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by

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1. Introduction:

In the traditional theory of the firm, it is customary to assume that the entrepreneur has perfect knowledge of the demand for his output, the supply of the factors and the technical conditions of production. Another common practice is to treat the factors as being perfectly variable and to consider that their productive services are proportional to their stocks. Under these conditions, the profit maximizing firm always operate at full capacity. This result contradicts the observed fact that capacity utilization varies not only when the demand for output falls but also in the case of constant and growing demand.

K. R. Smith [4,5] has recently proposed a model where he attempts to reconcile the theory with this observation. Optimal variations in capital utilization are made possible in the model by modifying some of the above assumptions with respect to the variability of the factors and the relationship between their stocks and the flow of productive services on the one hand, and by introducing uncertainty in the demand conditions on the other.

An important and rather general result obtained by Smith is that the optimum expected rate of capital utilization will be less when demand is uncertain than when demand is known with certainty. But his conclusions with

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respect to the optimum stock of capital are more ambiguous. When a Cobb-Douglas production function is used, it is found that the optimum stock under uncertainty \((K^*)\) is less than the optimum stock under certainty \((\bar{K})\). The author points out however, that if a linear homogeneous CES production function is used instead, it can be shown that for low values of the elasticity of substitution the reverse result will hold, i.e. \(K^* \geq \bar{K}\).

Smith's results suggest that there exists an inverse relationship between the ease of substitution between the factors and the optimal stock of capital under uncertainty. This conclusion appeals to common sense: if labor is a good substitute for capital it seems intuitively that the best policy, for the firm in the face of uncertainty, is to have a relatively small stock of the imperfectly variable factor (capital) and adjust the perfectly variable factor (labor) as needed. If on the other hand, labor is a poor substitute for capital, the cost of being "caught short" of capital may be high and a larger capital stock desirable.

The validity of the above conclusion may however depend on the relatively restrictive assumptions imposed by Smith on the production function. In this paper we intend to examine this question further by giving a more general form to the firm's production function. We shall assume that it is homogeneous of some degree \(\rho\), not necessarily equal to one, in order to allow for non constant returns to scale. This extension seems appropriate here, since the firm under consideration is a monopoly and that monopolies often arise when substantial gains in productivity can be reaped from large scale production. The ease of substitution between the factors is not the only element affecting the optimal stock of capital under uncertainty. We shall show in fact that \(K^*\) depends also on the returns to scale and the elasticity of the demand for output. Before we do so, however, we shall first give a brief description of the model.
2. **The Model**

Formally we consider a monopolist who faces a random isoelastic demand curve given by

\[ D = Ap^\varepsilon; \quad \varepsilon < -1, \ A > 0, \]  

(1)

where \( \varepsilon \) is the constant elasticity of demand and \( A \) is randomly distributed with mean \( \bar{A} \) and variance \( A \). The firm uses only two inputs: an imperfectly variable factor, capital (K) which is variable in the long run but fixed in the short run, and a perfectly variable factor, labor (L). The production function is given by

\[ Q = F(S,L) = F(U,K,L) \]  

(2)

where \( Q \), \( S \) and \( U \) are respectively the level of output, capital services and the rate of utilization of capital. Capital services (S) are assumed to be a multiplicative function of the capital stock (K) and the rate of utilization of capital (U). Labor services are always proportional to the amount of labor used.

The model distinguishes between two types of depreciation rates: the rate of time depreciation (d) which is fixed and exogenous to the firm, and the rate of depreciation in use (γ) which is assumed to be a quadratic function of the rate of utilization U:

\[ \gamma = \gamma(U) = aU^2, \ a > 0. \]  

(3)

Hence in the short-run (i.e. when K is fixed) only the depreciation in use depends on the decisions made by the firm.

The instantaneous profit of the monopoly is given by

\[ \pi = pD - WL - (r + d)GK - \gamma(U)GK \]

where \( W \), \( r \), and \( G \) are respectively the wage rate, the rate of interest and the price of new capital goods. They are all fixed and exogenously determined. The firm is assumed to maximize expected profits, given its stock of capital (K).
The decision variables are labor (L), capital utilization (U), and the price of output (p). It is assumed that, once the price (p) is determined, output (Q) must be equal to demand (D), or

$$D = Q$$ (4)

In Smith's paper L and U are determined after the actual demand curve is known, but two alternative situations are considered regarding p. In the first case, it is assumed that the price decision is a short-run decision (i.e., that it is made after the actual demand curve is known); in the second situation it is assumed to be a long-run decision (i.e., it is made before the actual demand curve is known).

