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Essays on Monetary Economics

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Abstract

My dissertation, which consists of three papers, is devoted to studying the implications of conventional and unconventional monetary policies for inflation, asset prices, and welfare.

The first paper examines the sustainability and effectiveness of negative nominal interest rates. I construct a model of multiple means of payment where the cost of holding paper currency—its storage and security costs—determines the effective rate of return on currency, which establishes the effective lower bound on nominal interest rates. I show that central banks can reduce the effective rate of return on currency, and thus the effective lower bound, by altering their policy on bank reserves. However, reducing the lower bound leads to welfare losses associated with individuals holding more currency. Moreover, sustaining a negative rate by reducing the lower bound has no stimulative effects. This occurs because this policy combination reduces both the rate of return on currency and interest rates on financial assets, leaving the relative interest rates between currency and financial assets unchanged.

In the second paper, I develop a two-country model with financial frictions to study how a central bank's unconventional asset purchases affect international asset prices and welfare. In the model, the key financial frictions are limited commitment, differential pledgeability of assets as collateral, and a scarcity of collateralizable assets. Due to the differential pledgeability of assets, financial intermediaries acquire different asset portfolios depending on their home country. I find that quantitative easing can reduce long-term bond yields and term premia internationally and depreciate the creditor country's currency. Foreign exchange intervention always depreciates the local currency, but it can improve welfare globally if implemented by the creditor country.

The third paper studies the implications of heterogeneous payment choices for monetary policy. I construct a model of money and credit where each consumer participates in a small-value or a large-value transaction depending on a preference shock. Financial intermediaries write deposit contracts for consumers to intermediate credit transactions. The preference shock is private information and is costly for intermediaries to observe. I find that, in equilibrium, financial intermediaries create state-contingent deposit contracts for consumers. However, private information and costly monitoring generate an incentive problem, so that the quantity of credit is constrained for consumers in large-value transactions. The effects of monetary policy on the allocation of means of payment vary depending on the size of transaction.

Keywords: Negative Interest Rate, Quantitative Easing, Foreign Exchange Intervention, Monetary Policy, Collateral Constraint, Money, Banking, Credit, Private Information, Costly Monitoring

Summary for Lay Audience

Typically, central banks choose a short-term interest rate as a target interest rate and determine the level of the target rate to control inflation. However, they have introduced many other tools to achieve their policy goal. For example, central banks in emerging market economies intervened in foreign exchange markets to stabilize exchange rate fluctuations. Some central banks in advanced economies purchased long-term government bonds (also known as quantitative easing) and promised to keep the short-term rate low for an extended period (forward guidance) to lower long-term interest rates. Some central banks introduced a negative short-term interest rate, the sustainability of which was unknown before. My dissertation is devoted to studying the implications of these non-traditional monetary policies for inflation, asset prices, and welfare.

The first paper examines the sustainability and effectiveness of negative interest rates. Without frictions, negative rates would not have been sustainable due to arbitrage: investors could borrow at the negative rate and hold zero-interest paper currency. However, holding paper currency is costly because of storage and security costs. This makes the effective rate of return on holding paper currency negative, implying that the lower bound on interest rates is also negative. I show that central banks can reduce the effective rate of return on currency, and thus the lower bound, by altering their policy on bank reserves. However, reducing the lower bound leads to welfare losses associated with individuals holding more currency. Moreover, sustaining a negative rate by reducing the lower bound has no stimulative effects. This occurs because this policy combination reduces both the rate of return on currency and interest rates on financial assets, leaving the relative interest rates between currency and financial assets unchanged.

In the second paper, I study how a central bank's unconventional asset purchases affect international asset prices and welfare. I find that a central bank's purchases of long-term government bonds, or quantitative easing, reduce long-term interest rates internationally. Also, quantitative easing relaxes financial constraints in the global economy and improves welfare globally. This occurs because quantitative easing eventually involves the central bank's swaps of short-term bonds for long-term bonds. Short-term bonds are typically more useful as collateral than long-term bonds, so this intervention increases the effective stock of collateral, relaxing financial constraints. The

international effects of foreign exchange intervention depend on the implementing country. It can relax financial constraints and improve welfare globally if implemented by the country that supplies more useful bonds in the global economy.

The third paper studies the implications of heterogeneous payment choices for monetary policy. Consumers typically use more cash and less credit in small-value transactions. Based on this observation, I construct a model of money and credit where each consumer participates in a small-value or a large-value transaction depending on a preference shock. Financial intermediaries write deposit contracts for consumers to intermediate credit transactions. A consumer's preference shock is private information and is costly for intermediaries to observe. I find that it is optimal for financial intermediaries to offer state-contingent deposit contracts for consumers. However, private information and costly monitoring generate an incentive problem, so the quantity of credit is constrained for consumers in large-value transactions. The effects of monetary policy on the allocation of means of payment vary depending on the size of the transaction.

Dedication

To my wife Hyunjung and my children Yuchan and Jimmy

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Table of Contents

List of Tables

List of Figures

List of Appendices

Chapter 1

Negative Nominal Interest Rates and Monetary Policy

1.1 Introduction

Some macroeconomists have argued that the existence of the lower bound on nominal interest rates can be a major obstacle for operating monetary policy and have suggested several policy tools that could potentially remove or reduce the lower bound.[1] Understanding how and why a central bank could control the lower bound with these policy tools requires uncovering the determinants of the lower bound on nominal interest rates. What economic fundamentals or frictions determine the lower bound? If a central bank attempts to reduce the lower bound by manipulating the frictions, what would be the welfare implications? The main goal of this paper is to study the implications of reducing the lower bound on nominal interest rates.

In mainstream macroeconomic theory, monetary policy is constrained by the zero lower bound on nominal interest rates.[2] This constraint arises because, at a negative interest rate, borrowers can exploit an arbitrage opportunity by investing in zero-interest currency. However, in practice it is clear that negative nominal short-term interest rates are feasible, as the European Central Bank, the Swiss National Bank, the Swedish Riksbank, the Bank of Japan, and the National Bank of Denmark, among others, have demonstrated. The implication is that, in practice, there exist significant frictions that inhibit arbitrage, so that the effective lower bound (ELB) on the nominal interest rate is negative.

What are the frictions that make the ELB negative? It is generally recognized that these frictions arise principally because of the costs associated with storing, transporting, and exchanging

[1]See, for example, Goodfriend (2016), Rogoff (2017a), and Rogoff (2017b).

[2]See, for example, Woodford (2003) and Curdia and Woodford (2010) for standard New Keynesian models.

large quantities of currency. For example, if a private bank is facing a negative nominal interest rate on reserves, it could hold currency instead of having reserve balances with the central bank. However, holding currency would entail costs of installing a large vault and hiring guards to watch it, and the currency would be of little or no use in making interbank transactions and online transactions.

Some macroeconomists have made the case that the ELB can be an important binding constraint on monetary policy in some circumstances. For some of these researchers, the frictions that determine the ELB are not welfare-reducing impediments, but frictions that should be enhanced. That is, greater friction means a lower ELB, which permits welfare-enhancing monetary policy, according to the argument (Goodfriend, 2016; Rogoff, 2017a,b). In this regard, several potential policy tools have been suggested to reduce the ELB. These policy tools range from abolishing paper currency to setting a quantitative limit on currency withdrawals.[3]

A market-based approach, and perhaps the most interesting policy tool presented in the literature, involves altering the one-to-one exchange rate between currency and reserves (Eisler, 1932; Buiter, 2010; Agarwal and Kimball, 2015; Goodfriend, 2016; Rogoff, 2017a,b). This policy tool allows the central bank to set a different exchange rate for private banks' current currency withdrawals from the one for their future currency deposits, and potentially creates a negative nominal rate of return on currency. In particular, Agarwal and Kimball (2019) consider setting a nonpar exchange rate as "the first-best approach with the fewest undesirable side-effects" because the nominal rate of return on currency in units of reserves created by the central bank can be naturally transmitted to the rate of return on currency in units of other interest-bearing assets.

To fully understand how introducing this novel policy tool can reduce the ELB, it is crucial to explicitly consider the costs of handling currency and the imperfect role of currency as a means of payment. So, I develop a general equilibrium model with two different types of means of payment—currency and bank deposits—and include costs of holding currency, to answer the following questions. How does introducing a nonpar exchange rate between currency and reserves affect the frictions that determine the ELB? If the ELB falls as a result of this unconventional intervention, what implications does this have for the allocation of means of payment and welfare?

In my model, *private banks* issue deposit contracts to consumers and acquire an asset portfolio of currency, government bonds, and reserves. Deposit contracts allow consumers to choose one of

[3]Complete elimination of paper currency with keeping coins outstanding, which would increase the storage cost of money, is discussed by Rogoff (2017a) and Rogoff (2017b). Goodfriend (2016) discusses introducing a quantitative limit to cash withdrawals at the central bank cash window. Interestingly, Correia et al. (2013) identify a tax policy that enables a negative interest rate, which requires a rising path of consumption taxes, a declining path of labor income taxes, and a temporary investment tax credit. Altering the one-to-one exchange rate between paper currency and reserves was first proposed by Eisler (1932) and later discussed by Buiter (2010), Agarwal and Kimball (2015), Goodfriend (2016), Rogoff (2017a), and Rogoff (2017b). Interested readers can refer to Agarwal and Kimball (2019) for a comprehensive survey of potential policy tools that enable substantially negative interest rates.

two options. If a consumer needs currency in transactions, he or she can withdraw currency from the private bank. Otherwise, the consumer uses bank deposits to make payments in transactions. However, private banks are subject to limited commitment. To resolve the limited commitment problem, private banks post their assets as collateral to back their deposit liabilities. Although bank deposits are useful in transactions, there is inefficiency in the banking system due to binding collateral constraints for private banks.[4] So, consumers cannot obtain the first-best quantity of consumption goods in those transactions.

The central bank, which conventionally determines the nominal interest rate on reserves, can set the exchange rate between currency and reserves off par to create a negative nominal rate of return on currency. In particular, withdrawing one unit of currency from the central bank requires more than one unit of reserves (a nonpar exchange rate applied), while depositing one unit of currency with the central bank provides only one unit of reserves (a one-to-one exchange rate). However, private banks can acquire currency from other private individuals to avoid costly currency withdrawals from the central bank cash window. In other words, there is a possibility of side trading in the private sector, which the central bank cannot observe. Due to side trades of currency, the rate of return on currency set by the central bank can differ from the actual rate of return on currency determined by market participants.

To create a negative rate of return on currency, there must be both currency deposits and withdrawals through the central bank cash window. If private individuals can participate in side trades of currency with no frictions, currency might not come back to the central bank cash window when side trading is more profitable than depositing currency with the central bank. So, it is possible that a nonpar exchange rate fails to create a negative rate of return on currency. However, in practice it is costly to transport currency due to the risk of theft. For instance, private banks hire armed security guards and use armored vehicles to transport a large volume of currency. To reflect potential friction in side trades of currency, I introduce endogenous theft into the model, as another cost of holding currency. Theft can take place if private individuals attempt to side trade currency. In particular, some individuals can acquire a theft technology at a cost and steal currency from those participating in side trading.[5]

A key result is that a nonpar exchange rate can indeed reduce the ELB on nominal interest rates, in line with the idea presented by Eisler (1932), Buiter (2010), and Agarwal and Kimball

[4]More precisely, the fiscal authority determines the total value of consolidated government debt outstanding. If the fiscal authority chooses the value that is sufficiently low, then there arises a shortage of collateralizable assets, leading to binding collateral constraints for private banks.

[5]There could also be other sources of inefficiency in a currency system stemming from side trades of currency. The side trades of currency, especially if occurring in a large volume, could encourage opportunistic behaviors such as fraud and counterfeiting as well as theft. In the literature, the risk of theft is considered in He et al. (2005), He et al. (2008), and Sanches and Williamson (2010), and counterfeiting of currency is introduced in Williamson (2002), Nosal and Wallace (2007), Li and Rocheteau (2011), and Kang (2017).

(2015). In the model, a nonpar exchange rate policy is effective if it can reduce the nominal rate of return on currency faced by both private banks and private individuals. In particular, if the cost of theft is sufficiently low (or equivalently, if there exists theft in equilibrium), the actual rate of return on currency perceived by private individuals tracks the one faced by private banks.[6] This occurs because the presence of theft acts to make some individuals deposit their currency with their banks, in which case one unit of currency is effectively exchanged for one unit of reserves.[7] Since private individuals deposit and withdraw currency at different exchange rates, the actual rate of return on currency is determined by the target rate of return set by the central bank.

Introducing a nonpar exchange rate between currency and reserves, however, is not innocuous for the economy. With a decrease in the rate of return on currency, private banks issue a deposit contract that offers a smaller quantity of currency withdrawal. As a result, the quantity of goods traded in transactions using currency (*currency transactions*, hereafter) decreases and the quantity traded in transactions using bank deposits (*bank deposit transactions*, hereafter) increases in equilibrium. Also, a nonpar exchange rate acts to increase the market price of currency by increasing private banks' demand for currency. As an increase in the market price of currency encourages both side trading and theft, the fraction of individuals bearing the cost of theft increases. As a consequence, the total cost of theft increases, decreasing welfare.

In contrast, if the cost of theft is sufficiently high (or equivalently, if no theft takes place in equilibrium), introducing a nonpar exchange rate does not reduce the rate of return on currency perceived by private individuals. With no risk of theft, individuals strictly prefer side trading and do not deposit their currency with the central bank. As the one-to-one exchange rate (applied to currency deposits) is no longer effective for them, the actual rate of return on currency for individuals does not fall in response to an increase in the nonpar exchange rate (applied to currency withdrawals). However, a nonpar exchange rate can reduce the ELB if the ELB is determined by the rate of return on currency for private banks. There is a range of nonpar exchange rates within which increasing the nonpar exchange rate leads to a fall in the ELB. In particular, an increase in the nonpar exchange rate decreases the ELB if the nonpar exchange rate is sufficiently low, but increases the ELB if the nonpar exchange rate is sufficiently high.[8]

If the cost of theft is sufficiently low, the optimal nominal interest rate on reserves can be negative for an unconventional reason. If currency is costly to store, consumers in currency transactions

[6] In the model, the central bank can observe the quantity of currency held by each private bank and ask private banks to deposit their currency with the central bank (one-to-one exchange rate).

[7] More precisely, private individuals deposit their currency with private banks and then the private banks are required to exchange the currency for reserves one-for-one with the central bank. Therefore, the private individuals indirectly exchange their currency for reserves at par.

[8] This result arises from a collateral role of currency. Private banks use currency as collateral while private individuals do not. As currency bears a liquidity premium due to its role as collateral, it is more costly for private individuals to invest in currency than private banks.

need to bring more currency to compensate for the cost that will be paid by their counterparts.[9] Then, a negative nominal interest rate can mitigate the inefficiency in currency transactions because it increases the rate of return on currency relative to reserves, which indirectly reduces the cost of holding currency. However, even if a negative nominal interest rate is constrained by the ELB, introducing a nonpar exchange rate between currency and reserves cannot increase welfare. This occurs because the optimal nominal interest rate falls in response to an increase in the nonpar exchange rate. So, implementing a deeper negative nominal interest rate with a nonpar exchange rate cannot attain the optimal equilibrium allocation while increasing costly theft. Therefore, optimal monetary policy is to set the nominal interest rate at the ELB and maintain the one-to-one exchange rate between currency and reserves.

To quantify the magnitude of the welfare cost incurred by a nonpar exchange rate, I calibrate my model to the U.S. economy. Three different scenarios are considered where the cost of theft is given by 2.5 percent, 5 percent, and 10 percent of consumption.[10] In each scenario, I measure the welfare cost of increasing the nonpar exchange rate by asking how much consumption private individuals would need to be compensated to endure the welfare loss. I find that the costs of introducing the nonpar exchange rate of 1.05 (or reducing the ELB by around 5 percentage points) are 0.11 percent, 0.22 percent, and 0.44 percent of consumption, respectively, in the three scenarios. Considering that estimates for the welfare cost of 10 percent inflation are typically around 1 percent of consumption, the cost of introducing a nonpar exchange rate seems significant.[11]

The frictions that create a negative ELB could also arise from potential disintermediation. Disintermediation happens when consumers opt to use currency rather than bank deposits to make transactions given a sufficiently low interest rate.[12] This is a practical concern because bank deposits serve as a primary and stable funding source for financing bank loans, and thus, disintermediation might cause long-run inefficiency in the financial system.[13] To understand the implications of introducing a nonpar exchange rate for potential disintermediation, I modify the assumption

[9]Retailers who accept currency usually hold currency by the end of the day or overnight. So, the cost of storing currency here can be interpreted as the cost of purchasing a safe, using a security system, or hiring a security guard.

[10]Since theft does not occur in the model given a one-to-one exchange rate, the cost of theft must be directly calibrated outside the model. However, to the best of my knowledge, there is no data that allows measuring the cost of theft.

[11]The welfare cost of increasing inflation from 0 percent to 10 percent is 0.62 percent of consumption in Chiu and Molico (2010), 0.87 percent in Lucas Jr (2000), and 1.32 percent (take-it-or-leave-it offer) in Lagos and Wright (2005), for example.

[12]In fear of disintermediation, private banks might not be able to actively adjust their deposit rates in response to monetary policy in negative territory. See Eggertsson et al. (2019) and Ulate (2021), who study the implication of negative interest rates for the transmission mechanism of monetary policy.

[13]Replacing physical currency with central bank digital currency (CBDC) can also help reduce the ELB. This is because the central bank can directly set a negative nominal interest rate on CBDC, which is impossible with physical currency. However, it has raised more concern about disintermediation and financial instability as CBDC is a type of electronic means of payment that can completely substitute for bank deposits. Interested readers could refer to Williamson (2022a), Williamson (2022b), and Keister and Sanches (2022), for example.

about available means of payment in transactions. In the modified version of the model, some individuals accept only currency, and others accept both currency and bank deposits as a means of payment. In other words, currency is a universally accepted means of payment, and therefore, potential depositors can opt out of bank deposit contracts and use only currency in transactions.[14]

A key finding is that a nonpar exchange rate between currency and reserves can allow the central bank to set a negative nominal interest rate without harming the banking system, consistent with the conventional view. That is, disintermediation does not take place if the central bank implements a negative nominal interest rate and a nonpar exchange rate so that the nominal rate of return on currency relative to other assets remains constant. However, introducing a nonpar exchange rate encourages costly theft and reduces social welfare as in the baseline model.

Interestingly, in an economy subject to disintermediation, the nominal interest rate must be sufficiently high to prevent a complete flight to currency. In this economy, a complete flight-to-currency episode cannot be supported in equilibrium if there is a shortage of government bonds. If all the depositors decided to opt out of banking contracts, the central bank would have to purchase a sufficient quantity of government bonds to issue a required quantity of currency. But, this would be infeasible if there is a shortage of government bonds. Therefore, the nominal interest rate must be sufficiently high to encourage banking activities. Also, note that the set of nominal interest rates that support banking activities differs from the set of rates that prevent arbitrage. In an economy subject to disintermediation, the former can determine the ELB rather than the latter.

The rest of the paper is organized as follows. I construct the baseline model in Section 2 and define and characterize an equilibrium in Section 3. In Section 4, optimal monetary policy is analyzed, and quantitative analysis is presented in Section 5. In Section 6, a modified version of the model is constructed to analyze the policy implication for disintermediation, and Section 7 is a conclusion.

1.1.1 Related Literature

This paper is related to the existing literature on how to make negative nominal interest rates implementable by removing or reducing the ELB. Among others, Agarwal and Kimball (2019) provide a comprehensive survey and discuss the pros and cons of policy tools suggested in the literature. The idea of a nonpar exchange rate between currency and reserves was first proposed by Eisler (1932) in the form of a dual currency system where one currency (physical currency) is

[14]Recall that, in the baseline model, some transactions require currency while other transactions require bank deposits as a means of payment. In practice, some transactions, especially online transactions, cannot be made with currency. In contrast, some transactions can be made only with currency because either the consumer or the retailer does not have access to the banking system or because they value privacy. Also, there are transactions where both means of payment can be accepted. Therefore, reality may fit somewhere between the two versions of the model.

used as a means of payment, and the other (electronic money) plays a unit-of-account role. Eisler's proposal was revived recently by Buiter (2010), who developed a simple model of a dual currency system where the central bank can reduce the ELB by adjusting the exchange rate between two currencies. Although the simple model helps understand how a dual currency system would work, the main limitation is that the central bank is assumed to frictionlessly determine the ELB with a proper exchange rate between the two currencies.

The key contribution of this paper is to explicitly consider the frictions that determine the ELB, including the imperfect substitutability between currency and bank deposits and inefficiency stemming from storing or transporting currency. Therefore, my paper can provide the implications of a nonpar exchange rate policy itself for the ELB, the allocation of means of payment, and welfare. In contrast, a nonpar exchange rate policy is neutral to the allocation of means of payment and welfare in Buiter (2010), aside from reducing the ELB. More recently, a nonpar exchange rate between currency and reserves has been favorably discussed, but without a formal model by Agarwal and Kimball (2015), Goodfriend (2016), Rogoff (2017a), and Rogoff (2017b). My paper complements these papers by providing a full-fledged theoretical model that helps understand under what conditions the policy works and why it can go wrong.

This paper is also related to a few theoretical papers studying the implications of a negative nominal interest rate. The most closely related paper is the work of He et al. (2008), who construct a model of two competing means of payment, currency and bank deposits, where using currency is relatively less safe due to the risk of theft. Given endogenous theft and banking, they show that the ELB can be negative, and a negative nominal interest rate can be optimal for some parameters. My paper departs from theirs by differentiating private banks' deposits with the central bank (reserves) from private individuals' bank deposits. This structure allows me to study the transmission of a nonpar exchange rate between currency and reserves into the actual rate of return on currency and the terms of bank deposit contracts. Another paper related to the current one is Brunnermeier and Koby (2019), who study the ELB on nominal interest rates in a model with private banks. They define "reversal interest rate" as the interest rate at which lowering the rate further becomes contractionary and interpret it as the ELB. In contrast, I define the ELB as the lower bound on implementable nominal interest rates and interpret their "reversal interest rate" as the optimal monetary policy rate. Relative to this literature, the main contribution of the current paper is not only to study the welfare implications of implementing a negative nominal interest rate but also to introduce a policy tool that can reduce the ELB.[15]

Search-theoretic models with currency (outside money) and bank deposits (inside money) have been analyzed in Cavalcanti et al. (1999), Williamson (1999), He et al. (2005), Li (2006), He et al.

[15] See also, Eggertsson et al. (2019), Jung (2019), and Ulate (2021), who study the implication of a negative nominal interest rate.

(2008), Li (2011), and Williamson (2012) among others. However, these papers do not answer questions such as how the currency-related frictions determine the ELB on nominal interest rates and how the ELB can be lowered by a policy tool. Endogenous theft is also not new in the literature. In the models of He et al. (2005) and He et al. (2008), theft can happen in bilateral meetings if one individual holds currency and the other is a thief. Due to the risk of theft, using currency in transactions is riskier than making a payment with bank deposits. Sanches and Williamson (2010) assume that theft can happen in a subset of bilateral meetings, and acquiring a theft technology incurs resource costs. A key difference is that, in my model, theft can happen in a meeting where one individual carries currency for side-trading purposes, not transaction purposes, and the other has invested in the theft technology.

1.2 Baseline Model

The basic structure of the model is similar to Lagos and Wright (2005) and Rocheteau and Wright (2005) with additional details incorporated to address the particular issues related to this problem. Time is indexed by t = 0, 1, 2, ..., and there are three subperiods in each period. The theft market (TM) opens in the first subperiod, the centralized market (CM) opens in the following subperiod, and the decentralized market (DM) opens in the last subperiod. There is a continuum of buyers with unit mass, each of whom maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[-\tilde{H}_t - H_t + u(x_t) \right], \tag{1.1}$$

where $0 < \beta < 1$, $\tilde{H}_t$ and H_t denote the buyer's labor supply in the TM and the CM respectively and x_t denotes his or her consumption in the DM. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u'(0) = \infty$, $u'(\infty) = 0$, and $-\frac{xu''(x)}{u'(x)} < 1$. There is also a continuum of sellers with unit mass, each of whom maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[-\tilde{H}_t^s + X_t^s - h_t^s \right], \tag{1.2}$$

where X_t^s denotes the seller's consumption in the CM, and $\tilde{H}_t^s$ and h_t^s denote his or her labor supply in the TM and the DM. Finally, there is a continuum of private banks each of which maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[X_t^b - H_t^b \right], \tag{1.3}$$

where X_t^b is the bank's consumption in the CM and H_t^b is its labor supply in the CM. Private banks are agents who are active only in the CM. In the CM or the DM, one unit of perishable consumption good can be produced with one unit of labor supply while no production takes place in the TM. Buyers cannot produce goods in the DM, while sellers cannot produce in the CM.

At the beginning of the TM, sellers holding currency can deposit the currency with the central bank in exchange for reserve balances.[16] At this stage, currency trades one-to-one for reserve balances, and then sellers can exchange reserve balances for goods in the following CM. After sellers make currency deposits with the central bank, buyers can incur κ units of labor to acquire a theft technology (e.g., producing a weapon). Then, buyers and sellers are randomly matched. If a seller with currency meets a buyer with the theft technology, the buyer steals all of the seller's currency.

At the beginning of the CM, debts are paid off, then production, consumption, and exchange take place in a perfectly competitive market. Private banks can obtain currency in three different ways—from a seller, from a buyer (stolen currency), or by acquiring reserves with the central bank and exchanging the reserves for currency. Also, private banks write deposit contracts with buyers before buyers learn their types. A type for a buyer is the type of seller he or she will meet in the following DM, as specified in what follows. Bank deposit contracts provide insurance, by allowing buyers to withdraw currency at the end of the CM when they learn their types. Buyers' types are publicly observable.

In the DM, each buyer is randomly matched with a seller and makes a take-it-or-leave-it offer to the seller. In any DM matches, a matched buyer and seller do not know each others' histories (no memory or record keeping) and are subject to limited commitment. This implies that no buyers' IOUs can be traded in the DM. There are two types of sellers. Fraction ρ of sellers accepts only currency, and fraction $1 - \rho$ accepts only claims on banks. The timing of events is summarized in Figure 1.1.

[16] In practice, private individuals cannot have a reserve account with a central bank. It could be interpreted that an individual deposits the currency with a private bank and then the private bank deposits the currency in exchange for reserve balances. Suppose the central bank can ask private banks to deposit their currency at the beginning of the TM. Alternatively, I could assume that, when an individual deposits the currency with the central bank, the central bank credits the payment to the corresponding private bank which in turn credits it to the individual's bank account.

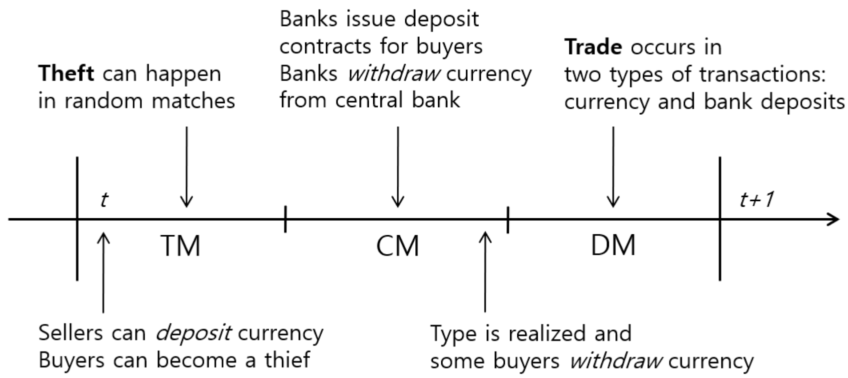

Figure 1.1: Timing of events

Some sellers who acquire currency in the DM may want to exchange the currency for goods in the following CM, instead of safely depositing it with the central bank. Let α_t^s denote the fraction of sellers who carry currency into the CM conditional on having acquired the currency in the previous DM and let α_t^b denote the fraction of buyers who acquire the technology to steal currency. Then, the probability that each seller meets a buyer with the theft technology is α_t^b and the probability that each buyer meets a seller with currency is $\rho\alpha_t^s$.

There are three underlying assets in this economy — currency, reserves, and nominal government bonds. Currency and reserves are issued by the central bank. Currency is perfectly divisible, portable, and storable, and bears a nominal interest rate of zero. Reserves are private banks' account balances with the central bank, and one unit of reserves acquired in the CM of period t pays off R_{t+1}^m units of reserves at the beginning of the CM in period $t+1$. Private banks visit the central bank if they want to withdraw currency from their reserve accounts. Following the ideas presented in Eisler (1932), Buiter (2010) and Agarwal and Kimball (2015) among others, the central bank can set an exchange rate between currency and reserves off par to create a negative nominal rate of return on currency. The exchange rate, denoted by $\eta_t \geq 1$, measures the units of reserves exchanged to withdraw one unit of currency in period t while deposited currency is always exchanged one-to-one for reserves. Nominal government bonds, issued by the fiscal authority, are one-period bonds with a gross nominal interest rate of R_t^b.

In addition to the underlying assets issued by the consolidated government, there are bank deposit claims that private banks create endogenously. I assume that private banks have a collateral technology that allows creditors to seize at least part of the asset if the bank defaults. This implies that private banks can issue bank claims that can be accepted in transactions by some sellers as the claims will be backed by collateral. Private banks hold government bonds and reserves as collateral. A bank could potentially hold currency in its portfolio from one CM until the next (also using it as collateral). But, in doing so it bears a cost γc_t, where $0 < \gamma < 1$ and c_t is the real value of currency held by the bank when it is acquired, and the cost is born in the CM of period $t+1$. Sellers have the same storage technology as banks, but buyers cannot hold currency across periods.[17] In addition to the proportional cost of storing currency across periods, there is a fixed cost μ of holding currency incurred by individuals at the beginning of the TM.

1.2.1 Government

Confine attention to stationary equilibria where all real variables and government policies are constant across periods. Assume that the consolidated government starts issuing its liabilities with no unsettled debt outstanding in period 0. Then, the consolidated government budget constraint at $t = 0$ can be written as

$$\eta\bar{c} + \bar{m} + \bar{b} = \tau_0, \tag{1.4}$$

where $\bar{c}$, $\bar{m}$, and $\bar{b}$ denote the real quantities of currency, reserves, and nominal government bonds outstanding at the end of period 0 (and in every following period). Also, τ_0 is the real quantity of the lump-sum transfer to each buyer. Assume that the fiscal authority can levy lump-sum taxes on buyers in equal amounts. So, the consolidated government budget constraint at $t = 1, 2, ...$ can be written as

$$\eta\bar{c} + \bar{m} + \bar{b} = \frac{\bar{c} + R^m\bar{m} + R^b\bar{b}}{\pi} + \tau, \tag{1.5}$$

where π is the gross inflation rate and τ is the real quantity of the lump-sum transfer (or the lump-sum tax if $\tau < 0$) to each buyer. The left-hand side of (1.5) represents the revenue of the consolidated government from issuing new liabilities, and the right-hand side represents the expenditure on repayments of government debt issued in the previous period and the transfer to buyers.

As in Andolfatto and Williamson (2015) and Williamson (2016), I assume that the fiscal authority determines the real value of the consolidated government debt outstanding, v, where

$$v = \eta\bar{c} + \bar{m} + \bar{b}, \tag{1.6}$$

for all periods.[18] So, the fiscal authority adjusts lump-sum transfers, in response to a change in

[17] This implies that an equilibrium nominal interest rate must satisfy no arbitrage conditions for both private banks and sellers from holding currency across periods.

[18] The central bank purchases government bonds by issuing currency and reserves in period 0 and transfers its profits to the fiscal authority in every following period. This implies that the real value of the central bank's assets must be equal to that of its liabilities in every period, that is,

$$\eta\bar{c} + \bar{m} = \hat{b},$$

monetary policy, to achieve the fiscal policy goal. Given the fiscal policy that targets the total value of the consolidated government debt, the central bank's monetary policy changes its composition. A key feature I can capture through this fiscal policy rule is the scarcity of collateral and resulting low real interest rates. That is, a low supply of consolidated government debt, or a low v, will lead to low real interest rates, and I will eventually analyze the effects of monetary policy given a sufficiently low v.

1.3 Equilibrium

In this section, I will describe how buyers and sellers make their decisions in the TM and how private banks choose their deposit contract and asset portfolio in the CM. Then, I will define and characterize an equilibrium and show why the effective lower bound on the target policy rate can be negative.

1.3.1 Side Trading and Theft

In the CM, private banks can withdraw one unit of currency from the central bank by paying η units of their reserve balances. If there are would-be sellers of currency in the CM, private banks would participate in side trades of currency to reduce the cost of acquiring currency. In equilibrium, private banks must be indifferent between withdrawing currency from the central bank and obtaining currency from any would-be sellers of currency. This implies that the price of currency in terms of reserves in the CM must be η in equilibrium.[19]

Side trading in currency between private banks and currency holders can take place only when there are sellers who choose not to deposit their currency with the central bank in the TM. However, in the TM, some buyers may incur κ units of labor supply to acquire the theft technology. Suppose that the representative currency-holding seller carries c^s units of currency in real terms into the TM. As acquiring the theft technology is costly, each buyer's decision on stealing currency must

where $\hat{b}$ denotes the real quantity of government bonds held by the central bank. Therefore, the fiscal authority determines the real quantity of total government bonds issued by the fiscal authority because

$$v = \hat{b} + \bar{b}.$$

[19]This happens if the real quantity of currency traded in the CM does not exceed the real quantity demanded by private banks. Here, I focus on stationary equilibria with sufficiently low v and γ so that there is always inflation $\pi > 1$ and the real value of currency decreases over time. See Appendix A.1 for the details on stationary equilibria with deflation.

be incentive compatible in equilibrium, that is,

$$\text{if} \quad \kappa > \rho\alpha^s\eta c^s, \quad \text{then} \quad \alpha^b = 0, \tag{1.7}$$

$$\text{if} \quad \kappa = \rho\alpha^s\eta c^s, \quad \text{then} \quad 0 \le \alpha^b \le 1, \tag{1.8}$$

$$\text{if} \quad \kappa < \rho\alpha^s\eta c^s, \quad \text{then} \quad \alpha^b = 1, \tag{1.9}$$

where $\rho\alpha^s$ is the probability of meeting a currency-holding seller and ηc^s is the real value of currency held by the seller. Conditions (1.7)-(1.9) state that no theft takes place if the buyer strictly prefers not to steal currency, the buyer sometimes steals if he or she is indifferent between two options, and the buyer always steals if theft is strictly preferred. Also, each seller's decision on carrying currency into the TM must be incentive compatible, that is,

$$\text{if} \quad (1-\alpha^b)\eta < 1, \quad \text{then} \quad \alpha^s = 0, \tag{1.10}$$

$$\text{if} \quad (1-\alpha^b)\eta = 1, \quad \text{then} \quad 0 \le \alpha^s \le 1, \tag{1.11}$$

$$\text{if} \quad (1-\alpha^b)\eta > 1, \quad \text{then} \quad \alpha^s = 1. \tag{1.12}$$

The seller does not carry the currency into the TM if the expected payoff from carrying one unit of currency $(1-\alpha^b)\eta$ is less than the payoff from depositing it with the central bank and obtaining one unit of reserves. If the seller is indifferent between two choices, he or she sometimes carries the currency into the TM. Otherwise, the seller always carries the currency into the TM.

1.3.2 Deposit Contracts

Private banks write deposit contracts for buyers in the CM before buyers learn their types. Deposit contracts provide insurance to buyers as in Williamson (2012, 2016, 2022b) by giving them an option to withdraw currency when they learn their types. Those buyers who do not exercise the option will use bank claims as a means of payment in DM meetings. Suppose a bank proposes a deposit contract (k, c', d), where k is the quantity of CM goods deposited by each buyer at the beginning of the CM, c' is the real quantity of currency that the buyer can withdraw at the end of the CM, and d is the quantity of claims to CM goods in the following period that the buyer can exchange in the DM if currency has not been withdrawn. Also, the bank acquires an asset portfolio (b, m, c), where b is the quantity of government bonds, m is the quantity of reserves, and c is the quantity of currency in real terms. In equilibrium, the bank's problem can be written as

$$\max_{k,c',d,b,m,c} \left\{ -k + \rho u\left([1-\alpha^s+\alpha^s(1-\alpha^b)\eta]\frac{\beta c'}{\pi} - \beta\mu \right) + (1-\rho)u(\beta d) \right\} \tag{1.13}$$

subject to

$$k - b - m - \eta c + \beta\left[-(1-\rho)d + \frac{R^m m + R^b b + c - \rho c'}{\pi} - \gamma(c - \rho c')\right] \geq 0, \tag{1.14}$$

$$-(1-\rho)d + \frac{R^m m + R^b b + c - \rho c'}{\pi} \geq \frac{\delta\left(R^m m + R^b b + c\right)}{\pi}, \tag{1.15}$$

$$k, c', d, b, m, c, c - \rho c' \geq 0. \tag{1.16}$$

The objective function (1.13) is the representative buyer's expected utility, implying that the bank chooses a contract that maximizes the buyer's expected utility in equilibrium. With probability ρ, the buyer realizes that, in the following DM, he or she will be matched with a seller who accepts only currency. In this case, the buyer visits the bank to withdraw c' units of currency at the end of the CM. In the following DM, the buyer makes a take-it-or-leave-it offer to the matched seller and acquires $[1 - \alpha^s + \alpha^s(1-\alpha^b)\eta]\frac{\beta c'}{\pi} - \beta\mu$ units of goods.[20] With probability $1-\rho$, the buyer learns that he or she will meet a seller who accepts a claim on the bank. As the buyer does not withdraw currency in this case, he or she receives a claim to d units of goods in the next CM. So, the buyer's take-it-or-leave-it offer implies that the buyer trades d deposit claims for βd units of goods.

Constraint (1.14) states that the bank earns a nonnegative discounted net payoff in equilibrium. In the CM, the bank receives k deposits from the buyer and acquires a portfolio of government bonds b, reserves m, and currency c. At the end of the CM, the bank pays off currency to the fraction ρ of buyers, each of whom withdraws c' currency. The remaining fraction $1-\rho$ of buyers exchange their deposit claims in the DM. So, in the following CM, the bank pays off d units of goods to each holder of the deposit claims. Notice that the bank stores the remaining $c - \rho c'$ units of currency until the next CM which incurs $\gamma(c - \rho c')$ units of labor supply. At the beginning of the next CM, the bank must deposit the remaining currency with the central bank at a one-to-one exchange rate.

As for any agents in the economy, the bank is subject to limited commitment. So, the bank's deposit liabilities must be backed by collateral and (1.15) is a collateral constraint. Assume that the bank can abscond with a fraction δ of its assets, pledged as collateral, when it defaults. Then, the collateral constraint tells us that the bank must weakly prefer to repay its deposit liabilities in the CM and in the next CM rather than absconding with collateral. If the bank were to default,

[20] As the buyer makes a take-it-or-leave-it offer to the seller, the buyer can extract all the surplus from trade. By accepting the buyer's offer, the seller receives c' units of currency (in real terms) from the buyer in the DM. Then, in the next period, the seller will be holding $\frac{c'}{\pi}$ units of currency and bear μ units of fixed cost (in terms of labor supply) at the beginning of the TM. The ex-ante expected payoff per unit of currency is $1 - \alpha^s + \alpha^s(1-\alpha^b)\eta$ because with probablity $1-\alpha^s$ the seller deposits the currency to receive one unit of reserves, and with probability $\alpha^s(1-\alpha^b)$ the seller successfully sells the currency in the CM at price η. Since the seller's surplus from trade is zero, the quantity of goods produced by the seller and transferred to the buyer (or equivalently, the disutility from producing goods) is equal to the seller's discounted expected net payoff from acquiring c' units of currency in the DM.

it would not let buyers withdraw currency as in Williamson (2022b). Finally, constraint (1.16) demonstrates that all real quantities must be nonnegative.

1.3.3 Definition of Equilibrium

Any contract that provides a positive discounted net payoff for the bank cannot be supported in equilibrium. If private banks were to earn a positive discounted net payoff, a bank would design an alternative contract that provides a slightly lower payoff per contract, but a higher total payoff by attracting all buyers. So, constraint (1.14) must hold with equility in equilibrium.

Let λ denote the Lagrange multiplier associated with the collateral constraint (1.15). Then, I can derive the first-order conditions for the bank's maximization problem, (1.13) subject to (1.14)-(1.16), as follows.

$$\frac{\beta[1-\alpha^s+\alpha^s(1-\alpha^b)\eta]}{\pi}u'\left([1-\alpha^s+\alpha^s(1-\alpha^b)\eta]\frac{\beta c'}{\pi}-\beta\mu\right)-\eta-\frac{\lambda\delta}{\pi}=0, \tag{1.17}$$

$$\beta u'(\beta d)-\beta-\lambda=0, \tag{1.18}$$

$$-1+\frac{\beta R^m}{\pi}+\frac{\lambda R^m(1-\delta)}{\pi}=0, \tag{1.19}$$

$$-1+\frac{\beta R^b}{\pi}+\frac{\lambda R^b(1-\delta)}{\pi}=0, \tag{1.20}$$

$$-\eta+\frac{\beta}{\pi}-\beta\gamma+\frac{\lambda(1-\delta)}{\pi}\leq 0, \tag{1.21}$$

$$\lambda\left[-(1-\rho)d+\frac{(1-\delta)\left(R^m m+R^b b+c\right)}{\pi}-\frac{\rho c'}{\pi}\right]=0. \tag{1.22}$$

A necessary condition for an equilibrium to exist is that sellers do not hold currency from the CM to the next CM. So, the expected payoff from holding currency across periods must be nonpositive at the margin. That is,

$$-\eta+\beta\left[\frac{1-\alpha^s+\alpha^s(1-\alpha^b)\eta}{\pi}-\gamma\right]\leq 0. \tag{1.23}$$

Also, in equilibrium, asset markets clear in that the demand for each asset is equal to the supply. That is,

$$c=\bar{c};\quad m=\bar{m};\quad b=\bar{b}. \tag{1.24}$$

For convenience, let x^c and x^d denote the consumption quantities in DM meetings, respectively, with currency and deposit claims being traded, i.e.,

$$x^c = [1 - \alpha^s + \alpha^s(1 - \alpha^b)\eta]\frac{\beta c'}{\pi} - \beta\mu, \tag{1.25}$$

$$x^d = \beta d. \tag{1.26}$$

I assume that the central bank conducts monetary policy under a floor system where a sufficiently large quantity of reserve balances are held by private banks and the central bank sets the nominal interest rate on reserves R^m. Under this system, private banks treat reserves and government bonds as identical assets at the margin, so the nominal interest rate on reserves pegs the nominal interest rate on government bonds in equilibrium, i.e., $R^m = R^b$ from (1.19) and (1.20).[21] Also, the central bank can expand its balance sheet through swaps of reserves for government bonds. Let $\omega = \eta\bar{c} + \bar{m}$ denote the size of the balance sheet, which is equivalent to the real value of the central bank's liabilities.[22]

Then, I can define an equilibrium as follows.

Definition *Given exogenous fiscal policy v and monetary policy (R^m, ω, η), a stationary equilibrium consists of DM consumption quantities (x^c, x^d), asset quantities (k, c', d, b, m, c), the fraction of buyers who choose to steal currency in the TM α^b, the fraction of sellers who choose to carry currency into the TM conditional on having acquired the currency in the previous DM α^s, transfers (τ_0, τ), gross inflation rate π, and gross nominal interest rate on government bonds R^b, satisfying the consolidated government budget constraints (1.4) and (1.5), the fiscal policy rule (1.6), the first-order conditions for the bank's problem (1.17)-(1.22), no arbitrage condition for sellers (1.23), the incentive compatibility conditions for buyers and sellers (1.7)-(1.12), and market clearing conditions (1.24).*

Notice that, according to the definition, the fiscal and monetary policies are given exogenously. The fiscal authority determines the total value of consolidated government debt, in real terms, while the central bank has three policy targets: (i) the nominal interest rate on reserves, (ii) the size of the central bank's balance sheet, and (iii) the exchange rate between currency and reserves. As the total value of consolidated government debt is exogenously set by the fiscal policy rule, the

[21] In practice, reserves are considered as a useful means of payment in intraday trading in the banking system. However, a key property of the U.S. financial system in the post-financial crisis period is that a large volume of reserves has been held by private banks without being used in intraday financial transactions. This observation allows us to simply assume that reserves and government bonds share the same properties by abstracting from the transaction role of reserves. Also, note that only reserves can be turned into currency through the central bank cash window. But, this property does not play an important role in equilibrium.

[22] The real value of liabilities is equal to that of assets because the central bank is assumed to transfer any profits/losses to the fiscal authority and the central bank's net worth is zero.

central bank's monetary policy effectively determines the composition of the debt. Also, note that the fiscal authority manipulates transfers, in response to monetary policy, so as to satisfy its fiscal policy target and the government budget constraints.

1.3.4 Characterization of Equilibrium

I first characterize the effective lower bound (ELB) on the gross nominal interest rate R^m. Note that inequality (1.21) represents no arbitrage for private banks from acquiring currency in the CM, holding it across periods, and redepositing it in the next CM. Substituting (1.19) into (1.21) gives

$$R^m \geq \frac{1}{\eta + \beta\gamma}.$$

Also, using (1.19), the no arbitrage condition for sellers from holding currency across periods (1.23) can be rewritten as

$$R^m \geq \frac{1 - \alpha^s + \alpha^s(1 - \alpha^b)\eta}{(\eta + \beta\gamma)[(1 - \delta)u'(x^d) + \delta]}.$$

So, the ELB on the gross nominal interest rate in equilibrium is given by

$$R^m \geq \max\left\{\frac{1}{\eta + \beta\gamma}, \frac{1 - \alpha^s + \alpha^s(1 - \alpha^b)\eta}{(\eta + \beta\gamma)[(1 - \delta)u'(x^d) + \delta]}\right\}. \tag{1.27}$$

Suppose the ELB is determined by the first argument in the above maximization problem. Then, the gross nominal interest rate on reserves R^m can be less than one, or the net nominal interest rate $R^m - 1$ can be negative, for two reasons. If the exchange rate for currency withdrawals is not one-to-one, $\eta > 1$, then a negative net nominal interest rate on reserves can be supported in equilibrium, as proposed by Eisler (1932), Buiter (2010), and Agarwal and Kimball (2015). Another reason why the net nominal interest rate can be negative comes from the proportional cost of storing currency. If holding currency across periods is costly and the cost is proportional to the quantity of currency, i.e., $\gamma > 0$, negative nominal interest rates can be supported in equilibrium. This result can explain why some central banks could implement negative nominal interest rates without causing a flight to currency.

Now, suppose the second argument in the above maximization problem determines the ELB on $R^m - 1$. Then, there is a nonstandard reason for a negative ELB. As I will show later, the term $(1 - \delta)u'(x^d) + \delta$ is higher than one due to low real interest rates on interest-bearing assets. Real interest rates are low because those assets are useful as collateral and therefore bear a liquidity premium. However, sellers do not use currency or any interest-bearing assets as collateral. As low real interest rates are accompanied by a high inflation rate, currency yields a low real return to sellers, which makes a negative nominal interest rate feasible. For analytical convenience, I will focus on cases where (1.27) holds with strict inequality. That is, the nominal interest rate on reserves is not constrained by the ELB.

Suppose the fraction ρ of sellers, who accept currency in the DM, holds c^s currency each in real terms at the beginning of the TM. Then, the following lemma shows the values of α^b and α^s that are consistent with optimal decisions of buyers and sellers in equilibrium.

Lemma 1 *Suppose that $\eta = 1$. Then, no theft occurs in equilibrium, i.e., $\alpha^b = 0$. Furthermore, $\alpha^s \in [0, 1]$ for all $\kappa \geq \rho\eta c^s$ and $\alpha^s \in [0, \bar{\alpha}^s]$ for all $\kappa < \rho\eta c^s$ where $\bar{\alpha}^s = \frac{\kappa}{\rho\eta c^s}$. Alternatively, suppose that $\eta > 1$. Then, no theft occurs in equilibrium with $\alpha^b = 0$ and $\alpha^s = 1$ for all $\kappa \geq \rho\eta c^s$ while theft exists in equilibrium with $\alpha^b = \frac{\eta-1}{\eta}$ and $\alpha^s = \frac{\kappa}{\rho\eta c^s}$ for all $\kappa < \rho\eta c^s$.*

Proof See Appendix □

According to Lemma 1, there is no theft in equilibrium if private banks can withdraw currency from their reserve accounts at par. In this case, the market price of currency is identical to the price of reserves, so sellers are indifferent between depositing the currency with the central bank and trading the currency with a private bank. If the central bank sets a nonpar exchange rate between currency and reserves, i.e., $\eta > 1$, then the policy tends to encourage sellers to trade currency with private banks. However, sellers become indifferent to depositing the currency at the central bank in equilibrium because there are some buyers trying to steal the currency, given that the cost of theft is sufficiently low.

From (1.25) and Lemma 1, I can show that

$$x^c = \frac{\beta c' \eta}{\pi} - \beta\mu, \qquad \forall \kappa \geq \frac{\rho\eta c'}{\pi} \tag{1.28}$$

$$x^c = \frac{\beta c'}{\pi} - \beta\mu. \qquad \forall \kappa < \frac{\rho\eta c'}{\pi} \tag{1.29}$$

In what follows, I will consider the case where the real value of the consolidated government debt outstanding v is sufficiently low, so as to confine attention to an equilibrium with binding collateral constraints.

1.3.5 Equilibrium with No Theft

In this section, I characterize an equilibrium where all currency-holding sellers trade currency with private banks in the CM with no threat of theft in the TM. Suppose that the cost of acquiring the theft technology is sufficiently high, so that condition (1.28) holds, $\alpha^b = 0$, and $\alpha^s = 1$ in equilibrium. Then, from (1.17)-(1.20) and (1.28), the inflation rate π and the nominal interest rates on reserves and government bonds R^m and R^b are given by

$$\pi = \frac{\beta}{\eta}\left[\eta u'(x^c) - \delta u'(x^d) + \delta\right], \tag{1.30}$$

$$R^m = R^b = \frac{\eta u'(x^c) - \delta u'(x^d) + \delta}{\eta\left[u'(x^d) - \delta u'(x^d) + \delta\right]}, \tag{1.31}$$

and the real interest rate is given by

$$r^m = r^b = \frac{1}{\beta\left[u'(x^d) - \delta u'(x^d) + \delta\right]}. \tag{1.32}$$

In equilibrium, the quantity of consumption in DM trades that involve using bank claims is inefficiently low due to a binding collateral constraint, leading to a low real interest rate.

