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ABSTRACT

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Public goods are often needed to facilitate private consumption. For example, highways are needed so that people can drive their cars. The provision of many public goods thus depends directly on the consumption of certain private commodities. This paper examines the implications of such "facilitating" public goods for the structure of indirect taxes. It is shown that many results on optimal taxation, derived in the literature, need to be modified when facilitating public goods are introduced. A Pigouvian tax, based on the externality generated by the consumption of private goods, turns out to be a simple solution to the problem.
PUBLIC GOODS, OPTIMAL TAXATION, AND ECONOMIC EFFICIENCY

by
Kul B. Bhatia*

1. Introduction

Much interest has been shown in the problem of optimal taxation in recent literature. Diamond and Mirrlees (1971), Stiglitz and Dasgupta (1971), and others have discussed various aspects of the question, dealing with cases of both private and public goods. One category of public goods, however, has not received much attention in this literature. These are goods whose provision depends directly on the consumption of certain private commodities. For instance, if people eat more nutritious food, fewer health facilities might be needed; safer cars might result in fewer accidents, and hence a smaller number of hospitals, etc. In some cases, these goods are needed primarily to facilitate the consumption of private goods. Thus an increase in the number of cars will necessitate more highways, and so on. Following Sen (1966), such goods may be called "permissive" or "facilitating" public goods.¹ Their presence significantly affects the efficient structure of commodity taxes and alters most of the results on optimal taxation derived in the literature. Moreover, many earlier studies in this area assume that the government raises a fixed revenue, without asking how this revenue is disposed of — [e.g., Atkinson and Stiglitz (1972), Dixit (1970), etc.]. The provision of public goods is one possible use of tax revenue. The analysis in this paper thus provides a more realistic look at the problem of optimal taxation.

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The model is outlined in Section 2. The optimality conditions are derived in Section 3, and their implications for the structure of indirect taxes are discussed in Section 4. The emphasis is on efficiency and the role of demand factors. Equity considerations, although very important in the financing of public goods, are not essential for the main argument of the paper, and are left out of the analysis.

2. The Model

2.1 General assumptions

We assume that there are \( n \) commodities \((x_1, x_2, \ldots, x_n)\) which the consumers buy for private consumption. The government provides \( m \) commodities \((g_1, g_2, \ldots, g_m)\) which do not directly enter individuals' utility functions, but "facilitate" private consumption of the \( x_i \)'s. It is also assumed that producer prices are fixed for all commodities and labour (the only factor supplied by households). Further, the \( x_i \)'s are subject to a tax per unit, \( t_i \). The prices paid by consumers \((q_i)\) therefore differ from the producer price \((p_i)\): \( q_i = (p_i + t_i) \). There is only one set of prices for public goods, however, because there is no tax on them. Without loss of generality, we assume that, besides the \( m \) public goods, one private good (leisure) is also not taxed. For ease of exposition, the wage rate and prices of public goods are set equal to unity by a suitable choice of units.

2.2 Assumptions about production

To focus on the interaction between public and private goods, we make the simplest possible assumptions about production. We assume constant returns to scale throughout the economy. Supply elasticities thus do not play a very important role in the analysis. The frontier of the convex
production set is given by:

$$P(x_1, x_2, \ldots, x_n, g_1, g_2, \ldots, g_m) = 0$$  \hspace{1cm} (2.1)

2.3 **Assumptions about consumption**

A consumer's budget constraint is given by

$$y = \sum_{i=1}^{n} q_i x_i = L,$$  \hspace{1cm} (2.2)

where $y$ denotes money income, and $x_i$ is the quantity of the $i^{th}$ good purchased by the consumer. A utility function $U(x, L)$ is maximized subject to (2.2). The usual first order conditions for each individual are

$$U_i = \alpha q_i$$  \hspace{1cm} (2.3a)

$$-U_L = \alpha$$  \hspace{1cm} (2.3b)

where $U_i = \partial U / \partial x_i$, and $\alpha$ is the marginal utility of income.

Since we do not deal with equity considerations, we assume that all consumers are identical. What is good for one is good for all, which suggests that we can focus on the welfare of a "typical" individual, $U(x, L)$, where $x$ is a vector of privately consumed goods $\{x_i\}$.

