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The Investigation of Non-STEM Undergraduate Students' Geometric Cognition Development within an Embodied Cognition Lens

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Abstract

According to Lakoff & Núñez (2000), the mind does not operate separately in the process of building mathematics concepts. Incorporating the human body and the material surroundings into mathematics learning may contribute to the development of students' mathematical minds, which is consistent with the theory of embodied cognition asserting that cognition rests on the close interaction between the human body and mind, also coupled with the environment. To seek the possibility of applying this theory to mathematics education, this study examines non-STEM undergraduate students' geometric cognition development as they took part in a geometry course that integrated the arts and the dance. The data comes from a previous study that was conducted at a university in New York, mainly including videos of non-STEM undergraduate students' classroom interactions, and student submissions of select assignments. This study can inform educators of embodied approaches in further mathematics education by analyzing how students develop their geometric cognition through an embodied theoretical lens.

Keywords

Embodied Cognition, Mathematics Education, Geometry, Cognition Development, Interdisciplinary Education.

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Table of Contents

List of Tables

List of Figures

List of Appendices

Chapter 1

1 Introduction

1.1 Status Quo of American Mathematics Education

Mathematical knowledge and thinking play an important role in our individual and social lives (Mutlu, 2019). For example, every financial or investment strategy can not be discussed without analytical mathematics. The demographic challenges facing the United States' future can be predicted by a mathematical model. As the leading set of important "processes and proficiencies" in the American Common Core State Standard for Mathematics (CCSSM), mathematical competencies, such as problem-solving, reasoning and proof, communication, representation, and connections, guide and influence our life experiences. Further, students' experiences and performances in mathematics classes directly affect which Science, Technology, Engineering, Mathematics (STEM) field they pursue as a career (Laursen & Rasmussen, 2019). Such many careers in STEM fields, require basic or advanced mathematics knowledge as a foundation since mathematics is the body of knowledge in the area of science and technology (Raj Acharya, 2017). Petrillo (2016) and Ogden (2015) cautioned that students' negative experiences in learning mathematics may discourage them from choosing careers in STEM fields (Lo et al., 2017). Moreover, knowledge of mathematics that a student has is often deemed as a proxy for intelligence and only ones who have sufficient mathematical knowledge gain access to high-level mathematics coursework which serves as a gateway to academic and economic opportunity linked especially to scientific and technological progress (U.S. Department of Education 2008, as cited in Martin, 2019). Thus, those factors imply that we need to pay more attention to students' important "processes and proficiencies" development in mathematics.

1.2 The Statement of Problem

The National Assessment of Educational Progress (NAEP) assesses student performance in mathematics in Grades 4, 8, and 12 in both public and private schools across the United States of America. NAEP achievement levels define what students should know and be able to do: Basic indicates partial mastery of fundamental skills, Proficient indicates solid academic performance and demonstrated competency over challenging subject matter, and Advanced indicates superior performance beyond proficient (National Center for Education Statistics, 2022). The percentage of students assessed at the basic level is around 40% in general from 1990 to 2022 grades 4 and 8, which is a relatively big portion (39% for grade 4 in 2022, 35% for grade 8 in 2022, 35% for grade 12 in 2019 due to the unavailable data for 2022). Even more surprising is that about 30% of students are below the basic level. Totally, up to 70% of students are only at basic mathematics achievement level or below basic level. Additionally, it is noticeable that the statistics of students at lower levels keeps escalating from grade 4 to grade 8, and 12 (59% for grade 4 in 2019, 66% for grade 8 in 2019, 75% for grade 12 in 2019), which indicates that students feel it more and more tricky to master mathematics knowledge and skills as they step into higher grades.

Based on the results of "The Associated Press/AOL POL: Back to School Study" conducted by Ipsos-Public Affairs (2005), math is the most unpopular school subject, especially among female students. Americans hold a belief that mathematics is difficult (Raj Acharya, 2017) and that people's mathematics abilities are fixed rather than malleable, adaptive, and able to grow (Dweck & Leggett, 1988). When studying mathematics, students have a high potential of feeling miserable (Sonnert et al., 2020). Sainio et al. (2019) carried out a study regarding the role of learning difficulties in adolescents' academic emotions and academic achievement in math domains among 845 Grade 6 adolescents, revealing that students who are suffering mathematics difficulties (MD) have lower enjoyment, lower hope, and higher anxiety than those without MD. Mutlu's (2019) research conducted among elementary school students reveals that there was a strong correlation ($r=-.597$) between the math anxiety and math achievement of the participants, which implies that the learning difficulties that students experienced are related to their negative attitudes and further affect their efficiency in learning mathematics, increasing the risks of dropping out of mathematics in their life. According to Mutlu's (2019) conclusion concerning the leading reasons of failing to recognize the importance of mathematics, the abstract and hierarchical structure of mathematics, methods and strategies in teaching mathematics, and the learning difficulties in mathematics lead to obstacles that prevent students from mathematics acquisition and enjoyment (Mutlu, 2019).

Making mathematics meaningful and applicable to students' lives is always full of challenges at the stages of K-12. Even if young children are perceived having a natural, spontaneous capabilities in mathematical attainments (Verbruggen et al., 2021), a wide variety of factors hinder their mathematical achievements. According to Thanheiser & Koestler (2021), prospective elementary school teachers who are assigned to teach a general range of subjects often hold a relatively negative attitude towards mathematics, seeing mathematics as a neutral, universal, procedural, daunting subject, and as a disconnected body of knowledge, not related to the real world. Those opinions kept in the teachers' minds subtly and unconsciously may impact children's learning. Consequently, both teachers' knowledge and beliefs have a significant role in mathematics instruction practice and therefore it is necessary to prepare adequately and support teachers successfully to bridge the gap between the expectations of national standards (National Council of Teachers of Mathematics [NCTM], 2018) and the status quo of students' mathematics learning difficulties.

As for other approaches aiming to enhance younger students' mathematics learning, many scholars reported that digital games (e.g., video games or computer games) serve as an effective instrument since they can boost students' motivation, attitudes and performance in learning mathematics (Byun & Joung, 2018). Likewise, Verbruggen et al. (2021) claim that educational technology (ET) can be an effective tool for supporting preschoolers' early mathematical development by interacting with the learner offering unique opportunities for supporting learning and instruction with immediate feedback and adaptivity. The fact that features of engagement and interaction in any assistant technology are increasingly attracting educators' attention indicates the demand and trend of improvements.

When it comes to undergraduate mathematics education, many educators or researchers are enacting explorations and innovations aiming to promote the quality of mathematics instruction and meeting students' needs. That is because there is a call to provide students with learning experiences that are rich and meaningful (Johnson et al., 2019). For instance, a special type of digital textbook that have embedded interactive features is designed and applied in the classroom so that it can get students engaged with mathematics. Another example is the application of a flipped classroom approach, which benefits students learning by increasing the students' interactions with their instructors and classmates during in-class sessions and preparing students for activities before class (Lo et al., 2017). Those initiatives reveal the fact that undergraduate mathematics instruction desires a more embodied and interactive method to activate students' enthusiasm in coping with the difficulties in the process of learning mathematics. As the statistics shown in a survey of mathematics tutoring centres conducted in the United States, undergraduate students have an overwhelmed need in attaining mathematics support in their school, with a statistic of 89.5% of the 118 universities offering calculus tutoring by undergraduates as an example (Mills et al., 2022). However, addressing the difficulties and facilitating students' engagement in the learning processes seems to be more practical and effective than remediation after learning difficulties arise so that it needs to be taken into account as early as possible.

Besides, the United States is working toward inclusive mathematics education with a faith that "Mathematics is not only for all but also of all, that is, it belongs to all learners (Padilla & Tan, 2019, p. 299)." Equity-oriented resources are supposed to be offered to all students who are not differentiated by dis/ability, a variety of race or backgrounds. With the goal of advancing inclusive mathematics education practices, individualized education that incorporates the characteristics of different students deserves more attention in the context of dynamic reforms on educational policies and teaching practices.

In terms of the problems of mathematics that are too ethereal to understand, some educators suggest connecting mathematics to a real-world problem. For instance, Abramovich et al. (2019) propose action learning in mathematics education that is combined with rote theory to bring mathematical topics to the real world. We are inclined to use the notion of embodied cognition, which is a theory of learning that highlights the combination and interaction between mind, body and environment. It assumes that cognition is shaped by our bodies and the way we act on and are acted on by the environment (Valentine & Kopcha, 2020).

In this regard, mathematics is not conceived as a mental activity purely operated by the human brain and limited by human mental capacities anymore; rather, it should be a product of the coordination of mind-body-environment interactions in which the body serves as a mediator and is capable of reifying abstract mathematical concepts in learners' mind. The use of embodied cognition converts mathematics to a concrete and tangible subject and engages students deeper into mathematics learning rather than merely knowledge input into their brains.

The development of the capability of transformation between abstract thoughts and concrete ideas in mathematics aligns with the requirements that Common Core State Standards for Mathematics (CCSSM) set up for K-12 students. Standards stress the operation of symbols in mathematics. Students are supposed to have the potential to abstract the real situation and represent it symbolically, which enlightens educators to think about how to construct a flexible mathematics curriculum that allows students to practice this skill and meet their needs.

To sum up, incorporating the idea of embodiment into the design and operation of mathematics instructions seems feasible but still needs to be evidential. The question at stake is how to exemplify the effectiveness of doing mathematics in an embodied way in a class.

1.2.1 The Statement of Purpose

To address this issue, we analyze an undergraduate interdisciplinary course that focused on Science, Technology, Engineering, the Arts and Mathematics (STEAM) learning in multi-model ways. This course was co-taught by faculty in STEM Education and Dance with unique specialties and had nine non-STEM students enrolled initially. This study uses secondary data collected from these students who consented to be participants in this research. By analyzing students' classroom performances and their assignments, this study intends to use an embodied cognition lens to examine how non-STEM students develop geometrical cognition.

1.2.2 Research Questions

RQ1: How do non-STEM students demonstrate geometrical concepts and patterns as they develop geometric cognition?

RQ2: How do non-STEM students leverage geometrical concepts and patterns as they develop geometric cognition?

1.3 The Significance of Research

First, the exploration of integrating dance into mathematics education is unique and rare within the context of non-STEM undergraduate students. It is important to investigate how learning takes place when students leverage their funds of dance knowledge and potential capacities like creativity to build mathematical understanding, which will be a fantastic supplement to the research of interdisciplinary education, especially in the STEAM fields. Second, in terms of its contribution to mathematics education in the United States of America which pursues the goal of "inclusive", this study brings an innovative and personal mathematics educational practice that was tailored for featured learners and examines whether it can mitigate the learning difficulties and facilitate students' engagement in the learning processes. Third, our research matters for the theoretical construct of embodied cognition by offering a novel and important lens to interpret bodily movements being operated in mathematics classes. Ample and profound empirical resources produced in this study not only can add to the limited knowledge of educators who are unfamiliar with theories of embodied cognition, but also benefit researchers for further analysis and appraisal of these theories.

1.4 Embodied Cognition Theory

Valentine & Kopcha (2020) states that embodied cognition is a theory of learning rooted in existential phenomenology and ecological psychology (Heidegger, 2008; Merleau-Ponty, 2002; Gibson, 1986). The theory asserts that the formation of concepts or the production of cognition rests on the close interaction between the human body and mind, also coupled with the environment. In this perspective, cognition is considered as strongly bodily based (Wilson, 2002, p.632, as cited in Arzarello & Robutti, 2008) and human beings use their bodies to “think” (Seitz, 2000).

This assertation has a significant bearing on mathematics education since it offers a novel and incredible lens to see how mathematical concepts are developed in students’ minds. Lakoff & Núñez (2000) took an early step in the application of embodied cognition in mathematics which is viewed as mind-based, limited and structured by human bodies and minds. According to their perspectives, human makes fundamental use of conceptual metaphor in characterizing mathematical concepts. Conceptual metaphor is a cognitive mechanism that belongs to the realm of thought, essential to mathematical thought, which can enable us to reason about an abstract concept as if it were another one that has existed in our common experiences, and thereby we can conceptualize abstract concepts in everyday concrete terms so that mathematics becomes much more accessible and comprehensible (Lakoff & Núñez, 2000). In this regard, ideas and modes of reasoning used in the operation of conceptual metaphors are grounded in the sensory-motor system where Gallese & Lakoff (2005) believe that conceptual knowledge is mapped and embodied by human bodies’ moving, seeing, touching, etc. The sensory-motor system responds to the way that we function with our bodies in the world and provides structure to conceptual content, as well as characterizes concepts (Gallese & Lakoff, 2005). With this system, our brains, our bodies, our evolution, our environment, our long social and cultural history and any other individual factors like previous knowledge and experiences jointly formulate our understanding of mathematics concepts.

In the class that we targeted, bodily movements are taken as conceptual metaphors for students to build and perform their understanding of several complicated and delicate geometric shapes. They were also producing their mathematical embodied cognition by acting with their body and the environment within the sensory-motor system. Our research will rest on embodied cognition theory to interpret students’ performances.

Chapter 2

2 Literature Review

This chapter focuses on the essential theory applied to our research-embodied cognition and its associated concept, the conceptual metaphor. These notions are presented with explicit definitions and detailed examples that were extracted from a wide range of literature that is concerned with this theory and related topics. Additionally, we present a general overview of the Laban/Bartenieff Movement Analysis which is innovatively applied to our observed target unit—the interdisciplinary undergraduate course.

2.1 Embodied Cognition

Embodied cognition is a relatively novel concept in cognitive science. Around three decades ago, mainstream cognitive scientists commonly conceived the mind as an abstract and symbolic information processing system where disembodied symbols are abstractly manipulated and constitute internal ‘‘representations of external reality’’ without serious mediation by the body and brain (Lakoff, 2012). Concepts, from this perspective, were conceived of as represented in some “language of thought” and made up of symbols (Fodor, 1975, 1987, as cited in Gallese & Lakoff, 2005). Inspired by the emergence of substantial empirical evidence that implicates the interaction of body and mind, cognitivists gradually realize that the formation of thoughts is always inextricably related to the work of perception and action systems (Barsalou, 1999; Barsalou, Niedenthal, Barbey, & Ruppert, 2003; Glenberg, 1997; Wilson, 2002, as cited in Markman & Brendl, 2005) and that the specifics of human physiology and interaction between the person and the material and social world in which the person thinks and acts greatly matter in cognitive development (Amin et al., 2015). In other words, the perception and action systems are considered actively involved in higher-level cognitive processes (Markman & Brendl, 2005), arguing against the idea of viewing our sensory-motor system as serving a peripheral, input/output role.

The birth of the idea that cognition takes place in the perceptual and motor systems can be traced to Gibson (1986)'s ecological theory of perception. He posits that perception could be understood in terms of sensory information—visual, kinesthetic, olfactory, etc. (as cited in Weisberg & Newcombe, 2017). Gallese & Lakoff (2005) subsequently argue that conceptual knowledge is embodied and mapped within human's sensory-motor system, which is conceived of as not only providing structure to conceptual content but also characterising the semantic content of concepts in terms of the way that we function with our bodies in the world. The two main presupposed mechanisms of how the sensory-motor system works are delineated in the next subsection.

2.1.1 The Sensory-Motor System

Firstly, Gallese & Lakoff's (2005) arguments heavily rely on a major finding in neuroscience: Imagining a thing and doing that thing uses a shared neural substrate. In other words, we usually use some of the same parts of the brain when we imagine an action or an item and when we manipulate it. Gallese & Lakoff (2005) show us an example to demonstrate: One can reason about grasping without grasping, yet one may still use the same neural substrate in the sensory-motor system. The hypothesis of the same neural substrate used in imagining and understanding implies the idea that the understanding of concepts has the same path as the imagination. They are both embodied and structured by our constant encounters and interaction with the world via our bodies and brains.

The second important assumption about the sensory-motor system is the coordination of bodies and brains. Based on the results of Gallese & Lakoff's (2005) exploitation in cognitive neuropsychology, we believe that the sensory-motor system is "multimodal", which links together many modalities by infusing each with properties of others. And on the other hand, the body provides the cognitive system with sensory inputs like sight, hearing touching, motor actions, etc. (Weisberg & Newcombe, 2017). Therefore, we can conclude as Atkinson (2010) said, bodies link minds to the world—we experience, understand, and act on the world through our bodies.

The current research literature supports the idea that cognition is embodied. The most common and straightforward claim on embodied cognition can be found in Wilson & Golonka's (2013, p. 1) article, which said “states of the body modify states of the mind.” Markman & Brendl (2005) claim that accounting for embodied phenomena requires understanding the complex interplay between perceptual and motor representations and people’s representations of their selves in space. Amin et al. (2015) suggest that models of cognition need to attend to the characteristics of human brains and bodies, and the material contexts in which thought is taking place after summarizing and synthesizing many trustworthy prior research results (e.g., Barsalou 2008; Clark & Chalmers, 1998; Shapiro, 2011; Varela, Thompson, & Rosch, 1991; Wilson, 2002). Likewise, Shapiro & Stolz (2019) review many neuroscientific findings regarding the connection between cognitive processing and brain areas with physical motion, concluding that the brain represents the world not through amodal symbols but, in effect, by trying to imagine a body in active engagement with the world. To develop a more concrete and specific idea of embodied cognition, we will present several dominant characteristics and applications of this theoretical concept in the next section.

2.1.2 Six Valuable Claims under Embodied Cognition

We draw the following claims from Wilson (2002), which can in the most effective way describe the nature of embodied cognition and assist in forming an outstanding general idea on this topic (as cited in Amin et al., 2015).

> (1) Cognitive processes are situated, varying depending on the real-world contexts in which they are carried out.
>
> (2) Cognitive processes must be understood with respect to the specific temporal constraints imposed on our brains by the environment when cognitive tasks are carried out.
>
> (3) Cognitive processes recruit the material, symbolic and social structure of the environment, reducing what needs to be performed in the mind itself.

(4) Cognitive systems can be viewed as extended, where there is no sharp divide between internal and external contributions to cognition.

(5) The function of cognition is not primarily to represent the external world but to guide action in it.

(6) Even cognition that takes place in the ‘mind’ properly relies on knowledge structures that emerge from body-based experiences.