From (1.6), (1.24)-(1.26), and (1.30)-(1.31), the binding collateral constraint (1.15) can be rewritten as

$$\left[u'(x^c) + \frac{\delta}{(1-\delta)\eta}\right]\rho(x^c + \beta\mu) + \left[u'\left(x^d\right) + \frac{\delta}{1-\delta}\right](1-\rho)x^d = v. \tag{1.33}$$

Equation (1.33) implies that the aggregate demand for collateral (the left-hand side) must be equal to the aggregate supply (the right-hand side) in equilibrium. From (1.28), a necessary condition for buyers to not invest in the theft technology is given by

$$\kappa \geq \frac{\rho(x^c + \beta\mu)}{\beta}. \tag{1.34}$$

Finally, the ELB on the gross nominal interest rate in equilibrium is given by

$$R^m \geq \max\left\{\frac{1}{\eta + \beta\gamma}, \frac{\eta}{(\eta + \beta\gamma)[(1-\delta)u'(x^d) + \delta]}\right\}. \tag{1.35}$$

If the nonpar exchange rate between currency and reserves η is sufficiently close to one and the real interest rate $r^m = r^b$ is sufficiently low so that the term $(1-\delta)u'(x^d) + \delta$ is sufficiently high, then the ELB is determined by the first argument. In this case, a higher η implies a lower ELB, consistent with the claims made by Eisler (1932), Buiter (2010), and Agarwal and Kimball (2015). However, if η is sufficiently larger than one and $r^m = r^b$ is sufficiently high, then the second argument governs the ELB on the nominal interest rate. In this case, an increase in η does not necessarily lower the ELB, and it can even increase the ELB. This happens because there are insufficient frictions to prevent sellers from carrying currency across periods. Sellers can exploit arbitrage by purchasing currency at price η and selling it at the same price with no risk of theft in the next period. Given a fixed real interest rate, an increase in η only increases the market price of currency, making arbitrage more profitable. Therefore, the ELB can even increase in response to an increase in η.

An interpretation is that, for the realized nominal rate of return on currency to be negative, there must be currency deposits and withdrawals at different exchange rates in equilibrium. If either one of those two activities does not occur, a nonpar exchange rate can fail to reduce the rate of return on currency and the ELB. In an equilibrium with no theft studied here, the absence of sellers' currency deposits breaks the link between the nonpar exchange rate and the nominal rate of return on currency. In contrast, if there is deflation in equilibrium, it is possible that private banks do not withdraw currency from the central bank cash window as can be seen in Appendix A.1.

Given the above equilibrium conditions, I can solve the model as follows. First, equations (1.31) and (1.33) solve for (x^c, x^d), given monetary policy (R^m, η) and fiscal policy v. Then, equation (1.30) solves for π, equation (1.32) solves for r^m and r^b, and inequalities (1.34) and (1.35) give necessary conditions for this equilibrium to exist.

1.3.5.1 Effects of Monetary Policy

Note that the size of the central bank's balance sheet ω is irrelevant to asset prices or consumption quantities. This occurs because an expansion in the size of the balance sheet involves central bank swaps of reserves for government bonds. As those assets are perfect substitutes for private banks at the margin, this only changes the composition of government bonds and reserves in bank asset portfolios, with no effects on other variables.

In what follows, I analyze the effects of monetary policy interventions given that the net nominal interest rate on reserves is close to zero and the exchange rate between currency and reserves is close to one. This implies that the ELB on the nominal interest rate is determined by the first argument in (1.35).

Proposition 1 *Suppose that inequalities (1.34) and (1.35) hold in equilibrium,* (R^m, η) *is sufficiently close to* $(1, 1)$*, and the fixed cost of holding currency* μ *is sufficiently close to zero. Then, an increase in* R^m *results in a decrease in* x^c*, an increase in* x^d*, an increase in real interest rates* (r^m, r^b)*, and an increase in* π*, with no effect on the ELB. In contrast, an increase in* η *results in an increase in* x^c *and a decrease in the ELB. Furthermore, there exists* $\hat{v}$ *such that an increase in* η *decreases* x^d *and* (r^m, r^b) *and increases* π *for* $v \in (0, \hat{v}]$*, while it increases* x^d *and* (r^m, r^b) *and decreases* π *for* $v \in (\hat{v}, \bar{v})$ *where* $\bar{v}$ *is the upper bound of the values of consolidated government debt that support an equilibrium with a binding collateral constraint.*

Proof See Appendix □

With the exchange rate between currency and reserves η held constant, an increase in the nominal interest rate on reserves R^m affects bank asset portfolios since it becomes more profitable to hold more reserves or government bonds rather than currency. With a larger quantity of reserves or government bonds, banks can provide a larger quantity of claims to buyers, so x^d rises. However, a smaller quantity of currency outstanding, in real terms, leads to a smaller quantity of consumption in DM trades using currency x^c. Also, the decrease in the real quantity of currency outstanding must be accompanied by a decrease in the real rate of return on currency in equilibrium, implying a rise in the inflation rate π with η held constant. As a larger quantity of reserves and government bonds makes collateral less scarce, a rise in R^m acts to increase real interest rates, r^m and r^b. But real rates increase by less than do nominal interest rates.

(For a sufficiently low v)

	∂x^c	∂x^d	$\partial \pi$	∂r^m	∂ELB
∂R^m	−	+	+	+	·
$\partial \eta$	+	−	+	−	−

(For a sufficiently high v)

	∂x^c	∂x^d	$\partial \pi$	∂r^m	∂ELB
∂R^m	−	+	+	+	·
$\partial \eta$	+	+	−	+	−

Table 1.1: Effects of monetary policy (R^m, η) in an equilibrium with no theft

A novel finding is that an increase in the exchange rate between currency and reserves η itself has real effects. In particular, an increase in η leads to an increase in the consumption quantity in DM trades using currency x^c, with R^m held constant. This occurs because an increase in η increases the price of currency in the CM, which in turn increases the value of currency in the DM. As currency is exchanged for a larger quantity of goods in DM trades, the quantity of consumption in those trades increases. The effect of an increase in η on the consumption quantity in DM trades using bank claims x^d depends on the value of consolidated government debt v. If v is sufficiently low or collateralizable assets are sufficiently scarce, then an increase in η decreases x^d and real interest rates (r^m, r^b), implying that a larger quantity of currency outstanding effectively decreases the stock of government bonds and reserves held by private banks. In contrast, if v is sufficiently high but not too high, then an increase in η increases x^d and real interest rates (r^m, r^b). In this case, a higher price of currency acts to decrease the real quantity of currency c' (the income effect dominates the substitution effect) which effectively relaxes the collateral constraint from (1.22). Therefore, x^c and x^d both increase. These results are summarized in Table 1.1.

Corollary 1 *If η is sufficiently close to one, an increase in η leads to a decrease in the ELB on the nominal interest rate. But if η is sufficiently high, an increase in η can increase the ELB.*

As mentioned earlier, the ELB on the nominal interest rate is determined by the first argument in (1.35) if the exchange rate between currency and reserves η is sufficiently close to one. This implies that there is no arbitrage opportunity for sellers from holding currency across periods as long as there is no such arbitrage opportunity for private banks. However, if η is sufficiently high, then the second argument in (1.35) can determine the ELB. In this case, no arbitrage for private banks from investing in currency does not prevent the sellers' opportunistic behavior. This happens because sellers trade currency at the market price η without threat of theft in the TM. So, an increase in η does not effectively reduce the rate of return on currency. Instead, it is possible that an increase in η leads to an increase in the rate of return on currency because the cost of storing currency becomes relatively smaller as the price of currency η rises. This relation between the exchange rate η and the ELB is illustrated by Figure 1.2.

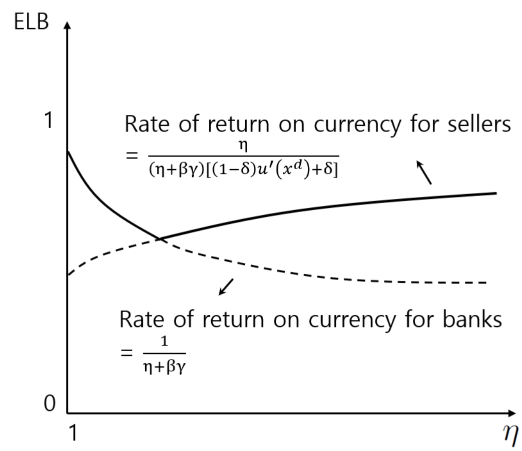

Figure 1.2: Exchange rate η and the effective lower bound (ELB)

Corollary 2 *Given that* (R^m, η) *is sufficiently close to* $(1, 1)$ *and the fixed cost of holding currency* μ *is sufficiently close to zero, suppose the central bank increases* η *and decreases* R^m *to hold* ηR^m *constant. Then, this policy increases* x^c *and decreases the ELB. Furthermore, if* $\nu \in (0, \hat{\nu}]$, *then* x^d *decreases and* (r^m, r^b) *decrease, while the effect on* π *is ambiguous. If* $\nu \in (\hat{\nu}, \bar{\nu})$, *then* π *falls while the effects on* x^d *and* (r^m, r^b) *are ambiguous.*

Suppose the ELB on the nominal interest rate is binding. If the central bank wishes to reduce the nominal interest rate R^m below the ELB, it could reduce the nominal rate of return on currency in the same magnitude (in percentage) as the nominal interest rate. Corollary 2 shows what happens if the central bank increases the exchange rate η and reduce the nominal interest rate on reserves R^m in the same magnitude, so that the relative rate of return on currency $\frac{1}{\eta R^m}$ remains constant. The policy leads to an increase in x^c and a decrease in the ELB, while its effects on other variables such as x^d, r^m, r^b, and π, depend on the value of consolidated government debt v. Most importantly, the policy is not neutral as it affects the DM consumption and real interest rates. These real effects arise in this equilibrium because the expected payoff for sellers from trading currency with private banks is higher than the payoff from depositing the currency with the central bank. In other words, the expected rate of return on currency perceived by sellers does not coincide with the target rate of return set by the central bank. Therefore, the policy serves to distort the allocation of currency and bank claims in DM transactions.

1.3.6 Equilibrium with Theft

In this section, I analyze an equilibrium where some sellers carry currency into the TM and some buyers steal currency. Suppose that the cost of acquiring the theft technology κ is sufficiently low, so that condition (1.29) holds in equilibrium. Then, from (1.17)-(1.20) and (1.29), I obtain

$$\pi = \frac{\beta}{\eta}\left[u'(x^c) - \delta u'(x^d) + \delta\right], \tag{1.36}$$

$$R^m = R^b = \frac{u'(x^c) - \delta u'(x^d) + \delta}{\eta\left[u'(x^d) - \delta u'(x^d) + \delta\right]}, \tag{1.37}$$

and the real interest rate is given by

$$r^m = r^b = \frac{1}{\beta\left[u'(x^d) - \delta u'(x^d) + \delta\right]}. \tag{1.38}$$

From (1.6), (1.24)-(1.26), and (1.36)-(1.37), the binding collateral constraint (1.15) can be rewritten as

$$\left[u'(x^c) + \frac{\delta}{1-\delta}\right]\rho(x^c + \beta\mu) + \left[u'\left(x^d\right) + \frac{\delta}{1-\delta}\right](1-\rho)x^d = v, \tag{1.39}$$

From Lemma 1 and (1.29), I can write the fraction of buyers who choose to acquire the theft techonology α^b and the fraction of sellers who choose to carry currency in the TM α^s as

$$\alpha^b = \frac{\eta - 1}{\eta}, \tag{1.40}$$

$$\alpha^s = \frac{\beta\kappa}{\rho\eta(x^c + \beta\mu)}, \tag{1.41}$$

and I can derive a necessary condition for this equilibrium to exist, which is given by

$$\kappa < \frac{\rho\eta(x^c + \beta\mu)}{\beta}. \tag{1.42}$$

From (1.27) and (1.40), no arbitrage from holding currency across periods is given by

$$R^m \geq \frac{1}{\eta + \beta\gamma}. \tag{1.43}$$

Solving the model is straighforward, as with an equilibrium with no theft. First, equations (1.37) and (1.39) solve for (x^c, x^d), given monetary policy (R^m, η) and fiscal policy v. Then, equation (1.36) solves for π, equation (1.38) solves for (r^m, r^b), equations (1.40)-(1.41) solve for (α^b, α^s), and inequalities (1.42) and (1.43) give necessary conditions for this equilibrium to exist.

1.3.6.1 Effects of Monetary Policy

The size of the central bank's balance sheet ω is irrelevant to asset prices or consumption quantities as in an equilibrium with no theft. So, in what follows I analyze the effects of monetary policy interventions with R^m (the nominal interest rate on reserves) and η (the nonpar exchange rate between currency and reserves).

Proposition 2 *Suppose that inequalities (1.42) and (1.43) hold in equilibrium. Then, an increase in R^m or η results in a decrease in x^c, an increase in x^d, and an increase in real interest rates (r^m, r^b). In addition, an increase in R^m increases π and α^s with no effects on the ELB and α^b. An increase in η decreases π and the ELB, and increases α^b, but its effect on α^s is ambiguous.*

Proof See Appendix □

With the exchange rate between currency and reserves η held constant, the effects of an increase in the nominal interest rate on reserves R^m on consumption quantities, real interest rates, and the inflation rate are qualitatively identical to those in an equilibrium with no theft, although the fraction of sellers who carry currency into the TM α^s increases in this equilibrium. In response to an increase in R^m, sellers receive a smaller quantity of real currency from buyers in the DM. As buyers have a lower incentive to invest in the theft technology when sellers hold a smaller quantity of real currency, sellers can increase the probability of carrying currency into the TM until the fraction of buyers with the theft technology α^b remains the same.

A key result is that an increase in the exchange rate between currency and reserves η (a decrease in the nominal/real rate of return on currency) leads to a decrease in the inflation rate π (an increase in the real rate of return on currency). So, the fall in the real rate of return on currency due to an increase in η is mitigated by a decrease in π in equilibrium. Also, an increase in η decreases the ELB. These results are consistent with Eisler (1932), Buiter (2010), and Agarwal and Kimball

(2015), in that an increase in the exchange rate between currency and reserves reduces the real rate of return on currency and the ELB on nominal interest rates.

Note that a higher η, or a higher price of currency in the CM, induces buyers to invest in the theft technology more often. Then, due to a higher risk of theft, sellers become indifferent between carrying currency into the TM and safely depositing it with the central bank, although they can sell it in the CM at a higher price.[23] Therefore, the central bank can successfully reduce the rate of return on currency and the ELB on nominal interest rates in this equilibrium owing to endogenous theft. However, reducing the ELB is costly because a larger fraction of buyers investing in the theft technology implies a larger welfare loss.

	∂x^c	∂x^d	$\partial \pi$	∂r^m	∂ELB	$\partial \alpha^b$	$\partial \alpha^s$
∂R^m	−	+	+	+	·	·	+
$\partial \eta$	−	+	−	+	−	+	?

Table 1.2: Effects of monetary policy (R^m, η) in an equilibrium with theft

As in an equilibrium with no theft, an increase in the exchange rate η itself has real effects as it decreases x^c and increases x^d, r^m and r^b. These effects occur because, with R^m held constant, an increase in η leads to a decrease in the rate of return on currency relative to government bonds and reserves $\frac{1}{\eta R^m}$. Note that the effects of an increase in the exchange rate η on consumption quantities and real interest rates are qualitatively the same as those of an increase in R^m. These results are summarized in Table 1.2.

Corollary 3 *Suppose the central bank increases η and decreases R^m to hold ηR^m constant. Then, this policy decreases π one-for-one, decreases α^s and the ELB, and increases α^b. However, consumption quantities (x^c, x^d) and real interest rates (r^m, r^b) remain unchanged.*

Suppose that the central bank increases the exchange rate η and reduces the interest rate on reserves R^m with $\frac{1}{\eta R^m}$, the relative rate of return on currency, held constant. In Corollary 3, the policy acts to decrease the inflation rate π one-for-one with an increase in η, implying no effect on the real rate of return on currency. Also, real interest rates do not change because a decrease in π offsets the decrease in R^m one-for-one (a pure Fisher effect). Notice that, although there are no effects on consumption quantities (x^c, x^d), the fraction of buyers who acquire the theft technology α^b increases in equilibrium and this has welfare implications.

[23] The effect of an increase in η on the sellers' behavior is ambiguous. Although a higher η implies a higher payoff from selling currency in the CM, a higher risk of theft tends to reduce the sellers' expected payoff. Therefore, the effect of an increase in η on the fraction of sellers who carry currency into the CM α^s is ambiguous.

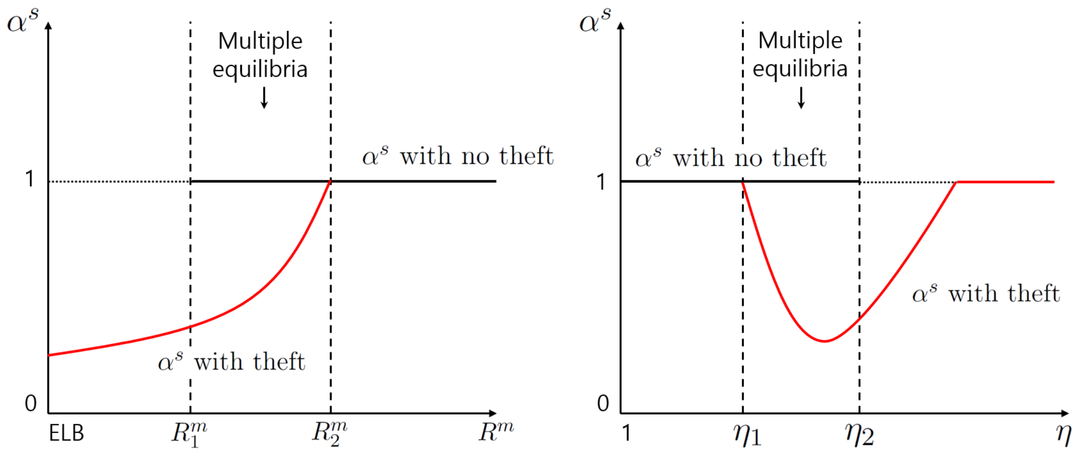

Figure 1.3: Equilibrium α^s with monetary policy (R^m, η)

With necessary conditions (1.34) and (1.42), Propositions 1 and 2 help us understand what type of equilibrium (or equilibria) may arise given monetary policies (R^m, η). With η held constant, an increase in R^m decreases x^c, so an equilibrium with no theft is more likely to arise for high R^m. Suppose η is sufficiently high or the cost of theft κ is sufficiently low, so that there exists theft for sufficiently low R^m. Then, as illustrated in the left panel of Figure 1.3, there may exist a range of nominal interest rates R^m that support equilibria with theft and with no theft. That is, for some parameters, multiple equilibria arise. In the figure, the upper bound on nominal interest rates that support an equilibrium with theft R_2^m is higher than the lower bound on nominal interest rates that support an equilibrium with no theft R_1^m. So, multiple equilibria exist for $R^m \in [R_1^m, R_2^m]$. However, for some parameters, the upper bound R_2^m can be lower than the lower bound R_1^m, so there does not exist an equilibrium for $R^m \in (R_2^m, R_1^m)$ in that case.

Propositions 1 and 2 also state that, with R^m held constant, an increase in η decreases x^c in an equilibrium with theft and increases x^c in an equilibrium with no theft. This implies that an equilibrium with no theft is more likely to arise for low η and high κ (the cost of theft). Suppose either R^m or κ is sufficiently high so that there is no theft for sufficiently low η. Then, for η sufficienly high but not too high, there may exist multiple equilibria as illustrated in the right panel of Figure 1.3. Notice that the effect of an increase in η on α^s (the fraction of sellers carrying currency in the TM) is ambiguous in an equilibrium with theft. The right panel of Figure 1.3 displays a special case where an increase in η decreases α^s for low η and increases α^s for high η in an equilibrium with theft. If η is sufficiently high, it is possible that all currency-holding sellers carry currency in the TM ($\alpha^s = 1$) although some buyers still currency in equilibrium ($0 < \alpha^b < 1$). In this section, I have focused on analyzing an equilibrium with theft where sellers are indifferent between depositing currency and not depositing, i.e., $0 < \alpha^b < 1$ and $0 < \alpha^s < 1$. Finally, Figure 1.4 shows how the nominal interest rate on reserves R^m and the nonpar exchange rate between currency and reserves η determine the existence of particular equilibria.

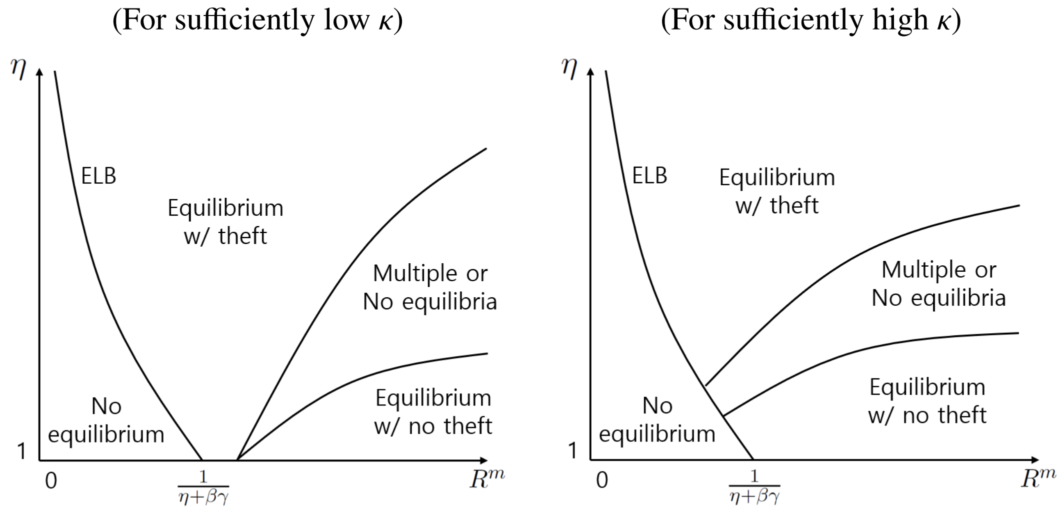

Figure 1.4: Equilibria with monetary policy (R^m, η)

1.4 Optimal Monetary Policy

I define welfare as

$$\mathcal{W} = \underbrace{0}_{\text{CM surpluses}} \underbrace{+\rho\left[u(x^c) - x^c + \beta\mu\right] + (1-\rho)\left[u(x^d) - x^d\right]}_{\text{DM surpluses}} \underbrace{-(\rho\beta\mu + \alpha^b\kappa)}_{\text{total cost in TM}}, \tag{1.44}$$

which is the sum of surpluses from trade in the CM and the DM, net of the total cost incurred in the TM. Welfare defined here is also equivalent to the sum of period utilities in equilibrium. I will discuss the optimal monetary policy using this measure in what follows.

Proposition 3 *If the cost of theft κ is sufficiently low and the fixed cost of holding currency μ is sufficiently close to zero, then the optimal monetary policy consists of $\eta = 1$ and $R^m \leq 1$ for given $\mu \geq 0$. However, if the optimal nominal interest rate on reserves is constrained by the ELB, then the optimal monetary policy is $\eta = 1$ and $R^m = ELB$.*

Proof See Appendix □

To understand the intuition behind the results, consider the case where there is no fixed cost of holding currency at the beginning of the TM ($\mu = 0$) as a benchmark. In Appendix A.2, I provide the proof of Proposition 3 in two steps. Taking α^b (the fraction of buyers who choose to steal) as given, an optimal monetary policy can be characterized by a *modified* Friedman rule. That is, the nominal interest rate on reserves relative to currency is zero (or equivalently, $\eta R^m = 1$) at the optimum. A modified Friedman rule achieves a social optimum, given α^b, by allowing buyers to perfectly smooth their consumption across different states of the world. Then, I argue that, among those policy alternatives, the optimal monetary policy consists of $\eta = 1$ and $R^m = 1$ because this eliminates costly investment in the theft technology.

Optimality is achieved when the exchange rate between currency and reserves is one-to-one, as in a traditional central banking system, and the nominal interest rate on reserves is zero (a Friedman rule). Since a zero nominal interest rate is optimal even though negative interest rates are available, the Friedman rule policy rate can be thought of as the "reversal interest rate", the interest rate at which lowering interest rate becomes contractionary (Brunnermeier and Koby, 2019).

Now, consider the case where there is a fixed cost of storing currency ($\mu > 0$). In this case, a negative nominal interest rate is optimal ($R^m < 1$) given a one-to-one exchange rate $\eta = 1$. As currency-holding buyers need to compensate for the storage cost incurred by sellers, they carry a larger quantity of currency than in an economy with no storage costs ($\mu = 0$). Then, it is welfare-improving to reduce the cost of holding currency by lowering the nominal interest rate R^m and the inflation rate π further from the one characterized by the Friedman rule.

Note that, even if the optimal nominal interest rate is constrained by the ELB, introducing a nonpar exchange rate to further reduce the nominal interest rate does not improve welfare. From Corollary 3, such a policy only increases costly theft without increasing surpluses from trade in the DM and the CM. Therefore, it is optimal to set the nominal interest rate at the ELB in this case.

Proposition 4 *If the cost of theft κ is sufficiently high, then the optimal monetary policy is given by $\eta = \bar{\eta}$ where $\bar{\eta}$ consists of the solution to (1.31) and (1.33) for $x^c = \beta(\frac{\kappa}{\rho} - \mu)$. In addition, if the fixed cost of holding currency μ is sufficiently close to zero, then the optimal nominal interest rate on reserves R^m satisfy that $R^m > 1/\bar{\eta}$.*

Proof See Appendix □

Interestingly, if the cost of theft κ is sufficiently high and theft does not take place in equilibrium, the optimal nominal interest rate can be higher than the one satisfying a modified Friedman rule. In Appendix A.2, I show that with a modified Friedman rule, or $\eta R^m = 1$, an increase in the interest rate on reserves R^m or the exchange rate η improves social welfare. An increase in R^m increases the level of welfare for the same reason as in the case with a sufficiently low cost of theft, by smoothing consumption quantities across states. However, increasing η from a modified Friedman rule improves welfare for a nonstandard reason. Note that in an equilibrium with no theft an increase in η does not effectively reduce the nominal rate of return on currency perceived by sellers but does increase the price of currency in the CM. A higher price of currency increases the quantity of consumption in DM meetings using currency, and the resulting increase in the buyer's utility from those DM meetings exceeds the potential decrease in the utility from DM meetings using bank claims. Therefore, a modified Friedman rule does not achieve a social optimum and the optimal monetary policy can be characterized by $\eta R^m > 1$. Also, as the level of welfare increases with η, it is optimal to set the exchange rate at the highest possible level that does not cause theft in equilibrium.

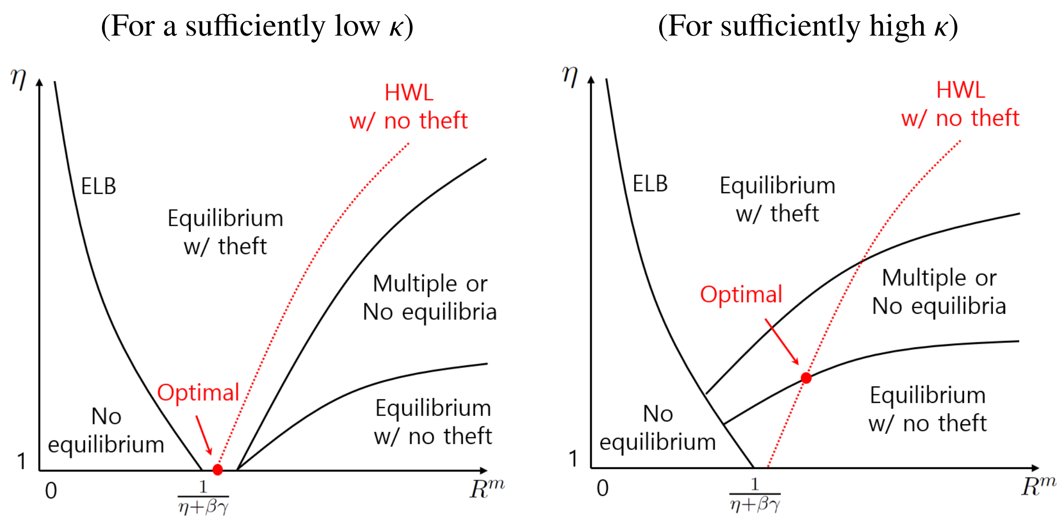

Figure 1.5: Optimal monetary policy

These results can be illustrated by Figure 1.5. In the figure, the HWL curve in each panel depicts the locus of nominal interest rates R^m that deliver the highest welfare given an exchange rate η in an equilibrium with no theft. If the cost of theft κ is sufficiently low, then welfare can be maximized at $\eta = 1$ and $R^m < 1$ as in the left panel. In contrast, if κ is sufficiently high, then welfare can be maximized at the highest possible level of η, with the corresponding R^m on the HWL curve, that supports an equilibrium with no theft as a unique equilibrium.

1.5 Quantitative Analysis

Theoretically, introducing a nonpar exchange rate between currency and reserves can decrease welfare by encouraging costly theft and distorting the equilibrium allocation. To understand the magnitude of the welfare cost, I calibrate the baseline model to the U.S. economy, and conduct a counterfactual analysis to evaluate the welfare cost of introducing a nonpar exchange rate between currency and reserves.

1.5.1 Calibration

I consider an annual model and assume that the utility function in the DM takes the form $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$. When calibrating the baseline model to data, I exclude the cost of theft κ because a one-to-one exchange rate between currency and reserves implies no theft in the model.[24] Then, there are eight parameters to calibrate: σ (the curvature of DM consumption), β (discount factor), ρ (the fraction of currency transactions in the DM), μ (the fixed cost of storing currency), γ (the proportional cost of storing currency), δ (the fraction of assets private banks can abscond with), R^m (the nominal interest rate on reserves), and v (the value of government liabilities held by the public).

Table 1.3 summarizes the calibration results along with the target moments. Most of the target moments are constructed from the U.S. data for 2013-2015. I consider the period 2013-2015 because it is proper to consider a time period when the policy rate was close to zero as the purpose of this exercise is to evaluate the welfare cost of reducing the ELB. Also, key variables such as the nominal interest rate on reserves and domestically-held public debt to GDP were stable during this period.

There are three parameters calibrated externally. The discount factor β is given by $\beta = 0.96$. From Federal Reserve Economic Data (FRED), the nominal interest rate on reserves was 0.25 percent over period 2013-2015 ($R^m = 1.0025$). Finally, the lowest target range for the federal funds rate has been between 0 and 0.25 percent since 1954. Although this does not imply that the proportional cost of storing currency is zero, the proportional cost γ is assumed to be zero for convenience.[25]

Parameters	Values	Calibration targets	Sources
β	0.96	Standard in literature	
R^m	1.0025	Avg. interest rate on reserves: 0.25%	FRED
γ	0.00	Lowest target range for fed funds rate: 0-0.25%	FRED
σ	0.17	Money demand elasticity (1959-2007): -4.19	FRED
ρ	0.17	Currency to M1 ratio: 17.22%	FRED; Lucas and Nicolini (2015)
v	1.13	Avg. locally-held public debt to GDP: 66.73%	FRED
δ	0.45	Avg. inflation rate: 1.06%	FRED
μ	0.01	Fixed storage cost: 2% of currency payments	Author's assumption

Table 1.3: Calibration results

[24] To quantify the welfare cost arising from an increase in theft, the cost of theft κ needs to be calibrated. Since theft does not occur in the model given a one-to-one exchange rate, parameter κ must be directly calibrated outside the model. However, to the best of my knowledge, there is no data that allows measuring the cost of theft.

[25] The proportional cost of storing currency implies that the ELB on the nominal interest rate can be negative. However, the Federal Reserve might have faced some legal and political issues of implementing negative nominal interest rates. As a negative rate has not been explored in the U.S., it seems difficult to calibrate the proportional cost of storing currency with this model.

Calibrating σ (the curvature of DM consumption) involves matching the elasticity of money demand in the model with the empirical money demand elasticity obtained from the data. Estimating the money demand elasticity requires a longer time-series data, so I choose the time period from 1959 to 2007.[26] Using data on currency in circulation and nominal GDP from FRED, I calculate the currency-to-GDP ratios. Then, the money demand elasticity can be estimated using Moody's AAA corporate bond yields from FRED and the currency-to-GDP ratios, and the estimated elasticity is -4.19.[27]

Then, I jointly calibrate four parameters: the curvature of DM consumption σ, the fraction of currency transactions in the DM ρ, the value of government liabilities held by the public v, the fraction of assets that can be absconded δ, and the fixed cost of storing currency μ. The curvature parameter σ is calibrated to match the estimated money demand elasticity. Using the currency-in-circulation data from FRED and the new M1 series from Lucas Jr and Nicolini (2015), I calibrate the fraction of currency transactions in the DM ρ until the model generates the currency-to-M1 ratio. I use domestically-held public debt to GDP from FRED to calibrate the value of publicly-held government liabilities v.[28] Another variable I use to calibrate parameters is the inflation rate. Together with other parameters, the fraction of assets that can be absconded δ is calibrated so that the model generates an inflation rate consistent with the observed rate of 1.06 percent. Finally, I calibrate the fixed cost of storing currency μ to be 2 percent of cash payments.[29]

1.5.2 Counterfactual Analysis

I consider three different environments where the fixed cost of theft κ is (i) 2.5 percent, (ii) 5 percent, and (iii) 10 percent of the current consumption level. Given the calibrated parameters and each κ, I vary the nonpar exchange rate η and find the corresponding nominal interest rate on reserves that maximizes welfare, denoted by R^*_η. As illustrated by Figure 1.6, an increase in η decreases the ELB on the nominal interest rate. But, the welfare level under the optimal nominal interest rate R^*_η decreases as η increases.

[26] In the aftermath of Global Financial Crisis in 2007-2008, the demand for currency has increased possibly due to non-transactional purposes. To calculate the elasticity of money demand only for transactions, I exclude post-crisis data as in Chiu et al. (2022) and Altermatt and Wang (2022), for example.

[27] The interest rate on liquid bonds (e.g., 3-month Treasury Bill rate) can fluctuate due to liquidity premium. I consider AAA corporate bond yield as the nominal interest rate on illiquid bonds and $\frac{\pi}{\beta}-1$ as its theoretical counterpart.

[28] I define domestically-held public debt by total public debt net of public debt held by foreign and international investors.

[29] This assumption implies that each buyer pays approximately 2 percent more to purchase goods in currency transactions, to compensate the seller's storage cost. The fixed storage cost μ could be lower or higher than 2 percent of cash payments, but varying μ from 0 percent to 10 percent does not make much difference for counterfactual analysis.

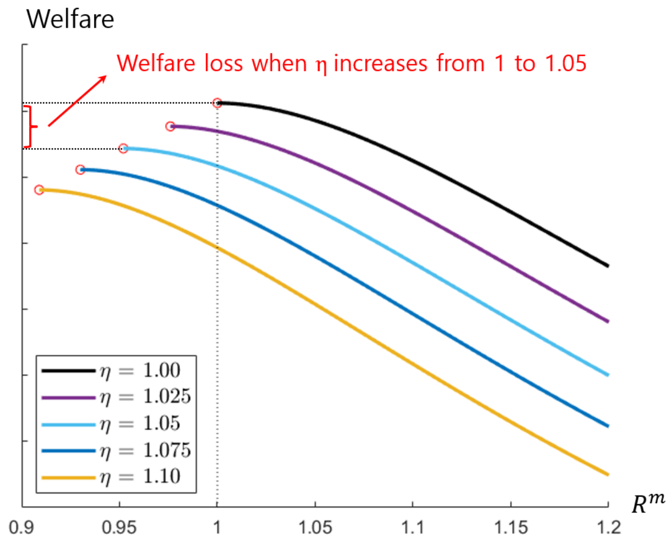

Figure 1.6: Monetary policy (R^m, η) and welfare

My approach to quantifying the welfare cost of increasing η is to measure how much consumption private individuals would need to be compensated to endure the nonpar exchange rate η. For any (R^m, η), the welfare measure is given by

$$\mathcal{W}(R^m, \eta) = \rho[u(x^c) - x^c] + (1 - \rho)[u(x^d) - x^d] - \alpha^b \kappa.$$

If I choose a nonpar exchange rate η but also adjust the quantities of consumption in the DM by a factor Δ, welfare is expressed as

$$\mathcal{W}_\Delta(R^m = R^*_\eta, \eta) = \rho[u(\Delta x^c) - x^c] + (1 - \rho)[u(\Delta x^d) - x^d] - \alpha^b \kappa.$$

Then, I can obtain the value Δ_η that solves $\mathcal{W}_{\Delta_\eta}(R^m = R^*_\eta, \eta > 1) = \mathcal{W}(R^m = R^*_\eta, \eta = 1)$. The welfare cost of introducing η can be measured as $\Delta_\eta - 1$ percent of consumption. If private individuals are compensated with this amount of consumption, they would be indifferent between the two policy choices: a one-to-one exchange rate and a nonpar exchange rate.

Table 1.4 presents the ELB, the optimal nominal interest rate R^*_η, and the welfare cost of introducing a nonpar exchange rate η given a fixed cost of theft κ. Specifically, an increase in η reduces both the ELB and the optimal interest rate regardless of the cost of theft. Recall that, given a fixed cost of storing currency close to zero ($\mu \approx 0$), the optimal monetary policy can be characterized by a modified Friedman rule ($\eta R^m \approx 1$). So, an increase in η decreases the optimal nominal interest rate R^*_η. As monetary policy is conducted optimally given a nonpar exchange rate η, there would be no distortion in the equilibrium prices and allocations.

η	ELB	R^*_η	$(\Delta_\eta - 1) \times 100$		
			$\kappa = 2.5\%$	$\kappa = 5\%$	$\kappa = 10\%$
1.00	1.000	1.000	-	-	-
1.025	0.976	0.976	0.0561	0.1118	0.2236
1.05	0.952	0.952	0.1095	0.2183	0.4367
1.075	0.930	0.930	0.1604	0.3199	0.6399
1.10	0.909	0.909	0.2091	0.4168	0.8339

Table 1.4: ELB, optimal interest rate, and the welfare cost of reducing the ELB

Introducing a nonpar exchange rate η, however, increases the aggregate cost of theft in equilibrium. If the fixed cost of theft κ is 2.5 percent of the current consumption level, increasing η by 5 percent and 10 percent costs, respectively, 0.11 percent and 0.21 percent of consumption. If κ is 5 percent of the current consumption level, increasing η by the same magnitudes costs, respectively, 0.22 percent and 0.42 percent of consumption. Finally, if the value of κ is the same as 10 percent of the current consumption level, increasing η by, respectively, 5 percent and 10 percent, costs 0.44 percent and 0.84 percent of consumption.[30]

How large is the welfare cost of introducing a nonpar exchange rate? To better understand its magnitude, the welfare cost computed here can be compared with estimates for the welfare cost of another policy that have been frequently discussed in the literature: the welfare cost of 10 percent inflation. As the estimates for the welfare cost of 10 percent inflation are typically around 1 percent of consumption, the welfare cost of introducing a nonpar exchange rate seems significant.[31] Note that the welfare cost of using a nonpar exchange rate critically depends on the fixed cost of investing in the theft technology κ. As κ increases, the welfare cost also increases proportionally.

1.6 Disintermediation

In the baseline model, only currency is accepted in some transactions while only bank claims are accepted in other transactions. In this case, no arbitrage conditions from holding currency across periods determine the effective lower bound (ELB) on nominal interest rates, as shown earlier. Another concern about implemening a negative interest rate is a possibility of disintermediation: consumers can choose to withdraw all currency from their deposit accounts.[32] However, as currency

[30]Assuming a different fixed cost of storing currency μ does not significantly change the result. For example, if μ is 10 percent of cash payments and κ is 5 percent of the current consumption level, increasing η by 5 percent and 10 percent costs, respectively, 0.2185 percent and 0.4172 percent of consumption.

[31]The welfare cost of increasing inflation from 0 percent to 10 percent is 0.62 percent of consumption in Chiu and Molico (2010), 0.87 percent in Lucas Jr (2000), and 1.32 percent (take-it-or-leave-it offer) in Lagos and Wright (2005), for example.

[32]Disintermediation is a practical concern. As a negative deposit rate might lead to massive cash withdrawals, private banks may not want to reduce their deposit rates below zero. See Eggertsson et al. (2019) for empirical

and bank deposits are not substitutable, disintemediation does not occur in the baseline model. To understand the implications of introducing a nonpar exchange rate for potential disintermediation, I modify the baseline model by assuming that currency can be acceptable in all DM transactions. Although this is an extreme assumption, it will help us understand how the ELB can be determined to prevent disintermediation.

As it is possible to use only currency in DM transactions, buyers might benefit from opting out of banking arrangements. In the previous sections, I assumed that a fraction $1-\rho$ of sellers accept only bank claims as a means of payment in DM transactions. Here, I will assume that those sellers accept both currency and bank claims, while a fraction ρ of sellers accept only currency as in the baseline model. Other than that, the model is the same as the one described in Section 2.

Let θ denote the fraction of buyers who choose to deposit with private banks in the CM. Each private bank's contracting problem and the first-order conditions for the bank's problem are identical to those in the baseline model. A fraction $1-\theta$ of buyers choose to opt out of banking arrangements and use only currency in DM transactions. Each of these buyers solves

$$\max_{c^o \geq 0} \left\{ -\eta c^o + u\left(\frac{\beta c^o \left[1-\alpha^s+\alpha^s(1-\alpha^b)\eta\right]}{\pi} - \beta\mu \right) \right\}, \tag{1.45}$$

where c^o is the quantity of currency in real terms. Although private banks do not write a deposit contract with these buyers, I assume that private banks withdraw currency from the central bank to sell it to these buyers whenever necessary. Then, a no arbitrage condition implies that the price of currency is η in equilibrium. The first-order condition for each of these buyer's problem is given by

$$-\eta + \frac{\beta\left[1-\alpha^s+\alpha^s(1-\alpha^b)\eta\right]}{\pi} u'\left(\frac{\beta c^o \left[1-\alpha^s+\alpha^s(1-\alpha^b)\eta\right]}{\pi} - \beta\mu \right) = 0, \tag{1.46}$$

and let x^o denote the consumption quantity in DM meetings for the buyer. Asset market clearing conditions are given by

$$\theta c + (1-\theta)c^o = \bar{c}; \quad \theta m = \bar{m}; \quad \theta b = \bar{b}. \tag{1.47}$$

Let U^b denote the expected utility for buyers who write banking contracts and let U^o denote the expected utility for buyers who opt out of banking contracts. Then, using (1.14), (1.17)-(1.19),

evidence on the malfunction of the transmission mechanism of monetary policy at negative territory.

and (1.46), U^b and U^o can be written as

$$U^b = \rho\left[u(x^c) - (x^c + \beta\mu)u'(x^c)\right] + (1-\rho)\left[u(x^d) - x^d u'(x^d)\right], \tag{1.48}$$

$$U^o = u(x^o) - (x^o + \beta\mu)u'(x^o). \tag{1.49}$$

In equilibrium, the fraction θ must be the solution to the following problem:

$$\max_{0\leq\theta\leq 1}\left[\theta U^b + (1-\theta)U^o\right], \tag{1.50}$$

which implies that θ must be consistent with each buyer's utility maximization problem.

Depending on who the seller meets in the DM, the quantity of currency can differ across currency-holding sellers in the TM. Some buyers write deposit contracts and withdraw c' units of real currency in equilibrium. This implies that sellers who meet these buyers in the DM hold $\frac{c'}{\pi}$ units of real currency in the following TM. Some buyers use only currency in the DM, so sellers who meet these buyers in the DM hold $\frac{c^o}{\pi}$ units of real currency in the following TM. For convenience, I assume that each seller decides whether to carry currency into the TM before realizing the type of buyer he or she meets in the DM.

The fraction of buyers who acquire the theft technology α^b and the fraction of sellers who choose to participate in side trades of currency α^s must be incentive compatible in equilibrium. Then,

$$\text{if} \quad \kappa > \frac{\left[\theta\rho c' + (1-\theta)c^o\right]\alpha^s\eta}{\pi}, \quad \text{then} \quad \alpha^b = 0, \tag{1.51}$$

$$\text{if} \quad \kappa = \frac{\left[\theta\rho c' + (1-\theta)c^o\right]\alpha^s\eta}{\pi}, \quad \text{then} \quad 0 \leq \alpha^b \leq 1, \tag{1.52}$$

$$\text{if} \quad \kappa < \frac{\left[\theta\rho c' + (1-\theta)c^o\right]\alpha^s\eta}{\pi}, \quad \text{then} \quad \alpha^b = 1. \tag{1.53}$$

With probability $\theta\rho\alpha^s$, each buyer is matched with a seller holding $\frac{c'}{\pi}$ units of real currency and with probability $(1-\theta)\alpha^s$ each buyer is matched with a seller holding $\frac{c^o}{\pi}$ units of real currency. Conditions (1.51)-(1.53) state that theft does not occur if theft is too costly, theft sometimes takes place if the buyer is indifferent between stealing and not stealing, and theft always takes place if theft is profitable. Also, each seller's decision on whether to carry currency into the TM must be incentive compatible. That is,

$$\text{if} \quad (1-\alpha^b)\eta < 1, \quad \text{then} \quad \alpha^s = 0, \tag{1.54}$$

$$\text{if} \quad (1-\alpha^b)\eta = 1, \quad \text{then} \quad 0 \leq \alpha^s \leq 1, \tag{1.55}$$

$$\text{if} \quad (1-\alpha^b)\eta > 1, \quad \text{then} \quad \alpha^s = 1. \tag{1.56}$$

In this section, I confine attention to an equilibrium with a sufficiently low cost of theft κ, implying that theft takes place in equilibrium.[33] Also, notice that I have focused on cases where collateralized assets are sufficiently scarce. Specifically in this section, I will assume that

$$v < x^* + \beta\mu, \tag{1.57}$$

where x^* is the efficient quantity of consumption in DM transactions that solves $u'(x) = 1$. This assumption implies that the consumption quantities in DM transactions for buyers choosing bank contracts are inefficiently low due to the shortage of collateral. But, this also implies that the central bank cannot support the efficient quantity of consumption for those opting out of banking contracts. This is because the quantity of currency outstanding is constrained by the size of the central bank's balance sheet, which can only be increased by purchasing scarce government debt. Then, the following proposition characterizes the effective lower bound (ELB) on nominal interest rates R^m.

Proposition 5 *Suppose that both the cost of theft κ and the fixed cost of holding currency μ are sufficiently low, and that the value of consolidated government debt outstanding v satisfies (1.57). Then, an equilibrium exists if and only if*

$$R^m \geq \frac{u'(x^o)}{\eta[(1-\delta)u'(x^d) + \delta]}, \tag{1.58}$$

where (x^o, x^d), together with x^c, are the solutions to $(x^o + \beta\mu)u'(x^o) = v$, $u'(x^o) = u'(x^c) - \delta u'(x^d) + \delta$, and $U^b = U^o$ from (1.48)-(1.49). Furthermore,

$$\frac{u'(x^o)}{(1-\delta)u'(x^d) + \delta} > 1.$$

Proof See Appendix □

Proposition 5 shows that nominal interest rates R^m below some threshold cannot be supported in equilibrium. This is because a sufficiently low R^m induces buyers to use only currency in DM transactions but there is a shortage of government debt to back the required quantity of currency. That is, the central bank cannot issue the required quantity of currency to meet the public demand, if R^m is sufficiently low. The right-hand side of (1.58) can be interpreted as the ELB on nominal

[33] I can provide equilibrium conditions given a sufficiently high cost of theft, but analyzing the effects of monetary policy in that case appears not to be straightforward.

interest rates. Therefore, introducing a nonpar exchange rate $\eta > 1$ can reduce the ELB as in Eisler (1932), Buiter (2010), and Agarwal and Kimball (2015).

The proportional cost of storing currency γ becomes irrelevant here as nominal interest rates that effectively encourage buyers to participate in banking arrangements are sufficiently high to prevent arbitrage opportunities from storing currency across periods. Interestingly, the ELB on nominal interest rates can be positive given a one-to-one exchange rate ($\eta = 1$). When the nominal interest rate on reserves is zero ($R^m = 1$), there is an incentive for buyers to opt out of deposit contracts because $x^o > x^c = x^d$. That is, there is inefficiency in the banking system due to a shortage of government debt and a binding collateral constaint, leading to lower consumption in the DM for contracting buyers $x^c = x^d$ than noncontracting buyers x^o. As a complete flight to currency cannot be supported in equilibrium, the nominal interest rate must be higher than zero to prevent buyers from opting out of deposit contracts.

From now on, I will consider cases where $R^m > \frac{u'(x^o)}{\eta[u'(x^d)-\delta u'(x^d)+\delta]}$ and assume that $\mu = 0$ for analytical convenience.[34] That is, there is no fixed cost of holding currency at the beginning of the TM. Then, from (1.6), (1.17)-(1.22), and (1.46)-(1.47), I obtain

$$\eta R^m = \frac{u'(x^c) - \delta u'(x^d) + \delta}{u'(x^d) - \delta u'(x^d) + \delta}, \tag{1.59}$$

$$(1-\rho)\theta x^d \left[u'(x^d) + \frac{\delta}{1-\delta}\right] + \rho\theta(x^c)\left[u'(x^c) + \frac{\delta}{1-\delta}\right] + (1-\theta)(x^o)u'(x^o) = v, \tag{1.60}$$

$$\pi = \frac{\beta\left[u'(x^c) - \delta u'(x^d) + \delta\right]}{\eta}. \tag{1.61}$$

Equations (1.59)-(1.61) come from the first-order conditions for a private bank's problem in equilibrium and equation (1.60) is the collateral constraint. The collateral constraint here is somewhat different from (1.39), the collateral constraint in the baseline model, as some government debt must be additionally held by the central bank to back its liabilities—currency held by buyers who opt out of banking arrangements. From (1.17), (1.18), (1.46), and (1.48)-(1.50),

$$u'(x^o) = u'(x^c) - \delta u'(x^d) + \delta, \tag{1.62}$$

$$\rho\left[u(x^c) - x^c u'(x^c)\right] + (1-\rho)\left[u(x^d) - x^d u'(x^d)\right] \geq u(x^o) - x^o u'(x^o). \tag{1.63}$$

Equation (1.62) is a necessary condition that guarantees positive quantities of consumption in DM transactions for both types of buyers (buyers who choose to participate in banking arrangements and buyers who choose not to do so). This equation implies that $x^c < x^o < x^d$ in equilibrium. Notice that there is inefficiency in deposit contracts arising from the shortage of collateral, which

[34]The results obtained here can be applied to cases with a sufficiently low μ.

not only constrains x^d but also constrains x^c. So, bank deposit contracts are useful only when $x^c < x^d$ because otherwise buyers would strictly prefer opting out of deposit contracts.[35] Equation (1.63) shows that, in equilibrium, buyers must weakly prefer participating in a banking arrangement to not participating. Finally, from (1.52) and (1.55),

$$\alpha^b = \frac{\eta - 1}{\eta}, \tag{1.64}$$

$$\alpha^s = \frac{\beta\kappa}{\eta\left[\theta\rho x^c + (1-\theta)x^o\right]}, \tag{1.65}$$

$$\kappa < \frac{\eta\left[\theta\rho x^c + (1-\theta)x^o\right]}{\beta}. \tag{1.66}$$

Equations (1.64) and (1.65) determine the fraction of buyers who choose to acquire the theft technology α^b and the fraction of sellers who choose to carry currency in the TM α^s. Inequality (1.66) is a necessary condition for this equilibrium to exist.

