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Flexural Behaviour of Superelastic Shape Memory Alloy Reinforced Concrete Beams During Loading and Unloading Stages

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ABSTRACT

Trend of using smart structures, which can adjust when exposed to severe unexpected loading, is increasing. One of the methods to achieve such structures relies on smart materials. For example, replacing conventional steel reinforcing bars in Reinforced Concrete (RC) structures with superelastic Shape Memory Alloy (SMA) bars significantly reduces the residual deformations caused by post-yielding behaviour. This paper provides in-depth understanding of the flexural behaviour of SMA RC beams. A sectional analysis method, which predicts the flexural behaviour of SMA RC beams during both loading and unloading stages, is adopted and validated using available experimental data. An extensive parametric study is then carried out to investigate the effect of different geometrical properties. Recommendations for the optimum amount and length of SMA bars are drawn based on results of this study.

Keywords: reinforced concrete (RC), shape memory alloys (SMAs), moment-curvature, load-displacement, sectional analysis, moment-area method.

1 INTRODUCTION

Reinforced Concrete (RC) beams are exposed to reversed cyclic bending moments during earthquake events. These moments can lead to permanent deformations and rotations, which complicate future retrofit efforts or make the damaged structure irreparable.

Superelastic Shape Memory Alloys (SMAs) are set of smart alloys that can undergo large deformations and return to their undeformed shapes upon unloading. They also have exceptional behaviour under cyclic loads (i.e. flag shaped stress-strain). They can undergo large number of inelastic loading/unloading cycles, while keeping zero or very small residual deformations upon load removal [1]. In addition, SMAs have exceptional resistance to corrosion and fatigue loads [2]. These unique properties have motivated researchers to utilize SMAs in different engineering applications. Yet, the use of SMAs in the structural engineering field is considered relatively new.

The use of SMAs as primary reinforcing bars in RC structures is a potential application. Up to now, this application is mainly covered in the research field with few field applications. For example, Saiidi et al. [3] tested simply supported beams reinforced with external SMA bars. Test results showed that the use of SMA bars significantly reduced residual deformations upon unloading as compared to conventional steel bars. Elbahy et al. [4, 5, 6, 7] conducted extensive analytical studies to develop design equations for SMA RC members. The developed equations serve both strength and serviceability requirements. In real applications, SMAs have been used in the rehabilitation of the S. Giorgio Church Bell-Tower in Italy [8] and a RC bridge in Michigan [9].

A 15 mm diameter SMA bar costs about \$1250 per meter [10]. The relatively high cost of SMAs as compared to conventional steel bars limit their use as primary reinforcement. To overcome this problem, researchers suggested limiting the length and position of SMA bars to critical regions of the structure [11, 12]. This suggestion facilitates using SMA bars in real applications. However, there are no guidelines or standards to estimate the needed amount and length of the SMA reinforcement. Thus, the aim of this study is to develop such guidelines

2 ANALYSIS METHOD

The flexural behaviour of steel and SMA RC cross-sections is analytically investigated in this paper. The analysis method is based on the sectional analysis approach, which uses fibre modelling [5, 13]. The studied cross-section is divided into discrete number of horizontal fibres. Utilizing the one-dimensional constitutive relationship of each fibre, and taking into account the cross-section equilibrium and kinematics, the mechanical behaviour of the cross-section is obtained.

A displacement-controlled loading technique is used in the analysis, where the cross-section is subjected to curvature values in an incremental way. During the unloading stage, the load is incrementally removed. Two main assumptions are made: (i) plane sections remain plane (i.e. linear strain distribution); and (ii) perfect bond exists between concrete and the reinforcement.

Four different materials models are implemented in the developed program. These models represent the behaviour of concrete, steel, and SMA under tensile and compressive loadings. Brief description of these models is introduced in the following subsections.

2.1 Concrete under Compression

The model developed by Scott et al. [14], shown in **Fig. 1(a)** and given by **Eq. 1**, is used to model the concrete behaviour under compression. This model represents a good balance between accuracy and simplicity. During the unloading stage, behaviour of concrete is assumed to follow the model proposed by Karsan and Jirsa [15], **Eq. 2**. When unloading starts, the material follows linear straight path that connects the strain at the unloading start, ε_r , to the unloading strain at zero-stress, ε_p . After reaching ε_p , the strains continue to reduce while keeping the stress value equal to zero. This continues till reaching the point of zero strain.

$$f_c = K_h f'_c \left[2.0 \left(\frac{\varepsilon_c}{0.002} \right) - \left(\frac{\varepsilon_c}{0.002} \right)^2 \right] \quad 0 \leq \varepsilon_c \leq 0.002 \quad [1a]$$

$$f_c = K_h f'_c [1 - Z(\varepsilon_c - 0.002)] \quad \varepsilon_c \geq 0.002 \quad \text{and} \quad f_c \geq 0.2 f'_c \quad [1b]$$

$$Z = \frac{0.5}{\frac{3 + 0.29 f'_c (\text{MPa})}{145 f'_c (\text{MPa}) - 1000} + 0.75 \rho_{stirrups} \sqrt{\frac{h'}{S_h}} - 0.002 K_h} \quad [1c]$$

$$K_h = 1 + \frac{\rho_{stirrups} f_y}{f'_c} \quad [1d]$$

where: f_c = concrete compressive stress, Z = slope of compressive strain softening branch, ε_c = concrete compressive strain, K_h = confinement factor, h' = width of the concrete core measured to the outside of ties, S_h = centre-to-centre spacing of the ties or hoop sets, $\rho_{stirrups}$ = ratio of volume of stirrup reinforcement to volume of concrete core measured to outside of the stirrups, and f_y = reinforcement yielding stress.

$$\frac{\varepsilon_p}{\varepsilon_0} = 0.145 \left(\frac{\varepsilon_r}{\varepsilon_0} \right)^2 + 0.13 \left(\frac{\varepsilon_r}{\varepsilon_0} \right) \left(\frac{\varepsilon_r}{\varepsilon_0} \right) < 2 \quad [2a]$$

$$\frac{\varepsilon_p}{\varepsilon_0} = 0.707 \left(\frac{\varepsilon_r}{\varepsilon_0} - 2 \right) + 0.834 \left(\frac{\varepsilon_r}{\varepsilon_0} \right) \geq 2 \quad [2b]$$

where: ε_p = concrete strain at which unloading stress first reaches zero, ε_0 = concrete strain at maximum compressive loading stress f'_c , ε_r = concrete strain when unloading starts.