Those claims under the general heading of embodied cognition have two main regards, the effect of the environment (external world) and the participation of the body. There are two conclusive perspectives of embodied cognition drawn from Goldinger et al. (2016) for easier understanding: (1) cognition is situated in the environment, and constrained by it; (2) cognitive processes are influenced by the body and exist in the service of action.

These arguments also line up with several classic cognitive developmental traditions, such as the Piagetian (Piaget, 1952, as cited in Wellsby & Pexman, 2014), which emphasizes our sensorimotor system as a basis for the development of abstract concepts, and Vygotsky’s (1978) recognition of the role of our interaction with physical and symbolic artifacts. In terms of the educational sciences, certain ideas of embodied cognition conform to pragmatic and progressive traditions, for example, those of Dewey (1916) which highlight the role of personal and physical experiences in learning (as cited in Amin et al., 2015). As Atkinson (2010) presents, learning is ultimately a matter of change in an individual’s internal mental state. Exploration of the extended implications of embodied cognition in the educational fields tends to make greater sense. Therefore, the inquiry regarding the integrated natures of mind, body and environment arises increasingly, also leading to the growth of extensive research concerning embodied cognition in the educational phenomenon.

2.1.3 Theoretical Applications of Embodied Cognition in Education

As Tran et al. (2017) claim, there are decades of research working on the relationship between embodied cognition and learning. We review the literature building on embodied learning and find many educators actively exploring this concept in their fields respectively, such as science education (e.g. Amin et al., 2015, Weisberg & Newcombe, 2017), language acquisition (e.g. Atkinson, 2010, Lan et al., 2015), as well as mathematics education (e.g. Núñez et al., 1999; Tran et al., 2017). The theory of embodied cognition is also employed to forge effective interdisciplinary inquiry (Osgood-Campbell, 2015). Accordingly, we will briefly discuss the reasons why embodied cognition is proper to be applied to learning and curriculum, the characteristics of embodied curriculum, and dive deeper into separate subject matters that combine embodied cognition.

2.1.3.1 Reasons Embodied Cognition is Involved in Learning and Curriculum

According to the theory of embodied cognition, the generation of the human mind closely relies on sensorimotor experience (Lan et al., 2015). A metaphor of the mind is promoted as a larger network, distributed throughout the body via the central nervous system, and in constant exchanges with its external surroundings (Osgood-Campbell, 2015). Therefore, the traditional instructions that purely impart knowledge mentally to shape students' recognition of the world have been questioned with the rise of the theory of embodied cognition. Many studies of embodied cognition in the fields of education and curriculum construction focus on how to transform conventional learning into embodied learning by promoting the integration of body, mind and the environment. Some research concentrate on the impact of a sensorimotor activity on enhancing academic competencies such as language comprehension, mathematics, and scientific thinking (Osgood-Campbell, 2015). Some seek to illustrate how physical activities may affect brain functions involved with processes of classroom learning. A popular trend in the research on embodied cognition is to investigate the use of representational gesture, which is a specific aspect of the embodied application, in a learning environment (e.g., Sullivan, 2018). Besides, the discussion about how these studies contribute to the development of new learning strategies with the principles of embodied cognition is common as well (e.g., Fugate et al., 2019).

2.1.3.2 Embodied Curriculum

As Wang & Zheng (2018) conclude, there are three basic dimensions in constructing an embodied curriculum: (1) returning to the real living world and promoting the integration of the curriculum narrative framework and the student's learning trajectory; (2) introducing the embodied experience and realizing the continuous interactions among the learner's body, mind, and environment; and (3) integrating the functions of technologies to provide a powerful mediating tool for the construction of embodied curriculum (Wang & Zheng, 2018, p. 217). Identifying and landing these considerations prior to the construction of embodied curriculum is strongly recommended. Next, we will take a glance at some representative research that integrates embodied cognition into separate subject matter learning.

2.1.3.3 Embodied Cognition in Language Acquisition

The theory of embodied cognition often participates in the studies of second language acquisition (SLA). That is because linguistic researchers recognize that language learning, like any other learning, is ultimately a matter of change in an individual's internal mental state. As such, research on SLA is frequently viewed as a branch of cognitive science (Atkinson, 2010). There is agreement that action enhances comprehension (Asher 1977; Glenberg and Goldberg 2011; Glenberg et al. 2004; Tellier 2008) and an indexical hypothesis which states that an understanding of language results from a simulation of the actions implied by the meaning of the sentence (Glenberg and Kaschak 2002, as cited in Lan et al., 2015). Lan et al. (2015) conduct a rigorous review process on eight high-quality research papers to verify that a person's bodily sensations and actions will impact how he/she comprehends language, which aligns with the above hypothesis. In terms of embodied language processing, empirical research is still prevalent. For example, Atkinson (2010) examines two alternative approaches, extended cognition and embodied cognition—for how they might help us conceptualize SLA.

2.1.3.4 Embodied Cognition in Science Education

According to Amin et al. (2015), there is a critical mass of studies in science education capitalizing on ideas from the theory of 'embodied cognition' in cognitive science. Embodied learning promotes an action-to-abstraction transition through gesture, sketching, and analogical mapping, which seemly has special effects in science, technology, engineering, and mathematics (STEM) education (Weisberg & Newcombe, 2017). Thus, articles seek to formalize and test the effectiveness of embodied learning in STEM (Weisberg & Newcombe, 2017). For example, Castro-Alonso et al. (2019) present accumulating evidence to verify that executing and observing manipulations and gestures can be effective for science learning and visuospatial (Castro-Alonso et al., 2019). Schuman et al. (2022) investigate how direct manipulation of data visualizations on a touchscreen table affords meaningful learning of science concepts and practices. Weisberg & Newcombe (2017) provide a broader context concerning mechanisms that can support embodied learning and make it well suited to the STEM disciplines.

2.1.3.5 Embodied Cognition in Mathematics Education

According to Lakoff & Núñez (2000), it is of special interest that the neural circuitry we have evolved for other purposes is an inherent part of mathematics, which suggests that embodied mathematics does not exist independently of other embodied concepts used in everyday life. Rather, mathematics makes use of our adaptive capacities—our ability to adapt other cognitive mechanisms for mathematical purposes (Lakoff & Núñez, 2000). Mathematics naturally connects human perception and action, and therefore embodiment in this domain seems especially helpful in understanding its abstract, complex nature (Tran et al., 2017). It appears that the cognitive structure of advanced mathematics makes use of the kind of conceptual apparatus that is the stuff of ordinary everyday thought. Therefore, the application of the theory of embodied cognition to mathematics education is more prevalent.

Núñez et al. (1999), in the same period as the book publishment of Lakoff & Núñez (2000), uncover the bodily-grounded nature of cognition. Their paper presents elements of a theory which focuses on how human cognition is bodily-grounded and examines the ways in which this embodiment helps to determine the nature of mathematical understanding and thinking (Núñez et al., 1999). They illustrate the nature of shared human bodily experience and action through basic embodied cognitive processes and conceptual systems, which empowers a reconceptualization of cognition and mathematics itself, and has a huge significance for mathematics education (Núñez et al., 1999).

Since then, mathematics education called for studies that answer questions such as “What is the role of the body in a domain with such seemingly abstract or imaginary entities and how is mathematics (or how could it be) taught and learned in ways that exploit phenomena of embodiment?” (Hall and Nemirovsky, 2012, p. 207-208 as cited in Valentine & Kopcha, 2020, p. 542). However, no single perspective is accepted in the literature surrounding embodied cognition: some value gestures, whereas others pay attention to less overt forms of embodiment (e.g., conceptual metaphors) (Valentine & Kopcha, 2020).

When people gradually accepted that thinking does not occur solely in the head, the notion of a multimodal “material” conception of thinking draws on scholars’ attention. They carried out more research to illustrate that thinking occurs in and through a sophisticated semiotic coordination of speech, body, gestures, symbols and tools (Radford, 2009). Gestures are increasingly valued in research that works on mathematical cognition development. For instance, Radford (2009) discusses the role of gestures through a Grade 10 mathematics classroom example and determined how gestures related to learning and thinking. Likewise, Nemirovsky & Ferrara (2009) analyze the gestures and words of a student in a classroom episode to characterize mathematical imagination with the perspectives of embodied cognition and intersubjectivity.

Besides the particular attention of researchers on the role of gestures, others seek to provide insight regarding the way that mathematics is taught and learned with the rooted consideration that mind, body, and environment are closely linked in the doing of mathematics. They are mostly looking for supporting evidence in teaching practices to examine the theory of embodied cognition.

Arzarello & Robutti (2008) analyze the elements of embodied cognition in mathematics education research in the trend of the prevalence of applying cognitive science to mathematics. They aim to find the appropriate paradigms of embodied mind to ultimately build a fresh theoretical frame for mathematics learning situations. Nemirovsky et al. (2013) argue that mathematics as an activity is built on the systematic interpenetration of perceptual and motor aspects of its implementation, which is called “perceptuomotor integration”. Mathematical imagination grows from this kind of integration and can be achieved in many cases of mathematical activities, like an interactive mathematics exhibit in a science museum, which is used as a primary unit of analysis in this research. Tran et al. (2017) focus on developing technologically enhanced embodied mathematical activities and argue that technologies may provide the largely untapped potential to increase learning effectiveness via an embodied cognition approach (Tran et al., 2017). To conclude, the notion of leveraging actions to bolster learning is widely influential in mathematics education through a lens of embodied cognition.

2.1.3.6 Embodied Cognition in Geometric Thinking

According to Valentine & Kopcha (2020), several recent studies suggest that geometric thinking manifests through both physical gestures (Alibali & Nathan, 2012) and imagining what is mathematically possible (Nemirovsky & Ferrara, 2009; Nemirovsky et al., 2012). However, research on embodied cognition and geometry is still in its infancy (Valentine & Kopcha, 2020). We will discuss two featured pieces of research concerning embodied geometrical thinking below.

Roth’s (2010) cube articulates an alternative to standard embodiment approaches to enhance the phenomenology of geometrical and spatial experience. It illustrates materialist approaches that assume human’s flesh auto-affects itself in moving prior to any intention to move. The flesh has the capacity of tact (i.e., sense of touch), contact (i.e., touched and being touched), and contingency that is the ground of all senses, sense-making efforts, and knowledge (Roth, 2010). This analysis shows that incarnation, the flesh, is the necessary condition for a world.

Valentine & Kopcha (2020) present a phenomenological study examining the ways in which eighth-grade learners problematized geometric space as an embodied phenomenon. The authors figure out the nature of middle school learners’ problematization activity, discuss how their activity related to the embodiment of mathematical concepts, and lastly address how students problematize geometric concepts by connecting their minds, bodies, and experiences to the world in which they exist (Valentine & Kopcha, 2020). This article strongly argues that cognitive structures are not abstracted away from experiences and offers a good guidepost for further research.

2.2 Conceptual Metaphor

Conceptual metaphor is also viewed as a theory of learning. There is a kind of overlapping area between embodied cognition and conceptual metaphor.

One of the principal arguments in cognitive science is that abstract concepts are hardly isolated in any human mind since they are intangible. The basic way to make abstract thought possible is to use metaphor. The use of conceptual metaphor enables abstract thoughts to be rooted in the mind by leveraging systematic metaphorical mappings that employ concrete concepts to interpret abstract ones. To be specific, each conceptual metaphor is a unidirectional mapping from entities in one conceptual domain to corresponding entities in another conceptual domain, which allows us to reason about relatively abstract domains using the inferential structure of relatively concrete domains.

In metaphor, conceptual cross-domain mapping is primary; metaphorical language is secondary, deriving from conceptual mapping. Many words for source-domain concepts also apply to corresponding target-domain concepts. When words for source-domain concepts do apply to corresponding target concepts, they do so systematically, not haphazardly. As such, abstract concepts are understood via metaphorical mapping in terms of more concrete concepts (Gibbs, 2005, as cited in Amin et al., 2015).

For example, affection is understood in terms of physical warmth, as in sentences like “she warmed up to me,” and “You’ve been cold to me all day.” As this example shows, the metaphor is not a matter of words, but of conceptual structure. Affection is conceptualized in terms of warmth and disaffection in terms of cold. (Lakoff &

Núñez, 2000, p. 41) From a brain and neural science perspective, it is the simultaneous activation of two distinct areas of our brain, each concerned with distinct aspects of our experience, like the physical experience of warmth and the emotional experience of affection. It happens in a conflation where the two kinds of experience occur inseparably. The coactivation of two or more parts of the brain generates a single complex experience--an experience of affection with warmth. When such conflations that neural links across domains are developed, one domain can be conceptualized in terms of the other, which results in result in "conceptual metaphor" (Lakoff & Núñez, 2000, p. 42)

The research building on conceptual metaphor is not unique in the educational literature. Amin et al. (2015) conducted an empirical study to identify image schemas invoked by students, teachers and scientists and how they map them metaphorically onto abstract scientific concepts they are thinking about and reasoning with.

To sum up, conceptual metaphors are part of our system of thoughts, which is used to comprehend relatively abstract concepts from metaphorical mapping. In our discussion of all these topics and more, conceptual metaphor is the principal cognitive mechanism of extension from the human body to sophisticated applications of geometry. Moreover, a sophisticated understanding of geometry itself requires conceptual metaphors using non-numerical mathematical source domains like a dance.

2.3 Learning Mathematics through Movements

2.3.1 Learning by Doing Movements

Learning by doing movements is not new to classrooms. We can witness significant growth in the literature base of incorporating physical activities or featured movements into school teaching and learning. Physical activities are employed to serve students as a pedagogical approach given the awareness of the importance of enacting movements in the learning process. Kuczala (2010) claims that movement is a necessity in today's classroom since it offers a stimulating classroom environment in which students can grow cognitively, physically, mentally, emotionally, and socially. For example, it allows students to get out of their seats to move while learning, which provides novelty so that students are actively engaged in the learning process (Kuczala, 2010).

The approach of conducting movements while learning has a neural basis although the research is inconclusive and promising. In general, integrative movements help students by forcing the hemispheres of the brain, each of which controls the opposite side of the body, to work together. Meanwhile, it promotes energy production and blood flow, decreasing muscle tension, and stimulating and focusing the brain to improve concentration (Dennison & Dennison, 1988; Hannaford, 1995; Promislow, 1999, as cited in Kuczala, 2010). Hence, many researchers and theorists believe that specific, directed physical movements enhance the brain's function and assist in preparing the brain for learning (Kuczala, 2010). Ratey (2008) presents evidence and claims that physical movements in which we participate influence the way we think, learn, and remember (as cited in Kuczala, 2010). In terms of mathematics education, according to Smith et al. (2014), many researchers state that body-based activities can benefit the development of mathematical understanding as well (Howison et al., 2011; Petrick & Martin, 2012; Wright, 2001).

Another paramount reason for learning by doing movements lies in its significant role in enhancing students' enjoyment and engagement, especially for mathematics classes, in which a low level of engagement is of concern globally (Riley et al., 2017). Incorporating movement-based learning into the mathematics class as an innovative intervention is verified strongly effective in addressing student disengagement (Riley et al., 2017). Further, Riley et al. (2017) report students' and teachers' perceptions of physical activity integration strategies to fill the void of research in this area.

In the research that concentrates on learning by doing movements, taking dance into classes as a pedagogical approach is innovative but not distinct. Relevant research is often conducted by some people who are interested in dance. Wood (2008), who has a long time of experience in dance, investigates the links between kinaesthetic and conceptual learning. Based on her observation of her classroom in northern Western

Australia, students were more engaged with ideas and learning if there was an element of movement involved. Wood (2008) also concludes that the movement gave the tasks an element of fun and that the students' enjoyment was obvious. Gerofsky (2013) dives deeper into the integration of mathematics and dance. In his article "Learning Mathematics Through Dance", Gerofsky (2013) states two integral points: doing mathematics makes extensive use of sensory representations (including visual, verbal and sonic imagery and kinesthetic gesture and movement), and dance has great potential for sensory, fully-embodied mathematical engagement. Hence, he analyzes the means of human bodies by which we experience dance and embodied mathematics, as well as its potential affordances and limitations. Lastly, he offers an example of how Sarah Chase dances aspects of Number Theory to support the above viewpoints. These findings all lend support to the idea of adopting innovative kinesthetic approaches that counter pedagogic traditions that only keep eyes on abstract and disembodied mathematical representations in mathematics learning. Now, we call into question why we incorporated Laban/Bartenieff Movement Analysis (L/BMA) into our class.

2.4 Laban/Bartenieff Movement Analysis (L/BMA)

2.4.1 What is L/BMA?

Generally, L/BMA offers a framework and a language for understanding and perceiving human movements. According to L/BMA, muscles and nerves constitute the biological mechanisms of movement, and movement in its most fundamental sense emerges from the impulses to meet needs for safety, food, sensation, curiosity, comfort, and love—for example, going toward or moving away something (Wahl, 2019). By making the process of observing, describing, and analyzing movement layered and precise, L/BMA improves how movement is perceived. Meanwhile, L/BMA recognizes the underpinnings of desire, intention, and expression as relevant to the study of human movements (Wahl, 2019). The specificity of the L/BMA framework heightens sensation to bring greater clarity in movement so that you can manifest what human intends.

On the other hand, L/BMA seeks to describe and make meanings from human movements. L/BMA provides the language and skill to fully describe the movement. L/BMA offers a specific lexicon for articulating human movements (Wahl, 2019). A basic goal of L/BMA is to promote objectivity in movement analysis, allowing the observer to address what is happening free of implicit bias toward certain preferences and interpretive meanings (Wahl, 2019). Hence, L/BMA is useful in the study of human movements, such as describing, observing, experiencing, coaching and teaching movement.

In addition, L/BMA enriches the pursuit of moving by increasing movement possibilities and providing options for greater clarity. The L/BMA framework scaffolds the elements of body, effort, shape, and space, which come together in Praise within a specific context in a unique and constantly changing way to create infinite relationships through and in movement (Wahl, 2019). You can shift the lens or focus of your perceptions by centering any element among Body, Effort, Shape, and Space, and therefore you can look at the same movement event in different ways. Through each lens, you will gain new recognitions and thoughts about the movement in the event you are perceiving. When perceiving movement through the various lenses provided by L/BMA, you will realize that movement is multifaceted.

