for the manufacturer’s optimization problem under \(E = Y\) we have

\[
\max_I \pi^Y_M (I) \text{ s.t. } \pi^N_A (I) \leq \pi^Y_A (I)
\]

\[\equiv\]

\[
\max_I \pi^Y_M (I) \text{ s.t. } I, b, q, F \in S^Y.
\]

We use the KKT method to solve optimization problems \[\text{A.74}\] and \[\text{A.76}\] It should be reminded that since the manufacturer’s profit functions are both strictly concave and continuous, when the interior solution is feasible it is always optimal as well. This gives us the following corollary for the no encroachment case.

**Corollary A.1.3** Under \(E = N\), optimization problem \[\text{A.74}\]

- has the interior solution of \(I^*_\text{int} = \frac{(a+1)^2}{4096}\) if and only if \([b \leq b_0) \lor (b_0 < b \leq b_1, q \geq q_0) \lor (b_0 < b \leq b_1, q_0 < q < q_1, F > F_1) \lor (b_0 < b \leq b_1, q \geq q_1) \lor (b_1 < b < 1, q \leq q_0) \lor (b_1 < b < 1, q > q_0, F > F_1)\],

- has the binding solution of \(I^*_\text{bin} = I_0\) if and only if \([b_0 < b \leq b_1, q_0 < q < q_1, F < F_1) \lor (b_1 \leq b < 1, q > q_0, F < F_1)\].

Similarly, for the case of encroachment we have the following corollary.
Corollary A.1.4 Under $E = Y$, optimization problem [A.76]

- has the interior solution of $I_{in}^* = \frac{(q-1)^2(q^4-10q^3+41q^2+4q^2-80q^2-16q+64q+16)^2}{64(q-2)^2(q^2-5q+8)^3}$ if and only if $[(b_0 < b < b_1, q_0 < q < q_1, F < F_2) \lor (b_1 < b < 1, q_0 < q, F < F_2)]$.

- never has the binding solution of $I_{bin}^* = I_0$.

Where

$$F_0 = \frac{\eta_1}{\eta_2} \quad (A.77)$$

$$F_1 = \frac{-\eta_1}{\eta_2} \quad (A.78)$$

$$\eta_1 = -27b^2q^3 + 207b^2q^2 - 576b^2q + 192b^2 + 27bq^4 - 207bq^3 + 576bq^2 - 192bq + 18q^4 - 90q^3 + 48q^2 \quad (A.79)$$

$$\eta_2 = 32768b^2q^2 - 163840b^2q + 262144b^2 - 32768bq^2 + 163840bq^2 - 262144bq + 8192q^4 - 40960q^3 + 65536q^2 \quad (A.80)$$

$$F_2 = \frac{\kappa_{10}}{\kappa_{11}} \quad (A.81)$$

$$\kappa_{10} = 9b^4q^7 - 159b^4q^5 + 1251b^4q^4 + 14608b^4q^3 - 22400b^4q^2 + 17408b^4q - 4096b^4 \quad (A.82)$$

$$18b^3q^8 + 318b^3q^7 - 2502b^3q^6 + 11642b^3q^5 - 35360b^3q^4 + 71488b^3q^3 - 84608b^3q^2 + 45056b^3q - 8192b^3 + 9b^2q^9 - 165b^2q^8 + 1341b^2q^7 - 6959b^2q^6 + 25694b^2q^5 - 65872b^2q^4 + 95296b^2q^3 - 60416b^2q^2 + 12288b^2q + 6bq^9 - 90bq^8 + 850bq^7 - 5326bq^6 + 19856bq^5 - 35648bq^4 + 26368bq^3 - 6144bq^2 + 192q^7 - 1536q^6 + 3776q^5 - 3456q^4 + 1024q^3 \quad (A.83)$$

$$\kappa_{11} = 32768b^3q^6 - 491520b^3q^5 + 3244032b^3q^4 - 11960320b^3q^3 + 25952256b^3q^2 - (A.84)$$

$$31457280b^3q + 16777216b^3 - 49152b^2q^7 + 737280b^2q^6 - 4866048b^2q^5 + 17940480b^2q^4 - 38928384b^2q^3 + 47185920b^2q^2 - 25165824b^2q + 24576bq^8 - 368640bq^7 + 243304bq^6 - 8970240bq^5 + 19464192bq^4 - 23592960bq^3 + 12582912bq^2 - 4096bq^9 + 61440q^8 - 405504q^7 + 1495040q^6 - 3244032q^5 + 3932160q^4 - 2097152q^3.$
The feasible space of all the parameters and the decision variable $I$ is divided according to corollaries A.1.3 and A.1.4. Therefore, for the manufacturer to make the optimal decision all the sub-spaces under $E = N$ must be compared against all the sub-spaces under $E = Y$. We perform all the pair-wise comparisons and obtain the necessary and sufficient conditions for $I_{int}^N$, $I_{int}^N$, $I_{int}^Y$, and $I_{bin}^Y$ to be the subgame perfect equilibria. Therefore, we obtain the following (Proposition 1).

**Proposition A.1.5** The classification of the potential subgame perfect equilibria and the corresponding conditions is as follows:

- $Y_{int}$ is the equilibrium if and only if one of the following sets of conditions hold:
  
  $[b_0 < b \leq b_1, q_0 < q < q_1, F < F_3], [b_1 < b < 1, q_0 < q < b, F < F_5]$.

- $N_{int}$ is the equilibrium if and only if one of the following sets of conditions hold:
  
  $[b \leq b_0], [b_0 < b \leq b_1, q \leq q_0], [b_0 < b \leq b_1, q_0 < q < q_1, F > F_4], [b_0 < b \leq b_1, q \geq q_1], [b_1 \leq b < 1, q \leq q_0], [b_1 \leq b < 1, q > q_0, F > F_6]$.

- Otherwise, $N_{bin}$ is the equilibrium.

Propositions 2.5.1, 2.5.2, and 2.5.3 in are direct results of Proposition 2.4.1. Proposition 2.5.1 states that if

$$(q_0 < q < q_1, F_5 < F < F_6, b = b_1),$$

then an increase in $b$ reduces the manufacturer’s profit. The conditions listed in A.84 describe the manufacturer’s indifference point between $N_{int}$ and $N_{bin}$.

Under $N_{int}$, the manufacturer’s optimal profit is

$$\pi_{M, N_{int}} = \frac{9}{16384},$$

(A.85)
and under $N_{bin}$ we have

$$\pi_{bin}^{N} = \frac{F^2((q - 5)q + 8)^2(q - 2b)^4}{(b^2(3q(q(3q - 23) + 64) - 64) + bq(64 - 3q(q(3q - 23) + 64)) - 2q^2(3(q - 5)q + 8))^2}$$

Such that

$$\frac{\partial \pi_{bin}^{N}}{\partial b} = \frac{9F^2(8192\kappa_{12} - 3)q^3((q - 5)q + 8)^3(2b - q)^3}{\kappa_{13}\kappa_{14}}$$

Where

$$\kappa_{12} = \sqrt{\frac{F^2((q - 5)q + 8)^2(q - 2b)^4}{(b^2(3q(q(3q - 23) + 64) - 64) + bq(64 - 3q(q(3q - 23) + 64)) - 2q^2(3(q - 5)q + 8))^2}}$$

$$\kappa_{13} = \sqrt{\frac{F^2((q - 5)q + 8)^2(q - 2b)^4}{(b^2(3q(q(3q - 23) + 64) - 64) + bq(64 - 3q(q(3q - 23) + 64)) - 2q^2(3(q - 5)q + 8))^2}}$$

$$\kappa_{14} = b^2(3q(q(3q - 23) + 64) - 64) + bq(64 - 3q(q(3q - 23) + 64)) - 2q^2(3(q - 5)q + 8).$$

Therefore, as $b$ increase beyond $b_1$ the manufacturer’s profit is decreased.