2.2 Concrete under Tension

Behaviour of concrete under tension is assumed to follow the model proposed by Stevens et al. [15] and simplified by Youssef and Ghobarah [17], **Fig. 1(b)** and **Eq. 3**. In the pre-cracking zone, the concrete behaves in a linear fashion up to the cracking stress f_{cr} . This is followed by significant reduction in the stress values.

$$f_t = f_{cr} \left[0.95 \times e^{-1000 \times (\varepsilon_c - \varepsilon_{cr})} + 0.05 \right] \quad \varepsilon_c \geq \varepsilon_{cr} \quad [3]$$

If unloading starts before reaching f_{cr} , the concrete behaves in a linear fashion similar to the loading stage. If unloading starts after reaching f_{cr} , the material follows a linear path with a slope equal to the modulus of elasticity of concrete. After reaching the zero-stress point, the strain continues to decrease while the stress is kept equal to zero. This continues until reaching the point of zero-strain.

2.3 Steel Bars

The behaviour of the steel material is assumed to follow a bilinear stress-strain relationship under both tension and compression loadings, **Fig. 1(c)**. The material behaves elastically until reaching its yielding strain, ε_{y-s} . Then, the modulus of elasticity is significantly reduced. The equations used to represent the model are:

$$f_s = E_{y-s} \varepsilon_s \quad 0 \leq \varepsilon_s \leq \varepsilon_{y-s} \quad [4a]$$

$$f_s = f_{y-s} + E_{u-s} (\varepsilon_s - \varepsilon_{y-s}) \quad \varepsilon_{y-s} \leq \varepsilon_s \leq \varepsilon_{u-s} \quad [4b]$$

Where: f_s = steel stress, f_{y-s} = steel yielding stress, f_{u-s} = steel ultimate stress, ε_s = steel strain, ε_{y-s} = steel yielding strain, ε_{u-s} = steel strain at failure, E_{y-s} = steel elastic modulus of elasticity, and E_{u-s} = steel plastic modulus of elasticity.

If unloading starts within the pre-yielding zone, the material behaves in an elastic manner similar to the loading stage with no residual deformations at complete unloading. If the unloading starts within the post-yielding zone, the material follows a linear unloading path until yielding on the other side (tension or compression).

2.4 Superelastic SMA Bars

The stress-strain model of Nickel-Titanium (Ni-Ti) SMAs consists of four linear branches that are connected by smooth curves [1], **Fig. 1(d)**. To simplify the modelling process, the smooth curves are ignored, and the linear branches are assumed to directly intersect. The material

behaves elastically until reaching the SMA critical stress f_{cr-SMA} , which represents the start of the martensite stress induced transformation. Exceeding this limit, the material stiffness significantly reduces to about 10% of its initial value. If loading continues until full transformation to martensite phase occurs, the material regains about 50% of its initial stiffness. Then another significant reduction in the material stiffness occurs at yielding. The behaviour of the SMA material during the different loading stages is given by **Eq. [5]**.

$$f_{SMA} = E_{cr-SMA} \epsilon_{SMA} \quad 0 \leq \epsilon_{SMA} \leq \epsilon_{cr-SMA} \quad [5a]$$

$$f_{SMA} = f_{cr-SMA} + E_{pl} (\epsilon_{SMA} - \epsilon_{cr-SMA}) \quad \epsilon_{cr-SMA} \leq \epsilon_{SMA} \leq \epsilon_{pl} \quad [5b]$$

$$f_{SMA} = f_{pl} + E_{y-SMA} (\epsilon_{SMA} - \epsilon_{pl}) \quad \epsilon_{pl} \leq \epsilon_{SMA} \leq \epsilon_{y-SMA} \quad [5c]$$

$$f_{SMA} = f_{y-SMA} + E_{u-SMA} (\epsilon_{SMA} - \epsilon_{y-SMA}) \quad \epsilon_{y-SMA} \leq \epsilon_{SMA} \leq \epsilon_{u-SMA} \quad [5d]$$

Where: f_{SMA} = SMA stress, f_{pl} = maximum recovery stress, f_{y-SMA} = SMA yielding stress, ϵ_{y-SMA} = SMA yielding strain, and ϵ_{u-SMA} = SMA strain at failure.

The behaviour of SMAs during the unloading stage is illustrated in fig. 1(d). if unloading starts before reaching SMA critical stress, the material behaves in an elastic manner (i.e. unloading path 1). If unloading starts when the stress in the material is in between the critical and yielding stresses, the material follows a flag shaped stress-strain relationship (i.e. unloading path 2). If unloading starts after the material reaches its yielding limit), the material follows a linear unloading path (i.e. unloading path 3). For simplicity, it is assumed that the behaviour of the SMA material under compression and tension is identical, and that the material does not exhibit any tension/compression asymmetry. Additionally, it is clear from the figure that allowing the

- 1 SMA material to reach yielding results in eliminating one of its main advantages, which is
- 2 superelasticity. Thus, it is not practical to allow SMAs to yield. This was accounted for, while
- 3 designing the SMA RC sections. To simplify the discussion in the following sections, the SMA
- 4 critical stress $f_{cr-smas}$ is referred to as the yielding stress.

(a) Concrete in compression

(b) Concrete in tension

(c) Steel in tension/compression

(d) SMAs in tension/compression

Fig. 1: Stress-strain models during loading and unloading stages

3 DEFLECTION CALCULATIONS

The moment-area method is utilized to calculate the rotation and deflection values. Steps involved in this method include: (i) perform moment-curvature analysis of the different cross-sections; (ii) obtain the curvature distribution along the length of the member; (iii) calculate rotation by integrating the area under the curvature distribution and deflection by calculating the moment of the integrated area.

4 EXPERIMENTAL VALIDATION

The experimental work performed by Abdulridha [18] and Abdulridha et al. [14] is used to validate the accuracy of using the developed program to predict the flexural behaviour of SMA RC beams. The beams have similar dimensions: 2800 mm length, 2400 mm span, 125 mm cross-section width, and 250 mm cross-section height. The beams differ in the type of flexural reinforcement at the mid-span section (steel or SMAs), and the type of applied loading (monotonic or cyclic). The experimental tests were performed under monotonic, cyclic, and reversed cyclic loadings. The developed analysis tool can capture the behaviour of steel and SMA RC beams during one loading-unloading cycle. Thus, the envelopes of the cyclic and reversed cyclic tests are used to validate the developed analytical tool.

