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Fraction Magnitude Understanding Across Learning Formats: an fMRI Study

Chloe A. Henry, *The University of Western Ontario*

Supervisor: Ansari, Daniel, *The University of Western Ontario*
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Abstract

Knowledge of fraction magnitudes are an important, but notoriously difficult mathematical concept to master. Behavioural work has begun to explore and compare the instructional tools used for fraction learning. However, how fraction instructional tools are processed in the brain remains an underexplored question. Therefore, in the present thesis, we used functional brain MRI methodology to examine the neural activity of adult participants while completing a fraction verification task using the number line and area model, two common methods of fraction learning. We found that both models commonly recruited fronto-parietal activity, the neural regions typically implicated in number processing. However, we also found specific clusters of activation in frontal and parietal regions that displayed a greater response to area models. Given that participants indicated a greater familiarity with the area model, we suggest this could arise due to differences in strategy employed when using the number line and area model formats.

Keywords

Functional Magnetic Resonance Imaging, Fraction Magnitude, Learning Models, Number Line, Area Model, Fronto-parietal Network, Intraparietal Sulcus, Numerical Cognition

Summary for Lay Audience

As the classic joke goes, “6 out of 5 people struggle to understand fractions”. Unfortunately, this joke holds some truth, as many studies have documented that both children and educated adults struggle considerably to understand fraction magnitude. As a result, this raises the question of whether methods used for teaching fractions are not effective instructional approaches. In the past, mathematics curriculums have predominantly relied on use of the area/pie model for teaching fraction magnitudes. However, recently, there has been a growing push to emphasize number line use in early fraction learning instead. Indeed, behavioural work has supported the notion that the number line is an effective model for fraction magnitude learning. However, it currently remains unknown how the brain processes magnitude in number line and area model formats. The inclusion of neuroimaging can complement our knowledge from the behavioural literature and can provide valuable insight towards best practices for fraction learning. Therefore, in this thesis, we used functional brain MRI methodology to explore commonalities and differences in how the brain processes fraction magnitude in number line and area model formats. Through this, we found that while the two models were processed in the brain highly similarly (both recruiting regions of the brain typically involved in magnitude processing), there were also regions within this network that were activated to a greater extent by the area model. We suggest that it is possible that these differences arose because the participants in our sample had a greater familiarity with the area model. Therefore, it is possible that there were different approaches taken when completing trials in number line and area model format. However, we encourage future work to explore this question more directly to obtain better insight towards the mechanisms that may be causing these differences between the models at the level of the brain. Given that our study is the first to directly explore how the brain processes fraction instructional models, we believe this can serve as a strong framework for future neuroimaging studies exploring fraction learning. Ultimately, we hope this will improve our knowledge regarding best practices for fraction learning.

Co-Authorship Statement

The present thesis represents my own work. All written work, data collection, and analyses for this thesis were conducted by myself, Chloe Henry. However, there were other members who made valuable contributions towards the completion of this project. My supervisor, Dr. Daniel Ansari played a role in all aspects of this study, including in the study design, the interpretations of results and the editing of written work. My colleague, Dr. Chenxi He contributed to the present work by aiding in the generation of the research question, data analysis and interpretations of results. Finally, Akshaj Jonnalagadda and Gurleen Badwal contributed to this study by aiding in participant recruitment and data collection.

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Chapter 1

1 Introduction

Every day, we rely on our numerical skills to help us understand and navigate through the world around us. Numerical knowledge is relied upon to complete many of the tasks that are encountered in daily life, whether that be calculating discounts at a sale, cooking a new recipe, determining your portion of a split dinner bill, or calculating medical dosages. Further, from an educational standpoint, previous work has identified early mathematical skills as one of the strongest predictors of later academic achievement (Duncan et al., 2007; Romano et al., 2010). Additionally, the significance of numerical competency is also apparent at the societal level. Notably, it has been shown that sizable socio-economic costs incur when poor numeracy rises, including increased rates of mental and physical illness, incarceration, and unemployment (Bynner & Parsons, 1996; Parsons & Bynner, 2005). Taken together, these findings illustrate the relevance of number knowledge for both individual and societal functioning.

1.1 Number Processing in the Brain

Given the importance of numerical skills, the question of how the brain processes and represents number has been a growing topic of interest. With the rise of neuroimaging techniques, such as functional magnetic resonance imaging (fMRI), the ability to characterize the neural regions implicated in number processing has improved markedly. These additions have complemented our knowledge from the behavioural literature and have provided meaningful insights on the mechanisms and processes that underlie number processing (Matejko & Ansari, 2018). From these studies, we have learned that number processing is associated with the recruitment of fronto-parietal regions of the brain (Arsalidou & Taylor, 2011, for meta-analysis). Specifically, neuroimaging work has consistently revealed activation associated with number processing in prefrontal regions, as well as in the bilateral parietal lobules, particularly around the intraparietal sulcus (IPS) (Appolonio et al., 1994; Dehaene et al., 1996; Dehaene et al., 2003; Roland & Friberg, 1985; from Nieder & Dehaene, 2009). The IPS is a parietal brain region that has consistently been implicated in numerical cognition. Indeed, the IPS has been documented to be activated in response to a multitude of different numerical tasks, such as in the presentation of Arabic digits, in number comparison tasks, in calculation or mental arithmetic, in response to written and spoken number words, as well as in the presentation of non-symbolic number (e.g., dot arrays) (e.g. Ansari et al., 2005; Chochon et al., 1999; Eger et al., 2003; Holloway et al., 2010; Pesenti et al., 2000; Piazza et al., 2004; Pinel et al., 2001). Taken together, frontal and parietal regions, predominately around the IPS, have been identified as key regions implicated in the processing of a wide array of numerical tasks (see Arsalidou & Taylor, 2011 for meta-analyses).

However, while our understanding of how the human brain processes numbers has undoubtedly improved, it is important to note that, currently, much of what we know about number processing comes from the whole/natural numbers literature. Here, we refer to whole numbers as a positive number that does not contain a decimal or fractional component (e.g., 7). While these insights have been meaningful, whole number reasoning only encompasses a subset of our daily processing and use of numerical information. Further, whole numbers are used relatively little in more complex mathematics (Sidney, Thompson, & Opfer, 2019). Therefore, interest has been directed towards better exploring how other forms of number are processed and understood. Among these, the study of fraction processing has emerged as a particular topic of interest within the numerical cognition field. Fractions, in particular, have provided an interesting case to explore because fraction knowledge is an important learning milestone (Bailey et al., 2012; Booth & Newton, 2012; Siegler et al., 2012; Siegler et al., 2013; Torbeyns et al., 2015). However, despite considerable instruction in early education, becoming proficient with fractions has been shown to be a notoriously difficult task for both children and educated adults (Bailey et al., 2015; Behr et al., 1985; National Mathematics Advisory Panel, 2008; Siegler et al., 2013; Stigler et al., 2010).

1.2 The Importance of Fraction Understanding

1.2.1 What is a Fraction?

A fraction is a numeral that is a notation of a rational number, which is a type of number that represents the ratio between two integers. The fraction notation takes on three elements, the numerator, denominator and the line that separates the two (e.g., $\frac{1}{7}$). In North America, fractions are introduced early on in primary school years with the expectation that by the end of grade school students will be fluent with fraction operations and application (CCSSI, 2010; Ontario Ministry of

Education, 2005). Specifically, curriculum standards suggest that fractions are first introduced to children in grade 1. By grade 4, it is expected that students have a solid grasp of fraction magnitude and can compare and order fraction numbers. Then, by grades 7 and 8, children should be able to carry out fraction arithmetic operations and successfully apply fractions in algebraic equations (CCSSI, 2010; Ontario Ministry of Education, 2005). Fraction instruction is included and emphasized in the early mathematics curriculum as developing a proficiency with fractions is a crucial component of our mathematical learning that is subsequently relied upon in many areas of life (Siegler et al., 2013).

1.2.2 Fraction Importance in the Classroom

Developing a sound understanding of fraction concepts is critically important for later math learning (Bailey et al., 2012; Booth & Newton, 2012; Siegler et al., 2012; Torbeyns et al., 2015). For instance, previous work has found that fraction magnitude knowledge is more strongly related to measures of algebra readiness than is knowledge of whole number magnitudes (Booth & Newton, 2012). Similarly, previous work has also identified relations between fraction knowledge and math achievement more generally. For example, Siegler and colleagues (2012) found that secondary school students' knowledge of fractions were strongly correlated with measures of overall math achievement. Further, this study also found that early fraction knowledge can serve as a key indicator of student's later achievement in mathematics. Indeed, this study demonstrated that student's elementary school fraction knowledge uniquely predicted later mathematics gains in high school, even after controlling for relevant educational factors such as general IQ, family education levels as well as other types of mathematical knowledge (Siegler et al., 2012). Taken together, these findings illustrate that fractions serve as a foundational building block that are necessary for more advanced mathematical learning.

1.2.3 Fraction Importance Beyond the Classroom

Though, while the educational implications of fraction knowledge are clear, the importance of understanding fractions extend beyond the classroom. Fractions are among the most ubiquitous forms of number encountered in daily life and are necessary for carrying out various day-to-day activities like baking (e.g., ¾ a cup of flour), shopping (e.g., ½ off sale), or expressing time (quarter after 7). Additionally, knowledge of fractions is also relied upon to decipher and understand important statistical and probability data, such as infection rates during COVID-19, changes in stock market price, or compound interest (Lortie-Forgues et al., 2015). Furthermore, fraction knowledge is relied upon in a wide range of occupations, in both STEM and non-STEM fields. For instance, the U.S labour survey reported that 68% of high-skill blue collar respondents indicated that they required fraction knowledge in their work (Handel, 2016). Therefore, being able to work with fraction concepts is an important cognitive skill, not only for academic achievement, but also for successful functioning in our everyday world.

1.3 Documented Difficulties in Fraction Understanding

Despite the fact that fractions are taught from an early age onwards, crucial gaps in fraction understanding are frequently documented (Bailey et al., 2015; Behr et al., 1985; Perie et al., 2005; Siegler et al., 2013; Stigler et al., 2010). For instance, in the 2004 National Assessment of Educational Progress (NAEP), a measure of educational achievement in the U.S, it was found that only 50% of 8th grade students were successful in accurately ordering the magnitude of three simple fractions (Lester, 2007). Unfortunately, similar trends in fraction difficulty have continued to persist in the NAEP data throughout the years. For instance, on the 2009 NAEP, it was found that only 25% of 4th grade students could identify the fraction closest to ½ out of a list of fractions. Further, in the 2017 NAEP data it was documented that only 32% of 4th grade students were able to accurately assess whether a fraction item was greater than, less than or equal to ½. This illustrates that while extensive efforts have been put forward towards advancing mathematics education, the challenges of learning fractions have persisted over time. As a result, these fundamental misconceptions in fraction knowledge have been identified as some of the most significant barriers to advanced mathematical learning by secondary school algebra teachers (Hoffer et al., 2007).

Further, it is important to note that these gaps in fraction knowledge are not exclusive to early learners. Rather, difficulties with fractions remain apparent into adulthood as well. For example, Stigler and colleagues (2010) found that only 33% of college level students were able to identify which of four simple fractions were the largest in magnitude (Stigler et al., 2010). Further, perhaps most concerningly, these gaps in fraction understanding are present among those responsible for teaching fractions as well. Indeed, previous work has shown that both pre-service and in-service teachers also display difficulties in fraction understanding (Copur-Gencturk, 2022; Ma, 2020; Newton, 2008; Olanoff et al., 2014). Specifically, it has been noted that even when teachers are able to apply procedural knowledge and compute with fractions, their conceptual understanding is largely lacking, as they struggle to provide an explanation for these procedures and when they should be applied (Newton, 2008; Olanoff et al., 2014; Son & Crespo, 2009). This finding is problematic given that it has been demonstrated that students learning outcomes are improved when their instructor holds a sound understanding of the material being taught (Hill et al., 2005). These findings demonstrate that while curriculum standards suggest that fraction concepts should be mastered in early education (CCSSI, 2010; Ontario Ministry of Education, 2005), this is often not the case.

Finally, it is worthwhile to note that fraction difficulty is not restricted to the classroom setting. Indeed, misconceptions in fraction knowledge manifest in the everyday decisions that we make. A common example of this comes from the popular media headline "How failing at fractions saved the Quarter Pounder" (CBC Radio, 2021). In 1980, A&W introduced the 'third-of-a-pound burger', a competitor to the famous quarter pounder burger from McDonald's. It was projected that this new addition would be a hit, given that A&W was offering a larger burger for the exact same cost as the quarter pounder. However, unexpectedly, sales on this burger were concerningly low. When A&W ran a focus group to explore why the burger had been so unsuccessful it became apparent that lack of fraction knowledge among consumers was the culprit. In fact, over 50% of consumers held the belief that one third was smaller than one fourth, and thus believed they were being ripped off by A&W by having to pay the same cost for a smaller burger (CBC Radio, 2021). While humorous, this real-life example illustrates how fundamental misconceptions in fraction knowledge can impact the day-to-day decisions that we make. Taken in sum, the above findings exemplify that, despite instruction in early education, difficulties with fractions are pervasive across development and exist in many different areas of life.

1.3.1 Understanding Fraction Magnitude: A Particular Challenge in Fraction Learning

Given that difficulty with fractions is far from rare, it has led researchers to consider explanations for why such a difficulty occurs. One leading explanation is that there is a fundamental difficulty in understanding that fractions represent single numerical magnitudes (Bonato et al., 2007; Kallai & Tzelgov, 2012; Stafylidou & Vosniadou, 2004). Indeed, a common theme that emerges in many of the documented examples of fraction difficulty, including those provided above, are examples that revolve around misconceptions in magnitude. Magnitude understanding refers to the specific ability to "comprehend, estimate, and compare the sizes of numbers" (Fazio et al., 2014, page 54). However, to do so, this requires an understanding of what a fraction represents. A fraction, in essence, represents the exact same thing as a whole number, a single numeric value. However, this realization is not always intuitive because fractions take on a different representation (numerator/denominator) than the magnitudes we are first introduced to (whole numbers). As a result, this can lead to componential processing, where fractions are viewed as parts of a whole, rather than as a holistic, single magnitude (Zhang et al., 2014).

Relatedly, difficulty in understanding fraction magnitude is often attributed to an inflexible fixation on whole number knowledge. In early education years, children work nearly exclusively with whole number magnitudes. Resultantly, a particular sensitivity towards whole number concepts is typically developed, which can lead to misconceptions and misapplications when children shift towards working with fractions – a phenomenon known as the 'whole number bias' (Ni & Zhou, 2005). Specifically, the whole number bias refers to the erroneous procedure whereby individuals attend to the whole number components of a fraction's numerator and denominator rather than processing the fraction as a single number value (Ni & Zhou, 2005). A clear example of this bias was documented by Mack (1990). Specifically, in this study it was reported that 6th grade students commonly claimed that $\frac{1}{8}$ was a larger fraction value than $\frac{1}{6}$ with the reasoning being that 8 is a larger number than 6 (Mack, 1990). Indeed, this explanation can also explain why sales on A&W's burger failed, as consumers believed that $\frac{1}{3}$ was less than $\frac{1}{4}$ because 3 is less than 4.

Evidently, this difficulty is especially problematic as magnitude understanding is a foundational component of our number knowledge (Siegler, 2016). Indeed, without an understanding of what a number represents it becomes difficult to reason with that number in any capacity. This logic is supported in the 'integrated theory of numerical development', where theorist Robert Siegler highlights that the feature that unifies all real numbers is the representation of magnitude (Siegler et al., 2011). Thus, learning to view fractions as single numerical values that can be ordered and compared, alike whole numbers, is integral for a genuine understanding of fractions (Siegler et al., 2011, 2013). More specifically, this difficulty is problematic because fraction magnitude knowledge has been shown to be crucial for fraction learning more generally. For instance, knowledge of fraction magnitudes is correlated with later achievement in fraction arithmetic (Siegler & Pyke, 2013). Further, previous interventional work has demonstrated that targeting magnitude understanding can yield transferable increases in knowledge of arithmetic and other fraction concepts (Fazio, Kennedy, et al., 2016; Fuchs et al., 2013; Moss & Case, 1999; Saxe et al., 2013). Moreover, understanding fraction magnitude also has critical implications for mathematical learning more broadly. For instance, the relationship between fraction magnitude knowledge and overall math achievement has been found in representative samples from North American, European, and Asian countries (Torbeyns et al., 2015). Furthermore, previous work has demonstrated that early fraction magnitude knowledge is a key indicator of performance in later advanced mathematics, particularly in algebra (Booth et al., 2014; Siegler et al., 2012).