I can solve the model differently depending on whether (1.63) holds with equality. If (1.63) holds with equality, equations (1.59), (1.62), and (1.63) solve for x^c, x^d, and x^o given monetary policy (R^m, η) and fiscal policy v. Then, equation (1.60) solves for θ, equation (1.61) solves for π, and equations (1.64) and (1.65) solve for α^b and α^s, respectively. If (1.63) holds with strict inequality, equations (1.59) and (1.60) with $\theta = 0$ solve for x^c and x^d. Then, equation (1.61) solves for π and equations (1.64) and (1.65) solve for α^b and α^s, respectively, with $\theta = 0$. Equation (1.62) solves for x^o, the would-be quantity of consumption in DM transactions if, off equilibrium, a buyer were to opt out of banking arrangements.

The following proposition shows how the fraction θ is determined in equilibrium and the effects of monetary policy (R^m, η) depending on the value of θ.

Proposition 6 *Suppose that the fixed cost of holding currency μ is zero. If ηR^m is sufficiently high but not too high, then $0 \leq \theta < 1$ in equilibrium and (1.63) holds with equality. In this case, an increase in ηR^m (an increase in R^m or η or both) leads to decreases in x^c and x^o and increases in x^d and θ along with an increase in real interest rates (r^m, r^b). Furthermore, an increase in R^m increases π and α^s but does not affect α^b. An increase in η decreases π and increases α^b but its effect on α^s is ambiguous. If ηR^m is very high, then $\theta = 1$ in equilibrium and (1.63) holds with strict inequality.*

Proof See Appendix □

[35] However, using currency in transactions is also inefficient due to the fixed cost of holding currency μ. With a sufficiently high μ, buyers could choose to use deposit contracts even though $x^d \leq x^c$.

In Proposition 6, a higher nominal interest rate R^m or exchange rate η implies a lower rate of return on currency relative to reserves and government bonds, leading to a substitution of bank claims for currency in DM transactions. If $0 \leq \theta < 1$ in equilibrium, the consumption quantity in DM transactions using bank claims x^d increases along with a rise in the fraction of buyers who participate in banking arrangements θ, while the consumption quantities in DM transactions using currency x^c and x^o decrease. So, given a traditional central banking system with $\eta = 1$, lowering the nominal interest rate R^m can contribute to disintermediation in that the fraction of buyers participating in banking arrangements θ falls.

The central bank, however, can introduce an appropriate nonpar exchange rate between currency and reserves η that helps hold the relative rate of return on currency $\frac{1}{\eta R^m}$ constant. This implies that the central bank can implement a negative nominal interest rate without causing a disruptive effect on the banking system, consistent with the conventional view, since the fraction θ as well as x^c, x^d, and x^o would remain unchanged with ηR^m held constant. But, the fraction of buyers who invest in the costly theft technology α^b would increase, which has welfare implications.

I can define a welfare measure for this economy as

$$\mathcal{W} = \rho\theta\,[u\,(x^c) - x^c] + (1-\rho)\,\theta[u(x^d) - x^d] + (1-\theta)\,[u\,(x^o) - x^o] - \alpha^b\kappa, \tag{1.67}$$

which is the sum of surpluses from trade in the CM and the DM, net of the total cost of theft in the TM. Then, the following proposition characterizes the optimal monetary policy.

Proposition 7 *Suppose that the fixed cost of holding currency μ is zero. The optimal monetary policy consists of $\eta = 1$ and $R^m = \frac{u'(x^o)}{u'(x^d) - \delta u'(x^d) + \delta} > 1$, where (x^o, x^d), together with x^c, are the solutions to $x^o u'(x^o) = v$, (1.62), and (1.63) with equality.*

Proof See Appendix □

Setting a one-to-one exchange rate between currency and reserves ($\eta = 1$), as in a traditional central banking system, is optimal because the central bank can eliminate costly theft without reducing welfare. The central bank can avoid a reduction in welfare by choosing an appropriate policy (η, R^m). Also, optimality is obtained when the central bank sets the nominal interest rate on reserves R^m at the ELB. As the ELB is higher than one, the quantity of consumption in DM transactions using bank claims is higher than the quantities in other DM transactions using currency for any given R^m. So, lowering the nominal interest rate always improves welfare as it allows buyers to better smooth their consumption across DM transactions.

Which model specification would represent reality better, the baseline model or the modified one? In the baseline model, I assume that only currency is used in some transactions while only bank claims are used in other transactions. In contrast, in the modified model, currency is accepted everywhere while bank deposits can be accepted only in some transactions. In practice, some transactions including online transactions cannot be made with currency. In contrast, some transactions can be made only with currency either because the consumer or the retailer does not have access to banking system or because they value privacy. Also, there are transactions where both means of payment can be accepted. Therefore, reality may be somewhere between the two versions of the model.

1.7 Conclusion

In the literature, a nonpar exchange rate between currency and reserves has been proposed as a potential policy instrument that could reduce the effective lower bound (ELB) on nominal interest rates. I have constructed a model with two means of payment, currency and bank deposits, and frictions associated with the storage and transportation of currency, to study the implications of introducing a nonpar exchange rate. A key finding is that a nonpar exchange rate can indeed reduce the ELB on nominal interest rates if there exist sufficient frictions that induce agents to exchange currency and reserves rather than avoiding the central bank.

Introducing a nonpar exchange rate, however, can be costly because lowering the ELB must be accompanied by an enhancement in the frictions that determine the ELB. In particular, a nonpar exchange rate can increase the market value of currency, and thus encourage socially undesirable behavior and decrease welfare. Even if the optimal interest rate is constrained by the ELB, a nonpar exchange rate does not help increase welfare because introducing a nonpar exchange rate leads to a fall in the optimal interest rate. As this only increases the resource cost to support such undesirable activities, the optimal monetary policy is to set the nominal interest rate at the ELB and maintain the one-to-one exchange rate between currency and reserves. With a modified version of the model, I have also shown that a nonpar exchange rate can help the central bank implement a negative interest rate without causing disintermediation, although this can decrease welfare.

Chapter 2

International Effects of Quantitative Easing and Foreign Exchange Intervention

2.1 Introduction

In the past two decades, central banks' large-scale asset purchases have drawn intense interest in academic and policy circles due to their domestic effects and potential spillovers. This paper develops a two-country general equilibrium model with financial frictions to explore the implications of central banks' unconventional domestic/foreign asset purchases for global asset prices and welfare. In the model, a central bank can affect domestic and foreign financial conditions because its asset swaps change the composition of assets traded in financial markets. In particular, when international financial markets are effectively integrated, a central bank's purchases of long-term local government debt can reduce long-term bond yields and term premia internationally and improve global welfare. Purchases of foreign government debt can relax the frictions in financial markets and improve welfare globally only if implemented by the central bank in the creditor country.

In most monetary systems, central banks choose a target interest rate (typically, an overnight rate or an interest rate on reserves) and intervene in financial markets (through open market operations, for example) to hit the target. This policy is commonly referred to as conventional monetary policy. However, in the aftermath of the 2007-08 global financial crisis, many central banks began to rely more on unconventional policies as their conventional policy rates became constrained at their respective lower bounds. Some central banks in advanced economies, such as the Federal Reserve, the European Central Bank, and the Bank of England, engaged in large-scale domestic asset purchases (also known as quantitative easing or QE) to lower long-term interest rates and spur economic activity. For example, the US Federal Reserve's holdings of US Treasury securities increased from around $500 billion in 2009Q1 to $2.5 trillion in 2014Q3, as illustrated in the left panel of Figure 2.1.

These unconventional policies seem to have substantial international spillover effects on other economies, including increased capital inflows, reductions in long-term bond yields, and exchange rate appreciations.[1] In response to large capital inflows, some central banks unconventionally intervened in foreign exchange markets to stabilize their currency appreciations. For instance, the Swiss National Bank (SNB) increased its holdings of foreign exchange reserves from roughly 10% of GDP in 2010 to more than 100% in 2016 to keep the exchange rate higher than 1.2 Swiss francs per Euro. The right panel of Figure 2.1 shows the trend increase in central banks' foreign reserve holdings. While the US Fed increased its holdings of US Treasury securities by $2.0 trillion between 2009Q1 and 2014Q3, other central banks increased their foreign reserve holdings in US dollars by roughly $2.9 trillion (from $4.7 trillion to $7.6 trillion) over the same period.

What are the consequences of central banks' large-scale purchases of local and foreign government bonds? Many researchers point out a consistent imbalance between the demand for and the supply of safe assets (or a shortage of safe assets) as a major force that has lowered global real interest rates.[2] So, it seems obvious that accumulating safe foreign government bonds by emerging market and developing economies would contribute to the shortage of safe assets, lowering real interest rates. However, the effect of the SNB's foreign exchange (FX) intervention might have been different because it not only purchased safe government bonds but also purchased risky corporate bonds and equities as foreign exchange reserves.[3] In addition, the SNB effectively supplied Swiss government liabilities (including sight deposits at the SNB) in exchange for foreign reserves. What are the implications of the SNB's FX intervention for global financial markets and real interest rates? Could the SNB's intervention help mitigate the shortage of safe assets in global financial markets? How does it affect welfare domestically and globally? If the shortage of safe assets is taken into account, the effect of QE on global financial markets also becomes less obvious as QE involves a central bank's swaps of domestic short-term government liabilities (for example, reserve balances at the US Fed) for long-term ones. Then, what are the effects of QE on global financial markets if the shortage of safe asset matters? If QE reduces long-term nominal interest rates and term premia, does it reduce real interest rates as well? What are its welfare implications?

[1] See, for example, Bauer and Neely (2014), Neely (2015), and Rogers et al. (2014), who find that the QE implemented by the US Federal Reserve decreased long-term bond yields internationally. See also Bhattarai et al. (2021), who find the spillover effects of the QE on emerging market economies.

[2] Del Negro et al. (2019) find that the secular decline in global real interest rates since the 1990s is driven primarily by the shortage of safe assets. Also, Jorda et al. (2017) find that global interest rates on safe assets have been declining over the past three decades, while risky returns have been roughly stable over this period, consistent with the shortage-of-safe-asset hypothesis. See, also, Caballero and Krishnamurthy (2009), Bernanke et al. (2011), Caballero (2010), and Caballero et al. (2016).

[3] At the end of 2021Q1, the SNB holds 66% of its foreign exchange reserves as government bonds, 11% as other bonds, and 23% as equities.

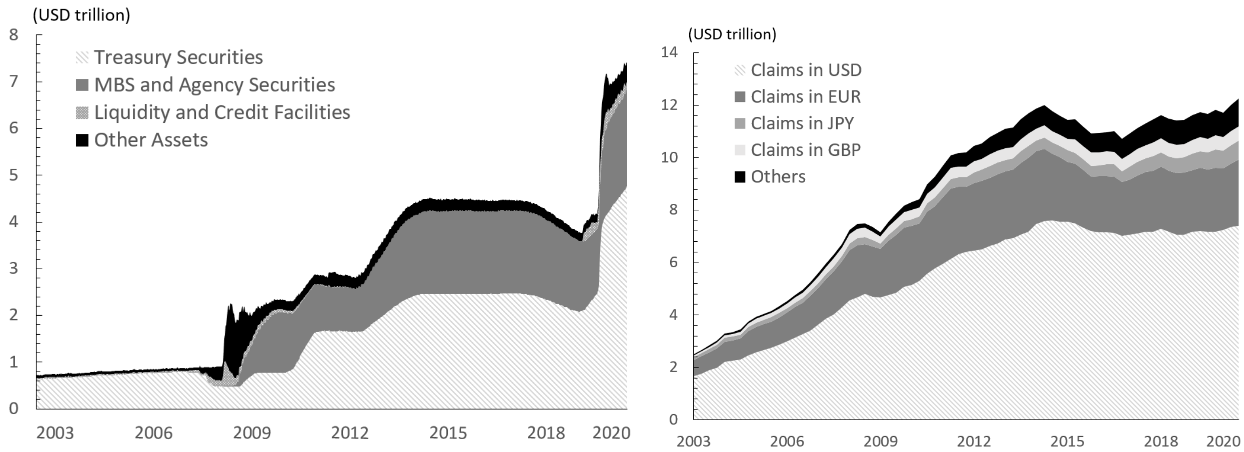

Figure 2.1: Central banks' foreign reserve holdings (left) and the US Fed's asset holdings (right)

Sources: Federal Reserve Board, IMF International Financial Statistics (IFS) and Currency Composition of Official Foreign Exchange Reserves (COFER), and author's calculations

Notes: The left chart reports end-of-period quarterly data and the right chart reports end-of-period weekly data. The currency composition of total foreign reserves is calculated using the IFS data on total foreign exchange reserves and shares of allocated reserves by currency that were reported under COFER.

Answering these questions has become even more important because many central banks in developed and emerging market economies implemented QE or QE-like interventions during the COVID-19 pandemic in 2020.[4] Therefore, I develop a two-country general equilibrium model with financial frictions where safe assets are useful as collateral, but the supply of safe assets is limited. The basic structure of the model comes from Lagos and Wright (2005). Details of the structure of financial intermediaries and fiscal policy are related to Williamson (2012, 2016, 2018, 2019, 2022b) and Andolfatto and Williamson (2015), while the structure of international trade is related to Gomis-Porqueras et al. (2013) and Gomis-Porqueras et al. (2017).

In the model, private financial intermediaries in each country play a liquidity transformation role for domestic economic agents, in line with Diamond and Dybvig (1983). However, intermediaries are subject to limited commitment, so their asset holdings must serve as collateral. A key assumption is that different assets have different degrees of "pledgeability", as in Kiyotaki and Moore (2005), Venkateswaran and Wright (2014), and Williamson (2016). In the model, a higher degree of pledgeability for short-term assets than long-term assets leads to a term premium (an upward-sloping nominal yield curve). A higher degree of pledgeability for local currency-denominated assets than foreign currency-denominated assets forms the "home bias" in asset portfolios of financial intermediaries. Also, collateralizable assets are collectively scarce due to suboptimal fiscal

[4]Developed and emerging market economies, including Australia, Canada, Columbia, Chile, Croatia, Hungary, India, Indonesia, Israel, Korea, Mexico, New Zealand, the Philippines, Poland, Romania, South Africa, Sweden, and Turkey, implemented QE amid the COVID-19 outbreak in 2020 (Hartley and Rebucci, 2020; Arslan et al., 2020).

policies.

In equilibrium, each asset is allocated to the country where the asset is most useful as collateral. In particular, differential and asymmetric pledgeability of assets, along with different degrees of asset scarcity across countries, determine which assets migrate to which country. If the degrees of asset scarcity are not significantly different across countries in equilibrium, then assets issued in each country (and thus denominated in the local currency) are held only by local intermediaries, implying no cross-border capital flows. If the degree of asset scarcity in one country is sufficiently high relative to the other country, then intermediaries in the *high-asset-scarcity* country choose to acquire some foreign assets as well as local assets. In contrast, intermediaries in the *low-asset-scarcity* country acquire only local assets. Therefore, capital flows from the high-asset-scarcity country to the other, leading to an integration of financial markets.

Relative inflation rates determine the exchage rate between two currencies as in the standard international asset pricing model of Lucas Jr (1982). However, the exchange rate is determined away from uncovered interest parity (UIP) condition. This occurs because safe assets carry liquidity premia due to their role as collateral, which in turn leads to low real interest rates. In this regard, the model is related to Lee and Jung (2020) and Bianchi et al. (2021), which provide a liquidity-based explanation on why a relatively higher interest-yielding currency can appreciate (or the UIP puzzle).[5]

A key result is that monetary policies can have different effects depending on the equilibrium asset portfolios of financial intermediaries. If all intermediaries voluntarily acquire only local assets in equilibrium, domestic asset markets are effectively segmented from foreign asset markets. In this case, QE affects only domestic asset prices and depreciates the local currency. If some intermediaries acquire foreign assets as well as local assets in equilibrium, QE reduces long-term bond yields and term premia internationally, as in Alpanda and Kabaca (2020) and Kolasa and Wesolowski (2020).[6] However, real bond yields rise as opposed to what occurs in their models. As a central bank's purchases of long-term government debt tend to reduce both the short-term and long-term nominal interest rates, the central bank conducts open market operations to hold the short-term rate at the target level. So, in equilibrium, QE involves swaps of better collateral (short-term government debt) for worse collateral (long-term government debt), leading to an increase in the effective stock of collateral held by the public. This then relaxes the collateral constraints of intermediaries, as in Dedola et al. (2013), and reduces liquidity premia, implying an increase in real

[5] In the literature, Fama (1984) find that higher interest-yielding currencies appreciate on average, which contradicts the foundation of the UIP. This empirical finding is still a puzzle because more recent studies also show deviations from the UIP and there has been very few theoretical explanations about it.

[6] Alpanda and Kabaca (2020) and Kolasa and Wesolowski (2020) each develops a two-country general equilibrium model with asset market segmentation. As opposed to their models that assume imperfect substitutability between any two types of bonds, asset substitutability is determined endogenously in my model.

bond yields.[7] In this equilibrium, the effect on the nominal exchange rate is qualitatively identical, no matter where QE is implemented. In particular, any QE leads to an immediate depreciation of the high-asset-scarcity currency (or the creditor country's currency), along with its expected appreciation.

In the model, FX intervention can be *beggar-thy-neighbor* as it can improve domestic welfare by decreasing welfare in the foreign country. This occurs because FX intervention involves the central bank's swaps of local government bonds for foreign government bonds in equilibrium. If all intermediaries hold only local assets in equilibrium, then the intervention acts to decrease the supply of collateral in the foreign country while increasing the collateral supply in the domestic economy. As a result, the intervention relaxes financial frictions and improves welfare domestically at the expense of tightened financial frictions and decreased welfare in the foreign country.[8]

There are also cases where FX intervention can be mutually beneficial. If global financial markets are effectively integrated and local government bonds are considered as more valuable collateral in financial markets, the central bank's FX intervention and resulting foreign asset purchases can effectively increase the stock of collateral held by intermediaries. In this case, the intervention serves to take worse collateral (foreign government debt) out of markets and replace it with better collateral (local government debt), relaxing financial frictions internationally. This finding shows that the Swiss National Bank (SNB)'s FX intervention can be a good thing for global financial markets if Swiss government liabilities are superior, on average, as collateral to the SNB's foreign exchange reserves.

In contrast to the case of Switzerland, FX intervention in some emerging market and developing economies can tighten global financial frictions by reducing the effective supply of collateral in global financial markets. This can occur when the government liabilities of those economies are considered less valuable as collateral than what they accumulate, mostly global safe assets. In this case, the Fed's overnight reverse repurchase agreement facility and liquidity swap lines, which effectively allow foreign central banks to hold reserve accounts at the Fed or to have options to withdraw US dollars, can act to mitigate global financial frictions by reducing foreign central banks' incentives to accumulate US Treasury securities.

An important contribution to the literature on the global implication of a safe-asset scarcity for government policies is the work by Caballero et al. (2020). They develop a two-country model

[7]Dedola et al. (2013) adopt limited commitment of financial intermediaries with limited pledgeability of assets, as I do, to capture the international transmission of QE. A key difference is that a monetary equilibrium model developed in my paper allows a further analysis on the behaviors of inflation rates and the nominal exchange rate, which are not addressed in Dedola et al. (2013).

[8]This finding is somewhat related to Fanelli and Straub (2020), which develops a multi-country model to show that countries over-accumulate foreign reserves in equilibrium, leading to reduced welfare and inefficiently low interest rates on safe assets.

where, compared to risky assets, safe assets are superior as a store of value but their nominal rate of return is constrained by the zero lower bound. Although my model is quite different in that the real rate of return on safe assets is low due to their role as collateral, some of my findings are consistent with theirs: (i) with an integrated financial market, any government policies that effectively increase the supply of safe assets is expansionary for the global economy and (ii) when the degrees of asset scarcity are significantly different across two countries, the high-asset-scarcity country becomes a net creditor and effectively exports its asset scarcity abroad. I complement their findings by deriving conditions under which local agents purchase foreign assets and the country becomes a net creditor. Also, my paper goes further by showing how different monetary policies affect the supply of safe assets differently. A key difference in my paper is that central banks conduct monetary policies through their asset swaps and intervene in the markets via open market operations. This helps improve our understanding of how different types of monetary policies affect domestic and foreign economies differently, especially when a shortage of safe assets matters.

While the model of Caballero et al. (2020) is silent on how the central bank manages to set the exchange rate, my model explains how a central bank's FX intervention is accompanied by a change in the central bank's balance sheet. In this respect, my paper is related to Amador et al. (2020) which develops a model of a small-open economy with binding balance sheet constraints of banking sector to rationalize the SNB's FX intervention. Their model shows that when nominal interest rates become zero in both domestic and foreign financial markets, international capital can flow into the domestic economy due to the expected future appreciation. As the central bank cannot further lower the nominal interest rates, it needs to absorb capital inflows through FX intervention. Differently, my model can apply to an economy away from the zero lower bound although the scarcity of safe assets becomes more severe when the nominal interest rate is closer to zero. Also, my model can explain different global implications of FX intervention depending on the type of equilibrium and the implementing country.

The rest of the paper is organized as follows. I describe the model in Section 2, and define and characterize a stationary equilibria with nonbinding collateral constraints in Section 3. In Section 4, two types of equilibria with binding collateral constraints are analyzed. Section 5 concludes, and Appendix contains all proofs as well as some additional details and discussions.

2.2 Model

Suppose there are two countries, *Home* and *Foreign*. In each country, there are three types of agents: *buyers*, *sellers*, and *banks*. For each type, there is a continuum of agents with unit mass. Home parameters and variables are denoted with a subscript h and without an asterisk, and Foreign ones with a subscript f and an asterisk. Time is discrete and indexed by $t = 0, 1, 2, ...$, and all agents discount the future at rate $\beta \in (0, 1)$. For convenience, I describe the model environment from the perspective of the Home country bearing in mind that there is a symmetric Foreign counterpart.

The model is based on Lagos and Wright (2005) and Rocheteau and Wright (2005). In each period, there are two subperiods—the centralized market (CM) followed by the decentralized market (DM). In the CM, all agents in both countries interact and debts are settled at the beginning. Then, both Home and Foreign agents produce and consume homogeneous perishable goods, and trade assets and goods internationally in a perfectly competitive market. Buyers and banks can produce goods, but sellers cannot in the CM. Specifically, buyers incur disutility H from producing H units of the CM good and sellers receive utility X from consuming X units of the CM good. Banks' per-period preferences are $X - H$ where X and H are units of the CM good consumed and produced, respectively. There is no difference in preferences between Home agents and Foreign agents. As goods and assets are traded internationally, the CM is referred to as the *tradeable sector*.[9]

In the DM, random matches happen between buyers and sellers within each country, where each buyer is matched with a seller. That is, trades take place exclusively between Home buyers and Home sellers, and between Foreign buyers and Foreign sellers. In this regard, the DM is considered as the *non-tradeable sector*.[10] Sellers can produce goods, but buyers cannot in the DM. Buyers receive utility $u(x)$ from consuming x units of the DM good and sellers incur disutility h from producing h units of the DM good. The function $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u'(0) = \infty$, $u'(\infty) = 0$, and $-\frac{xu''(x)}{u'(x)} < 1$. Denote the first-best quantity by $\hat{x} \in (0, \infty)$ that solves $u'(\hat{x}) = 1$.

In all DM matches, there is no memory or record keeping, and a matched buyer and seller are subject to limited commitment. As unsecured credit cannot be supported in equilibrium, some assets must be exchanged on the spot or posted as collateral for trade in the DM. However, there are two types of limitations on the available means of payment. First, in the DM, there is no technology that permits sellers to recognize liabilities issued by foreign institutions, including foreign currency and foreign bank deposits.[11] Second, some sellers have the information technology that enables

[9]The structure of the tradeable and the non-tradeable sector comes from Gomis-Porqueras et al. (2013) and Gomis-Porqueras et al. (2017). As mentioned in Gomis-Porqueras et al. (2013), the tradeable sector in the model is consistent with the one in existing international monetary models. See, for example, Schlagenhauf and Wrase (1995) and Chari et al. (2002).

[10]In this model, international trades take place only in the CM. In practice, however, the existence of an over-the-counter market for international trades can explain why foreign reserve assets are in high demand. Though introducing decentralized international trades is beyond the scope of this paper, interested readers can refer to Geromichalos and Simonovska (2014) and Geromichalos and Jung (2018), for example.

[11]Without this assumption, two currencies become perfect substitutes, so the nominal exchange rate is indeterminate as in Kareken and Wallace (1981). Alternatively, a special trading mechanism such as the one introduced in Zhu and Wallace (2007) could be adopted to generate the same result, as in Rocheteau and Nosal (2017, chapter 12.1.2) and Lee and Jung (2020). However, it would only make the model more complicated without adding any useful implications in this context. In contrast to the DM, I assume that there is a publicly available technology in the CM that helps verify liabilities issued by foreign institutions.

them to accept bank deposits as a means of payment while others do not, as in Williamson (2012, 2016, 2018, 2019). Specifically, in a fraction ρ of each country's DM meetings, the seller cannot verify the buyer's asset holdings other than local currency. So, only local currency can be accepted as a means of payment in those meetings, which I refer to as *currency transactions*. In a fraction $1-\rho$ of DM meetings, the seller can verify any liabilities issued by local institutions. This implies that local currency, local bank deposits, or both can be used in those meetings, which I refer to as *non-currency transactions*. In any matches, the buyer makes a take-it-or-leave-it offer to the seller.

The type of transaction buyers will involve in the following DM is unknown at the beginning of the CM when they write contracts with banks. Buyers observe this outcome at the end of the CM, after trades in goods and assets have taken place. A buyer's type of transaction is private information, and each buyer can meet with his or her bank after observing the type. Eventually, the interaction between a buyer and a bank at the end of the CM will only occur for the execution of the contract written earlier in the CM.[12]

In this model, there are three types of fundamental assets—currency, short-term government bonds, and long-term government bonds—issued by each country's government.[13] First, each country's central bank produces a perfectly divisible and storable currency. Home (Foreign) currency is issued by the Home (Foreign) central bank with a price ϕ (ϕ^*) in units of the CM good. The nominal exchange rate is denoted by e and measures the price of Foreign currency in units of Home currency. As agents can trade goods and assets, including currencies, without any frictions in the CM, the law of one price holds, i.e., $\phi^* = e\phi$.[14] Also, each country's fiscal authority issues local currency-denominated government bonds with two different maturities. I refer to Home currency-denominated government bonds as *Home bonds* and Foreign currency-denominated government bonds as *Foreign bonds*. A short-term Home (Foreign) bond sells at a price z_s (z_s^*) in units of Home (Foreign) currency in the CM and pays off one unit of Home (Foreign) currency in the following CM. A long-term Home (Foreign) bond sells at a price z_l (z_l^*) in units of Home (Foreign) currency in the CM and pays off one unit of Home (Foreign) currency in every future CM.[15]

As well as the fundamental liabilities issued by governments, there are bank liabilities that arise endogenously in the private sector. Like any other credit arrangements, bank liabilities must

[12] This assumption essentially imposes spatial separation between any agents at the end of the CM. For banks to play a Diamond and Dybvig (1983) insurance role, restrictions on side-trading is necessary because otherwise side trades would undo the banking arrangements (See Jacklin, 1987; Wallace, 1988).

[13] Private assets could also be introduced in the model. However, this would only make the model more complicated without changing the main results of the paper.

[14] It seems worthwhile to note that the law of one price holds in the tradeable sector but does not in the non-tradeable sector. Also, some frictions in the international market could be introduced to generate deviations from the law of one price. However, the main results of the paper would remain unchanged.

[15] These long-term government bonds can be thought of as Consols, which the British government first issued in 1751. All the remaining British Consols were fully redeemed in 2015.

be secured by collateral due to the lack of commitment and memory. However, at the beginning of the CM, any agent can abscond with some parts of his or her assets posted as collateral following Kiyotaki and Moore (2005), Venkateswaran and Wright (2014), and Williamson (2016). In particular, an agent in country $i = h, f$ can abscond with fraction θ_{is} (θ^*_{is}) of his or her holdings of Home (Foreign) currency and short-term Home (Foreign) bonds, and with fraction θ_{il} (θ^*_{il}) of long-term Home (Foreign) bonds. Agents can acquire both Home and Foreign assets in their portfolios, but the fraction of an asset that a Home agent can abscond with differs from the fraction that a Foreign agent can abscond with, i.e., $\theta_{hj} \neq \theta_{fj}$ and $\theta^*_{hj} \neq \theta^*_{fj}$ for $j = s, l$.

In practice, an asset with a higher volatility in its market value tends to receive a larger haircut when pledged as collateral. So, assets with longer maturity or assets denominated in foreign currency typically receive larger haircuts than assets with shorter maturity or assets denominated in local currency. For example, when the US Federal Reserve extends discount window lendings to depository institutions, 99% of the market value can be pledged as collateral for US Treasury securities with duration less than three years, and 95% can be pledged for US Treasury securities with duration more than ten years. For foreign government bonds maturing within three years, the percentage of the market value pledgeable as collateral is 97-98%, if denominated in the US dollar, and 94% if denominated in a foreign currency.[16]

In what follows, I will assume that (i) long-term bonds are less pledgeable, as collateral, than short-term bonds with the same denomination ($\theta_{il} > \theta_{is}$ and $\theta^*_{il} > \theta^*_{is}$, $i = h, f$), (ii) foreign currency-denominated bonds are less pledgeable than local currency-denominated bonds with the same maturity ($\theta^*_{hj} > \theta_{hj}$ and $\theta_{fj} > \theta^*_{fj}$, $j = s, l$), and (iii) the pledgeability of short-term bonds relative to long-term bonds is higher for foreign currency-denominated bonds ($\frac{1-\theta_{fs}}{1-\theta_{fl}} > \frac{1-\theta_{hs}}{1-\theta_{hl}}$ and $\frac{1-\theta^*_{hs}}{1-\theta^*_{hl}} > \frac{1-\theta^*_{fs}}{1-\theta^*_{fl}}$).[17]In the model, the different degrees of plegeability are given exogenously. Endogenizing the degrees of pledgeability by, for example, introducing country-specific aggregate shocks into the model would certainly be interesting but this is beyond the scope of this paper.

2.2.1 Private Banks

In this model, banks endogenously create a deposit contract that effectively allocates currency to currency transactions and other higher-yielding assets to non-currency transactions, as in Williamson (2012, 2016, 2018, 2019); **?**. Therefore, the role for the deposit contract here is similar to Diamond

[16]See Discount Window & Payment System Risk Collateral Margins Table, which is available at https://www.frbdiscountwindow.org/Home/Pages/Collateral/. See also Collateral Margin Requirements for the Bank of Canada's Standing Liquidity Facility, which is available at https://www.bankofcanada.ca/2020/04/assets-eligible-collateral-standing-liquidity-facility-090420/.

[17]The first two assumptions are empirically relevant. The last assumption implies that agents would prefer short-maturity bonds to long-maturity bonds if they need to use foreign currency-denominated assets as collateral.

and Dybvig (1983), in that it provides insurance to depositors.[18] Here, key differences are that agents in two countries interact internationally in the CM, and the pledgeability of each asset is different across countries. In this model, the differential pledgeability leads assets to be allocated to the country they are most useful as collateral.

Banks are agents who become active only in the CM. Like other agents, banks go through the settlement, production and consumption, and then asset-trading at the beginning of the CM. At the end of the CM, each buyer can meet with his or her bank. Consider a bank in the Home country (*Home bank*, hereafter) that issues deposit contracts for buyers in the Home country (*Home buyers*, hereafter).[19] At the beginning of the CM, the bank writes a deposit contract for buyers before they realize their type of transaction (currency or non-currency) in the following DM. The deposit contract allows each buyer to choose either one of two options. After learning the type at the end of the CM, the buyer can contact the bank to withdraw currency as specified in the contract, having no other claims left on the bank. Alternatively, if the buyer has chosen to not withdraw currency, the buyer receives a deposit claim that is redeemed in the following CM for a specified quantity of the CM good.

A deposit contract is essentially a debt contract that allows the bank to borrow from buyers in the CM. To settle this debt, the bank delivers currency at the end of the CM and goods in the following CM. However, due to the limited commitment problem, the bank's liabilities must be backed by its asset holdings. As for any agents in the Home country, the Home bank can abscond with fraction θ_{hs} (θ^*_{hs}) of its holdings of Home (Foreign) currency and short-term Home (Foreign) bonds, and fraction θ_{hl} (θ^*_{hl}) of long-term Home (Foreign) bonds.

Then, the Home bank's problem in equilibrium can be expressed as

$$\max_{k,c,d,b_{hs},b_{hl},b^*_{hs},b^*_{hl}} \left[-k + \rho u \left(\frac{\beta \phi_{+1} c}{\phi} \right) + (1-\rho) u(\beta d) \right] \tag{2.1}$$

[18]To understand how a deposit contract can improve social welfare in this model, suppose that banking activity was prohibited. Then, each buyer would acquire currency and government bonds in the CM. Then, the buyer would be holding idle government bonds in a DM currency transaction since only currency would be accepted as a means of payment. In a DM non-currency transaction, the buyer would be holding some currency that permits less consumption than do government bonds. Though buyers can opt out of deposit contracts in the model, they choose to participate in a deposit contract as it essentially allocates currency only to currency transactions and government bonds only to non-currency transactions, providing insurance to buyers.

[19]Recall that the Home bank liabilities are useless in the DM meetings between Foreign agents. So, Home banks issue deposit contracts only for Home buyers in equilibrium.

subject to

$$
\begin{aligned}
& k - \rho c - z_s b_{hs} - z_s^* b_{hs}^* - z_l b_{hl} - z_l^* b_{hl}^* - \beta(1-\rho)d \\
& + \beta \frac{\phi_{+1}}{\phi} \{b_{hs} + (1 + z_{l,+1}) b_{hl}\} + \beta \frac{\phi_{+1}^*}{\phi^*} \left\{b_{hs}^* + (1 + z_{l,+1}^*) b_{hl}^*\right\} \geq 0, \qquad (2.2) \\
& -(1-\rho)d + \frac{\phi_{+1}}{\phi} \{b_{hs} + (1 + z_{l,+1}) b_{hl}\} + \frac{\phi_{+1}^*}{\phi^*} \left\{b_{hs}^* + (1 + z_{l,+1}^*) b_{hl}^*\right\} \\
& \geq \frac{\phi_{+1}}{\phi} \{\theta_{hs}(\rho c + b_{hs}) + (1 + z_{l,+1}) \theta_{hl} b_{hl}\} + \frac{\phi_{+1}^*}{\phi^*} \left\{\theta_{hs}^* b_{hs}^* + (1 + z_{l,+1}^*) \theta_{hl}^* b_{hl}^*\right\}, \qquad (2.3) \\
& k, c, d, b_{hs}, b_{hl}, b_{hs}^*, b_{hl}^* \geq 0. \qquad (2.4)
\end{aligned}
$$

In the above problem, (k, c, d) is a deposit contract where k is the quantity of goods the buyer deposits in the CM, c is the real quantity of Home currency the buyer can withdraw at the end of the CM, and d is the quantity of claims to goods in the following CM that the buyer receives if currency has not been withdrawn. In addition, b_{hs} and b_{hl} (b_{hs}^* and b_{hl}^*) are, respectively, short-term and long-term Home (Foreign) bonds acquired by the Home bank.[20] All quantities in the above problem are denoted in terms of the current CM good except that d is denoted in terms of the following CM good.

The buyer's take-it-or-leave-it offer in each DM meeting implies that the buyer exchanges c units of Home currency for $\frac{\beta \phi_{+1} c}{\phi}$ units of goods in a currency transaction and d units of claims for βd units of goods in a non-currency transaction, as expressed in the objective function (2.1). The Home bank's discounted net payoff from the deposit contract is expressed as the value on the left-hand side of inequality (2.2). In the current CM, the bank receives k units of deposits from the buyer and obtains a portfolio of Home currency ρc, short-term Home bonds $z_s b_{hs}$, long-term Home bonds $z_l b_{hl}$, short-term Foreign bonds $z_s^* b_{hs}^*$, and long-term Foreign bonds $z_l^* b_{hl}^*$. At the end of the CM, the fraction ρ of buyers realize they will be in currency transactions and withdraw c units of Home currency. The fraction $1 - \rho$ of buyers will be in non-currency transactions and receive deposit claims. In the following CM, the bank pays off d units of goods to each holder of the deposit claims. The collateral constraint, inequality (2.3), demonstrates that the net payoff for the Home bank from repaying its liabilities must not be smaller than the payoff from absconding.[21] This implies that the Home bank does not default in equilibrium.

The problem (2.1) subject to (2.2)-(2.4) shows that, in equilibrium, the Home bank must choose a contract that maximizes the representative Home buyer's expected utility, subject to the bank's

[20] The Home bank does not hold Foreign currency across periods because holding Foreign bonds is always weakly preferred to holding Foreign currency.

[21] Following Williamson (2022b), if the bank defaults, it will not allow withdrawals of currency at the end of the CM, and will abscond in the next CM with a fraction of each asset acquired in the current CM.

nonnegative payoff constraint (2.2), the bank's collateral constraint (2.3), and the nonnegativity constraints (2.4). If the contract did not solve this problem, then there would exist an alternative contract that can attract all of the Home buyers and earn a higher expected payoff for the bank. Therefore, the solution to the problem (2.1) subject to (2.2)-(2.4) consists of a Nash equilibrium. A Foreign bank's problem is analogous to the Home bank's problem and relegated to Appendix.

2.2.2 Fiscal Authority and Central Bank

I will confine attention to stationary equilibria where all real variables are constant across periods. This implies that $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ and $\frac{\phi^*_{t+1}}{\phi^*_t} = \frac{1}{\mu^*}$ for all t where μ and μ^* are the gross inflation rates, respectively, in Home and Foreign countries. Each fiscal authority can levy lump-sum taxes on buyers in each country, in equal amounts, and issues short-term and long-term government bonds denominated in the local currency. Each central bank issues the local currency through open market purchases of local and foreign government bonds, and transfers any profits to the local fiscal authority. Then, the consolidated government budget constraints for two countries in period 0 are given by

$$\bar{c} + \sum_{i=s,l} \left[z_i \bar{b}_i - z_i^* a_i^* \right] = \tau_0,$$
$$\bar{c}^* + \sum_{i=s,l} \left[z_i^* \bar{b}_i^* - z_i a_i \right] = \tau_0^*,$$

where $\bar{c}$, $\bar{b}_s$, and $\bar{b}_l$ ($\bar{c}^*$, $\bar{b}_s^*$, and $\bar{b}_l^*$) denote the real quantities of Home (Foreign) currency, and short-term and long-term Home (Foreign) bonds outstanding. As well, a_s^* and a_l^* (a_s and a_l) denote the real quantities of short-term and long-term Foreign (Home) bonds held by the Home (Foreign) central bank, and τ_0 (τ_0^*) denotes the lump-sum transfer to each buyer in the Home (Foreign) country. Then, the consolidated government budget constraints for each succeeding period, $t = 1, 2, 3, ...$, are given by

$$\bar{c} + \sum_{i=s,l} \left[z_i \bar{b}_i - z_i^* a_i^* \right] = \frac{1}{\mu} \left[\bar{c} + \bar{b}_s + (1 + z_l) \bar{b}_l \right] - \frac{1}{\mu^*} \left[a_s^* + (1 + z_l^*) a_l^* \right] + \tau,$$
$$\bar{c}^* + \sum_{i=s,l} \left[z_i^* \bar{b}_i^* - z_i a_i \right] = \frac{1}{\mu^*} \left[\bar{c}^* + \bar{b}_s^* + (1 + z_l^*) \bar{b}_l^* \right] - \frac{1}{\mu} \left[a_s + (1 + z_l) a_l \right] + \tau^*,$$

where τ (τ^*) is the lump-sum transfer to each buyer in the Home (Foreign) country for $t = 1, 2, 3, ...$. In each of the above equations, the left-hand side is the total value of consolidated government liabilities issued in each period, while the right-hand side is the sum of the total value of consolidated government liabilities redeemed in each period and the transfers to buyers.

The behavior of fiscal authorities is important in determining an equilibrium, as it affects the aggregate supply of collateral. A key observation in reality is the low real interest rates on government liabilities, and in this model, real interest rates are low due to a scarcity of collateralizable assets and binding collateral constraints. As a low supply of consolidated government liabilities, in real terms, issued in both countries creates a scarcity of collateralizable assets, I assume that each fiscal authority determines the real value of the consolidated government liabilities of each country following Andolfatto and Williamson (2015) and Williamson (2016, 2018, 2019, 2022b). In particular, the Home and Foreign fiscal authorities set V and V^*, respectively, where

$$V = \bar{c} + \sum_{i=s,l} \left[z_i \bar{b}_i - z_i^* a_i^* \right], \tag{2.5}$$

$$V^* = \bar{c}^* + \sum_{i=s,l} \left[z_i^* \bar{b}_i^* - z_i a_i \right]. \tag{2.6}$$

Then, (2.5) and (2.6) imply that $\tau_0 = V$ and $\tau_0^* = V^*$ while τ and τ^* are determined endogenously in equilibrium. In other words, each fiscal authority passively determines the lump sum transfer to achieve the real value of the consolidated government liabilities at the target level. This fiscal policy rule can be interpreted as the debt ceiling or debt limit in the United States.[22] Since my interest lies in the cases where supplies of government liabilities are inefficiently low, I will eventually analyze the effects of monetary policy interventions given suboptimal fiscal policies, or sufficiently small V and V^*.

Fiscal authorities also determine the outstanding value of local government bonds of each maturity. Let V_s and V_l (V_s^* and V_l^*) denote the values of short and long-term Home (Foreign) bonds outstanding, respectively, where $V = V_s + V_l$ ($V^* = V_s^* + V_l^*$). Then, the following conditions must hold in equilibrium:

$$0 \le z_i \bar{b}_i \le V_i; \qquad 0 \le z_i^* \bar{b}_i^* \le V_i^*,$$

for $i = s, l$. Given fiscal policies (V, V_s, V_l) and (V^*, V_s^*, V_l^*), central banks' balance sheets are well defined as illustrated in Table 2.1. The fiscal policies help define conventional and unconventional monetary policies in a plausible way and allow a tractability for analyzing the effects of monetary policies.

Each central bank's monetary policy has four dimensions. First, the Home central bank determines the price of short-term Home bonds z_s or, equivalently, pegs the short-term nominal interest rate on Home bonds R_s (conventional monetary policy). Also, the Home central bank sets $\omega_l = z_l \bar{b}_l$,

[22] The equivalence between the fiscal policy rule in this model and a debt ceiling policy is presented in Appendix, which also contains more discussions on alternative fiscal and monetary policy rules.

the value of long-term Home bonds held by the public. By choosing ω_l, the Home central bank determines the value of its holdings of long-term Home bonds $V_l - \omega_l$ (quantitative easing or tightening). Lastly, the Home central bank determines $\kappa_s^* = z_s^* a_s^*$ and $\kappa_l^* = z_l^* a_l^*$, the values of its holdings of short-term and long-term Foreign bonds (foreign exchange reserves). The monetary policies of the Foreign central bank are similarly defined.

<Home Central Bank>

Assets	**Liabilities**
ST Home bonds $V_s - z_s\bar{b}_s$	Home
LT Home bonds $V_l - \omega_l$	currency
ST Foreign bonds κ_s^*	$\bar{c}$
LT Foreign bonds κ_l^*	

<Foreign Central Bank>

Assets	**Liabilities**
ST Foreign bonds $V_s^* - z_s^*\bar{b}_s^*$	Foreign
LT Foreign bonds $V_l^* - \omega_l^*$	currency
ST Home bonds κ_s	$\bar{c}^*$
LT Home bonds κ_l	

Table 2.1: Balance sheets of central banks

Notes: ST - short-term; LT - long-term

From the central banks' balance sheets in Table 2.1, the real value of short-term Home (Foreign) bonds acquired by the Home (Foreign) central bank can be expressed as $\bar{c} - V_l + \omega_l - \sum_{i=s,l} \kappa_i^*$ ($\bar{c}^* - V_l^* + \omega_l^* - \sum_{i=s,l} \kappa_i$) and cannot exceed the value of short-term Home (Foreign) bonds issued by the fiscal authority. Therefore, monetary policy variables $(\omega_l, \kappa_i^*, \omega_l^*, \kappa_i)$ for $i = s, l$ must satisfy the following:

$$\max\left[0, V_l - \bar{c} + \kappa_s^* + \kappa_l^*\right] \leq \omega_l \leq \min\left[V_l, V - \bar{c} + \kappa_s^* + \kappa_l^*\right], \tag{2.7}$$

$$\max\left[0, V_l^* - \bar{c}^* + \kappa_s + \kappa_l\right] \leq \omega_l^* \leq \min\left[V_l^*, V^* - \bar{c}^* + \kappa_s + \kappa_l\right]. \tag{2.8}$$

2.3 Equilibrium Characterization and Plentiful Collateral

In this section, I will define a stationary equilibrium and characterize an equilibrium with nonbinding collateral constraints. The collateral constraints of private banks do not bind in equilibrium if the supply of collateralizable assets in financial markets is sufficiently high. As fiscal policies V and V^* determine the value of collateralizable assets available in financial markets, a sufficiently large V and V^* leads to nonbinding collateral constraints in equilibrium.

2.3.1 Characterization of Equilibrium

Let λ and λ^* denote the multipliers related to the collateral constraints of Home and Foreign banks, respectively. Then, from the problem (2.1) subject to (2.2)-(2.4), noting that $\frac{\phi_{+1}}{\phi} = \frac{1}{\mu}$ and $\frac{\phi_{+1}^*}{\phi^*} = \frac{1}{\mu^*}$, the first-order conditions for the Home bank's problem can be written as

$$\frac{\beta}{\mu}u'\left(\frac{\beta c}{\mu}\right) - 1 - \frac{\lambda\theta_{hs}}{\mu} = 0, \tag{2.9}$$

$$\beta u'(\beta d) - \beta - \lambda = 0, \tag{2.10}$$

$$-z_s + \frac{\beta}{\mu} + \frac{\lambda(1-\theta_{hs})}{\mu} \leq 0, \tag{2.11}$$

$$-z_s^* + \frac{\beta}{\mu^*} + \frac{\lambda\left(1-\theta_{hs}^*\right)}{\mu^*} \leq 0, \tag{2.12}$$

$$-z_l + \frac{\beta(1+z_l)}{\mu} + \frac{\lambda(1-\theta_{hl})(1+z_l)}{\mu} \leq 0, \tag{2.13}$$

$$-z_l^* + \frac{\beta\left(1+z_l^*\right)}{\mu^*} + \frac{\lambda\left(1-\theta_{hl}^*\right)\left(1+z_l^*\right)}{\mu^*} \leq 0, \tag{2.14}$$

and

$$\lambda\left\{-(1-\rho)d + \frac{1}{\mu}\left[-\theta_{hs}\rho c + (1-\theta_{hs})\,b_{hs} + (1+z_l)(1-\theta_{hl})\,b_{hl}\right]\right.$$
$$\left.+\frac{1}{\mu^*}\left[(1-\theta_{hs}^*)\,b_{hs}^* + (1+z_l^*)(1-\theta_{hl}^*)\,b_{hl}^*\right]\right\} = 0. \tag{2.15}$$

Similarly, the first-order conditions for the Foreign bank's problem can be written as

$$\frac{\beta}{\mu^*}u'\left(\frac{\beta c^*}{\mu^*}\right) - 1 - \frac{\lambda^*\theta_{fs}^*}{\mu^*} = 0, \tag{2.16}$$

$$\beta u'(\beta d^*) - \beta - \lambda^* = 0, \tag{2.17}$$

$$-z_s + \frac{\beta}{\mu} + \frac{\lambda^*\left(1-\theta_{fs}\right)}{\mu} \leq 0, \tag{2.18}$$

$$-z_s^* + \frac{\beta}{\mu^*} + \frac{\lambda^*(1-\theta_{fs}^*)}{\mu^*} \leq 0, \tag{2.19}$$

$$-z_l + \frac{\beta(1+z_l)}{\mu} + \frac{\lambda^*\left(1-\theta_{fl}\right)(1+z_l)}{\mu} \leq 0, \tag{2.20}$$

$$-z_l^* + \frac{\beta\left(1+z_l^*\right)}{\mu^*} + \frac{\lambda^*\left(1-\theta_{fl}^*\right)\left(1+z_l^*\right)}{\mu^*} \leq 0, \tag{2.21}$$

and

$$\lambda^* \left\{ -(1-\rho)d^* + \frac{1}{\mu^*}\left[-\theta^*_{fs}\rho c^* + (1-\theta^*_{fs})b^*_{fs} + (1+z^*_l)(1-\theta^*_{fl})b^*_{fl} \right] + \frac{1}{\mu}\left[\left(1-\theta_{fs}\right) b_{fs} + (1+z_l)\left(1-\theta_{fl}\right) b_{fl} \right] \right\} = 0. \quad (2.22)$$

Also, it proves convenient to characterize an equilibrium using the quantities of consumption in DM transactions. In particular, let $x_1 = \frac{\beta c}{\mu}$ and $x^*_1 = \frac{\beta c^*}{\mu^*}$ denote the consumption quantities in DM currency transactions for Home and Foreign buyers, and let $x_2 = \beta d$ and $x^*_2 = \beta d^*$ denote the corresponding consumption quantities in DM non-currency transactions.

In equilibrium, asset markets clear, so that the sum of the demands for each asset is equal to the corresponding supply. That is,

$$\rho c = \bar{c}; \quad b_{hi} + b_{fi} + a_i = \bar{b}_i; \quad \rho c^* = \bar{c}^*; \quad b^*_{hi} + b^*_{fi} + a^*_i = \bar{b}^*_i, \quad (2.23)$$

for $i = s, l$. Then, an equilibrium can be defined as follows.

Definition *Given fiscal policies* (V, V_s, V_l) *and* (V^*, V^*_s, V^*_l) *and monetary policies* $(z_s, \omega_l, \kappa^*_s, \kappa^*_l)$ *and* $(z^*_s, \omega^*_l, \kappa_s, \kappa_l)$*, a stationary equilibrium consists of DM consumption quantities* (x_k, x^*_k) *for* $k = 1, 2$*, asset quantities* $(c, d, b_i, b_{ji}, c^*, d^*, b^*_i, b^*_{ji})$ *for* $i = s, l$ *and* $j = h, f$*, prices of long-term government bonds* (z_l, z^*_l)*, gross inflation rates* (μ, μ^*)*, and nominal depreciation rate of Home currency* $\frac{e_{+1}}{e}$*, satisfying (2.5)-(2.23).*

2.3.2 Equilibrium with Plentiful Collateral

What happens if fiscal authorities choose a sufficiently large V and V^* so that collateralizable assets are collectively plentiful in equilibrium? The following proposition characterizes such an equilibrium.

