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## Performance-based design of RC beams using an equivalent standard fire

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# Performance-Based Design of RC Beams Using an Equivalent Standard Fire

## Abstract

The design of buildings for fire events is essential to ensure occupant safety. Supplementary to simple prescriptive methods, performance-based fire design can be applied to achieve a greater level of safety and flexibility in design. To make performance-based fire design more accessible, a time equivalent method can be used to approximate a given natural fire event using a single standard fire with a specific duration. Doing so, allows for natural fire events to be linked to the wealth of existing data from the standard fire scenario. In this paper, the use of an existing time equivalent method is reviewed and assessed for application in the performance-based design of reinforced concrete (RC) beams. The assessment is established by computationally developing the moment-curvature response of RC beam sections during fire exposure. The sectional response due to natural fire and time equivalent fire are compared. It is shown that the examined time equivalent method is able to predict the sectional response with suitable accuracy for performance-based design purposes.

*Keywords:* Standard Fire, Natural Fire, Reinforced Concrete, RC Beams, Performance-Based Design, Time Equivalent, Bending Moment-Axial Load Relationship

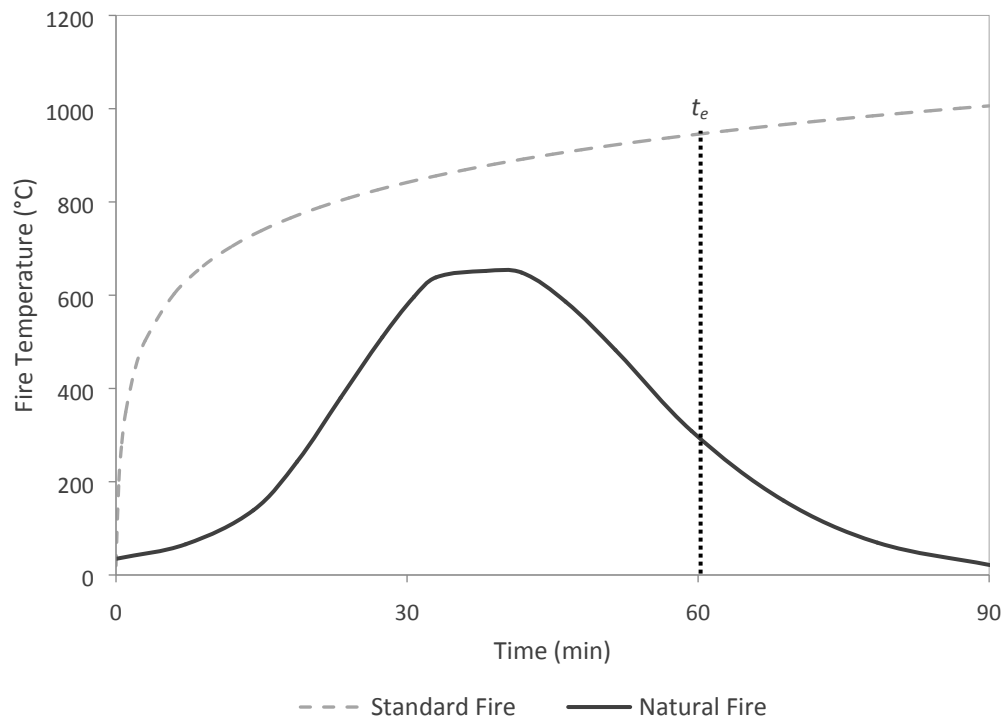
## 1. Introduction

In both Canada and the United States, buildings are primarily designed for fire events using prescriptive standards (NFC, 2015; NFPA 1, 2018), developed based on historic experience of real world and laboratory fires. Although these standards have been successful over the past decades in reducing fire related injuries, they provide minimal knowledge of structural integrity during the fire event, and as such, can only provide limited protection within the extent of the historic experience. To provide a better understanding for fire safety design, the most recent publication of ASCE-7 (2016) provides a framework for a performance-based approach. By implementing performance-based methodology in a fire safety analysis, structures can be designed using sound engineering principles to reduce construction costs and improve public safety.

### 1.1 Use of a Time Equivalent in Performance-Based Design

One of the first and more difficult steps in undertaking a performance-based fire design, lies in accurately defining the fire load applied to a structural element. Fire events are typically represented using a temperature-time relationship, depicting the change in a compartment's gas temperature over time (Fig. 1). Every fire event is different; however, a typical natural fire will follow the growth and decay profile identified in Fig. 1. The performance of a structural element varies greatly depending on a fire event's specific temperature-time relationship. For reinforced concrete (RC), it has been shown that a fire's heating rate, maximum temperature, cooling rate, and overall duration all influence strain development and material degradation (Mohamedbhai, 1986; Zhang et al., 2001). To simplify the complexity of natural fire events, the fire safety industry has defined a single standard fire, shown in Fig. 1 (ISO 834, 2014). The standard fire does not