2.4.2 What is L/BMA Used For?

On one hand, the guiding concepts and organizing themes stated in L/BMA, including Function-Expression, Inner-Outer, Part-Whole, Stability-Mobility, and Exertion-Recuperation, provide users with varied perspectives to consider and frame their use of L/BMA to shed light on their own interest. Unique and valuable perspectives are generated to see what is happening in the movement and bring new knowledge to individuals.

On the other hand, movement does not inherently present a specific meaning every time it is performed. Movement observation is a complex weaving together of what is occurring and the context in which it is occurring. The interpretation of movement is influenced by the observers’ lens. By learning to observe movement through the lens of L/BMA, users usually get better at interpreting movement based on what they see and what is actually taking place.

2.4.3 Why Apply L/BMA to This Course?

Students are both observers and performers of their movements in this classroom. The novel and detailed information gathered through the unique and varied L/BMA lens enables students to interpretate their movements as an observer at the center of their meaning making. L/BMA also attunes students to how movement makes sense to them when they perform movements. Therefore, throughout this application of L/BMA in the class, students are encouraged to think about and comprehend movements in greater depth. After learning L/BMA, they are capable to reflect on movement in their life and to expand their range of movement choices. They can fulfill these goals more fully if they wish.

Besides, in L/BMA, the words and their associated symbols represent elements of movement. The word is not more important than the movement; the word and symbol are in service to the movement, and the movement is an entity unto itself (Wahl, 2019, p.16). L/BMA can address the concern that words are not adequate to give a rich and distinct description of movement. Hence, L/BMA empowers students to share their movement insights (including through writing) by equipping students with richer movements and language to impart their message most effectively.

2.5 Geometry and Geometric Cognition

Geometry is one of the oldest branches of mathematics and a key area of mathematics, which is not only fundamental for the understanding of other areas of mathematics but also many scientific fields, such as physics and chemistry (Pantsar, 2022). Initially, based on the introduction of “What is Geometry” on the website of the Department of Pure Mathematics at the University of Waterloo (n.d., para. 1), geometry was studied to understand the physical world we live in, and the tradition continues to this day. It is always concerned with the shape of individual objects, spatial relationships among various objects, and the properties of surrounding space (Heilbron, n.d.). Another important reference arguing about geometry is from the website of Math Planet (n.d.) which is organized by Mattecentrum, a Swedish non-profit organization aiming to stimulate students' interest in STEM. It asserts that geometry will guide us through among other things points, lines, planes, angles, parallel lines, triangles, similarity, trigonometry, quadrilaterals, transformations, circles, and area (Math Planet, n.d., para. 1). In this thesis, we generally use these relatively common terms in our discussion of our participants' geometric cognition.

Geometric cognition involves a wide range of abilities from the simplest shape recognition and orientation to complicated visual or spatial reasoning. Foundations of geometric cognition show that basic geometric skills are deeply hardwired in the visuospatial cognitive capacities of our brains, namely spatial navigation and object recognition (Hohol, 2019). The survey on the theme of geometry education conducted by Sinclair et al. (2016) identified several major threads of contributions to geometry education, including advances in the understanding of visual spatial reasoning; the use and role of diagrams and gestures, advances in the understanding of the role of digital technologies; advances in the understanding of the teaching and learning of definitions; advances in the understanding of the teaching and learning of the proving process; and, moving beyond traditional Euclidean approaches.

According to NCTM (2018), Grade 9-12 students are expected to be able to analyze or determine attributes of two- and three-dimensional objects, prove theorems, use trigonometric relationships to determine lengths and angle measures, etc. As such, the geometry curriculum and teaching practice should equip students with a deeper understanding of geometry and stronger skills to reach it. Moreover, if we turn to the Ontario mathematics curriculum (The Ministry of Education, 2020), students are probably exposed to many challenging models or contents such as trigonometric functions and vectors, which require geometric cognition as an essential asset. To aid students in learning geometry, especially 3-D geometry which may be complicated for students, the use of technology is examined to facilitate the students' geometric cognitive growth. For example, Gargrish et al. (2020) develop Augmented Reality (AR)- based geometry learning for the Android and iOS platforms and deploy the applications among students for teaching 3-D geometry to high school students. In 2021, Gargrish et al. implemented experimental research to investigate the impact of

AR intervention on learners' memory retention abilities and learning.

2.6 Conclusion

Our research adopts embodied cognition as a theoretical lens, which assumes that cognition is shaped by our bodies and the ways we act on and are acted on by the environment. According to the theory of embodied cognition, mind and body are not separated but rather work together to support reasoning. The sensory-motor system embedded in the body provides the cognitive system with sensory inputs like sight, hearing touching, motor actions, etc. We experience, understand, and act on the world through our bodies, and thus, we say that bodies link minds to the world.

Conceptual metaphor is an essential notion adjacent to the theory of embodied cognition. Based on conceptual metaphors, image schemas can be mapped metaphorically to more abstract domains so that students have more potential to overcome learning difficulties, especially to build obscure concepts in the mind, by leveraging conceptual metaphors.

Our experimental undergraduate course employed L/BMA to enrich its content, as well as empower students to understand and interpret their movements with richer lexical learned from L/BMA. In the analysis of students' performance, we can verify the emergence and expansion of embodied cognition in students' minds. We assume that (1) students take advantage of the sensory-motor system to collect information and perform what they understand; (2) bodily movements work as a kind of conceptual metaphor assisting students in understanding three-dimensions geometry by mapping metaphorically from a concrete domain to an abstract domain; (3) and thus students who enrolled in this course develop their geometrical cognition through the perspective of embodied cognition. The relations described above can be seen in Figure 1.

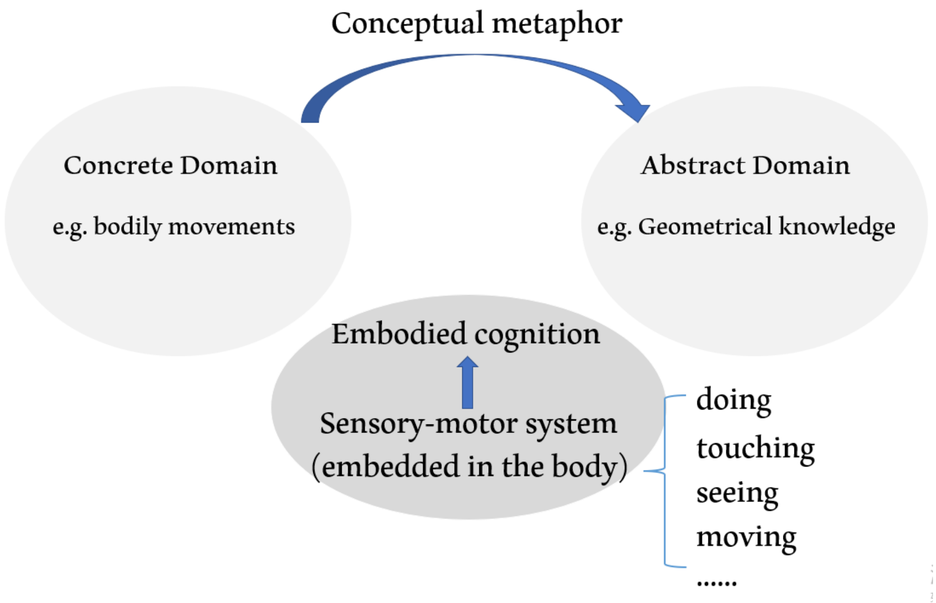

Figure 1: The Relationship Construction Used in This Research.

Chapter 3

3 Methodology

To fulfill the goal of examining how students develop their geometric cognition within an embodied cognition lens, my study employs qualitative interpretivism as a worldview and methodology. This section outlines the use of qualitative research, including the specific design being used in the study and its basic intent. It also states the overall data collection and recording procedures. It further expands on the data analysis steps and the methods used for presenting the data, interpreting it, and how to make it trustworthy. In qualitative research, comments by the researcher about their role are indispensable to convey as well.

3.1 Paradigm

The constructivist worldview, often combined with interpretivism, believes that individuals construct their understanding of the world based on their own experience, which leads to varied constructions of the meaning (Creswell, 2018). Thus, researchers are expected to look for the complexity of views by visiting participants' context and collecting information personally and to interpret data by considering their backgrounds and experience (Creswell, 2018).

This study employs interpretivism as a paradigm. As a researcher, I hold the opinion that how individuals understand the world heavily rests on the circumstances in which they live and work. The ways in which they engage with their world determine their subjective understanding and interpretation of their experience. In my study, students who participated in the same class altogether constructed what they learned in their minds in different ways and therefore obtained varied learning results as well as reflections. When we recognize and admit this kind of difference, it is understandable to incorporate participants' and researchers' backgrounds into our analysis and make sense of them eventually.

Gathering thick descriptive data and analyzing them in depth by taking into consideration the participants' context and experience is what this study does within the paradigm of qualitative interpretive research. As a result, we aim to gain a deeper understanding of how students develop their geometrical cognition in some embodied ways. Thus, the results may not be generalizable for the broad public but can assist in generating new and valuable viewpoints on our concerned topic.

3.2 My Role as a Researcher

I am a master's student who has two-year learning experiences in Canada. Thus, I am somewhat new to the North American mathematics education system. I know little about what mathematics classes look like here in Canada since I was born and primarily educated in China. Though I was a biology teacher for five years, which empowered me with teaching experience, relatively speaking, I have less mathematics-related background. When I had an opportunity to join the team that focuses on mathematics and STEAM education, I was fortunate to learn more about this discipline. It may bring certain bias to my study due to my degree of familiarity with of American mathematics education and north American mathematics education in general.

On the other hand, I am a dance enthusiast who has been learning and practicing dance for years. Therefore, I have developed my unique understanding of dance as well as body movements and the role dance plays in human cognition. From my perspective, dance is typically a type of language, which can effectively convey meanings and feelings to audiences with performers' bodies. Furthermore, I presume that dance has a great effect on changing the moods of both dancers and audiences. As such, it is invariable that my experiences in dance will influence the way I make interpretations of the data.

3.3 Bounding the Study

My study is focused on an undergraduate interdisciplinary course integrating mathematics and bodily movements, aiming to investigate using an embodied cognition lens, how students in this course develop their geometric cognition. The classroom and other sites involved in this course are the places where participants acquire the targeted experience under the study. The course also makes use of certain instruments and technology. Details about this course are presented below.

3.3.1 Setting

This is an elective interdisciplinary undergraduate course called "World of Pattern: Geometry, Body, and Beauty", which was offered at a private university in New York State, USA.

This course explored selected topics in body movement, arts, and world of patterns, and their underpinning mathematical ideas and geometrical cognitions. Class topics were separated into two general but integrated areas, including geometry topics like “Contortion of Space”, and arts topics like “Metaphors/Meaningful Associations with Space”, etc. Course contents progressed through multiple modalities with emerging knowledge and best practices from STEAM education, including dance and movement studies, the aesthetic nature of math, Islamic arts, and computational modelling.

The course was co-taught by a faculty member in Mathematics Education and another faculty member in Dance, both with unique specialties. One of the instructors of this course is an Assistant Professor of Dance, as well as a movement educator, scholar, author, and choreographer. She is also a certified Laban/Bartenieff Movement analyst through Integrated Movement Studies, a registered Somatic Movement educator through ISMETA (International Somatic Movement Educators and Therapists Association) and a certified personal trainer through NASM (National Academy of Sports Medicine). Moving body is at the forefront of her interests. The other instructor is an Assistant Professor working in Interdisciplinary and Inclusive Mathematics Education. Her passion is empowering underrepresented/underserved students through STEM-ed, CS, and Computational Thinking.

3.3.2 Participants

Initially, nine students were enrolled in this course. At the end of the course, six students finished the final project and were interviewed. Those students’ age ranged from 18 to 21 years. According to their responses in the survey, five of them identified themselves as female, and one of six preferred not to say their gender identity. In terms of ethnicity/race, besides one student who selected "prefer not to say", four students are reported as White, and one is African American.

3.3.3 Events

For the purpose of this research, some activities were implemented in the classroom, which consisted of recognizing, planning, drawing, and performing movements.

The most frequent one among diverse activities is performing body movements framed by Laban/Bartenieff Movement Analysis (we call these movements "Laban Dance"). The dance instructor led students to do some basic movements of Laban Dance, like touching every aspect of the space around the dancer's body (e.g., up and down, front and back, left and right). As the course proceeded, the movements they performed tend to be more complex, gradually involving more directions of that space such as upper-right and lower-left or involving the transition between different diagonals.

Besides dancing in the class, watching dance videos like figure skating videos, reading poems and other mathematical/geometrical exploration activities were undertaken to inspire students' artistic and geometrical thinking. Given that there was a certain topic for each class, such as "Platonic solids turn amorous", and "Making Individualized Movement Sequences", instructors employed varied activities to fulfill the learning goals in each class. All topics for classes can be seen in the syllabus attached in the appendix. They even took advantage of technology like Virtual Reality, Scratch, "Movement Space" mobile app, etc. to facilitate the class.

Three assignments with different themes and a final project were deployed as major approaches for evaluation. The assignments have individually specific orientations for assessing students' skills, imagination, creativity, the ability of making connections to the real world, and computational thinking. The final project asked students to creatively produce a piece of work (a dance, a video, a song, a Scratch program, and so on) based on what they have learned in the course and to finally present it at the last class. All details of assignments or final project requirements can be found in the syllabus attached in the appendix.

In the last class, researchers who are interested in this innovative course launched an online survey and focus group interview for the purpose of research. The results of the survey and the interview were not used for this study.

Overall, there are three main characteristics of this course that made it accessible to examine students' embodied cognition.

(1) Laban/Bartenieff Movement Analysis is integrated into the course, which brings a wide variety of body movements to learners. Laban/Bartenieff Movement Analysis provides students "a language" to describe their thoughts with their bodies. Students tend to use Laban/Bartenieff movements to generate and interpret their mathematical ideas. For example, students denoted geometrical shapes symbolically with their bodies. The transformation between the real geometrical shapes and body movements may imply students' capacity to reason abstractly.

(2) This course adopted a variety of class activities triggering students to present their embodied ideas. These activities include dance, photography, scratch programming, etc., taking advantage of multiple human senses (modalities) and inspiring students' creative thoughts by reifying geometric knowledge in an embodied way.

(3) The instructors of this course not only recorded students' performance in the classes but also collected their artifacts in their assignments which contain many valuable and stimulating ideas developed by students. It is beneficial to produce a persuasive series of evidence to back up our findings.

3.4 Ethical Considerations

All the data for this study is secondary data, which was originally collected by the mathematics education instructor during the classes and used for her research. The instructor was authorized to implement this study as of January 25th, 2021, with full approval by the Human Subjects Research Committee of the academic institution she worked for. I was a team member in this original study and my own study received approval from the Western University Non-Medical Research Ethics Board to use these secondary data.

This study is built on the principle of respecting the rights, needs, values, and desires of the participants. At the first class of this course, the instructors introduced to all students about our research purpose regarding this course and obtained all of the students' consent to collect and examine their capstone projects, videos recording their in-class interactions (whether online or in-session), focus group discussion and so on. Students' personal information always remains confidential by being de-identified and anonymized even if their position and institution may be highly visible.

My study does not include any information that will make it possible to identify the university where the classes happened as well as any participants. To ensure anonymity, I use pseudonyms for students. All videos, photos and copies of students' assignments are anonymous by showing no faces and other identifying features.

The records of this study are kept private. Research records are kept on password-protected devices and the secured Clouds; only the research team members have access to the records. All the data in NVivo software is password protected and stored confidentially.

3.5 Data Collection Strategies

Original data were collected from February 2022 through April 2022. The categories of data contain classroom videos, students' artifacts, survey results, final project presentations and a focus group interview. Observing how participants behave and act within their contexts and making analysis in the constructivist worldview is the main method for this qualitative research (Creswell, 2018).

Most of classes were video-recorded and stored by the mathematics education instructor. All participants agreed to record moments presenting their experiences and feelings, especially their distinguishing reactions to exclusive class activities. Unfortunately, some videos are incomplete due to the limitation of the technology. As a researcher, I commenced two-hour observations of a regular class every week through these videos and communicated with the math instructor from time to time throughout the data collection process.

Students' artifacts created in the classes or submitted in their assignments are another important data source, which mainly contains pictures that students drew or took with a camera, resource writing, videos where they recorded their dance, etc. These artifacts were gathered during or after the classes and subsequently archived by the math instructor and shared with me.

3.6 Data Analysis Procedures

Firstly, I catalogued all the data depending on the sources of information in the order of appearance when the classes proceeded.

After the course ended, I watched the classroom videos and all artifacts from students to develop a general sense of information and reflect on its overall meaning. I recorded details in a field notebook based on my observation and kept a field diary to chronicle my thinking, feeling, and perceptions.

I used the software NVivo to organize the data for assistance. To begin with, I almost transcribed all the videos with verbal sources, and I did the same thing to all kinds of artifacts that I have. I winnowed the data regarding the development of students' geometric cognition and coded them in NVivo. It involves taking text data, pictures, and videos gathered during data collection into two categories in line with my two research questions. Then, I separately listed all episodes that respond to the two research questions and briefly presented how embodied cognition takes effect in each episode. I finally selected 10 featured episodes to generate a detailed description of the settings, people and analysis for embodied cognition within a few narrative passages for each. Finally, I looked for common themes for summarizing the answers to the two research questions. The data analysis procedures are presented in Figure 2.