Proposition 1 *If the sum of* V *and* V^* *is sufficiently large, then there exists a stationary equilibrium where collateral constraints do not bind and no international trades take place. In this equilibrium, quantities of DM consumption are given by* $x_1 = (u')^{-1}[1/z_s]$, $x^*_1 = (u')^{-1}[1/z^*_s]$ *and* $x_2 = x^*_2 = \hat{x}$*. Gross inflation rates are* $\mu = \frac{\beta}{z_s}$ *and* $\mu^* = \frac{\beta}{z^*_s}$*, prices of long-term government bonds are* $z_l = \frac{\beta}{\mu-\beta}$ *and* $z^*_l = \frac{\beta}{\mu^*-\beta}$*, and the nominal depreciation rate of Home currency is given by* $\frac{e_{+1}}{e} = \frac{z^*_s}{z_s}$*.*

Proposition 1 shows that collateral constraints do not bind and no international trades take place in equilibrium if the supply of collateralizable assets is sufficiently high. It also provides equilibrium prices and allocations in such an equilibrium. The first thing to note is that there is no inefficiency in DM non-currency transactions because buyers can consume the first-best quantity of consumption good $\hat{x}$. Also, a change in total value of consolidated government debt outstanding (a change in V and V^*) has no effect on equilibrium prices and allocations.

Conventional monetary policies matter because they affect the quantities of consumption in DM currency transactions, as is standard in monetary models. For example, suppose that the Home central bank decreases z_s. This policy is equivalent to an increase in the short-term nominal interest rate on Home bonds R_s because $1 + R_s = \frac{1}{z_s}$. As the Home inflation rate can be expressed as $\mu = \beta(1 + R_s)$, an increase in R_s increases μ one-for-one (a pure *Fisher effect*). Then, the quantity of consumption in DM currency transactions x_1 falls as a higher μ leads to a decrease in the real quantity of Home currency held by the public ρc. In contrast, other types of monetary policies are irrelevant to equilibrium prices and allocations, if conventional policy variables z_s and z_s^* remain constant. This is because, in equilibrium, central banks can only change the composition of collateralizable assets—local and foreign government bonds with short and long maturity—held by the public, but the composition does not matter if collateralizable assets are plentiful in financial markets.[23]

In what follows, I will show how monetary policies work differently in an equilibrium with binding collateral constraints. Also, welfare measures for Home and Foreign countries can be defined as

$$W = \rho[u(x_1) - x_1 u'(x_1)] + (1-\rho)[u(x_2) - x_2 u'(x_2)] + \bar{X} - \bar{X}^*,$$
$$W^* = \rho[u(x_1^*) - x_1^* u'(x_1^*)] + (1-\rho)[u(x_2^*) - x_2^* u'(x_2^*)] - \bar{X} + \bar{X}^*,$$

where $\bar{X}$ is the quantity of CM goods produced by Foreign agents and consumed by Home agents and $\bar{X}^*$ is the quantity of CM goods produced by Home agents and consumed by Foreign agents. As is standard in monetary economics literature, each welfare measure is the sum of expected utilities across domestic agents in each country. In the CM, the sum of domestic agents' utilities from consumption may differ from the sum of their disutilities from production. If there are net imports in equilibrium, then the sum of utilities exceeds that of disutilities in the CM as domestic agents consume more than they produce. In contrast, if there are net exports, domestic agents produce more than they consume, so the sum of disutilities exceeds that of utilities in the CM. I will discuss the welfare implications of monetary policies using these measures.[24]

[23] A central bank's purchases of long-term local government bonds only result in swaps between short-term government bonds and long-term ones in equilibrium. This change in portfolio held by the public is irrelevant, similar to Wallace (1981). Analogously, a central bank's foreign asset purchases with a sterilized intervention only change the relative supplies of local and foreign currency-denominated bonds in equilibrium. So, sterilized interventions are irrelevant, in line with Backus and Kehoe (1989).

[24] Note that there is no transitional dynamics in this model, as in standard models based on Lagos and Wright (2005)

2.4 Equilibrium with Scarce Collateral

In this section, I consider cases where the total value of consolidated government liabilities in two countries (or the sum of V and V^*) is inefficiently small, so that collateral constraints bind in equilibrium. Also, for simplicity, I assume that central banks cannot hold long-term bonds as foreign exchange reserves, i.e., $\kappa_l = \kappa_l^* = 0$.[25] There are cases where local currency-denominated assets are held only by local agents, implying the absence of international capital flows. In other cases, private banks in one country acquire both local and foreign government bonds in equilibrium. This happens because, if the supply of collateralizable assets in a country is sufficiently low relative to the collateral supply in the foreign country, private banks in the *high-asset-scarcity* country are willing to pay a high price for foreign currency-denominated assets. Eventually, assets migrate to a country where they are valued highest as collateral.

As I will show later, binding collateral constraints imply a higher asset price than its fundamental price (the price determined by fundamentals such as the payoff structure and the preferences of agents). In this model, an asset becomes "scarce" if the supply of the asset is lower than the demand at its fundamental price due to binding collateral constraints. Since the degrees of asset scarcity can be different across countries, there arise a number of different cases depending on the relative degree of asset scarcity. For simplicity, I will confine attention to cases where the asset scarcity is more severe in the Home country. The degrees of asset scarcity can be measured by the multipliers to collateral constraints, λ and λ^*, which can be expressed as, from (2.10) and (2.17),

$$\lambda = \beta \left[u'(x_2) - 1 \right],$$
$$\lambda^* = \beta \left[u'(x_2^*) - 1 \right].$$

Then, a higher degree of asset scarcity in the Home country implies that $\lambda \geq \lambda^*$. This restriction also implies that Foreign banks do not purchase Home bonds while Home banks may be willing to purchase Foreign bonds.

With $\lambda \geq \lambda^*$, there are four remaining cases to study. If λ is not significantly high compared to λ^*, Home banks do not acquire Foreign bonds at prices Foreign banks are willing to pay. Similarly, Foreign banks do not purchase Home bonds at prices Home banks are willing to pay. However, as λ rises, Home banks would be willing to pay a higher price to acquire a Foreign bond. In particular, if λ becomes sufficiently high but not too high, then Home banks purchase short-term Foreign

and Rocheteau and Wright (2005). Due to quasi-linear preferences and unconstrained labor supplies, buyers and banks choose their asset portfolios in the CM (control variables) independent of their asset holdings they brought from the previous period (state variables). Relaxing one of those two assumptions would permit studying the out-of-steady-state dynamics, but that is beyond the scope of this paper.

[25] The implications of central banks' purchases of long-term foreign government bonds are discussed in Appendix.

bonds as well as Home bonds of all maturities. If Home banks and Foreign banks are willing to acquire a short-term Foreign bond at the same price, then short-term Foreign bonds are held in both countries in equilibrium. If λ rises further and Home banks are willing to pay a higher price for a short-term Foreign bond than Foreign banks, then short-term Foreign bonds are held only by Home banks in equilibrium. Finally, if λ becomes very high, then Home banks acquire all types of government bonds issued in two countries, while Foreign banks hold only long-term Foreign bonds. The relation between the degrees of asset scarcity and private banks' bond portfolios is summarized in Figure 2.2. I will focus on analyzing the first two cases in the following sections, and present the other two cases in Appendix.

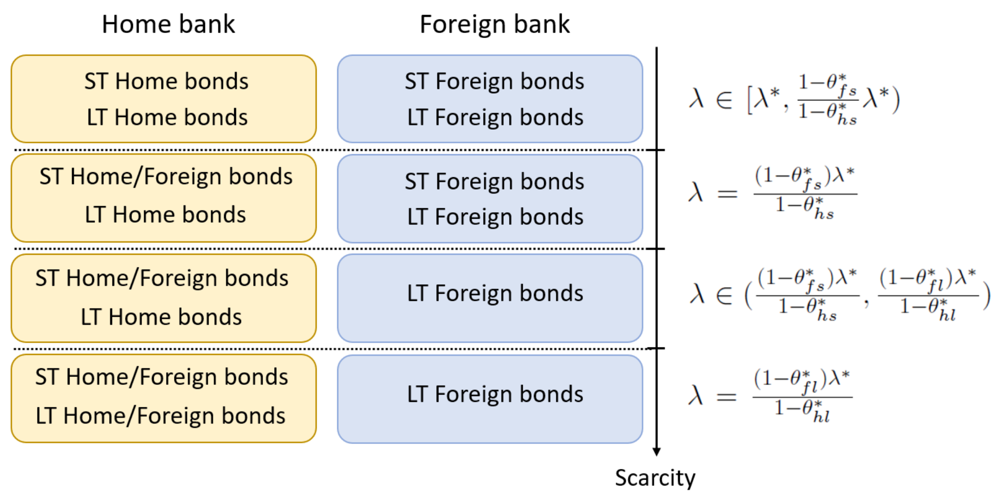

Figure 2.2: Asset scarcity and private banks' bond portfolios

Notes: ST - short-term; LT - long-term

2.4.1 Bond Yields and Term Premia

For convenience, I will restrict attention to Home bonds in describing the nominal/real yields and the corresponding liquidity premia. Foreign counterparts are analogously determined in equilibrium and described in Appendix. Noting that (2.11) and (2.13) hold with equality, the nominal bond yield of each maturity can be expressed as

$$R_j = \frac{\mu}{\beta\left[\left(1-\theta_{hj}\right)u'\left(x_2\right)+\theta_{hj}\right]} - 1, \tag{2.24}$$

for $j = s, l$. A term premium can be defined as the difference between long-term and short-term bond yields. So, the nominal term premium can be expressed as

$$R_l - R_s = \frac{\mu(\theta_{hl} - \theta_{hs})[u'(x_2) - 1]}{\beta[(1-\theta_{hl})u'(x_2) + \theta_{hl}][(1-\theta_{hs})u'(x_2) + \theta_{hs}]}, \tag{2.25}$$

The nominal term premium is strictly positive because long-term bonds receive larger haircuts and the quantity of consumption in DM non-currency transactions is inefficiently low. That long-term bonds receive larger haircuts (or equivalently, are less pledgeable) than short-term bonds is represented by $\theta_{hl} > \theta_{hs}$. Also, a collectively low supply of collateral and binding collateral constraints create inefficiencies in DM non-currency transactions, measured by $u'(x_2) > 1$. That is, the scarcity of collateralizable assets leads to a low quantity of goods exchanged in DM non-currency transactions, compared to the first best quantity $\hat{x}$.

In this model, the liquidity premium on a particular asset can be defined as the difference between the fundamental yield and the actual yield on the asset. The fundamental yield on each government bond can be determined by the payoff structure of the bond and the preferences of agents who acquire the bond in equilibrium. As quasi-linear preferences effectively make private agents risk-neutral with regard to payoffs on assets, the fundamental yield on Home bonds is given by $\frac{\mu}{\beta} - 1$. Therefore, the liquidity premium for Home bonds of each maturity can be expressed as

$$L_j = \frac{\mu}{\beta} - 1 - R_j = \frac{\mu\left(1-\theta_{hj}\right)[u'(x_2) - 1]}{\beta\left[\left(1-\theta_{hj}\right)u'(x_2) + \theta_{hj}\right]},$$

for $j = s, l$. Note that the liquidity premium increases with the associated pledgeability. For example, the liquidity premium for short-term bonds is higher than the premium for long-term bonds as $1 - \theta_{hs} > 1 - \theta_{hl}$. So, in this model, a term premium arises from the difference in liquidity premia across bonds with different maturities.

From (2.24), real bond yields and the real term premium can be expressed as

$$r_j = \frac{1}{\beta\left[\left(1-\theta_{hj}\right)u'(x_2) + \theta_{hj}\right]} - 1, \tag{2.26}$$

$$r_l - r_s = \frac{(\theta_{hl} - \theta_{hs})[u'(x_2) - 1]}{\beta[(1-\theta_{hl})u'(x_2) + \theta_{hl}][(1-\theta_{hs})u'(x_2) + \theta_{hs}]}, \tag{2.27}$$

for $j = s, l$. Since the fundamental real bond yield is $\frac{1}{\beta} - 1$ for all types of government bonds, the real liquidity premium for Home bonds of each maturity can also be calculated as

$$l_j = \frac{1}{\beta} - 1 - r_j = \frac{\left(1-\theta_{hj}\right)\left[u'\left(x_2\right)-1\right]}{\beta\left[\left(1-\theta_{hj}\right)u'\left(x_2\right)+\theta_{hj}\right]},$$

for $j = s, l$.

2.4.2 Nominal Exchange Rate

As the law of one price holds in the tradeable sector (CM), the nominal depreciation rate of the Home currency between the current and the future period can be written as

$$\frac{e_{+1}}{e} = \frac{\mu}{\mu^*}. \tag{2.28}$$

So, the depreciation rate of the Home currency is solely determined by the ratio between inflation rates in two countries. However, it seems important to note that the model works exactly the same as in the standard Lucas Jr (1982) model of international asset pricing, in that the nominal exchange rate is determined by

$$\frac{e_{+1}}{e} = \frac{m^*}{m}\frac{\mu}{\mu^*},$$

where m and m^* denote the intertemporal marginal rates of substitution (IMRS) of the representative Home buyer and the Foreign buyer. In this model, the IMRS is equal to a constant β since the quasi-linear preferences serve to make buyers risk-neutral with regard to payoffs on assets in the CM.

From (2.24), (2.26), and (2.28), the ratio of the gross nominal yield on Home bonds relative to the respective yield on Foreign bonds can be expressed as

$$\frac{1+R_j}{1+R_j^*} = \frac{e_{+1}}{e}\cdot\frac{1+r_j}{1+r_j^*} = \frac{e_{+1}}{e}\cdot\frac{(1-\theta_{fj}^*)u'\left(x_2^*\right)+\theta_{fj}^*}{\left(1-\theta_{hj}\right)u'\left(x_2\right)+\theta_{hj}}, \tag{2.29}$$

for each maturity, $j = s, l$. Equation (2.29) shows that uncovered interest parity (UIP) condition does not hold in general due to the different real interest rates on government bonds. The difference in real interest rates arises in this model because frictions, characterized by different degrees of pledgeability and binding collateral constraints, generate different liquidity premia on different government bonds. This result is closely related to Lee and Jung (2020), which finds the role of differential liquidity premia that causes a deviation from the UIP.[26]

[26] Lee and Jung (2020) develop a two-country model based on Lagos and Wright (2005), similar to mine, but they focus on the different roles of government bonds in transactions (direct means of payment or collateralizable assets) and abstract from international capital flows and banking activities. See, also, Bianchi et al. (2021) which builds a

2.4.3 Equilibrium with Segmented Asset Markets

If the degree of asset scarcity in the Home country is not significantly high compared to the Foreign country, then private banks in each country do not acquire assets denominated in the foreign currency, i.e., $b_{hi}^{*} = b_{fi} = 0$ for $i = s, l$. This implies that there is no international asset trade between private agents, and that international asset markets are effectively segmented. Then, first-order conditions (2.11), (2.13), (2.19), and (2.21) hold with equality, while (2.12), (2.14), (2.18), and (2.20) do not. From (2.12) and (2.19), a necessary condition for this equilibrium to exist is

$$\lambda^* \leq \lambda < \frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*}, \tag{2.30}$$

which implies that the degree of asset scarcity in the Home country is not sufficiently high relative to the asset scarcity in the Foreign country.

Substituting in the Home bank's collateral constraint (2.15) using (2.5), (2.9)-(2.11), (2.13), and (2.23) noting that $b_{fj} = b_{hj}^* = 0$ for $j = s, l$, I can rewrite the collateral constraint as

$$0 = \left[u'(x_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho x_1 + \left[u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)x_2 - \left\{V + \kappa_s^* - \kappa_s - \frac{(\theta_{hl}-\theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(x_2)+\theta_{hl}\right]}\right\}. \tag{2.31}$$

The first two terms on the right-hand side of (2.31) represent the demands for collateral, derived from the quantities of DM consumption in a currency transaction and a non-currency transaction. The last negative term represents the supply of collateral, so equation (2.31) effectively states that there is no excess demand for collateral in equilibrium. From (2.9)-(2.11), and (2.13), the first-order conditions for the Home bank's problem can be rewritten as

$$z_s = \frac{u'(x_2) - \theta_{hs}u'(x_2) + \theta_{hs}}{u'(x_1) - \theta_{hs}u'(x_2) + \theta_{hs}}, \tag{2.32}$$

$$z_l = \frac{(1-\theta_{hl})\,u'(x_2) + \theta_{hl}}{u'(x_1) - \theta_{hs}u'(x_2) + \theta_{hs} - \left[(1-\theta_{hl})\,u'(x_2)+\theta_{hl}\right]}, \tag{2.33}$$

$$\mu = \beta\left[u'(x_1) - \theta_{hs}u'(x_2) + \theta_{hs}\right]. \tag{2.34}$$

Similarly, using (2.6), (2.16), (2.17), (2.19), (2.21), and (2.23), the Foreign bank's collateral

two-country model to provide a liquidity-based explanation for the exchange rate dynamics.

constraint (2.22) can be rewritten as

$$0 = \left[u'(x_1^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right]\rho x_1^* + \left[u'(x_2^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right](1-\rho)x_2^* - \left\{V^* + \kappa_s - \kappa_s^* - \frac{(\theta_{fl}^* - \theta_{fs}^*)\omega_l^*}{(1-\theta_{fs}^*)[(1-\theta_{fl}^*)u'\left(x_2^*\right) + \theta_{fl}^*]}\right\}. \quad (2.35)$$

From (2.16), (2.17), (2.19), and (2.21), the first-order conditions for the Foreign bank's problem can be rewritten as

$$z_s^* = \frac{u'\left(x_2^*\right) - \theta_{fs}^* u'\left(x_2^*\right) + \theta_{fs}^*}{u'\left(x_1^*\right) - \theta_{fs}^* u'\left(x_2^*\right) + \theta_{fs}^*}, \quad (2.36)$$

$$z_l^* = \frac{(1-\theta_{fl}^*)u'\left(x_2^*\right) + \theta_{fl}^*}{u'\left(x_1^*\right) - \theta_{fs}^* u'\left(x_2^*\right) + \theta_{fs}^* - [(1-\theta_{fl}^*)u'\left(x_2^*\right) + \theta_{fl}^*]}, \quad (2.37)$$

$$\mu^* = \beta\left[u'(x_1^*) - \theta_{fs}^* u'(x_2^*) + \theta_{fs}^*\right]. \quad (2.38)$$

Note that, in addition to condition (2.30), unconventional monetary policy variables $(\omega_l, \kappa_s^*, \omega_l^*, \kappa_s)$ must satisfy conditions (2.7) and (2.8) for this equilibrium to exist. Another necessary condition is that the nominal interest rates on government bonds must be nonnegative in equilibrium. Since currencies always yield zero nominal interests and there is no friction that prevents arbitrage, a negative nominal interest rate on government bonds cannot be supported in equilibrium. This implies that central banks cannot choose short-term nominal interest rates lower than zero, i.e., $z_s \le 1$ and $z_s^* \le 1$.

Proposition 2 *There exists a nonempty set of parameter values that support a stationary equilibrium with binding collateral constraints that can be characterized by equations (2.31)-(2.38).*

Solving the model is quite straightforward. First, equations (2.31)-(2.32) solve for the quantities of Home DM consumption x_1 and x_2 given fiscal/monetary policies $(V, z_s, \omega_l, \kappa_s^*, \kappa_s)$. Then, (2.33) solves for the price of long-term Home bonds z_l, and (2.34) solves for the Home inflation rate μ. Similarly, equations (2.35)-(2.36) solve for x_1^* and x_2^* given fiscal/monetary policies $(V^*, z_s^*, \omega_l^*, \kappa_s, \kappa_s^*)$. Then, (2.37) solves for the price of long-term Foreign bonds z_l^*, (2.38) solves for the Foreign inflation rate μ^*, and (2.28) solves for the nominal depreciation rate of the Home currency $\frac{e_{+1}}{e}$. Lastly, conditions (2.7)-(2.8) put upper and lower bounds on x_1, x_2, x_1^*, and x_2^* noting that, from (2.9) and (2.16), $\bar{c} = \rho x_1\left[u'(x_1) - \theta_{hs}u'(x_2) + \theta_{hs}\right]$ and $\bar{c}^* = \rho x_1^*[u'\left(x_1^*\right) - \theta_{fs}^* u'\left(x_2^*\right) + \theta_{fs}^*]$.

2.4.3.1 Conventional Monetary Policy

In this section, I examine how conventional monetary policy works in an equilibrium where international asset markets are effectively segmented. Recall that I refer to the Home central bank's conventional monetary policy as setting z_s, the price of short-term Home bonds. Suppose that the Home central bank decreases z_s (or, equivalently, increases the short-term nominal interest rate R_s), with the value of long-term Home bonds outstanding ω_l and the value of short-term Foreign bonds acquired by the central bank κ_s^* held constant.[27]

Proposition 3 *Suppose that there is a decrease in z_s in an equilibrium characterized by (2.31)-(2.38). Then, x_1 decreases and x_2 increases. Also, μ, R_l, r_s, r_l, and $\frac{e_{+1}}{e}$ increase while $r_l - r_s$ and W fall. But, the effect on $R_l - R_s$ is ambiguous. Other variables remain unchanged.*

The effects of conventional monetary policy on the DM consumption quantities and asset prices in the Home country are consistent with the properties of the closed economy model in Williamson (2016), though these findings hold true only when the degrees of asset scarcity are not significantly different across countries in equilibrium. Specifically, as z_s decreases, the quantity of consumption in DM currency transactions x_1 falls and the quantity of consumption in DM non-currency transactions x_2 rises in equilibrium. This occurs because a decrease in z_s (or an increase in the short-term nominal interest rate R_s) is achieved by the Home central bank's open market sale of short-term Home bonds. Then, less currency outstanding, in real terms, leads to a lower consumption in currency transactions x_1. However, the quantity of Home bonds that can be used as collateral increases in the private sector, so this monetary intervention effectively relaxes the collateral constraints of Home banks, and increases consumption in non-currency transactions x_2.

The relaxation of the collateral constraints reduces the liquidity premium on Home bonds and increases the real interest rates r_s and r_l.[28] But, the increase in the real interest rate on Home bonds of each maturity does not outweigh the increase in the corresponding nominal interest rate, so the inflation rate μ rises. The nominal interest rate on long-term Home bonds R_l rises. However, the effect on the nominal term premium is ambiguous, as a rise in the inflation rate acts to increase the nominal term premium but the real term premium decreases. These results are summarized in Table 2.2.

[27]Note that monetary and fiscal policy variables are exogenously given in this model. In what follows, I discuss the effect of a certain monetary policy intervention based on a comparative statics analysis. That is, I study the effects of a marginal change in a policy variable with all other policy variables held constant.

[28]Note that these effects are permanent. Although permanent liquidity effects of monetary policy are prevalent in money-search literature, in some models including Lucas Jr (1990) liquidity effects are temporary.

x_1	x_2	μ	R_s	R_l	$R_l - R_s$	r_s	r_l	$r_l - r_s$	W	
↓	↑	↑	↑	↑	?	↑	↑	↓	↓	
x_1^*	x_2^*	μ^*	R_s^*	R_l^*	$R_l^* - R_s^*$	r_s^*	r_l^*	$r_l^* - r_s^*$	W^*	$\frac{e_{+1}}{e}$
·	·	·	·	·	·	·	·	·	·	↑

Table 2.2: Effects of an increase in R_s

The markets for Foreign currency-denominated assets are completely insulated from what happens in the markets for Home currency-denominated assets. In other words, asset markets are completely segmented depending on which currency the assets are denominated in. However, the rise in the short-term nominal interest rate R_s increases the expected depreciation of Home currency $\frac{e_{+1}}{e}$ in equilibrium. This finding is in line with the UIP relation, in that an increase in a nominal interest rate would appreciate the currency in the current period accompanied by an expected depreciation in the future. The UIP's prediction holds true since, from (2.29), real interest rates rise by less than do nominal interest rates, leading to an expected depreciation. In other words, there is a liquidity effect on real interest rates, but this liquidity effect (the rise in real interest rates) is not larger than the Fisher effect, so the currency is expected to depreciate.

2.4.3.2 Unconventional Monetary Policy

Unconventional monetary policy I study here is also called quantitative easing (QE) or tightening. In this model, the Home central bank implements QE by choosing the value of long-term Home bonds held by the public ω_l. Suppose that the Home central bank decreases ω_l with other policy variables, z_s and κ_s^*, held constant. This policy is effectively QE since the value of long-term Home bonds acquired by the Home central bank, $V_l - \omega_l$, increases.

Proposition 4 *Suppose that there is a decrease in ω_l in an equilibrium characterized by equations (2.31)-(2.38). Then, x_1, x_2, r_s, r_l, and W increase while μ, R_l, $R_l - R_s$, $r_l - r_s$, and $\frac{e_{+1}}{e}$ decrease. Other variables remain unchanged.*

As in the previous policy experiment, the effects of unconventional monetary policy on the Home country coincide with the findings of Williamson (2016). In particular, the quantities of consumption in Home DM transactions x_1 and x_2 rise in equilibrium. Initially, the Home central bank increases its holdings of long-term Home bonds $V_l - \omega_l$ through its swaps of Home currency for long-term Home bonds. However, open market purchases in general tend to decrease nominal interest rates. To maintain the short-term nominal interest rate R_s at the target level, the Home central bank must conduct open market sales of short-term Home bonds. As a result, the Home central bank effectively swaps short-term Home bonds and Home currency for long-term Home bonds in equilibrium. With a higher real quantity of Home currency outstanding, the quantity of consumption in DM currency transactions x_1 increases. Also, this unconventional intervention increases the effective stock of collateral in the private sector, as short-term bonds are better collateral than long-term bonds ($\theta_{hs} < \theta_{hl}$). Therefore, the quantity of consumption in DM non-currency transactions x_2 increases.

A larger stock of collateral held by Home banks relaxes their collateral constraints, leading to a reduction in liquidity premia and an increase in the real interest rates r_s and r_l. With no change in the short-term nominal interest rate R_s, an increase in the short-term real interest rate r_s implies a lower inflation μ in equilibrium. Although there are no effects on the prices of Foreign currency-denominated assets, a decrease in ω_l leads to a lower expected depreciation $\frac{e_{+1}}{e}$. This finding is also consistent with what the UIP predicts because a fall in the long-term nominal interest rate, caused by the central bank's purchase of long-term bonds, leads to a depreciation in the current period and an expected appreciation in the future. I summarize these results in Table 2.3.

x_1	x_2	μ	R_s	R_l	$R_l - R_s$	r_s	r_l	$r_l - r_s$	W	
↑	↑	↓	·	↓	↓	↑	↑	↓	↑	
x_1^*	x_2^*	μ^*	R_s^*	R_l^*	$R_l^* - R_s^*$	r_s^*	r_l^*	$r_l^* - r_s^*$	W^*	$\frac{e_{+1}}{e}$
·	·	·	·	·	·	·	·	·	·	↓

Table 2.3: Effects of a decrease in ω_l

Note that the Home central bank's conventional and unconventional monetary interventions do not affect the Foreign inflation rate or the nominal/real interest rates on Foreign bonds (no spillover effect). This is because international asset markets are completely segmented in that private banks participate only in local currency-denominated asset markets at their own choices. However, these policy interventions do affect the nominal depreciation rate (exchange rate effect) since there are no frictions in goods markets that prevent the law of one price.

2.4.3.3 Foreign Exchange Reserve Policy

Another interesting policy experiment that is available in this model is a central bank's foreign exchange reserve policy. Foreign asset purchases involve a central bank's swaps of local currency-denominated assets for foreign currency-denominated assets, and therefore, have global impacts if collateralizable assets are scarce. In an equilibrium with completely segmented asset markets across countries, a central bank's foreign asset purchases serve to increase the stock of collateral in the home country while reducing the stock of collateral in the foreign country.

Suppose that the Home central bank increases its holdings of short-term Foreign bonds κ_s^*, holding other policy variables z_s and ω_l constant. The following proposition presents the effects of such a policy.

Proposition 5 *Suppose that there is an increase in κ_s^* in an equilibrium characterized by equations (2.31)-(2.38). Then, x_1, x_2, and W increase while x_1^*, x_2^*, and W^* decrease. Also, r_s, r_l, μ^*, R_l^*, $R_l^* - R_s^*$, and $r_l^* - r_s^*$ increase while μ, R_l, $R_l - R_s$, $r_l - r_s$, r_s^*, r_l^*, and $\frac{e_{+1}}{e}$ decrease.*

Initially, the central bank needs to increase κ_s^* through its swap of Home currency for short-term Foreign bonds. Then, the increase in the real quantity of currency outstanding tends to increase x_1 proportionally, while x_2 remains constant. However, from (2.24) and (2.34), this tends to reduce the inflation rate μ and the short-term nominal interest rate R_s. So, the central bank must sell its holdings of short-term Home bonds to hold its policy rate R_s constant. As a result, the real quantities of Home currency and short-term Home bonds held by Home banks both increase, leading to an increase in x_1 and x_2. Also, the increase in the stock of collateral held by Home banks relaxes their collateral constraints and increases the real interest rates on Home bonds r_s and r_l (a liquidity effect). However, the rise in the long-term real interest rate does not outweigh the fall in the inflation rate, so the long-term nominal interest rate R_l and the nominal term premium $R_l - R_s$ both fall.

It seems important to note that the Home central bank conducts open market operations to "sterilize" the potential impact of Foreign asset purchases on the short-term nominal interest rate R_s. However, open market operations cannot fully sterilize the impacts of this intervention on other asset prices. This occurs as the Home central bank's intervention changes the relation between the short-term nominal interest rate and the inflation rate. So, nominal and real interest rates other than the short-term nominal interest rate change eventually. As opposed to the irrelevance result in an equilibrium with plentiful collateral, here sterilized foreign exchange market interventions matter due to the frictions associated with the differential pledgeability of scarce collateralizable assets.

x_1	x_2	μ	R_s	R_l	$R_l - R_s$	r_s	r_l	$r_l - r_s$	W	
↑	↑	↓	·	↓	↓	↑	↑	↓	↑	
x_1^*	x_2^*	μ^*	R_s^*	R_l^*	$R_l^* - R_s^*$	r_s^*	r_l^*	$r_l^* - r_s^*$	W^*	$\frac{e_{+1}}{e}$
↓	↓	↑	·	↑	↑	↓	↓	↑	↓	↓

Table 2.4: Effects of an increase in κ_s^*

In contrast to what happens in the Home country, consumption in currency transactions x_1^* and consumption in non-currency transactions x_2^* fall in the Foreign country, implying the opposite effects on the Foreign asset markets to those observed in the Home country. Note that the Home central bank's purchases of short-term Foreign bonds tend to reduce the short-term nominal interest rate on Foreign bonds R_s^*. Thus, the Foreign central bank must conduct an open market sale of short-term Foreign bonds to hold its policy rate R_s^* constant. This leads to a decrease in the quantity of currency outstanding, followed by a reduction in x_1^*. But, since the quantity of short-term Foreign bonds supplied by the Foreign central bank is smaller than the quantity purchased by the Home central bank, the supply of collateral in the Foreign country decreases, leading to a lower consumption in non-currency transactions x_2^*. Also, real bond yields r_s^* and r_l^* fall for all maturities as the decrease in the supply of collateral acts to tighten Foreign banks' collateral constraints.

An increase in κ_s^* reduces the nominal depreciation rate of Home currency $\frac{e_{+1}}{e}$. This finding is consistent with the conventional view in that a central bank's foreign asset purchases lead to a current depreciation of the local currency, followed by an expected appreciation. The Home country experiences a net export in the current CM since the Home central bank exchanges Home currency for Foreign bonds, and its counterpart (a Foreign bank) trades Home currency for CM goods produced by Home agents in equilibrium.[29] Though trade deficits are irrelevant to welfare in the Foreign country, the Home central bank's intervention tightens the collateral constraints of Foreign banks, which eventually reduces the quantities of DM consumption for Foreign buyers x_1^* and x_2^*. So, there is a *beggar-thy-neighbor* effect because a higher welfare in the Home country comes with a lower welfare in the Foreign country. These results are summarized in Table 2.4.

2.4.4 Equilibrium with Integrated Asset Markets

In this section, I consider an equilibrium with integrated asset markets, in particular an equilibrium where short-term Foreign bonds are held by private banks in both countries. Although Home bonds are held only by Home banks, and long-term Foreign bonds are held only by Foreign banks, the prices of Home bonds are responsive to those of Foreign bonds and vice versa. So, international asset markets are effectively integrated in this equilibrium.

In this equilibrium, first-order conditions (2.11)-(2.13), (2.19), and (2.21) hold with equality. From (2.10), (2.12), (2.17), and (2.19), a necessary condition for this equilibrium to exist is given by

$$\lambda = \frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*},$$

or

$$(1-\theta_{hs}^*)\, u'(x_2) + \theta_{hs}^* = \left(1-\theta_{fs}^*\right) u'(x_2^*) + \theta_{fs}^*, \tag{2.39}$$

which states that the degrees of inefficiency in DM non-currency transactions of two countries,

[29]From the following CM, however, the Home country experiences a net import as long as $z_s^* < 1$. Note that the increase in export of the Home country is not due to the currency depreciation, and that production/consumption in the CM is irrelevant with respect to welfare.

measured by $u'(x_2)$ and $u'(x_2^*)$, are positively correlated. Recall that inefficiencies in DM non-currency transactions come from the collectively low supply of collateralizable assets. So, what matters in determining an equilibrium is the integrated collateral constraint faced by both Home and Foreign banks. From (2.5), (2.6), and (2.9)-(2.23), noting that $b_{fj} = b_{hl}^* = 0$ for $j = s, l$, the collateral constraints can be rewritten as the form

$$\mathcal{F}(x_k, x_k^*, \omega_l, \omega_l^*, \kappa_s, \kappa_s^*, V, V^*) = \mathcal{D}(x_k, x_k^*) - \mathcal{S}(x_2, x_2^*, V, V^*, \omega_l, \omega_l^*, \kappa_s^*, \kappa_s) = 0, \tag{2.40}$$

for $k = 1, 2$ where $\mathcal{D}$ denotes the aggregate demand for collateral and $\mathcal{S}$ denotes the aggregate supply, implying that the excess demand in aggregate must be zero in equilibrium. The aggregate demand for collateral is given by

$$\mathcal{D} = \left[u'(x_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho x_1 + \left[u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)x_2 + \left[u'(x_1^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right]\Omega\rho x_1^* + \left[u'(x_2^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right]\Omega(1-\rho)x_2^*,$$

and the aggregate supply of collateral is given by

$$\mathcal{S} = V + \Omega V^* + (1-\Omega)(\kappa_s^* - \kappa_s) - \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'\left(x_2\right) + \theta_{hl}\right]} - \frac{\Omega(\theta_{fl}^* - \theta_{fs}^*)\omega_l^*}{(1-\theta_{fs}^*)[(1-\theta_{fl}^*)u'\left(x_2^*\right) + \theta_{fl}^*]}, \tag{2.41}$$

where

$$\Omega = \frac{u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}}{u'(x_2) + \frac{\theta_{hs}^*}{1-\theta_{hs}^*}}.$$

Note that the factor Ω augmented to the demand and supply from the Foreign country is less than one because $\theta_{hs} < \theta_{hs}^*$, that is, short-term Home bonds are more pledgeable than short-term Foreign bonds, if held by Home banks.

Other first-order conditions for private banks' problems are equivalent to those in an equilibrium with segmented asset markets, and are given by (2.32)-(2.34) and (2.36)-(2.38). Also, unconventional monetary policy variables $(\omega_l, \kappa_s^*, \omega_l^*, \kappa_s)$ must satisfy conditions (2.7) and (2.8).

Proposition 6 *There exists a nonempty set of parameter values that support a stationary equilibrium with binding collateral constraints that can be characterized by equations (2.32)-(2.34) and (2.36)-(2.40). Further, the function $\mathcal{F}$ in (2.40) is strictly increasing in the first seven arguments* *and strictly decreasing in the last three arguments.*

The model can be solved as follows. First, equations (2.32), (2.36), (2.39), and (2.40) solve for the DM consumption quantities x_k and x_k^* for $k = 1, 2$ given monetary/fiscal policies $(V, V^*, z_s, z_s^*, \omega_l, \omega_l^*, \kappa_s^*, \kappa_s)$. Then, (2.33) and (2.37) solve for z_l and z_l^*, (2.34) and (2.38) solve for μ and μ^*, and then (2.28) solves for $\frac{e_{+1}}{e}$. For graphical illustration, confine attention to cases where $\theta_{hs}^* = \theta_{fs}^*$ so that, from (2.39), $x_2 = x_2^*$ in equilibrium. In Figure 2.3, the integrated collateral constraint (2.40) in equilibrium is depicted by the curves IC in (x_1, x_2) space and IC^* in (x_1^*, x_2^*) space. The curves $z_s = z$ and $z_s^* = z^*$ are determined by equations (2.32) and (2.36), respectively, if $z_s = z < 1$ and $z_s^* = z^* < 1$. Therefore, the solution for x_k and x_k^* for $k = 1, 2$ is uniquely determined by the intersection of the curves IC and $z^s = z$, and the intersection of the curves IC^* and $z_s^* = z^*$. Finally, conditions (2.7) and (2.8) put upper and lower bounds on the DM consumption quantities.

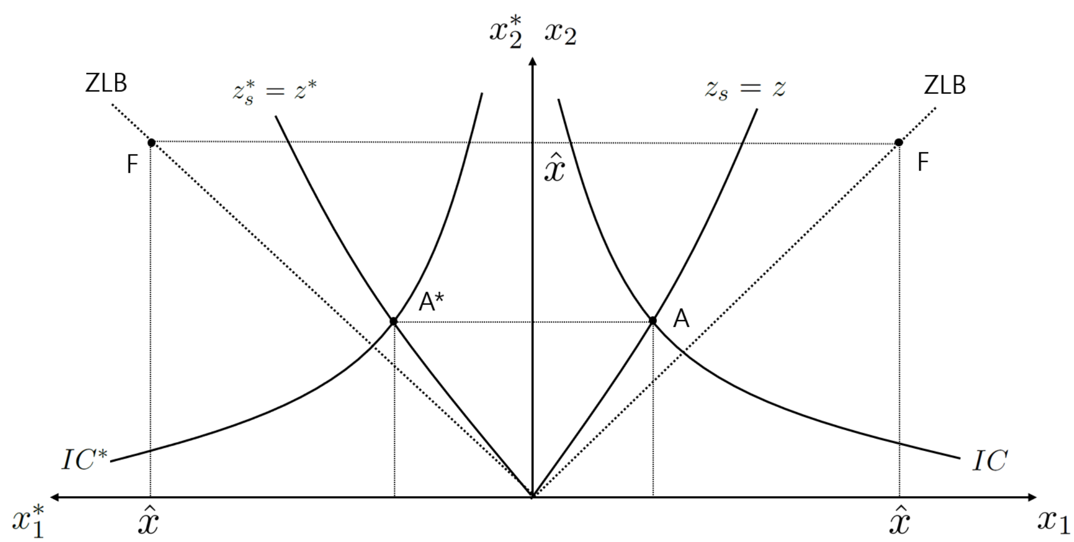

Figure 2.3: Equilibrium with short-term Foreign bonds held in both countries

As only Home banks purchase some Foreign bonds, there are international capital flows from the Home country to the Foreign country. In particular, Home banks receive deposits from Home buyers, and then exchange CM goods for b_{hs}^* units of short-term Foreign bonds, in real terms, at price $z_s^* \leq 1$. This implies that, from the Home country's perspective, there are net exports (a positive entry in the current account) and net capital outflows (a negative entry in the capital account) in the CM of period 0. In every following CM, Home banks receive b_{hs}^* units of Foreign currency from the Foreign fiscal authority, and then purchase the same quantity of short-term Foreign bonds at price $z_s^* \leq 1$. Therefore, there are net capital inflows with the value of $(1 - z_s^*)b_{hs}^*$, accompanied by net imports with the same value.

Proposition 7 *In an equilibrium characterized by (2.32)-(2.34) and (2.36)-(2.40), restricting international capital mobility can decrease W and increase W^*.*

Is the free movement of international capital mutually beneficial? Proposition 7 tells us that the answer is no if the degrees of asset scarcity are sufficiently different across countries. Consider a hypothetical equilibrium where international capital flows are strictly prohibited. Compared to this autarky equilibrium, the expected utility of Home buyers from consuming DM goods would be higher in an equilibrium with freely mobile international capital because Home banks can acquire a larger quantity of collateral and provide a larger quantity of deposit claims to buyers. Although Home agents would incur some disutility from producing exported CM goods, the higher expected utility would outweigh the higher disutility. Therefore, welfare in the Home country W would be higher in an equilibrium with international capital flows. However, for Foreign agents, the higher utility from consuming imported CM goods do not outweigh the lower expected utility from consuming DM goods. So, welfare in the Foreign country W^* would be lower with international capital flows. These effects arise because international asset markets effectively allow the Home country to export its asset scarcity to the Foreign country, as in Caballero et al. (2020).

2.4.4.1 Conventional Monetary Policy

In this section, I study the effects of the Home central bank's conventional monetary policy.[30] Specifically, suppose that the price of short-term Home bonds z_s decreases from z^0 to z^1 (or the short-term nominal interest rate R_s increases), with ω_l and κ_s^* held constant.

Proposition 8 *Suppose that there is a decrease in z_s in an equilibrium characterized by equations (2.32)-(2.34) and (2.36)-(2.40). Then, x_1 decreases and x_2 increases while x_1^* and x_2^* both increase. Also, μ, R_l, $R_l - R_s$, r_s, r_l, r_s^*, r_l^*, $\frac{e_{+1}}{e}$, and W^* increase while $r_l - r_s$, μ^*, R_l^*, $R_l^* - R_s^*$, $r_l^* - r_s^*$, and W decrease. But, the effect on $R_l - R_s$ is ambiguous.*

The effects of a decrease in z_s on the consumption quantities in DM transactions are depicted by Figure 2.4. From (2.36) and (2.40), the curves IC_0, IC_0^*, and $z_s^* = z^*$ remain unchanged with a decrease in z_s, but from (2.32) the curve $z_s = z^0$ shifts up to $z_s = z^1$. Thus, consumption in DM currency transactions x_1 falls and consumption in DM non-currency transactions x_2 rises (from point A to B) as in an equilibrium with segmented asset markets. However, this is accompanied by an increase in x_2^* and the shift of the curve IC^* from IC_0^* to IC_1^*. This occurs because the Home central bank can only increase R_s through its open market sale of short-term Home bonds, and this intervention tends to increase the supply of collateral in the global economy. But, points B

[30] Note that the effects of the Foreign central bank's conventional monetary intervention are qualitatively symmetric to those of the Home central bank's intervention.

and B^* do not appear to consist of an equilibrium because from (2.36) the increase in x_2^* tends to increase the short-term nominal interest rate on Foreign bonds R_s^*. To hold R_s^* at the target level, the Foreign central bank has to purchase short-term Foreign bonds (from point B^* to C^*), which shifts the curve IC_0 down to IC_1. This effect occurs because the open market purchase by the Foreign central bank reduces the quantity of collateral held in the global economy. Still, points C and C^* do not consist of an equilibrium as R_s tends to fall. So, the Home central bank must sell its holdings of short-term Home bonds more to control R_s at the target level, which again shifts up IC_1^*. This process continues until it reaches a new equilibrium at points E and E^*. Therefore, in the Home country, x_1 falls more and x_2 rises less than in an equilibrium with segmented asset markets (due to the effect from point B to E), and in the Foreign country, the consumption quantities in DM transactions x_1^* and x_2^* rise (from point A^* to E^*) due to the international spillover effects.

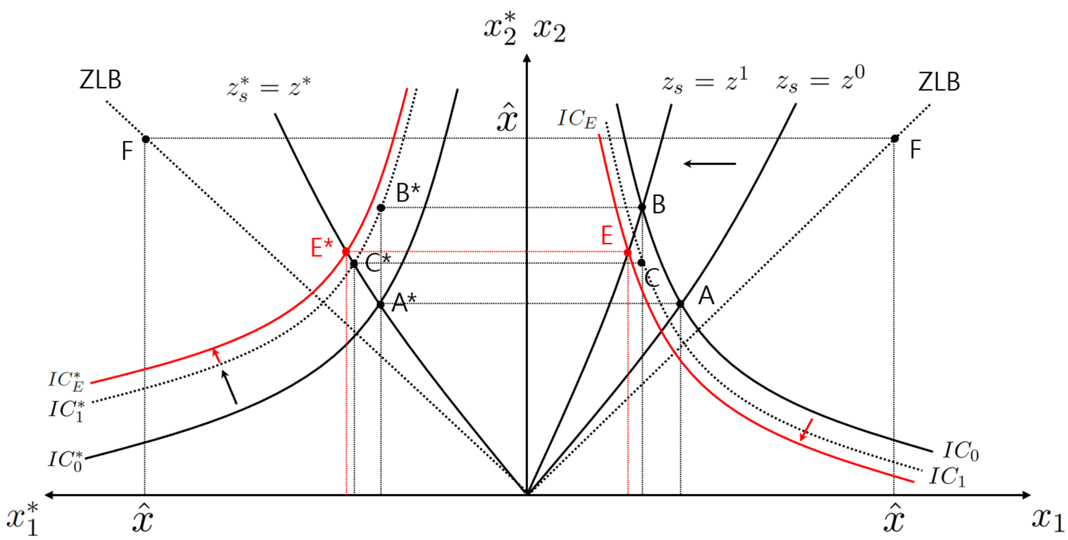

Figure 2.4: Conventional monetary policy: an increase in R_s

In this equilibrium, the effects of the conventional monetary intervention on prices of Home currency-denominated assets are qualitatively the same as those in an equilibrium with segmented asset markets. The Home inflation rate μ rises, the real interest rates on Home bonds r_s and r_l rise, and the real term premium $r_l - r_s$ falls. However, international spillovers cause a further increase in μ, as an increase in R_s requires a larger quantity of open market sales by the Home central bank. As there is a smaller quantity of Home currency outstanding compared to an equilibrium with segmented asset markets, the rate of return on currency must be lower (a higher μ). Also, international spillovers tend to decrease real interest rates r_s and r_l, because the Foreign central bank's conventional open market operation decreases the supply of short-term Foreign bonds, tightening the integrated collateral constraint. But, the decrease in r_s and r_l due to the Foreign central bank's conventional intervention does not outweigh their increase caused by the Home central bank's intervention. So, in equilibrium, r_s and r_l increase but to a smaller extent than in an equilibrium with segmented asset markets.

x_1	x_2	μ	R_s	R_l	$R_l - R_s$	r_s	r_l	$r_l - r_s$	W	
$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	?	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	
x_1^*	x_2^*	μ^*	R_s^*	R_l^*	$R_l^* - R_s^*$	r_s^*	r_l^*	$r_l^* - r_s^*$	W^*	$\frac{e_{+1}}{e}$
$\uparrow$	$\uparrow$	$\downarrow$	$\cdot$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$

Table 2.5: Effects of an increase in R_s

The Home central bank's conventional monetary intervention also affects asset prices in the Foreign country and the nominal exchange rate. The Foreign inflation rate μ^* falls, the real interest rates on Foreign bonds r_s^* and r_l^* rise, and the real term premium $r_l^* - r_s^*$ falls. The nominal interest rate on long-term Foreign bonds R_l^* and the nominal term premium $R_l^* - R_s^*$ both decrease and the depreciation rate of Home currency $\frac{e_{+1}}{e}$ rises. The effect on the nominal exchange rate is in line with the UIP's prediction in that an increase in R_s leads to a current appreciation of the Home currency, followed by an expected depreciation. However, the effect on R_l^* seems non-standard since capital outflows from the Foreign country tend to increase the nominal interest rate on long-term Foreign bonds. In this model, there is indeed upward pressure on the long-term nominal interest rate because of the increase in the short-term interest rate on Home bonds and resulting capital flows. But, there is also downward pressure due to the Foreign central bank's conventional open market operation, which increases the real quantity of Foreign currency outstanding and reduces the Foreign inflation rate. As the decrease in the Foreign inflation rate exceeds the increase in the long-term real interest rate, the long-term nominal interest rate decreases in equilibrium. These results are summarized in Table 2.5.

2.4.4.2 Unconventional Monetary Policy

In this section, I study the effects of the Home and Foreign central banks' unconventional monetary policies. First, consider a situation where the Home central bank implements quantitative easing (QE). What happens if the Home central bank decreases ω_l with z_s and κ_s^* held constant, so that the value of its holdings of long-term Home bonds $V_l - \omega_l$ increases?

Proposition 9 *Suppose that there is a decrease in ω_l in an equilibrium characterized by equations (2.32)-(2.34) and (2.36)-(2.40). Then, x_1, x_2, x_1^*, and x_2^* all increase. Also, μ, μ^*, R_l, R_l^*, $R_l - R_s$, $R_l^* - R_s^*$, $r_l - r_s$, $r_l^* - r_s^*$, and $\frac{e_{+1}}{e}$ decrease, while r_s, r_l, r_s^*, r_l^*, W, and W^* increase.*

The effects of a decrease in ω_l on the quantities of DM consumption are illustrated in Figure 2.5. From (2.32) and (2.36), the curves $z_s = z$ and $z_s^* = z^*$ remain fixed, but the curves that depict equation (2.40) shift up from IC_0 to IC_1 in (x_1, x_2) space, and from IC_0^* to IC_1^* in (x_1^*, x_2^*) space. As in an equilibrium with segmented asset markets, a decrease in ω_l acts to relax the collateral constraints of private banks because it involves the Home central bank's swaps of better collateral (short-term Home bonds) for worse collateral (long-term Home bonds) in equilibrium, leading to an increase in the effective stock of collateral held by the public. If the Foreign central bank did not intervene through its open market operation, x_1 and x_2 would increase from point A to B as in an equilibrium with segmented asset markets. However, there is upward pressure on the nominal interest rate on short-term Foreign bonds R_s^*, and hence, the Foreign central bank must swap Foreign currency for short-term Foreign bonds to hold its policy rate constant. The open market purchase by the Foreign central bank acts to tighten the collateral constraint, mitigating the original effects of a decrease in ω_l. In equilibrium, at points E and E^*, the quantities of DM consumption in the Home country x_1 and x_2 rise but less than they do in an equilibrium with segmented asset markets. The corresponding Foreign consumption quantities x_1^* and x_2^* also rise due to the international spillover effects.