In summary, knowledge of fraction magnitude is crucial towards later fraction learning as well as math achievement overall (Booth et al., 2014; Siegler et al., 2012; Siegler & Pyke, 2013; Torbeyns et al., 2015). As a result, current work in the field has suggested that focusing on enhancing magnitude knowledge should be the first step that is addressed in improving fraction proficiency and understanding overall (Torbeyns et al., 2015). Therefore, our study aimed to explore common methods in which fraction magnitudes are taught and understood to obtain better insight towards the best practices for fraction magnitude learning.

1.4 Instructional Tools Used to Teach Fractions

Thus far, we have seen that notable difficulties in comprehending fraction magnitude exist. Given that these difficulties have continued to be documented despite considerable instruction in school, it raises the question of whether the methods used to teach fraction magnitude are effective instructional approaches. Therefore, it is important to study the methods in which fraction magnitudes are taught in primary education. One of the most common approaches for teaching fraction magnitude is using visual models (Common Core Standards Writing Team, 2013). This approach, specifically, is recommended as a method of fraction learning by educational guides, such as the "IES Practice Guide for Developing Effective Fraction Instruction for Kindergarten through eighth Grade" (Institute of Education Studies, 2010). Moreover, visual models have been frequently implemented into fraction learning intervention studies and have yielded positive learning outcomes (e.g. Cramer et al., 1997; Fazio, Kennedy, et al., 2016; Fuchs et al., 2013). However, while this is true, it is important to acknowledge that the "fraction visual model" does not adopt a singular representation. Rather, different fraction models vary in many notable characteristics, including geometry and dimensionality. Consequently, the question of whether certain models yield better fraction learning outcomes, relative to others, has been raised (Rau et al., 2014). Specifically, here we consider two commonly used visual models for depicting fraction magnitude: the area/pie model and the number line model.

1.4.1 The Area Model Fraction Instructional Tool

In previous years, the area/pie model instructional tool has been the earliest and most emphasized model in fraction instruction (Common Core State Standards Initiative, 2015). This model consists of a two-dimensional circle, whereby a section of the circle is shaded. From this, magnitude is deciphered by assessing the amount of shading relative to the whole circle (Common Core State Standards Initiative, 2015) (Figure 1a). While this method has been widely implemented in the past, current theories have raised some concern about whether this model is actually exacerbating misconceptions in fraction magnitude (Hamdan & Gunderson, 2017). In particular, it has been suggested that the area model facilitates the notion that fractions are 'parts of a whole', thereby contributing to the lack in understanding that a fraction is a single numerical magnitude (Newcombe et al., 2015; Opfer & Siegler, 2012). Additionally, the two-dimensional feature of the area model has been criticized because it goes against the conception that all real numbers can be ordered in a single dimension (i.e., in a left to right unidimensional fashion) (Dehaene et al., 1993; Gunderson et al., 2019; Institute of Education Studies, 2010). This has raised the question of whether these features of the area model prevent this from being an effective fraction instructional tool.

1.4.2 The Number Line Fraction Instructional Tool

Given the criticism existing around the area model, interest has been directed towards utilizing other tools for fraction learning. Among these, the number line has gained traction as an effective tool for extracting fraction magnitude (for review see Abreu-Mendoza & Rosenberg-Lee, 2022). The number line is a unidimensional, linear model that allows for magnitude to be assessed by estimating the location of a tick mark along the length of the line (Hamdan & Gunderson, 2017) (Figure 1b). A critical feature that differentiates the number line from the area model is the difference in dimensionality. As a unidimensional model, the number line facilitates the understanding that fractions represent continuous magnitudes that can be sequentially ordered, similar to whole numbers (Gunderson et al., 2019; Siegler & Lortie-Forgues, 2014). Further, another beneficial feature of the number line is that its geometrical shape aligns with our conventional understanding of spatial and numerical associations, whereby smaller numbers are represented on the leftmost of the line and larger numbers on the rightmost of the line (e.g. Dehaene et al., 1993; Newcombe et al., 2015; Patro & Haman, 2012). Indeed, previous work has found that even preschool children expect numbers to be ordered in a increasing left-to-right fashion, suggesting that this mental organization of numerical magnitude emerges early in human development (Opfer et al., 2010). Further, this property is also analogous to how whole numbers are ordered and understood (Dehaene, 2011), thus reinforcing the idea that fractions are also single numeric magnitudes. For these reasons, it has been suggested that the number line may offer distinct advantages for learning and processing fraction magnitude (Gunderson et al., 2012, 2019; Hamdan & Gunderson, 2017; Siegler & Lortie-Forgues, 2014).

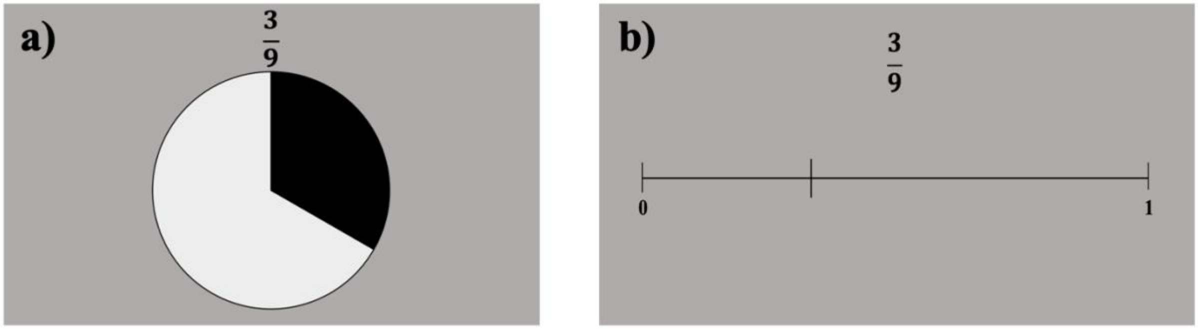

Figure 1: Example of the area/pie model format (a) and number line format (b). Magnitude in the area/pie model is assessed by examining the amount of black shading relative to the whole circle. Magnitude in the number line format is assessed by examining the location of the tick mark relative to the length of the 0-1 line.

While the literature on fraction visual models remains relatively limited, several studies have highlighted the effectiveness of the number line as a tool for fraction magnitude learning (for review see Abreu-Mendoza & Rosenberg-Lee, 2022). For example, in a 2020 study conducted by Barbieri and colleagues, 6th grade students with mathematics difficulties were evaluated in a fraction learning intervention study that had a number line centered approach. The authors found that the experimental intervention led to significant improvements in performance on magnitude comparison tasks, as well as on measures of general fraction concepts after a 7 week delay period (Barbieri et al., 2020). Similar findings to this have been obtained in intervention studies utilizing cohorts of 4th (Fuchs et al., 2013) and 5th grade students as well (Jayanthi et al., 2021). Therefore, this suggests that the number line can serve as an effective tool for enhancing fraction learning, especially in students with pre-existing math difficulties.

However, while the above studies indicate that the inclusion of the number line can yield gains in fraction learning, these do not speak to how the number line compares to other tools for teaching fraction magnitude. Therefore, to better explore potential differences in the effectiveness of the fraction instructional models, Hamdan and Gunderson (2017) compared the performance of 2nd and 3rd grade children who were trained on the number line with those trained on the area model. Specifically, in this study, the authors implemented a pre-test-training-post-test design to explore how training on each of the two models impacted students' performance on an untrained fraction magnitude task. During the untrained task, participants were presented with two fraction items and were instructed to select the one that was larger in magnitude. The authors found that in both training groups, students' performance estimating fractions using the model they were trained on improved in the post-test measure. However, interestingly, only children in the number line training group yielded transferable gains in fraction magnitude understanding. This was demonstrated as children who were trained on the number line performed significantly better on the novel magnitude comparison task than children who were trained on the area model. This study was the first to demonstrate that number line learning can causally impact knowledge of fraction magnitude. Further, this provides strong support towards the growing body of literature suggesting that the number line may be a more efficient model for representing fraction magnitude (Hamdan & Gunderson, 2017).

1.5 Fraction Processing in the Brain

1.5.1 Fraction Magnitudes Recruit Similar Regions as Whole Number Processing

In recent years, meaningful insights have begun to be drawn regarding why fraction difficulty occurs and methods for combatting these difficulties. However, the majority of work that has been done has approached this topic through a behavioural lens. The inclusion of neuroimaging can complement findings from the behavioural literature and can expand our knowledge of the mechanisms underlying fraction magnitude processing (Matejko & Ansari, 2018). Further, the addition of neuroimaging can be valuable for guiding new theories and testable questions which can ultimately provide a deeper understanding of fraction processing. However, to date, only a handful of neuroimaging studies have been devoted towards exploring how fractions are processed in the brain. Based on these studies, it has been found that the processing of fraction magnitudes recruits prefrontal and parietal regions of the brain, namely around the IPS (DeWolf et al., 2016; Ischebeck et al., 2009; Jacob & Nieder, 2009a, 2009b; Mock et al., 2018; Wortha et al., 2020). Thereby demonstrating that the neural regions responsible for processing fraction magnitude are largely similar to the regions responsible for whole number processing, as discussed in earlier sections.

Early evidence for this was obtained by Jacob and Nieder (2009) using an fMRI adaptation paradigm. In this experiment, the authors first adapted participants to a single fraction magnitude (i.e., 1:6). During adaptation, the neural response decreased in response to the prolonged exposure of the same stimulus magnitude (Larsson et al., 2016). Following adaptation, the authors then presented deviant stimuli, either symbolic fractions (e.g., $\frac{1}{9}$) or fraction words (e.g., one-ninth) that were different magnitudes from the one they were adapted to. Once the deviants were presented, a recovery in the BOLD signal was found between the deviant and adapted stimuli in bilateral prefrontal regions and the bilateral IPS. This activity was observed for both the symbolic fraction and fraction word stimuli (Jacob & Nieder, 2009a). Furthermore, other studies have documented a recruitment of similar regions during fraction magnitude processing. For instance, Ischebeck and colleagues (2009) recorded the neural activity of adult participants while completing a fraction magnitude comparison task. In response to this task, the authors observed clear fronto-parietal activation in regions including the inferior and superior parietal lobules as well as the right inferior and middle frontal regions (Ischebeck et al., 2009). Therefore, in summary, the current understanding is that the regions of the brain responsible for processing magnitude of whole numbers are also implicated in the processing of fraction magnitudes (see Wortha et al., 2022 for review).

1.5.2 The Neural Activity Associated with Learning using a Fraction Model

However, while these findings have been obtained, the question of how the brain processes fraction magnitude using the instructional models remains incredibly underexplored. To the best of our knowledge, only one study to date has explored the neural activity associated with learning using a fraction model (Wortha et al., 2020). In this study, the authors examined whether training on the number line could elicit changes in brain activation during fraction magnitude processing. To explore this question, the authors collected brain and behavioural measures from adult participants before and after 5 days of number line training. During each of these sessions, the participants were asked to complete a variety of tasks, including a symbolic fraction magnitude comparison task. In this study, differences in fraction magnitude processing were explored from pre to post-test, as modulated by the numerical distance effect. The numerical distance effect refers to the notion that, when completing a number comparison task, the task is performed quicker and more accurately when the two values are further apart from one another on the number line (e.g., 2 vs 9 is easier than 8 vs 9) (Moyer & Landauer, 1967). Importantly, the numerical distance effect has been shown to hold true for fraction magnitudes as well, with IPS activity being modulated by this effect when fractions are represented as a holistic numerical value (Ischebeck et al., 2009). As such, it has been suggested that the presence of a numerical distance effect when processing fraction magnitudes could be indicative of an active processing of the holistic magnitude, whereas a lack of distance effect could indicate less automatic access to magnitude or a difficulty processing the magnitude as a single value (Wortha et al., 2020).

Interestingly, Wortha and colleagues (2020) found that before training on the number line, there were no voxels that were significantly modulated by the numerical distance effect when completing the symbolic fraction magnitude comparison. However, following training on the number line there were significant increases in activity around the IPS that were modulated by the numerical distance effect. Additionally, behavioural performance on the magnitude comparison task also improved significantly from pre to post test. Taken together, the authors suggest that prior to training, participants had a difficulty accessing the holistic magnitude of the fractions being compared. However, training on the number line facilitated more efficient and automatic access to fraction magnitude. Therefore, this study provides some initial evidence of neural plasticity in fraction learning, whereby small-scale training on the number line, can improve how fraction magnitudes are understood and processed in the brain (Wortha et al., 2020).

This work has undoubtedly created an interesting avenue for exploring how fraction instructional models can facilitate magnitude learning on the neural level. However, there are still several questions that are left unanswered. Firstly, Wortha et al. (2020) explored how training on the number line impacted fraction magnitude understanding by using a fraction comparison task as a measure of magnitude knowledge. The use of the instructional model (i.e., training on the number line) took place outside of the scanner in between the pre and post-test scan. Therefore, how the brain processes and interprets fractional magnitude when presented in a fraction model has not yet been directly explored. Secondly, it is important to note that Wortha et al. (2020) only explored how number line training impacted brain and behavioural performance. Further, it should also be kept in mind that the study design did not include a control group in addition to the interventional group. Therefore, it cannot be said with certainty whether the results obtained from this study are truly number line-specific or whether improvements are the product of training alone. Relatedly, it is unclear whether training on another fraction instructional model, such as the commonly encountered area model, would yield a comparable finding. Therefore, another question that has been left open for discovery is directly comparing how the brain processes fraction magnitude using the number line and area model formats.

1.6 The Present Study

In summary, previous work has exemplified that understanding fraction magnitude is an especially difficult mathematical concept to grasp. One way to better understand the difficulties individuals experience when learning fractions is to examine the educational tools that are used to teach fraction magnitudes in early education. In many education curriculums, the area model is the first visual model that is introduced to children to teach fraction concepts (Common Core State Standards Initiative, 2015). However, a body of recent behavioural work has exemplified that the number line may actually be a more effective instructional tool for fraction learning (see Abreu-Mendoza & Rosenberg-Lee, 2022 for review). In addition to these behavioural findings, recent neuroimaging work has also demonstrated that training on the number line holds the potential to facilitate better access to fraction magnitude in the brain (Wortha et al., 2020). However, while these insights have been drawn, it currently remains unknown how fraction magnitudes are processed in the brain when presented in number line and area model formats.

Therefore, in the present study we sought to contribute to the currently limited body of literature by using functional brain imaging technology (fMRI) to explore the neural response of fractions presented in number line and area model formats. Based on the trends found in the behavioural literature (Hamdan & Gunderson, 2017), we hypothesized that the brain differentially processes fractional magnitude when presented in the different learning models. Specifically, we predicted that the number line would facilitate better access to fraction magnitude. Therefore, we expected to see greater activation in parietal regions, around the IPS, for fractions that were depicted in number line format. In contrast, we predicted that the area model would not efficiently facilitate access to holistic magnitude, thus we expected to see comparatively less activation in these regions responsible for processing magnitude.

Moreover, to complement our primary research aim, we also explored factors that may differentially impact magnitude processing in number line and area model format. Thus, as an exploratory addition, we also ran post-hoc analyses that examined whether different trial types were processed differently in number line and area model format. Specifically, we explored the processing of 'benchmark' fractions. In general, benchmark numbers are numbers that are highly familiar and easily identified (e.g. 10, ½) and thus can be used as a reference point when estimating the magnitudes of less familiar numbers (e.g. 13, $\frac{5}{8}$) (Obersteiner et al., 2020). This is a common strategy that is relied upon when performing whole number magnitude tasks (e.g. Peeters et al., 2016; Peeters, Sekeris, et al., 2017; Peeters, Verschaffel, et al., 2017; Sullivan et al., 2011). Further, while the evidence is limited, some work has suggested that this may be a beneficial strategy for fraction magnitude estimation as well (Liu, 2018; Obersteiner et al., 2020). Therefore, we explored whether there was evidence to suggest strategy differences between the models by examining how the brain processes benchmarks in number line and area model format.