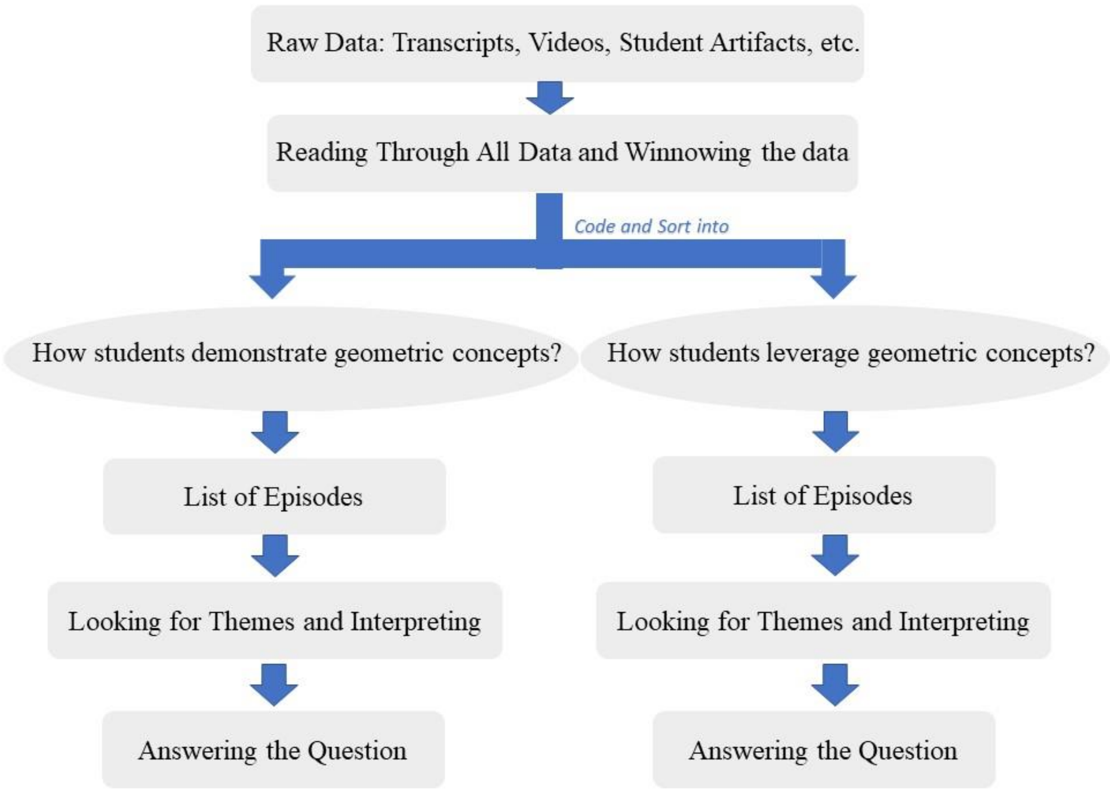

Figure 2: The Data Analysis Procedures.

When it comes to composing answers for the two research questions, both inductive and deductive approaches were used in my analysis process. To answer the first research question, how non-STEM students demonstrate geometrical concepts and patterns as they develop geometrical cognition, I preferred to use inductive ways for data analysis. Due to the unknown understanding of students regarding geometrical knowledge and embodied cognition, I had to work back and forth between the themes and the database so that I managed to figure out the most suitable set of themes for the research question. For the second research question, how non-STEM students leverage geometrical concepts and patterns as they develop geometrical cognition, we conduct deductive data analysis. That is mainly because we assume that students have developed their embodied cognition and so they are capable of "leveraging" them. When answering the second question, I looked for more evidence that can support the themes regarding what they have grasped about geometry and how they exemplify them. As such, the process adopts more deductive thinking rather than inductive ones. However, as the analysis moves forward, deductive and inductive methods always go hand in hand (Creswell, 2018).

3.7 Trustworthiness

This section presents how this study establishes validity and reliability. Qualitative validity is concerning how to check for the accuracy and credibility of my findings (Creswell, 2018). This study adopted multiple strategies as listed below to ensure qualitative validity.

3.7.1 Triangulate Different Data Sources

I incorporated classroom videos and all students' artifacts to build a coherent justification for the answers to my research questions. Students not only participated in classes but also generated various artifacts as their assignments after class. Aggregating more than two sources of data and perspectives from participants can add to the validity of my research.

3.7.2 Member Checking

Although it is impossible to reach out to the participants to take back part of this study to check for accuracy, I still shared it with the math instructor of this course and got her comments on the findings. Also, my supervisor reviewed the entire project and provided an objective assessment of this project through the process of this research to enhance its accuracy.

3.7.3 A Thick and Rich Description

In the sections of findings, I selected ten episodes to report how students demonstrate or leverage geometric concepts in embodied ways. In each selected episode, I offered detailed descriptions of the settings so that the readers can gain more and richer information about the reality.

3.7.4 Clarify the Bias

My interpretation of the findings is undeniably shaped by my background and experience. To clarify the bias brought by me and as reflected on my analysis as objectively as possible, I wrote notes about my individual experiences as memos during the observation of classroom videos and the analysis of all data sources to make sufficient reflexivity.

3.7.5 Spend Prolonged Time in This Field

I joined the original research team focusing on this course in February 2022 and observed the classes through videos or zooms during the whole academic term. I spent almost one year from my first participation in this course to the accomplishment of my research. The long time I spent in this field adds validity to an account.

We address the issue of reliability of our study by avoiding obvious mistakes in transcripts, keeping the consistency of the definition and the meaning of coding throughout the entire report, and holding a well-documented set of research procedures. Additionally, I make many of these steps visible and present my data with supporting evidence.

Chapter 4

4 Results

This chapter is generally divided into two sections. In the first section we present a summary of all instances from the data that can answer our two research questions (*demonstrating* and *leveraging* geometrical concepts and patterns as non-STEM students develop geometrical cognition) and characterize them with four elements: mind, body, environment and geometry. This kind of analysis is for a better understanding of how students make embodied cognition work with geometric concepts. All episodes are sorted by the data sources from which they are extracted. Additionally, we attach accordingly the specific introduction for each data source like the first assignment (Theme 1), the second assignment (Theme 2), the final project, classroom videos, etc. We skip the analysis of the third assignment that was using Scratch programming due to its being irrelevant to answer the research questions. In the second section, we select ten featured episodes from the first section for nuanced and in-depth analysis of the data and to illustrate our analysis with more details such as graphic clarifications and textual interpretations. This is done to offer readers sufficient descriptive granularity. The ten selected episodes include five episodes for research question 1: how non-students demonstrate geometrical concepts and patterns as they develop geometrical cognition and the other five episodes for research question 2: how non-STEM students leverage geometrical concepts and patterns as they develop geometrical cognition.

4.1 The Brief Analysis of Episodes for Answering Research Question 1

Table 1: The List of Episodes Responding to RQ1.

The First Assignment Description (Theme 1)

Theme 1: Reflection on aesthetic and mathematical presence in one’s life. *To create something: Decide on your medium of choice. 3-5 min video or a webpage or a visual representation, 500–800-word writing) and critically reflect on the topic (It could be a TikTok, Reel, YouTube (unlisted or public, your choice) Your reflection must be supported by at least 3 research articles (peer-reviewed), TEDTalks, or books) Online and website citation wouldn’t count. Your reflection must include both geometrical patterns and body movements.*
Episode 1. Violet-PowerPoint Presentation
Mind: she transforms the parts of the animals’ body into geometric figures. Body: visual information input. Environment: a clay sculpture of a small animal. Geometry: geometric figures recognized in the picture.
Episode 2. Fiona-Paragraph 1
Mind: she recognizes geometric elements in the process of drawing. Body: the face is a part of body, and drawing a circle and lines requires hands. Environment: the room where she was drawing or thinking of. Geometry: a circle, a horizontal line, a vertical line.
Episode 3. Fiona- Paragraph 5
Mind: she recognizes the use of geometric shapes and patterns and analyzes their functions in the pictures. Body: visual information input and the eyes tracing the lines or patterns. Environment: the place of her composition analysis

Geometry: triangles, symmetry, asymmetry, the golden ratio, the rule of thirds.
Episode 4. Helen- Paragraph 1
Mind: she recognizes the symmetry. Body: visual information input. Environment: a grocery store. Geometry: the symmetry of the establishment.
Episode 5. Helen- Paragraph 2
Mind: she recognizes the different shapes of geometry among fruits and vegetables and realizes people's preferred choice of the more symmetrical shaped fruits and vegetables. Body: visual information input and opt for the more symmetrical shaped fruits and vegetables. Environment: fruit shelves in a grocery store. Geometry: all sorts of shapes in the fruits and vegetables.
Episode 6. Helen- Paragraph 3
Mind: she recognizes geometric shape or patterns. Body: visual information input. Environment: the product shelves in the grocery store. Geometry: rectangles, square, round, symmetry.
Episode 7. Helen- Paragraph 4
Mind: she recognizes geometric shape or patterns.

Body: visual information input. Environment: the ceiling in the grocery store. Geometry: square pattern, symmetry.
Episode 8. Shay-Pictures and the Writing
Mind: she realizes the influence of external environment to her mind. So, she arranges all objects in a clear and tidy way with a consideration of geometric concepts. Body: visual information and her body moving among the objects in her room. Environment: her apartment and the objects in her apartment. Geometry: parallels, the space
Episode 9. Ruby-Video 1
Mind: she thinks of the chainmail jacket as a cylinder and measures her waist to determine the size of the chainmail jacket. Body: she measures her waist multiple times. Environment: she is making a chainmail jacket. Geometry: Cylinder, differential geometry
Episode 10. Cora-Pictures of Handicraft in a pdf
Mind: she recognizes geometric shape or patterns constituted by the parts of the body. Body: visual information input and the body's position. Environment: human activities like sweeping snow outdoor.

Geometry: rectangles, triangles, polygon.
The Second Assignment Description (Theme 2) Theme 2: Movement Sequence in Space, described in geometrical and metaphorical terms. *You will create, record, play, explain, artistic/creative movement. You may explain the metaphorical aspect of these movements through poem, play, sing a song, record a podcast for us to listen to as we walk through a space... You may bring awareness about equity, social justice, or cultural values into play. Be creative!*
Episode 11. Gianna-A Movement Scale Poem
Mind: she created this piece of work in imagination rather than in the reality. She opened her soul to the world around her and built the orientation of her body, interacting with this world. Body: she is doing imaginary movements that are interacting with the environment. Environment: the ground, the earth, the world. Geometry: alignment.
Episode 12. Shay-the Writing Named Spatial Planes in Cheerleading
Mind: she realizes the important role of expressive and transition movements in the motion of a dance by thinking of them within a geometric lens Body: her arms, legs, etc. Environment: Cheerleading dance and cheerleading playground in her imagination. Geometry: spatial planes.

The Final Project Description Based on what you know about geometry, dance, and how we move through and make meaning from the space around our bodies, create a dance style of your own. In a sense you are deciding that the dance world just doesn't understand how to use the space around the body and mathematics and programing to your satisfaction, so you are going to set out to teach us how to do just that. What is your dance style? What does it mean? This project has multiple components: Make a video of yourself doing the movement. Use the symbols of the Spatial Matrix based on the Laban/Bartenieff System Take photos using greenscreen (make a Scratch character) Use Scratch to write a program and perform it using loops, etc. Write a description of the movements and what they 'mean' in your dance technique
Episode 13. Fiona-Presentation
Mind: she recognizes the significance of the radius of the sphere. Body: dance movements in her presentation. Environment: the sphere, the studio. Geometry: the radius of the sphere.
Episode 14. Violet-pdf
Mind: she realizes that the shapes really did relate with the elements inside movement-wise while making the connections between the movement, shape, and

element. Body: her body movements work as a bridge connecting different platonic solids to elements of nature. Environment: the handout, the universe, the earth, the air, the water, and the fire. Geometry: platonic solids.
Classroom Videos
Episode 15. CV-IMG 0200
Mind: students recognize the geometric shapes like triangles created by the skating performers' bodies, and they conceive of the relationship among the weight, gravity and the positions of arms, legs and head. Body: the performers' body movements. Environment: Video of figure skating. Geometry: angles, triangles.
Episode 16. CV-03-Week 4 (5'18''-8'14'')
Mind: imagine the space in which they dance as marble tables, big round dresses, men in heels etc. Body: the students' movements in a round that consists of two or more students. Environment: the dance studio. Geometry: the round constituted by the student and her partner(s), the space around the student.
Episode 17. CV-17-Week 4 & 18-Week 4
Mind: students translate the space that is important to them onto the picture they

draw. Body: their hands that had drawn the picture, and their movements that aim to experience the space. Environment: the studio, the hallway, the stairs, the theatre. Geometry: the space.
Episode 18. CV-19-Week 4 (Tina)
Mind: Tina put herself in the center and all furniture pieces around her. She thinks of her cooking movements as the motion within a three-dimensional space. Body: the subject's movements like reaching up and going down, moving to the stove and going back, etc. Environment: the kitchen, cooking. Geometry: the space.
Episode 19. CV-April 6-3 (Gianna)
Mind: she recognizes that her body is forming a triangle with the floor. And she analyzes the geometrical relationship inside her body, like the spine is up the middle. Body: the pelvis, the spine, the legs, the head. Environment: a stick or a ball touching her body, the reflection asked by Mina. Geometry: a triangle.

4.2 The Brief Analysis of Episodes for Answering Research Question 2

Table 2: The List of Episodes Responding to RQ2.

The First Assignment Description (Theme 1) Theme 1: Reflection on aesthetic and mathematical presence in one's life. *To create something: Decide on your medium of choice. 3-5 min video or a webpage or a visual representation, 500–800-word writing) and critically reflect on the topic (It could be a TikTok, Reel, YouTube (unlisted or public, your choice) Your reflection must be supported by at least 3 research articles (peer-reviewed), TEDTalks, or books) Online and website citation wouldn't count. Your reflection must include both geometrical patterns and body movements.*
Episode 20. Tina-Video 1
Mind: she applies geometric shapes to make-up. Body: she operates make-up as she said. Environment: the face, the concealer. Geometry: triangles.
Episode 21. Tina-Video 2
Mind: she thinks of how to translate geometric patterns she had on her face into hand movements she is performing. Body: her hands express the same geometric patterns she had on her face. Environment: the bedroom, the make-up. Geometry: geometric patterns she had on her face.

Episode 22. Fiona-Paragraph 2
Mind: she capitalizes on geometric shapes assisting herself in drawing a figure's face. In this case, she connects different geometric shapes to the curve of an ear, an eye, a lip or a fall of hair. Body: she uses her hand to draw following the path of the shapes. Environment: the physical art, the drawing pencil. Geometry: the circle, the squares and rectangles, triangles, trapezoids
Episode 23. Fiona-Paragraph 4
Mind: she connects different shapes to different emotional connotations. Body: visual information input. Environment: character design. Geometry: round, circular, triangular, square shapes.
The Second Assignment Description (Theme 2) Theme 2: Movement Sequence in Space, described in geometrical and metaphorical terms. *You will create, record, play, explain, artistic/creative movement. You may explain the metaphorical aspect of these movements through poem, play, sing a song, record a podcast for us to listen to as we walk through a space... You may bring awareness about equity, social justice, or cultural values into play. Be creative!*
Episode 24. Fiona-Poem
Mind: she imagines the universe as a sphere and conceives its infinitum and endlessness with cycles of circles.

Body: her body movements imitate the infinitum of the universe. Environment: the universe, stardust, planet, each atom, world, etc. Geometry: spheres, circles.
Episode 25. Tina-Video
Mind: she lends sunrise and sunset to reflect on life. Body: her dance movements represent what she said. Environment: the bedroom, the music. Geometry: up and down, high and low, the rise
Episode 26. Ruby-Video
Mind: she creates a dance expressing the feature of the intersection of icosahedron and dodecahedron. Body: the dance movements she created. Environment: the dance studio, the physical frame of icosahedron in the studio. Geometry: the interaction between icosahedron and dodecahedron.
The Final Project Description Based on what you know about geometry, dance, and how we move through and make meaning from the space around our bodies, create a dance style of your own. In a sense you are deciding that the dance world just doesn't understand how to use the space around the body and mathematics and programing to your satisfaction, so you are going to set out to teach us how to do just that. What is your dance style? What does it mean? This project has multiple components:

Make a video of yourself doing the movement. Use the symbols of the Spatial Matrix based on the Laban/Bartenieff System Take photos using greenscreen (make a Scratch character) Use Scratch to write a program and perform it using loops, etc. Write a description of the movements and what they 'mean' in your dance technique
Episode 27. Gianna-Video & pdf
Mind: she imagines her limbs are in space relative to her body's midline. She makes the use of her body to understand and control the space of her body. Body: her limbs, her body's midline. Environment: the icosahedron as an extension of her body, and any backdrop she uses. Geometry: the planes and spatial pulls within the vertices, the A scale and B scale of the icosahedron.
Episode 28. Ruby-Video & pdf
Mind: she reflects on the events in her life and thinks of their relationship or retain memories when she moves within a physical icosahedron. Body: she uses her body and arm to reach a vertex of the icosahedron. Environment: her life, her energy, mental energy. Geometry: the space and vertices in the icosahedron.
Episode 29. Shay-pdf
Mind: she recognizes lines and directions implied in pictures and she discovers the

existence of eye tracking. Body: visual information input, and the movement of human's eyeballs Environment: visual compositions in pictures. Geometry: regular and balanced patterns or paths, horizontal lines, vertical lines, diagonals.
Episode 30. Fiona-Paragraph 2
Mind: she thinks of the sphere abstractly with the body and her personal intention. Body: the body is the central axis of the sphere, the radius then being equal to one step out from the center. Environment: dance in the physical space Geometry: the sphere, different geometric shapes and the relationship between shapes.
Episode 31. Fiona-Paragraph 4
Mind: she makes connections between shapes and emotions. Body: sensation and emotion, movements within the sphere. Environment: the actual boundaries of the sphere. Geometry: pointy shapes, curves, etc.
Episode 32. Fiona-Paragraph 5
Mind: she associates the sphere around the dancer to the star or planet in the universe, further associates to the human's relationship. Body: the movements of the dancer. Environment: the floor or stationery.

Geometry: the space.
Classroom Videos
Episode 33. CV-April 13 (Ruby)
Mind: she thinks of how a forward tuck can be executed and how to prepare herself. Body: her body does a forward tuck or a backward tuck. Environment: cheerleading activities. Geometry: the geometry in a forward tuck and the landing position.

4.3 Annotated Analysis for 10 Featured Episodes

4.3.1 Episode 1. Violet's PowerPoint Presentation

Violet demonstrates her geometric cognition by recognizing the geometric shapes within a clay sculpture of an animal. In her assignment, she took a series of pictures for a clay sculpture and denotes with colourful signs every shape she found from the general to the detailed (Figure 3). Here, I assume that visual information input is the main path to evoking Violet's geometric thinking with her sensory-motor system, which transforms the picture of the item into the geometric shapes. This process takes place in Violet's mind but depends on her perceptual experiences like viewing the picture and her existing knowledge about geometric shapes.