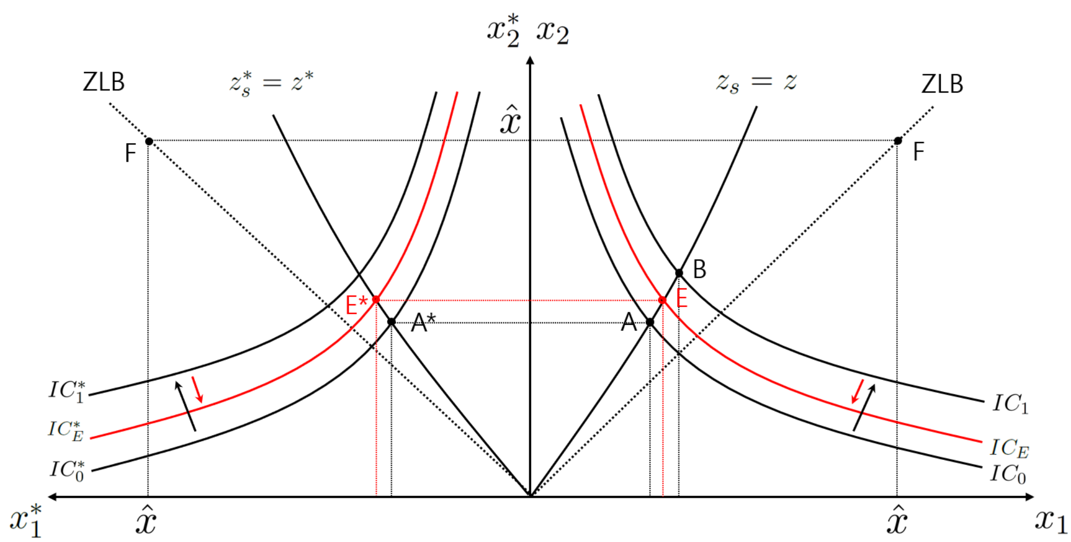

Figure 2.5: Quantitative easing or foreign asset purchases: an increase in $V_l - \omega_l$ or κ_s^*

In this equilibrium, the effects of a decrease in ω_l on the prices of Home currency-denominated assets are qualitatively the same as those in an equilibrium with segmented asset markets. The inflation rate μ falls, the real interest rates r_s and r_l rise, and the real term premium $r_l - r_s$ falls. Also, the long-term nominal interest rate R_l and the nominal term premium $R_l - R_s$ both decrease. However, the effects are quantitatively smaller than those in an equilibrium with segmented asset markets. This is because the Home central bank's unconventional intervention essentially leads to the Foreign central bank's purchases of short-term Foreign bonds, mitigating the original effects of a decrease in ω_l on the prices of Home currency-denominated assets.

The Home central bank's unconventional monetary intervention also changes the prices of Foreign currency-denominated assets due to the spillover effects. In particular, the prices of Foreign assets change to the same directions as do the prices of Home assets. That is, the real interest rates r_s^* and r_l^* rise, the long-term nominal interest rate R_l^* and the nominal term premium $R_l^* - R_s^*$ decrease, and therefore, the inflation rate μ^* falls. Further, the expected depreciation rate of Home currency $\frac{e_{+1}}{e}$ falls, implying a current depreciation accompanied by an expected appreciation. Though both central banks' open market purchases result in larger quantities of currencies outstanding, the Home central bank's open market purchases of long-term Home bonds (net of its sales of short-term Home bonds) have a larger effect on the Home inflation rate than the Foreign central bank's sales have on the Foreign inflation rate. As the Home inflation rate μ falls more than does the Foreign inflation rate μ^*, the depreciation rate $\frac{e_{+1}}{e}$ falls. I summarize these results in Table 2.6.

x_1	x_2	μ	R_s	R_l	$R_l - R_s$	r_s	r_l	$r_l - r_s$	W	
↑	↑	↓	·	↓	↓	↑	↑	↓	↑	
x_1^*	x_2^*	μ^*	R_s^*	R_l^*	$R_l^* - R_s^*$	r_s^*	r_l^*	$r_l^* - r_s^*$	W^*	$\frac{e_{+1}}{e}$
↑	↑	↓	·	↓	↓	↑	↑	↓	↑	↓

Table 2.6: Effects of an increase in $V_l - \omega_l$ or κ_s^*

Note that QE implemented by the Home central bank reduces the long-term nominal interest rates, flattens the yield curves internationally, and causes an immediate depreciation of Home currency. These results are consistent with Alpanda and Kabaca (2020) and Kolasa and Wesolowski (2020), who develop two-country DSGE models with some asset market segmentation using "preferred habitat" or "portfolio balance" theories to study the international effects of QE. However, as opposed to what occurs in their models, the real interest rates rise and the inflation rates fall in my model. An increase in real interest rates and an enhanced global welfare come from a relaxation of collateral constraints faced by Home and Foreign banks. This is in line with Dedola et al. (2013) in that QE mitigates the financial constraints of both domestic and foreign private banks, and therefore, has positive spillover effects.

A novel finding is that the effects of a Foreign central bank's unconventional monetary intervention are qualitatively symmetric to those of the Home central bank's intervention, except the effect on the nominal exchange rate. In particular, a decrease in ω_l^* (the Foreign central bank's QE) leads to a current appreciation in the Foreign currency. Since the degree of asset scarcity is higher in the Home country, in equilibrium, the Home central bank's open market purchases have a larger effect on the inflation rate than the Foreign central bank's purchases do on the associated inflation rate. Therefore, QE implemented by the Foreign central bank, or the relatively *low-asset-scarcity* central bank, leads to a current appreciation of the local currency followed by an expected depreciation. This finding is different from those in Alpanda and Kabaca (2020) and Kolasa and Wesolowski (2020) where QE always depreciates the local currency.

2.4.4.3 Foreign Exchange Reserve Policy

What are the effects of central banks' foreign asset purchases? Similar to the QE experiment, these interventions either relax or tighten the integrated collateral constraint. So, it is crucial to understand whether central banks' asset purchases increase the supply of collateralizable assets in financial markets. From (2.41) and Proposition 6, it is straightforward to obtain the following corollary.

Corollary 1 *The function $\mathcal{S}$ is strictly decreasing in $(\omega_l, \omega_l^*, \kappa_s)$ and strictly increasing in (κ_s^*, V, V^*).*

Corollary 1 states that both decreasing ω_l, ω_l^*, or κ_s and increasing κ_s^*, V, or V^* effectively increase the supply of collateral in the integrated asset market. Therefore, an increase in κ_s^* (the Home central bank's purchases of short-term Foreign bonds), a decrease in κ_s (the Foreign central bank's sales of short-term Home bonds), and a decrease in ω_l (the Home central bank's purchases of long-term Home bonds) have the same qualitative effects on equilibrium prices and allocations, provided that conventional policy rates R_s and R_s^* stay constant.

Consider the Home central bank's foreign exchange reserve policy. Suppose that there is an increase in κ_s^*, the value of short-term Foreign bonds held by the Home central bank. As in Figure 2.5, the curves that depict (2.40) shift up from IC_0 to IC_1 in (x_1, x_2) space, and from IC_0^* to IC_1^* in (x_1^*, x_2^*) space, with the curves $z_s = z$ and $z_s^* = z^*$ held constant. As opposed to an equilibrium with segmented asset markets where an increase in κ_s^* has a beggar-thy-neighbor effect, this intervention increases the effective supply of collateral in the global financial market. Also, the Foreign central bank must conduct open market operations to eliminate upward pressure on the short-term nominal interest rate on Foreign bonds, which shifts down IC_1 and IC_1^*. Identical to what happens in response to a decrease in ω_l, all consumption quantities in DM transactions increase and inflation rates fall. Long-term nominal interest rates fall and short-term and long-term real interest rates rise, so both nominal and real term premia fall. Finally, the depreciation rate of Home currency falls, implying its current depreciation followed by an expected appreciation.

In contrast, an increase in κ_s, the value of short-term Home bonds held by the Foreign central bank, leads to a decrease in the effective supply of collateral in the global economy, implying the opposite effects to those of an increase in κ_s^*. What makes the difference in the effects of foreign exchange (FX) intervention? It is important to note that the Home central bank's Foreign bond purchases involve its swaps of short-term Home bonds for short-term Foreign bonds in equilibrium. For Home banks, short-term Home bonds are better collateral than short-term Foreign bonds ($\theta_{hs} <$

θ^*_{hs}), and these two types of assets are substitutable. So, Home banks reduce their holdings of short-term Foreign bonds in equilibrium. This in turn increases the effective stock of collateral held by Foreign banks, which serves to relax the integrated collateral constraint. However, the Foreign central bank's Home bond purchases involve its swaps of short-term Foreign bonds for short-term Home bonds. As a result, a smaller quantity of Home bonds and a larger quantity of Foreign bonds are held in the private sector. Therefore, Home banks must acquire a larger quantity of short-term Foreign bonds, which tightens the integrated collateral constraint.

To properly evaluate the international effect of FX intervention, it seems important to assess the relative market value of assets, as collateral, that a central bank issues or sells in exchange for foreign assets. For example, at the end of the first quarter in 2021, the Swiss National Bank (SNB)'s foreign exchange reserves consist of government bonds (66%), other bonds (11%), and equities (23%). Among those fixed-income assets, AAA-rated asset holdings are 61%, AA-rated asset holdings are 20%, and assets rated below AA are 19%.[31] Considering that the credit rating for Swiss government bonds has been AAA, the SNB's swaps of local government liabilities for foreign assets are likely to mitigate the shortage of safe assets and global financial frictions.[32]

In contrast to the case of Switzerland, FX intervention in some emerging market and developing economies can exacerbate the global shortage of safe assets. This can happen if the government liabilities of those economies are considered less valuable as collateral in global financial markets than what they purchase, mostly liquid and safe assets. In this case, any policies that permit foreign central banks to hold reserve accounts at central banks in advanced economies (safe asset issuers) can mitigate the global shortage of safe assets. For example, the Fed's overnight reverse repurchase agreement facility and liquidity swap lines effectively allow foreign central banks to hold reserve accounts at the Fed or have options to withdraw US dollars. Those policies reduce foreign central banks' incentives to purchase US Treasury securities in financial markets, which serves to relax the collateral constraints of private banks.

2.5 Conclusion

I have constructed a two-country general equilibrium model with limited commitment, differential pledgeability of collateral, and a scarcity of collateralizable assets. Short-term government debt is more pledgeable as collateral than long-term government debt, leading to term premia. Also, in terms of financial intermediary asset portfolios, the home bias arises as local assets are more

[31]See Table for Investment Structure of Foreign Exchange Reserves and Swiss Franc Bond Investments at https://www.snb.ch/en/iabout/assets/id/assets_reserves.

[32]See Moody's Investors Service's 2020 rating outlook for Government of Switzerland bonds, which is available at https://www.moodys.com/research/Moodys-affirms-Switzerlands-Aaa-ratings-maintains-stable-outlook--PR_436243.

pledgeable than foreign assets.

Given a scarcity of collateralizable assets in interconnected financial markets, quantitative easing can reduce long-term bond yields and term premia and improve welfare internationally. As quantitative easing improves the quality of collateral held in the global financial market, it can serve to relax the collateral constraints of financial intermediaries, increasing real bond yields. Foreign exchange intervention can be beggar-thy-neighbor if financial markets are endogenously segmented in equilibrium. However, if financial markets are globally interconnected in equilibrium, foreign exchange intervention can improve global welfare in some cases. In general, a central bank's foreign asset purchases lead to swaps between local currency-denominated assets and foreign currency-denominated ones in equilibrium. Therefore, the intervention can increase the effective supply of collateral in financial markets if the former assets are considered more effective as collateral than the latter ones.

Although this paper sheds light on how domestic monetary policy affects global asset prices and welfare, some issues could be addressed in future research. For example, there are fundamental factors that could explain why long-term or foreign currency-denominated government bonds receive larger haircuts when pledged as collateral. To go deeper into modelling these factors is an important topic for future research.

Chapter 3

Money, Credit, and Financial Intermediation with Private Information and Costly Monitoring

3.1 Introduction

The main goal of this paper is to show how monetary and financial frictions affect the choice of means of payment in an economy where credit transactions between economic agents are intermediated by third-party financial intermediaries. I construct a model of money, credit, and financial intermediation in which a preference shock is private information for a depositor and is costly for intermediaries to monitor. Then, I characterize an equilibrium where there are roles for money and state-contingent deposit contracts, and analyze how monetary and financial frictions change the quantities of monetary and credit transactions.

One reason why intermediated credit arrangements are prevalent is related to the cost advantages of financial intermediaries in processing information. In the work of Diamond (1984) and Williamson (1986, 1987), financial intermediation arises in an economy with asymmetric information and costly monitoring. Since intermediaries achieve cost advantages due to portfolio diversification and asset transformation, depositors voluntarily delegate the task of monitoring information to intermediaries in those models. Another advantage of using intermediated credit is related to limited commitment and the higher credibility of intermediaries compared to anonymous agents. In particular, Gu et al. (2013) view financial intermediaries as more trustworthy agents who provide claims on deposits that can serve as a means of payment.

In this paper, the cost of using credit involves asymmetric information and costly monitoring, as in Diamond (1984) and Williamson (1986, 1987). Also, as in Gu et al. (2013), agents are subject to limited commitment, and financial intermediaries are more trustworthy than other agents. In particular, I assume that any direct credit arrangement between two agents is infeasible due to limited commitment, while any intermediated credit arrangement is feasible due to perfect enforceability by intermediaries. Then, I propose a model based on Lagos and Wright (2005) and Rocheteau and Wright (2005) incorporating preference shocks to depositors (*buyers*) and a costly monitoring technology to intermediaries. A preference shock, which is private information to each depositor, determines whether the depositor is willing to participate in a small transaction or a large transaction. Financial intermediaries are equipped with a monitoring technology that allows them to observe the shock at some cost.

As a benchmark, I characterize the equilibrium allocations in a pure credit economy, in which intermediaries write deposit contracts with buyers and credit contracts with retailers (*sellers*). The optimal deposit contract provides state-contingent claims on goods: a small quantity of goods for buyers with a low preference shock (*small-transaction buyers*) and a large quantity of goods for those with a high preference shock (*large-transaction buyers*). Following the spirit of Diamond and Dybvig (1983), the optimal contract serves an insurance role as buyers can be insured against the uncertainty about their preference shocks.[1] However, the quantity of claims each buyer receives depends on his or her report on the preference shock, which is costly to monitor. This asymmetric information and costly monitoring generate a standard incentive problem such that buyers with a low preference shock may claim to have a high preference shock. To make buyers report their shocks truthfully, intermediaries conduct monitoring with a positive probability when a buyer makes a report of a high shock. Consequently, the quantity of claims for large-transaction buyers is constrained while the quantity of claims for small-transaction buyers is larger than their first best quantity.

The optimal deposit contract, written by intermediaries, shares two important features with a conventional deposit contract and payment by debit card. First, like a conventional deposit contract, the deposit contract in the model gives the depositor one of two options. The depositor can either make a large transaction with no other claims left, or make a small transaction and have an unspent balance. The second practical feature is that a depositor who has a money balance at his or her bank account can make payments with a debit card, in which case the retailer does not immediately receive money from the issuing bank.[2] Due to this delayed settlement, the use of debit cards effectively involves a credit arrangement between the retailer and the bank. Thus, this model captures these sequential payment and settlement stages of transactions quite well, in

[1]The banking contract is similar to the one presented in Aiyagari and Williamson (2000), in that an individual receives a privately informed shock, and then makes a report to a financial intermediary on his or her state, following which the intermediary makes a transfer of consumption goods to the individual. The shock in their model is an individual's endowment, while in my model it is related to an individual's preferences.

[2]Instead, the retailer receives the money, usually within 24 hours.

that the deposit contract between the intermediary and the buyer turns into a credit arrangement between the intermediary and the seller at the time of transaction between the buyer and the seller.

Since the quantity of claims for buyers with a high shock is constrained, introducing fiat money can improve social welfare because buyers can trade money for additional goods in large transactions. Also, due to its portability and recognizability, money is assumed to be the only available means of payment in some small transactions. Then, in equilibrium, either money or credit is used in small transactions while money and credit are both used in large transactions. Intermediaries provide a deposit contract that plays an insurance role under two conditions: (i) the cost of monitoring is sufficiently low, and (ii) the inflation rate set by the government is sufficiently high. In this case, an increase in the inflation rate leads to a decrease in the use of money, an increase in the use of credit in large transactions, and a decrease in the use of credit in small transactions, along with a higher cost of monitoring. Therefore, money and credit are substitutes for buyers with a high shock but complements for those with a low shock.[3]

With regard to the effect of financial frictions on the economy, I show that there is an amplification effect of the monitoring cost on the quantity of credit for large transactions, which arises as follows. First, a decrease in the monitoring cost makes intermediaries increase the quantity of claims for large-transaction buyers, for any given quantity of money balances held by buyers. However, this change in the optimal deposit contract makes buyers adjust their asset portfolios. In particular, the representative buyer's expected utility from holding the previous quantity of money balances decreases at the margin. Therefore, buyers choose to reduce their money holdings at the optimum, which acts to further increase the quantity of claims for large-transaction buyers. Because of the change in buyers' money holdings, the increase in the use of credit in large transactions is amplified. This amplification effect helps in understanding how greater financial frictions result in a huge contraction of credit in a financial crisis.

This paper is related to literature that introduces financial intermediation in the framework of Lagos and Wright (2005) and Rocheteau and Wright (2005).[4] Among others, Williamson (2012) develops a model where trustworthy financial intermediaries write a deposit contract which gives each buyer one of two options. That is, the buyer can withdraw money with no other claims left on the bank, or have a tradeable claim if money is not withdrawn. Since the buyer faces a means-of-payment shock, i.e., credit is not accepted as a means of payment in some transactions, the deposit

[3] In most existing monetary models, money and credit are substitutes. However, the complementarity of money and credit is not new. For example, Freeman (1996), Ferraris (2010), and Dong and Huangfu (2021) find that money complements credit in an environment where money is the only means to settle credit. Li and Li (2013) and Ferraris and Mattesini (2020) also show that money and secured credit can be complements as higher inflation increases the cost of repaying debt.

[4] Examples in this literature include Berentsen et al. (2007), He et al. (2008), Gu et al. (2013), Li and Li (2013), Kang (2019), Altermatt and Wang (2022), Dong et al. (2021), Lee (2021), and Williamson (2012, 2016, 2018, 2019, 2022b).

contract efficiently allocates money to non-credit transactions and other assets to credit transactions. My model departs from Williamson (2012) by considering financial frictions that arise from asymmetric information and costly monitoring. Then, I show that the optimal deposit contract serves an insurance role for buyers against preference shocks and how a change in monetary or financial friction affects the trade quantities in small and large transactions.

This paper also falls within a strand of literature that studies the effect of monetary interventions in an economy where money and credit coexist.[5] Most models that have costs associated with credit arrangements assume a direct form of credit cost. For example, using credit is assumed to incur a fixed, proportional, or convex cost in Bethune et al. (2020) and Wang et al. (2020) and require the use of record-keeping technology with some fixed cost in Gomis-Porqueras and Sanches (2013) and Dong and Huangfu (2021). The only exception is Lotz and Zhang (2016) in that sellers can *ex ante* invest in a costly record-keeping technology, making the fraction of sellers who accept credit transactions endogenous. As opposed to those papers, the model presented in my paper assumes that the cost of using credit comes from information processing or information verification. Though the structure of asymmetric information and costly verification is standard in the banking literature, it has yet not been introduced into the money-credit literature and therefore can provide a novel approach to modelling money and credit.

The closest paper to mine is Dong and Huangfu (2021), in that money and credit are substitutes at low inflation rates and become complements at high inflation rates. Their model predicts an inverse U-shaped relationship between inflation and credit, which helps rationalize the difference in the inflation/credit relationships in countries with low inflation and those with high inflation. Similar to theirs, money and credit exhibit substitutability for low inflation rates and complementarity for high inflation rates in my model. However, this change in the relationship occurs only for buyers with a low preference shock, while money and credit are always substitutes for those with a high shock.

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 characterizes the optimal deposit contract in a pure credit economy. In Section 4, I introduce money into this economy and study the implications for the choice of payment means. Section 5 concludes.

[5]Recent papers in this literature include Aiyagari and Williamson (2000), Telyukova and Wright (2008), Sanches and Williamson (2010), Gomis-Porqueras and Sanches (2013), Gu et al. (2016), Lotz and Zhang (2016), Dong and Huangfu (2021), Araujo and Hu (2018), Bethune et al. (2020), and Wang et al. (2020).

3.2 Environment

The basic structure of the model is related to Lagos and Wright (2005) and Rocheteau and Wright (2005) incorporating the costly monitoring technology from Townsend (1979). Time is indexed by $t = 0, 1, 2, ...$, and each period is divided into two subperiods—the decentralized market (DM) followed by the centralized market (CM).

There are three types of economic agents, each with unit mass: buyers, sellers, and banks. Each buyer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\tilde{\theta}_t u(x_t) - H_t\right],$$

where $\beta \in (0, 1)$ is the discount factor, $\tilde{\theta}_t$ is the preference shock each buyer faces at the beginning of the DM, x_t is consumption during the DM, and H_t is labor supply in the CM. Assume that $u(\cdot)$ is a strictly increasing, strictly concave, and twice continuously differentiable function with $u'(0) = \infty$ and $u'(\infty) = 0$. The preference shock, $\tilde{\theta}_t$, can take one of two values, 1 and θ, with probabilities $1 - p$ and p, respectively, where $\theta > 1$.[6] Define x_l^* and x_h^* to be the solutions to $u'(x_l^*) = 1$ and $\theta u'(x_h^*) = 1$, respectively. Let buyers with $\tilde{\theta}_t = \theta$ be denoted as *large-transaction buyers*, and let buyers with $\tilde{\theta}_t = 1$ be denoted as *small-transaction buyers*. The realization of the preference shock is private information to a buyer. Each seller has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t (-h_t + X_t),$$

where h_t is labor supply in the DM and X_t is consumption in the CM. Each bank has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t (-\nu_t + X_t - H_t),$$

where ν_t is labor effort on monitoring in the DM, H_t is labor supply in the CM, and X_t is consumption in the CM.

One unit of labor supply produces one unit of perishable consumption good in both markets. Notice that buyers wish to consume but cannot produce in the DM, while they can produce but do not wish to consume in the CM. In contrast, sellers can produce but do not wish to consume in the DM while they wish to consume but cannot produce in the CM. Hence, there is a double coincidence problem. Banks can produce and consume in the CM, but cannot do so in the DM. A key assumption is that a bank can observe the realization of a buyer's preference shock with γ

[6] The preference shock is similar to those in the models of Bhattacharya et al. (2005) and Ennis (2008), in that the shock arrives to a buyer upon entering the DM and determines how much the buyer is willing to trade for consumption goods in that match.

units of labor effort on monitoring in the DM.

At the beginning of the CM, debts are paid off. Then, production and consumption take place and assets are traded in a centralized Walrasian market, and finally agents write contracts with banks. In the DM, there are random matches between buyers and sellers where each buyer is matched with a seller. In all DM matches, there is no memory or record keeping so that a matched buyer and seller have no knowledge of each others' histories including who these agents met and what was exchanged in those past meetings. Buyers and sellers are subject to limited commitment, and thus lack of memory implies that there can be no credit arrangement between the matched buyer and seller. If any seller were to make a loan to a buyer, the buyer would default. In contrast, I assume that there is a technology that allows record keeping of banks' actions, and that enforcement is feasible. That is, agents can force banks to repay liabilities. Therefore, the ability of banks to play an intermediary role in this economy not only comes from their monitoring technology but also from their ability to commit.[7]

3.3 A Pure Credit Economy

In this section, I describe the properties of an equilibrium in a pure credit economy. The pure credit economy is a useful benchmark to understand the role of banks. With no banking activity permitted in this economy, the agents would live in permanent autarky because there could not be any credit arrangements in the DM meetings. Banks can improve social welfare by providing deposit contracts to buyers. A deposit contract specifies the quantity of goods a buyer deposits at the bank in the current CM and the quantity of goods the buyer receives in the following CM. I show that the claims on deposits are state-contingent and can be used as a means of payment. That is, a large-transaction buyer receives a larger quantity of claims, while a small-transaction buyer receives a smaller quantity.[8] Therefore, the deposit contract serves an insurance role to buyers against their preference shocks, as in Diamond and Dybvig (1983).

3.3.1 Specification of a Contract and the Timeline

Let each buyer be indexed by $i \in [0, 1]$ and let each seller be indexed by $j \in [0, 1]$. Suppose that a bank writes a contract with buyer i in the CM of period $t-1$. The contract specifies the quantity

[7]Different levels of credibility across agents can explain who will act as banks in an economy, as shown in Gu et al. (2013).

[8]Although the quantity of unspent balances for a small-transaction buyer is assumed to be zero in the model, it does not harm the analysis as long as the quantity of unspent balances is constrained. If an intermediary can frictionlessly choose the quantity of the unspent balance, the intermediary can write a contract that provides the first best quantities for both types of transactions without incurring monitoring costs.

of consumption goods to be transferred from buyer i to the bank in the CM of period $t-1$, and the quantity of claims on goods that can be redeemed in the CM of period t. Notice that, in the CM of period $t-1$, the buyer and the bank do not know which preference shock the buyer will realize in the DM of period t. The quantity of claims on period-t CM goods is state-contingent. Since the shock remains private information to the buyer, the quantity is determined by the buyer's report of his or her preference shock.

At the beginning of the period-t DM, buyer i observes the preference shock and is matched with a seller, say seller j. The buyer makes a report to the bank, and the bank decides whether to monitor the buyer's state or not, based on its monitoring policy. I assume that monitoring is determined randomly. That is, there is a machine that gives the bank a lottery outcome given the specified probabilities and the bank makes a decision based on the outcome. I choose a stochastic monitoring policy, because it dominates the deterministic procedure as shown by Townsend (1979).

After the monitoring stage, the bank determines the buyer's holdings of claims and the buyer exchanges the claims for DM goods with the seller. So, the deposit contract between the buyer and the bank effectively turns into a credit arrangement between the bank and the seller. Finally, in the CM of period t, the bank produces consumption goods and transfers them to deposit holders to clear its obligations. The sequence of events is summarized in Figure 3.1.

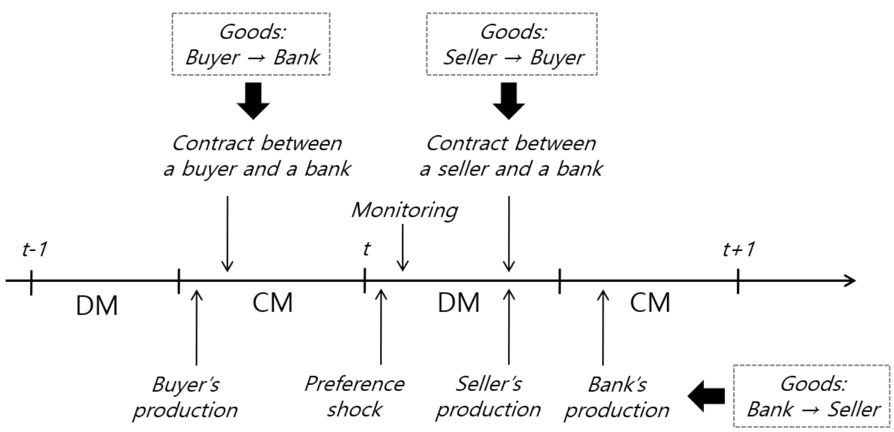

Figure 3.1: Sequence of Events

3.3.2 Equilibrium

I will confine attention to stationary equilibria. Let k denote the quantity of consumption goods that buyer i deposits at a bank in the current CM. Let $\omega \in \{l, h\}$ denote the actual state of the world in the following DM such that $\omega = l$ when $\tilde{\theta} = 1$, and $\omega = h$ when $\tilde{\theta} = \theta$. Let $\omega^s \in \{l, h\}$ denote the buyer's report of the preference shock. In addition, let x_ω denote the quantity of consumption goods to be transferred from the bank to the deposit holder in the following CM given that $\omega \in \{l, h\}$ is reported. I assume that the buyer receives no claims from the bank when ω^s is reported, but $\omega \neq \omega^s$ is detected through monitoring.[9] Finally, let π_ω denote the probability of monitoring when $\omega \in \{l, h\}$ is reported.

In equilibrium, a bank writes a deposit contract, $\{k, x_\omega, \pi_\omega\}_{\omega=l,h}$, with a buyer in the CM without knowing what preference shock the buyer realizes in the next period. At the beginning of the subsequent DM, the buyer observes the preference shock and truthfully reports the state to the bank. After the quantity of claims is determined through the monitoring stage, the buyer makes a take-it-or-leave-it offer to the seller using the claims as a means of payment.[10] Therefore, the bank chooses a contract $\{k, x_\omega, \pi_\omega\}_{\omega=l,h}$ that solves the following problem in the CM:

$$\max_{k,x_l,x_h,\pi_l,\pi_h} \left[-k + \beta\{(1-p)u(x_l) + p\theta u(x_h)\}\right] \tag{3.1}$$

subject to

$$k - \beta\{(1-p)(x_l + \pi_l\gamma) + p(x_h + \pi_h\gamma)\} \geq 0, \tag{3.2}$$
$$u(x_l) \geq (1-\pi_h)\, u(x_h), \tag{3.3}$$
$$\theta u(x_h) \geq (1-\pi_l)\, \theta u(x_l), \tag{3.4}$$
$$k, x_l, x_h \geq 0, \quad \pi_h, \pi_l \in [0,1]. \tag{3.5}$$

The maximization problem (3.1) states that the deposit contract must maximize the expected utility of the buyer subject to (3.2)-(3.5). Constraint (3.2) ensures that the bank's expected discounted payoff from writing the contract is nonnegative. The bank receives k units of goods from the buyer in the CM. Then, in the following DM, the bank conducts monitoring with probability π_ω, which costs γ units of labor effort. The buyer's take-it-or-leave-it offer implies that the buyer trades x_ω units of claims on CM goods for x_ω units of DM goods. So, the bank produces and delivers x_ω quantities of goods to the seller in the following CM. Constraints (3.3)-(3.4) require that the deposit contract is incentive compatible for the buyer to make a truthful report to the bank.[11] Constraint (3.3) is the incentive constraint for a small-transaction buyer, and constraint (3.4) is the incentive constraint for a large-transaction buyer. The left-hand sides of both constraints represent the expected utility from making a truthful report, while the right-hand sides represent the expected

[9] In general, I can let y_{ω,ω^s} denote the quantity of goods to be transferred in the following CM if ω^s is reported but $\omega \neq \omega^s$ is detected through monitoring. However, a decrease in y_{ω,ω^s} only relaxes the buyer's incentive compatibility constraints, so the bank can lower the probability of monitoring without changing the other terms of the contract. Therefore, the optimal y_{ω,ω^s} must be zero.

[10] This protocol enables us to focus on the friction associated with privately informed preference shocks and costly monitoring, as take-it-or-leave-it offers do not cause any bargaining inefficiencies.

[11] Similar constraints also appear in Aiyagari and Williamson (2000), in which consumers report their endowments to a financial intermediary.

utility from cheating.[12] Finally, constraint (3.5) is the set of feasibility constraints.

In equilibrium, the bank's expected discounted payoff must be zero. Otherwise, another bank could offer an alternative contract that earns a marginally smaller expected payoff per contract but makes buyers better off. With the alternative contract, the bank would attract all the buyers in the economy to earn higher payoffs. Therefore, given that (3.2) holds with equality, the bank's problem can be written as

$$\max_{x_l,x_h,\pi_l,\pi_h} \left[(1-p)\{u(x_l) - x_l - \pi_l\gamma\} + p\{\theta u(x_h) - x_h - \pi_h\gamma\}\right], \tag{3.6}$$

subject to

$$u\left(x_l\right) \geq \left(1-\pi_h\right) u\left(x_h\right), \tag{3.3}$$

$$\theta u\left(x_h\right) \geq \left(1-\pi_l\right) \theta u\left(x_l\right), \tag{3.4}$$

$$x_l, x_h \geq 0, \quad \pi_h, \pi_l \in [0,1]. \tag{3.5}$$

Lemma 3.1 *In equilibrium, a deposit contract must satisfy:*
(i) $x_h \geq x_l$,
(ii) $\pi_l = 0$, *and* $\pi_h = \{u\left(x_h\right) - u\left(x_l\right)\}/u\left(x_h\right)$.

Proof See Appendix □

The first property tells us that the quantity of claims offered to a large-transaction buyer must be at least as large as the quantity offered to a small-transaction buyer. This implies that an equilibrium contract serves an insurance role. That is, buyers can be insured against the uncertainty about their preference shocks. Thus, this model follows the spirit of Diamond and Dybvig (1983) in that depositors face privately informed risks concerning their liquidity needs, and banks play an insurance role.

The second property states that, when the bank receives a report of $\omega = l$, the probability of monitoring must be zero since only small-transaction buyers make such reports. It follows from the first property because $x_h \geq x_l$ implies that there is no incentive for a large-transaction buyer to cheat. However, the probability of monitoring when the bank receives a report of $\omega = h$ needs to be positive to induce a small-transaction buyer to report truthfully. Also, the incentive contraint must bind when $x_h > x_l$. Otherwise, a bank could lower the probability of monitoring and increase its payoffs without changing the other terms of the contract or violating the incentive constraints.

Using the properties given in Lemma 3.1, I can rewrite the bank's problem as follows:

[12] Notice that, for simplicity, I assume that the buyer would not be excluded from the banking system in the CM, even if he or she were to make an untruthful report and be detected in the previous DM.

$$\max_{x_l,x_h,\pi_h} \left[(1-p)\{u(x_l)-x_l\}+p\{\theta u(x_h)-x_h-\pi_h\gamma\}\right] \tag{3.7}$$

subject to

$$\pi_h = \frac{u(x_h)-u(x_l)}{u(x_h)}, \tag{3.8}$$

$$x_h \geq x_l. \tag{3.9}$$

The approach applied here is to assume interiority, characterize an equilibrium, and provide conditions that are consistent with the interiority. Substituting π_h from (3.8) into (3.7) gives the following first-order conditions:

$$u'(x_l) = 1 - \frac{p\gamma u'(x_l)}{(1-p)u(x_h)}, \tag{3.10}$$

$$\theta u'(x_h) = 1 + \frac{\gamma u'(x_h)\, u(x_l)}{[u(x_h)]^2}. \tag{3.11}$$

In equations (3.10) and (3.11), the left-hand sides are marginal utilities from increasing one unit of x_l and x_h, respectively, and the right-hand sides are associated marginal costs. The second term in the right-hand side in equation (3.10) is negative as one unit increase in x_l reduces the monitoring cost, while the corresponding term in eqaution (3.11) is positive as one unit increase in x_h does the opposite. Note that, if the monitoring cost γ is zero, the buyer receives the first best quantities in both states, i.e., $x_\omega = x^*_\omega$ for $\omega = l, h$. That is, if the bank can costlessly observe the buyer's state, then the buyer can be fully insured against the preference shock. However, if the monitoring cost is non-zero, the solution (x_l, x_h) to equations (3.10) and (3.11) must satisfy $x_l > x^*_l$ and $x_h < x^*_h$, which implies partial insurance to the buyer. This is due to the friction arising from the incentive problem, which is represented by the second terms in the right-hand sides in (3.10) and (3.11).

Further, it turns out that x_l increases and x_h decreases as the monitoring cost γ increases. This is because a higher γ implies a larger friction associated with the incentive problem, which tightens the incentive constraint. Then, there is a threshold value for the monitoring cost γ below which the solution to equations (3.10)-(3.11) satisfies the feasibility constraint (3.9), and above which it does not. Therefore, if γ is sufficiently small, then the optimal contract can be characterized by (x_l, x_h, π_h) that satisfy equations (3.8), (3.10), and (3.11).

If γ is sufficiently large, however, a state-contingent contract is not optimal because it is too costly. In this case, there is no monitoring regardless of the buyer's report. That is, $\pi_l = \pi_h = 0$ and $x_l = x_h = x$. So, an optimal contract can be characterized by x that solves the following auxiliary problem:

$$\max_{x\geq 0}\left[(1-p+p\theta)\,u(x)-x\right]. \tag{3.12}$$

Then, the first-order condition is

$$(1-p+p\theta)\,u'(x)=1. \tag{3.13}$$

The above discussion can be formally summarized by the following proposition:

Proposition 3.1 *Define the threshold monitoring cost as*

$$\bar{\gamma}\equiv\frac{(1-p)u(x)\left\{1-u'(x)\right\}}{pu'(x)}, \tag{3.14}$$

where $x\in(x_l^*,x_h^*)$ satisfies $p\left\{\theta u'(x)-1\right\}=(1-p)\left\{1-u'(x)\right\}$. Then, there exists an equilibrium contract (x_l,x_h,π_h) such that
(i) (x_l,x_h,π_h)=$(x_l^*,x_h^*,0)$ for $\gamma=0$;
(ii) (x_l,x_h,π_h) satisfies equations (3.8), (3.10), and (3.11) for $\gamma\in(0,\bar{\gamma})$;
(iii) $x=x_l=x_h$ satisfies equation (3.13) and $\pi_h=0$ for $\gamma\in[\bar{\gamma},\infty)$. .

Proof See Appendix □

Given that the monitoring cost is positive, an optimal contract specifies that the quantity of deposit claims for a large-transaction buyer is smaller than his or her first best quantity, $x_h<x_h^*$, while the quantity of claims for a small-transaction buyer is larger than the first best quantity, $x_l>x_l^*$. This is attributed to the incentive problem. The quantity of claims for a large-transaction buyer cannot be very large compared to the quantity of claims for a small-transaction buyer so as to prevent a small-transaction buyer from cheating. In other words, the amount of credit is constrained for large transactions, while it is not for small transactions.

3.4 An Economy with Money and Credit

In this section, I introduce fiat money into the economy described in Section 3. As the information friction tends to inefficiently reduce the size of large transactions, money can be used to increase the quantity of consumption for large-transaction buyers. However, introducing money can distort the incentive problem in credit contracts since the possibility of using money changes the expected payoffs for buyers from making an untruthful report.

Considering the relation between each buyer's asset portfolio and the associated expected utility, banks design an optimal contract that is contingent on the buyer's money holdings. In an economy with money, as I will show, the difference in the quantity of credit between two states is smaller and the probability of monitoring is lower (or equivalently, the cost of credit is smaller) compared to those in the economy without money. So, money is useful not only because it can be used in transactions, but also because it can reduce the monitoring cost. However, holding money is costly unless the monetary policy is efficient (or a Friedman rule). Therefore, buyers choose an optimal quantity of money holdings, considering the associated benefit and cost.

3.4.1 Introducing Money

I assume that fiat money is uniformly distributed across buyers at the beginning of period 0. Fiat money is perfectly divisible and durable, and the total money stock can be increased or decreased through a lump-sum transfer by the government to each buyer at the beginning of the CM.[13] Let τ_t denote the lump-sum transfer, in units of goods in the CM, to each buyer in the CM of period t. The quantity of the transfer is set so that the total money stock in the period t, denoted by M_t, grows at the gross rate μ. That is, $M_t = \mu M_{t-1}$. Let ϕ_t denote the price of money in terms of goods in the CM of period t. Then, it follows that $\tau_t = (\mu - 1)\,\phi_t M_{t-1}$. As in Lagos and Wright (2005), a necessary condition for an equilibrium to exist is $\phi_t/\phi_{t+1} \geq \beta$.

In addition, I assume throughout that small-transaction buyers face a means-of-payment shock in the following sense.[14] Suppose that some sellers, denoted as *connected sellers*, have a machine that connects sellers to the banking system in DM meetings, while others, denoted as *unconnected sellers*, do not have access to the banking system. Also, suppose that large-transaction buyers receive information about which sellers have the machine, while small-transaction buyers do not and have to search randomly. I assume that the mass of connected sellers is sufficiently high, so that each large-transaction buyer is matched with a connected seller. Then, only small-transaction buyers face a means-of-payment shock in that, with some positive probability, a credit arrangement is not available. In particular, a fraction δ of small-transaction buyers are matched with unconnected sellers who only accept money as a means of payment, while a fraction $1 - \delta$ of small-transaction buyers are matched with connected sellers who take any means of payment.

This type of means-of-payment shock is commonly used in the literature on money and credit.[15]

[13] For simplicity, I assume that there is perfect enforcement when it comes to taxation. This means that the government can force agents to pay lump-sum taxes to extract cash from the economy.

[14] I assume that the means-of-payment shock is asymmetric between small- and large-transactions buyers for analytical tractability. A symmetric means-of-payment shock complicates the analysis but does not change the main result of the paper. Also, the case without this payment shock is analyzed in Section 4.4.2.

[15] See, for example, Sanches and Williamson (2010), Williamson (2012), and Gomis-Porqueras and Sanches (2013).

As explained by Sanches and Williamson (2010), some degree of imperfect memory is necessary to give money a welfare-improving role in an environment with limited commitment, and the means-of-payment shock essentially introduces imperfect memory into the economy. In the context of this model, the absence of this assumption implies that money would not be valued if credit arrangements are state-contingent, while credit arrangements would not be state-contingent if money is valued, as we will see in Section 4.4.2.

3.4.2 Specification of a Contract and the Timeline

Because money is useful in transactions as described earlier, buyers want to carry some money balances from the CM to the next DM. So, some economic activities associated with money must be added in the economy described in the previous section. Specifically, in the current CM, each buyer produces and exchanges goods to acquire money, and then banks write contracts given the buyer's money holdings. For simplicity, I assume that the bank can observe the buyer's money holdings.[16] As described in the previous section, a deposit contract between a buyer and a bank consists of $\{k, x_\omega, \pi_\omega\}_{\omega=l,h}$ where k denotes the quantity of goods that the buyer deposits at the bank in the current CM, x_ω denotes the quantity of goods to be transferred from the bank to the depositor in the next CM, and π_ω denotes the probability of monitoring given that the buyer makes a report of $\omega \in \{l, h\}$ in the following DM. A fraction δ of small-transaction buyers are matched with unconnected sellers who only accept money, in which case, the buyer receives x_l units of goods from the bank in the following CM. In other cases, the buyer can exchange money for additional goods after credit transactions have been made. The buyer makes a take-it-or-leave-it offer to the seller. The rest is the same as described in the previous section and the sequence of events is summarized in Figure 3.2.

[16]In general, I can assume that the quantity of money balances held by a buyer is private information and that the buyer reports the quantity to a bank before the bank writes a contract. In this case, making a truthful report of his or her money holdings is incentive compatible, and there is no incentive for the buyer to adjust his or her money holdings *ex post* because the bank's objective and the buyer's objective are identical, which is the buyer's expected utility. Therefore, there is no loss of generality from simply assuming that the bank can observe the buyer's money holdings.

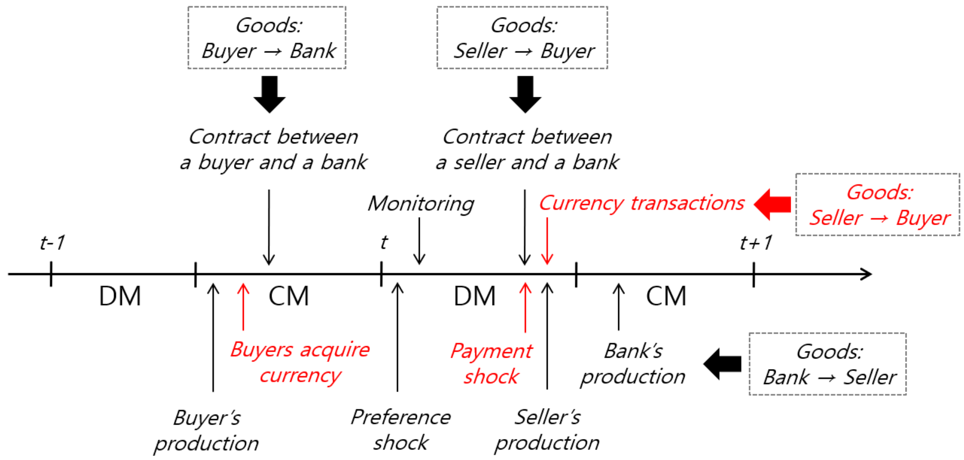

Figure 3.2: Sequence of Events

3.4.3 Optimization

The buyer's problem in the CM Let $V(m)$ denote the value function for the buyer at the beginning of the CM after all debts are paid off, and let $W_\omega(m, x_\omega, I)$ denote the value function for the buyer with preference shock $\omega \in \{l, h\}$ at the beginning of the DM, where m is the quantity of real money balances, x_ω is the quantity of claims on goods, and I is an indicator function such that $I = 0$ if the matched seller is unconnected to the banking system and accepts only cash, and $I = 1$ otherwise. Thus, $I = 0$ with probability δ and $I = 1$ with probability $1 - \delta$ if the buyer receives a low preference shock $\omega = l$, while $I = 1$ always if the buyer receives a high preference shock $\omega = h$. Notice that $V(m)$ does not have the subscript ω since the preference shock, which the buyer observes in the subsequent DM, is unknown in the current CM. Then, the Bellman equation associated with the buyer's optimization problem in the CM is

$$V(m) = \max_{H,m'} \{-H + \beta\mathbb{E}[W_\omega(m', x_\omega, I)]\} \tag{3.15}$$

subject to

$$\frac{\phi_t}{\phi_{t+1}} m' + k = H + m + \tau, \tag{3.16}$$

where m denotes the quantity of real money balances at the beginning of the CM, m' denotes the quantity of real money balances at the beginning of the following DM, and k denotes the price of the deposit contract. Then, substituting for H from (3.16) into (3.15), the value function can be rewritten as

$$V(m) = m + \tau - k + \max_{m'} \left\{ -\frac{\phi_t}{\phi_{t+1}} m' + \beta \mathbb{E}\left[W_\omega \left(m', x_\omega, I \right) \right] \right\}. \tag{3.17}$$

The buyer's problem in the DM Consider the terms of trade in the DM. Suppose that the buyer acquires and carries m units of real money balances from the CM to the next DM. After the stage of credit transactions, the buyer can make a monetary transaction with the seller. Let d denote the quantity of real money balances the buyer trades for z units of goods. Then, the buyer chooses z and d to maximize his or her surplus from trade subject to the money constraint and the seller's participation constraint, given that the buyer has received x_ω units of goods from a credit transaction. That is, d and z solve the following bargaining problem:

$$\max_{d,z} \left[\theta_\omega u\left(I x_\omega + z \right) - d + (1 - I) x_\omega \right], \tag{3.18}$$

subject to

$$d \leq m, \tag{3.19}$$

$$-z + d \geq 0, \tag{3.20}$$

for given $\theta_\omega \in \{1, \theta\}$, $x_\omega \in \{x_l, x_h\}$, and $I \in \{0, 1\}$. The problem can be rewritten as

$$\max_{d} \left[\theta_\omega u\left(I x_\omega + d \right) - d + (1 - I) x_\omega \right], \tag{3.21}$$

$$s.t. \quad d \leq m, \tag{3.19}$$

as the buyer's take-it-or-leave-it offer implies a binding participation constraint of the seller.

If the buyer is matched with a connected seller and receives goods more than x_ω^* units (the first best quantity) from the credit transaction, i.e., $x_\omega \geq x_\omega^*$, then additional monetary transactions would not be necessary for the buyer, i.e., $z = d = 0$.[17] In this case, the buyer carries all the money balances to the following CM. In constrast, if the buyer receives consumption goods less than x_ω^* units from the credit transaction, i.e., $x_\omega < x_\omega^*$, then there would be some monetary transactions between the buyer and the seller, but the trade volume would depend on the quantities x_ω and m. When x_ω and m are sufficiently large so that $x_\omega^* \leq x_\omega + m$, the buyer trades $x_\omega^* - x_\omega$ units of money balances to consume in total x_ω^* units of goods in the DM and carries $m - x_\omega^* + x_\omega$ units of real money balances to the subsequent CM. When x_ω and m are not sufficiently large so that $x_\omega + m < x_\omega^*$, the buyer uses all the money balances to trade for goods. In this case, the money constraint (3.19) binds, i.e., $z = d = m$.

[17] As defined in Section 2, the first best quantities x_l^* and x_h^* are solutions to $u'(x_l^*) = 1$ and $\theta u'(x_h^*) = 1$, respectively.

Finally, the value function for the buyer at the beginning of the DM can be defined as

$$W_\omega (m, x_\omega, I) = \max_d [\theta_\omega u (I x_\omega + d) + (1 - I)x_\omega + V(m - d)] \tag{3.22}$$

$$s.t. \quad d \leq m, \tag{3.19}$$

since the solution d to the maximization problem in the right-hand side of equation (3.22) subject to (3.19) is the same as the solution to the maximization problem (3.21) subject to (3.19) due to the quasilinearity in the buyer's preferences.

The bank's problem In equilibrium, banks write a deposit contract $\{k, x_l, x_h, \pi_l, \pi_h\}$ that solves the following problem for each buyer's money holdings $m \in [0, \infty)$:

$$\max_{k,x_l,x_h,\pi_l,\pi_h} \left\{ -k - \frac{\phi_t}{\phi_{t+1}} m + \beta \left[\begin{array}{c} (1-p)\{\delta W_l(m, x_l, 0) + (1-\delta) W_l(m, x_l, 1)\} \\ +p W_h(m, x_h, 1) \end{array} \right] \right\}, \tag{3.23}$$

subject to

$$k - \beta \{(1-p)(x_l + \pi_l \gamma) + p(x_h + \pi_h \gamma)\} \geq 0, \tag{3.24}$$

$$\delta W_l (m, x_l, 0) + (1-\delta) W_l (m, x_l, 1) \geq (1 - \pi_h) W_l (m, x_h, 1) + \pi_h V(m), \tag{3.25}$$

$$W_h (m, x_h, 1) \geq (1 - \pi_l) \{\delta W_h (m, x_l, 0) + (1-\delta) W_h (m, x_l, 1)\} + \pi_l V(m), \tag{3.26}$$

$$k, x_l, x_h \geq 0, \quad \pi_h, \pi_l \in [0, 1], \tag{3.27}$$

That is, a deposit contract must be chosen to maximize a buyer's expected utility given the buyer's money holdings, subject to the bank's nonnegative payoff constraint (3.24), the buyer's incentive constraints (3.25)-(3.26), and feasibility constraints (3.27).[18] Provided that the bank earns zero profit in equilibrium, the problem can be rewritten as

$$\max_{x_l,x_h,\pi_l,\pi_h} \left[\begin{array}{c} (1-p)\{\delta W_l(m, x_l, 0) + (1-\delta) W_l(m, x_l, 1) - x_l - \pi_l \gamma\} \\ +p\{W_h(m, x_h, 1) - x_h - \pi_h \gamma\} \end{array} \right], \tag{3.28}$$

subject to the constraints (3.25)-(3.27).

[18]For simplicity, I assume that the seller does not participate in a monetary transaction with the buyer if he or she has been caught cheating during the DM meeting. If the seller is allowed to participate in a monetary transaction with an untruthful buyer, this would further tighten the incentive constraint. For analytical tractability, I also assume that, if a small-transaction buyer claims to be a large-transaction buyer, then he or she receives information about connected sellers as other large-transaction buyers do. In contrast, if a large-transaction buyer claims to be a small-transaction buyer, then the buyer cannot have access to that information. The consequences of these assumptions are discussed in Footnote 20.

Definition 4.1 *An optimal contract is (x_l, x_h, π_l, π_h) that solves the problem (3.28) subject to (3.25)-(3.27) for each buyer's money holdings $m \geq 0$.*

Lemma 4.1 *In equilibrium, an optimal contract must satisfy:*
(i) $W_l(m, x_h, 1) \geq \delta W_l(m, x_l, 0) + (1-\delta) W_l(m, x_l, 1)$,
(ii) $\pi_l = 0$, and $\pi_h = \frac{W_l(m,x_h,1) - [\delta W_l(m,x_l,0) + (1-\delta) W_l(m,x_l,1)]}{W_l(m,x_h,1) - V(m)}$.