We shall only consider here the case when the price decision is a short-run decision and we shall essentially be concerned with the determination of the optimal stock of capital under uncertainty ($K^*$).

This completes our description of the model. Let us examine now how the optimal decision of the firm with respect to capital is affected by the ease of substitution between the factors, the returns to scale in production and the elasticity of the demand for output.
3. The Optimal Stock of Capital and Uncertainty

In the model described above, the firm faces two different kinds of capital costs: the interest and time depreciation cost \( c_1 \) and the capital utilization cost \( c_2 \). These two costs are respectively expressed here by

\[
c_1 = (r + d) GK
\]

and

\[
c_2 = \gamma(U) GK.
\]

\( c_1 \) is a fixed cost in the short-run since the stock of capital is given, while \( c_2 \) is variable through changes in \( U \).

When the firm makes its decision with respect to the stock of capital (i.e., in the long-run), it faces a trade-off between \( c_1 \) and \( c_2 \): if a relatively large stock of capital is chosen, the short-run fixed cost \( (c_1) \) will be relatively large but the rate of capital utilization \( (U) \) and therefore the utilization cost of capital \( (c_2) \) will be small. The opposite result would hold if the capital stock chosen was relatively small. We can therefore expect the firm to respond to anticipated higher utilization costs of capital by increasing its stock of capital.

Let us define \( \alpha \) the elasticity of the rate of capital utilization \( (U) \) with respect to the demand coefficient \( (A) \) when all the short-run decision variables \( (p, U \text{ and } L) \) are allowed to vary. We intend to show in what follows that \( \alpha \) and \( c_2 \) are positively related; hence that the larger the elasticity of capital utilization with respect to \( A \), the larger will be the optimum stock of capital with uncertainty.

The larger \( \alpha \) the greater will be the change in the optimal rate of capital utilization resulting from a given variation of \( A \). Hence, for any given random distribution of \( A \), the more dispersed will be the \( U \)'s around their mean.
But because the utilization cost function is quadratic in $U$, the more dispersed the $U$'s around their mean, the larger will be the expected utilization cost of capital. Let us show why. The expected cost of capital is given by

$$E(c_2) = E[\gamma(U)Gk]$$

$$= aGk E(U^2).$$

Hence, the larger the mean square of $U$, $E(U^2)$, the larger will be the expected utilization cost of capital $E(c_2)$. Now the mean square of $U$ can be expressed in terms of its mean ($\overline{U}$) and variance ($s$) as follows,\(^1\)

$$E(U^2) = s^2 + \overline{U}^2.$$

Thus, the more dispersed the $U$'s around their mean (i.e., the larger $s$) the larger will be the mean square of $U$ and therefore the utilization cost of capital.

In summary, the larger $\alpha$ the more dispersed will be the $U$'s for any distribution of the $A$'s, hence the larger will be the utilization cost of capital, hence the more the firm will be willing to substitute fixed cost of capital for variable costs and therefore the larger will be the optimum stock of capital under uncertainty.

If we assume the production function to be homogeneous of degree $\rho$, we can express $\alpha$ in terms of the elasticity of substitution between capital and labor services ($\sigma$), the returns to scale ($\rho$), the competitive shares of labor ($k$) and capital ($1-k$), and the elasticity of the demand for output ($\varepsilon$). In what follows, we shall first derive $\alpha$ as a function of $\sigma$, $\varepsilon$, $k$ and $\varepsilon$. We shall then examine how changes in these parameters affect $\alpha$ and therefore $K^*.$