Figure 3: Pictures from Violet's Assignment for Theme 1.

Most of the students present their recognition of geometric shapes in their assignments for theme 1. It seems that anything in the environment can bring up students' geometric cognition when they deliberately interpret them with geometric knowledge. What we need to realize is the importance of individual perceptual experience in constructing geometric cognition, which probably implies further mathematics education.

4.3.2 Episode 15. Figure Skating Video-CV-IMG 0200

This discussion takes place after students watched a figure skating video, which contains many athletes' continuous and elegant motions. The students' reflection predominately involves the momentum and the gravity of athletes, the curve or circles that athletes are drawing, the shape of the rink, the shapes of the athletes' body composition like triangles, and the angles. Every student participates in this round of discussion by presenting their innovative ideas and responding to others' presentations which inspire them.

On one hand, students illustrate their capacity to recognize the regular geometric shapes constituted by athletes' body parts in this round of discussion. This ability to recognize two-dimensional geometric shapes is obvious and can often be found in students' assignments and their daily performance in classes. In this discussion (Figure 4), the student sitting in the middle depicts many fluctuating triangles, which demonstrates her geometric cognition development. Visual information is input from the environment (the figure skating video) via the body's sensor system. The student's existing geometric knowledge in her mind is the basis of this reflection.

Figure 4: The Student in the Middle is Depicting Fluctuating Triangles.

On the other hand, more students focus on the movement of athletes because figure skating is a sport with gorgeous movements and music. The discussion around the movements reveals that students increase their awareness of three-dimensional geometry. Students are reflecting on how the movements happen, how they are created and what they look like. In this scenario, the rink and the music are the environments that impact students' thoughts. Students are primarily collecting visual information and processing the analysis of athletes' movements in their minds. Then, they present their ideas with their words and gestures (Figure 5). We can see the united operation of students' sensory-motor system, which connects figure skating to students' geometric cognition.

Figure 5: Students are Presenting Ideas with Words and Gestures.

4.3.3 Episode 16. Circle Dance-CV-03-Week 4 (5'18''-8'14'')

Developing a deep understanding of space is important for students studying three-dimensional geometry. As such, this class introduces general space and personal space to students. It starts with a circle dance that can lead to students' spatial cognition. In this dance, the dancer has one or more partners to stand in a circle, reaching out their hands to the center of the circle, and then they start to move as a circle in one direction with the music. After a few steps of dancing, they change to the opposite direction to move but still stay in the same circle. When they finish the two-direction movements, the circle breaks down and the dancers are supposed to look for new partners to start a new circle dance (Figure 6).

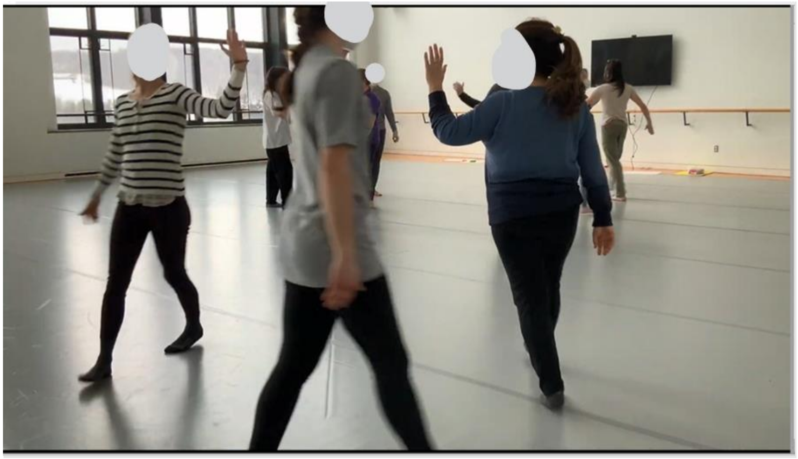

Figure 6: Students Circle Dance.

Students are asked to imagine what kind of space they did this dance in. Students' answers are good demonstrations that they are developing their understanding of the space. Firstly, their answers like marble tables, tables and carved-out chairs, and big round dresses almost involve objects that can be round or oval, which aligns with the circle their dance was drawing. Secondly, when they are dancing in the circle, they view their bodies as part of this circle. They not only understand what role they are playing in this dance but also develop an entire cognition about what they are constituting and what it looks like.

The imagination about the space derives from where they are dancing and what they are doing in this space. We can see students interacting with the studio, the partners, and the music to create this dance and operate their imagination. Despite students' reflecting thoughts being presented via their imagination, we assume that students are generating their geometric concepts based on the shape and the movements involved in the circle dance and their past lived experience, which helps them connect items like "marble tables" to their dance. Here, students' bodies are both executors of developing the circle and contributors to students' imaginations. Also, students' bodily movements in this dance become an important form of bodily experience that probably implies students' further thinking in these geometric concepts.

In addition, we can view the connection between "marble tables" and the circle dance as a type of conceptual metaphor, mapping from entities in one conceptual domain to corresponding entities in another conceptual domain.

4.3.4 Episode 18. Tina's Movements in the Kitchen-CV-19-Week 4

This class is talking about general space and personal space. They go through different kinds of general spaces like hallways, a black theatre, etc. Then, they are asked to draw a picture to present what they think about the space. Now, students are expected to reflect on what kind of personal space they have ever experienced.

Tina provides us with a perfect example to illustrate her personal space. She talks about her movements including going up and down, turning around the stove, and returning to the dining table in her grandma's kitchen. Those movements make up an all-direction space centering herself and her reflection demonstrates her strong cognition of this space (Figure 7).

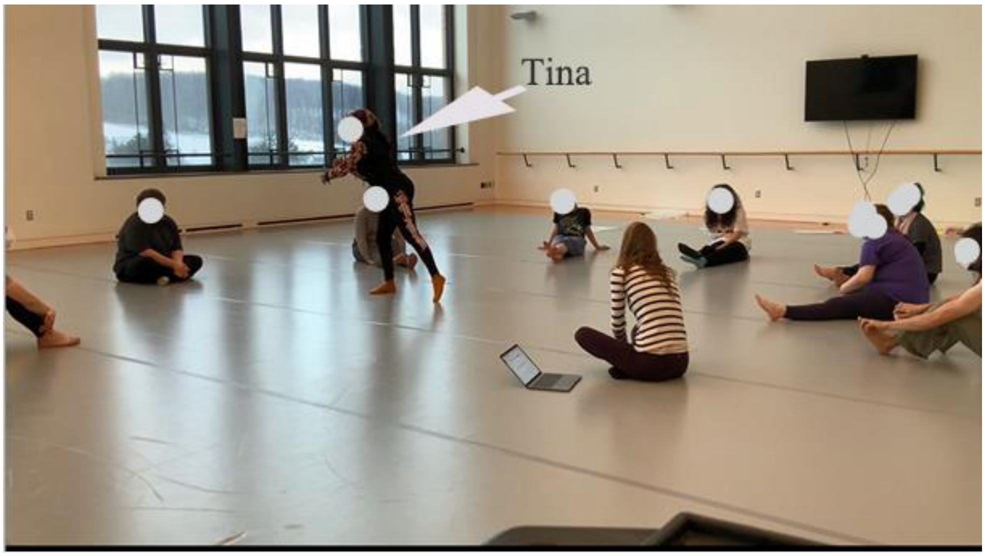

Figure 7: Tina is Performing Her Imaginative Kitchen Scenario.

Tina is a female student who has dance experience. She usually creates her dance to fill out her assignments for this course. In the snapshot of this classroom video, we can notice she is using body language to make the kitchen scenario she is describing reoccurring. In her presentation, the kitchen and all cooking tools are the environments for her. She combines them into her spatial cognition development. Her reflection is based on her real-life experience rather than pure imagination. Her geometric cognition is trigged and developed by expanding her understanding of the space and reflecting on her real life.

4.3.5 Episode 19. Geometrical Shapes Formed by the Body-CV-April 6-3

In this class, the instructor initiates an activity in which students need to use a stick to slightly touch the partner's body in a diagonal direction and then change to use the ball to do the same thing.

Gianna generates a great geometrical idea in terms of her body during this activity. When her partner uses the ball or the stick to click on her body, she shares how she thinks of geometric relationships between her body parts as followings (Figure 8).

> I guess it makes sense like in dance when you're talking about the pelvis being the center of the weight, then if your legs are the diagonal toward the pelvis, like, it's those diagonals toward the head, that are important because your spine is up the middle.

These statements illustrate that Gianna is demonstrating her understanding of geometric concepts like "center" and "diagonal" within her body. Here, her thoughts are closely related to her partner's touching the surface of her body. The touching from her partner as part of the environment stimulates Gianna's thoughts. We can say these touching actions evoke Gianna's reflection on the relationship between her body parts. Because touching is a type of tactile information sensed by her body, we assert that sensory information contributes to Gianna's geometric cognition development in her mind.

Figure 8: Gianna is Thinking of the Shapes formed by Her Body.

4.3.6 Episode 25. Tina Using the Sun as a Metaphor -Video

The second assignment asks students to create movement sequences in space and describe them in geometrical and metaphorical terms. Students can explain the metaphorical aspect of their movement through a poem, a play, a song or a podcast. Tina made a video recording of her movements and thoughts with background music. Through this video, we can see Tina leverages her geometric concepts to reflect on her life.

Tina dubs the video with the following (Transcript):

> I *rise* just as the sun does and set when it sets as days pass these events reset. Even when I am down, I love myself *up*, even when at times a life may suck, such near and far and high and low, for a glimpse of hope even when life says no. I continue to prosper and *rise* as the sun.

Tina said she rises as the sun does at the same time when she did an “up” movement (Figure 9). There is an evident linkage between her thought and movement and whereby her body works as a tool for expressing her mind. Further, her word in the video reveals her deep reflection “I continue to prosper and rise as the sun.” The geometric cognition like “the rise of the sun” is projected to her life experience like “prosper” in her mind. Prosper in this case represents a positive attitude in coordination with the feeling of “rise”. Tina is presenting her positive attitude with an "up" movement via the use of the conceptual metaphor of "the rise of the sun". Thus, this is a perfect illustration that Tina is leveraging her geometric cognition.

Figure 9: Tina Did an "Up" Movement.

4.3.7 Episode 26. Ruby Tracing the Intersecting Pentagons -Video

In this assignment, Ruby leverages her knowledge of the relationship between the dodecahedron and the icosahedron to create a dance. Her assignment is a video with dubbing, including two parts: one is her movement within the physical frame of the icosahedron, and the other is her dance in the studio.

In the video, we can see she is tracing the intersecting pentagons within the icosahedron shape. As we know, the icosahedron and the dodecahedron are duals, so connecting the centers of the faces of an icosahedron gives a dodecahedron and vice-versa (Figure 10). When the icosahedron and the dodecahedron interact and fit each other, the pentagons appear around each vertex of the icosahedron. Ruby recognizes this and stresses it through her movements (Figure 11).

Part of Ruby’s dubbing of the video being with movements (Figure 11):

> I want to think about the dodecahedron interacting with the icosahedron and think about the icosahedron is that the intersections between each part of dodecahedron causes like tiny triangles to peek out……（0:40）
>
> But I am taking my hand and I am tracing along each pentagon shape that is forming each point. (0:48) So each point has a pentagon, like triangular pyramid around it.

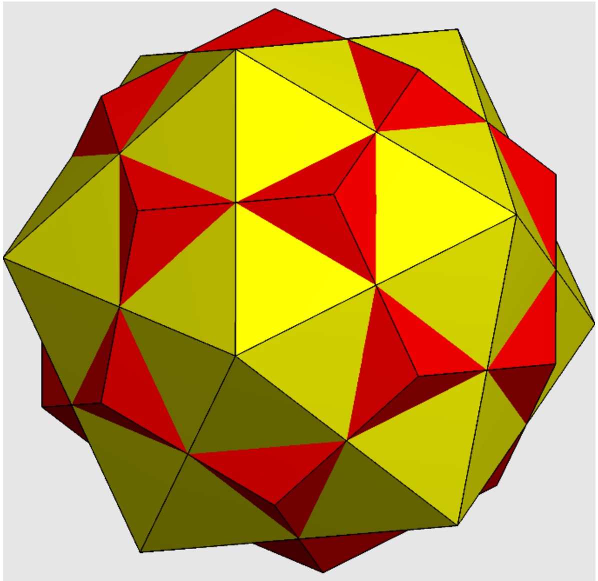

Figure 10: Compound of Dodecahedron and Icosahedron.

Figure 11: Ruby Tracing the Intersecting Pentagons within the Icosahedron Shape.

In this part, Ruby was interacting with the icosahedron frame and reflecting on the relationship between the icosahedron and the dodecahedron with her body and her tracing movements. The physical frame of the icosahedron not only provides Rudy with the space to move her body but also contributes to her thoughts regarding the pentagon shapes. At the same time, her arms make her thoughts regarding the pentagon shapes touchable and visible.

In the second part of Ruby's video, she dances lines and angles in a way that her body can move freely around the space (Figure 12). She dubs the second part of video with those words (Transcript):

> So, thinking about again the dodecahedron with icosahedron and how they interact with each other, and fit within each other kind of ties into that. So, this part of this video is, I want to recreate the dodecahedron laid out into the space. So basically, I am dancing half of it. I am starting with the base. It is kind of like a compressed version. So, I am starting with the base and then I am building up. And something that I have noticed about the dodecahedron is that there's a lot of, there is a top plane and a bottom plane, but the middle plane looks like zigzags as you

are going across it. That is kind of what I am doing here.

……

So, for this part, I am thinking about all of the intersecting pentagons within the icosahedron shape. So, I am finding them and then tracing them. I found it interesting because the points of the dodecahedron have the same kind of fractal on the ends of them when they are combined with the icosahedron. So, their natural form of shape is reminiscent of dodecahedron even though it is still, it is very much its own shape.

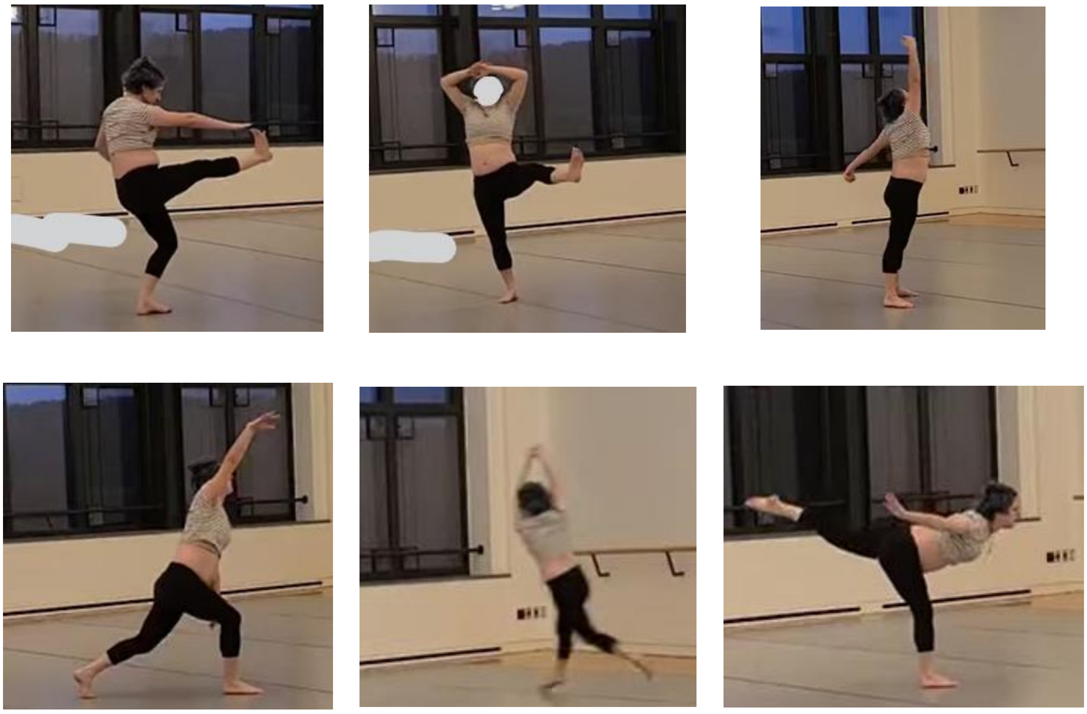

Figure 12: Some Snapshots of Ruby's Dance Representing the Dodecahedron with Icosahedron.

She wants to recreate the dodecahedron laid out into the space. Her movements are simulating the planes and angles that the dodecahedron has, especially the unique shapes that the dodecahedron looks like, such as "zigzags" as Ruby said. Incorporating her words into her dance, we acknowledge that she not only has an accurate and deep understanding of the dodecahedron but also, she grasps those features of the dodecahedron which are different from other shapes to create her dance.

In her dance, she is sculpting the space around her rather than being the space and embodying them. This idea may be raised using the physical frame of the icosahedron, but it is also significant and outstanding in her knowledge and creativity construction.

Overall, we conclude that the body movements that can dance lines and angles work as a representative for Ruby, especially when she illustrates her geometric cognitions like the relationship between the icosahedron and the dodecahedron. She draws on the features of geometric shapes and creates her unique dance.

4.3.8 Episode 27. Gianna's Balance and Gravity -Video & pdf

In her final project, Gianna has two keywords to show her entire idea. The first word is "gravity", and the other one is "balance", which both appear more than twice in her final writing and presentation.

As we know, "gravity" has an evident impact on the body and movements. Gianna paid sufficient attention to the influence of "gravity" in her movements and presented her understanding in her final class presentation. Here is part of transcript of her presentation:

> So feeling your weight down into the floor and then moving with (within) the impulse of that weight so if I felt if I was on the floor and felt that my arm wanted to go this way, I would keep that moment going in that direction and have my spine follow in that twist and then my pelvis and my legs however they want to follow and then keep following that impulse (Figure 13).

Figure 13: Gianna Is Keeping That Moment Going in That Direction.