To the best of our knowledge, this is the first study that explores how the brain processes fraction magnitude when presented in the different learning models. This is important because the inclusion of neuroimaging can contribute valuable insights toward assessing instructional methods used in education (see Seghier et al., 2019 for review). For instance, brain imaging can better explain common and discrete mechanisms that support cognitive functions, such as the processing of fraction magnitude (Mather et al., 2013). Specifically, for this reason, brain imaging can lend valuable insight in the context of comparing fraction learning models, by exploring whether the neural regions that are engaged when using the number line and area model instructional tools are largely overlapping or distinct. In conjunction with the behavioural literature, this line of investigation can contribute towards the question of which model is more effective for fraction learning by providing insight on whether the two models engage different processing mechanisms. Further, the addition of neuroimaging also holds the potential to inform how cognitive systems interact in the brain (Matejko & Ansari, 2018). Given that fraction magnitude understanding has been demonstrated to be a difficult task, this can be valuable for better understanding the cognitive processes and strategies that are engaged when processing fraction magnitude in each of the learning models. Above all, the current study is significant as it will contribute to our currently limited understanding of how fractions are processed in the brain. Specifically, for the first time, we will be able to characterize the neural underpinnings of fraction magnitude processing when using the different learning models.

Chapter 2

2 Methods

2.1 Participants

Healthy adult participants from London, Ontario were recruited to participate in the study. Participant recruitment occurred via recruitment emails and campus posting advertisements at Western University. Twenty-seven participants were recruited to participate in the study, however, due to motion exceeding the pre-registered head motion cut-off criteria, two participants were excluded from the final analysis (osf.io/ztjw7). Therefore, the final sample consisted of twenty-five healthy adult participants (14 female, 11 male; 21.04 ± 2.81 years old). All participants provided written consent to participate in the study prior to beginning the study session.

To be eligible for this study, it was required that participants were between 18 and 35 years of age, right-handed, a fluent English speaker, have normal or corrected-to-normal vision, and be MRI compatible (no non-removable metal on or in the body). Further, to be included in the final analysis, participants needed to have met the pre-registered head motion criteria (head movement that does not exceed 3 mm or 3 degrees across the entire run, with no sudden spikes that are greater than 1 mm or 1 degree). Functional runs that did not meet these parameters were excluded from the final analysis. Further, participants with head motion exceeding these criteria for more than two functional runs were excluded from the study completely. Data from this study was collected in a single session, which took an approximate duration of 1.5 hours. Participants were compensated a fixed amount of $30 for their participation. All procedures for this study were approved by the Non-Medical Research Ethics Board at Western University (Appendix 1).

2.2 Study Stimuli and Study Task

During the session, participants were presented with single-digit symbolic fractions (e.g., ¼) depicted in either number line or area/pie model format. In total, 36 fraction items were utilized, with numerators ranging from 1-9 and denominators ranging from 2-9 (see Appendix 2). Rather than more complex numerals, single digit fraction items were chosen as the aim of the study was to assess fundamental fraction magnitude knowledge using the different learning models. The study stimuli were created using the Excel and Word Microsoft Office applications. The model (number line or area model) was depicted in the center of the screen with the associated fraction depicted directly above. The model and associated fraction were depicted in black-and-white colouration against a grey background (Figure 2). The experimental paradigm was coded using the PsychoPy open-source software package, version 2021.2.3 (Pierce et al., 2019). These stimuli were projected onto a computer screen. This computer screen was made viewable for participants in the scanner via a mirroring system that was attached to the MRI head-coil.

2.2.1 Critical Trials

Stimuli pertaining to the critical trials included a fraction-model pair that was defined as either a “correct” or “incorrect” depiction. Correctly depicted trials involved a fraction model that accurately depicted the associated fraction (Figure 2a,c). In contrast, incorrectly depicted trials involved a fraction model that did not accurately depict the associated fraction (Figure 2b,d). Trials defined as incorrectly depicted were set to be a value of $\frac{1}{9}$ away from where the correct location was. Half of the incorrect trials were presented $\frac{1}{9}$ greater than where the correct depiction would be, and the remaining half were presented $\frac{1}{9}$ less than where the correct depiction would be. Setting incorrect trials to be depicted a value of $\frac{1}{9}$ away from the correct location was initially chosen through trial and error (i.e., self-testing using different methods of defining incorrect trials). However, this decision was later explored in a fully powered pilot study where it was determined that this parameter was neither indiscriminably difficult nor blatantly easy, and thus was determined a reasonable choice (Henry et al., unpublished). During the study, each of the 36 fraction items were both correctly and incorrectly depicted once in each model. Put simply, the item $\frac{1}{7}$, for instance, was presented four times during the session, once correctly depicted in number line format, once incorrectly depicted in number line format, once correctly depicted in area model format, and once incorrectly depicted in area model format.

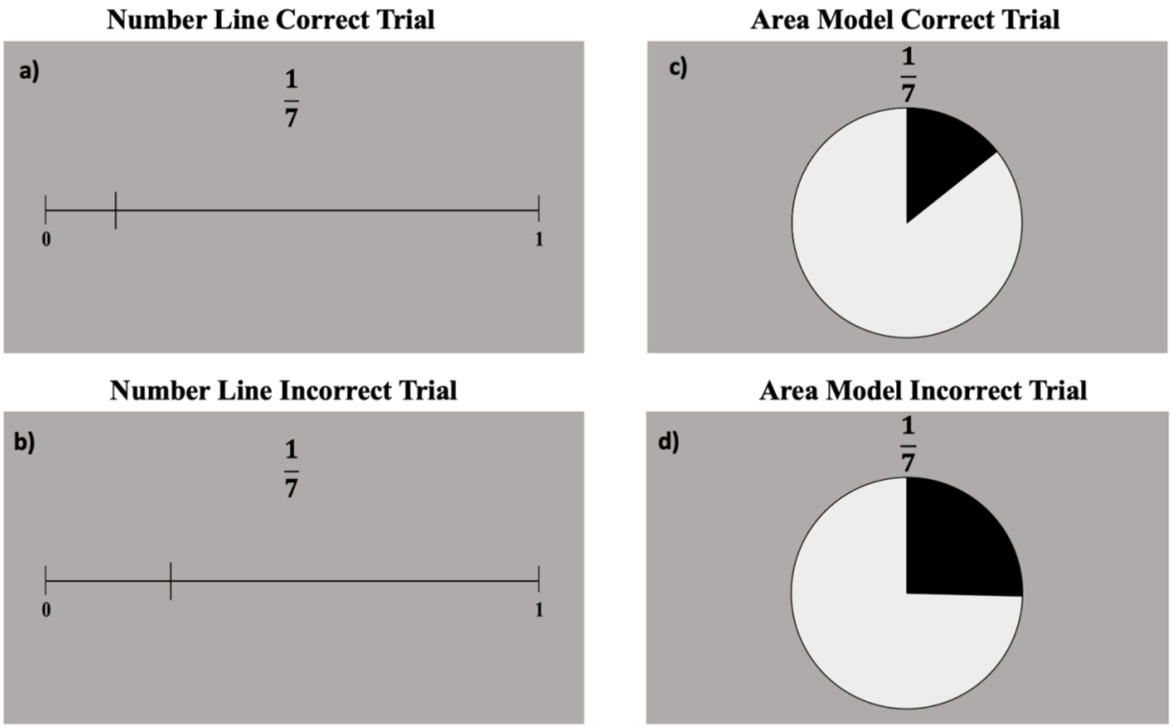

Figure 2: Example Critical Trial Stimuli. Panel a) displays an example of a correctly depicted number line trial. Panel b) displays an example of an incorrectly depicted number line trial. Similarly, on the right, panel c) displays a correctly depicted area model trial, and panel d) displays an incorrectly depicted area model trial.

Using these stimuli, participants were asked to perform a fraction verification task while in the scanner. This task required participants to view the fraction and associated model and make a judgement of whether the model accurately depicted that given fraction (e.g., using figure 2 as an example, does the model accurately depict the fraction $\frac{1}{7}$?). Specifically, participants were asked to press the button on the right when they believed the trial depicted a correct fraction-model match and to press the button on the left when they believed that trial depicted an incorrect fraction-model match. Responses were recorded using a response box that participants held in their right hand whilst in the scanner. Participants were instructed to respond as quickly and as accurately as possible but were not provided with any particular strategy on how to complete the task aside from this.

2.2.2 Control Trials

It is important to consider that the number line and area model formats differ from one another, quite substantially, in terms of visual features. Therefore, to control for the differences in visual stimulation between the models, a control task was implemented in the study design. During control trials, participants saw stimuli that resembled the critical trial stimuli (a fraction model with an associated single-digit fraction depicted directly above), however, now with colour included. Indeed, rather than black-and-white colouration, as in the critical trials, the fraction numerals as well as features of the model (tick from number line and shading in the area model) were one of three colours – black, red, or blue. Half of the control trials depicted a "colour match" whereby the colour of the fraction numeral was the same as the colour shown in the model (e.g., blue fraction numeral and blue tick on the number line) (Figure 3a,c). Further, the remaining half of the control trials depicted a "incorrect colour match" whereby the colour of the fraction numeral was different from the colour shown in the model (e.g., blue fraction numeral and black tick on the number line) (Figure 3b,d). During control trials, the magnitude of the fraction was never correctly depicted in the model as we did not want participants to attend to magnitude, rather only to the visual features of the model.

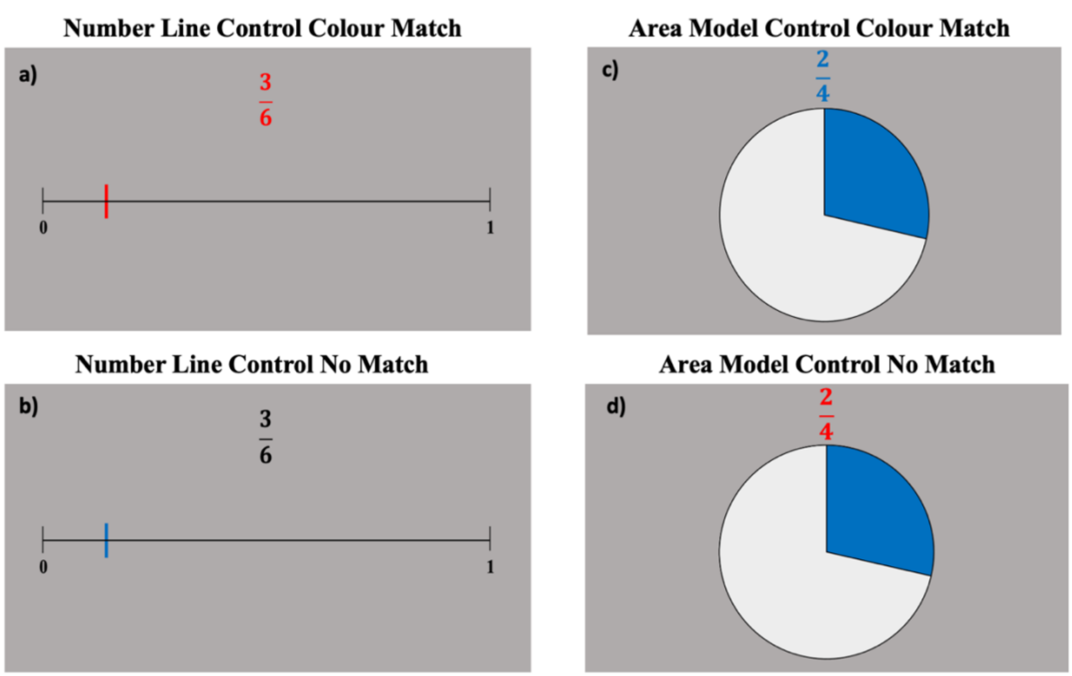

Figure 3: Example control trial stimuli. The left panels depict the number line control trials, with panel a) depicting an example of a colour match and panel b) depicting an example of an

incorrect colour match. Similarly, the right panels depict examples of area model control trials, with panel c) depicting an example of a colour match and panel d) depicting an example of an incorrect colour match.

Therefore, rather than performing a fraction magnitude verification, as in critical trials, during control trials participants were asked to perform a colour verification judgement. This required participants to view the fraction and associated model to determine whether the colour of the fraction numeral matched the colour depicted in the model or not (tick colour of the number line and shading colour of the area model). During the control trials, participants were instructed to only focus on whether the colour match was correct or incorrect and to not attend to whether the magnitude shown in the model was accurate. Participants were instructed to follow a similar response pattern as in the critical trials, whereby the right button corresponded to a correct fraction-model colour match and the left button corresponded to an incorrect fraction-model colour match. Once again, participants were asked to respond as quickly and as accurately as possible but were not provided with any specific strategy on how to complete these trials aside from this.

As a result, the study consisted of a total of 288 trials that were divided into the following conditions: **1) Number line critical trials** (36 correctly depicted trials, 36 incorrectly depicted trials); **2) Area model critical trials** (36 correctly depicted trials, 36 incorrectly depicted trials); **3) Number line control trials** (36 correct colour match trials, 36 incorrect colour match trials); **4) Area model control trials** (36 correct colour match trials, 36 incorrect colour match trials). These 288 trials were presented over four functional runs, with each run containing 72 trials (9 trials from each of the above conditions per run). The study was administered as a within-subjects design. Therefore, each participant saw and completed all trials from each of these conditions.

2.3 Experimental Procedure

The fMRI task was presented in a rapid, jittered event-related design, however, with grouping according to the task type. It is important to keep in mind that the judgement made during critical trials (fraction verification) differed from the judgement made during control trials (colour verification). Therefore, we deemed it beneficial to divide trials into small 'blocks/sections' of six trials, where all trials pertained to the same judgement (e.g., six trials of fraction verification then six trials of colour verification). Each sectioned group of trials commenced with a task cue which informed participants which of the two judgements should be made for the subsequent trials. Specifically, the word "Fraction" appeared on the screen to indicate that trials pertaining to the critical task would follow (fraction verification), or the word "Colour" appeared on the screen to indicate that trials pertaining to the control task would follow (colour verification). One of these two task cues was presented for a brief duration of 2 seconds, a randomized set of six trials pertaining to that cue would follow, then the next cue would be presented and so forth (Figure 4). Each trial had a stimulus duration of 3 seconds with a jittered fixation period of 2, 3, or 4 seconds following each individual trial. During this period, participants could respond to indicate whether they believed that trial was a correct fraction-model match/colour match. Participant responses made during the trial presentation and subsequent fixation period were recorded, therefore, participants could respond during either of these two periods. Participants were provided with instruction on how to complete the task, what each cue represented and how to make their responses prior to entering the scanner. In addition, during this instruction period, participants were provided the opportunity to complete a set of practice trials, to ensure that they were familiar and understood the task beforehand.

Structuring the fMRI runs in this manner, whereby trials were sectioned according to the judgement made, was done to reduce the cognitive load placed on participants. Once again, given that participants were making a different judgement in control trials than in critical trials, this allowed participants to be informed of which judgement they would be making prior to the trial presentation. This was deemed to be far less cognitively demanding than switching response judgement on a trial-by-trial basis. Furthermore, this structure was additionally beneficial to our study design, as it allowed us to include more breaks within each functional run. Indeed, following each section of trials, participants were allotted a 4 second baseline break before the next cue was presented. This was beneficial as it allowed the opportunity to provide our participants with more rest periods and additionally allowed us to obtain a better estimation of the baseline during each run.