\(^1\)See for instance E. Parzen (1960, p. 346).
Since the price of output is assumed to be determined in the short-run, only $A$ and $K$ are fixed when short-run decisions are made. The short-run first-order condition of profit maximization can therefore be expressed by

$$M_{RP_1} = W \quad (5)$$

$$M_{RP_2} = \gamma ' G \quad (6)$$

and

$$F(L, UK) = Ap^\varepsilon \quad (7)$$

where $M_{RP_1}$ and $M_{RP_2}$ represent respectively the marginal revenue product of labor ($L$) and the marginal revenue product of factor services ($S$). If we make use of (7) in (6) and (5) we can write

$$[A^{-1}F(L, UK)]^{1/\varepsilon} \left( \frac{\varepsilon + 1}{\varepsilon} \right) F_1(L, UK) = W \quad (8)$$

and

$$[A^{-1}F(L, UK)]^{1/\varepsilon} \left( \frac{\varepsilon + 1}{\varepsilon} \right) F_2(L, UK) = \gamma ' G. \quad (9)$$

Let us now take the logarithmic differential of (8) and (9) with respect to $L$, $U$, and $A$. We have,

$$(M_{RP_1})^*_* I_* + (M_{RP_1})^*_* U_* + (M_{RP_1})^*_* A_* = 0 \quad (10)$$

and

$$(M_{RP_2})^*_* I_* + (M_{RP_2})^*_* U_* + (M_{RP_2})^*_* A_* = (\gamma ')^*_* U_* , \quad (11)$$

---

2 In what follows we shall use "stars" to designate partial logarithmic derivatives and logarithmic differential:

$$X^* = \Delta \ln x \quad \text{and} \quad (X)^*_y = \frac{\partial \ln x}{\partial \ln y} .$$
We can solve (10) and (11) for \( \alpha \), the elasticity of \( U \) with respect to \( A \). We obtain:

\[
\alpha = \frac{U^*}{A^*} = \frac{\frac{1}{\epsilon} \left[ (\text{MRP}_2)_1^* - (\text{MRP}_1)_1^* \right]}{(\gamma')_U (\text{MRP}_1)_1^* + (\text{MRP}_1)_1^* (\text{MRP}_2)_2^* - (\text{MRP}_2)_1^* (\text{MRP}_1)_2^*}
\]

(12)

We shall now use some of the results derived in the appendix to express \( \alpha \) in terms of the elasticity of the demand for output (\( \epsilon \)), the elasticity of substitution between the factors (\( \sigma \)) and the degree of homogeneity of the production function (\( \rho \)).

Since in equilibrium the marginal revenue products of the factors \( \text{(MRP}_i \), \( i = 1, 2 \) are equal to the product of their marginal physical products \( \text{(F}_i \), \( i = 1, 2 \) with the marginal revenue \( \text{(MR)} \), we can write

\[
(\text{MRP}_i)_j^* = (\text{MR})_j^* + (\text{F}_i)_j^*, \quad i, j = 1, 2.
\]

In view of (7) the marginal revenue can be express as follows:

\[
\text{MR} = (QA^{-1})^\frac{1}{\epsilon} \left[ \frac{\epsilon + 1}{\epsilon} \right].
\]

Since the demand for output is isoelastic,

\[
(\text{MR})_Q^* = \frac{1}{\epsilon}
\]

Hence

\[
(\text{MR})_j^* = (\text{MR})_Q^* (\text{F}_j)^* = \frac{1}{\epsilon} (\text{F}_j)^*, \quad j = 1, 2
\]

If we now let

\[
k = \frac{(\text{F})_1^*}{\rho}
\]

and

\[
(1-k) = \frac{(\text{F})_2^*}{\rho},
\]

\[
(1-k) = \frac{(\text{F})_2^*}{\rho},
\]
where \( k \) and \( 1-k \) represent respectively the competitive shares of labor and capital services, we have:

\[
(MR)_1^* = \frac{\sigma k}{\epsilon},
\]

and

\[
(MR)_2^* = \frac{(1-k)\rho}{\epsilon}.
\]

as well, \((MR)_A^*\) is given by

\[
(MR)_A^* = (MRP_1)_A^* = -\frac{1}{\epsilon}.
\]

Furthermore, it is shown in the appendix that

\[
(F_1)_1^* = k \left[ \frac{1}{\sigma} + (\rho - 1) \right] - \frac{1}{\sigma}
\]

\[
(F_1)_2^* = (1-k) \left[ \frac{1}{\sigma} + (\rho - 1) \right]
\]

\[
(F_2)_1^* = k \left[ \frac{1}{\sigma} + (\rho - 1) \right]
\]

and,

\[
(F_2)_2^* = (1-k) \left[ \frac{1}{\sigma} + (\rho - 1) \right] - \frac{1}{\sigma}
\]