Her movements represent her awareness of gravity. Also, she takes the gravity of the weight of her body into account and demonstrates the function of "gravity" with a set of movements, which makes a difference in the dance field. Furthermore, she thinks of the "balance" of her dance between the weight and the space. She illustrated in her dance that her body holds a balance between her weight and the space within which she exists, which matters for the understanding and creation of dance.

The other "balance" mentioned in her presentation refers to the position she put herself in within the icosahedron. As her analysis regarding mathematical component in her final writing said:

> The reason I choose to use balance as the main element of my technique is that the positions, I put myself in are in response to the A scale and B scale of the icosahedron. Performing the scales can improve overall balance and perception of yourself in space. I used the information that the planes and spatial pulls within the vertexes provide to decide the positions and transitions between them, mainly because I view the icosahedron as an extension of my own body – all the space I could possibly inhabit.

The icosahedron, which is a balanced and regular three-dimensional geometric shape, is an important frame that is used to describe the space and directions around the dancer in the Laban dance. In regular classes, the instructor leads students to do a dance using Laban scales that involves the planes and spatial pulls within the icosahedron. When Gianna thinks of her dance, the space around her as a dancer as well as the movements framed by the A scale or B scale that she was taught by the instructor are both connected to the icosahedron. Therefore, we can see the icosahedron is both the frame that surrounds her as the environment and the tool that triggers her thoughts of geometry with her body. Gianna deepens her understanding of the use of Laban scales in the dance and sparks her idea of “balance”.

Overall, in Gianna’s final presentation, we can see the thinking of “gravity” and “balance” is significant to Gianna’s creative dance and her understanding of geometry. This understanding is achieved via the dance Gianna created for illustration of “gravity” and “balance” and her typical reflection in her mind during the dance. Thus, we conclude that Gianna was leveraging her knowledge regarding space, the icosahedron, the A scale or the B scale in Laban dance to gain an understanding of “gravity” and “balance” and create dance in an embodied way.

4.3.9 Episode 29. Shay’s Eye Tracking Duck, Shay-pdf

Shay’s final project writing illustrates that she is leveraging geometrical cognition developed throughout the course. In the class, the instructor introduced what Laban dance is and how Laban created this system, which inspired Shay. Given what Shay learned and reflected on her own life, she is aware that our eye-tracking tendencies may have the same mechanism as Laban Dance. Then, in her final project report, Shay presents a few animations of a ducky, butterfly, and dinosaur taking a walk on different paths, followed by other complex pictures that involve similar paths as the previous ones (Figure 14; Figure 15). Those animations and pictures are telling the audience how eye tracking happens.

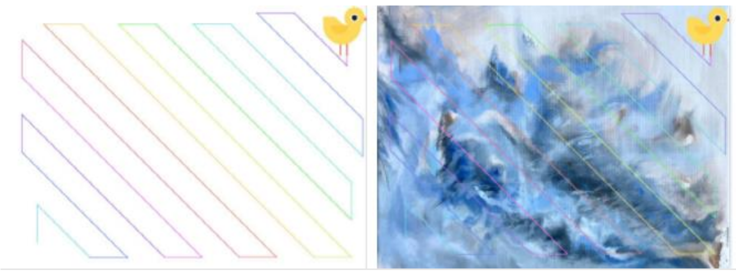

Figure 14: A Snapshot of Animations of a Ducky.

Figure 15: A Snapshot of Animations of a Butterfly.

Firstly, eye tracking relies on the function and movement of human eyeballs, which are part of our body. It attests to the work of the sensory-motor system within our body. As Shay said, “Our eyes are attracted to certain paths in visual compositions”. The reason why our eyes can be attracted by certain paths is because the eyes are one of the most important sensory organs in our body. The fact that the eyes following certain paths roll and move verifies that sensory function leads to motion. Based on Shay’s writing and presentation in the final class, we believe Shay has recognized the mechanism of eye-tracking, which is an excellent illustration of the sensory-motor system. Unlike other students who use their bodies to create dance, Shay reflects on the motion of eyeballs and explains her geometrical cognition in a visualized way, although it is two-dimensional.

In her writing, Shay mentions,

> Horizontal lines to be stabilizing and balanced. Vertical lines are reaching and draw focus upward. Diagonals are the most dynamic for their imbalance, leading our eye to find stability.

Shay has generated an awareness of these lines and directions and mastered their basic characteristics. Further, she combines these thoughts with eye tracking and reflects on more pictures to show the application of her geometrical cognition. There is no doubt that she offers several good examples of eye tracking, which also supports her good leverage of geometric concepts.

4.3.10 Episode 30-32. Fiona’s Reflection on Relationship-Videos & pdf

Fiona's final project has three segments—a report (writing), 3 videos, and a presentation in the last class. Her videos showcase two dances. The first one is ballet (Figure 16). In her final project report (the writing), she starts with two paragraphs describing the movements taking place in her dance, mainly from ballet. The writing illustrations of her dances are separated into two parts based on the space that the dance is occupying, the upper hemisphere and the lower hemisphere. She subsequently elaborates on her understanding and opinions regarding space and bodily movements. In terms of her interpretation of her dance, we can conclude some perspectives below.

Figure 16: Fiona's First Dance.

Firstly, Fiona adopts an embodied approach to understanding the space around her. She thinks of the human body as the center of a coordinate system and the sphere around the body is made of infinite points that can be posited in this coordinate system. Her limbs can take the initiative to create the movements of reaching any point within this space. Her body and movements working as conceptual metaphors provide the information input to her cognition system. This demonstrates her outstanding geometric cognition of the sphere as well as the use of a coordinate system. Then, based on what she has obtained from the envisioning of the space crafted with her body, Fiona demonstrates a good understanding of all geometrical shapes able to be presented in this space around the body. As she said, “Everything is possible with the sphere; it is infinitely individualizable.”

Further, she connects the individual sphere and multiple dancers’ space to the planets in the universe, which have similar features in a metaphorical lens. She expands her understanding of the sphere in the geometrical field to discuss the human relationship which is like two or more than two individual spheres close to each other and colliding with each other. Fiona involves many elements from the environment in her understanding of space and then projects her understanding onto more things in real life. This is not only about how her cognition is built within the interaction of her mind, body and the environment but also an excellent use of conceptual metaphor, which employs a concrete concept to comprehend an abstract one. Also, it is a good illustration to show she is leveraging her geometrical cognition to foster her bigger thoughts concerning the beauty of dance, the universe, and more things that may be ignored previously.

In conclusion, this chapter answers the two research questions with thirty-three episodes extracted from the raw data and presents ten featured episodes with details. Throughout the analysis of how non-STEM students demonstrate or leverage geometric concepts within a lens of embodied cognition, we can see various ways of embodied cognition taking effect in building students' knowledge and promoting their lives. We will review what we found in these results and discuss their implications in the next chapter.

Chapter 5

5 Discussion and Conclusion

In this chapter, we summarize the results of our analysis in Chapter 4, explore how to further expand students' geometric knowledge conceptually, review the use of embodied cognition in this research, and discuss its profound implications on mathematics education with specific executable cases, as well as interdisciplinary education. At last, we analyze the limitations of this study and forward the possible directions that future studies can take.

5.1 Summary and Discussion of Results

To clarify the main answers to the two research questions, we present a summary of the results we obtained from the previous chapter. Additionally, we discuss the potential conceptual limitations of the geometry that was performed in this course based on the NCTM expectations for students and propose several suggestions for the improvement of this course.

5.1.1 How Non-STEM Students Demonstrate Geometric Concepts

5.1.1.1 Summary of the Answer to Research Question 1

Students demonstrate geometric concepts in multiple ways. According to our data, the most common way for students to demonstrate geometric cognition is to recognize geometric shapes in their daily lives. For example, Violet recognizes circles and triangles in the picture of a clay sculpture (Episode 1). Fiona points out geometric elements in the process of drawing (Episode 2). Helen realizes the symmetry of the establishment when walking into a grocery store (Episode 4). Ruby thinks of the chainmail jacket as a cylinder as she is making a chainmail jacket (Episode 9). In addition to pictures, other things stimulating students' geometric thinking include videos like the figure skating video students watched in Episode 15 and dance like the paired circle dance in Episode 16, etc.

Visual information captured by the subject's eyes is transited to the brain to shape their perceptual experience. In this process, the work of the sensorimotor system is affected by both the environment and the subject's intention of looking for geometry. The brain finally generates a series of cognition that integrate original visual information and recognized geometric shapes or elements. That is a prominent path of how embodied cognition works for the demonstration of geometric cognition based on my analysis.

On the other hand, embodied cognition can collaborate with imagination or memory. Gianna creates a movement-scale poem in imagination (Episode 11). Tina recalls her memory of working in her grandmother's kitchen (Episode 18). The previous bodily experience is stored in the brain and can be activated by the current environment (Gentsch & Kuehn, 2022). The interaction between the two things can process in the mind in an imaginary manner without performing real physical movements. The statement from neuroscience that imagination of certain actions will activate the same neural area with the real physical action supports this phenomenon (Mulder, 2007). Meanwhile, in terms of the role of the body, it is not only the container of the sensory-motor system providing sensory inputs to cognition but also a resource that can be sensed and analyzed by the host herself. We can find evidence illustrating the dual role of the body in Episode 19 where Gianna analyzes that her body is forming a triangle with the floor.

5.1.1.2 Discussion of Conceptual Limitations and Improvement Possibilities in Terms of the Geometric Concepts Demonstrated

Although students demonstrate their geometry cognition in multiple pathways, the geometric concepts and patterns may be deepened and enhanced further if the following strategies or activities were applied to this course. We employ three students' performances as examples to show how to improve them.

5.1.1.2.1 Helen's Symmetry Thinking

In Episodes 4-7, Helen shows her strong recognition of symmetry when she hangs out in a grocery store. According to Helen, people prefer more symmetrical shaped fruits and vegetables rather than misshapen ones. The layout of the shelves and products are symmetric, as well as the patterns constituted by the tiles on the floor and on the ceiling. The sense of symmetry is rooted in Helen's mind and essential to her understanding of more complicated geometric concepts like common types of transformation including translation, rotation, and reflection. If we break the limitations of geometric concepts Helen developed in the grocery store and explore more important mathematics ideas based on symmetry, the student can reap more through expanding her knowledge. We will illustrate the possibility of expanding geometric knowledge by continuing the scene that the student has engaged in.

The understanding of common types of transformation like translation, rotation, and reflection strongly relied on the sense of symmetry. In other words, the specific example of symmetry Helen caught in the grocery store is a good source for her to understand transformation. For example, the student can change the layout of fruits to verify how reflection works and to differentiate reflection and rotation. Students can learn how to use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure by treating the layout of fruits as a figure and manipulating fruits. Additionally, if she can place a coordinate system in the layout of fruits, she may learn what the vector is when she moves the location of a certain apple. The concept of vectors is extremely important for students to learn about matrices in linear algebra, which can be experienced within a geometric context by moving items and recording their movements just like moving an apple on the shelf in a grocery store.

Further, the grocery store is a wonderful place for students to experience geometric transformations. Students can establish a model on paper to represent the symmetry appearing in the store and analyze how to organize the grocery with more significant geometric cognition. When the environment like the grocery store participates in the development of cognition, abstract geometry becomes more tangible and accessible for students. Using the theory of embodied cognition to facilitate the analysis and design of a geometric curriculum is full of potential. We will show more examples of expanding geometric knowledge in the original scene to achieve more mathematics depth in an embodied way in the following discussions.

5.1.1.2.2 Ruby's Chainmail Jacket

Rudy designed a chainmail jacket with her knowledge of cylinders and made a video recording of the processes of making a chainmail jacket as her Theme 2 assignment. We analyze her geometric cognition development in Episode 9. Rudy mentions that she measures around her waist multiple times to make the size of the chainmail jacket more precise. Based on her description, she used differential geometry to create this outfit. We appreciate her hard work on this time-consuming task. In addition to the production of a beautiful chainmail jacket, she also demonstrates her excellent understanding of the area and volume of the jacket which is similar to a cylinder but more complicated than the cylinder.

Giving an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone is suggested for American high school students according to CCSSM (n.d.). Geometric measurement and dimension are also fundamental for students to solve real-life problems. Thus, how to help students practice and enhance their skills in this area is important for educators to consider. Rudy has given us a good example in real life, and it can be expanded to more daily classroom practices by customizing individually for each student. For example, students can be asked to estimate the amount of wood needed to frame a sloping roof, manufacture, and calculate the capacity or volume of ice-cream cones or funnels, etc. To enhance their experience, students can even design a shopping cart for bigger capacity but more convenient use. When students are participating in any real-world problem-solving by designing or producing something, they are more engaged since the deeper interaction among their mind, body and the environment is based on the theory of embodied cognition.

5.1.1.2.3 Cora's Handicraft

In Episode 10. Cora-Pictures of Handicraft, Cara presents a few pictures with an analysis of geometric shapes she recognized, mainly triangles (Figure 17). The recognition of triangles relies on her sensory-motor system reacting to the visual information input and her existing geometric cognition regarding triangles. Based on her knowledge and awareness of triangles, further expanding her knowledge of similarity, right triangles and trigonometry can be operated here. Understanding similarity in terms of similarity transformations, proving theorems, and applying trigonometry are essential for students in making sense of problems and solving them (CCSSM, n.d.).

Figure 17: Cora's Pictures of Handicraft in Theme 1 Assignment.

To expand the student's knowledge, we can further discuss these pictures with the student and ask her to look for congruent triangles and similar triangles from what she has found. In this practice, students need to understand the determination of congruent triangles and similar triangles. If she cannot find these special triangles from what she has marked in the pictures, she is supposed to recognize new triangles to meet the requirements of congruent triangles or similar triangles. She may consider the same distance along the human's limbs or the same ratio of the arm to the leg to determine congruent triangles.

Additionally, the instructor can order the student to make a similar triangle with her body, which check the student's understanding of similarity. When students imitate the posture of the figure in the picture, she has to explain why the shape she creates with her body is a similar triangle to the one recognized in the picture. In the process of the student controlling the angles and the distances formed by her limbs, she is learning that if the measures of two sides in one triangle are proportional to the corresponding sides in another triangle and the including angles are congruent then the triangles are similar (Math Planet, n.d., Similarity section). This practice can be applied to the learning of right triangles and expanded to learn similar polygons as well.

5.1.2 How Non-STEM Students Leverage Geometric Concepts

5.1.2.1 Summary of the Answer to Research Question 2

After being aware of geometry, students start to apply geometric knowledge to all aspects of life. For example, Fiona capitalizes on geometric shapes assisting herself in drawing a figure's face (Episode 22). Tina uses geometric shapes to operate a make-up in her video (Episode 20). Tina (Episode 21) and Ruby (Episode 26), translate geometric patterns in their mind into hand movements or body movements. These pieces of evidence show that the human body works as a representing tool dominated by the brain to enact students' cognition. Thus, the result of our research enriches the exploration of the body's functions.

Further, several students use conceptual metaphors to project the knowledge or feelings of geometry they gained to another layer of thinking regarding life like eye tracking (Episode 29), mental energy (Episode 28), and human relationship (Episode 32). We can thereby conclude that leveraging geometric concepts benefits students' individual reflection. In our research, conceptual metaphor has a significant role in the mind, body, and environment interactions. That is, embodied geometric concepts are rooted in conceptual metaphors in that understanding (conceptualization) of abstract geometrical concepts is facilitated by making metaphoric connections to concepts in the environment perceived by the body (sensory motor). The use of conceptual metaphor involves the participation and interaction of the body and the environment, which facilitates the enactment of embodied cognition in this course. We can conclude that conceptual metaphors make great sense for students when they are leveraging geometric concepts. This finding might have implications for how mathematics is made accessible to a wider range of learners.

5.1.2.2 Discussion of Conceptual Limitations and Improvement Possibilities in Terms of the Geometric Concepts Leveraged

5.1.2.2.1 Shay's Parallel Lines

In Episode 29. Shay's Eye Tracking Duck, Shay asserted that our eyes are attracted to certain paths in visual compositions. Then, she illustrated how our eyes track certain patterns with three examples, which separately include horizontal lines, vertical lines, and diagonals. We have discussed how her geometric cognition was developed through an embodied lens. Here, in terms of the geometric concepts she had recognized, we intend to find a way to expand or deepen the student's understanding of parallel lines with intersections and angles.

Two lines that are stretched into infinity and still never intersect are called coplanar lines and are said to be parallel lines (Math Planet, n.d., Perpendicular and parallel section). Parallel lines appear in each picture that Shay presents in her assignment. She referred to these parallel lines to show how human attention is attracted via the eyes. Due to the regularity, parallel lines are easily caught by our eyes and impress us. Shay has recognized the features of parallel lines, which is a good opportunity for her to expand her knowledge. Students usually study perpendicular and parallel coming with angles, parallel lines, and transversals in the context of high school. The discussion of certain pairing angles like interior angles, exterior angles, and alternative angles helps students to deepen their understanding of the features of parallel lines and other relevant concepts, which may benefit the proof of some theorems.

To expand the conceptual depth of mathematics in this case, there are other examples presenting the important role of intersections where two or more than two lines meet. The following picture is from photography aiming to tell photographers how the photo draws the audience's attention (Figure 18). The focus of those pictures is usually the intersections of two perpendicular lines, which are usually divided by each other in a golden ratio. The golden ratio that works in these pictures catches the attention and it helps the students to pay attention to the intersections of lines. The instructor may change how two lines intersect to illustrate a variety of pairs of corresponding angles formed by the intersecting lines. On the other hand, students can take the initiative to manipulate the lines to produce different types of intersections of lines so that they gain more embodied experience in learning perpendicular and parallel.