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3 follow the typical profile of a natural fire event; instead, it was intended to describe a severe  
4 heating scenario that could be easily recreated using laboratory furnaces. Because the standard  
5 fire is not capable of representing a natural fire event, it lacks direct suitability for performance-  
6 based design purposes.  
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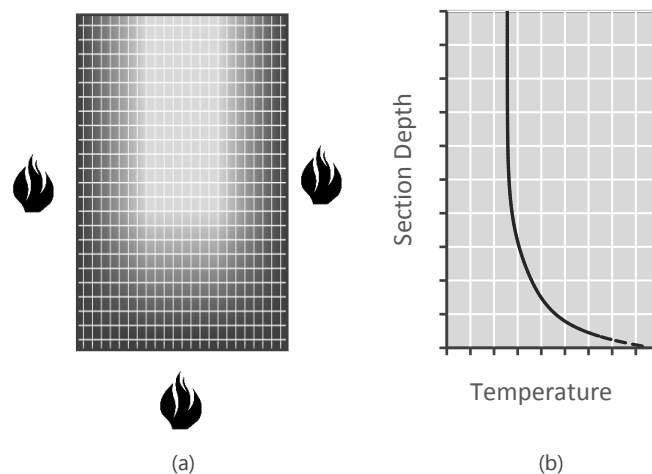
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39 **Fig. 1: Temperature-Time Relationship for Typical Natural and Standard Fire**

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44 A time equivalent ( $t_e$ ), is a means by which the severity of a natural fire event can be approximated  
45 as a single standard fire duration. Using a time equivalent method, the wealth of existing data  
46 and material models developed with the standard fire, can be related to a natural fire event, and  
47 in turn, applied to a performance-based analysis. In a previous publication by the first two authors  
48 (Kuehnen and Youssef, 2019), a time equivalent (AITP  $t_e$ ) was developed specifically for RC beams.  
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In this paper, the AITP  $t_e$  is first summarized, and then further evaluated in view of the flexural response of RC beams during natural fire exposure.

## 2. AITP Time Equivalent

During fire exposure, a typical RC beam undergoes heating from the two sides and lower face, resulting in an internal temperature gradient with hotter temperatures at the surface and cooler temperatures towards the core (Fig. 2a). To simplify an internal temperature gradient, the average internal temperature profile (AITP) can be calculated. This is achieved by dividing the section into a fine mesh and averaging the temperature across each horizontal mesh layer (Fig. 2b). The subsequent AITP of a section reflects the variation of the average temperature with depth. Application of AITP's in performance-based design has been proven in such publications as El-Fitiyany et al. (2017) and El-Fitiyany and Youssef (2009).



**Fig. 2: Typical Section Internal Temperatures: (a) 2D Temperature Gradient and (b) AITP**

The AITP  $t_e$  is defined as the duration of standard fire required to generate the equivalent AITP in an RC section as experienced by a selected natural fire (Kuehnen and Youssef, 2019). Equivalence is defined based on either mean or conservative criteria. A standard fire with a mean AITP  $t_e$  produces an internal temperature profile closely matching that of the natural fire. While a standard fire with a conservative AITP  $t_e$  results in a profile with equal or larger temperatures at every layer. A size adjustment factor ( $\psi_{size}$ ) was also proposed in the original publication to account for the influence of variable beam width ( $b_c$ ) and height ( $h_c$ ) on the AITP  $t_e$  duration. The general equations to calculate the AITP  $t_e$  and  $\psi_{size}$  are provided in Equation 1, with coefficients and the valid ranges in Table 1 (Kuehnen and Youssef, 2019). To best characterize the general form of a natural fire, the equations were derived in terms of the maximum temperature ( $T_{max}$ ), time of maximum temperature ( $t_{max}$ ), and overall duration ( $t_{final}$ ). For a worked example of the AITP  $t_e$ , refer to Kuehnen et al. (2019).

$$t_e = A + Bt_{max} + Ct_{final} + DT_{max} + Et_{max}^2 + Ft_{final}^2 + GT_{max}^2 + Ht_{max}t_{final} + It_{max}T_{max} + Jt_{final}T_{max} \quad (1a)$$

$$\psi_{size} = \begin{cases} 1.0 & \left\{ \begin{array}{l} \text{for } bc < 300 \text{ m} \\ \text{for conservative } t_e \text{ when } T_{max} > 1150^\circ\text{C} \\ \text{for conservative } t_e \text{ when } t_e > 180 \text{ min} \end{array} \right. \\ \frac{A + Bt_{max} + Ct_{final} + DT_{max} + b_c(E + Ft_{max} + Gt_{final} + HT_{max})}{b_c(E + Ft_{max} + Gt_{final} + HT_{max})} \geq 1.0 \end{cases} \quad (1b)$$