Proposition 2.5.2 focuses on the effect of an increase in $F$ on the equilibrium. In particular, if

$$(b_1 < b < 1, q_0 < q, F_5 < F < F_6),$$

then an increase in $F$ improves supplier A’s profit by shifting the equilibrium into $N_{bin}$. 
Under $N_{bin}$, supplier $A$’s optimal profit is

$$\pi_{A}^{N, bin} = \frac{6F^2((q - 5)q + 8)^2(q - 2b)^4}{(b^2(3q(q(3q - 23) + 64) - 64) + bq(64 - 3q(3q - 23) + 64)) - 2q^2(3(q - 5)q + 8))^2},$$

where

$$\frac{\partial \pi_{A}^{N, bin}}{\partial F} = \frac{6\sqrt{F^2((q - 5)q + 8)^2(q - 2b)^4}}{(b^2(3q(q(3q - 23) + 64) - 64) + bq(64 - 3q(3q - 23) + 64)) - 2q^2(3(q - 5)q + 8))^2} \cdot (A.94)$$

$$\implies \frac{\partial \pi_{A}^{N, bin}}{\partial F} > 0. \tag{A.95}$$

Therefore, $\pi_{A}^{N, bin}$ is increasing in $F$, and an increase in the market entry cost improves the supplier’s profit within this region.

Proposition 2.5.3 states that if

$$(b_1 < b < 1, q_0 < q, F = F_6), \tag{A.96}$$

then an increase in $q$ reduces the copycatting supplier’s profit. The conditions laid out in $A.96$ refer to the equilibrium shift from $N_{int}$ to $N_{bin}$. Under $N_{int}$ the supplier’s optimal profit is

$$\pi_{A}^{N, int} = \frac{9}{4096}, \tag{A.97}$$

which is independent of $q$ and therefore not affected by changes in the supplier’s quality. However, under $N_{bin}$ the supplier’s profit is

$$\pi_{A}^{N, bin} = \frac{6F^2((q - 5)q + 8)^2(q - 2b)^4}{(b^2(3q(q(3q - 23) + 64) - 64) + bq(64 - 3q(3q - 23) + 64)) - 2q^2(3(q - 5)q + 8))^2}, \tag{A.98}$$
such that
\[
\frac{\partial \pi_{A,bin}^N}{\partial q} = -\frac{\kappa_{15}}{\kappa_{16}\kappa_{17}^{3}}
\]
(A.99)

\[\implies \frac{\partial \pi_{A,bin}^N}{\partial q} < 0, \forall q > q_0,\]  
(A.100)

where

\[\kappa_{15} = 6F^2((q - 5)q + 8)(2b - q)^3(q(9q((q - 10)q + 41) - 976) + 1216) -
3b^2q(q(9q((q - 10)q + 41) - 976) + 1216) + 192bq^2(5 - 2q) + 32q^3(2q - 5))\]  
(A.101)

\[
\kappa_{16} = \sqrt{\frac{F^2((q - 5)q + 8)^2(q - 2b)^4}{(b^2(3q(3q - 23) + 64) - 64) + bq(64 - 3q(q(3q - 23) + 64)) - 2q^2(3(q - 5)q + 8))}}
\]
(A.102)

\[\kappa_{17} = b^2(3q(q(3q - 23) + 64) - 64) + bq(64 - 3q(q(3q - 23) + 64)) - 2q^2(3(q - 5)q + 8).\]  
(A.103)

which implies the supplier A’s optimal profit is decreasing in the quality of the copycat product it produces.

\[\blacksquare\]
Appendix B

Companion for Navigating Supplier Encroachment: Game-Theoretic Insights for Outsourcing Strategies

B.1 Definition of the Thresholds

Lemma B.1.1

(a) \( F_1 = \sqrt{\frac{2\lambda^2(\lambda^2 - 10\lambda + 25)}{64\lambda^8 - 640\lambda^6 + 2624\lambda^4 - 5120\lambda + 4096}} \).

(b) \( b_1 = \frac{32\lambda^2 - 96\lambda + 64}{\lambda^4 - 10\lambda^3 + 41\lambda^2 - 80\lambda + 64} \).

(c) \( \phi_1 = -\frac{\lambda^2 + 5\lambda}{8} \).

(d) \( F_2 = \frac{-(\lambda^2 - 5\lambda + 4\phi)}{4(\lambda^2 - 5\lambda + 8)} \).

(e) \( b_2 = \frac{1 - \frac{32(2 - \lambda)(1 - \lambda)(2 - \phi)}{(\lambda^2 - 5\lambda + 8)^2}}{\phi - 1} \).

B.2 Solution Method and Mathematical Proofs

In this section we present the solution method and the mathematical proofs. We start with the preliminaries—which includes introduction of notations and an illustration of the backward induction process, and sheds light on the probabilistic nature of the game
and its effect on the firms’ profits. We then proceed with the details of the solution method and the proofs of propositions.

B.2.1 Preliminaries

As marked by the dashed boxes in Figure B.1 there are 10 cases in the extensive form of the game. Each case is comprised of a sequence of decisions starting with the buyer choosing a supplier in \( t = 1 \). For ease of exposition, we introduce a new notation to describe case-specific profit functions, such that \( \pi_k(l), k \in \{M, A, B\}, t \in \{1, 2\}, l \in \{1, 2, \ldots, 10\} \), denotes the firm \( k \)’s profit in period \( t \) of the subgame \( l \); and \( \pi_k(l) = \pi_k(l) + \pi_k(l) \), denotes the firm \( k \)’s total profit under case \( l \). For example, if \( M \) outsources to supplier \( B \) in \( t = 1 \) and to supplier \( A \) in \( t = 2 \), then in \( t = 2 \) supplier \( A \) will compare \( \pi_A(8) \) against \( \pi_A(9) \) to decide on second-period market entry.

As mentioned earlier, we use the standard solution method for vertically differentiated games (i.e., backward induction) to solve the game. Using backward induction means that the firms will first optimize their second-period profits, \( \pi_k(l) \), and then their total profits, \( \pi_k(l) \). We illustrate this process by solving the second period of case 5. We start by using the utility functions \( U_M(\psi, p_M) = \psi - p_M \) and \( U_A(\psi, p_A) = \lambda \psi - p_A \) to solve for the indifferent consumers \( \psi_M \) and \( \psi_A \); so that we get the demand functions \( D_M = (1 - \psi_M) \frac{1}{2} \) and \( D_A = (\psi_M - \psi_A) \frac{1}{2} \). The second-period profit functions will therefore be: \( \pi_{M2}(5) = D_{M2} (p_{M2} - w_{A2}), \pi_{A2}(5) = D_{M2} w_{A2} + D_{A2} p_{A2} - F \), and \( \pi_{B2}(5) = 0 \). We check the second-order conditions for \( \pi_{A2}(5) \), and since \( \frac{\partial^2 \pi_{A2}(5)}{\partial p_{A2}^2} < 0 \), we proceed to use first-order conditions to solve for the last decision in case 5, i.e., \( p_A \), where we obtain \( p_A = \frac{1}{2} (p_{M2} + w_{A2}) \) \( \lambda \). Next, we solve for the penultimate decision of case 5, the \( M \)’s market price \( (p_{M2}) \). After plugging in \( p_A \) for \( p_{A2} \), we check the second-order conditions for \( \pi_{M2}(5) \), and since \( \frac{\partial^2 \pi_{M2}(5)}{\partial p_{M2}^2} < 0 \), we solve for the optimal market price and get \( p_{M2} = \frac{w_{A2} + \lambda + 1}{2 - \lambda} \). Lastly, the first decision in the second period of case 5 is the wholesale price. So we plug in \( p_{M2} \) for \( p_{M2} \), and check the second-order condition of \( \pi_{A2}(5) \). Since \( \frac{\partial^2 \pi_{A2}(5)}{\partial w_{A2}^2} < 0 \), we solve for the optimal wholesale price and obtain \( w_{A2} = \frac{\lambda^2 - 3\lambda + 4}{\lambda^2 - 8\lambda + 8} \). Finally, by plugging \( p_{M2}^*, p_{M2}^*, \) and \( w_{A2}^* \) into the profit functions, we
obtain the following optimal profit functions: \( \pi^*_M(5) = 2(\lambda^2 - 3\lambda + 2) \), \( \pi^*_A(5) = \lambda^2 - 3\lambda + 2 - F \), and \( \pi^*_B(5) = 0 \). The optimal second-period profit functions of the three firms for all the 10 cases are tabulated in Table B.1. Then, to determine supplier A’s optimal market entry decision after being chosen by the buyer and not encroaching in \( t = 1 \) (i.e., A’s choice in the second period between subgame 5 and subgame 6), we compare \( \pi^*_A(5) \) against \( \pi^*_A(6) = \frac{1}{8} \) and solve the following system of inequalities: \( \left\{ \frac{1}{\lambda^2 - 3\lambda + 2} - F \geq \frac{1}{8}, \right. \\
0 < b < 1, 0 < \lambda < 1, F > 0 \}. \) This is how the classification of the potential subgame perfect equilibria and the corresponding conditions are obtained, which will be further explained in the next subsection.