The SMA RC beams are reinforced with 12.7 mm SMA bars over a length of 600 mm centred at the mid-span, and 15M steel bars elsewhere. Mechanical couplers connect the steel and SMA bars. The diameter of the middle 300 mm of the SMA bars is reduced to 9.50 mm. Details of the tested beams are summarized in **Table 1** and are shown in **Figs. 2** and **3**. All beams are tested

under two point loads spaced at 125 mm around mid-span. The average concrete compressive strength is 32.7 MPa for the SMA RC beams and 34.6 MPa for the steel RC beam. Envelopes of SMA and steel stress-strain test results are used to develop idealized stress-strain relationships as shown in **Fig. 4**. Values of $\epsilon_{cr-SMAs}$ and ϵ_{pl} are taken equal to 0.007 mm/mm and 0.071 mm/mm, respectively.

Table 1: Details of beams tested by Abdulridha [18]

Specimen	Loading type	Reinforcement type	Longitudinal Reinforcement	
			Bottom	Top
B1-SM	Monotonic	Steel	2 bars, 10M	2 bars, $\phi = 6.35$ mm
B2-SC	Cyclic	Steel	2 bars, 10M	2 bars, $\phi = 6.35$ mm
B3-SR	Cyclic	Steel	2 bars, 10M	2 bars, 10M
B4-NM	Monotonic	SMAs	2 bars, $\phi = 9.5$ mm	2 bars, $\phi = 6.35$ mm
B6-NR	Cyclic	SMAs	2 bars, $\phi = 9.5$ mm	2 bars, $\phi = 9.5$ mm
B7-NCM	Cyclic	SMAs	2 bars, $\phi = 9.5$ mm	2 bars, $\phi = 9.5$ mm

The developed program is used to perform moment-curvature analysis of the different cross-sections of the studied beams. The moment-curvature diagrams for three different cross-sections of B6-NR are shown in **Fig. 5**: (i) 2-9.5 mm SMA RC cross-section; (ii) 2-12.7 mm SMA RC cross-section; and (iii) 2-15M steel RC cross-section. The moment-curvature analysis of each cross-section is performed up to the experimental unloading moment. Experimental load-displacement results are plotted versus the analytically obtained results for all beams in **Fig. 6**. Very good agreement between the experimental and analytical results is obtained for all beams.

(a) Elevation

Fig. 2: Details of SMA RC beams; (a) Elevation, (b) Cross-section

Fig. 3: Test setup

Fig. 4: Idealized stress-strain models for steel and SMAs bars.

Fig. 5: Moment-curvature curves for steel and SMA RC sections.

(a) B1-SM

(b) B4-NM

(c) B2-SC

(d) B7-NC

(e) B3-SR

(f) B6-NR

Fig. 6: Experimental vs. Analytical load-displacement relationships

5 PARAMETRIC STUDY

A parametric study is carried out to investigate the effect of different geometrical and cross-sectional parameters on the overall behaviour of SMA RC beams during loading/unloading stages. Studied parameters are: (i) cross-section reinforcement ratio (ρ_{SMA_s}); (ii) ratio between the amount of SMA reinforcement to the amount of steel reinforcement (A_{SMA_s}/A_s); (iii) cross-section height-to-width ratio (h/b); (iv) beam span-to-depth ratio (L/h); and (v) concrete compressive strength (f'_c).

The parametric study is performed on cantilever beams, **Fig. 7**. The beams are reinforced with SMA bars at the plastic hinge region (near the fixed end of the beams) and regular steel bars are used elsewhere. Steel couplers are assumed to connect the SMA and steel bars. For each of the studied parameters, nine different lengths are considered for the SMA bars. The considered lengths are: $0.050 L$, $0.100 L$, $0.125 L$, $0.167 L$, $0.250 L$, $0.333 L$, $0.500 L$, and $1.00 L$, where L is the full length of the studied beam. The clear cover is assumed to be 20 mm. The beams are loaded with single point loads applied at their free ends. Details of the studied beams and their cross-sections are given in **Table 2**.

For each of the studied beams, the developed program is used to obtain the moment-curvature relationships of its sections. The moment-area method is then used to obtain the load-displacement relationship. Results are then assessed based on: (i) load-displacement relationship; (ii) amount of residual deformations; (iii) change in flexural stiffness; and (iv) amount of dissipated energy. Detailed discussion of the parametric study results is given in the following section.

1

Table 2: Details of studied beams

Studied parameter	L/h	h/b	f'_c	A_{SMAs}/A_s	ρ_{SMAs}/ρ_{s-min}	ρ_{SMAs}/ρ_{s-b}	Loading level (δ_{max}/δ_y)
ρ_{SMAs}	4	2.0	40	1.0	1.0	0.09	10
	4	2.0	40	1.0	5.5	0.50	10
	4	2.0	40	1.0	11.0	1.0	10
A_{SMAs}/A_s	3	2.3	50	0.5	0.88	0.075	6
	3	2.3	50	1.0	1.76	0.15	8
	3	2.3	50	2.0	3.52	0.30	8
	3	2.3	50	4.0	7.04	0.60	8
h/b	5	1.0	35	1.0	1.4	0.13	6
	5	2.0	35	1.0	2.8	0.27	6
	5	3.0	35	1.0	4.2	0.40	6
L/h	3	3.5	30	1.0	4.0	0.42	7
	6	3.5	30	1.0	4.0	0.42	7
	9	3.5	30	1.0	4.0	0.42	7
f'_c	6	1.5	20	1.0	7.0	0.88	5
	6	1.5	45	1.0	7.0	0.88	5
	6	1.5	65	1.0	7.0	0.88	5

2

3 Where: $\rho_{SMAs} / \rho_{s-min}$ = ratio between the SMA reinforcement ratio and the minimum steel
4 reinforcement ratio; ρ_{SMAs} / ρ_{s-b} = ratio between the reinforcement ratio of the SMA RC cross-
5 section and the reinforcement ratio of the balanced steel RC cross-section; δ_{max}/δ_y = the ratio
6 between the maximum applied displacement at the beam tip and the displacement at which
7 reinforcement starts to yield.