Table 1: Valid Ranges and Coefficients for Equation 1

Valid Range		$t_e$ (Eq. 1a)					$\psi_{size}$ (Eq. 1b)	
		Mean	Conservative				Mean	Conservative
Valid Range	$b_c$ (mm)	250	250				200 - 800	200 - 800
	$h_c$ (mm)	500	500				300 - 800	300 - 800
	$t_{max}$ (min)	15 - 115	15 - 115				15 - 115	15 - 115
	$t_{final}$ (min)	20 - 240	20 - 240				20 - 240	20 - 240
	$T_{max}$ (°C)	350 - 1200	350 - 750	750 - 950	950 - 1100	1100 - 1200	350 - 1200 <sup>1</sup>	350 - 1200
Coefficients	A	8.124	8.685	2.370	566.30	4404.0	1.022	0.819
	B	-0.153	-0.0829	-0.0893	-0.465	-5.745	-2.57 x10 <sup>-4</sup>	3.78 x10 <sup>-4</sup>
	C	0.0384	0.0324	0.0446	1.188	1.039	2.69 x10 <sup>-4</sup>	-2.23 x10 <sup>-4</sup>
	D	-0.0431	-0.0428	-0.0186	-1.332	-8.177	-0.22 x10 <sup>-4</sup>	1.82 x10 <sup>-4</sup>
	E	-8.53 x10 <sup>-4</sup>	-4.74 x10 <sup>-4</sup>	-9.42 x10 <sup>-4</sup>	-20.00 x10 <sup>-4</sup>	-80.87 x10 <sup>-4</sup>	0.113	1.037
	F	-6.46 x10 <sup>-4</sup>	-4.16 x10 <sup>-4</sup>	-7.39 x10 <sup>-4</sup>	0.0	2.99 x10 <sup>-4</sup>	-8.23 x10 <sup>-4</sup>	-27.00 x10 <sup>-4</sup>
	G	0.50 x10 <sup>-4</sup>	0.66 x10 <sup>-4</sup>	0.35 x10 <sup>-4</sup>	7.95 x10 <sup>-4</sup>	38.36 x10 <sup>-4</sup>	14.01 x10 <sup>-4</sup>	27.15 x10 <sup>-4</sup>
	H	3.44 x10 <sup>-4</sup>	1.57 x10 <sup>-4</sup>	4.77 x10 <sup>-4</sup>	-3.07 x10 <sup>-4</sup>	-17.80 x10 <sup>-4</sup>	-1.93 x10 <sup>-4</sup>	-10.75 x10 <sup>-4</sup>
	I	6.55 x10 <sup>-4</sup>	5.33 x10 <sup>-4</sup>	5.40 x10 <sup>-4</sup>	12.05 x10 <sup>-4</sup>	69.36 x10 <sup>-4</sup>	---	---
	J	4.52 x10 <sup>-4</sup>	3.70 x10 <sup>-4</sup>	4.71 x10 <sup>-4</sup>	-9.00 x10 <sup>-4</sup>	-8.40 x10 <sup>-4</sup>	---	---

<sup>1</sup> Excluding  $T_{max} < 750$  °C reached during  $t_{max} < 60$  min



### 3. Moment-Curvature Assessment of the AITP $t_e$

Performance-based requirements are typically divided into serviceability and ultimate limit states, which can be measured using deflection and load capacity (Purkiss, 2007). In the case of RC beams, both of these requirements can be best represented by the sectional bending moment-curvature ( $M-\varphi$ ) response. To evaluate the AITP  $t_e$  in the application of performance-based design, a study was undertaken to develop the fire exposed  $M-\varphi$  relationship for a variety of RC beam sections and a range of natural fires. In this section, the sectional analysis method, study parameters, and evaluated results are presented to demonstrate the suitability of the AITP  $t_e$ .

#### 3.1 Sectional Analysis Method

A structural analysis program developed by El-Fitiany and Youssef (2009) was used to produce the  $M-\varphi$  response of beams during fire exposure. The program has three main steps: (1) determine the internal temperatures of the section, (2) evaluate the thermal strains at elevated temperature and (3) iteratively simulate an applied load to approximate the full sectional response. Section internal temperatures are calculated using the finite difference method (FDM) presented by Lie (1992). Three-sided fire exposure is applied to the RC section from the two sides and lower face. Concrete thermal strains ( $\varepsilon_T$ ) are estimated using the equations provided by Youssef and Moftah (2007). Sectional analysis is then carried out iteratively to determine the  $M-\varphi$  relationship. The program makes the following assumptions: (1) plane sections remain plane during fire exposure, as previously validated up to 1200°C by El-Fitiany and Youssef (2011); (2) perfect bond exists between steel and concrete; (3) normal strength concrete (NSC) is used, and

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3 thus, explosive spalling can be ignored; (4) influence of concrete tensile cracks on heat flow is  
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5 ignored; and (5) geometrical nonlinearity is not considered.  
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## 10 **3.2 Study Methodology**

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12 For a given RC section, the  $M-\varphi$  response was calculated for both natural and standard fire  
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14 exposure. The natural fire was assembled based on experimentally recorded temperature-time  
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16 relationships and theoretical profiles developed using the Eurocode approach (EN 1991-1-2,  
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18 2002). The standard fire was applied following the ISO profile (ISO 834, 2014) for a given mean  
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20 or conservative AITP  $t_e$  duration. Each cross-section was evaluated for three fire events: the design  
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22 fire, the AITP mean standard fire, and the AITP conservative standard fire. Due to the impracticality  
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24 of displaying the full  $M-\varphi$  diagram for a large range of design fires and cross-sections, three key  
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26 responses are identified for comparison. They are: the maximum moment at elevated temperature  
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28 ( $M_{iT}$ ), the initial curvature at elevated temperature ( $\varphi_{iT}$ ), and the initial stiffness at elevated  
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30 temperature ( $El_{iT}$ ). These three responses are crucial to defining the  $M-\varphi$  relationship, and in turn,  
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32 the serviceability and ultimate limit states needed for performance-based design.  
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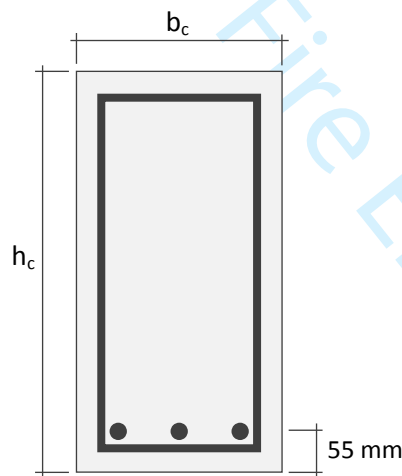
## 43 **3.3 Sample RC Sections and Fire Exposures**