As explained earlier in Section 3.4, the buyer may drop the encroaching supplier at the beginning of the second period with probability \( \phi \). In other words, if \( A^E \) is the realized subgame in the first period, the equilibrium strategy will be \( A^EB \) (i.e., case 1) with probability \( \phi \), while with probability \( 1 - \phi \) one of the three cases 2, 3, or 4 will be
realized in $t = 2$. We model this probabilistic node using expected value; such that the firms’ profit functions are a weighted average of case 1 (with weight $\phi$) and one of the cases 2 to 4 (with weight $1 - \phi$). Therefore, when the buyer outsources to supplier $A$ in $t = 1$ and $A$ encroaches into the market ($E_1 = 1$), the second-period profit functions to be used in the total profit functions are as follows

$$\pi_{k2} = \phi \pi_{k2}(1) + (1 - \phi) \pi_{k2}(i), \ k \in \{M, A, B\}, i \in \{2, 3, 4\},$$

where $i$ is determined by the buyer’s outsourcing decision and supplier $A$’s encroachment decision in $t = 2$ under the equilibrium. It is noteworthy that the probabilistic node occurs at the beginning of $t = 2$, thus it only affects the total profits of subgames $AE_A$, $AE_A$, and $AE_B$. In other words, the backward induction process to solve the second periods of cases 1 to 4 will be similar to that of case 5 (illustrated above).

### B.2.2 Mathematical Proofs

We present the step-by-step solution method of the game in this section. We start by solving the left-side of the extensive form of the game, i.e., case 1 to case 7, and then proceed to solve the right-side of the extensive form, i.e., case 8 to case 10. For ease of exposition we define $G_i(\cdot)$ functions based on Lemma B.1.1:

(a) $G_1(F, \lambda) = \frac{64F^2\lambda^4 - 640F^2\lambda^3 + 2624F^2\lambda^2 - 5120F\lambda^2 + 4096F^2}{\lambda^2(\lambda^2 - 10\lambda + 28)}$.

(b) $G_2(b, \lambda) = \frac{32\lambda^2 - 96\lambda + 64}{\lambda^2 - 10\lambda + 41\lambda^2 - 80\lambda + 64} - b$.

(c) $G_3(\phi, \lambda) = \lambda^2 - 5\lambda + 8\phi$.

(d) $G_4(F, \lambda, \phi) = 4F(\lambda^2 - 5\lambda + 8) + (\lambda^2 - 5\lambda + 4\phi)$.

(e) $G_5(b, \lambda, \phi) = \frac{1}{16}\left(b\phi - b + \frac{32(2-\lambda)(1-\lambda)(2-\phi)}{(\lambda^2 - 5\lambda + 8)^2} - 1\right)$.

**Proof of Proposition 3.3.1**: The logic is the same as the second periods of cases 5–6–7 and we omit the details to avoid redundancy. ■

**Proof of Proposition 3.5.1**: Let us begin with supplier $A$’s market entry decision in $t = 2$ after having encroached in $t = 1$, i.e., $A$’s choice in the second period between cases 2 and 3. We compare $A$’s optimal profit (reported in Table [B.1]) under case 2 against
Table B.1: The optimal second-period profit functions of the three firms in each subgame.

<table>
<thead>
<tr>
<th>Case</th>
<th>Profit Functions ((t = 2))</th>
<th>Case</th>
<th>Profit Functions ((t = 2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\pi^<em>_M(1) = \frac{b}{16}) (\pi^</em>_A(1) = 0) (\pi^*_B(1) = \frac{b}{8})</td>
<td>6</td>
<td>(\pi^<em>_M(6) = \frac{1}{16}) (\pi^</em>_A(6) = \frac{1}{8}) (\pi^*_B(6) = 0)</td>
</tr>
<tr>
<td></td>
<td>(\pi^<em>_M(2) = \frac{2(\lambda^2 - 3\lambda + 2)}{\lambda^2 - 5\lambda + 8}^2) (\pi^</em>_A(2) = \frac{1}{\lambda^2 - 5\lambda + 8}) (\pi^*_B(2) = 0)</td>
<td>7</td>
<td>(\pi^<em>_M(7) = \frac{b}{16}) (\pi^</em>_A(7) = 0) (\pi^*_B(7) = \frac{b}{8})</td>
</tr>
<tr>
<td>2</td>
<td>(\pi^<em>_M(3) = \frac{1}{16}) (\pi^</em>_A(3) = \frac{1}{8}) (\pi^*_B(3) = 0)</td>
<td>8</td>
<td>(\pi^<em>_M(8) = \frac{2(\lambda^2 - 3\lambda + 2)}{\lambda^2 - 5\lambda + 8}^2) (\pi^</em>_A(8) = \frac{1}{\lambda^2 - 5\lambda + 8} - F) (\pi^*_B(8) = 0)</td>
</tr>
<tr>
<td>3</td>
<td>(\pi^<em>_M(4) = \frac{b}{16}) (\pi^</em>_A(4) = 0) (\pi^*_B(4) = \frac{b}{8})</td>
<td>9</td>
<td>(\pi^<em>_M(9) = \frac{1}{16}) (\pi^</em>_A(9) = \frac{1}{8}) (\pi^*_B(9) = 0)</td>
</tr>
<tr>
<td>4</td>
<td>(\pi^<em>_M(5) = \frac{2(\lambda^2 - 3\lambda + 2)}{\lambda^2 - 5\lambda + 8}^2) (\pi^</em>_A(5) = \frac{1}{\lambda^2 - 5\lambda + 8} - F) (\pi^*_B(5) = 0)</td>
<td>10</td>
<td>(\pi^<em>_M(10) = \frac{b}{16}) (\pi^</em>_A(10) = 0) (\pi^*_B(10) = \frac{b}{8})</td>
</tr>
</tbody>
</table>

Case 3—that is \(\pi^*_A(2)\) versus \(\pi^*_A(3)\)—and we solve the following system of inequalities: \(\frac{\lambda^2 - 5\lambda + 8}{\lambda^2 - 3\lambda + 2} \geq \frac{1}{8}, 0 < \lambda < 1\). We conclude that \(\pi^*_A(2) \geq \pi^*_A(3)\) for all values of \(\lambda\) within the specified range. Therefore, subgame \(A^E A\) is never the equilibrium. Next step is the buyer’s outsourcing decision in \(t = 2\) after supplier \(A\) has encroached in \(t = 1\), i.e., \(M\)’s second-period choice between cases 2 and 4. For which, we compare \(\pi^*_M(2)\) and \(\pi^*_M(4)\), and solve the following system of inequalities: \(\frac{2(\lambda^2 - 3\lambda + 2)}{\lambda^2 - 5\lambda + 8} \geq \frac{b}{4}, F \geq 0, 0 < \lambda < 1, 0 < b < 1\). We conclude that \(\pi^*_M(2) \geq \pi^*_M(4) \iff b \leq \frac{32\lambda^2 - 96\lambda + 64}{\lambda^2 - 10\lambda^2 + 41\lambda - 80\lambda + 64}\), and
define \( G_2(b, \lambda) = \frac{32\lambda^2 - 96\lambda + 64}{\lambda^2 - 10\lambda^3 + 41\lambda^2 - 80\lambda + 64} - b \), such that

\[
\begin{align*}
\pi^*_{M2}(2) & \geq \pi^*_{M2}(4) \iff G_2(b, \lambda) \geq 0 \iff b \leq b_1, \\
\pi^*_{M2}(2) & < \pi^*_{M2}(4) \iff G_2(b, \lambda) < 0 \iff b > b_1.
\end{align*}
\]