2.4 **Government behaviour**

It is assumed that the commodity taxes are levied to pay for the public goods at producer prices. The government's demand for public goods ($g_k$) depends on the consumption of private goods.

$$g_k = G_k(x_1, x_2, \ldots, x_n) \hspace{1cm} k=1, \ldots, m$$  \hspace{1cm} (2.4)

The government's budget can be written as

$$\sum_{k=1}^{m} g_k = \sum_{i=1}^{n} t_i x_i = \sum_{i=1}^{n} (q_i - p_i) x_i = L - \sum_{i=1}^{n} p_i x_i$$  \hspace{1cm} (2.5)

Any surplus or deficit in the government budget is ruled out by (2.5).
3. Optimality Conditions

Ignoring externalities in production and consumption, the government selects \( t_i (i=1, \ldots, n) \) to maximize \( U(x,L) \) subject to (2.5), the balanced budget restriction, and (2.2) and (2.3), the conditions for individual utility maximization derived in Section 2. Take the Lagrangian multiplier expression:

\[
U(x,L) + \delta \left[ \sum_{i=1}^{n} q_i x_i - L \right] - \lambda \left[ \sum_{k=1}^{m} g_k + \sum_{i=1}^{n} p_i x_i - L \right]
\]  \hspace{1cm} (3.1)

We can substitute for \( q_i \) from (2.3a), and defining \(-L\) (leisure) as good 0, rewrite the Lagrangian in vector notation:

\[
U + \frac{\delta}{\alpha} U'x - \lambda (g + px)
\]  \hspace{1cm} (3.2)

Where \( U' \) is the vector of the first partial derivatives of the utility function \( \{U_i\} \), \( p \) is the vector of product prices \( \{p_i\} \), \( i=0, \ldots, n \), and \( g \) is the vector of public goods \( \{g_k\} \). The first order conditions are

\[
(1 + \frac{\delta}{\alpha})U' + \frac{\delta}{\alpha} U''x - \lambda g' = \lambda p
\]  \hspace{1cm} (3.3)

where \( U'' \) is the vector of second partial derivatives of \( U \) and \( g' \) is \( \partial g / \partial x \).

The \( ij \)th element of the \( nxm \) matrix \( g' \) is \( \partial g_j / \partial x_i \).

Following Atkinson and Stiglitz (1972), let us define

\[
H_j^n = \sum_{i=0}^{n} \frac{(-U_{ij})}{U_j}
\]

where \( H_j^n \) is the sum of the elasticities of the marginal utility of \( x_j \) with respect to each of the other commodities in the utility function. Substituting \( H_j^n \) in (3.3) we have:

\[
q_j [\alpha + \delta (1-H_j^n)] = \lambda (p_j + g'_j) \quad j=0, \ldots, n
\]  \hspace{1cm} (3.4)
Since there is no tax on leisure, \( q_0 = p_0 \). Further, from (2.4) it follows that \( g_0' = 0 \). Therefore,

\[
\delta = \frac{\lambda - \alpha}{1 - H^0}.
\]  \hspace{1cm} (3.5)

Substituting for \( \delta \) in (3.4) and setting \( g_0' = 0 \), we derive

\[
1 - \frac{p_j}{q_j} = \frac{t_j^*}{p_j + t_j^*} = \frac{\lambda - \alpha}{\lambda} \frac{H_j - H^0}{1 - H^0} + \frac{g_j}{\lambda q_j}
\]  \hspace{1cm} (3.6)

The optimal tax rates, \( t_j^* \), are expressed as a percentage of consumer prices of private goods in the result (3.6). It is the same as (3.5) in Atkinson and Stiglitz (1972), except for the last term on the right-hand side, which has some important implications for optimal structure of taxation. These implications are discussed in the next section. In particular, most of the conditions in which uniform taxation was previously optimal would now call for differential taxation. \(^5\)

4. Implications for Optimal Taxation

In the absence of facilitating public goods, efficiency considerations require uniform taxation in the following cases:

i) The indifference map is homothetic, which implies that by a suitable transformation, it can be represented by a function which is homogeneous of degree 1, \( U_i \), thus is homogeneous of degree zero, so \( \sum_i U_{ij}x_j = 0 = H_j^j \) for all \( j \).

ii) The supply of labour is completely inelastic. Then \((- H^0) \) tends to infinity, and

iii) The marginal utility of leisure is independent of the consumption of every other commodity, and \( H^k = H^j \), which implies equal, and hence unitary expenditure elasticities for all commodities. \(^6\)
In all these cases, uniform taxation would not be optimal when facilitating public goods are introduced unless either $g'_j = 0$ for all $j$, or $g'_j / q_j = g'_k / q_k$, i.e., for each commodity entering the utility function, the amount of facilitating goods required equals the same proportion of its consumer price.