Proof See Appendix □

Notice that, if buyers hold zero money balances, $m = 0$, and all sellers are connected to the banking system, $\delta = 0$, Lemma 4.1 is equivalent to Lemma 3.1. However, with the presence of unconnected sellers, i.e., $\delta > 0$, a deposit contract that fully economizes on the monitoring cost, i.e., $\pi_l = \pi_h = 0$, does not deliver a fixed quantity of claims on goods across states. To illustrate this, suppose a buyer acquires a contract that guarantees noncontingent claims on goods, i.e., $x_l = x_h$. If the probability of meeting an unconnected seller were to be zero, $\delta = 0$, then there would be no incentive for a small-transaction buyer to make an untruthful report, leading to $\pi_h = 0$. However, with a positive probability of meeting an unconnected seller, i.e., $\delta > 0$, a small-transaction buyer would make an untruthful report because, by doing so, the buyer would be able to meet a connected seller with no uncertainty and enjoy a higher utility. Therefore, fully economizing on the monitoring cost, $\pi_l = \pi_h = 0$, requires a larger quantity of claims for a small-transaction buyer than the quantity for a large-transaction buyer, as the first property in Lemma 4.1 indicates.

The probability of monitoring in a deposit contract depends on the buyer's money holdings as well as the cost of monitoring. If the monitoring cost is very high, then a deposit contract that specifies a positive probability of monitoring would never be optimal for any quantities of the buyer's money holdings. So, there would be no monitoring regardless of the buyer's money holdings in this case. A more interesting case would be when the monitoring cost is sufficiently low. In this case, if the buyer increases the quantity of money holdings, he or she can acquire more goods in large transactions. Then, the optimal deposit contract would specify a smaller quantity of claims for large-transaction buyers because an increase in the buyer's money holdings decreases the marginal utility of consumption for a large-transaction buyer. Since a smaller quantity of claims for large-transaction buyers implies a lower incentive for a small-transaction buyer to misreport, the optimal deposit contract would specify a lower probability of monitoring for large-transaction buyers. Therefore, the optimal probability of monitoring would decrease as the quantity of buyer's money holdings increases.[19]

[19] With different assumptions, it is possible that a larger quantity of money holdings tightens the incentive constraint

First, consider the case where the buyer carries a sufficiently small quantity of real money balances and exchanges all the money for goods in a connected large transaction or in an unconnected small transaction. Then, the bank's problem can be written as

$$\max_{x_l, x_h, \pi_h} \left\{ \begin{array}{c} (1-p)[\delta\{u(m)+x_l\}+(1-\delta)\{u(x_l)+m\}-x_l] \\ +p[\theta u(x_h+m)-x_h-\pi_h\gamma] \end{array} \right\}, \tag{3.29}$$

subject to

$$\pi_h = \frac{u(x_h)-u(x_l)+\delta\{u(x_l)-x_l-u(m)+m\}}{u(x_h)}, \tag{3.30}$$

$$\pi_h \in [0,1], \tag{3.31}$$

given the buyer's money holdings m. Substituting π_h from (3.30) into (3.29) yields the following first-order conditions:

$$u'(x_l) = 1 - \frac{p\gamma\{(1-\delta)u'(x_l)+\delta\}}{(1-p)(1-\delta)u(x_h)}, \tag{3.32}$$

$$\theta u'(x_h+m) = 1 + \frac{\gamma u'(x_h)[(1-\delta)u(x_l)+\delta x_l+\delta\{u(m)-m\}]}{\{u(x_h)\}^2}. \tag{3.33}$$

Equations (3.32) and (3.33) correspond to equations (3.10) and (3.11), which are optimality conditions for a deposit contract in a pure credit equilibrium. Similarly to equations (3.10) and (3.11), the left-hand sides in equations (3.32) and (3.33) are marginal utilities from increasing one unit of x_l and x_h, respectively, and the right-hand sides are associated marginal costs. Also, the second term in the right-hand side in equation (3.32) is negative as one unit increase in x_l reduces the monitoring cost, while the corresponding term in equation (3.33) is positive as one unit increase in x_h does the opposite. Note that, if the monitoring cost γ is zero, the buyer receives sufficient quantities of claims on goods in both states to consume first best quantities, i.e., $x_\omega = x_\omega^* - m$ for $\omega = l, h$. However, if the monitoring cost is strictly positive, i.e., $\gamma > 0$, then the solution (x_l, x_h) must specify that $x_l > x_l^*$ and $x_h < x_h^* - m$. That is, the quantity of consumption in a connected small transaction is inefficiently large, while the quantity of consumption in a large transaction is inefficiently small. Again, this inefficiency in DM transactions comes from the incentive problem that is related to asymmetric information and costly monitoring, as represented in the right-hand sides in equations (3.32) and (3.33).

The optimal contract, which is captured by equations (3.30), (3.32), and (3.33), shows two

of small-transaction buyers. Related issues are discussed in Footnote 20.

important roles of money as a means of payment. First, given (x_l, x_h, π_h), a larger m implies a larger consumption quantity in an unconnected small transaction, increasing the expected utility of a small-transaction buyer from making a truthful report. So, money effectively mitigates the incentive problem, leading to a decrease in the monitoring probability π_h as shown in equation (3.30).[20] Second, given (x_l, x_h, π_h), a larger m implies a lower marginal utility of consumption in a large transaction relative to the marginal cost, as described in equation (3.33). This tends to decrease the quantity of claims on goods x_h for a large-transaction buyer. Since a decrease in x_h mitigates the incentive problem, the monitoring probability π_h decreases.

Keeping in mind the roles of money described above, I can examine how the optimal contract (x_l, x_h, π_h) changes as the buyer's money holdings m increases. First, from equation (3.33), an increase in m decreases the marginal utility in the left-hand side and increases the marginal cost in the right-hand side. Thus, the quantity of consumption $x_h + m$ must decrease, implying that a decrease in x_h must be larger, in absolute value, than an increase in m. Then, from (3.32), a decrease in x_h increases x_l, and thus, x_h must decrease further from (3.33). Therefore, x_l increases and x_h decreases at an optimum, leading to a decrease in π_h from (3.30). These results are summarized by the following lemma:

Lemma 4.2 *Given a sufficiently small quantity of money holdings m, an increase in m leads to a decrease in x_h and an increase in x_l.*

Proof See Appendix □

As the quantity of buyer's money holdings m increases, the quantity of claims for a small-transaction buyer x_l increases and the quantity of claims for a large-transaction buyer x_h decreases, provided that the monitoring cost γ is sufficiently low. This implies that there exists a threshold quantity of money holdings $\hat{m}(\gamma) > 0$ below which the solution (x_l, x_h) to equations (3.32)-(3.33) satisfies the feasibility constraint (3.31), and above which the solution is not consistent with (3.31). Also, x_l increases and x_h decreases as γ increases similarly to what occurs in a pure credit economy,

[20] Here, an increase in the quantity of buyer's money holdings tends to mitigate the incentive problem in a deposit contract. This result critically depends on two assumptions: (i) only small-transaction buyers are subject to means-of-payment shocks, and (ii) sellers do not participate in monetary transactions with a perceived untruthful buyer. Relaxing the first assumption does not change the result. However, relaxing both assumptions does lead to the opposite result. If both types of buyers face means-of-payment shocks and sellers can make a monetary transaction with untruthful buyers, then an increase in the buyer's money holdings would eventually tighten the incentive problem, leading to an increase in the monitoring probability. In this case, when the optimal deposit contract provides an insurance role to buyers, money would be too costly to hold, and thus, would not be valued in equilibrium. Money would be valued only when the optimal deposit contract does not provide any insurance role in equilibrium (See Corollary 4.1). Since it is more interesting to analyze an equilibrium where both money and credit are useful, I restrict attention to cases under the above two assumptions.

given a sufficiently small m. This implies that there exists a threshold monitoring cost $\hat{\gamma}(m) > 0$ such that the optimal contract can be characterized by equations (3.30), (3.32), and (3.33) for $\gamma \leq \hat{\gamma}(m)$. Consequently, the optimal contract can be characterized by equations (3.30), (3.32), and (3.33) if and only if $\gamma \leq \hat{\gamma}(m)$ and $m \leq \hat{m}(\gamma)$.

What if the buyer's money holdings are sufficiently large? Suppose now that the buyer's money holdings are sufficiently large, $m > \hat{m}(\gamma)$. Then, any deposit contracts with a positive probability of monitoring would not be optimal. With no monitoring (or $\pi_h = 0$), the bank's problem (3.28) subject to (3.25)-(3.27) can be written as

$$\max_{x_l, x_h} \left[\begin{array}{c} (1-p)\{\delta W_l(m, x_l, 0) + (1-\delta) W_l(m, x_l, 1) - x_l\} \\ +p\{W_h(m, x_h, 1) - x_h\} \end{array} \right], \tag{3.34}$$

subject to

$$W_l(m, x_h, 1) = \delta W_l(m, x_l, 0) + (1-\delta) W_l(m, x_l, 1), \tag{3.35}$$

given the buyer's money holdings m.

If the quantity of buyer's money holdings is larger than the first best quantity for a large transaction, $m \in [x_h^*, \infty)$, then the buyer can trade money for x_l^* units of goods in a small transaction and x_h^* units in a large transaction. Although deposit contracts cannot make buyers better off in equilibrium, holding a deposit contract is innocuous for buyers because the associated net cost is essentially zero in this case. In particular, any contracts that specify $x = x_l = x_h$ and $x \in [0, x_l^*]$ is optimal. If the quantity of buyer's money holdings is smaller than the first best quantity for a large transaction but not smaller than the first best quantity for a small transaction, $m \in [x_l^*, x_h^*)$, an optimal contract can be characterized by $x = x_l = x_h$ and $x \in [x_h^* - m, x_l^*]$. In the above two cases, the buyer consumes x_ω^* units of goods and carries $m - x_\omega^* + x$ units of money balances to the following CM, for $\omega \in \{l, h\}$. Therefore, if the buyer's money holdings are very large, or $m \in [x_l^*, \infty)$, then any contracts with $x = x_l = x_h$ that satisfies the following condition are optimal:

$$\max[x_h^* - m, 0] \leq x \leq x_l^*. \tag{3.36}$$

Let $\bar{m}$ denote the lower bound of quantities of money balances that allow buyers to consume the first best quantities x_ω^* of goods for both $\omega = l, h$. That is, $\bar{m} \equiv x_l^*$.

Finally, consider the case where the quantity of buyer's money holdings is sufficiently large but not very large, i.e., $m \in (\hat{m}(\gamma), \bar{m})$. Since the quantity of money holdings is smaller than the first best quantity for a small-transaction buyer, i.e., $m \leq \bar{m} = x_l^*$, the buyer trades all the money for goods in an unconnected small transaction. Hence, the maximization problem (3.34) subject to

(3.35) can be written as

$$\max_{x_l, x_h} \left\{ \begin{array}{c} (1-p)[\delta\{u(m)+x_l\}+(1-\delta)\{u(x_l)+m\}-x_l] \\ +p[\theta u\,(x_h+m)-x_h] \end{array} \right\}, \tag{3.37}$$

subject to

$$u(x_h) = (1-\delta)u(x_l) + \delta x_l + \delta\{u(m) - m\}. \tag{3.38}$$

Then, the first-order condition is given by

$$\frac{(1-p)(1-\delta)\{1-u'(x_l)\}}{\delta+(1-\delta)u'(x_l)} = \frac{p\{\theta u'(x_h+m)-1\}}{u'(x_h)}. \tag{3.39}$$

Hence, an optimal contract can be characterized by (x_l, x_h) that satisfies equations (3.38) and (3.39).

Define a threshold monitoring cost as

$$\bar{\gamma} \equiv \frac{u(\dot{x}_h)\{\theta u'(\dot{x}_h)-1\}}{u'(\dot{x}_h)}, \tag{3.40}$$

where $\dot{x}_h \in (x_l^*, x_h^*)$, together with $\dot{x}_l \in (x_l^*, x_h^*)$, is the solution to

$$(1-\delta)(1-p)u'(\dot{x}_h)\{1-u'(\dot{x}_l)\} = p\{(1-\delta)u'(\dot{x}_l)+\delta\}\{\theta u'(\dot{x}_h)-1\}, \tag{3.41}$$

$$u(\dot{x}_h) = (1-\delta)u(\dot{x}_l) + \delta \dot{x}_l. \tag{3.42}$$

With a monitoring cost higher than the threshold, zero monitoring is optimal for all $m \in [0, \infty)$. Then, the above discussion can be summarized by the following proposition.

Proposition 4.1 *(i) Suppose $\gamma \in (0, \bar{\gamma})$. Then, there exists $\hat{m} \in [0, \bar{m})$ such that an optimal contract (x_l, x_h, π_h) can be characterized by a) equations (3.30), (3.32), and (3.33) for $m \in [0, \hat{m})$; b) equations (3.38) and (3.39) with $\pi_h = 0$ for $m \in [\hat{m}, \bar{m})$; and c) condition (3.36) with $x = x_l = x_h$ and $\pi_h = 0$ for $m \in [\bar{m}, \infty)$.*
(ii) Suppose $\gamma \in [\bar{\gamma}, \infty)$. Then, an optimal contract (x_l, x_h, π_h) can be characterized by a) equations (3.38) and (3.39) with $\pi_h = 0$ for $m \in [0, \bar{m})$; and b) condition (3.36) with $x = x_l = x_h$ and $\pi_h = 0$ for $m \in [\bar{m}, \infty)$.

Proof See Appendix □

The buyer's problem in the CM Recall the value function for the buyer at the beginning of the CM, or

$$V(m) = m + \tau - k + \max_{m'} \left\{ -\frac{\phi_t}{\phi_{t+1}} m' + \beta \mathbb{E}\left[W_\omega(m', x_\omega, I)\right] \right\}. \tag{3.17}$$

Considering that the bank writes an optimal contract contingent on the buyer's money holdings, the buyer chooses an optimal quantity of money holdings. So, the buyer's problem in the CM can be written as

$$\max_{m \geq 0} \left\{ -\frac{\phi_t}{\phi_{t+1}} m + \beta \left[\begin{array}{c} (1-p)\{\delta W_l(m, x_l, 0) + (1-\delta) W_l(m, x_l, 1) - x_l\} \\ +p\{W_h(m, x_h, 1) - x_h - \gamma \pi_h\} \end{array} \right] \right\}, \tag{3.43}$$

subject to

$$(x_l, x_h, \pi_h) = \arg\max_{x_l, x_h, \pi_h} \left\{ \begin{array}{c} (1-p)[\delta W_l(m, x_l, 0) + (1-\delta) W_l(m, x_l, 1) - x_l] \\ +p[W_h(m, x_h, 1) - x_h - \gamma \pi_h], \end{array} \right\}, \tag{3.44}$$

$$s.t. \quad \pi_h = \frac{W_l(m, x_h, 1) - [\delta W_l(m, x_l, 0) + (1-\delta) W_l(m, x_l, 1)]}{W_l(m, x_h, 1) - V(m)}$$

$$\pi_h \in [0, 1]$$

That is, the deposit contract (x_l, x_h, π_h) is now a function of the buyer's choice on m.

3.4.4 Equilibrium

In this subsection, I characterize a stationary equilibrium where all real quantities are constant over time. Then, from money market clearing, $\phi_t M_{t-1} = m$ for all t. This, in turn, implies that the inflation rate equals the money growth rate, i.e., $\phi_t/\phi_{t+1} = \mu$ for all t. It follows that $\mu \geq \beta$, as otherwise monetary equilibrium does not exist. Confining attention to the case with $\gamma > 0$, I can define an equilibrium as follows.

Definition 4.2 *Given a monitoring cost $\gamma > 0$ and a monetary policy $\mu \geq \beta$, a stationary equilibrium consists of the buyer's money holdings m and the deposit contract (x_l, x_h, π_h) such that*
(i) m solves the maximization problem (3.43) subject to (3.44), and
(ii) (x_l, x_h, π_h) solves the maximization problem (3.28) subject to (3.25)-(3.27).

Proposition 4.2 *If $\mu = \beta$, there exists a continuum of equilibria characterized by (x, m) that satisfies $m \geq \bar{m}$, $x_l = x_h = x$, $\pi_h = 0$, and condition (3.36).*

If the monetary policy is a Friedman rule, i.e, $\mu = \beta$, then the buyer can choose any $m \geq \bar{m}$ to consume x^*_ω units of goods in each state $\omega \in \{l, h\}$. Although money does not completely crowd out credit, credit is irrelevant in this case. Since money works perfectly well, there is no need for banks to incur monitoring costs to make their deposit contracts play an insurance role. However, if the monetary policy is away from a Friedman rule, i.e., $\mu > \beta$, holding money becomes costly. So, it is not optimal for buyers to consume their first best quantities of goods in both states.

3.4.4.1 Equilibrium with Unconnected Sellers ($\delta > 0$)

Consider the case where there are some unconnected sellers in the economy, i.e., $\delta > 0$. Also, suppose that μ is sufficiently low, implying that the buyer's money holdings are sufficiently large, i.e., $m \in [\hat{m}(\gamma), \bar{m})$, in equilibrium. Then, the equilibrium can be characterized by (m, x_l, x_h) that satisfies

$$u(x_h) = (1-\delta)u(x_l) + \delta x_l + \delta\{u(m) - m\}, \tag{3.38}$$

$$\frac{(1-p)(1-\delta)\{1-u'(x_l)\}}{\delta + (1-\delta)u'(x_l)} = \frac{p\{\theta u'(x_h + m) - 1\}}{u'(x_h)}, \tag{3.39}$$

$$\beta[(1-p)\{\delta u'(m) + 1 - \delta\} + p\theta u'(x_h + m)] = \mu. \tag{3.45}$$

Equation (3.45) is the first-order condition for the buyer's problem (3.43) subject to (3.44), and equations (3.38)-(3.39) are the optimality conditions for the bank's problem. Define μ_1 as the money growth rate that satisfies equations (3.38),(3.39), and (3.45) at $m = \hat{m}(\gamma)$. As m is decreasing in μ, a necessary condition for such equilibrium to exist is $\mu \in (\beta, \mu_1]$. Notice that $\hat{m}(\gamma)$ is defined only for $\gamma \in (0, \bar{\gamma})$, as shown in Proposition 4.1. Therefore, if the monitoring cost is very high, $\gamma \geq \bar{\gamma}$, then a necessary condition for the existence of such equilibrium is $\mu \in (\beta, \infty)$.

Now, suppose that $\gamma \in (0, \bar{\gamma})$ and μ is sufficiently high, implying that the buyer's money holdings are sufficiently small, i.e., $m \in [0, \hat{m}(\gamma))$, in equilibrium. Then, the equilibrium can be characterized by (m, x_l, x_h, π_h) that satisfies

$$\pi_h = \frac{u(x_h) - u(x_l) + \delta\{u(x_l) - x_l - u(m) + m\}}{u(x_h)}, \tag{3.30}$$

$$u'(x_l) = 1 - \frac{p\gamma\{(1-\delta)u'(x_l) + \delta\}}{(1-p)(1-\delta)u(x_h)}, \tag{3.32}$$

$$\theta u'(x_h + m) = 1 + \frac{\gamma u'(x_h)[(1-\delta)u(x_l) + \delta x_l + \delta\{u(m) - m\}]}{\{u(x_h)\}^2}, \tag{3.33}$$

$$\beta\left\{(1-p)[\delta u'(m) + 1 - \delta] + p[\theta u'(x_h + m) + \frac{\gamma\delta\{u'(m) - 1\}}{u(x_h)}]\right\} = \mu. \tag{3.46}$$

Equation (3.46) is the first-order condition for the buyer's problem (3.43) subject to (3.44), and equations (3.30), (3.32)-(3.33) are the optimality conditions for the bank's problem. Define μ_2 as the money growth rate that satisfies the above four equations at $m = \hat{m}(\gamma)$. Since m is decreasing in μ in this case, a necessary condition for such equilibrium to exist is $\mu \in (\mu_2, \infty)$. These results are summarized by the following proposition.

Proposition 4.3 *(i) Suppose $\gamma \in (0, \bar{\gamma})$. Then, there exist $\mu_1 \in (\beta, \infty)$ and $\mu_2 \in (\mu_1, \infty)$ such that a stationary equilibrium can be characterized by*
a) equations (3.38), (3.39) and (3.45) for $\mu \in (\beta, \mu_1]$; b) equations (3.38) and (3.39) with $m = \hat{m}(\gamma)$ for $\mu \in (\mu_1, \mu_2]$; and c) equations (3.30), (3.32)-(3.33), and (3.46) for $\mu \in (\mu_2, \infty)$.
(ii) Suppose $\gamma \in [\bar{\gamma}, \infty)$. Then, a stationary equilibrium can be characterized by equations (3.38), (3.39) and (3.45) for $\mu \in (\beta, \infty)$.

Proof See Appendix □

Proposition 4.3 exhibits how the monitoring cost γ and the money growth rate μ determine the existence of particular equilibria, which is presented in Figure 3.3. The left panel in Figure 3.3 depicts how the parameter space is subdivided with μ on the horizontal axis and γ on the vertical axis, focusing on whether monitoring occurs in equilibrium. From equation (3.46), the critical value of the money growth rate μ_2, that determines the existence of monitoring in equilibrium, is increasing in the monitoring cost γ. If γ is sufficiently high $\gamma \in [\bar{\gamma}, \infty)$, however, then the critical value μ_2 does not exist. Note that there is always a maximum value $\hat{\gamma}(\mu)$ of the monitoring cost γ, given the money growth rate μ, below which the probability of monitoring is positive.

Proposition 4.3 also shows how the monitoring cost γ and the money growth rate μ characterize an equilibrium. In particular, the right panel in Figure 3.3 depicts how the buyer's money holdings m is determined by μ given $\gamma \in (0, \bar{\gamma})$ in equilibrium. Interestingly, if the money growth rate is given by $\mu \in (\mu_1, \mu_2]$, the buyer's money holdings m is fixed at $\hat{m}(\gamma)$. From the buyer's perspective, decreasing m involves switching the contract from one with $\pi_h = 0$ to one with $\pi_h > 0$. A decrease in m leads to an increase in the cost of monitoring because it acts to tighten the incentive problem. This occurs because a decrease in m reduces the small-transaction buyer's payoff from reporting truthfully. As the cost of switching the contract is higher than the cost of reducing the quantity of money holdings at the margin in this case, the buyer facing an increase in μ does not reduce m. If the money growth rate becomes sufficiently high, $\mu > \mu_2$, then the marginal cost of holding money becomes higher than the cost of switching the contract, causing the buyer to decrease m.

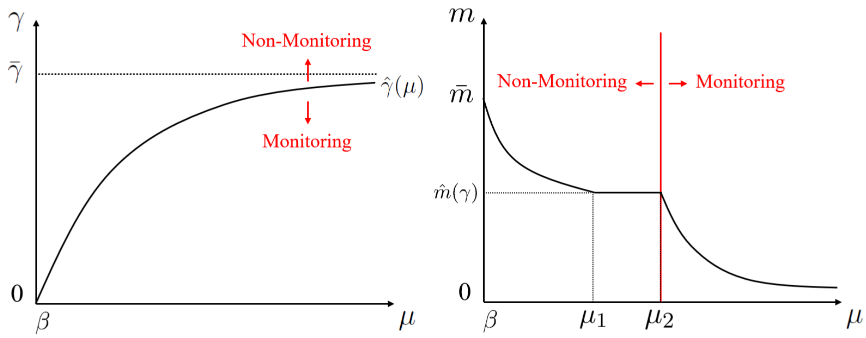

Figure 3.3: Equilibria with monitoring cost γ and monetary policy μ

Comparative statics Here, I examine the effects of the monitoring cost γ (or financial friction) and monetary policy μ (or monetary friction) on the deposit contract (x_l, x_h, π_h) and the buyer's money holdings m in equilibrium. These findings are summarized in Table 3.1. The comparative statics analyses for an economy with $\gamma \in (0, \bar{\gamma})$ and $\mu \in (\beta, \mu_1]$ or an economy with $\gamma \in [\bar{\gamma}, \infty)$ are straightforward. Any changes in γ do not affect the economy due to the absence of monitoring. In contrast, an increase in μ leads to a decrease in m and increases in both x_l and x_h. That is, there is a substitution of credit for money as the money growth rate μ rises, which is consistent with the findings of standard models.[21] However, here credit increases mainly due to its substitution for money in large transactions, while the increase in credit in standard models comes from a higher incentive for buyers to repay their debts.

Table 3.1: Effects of money policy and monitoring cost

	(i) $\gamma \in (0, \bar{\gamma})$ and $\mu \in (\beta, \mu_1]$; or $\gamma \in [\bar{\gamma}, \infty)$				(ii) $\gamma \in (0, \bar{\gamma})$ and $\mu \in (\mu_1, \mu_2]$				(iii) $\gamma \in (0, \bar{\gamma})$ and $\mu \in (\mu_2, \infty)$			
	∂m	∂x_l	∂x_h	$\partial \pi_h$	∂m	∂x_l	∂x_h	$\partial \pi_h$	∂m	∂x_l	∂x_h	$\partial \pi_h$
$\partial \mu$	-	+	+	·	·	·	·	·	-	-	+	+
$\partial \gamma$	·	·	·	·	·	·	·	·	+	+	-	-

If the money growth rate μ is sufficiently high $\mu > \mu_1$, then the effects of monetary policy on money and credit depart from the standard results. In particular, if $\gamma \in (0, \bar{\gamma})$ and $\mu \in (\mu_1, \mu_2]$, the buyer does not change his or her money holdings m in response to γ or μ. So, monetary policy μ and the monitoring cost γ are irrelevant at the margin in equilibrium. Monetary policy is irrelevant

[21]See, for example, Sanches and Williamson (2010), Williamson (2012), and Gomis-Porqueras and Sanches (2013). In those models, an endogenous credit constraint arises from limited commitment and the buyer's incentive to default on credit arrangements. In contrast, here the buyer's incentive to misreport his or her preference shock and costly monitoring jointly generate a credit constraint. Also, unlike standard models, credit in my model has two dimensions: the quantity of credit for a small transaction x_l and the quantity for a large transaction x_h.

because μ is not high enough, or the benefit of decreasing m is not high enough, to induce buyers to switch from a non-monitoring deposit contract to one with monitoring. Also, notice that the policy irrelevance arises away from a Friedman rule, as opposed to what happens in standard models.

Most interestingly, if $\gamma \in (0, \bar{\gamma})$ and $\mu \in (\mu_2, \infty)$, monitoring takes place in equilibrium, i.e., $\pi_h > 0$ and deposit contracts serve an insurance role for buyers. In this case, an increase in μ leads to decreases in m and x_l, and increases in x_h and π_h. The intuition for this novel result is as follows. In equilibrium, buyers equate the marginal cost of holding money and the corresponding marginal benefit. While the marginal cost of holding money is determined by μ only, the marginal benefit is mainly related to two factors: (i) a higher consumption in unconnected small transactions and connected large transactions and (ii) a reduced monitoring cost due to a higher incentive for small-transaction buyers to report truthfully. As an increase in μ implies a higher marginal cost of holding money relative to the marginal benefit in equilibrium, buyers choose to acquire a smaller m, leading to a smaller x_l and a larger x_h, in order to equate between the marginal cost and benefit.

In this case, an increase in the monitoring cost γ leads to increases in m and x_l and decreases in x_h and π_h. In other words, when financial friction increases, it becomes more expensive to use credit in transactions, and thus, there is a substitution of money for credit. Another interesting finding is that there is an amplification effect of financial friction on the credit quantities in the following sense. Suppose there is an increase in γ. The increase in γ makes the bank decrease the quantity of credit for large-transaction buyers x_h, for any quantity of the buyers' money holdings. However, this change in the optimal deposit contract makes buyers adjust their asset portfolios. In particular, each buyer's expected utility from holding the previous quantity of money balances increases relative to the cost at the margin. So, the buyer chooses to increase m at the optimum, which further reduces x_h. Because of the change in the buyer's money holdings, the decrease in the use of credit in large transactions is amplified.

3.4.4.2 Equilibrium without Unconnected Sellers ($\delta = 0$)

Consider a special case where all sellers are connected to the banking system, i.e., $\delta = 0$. In this case, the binding incentive contraint (3.30) becomes

$$\pi_h = \frac{u(x_h) - u(x_l)}{u(x_h)}, \tag{3.47}$$

so the buyer's money holdings are irrelevant to the incentive problem in a credit contract. Then, a stationary equilibrium can be characterized by the following proposition.

Proposition 4.4 *(i) Suppose $\gamma \in (0, \bar{\gamma})$ and $\delta = 0$. Then, there exists $\bar{\mu} \in (\beta, \infty)$ such that a stationary equilibrium can be characterized by*
a) $m \in (0, \bar{m})$, $x_l = x_h = x$, and $\pi_h = 0$ that satisfy

$$(1 - p)\{1 - u'(x)\} = p\{\theta u'(x + m) - 1\}, \tag{3.48}$$

$$\beta[1 - p + p\theta u'(x + m)] = \mu, \tag{3.49}$$

for $\mu \in (\beta, \bar{\mu}]$;
b) $m = 0$ and equations (3.8), (3.10) and (3.11) for $\mu \in (\bar{\mu}, \infty)$.
(ii) Suppose $\gamma \in [\bar{\gamma}, \infty)$ and $\delta = 0$. Then, a stationary equilibrium can be characterized by $m \in (0, \bar{m})$, $x_l = x_h = x$, and $\pi_h = 0$ that satisfy equations (3.48) and (3.49) for $\mu \in (\beta, \infty)$.

Proof See Appendix □

Proposition 4.4 states that, when money is useful in equilibrium, credit arrangements are not state-contingent and thus do not provide insurance to buyers against preference shocks. In contrast, when credit arrangements play an insurance role in equilibrium, money is not valued. This is because it would be too costly to hold state-contingent credit arrangements when money is a useful means of payment, while it would be too costly to hold money when credit arrangements provide state-contingent claims on goods to buyers. Money becomes useless if the cost of using money (the inflation rate) is too high. This occurs because credit arrangements can completely substitute for money as a means of payment although they are also costly. In other words, credit can drive out money in this case.

Note that, from (3.47), money is neutral to the incentive problem in credit arrangements if credit is always available. This leads to the following result.

Corollary 4.1 *If holding money is neutral to or tighten the incentive problem in credit contracts, then there does not exist an equilibrium where money and credit are both valued and credit contracts play an insurance role.*

In standard models, money usually tightens the incentive problem since a larger quantity of money holdings implies a lower incentive for borrowers to repay their debts. If that is the case in my model, money becomes too costly to use when credit arrangements provide insurance to buyers. In order to make both money and credit useful in the environment presented here, it is necessary that money mitigates the incentive problem in credit arrangements. Introducing an asymmetric means-of-payment shock provides one way to generate this feature in the model.

In any case, the optimal monetary policy is a Friedman rule, i.e, $\mu = \beta$. Under a Friedman rule, each buyer can consume x_l^* and x_h^* units of goods when the preference shock is low and high, respectively. However, the optimal monetary policy does not completely drive out credit, although credit is not essential in that the use of credit does not expand the set of feasible allocations in the economy when $\mu = \beta$.

3.5 Conclusion

As the use of credit in transactions is mostly available through third-party financial intermediation, it is important to understand the role of financial intermediation for the allocation of credit and its implications for monetary policy. To that end, I have built a model of money, credit, and financial intermediation that integrates two key frictions—asymmetric information and costly monitoring—and have analyzed how monetary policy or financial friction affects the functioning of money and credit as a means of payment.

Unlike standard models in which credit arrangements are constrained by the borrowers' inability to commit to repaying their debts, credit arrangements in my model are constrained by the agents' incentive to misreport their states and the costly monitoring technology. I have characterized different types of equilibria given different degrees of monetary and financial frictions. In an equilibrium where money is valued and credit arrangements provide an insurance role, a higher money growth rate implies a smaller quantity of money holdings of buyers, a smaller quantity of credit for small transactions, and a larger quantity of credit for large transactions. Also, I have shown that there is an amplification effect of financial friction on the quantity of credit for large transactions in equilibrium.

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Appendix A

Appendices to Chapter 1

A.1 Equilibrium with Deflation and No Theft

In this section, I confine attention to stationary equilibria where there is deflation and the cost of theft is sufficiently high. Due to deflation, the real value of currency increases over time and private banks can acquire a sufficient amount of currency in the CM. Private banks are no longer indifferent between acquiring currency from other private individuals and withdrawing currency from the central bank because they do not need to bear the cost of withdrawing currency. Instead, sellers must be indifferent between depositing their currency with the central bank and side trading with private banks. So, one unit of real currency must be exchanged for one unit of good in the CM in equilibrium. From (1.7) and (1.28), a necessary condition for theft to not take place is given by

$$\kappa \geq \frac{\rho(x^c + \beta\mu)}{\beta}.$$

As the price of real currency is one instead of η in equilibrium, η's in equations (1.6)-(1.12), (1.17)-(1.22) must be replaced by one. Then, from (1.19) and (1.21), no arbitrage condition for private banks from holding currency across periods can be expressed by

$$R^m \geq \frac{1}{1 + \beta\gamma}.$$

That is, the nonpar exchange rate η is irrelevant to the effective lower bound (ELB). This occurs because there are no currency withdrawals from the central bank cash window. Also, it is obvious that the nonpar exchange rate does not affect equilibrium prices and allocations.

The inflation rate in this equilibrium can be written as

$$\pi = \beta\left[u'(x^c) - \delta u'(x^d) + \delta\right].$$

To observe deflation in equilibrium, β and $u'(x^c)$ must be sufficiently low while δ and $u'(x^d)$ must be sufficiently high. It turns out that v must be sufficiently high and R^m is sufficiently low to support deflation in equilibrium. Note that in the body of the paper I focus on cases with sufficiently low v and γ so that there is no deflation in equilibrium for any R^m that is higher than the ELB.

A.2 Omitted Proofs

Proof of Lemma 1: First, consider the case where $\eta = 1$. In this case, sellers are indifferent between depositing the currency with the central bank and trading it with a private bank only if there is no theft, i.e., $\alpha^b = 0$. Also, $\alpha^s = 0$ is optimal for all $\alpha^b \in (0, 1]$. Suppose that $\alpha^b \in (0, 1]$ in equilibrium. Then, sellers would always choose to deposit their currency with the central bank, implying $\alpha^s = 0$. Then, there would be no incentives for buyers to acquire the theft technology by incurring κ units of labor, which contradicts with $\alpha^b \in (0, 1]$. Therefore, there must be no theft in equilibrium, if an equilibrium exists. Now, suppose that $\alpha^b = 0$ in equilibrium. A necessary condition for this equilibrium to exist is $\kappa \geq \rho\alpha^s\eta c^s$. If $\kappa > \rho\eta c^s$, then $\alpha^b = 0$ is optimal for buyers for any given $\alpha^s \in [0, 1]$. If $\kappa = \rho\eta c^s$, buyers are indifferent between acquiring the theft technology and not doing anything in the TM. In this case, an equilibrium exists only if $\alpha^b = 0$. If $\kappa < \rho\eta c^s$, then equilibrium exists only if the fraction of sellers who carry currency in the TM is sufficiently low. Since a necessary condition for the absence of theft is $\kappa \geq \rho\alpha^s\eta c^s$, an equilibrium exists with $\alpha^s \in [0, \bar{\alpha}^s]$ where $\bar{\alpha}^s = \frac{\kappa}{\rho\eta c^s}$. Therefore, given that $\eta = 1$, there exist a continuum of equilibria with $\alpha^b = 0$ and $\alpha^s \in [0, 1]$ for $\kappa \geq \rho\eta c^s$ and a continuum of equilibria with $\alpha^b = 0$ and $\alpha^s \in [0, \bar{\alpha}^s]$ for $\kappa < \rho\eta c^s$ where $\bar{\alpha}^s = \frac{\kappa}{\rho\eta c^s}$.

Next, I consider the case where $\eta > 1$. Suppose that $\alpha^b = 0$ in equilibrium. Then, this leads to $\alpha^s = 1$ as sellers would strictly prefer to trade currency with a private bank rather than depositing it with the central bank. A necessary condition for this equilibrium to exist is $\kappa \geq \rho\eta c^s$. Therefore, a unique equilibrium exists with $\alpha^b = 0$ and $\alpha^s = 1$ for $\kappa \geq \rho\eta c^s$. If $\kappa < \rho\eta c^s$, then $\alpha^b = 0$ cannot be supported in equilibrium as there are incentives for buyers to acquire the theft technology, given that $\alpha^s = 1$. However, $\alpha^b = 1$ cannot be an equilibrium as well because $\alpha^b = 1$ would lead to $\alpha^s = 0$, and then, there would be no incentives for buyers to acquire the theft technology. An equilibrium exists if and only if buyers and sellers are both indifferent between their own options. This implies that, from (1.8) and (1.11),

$$\alpha^b = \frac{\eta - 1}{\eta}, \tag{A.1}$$

$$\alpha^s = \frac{\kappa}{\rho\eta c^s}. \tag{A.2}$$

Therefore, there exists a unique equilibrium with (A.1) and (A.2) for $\kappa < \rho\eta c^s$.

Proof of Proposition 1: Note that, in equilibrium, equations (1.31) and (1.33) solve for x^c and x^d. Confine attention to the comparative statics analysis with respect to R^m. I totally differentiate (1.31) and (1.33) and evaluate the derivatives of x^c and x^d for $(R^m, \eta) = (1, 1)$ and $\mu = 0$ to obtain

$$\frac{dx^c}{dR^m} = \frac{(1-\rho)[(1-\delta)u'(x)+\delta][(1-\delta)(1-\sigma)u'(x)+\delta]}{u''(x)[(1-\delta)(1-\sigma)u'(x)+\delta+\rho\beta\mu(1-\delta)u''(x)]} < 0,$$
$$\frac{dx^d}{dR^m} = \frac{-\rho[(1-\delta)u'(x)+\delta][(1-\delta)(1-\sigma)u'(x)+\delta+\beta\mu(1-\delta)u''(x)]}{u''(x)[(1-\delta)(1-\sigma)u'(x)+\delta+\rho\beta\mu(1-\delta)u''(x)]} > 0,$$

where $x^c = x^d = x$. Then, it is immediate that from (1.30) and (1.32) r^m, r^b, and π increase, and from the first argument in (1.35) the ELB remains unchanged.

Now, I turn my attention to the comparative statics analysis with respect to η. For convenience, let $\sigma = -\frac{xu''(x)}{u'(x)}$ so that $\sigma \in (0, 1)$. Then, evaluating the derivatives of x^c and x^d with respect to η for $(R^m, \eta) = (1, 1)$ and $\mu = 0$ yields

$$\frac{dx^c}{d\eta} = -\frac{\delta\{\rho\sigma u'(x)+(1-\rho)[(1-\delta)(1-\sigma)u'(x)+\delta][u'(x)-1]-\beta\mu\rho u''(x)\}}{u''(x)[(1-\delta)(1-\sigma)u'(x)+\delta+\rho\beta\mu(1-\delta)u''(x)]} > 0,$$
$$\frac{dx^d}{d\eta} = \frac{\rho\delta[(1-\sigma)u'(x)-1+\beta\mu u''(x)][(1-\delta)u'(x)+\delta]}{u''(x)[(1-\delta)(1-\sigma)u'(x)+\delta+\rho\beta\mu(1-\delta)u''(x)]}.$$

Also, I evaluate equation (1.33) for $(R^m, \eta) = (1, 1)$ and $\mu = 0$ to obtain

$$\left[u'(x) + \frac{\delta}{1-\delta}\right](x + \rho\beta\mu) = v, \tag{A.3}$$

where x is increasing in v. Collateral constraint (A.3) does not bind in equilibrium if

$$v \geq \frac{x^* + \rho\beta\mu}{1-\delta}. \tag{A.4}$$

Let $\bar{v}$ denote the right-hand side of inequality (A.4), $\hat{x}$ denote the solution to $u'(x) = \frac{1}{1-\sigma}$, and $\hat{v}$ denote the solution to (A.3) when $x = \hat{x}$. Then, I can write the derivatives of x^d with respect to η as

$$\frac{dx^d}{d\eta} \leq 0, \qquad \text{if} \quad v \in (0, \hat{v}]$$
$$\frac{dx^d}{d\eta} > 0. \qquad \text{if} \quad v \in (\hat{v}, \bar{v})$$

So, from (1.30), an increase in η decrease r^m and r^b for $v \in (0, \hat{v}]$ and increase r^m and r^b for

$v \in (\hat{v}, \bar{v})$. From (1.32) or

$$\pi = \beta R^m \left[u'(x^d) - \delta u'(x^d) + \delta \right],$$

an increase in η increases π for $v \in (0, \hat{v}]$ and decreases π for $v \in (\hat{v}, \bar{v})$. Finally, from the first argument in (1.35), the ELB falls.

Proof of Proposition 2: Notice that the consumption quantities in the DM, x^c and x^d, are determined by equations (1.37) and (1.39). I can rewrite equation (1.39) in the form

$$F(x^c, x^d) = v, \tag{A.5}$$

and show that the function $F(\cdot, \cdot)$ is strictly increasing in both $0 \leq x^c < x^*$ and $0 \leq x^d < x^*$ because $-x\frac{u''(x)}{u'(x)} < 1$. This property implies that equation (A.5) can be depicted by a downward-sloping locus in (x^c, x^d) space, given v. Also, equation (1.37) can be depicted by an upward-sloping locus in (x^c, x^d) space, given (R^m, η).

For the comparative statics, suppose there is an increase in R^m with η held constant. It is straightforward that an increase in R^m decreases x^c and increases x^d from (1.37) and (1.39). Then, from (1.36) and (1.38), π rises and real interest rates (r^m, r^b) rise. From (1.27), (1.40), and (1.41), α^s increases but α^b and the ELB do not change. Next, suppose that there is an increase in η with R^m remaining constant. Then, from (1.37) and (1.39), x^c decreases and x^d increases. From (1.38), real interest rates (r^m, r^b) rise. Using (1.37), (1.36) can be written as

$$\pi = \beta R^m \left[u'(x^d) - \delta u'(x^d) + \delta \right],$$

so π falls. From (1.27) and (1.40), α^b increases and the ELB falls. However, from (1.41), the effect on α^s is ambiguous since ηx^c can increase or decrease depending on parameters.

Proof of Proposition 3: The proof involves two steps. First, I will search for monetary policies that maximize the welfare measure $\mathcal{W}$, taking α^b as exogenously given. Then, I will determine the optimal monetary policy considering that α^b is endogenously determined in response to a change in monetary policy.

In the first step, I solve the following maximization problem given $\alpha^b \in [0, 1]$:

$$\max_{(R^m, \eta)} \rho \left[u\left(x^c\right) - x^c \right] + (1 - \rho) \left[u\left(x^d\right) - x^d \right] - \alpha^b \kappa \tag{A.6}$$

subject to

$$\eta R^m = \frac{u'\left(x^c\right) - \delta u'\left(x^d\right) + \delta}{u'\left(x^d\right) - \delta u'\left(x^d\right) + \delta}, \tag{A.7}$$

$$\left[u'(x^c) + \frac{\delta}{1-\delta}\right]\rho(x^c + \beta\mu) + \left[u'\left(x^d\right) + \frac{\delta}{1-\delta}\right](1-\rho)x^d = v, \tag{A.8}$$

$$R^m \geq \frac{1}{\eta + \beta\gamma}, \quad \eta \geq 1 \tag{A.9}$$

Note that a monetary policy measure that is relevant to welfare in equilibrium is ηR^m. Let $\Omega \equiv \eta R^m$ denote the policy measure. Then, from (A.9), Ω must satisfy that $\Omega \geq \frac{\eta}{\eta+\beta\gamma}$. Differentiating the objective (A.6) with respect to Ω gives

$$\frac{d\mathcal{W}}{d\Omega} = \rho\left[u'(x^c) - 1\right]\frac{dx^c}{d\Omega} + (1-\rho)\left[u'(x^d) - 1\right]\frac{dx^d}{d\Omega}. \tag{A.10}$$

Let $\sigma = -\frac{xu''(x)}{u'(x)}$. Then, from totally differentiating (A.7) and (A.8), I obtain

$$\frac{dx^c}{d\Omega} = \frac{(1-\rho)\left[(1-\sigma)u'(x^d) + \frac{\delta}{1-\delta}\right]\left[(1-\delta)u'(x^d) + \delta\right]}{\Phi} < 0, \tag{A.11}$$

$$\frac{dx^d}{d\Omega} = \frac{-\rho\left[(1-\sigma)u'(x^c) + \frac{\delta}{1-\delta} + \beta\mu u''(x^c)\right]\left[(1-\delta)u'(x^d) + \delta\right]}{\Phi} > 0, \tag{A.12}$$

where

$$\Phi = (1-\rho)u''(x^c)\left[(1-\sigma)u'(x^d) + \frac{\delta}{1-\delta}\right] + \rho u''(x^d)\left[(1-\delta)\Omega + \delta\right]\left[(1-\sigma)u'(x^c) + \frac{\delta}{1-\delta} + \beta\mu u''(x^c)\right] < 0,$$

for a sufficiently low μ. Note that a monetary policy Ω attains a local optimum if the resulting consumption allocation x^c and x^d satisfy $\frac{d\mathcal{W}}{d\Omega} = 0$. From (A.10)-(A.12), I can characterize the optimal allocation x^c and x^d as follows:

$$\begin{aligned}
&\frac{d\mathcal{W}}{d\Omega} = 0, \\
&\Leftrightarrow \left[u'(x^c) - 1\right]\left[(1-\sigma)u'(x^d) + \frac{\delta}{1-\delta}\right] - \left[u'(x^d) - 1\right]\left[(1-\sigma)u'(x^c) + \frac{\delta}{1-\delta} + \beta\mu u''(x^c)\right] = 0, \\
&\Rightarrow \left[u'(x^c) - 1\right]\left[(1-\sigma)u'(x^d) + \frac{\delta}{1-\delta}\right] \leq \left[u'(x^d) - 1\right]\left[(1-\sigma)u'(x^c) + \frac{\delta}{1-\delta}\right], \\
&\Leftrightarrow \frac{u'(x^c) - 1}{(1-\sigma)u'(x^c) + \frac{\delta}{1-\delta}} \leq \frac{u'(x^d) - 1}{(1-\sigma)u'(x^d) + \frac{\delta}{1-\delta}}.
\end{aligned}$$

Since the function $F(x) = \frac{u'(x)-1}{(1-\sigma)u'(x)+\frac{\delta}{1-\delta}}$ is strictly decreasing in x, the above inequality is equivalent to $x^c \geq x^d$. Note that $x^c = x^d$ if $\Omega = 1$ and that x^c decreases and x^d increases as Ω rises. Therefore, the optimal monetary policy Ω must satisfy $\Omega \leq 1$ where the inequality holds with equality if and only if $\mu = 0$.

From the first step, I have shown that an optimal monetary policy must be a combination of (R^m, η) such that $\eta R^m \leq 1$. All the optimal combinations of monetary policy lead to the same gains from trade in the DM, that is, $\rho\left[u\left(x^c\right) - x^c\right] + (1-\rho)\left[u\left(x^d\right) - x^d\right]$. However, from (1.40), the fraction of buyers who choose to steal currency in the TM α^b increases as the exchange rate η rises. This implies that the welfare measure $\mathcal{W}$ is maximized if and only if $\eta = 1$. Therefore, the optimal monetary policy is given by $\eta = 1$ and $R^m \leq 1$.

Proof of Proposition 4: Suppose that the cost of theft κ is sufficiently high. Then, there is no theft or $\alpha^b = 0$ in equilibrium. To show that social welfare is increasing in η, suppose that the central bank sets the nominal interest rate on reserves R^m to obtain $x^c = x^d = x$ given an exchange rate between currency and reserves η. Such policy needs not be optimal but it helps understand the optimal level of the exchange rate η. Equations (1.31) and (1.33) can be rewritten as

$$R^m = R^b = \frac{\eta u'(x) - \delta u'(x) + \delta}{\eta\left[u'(x) - \delta u'(x) + \delta\right]}, \tag{A.13}$$

$$u'(x)\left[x + \rho\beta\mu\right] + \frac{\left[\rho + (1-\rho)\eta\right]\delta x + \rho\delta\beta\mu}{(1-\delta)\eta} = v. \tag{A.14}$$

In this case, equation (A.14) solves for x and then equation (A.13) solves for R^m. If the value of the consolidated government debt is sufficiencly low, or

$$v \leq \frac{\left[(1-\delta\rho)\eta + \delta\rho\right]x^* + \rho\beta\mu\left[(1-\delta)\eta + \delta\right]}{(1-\delta)\eta},$$

then x increases with η for a sufficiently low μ. Since the level of welfare is given by $\mathcal{W} = u(x) - x$ in this equilibrium, an increase in η effectively increases the level of welfare as long as the nominal interest rate R^m can be chosen to achieve $x^c = x^d = x$. But, from Proposition 1, an increase in η must be accompanied by an increase in R^m to attain the same consumption quantities across two types of DM transactions, implying that choosing R^m is not constrained by the ELB. Although the optimal R^m may not satisfy $x^c = x^d$, that the social welfare is increasing in η remains unchanged. Finally, η must be sufficiently low so that buyers do not have incentives to steal currency. Therefore, at the optimum, η is chosen so that buyers are indifferent between stealing currency and not stealing.

Now, suppose that there is no fixed cost of holding currency at the beginning of the TM, i.e., $\mu = 0$. Consider the following maximization problem given $\eta \geq 1$:

$$\max_{(R^m,\eta)} \rho\left[u\left(x^c\right)-x^c\right]+(1-\rho)\left[u\left(x^d\right)-x^d\right] \tag{A.15}$$

subject to

$$\eta R^m = \frac{\eta u'\left(x^c\right)-\delta u'\left(x^d\right)+\delta}{u'\left(x^d\right)-\delta u'\left(x^d\right)+\delta}, \tag{A.16}$$

$$\left[u'(x^c)+\frac{\delta}{(1-\delta)\eta}\right]\rho x^c+\left[u'\left(x^d\right)+\frac{\delta}{1-\delta}\right](1-\rho)x^d = v, \tag{A.17}$$

$$R^m \geq \max\left\{\frac{1}{\eta+\beta\gamma}, \frac{\eta}{(\eta+\beta\gamma)[(1-\delta)u'(x^d)+\delta]}\right\}. \tag{A.18}$$

I differentiate the objective (A.15) with respect to R^m to obtain

$$\frac{d\mathcal{W}}{dR^m} = \rho\left[u'(x^c)-1\right]\frac{dx^c}{dR^m}+(1-\rho)\left[u'(x^d)-1\right]\frac{dx^d}{dR^m}. \tag{A.19}$$

Letting $\sigma = -\frac{xu''(x)}{u'(x)}$ and totally differentiating (A.16) and (A.17) gives

$$\frac{dx^c}{dR^m} = \frac{\eta(1-\rho)\left[(1-\sigma)u'(x^d)+\frac{\delta}{1-\delta}\right]\left[(1-\delta)u'(x^d)+\delta\right]}{\Lambda} < 0,$$
$$\frac{dx^d}{dR^m} = \frac{-\eta\rho\left[(1-\sigma)u'(x^c)+\frac{\delta}{(1-\delta)\eta}\right]\left[(1-\delta)u'(x^d)+\delta\right]}{\Lambda} > 0,$$

where

$$\Lambda = (1-\rho)\eta u''(x^c)\left[(1-\sigma)u'(x^d)+\frac{\delta}{1-\delta}\right] + \rho u''(x^d)\left[(1-\sigma)u'(x^c)+\frac{\delta}{(1-\delta)\eta}\right]\left[\eta R^m(1-\delta)+\delta\right] < 0.$$

Then, I can evaluate the derivative of $\mathcal{W}$ or equation (A.19) for $\eta R^m = 1$. Noting that $\eta u'(x^c) = u'(x^d)$ from (A.16), I obtain

$$\left.\frac{d\mathcal{W}}{dR^m}\right|_{\eta R^m=1} = \frac{\eta(1-\eta)\rho(1-\rho)\left[(1-\delta)u'(x^d)+\delta\right]}{\eta^2(1-\rho)u''(x^c)+\rho u''(x^d)} \geq 0, \tag{A.20}$$

implying that $\eta R^m \geq 1$ at an optimum.