Following the backward induction logic, we move beyond the second period and form the total profit functions so that we can respectively solve for \( p_{A1}, p_{M1}, \) and \( w_{A1} \). The firms’ total profits are of the form \( \pi_k = \pi_{k1} + \pi_{k2}, k \in \{M, A\} \). As explained above, in the specific case of cases 1 to 4, we have \( \pi_{k2} = \phi \pi_{k2}(1) + (1 - \phi) \pi_{k2}(i), i \in \{2, 3, 4\} \); since we have shown subgame 3 is never the equilibrium, this leads to two different cases.

1. \( G_2(b, \lambda) \geq 0 \): in this case subgame 2 is the equilibrium and we have \( \pi_{k2} = \phi \pi_{k2}(1) + (1 - \phi) \pi_{k2}(2), k \in \{M, A\} \). We calculate \( \pi_M, \pi_A, \) and \( \pi_B \), and using the second- and first-order conditions, solve for \( p_{A1}^*, p_{M1}^*, \) and \( w_{A1}^* \). By plugging the optimal prices into the total profit functions, we obtain the following total profit functions for the firms under cases 1 to 4: \( \pi_M^* = \frac{1}{16} (b \phi - \frac{32(\lambda - 2)(\lambda - 1)(\lambda - 2)}{(\lambda - 5)\lambda + 8}) \), \( \pi_A^* = \frac{1}{8}(\phi - 2)(\lambda - 5)\lambda + 8 - F \), and \( \pi_B^* = \frac{1}{8} b \phi \).

2. \( G_2(b, \lambda) < 0 \): in this case subgame 4 is the equilibrium and we have \( \pi_{k2} = \phi \pi_{k2}(1) + (1 - \phi) \pi_{k2}(4), k \in \{M, A\} \). Similar to the previous case, we calculate \( \pi_M, \pi_A, \) and \( \pi_B \), and solve for \( p_{A1}^*, p_{M1}^*, \) and \( w_{A1}^* \). By plugging the optimal prices into the total profit functions, we obtain the following total profit functions for the firms under subgames 1 to 4: \( \pi_M^* = \frac{2(\lambda - 2)(\lambda - 1)}{(\lambda - 5)\lambda + 8} + \frac{b}{16}, \pi_A^* = \frac{1}{8}\lambda - 5 - \frac{1}{(\lambda - 5)\lambda + 8} - F \), and \( \pi_B^* = \frac{b}{8} \).

The summary of the cases 1 to 4 is presented in Table \[B.2\]

After solving the first four cases, we now present the solution method for subgames 5 to 7. We begin with supplier A’s market entry decision in \( t = 2 \) after deciding against encroachment in \( t = 1 \), i.e., A’s choice between subgames 5 and 6 in the second period. We compare \( \pi^*_{A2}(5) \) and \( \pi^*_{A2}(6) \) and conclude that \( \pi^*_{A2}(5) \geq \pi^*_{A2}(6) \iff \frac{64F^2\lambda^4 - 640F^2\lambda^3 + 2624F^2\lambda^2 - 5120F^2\lambda + 4096F^2}{\lambda^2(\lambda^2 - 10\lambda + 25)} \leq 0 \), and define

\[
G_1(F, \lambda) = \frac{64F^2\lambda^4 - 640F^2\lambda^3 + 2624F^2\lambda^2 - 5120F^2\lambda + 4096F^2}{\lambda^2(\lambda^2 - 10\lambda + 25)}, \tag{B.1}
\]
Following the backward induction, the next decision is the buyer’s choice of supplier in $t = 2$. While the condition we found on $G_1(F, \lambda)$ generates two separate scenarios:

1. $G_1(F, \lambda) \leq 0$: in this scenario subgame 5 is the equilibrium, so $\pi^*_M(5)$ is compared with $\pi^*_M(7)$. We solve the system of inequalities \( \frac{1}{\lambda^2 - 8\lambda + 8} - F \geq \frac{b}{10}, 0 < \lambda < 1, F > 0, 0 < b < 1, G_1(F, \lambda) \leq 0 \), and conclude that $\pi^*_M(5) \geq \pi^*_M(7) \iff b \leq \frac{32\lambda^2 - 96\lambda + 64}{\lambda^3 - 10\lambda^2 + 41\lambda^2 - 80\lambda + 64}$, that is, $\pi^*_M(5) \geq \pi^*_M(7) \iff G_2(b, \lambda) \geq 0$. Therefore we have

\[
\begin{align*}
\pi^*_M(5) &\geq \pi^*_M(7) \iff G_2(b, \lambda) \geq 0 \iff b \leq b_1, \\
\pi^*_M(5) &< \pi^*_M(7) \iff G_2(b, \lambda) < 0 \iff b > b_1.
\end{align*}
\]

Hence, to form the total profit function and respectively solve for $p^*_M$ and $w^*_A$, we study two new scenarios within this case:

a. $G_2(b, \lambda) \geq 0$: in this case, between subgames 5–6–7, case 5 is the equilibrium. Using the second- and the first-order conditions, we solve for the optimal prices and obtain the optimal profits. The total profit functions in this case are $\pi^*_M = \frac{2(\lambda - 2)(\lambda - 1)}{(\lambda - 5)(\lambda + 8)^2} + \frac{1}{16}, \pi^*_A = \frac{1}{8} \left( \frac{8}{(\lambda - 5)\lambda^8} + 1 \right) - F$, and $\pi^*_B = 0$.

b. $G_2(b, \lambda) < 0$: in this case, subgame 7 is the equilibrium between subgames 5–
6–7. Using the same logic as above we solve for the optimal first-period decision variables, and obtain the optimal profits: \( \pi^*_M = \frac{1}{16}(1 + b), \pi^*_A = \frac{1}{8}, \) and \( \pi^*_B = \frac{b}{8}. \)

2. \( G_1(F, \lambda) > 0 \): in this case, \( \pi^*_M(6) \) is compared with \( \pi^*_M(7) \). We solve the system of inequalities \( \left\{ \frac{1}{16} \geq \frac{b}{16}, 0 < b < 1, G_1(F, \lambda) > 0 \right\} \), and conclude that \( \pi^*_M(6) \geq \pi^*_M(7) \) for all values of parameters within the specified range. Solving for the first-period decision variables and plugging the optimal solutions back into the profit functions we obtain: \( \pi^*_M = \frac{1}{8}, \pi^*_A = \frac{1}{4}, \) and \( \pi^*_B = 0. \)

We can now find supplier \( A \)'s equilibrium encroachment decision in \( t = 1 \) and complete the solution of subgames 1 to 7. There are four scenarios to be considered:

1. \( G_1(F, \lambda) \leq 0, G_2(b, \lambda) \geq 0 \): supplier \( A \)'s choice here is between subgames 2 and 5. We solve the system of inequalities \( \left\{ -\frac{(\phi-2)}{(\lambda^2-5\lambda+8)} - F \geq \frac{1}{8}, 0 < \lambda < 1, \right\} \) for all values of parameters within the specified range. Solving for the first-period decision variables, and obtain the optimal profits:

\[
\begin{align*}
  &\pi^*_A(2) \geq \pi^*_A(5) \iff G_3(\phi, \lambda) \leq 0 \iff \phi \leq \phi_1, \\
  &\pi^*_A(2) < \pi^*_A(5) \iff G_3(\phi, \lambda) > 0 \iff \phi > \phi_1.
\end{align*}
\]