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45 Seven rectangular sections were selected to examine the  $M-\varphi$  response. Table 2 displays the  
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47 section properties. The studied parameters are: concrete strength ( $f'_c$ ), section width ( $b_c$ ), section  
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49 height ( $h_c$ ), tension reinforcement ratio ( $\rho_s$ ), and aggregate type (*agg.*) of either siliceous (*sil.*) or  
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51 calcareous (*cal.*). Fig. 3 exhibits general details of the studied cross-sections. At ambient  
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53 conditions, the value of  $f'_c$  is specified as either 30 or 40 MPa, and the steel yield strength ( $F_y$ ) is  
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held constant at 400 MPa. Longitudinal steel area was equally split into 3 bars, spaced at 55 mm from center to concrete face. Thermal properties for normal strength concrete (NSC) with siliceous and calcareous aggregate were applied from Lie (1992). The consideration of compression reinforcement and stirrup confinement was neglected for simplicity.

**Table 2: Parametric Study Beam Properties**

Beam #	$F_y$ MPa	$f'_c$ MPa	$b_c$ mm	$h_c$ mm	$\rho_s$ %	agg.	Studied Parameter
B1	400	30	250	500	1.0	sil.	$\rho$
B2					1.5		$\rho$
B3					2.0		$\rho$
B4		40	400	800	1.0	cal.	agg.
B5					1.0	$f'_c$	
B6		30	600	800	1.0	sil.	$b_c, h_c$
B7					1.0		$b_c, h_c$

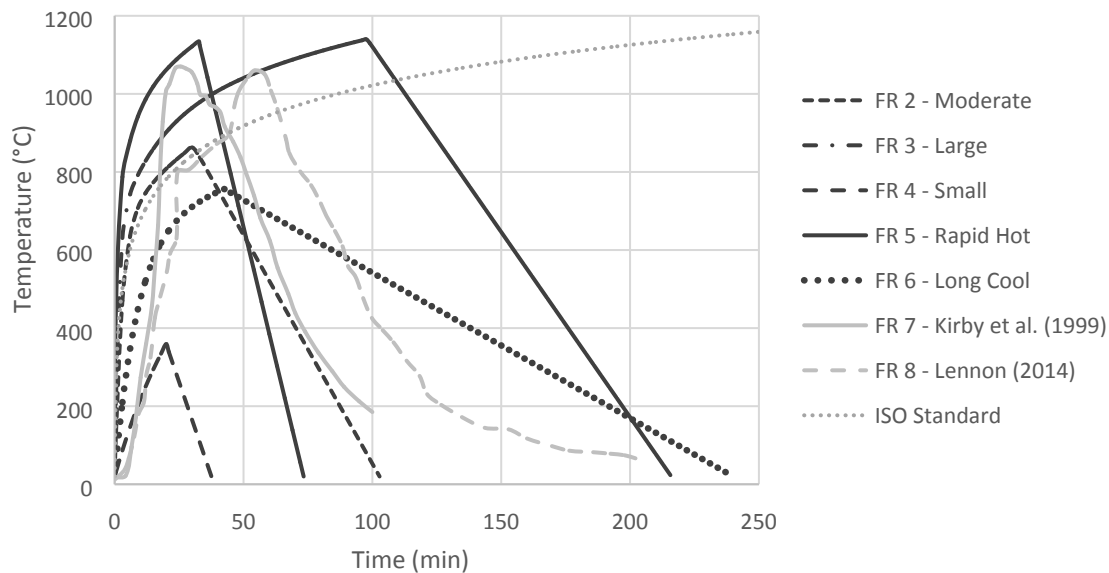


**Fig. 3: Cross Section of Parametric Study RC Beam**

Seven design fires were specified for the study (Fig. 4). The first five were developed using the Eurocode approach to demonstrate a range of possible natural fire events (EN 1991-1-2, 2002).

They can be broadly categorized as moderate, large, small, rapid hot, and long cool. The remaining two fires were taken from the experimental literature presented by Kirby et al. (1994) and Lennon (2014). The two experimental programs provide a good representation of typical natural fires that can occur in a concrete structure. The AITP  $t_e$  can be determined using Equation 1 based on the graphical interpretation of the fire profiles in Fig. 4.