Fig. 7: Details of the RC beams used in the parametric study.

6 RESULTS AND DISCUSSION

6.1 Reinforcement Ratio (ρ_{SMAs})

Three different reinforcement ratios are used: (i) minimum reinforcement ratio (ρ_{min}); (ii) balanced reinforcement ratio (ρ_b); and (iii) intermediate value between the ρ_{min} and ρ_b ($\rho_{int} = 0.5 \rho_b$). Values of the ρ_{min} and ρ_b are determined using the Canadian standards CSA-A23.3-14 for steel RC cross-sections.

Obtained load-displacement relationships of the three beams are shown in **Fig. 8**. Similar behaviour is observed for the three ρ_{SMAs} . As expected, a significant increase in the cross-section yield and maximum capacity is achieved by increasing the cross-section reinforcement ratio.

Fig. 9(a) shows the ratio of residual displacement to maximum applied displacement (δ_r/δ_{max}) plotted versus the ratio of the SMA bar length to the total cantilever beam length (L_{SMAs}/L). Increasing in the SMA bar length reduces the amount of residual displacement at complete

unloading. However, the rate of reduction is not constant. For example, significant reduction in the residual displacement (80%) can be observed with the increase of the SMA bar length from 0.00 L to 0.167 L . Increasing the SMA bar length beyond the 0.167 L has very minor effect on the amount of residual deformations.

Effect of varying the SMA bar length on the flexural stiffness of the SMA RC cantilever beams is shown in **Fig. 9(b)**. The flexural stiffness of the SMA RC beams relative to the steel RC beams is represented by the ratio of δ_{y-s} to $\delta_{cr-SMAs}$. As the SMA bar length increases, flexural stiffness of the cantilever beams decreases. For example, increasing the SMA bar length from 0.00 L to 0.50 L decreased the flexural stiffness of the SMA RC beam by 60% compared to the steel RC beam. This significant reduction in stiffness is attributed to the difference between the modulus of elasticity of steel and SMAs.

Fig. 9(c) shows the ratio between the dissipated energy by the SMA RC beam to that by the steel RC beam (EN_{SMAs}/EN_s) plotted versus the ratio of the SMA bar length to the total cantilever length (L_{SMA}/L). Significant reduction in the amount of dissipated energy (40%) is observed when the SMA bar length increases from 0.00 L to 0.20 L . The rate of reduction in amount of dissipated energy is then reduced. Changing the SMA bar length from 0.20 L to 1.00 L (80%) results in only 15% additional reduction in the amount of dissipated energy.

(a) $\rho = \rho_{s-min}$

(b) $\rho = \rho_{s-int}$

(c) $\rho = \rho_{s-b}$

Fig. 8: Effect of varying the cross-section reinforcement ratio on the load-displacement relationships of steel and SMA RC beams for $L_{SMA_s} = 0.00 L, 0.05 L, 0.25 L, 0.50 L$, and $1.0 L$

(a) Residual displacements

(b) Flexural stiffness

(c) Dissipated energy

Fig. 9: Effect of varying the cross-section reinforcement ratio on: (a) residual displacements; (b) flexural stiffness; (c) dissipated energy

6.2 Ratio between the Amount of SMA Bars and the Amount of Steel Bars (A_{SMAs}/A_s)

Four cantilever beams with four different A_{SMAs}/A_s ratios (0.5, 1.0, 2.0, and 4.0) are considered in the analysis. Obtained load-displacement relationships of the four beams are presented in **Fig. 10**. Varying the A_{SMAs}/A_s ratio significantly affects the load-displacement behaviour. The effect is more pronounced during the unloading stage. Flexural capacity of the steel RC beam ($L_{SMAs} = 0.0$) is the highest. As the length of the SMA bar increases, the flexural capacity decreases.

Different load-displacement behaviour is observed for $A_{SMAs}/A_s = 2.0$ and $A_{SMAs}/A_s = 4.0$. Yielding load of the beam increases when the length of the SMA bar increases. This different trend in behaviour as compared to cases of $A_{SMAs}/A_s = 0.5$ and $A_{SMAs}/A_s = 1.0$ is attributed to the significant increase in the amount of SMA reinforcement.

Amount of residual displacement at complete unloading is significantly affected by varying A_{SMAs}/A_s , as shown in **Fig. 11(a)**. SMA RC beams with $A_{SMAs}/A_s = 0.5$ recover high portion of the maximum applied displacement at complete unloading. When 0.05 L of the steel bars are replaced with SMA bars, residual displacement is reduced by 88%. This reduction is attributed to the intense plasticity happening in the SMA bars at $A_{SMAs}/A_s = 0.5$. For higher A_{SMAs}/A_s ratios, the percentage decrease in the residual displacement is reduced. Flexural stiffness of the studied beams is also affected by changing the A_{SMAs}/A_s ratio. As shown in **Fig. 11(b)**, significant reduction in the flexural stiffness of the beams is observed when the $A_{SMAs}/A_s = 0.5$ and $A_{SMAs}/A_s = 1.0$ are used. For example, using SMA bar length equal to 0.33 L causes a 300% reduction in the flexural stiffness in case of $A_{SMAs}/A_s = 0.5$ and a 240% in case of $A_{SMAs}/A_s = 1.0$.

As shown in **Fig. 11(b)**, increasing A_{SMAs}/A_s significantly lowers the reduction in the flexural stiffness of the beams. For example, using SMAs bar length equal to $0.33 L$ reduces the flexural stiffness by 55% in case of $A_{SMAs}/A_s = 2.0$ and by 15% in case of $A_{SMAs}/A_s = 4.0$.