2. \( G_1(F, \lambda) > 0, G_2(b, \lambda) \geq 0 \): supplier \( A \) chooses between subgames 2 and 6 in this case. We solve the following system of inequalities \( \left\{ -\frac{(\phi-2)}{(\lambda^2-5\lambda+8)} - F \geq \frac{1}{4}, \right\} \) for all values of parameters within the specified range. Solving for the first-period decision variables, and obtain the optimal profits:

\[
\begin{align*}
  &\pi^*_A(2) \geq \pi^*_A(6) \iff 4F(\lambda^2 - 5\lambda + 8) + (\lambda^2 - 5\lambda + 4\phi) \leq 0, \iff G_4(F, \lambda, \phi) \leq 0 \iff F \leq F_2, \\
  &\pi^*_A(2) < \pi^*_A(6) \iff G_4(F, \lambda, \phi) > 0 \iff F > F_2.
\end{align*}
\]

3. \( G_1(F, \lambda) \leq 0, G_2(b, \lambda) < 0 \): in this case, supplier \( A \)'s choice is between subgames 4 and 7. The system of inequalities \( \left\{ \frac{1}{(\lambda^2-5\lambda+8)} - F \geq \frac{1}{8}, \right\} \) for all values of parameters within the specified range. Solving for the first-period decision variables, and obtain the optimal profits: \( \pi^*_A(3) \geq \pi^*_A(4) \implies \phi < \phi_2. \)
\( b < 1, G_1(F, \lambda) \leq 0, G_2(b, \lambda) < 0 \) is solved, and it is concluded that \( \pi^*_A(4) \geq \pi^*_A(7) \) for all values of parameters specified above.

4. \( G_1(F, \lambda) > 0, G_2(b, \lambda) < 0 \): lastly, in this case, supplier \( A \) chooses between subgames 4 and 6. We solve the system of inequalities \( \left\{ \frac{1}{(\lambda-5)\lambda+8} - F \geq \frac{1}{4}, F > 0, 0 < \lambda < 1, 0 < \phi < 1, 0 < b < 1, G_1(F, \lambda) > 0, G_2(b, \lambda) < 0 \right\} \), and conclude that \( \pi^*_A(4) < \pi^*_A(6) \) for all values of parameters within the predefined range.

Subgames 8–9–10, which are less complicated, are solved using the same approach. To maintain a reasonable length for the proofs, we summarize the solution of subgames 8–9–10 in the following three scenarios:

1. \( G_1(F, \lambda) \leq 0, G_2(b, \lambda) \geq 0 \): in this case subgame 8 is the equilibrium of the right-side of the game, and we have \( \pi^*_M = \frac{2(\lambda-2)(\lambda-1)}{(\lambda-5)\lambda+8} + \frac{b}{16}, \pi^*_A = \frac{1}{(\lambda-5)\lambda+8} - F, \) and \( \pi^*_B = \frac{b}{8} \).

2. \( G_1(F, \lambda) \leq 0, G_2(b, \lambda) > 0 \): in this case subgame 10 is the equilibrium of the right-side of the game, and we have \( \pi^*_M = \frac{b}{8}, \pi^*_A = 0, \) and \( \pi^*_B = \frac{b}{2} \).

3. \( G_1(F, \lambda) > 0 \): finally, in this case subgame 9 is the equilibrium of the right-side of the game, and we have \( \pi^*_M = \frac{1}{16} (1 + b), \pi^*_A = \frac{1}{8}, \) and \( \pi^*_B = \frac{b}{8} \).

Finally, we can solve the first stage of the game, i.e., the buyer’s outsourcing decision at the beginning of \( t = 1 \). We summarize in Table \ref{table:B.3} the cases that exist for this decision. If \( G_1(F, \lambda) > 0, G_2(b, \lambda) \geq 0, \) and \( G_4(F, \lambda, \phi) \leq 0 \)—i.e., cases 4 and 5 in Table \ref{table:B.3}—the buyer’s choice is between subgames 2 and 9, and we solve the system of inequalities \( \left\{ \frac{1}{16} \left( b \phi - \frac{32(\lambda-2)(\lambda-1)(\phi-2)}{(\lambda-5)\lambda+8} \right) \geq \frac{1}{16} (1 + b), F > 0, 0 < \lambda < 1, 0 < \phi < 1, 0 < b < 1, G_1(F, \lambda) > 0, G_2(b, \lambda) \leq 0, G_4(F, \lambda, \phi) \leq 0 \right\} \). We conclude that \( \pi^*_M(2) \geq \pi^*_M(9) \iff \frac{1}{16} \left( b \phi - \frac{32(2-\lambda)(1-\lambda)(2-\phi)}{(\lambda^2-5\lambda+8)^2} - 1 \right) \geq 0 \), hence we define

\[
G_5(b, \lambda, \phi) = \frac{1}{16} \left( b \phi - \frac{32(2-\lambda)(1-\lambda)(2-\phi)}{(\lambda^2-5\lambda+8)^2} - 1 \right). \tag{B.2}
\]

\textbf{Proof of Propositions 3.5.2 and 3.5.3} Table \ref{table:B.3} also shows the necessary and sufficient conditions for the equilibria (Proposition 2). Propositions 3 and 4 are shown by tak-
Table B.3: Buyer’s outsourcing decision in $t = 1$. The equilibrium case in each condition is underlined.

<table>
<thead>
<tr>
<th>Condition #</th>
<th>$M$’s Choice</th>
<th>$G_1(\cdot)$</th>
<th>$G_2(\cdot)$</th>
<th>$G_3(\cdot)$</th>
<th>$G_4(\cdot)$</th>
<th>$G_5(\cdot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Subgame 2 v. 8</td>
<td>$G_1(\cdot) \leq 0$</td>
<td>$G_2(\cdot) \geq 0$</td>
<td>$G_3(\cdot) \leq 0$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>2</td>
<td>Subgame 5 v. 8</td>
<td>$G_1(\cdot) \leq 0$</td>
<td>$G_2(\cdot) \geq 0$</td>
<td>$G_3(\cdot) &gt; 0$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>3</td>
<td>Subgame 4 v. 10</td>
<td>$G_1(\cdot) \leq 0$</td>
<td>$G_2(\cdot) &lt; 0$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>4</td>
<td>Subgame 2 v. 9</td>
<td>$G_1(\cdot) &gt; 0$</td>
<td>$G_2(\cdot) \geq 0$</td>
<td>$-$</td>
<td>$G_4(\cdot) \leq 0$</td>
<td>$G_5(\cdot) \geq 0$</td>
</tr>
<tr>
<td>5</td>
<td>Subgame 2 v. 9</td>
<td>$G_1(\cdot) &gt; 0$</td>
<td>$G_2(\cdot) \geq 0$</td>
<td>$-$</td>
<td>$G_4(\cdot) \leq 0$</td>
<td>$G_5(\cdot) &lt; 0$</td>
</tr>
<tr>
<td>6</td>
<td>Subgame 6 v. 9</td>
<td>$G_1(\cdot) &gt; 0$</td>
<td>$G_2(\cdot) \geq 0$</td>
<td>$-$</td>
<td>$G_4(\cdot) &gt; 0$</td>
<td>$-$</td>
</tr>
<tr>
<td>7</td>
<td>Subgame 6 v. 9</td>
<td>$G_1(\cdot) &gt; 0$</td>
<td>$G_2(\cdot) &lt; 0$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Proof of Propositions 3.6.1 and 3.6.2. We have shown that only subgames 2, 5, 6, 9, and 10 can be the equilibrium of the game. For completeness, the profit functions of these subgames are listed in what follows: $\pi^*_M(2) = 1/16 (b\phi - 32(\lambda-2)(\lambda-1)(\phi-2)/(\lambda-3)^2, \pi^*_A(2) = -((\phi-2)/(\lambda-3)) - F, \pi^*_B(2) = \frac{1}{8}b\phi, [\pi^*_M(5) = \frac{2(\lambda-2)(\lambda-1)}{(\lambda-3)^2} + \frac{1}{16}, \pi^*_A(5) = \frac{1}{8} (\lambda-3) + 1) - F, \pi^*_B(5) = 0, [\pi^*_M(6) = \frac{1}{8}, \pi^*_A(6) = \frac{1}{4}, \pi^*_B(6) = 0, [\pi^*_M(9) = \frac{1}{16}(1+b), \pi^*_A(9) = \frac{1}{8}, \pi^*_B(9) = \frac{b}{8}]$, and $[\pi^*_M(10) = \frac{b}{8}, \pi^*_A(10) = 0, \pi^*_B(10) = \frac{b}{4}]$. Propositions 3.6.1 and 3.6.2 are shown by taking $G_z(\cdot) = 0, z \in \{1,2,\ldots,5\}$, and their equivalent conditions from Lemma B.1.1 as the buyer’s indifference points. In Proposition 3.6.1 we have $\{F > F_1, b \leq b_1, F \leq F_2, b = b_2\}$, where an increase in $\lambda$ causes an equilibrium shift from subgame 2 ($A^E A^E$) to subgame 9 ($BA$), thus improving buyer’s profit from $\pi^*_M(2)$ to $\pi^*_M(9)$ (Proposition 3.6.1a). If $\{F > F_1, b = b_1, F \leq F_2, b > b_2\}$, as specified in Proposition 3.6.2, an increase in $b$ will flip the sign of second threshold function from $b \leq b_1$ to $b > b_1$, which
consequently shifts the equilibrium strategy from subgame 9 (BA) to subgame 6 (AA), and reduces supplier B’s profit from $\pi^*_B(9)$ to zero. ■