Finally, since \( \gamma \) is a quadratic function of \( U \)

\[
(\gamma')_U^* = \frac{2aU}{2aU} = 1.
\]

It follows therefore that in expression (3.12.8)

\[
(MRP_1)_1^* = \frac{\sigma k}{\epsilon} + k \left[ \frac{1}{\sigma} + \rho - 1 \right] - \frac{1}{\sigma}
\]

\[
(MRP_1)_2^* = \frac{(1-k)\rho}{\epsilon} + (1-k) \left[ \frac{1}{\sigma} + \rho - 1 \right]
\]

(14)

\[
(MRP_2)_1^* = \frac{\sigma k}{\epsilon} + k \left[ \frac{1}{\sigma} + \rho - 1 \right]
\]
and
\[(\text{MP}_2')^*_{2} - (\gamma')^*_{U} = \frac{\sigma(1 - k)}{\varepsilon} + (1 - k) \left[ \frac{1}{\sigma} + \rho - 1 \right] - \frac{1}{\sigma} - 1\]

It is sufficient for the second order conditions of profit maximization to hold that
\[\text{(MP}_1')^*_{1} < 0, \quad \text{(MP}_2')^*_{2} - (\gamma')^*_{U} < 0\]

and
\[\left(\text{MP}_1')^*_{1} \left[ (\text{MP}_2')^*_{2} - (\gamma')^*_{U} \right] - (\text{MP}_1')^*_{2} (\text{MP}_2')^*_{1} > 0\]

Thus, the denominator of \(\alpha\) in (13) must be positive. Furthermore it follows from (14) that
\[\text{(MP}_2')^*_{1} - (\text{MP}_1')^*_{1} = \frac{1}{\sigma} > 0.\]

These results imply that \(\alpha\) must always be positive.

If we now solve (13) for \(\alpha\) making use of the relation (14) we finally obtain,
\[\alpha = \frac{1}{[\varepsilon(p - 1) + \rho][\sigma k + 1] - \varepsilon(1 - k)}.\]  \hspace{1cm} (15)

The value of \(\alpha\) in (15) depends on \(\sigma\), \(\varepsilon\) and \(\rho\). If we take the partial derivatives of \(\alpha\) with respect to these parameters we obtain the following results:
\[\frac{\partial \alpha}{\partial \sigma} = -k[\varepsilon(p - 1) + \rho] \quad \text{[square term]} \]  \hspace{1cm} (16)

\[\frac{\partial \alpha}{\partial \varepsilon} = \frac{-(\rho - 1)(\sigma k + 1) - (1 - k)}{[\text{square term}]} \]  \hspace{1cm} (17)

and
\[\frac{\partial \alpha}{\partial \rho} = \frac{-(\varepsilon + 1)(\sigma k + 1)}{[\text{square term}]} \quad .\]  \hspace{1cm} (18)
In order to make the interpretation of equations (16)-(18) easier, it is useful to derive first a formula for the elasticity of the marginal cost curve as a function of \( k, \rho \) and \( \sigma \). Let us call \( \lambda \) the marginal cost of output and let us define \( \beta \), the elasticity of the marginal cost with respect to output as follows:

\[
\beta \equiv \frac{\lambda^*}{Q^*}
\]

The optimum use of the two factors requires that

\[
\lambda = \frac{W}{F_1} = \frac{\gamma^* G}{F_2}
\]  \hspace{1cm} (19)

Furthermore, the output \( Q \), must be such that

\[
Q = F(L, UK)
\]  \hspace{1cm} (20)

If we take the total logarithmic differential of (19) and (20) with respect to \( U \) and \( L \) and solve for \( \beta \) we obtain

\[
\beta = \rho \frac{k[(F_2^*)_2 - (F_1^*)_2] + (1 - k)[(F_1^*)_1 - (F_2^*)_1 - 1]}{(F_2^*)_1 (F_1^*)_2 - (F_2^*)_2 (F_1^*)_1 + (F_2^*)_2}
\]