4.1 Differential taxation

In the literature on tax optimality, three situations have been widely recognized in which different commodities are taxed at different rates. Thus, under certain conditions, commodities with low price elasticity should be taxed more heavily (the inverse price-elasticity rule), luxuries should be taxed lightly (the inverse income-elasticity rule), and heavier taxes should be levied on commodities which are complementary with leisure. All these decision rules have to be changed when facilitating public goods are present.

i) The inverse price-elasticity rule. Following the work of Ramsey (1927) and others, it is well known that (for small taxes) if all commodities are independent ($U_{ij} = 0, i \neq j$), and the marginal disutility of labour is constant ($h^0 = 0$), commodities should be taxed in inverse proportion to their price elasticities. From differentiating (2.3a), we derive

$$U_{jj}(\partial x_j / \partial q_j) = \alpha.$$

Since $U_{ij} = 0$ ($i \neq j$),

$$h^j = \frac{-U_{jj} x_j}{U_j} = \frac{1}{\varepsilon_d^j}$$

where $\varepsilon_d^j$ is the own-price elasticity of demand for the $j^{th}$ good. When $h^0 = 0$, (3.6) reduces to (4.1):
\[
\frac{t^*_j}{p_j + t^*_j} = \frac{\lambda - \alpha}{\lambda} \frac{1}{\bar{c}_d^j} + \frac{1}{\bar{g}_j} \tag{4.1}
\]

If \( g_j' = 0 \), we obtain the inverse-elasticity rule. Otherwise, optimal tax
on a commodity has to be proportional not only to the reciprocal of its
price elasticity, but also to the marginal cost of public goods required
for the consumption of that commodity. A corollary of this result is that
even when \( \bar{c}_d^j = \infty \), the optimal tax on a commodity will not be zero unless
no complementary public goods are needed for its consumption.

ii) The inverse income-elasticity rule. When utility function is
directly additive, another familiar result arises: that optimal tax rates
will depend inversely on income elasticity of demand.

In a directly additive utility function, one commodity's consumption
does not affect the utility derived from another good: i.e., the utility
function can be transformed such that \( U_{ij} = 0 \) for \( i \neq j \). Therefore, as
derived above, once again

\[
H^i_j = \frac{-U_{ij}x_j}{U_j^i}.
\]

By differentiating (2.3a) with respect to money income (\( y \)) and using (2.3b)
we obtain:

\[
U_{jj} \frac{\partial x_j}{\partial y} = \frac{1}{\alpha} \frac{\partial \alpha}{\partial y}
\]
or

\[
H^i_j \bar{c}_y^j = -\frac{\bar{c}_y^j}{\frac{\alpha}{\partial y}}, \tag{4.2}
\]

where \( \bar{c}_y^j \) is the income elasticity of the \( j^{th} \) good.

By substituting (4.2) in (3.6), we find that the inverse income
elasticity rule holds in the absence of public goods. If private goods
require facilitating public goods, this rule also has to be changed, much like the inverse price elasticity formula in (4.1).

This analysis has some interesting implications for the conflict between equity and efficiency considerations which often arises in matters of taxation. For efficiency, commodities with a low income elasticity of demand (necessaries) should be taxed more heavily than those with a high income elasticity (luxuries). Inasmuch as lower income groups spend a large part of their income on necessaries, efficiency considerations will result in regressive taxation, which is not very equitable. It is possible, however, that some luxuries might require more facilitating public goods than necessaries (e.g., private aircrafts will require new runways or new airports, whereas tennis racquets may necessitate only a few tennis courts). In such cases, taxes levied on efficiency grounds will also be equitable. 7

iii) Complementarity with leisure. Following the work of Corlett and Hague (1953), and Harberger (1964), we know that goods which are complementary with leisure should be taxed more heavily than other commodities. Complementarity, in this context, can be defined either in terms of elasticity of the marginal utility of a good with respect to leisure \( H_{10} \) in our notation, or in terms of the Hicksian compensated elasticity of demand. 8

As noted above, in our analysis, leisure \textit{per se} has no effect on the demand for public goods. But leisure can have indirect effects, by altering the consumption of other private goods which require facilitating public goods. The rule for optimal taxation, accordingly, will have to be expanded in scope. Commodities which are complementary with leisure, \textit{ceteris paribus}, should be taxed more heavily than others if the former are not negatively related to facilitating public goods. By the same token, in certain situations, if leisure's substitutes require more public goods than leisure's complements, optimal taxation might well require heavier taxation of the former than the latter. 9
4.2 The solution