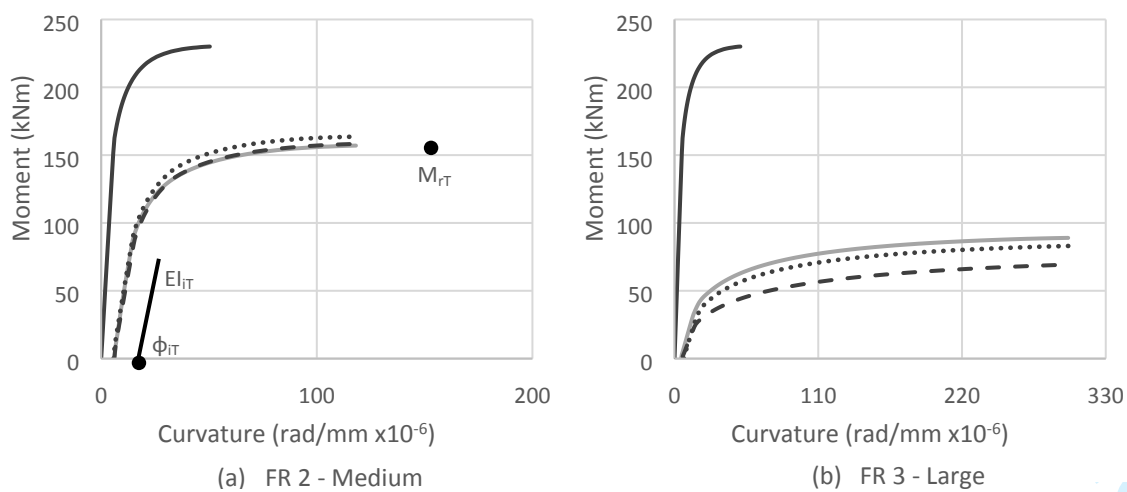
In total, the study consists of 147 test cases using the seven cross-sections, seven design fires, seven AITP mean standard fires, and seven AITP conservative standard fires. It should be noted, that B6 and B7 possess a  $b_c$  greater than 350 mm, and therefore do not meet the condition of the mean  $\psi_{size}$  in the case of FR 4, when  $T_{max}$  is less than 600°C. These two non-valid cases were excluded from the study for the mean  $t_e$ .



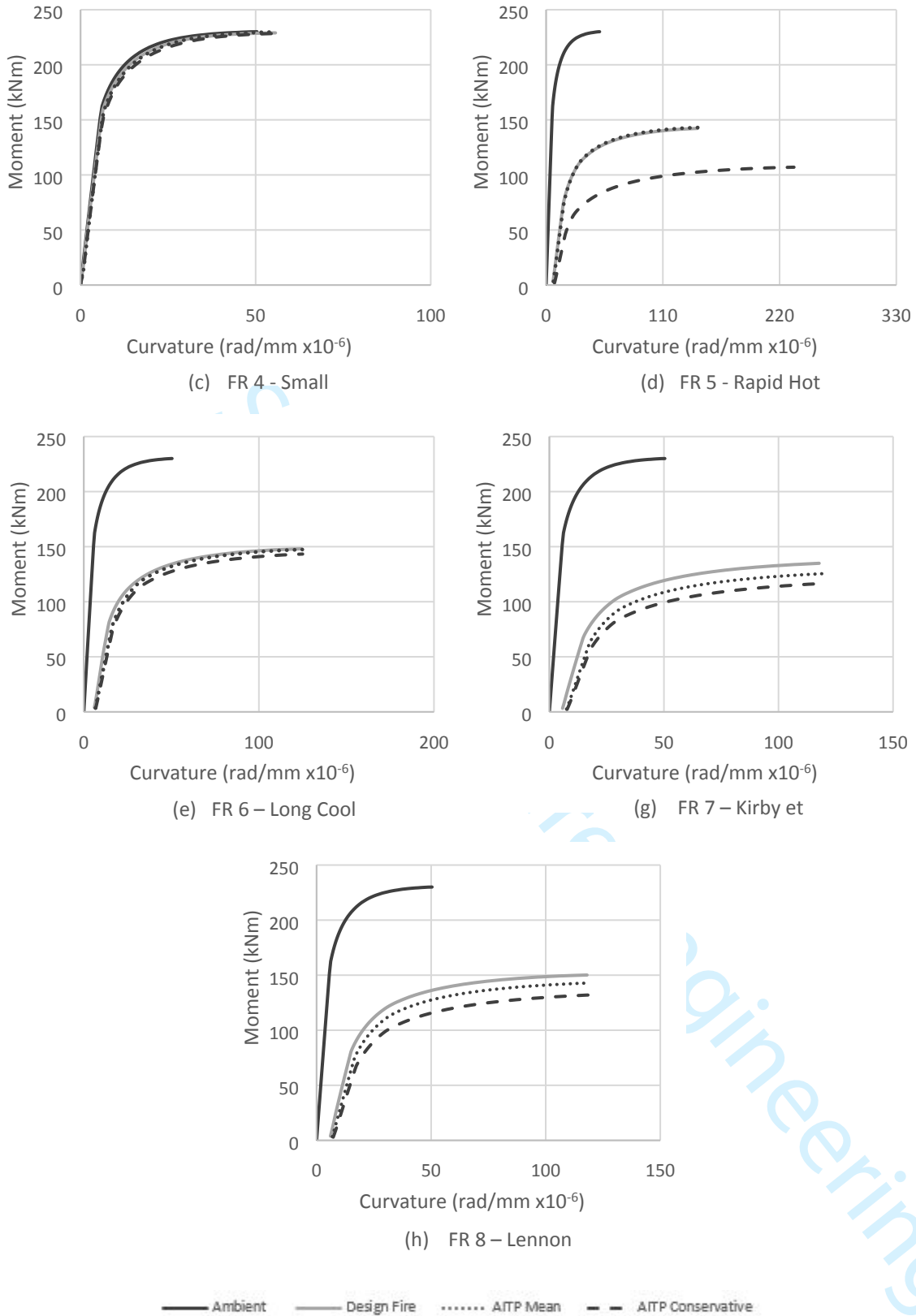
**Fig. 4: Representative Design Fire Profiles**