If we now substitute for \( (F_i^*)_j \), \( i, j = 1, 2 \), we finally obtain

\[
\beta = \frac{(1 - k) - (\rho - 1) [\sigma k + 1]}{\rho [1 + \sigma (1 - k)]}
\]  \hspace{1cm} (21)

We can see in relation (21) that the marginal cost curve will be upward sloping or downward sloping depending on whether

\[
(1 - k) - (\rho - 1) [\sigma k + 1] \gtrless 0.
\]

One can easily check also that the second order conditions derived earlier require that the marginal cost increases faster than the marginal revenue in the neighbourhood of the position of equilibrium. Furthermore, it can be shown by taking the partial derivative of \( \beta \) with respect to \( \rho \) and \( \sigma \) that the larger the
returns to scale and the greater the ease of substitution between the factors, the more elastic will be the marginal cost curve.

Let us turn now to the interpretation of equations (16)-(18). It was pointed out in the introduction that Smith's results as well as "common sense" suggest that there should be an inverse relationship between \( \sigma \) and \( K^* \). We would therefore expect \( \partial \alpha / \partial \sigma \) to be negative. However we can see in relation (16) that

\[
\frac{\partial \alpha}{\partial \sigma} > 0 \quad \text{as} \quad \varepsilon (\rho - 1) + \frac{\rho}{\varepsilon} < 0
\]

(22)

Since \( \varepsilon \) is strictly negative, we could as well express condition (22) as follows:

\[
\frac{\partial \alpha}{\partial \sigma} \geq 0 \quad \text{as} \quad (\rho - 1) + \frac{\rho}{\varepsilon} \geq 0
\]

It can easily be checked that \( (\rho - 1) + \frac{\rho}{\varepsilon} \) represents the elasticity of the marginal revenue products of both factors when a change in output takes place along an expansion path, that is:

\[
\rho - 1 + \frac{\rho}{\varepsilon} = \frac{\text{MMP}_1}{L^*} \quad \text{d}(\frac{L}{U^*}) = 0 \quad \frac{\text{MMP}_2}{U^*} \quad \text{d}(\frac{L}{U^*}) = 0
\]

Hence, whether a high degree of substitutability between the factors will make for a large or small optimal stock of capital under uncertainty, depends on whether a positive movement along an expansion path leads to an increase or a decline in the marginal revenue products of both factors. This result can be explained as follows: the larger the elasticity of substitution between the factors, the easier it is for the firm to substitute \( L \) for \( U \) when output expands. Hence the smaller need be the relative increase in \( U \) for any given relative increase in output. On the other hand, and as was pointed out above, the larger \( \sigma \) the more elastic will be the marginal cost curve, hence the larger will be the increase in output resulting from a given shift of the demand curve.
Consequently, the larger will be the increase in $U$ and $L$ necessary to accommodate this expansion in output. Depending on which one of these effects dominates, $\frac{\partial \alpha}{\partial \sigma}$ will be positive or negative. It turns out here, that the first effect, or "substitution effect," dominates when $\varepsilon(\rho - 1) + \rho > 0$, but that the second, or "output effect," dominates when $\varepsilon(\rho - 1) + \rho < 0$.

Let us turn now to the role of the elasticity of the demand curve. We can see in relation (17) that

$$\frac{\partial \alpha}{\partial \varepsilon} < 0$$

as

$$(1 - k) - (\rho - 1)(\sigma k + 1) > 0.$$  \hspace{1cm} (23)

This is precisely the condition which was found earlier for the marginal cost curve to be upward sloping or downward sloping. Hence when the marginal cost curve is upward sloping, a given horizontal shift of the demand for output have a larger effect on the use of the factors when the demand curve is elastic than when it is inelastic. The opposite is true however when the marginal cost curve is upward sloping. This result is illustrated in figure 1.

