

Modelling non-linear Functional Responses in Competitive Biological Systems.

Nickolas Goncharenko

January 25, 2019

Abstract

One of the most versatile and well understood models in mathematical biology is the Competitive Lotka Volterra (CLV) model, which describes the behaviour of any number of exclusively competitive species (that is each species competes directly with every other species). Despite its success in describing many phenomena in biology, chemistry and physics the CLV model cannot describe any non-linear environmental effects (including resource limitation and immune response of a host due to infection). The reason for this is the theory of monotone dynamical systems, which was codeveloped with the CLV model, does not apply when this non-linear effect is introduced. For the first time we propose a model extension, by incorporating a functional response, which solves this long standing problem. We name this new model a Functional Competitive Lotka Volterra (FCLV) model. We use a number of techniques most notably topology and monotone theory to perform a global analysis. The most important result obtained from the analysis is the existence of a direct mapping between the solutions of CLV and FCLV models. This means that tools and results from CLV analysis directly apply to FCLV models. We use this result to analyse a model which describes within host competition between different organisms in the midgut of the mosquito. The implications of this model to malaria elimination in wild mosquito populations is discussed as well.

Keywords— ODE, Dynamical Systems, Modelling, Competitive Lotka Volterra, Method of Fundamental Domains, Max-Separable Lyapunov Functions