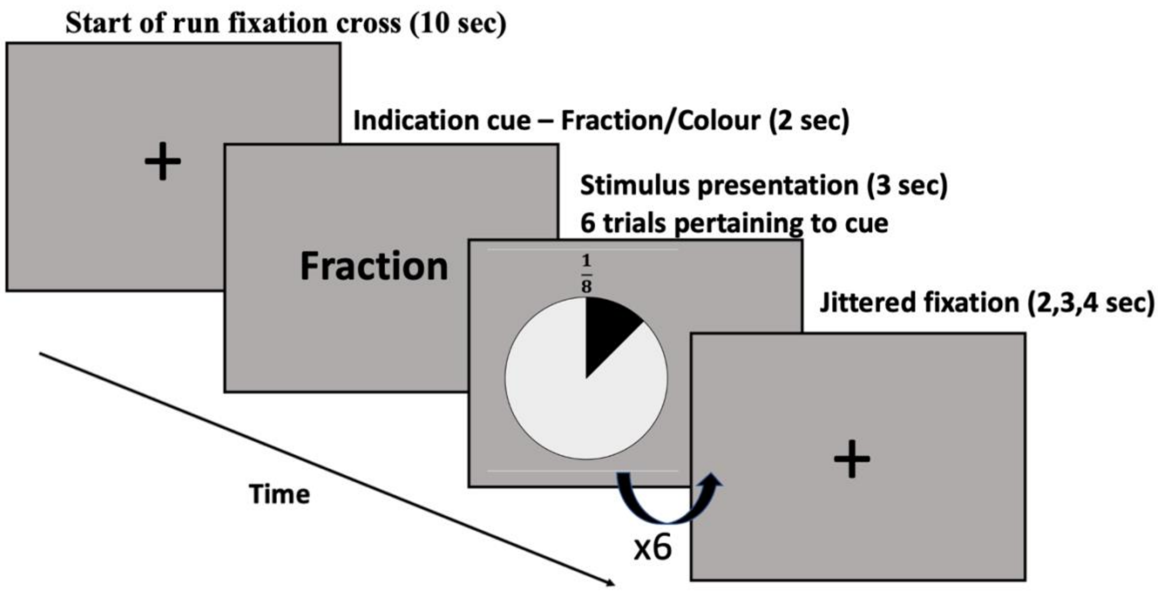

Figure 4: Overview of the procedure in which stimuli were presented during functional runs. Each run commenced with a 10 second fixation cross. Following this, an indication cue (Fraction or Colour) would briefly appear (2 seconds) to inform participants of which task to perform for the next 6 trials. Each individual trial was presented with a stimulus duration of 3 seconds and was followed by a fixation period of jittered interval (2,3,4 seconds). Following each section of 6 trials, participants were allotted a 4 second break period before the onset of the next cue.

2.4 Supplementary Behavioural Measures

Following the scanning session, participants were brought into a nearby testing room at the Robarts Research Institute to complete two brief supplementary behavioural measures. The first of these measures was a fraction concepts measure, which was an online measure, distributed to participants through a Qualtrics survey (Qualtrics, Provo, UT). The fraction concepts measure consisted of a subset of 15 multiple choice and open-ended items taken from the National Assessment of Educational Progress, a United States department of education program that serves as a measure of student achievement (NAEP; U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress, 1990–2009). This measure assessed knowledge of different fraction concepts, including understanding of fraction equivalence, fraction estimation, fraction ordering, fraction arithmetic, and fraction reasoning (Appendix 3). This supplementary measure was included in the study to obtain insight into our participants' general fraction knowledge and abilities. Participants were allotted three minutes to complete this measure. If participants were not able to complete the measure in this duration, the Qualtrics system automatically re-directed participants to the next measure.

The second and final supplementary measure participants were asked to complete during the study was a debrief form. Once again, this measure was completed online and was provided to participants through a Qualtrics survey (Qualtrics, Provo, UT). The debrief measure consisted of seven open-ended questions regarding participants thoughts and experiences pertaining to the study, as well as their previous math learning experiences. The debrief questions included: 1) What strategy or strategies were used to complete the task (if any)? 2) What did you find most difficult about this task? 3) Did you find one format more difficult to complete than the other? Or did you find the formats equal in difficulty? 4) Do you feel confident that you were able to judge whether the fraction was accurately depicted in the model? 5) What is your program of study or occupation? Would you say you use math often in your program/work? 6) Do you recall which fraction model you learned in school? Is one fraction model more familiar to you? Are they both familiar? Or neither familiar? 7) In general, how difficult do you find fractions? For this measure, no time limit was imposed, and participants were invited to provide whatever level of detail they deemed necessary.

2.5 fMRI Data Acquisition

The fMRI data was acquired at the Centre for Functional and Metabolic Mapping at the Robarts Research Institute at Western University. Structural and functional MRI scans were collected using a 3T Siemens Prisma Fit whole-body MRI scanner with a 32- channel head coil (Siemens, Erlangen, Germany). One high resolution T1-weighted anatomical scan was collected at the whole-brain level, where 192 slices were collected using an MPRAGE sequence in the sagittal plane (voxel size = 1 mm x 1 mm x 1 mm, TR = 2300 ms, TE = 2.98 ms , TI = 900 ms, flip angle = 9º). Additionally, functional data was acquired during four runs using a BOLD (blood oxygenation level dependent) sensitive T2*-weighted single echo-planar sequence. During each functional run, 536 volumes were collected, with each volume being composed of 60 slices that were acquired through multi-band imaging (voxel size = 2 mm x 2 mm x 2 mm, TR = 1000 ms, TE = 30 ms, flip angle = 40º, multi-band acceleration factor = 5). In accordance with these parameters, each functional run took approximately nine minutes to complete.

2.6 fMRI Preprocessing

The anatomical and functional MRI data was largely preprocessed using the fMRIPrep preprocessing pipeline, version 20.2.6 (Esteban et al., 2018), which is a software based off of Nipype 1.7.0 (Gorgolewski et al., 2011). The fMRIPrep software generates a citation boilerplate that outlines all preprocessing steps performed. The developers of fMRIPrep suggest using this boilerplate verbatim in written work, as the boilerplate is public domain. According to this output, these steps are summarized here below. During anatomical preprocessing, the T1-weighted (T1w) scan was corrected for intensity non-uniformity using a N4BiasFieldCorrection (Tustison et al., 2010). Further, the T1w image was then skull stripped using OASIS30ANTS as the target template. Following this, the brain extracted T1w image underwent tissue segmentation into cerebrospinal fluid, white matter, and grey matter (Zhang et al., 2001). Finally, the anatomical data was spatially normalized to standard space via non-linear registration with antsRegistration (ANTS 2.3.3), using the ICBM 152 Nonlinear Asymmetrical template version 2009c [Fonov et al., (2009), RRID:SCR_008796; TemplateFlow ID: MNI152NLin2009cAsym].

Additionally, preprocessing was performed on the data from the four BOLD-sensitive functional runs. Firstly, a reference volume, along with a skull-stripped version, were generated through custom fMRIPrep methodology. Susceptibility distortion correction of the reference volume was then performed using a field-map based phase-difference map. According to the level of susceptibility distortion from this, a more accurate BOLD reference was calculated and was then co-registered with the T1w reference using the FreeSurfer function bbregister (Greve & Fischl, 2009). Following this, the six head motion rotation and translation parameters were estimated with respect to the BOLD reference using the mcflirt motion correction tool from FSL (version 5.0.9) (Jenkinson et al., 2002). Following this step, the data was then slice-scan time corrected using

AFNI (Cox & Hyde, 1997). The functional data was then resampled into native space by applying a single composite transform to correct for head motion and susceptibility distortions. Next, the functional data was resampled into standard space, MNI152NLin2009Asym, in a single interpolation step that applied all the previous transformations (i.e., head motion correction, susceptibility distortion correction, etc) using the antsApplyTransforms command from ANTS configured from Lanczos interpolation. Finally, following initial preprocessing in fMRIPrep, the functional data was then spatially smoothed using a 6 mm FWHM Gaussian kernel, high-pass filtered and corrected for temporal autocorrelation assuming an AR(1) in the SPM 12 software (Ashburner et al., 2021).

2.7 Data Analysis

2.7.1 Statistical Threshold

The preprocessed MRI data was then analyzed using the SPM 12 software package (Ashburner et al., 2021). To begin, first-level analyses were carried out at the individual level for each of the twenty-five subjects. The first-level analyses were based on a general linear model using the experimental conditions and task cue as predictors of interest and including regressors for the six motion parameters (three translation and three rotation), white matter, and cerebrospinal fluid signals. Once the first-level analysis was specified and estimated for each participant, the second-level group analysis was performed at the whole-brain level. Second level analyses were then run with an uncorrected threshold of $p<0.001$. Whole-brain statistical maps were then corrected for multiple comparisons using a Gaussian Random Field (GRF) correction. For each statistical contrast, the minimum cluster size was simulated using the DPABI toolbox at the level of significance voxel $p<0.001$, cluster $p<0.05$ (Yan et al., 2016). These statistical threshold parameters, along with the minimum cluster sizes are reported with each individual contrast map.

2.7.2 Whole-Brain Contrasts

The primary question in this study was to explore how the brain processes fraction magnitude when presented in number line and area model format. Therefore, to explore this question, whole-brain random effects analyses were run to examine neural regions that were common and distinct for fractions depicted in number line and area model format. This was done through running the following contrasts: Firstly, to ensure that our task was functioning as intended we ran a sanity check t-contrast between the critical and control trials (Number Line and Area Model critical trials) – (Number Line and Area Model control trials). Then, to explore the regions of the brain that displayed common activation for both the number line and area model, a conjunction of the models was run (Number Line – Number Line Control) & (Area Model – Area Model Control). Finally, to explore potential differences in the neural response between the two fraction models, a t-contrast of the models was performed, with each model being subtracted by its respective visual control (Number Line – Number Line Control) – (Area Model – Area Model Control). In each contrast, the contrast vectors were weighted such that the contrasts were always balanced, or in other words summed to zero.

In addition to our primary analyses, we also conducted a set of exploratory analyses to examine whether there was evidence to suggest potential processing differences between the models. In these exploratory analyses, we examined the role of benchmark fractions in each of the models. The use of benchmarks or reference fractions is a strategy that can be implemented to estimate the magnitude of other fractions. Thus, we were interested in exploring how the brain processes benchmark versus non-benchmark fractions and whether the brain processes these trial types differently in number line and area model format. In this exploratory analysis, we only explored trials from the critical fraction magnitude verification task. For the purpose of our study, the fraction items $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{4}$, as well as all fractions equivalent to these (e.g., $\frac{2}{4}$, $\frac{6}{8}$) were considered as benchmark fractions. All remaining items were considered non-benchmark fractions (e.g., $\frac{1}{7}$, $\frac{5}{9}$) (Appendix 4). Using this design, we ran a contrast of benchmark versus non-benchmark fractions for both number line (Number Line Benchmark – Number Line Non-benchmark) and area model (Area Model Benchmark – Area Model Non-benchmark). All contrast vectors in the exploratory analysis were also weighted such that the contrast would be balanced. The primary and exploratory analyses conducted in this study were pre-registered on OSF prior to the study onset (osf.io/ztjw7).

2.7.3 Behavioural Data Analyses

In addition to the imaging data, behavioural data was also obtained from participants while in the scanner. Participant's response to each trial, along with the time in which it took for participants to make their responses was recorded by the PsychoPy open-source software package, version 2021.2.3 (Pierce et al., 2019). This behavioural data was processed using the tidyverse programming package (version 1.3.1, Wickham et al., 2019) in R studio (version 2022.07.1, RStudio, PBC, Boston). This allowed for trials to be filtered by condition so that response accuracy and response time could be explored across each of the experimental conditions at the group level. From this, descriptive statistics for response accuracy and response time were then calculated for each of the experimental conditions. In addition to this, for our primary analysis, a 2x2 repeated measures ANOVA, with task (critical vs control) and condition (Number line vs Area model) as the within-subjects factors was explored for both response time and response accuracy as the dependant variable. Similarly, for the exploratory analyses, a 2x2 repeated measures ANOVA with trial type (benchmark vs non-benchmark) and condition (Number line vs Area model) was explored for both response time and response accuracy as the dependent variable. Note that in the exploratory analysis, task was not included as a factor as only trials from the critical fraction magnitude task were explored. Where necessary, post-hoc tests from these analyses were corrected for multiple comparisons using a Bonferroni correction.

Chapter 3

3 Results

Analyses for the present study were pre-registered prior to data collection. The pre-registration can be found here osf.io/ztjw7. In this section, I begin by discussing the primary pre-registered analyses. The core research question in this study was to explore differences and commonalities in how the brain processes fraction magnitude in number line and area model format. Thus, presented first are the behavioural and imaging data that explore this question. Secondly, we also pre-registered an exploratory analysis that investigated whether trial type (benchmark versus non-benchmark) is differentially processed in number line and area model format. This exploratory analysis was included to provide insight into factors that may impact how each of these models are processed. Behavioural and imaging results from this exploratory analysis are presented last.

3.1 Primary Analysis: Behavioural Results

3.1.1 Response Accuracy

Descriptive statistics for response accuracy on the neuroimaging task and the fraction concepts measure are displayed in Table 1. Further, a 2x2 repeated measures ANOVA was conducted with task (critical vs control) and condition (number line vs area model) as the within-subjects factors and response accuracy as the dependent variable. This analysis revealed only a main effect of task $F(1, 24) = 140.42$, $p < 0.001$, whereby participants performed significantly more accurately when completing control trials (the colour judgement) in comparison to critical trials (the fraction verification judgement). The 2x2 repeated measures ANOVA revealed no main effect of condition $F(1, 24) = 2.01$, $p = 0.17$, and no statistically significant interaction between task and condition $F(1, 24) = 2.24$, $p = 0.15$.

Table 1. Response Accuracy Descriptive Statistics on fMRI Task and Fraction Concepts Measure.

Condition	*N*	*M* (% correct)	*SD* (% correct)
Number Line Critical	25	82.83	8.17

Area Model Critical	25	84.72	6.19
Number Line Control	25	99.28	0.99
Area Model Control	25	99.22	1.07
Fraction Concepts Measure	25	87.20	2.18

Table 1. This table demonstrates descriptive data pertaining to response accuracy from the task completed in the scanner and the fraction concepts measure completed after the scanning session. The first four rows display response accuracy descriptive statistics on the neuroimaging task by each condition. The final row displays response accuracy descriptive statistics on the fraction concepts supplementary measure, a general measure of fraction understanding.

3.1.2 Response Time

Descriptive statistics for response time on the neuroimaging task are displayed in Table 2. A 2x2 repeated measures ANOVA was conducted with task (critical vs control) and condition (number line vs area model) as the within-subjects factors and response time as the dependent variable. This analysis revealed a significant interaction between the effects of task and condition on response time $F(1, 24) = 10.14$, $p = 0.004$. Main effects analyses revealed a main effect of task $F(1, 24) = 216.25$, $p < 0.001$ on response time, however, no main effect of condition $F(1, 24) = 0.50$, $p = 0.49$. Given the significant interaction, post-hoc t-tests with Bonferroni correction were run. The results of the post-hoc comparison revealed that, across both models, trials in the control conditions were responded to significantly more quickly than trials in the critical conditions. In addition, the post-hoc comparison revealed that critical trials in number line format ($M = 2.00$, $SD = 0.43$) were responded to significantly more quickly than critical trials in area model format ($M = 2.06$, $SD = 0.48$) ($p = 0.031$). There were no significant differences in response time between the number line and area model control conditions.

Table 2: Response Time Descriptive Statistics by Condition from fMRI Task.

Condition	***N***	***M* (seconds)**	***SD* (seconds)**
Number Line Critical	25	2.00	0.43

Area Model Critical	25	2.06	0.48
Number Line Control	25	1.03	0.25
Area Model Control	25	0.99	0.22

Table 2. This table shows the response time descriptive data from the fMRI task. Each individual trial was displayed for three seconds, followed by a jittered fixation period of two, three or four seconds. Participants could respond to each trial at any point during the stimulus duration or subsequent fixation period. Response time values were recorded for each trial according to the time in which participants pressed the button box to make their response.

3.2 Primary Analysis: Imaging Results

3.2.1 Sanity Check Contrast (Critical Trials vs Control Trials)

We first ran a sanity check contrast to verify that our task was functioning as intended. To do so, we ran a contrast of all critical trials (number line and area model) > than all control trials (number line and area model) to verify that activation was present in the regions of the brain that are characteristic of typical number processing (Figure 5). Indeed, in this contrast, clusters of activation were observed in regions such as the inferior and superior parietal lobules, middle frontal gyrus, and the insula. These are specific regions of the brain that have been consistently documented to be recruited when processing number information (see Arsalidou & Taylor, 2011 for meta-analyses). In contrast, we observed activation in the opposite direction (where control trials > critical trials) in regions that are consistent with the default mode network (see Menon, 2023 for review). For instance, activation peaks were jointly observed in regions including the medial prefrontal cortex, the posterior cingulate gyrus, the precuneus and the middle temporal gyrus. These are regions of the brain that have been identified to be part of the default mode network, which are neural regions that are more active at rest than they are during an externally engaging task (Mazoyer et al., 2001; Shulman et al., 1997). This provided a reasonable sanity check as the task performed in the control trials (colour verification) was an easier and less cognitively demanding task than the task performed in critical trials (fraction magnitude verification).