**Proof of Proposition 3.7.1:** Lastly, let us show why dual sourcing is never the equilibrium, where the buyer can outsource a portion $\gamma \in [0, 1]$ of production to supplier A, and the rest $(1 - \gamma)$ to supplier B. When dual sourcing is available to the buyer, on top of the two existing options (i.e., $\gamma = 1$ and $\gamma = 0$), the buyer can have $0 < \gamma < 1$. Therefore, the extensive form of the game will further expand to accommodate the buyer’s new alternative in both periods. Since (1) the same solution approach as above is used to solve the dual sourcing game and (2) majority of the game (e.g., when the whole production is outsourced to only one of the two suppliers) are similar to the single sourcing game, we focus on the effect of dual sourcing on the extensive form and the optimal solution.

Under dual sourcing, after $\gamma$ is set, both suppliers set their wholesale prices at the same time, and then the buyer sets the market price. We assume that supplier A can encroach into the market if $\gamma > 0$. The profit functions in this case are $\pi_M = D_M (p_M - \gamma w_A - (1 - \gamma) w_B)$, $\pi_A = \gamma D_M w_A$, and $\pi_B = (1 - \gamma) D_M w_B$. After checking the second-order condition, we use first-order condition to find the optimal market price, where we have $p^*_M = \frac{b + w_B + \gamma - b\gamma + \gamma w_A - \gamma w_B}{2}$. Then, we simultaneously solve for the optimal wholesale prices and we obtain $w^*_A = \frac{b + \gamma - b\gamma}{3\gamma}$ and $w^*_B = \frac{-b - \gamma + b\gamma}{3(\gamma - 1)}$. By plugging the optimal prices into the buyer’s profit function we get $\pi^*_M = \frac{1}{36} (\gamma + b(\gamma) + b - 36)$. Since the dual-sourcing $\pi^*_M$ is linear in $\gamma$, the buyer’s profit will be always maximized at the boundary points (i.e., $\gamma = 1$ or $\gamma = 0$). Therefore, partially outsourcing the production to the suppliers is never optimal. ■

**B.3 Supplier A Retains Access to the Market after Being Dropped**

In the main model, we assume that supplier A loses access to the market and cannot continue selling its own independent product if it is dropped by the buyer. In this ex-
tension, we demonstrate the robustness of our results when the encroaching supplier retains access to the market after it is dropped by the buyer. At the 10th and 11th steps of the game sequence, instead of “The buyer sets the market price \( p_{M2} \)” and “Then, if \( E_2 = 1 \), supplier A decides on \( p_{A2} \),” we will have “The buyer sets the market price \( p_{M2} \). Then, if \( E_1 = 1 \) or \( E_2 = 1 \), supplier A decides on \( p_{A2} \).” To show the analytical results of this game we define a new set of threshold functions \( H_r, r \in \{1, 2, \ldots, 10\} \), where \( H_1(\cdot) \) and \( H_2(\cdot) \) coincide with their \( G(\cdot) \) counterparts, i.e., \( H_1(\cdot) = G_1(\cdot) \) and \( H_2(\cdot) = G_2(\cdot) \). Using the \( H(\cdot) \) functions—introduced below—Proposition B.3.1 provides the analytical classification of the potential subgame perfect equilibria and the corresponding conditions. The proof of this proposition closely follows the logic of the proof of Proposition 3.5.1.

**Proposition B.3.1** The following enumerates the necessary and sufficient conditions for the equilibria:

(a) \( A^E A^E \) is the equilibrium if and only if \( [H_1(\cdot) \leq 0 \text{ and } H_2(\cdot) \geq 0 \text{ and } H_3(\cdot) \geq 0 \text{ and } H_4(\cdot) \geq 0] \) or \( [H_1(\cdot) > 0 \text{ and } H_2(\cdot) \geq 0 \text{ and } H_7(\cdot) \geq 0 \text{ and } H_8(\cdot) \geq 0] \).

(b) \( A^E B \) is the equilibrium if and only if \( [H_1(\cdot) \leq 0 \text{ and } H_2(\cdot) < 0 \text{ and } H_5(\cdot) \geq 0 \text{ and } H_6(\cdot) \geq 0] \) or \( [H_1(\cdot) > 0 \text{ and } H_2(\cdot) < 0 \text{ and } H_9(\cdot) \geq 0 \text{ and } H_{10}(\cdot) \geq 0] \).

(c) \( A A^E \) is the equilibrium if and only if \( [H_1(\cdot) \leq 0 \text{ and } H_2(\cdot) \geq 0 \text{ and } H_3(\cdot) < 0] \).

(d) \( AA \) is the equilibrium if and only if \( [H_1(\cdot) > 0 \text{ and } H_2(\cdot) \geq 0 \text{ and } H_7(\cdot) < 0] \) or \( [H_1(\cdot) > 0 \text{ and } H_2(\cdot) < 0 \text{ and } H_9(\cdot) < 0] \).

(e) \( AB \) is the equilibrium if and only if \( [H_1(\cdot) \leq 0 \text{ and } H_2(\cdot) < 0 \text{ and } H_5(\cdot) < 0] \).

(f) \( B A^E \) is the equilibrium if and only if \( [H_1(\cdot) \leq 0 \text{ and } H_2(\cdot) \geq 0 \text{ and } H_3(\cdot) \geq 0 \text{ and } H_4(\cdot) < 0] \).

(g) \( BA \) is the equilibrium if and only if \( [H_1(\cdot) > 0 \text{ and } H_2(\cdot) \geq 0 \text{ and } H_7(\cdot) \geq 0 \text{ and } H_8(\cdot) \geq 0] \) or \( [H_1(\cdot) > 0 \text{ and } H_2(\cdot) < 0 \text{ and } H_9(\cdot) \geq 0 \text{ and } H_{10}(\cdot) < 0] \).