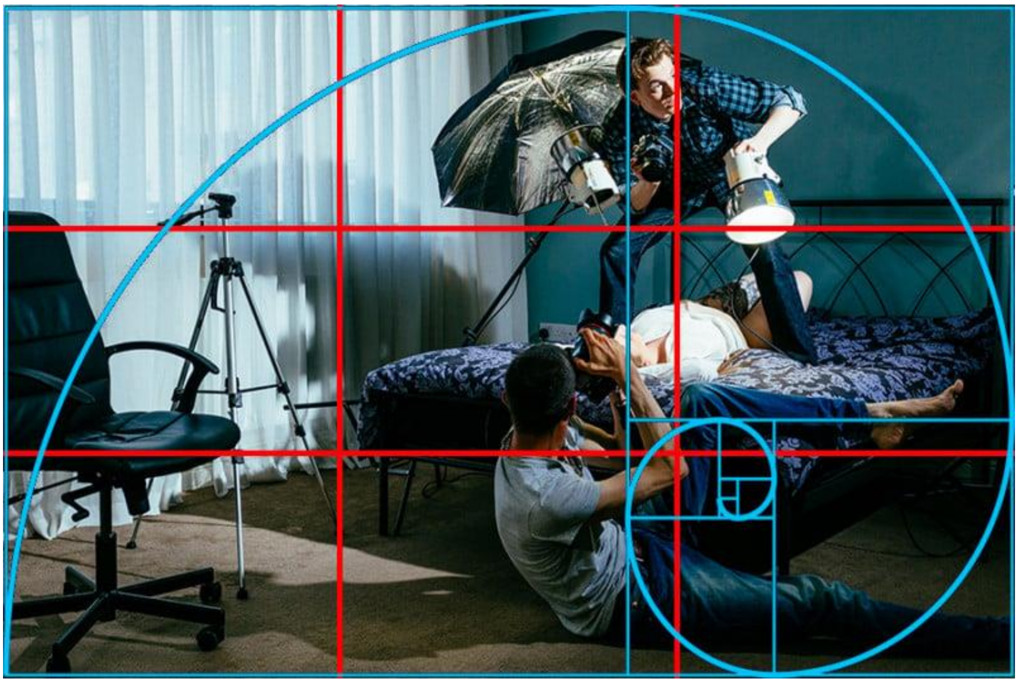

Figure 18: The Golden Ratio and the Rule of Thirds in Photography.

Note. Credit: https://petapixel.com/golden-ratio/

5.1.2.2.2 Tina's Spatial Reasoning Ability

Tina recalled her memory regarding her grandmother's motions in the kitchen when she was engaged in the class activity discussing the space. The process of reaching up to a big shelf, going down to the cabinet and then moving to the stove, etc. happens in a space that centers Tina or her grandmother. Tina describes this space and all motions within it in the class, combining her movements and recapping the kitchen scene. We can recognize Tina's spatial reasoning abilities, which include seeing and reflecting on spatial objects, representing the spatial relationships between parts of objects, the location of objects in space or their movements (Hegarty & Waller, 2005, as cited in Ramful et al., 2017).

Spatial reasoning abilities not only have a strong association with mathematics but also with STEM disciplines (Uttal & Cohen, 2012; Wai, et al., 2009). Thus, how to measure and develop spatial reasoning ability has been of sustained concern to educators and researchers (Ramful et al., 2017). Based on the spatial ability requirements at the middle school level, the boundaries of spatial reasoning ability should at least involve three dimensions including mental rotation, spatial orientation, and spatial visualization. Non-STEM undergraduate students are expected to develop these three spatial abilities to a greater extent.

Tina's kitchen work offers a practical opportunity to implement mental rotation. The instructor can ask Tina to determine the outcome of a rotation of a kettle or a rice cooker, which are 3D objects, with clockwise or anticlockwise turns. Also, she may try to differentiate between reflection and rotation in her mind rather than manipulating them with her hands. For practicing spatial orientation, which is usually used for reading maps, the instructor can require students to choose the shortest route to locate spots or vehicles as policewomen. They can increase the level of difficulty of this activity by setting that the north is not in the vertical upright direction and reading maps from different perspectives. Finally, students can try to visualize the folding and unfolding of a piece of paper with punched holes or join the parts of a polygon to construct a whole in their mind to achieve spatial visualization though it is not easy. The embodied approach can help any difficulties in imagination be resolved by landing them into real-life situations, but it is still hard to directly promote mental ability. Thus, connecting any tricky spatial tasks to real-world problems may benefit students by leading them to be accustomed to thinking in the mind with body assistance.

5.1.2.2.3 Fiona's Sphere

Fiona discussed a lot about spheres in her final presentation, which illustrates her outstanding understanding of spheres and three-dimensional geometry. She argues that spheres are intriguing because they have unlimited potential. She also mentions that all points of all shapes can be created on a sphere or within one. These two statements are significant in examining her geometric cognition. Based on our observation of Fiona's presentation, we can assume that she has a preliminary awareness of the relationship between the sphere and other polyhedrons. If she did not recognize it enough, we tend to enhance her knowledge further by bringing the thought and relevant concepts of limits which is commonly used in calculus or mathematical analysis.

How to introduce the thought and concepts of limits in mathematics classes is always challenging but essential. The pair of the polyhedron and the sphere may be an excellent example for students to understand the study of change, which is the foundation of calculus. Sometimes the sphere is used as an approximation tool to estimate the volume of a certain polyhedron. As the number of faces, edges, and vertices of the polyhedron increases, its shape tends to become smoother and more sphere-like. The thought of approximation can inspire students to understand the concept of limits in calculus. For this to work, the instructor can lead Fiona to put her favourite polyhedron in a sphere and visualize the relationship between them. Then, Fiona can take another regular polyhedron into the sphere to observe and figure out the relationship among two or more polyhedrons within the same sphere. The practice not only validates Fiona's statement that all points of all shapes can be created on a sphere or within one but also contributes to her deeper understanding of a set of regular polyhedrons which have the same centre.

In learning three-dimensional objects or visualizing relationships between two-dimensional and three-dimensional objects, understanding the concept of spheres is essential and useful, especially for learning regular polyhedrons. We can also explore the cross-sections of three-dimensional objects because Fiona mentions the intersection or the unity of different spheres where the dancers keep themselves as the central axis.

5.2 Embodied Cognition in our Research

Before discussing the results within an embodied cognition lens, we need to restate what embodied cognition posits.

Embodied cognition is a theory concerning cognitive processes, emphasizing the inextricable relationship between the mind, the body and the environment. According to Wilson's (2002) six distinct claims that fall under embodied cognition, human cognitive processes derive from the real-world environment, heavily relying on their body-based experiences (or say body-based actions) and being viewed as "extended" rather than dividing the sharply internal and external world. The theory of embodied cognition, as Markman & Brendl (2005) state, assumes that the formation of thoughts is closely related to the perception and action systems and there is an interaction between the perception and actions. The human body provides sensory inputs to cognition and characterizes the semantic content of concepts through the sensory-motor system.

The following discussion will expand this theory with separate discussions on important segments like the sensory-motor system, body-based experiences, and the interaction of body and environment, based on the empirical results of how non-STEM students construct and express their geometric knowledge through sight, perception and motion.

5.2.1 Visual Inputs Contributing to the Cognition

When the body interacts with the outside world, vision is a primary part of working within the sensory-motor system. In response to our two research questions, we can find many cases where students accept visual information input and eventually reflect on what they have seen. When students point out assumed geometric shapes in a photo where no certain regular geometry appears, we need to think of the mechanism that makes students connect the content of the photo to the recognized geometry.

Vision, according to the sensorimotor approach posted by O'Regan and Noë (2001), “is something we do, rather than something that happens in us” (as cited in Shapiro, 2014). Seeing is not just producing a “faithful metric-preserving replica of the outside world inside the head” (O'Regan, 1992, as cited in Shapiro, 2014). Rather, the process of seeing is like an exploring activity in which the finding depends on what the person is doing and thinking. For example, the reason why students can recognize the geometric shapes in the photos is that they keep geometry in their minds, and they are looking for shapes when they look at photos. This piece of evidence indicates that the product of the sensory-motor system is attuned to the mind.

We can see more exemplary evidence in our life. I believe the majority of people can recognize a facial shape in the below picture (Figure 19). This is mainly because most of us have had an impression of the face in our mind and the mind affects how we interpret the content of this picture. In these cases, the sensory-motor system is not only providing visual information inputs, but it can also be mediated by the existing knowledge that the subject has.

Figure 19: A Recognizable Facial Shape inside the Complex of Branches.

Note. Credit: TikTok/itsme.fuzz, as cited in Hayes, 2022.

5.2.2 The Importance of Body-Based Experiences

Based on the theory of embodied cognition, the role of body-based experiences is extremely salient in the process of forming cognition. Embodied cognition literature suggests that the physical actions we perform with the use of our body, as well as the actions being performed around us, shape our mental experience (Barsalou, 1999; Lakoff & Johnson, 1999; Niedenthal, 2007, as cited in Sullivan, 2018). Ratey (2008) presents evidence showing that physical movements in which we participate influence how we think, learn, and remember (as cited in Kuczala, 2010). Smith et al's. (2014) review summarizes that body-based activities can benefit the development of mathematical understanding.

We can find many cases in which students were gaining and utilizing their body-based experiences to develop their geometric cognition in class. For example, in episode 16 where the class topic was “space”, an important concept for students to frame three-dimensional geometry, students performed a dance in pairs and used their bodies to constitute a sensible “space”, and then employed various conceptual metaphors to describe that space. Their moving bodies are part of the space and contribute to their imagination and cognition of the space.

Another example is from Episode 19. Gianna started to recognize the geometric shape within her body and to think of the relationship of her pelvis and legs to her body when her partner used the ball or the stick to touch her body surface in a diagonal direction. The touching from her partner results in Gianna’s body to sense and constitutes her body-based experience, contributing to her final geometric reflection regarding her body parts.

These examples highlight the pivotal status of body-based experience, denying the old statement that cognition is solely formed in the mind (Lakoff & Núñez, 2000). The human body is not only the container of the sensory-motor system, but it also produces countless valuable experiences influencing the formation of cognition. When we look at our lives, the joint work between body-based experience and cognition appears frequently. For instance, when you want to purchase a new dress, putting it on your body in a fitting room is much easier to evoke your desire to buy it than shopping online. The feeling of touching the material of that dress and looking at your pretty figure in the mirror is an important type of body-based experience that persuades your brain to pay for it. It might also work in the opposite direction. Trying on a dress might also dissuade you from buying, highlighting that the body plays an integral and embodied role in cognition.

5.2.3 The Interaction of Body and Environment

Against the argument that cognition is an activity of the mind alone, how the environment participates in the generation of thoughts is the focus of the theory of embodied cognition. The literature on embodied cognition usually presents examples to illustrate that humans reduce their cognitive load by off-loading work on the environment as a cognitive strategy. One example comes from the game Tetris, in which the player needs to decide how to rotate or horizontally translate falling block shapes to fit the shapes that have already fallen. The time for the decision of how to orient and place each block is limited due to the necessary movements of falling blocks. According to Wilson (2002), the data suggest that players use actual rotation and translation movements to simplify the problem to be solved, rather than mentally computing a solution, and then executing it. Here, the environment and real-world objects not only afford valuable information but also undertake the cognitive function when the human subject interacts with them. The participation of external resources for embodied reasoning allows the human subject to solve the problem without conscious, deliberate thinking.

Episode 25 and Episode 26 provide good illustrations verifying the role of the environment. In Episode 25, Tina performed a dance in which she connected the sunrise to her lived experience and expressed her “up” spirit in bodily movements. The sunrise is one of the elements in her environment that has an impact on her cognition through the integration of her mind, body and environment. Her reflection of “up” and “rise” relies on the repetitive sunrise rather than happening independently in her mind. Likewise, Ruby generated her reflection within a physical frame of icosahedron posited in the studio in Episode 26. In Ruby’s video for the Theme 2 assignment, she was tracing the intersecting pentagons in the icosahedron shape with her arms and other parts of her body and reflected on the dual relationship between the icosahedron and the dodecahedron in the mind.

Overall, we can reasonably conclude that the environment contributes indispensably to the formation of cognition through the sensory-motor system and other more complex mechanisms embedded in the body and this is how the environment and the body get interacted. Meanwhile, the use of conceptual metaphors is a common method of translating different types of information and shaping new cognition. Taking good advantage of the interaction of the body, mind and the environment according to the theory of embodied cognition in an educational context will make a big difference and inspire tons of teachers, students and scholars.

5.3 Implications

5.3.1 For Mathematics Education

As we argued above, the theory of embodied cognition may benefit education if we seize its interactive nature and apply it to real scenarios. Here, we will go back to American different-level mathematics classes to seek outstanding cases of the application of the theory for addressing the problem of "abstract" mathematics.

5.3.1.1 Example 1 (Elementary School Level)

Co-Splitting Fraction and Angle Measures of a Circle to Reason about Lunar Phases

This case is extracted from Provost & Panorkou's (2023) study at Montclair State University, presented on the Grad Student Research-in-Progress Roundtables in the 2023 AERA annual meeting. In this case, an embodied action-based design is used to bridge the fraction and angle measures of a circle by employing a physical phenomenon of Lunar Phases. As Figure 20 shows, the whole circle of the Earth surrounded by the moon is 360°. The sunlight shines on the moon from a fixed direction, but the moon moves around the Earth as time goes on. What humans can see of the moon from the Earth depends on the location of the moon in orbit and the angle between the moon and the sun. Students can manipulate the moon to change its location or input the degree of the angle between the moon and the sun on a virtual simulating platform on the website (https://acmes.online/htmls/moonpie/moonpie.html) to observe the change in Lunar Phase. Furthermore, they are expected to use fractions to represent the location of the moon and find the relationship between the co-splitting fraction and the angle of a circle.

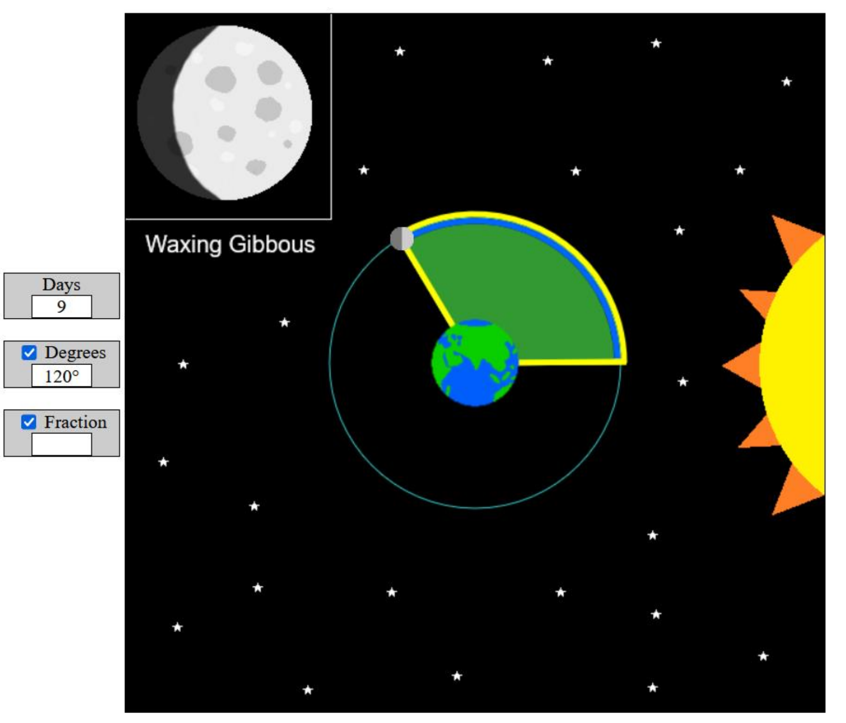

Figure 20: A Manipulatable Platform of the Moon Location and Lunar Phase.

(Provost & Panorkou, 2023)

In terms of the teaching of fractions, this case makes the fraction embodied by two steps, transiting the division of a circle into a visible explanation of fraction and employing an action-based technology to make the process manipulatable. Also, it connects with a common astronomical phenomenon that students are most familiar with and curious about. The visual information shown on the website and the actions students perform contribute to students' understanding of fractions by activating their sensory-motor system. The astronomical context in this case is a good environment engaging students and triggering their thinking specifically.

5.3.1.2 Example 2 (Middle School Level)

Mathematics Imagery Trainer for Proportion

This case is from an original research article, which is part of the research topic: Future of STEM Education: Multiple Perspectives from Researchers (Abrahamson et al., 2020).

Mathematics Imagery Trainer for Proportion is a computer program designed for the teaching of proportion. The system can be set at a particular ratio and students are asked to move two cursors up and down to find locations that make the screen green (Figure 21). When the student's right and left hands' respective heights above the base relate by that particular ratio, the screen becomes green. For example, the system is set at a 1:2 ratio, so that green feedback is activated only when the right hand is twice as high along the monitor as the left hand (see Abrahamson et al., 2014, for the case of other ratios). Once students succeed, they are asked to move both hands, keeping the screen green.

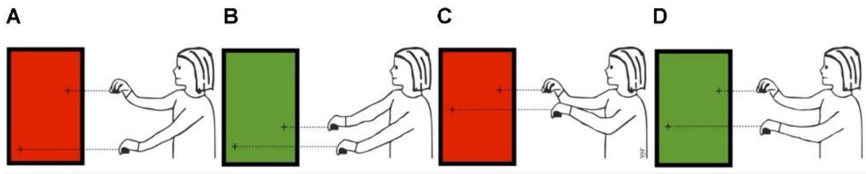

Figure 21: The Mathematics Imagery Trainer for Proportion: Schematic Activity Sequence.

Note. (A) while exploring, the student first positions the hands incorrectly (red feedback); (B) stumbles upon a correct position (green); (C) raises the hands, maintaining a fixed interval between them (red); and (D) corrects the position (green). Note in B and D the different spatial intervals between the cursors or hands. (Abrahamson et al., 2020)

This case is a good, embodied action-based design facilitating students' understanding of arithmetic and algebra. The mathematical concept is presented in the form of a sensory-motor task, as the student is asked to find and maintain continuous green feedback from the screen. Students are establishing new sensory-motor coordination to multimodal expressions of their reflections on the performance. The Mathematics Imagery Trainer acts as a transiting system providing students both the opportunity of developing a perception-action loop and the feedback contributing to the formation of cognition.

5.3.1.3 Example 3 (High School Level)

Understanding the Relationship between Radian and Angle in a Unit Circle

This is another example of embodied action-based design employing an artifact and a computer program where the screen becomes green when the student achieves the instrumented criteria (Shvarts et al., 2021). The design contains two tasks.