Figure 1.
Starting from the original position of equilibrium E, a given horizontal shift of the demand curve induces a movement from E to $E_1$ along the upward sloping marginal cost curve $MC_1$, when the demand curve is inelastic (marginal revenue curve $MR_1$ and $MR'_1$). If the demand curve is elastic however (marginal revenue curve $MR_2$ and $MR'_2$), the same horizontal shift induces only a movement from E to $E_2$ along $MC_1$. On the other hand, if the marginal cost curve is downward sloping (curve $MC_2$), a given horizontal shift of the demand curve will have a stronger effect when the demand is elastic (movement from E to $E'_2$) than when it is inelastic (movement from E to $E'_1$). In summary, whether an elastic demand will make for a large stock of capital under uncertainty will depend on whether the returns to scale and the ease of substitution between the factors are large enough to make the marginal cost curve downward sloping.

Large returns to scale tend to make $\alpha$ larger as can be seen in (18). Thus the larger the returns to scale in production, the larger will be the optimum stock of capital under uncertainty. This results from the fact that large returns to scale make the marginal cost curve more elastic and a given change in $A$ has a larger effect on the optimum level of output, hence on the use of the factors.
Summary and Conclusions:

The main objective of this paper has been to examine how the optimal stock of capital for a firm facing a random demand curve is related to the production function's parameters (elasticity of substitution between the factors and returns to scale) and the elasticity of the demand for output when the rate of capital utilization is allowed to vary in the short run (the stock of capital being fixed), subject to changes in the rate of depreciation. Throughout the production function of the firm is assumed to be homogeneous of some positive degree $\rho$ not necessarily equal to one.

The following results have been obtained:

1) Other things being equal, the larger the elasticity of substitution between the factor the larger (smaller) will be the optimal stock of capital under uncertainty if a positive movement along an expansion path always lead to an increase (a decline) in the marginal revenue products of the factors.

2) Other things being equal, the more elastic the demand for output, the smaller (larger) will be the optimal stock of capital under uncertainty when the marginal cost curve is upward (downward) sloping.

3) Other things being equal, the larger the return to scale in production, the larger will be the optimal stock of capital under uncertainty.
Appendix

Some Properties of Homogeneous Production Functions

Throughout this appendix we shall consider a production function homogeneous of degree \( \rho \) given by

\[
q = F(L, S)
\]

where

\[
S = U_k, \quad F_1 = \frac{\partial F}{\partial S}, \quad F_2 = \frac{\partial F}{\partial L}, \quad F_{12} = \frac{\partial^2 F}{\partial L \partial S}.
\]

**Proposition 1:** The elasticity of substitution between \( L \) and \( S \) is given by

\[
\sigma = \frac{F_1 F_2}{F_{12} F - (\rho - 1) F_1 F_2}
\]

**Proof:** From Euler's theorem, the partial derivative of \( F \) with respect to \( L \) and \( S \) is homogeneous of degree \( (\rho - 1) \). Their ratio will therefore be homogeneous of degree zero.

Let

\[
g(L, S) = \frac{F_2}{F_1}
\]

Since \( g \) is homogeneous of degree zero we have

\[
\frac{\partial g}{\partial L} L + \frac{\partial g}{\partial S} S = 0
\]

If we differentiate \( g \) totally with respect to \( L \) and \( S \) we have

\[
dg = \frac{\partial g}{\partial S} dS + \frac{\partial g}{\partial L} dL
\]

If we now make use of the homogeneity property of \( g \) we can write

\[
dg = \frac{\partial g}{\partial L} (L/S)^*\]

From which we can obtain

\[
\sigma = \frac{F_1 F_2}{F_{12} F - (\rho - 1) F_1 F_2}
\]
where

\[
\sigma = \frac{(L/S)^*}{g},
\]

is the elasticity of substitution between L and S.

Relation II reduces to Allen's result (1938, p. 343):

\[
\sigma = \frac{F_1 F_2}{F_{12}},
\]

when \( \rho \) is equal to one.

Relation II can also be expressed as

\[
F_{12} = \frac{[1 + \sigma (\rho - 1)]}{\rho F \sigma}.
\]

From III we can easily derive that

\[
(F_2)_1^* = k \left[ \frac{1}{\sigma} + (\rho - 1) \right]
\]

\[
(F_2)_2^* = -\frac{k}{\sigma} + (1 - k) (\rho - 1)
\]

\[
(F_1)_2^* = (1 - k) \left[ \frac{1}{\sigma} + (\rho - 1) \right]
\]

and

\[
(F_1)_1^* = -\frac{(1 - k)}{\sigma} + k(\rho - 1).
\]
References