(h) \( BB \) is the equilibrium if and only if \( [H_1(\cdot) \leq 0 \text{ and } H_2(\cdot) < 0 \text{ and } H_5(\cdot) \geq 0 \text{ and } H_6(\cdot) < 0] \).
Proposition B.3.1 is essentially similar to Proposition 3.5.1 in that (1.) the necessary and sufficient conditions for the subgame perfect equilibria are stated in terms of a list of (threshold) functions, (2.) all subgame-perfect equilibria stay an equilibrium, and (3.) subgame $A^E A$ is never an equilibrium. The difference between the two propositions is that subgames $A^E B$, $AB$, and $BA^E$ can be the equilibrium under Propositions B.3.1.

In what follows, we show that the main results hold under this extension, and we derive further managerial insights in Propositions B.3.2 and B.3.3. We first consider the effects of the quality of supplier A’s independent product ($\lambda$) and then the quality of supplier B’s product on the equilibrium of the game. The proof of the following propositions closely follows the proof of Propositions 3.5.2 and 3.5.3, respectively.

**Proposition B.3.2**

(a) If $[H_1(\cdot) = 0, H_2(\cdot) \geq 0, H_3(\cdot) \geq 0, H_4(\cdot) \geq 0, H_7(\cdot) \geq 0, \text{and } H_8(\cdot) < 0]$, then an increase in the quality of supplier A’s independent product ($\lambda$) would make $M$ switch from supplier B to supplier A.

(b) If $[H_1(\cdot) > 0, H_2(\cdot) \geq 0, H_7(\cdot) \geq 0, \text{and } H_8(\cdot) = 0]$, then an increase in the quality of supplier A’s independent product ($\lambda$) would make supplier A switch from market-entry ($E_2 = 1$) to no-entry ($E_2 = 0$).

**Proposition B.3.3**

(a) If $[H_1(\cdot) > 0, H_2(\cdot) = 0, H_7(\cdot) \geq 0, H_8(\cdot) < 0, \text{and } H_9(\cdot) < 0]$, then an increase in the quality of supplier B’s product ($b$) would make $M$ switch from supplier B to supplier A.

(b) If $[H_1(\cdot) > 0, H_2(\cdot) \geq 0, H_7(\cdot) = 0, \text{and } H_8(\cdot) \geq 0]$, then an increase in the quality of supplier B’s product ($b$) would make supplier A switch from market entry in both periods to no entry.
B.3.1 Threshold Functions $H_i(\cdot)$

\[ H_1(F, \lambda) = \frac{64F^2\lambda^4 - 640F^2\lambda^3 + 2624F^2\lambda^2 - 5120F^2\lambda + 4096F^2}{\lambda^4 - 10\lambda^3 + 25\lambda^2} \quad (B.3) \]

\[ H_2(b, \lambda) = \frac{32\lambda^2 - 96\lambda + 64}{\lambda^4 - 10\lambda^3 + 41\lambda^2 - 80\lambda + 64} - b \quad (B.4) \]

\[ H_3(b, \lambda, \phi) = \frac{\phi(b^2(9\lambda((\lambda - 5)\lambda + 8) - 64) + b\lambda(64 - 9\lambda((\lambda - 5)\lambda + 8)) - 16\lambda^2)}{(\lambda - 2b)^2} - (B.5) \]

\[ H_4(b, \lambda, \phi) = \frac{2b^2(\lambda^2 - 5\lambda + 8)^2(\phi - 1) + 32\lambda(\lambda^2 - 3\lambda + 2)(\phi - 1)}{2b - \lambda} + \frac{b(\lambda^5(1 - 2\phi) + 10\lambda^4(2\phi - 1) + \lambda^2(41 - 82\phi) + 16\lambda^2(6\phi - 1))}{2b - \lambda} + b(64\lambda(\phi - 2) - 128(\phi - 1)) \]

\[ H_5(b, F, \lambda, \phi) = \frac{1}{16} \lambda \left( \frac{9b\phi(b - \lambda)}{(\lambda - 2b)^2} - \frac{2(\lambda - 5)}{(\lambda - 5)\lambda + 8} \right) - F \quad (B.7) \]

\[ H_6(b, \lambda, \phi) = \frac{1}{8} \left( \frac{b(2b - \lambda(\phi + 1))}{4b - 2\lambda} - b + \frac{16(\lambda - 2)(\lambda - 1)}{((\lambda - 5)\lambda + 8)^2} \right) \]

\[ H_7(b, F, \lambda, \phi) = \frac{\phi(b^2(9\lambda((\lambda - 5)\lambda + 8) - 64) + b\lambda(64 - 9\lambda((\lambda - 5)\lambda + 8)) - 16\lambda^2)}{(\lambda - 2b)^2} + 32 \right) - F - \frac{1}{4} \quad (B.9) \]

\[ H_8(b, \lambda, \phi) = \frac{1}{16} \left( -1 - b + \frac{2b(\lambda^2 - 5\lambda + 8)^2}{(\lambda^2 - 5\lambda + 8)^2} \phi + 16\lambda(\lambda^2 - 3\lambda + 2)(\phi - 2) \right) \]

\[ \frac{2b(\lambda^5(-\phi) + 10\lambda^4\phi - 41\lambda^3\phi + 16\lambda^2(3\phi + 4) + 32\lambda(\phi - 6) - 64(\phi - 2))}{(\lambda^2 - 5\lambda + 8)^2(2b - \lambda)} \]

\[ H_9(b, F, \lambda, \phi) = \frac{1}{16} \left( \frac{9b\lambda\phi(b - \lambda)}{(\lambda - 2b)^2} + \frac{16}{(\lambda - 5)\lambda + 8} - 4 \right) - F \quad (B.12) \]

\[ H_{10}(b, \lambda, \phi) = \frac{1}{16} \left( \frac{b(2b - \lambda(\phi + 1))}{2b - \lambda} - b + \frac{32(\lambda - 2)(\lambda - 1)}{((\lambda - 5)\lambda + 8)^2} - 1 \right) \quad (B.13) \]
B.4 The Delayed Introduction of Supplier A’s Product to the Market

Here we demonstrate the robustness of the results of Section 3.3 when supplier A’s independent product does not appear in the market immediately after the supplier decides to encroach. The time horizon of the one-period benchmark game in Section 3.3 is extended, such that if supplier A decides to encroach in $t = 1$, its independent product will be available to the customers in $t = 2$. The revised game sequence is as follows:

1. As the Stackelberg leader, the buyer chooses a supplier $j \in \{A, B\}$ to outsource the production to.

2. If supplier A is selected, the supplier decides on whether to encroach into the market ($E$). *This independent product will be available to the customers only in the second period.*

3. The selected supplier sets the first-period wholesale price $w_{j1}$.

4. The buyer sets the first-period market price $p_{M1}$.

5. Customers make the purchase decision.

6. The selected supplier sets the second-period wholesale price $w_{j2}$.

7. The buyer sets the second-period market price $p_{M2}$.

8. If supplier A has decided to encroach in step 2 of the game ($E = 1$), then it decides on the market price $p_A$.

9. Customers make the second-period purchase decision.

When the buyer chooses supplier A and the supplier decides to encroach, the buyer’s and the supplier’s first-period profits are $\pi_{M1} = D_{M1}(p_{M1} - w_{A1})$ and $\pi_{A1} = D_{M1}w_{A1} - F$, respectively. Note that the supplier incurs the market-entry investment cost ($F$) but does not benefit from encroachment in the first period. In the second period, however,
we have \( \pi_{M2} = D_{M2}(p_{M2} - w_{A2}) \) and \( \pi_{A2} = D_{M2}w_{A2} + D_{A2}p_{A2} \). This is when the supplier’s independent product is available in the market, and thus, the supplier sees its profit increased by \( D_{A2}p_{A2} \). The profit functions for the other possibilities (e.g., when supplier A does not encroach or when the buyer outsources to supplier B) follow what we have in the paper, and we omit the details.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Optimal Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( \pi_M = \frac{1}{8} )</td>
</tr>
<tr>
<td></td>
<td>( \pi_A = \frac{1}{4} )</td>
</tr>
<tr>
<td></td>
<td>( \pi_B = 0 )</td>
</tr>
<tr>
<td>( A^E )</td>
<td>( \pi_M = \frac{2(2 - \lambda(1 - \lambda))}{(\lambda^2 - 5\lambda + 8)} + \frac{1}{16} )</td>
</tr>
<tr>
<td></td>
<td>( \pi_A = \frac{1}{8} \left( \frac{8\lambda^2 - 5\lambda + 8}{\lambda^2 - 5\lambda + 8} + 1 \right) - F )</td>
</tr>
<tr>
<td>( B )</td>
<td>( \pi_M = \frac{b}{8} )</td>
</tr>
<tr>
<td></td>
<td>( \pi_A = 0 )</td>
</tr>
<tr>
<td></td>
<td>( \pi_B = \frac{b}{4} )</td>
</tr>
</tbody>
</table>

Table B.4: The optimal profits under the two-period benchmark.