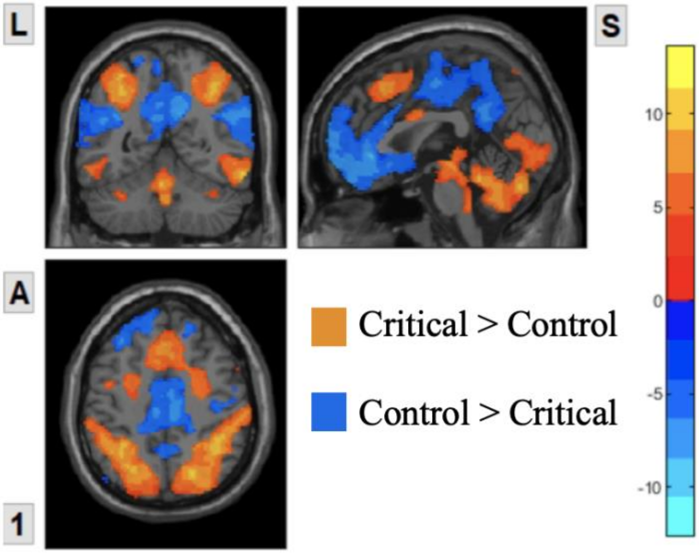

Figure 5: Sanity Check Contrast. Clusters of activation from the contrast of all critical trials – all control trials. Only significant clusters of activation are shown here at the threshold of voxel $p < 0.001$, cluster $p < 0.05$, GRF corrected, $k = 247$.

3.2.2 Common Neural Regions Activated for Number Line and Area Model

Common regions activated by magnitude processing in both number line and area model format were identified through a conjunction analysis (Number line – Number line control) & (Area model – Area model control). The conjunction analysis revealed a high degree of overlap between the two models (Figure 6). Notably, a range of frontal and parietal regions were significantly activated by both the number line and area model. Activation was seen in regions including the bilateral superior and inferior parietal lobules, the inferior frontal gyrus, the right middle frontal gyrus and the insula. These are regions that are commonly implicated in number processing more generally (see Arsalidou & Taylor, 2011 for meta-analyses), as well as in fraction magnitude tasks in specific (Ischebeck et al., 2009). This high degree of overlap in these frontal and parietal regions suggests that there are many similarities in how fraction magnitude is processed in number line and area model format. In particular, this demonstrates that both number line and area model formats recruit brain regions that are characteristic of typical number magnitude processing.

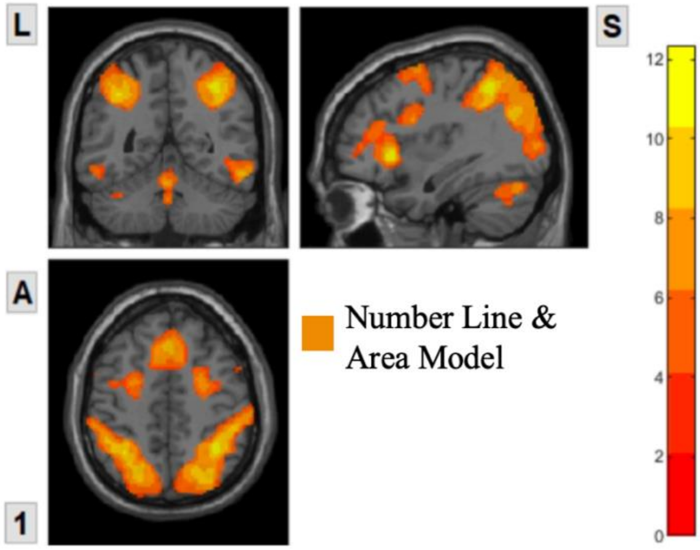

Figure 6: Common Areas Activated by Both Number Line and Area Model Formats (Conjunction Analysis). Only significant clusters of activation are shown here at the threshold of voxel $p < 0.001$, cluster $p < 0.05$, GRF corrected, $k = 251$.

3.2.3 Distinct Neural Regions Activated Between Number Line and Area Model Formats

However, while we observed evidence for many similarities in how the fraction models were processed in the brain, we were also interested in exploring whether there are differences. Therefore, to examine whether magnitude processing in number line and area model formats engaged distinct neural regions a t-contrast between the models was conducted at the whole-brain level. In this contrast, the difference between the models was explored with each model having its model-respective visual control subtracted out (Number line – Number line control) – (Area model – Area model control). In this contrast, we found neural regions where magnitude processing in number line and area model format were significantly different (Figure 7). However, we observed clusters of activation that did not match our predictions (Table 3). Two significant clusters of activation were identified where the area model was significantly more active than the number line (voxel p value < 0.001, cluster p value < 0.05, GRF corrected). These clusters of activation were found in the left inferior frontal lobe and in the right parietal lobe around the IPS. Moreover, only one cluster of activation was found where the number line was significantly more active than the area model (voxel p value < 0.001, cluster p value < 0.05, GRF corrected). This cluster was found in the primary visual cortex, around the calcarine sulcus. The results from this contrast demonstrate that while magnitude processing using the two fraction models recruit many overlapping regions, as seen in the conjunction contrast, there are also differences in how the brain processes fraction magnitude in number line and area model formats.

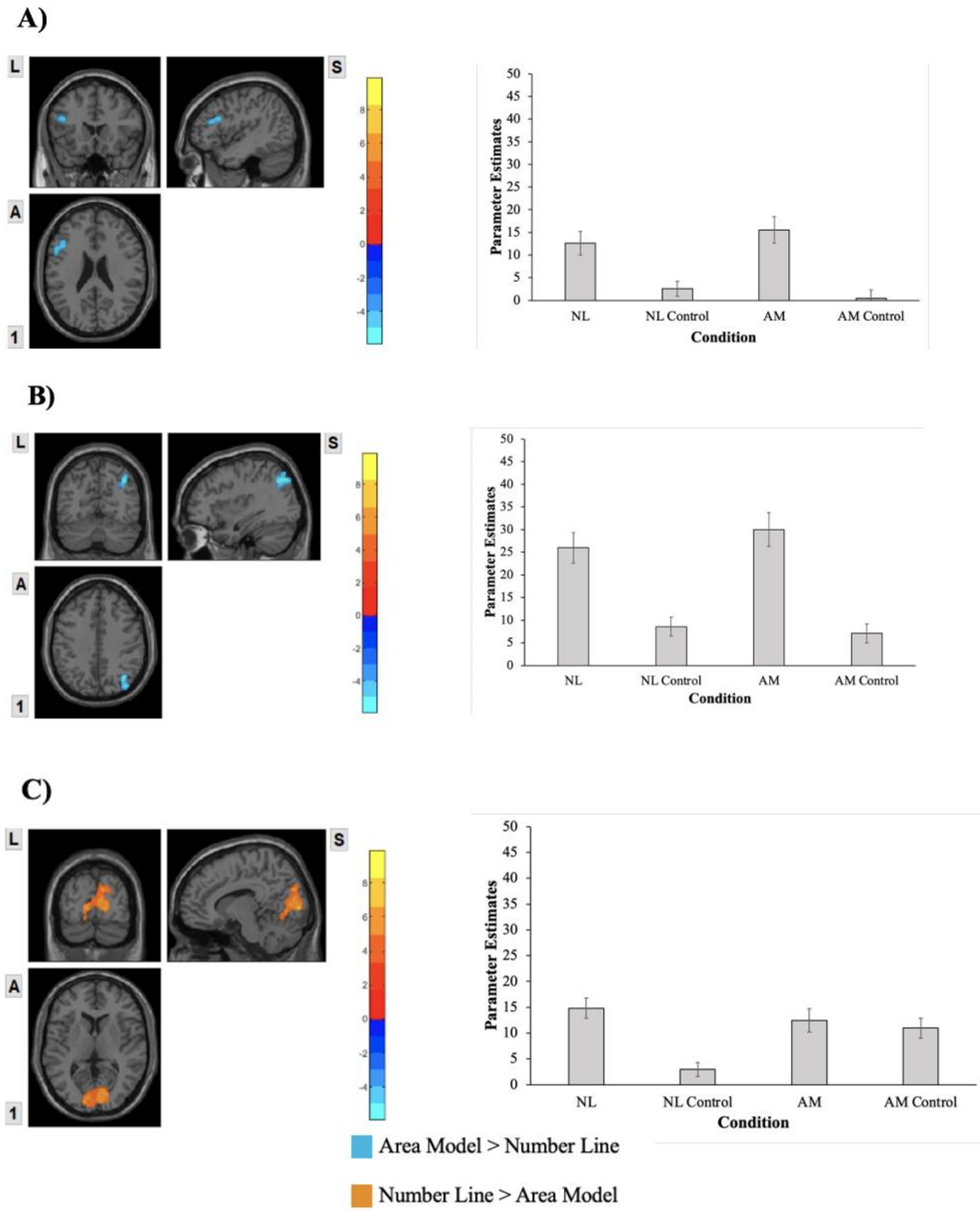

Figure 7: Neural Differences Between the Number Line and Area Model. Clusters of activation are from the contrast (Number Line – Number Line Control) – (Area Model – Area Model

Control). NL corresponds to Number Line, AM corresponds to Area Model. Only significant clusters are depicted here at the threshold p voxel < 0.001, p cluster < 0.05, GRF corrected, *k*=156. Each cluster of activation is displayed on the left of the figure, with panel A and B displaying regions where AM>NL and panel C displaying the region where NL>AM. To the right of each contrast map is a plot displaying the parameter estimate from that cluster across each condition. Parameter estimates were extracted by drawing a 6 mm sphere around the peak coordinate value from each cluster. The error bars in these plots reflects the standard error.

Table 3: Peak Neural Regions Identified in Contrast Between the Number Line and Area Model.

Contrast Direction	Brain Region	*x*	*y*	*z*	*k*
Area Model > Number Line	Right Intraparietal Sulcus	36	-67	40	334
	Left Inferior Frontal Gyrus	-45	20	24	204
Number Line > Area Model	Left Visual Cortex/Calcarine	-13	-95	-3	2692

Table 3. Coordinates provided correspond to the peak MNI coordinates from each significant cluster. Cluster size (k) corresponds to the number of voxels. Regions were identified using the Jülich Histological Atlas (Amunts et al., 2020) and Harvard-Oxford Cortical Structural Atlas.

3.3 Exploratory Analysis: The role of Benchmark Fractions

To further explore the differences in how magnitude is processed using the number line and area model formats, we ran a set of exploratory analyses to examine whether there was evidence of strategy differences when making magnitude judgements in each of the models. To do so, we ran analyses looking at trial type in the critical fraction magnitude verification trials. Specifically, we explored the role of benchmark versus non-benchmark fractions in both number and area model format. The behavioural and imaging results from these exploratory analyses are presented below.

3.3.1 Behavioural Results of Benchmark vs Non-Benchmark Fractions (Response Accuracy)

Descriptive statistics for response accuracy on benchmark versus non-benchmark fractions across both models are displayed in Table 4. A 2x2 repeated measures ANOVA was conducted with trial type (benchmark vs non-benchmark fraction) and condition (number line vs area model) as the within-subjects factors and response accuracy as the dependent variable. This analysis revealed only a main effect of trial type $F(1, 24) = 66.23$, $p < 0.001$, whereby participants performed significantly more accurate when completing benchmark fraction trials in comparison to non-benchmark fraction trials. In this analysis, there was no main effect of condition $F(1, 24) = 2.06$, $p = 0.16$, and no statistically significant interaction between trial type and condition $F(1, 24) = 0.67$, $p = 0.42$.

Table 4: Response Accuracy Descriptive Statistics for Benchmark vs. Non-benchmark Fractions.

Condition	*M (% correct)*	*SD (% Correct)*
Number Line Benchmark	89.16	7.69
Area Model Benchmark	89.77	7.12
Number Line Non-benchmark	78.80	9.97
Area Model Non-benchmark	81.49	7.68

Table 4. Response accuracy descriptive statistics from the neuroimaging task. In the exploratory analysis, trials from the critical task (fraction verification task) are divided into benchmark versus non-benchmark trials across both models.

3.3.2 Behavioural Results of Benchmark vs Non-Benchmark Fractions (Response Time)

Descriptive statistics for the response time of benchmark versus non-benchmark trials across the models is displayed in Table 5. Using a 2x2 repeated measures ANOVA, we found a significant interaction between the effects of trial type and condition on response time $F(1, 24) = 11.45$, $p = 0.002$. Main effects analyses revealed a main effect of trial type $F(1, 24) = 75.74$, $p < 0.001$ on response time, however, no main effect of condition $F(1, 24) = 1.89$, $p = 0.18$. Given the significant interaction, post-hoc t-tests with Bonferroni correction were run. The results of the post-hoc comparison revealed that, across both models, benchmark fraction trials were responded to significantly more quickly than non-benchmark fraction trials. However, in addition, non-benchmark trials in number line format (*M* = 2.07, *SD* = 0.45) were responded to significantly more quickly than non-benchmark trials depicted in area model format (*M* = 2.19, *SD* = 0.49) (p = 0.016). There were no significant differences in response time between the number line and area model benchmark fraction conditions.

Table 5: Response Time Descriptive Statistics for Benchmark vs Non-benchmark Fractions.

Condition	*M* (seconds)	*SD* (seconds)
Number Line Benchmark	1.89	0.41
Area Model Benchmark	1.85	0.49
Number Line Non-benchmark	2.07	0.45
Area Model Non-benchmark	2.19	0.49

Table 5. Response time descriptive statistics from the neuroimaging task. In the exploratory analysis, trials from the critical task (fraction verification task) are divided into benchmark versus non-benchmark trials across both models.

3.3.3 Exploratory Analysis: Imaging Results

To explore whether there are differences in how benchmark versus non-benchmark fractions are processed in the brain we ran a t-contrast for both the area model (Area model benchmark – Area model non-benchmark) and number line (Number line benchmark – Number line non-benchmark). Interestingly, we observed differences in how trial type impacted magnitude processing in number line and area model format, as these two contrasts revealed distinct clusters of activation. In the area model contrast (Area Model Benchmark – Area Model Non-benchmark), we only found significant clusters of activation where area model benchmark > area model non-benchmark (Figure 8) (Appendix 5). These significant clusters of activation were found in regions that are characteristic of the default mode network (see Menon, 2023 for review). For instance, activation was jointly observed in regions including anterior medial prefrontal regions, posterior division of the cingulate gyrus and in the bilateral inferior parietal lobules (Table 6). There were no clusters of activation in this contrast that were significantly more active for area model non-benchmark trials than benchmark trials.

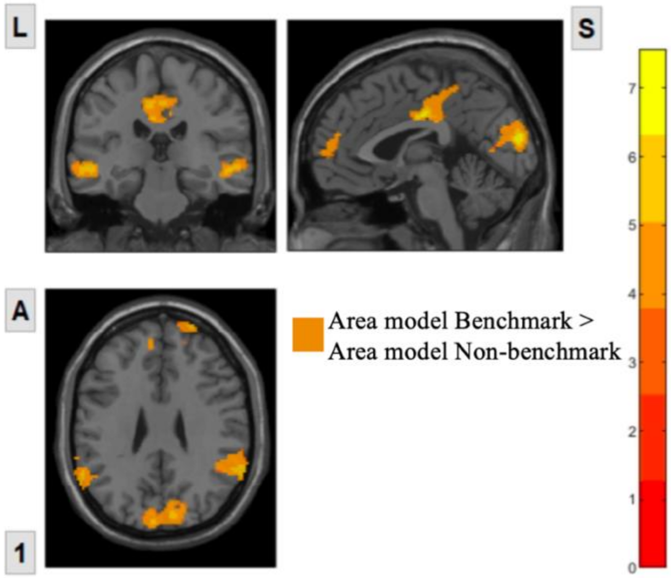

Figure 8: Contrast Map of Area Model Benchmark – Area Model Non-benchmark. Only significant clusters of activation are depicted here at the threshold p voxel < 0.001, p cluster < 0.05, GRF corrected, *k*=166. In this contrast, there were only significant clusters of activation revealed where area model benchmark > area model non-benchmark.