The optimal tax formula (3.6) suggests a simple solution to the problem: the tax on each commodity should reflect the cost of public goods provided to facilitate its consumption. When society provides these goods, the social cost of consuming a private commodity is greater than its private cost. It is a case of joint consumption, but only one component is paid for. The classic Pigouvian tax, equal to the difference between the social and private cost, will ensure optimality. In this framework, optimal commodity taxes will thus be determined by considerations based on utility functions (and summarized by \( H_j \), \( j=0,\ldots,n \)), and the amount of facilitating public goods required for the commodities which directly enter utility functions (\( g_j' \)).

5. Conclusion

We have discussed the implications of "facilitating" public goods for optimal taxation. Demand for such goods often depends directly on the consumption of certain private commodities. Public goods, however, are mostly provided free, or from the general revenues of the government. For efficiency, consumer prices of private goods must include the cost of the requisite public goods. The traditional rules for optimal taxation, therefore, have to be modified. A Pigouvian tax, imposed on the consumer of private goods provides a simple solution to the problem.

To facilitate tax administration, and for other reasons, commodities are often grouped together. For example, necessaries and luxuries might be classified separately and taxed at different rates. In terms of the concerns of this paper, commodities could be divided into two groups—those requiring public goods, and others. The results of our analysis can be
applied to the first group whereas the rules for optimal taxation derived previously in the literature will hold for the second group. Tax policy, in practice, does reflect some of these considerations. In many cases, for example, gasoline and automobile taxes are earmarked for building and maintaining highways, which is one way of charging automobile owners for the facilitating public goods required by them.
References


Footnotes

1 Sen shows that in the presence of such goods, indirect taxes will be invariably superior to direct taxes.

2 Dixit (1970) and Lerner (1970) discussed the problem of optimal commodity taxes with an untaxable sector. In their case, however, the untaxable commodities are all private goods in consumers' utility functions. Thus some elements of the vector \( \{x_i\} \), besides leisure, also are not taxable.

3 The assumption of a fixed revenue (\( \bar{R} \)), used in many earlier studies, can be easily incorporated here. The government's budget constraint (2.5) can be rewritten as (2.5'):

\[
\overline{R} + \sum_{k=1}^{m} g_k = \sum_{i=1}^{n} t_i x_i = \sum_{i=1}^{n} (q_i - p_i) x_i = L - \sum_{i=1}^{n} p_i x_i
\]

(2.5')

None of the results in the paper will be altered because \( \bar{R} \) is fixed. In this connection, it is interesting to note that if there are no public goods, and there are taxes and subsidies such that the net revenue is zero, the Atkinson-Stiglitz type of analysis cannot be carried out unless the assumption of a homogeneous population is dropped.

4 Note that \( x_0 \) does not appear as an argument in (2.4). Increased consumption of leisure will not directly affect the demand for facilitating public goods. Therefore, \( g_0' = 0 \). The specification in (2.4), however, is general enough to include indirect effects of leisure. For example, if increased leisure leads to greater consumption of tennis racquets, more (public) tennis courts may have to be provided.

5 Sen, for instance, proves that when such public goods exist, a direct tax, which is equivalent to uniform taxation on all commodities but leisure, will not be optimal, and indirect taxes will have to be applied. Sen, however, does not deal with differential taxation, or other aspects of the structure of indirect taxes.

6 For a detailed derivation of these results, see Atkinson and Stiglitz (1972).

7 The argument could of course be easily reversed. There is no stipulation that in general the demand for facilitating public goods will increase with income elasticity of private goods. Cheaper brands of tobacco might increase the incidence of cancer, more than the superior varieties, thus requiring more cancer hospitals.

8 For the latter approach, see Harberger (1964), pp. 52-53.
Good examples of this sort are hard to find. The point can be illustrated, however, by comparing two "complements" of leisure--television watching and driving for pleasure. If the former is more complementary than the latter (which presumably is the case in many North American cities where highways are congested and there is football on TV), televisions (or television watching) should be taxed more heavily than cars (or driving). Increased driving, however, will sooner or later require more highways. For optimal taxation, therefore, it might be necessary to tax cars more than television sets even though TV watching is more complementary with leisure than pleasure-driving.