### 3.4 Mechanical Response Assessment

Figure 5 displays the full  $M-\varphi$  diagrams for B1 during the various fire exposure regimes. The ambient temperature profile is also provided as a baseline. All of the fire events led to the expected response of lowering the moment capacities and increasing the curvatures. The small fire (FR 4) resulted in only minimal internal temperatures within the section, and as such, virtually no visible change occurs to the  $M-\varphi$  diagram during fire exposure. For all seven fire events, the mean  $t_e$  presents a good fit with the design response. The highest deviation occurs for the large fire (FR 3), but the accuracy of the moment capacity remains at most within 6.7% of the actual. The conservative  $t_e$  produced a conservative profile, with lower moment capacity and larger curvatures for all seven design fires. For the rapid hot fire (FR 5), the conservative  $t_e$  is significantly longer in duration than the mean  $t_e$ , allowing it to capture the high surface temperatures that occur during rapid hot events. The  $M-\varphi$  response of FR 5 reflects this fact, showing a very conservative estimate for the conservative  $t_e$ . The experimental design fires of FR 7 and FR 8 likewise correlate well with the AIT  $t_e$  approximations.



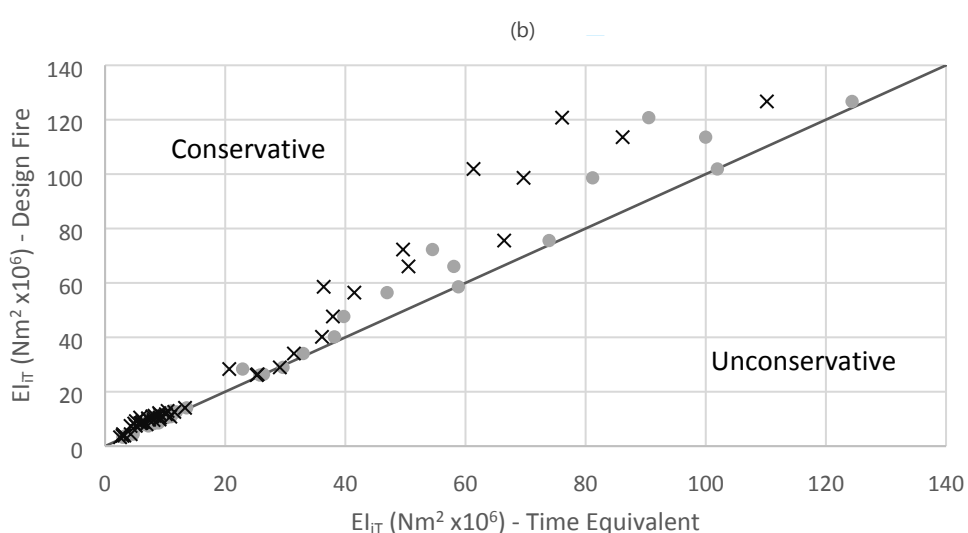
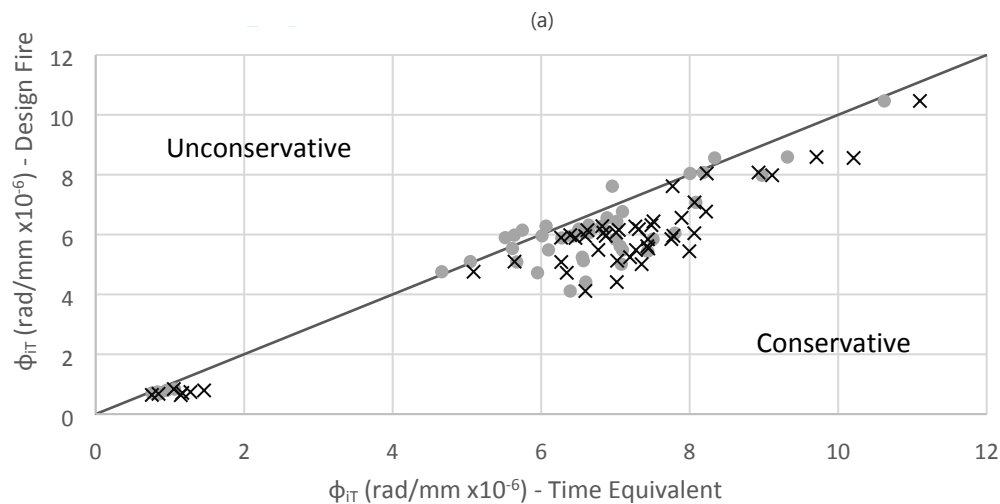
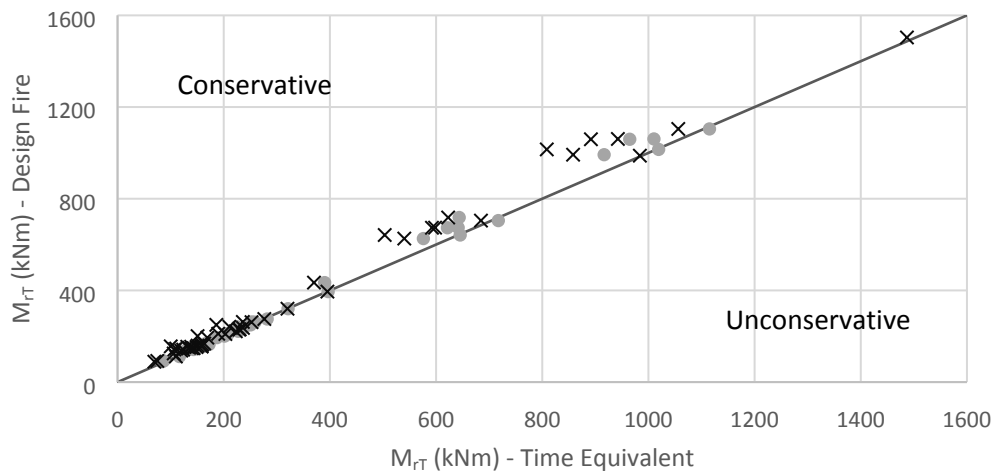
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**Fig. 5: Moment-Curvature Diagrams for B1**