Table 6: Peak Coordinates where Area Model Benchmark > Area Model Non-benchmark.

Brain Region	***x***	***y***	***z***	***k***
Left Visual Cortex/V2	-3	-91	16	1208
Left Posterior Cingulate Gyrus	-1	-13	34	800
Right Inferior Parietal Lobule	64	-51	30	735
Left Inferior Parietal Lobule	-61	-53	26	546
Left Anterior Cingulate Cortex	-9	58	14	497
Right Superior Temporal Sulcus	62	-25	-5	392

Left Superior Temporal Sulcus	-55	-27	-7	384
Right Frontal Pole	26	62	30	256
Left Cerebellum	-25	-79	-37	167

Table 6. Coordinates provided correspond to the peak MNI coordinates from each cluster. Cluster size (k) corresponds to the number of voxels. Regions were identified using the Jülich Histological Atlas (Amunts et al., 2020) and Harvard-Oxford Cortical Structural Atlas.

On the other hand, in the number line exploratory contrast (Number line benchmark – Number line non-benchmark) we observed different clusters of activation. For the number line, we only observed significant clusters of activation where number line non-benchmark > number line benchmark (Figure 9). Specifically, four significant clusters of activation were revealed where number line non-benchmark > number line benchmark (Appendix 6). These clusters of activation were found in frontal and parietal regions, including the right inferior parietal lobule around the IPS, and the right middle frontal gyrus (Table 7). There were no clusters of activation in this contrast that were significantly more active for number line benchmark fractions than number line non-benchmark fractions.

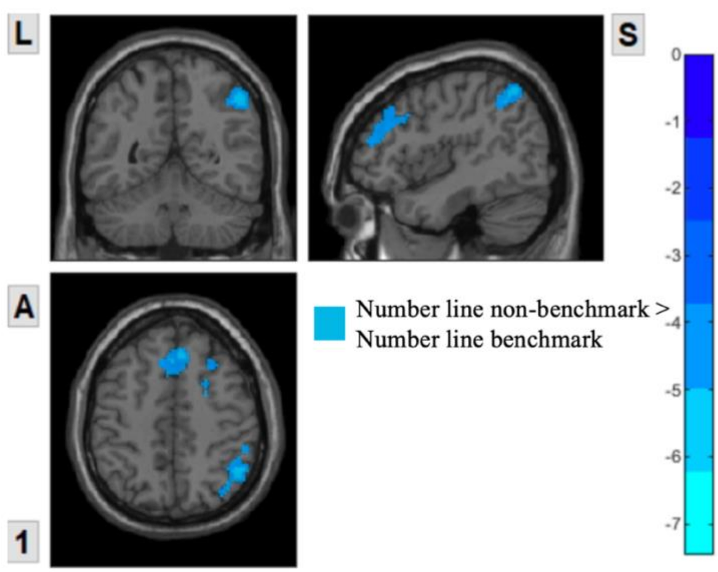

Figure 9: Contrast Map of Number Line Benchmark – Number Line Non-benchmark. Only significant clusters of activation are depicted here at the threshold p voxel < 0.001, p cluster <

0.05, GRF corrected, *k*=166. In this contrast, there were only significant clusters of activation revealed where number line non-benchmark > number line benchmark.

Table 7: Peak Coordinates where Number Line Non-benchmark > Number Line Benchmark**.**

Brain Region	***x***	***y***	***z***	***k***
Right Intraparietal Sulcus	48	-59	52	498
Right Middle Frontal Gyrus	38	38	34	461
Left Superior Frontal Gyrus	-1	30	40	431
Right Superior Frontal Sulcus	30	6	56	399

Table 7. Coordinates provided correspond to the peak MNI coordinates from each cluster. Cluster size (k) corresponds to the number of voxels. Regions were identified using the Jülich Histological Atlas (Amunts et al., 2020) and the Harvard-Oxford Cortical Structural Atlas.

Chapter 4

4 Discussion

The primary goal of the present study was to gain insight into how the brain processes fraction magnitude using the number line and area model instructional tools. Behavioural work has provided data to support the notion that number lines are an effective tool for fraction magnitude learning (see Abreu-Mendoza & Rosenberg-Lee, 2022 for review). Further, recent neuroimaging work has generated some initial evidence that training on the number line facilitates better access to fraction magnitude in the brain (Wortha et al., 2020). However, it currently remains unknown how the brain processes fraction magnitudes when presented in visual models, a common method of fraction instruction. Further, how magnitude is processed in the number line format compares to how magnitude is processed in the commonly used area model format remains an underexplored question. These are questions that we aimed to address in the present study.

To address our research question, we collected brain and behavioural measures from adult participants while they completed a fraction magnitude verification task. During this task, participants were presented with a single digit symbolic fraction alongside a fraction model (either a number line or area model). Participants were then asked to assess whether the fraction model was an accurate depiction of the fraction above it or not. To the best of our knowledge, our study is the first to explore the neural activity associated with magnitude processing using fraction instructional models. Therefore, given the novelty in this line of work, we conducted whole-brain analyses so that we could map out all the brain regions involved in our task. From this, we were able to identify the common and distinct neural regions that are involved in processing magnitude using these fraction instructional models for the first time.

4.1 The Number Line and Area Model Elicit Common Fronto-Parietal Activation

A conjunction analysis was run to determine the areas of the brain that revealed significantly common activation for both the number line and area model formats. The conjunction analysis revealed a high degree of overlapping regions that were commonly activated by both the number line and area model. Specifically, these common clusters of activation were largely located in bilateral frontal and parietal regions of the brain. These are neural regions that have been documented to be implicated in the processing of fraction magnitude tasks in specific (Ischebeck et al., 2009) as well as in the processing of a wide array of numerical tasks more generally (see Arsalidou & Taylor, 2011 for meta-analyses). Therefore, contrary to our predictions, this finding suggests that, in the brain, there are many similarities in how fraction magnitude is processed in number line and area model format.

However, it is important to note that while fronto-parietal activity is consistently documented in response to numerical tasks, fronto-parietal activity is by no means exclusive to number processing. Indeed, frontal and parietal regions are associated with a wide variety of cognitive tasks (e.g., Dosenbach et al., 2008; Zanto & Gazzaley, 2013) such as attentional demand (see Buschman & Kastner, 2015 for review) and executive functioning (see Ardila et al., 2018 for meta-analyses). In addition to processing magnitude, these are cognitive processes that were also likely engaged in the critical task, that contribute to the strong fronto-parietal activity observed in the conjunction of the models. We suggest that this is probable given that our control task was a simpler task that required less cognitive demand relative to the critical task (as evidenced by much faster reaction times and the significantly higher response accuracy for the control task compared to the critical task). Nevertheless, this does yield some initial evidence that there are many commonalities in how magnitude is processed in number line and area model format, whereby both models recruit regions that are typically involved in magnitude processing.

4.2 Differences Between Number Line and Area Model Processing in the Brain

While we did find evidence to suggest that there are likely many similarities in how adults process fractions in number line and area model format, we also found evidence to suggest that there are differences as well. Our critical contrast revealed three areas where the neural response significantly differed between the two fraction models. Contrary to our predictions, the only cluster of activation found where the number line was significantly more active than the area model was in primary visual areas of the brain. This result was unexpected given that control conditions were included in the study design with the aim of controlling for the low-level visual differences that are present between the number line and area model. When examining the parameter estimates across conditions from this specific cluster it was found that this activation was being driven by a significant difference in signal among the two control conditions (Figure 7). More specifically, this led to a notably larger difference in signal between the number line and its control than between the area model and its control. While it was not anticipated that such a large difference in signal would arise between the number line and the number line control condition, unexpected primary visual activation is not uncommon. Other studies, including Ischebeck et al. (2009) have identified similar activation in a seemingly visually-matched number paradigm and have speculated that this result could arise due to processes such as mental imagery or reanalysis of the stimuli (Kosslyn et al., 1995; Somers et al., 1999). Further, it is also possible that because our control stimuli did not specifically require all components of the stimulus to be examined in order to make the response, differential visual processing could arise (see Gilbert & Li, 2013, for review). In other words, it is possible that when completing critical trials in number line format participants were processing the entire visual stimulus, whereas in number line control trials only a narrow band of the visual stimulus was being processed. We speculate that it is possible that this difference in how the visual information was being processed in critical versus control conditions may not have arisen to the same degree for the area model because of some of the differences in visual properties between the number line and area model control stimuli (e.g., larger region of colouration for the area model controls) (see Gilbert & Li, 2013, for review). Resultingly, this may have encouraged participants to process the entire visual stimulus more during area model control trials than in number line control trials, thereby leading to a smaller difference in visual signal between the area model critical and control conditions. Regardless, for these reasons, we do not believe this specific cluster of activation is informative with respect to our hypotheses.

The remaining two clusters of activation identified through our critical contrast were regions where the area model displayed a greater neural response relative to the number line. Interestingly, these clusters of activation were located in frontal and parietal regions of the brain, specifically the left inferior frontal gyrus (IFG) and the right parietal lobe around the IPS. This indicates that while both the number line and area model formats commonly recruit regions of the brain that are implicated in magnitude processing, there are regions within this network that are activated to a comparatively greater extent when performing trials in area model format.

4.3 Possible Explanation for Greater Activation in Area Model

To summarize, data from the preliminary analyses revealed that both the number line and area model recruit regions of the brain that are involved in processing numerical magnitude. However, there were clusters within this region that were recruited to a greater extent for the area model relative to the number line. It is important to keep in mind that our study is the first of this kind in a largely novel field of research. Thus, from this study alone, it is not possible to draw definitive conclusions about what these results mean with absolute certainty. However, here, we aim to converge our behavioural and imaging findings to put forward a possible explanation for these results.

While it is possible that this finding indicates that the area model facilitates more magnitude processing in the brain, trends observed in our behavioural data have led us to suggest an alternative explanation that we believe may be more likely. Specifically, we suggest that one potential explanation for this pattern of results is that there are differences in the strategy that is employed when verifying magnitude in each of the two formats. Here, we use the term strategy to refer to "a procedure or set of procedures for achieving a higher-level goal or task" (Lemaire & Reder, 1999, page 365). Previous work has demonstrated that strategy use is prevalent and non-uniform in fraction tasks. For instance, various behavioural studies have demonstrated that feature differences of a fraction task yield within-subject variability in the strategies that are employed to complete the task (Alibali & Sidney, 2015; Fazio, DeWolf, et al., 2016; Schneider & Siegler, 2010; from Sidney, Thompson, & Rivera, 2019). Further, it has been demonstrated that differences in strategy can elicit differences in brain activity (e.g., Polspoel et al., 2017). For instance, neuroimaging work by Mock and colleagues (2018) recorded the neural response of participants while completing proportional magnitude tasks (e.g., magnitude comparison of symbolic fractions, pie chart magnitude comparison, etc.). The authors found that all these tasks elicited frontal lobe activation, regions of the brain typically implicated in strategy choice and procedural planning, in their individual contrasts (Collins & Koechlin, 2012; Grabner et al., 2009; Klein et al., 2016; Park et al., 2019). However, a conjunction analysis of all the tasks did not reveal any significant clusters of common frontal activation (Mock et al., 2018). Among other regions, it has been shown that shifts in strategy and context can modulate differences in activation in the frontal areas of the brain (Wagner et al., 1998). Therefore, the authors interpreted this finding to suggest that these differences in frontal activation could be indicative of different strategies that are applied when processing magnitude in different presentation formats (Mock et al., 2018). Using a similar rationale, we therefore suggest that it is plausible that the differences in activation we are observing between the number line and area model could possibly be reflective of different problem-solving approaches.

Specifically, we put forward the possibility that, perhaps when completing area model trials, a more consistent, deliberative strategy is applied, whereas number line trials are completed with more magnitude guided approximation. Our reasoning for this potential explanation stems from trends observed in our data, as well as in related work. Firstly, self-reported data from the debrief measure indicated that the majority of the participants in our sample learned using the area model, and thus were far more familiar with the area model format than with the number line format. Further, previous work within our lab explored this exact paradigm in a large-scale behavioural pilot study (n = 100) (Henry et al., unpublished). This study revealed a significant speed-accuracy trade-off, whereby participants responded significantly quicker on the number line but significantly more accurate on the area model. Similar trends were also observed in our behavioural fMRI data as well, where number line trials were responded to significantly quicker and area model trials were responded to with greater accuracy (though the response accuracy difference in our behavioural fMRI data did not reach significance, p = 0.17). Taken together, we speculate that due to greater familiarity, participants may have a better idea of how to approach using the area model. This could involve employing a more consistent strategy (less differences in strategy use both within and between participants), an approach that requires greater effort then just rough estimation, and thus takes longer to do but ultimately yields a more accurate result (Wickelgren, 1977).

This proposed explanation could align with the finding that the area model recruits the left inferior frontal gyrus and right IPS to a greater degree than the number line. For instance, neuroimaging work has proposed that the inferior frontal gyrus is implicated during rule-based cognitive operations when problem solving (see Arsalidou & Taylor, 2011 for meta-analyses). Similarly, other work has found that this region plays a role in retrieving stored conceptual representations within the brain (Badre & Wagner, 2007; Becker et al., 2020), and elicits a greater response for tasks that require more effortful processing to execute (Fedorenko et al., 2012). Similarly, increased activity in the right IPS could suggest a greater reliance on manipulating numerical information and semantic representation of the number information (Menon, 2014; Menon & Chang, 2021). Further, greater recruitment of the IPS could also suggest an increased reliance on attention due to greater effort required to complete the task (Mock et al., 2018; Shuman & Kanwisher, 2004). This therefore could align with the hypothesis that participants engage in a more systematic, consistent strategy when completing area model trials, which thus requires more cognitive effort than purely estimation. This could serve as a possible explanation for why certain frontal and parietal regions were recruited more when completing trials in area model format.

4.4 Differences in how Benchmark vs Non-Benchmark Fractions are Processed

To further explore the differences in how the two models are processed, we ran a set of exploratory analyses to examine whether there was evidence to suggest possible strategy differences between the models. To do so, one particular strategy that we assessed was the use of benchmarks. Benchmarks are familiar, commonly encountered numbers that can be used as a reference point when estimating the magnitude of other numbers (Obersteiner et al., 2020). The utilization of benchmark references is a common strategy used in whole number estimation tasks (e.g. Peeters et al., 2016; Peeters, Sekeris, et al., 2017; Peeters, Verschaffel, et al., 2017; Sullivan et al., 2011). While benchmark use in fractions tasks has not yet received much attention in the literature, 48% of our participants freely indicated that they employed a strategy that is consistent with benchmark use during the fraction verification task (e.g., “I tried to compare the fractions to simple fractions that I knew, like one-fourth or one-half”). Further, in our behavioural pilot data (Henry et al., unpublished), we obtained evidence that suggested that the influence of benchmark fractions was greater in area model format then in number line format. Therefore, we explored benchmark fraction processing to see whether the brain processes these similarly or differently in number line and area model format and whether this could hint towards differences in strategy use between the models.

The results of these contrasts demonstrated that the brain processes benchmarks differently in number line and area model formats. In the contrast of area model benchmarks (area model benchmark – area model non-benchmark), we only found significant clusters of activation where benchmarks > non-benchmarks. These clusters of activation were observed in regions within the so-called default mode network (DMN). Given that the default mode network is a task-negative network that shows greater response to easier tasks compared to harder tasks (Buckner et al., 2008; Gilbert et al., 2012; Shulman et al., 1997) we interpret this finding to indicate a highly intuitive, automatic access to area model benchmarks. Given that our participants identified a clear familiarity with the area model, this finding was not surprising to us as these items, in particular, were likely very recognizable and did not rely on too much engagement of magnitude processing.