Next, differentiate the objective (A.15) with respect to η to obtain

$$\frac{d\mathcal{W}}{d\eta} = \rho\left[u'(x^c)-1\right]\frac{dx^c}{d\eta}+(1-\rho)\left[u'(x^d)-1\right]\frac{dx^d}{d\eta}. \tag{A.21}$$

Totally differentiate (A.16) and (A.17) to get $\frac{dx^c}{d\eta}$ and $\frac{dx^d}{d\eta}$ and then evaluate the derivatives for $\eta R^m = 1$. This gives

$$\frac{dx^c}{d\eta} = \frac{\delta\rho x^c u''(x^d) - \delta\eta(1-\rho)\left[u'(x^d)-1\right]\left[(1-\delta)(1-\sigma)u'(x^d)+\delta\right]}{\eta\left[(1-\delta)(1-\sigma)u'(x^d)+\delta\right]\left[\rho u''(x^d)+\eta^2(1-\rho)u''(x^c)\right]} > 0,$$
$$\frac{dx^d}{d\eta} = \frac{\delta\rho\left\{\eta x^c u''(x^c) + \left[u'(x^d)-1\right]\left[(1-\delta)(1-\sigma)u'(x^d)+\delta\right]\right\}}{\eta\left[(1-\delta)(1-\sigma)u'(x^d)+\delta\right]\left[\rho u''(x^d)+\eta^2(1-\rho)u''(x^c)\right]}.$$

Using (A.21), I obtain

$$\left.\frac{d\mathcal{W}}{d\eta}\right|_{\eta R^m=1} = \frac{\Gamma + \rho\delta x^c\left\{\rho u''(x^d)\left[u'(x^c)-1\right] + \eta(1-\rho)u''(x^c)\left[u'(x^d)-1\right]\right\}}{\eta\left[(1-\delta)(1-\sigma)u'(x^d)+\delta\right]\left[\rho u''(x^d)+\eta^2(1-\rho)u''(x^c)\right]}, \tag{A.22}$$

where

$$\Gamma = \rho\delta(1-\rho)(\eta-1)\left[u'(x^d)-1\right]\left[(1-\delta)(1-\sigma)u'(x^d)+\delta\right] \geq 0.$$

From (A.22), the derivative of $\mathcal{W}$ is strictly positive if $\eta = R^m = 1$, i.e., $\left.\frac{d\mathcal{W}}{d\eta}\right|_{\eta=R^m=1} > 0$. This implies that the monetary policy at $\eta = R^m = 1$ is not optimal. Therefore, from (A.20) and (A.22), I conclude that the optimal monetary policy is away from a modified Friedman rule or $\eta R^m > 1$.

Proof of Proposition 5: Suppose that $\theta = 0$ in equilibrium. From (1.17)-(1.19) and (1.46),

$$\eta R^m\left[u'(x^d) - \delta u'(x^d) + \delta\right] = u'(x^o), \tag{A.23}$$
$$u'(x^o) = u'(x^c) - \delta u'(x^d) + \delta, \tag{A.24}$$

where (x^c, x^d) are the off-equilibrium consumption quantities in DM transactions, if a buyer were to participate in banking contracts. It can be shown that $\left|\frac{d[u'(x^d)-\delta u'(x^d)+\delta]}{d[\eta R^m]}\right| < 1$, so from (A.23) x^o increases with a decrease in ηR^m. However, the limited quantity of collateral $v < x^* + \beta\mu$ implies that, from (1.60), the highest possible quantity for x^o is $\bar{x}$ that solves $(\bar{x} + \beta\mu)u'(\bar{x}) = v$ and $\bar{x} < x^*$. So, any ηR^m that leads to x^o higher than $\bar{x}$ cannot be supported in equilibrium, implying that, from (A.23),

$$R^m \geq \frac{u'(\bar{x})}{\eta\left[u'(x^d) - \delta u'(x^d) + \delta\right]}, \tag{A.25}$$

where x^d is the off-equilibrium consumption quantity in DM transactions using bank claims that is consistent with (A.24). Also, any R^m higher than the right-hand side of (A.25) implies that

$0 < \theta \leq 1$ and $U^b \geq U^o$. So, by continuity, the off-equilibrium consumption quantities (x^c, x^d) when $R^m = \frac{u'(\bar{x})}{\eta\left[u'(x^d)-\delta u'(x^d)+\delta\right]}$ must satisfy (A.24) and $U^b = U^o$ from (1.48)-(1.49) given $x^o = \bar{x}$.

Recall that the nominal interest rate R^m must satisfy (1.43). That is, there must be no arbitrage opportunities from carrying currency across periods in equilibrium. To prove that the lower bound on the nominal interest rate in inequality (A.25) is always higher than the lower bound in (1.43), I claim that the following condition holds in equilibrium if the fixed cost of holding currency μ is close to zero:

$$\frac{u'(\bar{x})}{u'(x^d) - \delta u'(x^d) + \delta} > 1. \tag{A.26}$$

Suppose $\eta R^m = 1$ in equilibrium, so that deposit contracts effectively allow buyers to consume the same quantity of goods across two types of DM transactions, i.e., $x^c = x^d = x$. Then, the quantity of DM consumption for buyers opting out of deposit contracts is higher than the quantity for buyers holding deposit contracts $(x^o > x)$ since, from (A.23)-(A.24),

$$u'(x^o) = (1 - \delta)u'(x) + \delta.$$

This implies that the expected utility for buyers opting out of contracts is higher than the expected utility for buyers opting in because from (1.48)-(1.49),

$$U^o - U^b = \left[u(x^o) - x^o u'(x^o)\right] - \left[u(x) - xu'(x)\right] - \beta\mu\left[u'(x^o) - \rho u'(x)\right] > 0,$$

for a sufficiently low μ. So, $\eta R^m = 1$ cannot be supported in equilibrium (a contradiction) as this policy would lead to a complete disintermediation, i.e., $\theta = 0$. To encourage banking activities, $\eta R^m > 1$ must be satisfied so that (A.26) must hold in equilibrium. Also, $\frac{u'(\bar{x})}{\eta\left[u'(x^d)-\delta u'(x^d)+\delta\right]} > \frac{1}{\eta+\beta\gamma}$ for any $\eta \geq 1$ because $\eta + \beta\gamma$ increases more than $\eta\left[u'(x^d) - \delta u'(x^d) + \delta\right]$ as η rises. Therefore, the effective lower bound on the nominal interest rate is determined by (A.25).

Proof of Proposition 6: First, note that an equilibrium with $\theta = 0$ exists only if R^m is set at the effective lower bound (ELB) defined in (1.58). As mentioned in Proof of Proposition 5, any R^m higher than the ELB implies that $x^o < \bar{x}$ where $\bar{x}u'(\bar{x}) = v$. Since $x^o u'(x^o) < v$ and the collateral constraint must bind in an equilibrium where v is sufficiently low, there must be some buyers participating in banking contracts, i.e., $\theta > 0$. So, for any R^m that is higher than the ELB, $\theta > 0$ in equilibrium.

Next, consider an equilibrium with $0 < \theta < 1$. Then, (x^c, x^d, x^o, θ) must satisfy:

$$\eta R^m = \frac{u'(x^c) - \delta u'(x^d) + \delta}{u'(x^d) - \delta u'(x^d) + \delta}, \tag{A.27}$$

$$(1-\rho)\theta x^d \left[u'(x^d) + \frac{\delta}{1-\delta}\right] + \rho\theta x^c \left[u'(x^c) + \frac{\delta}{1-\delta}\right] + (1-\theta)x^o u'(x^o) = v, \tag{A.28}$$

$$u'(x^o) = u'(x^c) - \delta u'(x^d) + \delta, \tag{A.29}$$

$$\rho\left[u(x^c) - x^c u'(x^c)\right] + (1-\rho)\left[u(x^d) - x^d u'(x^d)\right] = u(x^o) - x^o u'(x^o). \tag{A.30}$$

Suppose that there is an increase in ηR^m. Then, from (A.27), x^c decreases and x^d increases as ηR^m rises. From (A.29), x^o decreases as x^c decreases and x^d increases. Also, from (A.29), a necessary condition for this equilibrium to exist is $x^c < x^d$, which implies that U^b decreases as x^c falls and x^d rises. Since the left-hand side of (A.30) decreases as x^c falls and x^d rises, x^o must fall in equilibrium. Then, from (A.28), θ must rise in equilibrium. The effects of an increase in R^m or an increase in η on $(\pi, \alpha^b, \alpha^s)$ are straightforward from (1.61), (1.64), and (1.65).

As an increase in ηR^m decreases x^c and x^o and increases x^d and θ, there exists $\Omega = \eta R^m$ that satisfies equation (A.27) where x^c and x^d are the solutions to equations (A.28)-(A.30) when $\theta = 1$. Therefore, I can conclude that, in equilibrium, $0 \leq \theta < 1$ if $\frac{u'(\bar{x})}{u'(\underline{x}^d) - \delta u'(\underline{x}^d) + \delta} \leq \eta R^m < \Omega$ and $\theta = 1$ if $\eta R^m \geq \Omega$ where $\bar{x}$ and $\underline{x}^d$ are the quantities defined in Proof of Proposition 5.

Proof of Proposition 7: In Proof of Proposition 6, I have shown that, for any $\eta R^m > \frac{u'(\bar{x})}{u'(\underline{x}^d) - \delta u'(\underline{x}^d) + \delta}$, the fraction θ is positive and $x^c < x^d$ in equilibrium. This implies that the left-hand side and the right-hand side of (A.30) both increase as x^c and x^o rise and x^d falls. So, given η, lowering R^m increases the welfare measure $\mathcal{W}$ because it increases x^c and x^o and decreases x^d and θ. Then, by continuity, the maximum $\mathcal{W}$ can be obtained when $\eta R^m = \frac{u'(\bar{x})}{u'(\underline{x}^d) - \delta u'(\underline{x}^d) + \delta}$ given η. However, if the central bank conducts monetary policy (R^m, η) such that $\eta R^m = \frac{u'(\bar{x})}{u'(\underline{x}^d) - \delta u'(\underline{x}^d) + \delta}$, a higher η only implies a higher α^b without increasing the sum of surpluses from trade in the CM and the DM. As a higher α^b leads to a larger total cost of theft, the welfare measure $\mathcal{W}$ is maximized if and only if $\eta = 1$ and $R^m = \frac{u'(\bar{x})}{u'(\underline{x}^d) - \delta u'(\underline{x}^d) + \delta}$.

Appendix B

Appendices to Chapter 2

B.1 A Foreign Bank's Problem

Similarly to problem (2.1) subject to (2.2)-(2.4), a Foreign bank's problem in equilibrium can be expressed as

$$\max_{k^*,c^*,d^*,b_{fs},b_{fl},b^*_{fs},b^*_{fl}} -k^* + \rho u\left(\frac{\beta\phi^*_{+1}c^*}{\phi^*}\right) + (1-\rho)u\left(\beta d^*\right)$$

subject to

$$\begin{aligned}
&k^* - \rho c^* - z_s b_{fs} - z^*_s b^*_{fs} - z_l b_{fl} - z^*_l b^*_{fl} - \beta(1-\rho)d^* \\
&+ \beta\frac{\phi_{+1}}{\phi}\left\{b_{fs} + (1+z_{l,+1})b_{fl}\right\} + \beta\frac{\phi^*_{+1}}{\phi^*}\left\{b^*_{fs} + (1+z^*_{l,+1})b^*_{fl}\right\} \geq 0, \\
&-(1-\rho)d^* + \frac{\phi_{+1}}{\phi}\left\{b_{fs} + (1+z_{l,+1})b_{fl}\right\} + \frac{\phi^*_{+1}}{\phi^*}\left\{b^*_{fs} + (1+z^*_{l,+1})b^*_{fl}\right\} \\
&\geq \frac{\phi_{+1}}{\phi}\left\{\theta_{fs}b_{fs} + (1+z_{l,+1})\theta_{fl}b_{fl}\right\} + \frac{\phi^*_{+1}}{\phi^*}\left\{\theta^*_{fs}(\rho c^* + b^*_{fs}) + (1+z^*_{l,+1})\theta^*_{fl}b^*_{fl}\right\}, \\
&k^*, c^*, d^*, b_{fs}, b_{fl}, b^*_{fs}, b^*_{fl} \geq 0,
\end{aligned}$$

where (k^*, c^*, d^*) is the deposit contract of the Foreign bank, which is analogous to (k, c, d) of the Home bank, and b^*_{fs} and b^*_{fl} (b_{fs} and b_{fl}) are, respectively, short-term and long-term Foreign (Home) bonds acquired by the Foreign bank.

B.2 Discussions on Alternative Fiscal and Monetary Policies

The fiscal policy rule presented in this paper can be interpreted as the debt ceiling or debt limit in the United States. To understand this, let $\hat{b}_s$ and $\hat{b}_l$ denote the Home central bank's holdings of short-term and long-term Home bonds, respectively, in period 0. The Home central bank purchases Home and Foreign bonds by issuing Home currency in period 0 and then transfers its profits to the Home fiscal authority in every following period. As central bank capital is zero, the real value of the central bank's assets must be equal to that of liabilities, that is,

$$\sum_{i=s,l} \left[z_i \hat{b}_i + z_i^* a_i^* \right] = \bar{c},$$

where a_i^* is the real quantity of Foreign bonds held by the central bank for $i = s, l$ and $\bar{c}$ is the real value of Home currency outstanding. Then, using equation (2.5), the fiscal policy rule in the Home country can be rewritten as

$$V = \sum_{i=s,l} z_i \left[\bar{b}_i + \hat{b}_i \right].$$

That is, the Home fiscal authority sets the total value of Home bonds issued in period 0. If $\bar{V}$ is the level of debt ceiling, then I can show that a welfare-maximizing fiscal authority would set the value of Home bonds at $\bar{V}$ in equilibrium, provided that $\bar{V}$ is sufficiently small.

In practice, central banks set a short-term nominal interest rate (typically, an overnight rate or an interest rate on reserves) to achieve a desired rate of inflation. So, conventional monetary policy in this paper is consistent with reality because central banks choose exogenously short-term nominal interest rates R_s and R_s^* so that inflation rates μ and μ^* are endogenously determined. This setup also allows me to study the effects of other types of monetary policies—quantitative easing (QE) and foreign exchange (FX) intervention—given conventional monetary policy rates. For example, it enables analyzing the international effect of QE when short-term nominal interest rates are contrained by the zero lower bound, as in Alpanda and Kabaca (2020) and Kolasa and Wesolowski (2020). Also, using the current setup, I can differentiate between a sterilized FX intervention and a nonsterilized one where the former requires a change in the short-term nominal interest rate to hold the inflation rate fixed and the latter does not. Nevertheless, there may be readers interested in the effects of QE and FX intervention in an economy where inflation rates are exogenously set by central banks (and thus, R_s and R_s^* are endogenous). In this model, doing so is straightforward as constant inflation rates imply constant consumption quantities in DM currency transactions x_1 and x_1^*. Therefore, the effects on asset prices discussed in the paper will be amplified given constant inflation rates.

In this model, fiscal authorities exogenously determine V and V^*, the real values of consolidated government liabilities held by the public. So, levels of taxes τ and τ^* are determined endogenously. However, fiscal authorities might choose the quantity of government debt outstanding given the tax levels. So, one may want to think of τ and τ^* as exogenous variables, and V and V^* as endogenous variables following the fiscal-theory-of-the-price-level literature (See Leeper, 1991). A main advantage of the current setup is that, given V and V^*, the balance sheets of central banks and the real values of government bonds held by private banks are well defined. This in turn allows me to analyze how monetary policies change the composition of assets held by private banks given V and V^* in a tractable way.

B.3 Foreign Bond Yields and Term Premia

Note that (2.19) and (2.21) hold with equality. So, the nominal yield on Foreign bonds of each maturity can be expressed as

$$R_j^* = \frac{\mu^*}{\beta[(1-\theta_{fj}^*)u'\left(x_2^*\right)+\theta_{fj}^*]} - 1, \tag{B.1}$$

for $j = s, l$. As a term premium is the difference between long-term and short-term bond yields, the nominal term premium for Foreign bonds can be expressed as

$$R_l^* - R_s^* = \frac{\mu^*(\theta_{fl}^* - \theta_{fs}^*)\left[u'\left(x_2^*\right)-1\right]}{\beta[(1-\theta_{fl}^*)u'\left(x_2^*\right)+\theta_{fl}^*][(1-\theta_{fs}^*)u'\left(x_2^*\right)+\theta_{fs}^*]}. \tag{B.2}$$

A liquidity premium is the difference between the fundamental yield and the actual yield on a particular asset. Since the fundamental yield on Foreign bonds is given by $\frac{\mu^*}{\beta} - 1$, the liquidity premia for Foreign bonds can be expressed as

$$L_j^* = \frac{\mu^*}{\beta} - 1 - R_j^* = \frac{\mu^*(1-\theta_{fj}^*)\left[u'\left(x_2^*\right)-1\right]}{\beta[(1-\theta_{fj}^*)u'\left(x_2^*\right)+\theta_{fj}^*]},$$

for $j = s, l$. The associated real bond yields, real term premium, and real liquidity premia can be expressed as

$$r_j^* = \frac{1}{\beta[(1-\theta_{fj}^*)u'\left(x_2^*\right)+\theta_{fj}^*]} - 1, \tag{B.3}$$

$$r_l^* - r_s^* = \frac{(\theta_{fl}^* - \theta_{fs}^*)\left[u'\left(x_2^*\right)-1\right]}{\beta[(1-\theta_{fl}^*)u'\left(x_2^*\right)+\theta_{fl}^*][(1-\theta_{fs}^*)u'\left(x_2^*\right)+\theta_{fs}^*]}, \tag{B.4}$$

$$l_j^* = \frac{1}{\beta} - 1 - r_j^* = \frac{(1-\theta_{fj}^*)\left[u'\left(x_2^*\right)-1\right]}{\beta[(1-\theta_{fj}^*)u'\left(x_2^*\right)+\theta_{fj}^*]},$$

for $j = s, l$.

B.4 Accumulating Long-Term Bonds as Foreign Exchange Reserves

Suppose that central banks can purchase long-term foreign government bonds as foreign exchange reserves, i.e., $\kappa_l^* > 0$ and $\kappa_l > 0$. Also, confine attention to cases discussed in Section 4: an equilibrium with segmented asset markets and an equilibrium with integrated asset markets where short-term Foreign bonds are held in both countries.

First, consider an equilibrium with segmented asset markets. I can obtain the following collateral constraints:

$$0 = \left[u'(x_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho x_1 + \left[u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)x_2 - \left\{V + \kappa_s^* - \kappa_s + \kappa_l^* - \Gamma\kappa_l - \frac{(\theta_{hl}-\theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(x_2)+\theta_{hl}\right]}\right\}, \quad \text{(B.5)}$$

for Home banks where

$$\Gamma = \frac{u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}}{u'(x_2) + \frac{\theta_{hl}}{1-\theta_{hl}}} < 1,$$

and

$$0 = \left[u'(x_1^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right]\rho x_1^* + \left[u'(x_2^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right](1-\rho)x_2^* - \left\{V^* + \kappa_s - \kappa_s^* + \kappa_l - \Gamma^*\kappa_l^* - \frac{(\theta_{fl}^*-\theta_{fs}^*)\omega_l^*}{(1-\theta_{fs}^*)\left[(1-\theta_{fl}^*)u'\left(x_2^*\right)+\theta_{fl}^*\right]}\right\}, \quad \text{(B.6)}$$

for Foreign banks where

$$\Gamma^* = \frac{u'(x_2) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}}{u'(x_2) + \frac{\theta_{fl}^*}{1-\theta_{fl}^*}} < 1.$$

Then, equations (2.32) and (B.5) determine x_1 and x_2 in equilibrium, while (2.36) and (B.6) determine x_1^* and x_2^*. In this case, an increase in κ_l^*, the value of long-term Foreign bonds held by the Home central bank, has the same qualitative effects as does an increase in κ_s^*, the value of short-term Foreign bonds held by the Home central bank. That is, x_1, x_2, W, r_s, r_l, μ^*, R_l^*,$R_l^* - R_s^*$, and $r_l^* - r_s^*$ increase while x_1^*, x_2^*, W^*, μ, R_l, $R_l - R_s$, $r_l - r_s$, r_s^*, r_l^*, and $\frac{e_{+1}}{e}$ decrease. Notice that the effects of an increase in κ_l^* on the Home country are quantitatively identical to those of an increase in κ_s^*, but the effects on the Foreign country are quantitatively smaller than those of an increase in κ_s^* because $\Gamma^* < 1$ in (B.6).

Next, consider an equilibrium where short-term Foreign bonds are held in both countries. In this equilibrium, the integrated collateral constraint can be expressed as

$$\mathcal{F}(x_k, x_k^*, \omega_l, \omega_l^*, \kappa_i, \kappa_i^*, V, V^*) = \mathcal{D}(x_k, x_k^*) - \mathcal{S}(x_2, x_2^*, V, V^*, \omega_l, \omega_l^*, \kappa_i^*, \kappa_i) = 0, \quad \text{(B.7)}$$

for $i = s, l$ and $k = 1, 2$ where $\mathcal{D}$ is the aggregate demand for collateral, identical to the one in Section 4.4, and $\mathcal{S}$ is the aggregate supply of collateral given by

$$\mathcal{S} = V + \Omega V^* + (1 - \Omega)(\kappa_s^* - \kappa_s) - (\Gamma - \Omega)\kappa_l + (1 - \Omega\Gamma^*)\kappa_l^* \\ - \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1 - \theta_{hs})\,[(1 - \theta_{hl})u'\,(x_2) + \theta_{hl}]} - \frac{\Omega(\theta_{fl}^* - \theta_{fs}^*)\omega_l^*}{(1 - \theta_{fs}^*)[(1 - \theta_{fl}^*)u'\left(x_2^*\right) + \theta_{fl}^*]}. \quad \text{(B.8)}$$

Then, equations (2.32), (2.36), (2.39), and (B.8) determine the DM consumption quantities in equilibrium, i.e., x_k and x_k^* for $k = 1, 2$. In this case, an increase in κ_l^* leads to an increase in x_k, x_k^*, r_i, r_i^*, W, and W^* and a decrease in μ, μ^*, R_l, R_l^*, $R_l - R_s$, $R_l^* - R_l^*$, $r_l - r_s$, $r_l^* - r_s^*$, and $\frac{e+1}{e}$ for $k = 1, 2$ and $i = s, l$. So, the effects of an increase in κ_l^* are qualitatively the same as, but quantitatively larger than, those of an increase in κ_s^* since $1 - \Omega\Gamma^* > 1 - \Omega$. However, the effects of an increase in κ_l (the Foreign central bank's holdings of long-term Home bonds) depend on θ_{hs}^* and θ_{hl}. If long-term Home bonds are more pledgeable than short-term Foreign bonds for Home banks ($\theta_{hs}^* > \theta_{hl}$), then $\Gamma - \Omega > 0$. In this case, an increase in κ_l decreases the supply of collateral in the global economy as does an increase in κ_s. So, x_k, x_k^*, r_i, r_i^*, W, and W^* decrease while μ, μ^*, R_l, R_l^*, $R_l - R_s$, $R_l^* - R_l^*$, $r_l - r_s$, $r_l^* - r_s^*$, and $\frac{e+1}{e}$ increase for $k = 1, 2$ and $i = s, l$. In contrast, if short-term Foreign bonds are more pledgeable than long-term Home bonds for Home banks ($\theta_{hs}^* < \theta_{hl}$), then $\Gamma - \Omega < 0$ and an increase in κ_l effectively increases the supply of collateral in the global economy. Therefore, the effects of an increase in κ_l are opposite to those of an increase in κ_s.

B.5 Other Types of Equilibrium with Integrated Asset Markets

B.5.1 Equilibrium with $\lambda \in (\frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*}, \frac{(1-\theta_{fl}^*)\lambda^*}{1-\theta_{hl}^*})$

In this equilibrium, Home bonds and short-term Foreign bonds are held only by Home banks, while long-term Foreign bonds are held only by Foreign banks. Then, first-order conditions (2.11)-(2.13), and (2.21) must hold with equality in equilibrium. These equations can be rewritten as (2.32)-(2.34), (2.37), and (2.38). From (2.10), (2.12), (2.16), and (2.17), I obtain the following equation:

$$z_s^* = \frac{u'(x_2) - \theta_{hs}^* u'(x_2) + \theta_{hs}^*}{u'\left(x_1^*\right) - \theta_{fs}^* u'\left(x_2^*\right) + \theta_{fs}^*}. \tag{B.9}$$

Also, from (2.5), (2.6), (2.15), (2.23), (2.32)-(2.34), (2.37)-(2.38), and (B.9), noting that $b_{fj} = b_{fs}^* = b_{hl}^* = 0$ for $j = s, l$, the Home bank's collateral constraint can be rewritten as

$$\begin{aligned} 0 = &\left[u'(x_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho x_1 + \left[u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)x_2 - \{V + \Omega V^* + (1-\Omega)(\kappa_s^* - \kappa_s) \\ &-\Omega\omega_l^* - \left[u'(x_1^*) - \theta_{fs}^* u'(x_2^*) + \theta_{fs}^*\right]\Omega\rho x_1^* - \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(x_2) + \theta_{hl}\right]}\Bigg\}, \end{aligned} \tag{B.10}$$

where

$$\Omega = \frac{u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}}{u'(x_2) + \frac{\theta_{hs}^*}{1-\theta_{hs}^*}} < 1.$$

Finally, from (2.22), (2.37)-(2.38), the Foreign bank's collateral constraint can be rewritten as

$$\theta_{fs}^*\rho x_1^* + (1-\rho)x_2^* = \frac{\left(1-\theta_{fl}^*\right)\omega_l^*}{(1-\theta_{fl}^*)u'\left(x_2^*\right) + \theta_{fl}^*}. \tag{B.11}$$

As first-order conditions (2.14) and (2.18)-(2.20) do not hold with equality, a necessary condition for this equilibrium to exist is given by

$$\frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*} < \lambda < \frac{(1-\theta_{fl}^*)\lambda^*}{1-\theta_{hl}^*}. \tag{B.12}$$

Therefore, if (B.12) holds, an equilibrium can be characterized by equations (2.32)-(2.34), (2.37)-(2.38), and (B.9)-(B.11).

Proposition A.1 *There exists a nonempty set of parameter values that support a stationary equilibrium with binding collateral constraints that can be characterized by equations (2.32)-(2.34), (2.37)-(2.38), and (B.9)-(B.11).*

B.5.2 Equilibrium with $\lambda = \frac{(1-\theta_{fl}^*)\lambda^*}{1-\theta_{hl}^*}$

In this equilibrium, Home banks acquire all types of government bonds issued in two countries while Foreign banks acquire only long-term Foreign bonds. Then, first-order conditions (2.11)-

(2.14), and (2.21) must hold with equality in equilibrium. This leads to equations (2.32)-(2.34), (2.37)-(2.38), and (B.9). From (2.14) and (2.21), a necessary condition for this equilibrium to exist is given by

$$(1-\theta_{hl}^{*})\,u'(x_2)+\theta_{hl}^{*}=\left(1-\theta_{fl}^{*}\right)u'(x_2^{*})+\theta_{fl}^{*}. \tag{B.13}$$

Also, from (2.5), (2.6), (2.23), (2.32)-(2.34), (2.37), (2.38), (B.9), and (B.13), noting that $b_{fj}=b_{fs}^{*}=0$ for $j=s,l$, I can rewrite the Home and Foreign banks' collateral constraints as the form

$$\mathcal{D}(x_1,x_2,x_1^{*},x_2^{*})-\mathcal{S}(x_2,x_2^{*},V,V^{*},\omega_l,\omega_l^{*},\kappa_s^{*},\kappa_s)=0, \tag{B.14}$$

where $\mathcal{D}$ denotes the aggregate demand for collateral and $\mathcal{S}$ denotes the aggregate supply, implying that the excess demand in aggregate is zero in equilibrium. The aggregate demand for collateral is given by

$$\begin{aligned}\mathcal{D}=&\left[u'(x_1)+\frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho x_1+\left[u'(x_2)+\frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)x_2\\&+\left[u'(x_1^{*})-\theta_{fs}^{*}u'(x_2^{*})+\theta_{fs}^{*}\right]\Omega\rho x_1^{*}+\left[u'(x_2)+\frac{\theta_{hs}^{*}}{1-\theta_{hs}^{*}}\right]\left(\frac{1-\theta_{hl}^{*}}{1-\theta_{fl}^{*}}\right)\Omega\left[(1-\rho)x_2^{*}+\theta_{fs}^{*}\rho x_1^{*}\right],\end{aligned} \tag{B.15}$$

and the aggregate supply of collateral is given by

$$\begin{aligned}\mathcal{S}=&\,V+\Omega V^{*}+(1-\Omega)(\kappa_s^{*}-\kappa_s)\\&-\frac{(\theta_{hl}-\theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(x_2)+\theta_{hl}\right]}-\frac{\Omega(\theta_{hl}^{*}-\theta_{hs}^{*})\omega_l^{*}}{(1-\theta_{hs}^{*})[(1-\theta_{hl}^{*})u'(x_2)+\theta_{hl}^{*}]},\end{aligned} \tag{B.16}$$

where

$$\Omega=\frac{u'(x_2)+\frac{\theta_{hs}}{1-\theta_{hs}}}{u'(x_2)+\frac{\theta_{hs}^{*}}{1-\theta_{hs}^{*}}}.$$

Therefore, an equilibrium can be characterized by equations (2.32)-(2.34), (2.37)-(2.38), (B.9), and (B.13)-(B.14).

Proposition A.2 *There exists a nonempty set of parameter values that support a stationary equilibrium with binding collateral constraints that can be characterized by equations (2.32)-(2.34), (2.37)-(2.38), (B.9), and (B.13)-(B.14).*

B.6 Omitted Proofs

Proof of Proposition 1: Suppose that the sum of V and V^* is sufficiently large so that collateral constraints do not bind in equilibrium. In this case, from (2.15) and (2.22), the Lagrange multipliers to the collateral constraints must be zero, that is, $\lambda = \lambda^* = 0$. Then, from (2.9)-(2.14) and (2.16)-(2.21), I can obtain

$$\begin{aligned} x_1 &= (u')^{-1}\left[\frac{1}{z_s}\right], \\ x_2 &= \hat{x}, \\ \mu &= \frac{\beta}{z_s}, \\ z_l &= \frac{\beta}{\mu - \beta}, \end{aligned}$$

for the Home country and

$$\begin{aligned} x_1^* &= (u')^{-1}\left[\frac{1}{z_s^*}\right], \\ x_2^* &= \hat{x}, \\ \mu^* &= \frac{\beta}{z_s^*}, \\ z_l^* &= \frac{\beta}{\mu^* - \beta}, \end{aligned}$$

for the Foreign country. Also, the law of one prices must hold, implying that

$$\frac{e_{+1}}{e} = \frac{\mu}{\mu^*}.$$

Finally, a necessary condition for collateral constraints to not bind is given by

$$\begin{aligned} -(1-\rho)d - (1-\rho)d^* + \frac{1}{\mu}\left[-\theta_{hs}\rho c + (1-\theta_{hs})\, b_{hs} + (1+z_l)\,(1-\theta_{hl})\, b_{hl}\right] \\ + \frac{1}{\mu^*}\left[-\theta_{fs}^*\rho c^* + (1-\theta_{fs}^*)b_{fs}^* + (1+z_l^*)(1-\theta_{fl}^*)b_{fl}^*\right] \geq 0. \end{aligned}$$

Using the equilibrium consumption quantities and asset prices, together with the fiscal policies given by (2.5)-(2.6), the above inequality can be rewritten as

$$V + V^* \geq \rho\left(\frac{1}{z_s} + \frac{\theta_{hs}}{1-\theta_{hs}}\right)(u')^{-1}\left[\frac{1}{z_s}\right] + \rho\left(\frac{1}{z_s^*} + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right)(u')^{-1}\left[\frac{1}{z_s^*}\right] + \frac{(1-\rho)\hat{x}}{1-\theta_{hs}} + \frac{(1-\rho)\hat{x}}{1-\theta_{fs}^*} + \frac{(\theta_{hl}-\theta_{hs})\omega_l}{1-\theta_{hs}} + \frac{(\theta_{fl}^*-\theta_{fs}^*)\omega_l^*}{1-\theta_{fs}^*}. \quad \text{(B.17)}$$

Therefore, for an equilibrium with nonbinding collateral constraints to exist, the sum of V and V^* must be sufficiently large to satisfy the above inequality. □

Proof of Proposition 2: In order for equations (2.31)-(2.38) to characterize an equilibrium, V and V^* must be sufficiently small so that (B.17) does not hold. That is,

$$V + V^* < \rho\left(\frac{1}{z_s} + \frac{\theta_{hs}}{1-\theta_{hs}}\right)(u')^{-1}\left[\frac{1}{z_s}\right] + \rho\left(\frac{1}{z_s^*} + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right)(u')^{-1}\left[\frac{1}{z_s^*}\right] + \frac{(1-\rho)\hat{x}}{1-\theta_{hs}} + \frac{(1-\rho)\hat{x}}{1-\theta_{fs}^*} + \frac{(\theta_{hl}-\theta_{hs})\omega_l}{1-\theta_{hs}} + \frac{(\theta_{fl}^*-\theta_{fs}^*)\omega_l^*}{1-\theta_{fs}^*}. \quad \text{(B.18)}$$

Also, a necessary condition for this equilibrium to exist is given by

$$\lambda^* \leq \lambda < \frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*}.$$

Let $(\bar{x}_1, \bar{x}_2)$ denote the solution to (2.31)-(2.32) and $(\bar{x}_1^*, \bar{x}_2^*)$ denote the solution to (2.35)-(2.36). Then, from (2.10) and (2.17), the above condition can be rewritten as

$$\bar{x}_2 \leq \bar{x}_2^*,$$

and

$$u'(\bar{x}_2) < \frac{[(1-\theta_{fs}^*)u'(\bar{x}_2^*) + \theta_{fs}^*] - \theta_{hs}^*}{1-\theta_{hs}^*}.$$

The first inequality implies that

$$V \leq \left[u'(\dot{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\dot{x}_1 + \left[u'(\dot{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\dot{x}_2 - \kappa_s^* + \kappa_s + \frac{(\theta_{hl}-\theta_{hs})\,\omega_l}{(1-\theta_{hs})\,[(1-\theta_{hl})u'(\dot{x}_2) + \theta_{hl}]}, \quad \text{(B.19)}$$

where $(\dot{x}_1, \dot{x}_2)$ is the solution to

$$\dot{x}_2 = \bar{x}_2^*,$$
$$z_s = \frac{u'(\dot{x}_2) - \theta_{hs} u'(\dot{x}_2) + \theta_{hs}}{u'(\dot{x}_1) - \theta_{hs} u'(\dot{x}_2) + \theta_{hs}}.$$

The second inequality implies that

$$V > \left[u'(\tilde{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\tilde{x}_1 + \left[u'(\tilde{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\tilde{x}_2 - \kappa_s^* + \kappa_s + \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(\tilde{x}_2) + \theta_{hl}\right]}, \quad \text{(B.20)}$$

where $(\tilde{x}_1, \tilde{x}_2)$ is the solution to

$$u'(\tilde{x}_2) = \frac{[(1-\theta_{fs}^*)u'(\bar{x}_2^*) + \theta_{fs}^*] - \theta_{hs}^*}{1-\theta_{hs}^*},$$
$$z_s = \frac{u'(\tilde{x}_2) - \theta_{hs} u'(\tilde{x}_2) + \theta_{hs}}{u'(\tilde{x}_1) - \theta_{hs} u'(\tilde{x}_2) + \theta_{hs}},$$

Therefore, given V and V^* that satisfy (B.18), (B.19), and (B.20), there exists an equilibrium that can be characterized by equations (2.31)-(2.38). □

Proof of Proposition 3: The collateral constraints (2.31) and (2.35) can be expressed, respectively, as

$$C_{\mathcal{H}}(x_1, x_2, a_l, \kappa_s, \kappa_s^*, V) = 0, \quad \text{(B.21)}$$
$$C_{\mathcal{F}}(x_1^*, x_2^*, a_l^*, \kappa_s^*, \kappa_s, V^*) = 0, \quad \text{(B.22)}$$

where both functions $C_{\mathcal{H}}$ and $C_{\mathcal{F}}$ are strictly increasing in the first four arguments and strictly decreasing in the last two arguments. In equation (2.32), x_1 increases with an increase in x_2 and similarly, x_1^* increases with x_2^* in (2.36). Notice that, given fiscal/monetary policies, these four equations characterize equilibrium consumption quantities x_1, x_2, x_1^*, and x_2^*, and are illustrated in Figure B.1.

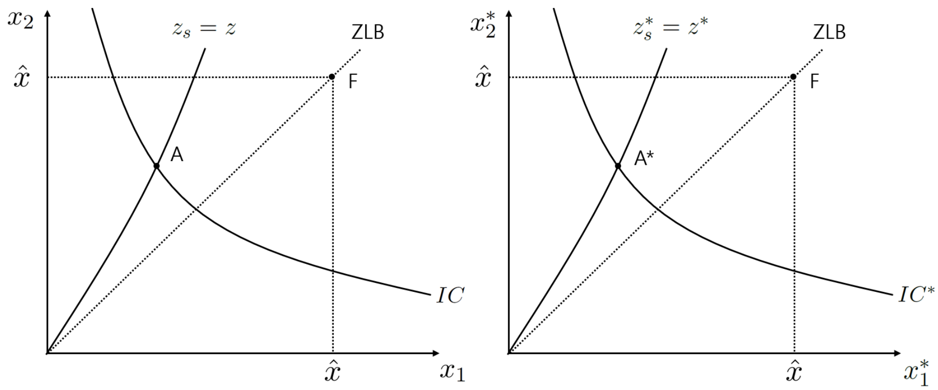

Figure B.1: Equilibrium with no international capital flows

In the left panel of the figure, the locus IC is generated by (B.21), and the locus $z_s = z$ is generated by (2.32). Analogously, the locus IC^* in the right panel is generated by (B.22), and the locus $z_s^* = z^*$ is generated by (2.36). Therefore, the solution for (x_1, x_2) is determined by the intersection, at point A, of the curve IC and the curve $z^s = z$, and the solution for (x_1^*, x_2^*) is determined at point A^*, the intersection of the curve IC^* and the curve $z_s^* = z^*$.

Suppose that there is a decrease in z_s from z^0 to z^1, with (ω_l, κ_s^*) held constant. Then, in Figure B.2, the curves $z_s^* = z^*$, IC, and IC^* do not shift, but the curve $z_s = z^0$ shifts up to $z_s = z^1$. So, x_1 falls and x_2 rises in equilibrium. Then, from (2.34) μ rises, and from (2.24) and (2.34) R_l rises. From (2.26), r_s and r_l rise, and from (2.27) the real term premium $r_l - r_s$ falls. From (2.25), the effect on the nominal term premium $R_l - R_s$ is ambiguous, and from (2.28) $\frac{e_{+1}}{e}$ rises.

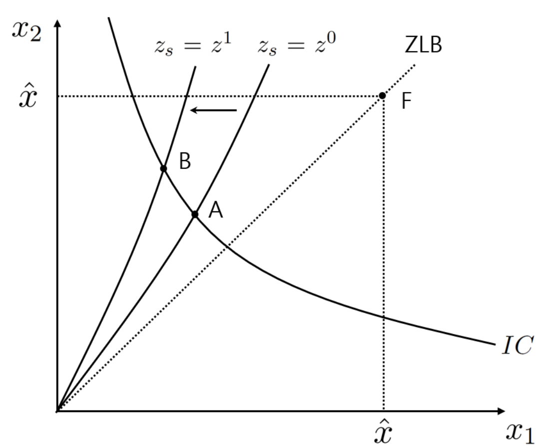

Figure B.2: Conventional monetary policy: a decrease in z_s

To find the welfare implication of a decrease in z_s, differentiate the welfare measure W with respect to z_s. As $\bar{X} = \bar{X}^* = 0$ in an equilibrium with completely segmented asset markets, the derivative of W is given by

$$\frac{dW}{dz_s} = -\rho x_1 u''(x_1)\frac{dx_1}{dz_s} - (1-\rho)x_2 u''(x_2)\frac{dx_2}{dz_s}.$$

For convenience, let $\sigma = -\frac{xu''(x)}{u'(x)}$ where $0 < \sigma < 1$. Totally differentiating (2.31) and (2.32) with respect to z_s gives

$$\frac{dx_1}{dz_s} = \frac{-(1-\rho)\left[(1-\sigma)(1-\theta_{hs})u'(x_2)+\theta_{hs}\right]\left[u'(x_1)-\theta_{hs}u'(x_2)+\theta_{hs}\right]^2}{\left\{\begin{array}{c}\rho u''(x_2)\left[(1-\sigma)(1-\theta_{hs})u'(x_1)+\theta_{hs}\right]\left[(1-\theta_{hs})u'(x_1)+\theta_{hs}\right]\\ +(1-\rho)u''(x_1)\left[(1-\sigma)(1-\theta_{hs})u'(x_2)+\theta_{hs}\right]\left[(1-\theta_{hs})u'(x_2)+\theta_{hs}\right]\end{array}\right\}},$$

$$\frac{dx_2}{dz_s} = \frac{\rho\left[(1-\sigma)(1-\theta_{hs})u'(x_1)+\theta_{hs}\right]\left[u'(x_1)-\theta_{hs}u'(x_2)+\theta_{hs}\right]^2}{\left\{\begin{array}{c}\rho u''(x_2)\left[(1-\sigma)(1-\theta_{hs})u'(x_1)+\theta_{hs}\right]\left[(1-\theta_{hs})u'(x_1)+\theta_{hs}\right]\\ +(1-\rho)u''(x_1)\left[(1-\sigma)(1-\theta_{hs})u'(x_2)+\theta_{hs}\right]\left[(1-\theta_{hs})u'(x_2)+\theta_{hs}\right]\end{array}\right\}},$$

so the derivative of W can be written as

$$\frac{dW}{dz_s} = \frac{\sigma\rho(1-\rho)\theta_{hs}\left[u'(x_2)-u'(x_1)\right]\left[u'(x_1)-\theta_{hs}u'(x_2)+\theta_{hs}\right]^2}{\left\{\begin{array}{c}\rho u''(x_2)\left[(1-\sigma)(1-\theta_{hs})u'(x_1)+\theta_{hs}\right]\left[(1-\theta_{hs})u'(x_1)+\theta_{hs}\right]\\ +(1-\rho)u''(x_1)\left[(1-\sigma)(1-\theta_{hs})u'(x_2)+\theta_{hs}\right]\left[(1-\theta_{hs})u'(x_2)+\theta_{hs}\right]\end{array}\right\}}.$$

From (2.32), the zero lower bound constraint, or $z_s \leq 1$, implies that $u'(x_2) \leq u'(x_1)$ in equilibrium. This in turn implies that $\frac{dW}{dz_s} \geq 0$. Therefore, a decrease in z_s leads to a decrease in W. □

Proof of Proposition 4: The effects of a decrease in ω_l on the DM consumption quantities x_1 and x_2 in the Home country are illustrated in Figure B.3. Note that the curves IC^* and $z_s^* = z^*$ in Figure B.1 do not shift in response to a change in ω_l. From (2.32), the curve $z_s = z$ remains fixed, but the curve that depicts (B.21) shifts upward from IC_0 to IC_1. As a result, both x_1 and x_2 rise in equilibrium.

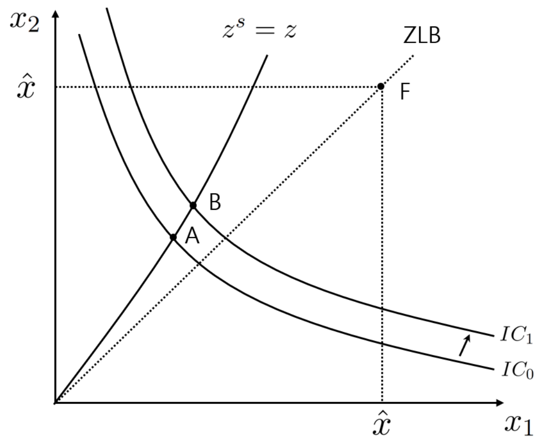

Figure B.3: Quantitative easing: a decrease in ω_l

Then, from (2.26) r_s and r_l rise, and from (2.27) the real term premium $r_l - r_s$ falls. From (2.24), (2.25), (2.32), and (2.34), the nominal interest rate on long-term Home bonds and the nominal term premium can be rewritten as, respectively,

$$R_l = \frac{(1-\theta_{hs})\, u'(x_2) + \theta_{hs}}{z_s\left[(1-\theta_{hl})\, u'(x_2) + \theta_{hl}\right]} - 1, \tag{B.23}$$

$$R_l - R_s = \frac{(\theta_{hl} - \theta_{hs})\left[u'(x_2) - 1\right]}{z_s\left[(1-\theta_{hl})\, u'(x_2) + \theta_{hl}\right]}. \tag{B.24}$$

Since each differentiation of the right-hand sides of (B.23) and (B.24) with respect to x_2 are both negative, R_l and $R_l - R_s$ both decrease. Totally differentiating (2.32) and (2.34) gives

$$\frac{d\mu}{dx_1} = \frac{\beta(1-\theta_{hs})u''(x_1)}{z_s\theta_{hs} + (1-\theta_{hs})} < 0, \tag{B.25}$$

so μ falls with an increase in x_1. From (2.28), $\frac{e+1}{e}$ falls, and finally, W increases as both x_1 and x_2 increase. □

Proof of Proposition 5: The effects of an increase in κ_s^* on the DM consumption quantities x_1, x_2, x_1^*, and x_2^* are illustrated by Figure B.4. In the figure, the curves $z_s = z$ and $z_s^* = z^*$ that depict, respectively, (2.32) and (2.36) remain fixed, while the curve that represents (B.21) shifts up from IC_0 to IC_1, and the curve that represents (B.22) shifts down from IC_0^* to IC_1^*. Therefore, x_1 and x_2 increase, but x_1^* and x_2^* decrease.

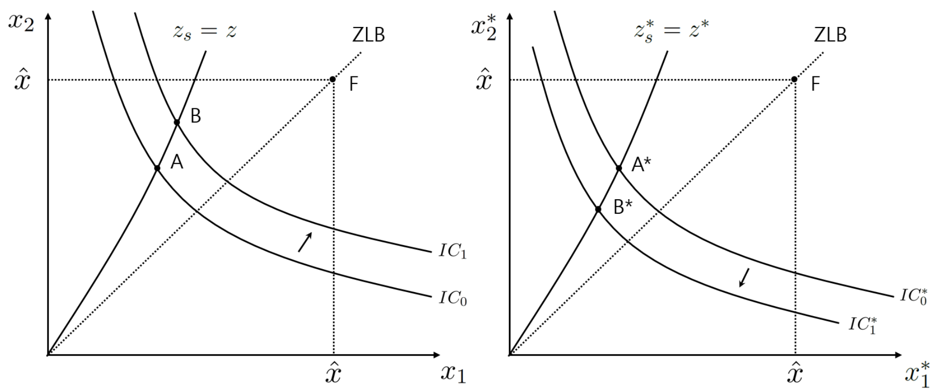

Figure B.4: Foreign asset purchases by the Home central bank: an increase in κ_s^*

Then, from (2.24) and (2.25), R_l and $R_l - R_s$ both decrease. From (2.26), r_s and r_l rise, and from (2.27) $r_l - r_s$ falls. From (B.1) and (B.2), R_l^* and $R_l^* - R_s^*$ both increase. From (B.3) and (B.4), r_s^* and r_l^* fall and $r_l^* - r_s^*$ rises. From (2.34), μ falls, and from (2.38) μ^* rises, so from (2.28), $\frac{e_{+1}}{e}$ falls. Finally, W increases as both x_1 and x_2 increase, but W^* decreases as both x_1^* and x_2^* decrease. □

Proof of Proposition 6: Suppose that V and V^* are sufficiently small to satisfy (B.18). From (2.23), a necessary condition for an equilibrium where short-term Foreign bonds are held in both countries to exist is given by

$$0 < b_{hs}^* < \bar{b}_s^* - \frac{\kappa_s^*}{z_s^*},$$

that is, both Home and Foreign banks must hold positive quantities of short-term Foreign bonds in equilibrium. Note that, if $b_{hs}^* = 0$, then the equilibrium becomes the one with segmented asset markets described in Section 4.3. If $b_{hs}^* = b_s^* - \frac{\kappa_s^*}{z_s^*}$, then $b_{fs}^* = 0$ and $\lambda \in (\frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*}, \frac{(1-\theta_{fl}^*)\lambda^*}{1-\theta_{hl}^*})$ must hold in equilibrium. Then, as shown in Appendix A.5.1, an equilibrium can be characterized by equations (2.32)-(2.34), (2.37)-(2.38), and (B.9)-(B.11). For $b_{hs}^* > 0$ in equilibrium, it must be satisfied that, from (B.20),

$$V \leq \left[u'(\tilde{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\tilde{x}_1 + \left[u'(\tilde{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\tilde{x}_2$$
$$- \kappa_s^* + \kappa_s + \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(\tilde{x}_2) + \theta_{hl}\right]}, \quad \text{(B.26)}$$

where $(\tilde{x}_1, \tilde{x}_2)$ is the solution to

$$u'(x_2) = \frac{[(1-\theta^*_{fs})u'(\bar{x}^*_2) + \theta^*_{fs}] - \theta^*_{hs}}{1-\theta^*_{hs}},$$
$$z_s = \frac{u'(x_2) - \theta_{hs}u'(x_2) + \theta_{hs}}{u'(x_1) - \theta_{hs}u'(x_2) + \theta_{hs}}.$$

Similarly, for $b^*_{fs} > 0$ or $b^*_{hs} < b^*_s - \frac{\kappa^*_s}{z^*_s}$, it must be satisfied that, from (B.10),

$$\begin{aligned} V \geq & \left[u'(\ddot{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\ddot{x}_1 + \left[u'(\ddot{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\ddot{x}_2 - \Omega V^* - (1-\Omega)(\kappa^*_s - \kappa_s) \\ & + \Omega\omega^*_l + \left[u'(\ddot{x}^*_1) - \theta^*_{fs}u'(\ddot{x}^*_2) + \theta^*_{fs}\right]\Omega\rho\ddot{x}^*_1 + \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(\ddot{x}_2) + \theta_{hl}\right]}, \end{aligned} \tag{B.27}$$

where $(\ddot{x}^*_1, \ddot{x}^*_2)$ is the solution to

$$z^*_s = \frac{u'\left(x^*_2\right) - \theta^*_{fs}u'\left(x^*_2\right) + \theta^*_{fs}}{u'\left(x^*_1\right) - \theta^*_{fs}u'\left(x^*_2\right) + \theta^*_{fs}}, \tag{B.28}$$

$$\theta^*_{fs}\rho x^*_1 + (1-\rho)x^*_2 = \frac{\left(1-\theta^*_{fl}\right)\omega^*_l}{(1-\theta^*_{fl})u'\left(x^*_2\right) + \theta^*_{fl}}, \tag{B.29}$$

and $(\ddot{x}_1, \ddot{x}_2)$ is the solution to (2.32) and (B.10) given $(\ddot{x}^*_1, \ddot{x}^*_2)$. Therefore, given V and V^* that satisfy (B.18), (B.26), and (B.27), there exists an equilibrium where short-term Foreign bonds are held in both countries and the equilibrium can be characterized by (2.32)-(2.34), (2.36)-(2.38), (2.39), and (2.40).