Fig. 10: Effect of varying the A_{SMAs}/A_s ratio on the load-displacement relationships of steel and SMA RC beams for $L_{SMAs} = 0.00 L, 0.05 L, 0.25 L, 0.50 L$, and $1.0 L$

Effect of varying the A_{SMAs}/A_s ratio on the amount of dissipated energy is illustrated in **Fig. 11(c)**. As shown in the figure, the amount of dissipated energy significantly decreases with the increase in the SMA bar length for $A_{SMAs}/A_s = 0.5$ and $A_{SMAs}/A_s = 1.0$. Reduction in dissipated energy

reaches a value of almost 50%, when the steel bars are replaced with full length SMA bars. For $A_{SMAs}/A_s = 2.0$, the amount of dissipated energy is increased to 125% by increasing the length of SMA bars up to $0.33 L$. Then, it started to decrease with any further increase in the SMA bar length. It reached a value of 105% at SMA bar length of $0.5 L$ and 90% at $1.0 L$. Similar behaviour is observed for $A_{SMAs}/A_s = 4.0$. The increase in the amount of dissipated energy in case of $A_{SMAs}/A_s = 2.0$ and $A_{SMAs}/A_s = 4.0$ is attributed to the increase in the area bounded by the load-displacement curve for small lengths of SMA bars.

6.3 Cross-section Height-to-Width Ratio (h/b)

Three beams with three different h/b values are used in this study: $h/b = 1.0$, $h/b = 2.0$, and $h/b = 3.0$. Calculated load-displacement relationships of the three beams considering different SMA bar lengths are shown in **Fig. 12**. Residual displacement at complete unloading is almost identical for the three beams, **Fig. 13(a)**. It is clear from the figures that the cross-section dimensions do not have a noticeable effect on the load-displacement behaviour of the beams. Increasing the SMA bar length significantly reduces the amount of residual displacements. For example, residual displacement at complete unloading is reduced from 80% to 10% of the maximum applied displacement, when the length of the SMA bars increased from $0.0 L$ to $0.167 L$. Extending the SMA bars beyond the $0.167 L$ length limit has a very minor effect on the residual displacement.

Fig. 13(b) shows no effect for varying the h/b ratio on the flexural stiffness of the three beams. Increasing the length of the SMA bars reduces the flexural stiffness of the cantilever beams. This is attributed to the difference in the modulus of elasticity between steel and SMA materials.

Beams with $h/b = 2.0$ and $h/b = 3.0$ have almost identical change in stiffness when the SMA bar length changes. For $h/b = 1.0$, similar trend in stiffness is observed. However, more pronounced reduction in flexural stiffness is observed when the steel bars are replaced with full length SMA bars. When SMA bar length of $1.0 L$ is used, flexural stiffness is reduced by 400% in case of $h/b = 1.0$ compared to 300% and 310% in case of $h/b = 3.0$ and 2.0 , respectively.

Trend of variation in the amount of dissipated energy with the change in the h/b ratio is plotted in **Fig. 13(c)**. It is clear that h/b ratio does not affect the amount of dissipated energy. Increasing the length of the SMA bars decreases the amount of dissipated energy. The trend of reduction is identical for the three studied beams. Increasing the SMA bar length from $0.0 L$ to $0.20 L$ decreases the amount of dissipated energy by 40%. Replacing steel bars with full length ($1.0 L$) SMA bars results in 55% reduction in the amount of dissipated energy.

(a) Residual displacements

(b) Flexural stiffness

(c) Dissipated energy

Fig. 11: Effect of varying the A_{SMA_s}/A_s ratio on: (a) residual displacements; (b) flexural stiffness; (c) dissipated energy

(a) $h/b = 1.00$

(b) $h/b = 2.00$

(c) $h/b = 3.00$

Fig. 12: Effect of varying the h/b ratio on the load-displacement relationships of steel and SMA RC beams for $L_{SMA} = 0.00 L, 0.05 L, 0.25 L, 0.50 L, \text{ and } 1.0 L$

(a) Residual displacements

(b) Flexural stiffness

(c) Dissipated energy

Fig. 13: Effect of varying the cross-section height-to-width ratio (h/b) on: (a) residual displacements; (b) flexural stiffness; (c) dissipated energy

6.4 Beam Span-to-Depth Ratio (L/h)

Three cantilever beams with: $L/h = 3.0$, $L/h = 6.0$, and $L/h = 9.0$ are considered. **Fig. 14** illustrates the load-displacement response of the three beams when different lengths of the SMA bars are considered. Increasing the L/h ratio significantly reduces the yielding and maximum capacity of the beams.

Fig. 15(a) shows that L/h ratio does not affect the residual displacement. Increasing the length of the SMA bars significantly reduces the amount of residual displacements. Since the reduction in the amount of residual displacement is similar for the three beams, it can be concluded that varying the L/h ratio has no effect on changing the amount of residual displacements at complete unloading. Using SMA bar length of $0.167 L$ reduces the amount of residual displacement at complete unloading by 90%. Increasing the SMA bar length beyond $0.167 L$ is found to have a negligible effect on the residual displacements.

Fig. 15(b) shows that the L/h ratio does not affect the flexural stiffness of the beams. However, increasing the length of the SMA bar significantly decreases the flexural stiffness. Replacing the steel bars with full length ($1.0 L$) SMA bars reduces the flexural stiffness by 200%. Increasing the SMA bar length significantly reduces the effect of L/h ratio on the amount of dissipated energy, **Fig. 15(c)**.

6.5 Concrete Compressive Strength (f'_c)

Three cantilever beams with: $f'_c = 20$ MPa, $f'_c = 45$ MPa, and $f'_c = 65$ MPa are used. Load-displacement responses of the three beams with different lengths of SMA bars are plotted in **Fig. 16**. Varying the concrete compressive strength has a minor effect on the yielding and maximum capacities of the beams. Maximum displacement is reduced by almost 15% when the f'_c value increased from 20 MPa to 65 MPa. The amount of residual displacement is also not affected by varying the f'_c as shown in **Fig. 17(a)**. The residual displacement changes when the length of the SMA bars is increased. Trend of change is identical for the three beams. Increasing the length of the SMA bars from $0.0 L$ to $0.167 L$ reduces the residual displacements by 90%

Fig. 17(b) shows that varying the f'_c value has no effect on the reduction happening in the flexural stiffness. As shown in the figure, using SMA bars with length $0.5 L$ reduces the flexural stiffness by 60%, while replacing the steel bars with length L SMA bars reduces the flexural stiffness by 65%.