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3 Comparison of the remaining test cases is conducted based on the  $M_{rT}$ ,  $\varphi_{iT}$ , and  $EI_{iT}$  (graphically  
4 depicted on Fig. 5a). Fig. 6 displays the results of the design fire predictions versus the time  
5 equivalent fire predictions. The conservative criterion achieves its intended objective, resulting in  
6 conservative approximations of lower moment capacity, larger curvature, and lower stiffness for  
7 every test case. The mean criterion presents a reasonable fit along the line of equality. The  $M_{rT}$  is  
8 captured with a high degree of accuracy by the mean  $t_e$ , with error less than 10 % for every section  
9 and design fire. The  $\varphi_{iT}$  and  $EI_{iT}$  generally fall within 10 % error; however, because both responses  
10 are highly sensitive to small changes in thermal strains, some outliers yield higher errors.  
11 Furthermore, in contrast with moment capacity, curvature and stiffness calculations at ambient  
12 and elevated temperatures are far more approximate (Concrete Design Handbook, 2016). Given  
13 the approximate and sensitive nature of the calculations, it is difficult for the AITP  $t_e$  to provide  
14 highly accurate predictions for  $\varphi_{iT}$  and  $EI_{iT}$ . It should be noted however, that the higher error  
15 predictions of  $\varphi_{iT}$  and  $EI_{iT}$  associated with the mean  $t_e$  are on the conservative side. The maximum  
16 error for the unconservative mean  $t_e$  predictions are always within 10 %.

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● AITP Mean    × AITP Conservative

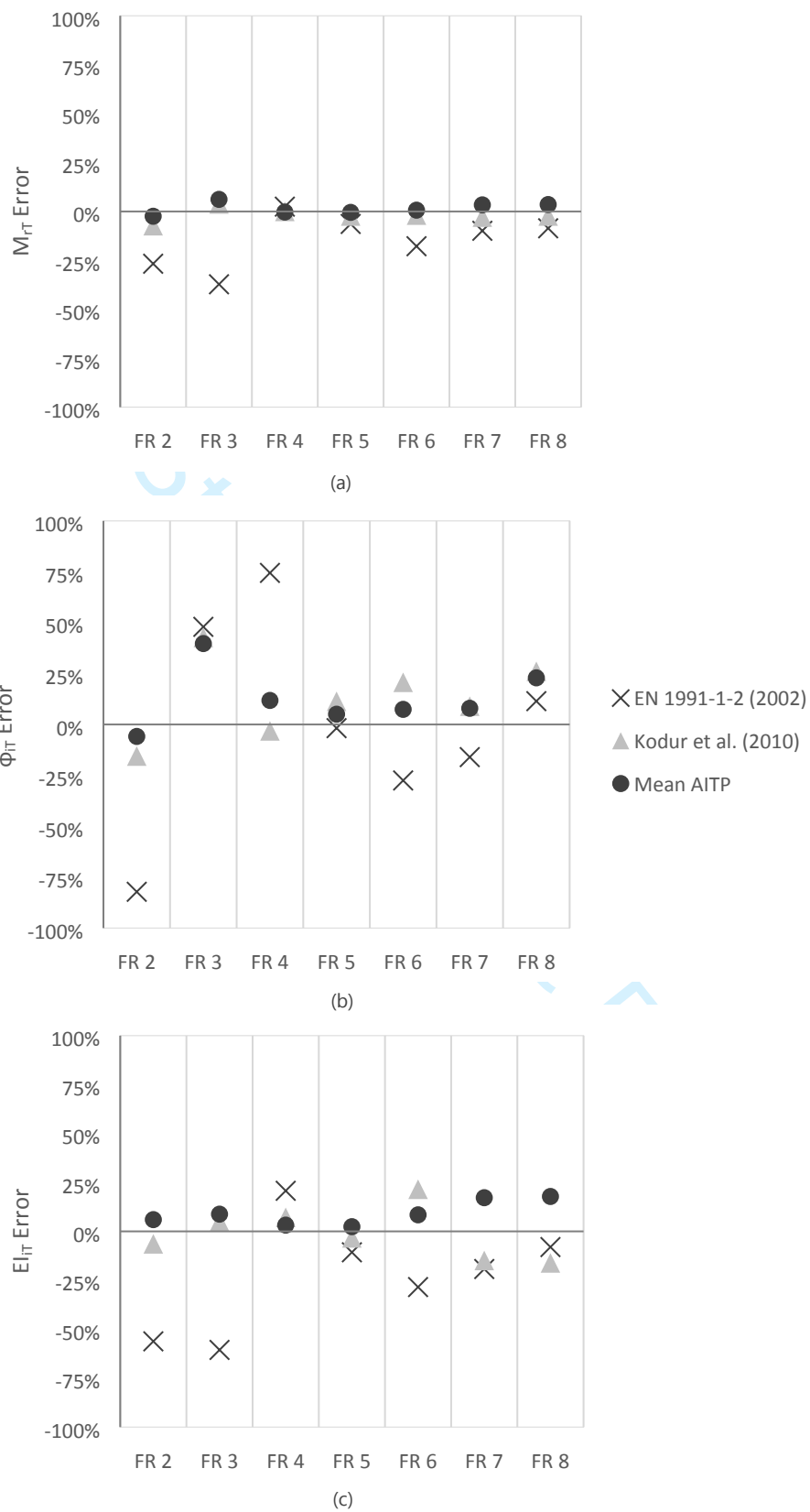
**Fig. 6: Design vs. AITP  $t_e$  Response for: (a)  $M_{rT}$ , (b)  $\phi_{iT}$ , and (c)  $E_{iT}$**