The contrast of number line benchmarks (number line benchmark – number line non-benchmark) revealed a different finding. Interestingly, in this contrast the only significant clusters of activation revealed were where non-benchmarks > benchmarks. Therefore, contrary to what was seen in the area model benchmark contrast, there were no clusters of activation that were significantly activated more by number line benchmarks. In fact, even when we lowered our statistical threshold down to $p < 0.05$, for exploratory purposes, we still did not yield activity consistent with the DMN for benchmarks, as we did in the area model contrast. This suggests that the brain does not process benchmark fractions in number line format in the same straightforward, automatic fashion that it does in area model format. Moreover, the clusters of activation observed where number line non-benchmark > benchmark were localized in frontal and parietal regions of the brain, including in the right IPS and the right frontal areas. These are areas that are commonly recruited when magnitude processing is engaged (see Arsalidou & Taylor, 2011 for meta-analyses). Further, the behavioural data from this analysis demonstrated that non-benchmarks in number line format were responded to significantly quicker than non-benchmark fractions in area model format. Together, this suggests that, in number line format, when participants do not have the same level of access to recognizable reference points, participants may rely more heavily on a magnitude-guided estimation approach, rather than a consistent strategy.

Taken together, the results of the exploratory benchmark contrasts do provide clear evidence that benchmark and non-benchmark fractions influence the number line and area model formats very differently. Specifically, within the area model format, we obtained evidence to suggest that benchmarks are very easily accessed and are processed in a straightforward fashion in the brain. In contrast, in number line format, no such strong differential seems to exist in the brain for benchmark items. Rather, in the absence of these clear reference points more quantitative strategies, typical of approximate magnitude processing seem to be engaged. Knowing that these differences exist, it is imperative to keep in mind that, in our critical contrast between the models, these trial types (benchmark and non-benchmark) were averaged within the number line and area model format conditions. Therefore, it is possible that these notable differences in how benchmarks influence the two formats may be driving, or at least contributing towards the findings in our critical contrast. Thus, it is possible that there are differences between the two models when true magnitude engagement must be executed that are being washed out due to the disproportionate influence of benchmarks in the area model format. This could explain why activity around the IPS was observed in the critical contrast where area model > number line but also in the number line benchmark contrast where non-benchmark > benchmark. To better understand whether this is the case, we suggest future work should remove the influence of benchmarks and compare only trials that require magnitude estimation to solve (i.e., non-benchmark trials). We suggest that by doing so, this will lend better insight towards whether the clear differences in how trial types are processed between the formats can explain trends seen in our critical contrast.

In summary, the results of the exploratory benchmark contrasts do provide evidence that benchmark fractions influence the number line and area model formats differently. We suggest that these differences could possibly indicate that different strategies are engaged in when completing trials in the two formats, which can potentially contribute to findings seen in our critical contrast. However, it is important to preface that our paradigm was specifically designed to just assess the commonalities and differences in the neural response between the number line and area model formats. This study was not designed to assess the specific mechanisms that are engaged when completing magnitude tasks in number line and area model format. Put differently, our paradigm was not designed to definitively answer the question of whether different strategies are engaged or relied upon when using the number line versus the area model. Therefore, it is imperative to keep in mind that we are simply suggesting one particular factor that may be at play based on the trends seen in our data. To gain more direct insight into whether strategy differences can truly account for the differences in brain activity observed between the models, we encourage future work to build off this study and intentionally design a paradigm with this goal. We believe that future exploration into this question is required to better understand what the differences in neural activity between the number line and area model correspond to.

4.5 Limitations and Future Directions

Our study has contributed to the, currently, very limited understanding of how fractions are processed in the brain. Specifically, we have been the first to explore and compare the neural activity associated with fraction instructional models. Accordingly, we believe that our study can serve as a strong baseline framework for future neuroimaging studies exploring methods of fraction learning. Notably, this is possible because we have taken the measures to create a fully reproducible workflow through pre-registration, utilizing a standard preprocessing pipeline and providing open access to our study paradigm and stimuli. However, there are clear limitations that we have identified in our current design that are worth taking note of. In this section, I review some of these limitations and follow each up with a specific recommendation of how future work can better explore this overarching question.

1) The role of familiarity and learning history in our sample

Firstly, it is important to note that our sample was composed of adult participants. We focused on adults as this is the first study to explore how the brain processes fraction magnitude using the different learning models. Thus, we aimed to get a baseline understanding of how this task functions in a neuroimaging study before running this paradigm in children. Nevertheless, examining adults in this context does introduce some caveats. First and foremost, fractions are learned and worked with early on in primary school education and throughout formal learning years (CCSSI, 2010; Ontario Ministry of Education, 2005). Therefore, it is expected that at this point all participants in our sample will have already had extensive experience with fraction concepts. Given that there were disparities in familiarity between the models, whereby individuals were more familiar and had more learning experiences with the area model format, it is possible that learning history may impact trends seen in both the behavioural and imaging data. Indeed, previous work has demonstrated that learning history does play a role at the level of the brain, and that the role of instruction can remain evident beyond training (e.g., Yoncheva et al., 2015). Thus, it is very possible that previous experience is a factor that could be influencing the results we observed. Therefore, replicating this study in a developmental population or in a sample with a different learning background may yield different results than what we obtained here.

To address the impact that learning history may play, we encourage future work to expand off this study and conduct a related study in a developmental population. Specifically, we suggest exploring a similar paradigm using a sample of children who are at the age when fractions are being taught for the first time. We suggest that by doing so, the role of previous learning experiences will be largely reduced, thereby mitigating the role of potential confounds in the behavioural and imaging data. One possible method for doing so could simply be conducting a complete replication of our study using a sample of school-aged children. A second possible method could involve employing a similar design as Wortha et al. (2020), a pre-test-training-post test design, whereby one group is trained on the number line and a second group on the area model. Regardless, we believe addressing this limitation can lend more fundamental insight toward which model yields the most effective fraction learning outcomes.

2) Our sample contained predominantly educated STEM students

Secondly, it should be kept in mind that our sample consisted exclusively of university students. Therefore, all our participants have completed, or are working towards completing a form of higher-level education. Further, the majority of participants in our study (92%) self-reported that they were completing a degree in a STEM-related program (e.g., Neuroscience, medical biophysics, computer science, biology, etc.). This means that the majority of our sample has likely completed at least some level of higher education mathematics, as this is often pre-requisite for university STEM programs (Dooley et al., 2016). Indeed, mathematical proficiency and competency with fractions was demonstrated by the high scores obtained on the fraction concepts measure as well. Therefore, it is worthwhile to note that our results come from a sample of educated adults who are likely proficient with math in a manner that may not be reflective of the general population. Level of education is a factor that can account for variability in brain activity. For instance, individuals with different levels of education may recruit different strategies when completing cognitive tasks (Springer et al., 2005; Stern et al., 1999). Thus, it is possible that having a highly educated sample could impact the trends observed in our data as well.

Accordingly, we suggest that future work exploring similar questions should aim to obtain more diversity in the sample collected. We encourage future work to incorporate more thorough pre-screening in the inclusion criteria and systematically recruit participants from more diverse areas to ensure that these findings are generalizable. It would be interesting to see how these results compare to those obtained in our study using an educated sample.

3) Fraction models are produced not verified in school

Finally, it should be kept in mind that our study was a first attempt in trying to understand how magnitude processing using fraction models are processed in the brain. To do so we designed a magnitude verification task so that we could explore fraction model processing whilst balancing the restrictions of running a cognitive task in an MRI scanner. Put differently, we took consideration of what could be done that would not require too much movement of the head or body, an important component of collecting viable neuroimaging data (Friston et al., 1996; Power et al., 2012; Wylie et al., 2014). While this study undoubtedly provides meaningful insight, it should be noted that verification is not truly the process that is engaged in when learning and using fraction models (Doğan & Işik Tertemiz, 2020). Indeed, most often in school, children do not examine a model and fraction to verify that $\frac{4}{8}$ is accurately shown on the number line, but rather children are asked to draw a tick where $\frac{4}{8}$ would be on a blank line (Doğan & Işik Tertemiz, 2020). Therefore, by asking participants to verify rather than produce we introduce some caveats in terms of external validity.

To address this, we challenge future work to design a paradigm that can assess how fraction models are used more typically in school. Specifically, this would require the act of production rather than verification. For instance, this could involve a paradigm that requires placing the tick on the number line or shading in the circle according to a given fraction item. To support this, we suggest future work should consider making use of a more advanced hand-held response system that includes features beyond button presses (i.e. scrolling wheel, joystick, etc.) (e.g., Jarrahi et al., 2013). While we recognize that this task will require intentional creative design and more post-study scoring of responses, we believe that this would provide a better estimate of the true brain response that is engaged when completing fraction tasks in each of these instructional models. In addition, previous work has exemplified that the translational potential of fMRI research is improved when the paradigm resembles the real-world context as closely as possible (Lowe, 2012; Maguire, 2012; Seghier et al., 2019). Thus, we suggest that incorporating a design that is more reflective of the true fraction learning experience may be more effective for guiding changes in the classroom.

4.6 Conclusion

Fraction knowledge is an important educational milestone but is unfortunately associated with many learning challenges. Therefore, better understanding the tools used to teach fractions is an important educational question. To date there has been very little work that has explored how fraction magnitudes are processed in the brain and virtually no work that has directly explored methods of fraction learning in the brain. In this study, we were able to contribute to this very limited body of research and provide valuable insight towards how the brain processes fraction magnitude in two of the most common formats for fraction learning: the number line and area model. While we did not yield results that were consistent with our predictions, we were able to successfully identify commonalities and differences in how the brain processes fraction magnitude using these two learning models.

Our study provides evidence to suggest that, at least in adult participants, fraction magnitude processing in number line and area model format are processed highly similar in the brain. Both the number line and area model recruited fronto-parietal regions of the brain, consistent with typical number magnitude processing. However, while the two formats are processed highly similarly, the brain does not process these two models identically. Indeed, we found clusters of activation in frontal and parietal regions that were recruited to a greater extent in the area model format. Post-hoc exploratory analyses of this data generated evidence that certain fraction types (i.e., benchmarks) are processed differently in number line and area model format. We therefore take these findings to suggest that this could hint towards differences in strategies that are engaged when using these two models. However, future work would need to design a paradigm to address this question directly to verify if this is what is driving the differences in activation.

Taken together, the results from this study provide a baseline understanding of the neural activity that supports fraction processing in number line and area model formats. Further, our study provides a reference framework that can be used to generate testable hypotheses about the mechanisms that support fraction processing using different learning methods. Our study, in conjunction with future neuroimaging work on this topic, will lend valuable insight towards which method of fraction magnitude learning yields the best learning outcomes. Ultimately, we hope this work can serve as a step forward in the quest to remediate the challenges in fraction learning.

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Appendices

Appendix 1: NMREB Ethics Approval

Western Research

Date: 14 April 2022

To: Prof. Daniel Ansari

Project ID: 120528

Study Title: Fraction Magnitude Understanding Across Learning Formats: An fMRI Study

Short Title: Fraction Magnitude Understanding Across Learning Formats

Application Type: NMREB Initial Application

Review Type: Delegated

Full Board Reporting Date: 06/May/2022

Date Approval Issued: 14/Apr/2022 10:03

REB Approval Expiry Date: 14/Apr/2023

Dear Prof. Daniel Ansari

The Western University Non-Medical Research Ethics Board (NMREB) has reviewed and approved the WREM application form for the above mentioned study, as of the date noted above. NMREB approval for this study remains valid until the expiry date noted above, conditional to timely submission and acceptance of NMREB Continuing Ethics Review.

This research study is to be conducted by the investigator noted above. **All other required institutional approvals and mandated training must also be obtained prior to the conduct of the study.**

Documents Approved:

Document Name	Document Type	Document Date	Document Version
Fraction Measure	Paper Survey		
Example_Stimuli_2022	Other Data Collection Instruments		
Demographics Form_April2022	Other Data Collection Instruments		
Poster_Clean_April 2022	Recruitment Materials		
Recruitment_Email_Script_Clean_April2022	Recruitment Materials		
Letter of Information and Consent_Clean_April2022	Written Consent/Assent		

Documents Acknowledged:

Document Name	Document Type	Document Date	Document Version
fMRI_screening_form	Screening Form/Questionnaire		

No deviations from, or changes to the protocol should be initiated without prior written approval from the NMREB, except when necessary to eliminate immediate hazard(s) to study participants or when the change(s) involves only administrative or logistical aspects of the trial.

The Western University NMREB operates in compliance with the Tri-Council Policy Statement Ethical Conduct for Research Involving Humans (TCPS2), the Ontario Personal Health Information Protection Act (PHIPA, 2004), and the applicable laws and regulations of Ontario. Members of the NMREB who are named as Investigators in research studies do not participate in discussions related to, nor vote on such studies when they are presented to the REB. The NMREB is registered with the U.S. Department of Health & Human Services under the IRB registration number IRB 00000941.

Please do not hesitate to contact us if you have any questions.

Sincerely,

Appendix 2: List of Fraction Items used in Study

Fraction Items		
$\frac{1}{2}$	$\frac{2}{7}$	$\frac{4}{8}$
$\frac{1}{3}$	$\frac{2}{8}$	$\frac{4}{9}$
$\frac{1}{4}$	$\frac{2}{9}$	$\frac{5}{6}$
$\frac{1}{5}$	$\frac{3}{4}$	$\frac{5}{7}$
$\frac{1}{6}$	$\frac{3}{5}$	$\frac{5}{8}$
$\frac{1}{7}$	$\frac{3}{6}$	$\frac{5}{9}$
$\frac{1}{8}$	$\frac{3}{7}$	$\frac{6}{7}$
$\frac{1}{9}$	$\frac{3}{8}$	$\frac{6}{8}$
$\frac{2}{3}$	$\frac{3}{9}$	$\frac{6}{9}$
$\frac{2}{4}$	$\frac{4}{5}$	$\frac{7}{8}$
$\frac{2}{5}$	$\frac{4}{6}$	$\frac{7}{9}$
$\frac{2}{6}$	$\frac{4}{7}$	$\frac{8}{9}$

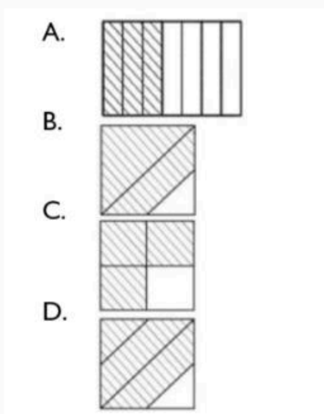
Note. List of fraction items presented in the study. Fraction items presented were all single-digit symbolic fractions with numerators ranging from one to nine and denominators ranging from two to nine.

Appendix 3: Questions Included on the Fraction Concepts Measure

Fraction Concepts Understanding Measure

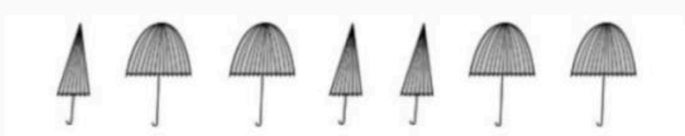
Please complete the following 15 fraction questions as accurately as possible. You will have three minutes to complete this measure.

Which shows 3/4 of the picture shaded?



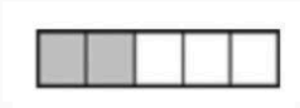
○ A
○ B
○ C
○ D

What fraction of the group of umbrellas is closed?



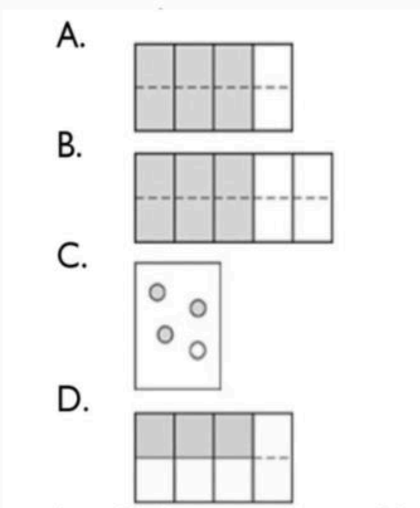
○ 1/3
○ 3/7
○ 4/7
○ 3/4

What fraction of the figure below is shaded?



These three fractions are equivalent. Write two more fractions that are equivalent to these.

Which picture shows that 3/4 is the same as 6/8?