(a) $L/h = 3.00$

(b) $L/h = 6.00$

(c) $L/h = 9.00$

Fig. 14: Effect of varying the L/h ratio on the load-displacement relationships of steel and SMA RC beams for $L_{SMAs} = 0.00 L, 0.05 L, 0.25 L, 0.50 L$, and $1.0 L$

(a) Residual displacements

(b) Flexural stiffness

(c) Dissipated energy

Fig. 15: Effect of varying the beam span-to-depth ratio (L/h) on: (a) residual displacements; (b) flexural stiffness; (c) dissipated energy

Fig. 17(c) clearly indicates the negligible effect of f'_c on the amount of dissipated energy. The three beams have a sudden drop (40%) in the amount of dissipated energy when the SMA bar length increased from $0.0 L$ to $0.167 L$. Increasing the SMA bar length beyond the $0.167 L$ has minor effect on the amount of dissipated energy. For instance, replacing the steel bars with full length SMA bars results in only 63% reduction in the amount of dissipated energy as compared to the steel RC beams.

7 CHOICE OF SMA LENGTH

There are only few real applications that utilized SMA bars as primary reinforcing bars. This section provides equations that can predict the residual displacements at complete unloading, the change in the beam stiffness, and the dissipated energy when regular steel bars are replaced with SMA bars. Multiple linear regression technique is used to determine these equations. Linear, quadratic power and logarithmic models are examined. The backward elimination stepwise regression is adopted [19]. All explanatory variables (inputs) are included at the beginning. The non-significant variables are then eliminated one at a time. By the end of the analysis, the remaining variables are only the statistically significant ones.

The database used in the analysis is created from the results obtained from the parametric study. The inputs used in the analysis are: L_{SMAs}/L , A_{SMAs}/A_s , $\rho_{SMAs} / \rho_{s-min}$, ρ_{SMAs} / ρ_{s-b} , f'_c , h/b , LL , L/h . The outputs used in the analysis are: δ_r/δ_{max} , $(\delta_{y-s}/\delta_{cr-SMAs})$, and EN_{SMAs}/EN_s . Descriptive statistics of the used data are presented in **Table 3**

(a) $f_c = 20$ MPa

(b) $f_c = 45$ MPa

(c) $f_c = 65$ MPa

Fig. 16: Effect of varying the concrete compressive strength on the load-displacement relationships of steel and SMA RC beams for $L_{SMAs} = 0.00 L$, $0.05 L$, $0.25 L$, $0.50 L$, and $1.0 L$

(a) Residual displacements

(b) Flexural stiffness

(c) Dissipated energy

Fig. 17: Effect of varying the concrete compressive strength on: (a) residual displacements; (b) flexural stiffness; (c) dissipated energy

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Table 3: Descriptive statistics of the used data

Variable	Number of Observations	Mean	Standard Deviation	Minimum value	Maximum value
δ_r/δ_{max}	144	0.2455369	0.2872124	0.0228764	0.8367937
$\delta_{y-s}/\delta_{cr-SMAs}$	144	0.7231725	0.2538049	0.3590281	2.156095
EN_{SMAs}/EN_s	144	0.5904093	0.2191969	0.1694713	1.058578
L_{SMAs}/L	144	0.2805556	0.2937315	0	1
A_{SMAs}/A_s	144	1.21875	0.7726051	0.5	4
ρ_{SMAs}/ρ_{s-min}	144	4.112833	2.506175	0.83077	10.958
ρ_{SMAs}/ρ_{s-b}	144	0.4028801	0.2516064	0.07538	1
f'_c	144	40.3125	10.71269	20	65
h/b	144	2.270833	0.7429741	1	3.5
Load level (δ_{max}/δ_y)	144	6.6	1.0209	5	8
L/h	144	4.6875	1.614916	3	9

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3 The analysis starts with investigating the correlation between each pair of the variables. Highly
4 correlated pairs and their signs are noted. Correlation matrix is determined and is shown in **Table**
5 **4**. **Tables 5 to 10** present the final regression models for the three outputs. All the variable
6 coefficients reported in these tables are statistically significant from zero at 95% confidence
7 level, as the associated p-values of all the coefficients are less than 0.05.

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Table 4: Correlation coefficients between all variables

	δ_r/δ_{max}	$\delta_{y-s}/\delta_{cr-SMAs}$	EN_{SMAs}/EN_s	L_{SMAs}/L	A_{SMAs}/A_s	ρ_{SMAs}/ρ_{s-min}	ρ_{SMAs}/ρ_{s-b}	f'_c	h/b	LL	L/h
δ_r/δ_{max}	1.00										
$\delta_{y-s}/\delta_{cr-SMAs}$	0.79	1.00									
EN_{SMAs}/EN_s	0.83	0.60	1.00								
L_{SMAs}/L	-0.46	-0.38	-0.76	1.00							
A_{SMAs}/A_s	0.51	0.70	0.23	0.00	1.00						
ρ_{SMAs}/ρ_{s-min}	0.10	0.09	0.10	0.00	0.27	1.00					
ρ_{SMAs}/ρ_{s-b}	0.07	0.04	0.08	0.00	0.21	0.97	1.00				
f'_c	0.16	0.25	0.06	0.00	0.26	-0.18	-0.32	1.00			
h/b	0.05	0.04	0.02	0.00	0.02	-0.02	-0.02	-0.3	1.00		
LL	0.32	0.42	0.08	0.00	0.47	0.00	-0.08	0.06	0.52	1.00	
L/h	-0.21	-0.32	-0.06	0.00	-0.30	0.09	0.17	-0.36	0.09	-0.44	1.00

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The overall model fitness is also reported with the model results. R-squared is the coefficient of determination that represents the proportions of the variance in the dependent variable explained by the independent variables. R-squared is used to assess the overall model goodness-of-fit. The closer the R-squared value to 1.0 the better the model fitness is.

Adjusted R-squared is an adjustment of the R-squared to account for the addition of the explanatory variables and their effect on the model fitness. Root Mean Square Error (Root MSE) is another measure of the model goodness-of-fit. It is the standard deviation of residuals to indicate how close the linear regression model is to the measured data points. The smaller the Root MSE is, the closer the model fit to the observed data. All the reported models are considered to be with very good fit as their R-squared values vary from 0.7 to 0.9. Furthermore, the values of MSE range from 0.05 to 0.39 confirming also a very good model fit [20].