#### 4. Comparison with Existing Methods

There are two existing time equivalent methods that are specifically applicable for RC beams. The first is presented in the Eurocode (EN 1991-1-2, 2002) and the second by Kodur et al. (2010). The Eurocode method was derived by equating the maximum internal temperature that arises during natural and standard fire exposure. The experimental work for its derivation focused on steel sections, and as such, its applicability to evaluate the load capacity of RC elements has been disputed by Thomas et al. (1997) and Xie et al. (2017). However, given the Eurocode's clear statement of applicability for concrete elements and its prominent standing as a design standard, it serves as a solid method for comparison. Kodur et al.'s (2010) method was derived based on equating the energy transfer of a natural fire to that of a standard fire in RC beams. This method was selected for comparison as it was derived specifically for RC beams and has previously been shown to reasonably approximate internal temperatures (Kuehnen and Youssef, 2019).

Fig. 7 shows the comparison between the mean AITP  $t_e$ , the Eurocode, and Kodur et al. (2010). The comparison is made based on the moment-curvature responses of  $M_{rT}$ ,  $\varphi_{iT}$ , and  $EI_{iT}$ . The three responses are recorded as a percentage error from the value calculated using the design fire. A positive error indicates the time equivalent results in a conservative estimate of the actual design fire response, and a negative error indicates the opposite. The evaluation was undertaken for the beam section B2. Fire exposure was applied consistent with the seven design fires in Fig. 4, allowing for assessment of the methods over a range of possible natural fire events.



**Fig. 7: Flexural Response of B2 for Existing Time Equivalent Methods**

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3 From Fig. 7, the mean AITP  $t_e$  presents a high degree of accuracy in comparison with the existing  
4 methods. The Eurocode approach produces significantly deviant results across all three  
5 responses. The  $\varphi_{iT}$  in particular is poorly approximated by the Eurocode, with results ranging from  
6 82 % unconservative for FR 1, to 74 % conservative for FR 4. Considering the inaccuracy of the  
7 Eurocode, it is evident that the consideration of internal concrete temperatures is critical to the  
8 determination of a  $t_e$  for RC elements.  
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19 Kodur et al.'s method presents a good level of accuracy, often producing comparable results to  
20 those developed by the AITP  $t_e$ . Although in general, the AITP  $t_e$  produces slightly more accurate  
21 results. It should also be noted that the AITP  $t_e$  is often conservative when compared to  
22 Kodur et al.'s results. This is most evident for the  $El_{iT}$  approximation during exposure to the  
23 experimental fires of FR 7 and FR 8. In this case, both methods record errors greater than 10 %,  
24 but the predictions of the AITP  $t_e$  are conservative, while those of Kodur et al.'s method are  
25 unconservative.  
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37 The conservative AITP  $t_e$  is not displayed on the figures. However, it should be noted, that given  
38 the same testing parameters, the conservative AITP  $t_e$  is the only method that consistently  
39 recorded conservative results for all responses and fire exposures.  
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## 47 **6. Conclusion**

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50 The AITP  $t_e$  method was assessed based on the flexural response of RC beam sections. Using a  
51 finite difference software developed by El-Fitiany and Youssef (2009), the  $M-\varphi$  relationship of RC  
52 beams during fire exposure was developed. A parametric study was undertaken to compare the  
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3  $M-\varphi$  response of beams exposed to a range of design fires and standard fires with an AITP  $t_e$   
4 duration. To assess the AITP  $t_e$  for a larger number of cases, three key responses from the  $M-\varphi$   
5 relationship were selected for evaluation: maximum moment, initial curvature, and initial stiffness.  
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7 Evaluation of the key responses displayed good correlation between the AITP mean  $t_e$  and the  
8 design fire. Additionally, the conservative time equivalent produced a  $M-\varphi$  profile with lesser  
9 moments and larger curvatures for every test case. Further comparison was undertaken with  
10 relation to existing time equivalent methods, demonstrating the improved accuracy of the AITP  $t_e$   
11 in approximating the flexural response of RC beams.  
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## 25 **Conflicts of interest**

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28 Robert Kuehnen, Maged Youssef, and Salah El-Fitiany declare that they have no conflict of interest.  
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38 Multidisciplinary Partnerships to Accelerate Community Transformation and Sustainability  
39 (ICIMFACTS), and Western University.  
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