Table B.4 summarizes the optimal profits of the buyer and the two suppliers under this extension. In Figure B.2 we illustrate the impacts of a two-period horizon in the benchmark scenario. The results of the left and the right panels of Figure B.2 are consistent in three significant ways: (1.) supplier B is chosen only when its quality (\( b \)) is sufficiently large, (2.) supplier A would only encroach if a large \( \lambda \) (i.e., the quality of the independent product) makes encroachment profitable, and (3.) a combination of supplier B’s low quality and low \( \lambda \) makes outsourcing to supplier A the optimal strategy. The expansion of region B in the right panel highlights the necessity of incorporating the risks of encroachment (i.e., Section 3.4). When the ramifications of encroachment are not taken into account, the buyer has to act conservatively to avoid losing market share to the encroaching supplier; hence, supplier B will be chosen even when encroachment is not highly profitable (i.e., low \( \lambda \)).
Chapter B. Supplier Encroachment Appendix

(a) One-period benchmark (Section 3.3).
(b) Two-period benchmark.

Figure B.2: The impact of the two-period benchmark on the equilibrium strategy.

B.5 The Presence of Long-Term Outsourcing Contracts

In the main model, the buyer chooses a supplier at the beginning of each period and outsources the production for a duration of one period. Here we extend the main model and demonstrate the robustness of the results when the buyer signs a long-term (i.e., two-period) contract with the chosen supplier. We structure the long-term contract such that the buyer trades with the chosen supplier under the same wholesale price, unless an encroachment occurs. Therefore, if production is outsourced to supplier $B$ in $t = 1$, then supplier $B$ will be the second-period supplier as well. But if supplier $A$ is chosen in $t = 1$ and encroachment occurs ($E_1 = 1$), the buyer will outsource the second-period production to supplier $B$ upon dropping the encroaching supplier. The revised game sequence under this extension is as follows:

1. The buyer chooses a supplier $j \in \{A, B\}$ to outsource the production to.

2. If supplier $A$ is selected, it decides on whether to encroach into the market with its own independent product ($E_1$).

3. The chosen supplier sets the wholesale price, $w_{j1}$.
4. The buyer sets the market price, \( p_{M1} \). Then, if supplier \( A \)'s decision has been to encroach in the first period \( (E_1 = 1) \), supplier \( A \) decides on \( p_{A1} \).

5. Customers make the purchase decision.

6. If \( E_1 = 1 \), the buyer decides on keeping or dropping the encroaching supplier. There is a probability \( \phi \) that the encroaching supplier is dropped by the buyer if it has encroached in \( t = 1 \).

7. If supplier \( A \) has encroached in \( t = 1 \) \( (E_1 = 1) \) and is dropped by the buyer in the previous step, then the buyer outsources the second-period production to supplier \( B \). *Otherwise, the first-period supplier will keep the buyer’s business in the second period as well.*

8. If supplier \( A \) is chosen in \( t = 2 \), it decides on whether to encroach \( (E_2) \).

9. If supplier \( A \) has encroached in \( t = 1 \) and is dropped by the buyer in step 6, then supplier \( B \) sets the second-period wholesale price \( w_{B2} \). *Otherwise, the first-period wholesale price \( w_{j1} \) applies to the second period as well.*

10. The buyer sets the market price, \( p_{M2} \). Then, if supplier \( A \)'s decision has been to encroach in the second period \( (E_2 = 1) \), it decides on \( p_{A2} \).

11. Customers make the purchase decision.

Consider the case where the buyer outsources the production to supplier \( A \) in both periods, and supplier \( A \) encroaches in the second period only. In that case, the buyer’s and the supplier’s first-period profits are, respectively, \( \pi_{M1} = D_{M1}(p_{M1} - w_{A1}) \) and \( \pi_{A1} = D_{M1}w_{A1} \). Since supplier \( A \) has not yet encroached when the buyer makes the outsourcing decision in the second period, the wholesale price set in the first-period contract \( (w_{A1}) \) would be valid in the second period as well, and we have \( \pi_{M2} = D_{M2}(p_{M2} - w_{A1}) \) and \( \pi_{A2} = D_{M2}w_{A1} + D_{A2}p_{A2} - F \). The profit functions for the other possibilities follow what we have in the paper, and we omit the details.
(a) The main model.  
(b) Long-term contract.

Figure B.3: The impact of long-term outsourcing contract on the equilibrium strategy.

We solve this game analytically and tabulate the optimal profits in Table B.5. For demonstration purposes, we illustrate the results in Figure B.3 using the same parameter setting as in Section 3.4. The left panel is the equilibrium strategy corresponding to the main model (short-term contract), and the right panel is the equilibrium strategy corresponding to this extension (long-term contract). Similar to the results of Section 3.5 (1.) supplier A would only encroach if $\lambda$ is sufficiently large, (2.) the buyer’s choice of supplier switches from A to B and from B to A as $\lambda$ increases, and (3.) an increase in $\lambda$ can harm supplier B’s profits. However, knowing that the only way to lose the second-period outsourcing contract has a probability of $\phi$ if $E_1 = 1$, supplier A becomes more inclined to encroach in both periods. To counter this, the buyer adopts a defensive outsourcing strategy and outsources to supplier B even at lower values of $b$, which harms the buyer’s profit.

We use the short-term contract in the main math model for the following two reasons. (1) The buyer periodically evaluates and assesses its suppliers (Carr & Pearson, 1999; Brown, 2010). We assume that this investigation happens at the beginning of period two. The buyer retains the right to drop a supplier that lacks commitment and
### Table B.5: The optimal total profits under the long-term contract.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Optimal Profits</th>
</tr>
</thead>
</table>
| AA       | $\pi_M = \frac{1}{3}$  
          | $\pi_A = \frac{1}{4}$  
          | $\pi_B = 0$  |
| AE       | $\pi_M = \frac{1}{16} \left( b\phi + \frac{32(2-\lambda)(1-\lambda)(2-\phi)}{(\lambda^2-5\lambda+8)^2} \right)$  
          | $\pi_A = \frac{(2-\phi)}{\lambda^2-5\lambda+8} - F$  
          | $\pi_B = \frac{b\phi}{8}$  |
| BB       | $\pi_M = \frac{b}{8}$  
          | $\pi_A = 0$  
          | $\pi_B = \frac{b}{4}$  |

loyalty in the first period, and to seek a new supplier in the second period. We also assume that if the same supplier is to be chosen for both periods, the wholesale price will be decided again. (2) Since the buyer cannot freely switch from one supplier to another under the long-term contract, certain dynamics of the supplier encroachment phenomenon cannot be captured. For example, subgame $BA$ cannot occur with a long-term contract, therefore, the buyer has to outsource to supplier $B$ in both periods to prevent supplier $A$ from encroachment. This inflexibility is not observed frequently in the real world.