- ○ A
- ○ B
- ○ C
- ○ D

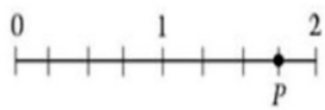
Luis had two apples and he cut each apple into fifths. How many pieces of apple did he have?

- ○ 2/5
- ○ 2
- ○ 5
- ○ 10

$$\frac{4}{6} - \frac{1}{6} =$$

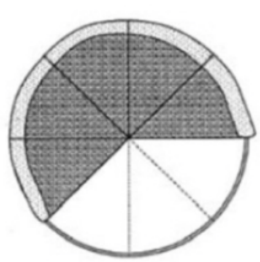
How many fourths make a whole?

On the number line below, what number does P represent?



- ○ 2/3
- ○ 3/4
- ○ 1 2/3
- ○ 1 3/4

The figure below shows that part of a pizza has been eaten. What part of the pizza is still there?



- ○ 3/8
- ○ 3/5
- ○ 5/8
- ○ 5/3

Students in Mrs. Johnson's class were asked to tell why 4/5 is greater than 2/3. Whose reason is best?

- ○ "Because 4 is greater than 2"
- ○ "Because 5 is greater than 3"
- ○ "Because 4/5 is closer than 2/3 to 1"
- ○ "Because 4+5 is larger than 2+3"

Which fraction has a value closest to 1/2?

- ○ 5/8
- ○ 1/6
- ○ 2/2
- ○ 1/5

What fraction of the figure below is shaded?

- ○ 1/4
- ○ 3/10
- ○ 1/3
- ○ 3/7

Mark says 1/4 of his candy bar is smaller than 1/5 of the same candy bar. Is Mark right? Explain why you think Mark is right or wrong.

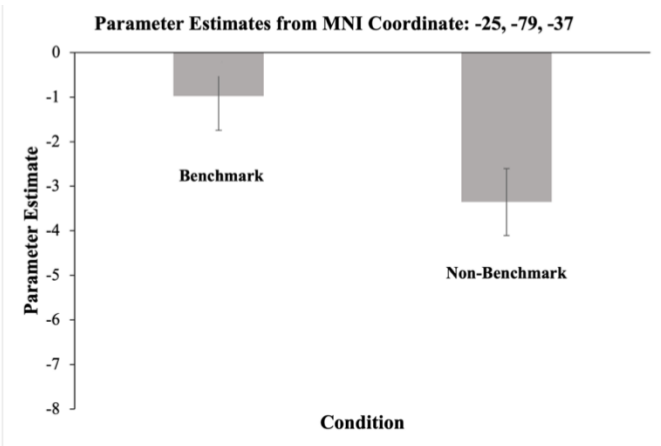
In which of the following are the three fractions arranged from least to greatest?

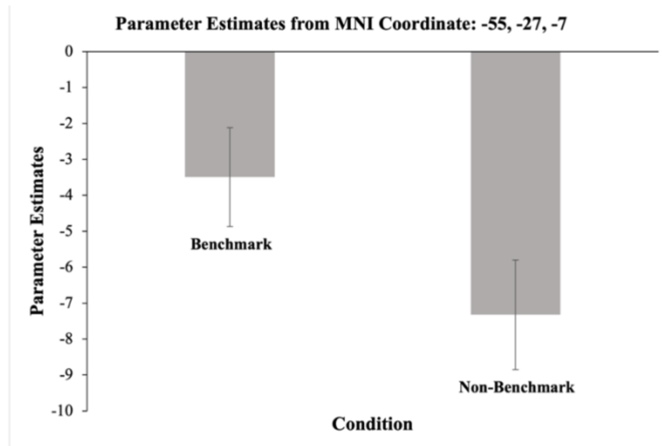
- ○ 2/7, 1/2, 5/9
- ○ 1/2, 2/7, 5/9
- ○ 5/9, 1/2, 2/7
- ○ 5/9, 2/7, 1/2

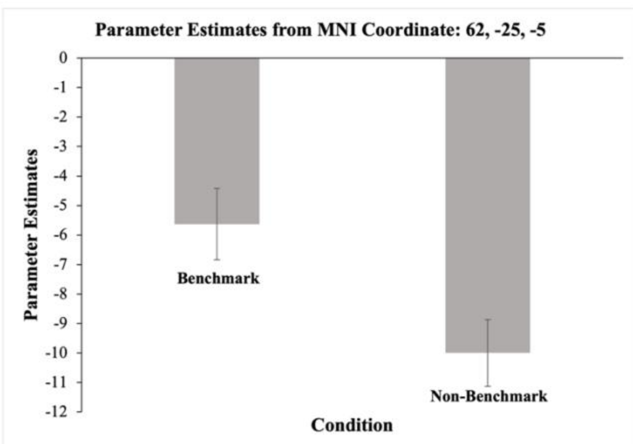
Appendix 4: Fractions Included in the Benchmark vs Non-Benchmark Categories in Exploratory Analysis

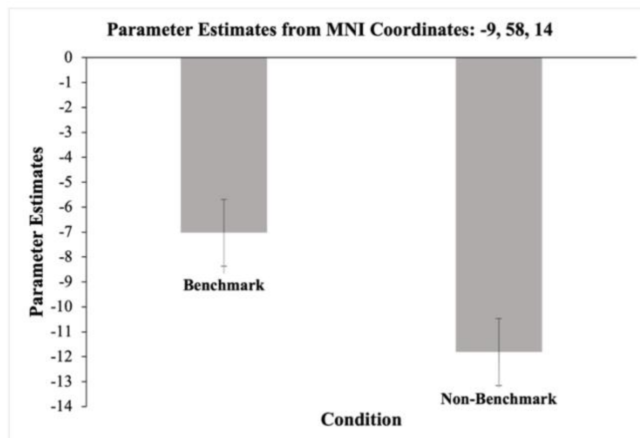
Trial Type	
Benchmark	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{3}{4}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{2}{6}, \frac{3}{9}, \frac{2}{8}, \frac{4}{6}, \frac{6}{9}, \frac{6}{8}$
Non-Benchmark	$\frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{2}{5}, \frac{2}{7}, \frac{2}{9}, \frac{3}{5}, \frac{3}{7}, \frac{3}{8}, \frac{4}{5}, \frac{4}{7}, \frac{4}{9}, \frac{5}{6}, \frac{5}{7}, \frac{5}{8}, \frac{5}{9}, \frac{6}{7}, \frac{7}{8}, \frac{7}{9}, \frac{8}{9}$

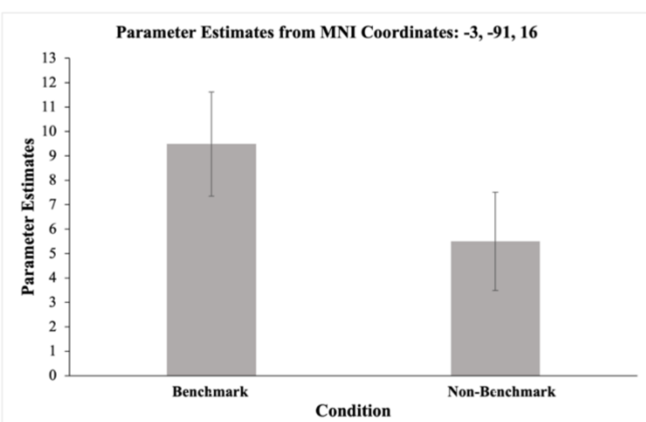
Appendix 5: Parameter Estimates from Area Model Benchmark Contrast

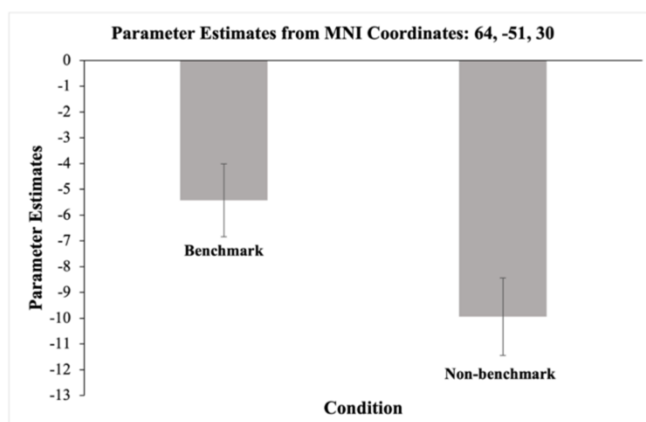


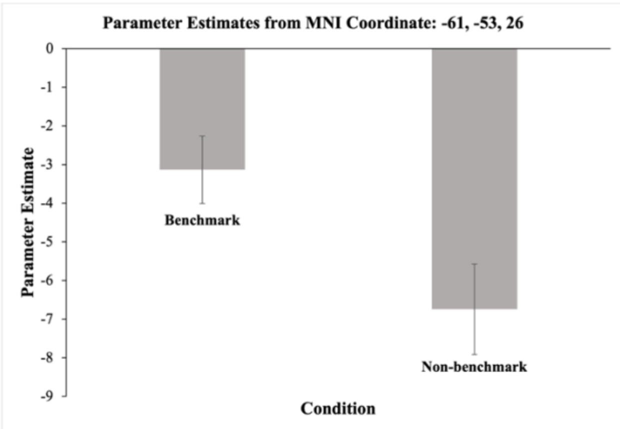


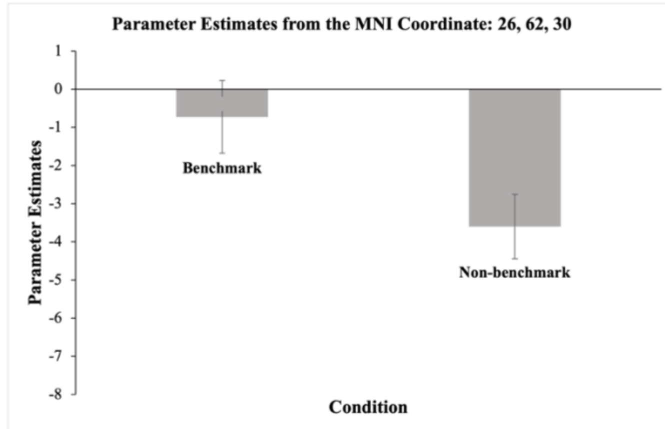


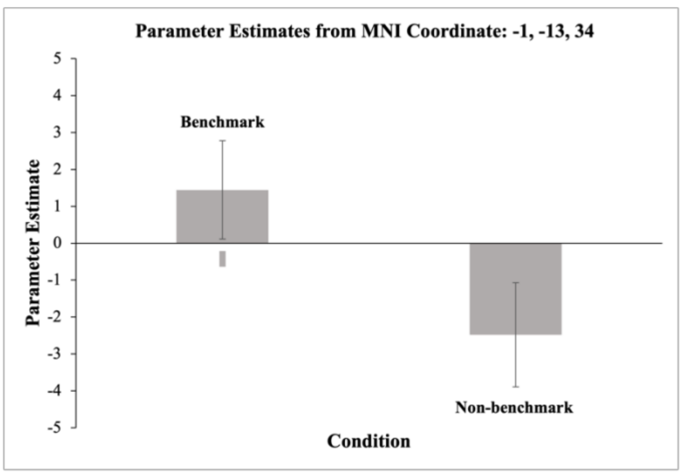






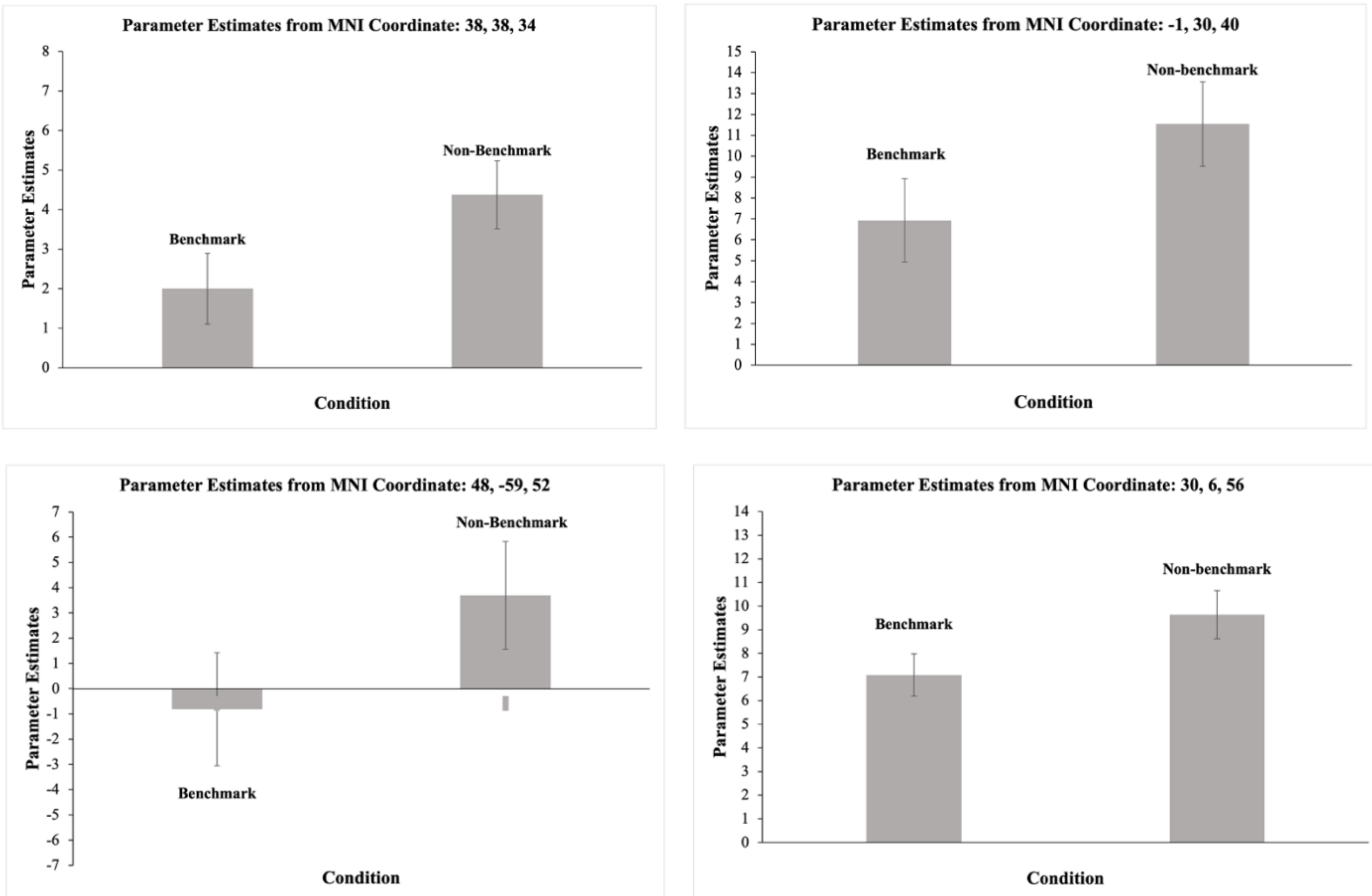






Note. Error bars represent standard error. Parameter estimates were obtained by drawing a 6mm sphere around the peak coordinate from each cluster.

Appendix 6: Parameter Estimates from Number Line Benchmark Contrast



Note. Error bars represent standard error. Parameter estimates were obtained by drawing a 6mm sphere around the peak coordinate from each cluster.

Curriculum Vitae

Name:	Chloe Henry
Post-secondary Education and Degrees:	The University of Western Ontario London, Ontario, Canada 2017- 2021 BSc (Honours) The University of Western Ontario London, Ontario, Canada 2021 - 2023 MSc, Neuroscience
Honours and Awards:	The Western Scholarship of Excellence 2017 - 2018
Related Work Experience:	Teaching Assistant The University of Western Ontario 2021 – 2023 USRI Summer Research Intern The University of Western Ontario Supervisor: Dr. Scott MacDougall-Shackleton 2020
Research Experience:	Advanced Facility for Avian Research The University of Western Ontario Supervisor: Dr. Scott MacDougall-Shackleton 2020-2021 Numerical Cognition Lab The University of Western Ontario Supervisor: Dr. Daniel Ansari 2021-2023