For example, **Table 5** presents a linear regression model of δ_r/δ_{max} when $L_{SMAs}/L \leq 0.14$. The model defines the linear relationship between the dependent variable δ_r/δ_{max} and the second order transformation of the three independent variables (L_{SMAs}/L , A_{SMAs}/A_s , and L/h). The coefficient for each independent variable is estimated. The final suggested regression models are summarized in **Equations [6] to [8]**.

$$\delta_r/\delta_{max} = a_1 \times (L_{SMAs}/L) + a_2 \times (L_{SMAs}/L)^2 + a_3 \times (A_{SMAs}/A_s) + a_4 \times (A_{SMAs}/A_s)^2 + a_5 \times (L/h) + a_6 \times (L/h)^2 + a_7 \quad L_{SMAs}/L \leq 0.14 \quad [6a]$$

$$\ln(\delta_r/\delta_{max}) = b_1 \times (L_{SMAs}/L) + b_2 \times (A_{SMAs}/A_s) + b_3 \times (L/h) + b_4 \times (L/h)^2 + b_5 \quad L_{SMAs}/L > 0.14 \quad [6b]$$

$$\ln(\delta_{y-s} / \delta_{cr-SMA}) = c_1 \times (L_{SMA}/L) + c_2 \times (A_{SMA}/A_s) + c_3 \times (A_{SMA}/A_s)^2 + c_4 \times (L/h) + c_5 \times (L/h)^2 + c_6$$

$$L_{SMA}/L \leq 0.14 \quad [7a]$$

$$\ln(\delta_{y-s} / \delta_{cr-SMA}) = d_1 \times (L_{SMA}/L)^2 + d_2 \times (A_{SMA}/A_s) + d_3 \times (\rho_{SMA} / \rho_{s-b}) + d_4 \times (\rho_{SMA} / \rho_{s-b})^2 + d_5$$

$$L_{SMA}/L > 0.14 \quad [7b]$$

$$EN_{SMA}/EN_s = e_1 \times (L_{SMA}/L) + e_2 \times (L_{SMA}/L)^2 + e_3 \times (A_{SMA}/A_s)^2 + e_4 \quad L_{SMA}/L \leq 0.14$$

$$[8a]$$

$$EN_{SMA}/EN_s = f_1 \times (L_{SMA}/L) + f_2 \times (L_{SMA}/L)^2 + f_3 \times (A_{SMA}/A_s)^2 + f_4 \times (\rho_{SMA} / \rho_{s-b}) + f_5$$

$$L_{SMA}/L > 0.14 \quad [8b]$$

For example, if only 10% of the length of the steel bars of a cantilever beam ($L/h = 5.0$) is replaced with SMA bars (i.e. $L_{SMA}/L = 0.10$), and the cross-sectional areas of the bars are the same ($A_{SMA}/A_s = 1.0$), then this beam will keep only 15.5% of the applied displacement (i.e. $\delta_r/\delta_{max} = 15.5\%$). The stiffness of the beam will be reduced by 29.3% (i.e. $\delta_{y-s} / \delta_{cr-SMA} = 70.7\%$), while the amount of dissipated energy by the beam will be reduced by 33.6% (i.e. $EN_{SMA}/EN_s = 66.4\%$) compared to the steel RC beam.

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Table 5: Regression model for δ_r/δ_{max} when $L_{SMAs}/L \leq 0.14$

Number of obs	=	64
R-squared	=	0.9012
Adj R-squared	=	0.8908
Root MSE	=	0.10197

δ_r/δ_{max}	Coef.	Coef. Value
L_{SMAs}/L	a_1	-9.38644
$(L_{SMAs}/L)^2$	a_2	35.56246
A_{SMAs}/A_s	a_3	0.6020821
$(A_{SMAs}/A_s)^2$	a_4	-0.0932349
L/h	a_5	-0.1102018
$(L/h)^2$	a_6	0.0085443
Constant	a_7	0.5670711

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Table 6: Regression model for δ_r/δ_{max} when $L_{SMAs}/L > 0.14$

Number of obs	=	80
R-squared	=	0.7366
Adj R-squared	=	0.7226
Root MSE	=	0.39181

$\ln(\delta_r/\delta_{max})$	Coef.	Coef. Value
L_{SMAs}/L	b_1	-0.9008659
$\ln(A_{SMAs}/A_s)$	b_2	1.213188
L/h	b_3	-0.4609012
$(L/h)^2$	b_4	0.0353669
Constant	b_5	-1.027448

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Table 7: Regression model for $\delta_{y-s} / \delta_{cr-SMA_s}$ when $L_{SMA_s}/L \leq 0.14$

Number of obs	=	64
R-squared	=	0.83
Adj R-squared	=	0.8154
Root MSE	=	0.08098
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$\ln(\delta_{y-s} / \delta_{cr-SMA_s})$	Coef.	Coef. Value
L_{SMA_s}/L	c_1	-2.871903
A_{SMA_s}/A_s	c_2	0.3076423
$(A_{SMA_s}/A_s)^2$	c_3	-0.0438487
L/h	c_4	-0.0896358
$(L/h)^2$	c_5	0.0066686
Constant	c_6	-0.0422848

Table 8: Regression model for $\delta_{y-s} / \delta_{cr-SMA_s}$ when $L_{SMA_s}/L > 0.14$

Number of obs	=	80
R-squared	=	0.8768
Adj R-squared	=	0.8703
Root MSE	=	0.11583
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$\ln(1/(\delta_{cr-SMA_s}/\delta_{y-s}))$	Coef.	Coef. Value
$(L_{SMA_s}/L)^2$	d_1	-0.2967487
A_{SMA_s}/A_s	d_2	0.3830598
ρ_{SMA_s}/ρ_{s-b}	d_3	-0.7221479
$(\rho_{SMA_s}/\rho_{s-b})^2$	d_4	0.4479831
Constant	d_5	-0.6942598

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Table 9: Regression model for EN_{SMAs}/EN_s when $L_{SMAs}/L \leq 0.14$

Number of obs	=	64
R-squared	=	0.8961
Adj R-squared	=	0.8909
Root MSE	=	0.05337

EN_{SMAs}/EN_s	Coef.	Coef. Value
L_{SMAs}/L	e_1	-4.246152
$(L_{SMAs}/L)^2$	e_2	10.20425
$(A_{SMAs}/A_s)^2$	e_3	0.0130695
Constant	e_4	0.9730266

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Table 10: Regression model for EN_{SMAs}/EN_s when $L_{SMAs}/L > 0.14$

Number of obs	=	80
R-squared	=	0.7995
Adj R-squared	=	0.7888
Root MSE	=	0.05384

EN_{SMAs}/EN_s	Coef.	Coef. Value
L_{SMAs}/L	f_1	-0.8652805
$(L_{SMAs}/L)^2$	f_2	0.4895451
$(A_{SMAs}/A_s)^2$	f_3	0.0141995
ρ_{SMAs}/ρ_{s-b}	f_4	0.057189
Constant	f_5	0.6334503

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8 CONCLUSIONS

In this study, flexural behaviour of SMA RC beams during loading/unloading stages is investigated. Analysis method, which is based on the sectional analysis approach, is used to investigate the flexural behaviour of steel and SMA RC beams. First, the applicability of using the moment-area method with SMA RC beams is validated using available experimental results.