It seems obvious that the function is $\mathcal{F}$ is strictly increasing in ω_l, ω^*_l, and κ_s, and strictly decreasing in κ^*_s, V, and V^*. But, with respect to x_1, x_2, x^*_1, and x^*_2, it seems less obvious how $\mathcal{F}$ moves with these arguments. The derivatives of $\mathcal{F}$ with respect to x_1, x^*_1, and x^*_2 are given by

$$\begin{aligned} \frac{\partial\mathcal{F}}{\partial x_1} &= \rho\left[u'(x_1)\left\{1 + \frac{x_1u''(x_1)}{u'(x_1)}\right\} + \frac{\theta_{hs}}{1-\theta_{hs}}\right] > 0, \\ \frac{\partial\mathcal{F}}{\partial x^*_1} &= \rho\Omega\left[u'(x^*_1)\left\{1 + \frac{x^*_1u''(x^*_1)}{u'(x^*_1)}\right\} + \frac{\theta^*_{fs}}{1-\theta^*_{fs}}\right] > 0, \\ \frac{\partial\mathcal{F}}{\partial x^*_2} &= (1-\rho)\,\Omega\left[u'(x^*_2)\left\{1 + \frac{x^*_2u''(x^*_2)}{u'(x^*_2)}\right\} + \frac{\theta^*_{fs}}{1-\theta^*_{fs}}\right] - \frac{(1-\theta^*_{fl})(\theta^*_{fl} - \theta^*_{fs})\Omega\omega^*_lu''(x^*_2)}{(1-\theta^*_{fs})\left[(1-\theta^*_{fl})u'(x^*_2) + \theta^*_{fl}\right]^2} > 0. \end{aligned}$$

The derivative of $\mathcal{F}$ with respect to x_2 is given by

$$\frac{\partial \mathcal{F}}{\partial x_2} = (1-\rho)\left[u'(x_2)\left\{1 + \frac{x_2 u''(x_2)}{u'(x_2)}\right\} + \frac{\theta_{hs}}{1-\theta_{hs}}\right] - \frac{(1-\theta_{hl})(\theta_{hl}-\theta_{hs})\omega_l u''(x_2)}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(x_2)+\theta_{hl}\right]^2}$$
$$+ \frac{\partial \Omega}{\partial x_2}\left[\left\{u'(x_1^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right\}\rho x_1^* + \left\{u'\left(x_2^*\right) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right\}(1-\rho)x_2^*\right.$$
$$\left. - \left\{V^* + \kappa_s - \kappa_s^* - \frac{(\theta_{fl}^*-\theta_{fs}^*)\omega_l^*}{(1-\theta_{fs}^*)[(1-\theta_{fl}^*)u'\left(x_2^*\right)+\theta_{fl}^*]}\right\}\right] > 0,$$

where

$$\frac{\partial \Omega}{\partial x_2} = \frac{(\theta_{hs}^* - \theta_{hs})u''(x_2)}{(1-\theta_{hs}^*)(1-\theta_{hs})} < 0.$$

If asset markets were segmented, Foreign banks' demand for collateral would be equal to the supply of Foreign collateral, so the last term in the above derivative would be zero. However, in this equilibrium, Home banks purchase some Foreign collateral implying that Foreign bank's holdings of collateral must be lower than the supply of Foreign collateral. So, the last term must be positive. □

Proof of Proposition 7: Without loss of generality, assume that $z_s = z_s^* = 1$. Then, $\bar{X}^* = -\bar{X} = (1-\beta)b_{hs}^*$, $x_1 = x_2 = x$ and $x_1^* = x_2^* = x^*$ in equilibrium. Let x_0 and x_0^* denote the would-be quantities of DM consumption in the Home and Foreign countries, if capital did not flow across countries. Then, from (2.22) and (2.40),

$$b_{hs}^* = x_0^* u'(x_0^*) + \frac{\theta_{fs}^* x_0^*}{1-\theta_{fs}^*} - x^* u'(x^*) - \frac{\theta_{fs}^* x^*}{1-\theta_{fs}^*},$$
$$= \frac{1}{\Omega}\left[x u'(x) + \frac{\theta_{hs} x}{1-\theta_{hs}} - x_0 u'(x_0) + \frac{\theta_{hs} x_0}{1-\theta_{hs}}\right],$$

where $x > x_0$, $x_0^* > x^*$, and

$$\Omega = \frac{u'(x) + \frac{\theta_{hs}}{1-\theta_{hs}}}{u'(x) + \frac{\theta_{hs}^*}{1-\theta_{hs}^*}}.$$

So, the welfare measures for two countries can be written as

$$W = u(x) - xu'(x) - \frac{1-\beta}{\Omega}\left[xu'(x) + \frac{\theta_{hs}x}{1-\theta_{hs}} - x_0u'(x_0) + \frac{\theta_{hs}x_0}{1-\theta_{hs}}\right],$$
$$W^* = u(x^*) - x^*u'(x^*) + (1-\beta)\left[x_0^*u'(x_0^*) + \frac{\theta_{fs}^*x_0^*}{1-\theta_{fs}^*} - x^*u'(x^*) - \frac{\theta_{fs}^*x^*}{1-\theta_{fs}^*}\right].$$

The derivatives of the measures with respect to x and x^* are given by

$$\frac{dW}{dx} = -xu''(x) - \frac{1-\beta}{\Omega}\left[xu''(x) + u'(x) + \frac{\theta_{hs}}{1-\theta_{hs}}\right], \tag{B.30}$$
$$\frac{dW^*}{dx^*} = -x^*u''(x^*) + (1-\beta)\left[x^*u''(x^*) + u'(x^*) + +\frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right] > 0. \tag{B.31}$$

Since international capital flows lead to a decrease in x^*, the welfare measure for the Foreign country W^* decreases. For a sufficiently high β, the welfare measure for the Home country W always increases with x. As international capital flows increase x, the welare measure for the Home country W increases. □

Proof of Proposition 8: The equilibrium quantities of DM consumption in two countries x_1, x_2, x_1^*, and x_2^* are determined by equations (2.32), (2.36), (2.39), and (2.40). These four equations can be expressed, respectively, by

$$\mathcal{Z}(x_1, x_2; z_s) = 0, \tag{B.32}$$
$$\mathcal{Z}^*(x_1^*, x_2^*; z_s^*) = 0, \tag{B.33}$$
$$\mathcal{G}(x_2, x_2^*) = 0, \tag{B.34}$$
$$\mathcal{F}(x_1, x_2, x_1^*, x_2^*; \omega_l, \omega_l^*, \kappa_s, \kappa_s^*, V, V^*) = 0, \tag{B.35}$$

where

$$\mathcal{Z}(x_1, x_2; z_s) = \frac{u'(x_2) - \theta_{hs}u'(x_2) + \theta_{hs}}{u'(x_1) - \theta_{hs}u'(x_2) + \theta_{hs}} - z_s,$$
$$\mathcal{Z}^*(x_1^*, x_2^*; z_s^*) = \frac{u'\left(x_2^*\right) - \theta_{fs}^*u'\left(x_2^*\right) + \theta_{fs}^*}{u'\left(x_1^*\right) - \theta_{fs}^*u'\left(x_2^*\right) + \theta_{fs}^*} - z_s^*,$$
$$\mathcal{G}(x_2, x_2^*) = (1-\theta_{hs}^*)\,u'(x_2) + \theta_{hs}^* - \left(1-\theta_{fs}^*\right)u'(x_2^*) - \theta_{fs}^*.$$

Then, it is straightforward to obtain the following:

$$\begin{array}{lll} \frac{\partial \mathcal{Z}}{\partial x_1} > 0, & \frac{\partial \mathcal{Z}}{\partial x_2} < 0, & \frac{\partial \mathcal{Z}}{\partial z_s} < 0, \\ \frac{\partial \mathcal{Z}^*}{\partial x_1^*} > 0, & \frac{\partial \mathcal{Z}^*}{\partial x_2^*} < 0, & \frac{\partial \mathcal{Z}^*}{\partial z_s^*} < 0, \\ \frac{\partial \mathcal{G}}{\partial x_2} < 0, & \frac{\partial \mathcal{G}}{\partial x_2^*} > 0. & \end{array}$$

Noting that the function $\mathcal{F}$ is increasing in x_1, x_2, x_1^*, and x_2^*, implicitly differentiating the above four equations with respect to z_s gives

$$\frac{dx_1}{dz_s} = \frac{\frac{\partial \mathcal{Z}}{\partial z_s}\Phi}{\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi} > 0,$$

$$\frac{dx_2}{dz_s} = -\frac{\frac{\partial \mathcal{Z}}{\partial z_s}\frac{\partial \mathcal{F}}{\partial x_1}}{\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi} < 0,$$

$$\frac{dx_1^*}{dz_s} = -\frac{\frac{\partial \mathcal{Z}^*}{\partial x_2^*}\frac{\partial \mathcal{G}}{\partial x_2}\frac{\partial \mathcal{Z}}{\partial z_s}\frac{\partial \mathcal{F}}{\partial x_1}}{\frac{\partial \mathcal{Z}^*}{\partial x_1^*}\frac{\partial \mathcal{G}}{\partial x_2^*}\left[\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi\right]} < 0,$$

$$\frac{dx_2^*}{dz_s} = \frac{\frac{\partial \mathcal{G}}{\partial x_2}\frac{\partial \mathcal{Z}}{\partial z_s}\frac{\partial \mathcal{F}}{\partial x_1}}{\frac{\partial \mathcal{G}}{\partial x_2^*}\left[\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi\right]} < 0,$$

where

$$\Phi = \frac{\partial \mathcal{F}}{\partial x_2} - \frac{\frac{\partial \mathcal{G}}{\partial x_2}}{\frac{\partial \mathcal{G}}{\partial x_2^*}}\left[\frac{\partial \mathcal{F}}{\partial x_2^*} - \frac{\frac{\partial \mathcal{Z}^*}{\partial x_2^*}}{\frac{\partial \mathcal{Z}^*}{\partial x_1^*}}\right] > 0.$$

Therefore, a decrease in z_s decreases x_1 and increases x_2, x_1^*, and x_2^*. Then, from (2.34) μ rises, from (2.26) r_s and r_l rise, and from (2.27) the real term premium $r_l - r_s$ falls. From (2.24) and (2.34) R_l rises, but from (2.25) the effect on the nominal term premium is ambiguous. Further, from (2.38) μ^* falls, and from (B.3) and (B.4) r_s^* and r_l^* rise while $r_l^* - r_s^*$ falls. From (2.36), (2.38), (B.1), and (B.2), the nominal interest rate on long-term Foreign bonds and the nominal term premium can be rewritten, respectively, as

$$R_l^* = \frac{(1-\theta_{fs}^*)u'\left(x_2^*\right) + \theta_{fs}^*}{z_s^*[(1-\theta_{fl}^*)u'\left(x_2^*\right) + \theta_{fl}^*]} - 1, \tag{B.36}$$

$$R_l^* - R_s^* = \frac{(\theta_{fl}^* - \theta_{fs}^*)\left[u'\left(x_2^*\right) - 1\right]}{z_s^*[(1-\theta_{fl}^*)u'\left(x_2^*\right) + \theta_{fl}^*]}, \tag{B.37}$$

so R_l^* and $R_l^* - R_s^*$ both decrease with the increase in x_2^*. Then, from (2.28), $\frac{e_{+1}}{e}$ rises. Proposition 3 shows that W decreases in response to a decrease in z_s in an equilibrium with segmented asset markets. That is, in Figure 2.4, W decreases if the equilibrium moves from point A to B. However, W is lower at point E than B because x_1 and x_2 are both smaller at E. Therefore, as z_s decreases, W decreases to a larger extent than it does in an equilibrium with segmented asset markets. Finally, from (B.31) W^* increases as both x_1^* and x_2^* increase. □

Proof of Proposition 9: Implicitly differentiate equations (B.32)-(B.35) with respect to ω_l to obtain

$$\frac{dx_1}{d\omega_l} = -\frac{\frac{\partial \mathcal{F}}{\partial \omega_l}\frac{\partial \mathcal{Z}}{\partial x_2}}{\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi} < 0,$$
$$\frac{dx_2}{d\omega_l} = \frac{\frac{\partial \mathcal{F}}{\partial \omega_l}\frac{\partial \mathcal{Z}}{\partial x_1}}{\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi} < 0,$$
$$\frac{dx_1^*}{d\omega_l} = \frac{\frac{\partial \mathcal{Z}^*}{\partial x_2^*}\frac{\partial \mathcal{G}}{\partial x_2}\frac{\partial \mathcal{F}}{\partial \omega_l}\frac{\partial \mathcal{Z}}{\partial x_1}}{\frac{\partial \mathcal{Z}^*}{\partial x_1^*}\frac{\partial \mathcal{G}}{\partial x_2^*}\left[\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi\right]} < 0,$$
$$\frac{dx_2^*}{d\omega_l} = -\frac{\frac{\partial \mathcal{G}}{\partial x_2}\frac{\partial \mathcal{F}}{\partial \omega_l}\frac{\partial \mathcal{Z}}{\partial x_1}}{\frac{\partial \mathcal{G}}{\partial x_2^*}\left[\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi\right]} < 0.$$

Therefore, a decrease in ω_l increases all x_1, x_2, x_1^*, and x_2^*. Then, from (2.34) and (2.38), both μ and μ^* fall, and from (2.26) and (B.3) r_s, r_l, r_s^*, and r_l^* all rise. From (2.27) and (B.4), both $r_l - r_s$ and $r_l^* - r_s^*$ fall. Also, from (B.23) and (B.24), R_l and $R_l - R_s$ decrease, and from (B.36) and (B.37) R_l^* and $R_l^* - R_s^*$ decrease. From (2.29) and (2.39), I obtain

$$\frac{e_{+1}}{e} = \frac{z_s^*\left[(1-\theta_{hs})u'(x_2) + \theta_{hs}\right]}{z_s\left[(1-\theta_{hs}^*)u'(x_2) + \theta_{hs}^*\right]},$$

and differentiating the above equation gives

$$\frac{d\left(e_{+1}/e\right)}{dx_2} = \frac{z_s^* u''(x_2)(\theta_{hs}^* - \theta_{hs})}{z_s[(1-\theta_{hs}^*)u'(x_2) + \theta_{hs}^*]^2} < 0,$$

which implies that, as x_2 rises in response to a decrease in ω_l, $\frac{e_{+1}}{e}$ falls. Finally, from (B.30) and (B.31) both W and W^* increase because x_1, x_2, x_1^*, and x_2^* all increase. □

Proof of Proposition A.1: Suppose that V and V^* are sufficiently small to satisfy (B.18). A necessary condition for this equilibrium to exist is given by

$$\frac{(1-\theta^*_{fs})\lambda^*}{1-\theta^*_{hs}} < \lambda < \frac{(1-\theta^*_{fl})\lambda^*}{1-\theta^*_{hl}}.$$

For $\lambda > \frac{(1-\theta^*_{fs})\lambda^*}{1-\theta^*_{hs}}$ to hold in equilibrium, from (B.10) the following inequality must be satisfied:

$$V < \left[u'(\ddot{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\ddot{x}_1 + \left[u'(\ddot{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\ddot{x}_2 - \Omega V^* - (1-\Omega)(\kappa^*_s - \kappa_s)$$
$$+ \Omega\omega^*_l + \left[u'(\ddot{x}^*_1) - \theta^*_{fs}u'(\ddot{x}^*_2) + \theta^*_{fs}\right]\Omega\rho\ddot{x}^*_1 + \frac{(\theta_{hl}-\theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(\ddot{x}_2) + \theta_{hl}\right]}, \quad \text{(B.38)}$$

where $(\ddot{x}^*_1, \ddot{x}^*_2)$ is the solution to (B.28) and (B.29), and $(\ddot{x}_1, \ddot{x}_2)$ is the solution to (2.32) and (B.10) given $(\ddot{x}^*_1, \ddot{x}^*_2)$. Similarly, for $\lambda < \frac{(1-\theta^*_{fl})\lambda^*}{1-\theta^*_{hl}}$ to hold in equilibrium, it must be satisfied that, from (B.10),

$$V > \left[u'(\dddot{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\dddot{x}_1 + \left[u'(\dddot{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\dddot{x}_2 - \Omega V^* - (1-\Omega)(\kappa^*_s - \kappa_s)$$
$$+ \Omega\omega^*_l + \left[u'(\dddot{x}^*_1) - \theta^*_{fs}u'(\dddot{x}^*_2) + \theta^*_{fs}\right]\Omega\rho\dddot{x}^*_1 + \frac{(\theta_{hl}-\theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(\dddot{x}_2) + \theta_{hl}\right]}, \quad \text{(B.39)}$$

where $(\dddot{x}_1, \dddot{x}_2, \dddot{x}^*_1, \dddot{x}^*_2)$ is the solution to (2.32), (B.9), (B.11), and (B.13). Therefore, given V and V^* that satisfy (B.18), (B.38), and (B.39), there exists an equilibrium that can be characterized by equations (2.32)-(2.34), (2.37)-(2.38), and (B.9)-(B.11). □

Proof of Proposition A.2: Suppose V and V^* satisfy (B.18). From (2.14), a necessary condition for equations (2.32)-(2.34), (2.37)-(2.38), (B.9), and (B.13)-(B.14) to characterize an equilibrium is

$$0 < b^*_{hl} < b^*_l,$$

that is, both Home and Foreign banks must hold positive quantities of long-term Foreign bonds in equilibrium. Note that, if $b^*_{hl} = 0$, the economy is in an equilibrium with $\lambda \in \left(\frac{(1-\theta^*_{fs})\lambda^*}{1-\theta^*_{hs}}, \frac{(1-\theta^*_{fl})\lambda^*}{1-\theta^*_{hl}}\right)$. Also, note that $b^*_{hl} = b^*_l$ cannot be supported as an equilibrium since the asset scarcity in the Foreign country diverges to infinity as b^*_{fl} gets close to zero. For $b^*_{hl} > 0$, it must be satisfied that, from

(B.10),

$$V \leq \left[u'(\ddot{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\ddot{x}_1 + \left[u'(\ddot{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\ddot{x}_2 - \Omega V^* - (1-\Omega)(\kappa_s^* - \kappa_s)$$
$$+ \Omega\omega_l^* + \left[u'(\ddot{x}_1^*) - \theta_{fs}^* u'(\ddot{x}_2^*) + \theta_{fs}^*\right]\Omega\rho\ddot{x}_1^* + \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'\left(\ddot{x}_2\right) + \theta_{hl}\right]}, \quad \text{(B.40)}$$

where $(\ddot{x}_1, \ddot{x}_2, \ddot{x}_1^*, \ddot{x}_2^*)$ is the solution to (2.32), (B.9), (B.11), and (B.13). Therefore, given V and V^* that satisfy (B.18) and (B.40), there exists an equilibrium that can be characterized by equations (2.32)-(2.34), (2.37)-(2.38), (B.9), and (B.13)-(B.14). □

Appendix C

Appendices to Chapter 3

C.1 Omitted Proofs

Proof of Lemma 3.1 Suppose that $\hat{x}_l$ and $\hat{x}_h$, with $\hat{x}_l > \hat{x}_h$, consist of an equilibrium contract. This implies that the incentive constraint (3.3) holds for any $\pi_h \in [0, 1]$. Then, the optimal $\hat{\pi}_h$ must be zero as the lower π_h, the higher the buyer's expected utility will be. In addition, the optimal $\hat{\pi}_l$ must satisfy the incentive constraint (3.4) with equality, or

$$\hat{\pi}_l = \frac{u(\hat{x}_l) - u(\hat{x}_h)}{u(\hat{x}_l)}, \tag{C.1}$$

since the lower π_l, the higher the buyer's expected utility will be.

Now, suppose further that (i) $\hat{x}_h < \hat{x}_l \leq x_h^*$ in an equilibrium contract, and consider an alternative contract with $\tilde{x}_l = \tilde{x}_h = \hat{x}_l$. The alternative contract allows the bank to set $\tilde{\pi}_l = \tilde{\pi}_h = 0$ since this does not violate the incentive constraints (3.3) and (3.4). Then, the change in the buyer's expected utility in changing contract from $\{\hat{x}_\omega, \hat{\pi}_\omega\}_{\omega=l,h}$ to $\{\tilde{x}_\omega, \tilde{\pi}_\omega\}_{\omega=l,h}$ is

$$\beta \left[p \left\{ \theta u\left(\hat{x}_l\right) - \hat{x}_l - \theta u(\hat{x}_h) + \hat{x}_h \right\} + (1-p)\hat{\pi}_l \cdot \gamma \right] > 0, \tag{C.2}$$

which is a contradiction. Instead, suppose that (ii) $\hat{x}_l > x_h^*$ and $\hat{x}_l > \hat{x}_h$ in an equilibrium contract, and then I can suggest an alternative contract with $\tilde{x}_l = \tilde{x}_h = x_h^*$ which makes buyers better off, a contradiction as well.

Therefore, the solution must satisfy $x_l \leq x_h$. Then, it follows that the optimal π_l must be zero, and the optimal π_h must satisfy the incentive constraint (3.3) with equality. Otherwise, without changing the other terms of the contract, a bank could lower π_l or π_h and make buyers better off, which contradicts with the optimality. □

Proof of Proposition 3.1 The second-order conditions are given by[1]:

$$u''(x_l)\left\{1-p+\frac{p\gamma}{u(x_h)}\right\}<0, \tag{C.3}$$

$$p\left[\theta u''(x_h)-\frac{\gamma u(x_l)}{\{u(x_h)\}^2}(u''(x_h)-\frac{2\{u'(x_h)\}^2}{u(x_h)})\right]\left[(1-p)u''(x_l)+\frac{p\gamma u''(x_l)}{u(x_h)}\right]$$
$$-\left[\frac{p\gamma u'(x_h)u'(x_l)}{\{u(x_h)\}^2}\right]^2>0. \tag{C.4}$$

Let Ψ denote the left-hand side of (C.4). Then, implicitly differentiating equations (3.10) and (3.11) with respect to γ yields the following:

$$\frac{dx_h}{d\gamma}=\frac{u'(x_h)[u(x_l)u''(x_l)\{(1-p)u(x_h)+p\gamma\}-p\gamma\{u'(x_l)\}^2]}{\Psi\{u(x_h)\}^3}<0, \tag{C.5}$$

$$\frac{dx_l}{d\gamma}=\frac{pu'(x_l)}{u''(x_l)\{(1-p)u(x_h)+p\gamma\}}\left\{\frac{dx_h}{d\gamma}\frac{\gamma u'(x_h)}{u(x_h)}-1\right\}>0. \tag{C.6}$$

As $x_l = x_l^*$ and $x_h = x_h^*$ at $\gamma = 0$, (C.5) and (C.6) imply that there exists a monitoring cost $\bar{\gamma} > 0$ such that the solution to equations (3.10)-(3.11) satisfies that $x_l = x_h$. With $\gamma = \bar{\gamma}$ and $x_l = x_h = x$, the first-order conditions (3.10)-(3.11) can be written as

$$u'(x)=1-\frac{p\bar{\gamma}u'(x)}{(1-p)u(x)}, \tag{C.7}$$

$$\theta u'(x)=1+\frac{\bar{\gamma}u'(x)}{u(x)}. \tag{C.8}$$

Then,

$$\bar{\gamma}=\frac{(1-p)u(x)\{1-u'(x)\}}{pu'(x)}, \tag{C.9}$$

where x is the solution to $p\{\theta u'(x)-1\}=(1-p)\{1-u'(x)\}$. Therefore, a necessary condition for equations (3.8), (3.10)-(3.11) to characterize an equilibrium is

$$\gamma<\bar{\gamma}. \ \square$$

[1]These conditions guarantee that the Hessian matrix is negative definite.

Proof of Lemma 4.1 An equilibrium contract must solve the following problem:

$$\max_{x_l,x_h,\pi_l,\pi_h}\begin{bmatrix}(1-p)\{\delta W_l(m,x_l,0)+(1-\delta)W_l(m,x_l,1)-x_l-\pi_l\gamma\}\\ +p\{W_h(m,x_h,1)-x_h-\pi_h\gamma\}\end{bmatrix}, \tag{3.28}$$

subject to

$$\delta W_l(m,x_l,0)+(1-\delta)W_l(m,x_l,1)\geq(1-\pi_h)W_l(m,x_h,1)+\pi_h V(m), \tag{3.25}$$

$$W_h(m,x_h,1)\geq(1-\pi_l)\{\delta W_h(m,x_l,0)+(1-\delta)W_h(m,x_l,1)\}+\pi_l V(m), \tag{3.26}$$

$$k,x_l,x_h\geq 0,\quad \pi_h,\pi_l\in[0,1].\quad m\ given \tag{3.27}$$

The proof of Lemma 4.1 is somewhat similar to the proof of Lemma 3.1. Suppose that $\hat{x}_l$ and $\hat{x}_h$ consist of an equilibrium such that

$$\hat{\pi}_l=\frac{\delta W_h(m,x_l,0)+(1-\delta)W_h(m,x_l,1)-W_h(m,x_h,1)}{\delta W_h(m,x_l,0)+(1-\delta)W_h(m,x_l,1)-V(m)}>0. \tag{C.10}$$

Then, $\hat{\pi}_h=0$ if $W_l(m,x_h,1)-\delta W_l(m,x_l,0)-(1-\delta)W_l(m,x_l,1)\leq 0$ and $\hat{\pi}_h>0$ otherwise. First, suppose that $\hat{\pi}_h=0$. If $\hat{x}_h+m<x_h^*$, then consider an alternative contract with $\tilde{x}_l=\hat{x}_l$ and $\tilde{x}_h>\hat{x}_h$ such that $\tilde{\pi}_h=0$. Then, the change in the buyer's expected utility in changing contract from $\{\hat{x}_\omega,\hat{\pi}_\omega\}_{\omega=l,h}$ to $\{\tilde{x}_\omega,\tilde{\pi}_\omega\}_{\omega=l,h}$ is

$$p\{W_h(m,\tilde{x}_h,1)-\tilde{x}_h-W_h(m,\hat{x}_h,1)+\hat{x}_h\}+(1-p)(\hat{\pi}_l-\tilde{\pi}_l)\gamma>0, \tag{C.11}$$

since $W_h(m,\tilde{x}_h,1)-\tilde{x}_h>W_h(m,\hat{x}_h,1)-\hat{x}_h$ and $\hat{\pi}_l>\tilde{\pi}_l$. This contradicts with the optimality of the original contract. If $\hat{x}_h+m\geq x_h^*$, then consider an alternative contract with $\tilde{x}_l<\hat{x}_l$ and $\tilde{x}_h=\hat{x}_h$ such that $\tilde{\pi}_h=0$. Then, with noting that (C.10) and $\hat{x}_h+m\geq x_h^*$ imply $\hat{x}_l+m\geq x_h^*>x_l^*$, the change in the buyer's expected utility in changing contract from $\{\hat{x}_\omega,\hat{\pi}_\omega\}_{\omega=l,h}$ to $\{\tilde{x}_\omega,\tilde{\pi}_\omega\}_{\omega=l,h}$ is

$$(1-p)\left\{\begin{array}{c}\delta W_l(m,\tilde{x}_l,0)+(1-\delta)W_l(m,\tilde{x}_l,1)-\tilde{x}_l\\ -\delta W_l(m,\hat{x}_l,0)-(1-\delta)W_l(m,\hat{x}_l,1)+\hat{x}_l+(\hat{\pi}_l-\tilde{\pi}_l)\gamma\end{array}\right\}>0, \tag{C.12}$$

since $\delta W_l(m,\tilde{x}_l,0)+(1-\delta)W_l(m,\tilde{x}_l,1)-\tilde{x}_l\geq\delta W_l(m,\hat{x}_l,0)+(1-\delta)W_l(m,\hat{x}_l,1)-\hat{x}_l$ and $\hat{\pi}_l>\tilde{\pi}_l$. This is a contradiction as well.

Suppose instead that $\hat{\pi}_h>0$. If $\hat{x}_h+m<x_h^*$, then there exist $\tilde{x}_l=\hat{x}_l$ and $\tilde{x}_h>\hat{x}_h$ such that the change in the buyer's expected utility in changing contract from $\{\hat{x}_\omega,\hat{\pi}_\omega\}_{\omega=l,h}$ to $\{\tilde{x}_\omega,\tilde{\pi}_\omega\}_{\omega=l,h}$ is

$$\begin{bmatrix}p\{W_h(m,\tilde{x}_h,1)-\tilde{x}_h-W_h(m,\hat{x}_h,1)+\hat{x}_h-(\tilde{\pi}_h-\hat{\pi}_h)\gamma\}\\ +(1-p)(\hat{\pi}_l-\tilde{\pi}_l)\gamma\end{bmatrix}>0, \tag{C.13}$$

which is a contradiction. If $\hat{x}_h + m \geq x_h^*$, then there exist $\tilde{x}_l < \hat{x}_l$ and $\tilde{x}_h = \hat{x}_h$ such that the change in the buyer's expected utility in changing contract from $\{\hat{x}_\omega, \hat{\pi}_\omega\}_{\omega=l,h}$ to $\{\tilde{x}_\omega, \tilde{\pi}_\omega\}_{\omega=l,h}$ is

$$\begin{bmatrix} -p\gamma(\tilde{\pi}_h - \hat{\pi}_h) + (1-p)\{\delta W_l(m, \tilde{x}_l, 0) + (1-\delta)W_l(m, \tilde{x}_l, 1) - \tilde{x}_l\} \\ -(1-p)\{\delta W_l(m, \hat{x}_l, 0) + (1-\delta)W_l(m, \hat{x}_l, 1) - \hat{x}_l\} + (1-p)\gamma(\hat{\pi}_l - \tilde{\pi}_l) \end{bmatrix} > 0, \tag{C.14}$$

which is a contradiction. Therefore, it must be true that $\pi_l = 0$ and $\pi_h \geq 0$ in equilibrium. The probability of monitoring for a large-transaction buyer is given by

$$\hat{\pi}_h = \frac{W_l(m, x_h, 1) - \delta W_l(m, x_l, 0) - (1-\delta)W_l(m, x_l, 1)}{W_l(m, x_h, 1) - V(m)}, \tag{C.15}$$

so it must be satisfied that $W_l(m, x_h, 1) \geq \delta W_l(m, x_l, 0) + (1-\delta)W_l(m, x_l, 1)$. □

Proof of Lemma 4.2 See Proof of Proposition 4.1.

Proof of Proposition 4.1 I begin with an argument similar to that in the proof of Proposition 3.1. Suppose that the monitoring cost is sufficiently low and the buyer's money holdings are sufficiently small, so that the optimal contract can be characterized by equations (3.30), (3.32)-(3.33). The second-order conditions are given by[2]:

$$(1-\delta)u''(x_l)\left\{1 - p + \frac{p\gamma}{u(x_h)}\right\} < 0, \tag{C.16}$$

$$\begin{aligned} &p\left(\theta u''(x_h + m) - \frac{\gamma[(1-\delta)u(x_l) + \delta x_l + \delta\{u(m) - m\}]}{\{u(x_h)\}^2}\left[u''(x_h) - \frac{2\{u'(x_h)\}^2}{u(x_h)}\right]\right) \\ &\cdot (1-\delta)u''(x_l)\left\{1 - p + \frac{p\gamma}{u(x_h)}\right\} - \left[\frac{p\gamma u'(x_h)\{(1-\delta)u'(x_l) + \delta\}}{\{u(x_h)\}^2}\right]^2 > 0. \end{aligned} \tag{C.17}$$

Let Ψ denote the left-hand sides of (C.17) and let

$$\psi_1 \equiv (1-\delta)u''(x_l)\left\{1 - p + \frac{p\gamma}{u(x_h)}\right\} < 0, \tag{C.18}$$

$$\psi_2 \equiv p\theta u''(x_h + m) - \frac{p\gamma[(1-\delta)u(x_l) + \delta x_l + \delta\{u(m) - m\}]}{\{u(x_h)\}^2}\left[u''(x_h) - \frac{2\{u'(x_h)\}^2}{u(x_h)}\right] < 0, \tag{C.19}$$

$$\psi_3 \equiv -\frac{p\gamma u'(x_h)\{(1-\delta)u'(x_l) + \delta\}}{\{u(x_h)\}^2} < 0. \tag{C.20}$$

Then, the second-order condition (C.17) can be written as

$$\psi_1\psi_2 - \psi_3^2 = \Psi > 0.$$

[2] Again, these conditions guarantee that the Hessian matrix is negative definite.

Implicitly differentiating equations (3.32) and (3.33) with respect to γ yields the following:

$$\frac{dx_h}{d\gamma} = \frac{p}{\Psi\{u(x_h)\}^2} \cdot \left\{ \begin{array}{c} \psi_1 u'(x_h)\,[(1-\delta)u(x_l) + \delta x_l + \delta\{u(m) - m\}] \\ +\psi_3 u(x_h)[(1-\delta)u'(x_l) + \delta] \end{array} \right\} < 0, \tag{C.21}$$

$$\frac{dx_l}{d\gamma} = -\frac{\psi_3}{\psi_1}\frac{dx_h}{d\gamma} - \frac{p\{(1-\delta)u'(x_l) + \delta\}}{\psi_1 u(x_h)} > 0. \tag{C.22}$$

This implies that, given a sufficiently small m, there exists $\hat{\gamma}(m)$ such that the solution (x_l, x_h) to equations (3.32)-(3.33) satisfies $\pi_h = 0$. That is, with $\gamma = \hat{\gamma}(m)$, x_l and x_h satisfy

$$u'(x_l) = 1 - \frac{p\gamma\{(1-\delta)u'(x_l) + \delta\}}{(1-\delta)(1-p)u(x_h)}, \tag{3.32}$$

$$\theta u'\,(x_h + m) = 1 + \frac{\gamma u'(x_h)}{\{u(x_h)\}^2}[(1-\delta)u(x_l) + \delta x_l + \delta\{u(m) - m\}]. \tag{3.33}$$

$$u(x_h) = (1-\delta)u(x_l) + \delta x_l + \delta\{u(m) - m\}. \tag{3.38}$$

Hence, given a sufficiently small m, $\hat{\gamma}(m)$ can be expressed as

$$\hat{\gamma}(m) = \frac{u(x_h)\{\theta u'(x_h + m) - 1\}}{u'(x_h)}, \tag{C.23}$$

where $x_h \in (x_l^*, x_h^*)$, together with $x_l \in (x_l^*, x_h^*)$, is the solution to the following equations.

$$(1-\delta)(1-p)u'(x_h)\{1 - u'(x_l)\} = p\{(1-\delta)u'(x_l) + \delta\}\{\theta u'(x_h + m) - 1\}, \tag{C.24}$$

$$u(x_h) = (1-\delta)u(x_l) + \delta x_l + \delta\{u(m) - m\}. \tag{3.38}$$

Since $\hat{\gamma}(m)$ is decreasing in m, the highest possible value for $\hat{\gamma}(m)$ is $\hat{\gamma}(0)$. If $\gamma \geq \hat{\gamma}(0)$, then there does not exists $m \geq 0$ that generates $\pi_h > 0$ in the deposit contract. That is, the optimal contract features no monitoring $\pi_h = 0$ for all $m \geq 0$ if $\gamma \geq \hat{\gamma}(0)$. Therefore, a necessary condition, for the existence of $m \geq 0$ such that for given m equations (3.30), (3.32)-(3.33) characterize the optimal deposit contract, is

$$\gamma < \hat{\gamma}(0).$$

Suppose $\gamma < \hat{\gamma}(0)$. Then, the optimal contract can be characterized by equations (3.30), (3.32)-(3.33) for $m = 0$. Implicitly differentiating equations (3.32) and (3.33) with respect to m yields the following:

$$\frac{dx_h}{dm} = -\frac{p\theta u''\,(x_h + m)\,\psi_1}{\Psi} + \frac{p\gamma\delta u'(x_h)\{u'(m) - 1\}\psi_1}{\Psi\{u(x_h)\}^2} < 0, \tag{C.25}$$

$$\frac{dx_l}{dm} = -\frac{\psi_3}{\psi_1} \cdot \frac{dx_h}{dm} > 0, \tag{C.26}$$

since $\psi_1, \psi_3 < 0$ and $\Psi > 0$. This implies that there exists $\hat{m}(\gamma)$ such that the solution (x_l, x_h) to equations (3.32)-(3.33) satisfies $\pi_h = 0$ given $m = \hat{m}(\gamma)$. Similarly to the case where $\gamma = \hat{\gamma}(m)$ explained above, $\hat{m}(\gamma)$ solves equations (3.32), (3.33), and (3.38), together with x_l and x_h given $\gamma < \hat{\gamma}(0)$. Therefore, the optimal contract can be characterized by equations (3.30), (3.32)-(3.33) for $m \in [0, \hat{m}(\gamma)]$. For any $m > \hat{m}(\gamma)$, any contracts that satisfy equations (3.32)-(3.33) do not satisfy the feasibility constraint $\pi_h \in [0, 1]$. Then, the optimal contract can be characterized by equations (3.38) and (3.39) for $m \in [\hat{m}(\gamma), x_l^*)$. Finally for $m \geq x_l^*$, any contracts that satisfiy condition (3.36) are optimal.

Suppose $\gamma \geq \hat{\gamma}(0)$. Then, there does not exist $\hat{m}(\gamma) > 0$ such that the solution to equations (3.32)-(3.33) satisfies that $\pi_h = 0$. Also, for any $m \geq 0$, the solution to equations (3.32)-(3.33) does not satisfy the feasibility constraint $\pi_h \in [0, 1]$. In this case, an optimal contract can be characterized by equations (3.38) and (3.39) for $m \in [0, x_l^*)$ and by condition (3.36) for $m \in [x_l^*, \infty)$.
□

Proof of Proposition 4.3 To derive the ranges of money growth rates that support each type of equilibria, it is convenient to show that m is decreasing in μ for all $\mu \geq \beta$. As it is straightforward to show this relation for equilibria with $m \in [\hat{m}(\gamma), \bar{m})$, I focus on the case where $\gamma \in (0, \bar{\gamma})$ and $m \in [0, \hat{m}(\gamma))$. In this equilibrium, (x_l, x_h, m) satisfies

$$u'(x_l) = 1 - \frac{p\gamma\{(1-\delta)u'(x_l) + \delta\}}{(1-p)(1-\delta)u(x_h)}, \tag{3.32}$$

$$\theta u'(x_h + m) = 1 + \frac{\gamma u'(x_h)[(1-\delta)u(x_l) + \delta x_l + \delta\{u(m) - m\}]}{\{u(x_h)\}^2}, \tag{3.33}$$

$$\beta\left\{(1-p)[\delta u'(m) + 1 - \delta] + p[\theta u'(x_h + m) + \frac{\gamma\delta\{u'(m) - 1\}}{u(x_h)}]\right\} = \mu. \tag{3.46}$$

I will use the notations defined in Proof of Proposition 4.1. That is,

$$\psi_1 \equiv (1-\delta)u''(x_l)\left\{1 - p + \frac{p\gamma}{u(x_h)}\right\} < 0, \tag{C.18}$$

$$\psi_2 \equiv p\theta u''(x_h + m) - \frac{p\gamma[(1-\delta)u(x_l) + \delta x_l + \delta\{u(m) - m\}]}{\{u(x_h)\}^2}\left[u''(x_h) - \frac{2\{u'(x_h)\}^2}{u(x_h)}\right] < 0, \tag{C.19}$$

$$\psi_3 \equiv -\frac{p\gamma u'(x_h)\{(1-\delta)u'(x_l) + \delta\}}{\{u(x_h)\}^2} > 0, \tag{C.20}$$

$$\Psi \equiv \psi_1\psi_2 - \psi_3^2 > 0. \tag{C.17}$$

In addition, the second-order condition for the buyer's problem is given by

$$(1-p)\delta u''(m) + p\theta u''(x_h + m) + \frac{p\gamma\delta u''(m)}{u(x_h)} - \frac{\psi_1}{\Psi}\left[p\theta u''(x_h + m) - \frac{p\gamma\delta u'(x_h)\{u'(m) - 1\}}{\{u(x_h)\}^2}\right]^2 < 0. \tag{C.27}$$

For notational simplicity, let

$$\psi_4 \equiv (1-p)\delta u''(m) + p\theta u''(x_h + m) + \frac{p\gamma\delta u''(m)}{u(x_h)} < 0, \tag{C.28}$$

$$\psi_5 \equiv p\theta u''(x_h + m) - \frac{p\gamma\delta u'(x_h)\{u'(m) - 1\}}{\{u(x_h)\}^2} < 0. \tag{C.29}$$

Then, the second-order condition (C.27) can be written as

$$\psi_4 - \psi_1\psi_5^2/\Psi < 0. \tag{C.30}$$

Implicitly differentiating equations (3.32), (3.33) and (3.46) with respect to μ yields

$$\frac{dm}{d\mu} = \frac{1}{\beta[\psi_4 - \psi_1\psi_5^2/\Psi]} < 0, \tag{C.31}$$

so m is decreasing in μ in this type of equilibria.

To derive the cutoff money growth rates μ_1 and μ_2, let $(\hat{x}_l, \hat{x}_h)$ denote the optimal deposit contract given $m = \hat{m}(\gamma)$. That is, $(\hat{x}_l, \hat{x}_h)$ satisfies equations (3.32) and (3.33) at $m = \hat{m}(\gamma)$. As m is decreasing in μ for all $\mu \in [\beta, \infty)$, μ_1 is the upper bound on μ that support equilibria characterized by (3.38), (3.39), and (3.45). From (3.45), μ_1 is given by

$$\mu_1 = \beta\{(1-p)(\delta u'[\hat{m}(\gamma)] + 1 - \delta) + p\theta u'[\hat{x}_h + \hat{m}(\gamma)]\}. \tag{C.32}$$

In addition, μ_2 is the lower bound on μ that support equilibria characterized by (3.30), (3.32)-(3.33), and (3.46). From (3.46), μ_2 is given by

$$\mu_2 = \beta\left[(1-p)\{\delta u'[\hat{m}(\gamma)] + 1 - \delta\} + p\theta u'[\hat{x}_h + \hat{m}(\gamma)] + \frac{p\gamma\delta\{u'[\hat{m}(\gamma)] - 1\}}{u(\hat{x}_h)}\right]. \tag{C.33}$$

Therefore,

$$\mu_2 = \mu_1 + \frac{\beta p\gamma\delta\{u'[\hat{m}(\gamma)] - 1\}}{u(\hat{x}_h)} > \mu_1. \quad \square \tag{C.34}$$

Proof of Proposition 4.4 Consider an equilibrium where money is valued and deposit contracts are state-contingent. In this case, the representative buyer's problem in the CM, the maximization problem (3.43) subject to (3.44) can be written as

$$\max_{m\geq 0}\left\{-\mu m+\beta\left[\begin{array}{c}(1-p)\{u(x_l)+m-x_l\}\\ +p\{\theta u(x_h+m)-x_h-\gamma\pi_h\}\end{array}\right]\right\}, \tag{C.35}$$

subject to

$$(x_l,x_h,\pi_h)=\arg\max_{x_l,x_h,\pi_h}\left\{\begin{array}{c}(1-p)[u(x_l)+m-x_l]\\ +p[\theta u(x_h+m)-x_h-\gamma\pi_h],\end{array}\right\}, \tag{C.36}$$

$$\begin{aligned} s.t.\quad \pi_h &= \frac{u(x_h)-u(x_l)}{u(x_h)}\\ \pi_h &\in [0,1]\end{aligned}$$

Assuming an interior solution, the first-order condition is given by

$$-\mu+\beta\{1-p+p\theta u'(x_h+m)\}=0, \tag{C.37}$$

and the second-order condition is given by

$$\beta p\theta u''(x_h+m)\left[\frac{\partial x_h}{\partial m}+1\right]<0. \tag{C.38}$$

However, it turns out that

$$\begin{aligned}&\beta p\theta u''(x_h+m)\left[\frac{\partial x_h}{\partial m}+1\right]\\ =&\frac{\beta p\theta u''(x_h+m)}{\Psi}\left(-\frac{p\gamma u(x_l)\psi_1}{[u(x_h)]^2}\left[u''(x_h)-\frac{2\{u'(x_h)\}^2}{u(x_h)}\right]-\psi_3^2\right)>0,\end{aligned}$$

for all $m \geq 0$ and $\gamma > 0$ where

$$\psi_1 \equiv u''(x_l)\left\{1 - p + \frac{p\gamma}{u(x_h)}\right\} < 0, \tag{C.39}$$

$$\psi_2 \equiv p\theta u''\left(x_h + m\right) - \frac{p\gamma\left[(1-\delta)u(x_l)\right]}{\{u(x_h)\}^2}\left[u''(x_h) - \frac{2\{u'(x_h)\}^2}{u(x_h)}\right] < 0, \tag{C.40}$$

$$\psi_3 \equiv \frac{p\gamma u'(x_h)u'(x_l)}{\{u(x_h)\}^2} > 0, \tag{C.41}$$

$$\Psi \equiv \psi_1\psi_2 - \psi_3^2 > 0. \tag{C.42}$$

Note that the last condition comes from the properties of the optimal contract. This result implies that there does not exist an interior solution to the maximization problem (C.35) subject to (C.36). So, in equilibrium, the quantity of the buyer's money holdings and the probability of monitoring are either $m > 0$ and $\pi_h = 0$ or $m = 0$ and $\pi_h > 0$.

Consider the cases where the monitoring cost is sufficiently low, i.e., $\gamma \in (0, \bar{\gamma})$.[3] Since a state-contingent deposit contract is always suboptimal if $m > 0$, the buyer chooses to acquire either a positive quantity of money balances with a noncontingent deposit contract or zero money balances with a state-contingent contract. Let Π_m denote the expected discounted payoff for the buyer from acquiring an optimal asset portfolio consisting of a positive quantity of money balances and a noncontingent deposit contract. Then, the buyer's expected discounted payoff can be written as

$$\Pi_m = \max_{m>0}\left\{-\mu m + \beta\left[(1-p)\left\{u\left(x\right) + m\right\} + p\theta u\left(x + m\right) - x\right]\right\}, \tag{C.43}$$

subject to

$$x = \arg\max_{x\geq 0}\left\{(1-p)\left[u\left(x\right) + m\right] + p\theta u\left(x + m\right) - x\right\},$$

or equivalently,

$$\Pi_m = -\mu\dot{m} + \beta\left[(1-p)\left\{u\left(\dot{x}\right) + \dot{m}\right\} + p\theta u\left(\dot{x} + \dot{m}\right) - \dot{x}\right], \tag{C.44}$$

where $(\dot{m}, \dot{x})$ is the solution to

$$(1-p)u'\left(x\right) + p\theta u'\left(x + m\right) - 1 = 0, \tag{3.48}$$

$$-\mu + \beta\{1 - p + p\theta u'\left(x + m\right)\} = 0. \tag{3.49}$$

Let Π_0 denote the buyer's expected discounted payoff, at an optimum, from acquiring zero money

[3]Characterizing equilibria with $\gamma \in [\bar{\gamma}, \infty)$ is straightforward, and hence, omitted.

balances and a state-contingent deposit contract. Then, Π_0 can be written as

$$\Pi_0 = (1-p)\{u(\ddot{x}_l) - \ddot{x}_l\} + p\{\theta u(\ddot{x}_h) - \ddot{x}_h - \ddot{\pi}_h \gamma\}, \tag{C.45}$$

where $(\ddot{x}_l, \ddot{x}_h, \ddot{\pi}_h)$ is the solution to

$$\pi_h = \frac{u(x_h) - u(x_l)}{u(x_h)}, \tag{3.8}$$

$$u'(x_l) = 1 - \frac{p\gamma u'(x_l)}{(1-p)u(x_h)}, \tag{3.10}$$

$$\theta u'(x_h) = 1 + \frac{\gamma u'(x_h)\, u(x_l)}{[u(x_h)]^2}. \tag{3.11}$$

Note that, from equation (C.44), the buyer's expected discounted payoff from acquiring a positive quantity of money balances is maximized at $\mu = \beta$. In this case,

$$\Pi_m|_{\mu=\beta} = \beta\left[(1-p)\{u(x_l^*) - x_l^*\} + p\{\theta u(x_h^*) - x_h^*\}\right], \tag{C.46}$$

where x_ω^* is the first best quantity for $\omega \in \{l, h\}$. Then, it seems straighforward that $\Pi_m|_{\mu=\beta} > \Pi_0$ for all $\gamma > 0$. Since the optimal quantity of the buyer's money holdings m decreases as the money growth rate μ increases, the optimal m converges to zero as the money growth rate goes to a certain threshold level. Specifically, $m \to 0$ as $\mu \to \tilde{\mu}$ where

$$\tilde{\mu} = \beta\{1 - p + p\theta u'(\tilde{x})\}, \tag{C.47}$$

and $\tilde{x}$ is the solution to

$$(1-p)u'(x) + p\theta u'(x) - 1 = 0. \tag{C.48}$$

Then, the buyer's expected discounted payoff at the limit can be written as

$$\lim_{\mu\to\tilde{\mu}} \Pi_m = \beta\left[(1-p)u(\tilde{x}) + p\theta u(\tilde{x}) - \tilde{x}\right]. \tag{C.49}$$

It is also straightfoward that

$$\lim_{\mu\to\tilde{\mu}} \Pi_m < \Pi_0 \qquad \forall\gamma \in (0, \bar{\gamma}), \tag{C.50}$$

$$\lim_{\mu\to\tilde{\mu}} \Pi_m = \Pi_0 \qquad \forall\gamma \in [\bar{\gamma}, \infty). \tag{C.51}$$

Since $\Pi_m|_{\mu=\beta} > \Pi_0$ and $\lim_{\mu\to\tilde{\mu}} \Pi_m < \Pi_0$, there exists $\bar{\mu} \in (\beta, \tilde{\mu})$ such that $\Pi_m|_{\mu=\bar{\mu}} = \Pi_0$ for $\gamma \in (0, \bar{\gamma})$.

Therefore, an equilibrium can be characterized by (m, x) satisfying equations (3.48) and (3.49) for $\mu \in (\beta, \bar{\mu})$, and (x_l, x_h, π_h) satisfying equations (3.8), (3.10), and (3.11) for $\mu \in [\bar{\mu}, \infty)$. $\square$

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