The first task concerns the coordination of the distances along the circumference of the unit circle and along the x-axis. Students can move point C around the circle and point X on the x-axis (Figure 22). The feedback frame turns green when these two target distances are equal. Here, the origin on the Cartesian plane is deliberately aligned with the counting origin on the unit circle. The student is required to find green locations and then maintain—while moving both hands continuous green feedback from the screen without knowing ahead of the enactment the rule that determines the feedback.

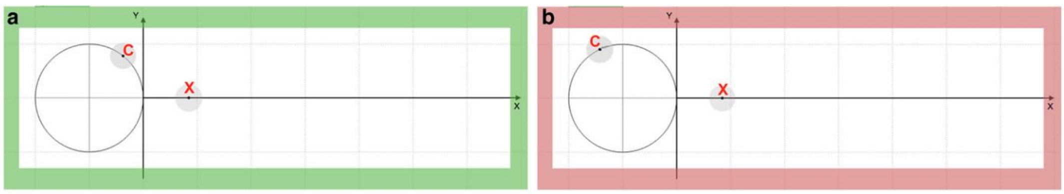

Figure 22: Coordinating the Distances along the Circumference of the Unit Circle and along the X-Axis.

Note. The frame turns green when two distances from the origin are the same. Red notations of the points are added for clarity and were not shown to the students. (Shvarts et al., 2021)

When the student explores the embodied task, they may iteratively start from the origin when their right hand moves at a different speed from the left hand, thus making the frame red. The student's eyes must concentrate on the frame most of the time. After a few minutes, they may be able to sustain green feedback. If the student reflects on his strategy, we can find a new level of intentionality and a new level of the functional system is established as a result of their perception of a one-to-one correspondence between the points, which is viewed as shaping body potentialities.

The constant feedback from the interactive design facilitated the development of a new functional system of body regulation in the digital environment. This embodied experience not only helps the student to understand the relationship between an arc on the unit circle and an x-coordinate on the Cartesian plane, but it also becomes naturally activated when they encounter the next artifact's mathematic task: to find the corresponding angles and radians using an artifact in which only the point C on the circle was manipulatable and the sum of green arc and the green segment was equal to the entire circle (Figure 23). It is not tricky to recognize the artifact's affordances for aligning two different measurements of the arc: a possibility to measure the arch length in degrees on the unit circle and to receive the length in radians on the x-axis.

Previously elaborated coordination is preserved in the student's body potentialities and now was activated within the new body-artifact functional system and helped in anticipation of the automatically moving point along the x-axis.

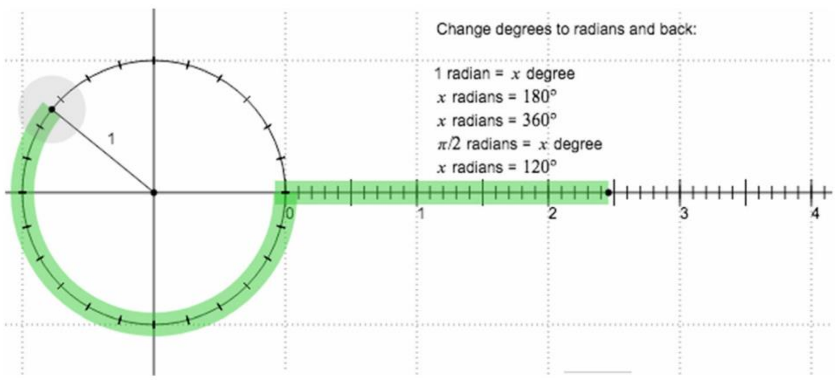

Figure 23: Mathematical Task and Digital Artifact for Aligning the Distance on the X-Axis and the Distance on the Unit Circle.

(Shvarts et al., 2021)

In this case, the instrumented action was regulated by the system of the body (including eye movements) and the artifact: the perception-action loop is mediated—in a very physical, not semiotic sense—by the green automatically moved strip which physically connects the arm movement and gaze. Digital artifact, the student's motor action and focus of visual perception are physically interlocked into a loop, thus contributing to the solution of the target task and the understanding of the concept.

5.3.1.4 Example 4 (College or University Level)

Bottle-filling Problem

This case works for the understanding of the rate of change, which is a basis of calculus for undergraduate students (Yu & Uttal, 2022). Participants are presented with a funnel-shaped container (Figure 24) and are told that water is being added into the container at a constant rate (α ml/s). Then they are allowed to fill the water with their hands and measure and graph the height of water in the bottle as a function of time and the rate of change of height as a function of time. That is, h(t) and h'(t). Given that we ask students to graph not only the height h(t) but also its derivative h'(t), the rate at which water level increases with time, they have to go beyond intuitively conceiving the rate of change in terms of going "faster" or "slower," "increasing" or "decreasing." Instead, they need to coordinate the relationship between multiple variables: the volume of water being added into the container per second, which is a constant (α ml/s), and the change in the container's volume with every small increase in height; this value remains a constant within the cylindrical section but increases quadratically with height in the conical section. Students can formulate and solve calculus functions or make concrete representations that show how the variables are related. Successfully conceptualizing and coordinating the multiple variables in the problem will lead to finding the h'(t) to be steady in the cylindrical section and decreasing in the conical section as shown in Figure 25.

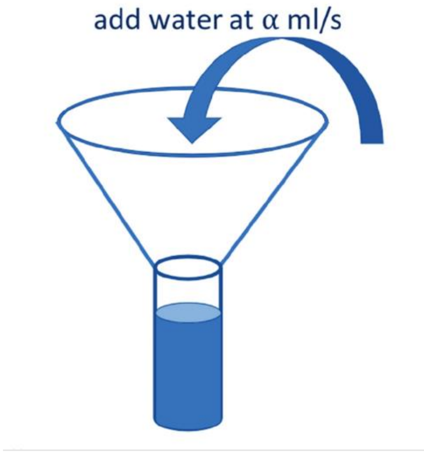

Figure 24: Diagram of the Problem.

(Yu & Uttal, 2022)

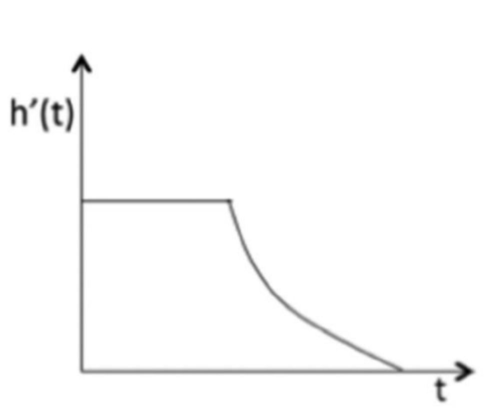

Figure 25: The Graph Most Closely Represents the Actual Trend of the Rate of Change of the Height.

(Yu & Uttal, 2022)

In this case, it is effortless for students to recognize the rate of change as going “faster” or “slower,” “increasing” or “decreasing.” The focus of this design is nevertheless to build the relationship between the shape of the water container and the change of height as a function of time. The design of this model along with a real-world problem is an excellent representation of the basic calculus. Students can get bodily experience by pouring water into the container and measuring the height of the water by themselves. In the process of students pouring water and observing the height, they are not only intuitively impressed by the change of height going slower, but they are also possibly brought to conceive of the change of cross-section of the container with water. Here the interaction between students and the funnel-shaped container shapes particular bodily experiences contributing to the cognition of the rationale of calculus.

Overall, the embodied approach enriches mathematics teaching by learning through movements. Mobilizing students' sensory-motor system, increasing their bodily experience and interacting with the environment can effectively expose students to both knowledge and enjoyment, which may mediate students' mathematics difficulties to some extent. In this sense, enacting the student's body and connecting it to the class content is a good idea for mathematics teachers to design their classes. For students, embodied mathematics learning provides them with new options to understand abstract concepts instead of root learning.

5.3.2 For Interdisciplinary Education

According to Vasquez (2014), the definition of interdisciplinary education is where students learn concepts and skills from two or more disciplines that are tightly linked so as to deepen knowledge and skills.

Dance and mathematics are not closely related to each other, but they can get connected through specific aspects like the connection between geometry and Laban dance. Students are aware of and dominate their bodies when performing the Laban dance, and they develop the innovative idea of putting themselves in the center of the coordination system to understand three-dimension geometry. What we find in this disciplinary class suggests that the connecting point among different disciplines is extremely significant for students' understanding of knowledge, which implies the importance of grounding the connecting points in an interdisciplinary curriculum.

In addition, an interdisciplinary course that integrates different subjects can provoke students' interest in learning by starting with the subject that students are familiar with or interested in. For example, Fiona and Gianna had dance-related experience but they are non-STEM students. They attended this class with their interest in dance and fill their dance ideas into their assignments representing geometry. Here, the learning of geometry is effectively promoted through students' understanding of dance. Therefore, we assert that interdisciplinary education can take effect by evoking students' interest, which is another minor implication of our study.

Interdisciplinary education increases the possibility of applying embodied cognition to teaching and learning, which may enable students to access knowledge that used to be inaccessible. In the process of students gaining embodied experiences in an interdisciplinary context, they are empowered to think of what they are doing in more than one way. Multiple ways of thinking can supplement each other to make sense of difficult knowledge. Also, the application of embodied cognition encourages students to think of their behaviors and reflect on the contents of their experience, which is called meta-awareness (Chin & Schooler, 2009). When students think with purpose, like what is happening when they are processing the knowledge, they tend to understand that knowledge. In our data, students reflected on the way their body represented mathematics, then being aware of their body, the space and their mind. The meta-awareness that students undertook helps them be aware of their situation, like how their bodies interact with the objects around them, facilitating them to make sense of abstract concepts. Those embodied experiences in turn promote meta-awareness.

5.4 Limitations of This Study

As qualitative research, my interpretation is relatively subjective, which means that the results are highly based on my interpretations. My background, experience and individual views have a great impact on my analysis. For example, what I interpreted on students' movements is based on my dance experience, which may automatically recognize dance movements when a student did a similar one. Instead, due to the lack of mathematics knowledge and experience, my interpretation of embodied mathematic concepts may be different from other scholars. In this case, you can learn from this study and the findings, but you are unable to apply it to every similar situation since it is not generalizable enough. The subjective interpretation of the sole author is a common problem in qualitative research.

To mediate the subjective interpretation and make my findings more accurate, I should interview participants to validate their thoughts when they were performing those embodied experiences analyzed by us. Or I should share with them our interpretation of their performances to do a member check. Besides, I wish I can know more about the participants and their backgrounds. It can be a strengthened analysis if I have an opportunity to speak with students after analyzing or examining their data. However, we didn't do it since we are using secondary data that is untraceable. In this sense, the use of secondary data and untraceable participants leads to more limitations to this research.

Last but not least, I was not there physically to make observations. The videos I watched and drew on are just some clips rather than the recording of the whole class. I may have missed some important interactions that may have occurred. Or it may result in a biased understanding of students' performances without an entire observation.

5.5 Future Directions

The application of embodied cognition to education is a growing area of research over the last decade. The exploration and discussion of how to employ embodied cognition in curriculum and relevant fields continue to evoke the researcher's interest. To advance this research or inspire other research, a few measures can be considered.

Expanding the group of participants is good for gaining rich information regarding the functions of embodied cognition in learning. In this study, students participating in this class usually had a dance background or experience, which may promote their embodied behaviors. Further study on this topic can investigate more students who may not have much experience in dance or sports to collect generalized data. Likewise, we can also expand the age range of participants to kindergarten level or older adults.

This study builds on a coincidental intersection between three-dimensional geometry and Laban Dance, in which the results may not fit each case that wants to employ embodied cognition. Future studies should explore more interdisciplinary courses or programs to examine embodied cognition so that the research results can broaden the scope of the application of embodied cognition.

The idea of interdisciplinary education and the use of embodied cognition can be widely found outside of school or campus. Conducting research to investigate how embodied cognition works in after-school activities or in informal learning sites is also a good idea. Or we can design experimental activity for students purposely to evoke their embodied cognition and further gather the specific strategies of enacting embodied cognition.

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Appendices

Appendix A: Ethics Approval Form.

Western Research

Date: 14 April 2023

To: Dr. Anton Puvirajah

Project ID: 120949

Study Title: The Investigation of Non-STEM Undergraduate Students' Geometric Cognition Development within an Embodied Cognition Lens

Short Title: Mathematics and Dance Education

Application Type: NMREB Initial Application

Review Type: Delegated

Full Board Reporting Date: 05/May/2023

Date Approval Issued: 14/Apr/2023 10:41

REB Approval Expiry Date: 14/Apr/2024

Dear Dr. Anton Puvirajah

The Western University Non-Medical Research Ethics Board (NMREB) has reviewed and approved the WREM application form for the above mentioned study, as of the date noted above. NMREB approval for this study remains valid until the expiry date noted above, conditional to timely submission and acceptance of NMREB Continuing Ethics Review.

This research study is to be conducted by the investigator noted above. **All other required institutional approvals and mandated training must also be obtained prior to the conduct of the study.**

Documents Approved:

Document Name	Document Type	Document Date	Document Version
Masters Thesis Proposal	Protocol	31/Mar/2023	1

The Western University NMREB operates in compliance with the Tri-Council Policy Statement Ethical Conduct for Research Involving Humans (TCPS2), the Ontario Personal Health Information Protection Act (PHIPA, 2004), and the applicable laws and regulations of Ontario. Members of the NMREB who are named as Investigators in research studies do not participate in discussions related to, nor vote on such studies when they are presented to the REB. The NMREB is registered with the U.S. Department of Health & Human Services under the IRB registration number IRB 00000941.

Please do not hesitate to contact us if you have any questions.

Sincerely,

Ms. Zoë Levi, Research Ethics Officer on behalf of Dr. Randal Graham, NMREB Chair

Note: This correspondence includes an electronic signature (validation and approval via an online system that is compliant with all regulations).

Page 1 of 1

Appendix B: Part of the Course Syllabus.

World of Pattern: Geometry, Body, and Beauty – DANC 200.03

Spring 2022

*** Students must be opened to learn and explore new technology, making errors/mistakes and become self-regulated learners. There is a learning curve.

Course Description
This course explores selected topics in body movement, arts, and world of patterns, and their underpinning mathematical ideas and geometrical patterns. Course content will progress through multiple modalities with emerging knowledge and best practices from STEAM education including dance and movement studies, aesthetic nature of math, Islamic arts, and computational modeling.

Student Learning Outcomes

- Students will explore and identify mathematical and geometrical underpinning for creativity and making with hands-on activities
- Students will investigate patterned human movement and 'movement' scales to train the moving body, with respect to geometrical patterns
- Students will apply and synthesize their learning in creative and computational studies, and through problem solving activities.

Grade Distribution		
Assignments/Projects[1]	**Due date**	**%**
Active Participation *Being present in every class is required but does not get you any points; rather, you must passionately contribute to the course activities. You should also actively being engage and take verbal and physical risks in class.*	Ongoing through the semester	15%
Theme 1: Human Movement and geometrical patterns Theme 1: Reflection on aesthetic and mathematical presence in one's life. *Create something: Decide on your medium of choice. 3-5 min video or a webpage or a visual representation, 500-800 word writing) and critically reflect on the topic (It could be a TikTok, Reel, YouTube (unlisted or public, your choice) Your reflection must be supported by at least 3 research articles (peer-reviewed), TEDTalks, or books) Online and website citation wouldn't count. Your reflection must include both geometrical patterns and body movements.*	Due Feb 13th	15%
Theme 2: Creative geometrical movements Theme 2: Movement Sequence in Space, described in geometrical and metaphorical terms. *You will create, record, play, explain, artistic/creative movement. You may explain the metaphorical aspect of these movement through poem, play, sing a song, record a podcast for us to listen to as we walk through a space... You may bring awareness about equity, social justice, or cultural values into play. Be creative!*	Due Mar 6th	15%

Theme 3: Aesthetic nature of mathematics and computational arts modeling	Due Apr 3rd	15%
Theme 3: Program a movement sequence on Scratch. *Through Scratch block-based programing, you will design an artistic movement/pattern/game/ etc.*		
Final Project. Details will be provided in February.	Due Apr 27th	40%

Weekly Schedule

Sections	Week	Geometry/CS topic	Arts Topics	Assignments
1	Week 1 Jan 19	Intro and warm-up activities	Mass, Grounding, Kinesphere	
	Week 2 Jan 26	Geometry, the gem of mathematics. Identify geometrical shapes		
2	Week 3 Feb 2		Components of Kinesphere and Movement	
	Week 4 Feb 9		Metaphors/Meaningful Associations with Space	Theme 1 Due Feb 13
3	Week 5 Feb 16	The sexist's rectangle; Islamic Arts		
	Week 6 Feb 23	Platonic solids turn amorous		
4	Week 7 March 2		Spatial Pulls, Transversals, CPT	Theme 2 Due Mar 3
	Week 8 March 9	SPRING BREAK		
	Week 9 March 16		Neurological Connections to Space	
5	Week 10 March 23	Creative modeling Programing		
	Week 11 March 30		Making Individualized Movement Sequences	Theme 3 Apr 3
6	Week 12 April 6	Special Ability workout	Teaching Movement Sequences from Space and Metaphor	
	Week 13 April 13	Soothing symmetry. Contortion of space		
7	Week 14 April 20	Whatever is most needed		
	Week 15 April 27	Application Project Presentations	Application Project Presentations	Final project
	May 4th	Continue of the project presentation	Application Project Presentations	Final project

Curriculum Vitae

Name:	Chen Lin
Post-secondary Education and Degrees:	Northeast Normal University Changchun, Jilin, China 2012-2016 B.Sc.
	Northeast Normal University Changchun, Jilin, China 2017-2019 M.Ed.
	The University of Western Ontario London, Ontario, Canada 2021-2023 M.A.
Honors and Awards:	Art Geddis "Learning About Teaching" Memorial Award 2022-2023
Related Work Experience	Biology Teacher Harbin No. 6 High School 2016-2021

Presentations:

Lin, C (2023). Body, Mind, and Integration in the Development of Mathematics Concepts. Poster presented at the 2023 American Educational Research Association (AERA) Annual Meeting, Chicago, Illinois, U.S., April 14, 2023

Sedaghatjou, M., Wahl, C., **Lin, C** (2022). World of Pattern: Geometry, Body, and Beauty. Poster presented at the Forty-Fourth Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education, Nashville, Tennessee, U.S., November 19, 2022.