An extensive parametric study is then carried out to investigate the effect of different geometrical and cross-sectional parameters on the flexural behaviour of SMA RC beams. Studied parameters are: (i) cross-section reinforcement ratio; (ii) ratio between the amounts of SMA reinforcement to the amount of steel reinforcement; (iii) cross-section height-to-width ratio; (iv) beam span-to-depth ratio; and (v) concrete compressive strength. For each of the studied parameters, nine load-displacement responses are calculated assuming different lengths of the SMA bars.

Increasing the SMA bar length significantly reduces the amount of residual displacements at complete unloading. In addition, the flexural stiffness is found to decrease significantly by increasing the SMA bar length. Amount of dissipated energy is also found to be dependent on the length of the SMA bars. However, it is noted that the rate of reduction of the flexural stiffness and the dissipated energy reduces as the length of SMA bars increase.

Four out of the five considered parameters in the parametric study are found to have minor or negligible effect on the beams behaviour. The ratio between the amounts of SMA reinforcement to the amount of steel reinforcement is the only parameter that caused a significant change in

behaviour in terms of amount of residual displacement, change in flexural stiffness, and change in amount of dissipated energy.

Results of the parametric study are then used for multiple linear regression analysis to develop equations to help designers address the change occurring in the beam behaviour when regular steel reinforcing bars are replaced with SMA reinforcing bars. The developed equations can predict the behaviour of SMA RC beams for one complete cycle of loading/unloading. Thus, it can be used to predict the behaviour of SMA RC beams under monotonic loading, and the overall behaviour under cyclic and reversed cyclic loading. The developed equations can predict the changes happening in the amounts of residual displacements, flexural stiffness and dissipated energy.

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1 10 LIST OF SYMBOLS

A_{SMA_s}/A_s	Ratio between the amounts of SMA reinforcement to the amount of steel reinforcement.
E_{cr-SMA}	SMA austenite modulus of elasticity.
EN_s	Amount of dissipated energy by the steel RC beam.
EN_{SMA_s}	Amount of dissipated energy by the SMA RC beam.
EN_{SMA_s}/EN_s	Ratio between the amount of dissipated energy by the SMA RC beam and the amount of dissipated energy by the steel RC beam.
E_{pl}	SMA modulus of elasticity during the stress induced transformation from austenite to martensite phase.
E_{y-SMA}	SMA martensite modulus of elasticity.
E_{u-s}	Post-yielding modulus of elasticity of steel.
E_{u-SMA}	Post-yielding modulus of elasticity of SMAs.
E_{y-s}	Pre-yielding modulus of elasticity of steel.
f'_c	Concrete compressive strength.
f_c	Concrete compressive stress.
f_{cr}	Concrete cracking stress.
f_{cr-SMA}	SMA critical stress at which stress induced transformation from austenite to martensite phase starts.
f_t	Concrete tensile stress.
f_y	Reinforcement yield stress.
h/b	Cross-section height-to-width ratio.
L	Full length of the studied beam.
L/h	Beam span-to-depth ratio.
L_{SMA_s}	SMA bar length.
L_{SMA_s}/L	Ratio between the SMA bar length and the total cantilever beam length.
Ni-Ti	Nickel-Titanium.
RC	Reinforced Concrete.
SMAs	Shape Memory Alloys.
Z	Slope of compressive strain softening branch.
δ_{cr-SMA_s}	SMA RC beam critical displacement at which SMA bars reach their critical stress value.
δ_{max}	Maximum applied displacement at the cantilever beam free end.
δ_{max}/δ_y	Ratio between the maximum applied displacement at the beam tip and the displacement at which beam reinforcement starts to yield.
δ_r	Residual displacement.
δ_r/δ_{max}	Ratio between residual displacement and maximum applied displacement.
δ_y	Displacement at which beam reinforcement starts to yield.

δ_{y-s}	Steel RC beam yielding displacement at which steel bars reach their yield stress.
$\delta_{y-s} / \delta_{cr-SMA}$	Inverse of the ratio between the SMA RC beams critical displacement and the steel RC beam yield displacement.
ϵ_0	Concrete strain at maximum compressive loading stress f'_c .
ϵ_c	Concrete compressive strain.
ϵ_{cr}	Concrete cracking strain.
ϵ_{cr-SMA}	SMA critical strain.
ϵ_{cu}	Ultimate concrete compressive strain.
ϵ_p	Concrete strain at which first zero-stress value is achieved after unloading starts.
ϵ_{pl}	SMA strain after complete stress induced transformation from austenite to martensite phase.
ϵ_r	Concrete strain at which unloading starts.
ϵ_{un}	Strain at which unloading start.
ϵ_{u-s}	Steel rupture strain.
ϵ_{y-s}	Yield strain of steel.
ϵ_{y-SMA}	SMA real yielding strain.
ρ_b	Balanced cross-section reinforcement ratio.
ρ_{int}	Intermediate cross-section reinforcement ratio ($\rho_{int} = 0.5 \rho_b$).
ρ_{min}	Minimum cross-section reinforcement ratio.
ρ_{s-b}	Reinforcement ratio of balanced steel RC cross-section.
ρ_{SMA}	Reinforcement ratio of SMA RC cross-section.
ρ_{SMA} / ρ_{s-b}	Ratio between the reinforcement ratio of the SMA RC cross-section and the reinforcement ratio of the balanced steel RC cross-section steel.
$\rho_{SMA} / \rho_{s-min}$	the ratio between the SMA reinforcement ratio and the minimally reinforced cross-section steel reinforcement ratio
ρ_{s-min}	Minimum reinforcement ratio of steel RC cross-section.
$\emptyset$	Bar diameter.